# NUMERİCAL METHODS/MATH214

***PROJECT 5***

Approximation Theory

- Linear least squares polynomial,
- Least squares polynomial of degree 2,
- Least squares polynomial of degree 3

Muhammed Cemal Eryiğit

1801022024

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## 1- Problem definition and introduction formulas

The problem is to fit the given values on a polynomial with the least error. Least squares method is used to fit with the least error. The least squares method is the most convenient procedure for determining best linear approximations. The least squares method is applied with this formula in order to minimize the data with the least error.

$$E(a_0, a_1) = \sum_{i=1}^{m} [y_i - (a_1 * x_i + a_0)]^2$$

The derivative of equation E according to a0 and a1, equal to 0.

$$0 = 2\sum_{i=1}^{m} (y_i - a_1 * x_i - a_0) * (-1) = 2\sum_{i=1}^{m} (y_i - a_1 * x_i - a_0) * (-x_i)$$

The solution to this system of equations is

$$a_0 = \frac{(\sum_{i=1}^{m} x_i^2 * \sum_{i=1}^{m} y_i - \sum_{i=1}^{m} x_i * y_i)}{m * (\sum_{i=1}^{m} x_i^2) - (\sum_{i=1}^{m} x_i)^2}$$

$$a_1 = \frac{(m * \sum_{i=1}^{m} x_i * y_i - \sum_{i=1}^{m} y_i * \sum_{i=1}^{m} x_i)}{m * (\sum_{i=1}^{m} x_i^2) - (\sum_{i=1}^{m} x_i)^2}$$

When these values are applied for a linear equation: $y_i = (a_1 * x_i + a_0)$ was obtained.

The coefficient values of x for a quadratic or higher polynomial are obtained by the following formula:

$$a_0 * \sum_{i=1}^{m} x_i^n + a_1 * \sum_{i=1}^{m} x_i^{n+1} + a_2 * \sum_{i=1}^{m} x_i^{n+2} + \ldots\ldots\ldots\ldots\ldots + a_n * \sum_{i=1}^{m} x_i^{2n} = a_0 * \sum_{i=1}^{m} y_i x_i^n$$

**2- Code description and inputs**

All result and figures are generated using the code, which is given at the appendix section. The input data given by pr5data.dat are imported and column of the imported data are assigned to [xi], [yi] respectively. Length of arrays assigned to m as 11. a0 and a1 values assigned to plsa0 and plsa1 for linear least square. Other a values are assigned to plsa2 and plsa3 arrays for the polynomial least square. The sum of xi values is defined as txi(number)n. The coefficients of the values a in the equation with lsd (number) _1 are written in matrix form. In lsd (number) _2 the values of a are expressed as an empty. In lsd (number) _3, the results of the equation were written in matrix form. Calculated matrix in lsd (number) _1 with inv(.) command and saved as invlsd3 then the values in invlsd3 are resave to lsd (number) _2. The differences with result (number) are calculated.

The values converged after the operation are shown in the table. (Table-1)

## 3- Project Result and Analysis

Comment on error rates and methods will be evaluate in this section

A-Error Rate

The polynomial with the least error is fitted. The table can be examined to compare. (Table-1), (Table-2), (Table-3)

Linear Least Squares

| Xi | Yi | P(x) | Yi-P(x) |
|---|---|---|---|
| 1000 | 0.030495642 | 0.0285561288181819 | 0.00193951318181814 |
| 1100 | 0.027594421 | 0.0270311761636364 | 0.000563244836363594 |
| 1200 | 0.025538733 | 0.0255062235090909 | 3,25E+09 |
| 1300 | 0.02299776 | 0.0239812708545455 | -0.000983510854545493 |
| 1400 | 0.02134025 | 0.0224563182000000 | -0.00111606820000004 |
| 1500 | 0.020604664 | 0.0209313655454546 | -0.000326701545454576 |
| 1600 | 0.017475806 | 0.0194064128909091 | -0.00193060689090912 |
| 1700 | 0.017344306 | 0.0178814602363637 | -0.000537154236363666 |
| 1800 | 0.01590431 | 0.0163565075818182 | -0.000452197581818206 |
| 1900 | 0.015559922 | 0.0148315549272728 | 0.000728367072727248 |
| 2000 | 0.015389207 | 0.0133066022727273 | 0.00208260472727271 |

(Table-1)

Polinomial Least Squares degree 2

| Xi | Yi | P(x) | Yi-P(x) |
|---|---|---|---|
| 1000 | 0.030495642 | 0.0304496989230778 | 4,59E+09 |
| 1100 | 0.027594421 | 0.0277886042055953 | -0.000194183205595305 |
| 1200 | 0.025538733 | 0.0253799855020988 | 0.000158747497901186 |
| 1300 | 0.02299776 | 0.0232238428125884 | -0.000226082812588381 |
| 1400 | 0.02134025 | 0.0213201761370640 | 2,01E+09 |
| 1500 | 0.020604664 | 0.0196689854755256 | 0.000935678524474362 |
| 1600 | 0.017475806 | 0.0182702708279733 | -0.000794464827973348 |
| 1700 | 0.017344306 | 0.0171240321944071 | 0.000220273805592901 |
| 1800 | 0.01590431 | 0.0162302695748269 | -0.000325959574826892 |
| 1900 | 0.015559922 | 0.0155889829692327 | -2,91E+09 |
| 2000 | 0.015389207 | 0.0152001723776246 | 0.000189034622375362 |

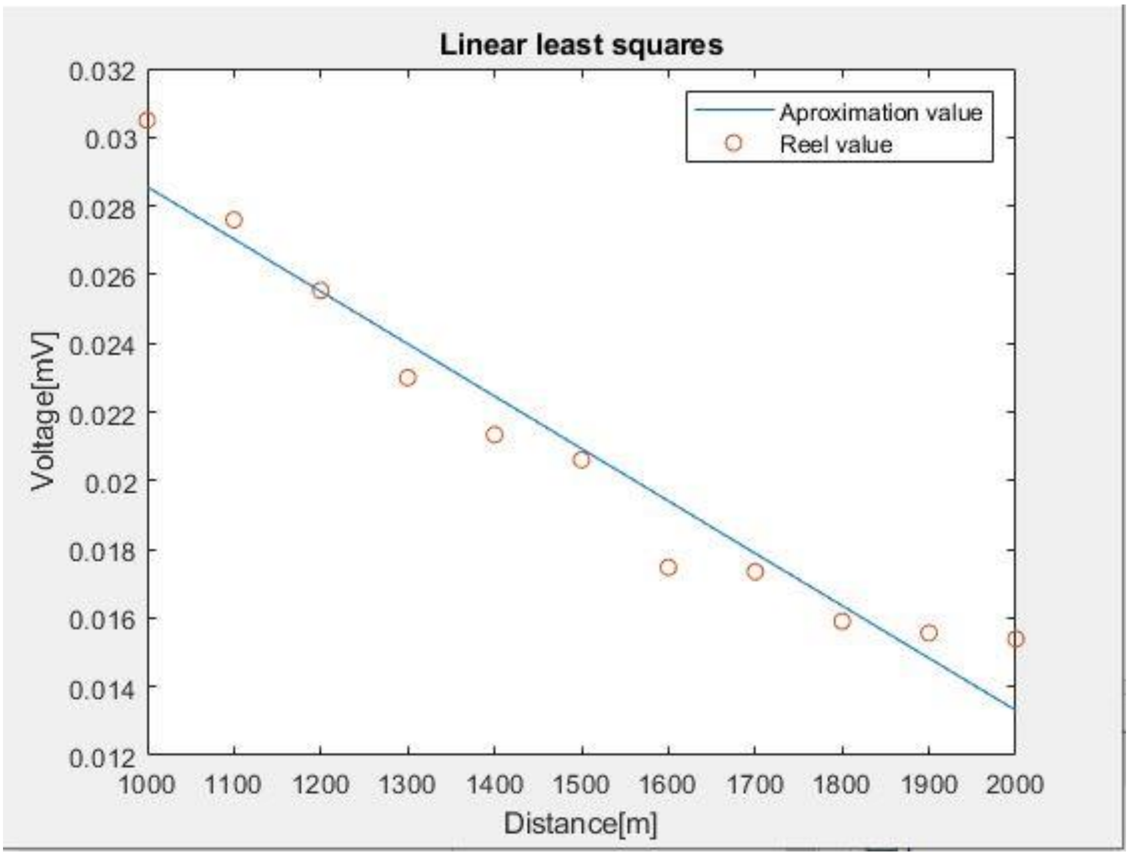
(Table-2)

Polinomial Least Squares degree 3

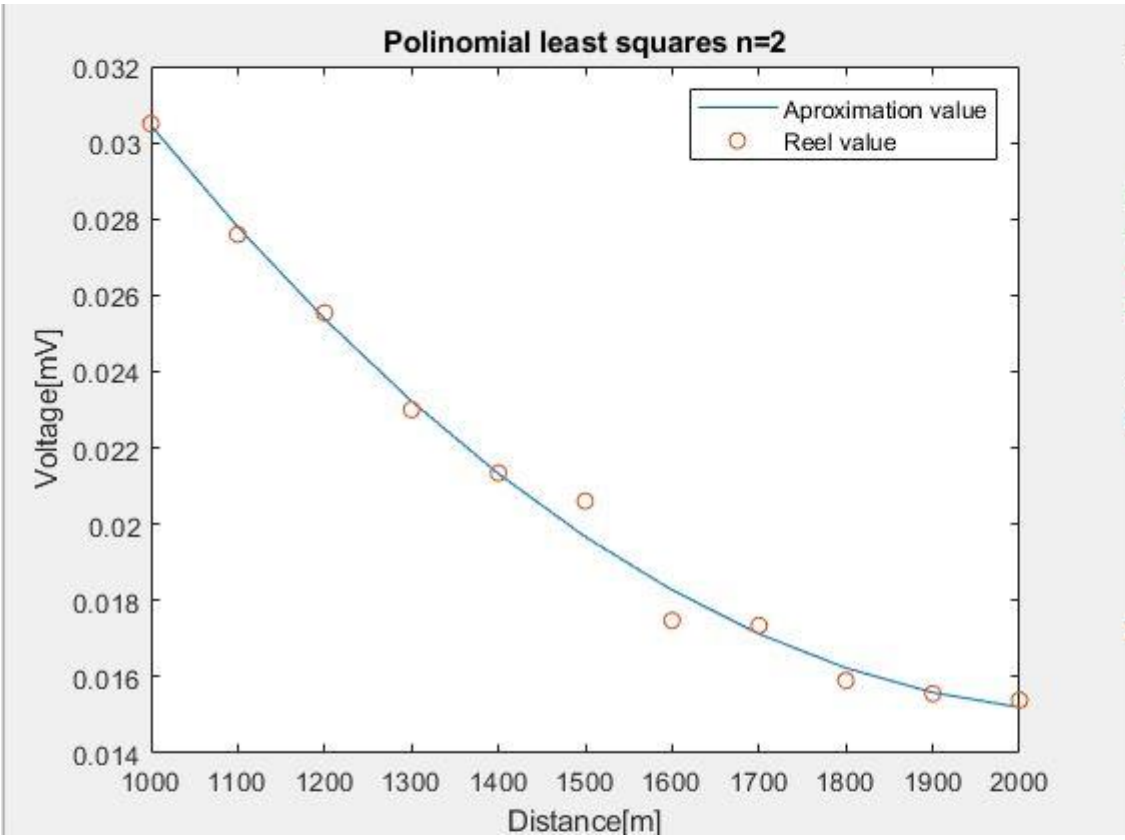
| Xi | Yi | P(x) | Yi-P(x) |
|---|---|---|---|
| 1000 | 0.030495642 | 0.0285561288181819 | 0.000151557818893752 |
| 1100 | 0.027594421 | 0.0270311761636364 | -0.000215306152997329 |
| 1200 | 0.025538733 | 0.0255062235090909 | 8,13E+09 |
| 1300 | 0.02299776 | 0.0239812708545455 | -0.000307054113065482 |
| 1400 | 0.02134025 | 0.0224563182000000 | -2,92E+09 |
| 1500 | 0.020604664 | 0.0209313655454546 | 0.000935678525977507 |
| 1600 | 0.017475806 | 0.0194064128909091 | -0.000745177947024649 |
| 1700 | 0.017344306 | 0.0178814602363637 | 0.000301245109122588 |
| 1800 | 0.01590431 | 0.0163565075818182 | -0.000248508762456915 |
| 1900 | 0.015559922 | 0.0148315549272728 | -7,94E+08 |
| 2000 | 0.015389207 | 0.0133066022727273 | 8,34E+09 |

(Table-3)

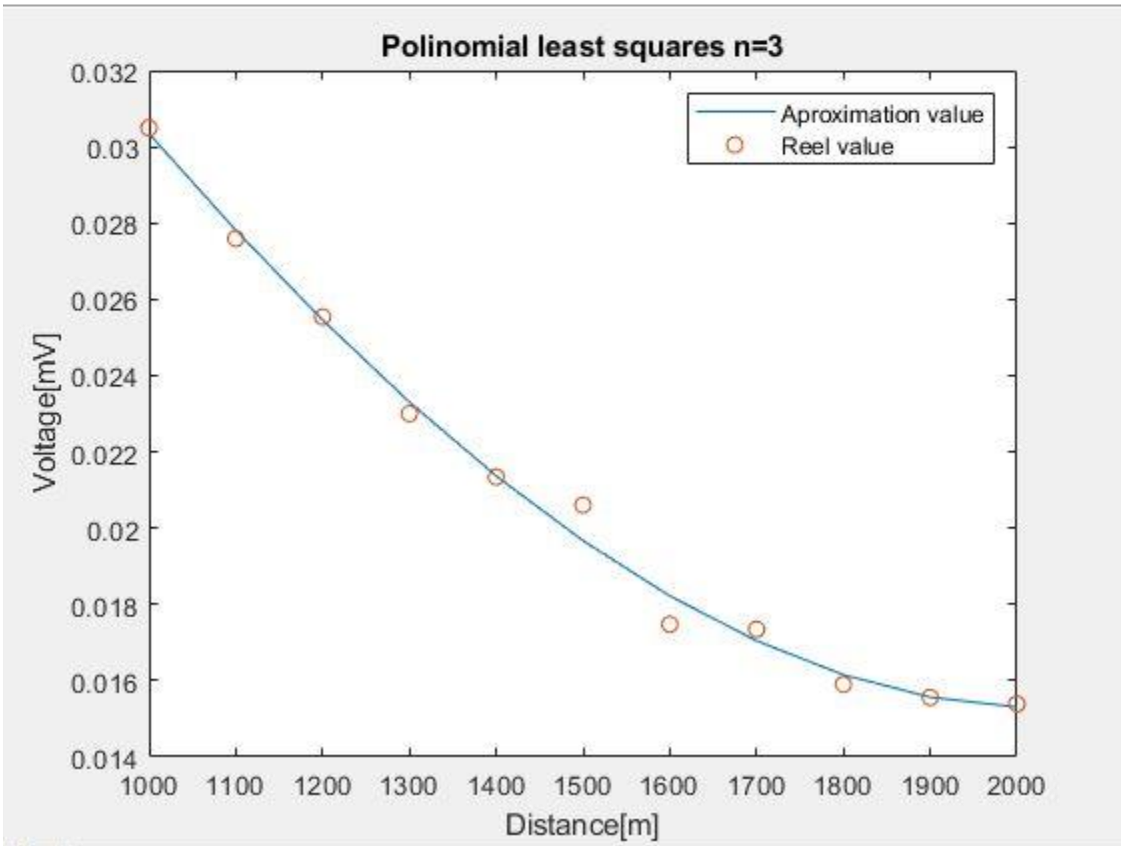
If we plot the data we show in the table:




(Plot-1)


Polinomial least squares n=2
Aproximation value
Reel value
Voltage[mV]
0.032
0.03
0.028
0.026
0.024
0.022
0.02
0.018
0.016
0.014
1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
Distance[m]

(Plot-2)


Polinomial least squares n=3
Aproximation value
Reel value
Voltage[mV]
0.032
0.03
0.028
0.026
0.024
0.022
0.02
0.018
0.016
0.014
1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000
Distance[m]

(Plot-3)

B-Analysis

When the graphics and tables are examined, the placement is made with the least error according to the polynomials.

Matlab gives a warning because the numbers reach very small values.

## 4- Appendix Section

```
clc
clear

load pr5data.dat;
xi=pr5data(:,1);  yi=pr5data(:,2);
%Linear least squares
m=length(xi);
txi=0;
tyi=0;
txi2=0;
txiyi=0;
plsa0=0;
plsa1=0;
for i=1 : m
    txi=xi(i,1)+txi;
    tyi=yi(i,1)+tyi;
    txi2=(xi(i,1)^2)+txi2;
    txiyi=(xi(i,1)*yi(i,1))+txiyi;
end
plsa0=(txi2*tyi-txiyi*txi)/(m*txi2-(txi^2));
plsa1=(m*txiyi-txi*tyi)/(m*txi2-(txi^2));

plsPx = @ (x) plsa1*x+plsa0;
plsy=zeros(m,1);
for i=1 : m
   plsy(i,1)=plsPx(xi(i,1));
end
result1=zeros(m,1);
for i=1 : m
    result1(i,1)=yi(i,1)-plsy(i,1);
end
plot(xi,plsy);
hold on
plot(xi,yi,'o');
xlabel('Distance[m]');
ylabel('Voltage[mV]');
title('Linear least squares');
legend('Aproximation value','Reel value');

%polinomial least squares degree 2
txin=0;
txi2n=0;
txi3n=0;
txi4n=0;
txiyi2n=zeros(3,1);
```

```
plsa2=zeros(3,1);
n=0;
n1=2;
for i=1 : m
    txin=xi(i,1)+txin;
    txi2n=(xi(i,1)^2)+txi2n;
    txi3n=(xi(i,1)^3)+txi3n;
    txi4n=(xi(i,1)^4)+txi4n;
end
for n=0 : 2
for i=1 : m
    txiyi2n(n+1,1)=((xi(i,1)^n)*yi(i,1))+txiyi2n(n+1,1);
end
end
                         %equation system definition
%     plsa2(1,1)*m+plsa2(2,1)*txin+plsa2(3,1)*txi2n==txiyi2n(1,1);
%     plsa2(1,1)*txin+plsa2(2,1)*txi2n+plsa2(3,1)*txi3n==txiyi2n(2,1);
%     plsa2(1,1)*txi2n+plsa2(2,1)*txi3n+plsa2(3,1)*txi4n==txiyi2n(3,1);

lsd2_1=[m txin txi2n; txin txi2n txi3n; txi2n txi3n txi4n];
lsd2_2=[plsa2(1,1);plsa2(2,1);plsa2(3,1)];
lsd2_3=[txiyi2n(1,1);txiyi2n(2,1);txiyi2n(3,1)];

invlsd2=inv(lsd2_1);
lsd2_2=invlsd2*lsd2_3;

plsPx2 = @ (x) lsd2_2(3,1)*x^2+lsd2_2(2,1)*x+lsd2_2(1,1);
plsy2=zeros(m,1);
for i=1 : m
   plsy2(i,1)=plsPx2(xi(i,1));
end
result2=zeros(m,1);
for i=1 : m
    result2(i,1)=yi(i,1)-plsy2(i,1);
end
figure
plot(xi,plsy2);
hold on
plot(xi,yi,'o');
xlabel('Distance[m]');
ylabel('Voltage[mV]');
title('Polinomial least squares n=2');
legend('Aproximation value','Reel value');

%polinomial least squares degree 3
txin=0;
txi2n=0;
txi3n=0;
txi4n=0;
txi5n=0;
txi6n=0;
txiyi3n=zeros(4,1);
plsa3=zeros(4,1);
n=0;
n2=3;
for i=1 : m
    txin=xi(i,1)+txin;
    txi2n=(xi(i,1)^2)+txi2n;
    txi3n=(xi(i,1)^3)+txi3n;
    txi4n=(xi(i,1)^4)+txi4n;
    txi5n=(xi(i,1)^5)+txi5n;
```

```
    txi6n=(xi(i,1)^6)+txi6n;
end
for n=0 : 3
for i=1 : m
        txiyi3n(n+1,1)=((xi(i,1)^n)*yi(i,1))+txiyi3n(n+1,1);
end
end
                                %equation system definition
%
plsa3(1,1)*m+plsa3(2,1)*txin+plsa3(3,1)*txi2n+plsa3(4,1)*txi3n==txiyi3n(1,1
);
%
plsa3(1,1)*txin+plsa3(2,1)*txi2n+plsa3(3,1)*txi3n+plsa3(4,1)*txi4n==txiyi3n
(2,1);
%
plsa3(1,1)*txi2n+plsa3(2,1)*txi3n+plsa3(3,1)*txi4n+plsa3(4,1)*txi5n==txiyi3
n(3,1);
%
plsa3(1,1)*txi3n+plsa3(2,1)*txi4n+plsa3(3,1)*txi5n+plsa3(4,1)*txi6n==txiyi3
n(4,1);

lsd3_1=[m txin txi2n txi3n; txin txi2n txi3n txi4n; txi2n txi3n txi4n txi5n
; txi3n txi4n txi5n txi6n];
lsd3_2=[plsa3(1,1);plsa3(2,1);plsa3(3,1);plsa3(4,1)];
lsd3_3=[txiyi3n(1,1);txiyi3n(2,1);txiyi3n(3,1);txiyi3n(4,1)];

invlsd3=inv(lsd3_1);
lsd3_2=invlsd3*lsd3_3;

plsPx3 = @ (x) lsd3_2(4,1)*x^3+lsd3_2(3,1)*x^2+lsd3_2(2,1)*x+lsd3_2(1,1);
plsy3=zeros(m,1);
for i=1 : m
   plsy3(i,1)=plsPx3(xi(i,1));
end
result3=zeros(m,1);
for i=1 : m
    result3(i,1)=yi(i,1)-plsy3(i,1);
end
figure
plot(xi,plsy3);
hold on
plot(xi,yi,'o');
xlabel('Distance[m]');
ylabel('Voltage[mV]');
title('Polinomial least squares n=3');
legend('Aproximation value','Reel value');
```