STAT 705 HW 4

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2.1.

We are asked to find the MLE of the Cauchy(theta,1) distribution, which is analytically intractable. Thus, the usage of numerical algorithms to approximate it is required.

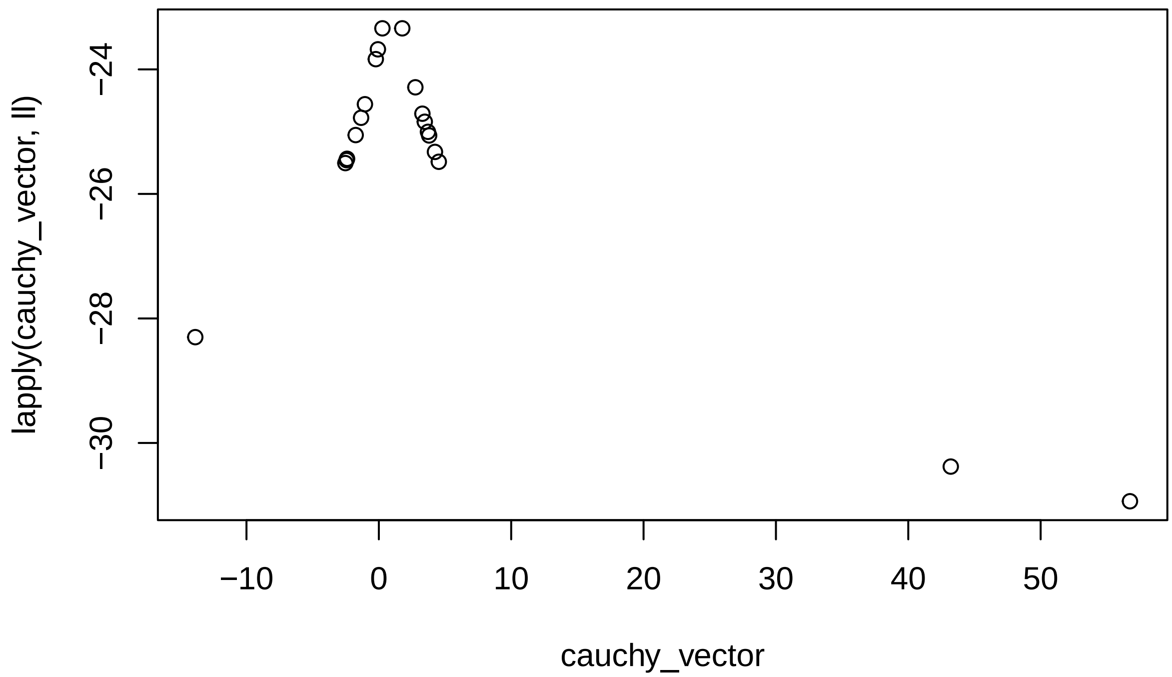
2.1.a.

The log-likelihood for the Cauchy(theta,1) distribution is given by: l(theta,x) = -nlog(pi) - sum(log(1 + (x - theta)^2))

```
#Values from Cauchy(theta,1) distribution
cauchy_vector <- c(1.77,-0.23,2.76,3.80,3.47,56.75,-1.34,4.24,-2.44,3.29,3.71,-2.40,4.53,-0.07,-1.05,-13

#Length and mean of Cauchy value vector
n <- length(cauchy_vector)
avg <- mean(cauchy_vector)
theta <- median(cauchy_vector)

#log-likelihood of Cauchy(theta, 1) distribution
ll <-  function(x){ -n * log(pi) - sum(log(1 + (x - theta)^2))}
#Plot of log-likelihood of Cauchy(theta,1)
plot(cauchy_vector, lapply(cauchy_vector,ll))
```




```r
#derivative of log-likelihood
llprime <- function(x){sum((2*(x - theta))/(1 + (x - theta)^2))}
```

```r
library(NLRoot)
library(cmna)

starting_points <- c(-11,-1,0,1.5,4,4.7,7,8,38,avg)
```

```r
Gradmat<-function(parvec, infcn, eps = 1e-06)
{
# Function to calculate the difference-quotient
# approx gradient (matrix) of an arbitrary input
# (vector) function "infcn" at "parvec"
  dd = length(parvec)
  aa = length(infcn(parvec))
  epsmat = (diag(dd) * eps)/2
  gmat = array(0, dim = c(aa, dd))
  for(i in 1:dd){
    gmat[, i] <- (infcn(parvec + epsmat[, i]) -
                    infcn(parvec - epsmat[, i]))/eps}
  if(aa > 1) gmat else c(gmat)
}

NR.MLE<-function(par.0, infcn,it.max=25, tol = 1e-05, eps=1e-06){
  gradfunc = function(x) Gradmat(x, infcn, eps)
```

```r
  hessfunc = function(x) Gradmat(x, gradfunc, eps)
  oldpar = par.0
  newpar <- oldpar - solve(hessfunc(oldpar), gradfunc(oldpar))
  it <- 1
  while(it < it.max & sqrt(sum((newpar - oldpar)^2)) >tol){
    oldpar <- newpar
    newpar <- oldpar - solve(hessfunc(oldpar), gradfunc(oldpar))
    it <- it + 1
    }
  list(nstep = it, initial = par.0, final = newpar,fval = infcn(newpar))}

#for (starting_point in starting_points){
#  print(NR.MLE(starting_point,ll))
#}
#
```

For this problem, I would get this error: Error in solve.default(hessfunc(oldpar), gradfunc(oldpar)) : Lapack routine dgesv: system is exactly singular: U[1,1] = 0 for all starting values except for 1.5. I am unsure why. Here is my result for 1.5:

```r
print(NR.MLE(1.5,ll))
```

```
## $nstep
## [1] 5
##
## $initial
## [1] 1.5
##
## $final
## [1] 1.02
##
## $fval
## [1] -22.8946
```

2.1.b.

Bisection method for finding the MLE

```r
BFfzero(llprime,-1,1)
```

```
## [1] 1
## [1] 1.019998
## [1] -3.662109e-06
## [1] "finding root is successful"
```

2.1.c.

Fixed point method for finding the MLE

```r
library(SQUAREM)
fpiter(-1,ll,llprime)
```

```
## $par
## [1] -29.74865
##
## $value.objfn
## [1] -0.06493265
##
## $fpevals
## [1] 9
##
## $objfevals
## [1] 0
##
## $convergence
## [1] TRUE
```

2.1.d.

Secant method for finding the MLE

```r
SMfzero(llprime, -2, -1)
```

```
## [1] -1.42563e+154
## [1] 0
## [1] "finding root is successful"
```

```r
SMfzero(llprime, -3, 3)
```

```
## [1] 1.02
## [1] 6.261658e-14
## [1] "finding root is successful"
```

2.1.e.

Compare runtime speeds for the various algorithms

```r
#rt for Newton-Raphson
rt <- proc.time()
NR.MLE(1.5,ll)
```

```
## $nstep
## [1] 5
##
## $initial
## [1] 1.5
##
## $final
```

```
## [1] 1.02
##
## $fval
## [1] -22.8946
```

```r
proc.time() - rt
```

```
##    user  system elapsed
##       0       0       0
```

```r
#rt for bisection method
rt <- proc.time()
BFfzero(llprime,-1,1)
```

```
## [1] 1
## [1] 1.019998
## [1] -3.662109e-06
## [1] "finding root is successful"
```

```r
proc.time() - rt
```

```
##    user  system elapsed
##    0.00    0.00    0.01
```

```r
#rt for fixed point iterations method
rt <- proc.time()
fpiter(-1,ll,llprime)
```

```
## $par
## [1] -29.74865
##
## $value.objfn
## [1] -0.06493265
##
## $fpevals
## [1] 9
##
## $objfevals
## [1] 0
##
## $convergence
## [1] TRUE
```

```r
proc.time() - rt
```

```
##    user  system elapsed
##       0       0       0
```

```r
#rt for secant method
rt <- proc.time()
SMfzero(llprime, -3, 3)
```

```
## [1] 1.02
## [1] 6.261658e-14
## [1] "finding root is successful"
```

```r
proc.time() - rt
```

```
##    user  system elapsed
##       0       0       0
```

Now compare the runtime speeds for 20 Normal(theta, 1) samples

```r
normal_samples <- rnorm(20,theta,1)
n <- length(normal_samples)
theta <- mean(normal_samples)
#Need to find the log-likelihood and its derivative for N(theta,1)
ll_normal <- function(x) {-0.5 * (n*theta^2 - 2*theta*sum(x))}
ll_normal_prime <- function(x) {-n*theta + sum(x)}

#rt for Newton-Raphson
rt <- proc.time()
#NR.MLE(normal_samples,ll_normal)
proc.time() - rt
```

```
##    user  system elapsed
##       0       0       0
```

```r
#rt for bisection method
rt <- proc.time()
BFfzero(ll_normal_prime,-1,1)
```

```
## [1] "finding root is fail"
```

```r
proc.time() - rt
```

```
##    user  system elapsed
##       0       0       0
```

```r
#rt for fixed point iterations method
rt <- proc.time()
#fpiter(-1,ll_normal,ll_normal_prime)
proc.time() - rt
```

```
##    user  system elapsed
##       0       0       0
```

```r
#rt for secant method
rt <- proc.time()
SMfzero(ll_normal_prime, -3, 3)
```

```
## [1] 17.55926
## [1] 0
## [1] "finding root is successful"
```

```r
proc.time() - rt
```

```
##    user  system elapsed
##       0       0       0
```

2.2.

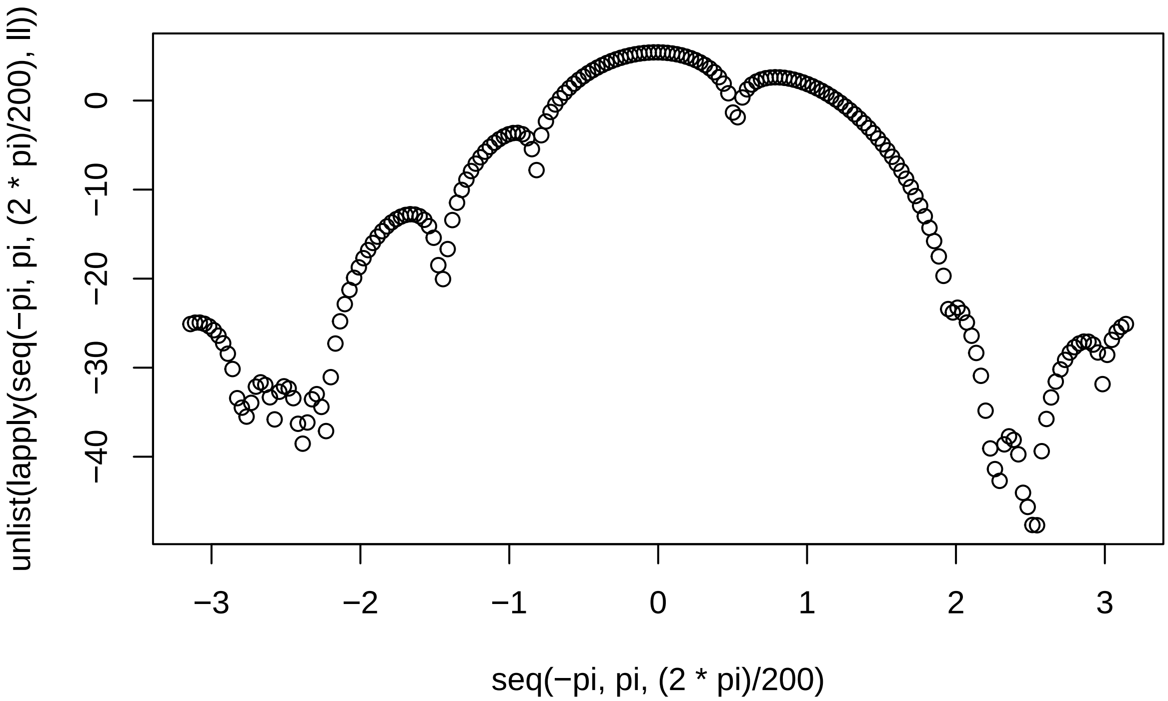
Now, we have the function f(x) = (1 - cos(x - theta))/2pi

2.2.a.

```r
#Data generated from above function
data <- c(3.91,4.85,2.28,4.06,3.70,4.04,5.46,3.53,2.28,1.96,2.53,3.88,2.22,3.47,4.82,2.46, 2.99,2.54, 0

#log-likelihood of given function. We wish to estimate theta
ll <- function(theta) {sum(log(1 - cos(data - theta)))}
llprime <- function(theta) {-sum((sin(x - theta))/(1 - cos(data - theta)))}


#Plot of log-likelihood for -pi < theta < pi
plot(seq(-pi,pi,(2*pi)/200),unlist(lapply(seq(-pi,pi, (2*pi)/200),ll)))
```



2.2.b.

The method of moments is theta = arcsin(pi - avg(x))

```
avg <- mean(data)
method_of_moments <- asin(pi - avg)
print(method_of_moments)
```

```
## [1] -0.05844061
```

2.2.c.

```
NR.MLE(method_of_moments,ll)
```

```
## $nstep
## [1] 3
##
## $initial
## [1] -0.05844061
##
## $final
## [1] -0.011972
```

```
##
## $fval
## [1] 5.414629
```

```
NR.MLE(-2.7,ll)
```

```
## $nstep
## [1] 4
##
## $initial
## [1] -2.7
##
## $final
## [1] -2.6667
##
## $fval
## [1] -31.64165
```

```
NR.MLE(2.7,ll)
```

```
## $nstep
## [1] 5
##
## $initial
## [1] 2.7
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
```

2.2.d.

I do not understand this part.

```
for (value in seq(-pi, pi, (2*pi)/200)){
  print(NR.MLE(value,ll))
}
```

```
## $nstep
## [1] 4
##
## $initial
## [1] -3.141593
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
```

```
##
## $nstep
## [1] 3
##
## $initial
## [1] -3.110177
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 3
##
## $initial
## [1] -3.078761
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 3
##
## $initial
## [1] -3.047345
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 4
##
## $initial
## [1] -3.015929
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 4
##
## $initial
## [1] -2.984513
```

```
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 4
##
## $initial
## [1] -2.953097
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.921681
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.890265
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 6
##
## $initial
## [1] -2.858849
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
```

```
##
## $nstep
## [1] 7
##
## $initial
## [1] -2.827433
##
## $final
## [1] -3.093092
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 4
##
## $initial
## [1] -2.796017
##
## $final
## [1] -2.786167
##
## $fval
## [1] -34.23217
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.764602
##
## $final
## [1] -2.786167
##
## $fval
## [1] -34.23217
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.733186
##
## $final
## [1] -2.6667
##
## $fval
## [1] -31.64165
##
## $nstep
## [1] 4
##
## $initial
## [1] -2.70177
```

```
##
## $final
## [1] -2.6667
##
## $fval
## [1] -31.64165
##
## $nstep
## [1] 3
##
## $initial
## [1] -2.670354
##
## $final
## [1] -2.6667
##
## $fval
## [1] -31.64165
##
## $nstep
## [1] 4
##
## $initial
## [1] -2.638938
##
## $final
## [1] -2.6667
##
## $fval
## [1] -31.64165
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.607522
##
## $final
## [1] -2.6667
##
## $fval
## [1] -31.64165
##
## $nstep
## [1] 7
##
## $initial
## [1] -2.576106
##
## $final
## [1] -2.507613
##
## $fval
## [1] -32.08022
```

```
##
## $nstep
## [1] 4
##
## $initial
## [1] -2.54469
##
## $final
## [1] -2.507613
##
## $fval
## [1] -32.08022
##
## $nstep
## [1] 3
##
## $initial
## [1] -2.513274
##
## $final
## [1] -2.507613
##
## $fval
## [1] -32.08022
##
## $nstep
## [1] 3
##
## $initial
## [1] -2.481858
##
## $final
## [1] -2.507613
##
## $fval
## [1] -32.08022
##
## $nstep
## [1] 4
##
## $initial
## [1] -2.450442
##
## $final
## [1] -2.507613
##
## $fval
## [1] -32.08022
##
## $nstep
## [1] 6
##
## $initial
## [1] -2.419026
```

```
##
## $final
## [1] -2.507613
##
## $fval
## [1] -32.08022
##
## $nstep
## [1] 2
##
## $initial
## [1] -2.38761
##
## $final
## [1] -2.3882
##
## $fval
## [1] -38.53416
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.356194
##
## $final
## [1] -2.297256
##
## $fval
## [1] -32.96188
##
## $nstep
## [1] 4
##
## $initial
## [1] -2.324779
##
## $final
## [1] -2.297256
##
## $fval
## [1] -32.96188
##
## $nstep
## [1] 3
##
## $initial
## [1] -2.293363
##
## $final
## [1] -2.297256
##
## $fval
## [1] -32.96188
```

```
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.261947
##
## $final
## [1] -2.297256
##
## $fval
## [1] -32.96188
##
## $nstep
## [1] 3
##
## $initial
## [1] -2.230531
##
## $final
## [1] -2.232167
##
## $fval
## [1] -37.06318
##
## $nstep
## [1] 7
##
## $initial
## [1] -2.199115
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.167699
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 6
##
## $initial
## [1] -2.136283
```

```
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 6
##
## $initial
## [1] -2.104867
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.073451
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 6
##
## $initial
## [1] -2.042035
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 5
##
## $initial
## [1] -2.010619
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
```

```
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.979203
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.947787
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.916372
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.884956
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.85354
```

```
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.822124
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.790708
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.759292
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.727876
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
```

```
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.69646
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 3
##
## $initial
## [1] -1.665044
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.633628
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.602212
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.570796
```

```
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.53938
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 6
##
## $initial
## [1] -1.507964
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 8
##
## $initial
## [1] -1.476549
##
## $final
## [1] -1.658283
##
## $fval
## [1] -12.76318
##
## $nstep
## [1] 3
##
## $initial
## [1] -1.445133
##
## $final
## [1] -1.447479
##
## $fval
## [1] -20.00273
```

```
##
## $nstep
## [1] 7
##
## $initial
## [1] -1.413717
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
##
## $nstep
## [1] 6
##
## $initial
## [1] -1.382301
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
##
## $nstep
## [1] 7
##
## $initial
## [1] -1.350885
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.319469
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
##
## $nstep
## [1] 6
##
## $initial
## [1] -1.288053
```

```
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
##
## $nstep
## [1] 6
##
## $initial
## [1] -1.256637
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.225221
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
##
## $nstep
## [1] 4
##
## $initial
## [1] -1.193805
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
##
## $nstep
## [1] 5
##
## $initial
## [1] -1.162389
##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
```

```
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## $nstep
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##
## $final
## [1] -0.9533363
##
## $fval
## [1] -3.620938
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## $nstep
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## $final
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##
## $fval
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## $nstep
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```

```
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## $nstep
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##
## $fval
## [1] -3.620938
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## $nstep
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## $final
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## [1] -3.620938
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```
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## $nstep
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## $initial
## [1] -0.8168141
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## $final
## [1] -0.011972
##
## $fval
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## $nstep
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## $final
## [1] -0.011972
##
## $fval
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## $nstep
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## $nstep
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```
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## $final
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## $nstep
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## $final
## [1] -0.011972
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## $nstep
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## $final
## [1] -0.011972
##
## $fval
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## $nstep
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##
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```
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##
## $nstep
## [1] 4
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```
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##
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##
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```
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## $nstep
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##
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## $nstep
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##
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## $nstep
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##
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```
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##
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## $nstep
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## $nstep
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##
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```
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## $nstep
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##
## $fval
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```

```
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```
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## $fval
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##
## $nstep
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## $final
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##
## $fval
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## $nstep
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## $nstep
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## $final
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##
## $fval
## [1] 2.616191
```

```
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## $nstep
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## $nstep
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## $final
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## $fval
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## $nstep
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## $initial
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## $final
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## $fval
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##
## $nstep
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```
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## $nstep
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## $nstep
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## $fval
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## $nstep
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## $fval
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##
## $nstep
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## $final
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##
## $fval
## [1] 2.616191
```

```
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## $nstep
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## $fval
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##
## $nstep
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## $fval
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## $nstep
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```
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## $nstep
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## $nstep
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##
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```
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```
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```
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```
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```
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## $nstep
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## $nstep
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```

```
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## $fval
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## $nstep
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```
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##
## $fval
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##
## $nstep
## [1] 9
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##
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##
## $fval
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##
## $nstep
## [1] 4
##
## $initial
## [1] 2.230531
##
## $final
## [1] 2.236219
##
## $fval
## [1] -38.87625
```

```
##
## $nstep
## [1] 5
##
## $initial
## [1] 2.261947
##
## $final
## [1] 2.236219
##
## $fval
## [1] -38.87625
##
## $nstep
## [1] 6
##
## $initial
## [1] 2.293363
##
## $final
## [1] 2.360718
##
## $fval
## [1] -37.70018
##
## $nstep
## [1] 4
##
## $initial
## [1] 2.324779
##
## $final
## [1] 2.360718
##
## $fval
## [1] -37.70018
##
## $nstep
## [1] 3
##
## $initial
## [1] 2.356194
##
## $final
## [1] 2.360718
##
## $fval
## [1] -37.70018
##
## $nstep
## [1] 3
##
## $initial
## [1] 2.38761
```

```
##
## $final
## [1] 2.360718
##
## $fval
## [1] -37.70018
##
## $nstep
## [1] 3
##
## $initial
## [1] 2.419026
##
## $final
## [1] 2.360718
##
## $fval
## [1] -37.70018
##
## $nstep
## [1] 6
##
## $initial
## [1] 2.450442
##
## $final
## [1] 2.360718
##
## $fval
## [1] -37.70018
##
## $nstep
## [1] 2
##
## $initial
## [1] 2.481858
##
## $final
## [1] 2.475374
##
## $fval
## [1] -45.38483
##
## $nstep
## [1] 2
##
## $initial
## [1] 2.513274
##
## $final
## [1] 2.513593
##
## $fval
## [1] -47.66705
```

```
##
## $nstep
## [1] 9
##
## $initial
## [1] 2.54469
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 7
##
## $initial
## [1] 2.576106
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 6
##
## $initial
## [1] 2.607522
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 5
##
## $initial
## [1] 2.638938
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 5
##
## $initial
## [1] 2.670354
```

```
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 5
##
## $initial
## [1] 2.70177
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 4
##
## $initial
## [1] 2.733186
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 4
##
## $initial
## [1] 2.764602
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 4
##
## $initial
## [1] 2.796017
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
```

```
##
## $nstep
## [1] 4
##
## $initial
## [1] 2.827433
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 3
##
## $initial
## [1] 2.858849
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 4
##
## $initial
## [1] 2.890265
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 4
##
## $initial
## [1] 2.921681
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 6
##
## $initial
## [1] 2.953097
```

```
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 8
##
## $initial
## [1] 2.984513
##
## $final
## [1] 2.873095
##
## $fval
## [1] -27.04811
##
## $nstep
## [1] 6
##
## $initial
## [1] 3.015929
##
## $final
## [1] 3.190094
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 5
##
## $initial
## [1] 3.047345
##
## $final
## [1] 3.190094
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 5
##
## $initial
## [1] 3.078761
##
## $final
## [1] 3.190094
##
## $fval
## [1] -24.92414
```

```
##
## $nstep
## [1] 4
##
## $initial
## [1] 3.110177
##
## $final
## [1] 3.190094
##
## $fval
## [1] -24.92414
##
## $nstep
## [1] 4
##
## $initial
## [1] 3.141593
##
## $final
## [1] 3.190094
##
## $fval
## [1] -24.92414
```

2.2.e.

Find two nearly identical numbers for which the Newton-Raphson algorithm converges to two different solutions. Here, we want to look at the above plot to find a local maxima and pick two values near its peak so that the algorithm rolls into two different local minima. Theta of approximately 2.2 and 2.25 looks promising.

```r
NR.MLE(2.2,ll)
```

```
## $nstep
## [1] 9
##
## $initial
## [1] 2.2
##
## $final
## [1] 2.003645
##
## $fval
## [1] -23.23912
```

```r
NR.MLE(2.25,ll)
```

```
## $nstep
## [1] 4
##
## $initial
```

```
## [1] 2.25
##
## $final
## [1] 2.236219
##
## $fval
## [1] -38.87625
```