Extragalactic Astro, HW 4

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1 Stellar populations, Order of Magnitude Exercise 4

What is difference between the mass-to-light ratios of two 10-billion-year-old stellar populations, one with a Salpeter IMF and one with an IMF which is Salpeter above $M = 0.7M_\odot$ but has $\Phi(M) \propto$constant at lower masses?.

Solution: In the following I use a combination of Schneider's Extragalactic Astronomy and Cosmology and Gary Glatzmaier's class notes for the physics of stars https://websites.pmc.ucsc.edu/~glatz/astr_112/lectures/notes19.pdf as sources.

The initial mass function (IMF) is the initial mass distribution of stars at their birth. I can write down the normalization conditions of this distribution

$$1M_\odot = \int_{M_{min}}^{M_{max}} M\Phi(M)dM \tag{1}$$

and

$$1 = \int_{M_{min}}^{M_{max}} \Phi(M)dM \tag{2}$$

where $\Phi(M)$ is the IMF, and M_{min}, M_{max} are the minimum and maximum observed initial star masses, respectively.

For main sequence stars, let's assume that

$$L \propto M^3 \tag{3}$$

and therefore the mass to light ratio is

$$\frac{M}{L} \propto L^{-2/3}. \tag{4}$$

To find the mass to light ratio for a population of stars given its IMF, I therefore just have to find its luminosity. The initial luminosity L_0 can be found from a IMF:

$$L_0 = N\int_{M_{min}}^{M_{max}} L(M)\Phi(M)dM \tag{5}$$

where N is the number of stars in the population and $L(M)$ is the star luminosity as a function of mass M.

However, suppose we don't want the star population's initial luminosity, we want its luminosity 10 billion years later. This makes this population of stars pretty old, and during these 10 billion years stars have peeled off the main sequence. Because star lifetime decreases with mass, as time passes lower and lower mass stars break off from the main sequence. Let's assume that at 10 billion years since the birth of our population, stars of $1M_\odot$ have just departed the main sequence. Then the current luminosity of the population will be

$$L_1 = N\int_{M_{min}}^{M_t} L(M)\Phi(M)dM. \tag{6}$$

By taking the ratio of L_1 to L_0, I can calculate by how much the population luminosity has decreased from the initial luminosity

$$\frac{L_1}{L_0} = \frac{\int_{M_{min}}^{M_t} M^3\Phi(M)dM}{\int_{M_{min}}^{M_{max}} M^3\Phi(M)dM} \tag{7}$$

where I have used the Eq. 3 to get the approximation

$$L(M) = L_\odot\left(\frac{M}{M_\odot}\right)^3. \tag{8}$$

From Eq. 7 and Eq. 4, if the luminosity of a population decreases by L_1/L_0, its mass to light ratio will increase by $(L_1/L_0)^{-2/3}$.

Now consider 2 populations of stars that are both 10 billion years old, except one has the IMF

$$\Phi(M) \propto M^{-2.35} \tag{9}$$

and the other has the IMF

$$\Phi'(M) \propto \begin{cases} 1, \; M < M' \\ M^{-2.35} \text{ otherwise.} \end{cases} \tag{10}$$

The former equation is known as the Salpeter IMF. Here, I set $M' = 0.7M_\odot$, $M_{min} = 0.1M_\odot$, $M_{max} = 100M_\odot$, and $M_t = 1M_\odot$. In the following, I won't worry about finding the proportionality constants for the IMFs because I will just end up dividing them out anyway when I take luminosity ratios.

The Salpeter population's decrease in luminosity after 10 billion years is

$$\frac{L_1}{L_0} = \frac{\int_{M_{min}}^{M_t} M^{0.65} dM}{\int_{M_{min}}^{M_{max}} M^{0.65} dM} = \frac{M_t^{1.65} - M_{min}^{1.65}}{M_{max}^{1.65} - M_{min}^{1.65}} \sim 5 \times 10^{-4} \tag{11}$$

The non-Salpeter population's decrease in luminosity after 10 billion years is

$$\frac{L_1'}{L_0'} = \frac{\int_{M_{min}}^{M'} M^3 dM + \int_{M'}^{M_t} M^{0.65} dM}{\int_{M_{min}}^{M'} M^3 dM + \int_{M'}^{M_{max}} M^{0.65} dM} = \frac{\frac{1}{4}(M'^4 - M_{min}^4) + \frac{1}{1.65}(M_t^{1.65} - M'^{1.65})}{\frac{1}{4}(M'^4 - M_{min}^4) + \frac{1}{1.65}(M_{max}^{1.65} - M'^{1.65})} \sim 3 \times 10^{-4} \tag{12}$$

Therefore, the increase in mass to light ratio after 10 billion years for the Salpeter population is by a factor of ~ 220, while the increase in mass to light ratio for the non-Salpeter population is by a factor of ~ 160. If the 2 populations started out with roughly the same mass to light ratio at their birth, then the Salpeter population will have a mass to light ratio roughly 1.3 times larger than that of the non-Salpeter population after 10 billion years.