Extragalactic Astro, HW 3

Mia Morrell

October 1, 2022

1 Detectors, Analytic Exercise 2

The noise in a data set is determined by the Poisson noise in the expectation value of the signal. However, observationally we often only have access to one realization of the signal, and often the quoted errors are based on the Poisson noise in that signal. If we take multiple observations and combine them together weighting by the inverse variance estimated in this way, it leads to a bias. Ignoring any background contribution to the noise, estimate this bias as a function of n_e.

2 Solution

Here, n_e is the number of electrons recorded by the detector. Recall that the digital numbers reported by the detector are $N_{DN} = n_e/G$ where G is gain. Gain is the number electrons per digital number reported by the detector. (See Detector notes for this class for this info.)

In the absence of background noise, the noise of N_{DN} is Poisson distributed such that $\sigma^2_{DN} = \frac{n_e}{G^2}$. Let N_{DN} be drawn from a Poisson distribution with a true mean of $\lambda = \frac{n_e}{G^2}$ and a variance of $\lambda = \frac{n_e}{G^2}$.

Suppose we take multiple observations of N_{DN} and combine them together in a weighted average. Then the weighted average of the observed digital number reported by the detector is

$$\bar{\lambda} = \frac{\sum_{i=1}^{N} n_i/(G\sigma_i^2)}{\sum_{i=1}^{N}(1/\sigma_i^2)} = \frac{\sum_{i=1}^{N} G}{\sum_{i=1}^{N}(G^2/n_i)} = \frac{N}{G\sum_{i=1}^{N}(1/n_i)} \tag{1}$$

where n_i is the number of electrons reported by the detector at observation i and σ_i^2 is the variance of that observation. In the above I used the fact that the variance of a Poisson distribution is equal to its mean.

Then the bias observed between the weighted mean and the true mean is

$$\Phi(n_e) = \frac{(\bar{\lambda} - \lambda)}{\lambda} = \frac{G^2}{n_e}\left(\frac{N}{G\sum_{i=1}^{N}(1/n_i)} - \frac{n_e}{G^2}\right) = \frac{GN}{n_e\sum_{i=1}^{N}(1/n_i)} - 1. \tag{2}$$