Resampling Techniques and their Application

-Class 5-

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Types of Error in Hypothesis Tests

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- **Type I error:** Reject H_0 when H_0 is actually true.
- **Type II error:** Not reject H_0 when H_1 is actually true.
- **Power** of a test: 1-Type-II error = ”correct decision to reject H_0”
- These errors are defined conditional on the true status (H_0 or H_1).

Power of a Test

- Based on data, we either reject or not reject the hypothesis
- In simulation, we condition on H_0 or H_1

- **Type-1 error**
  - Assume H_0 is true
  - All operations in first row of the table
  - Data generations always under H_0

- **Type-II error**
  - Assume H_1 is true
  - All operations in second row of the table
  - Data generations under H_1

Power of a Test

- Let $X_1, \ldots, X_n$ be a sample from F with $E(X_k) = \mu$ and $Var(X_k) = \sigma^2$. We test the null hypothesis $H_0 : \mu = 0$ versus $H_1 : \mu \neq 0$ at level $\alpha = 5\%$. Simulate the power of the t-statistic

$$T = \sqrt{n}\frac{\overline{X}.}{\widehat{\sigma}}$$

  to detect the alternative $H_1 : \mu = \delta$.
- Use $n = 10, 20, 30$ and $\delta = 0, 0.1, 0.2, \ldots, 2$

Power of a Test

```
set.seed(1)
myPower<-function(n,nsim,Distribution,delta){
erg=c()
if(Distribution=="Normal"){
x<-matrix(rnorm(nsim*n),ncol=nsim)}
if(Distribution=="Exp"){
x<-matrix(rexp(nsim*n)-1,ncol=nsim)}
x<-x+delta #Expectation of x is delta
```

```
mx<-colMeans(x)
sdx<-sqrt((colSums(x^2)-n*mx^2)/(n-1))
T<-sqrt(n)*mx/sdx
result<-data.frame(n=n,Dist=Distribution,delta=delta,
tTest=mean(abs(T)>=qt(0.975,n-1)))
result}
myPower(10,10000,"Exp",0.5)
```

Power Curve

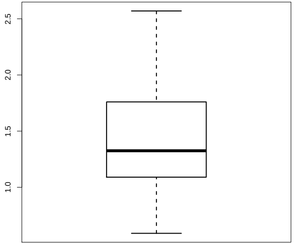
- **Power curve**: plot the power to detect δ

```
delta <- seq(0,2,0.1)
power <-c()

for(h in 1:length(delta)){
power[h]<-myPower(10,10000,"Normal",delta[h])[4]
}
plot(delta,power,type="l",lwd=3,col="blue",cex.lab=1.7,cex.axis=1.7)
abline(h=0.05,lwd=2,col="red")
```

Motivation and Examples-II

The diameter of cork of a Champagne bottle is supposed to be 1.5 cm. If the cork is either too large or too small, it will not fit in the bottle. The manufacturer measures the diameter in a random sample of **n=36** bottles and obtains:




```
X=c(
0.59, 1.23, 1.00, 0.84, 0.88, 1.71,
1.81, 1.84, 2.03, 1.39, 1.30, 1.31,
1.96, 1.33, 2.57, 1.19, 1.01, 2.06,
1.32, 1.55, 1.28, 0.93, 1.63, 1.24,
1.83, 1.81, 0.94, 1.46, 1.25, 1.56,
0.61, 0.83, 1.17, 2.24, 1.68, 1.51)
```

- Estimate the mean and **the median** and their characteristics

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- Think about the following cases
  - Mean
  - Variance
  - Variance of an estimator (e.g. mean)
  - Variance of correlation coefficients
  - Overdispersion parameters, variance of their estimators,...

Parameter Estimation - II

- Statistical model

$$X_1, \ldots, X_n \sim F(\theta)$$

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$$X_1, \ldots, X_n \sim F(\theta)$$

- F is a distribution
- θ are parameters of this distribution
- How to estimate $f(\theta)$?

Parameter Estimation - III

Estimation of θ	Properties
Maximum-Likelihood	F must be known
	Algorithm can be difficult
	Algorithm might not converge
	Large sample for distribution
Moment based	F can be unknown
	Computation usually feasible
	Usually exist (no converging issues)
	Small sample approximations
Resampling Methods	

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$$\widehat{F}_n \to F, n \to \infty$$

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  - Basically, we simulate data from $\widehat{F}_n$

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  - Repeat the previous steps n_{boot} times

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  - Compute the estimator $\widehat{\theta}^*$ and safe this value
  - Repeat the previous steps n_{boot} times
  - Estimate the parameter of interest using the values of $\widehat{\theta}_1, \ldots, \widehat{\theta}_{n_{boot}}$

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```
X#cork diameter data
n <- 36
nboot <-10000
B<- apply(matrix(1:n,
ncol=nboot,nrow=n),
2,sample,replace=TRUE)
xstar <- matrix(X[B],
ncol=nboot,nrow=n)
mxstar <- colMeans(xstar)
tauhat2 <- var(mxstar)
```

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  - Compute the estimator upon the sample and safe the value in $\widehat{\theta}_s$
  - Assess the bias and MSE

$$Bias = \frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} (\widehat{\theta}_s - \theta) \text{ and } MSE = \frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} (\widehat{\theta}_s - \theta)^2$$

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- How to get an idea about the true variance?
- Is bootstrap a good way?

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- Sampling strategies

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```
set.seed(1)
n<-50
erg <-c()
for(i in 1:10000){
x <-rnorm(n)
erg[i]<-median(x)}
hist(erg)
mean(erg)
```

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set.seed(1)
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erg <-c()
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  3. Repeat the above n_{boot} times and obtain $\widehat{\nu}_1^*, \ldots, \widehat{\nu}_{n_{boot}}^*$

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  3. Repeat the above n_{boot} times and obtain $\widehat{\nu}_1^*, \ldots, \widehat{\nu}_{n_{boot}}^*$
  4. Sort the values from smallest to largest: $\widehat{\nu}_{[1]}^*, \ldots, \widehat{\nu}_{[n_{boot}]}^*$

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- Compute a confidence interval for ν
  1. Sample $X_1^*, \ldots, X_n^*$ with replacement from **X**
  2. Compute $\widehat{\nu}^*$ from $X_1^*, \ldots, X_n^*$ and safe this value in $\widehat{\nu}_\ell^*$
  3. Repeat the above n_{boot} times and obtain $\widehat{\nu}_1^*, \ldots, \widehat{\nu}_{n_{boot}}^*$
  4. Sort the values from smallest to largest: $\widehat{\nu}_{[1]}^*, \ldots, \widehat{\nu}_{[n_{boot}]}^*$
  5. Estimate the $(1-\alpha)$ confidence interval for ν by

$$CI_\nu = \left[\widehat{\nu}^*_{[0.025*nboot]}, \widehat{\nu}^*_{[0.975*nboot]}\right]$$

Simulation Study: Coverage Probability

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- Simulation with $n = 50$, $n_{sim} = 1K$, $n_{boot} = 1K$

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- Simulation with $n = 50$, $n_{sim} = 1K$, $n_{boot} = 1K$
- Normal and Exponential distributions

Simulation Study: Coverage Probability

```
myCI<-function(n,nsim,nboot,Distribution){
erg<-c()
B<- apply(matrix(1:n,ncol=nboot,nrow=n),2,sample,replace=TRUE)
if(Distribution=="Normal"){
x<-matrix(rnorm(n*nsim),ncol=nsim)
nu<-0}
if(Distribution=="Exp"){
x<-matrix(rexp(n*nsim),ncol=nsim)
nu<-0.693}
for(i in 1:nsim){
xstar = matrix(x[,i][B],ncol=nboot,nrow=n)
nustar<-apply(xstar,2,median)
nustarS<-sort(nustar)
lower<-nustarS[0.025*nboot]; upper<-nustarS[0.975*nboot]
erg[i]<-(lower <nu && upper>nu)}
result <- data.frame(nsim=nsim, nboot=nboot,nu=nu,
CI=mean(erg))
result}
myCI(50,1000,1000,"Exp")
```

Parameter Estimation: Uncertainty of Median

- Revise the cork diameter example

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- $n_{boot} = 100K$

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```
hist(X,freq=F)
nustar <-c()
nboot<-100000
set.seed(1)
for(i in 1:nboot){
xB<- sample(X,36,replace=TRUE)
nustar[i]<-median(xB)}
hist(nustar,freq=F)
nustarS<-sort(nustar)
lower<-nustarS[0.025*nboot]; upper<-nustarS[0.975*nboot]
c(lower,upper)
```

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- Often, the width of a distribution is described by the Interquartile Range (IQR), which is defined as the IQR=75%-quantile - 25% quantile
- Using sampling, find the true IQR of $N(0, 1)$ and $Exp(1)$ distributions and illustrate its variability. Check using the functions $qnorm()$ and $qexp()$

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- Often, the width of a distribution is described by the Interquartile Range (IQR), which is defined as the IQR=75%-quantile - 25% quantile
- Using sampling, find the true IQR of $N(0, 1)$ and $Exp(1)$ distributions and illustrate its variability. Check using the functions $qnorm()$ and $qexp()$
- Using resampling, find a 95% confidence interval for the IQR

Project: Width of a Distribution

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- Often, the width of a distribution is described by the Interquartile Range (IQR), which is defined as the IQR=75%-quantile - 25% quantile
- Using sampling, find the true IQR of $N(0, 1)$ and $Exp(1)$ distributions and illustrate its variability. Check using the functions $qnorm()$ and $qexp()$
- Using resampling, find a 95% confidence interval for the IQR
- Investigate its coverage probability in a simulation study with $n_{sim} = 1000$ and $n_{boot} = 1000$ runs for normal and exponential distributions with sample sizes $n \in 10, 20, 30, 40$

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- Compute the precision interval for the empirical coverage probability and state your conclusion