```
//autor: Saeed
method Max(arr: array<int>) returns (result: int)
requires arr != null && arr.Length > 0;
ensures forall k: int :: 0 <= k < arr.Length ==> result >= arr[k];
 var i : int := 1;
 var max : int := arr[0];
 while i < arr.Length
 invariant i >= 0;
 invariant i <= arr.Length;
 invariant max >= arr[i-1];
 invariant forall j: int :: 0 \le j \le i ==> \max >= arr[j];
 decreases arr.Length - i;
  if (max < arr[i])
  max := arr[i];
 i := i+1;
 result := max;
method Search(a: array<int>, x: int) returns (i: int)
requires a != null;
ensures 0 \le i = > i \le a.Length && a[i] == x;
ensures i < 0 ==> forall k :: 0 <= k < a.Length ==> a[k] != x;
{
 i := 0;
 while (i < a.Length)
 invariant 0 <= i <= a.Length;
 invariant forall k :: 0 \le k \le i ==> a[k] != x;
  if (a[i] == x) { return i; }
  i := i + 1;
 }
 i := -1;
// Übungsaufgabe 3.3*****************************
```

```
predicate sorted(a: array<int>, l:int, h:int) // prädikat - zum überprüfung
requires a != null;
requires a.Length >= h >= 0;
requires a.Length >= I >= 0;
reads a; //ohne das kann er "a" nicht lesen
 forall j, k :: l \le j < k < h ==> a[j] \le a[k]
}
method MaxSort(a: array<int>) returns (b: array<int>)
requires a != null;
modifies a;
ensures b != null;
ensures sorted(b, 0, b.Length);
ensures multiset(b[..]) == multiset(old(b[..]));
{
  b := a;
 var i := b.Length;
  var m := 0;
  while (i > 0)
  decreases i;
  invariant 0 <= m <= i <= b.Length;
  invariant sorted(b, i, b.Length);
  invariant forall k, I :: 0 \le k \le i - 1 \le l \le b. Length ==> b[k] <= b[l];
  invariant multiset(b[..]) == multiset(old(b[..]));
  //für Test
   print multiset(b[..]);
   print "\n";
   //für Test
   var m := MaxIndex(b, i - 1); //die maximale erreiichbare index wird gesucht
   b[m], b[i-1] := b[i-1], b[m];
  i := i - 1;
 }
}
method MaxIndex(a: array<int>, j: int) returns (imax: int)
requires a != null;
requires 0 <= j < a.Length;
ensures 0 <= imax <= j;
ensures forall k :: 0 \le k \le j \Longrightarrow a[k] \le a[imax];
 imax := 0;
 var i := 0;
 while (i \le j)
 decreases j - i;
 invariant 0 <= j < a.Length;
 invariant 0 \le i \le j + 1;
```

```
invariant 0 <= imax <= j;
 invariant forall k :: 0 \le k \& k \le i \Longrightarrow a[imax] >= a[k];
  if (a[i] > a[imax]) { imax := i; }
  i := i + 1;
}
// Übungsaufgabe 3.4**************
method InsertionSort(a: array<int>) returns (b: array<int>)
requires a != null;
modifies a; // veränderung der variable "a" ist erlaubt
ensures b != null;
ensures sorted(b, 0, b.Length);
ensures multiset(old(b[..])) == multiset(b[..])
{
 b := a;
 if (b.Length < 2) { return; }</pre>
 var i, j := 1, 0;
 while i < b.Length
 invariant i <= b.Length;
 invariant sorted(b, 0, i);
 invariant multiset(b[..]) == multiset(old(b[..]));
 decreases b.Length - i;
 {
  //für Test
  print multiset(b[..]);
  print "\n";
  //für Test
  j := i;
  while j > 0 \&\& b[j] \le b[j - 1]
  invariant forall k, I :: 0 \le k \le j \le l \le i ==> b[k] \le b[l];
  invariant sorted(b, 0, j) && sorted(b, j, i + 1);
  invariant multiset(b[..]) == multiset(old(b[..]));
  decreases j;
           b[j], b[j-1] := b[j-1], b[j];
   j := j - 1;
  i := i + 1;
 }
}
```

```
method Main()
 print "********MaxTest********\n";
 var a := new int[10];
 var i := 0;
 while i < a.Length
  a[i] := i + 1;
  i := i + 1;
 }
 var result := Max(a);
 print "maximalen Element in Array: ";
 print result;
 print "n\n\n";
 print "*******SearchTest********\n";
 var x := 5;
 i := 0;
 a := new int[10];
 while i < a.Length
 {
  a[i] := i + 1;
  i := i + 1;
 }
 result := Search(a, x);
 print "Index der gesuchte Element ist: ";
 print result;
 print "n\n\n";
 print "*******MaxSortTest********\n";
 a := new int[10];
 i := 10;
 while i > 0
  a[a.Length - i] := i;
  i := i - 1;
 }
 print "Array VOR MaxSort:\n[";
 i := 0;
 while i < a.Length
  print a[i];
```

```
if i + 1 != a.Length
  print ", ";
 }
i := i + 1;
}
print "]";
print "n\n";
var result_2 := MaxSort(a);
if result_2 != null
 print "\nArray NACH MaxSort:\n[";
 while i < result_2.Length
  print result_2[i];
  if i + 1 != result_2.Length
  {
   print ", ";
 i := i + 1;
 print "]";
 print "\n\n\n";
print "********InsertionSortTest*********\n";
a := new int[10];
i := 10;
while i > 0
 a[a.Length - i] := i;
i := i - 1;
}
print "Array VOR InsertionSort:\n[";
i := 0;
while i < a.Length
 print a[i];
 if i + 1 != a.Length
  print ", ";
```

```
}
 i := i + 1;
 }
 print "]";
 print "\n\n";
 var result_3 := InsertionSort(a);
 if result_3 != null
  print "\nArray NACH InsertionSort:\n[";
  i := 0;
  while i < result_3.Length
   print result_3[i];
   if i + 1 != result_3.Length
   {
    print ", ";
   i := i + 1;
  print "]";
  print "n\n\n";
}
```