Extragalactic Astro, HW 8

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1 Dynamics, Order-of-magnitude Exercises, Exercise 3

What are the tidal radii for:

1. A Milky Way mass galaxy (1012 $M_\odot$ total) inside a Coma-mass cluster of galaxies (1015 $M\odot$), at 500 kpc from the cluster center.
2. An LMC-mass galaxy (1011 $M_\odot$ total) inside a Milky Way-mass galaxy, at 50 kpc from the galactic center.
3. A relatively massive globular cluster (105 $M_\odot$) inside a Milky Way mass galaxy, at 10 kpc from the galactic center.

Solution: Consider a system of 2 masses m and M interacting gravitationally. Let $m < M$. Then the largest orbit radius possible is given by the tidal radius r. For radii larger than the tidal radius, mass m becomes unbound from mass M. The expression for tidal radius is

$$r = (\frac{m}{3M})^{1/3} D \tag{1}$$

where D is the separation between the masses.

Plugging in the above numbers into this equation, the tidal radii for the above cases are as follows:

1. $r \sim 346$ kpc
2. $r \sim 34.7$ kpc
3. $r \sim 3.26$ kpc

2 Dynamics, Order-of-magnitude Exercises, Exercise 4

What is the dynamical friction time scale for:

1. A Milky Way mass galaxy (1012 $M_\odot$ total) inside a Coma-mass cluster of galaxies ($\sigma \sim 1000$ km/s), at 500 kpc from the cluster center.
2. An LMC-mass galaxy (1011 $M_\odot$ total) inside a Milky Way-mass galaxy ($\sigma \sim 150$ km/s), at 50 kpc from the galactic center.
3. A relatively massive globular cluster (105 $M_\odot$) inside a Milky Way mass galaxy, at 10 kpc from the galactic center.

Solution: Consider a system in which a body mass M is moving through a system with density ρ with an isothermal distribution function of velocity with width σ. At at large radius $\rho \propto r^{-2}$ for this isothermal model, and at these large radii the circular velocity is $v_c = \sqrt{2}\sigma$. The total mass of the isothermal system is M_{tot}, and the initial separation of mass M from the center of the isothermal system is r_i. Then a drag force is exerted on the moving body, and the time scale of this drag force is

$$t_f = \frac{2 \times 10^{11}}{\ln(\frac{M_{tot}}{M})} [\frac{r_i}{2\text{kpc}}][\frac{v_c}{250\text{km/s}}][\frac{10^6 M_\odot}{M}] \tag{2}$$

Plugging in the above numbers into this equation, the tidal radii for the above cases are as follows:

1. $r_i = 500\text{kpc}$, $v_c = \sqrt{2} \times 1000\text{km/s}$, $M_{tot} = 1015 M_\odot$, $M = 1012 M_\odot$, $\implies t_f \sim 1 \times 10^{20}$ yr
2. $r_i = 50\text{kpc}$, $v_c = \sqrt{2} \times 150\text{km/s}$, $M_{tot} = 1012 M_\odot$, $M = 1011 M_\odot$, $\implies t_f \sim 2 \times 10^{11}$ yr
3. $r_i = 10\text{kpc}$, $v_c = \sqrt{2} \times 150\text{km/s}$, $M_{tot} = 1012 M_\odot$, $M = 105 M_\odot$, $\implies t_f \sim 1 \times 10^8$ yr