NUMERİCAL METHODS/MATH214

PROJECT 2

Inductions Value Calculation with

A) Forward difference formula
B) Backward difference formula
C) Centered difference formula

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Explanation: In a circuit with impressed voltage $\mathcal{E}(t)$, inductance L, and resistance R, as shown in Figure 1, Kirchhoff's voltage law gives the relationship

$$\mathcal{E}(t) = L\frac{d}{dt}i(t) + Ri(t),$$

where $i(t)$ is the current. This is due to the voltage difference induces on an inductance is proportional to the inductance value and the time derivative of the current flows through the inductance, i.e. $V_L(t) = L\frac{d}{dt}i(t)$. On the other hand, the voltage difference on a resistance can be determined as $V_R(t) = Ri(t)$.

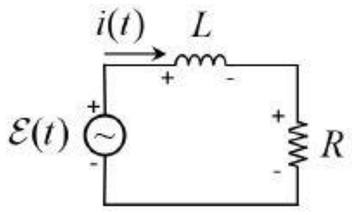

Figure 1: A RL circuit.

Question: The current values of the circuit, where $L = 0.98$ H (Henries) and $R = 14.2\ \Omega$ (Ohms), is measured using a current meter for different time step sizes at the time interval $t \in [0, 600]$ ms. The measured data sets of the current with time step sizes Δt are plotted in Figure 2 for $t \in [0, 500]$ ms and relations are specified in Table 1. In the measurement data files, the first column is the time in seconds (s) and second column is the current value in Amperes (A).

Table 1: Data sets.

Data File	Time Step Size Δt
current1.dat	75 ms
current2.dat	50 ms
current3.dat	25 ms
current4.dat	10 ms

Solution: Firstly calcule the derivative of each current. The derivative is calculetad in 3 different ways

a) Forward difference formula: For small values of h , if h > 0 , it is tried to converge using this formula:

$$\frac{f(x+h) - f(x)}{h} = \frac{\Delta_h[f](x)}{h}.$$

b) Backward difference formula: For small values of h , if h < 0 , it is tried to converge using this formula:

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

Note_1: Forward and backward difference formula not exacly approximate because the function does not have any other value to calculate the right or left endpoint.

c) Centered difference formula: Using a point as the center point its derivatives can be approximated, so if x is our center point, we use (x – h) and (x + h). Central difference approach in this case

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}.$$

Note_2: To calculate the first and last point , is used this formula (endpoint formula)

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)] \cdot$$

-Inductance value is calculated according to the formula with the calculated derivative

Analysis-1:

When the value of h decrease, we get a more accurate result for the convergence value in the divergence method. the derivative of current values calculated with different measures in the same time interval converges more as the time interval gets decreased in the forward difference method. Let's compare exaples for this question:

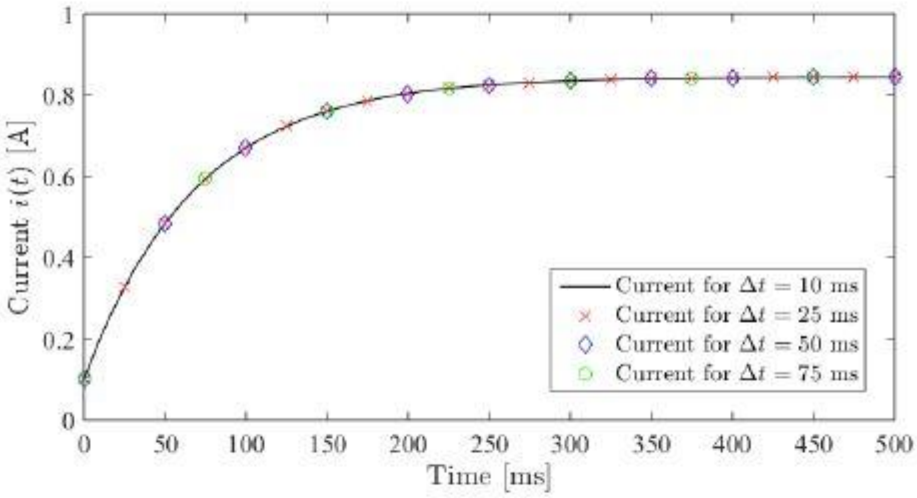

Figure-2: Time , current graph

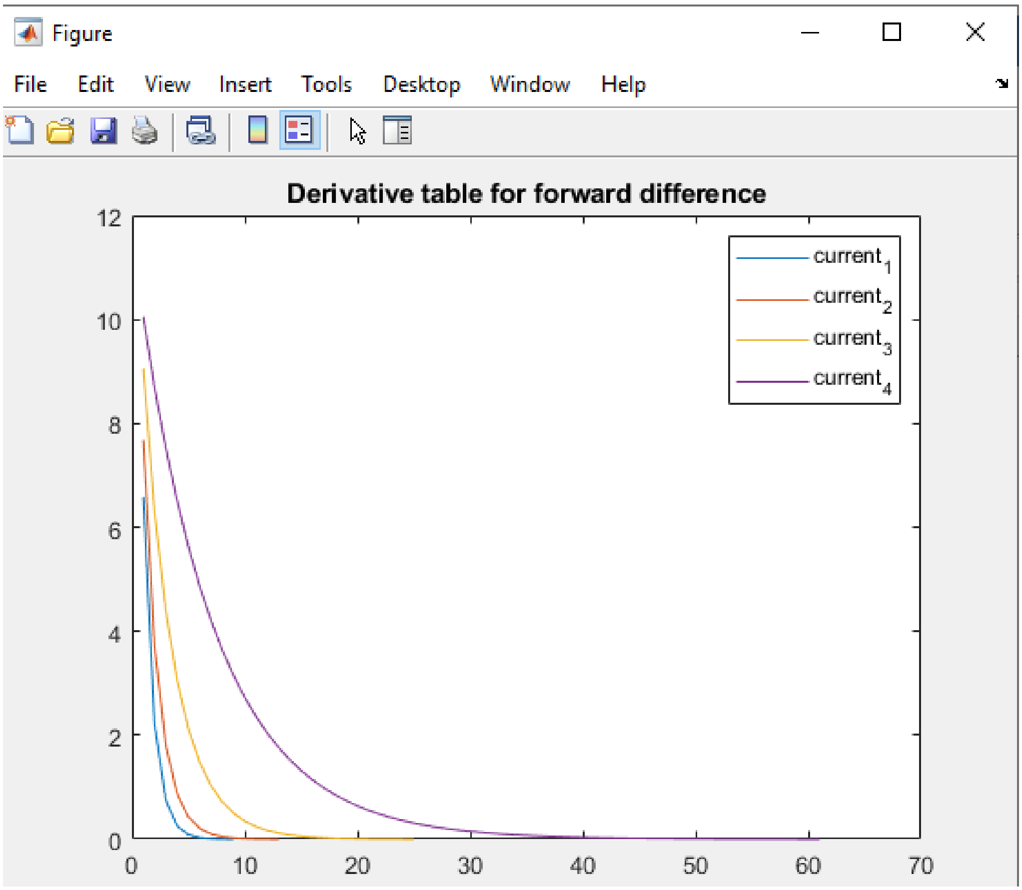

Figure-3: Derivative graph

The graph of the current increase amount from the current / time graph in Figure 2 is shown in Figure 3. When we compare the derivative graph of current 4 with the derivative graph of current number 1, the data measured at more frequent time (h_1=75 ms, h_4= 10 ms) intervals enabled us to reach a more real result.

The bacward difference method converge more as the time interval gets shorter like forward difference method., Difference between the two method, forward difference method is increasingly converged and backward difference method is decreasingly converged

Note_3: First point of the derivative is not shown correctly in Figure 4. The reason is indicated in note_1

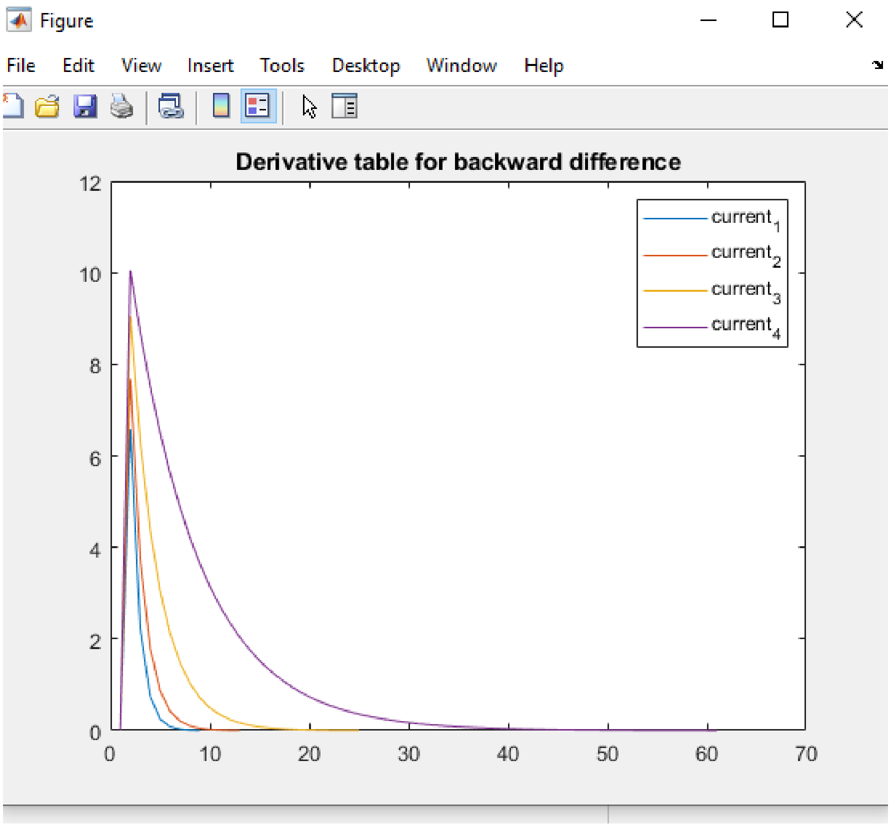

Figure-4: Derivative graph

The smaller the value of h, the more real result is calculated for current 4, which has a lot of data. because the current 4 measured at intervals of less than 1, 2, 3

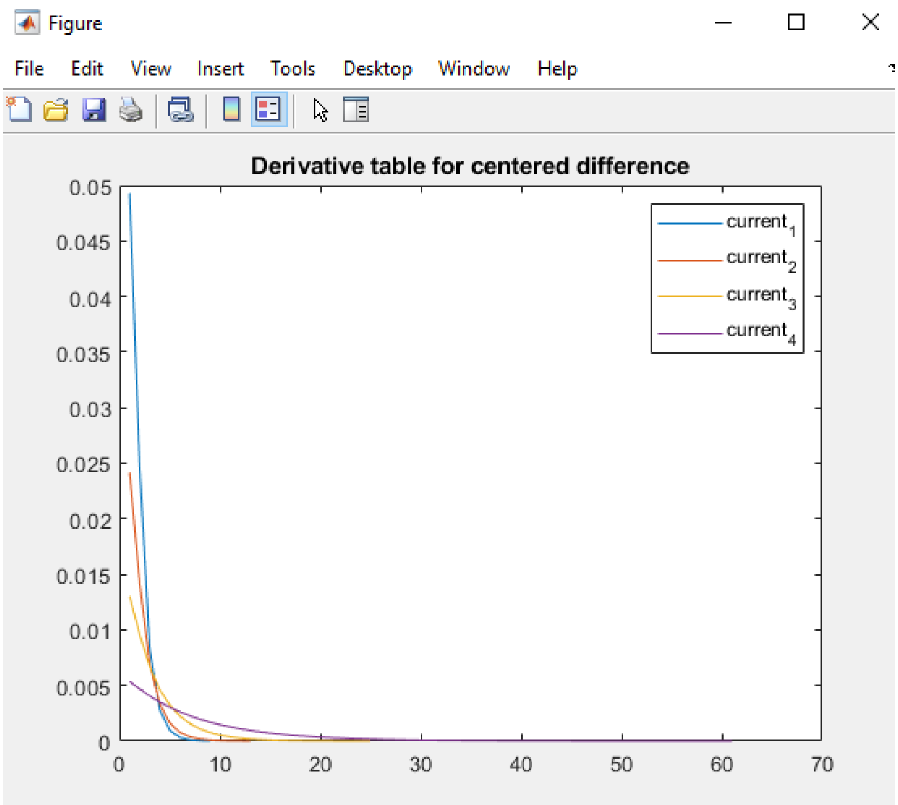

Figure-5: Derivative graph

Figure 5 shows the derivative of the current over time using the centered difference method. This methods graph is not similar to the other two methods because intermediate values converge faster with more time/current data (with a smaller h value) due to the smaller difference (like current 4 approaching 0 faster than others)

Note_4: We use three point endpoint formula for the first and last points because it cannot calculated with three point midpoint formula
Note_5: Endpoint not calculated because three point endpoint formula not used correctly. So the endpoint is assigned 0.

Analysis-2:

The induction value was calculated with the formula in the explanation part of the problem. We use current derivative calculated by 3 different methods and L constant and R value given in question while calculating

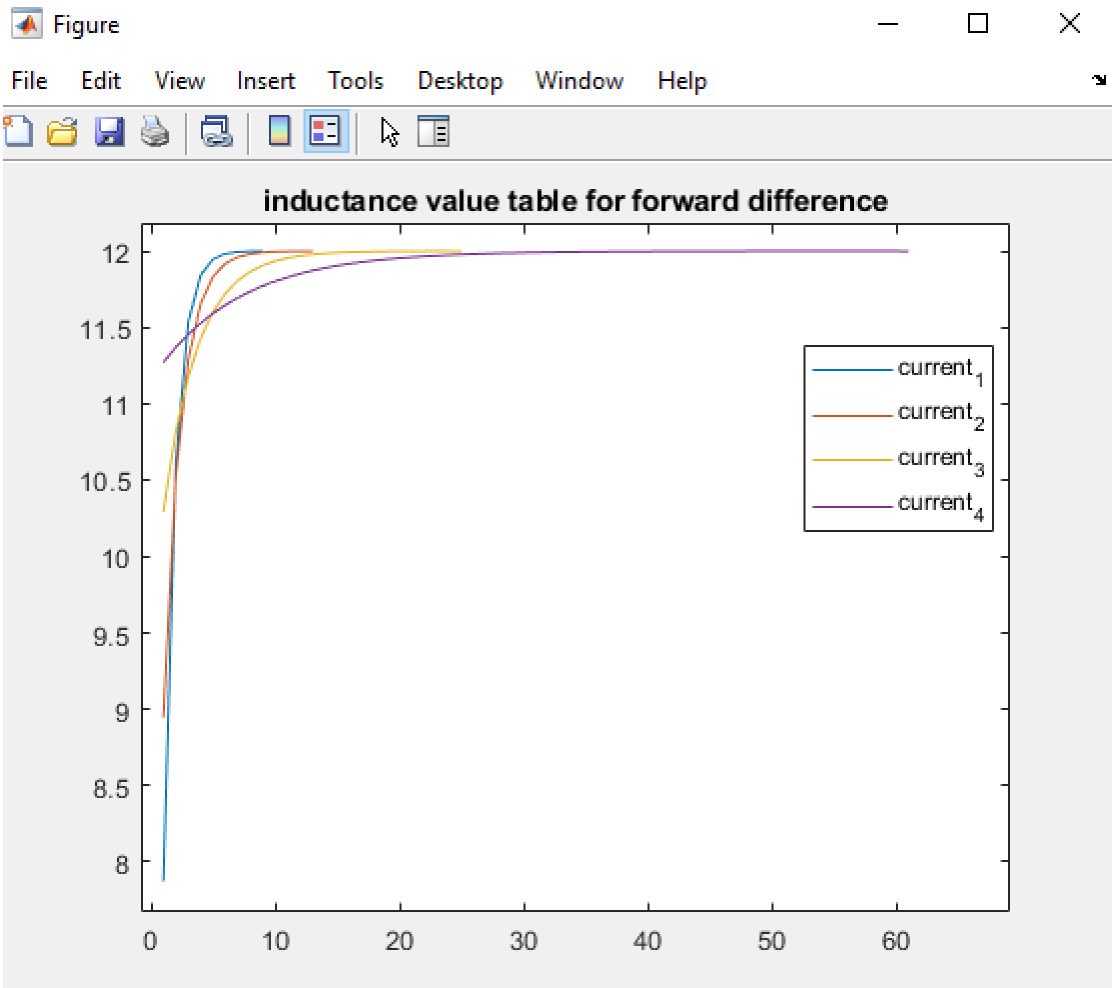

Figure-6: Inductance value graph

Figure 6 shows the convergence graph of the induction value over time. The derivative value of the more correctly calculated current 4 (because h value more than small other current values) begins to converge more closely to the induction value.

The induction value increased and converged due to the derivative value increased with the forward difference method (i.e figure 6)

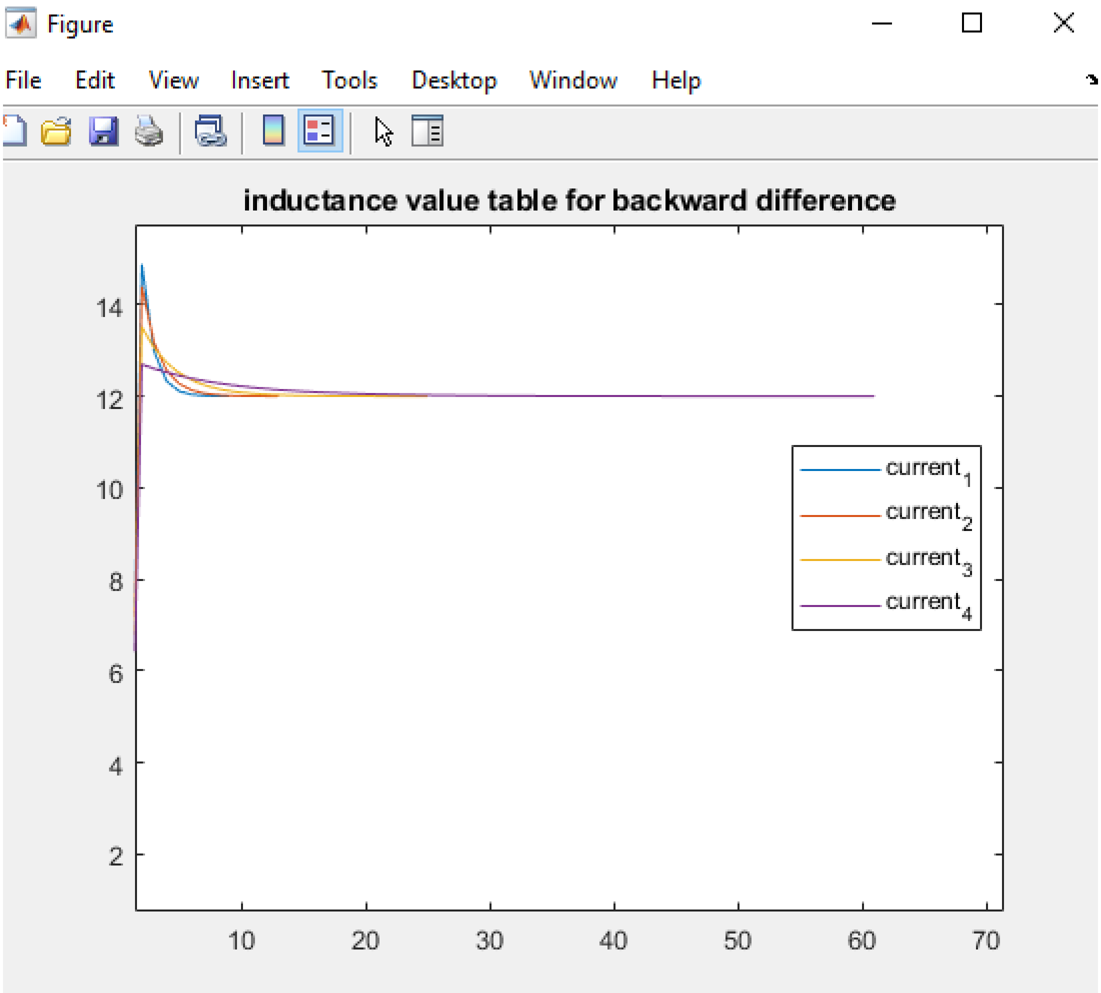

Figure-7: Inductance value graph

The induction value calculated by the derivative of the current with a small h value approaches the value that should converge in figure 7 like figure 6.

The induction value decreased and converged due to the derivative value decreased with the backward difference method (i.e figure 7)

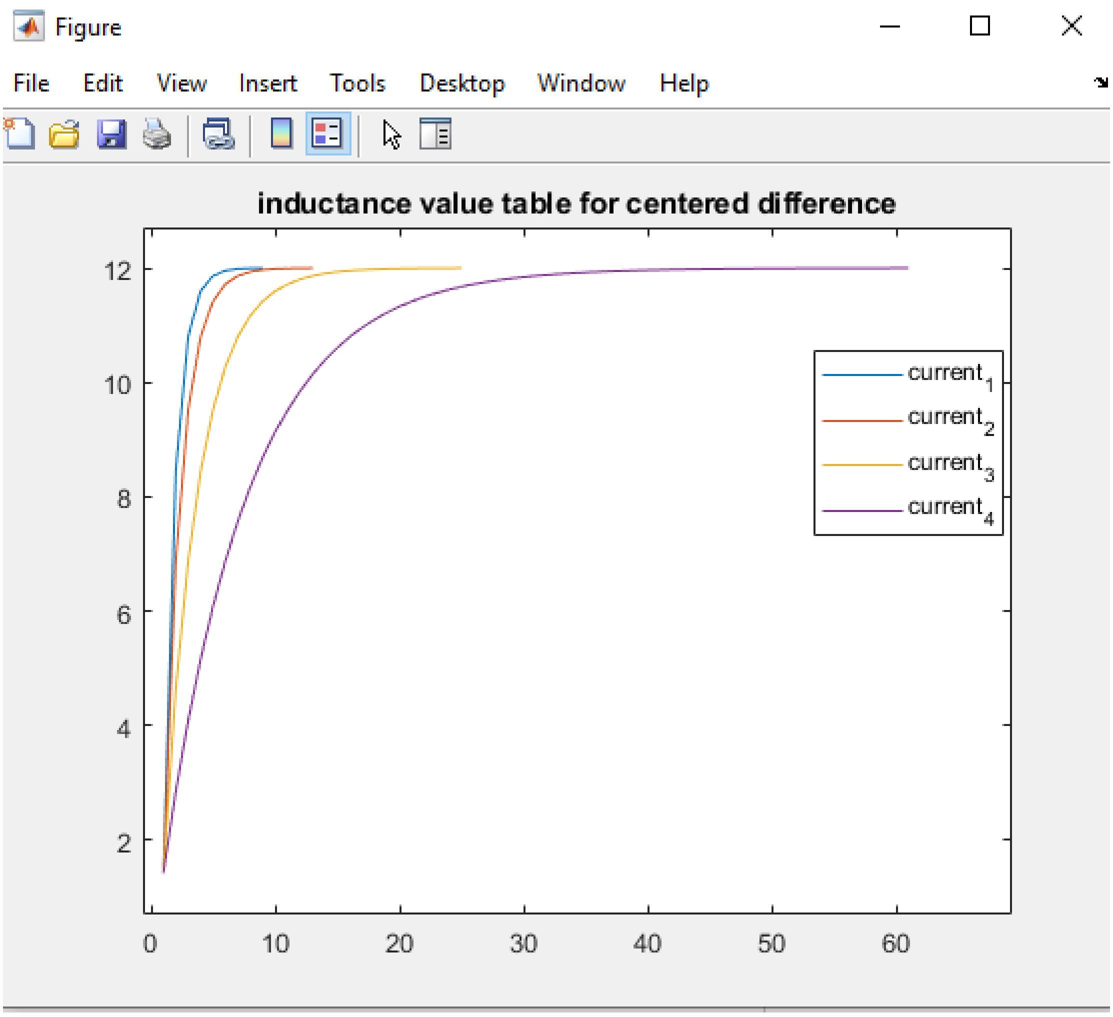

Figure-8: Inductance value graph

Since the derivative value of the 4th current converges (Figure 5), it allows us to make a more accurate calculation for the induction value centered differences in figure 8 because time intervals are shorter in 4th current.

Finally:
Numerical differentiation describes algorithms for estimating the derivative of a mathematical function or function subroutine using values of the function and perhaps other knowledge about the function. Induction value is calculated with time varying currents using one of the numerical analysis methods, in this project. When calculating, it was observed how time intervals affect the final value and the reasons for the errors was commanted. It was concluded that the increase in time interval caused it not to approach the correct values for all methods. Therefore, if we had the equation for current, the error rate could be calculated for each time interval, but it is concluded that the error rate would be high for currents with high **Δt.**
The endpoint formula is used for the first value of the centered difference method and it is considered to be converging correctly. However, it could not be calculated for the last point.

Information:

- The value expressed with h in the explanations indicates the time interval. **(Δt)**
- Time intervals in the graphs
    - 0.010 s for current 4,
    - 0.025s for current 3
    - 0.050s for current 2
    - 0.075s for current 1
- Calculation errors and information that needs attention are explained with notes.
- Numerical Analysis Richard L. Burden - J. Douglas Faires book and numerical analysis lecture notes were used as references.

Code

```
clear all
clear
clc
load current1.dat
load current2.dat
load current3.dat
load current4.dat
%definition

c1=zeros(9,2);
c2=zeros(13,2);
c3=zeros(25,2);
c4=zeros(61,2);
R=14.2;
L=0.98;

for i=1 :9
for j=1 :2
c1(i,j)=current1(i,j);
end
end

for i=1 :13
for j=1 :2
c2(i,j)=current2(i,j);
end
end
```

```
for i=1 :25
for j=1 :2
c3(i,j)=current3(i,j);
end
end

for i=1 :61
for j=1 :2
c4(i,j)=current4(i,j);
end
end

%forward difference
for k=1 :8
    h=c1(2,1);
    f1(k,1)=(c1((k+1),2)-c1(k,2))/h;
    f1(9,1)=0;
end

for k=1 :12
    h=c2(2,1);
    f2(k,1)=(c2((k+1),2)-c2(k,2))/h;
    f2(13,1)=0;
end

for k=1 :24
    h=c3(2,1);
    f3(k,1)=(c3((k+1),2)-c3(k,2))/h;
    f3(25,1)=0;
end

for k=1 :60
    h=c4(2,1);
    f4(k,1)=(c4((k+1),2)-c4(k,2))/h;
    f4(61,1)=0;
end

%backward difference
for k=9:-1:2
    h=c1(2,1);
    b1(k,1)=(c1((k),2)-c1(k-1,2))/h;

end

for k=13:-1 :2
    h=c2(2,1);
    b2(k,1)=(c2((k),2)-c2(k-1,2))/h;
end

for k=25:-1 :2
    h=c3(2,1);
    b3(k,1)=(c3((k),2)-c3(k-1,2))/h;
end

for k=61:-1:2
    h=c4(2,1);
    b4(k,1)=(c4((k),2)-c4(k-1,2))/h;
end
```

```
%centered difference
for k=2 :8
    h=c1(2,1);
    cd1(1,1)=((-3*c1(1,2))+4*c1(1+1,2)-c1(1+2,2))/2*h; %endpoint formula
    cd1(k,1)=(c1((k+1),2)-c1(k-1,2))/2*h; %midpoint formula
    cd1(9,1)=0;
end

for k=2 :12
    h=c2(2,1);
    cd2(1,1)=((-3*c2(1,2))+4*c2(1+1,2)-c2(1+2,2))/2*h; %endpoint formula
    cd2(k,1)=(c2((k+1),2)-c2(k-1,2))/2*h; %midpoint formula
    cd2(13,1)=0;
end

for k=2 :24
    cd3(1,1)=((-3*c3(1,2))+4*c3(1+1,2)-c3(1+2,2))/2*h; %endpoint formula
    cd3(k,1)=(c3((k+1),2)-c3(k-1,2))/2*h; %midpoint formula
    cd3(25,1)=0;
end

for k=2 :60
    cd4(1,1)=((-3*c4(1,2))+4*c4(1+1,2)-c4(1+2,2))/2*h; %endpoint formula
    cd4(k,1)=(c4((k+1),2)-c4(k-1,2))/2*h; %midpoint formula
    cd4(61,1)=0;
end
%inductance value for forward difference
for e=1 :9
Ef1(e,1)=L*f1(e,1)+R*c1(e,2);
end
for e=1 :13
Ef2(e,1)=L*f2(e,1)+R*c2(e,2);
end
for e=1 :25
Ef3(e,1)=L*f3(e,1)+R*c3(e,2);
end
for e=1 :61
Ef4(e,1)=L*f4(e,1)+R*c4(e,2);
end
%inductance value for backward difference
for e=1 :9
Eb1(e,1)=L*b1(e,1)+R*c1(e,2);
end
for e=1 :13
Eb2(e,1)=L*b2(e,1)+R*c2(e,2);
end
for e=1 :25
Eb3(e,1)=L*b3(e,1)+R*c3(e,2);
end
for e=1 :61
Eb4(e,1)=L*b4(e,1)+R*c4(e,2);
end
%inductance value for centered difference
for e=1 :9
Ec1(e,1)=L*cd1(e,1)+R*c1(e,2);
end
for e=1 :13
Ec2(e,1)=L*cd2(e,1)+R*c2(e,2);
end
for e=1 :25
```

```
Ec3(e,1)=L*cd3(e,1)+R*c3(e,2);
end
for e=1 :61
Ec4(e,1)=L*cd4(e,1)+R*c4(e,2);
end

%plot// if you want to view any table use the ctrl+r key combination for
comment line


% plot(f1)
% hold on
% plot(f2)
% hold on
% plot(f3)
% hold on
% plot(f4), title('Derivative table for forward difference')
,legend('current_1', 'current_2', 'current_3', 'current_4');
% hold on
%
% plot(b1)
% hold on
% plot(b2)
% hold on
% plot(b3)
% hold on
% plot(b4), title('Derivative table for backward difference')
,legend('current_1', 'current_2', 'current_3', 'current_4');
% hold on
%
% plot(cd1)
% hold on
% plot(cd2)
% hold on
% plot(cd3)
% hold on
% plot(cd4), title('Derivative table for centered difference')
,legend('current_1', 'current_2', 'current_3', 'current_4');
% hold on
%
% plot(Ef1)
% hold on
% plot(Ef2)
% hold on
% plot(Ef3)
% hold on
% plot(Ef4), title('inductance value table for forward difference')
,legend('current_1', 'current_2', 'current_3', 'current_4');
% hold on
%
% plot(Eb1)
% hold on
% plot(Eb2)
% hold on
% plot(Eb3)
% hold on
% plot(Eb4), title('inductance value table for backward difference')
,legend('current_1', 'current_2', 'current_3', 'current_4');
% hold on
%
% plot(Ec1)
```

```
% hold on
% plot(Ec2)
% hold on
% plot(Ec3)
% hold on
% plot(Ec4), title('inductance value table for centered difference')
,legend('current_1', 'current_2', 'current_3', 'current_4');
% hold on
```