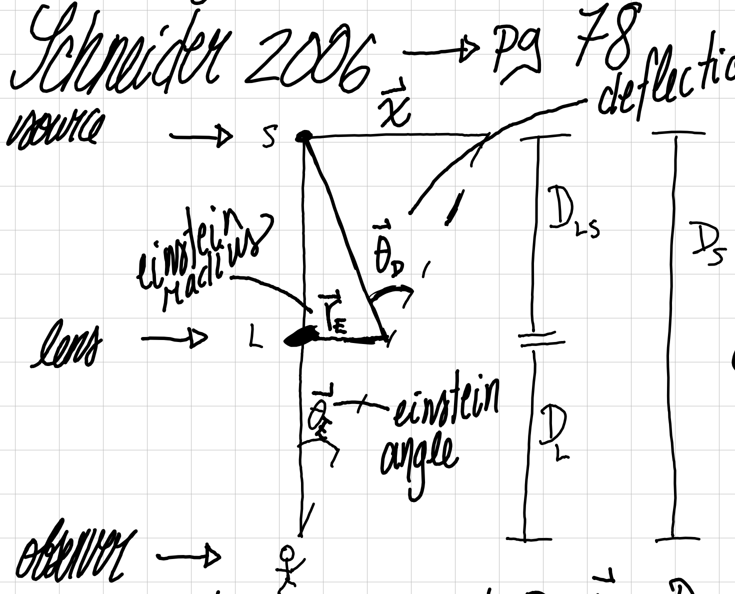
# Extragalactic Astro HW#10

## 1) Lensing, Analytic exercise #2

For a symmetric point mass, derive the angular radius of the image that is formed, called the Einstein angle

↳ here we'll follow the approach of Schneider 2006 → pg 78




$$\vec{\alpha} = \frac{4GM}{c^2 r_E}$$

$$\vec{r}_E = D_L \vec{\theta}_E$$

↳ $\vec{\theta}_E = \frac{\vec{r}_E}{D_L}$

$$D_{LS}\vec{\theta}_D = D_S \vec{\theta}_E = \frac{D_S}{D_L}\vec{r}_E = \vec{x} \to \vec{\theta}_E = \frac{D_{LS}}{D_S}\vec{\theta}_D = \frac{D_{LS}}{D_S}\frac{4GM}{c^2 D_L |\vec{\theta}_E|} = \frac{D_{LS}}{D_S}\frac{4GM\vec{\theta}_E}{c^2 D_L |\vec{\theta}_E|^2}$$

$$\Rightarrow 1 = \frac{D_{LS}}{D_S}\frac{4GM}{c^2 D_L |\vec{\theta}_E|^2} \to |\vec{\theta}_E|^2 = \frac{D_{LS}}{D_S}\frac{4GM}{c^2 D_L}$$

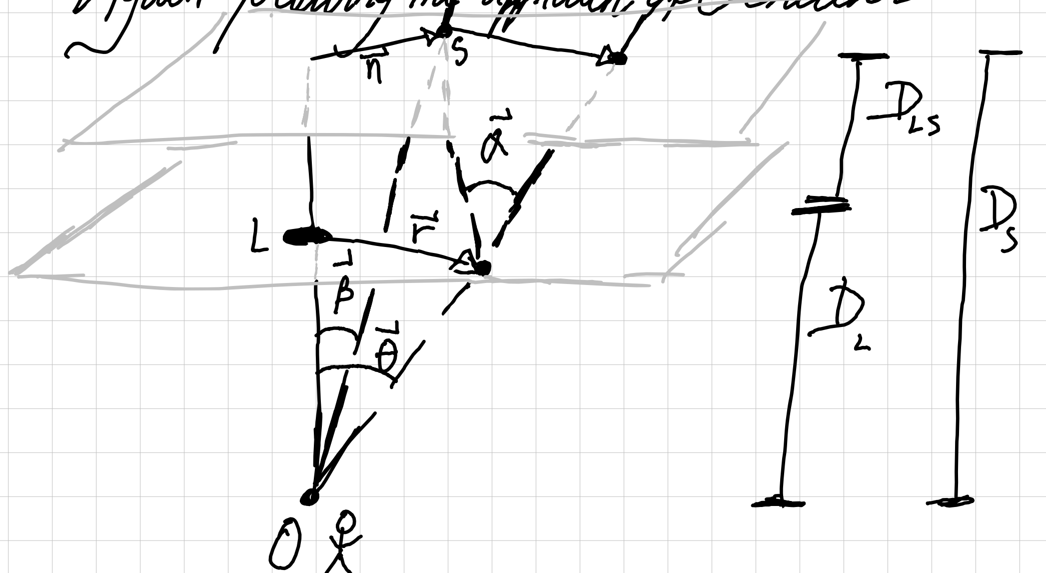
$$\Rightarrow \boxed{|\vec{\theta}_E| = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}} = \theta_E} \checkmark$$

2) Gravitational lensing, Analytical

Exercise 3

Calculate the location of the images for the offset point mass case

→ Again following the approach of Schneider:




$D_L\vec{\theta} = \vec{r}$, $D_S\vec{\beta} = \vec{\eta}$, $\vec{\eta} = D_S\vec{\theta} - D_{LS}\vec{\alpha} = \frac{D_S}{D_L}\vec{r} - D_{LS}\vec{\alpha}$

$\vec{\beta} = \frac{\vec{\eta}}{D_S} \rightarrow \vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_S}\vec{\alpha}$

Define the deflection angle $\vec{\theta}_D$:

$$\vec{\theta}_D \equiv \frac{D_{LS}}{D_S}\vec{\alpha} \rightarrow \vec{\beta} = \vec{\theta} - \vec{\theta}_D$$

$$\Rightarrow \vec{\theta}_D = \frac{4GM}{c^2}\frac{D_{LS}}{D_S D_L}\frac{\vec{\theta}}{|\vec{\theta}|^2} = \theta_E^2\frac{\vec{\theta}}{|\vec{\theta}|^2}, \quad \theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}}$$

$$\Rightarrow \vec{\beta} = \vec{\theta} - \theta_E^2 \frac{\vec{\theta}}{|\vec{\theta}|^2}$$

define coordinates $\vec{x}$ and $\vec{y}$ in terms of the characteristic angle $\theta_E$:

$$\vec{y} \equiv \frac{\vec{\beta}}{\theta_E}, \quad \vec{x} \equiv \frac{\vec{\theta}}{\theta_E}$$

$$\Rightarrow \vec{y} = \vec{x} - \frac{\vec{x}}{|\vec{x}|^2}$$

↳ this is a quadratic eq:

$$\vec{x} = \frac{1}{2}\left(|\vec{y}| \pm \sqrt{4 + |\vec{y}|^2}\right)\frac{\vec{y}}{|\vec{y}|}$$

↳ this yields 2 images for $\vec{y} \neq 0$

the coordinates $(\vec{x}^{(+)}, \vec{y}^{(+)})$ and $(\vec{x}^{(-)}, \vec{y}^{(-)})$ denote their location.