How Does Household Earnings Risk Influence Aggregate Investment?*

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Abstract

When households face both idiosyncratic and aggregate risk under incomplete markets, each household will have different relative valuations of each future aggregate state. Given this heterogeneity, how does a firm owned by these households value payoffs in each state? I develop a method of aggregating shareholder preferences that is disciplined by the threat of an outside manager. This method has two useful properties relative to other approaches. First, the firm's value function is its cum-dividend share price. Second, it doesn't matter exactly who holds the median share. I find firms engage in more precautionary savings behavior as their shareholders are exposed to increasing uninsurable idiosyncratic risk. Models where firms do not account for household earnings risk will invest too little because they undervalue payouts in low aggregate productivity states. I also show that inequality in productivity or wealth generates an increase in investment and a decline in interest rates.

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1 Introduction

All dynamic economic models require agents to assign value to payoffs across future states. Households maximize lifetime utility by balancing marginal utility today against expected discounted marginal utility in the future. These intertemporal marginal rates of substitution determine how much a household is willing to save, invest, or borrow given a set of prices. When markets are incomplete, this leads to heterogeneity in marginal rates of substitution across households. Asset rich or high productivity households will value payoffs in good states relatively more than poorer or lower productivity households value those states (Aiyagari, 1994).

Firms also need to discount the future when they are held by risk-averse households. In a representative household setting, firms maximize shareholder value if they value future aggregate states by the discounted expected marginal rates of substitution of the representative household (Khan and Thomas, 2008). This effectively passes the household's risk-aversion to the firm.

Alternatively, some models assume that firms discount by a fixed interest rate r (Bloom, 2009). This approach simplifies modeling by removing a price from the problem, but fails to account for risk aversion by the shareholders who own the firm.

When firms are owned by heterogeneous shareholders in incomplete markets, however, it is not immediately clear how a firm should value dividends in future states. Wealthy households are on a relatively flat part of their utility function, so their marginal rates of substitution across aggregate states will be relatively closer together than the marginal rates of substitution of poorer households. In other words, poorer households value payouts in low states much more than they value payouts in good states.

Some models account for this discrepancy by allowing entrepreneur households to own and operate their own firms, investing as they see fit (Boar, 2018). However, corporate firms are responsible for 60-65% of private fixed investment in the US[1], so this modeling approach fails to consider investment decisions of the largest group of investors. In all three of the settings above, there is not explicit aggregation of shareholder preferences.

With these existing methods and challenges in mind, I propose an equilibrium concept that aggregates households' heterogeneous discount rates into a unified discount factor for firms to use in making intertemporal decisions. I follow an approach similar to Grossman and Hart (1979), which calculates the firm's objective function using a share-weighted sum of marginal rates of substitution. While their approach was written for a firm deciding between producing different products, I demonstrate that it can be applied to an intertemporal setting.

I find that the share-weighted discounting approach results in firms that engage in precautionary savings on behalf of their shareholders beyond what a similar firm would do in a representative shareholder setting. As idiosyncratic household productivity risk increases, share price increases and firms invest more. This result suggests that individual risk plays a meaningful role in determining asset prices and investment.

I also find that the distribution of households over both productivity and shareholding can influence the firm's investment behavior. More inequality in either of these distributions will cause the firm to increase investment relative to an equality baseline. However, the interaction of these distributions can cause non-monotone investment changes. If asset rich households are also endowed with higher productivity, then investment falls. This suggests that the joint distribution of households over productivity and shareholding matters when trying to understand how firms invest.

The closest papers to my work are from Carceles-Poveda (2009) and Anagnostopou-

[1] Small businesses account for about 10% of investment in private fixed assets, households account for around 20%, and nonprofit or tax-exempt organizations account for the remaining 5%. U.S. Bureau of Economic Analysis, "Table 6.7. Investment in Private Fixed Assets by Industry Group and Legal form of Organization" (accessed July 14, 2021)

los, Cárceles-Poveda and Mihalache (2021). In those papers, the discount factor is either unanimous or set by the median shareholder. However, each model features somewhat restrictive assumptions that my model can work around. They require constant returns production while I permit decreasing or constant returns technologies. Additionally, their model is written with only two types of households. My model can permit any number of types of households. Finally, the firm price in these models is found computationally. In my approach, the price of any firm is an analytic expression. This feature allows my model to extended to environments with heterogeneous firms, including those that face investment frictions.

Another approach to aggregating discount factors of heterogeneous shareholders comes from Another similar approach that backs out aggregate discount factors is Krusell, Mukoyama and Smith (2011). They also look at a model with both aggregate and idiosyncratic risk. They require a maximally tight savings rule. This approach has the benefit of resulting in a closed form analytical solution for asset prices with the significant limitation that households are not permitted meaningful shareholding choice. In contrast, my model features discount factors that are consistent with a stock market equilibrium and non-binding savings constraint. However, if I impose the same maximally tight savings constraints, my model nests their approach.

This methodology can also be used to provide additional discipline to the common ownership literature, recently popularized by Azar, Schmalz and Tecu (2018). That literature currently relies on assumptions about firm behavior to construct the firm's problem. The approach I describe in this paper could be adapted to an environment where some shareholders prefer industry profit maximization while other shareholders want firm profit maximization.

The remainder of this paper proceeds as follows: Section 2 describes the basic setting of the three-period model environment, section 3 discusses parameterization and results, section 4 compares my model's approach to alternate settings, and section 5 concludes.

2 Environment and Model

This model features three types of agents: production firms, households, and a management firm. The production firm is owned by shareholders. It invests, produces, and pays dividends. The management firm exists solely as an off-equilibrium threat that pins down the share price of the firm. The aggregate state is a vector of aggregate productivity z, the distribution of firms over productivity and capital μ_f, and the distribution of households over labor productivity and shareholding μ_h. This is a finite horizon model which can illustrate the mechanism more simply than an infinite horizon model could. While this is a finite problem, I write each dynamic agent's problem recursively to show that the same methodology can be applied to an infinite horizon setting.

2.1 Production Firms

There are N_f ex-ante identical firms in this economy. Each firm is defined by its capital k. After observing the aggregate productivity level z and the wage w, firms choose labor n that maximizes period profits. I assume a Cobb-Douglas production function of the form $y = zk^{\alpha}n^{\gamma}$. The production technology is assumed to be non-increasing, $\alpha + \gamma \in (0, 1]$. This is a single good economy, and that good is used for both consumption and investment. For convenience, I assume that z follows a Markov chain with $z \in \{z_1, \ldots, z_{N_z}\}$ with $\Pr(z' = z_n | z = z_m) \equiv \pi_{mn} \geq 0$ and $\sum_{n=1}^{N_z} \pi_{mn} = 1 \; \forall \; m$.

After producing, firms choose investment and pay any remaining real output to shareholders as dividends. I assume capital accumulation follows a linear technology $k' = (1 - \delta)k + i$. Dividends are given as $d = y - wn - i$.

These production firms are owned by households, who buy shares in the firm through a stock market. Firms choose dividend payments in each period and discount the future by a vector of discount factors $\{\chi\}_{N_z}$ which depend on both the current and future aggregate state. The firm takes these discount factors as given. I describe these discount factors in more detail after introducing the household. In the final period, firms produce and can return any undepreciated capital for dividend payments.

Each firm is characterized only by its start of period capital. The aggregate state of the economy is made up of the distribution of households over shareholding and productivity μ_h, the distribution of firms over capital μ_f,[2] and aggregate productivity z. I collectively refer to these states with the shorthand $\mathbb{Z} \equiv \{z, \mu_f, \mu_h\}$. Because the discount factor depends on the current and each future aggregate state, I write it as $\chi(\mathbb{Z}; z_n, \mu'_f, \mu'_h)$.

The recursive version of the firm's problem is:

$$F(k;\mathbb{Z}) = \max_{d,n,k'} d + \sum_{n=1}^{N_z} \chi(\mathbb{Z}; z_n, \mu'_f, \mu'_h) F(k'; z_n, \mu'_f, \mu'_h) \tag{1}$$

$$\begin{aligned} \text{s.t. } & k' + d \leq z k^{\alpha} n^{\gamma} - wn + (1-\delta)k && t < T \qquad (2) \\ \& \quad & d \leq z k^{\alpha} n^{\gamma} - wn + (1-\delta)k && t = T \\ \& \quad & \mu'_h = \Gamma_h(\mathbb{Z}) \\ \& \quad & \mu'_f = \Gamma_f(\mathbb{Z}) \end{aligned}$$

The functions Γ_h & Γ_f describe the perceived law of motion for distributions of households and firms, respectively. The firm uses TFP and the distribution of firms over capital to anticipate future labor price. The distribution of households matters to the firm in anticipating the discount factor in future periods. I also assume that the firm's value after the final period is equal to zero.

[2]For most of this paper, the distribution will be degenerate. Explicitly tracking the distribution of firms allows for analysis of off-equilibrium deviations, which will be used to pin down the aggregate discount factor.

In equilibrium, the perceived law of motion will be correct. The discount factor can then be simplified to:

$$\chi(\mathbb{Z}; z_n, \mu'_f, \mu'_h) = \chi(\mathbb{Z}; z_n, \Gamma_f(\mathbb{Z}), \Gamma_h(\mathbb{Z})) \equiv \chi(\mathbb{Z}; z_n) \tag{3}$$

2.2 Households

A unit measure of households maximize discounted lifetime expected utility, subject to a budget constraint in each period. I start by assuming that all households are endowed in the first period with identical shareholding and productivity. I relax this assumption in section 3.1 to see how firms change their behavior when owned by households that more closely represent the US distribution of wealth and income. At the beginning of periods two and three, households receive an idiosyncratic productivity shock η following an exogenously given Markov process. I assume that $\eta \in \{\eta_1, \ldots, \eta_{N_\eta}\}$ with $\Pr(\eta' = \eta_j | \eta = \eta_i) \equiv \pi_{ij} \geq 0$ and $\sum_{j=1}^{N_\eta} \pi_{ij} = 1 \; \forall \; i$. I also set the shock process to keep aggregate labor supply fixed across aggregate states.

Households do not value leisure and therefore inelastically supply their productivity endowment to the labor market. Utility is a time separable, differentiable, continuous, concave function of consumption. I assume period utility is CRRA and takes the form $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, $\sigma > 0$. Households can save by buying shares of production firms.

Assumption 1 Households save in aggregate equity rather than investing in individual firms. This keeps the problem tractable by eliminating the portfolio choice when firms have different levels of capital. Most household wealth is in a pooled investment or retirement account, so this is not too extreme of an assumption.[3] With this simplification, I

[3]According to the 2019 SCF, roughly 64.9% of financial wealth (excluding transaction accounts) is held in pooled investment funds and retirement accounts, compared to only 18.7% held as direct stocks and bonds. The remaining 14.4% is held in trusts, CDs, and life insurance.

define aggregate price and dividend as:

$$D = D(\mathbb{Z}) \equiv \sum^{N_f} \frac{1}{N_f} d(k;\mathbb{Z}) \tag{4}$$

$$P = P(\mathbb{Z}) \equiv \sum^{N_f} \frac{1}{N_f} p(k;\mathbb{Z}) \tag{5}$$

where $d(k;\mathbb{Z})$ are the dividend choices of a firm with capital level k and aggregate state $\mathbb{Z}$ and $p(k;\mathbb{Z})$ are the share prices of the firms. I will discuss firm share price in Section 2.5.1. If all firms are identical, then the price and dividends of one firm will be equal to the aggregate price and dividend levels.

The household's problem is written recursively as:

$$V(s,\eta_i;\mathbb{Z}) = \max_{c,s'} u(c) + \beta \sum_{j=1}^{N_\eta} \sum_{n=1}^{N_z} \pi_{ij}\pi_{mn} V(s',\eta_j;z_n,\mu'_f,\mu'_h) \tag{6}$$

$$\text{s.t. } c + Ps' \leq (P+D)s + w\eta \tag{7}$$

$$\& \ \underline{s} \leq s' \tag{8}$$

$$\& \ \mu'_h = \Gamma_h(\mathbb{Z})$$

$$\& \ \mu'_f = \Gamma_f(\mathbb{Z})$$

where $\underline{s}$ is a minimum shareholding constraint. If it takes the value of 0, then it disallows short sales. If it is negative, some short sales are allowed. If it is set to 1, then all households must hold the median number of shares. I mention this special case because it allows my model to nest Krusell, Mukoyama and Smith (2011). In all cases, I assume that $\underline{s}$ is at least the natural borrowing limit. I also assume an arbitrarily large maximum savings level s_{max} to create a closed set.

2.3 Management Firm

A management firm exists as an off-equilibrium threat to discipline the discount factor used by the production firm. It operates for one period at a time and does not act if it cannot earn a positive real profit.

After households trade in the stock market, the management firm attempts to intervene by proposing an alternate investment scheme for one of the production firms. It proposes some alternate future investment $\hat{k}'$, collects funds ($\xi < 0$) from shareholders who are made better off by the new investment scheme, and compensates shareholders ($\xi > 0$) who are made worse off by the new investment scheme. These side payments must satisfy a participation constraint for all interior[4]shareholders.

A manager only makes side payments if the manager is able to collect more revenue from side payments than she has to pay out in compensation. If this is the case, she collects funds, makes payments, consumes excess revenue collected, and implements a new investment plan $\hat{k}'$. While this is a single period deviation, the investment deviation causes the production firm to begin the next period with a different starting capital than it originally planned. This deviation will have a lasting share price and dividend impacts into future periods through the new capital level.

If the manager cannot find a profitable alternate investment scheme, there are no side payments and the firm continues with its original investment and dividend plan.

Assumption 2 When a manager implements an alternate investment plan, the share price doesn't change. I relax this assumption in Appendix A where I discuss consistent price perceptions. There, I will show that my method still holds when allowing share price to change in response to an investment deviation.

[4]The firm does not make side payments to or collect payments from households that choose $s' = \underline{s}$.

2.3.1 Deviation Dividends and Future Prices

Before formally discussing the management firm's problem, I discuss how a deviation in capital will change current dividends, future expected dividends, and future expected equity price. From Equations 4 and 5, the aggregate dividend and price are the sums of firm dividends and prices, respectively. If a single firm chooses an off-equilibrium level of future capital $\hat{k}'$, the dividends it pays are:

$$\hat{d}(k;\mathbb{Z}) = d(k;\mathbb{Z}) - (\hat{k}' - k') \tag{9}$$

The dividends of a deviating firm are the dividends it would have paid less the change in investment. The aggregate dividend payments under a deviation are:

$$\hat{D}(\mathbb{Z}) \equiv D(\mathbb{Z}) - \frac{1}{N_f}(\hat{k}' - k') \tag{10}$$

By Assumption 2, the aggregate price doesn't change in this period because the price of the deviating firm doesn't change.

In the next period, the production firm returns to its normal operations. The only change to the firm is that it starts the period with $\hat{k}$ instead of k. This new level of capital slightly shifts the aggregate state. I refer to the new distribution of firms as $\hat{\mu}_f$, which reflects that a single firm deviated in the previous period. The new aggregate state after an investment deviation is $\hat{\mathbb{Z}} \equiv \{z, \hat{\mu}_f, \mu_h\}$.

Assumption 3 N_f is sufficiently large that deviation by a single firm in the current period will not change the wage or aggregate discount factor in the next period. Therefore, a deviation by a single firm will not influence any other firms in the next period.

A deviation does, however, change the aggregate dividend payments and aggregate price in the next period.

$$D(\hat{\mathbb{Z}}) = D(\mathbb{Z}) + \frac{1}{N_f}\left(d(\hat{k};\mathbb{Z}) - d(k;\mathbb{Z})\right) \tag{11}$$

$$P(\hat{\mathbb{Z}}) = P(\mathbb{Z}) + \frac{1}{N_f}\left(p(\hat{k};\mathbb{Z}) - p(k;\mathbb{Z})\right) \tag{12}$$

If the investment deviation was positive ($\hat{k}' > k'$), then dividends paid during the first period are lower than expected while future price and dividends are higher than expected.

2.3.2 Management Firm's Problem

The management firm chooses an investment deviation $\hat{k}' \neq k'$ and a vector of side payments $\xi(s,\eta)$ to each household of type s,η. The management firm's problem is written as:

$$\max_{\{\xi(s,\eta)\},\hat{k}'} \int_{(\underline{s},s_{max}]}\int_{\eta} -\xi(s,\eta)\mu_h(s,\eta)d\eta ds \tag{13}$$

$$\begin{aligned} \text{s.t. } u(c) + &\beta\mathbb{E}V(s',\eta';z',\mu_f',\mu_h') \leq u(\hat{c}) + \beta\mathbb{E}V(s',\eta';z',\hat{\mu}_f',\mu_h') \\ \hat{c} \equiv &c + \xi(s,\eta) - s\frac{1}{N_f}(\hat{k}' - k') \end{aligned} \tag{14}$$

where ˆ variables denote those that have changed due to the alternate investment scheme. Equation 14 holds for all households that chose $s' > \underline{s}$. $\xi(s,\eta)$ is the manager's choice of side payment to each type of household indexed by each household's current shareholding and productivity. When $\xi > 0$, the manager is making a payment because the household is made worse off (and vice-versa).

The $\xi(s,\eta)$ values are pinned down by Equation 14 which requires shareholders to be compensated for any deviation. Because of the concavity and monotonicity of the utility function, this constraint will always bind. If the management firm cannot find a profitable deviation ($\int\int -\xi(s,\eta)d\eta ds \leq 0$), no payments are made and the management firm will not operate. In equilibrium, there are no side-payments. Failure of the management firm also results in the perceived law of motion for the distribution of firms over capital will be correct ($\Gamma_f = \mu'_f(k)$).

Market power I assume that the management firm is only able to change the investment decisions of a single atomistic firm. Off the equilibrium path, this is an unrealistic assumption. If it is beneficial for a single firm to deviate, then every firm would deviate. However, this assumption prevents the management firm from being able to exercise market power in excess of what the independent production firm could have on its own. If the management firm knows it can change every firm's capital, it could optimize firm value in a way that an individual firm could never achieve. A management firm that operates all production firms could conceivably introduce monopsonist wage power in the labor market or add markups to the product market.

2.4 Equilibrium

A competitive equilibrium is defined as:

1. (Labor market) Wages w clear the labor market in each aggregate state.
   $\int_k n\mu_f(k)dk = 1 = \int_\eta \int_s \eta\mu_h(s,\eta)dsd\eta$
2. (Stock market) The price for aggregate equity $P(\mathbb{Z})$ clears the stock market in each aggregate state. $\int_\eta \int_s s'\mu_h(s,\eta)dsd\eta = 1$

3. (Household choices) Consumption and shareholding decisions solve the household's value function described in Equation 6.

4. (Firm choices) Labor, dividend, and investment choices solve the production firm's value function described in Equation 1.

5. (Goods market) The goods market clears.
$\int_\eta \int_s c_{s,\eta}\mu_h(s,\eta)dsd\eta + \int_k (k' - (1-\delta)k)\mu_f(k)dk = Y = \int_k (zk^\alpha n^\gamma)\mu_f(k)dk$

6. (Proxy survival) A vector of discount factors $\chi(\mathbb{Z}; z_n)$ for each aggregate state that does not allow profits by the management firm. This is the case if Equation 13=0 in each aggregate state.

7. (Laws of motion) The perceived laws of motion for the household and firm distributions must be accurate. $\mu'_f = \Gamma_f(\mathbb{Z}), \ \ \mu'_h = \Gamma_h(\mathbb{Z})$

2.5 Characterizing the solution

The solution to the household's shareholding problem is standard, yielding:

$$[s'] : \qquad P = \beta \mathbb{E}_{\eta', z'} \left(\frac{u'(c')}{u'(c)} (P' + D') \right) + \lambda_s \tag{15}$$

where λ_s is the Lagrange multiplier on the borrowing constraint. Equation 15 holds for each household indexed by $\{s, \eta\}$ and in each aggregate state $\mathbb{Z}$.

For convenience, I define each household's expected marginal rate of substitution between aggregate states as:

$$q^n_{s,\eta} \equiv \beta \pi_{mn} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} \left(u'(c') | s', \eta_j; k', \mu', z' = z_n \right)}{u'(c_i)} \tag{16}$$

The $q^n_{s,\eta}$ term emphasizes that each shareholder will have an expected valuation of each future state z_n, and that this expectation depends on the shareholder's current state s and η. Using this shorthand, I rewrite the household's first order conditions (for households who choose interior s') as:

$$P = \sum_{n=1}^{N_z} q^n_{s,\eta}(P'_n + D'_n) \tag{17}$$

The firm's dividend and investment choices are characterized by the first order condition:

$$1 = \sum_{n=1}^{N_z} \chi(\mathbb{Z}, z_n) \frac{\partial F(k'; z_n, \mu'_f, \mu'_h)}{\partial k'} \tag{18}$$

The goods market will clear via Walras's Law.

2.5.1 Equilibrium Discount Factor

The management firm solves its maximization problem by choosing the best alternate investment scheme ($\hat{k}$) that satisfies the participation constraint condition in Equation 14. First order and Benveniste-Scheinkman conditions are below:

$$[\xi(s,\eta)] : \qquad \lambda_{s,\eta} = \frac{\mu(s,\eta)}{u'(c)} \qquad \forall\ s,\eta \tag{19}$$

$$[\hat{k}'] : \qquad \lambda_{s,\eta}\frac{1}{\mathrm{N}_f}u'(c)s = \lambda_{s,\eta}\beta\mathbb{E}\frac{\partial V(s',\eta';z',\hat{\mu}'_f,\mu'_h)}{\partial \hat{k}'} \qquad \forall\ s,\eta \tag{20}$$

$$[\text{B-S}] : \qquad \frac{\partial V(s,\eta;z,\hat{\mu}_f,\mu_h)}{\partial \hat{k}} = \frac{1}{N_f}s\left(\frac{\partial P(\hat{k})}{\partial \hat{k}} + \frac{\partial D(\hat{k})}{\partial \hat{k}}\right)u'(c) \qquad \forall\ s,\eta \tag{21}$$

where $\lambda_{s,\eta}$ is the Lagrange multiplier on the participation constraint in Equation 14.

Combining equations 19-21, substituting the household's vector of expected marginal rates of substitution $q^n_{s,\eta}$, and aggregating over all interior households yields:

$$\underbrace{\int_\eta \int_{(\underline{s}, s_{max}]} s\mu(s,\eta) ds' d\eta}_{=1} = \int_\eta \int_{(\underline{s}, s_{max}]} \left(s'\mu(s,\eta) \sum_{n=1}^{N_z} q^n_{s,\eta} \left(\frac{\partial P'(\hat{k}')}{\partial \hat{k}'} + \frac{\partial D'(\hat{k}')}{\partial \hat{k}'} \right) \right) ds' d\eta \tag{22}$$

The term on the left hand side of Equation 22 is equal to one only if there are not any shareholders who choose $s' = \underline{s}$. I discuss the case where some households are constrained in Appendix B.

Finally, I define an aggregate share-weighted state-contingent discount factor and substitute it in to the management firm's solution:

$$\bar{q}^n \equiv \int_\eta \int_{(\underline{s}, s_{max}]} s'\mu(s,\eta) q^n_{s,\eta} ds' d\eta \tag{23}$$

$$1 = \sum_{n=1}^{N_z} \bar{q}^n \left(\frac{\partial P'(\hat{k}')}{\partial \hat{k}'} + \frac{\partial D'(\hat{k}')}{\partial \hat{k}'} \right) \tag{24}$$

Lemma 1. *The production firm survives a proxy challenge if* $\chi(\mathbb{Z}, z_n) = \bar{q}^n$.

Suppose $F(\cdot) = D(\cdot) + P(\cdot)$. Then the solution to the firm's problem in Equation 18 is rewritten as $1 = \sum_{n=1}^{N_z} \chi(\mathbb{Z}, z_n) \left(\frac{\partial P'(\cdot)}{\partial k'} + \frac{\partial D'(\cdot)}{\partial k'} \right)$. If $\chi(\mathbb{Z}, z_n) = \bar{q}^n$ for each aggregate state, then the production firm's capital choice is the same as the management firm's proposed deviation. The management firm fails in its takeover attempt and the discount factor satisfies the proxy survival equilibrium condition.

A convenient property of this discount factor is that the ex-dividend share price is equal to the discounted future value of the firm. That is:

$$P = \sum_{n=1}^{N_z} \chi(\mathbb{Z}, z_n) \left(P' + D' \right) \tag{25}$$

Equation 25 will hold even if there is heterogeneity in firms. In a setting with multiple types of firms and where households can only invest in aggregate equity, all firms will use the same vector of stochastic discount factors $\chi(\mathbb{Z}, z_n)$ to value future prices and dividends.

The firm discounting by q^n is a sufficient condition to survive a proxy challenge, but not necessary. To survive a challenge, the share price and investment choice are the only things pinned down by the proxy challenge. If there are more than two aggregate states then there are trivially many discount factors that could be used in the firm's problem to value the future that would be consistent with surviving a proxy challenge.

It may seem counterintuitive that a firm's current valuation of the future depends on who its shareholders will be, not who its shareholders are. In equilibrium, firms will know who their future shareholders will be by observing the law of motion $\mu'_h = \Gamma_h(\mathbb{Z})$.

A convenient note is that this perfectly nests the representative household case. If I eliminate all idiosyncratic risk, every household would have have the same intertemporal discount factor, $q^n_{\text{representative}} = \bar{q}^n$.

Unanimity and side payments A common issue in the literature is that shareholders are not unanimous in what they want the firm to do. Some (or all) shareholders would choose different investment, production, or dividend choices if they were solely in charge of running the firm. If asked to vote on a choice of investment, each type of shareholder would vote for a slightly different plan.

One method of resolving the problem of unanimity is via side payments (Drèze, 1974). Shareholders who prefer dividends could be compensated with side-payments by investors who prefer more investment. The fact that we generally do not see these side payments suggests that this is not a feasible equilibrium concept.

The proxy survival approach deals with both of these problems cleanly. Unanimity is not a problem because the firm is disciplined by the threat of a proxy battle, not by direct shareholder involvement. Shareholders in this model who would prefer more dividends will sell some of their shares, thereby increasing their current consumption. It also avoids the problem of unrealistic side payments. Shareholders in this model are only compensated when there is a successful proxy takeover, which is empirically a rare event (Anton, Gine and Schmalz, 2016). Most of the time when a proxy fight is successful, there is a change to a firm's board of directors. In my model, this would be viewed as changing an incorrect guess of χ to be closer to the actual shareholder discounting $\bar{q}$.

3 Parameterization and Results

Parameters are chosen arbitrarily, though they are all generally reasonable within existing literature. Externally assigned parameters are listed in Table 1.

Variable	Value	Variable	Value
δ (Depreciation)	0.075	z (TFP shock)	$\pm 5\%$
σ (Risk aversion)	3.0	β (Discounting)	$\frac{1}{1.07}$
α (Capital share)	0.30	γ (Labor share)	0.65

Table 1: Initial Parameters

TFP and labor shocks are cumulative across periods. For example, A negative shock in periods two and three would result in a final TFP of $0.9025 (= 0.95 \times 0.95)$. These are large productivity shocks, but they illustrate the discounting mechanism well. There is a 1/3 probability of drawing each the low, neutral (100%), and high productivity shocks.

If there are no idiosyncratic shocks, my model nests a representative household case. To illustrate the role of idiosyncratic labor risk, I compare moments of three economies to the baseline case without individual risk.

In Table 2, I show that as idiosyncratic risk increases, firms engage in precautionary savings on behalf of the households. This is consistent with models where households save

Measure	σ_η		
	0.0% (rep HH)	10%	20%
I/Y:	11.69%	11.91%	12.56%
$q^{0.95}$	0.1852	0.1857	0.1871
$q^{1.00}$	0.1616	0.1621	0.1634
$q^{1.05}$	0.1418	0.1423	0.1436
$\mathbb{E}\frac{P'+D'}{P}$:	1.052	1.046	1.029
P_1/K_2:	1.1355	1.1353	1.1350

Table 2: Firm statistics in the first period in economies with increasing levels of employment risk. Measures are investment as a share of total GDP, expected return on equity, equity price divided by future capital stock, and the SDF used by the firm for each draw of z_2.

in capital directly (Aiyagari, 1994). As individual risk increases, the stochastic discount factors q increase, which causes firms to save more for the future. Firms cut dividends in the first period in favor of additional investment to smooth household risk across periods. And as capital increases, the expected equity rate on return falls. Households are willing to accept the lower equity rate of return because savings become more important as individual risk increases.

I include the measure P_1/K_2 to contrast my work from Carceles-Poveda (2009). In a constant returns environment, shareholders agree about the firm's investment level when the share price is equal to the future capital level. I can nest that model by introducing constant returns to my environment, which would result in $P_1/K_2 = 1$.

	$z_2 = z^{0.95}$			
Measure	Agg	η_ℓ	η_m	η_h
Shares	100%	32.73%	33.33%	33.94%
$q^{0.95}$	0.1860	0.1858	0.1861	0.1863
$q^{1.0}$	0.1647	0.1647	0.1647	0.1647
$q^{1.05}$	0.1465	0.1467	0.1465	0.1463

Table 3: The shareholding choice s_3 for each date 2 productivity draw η, conditional on z_2 being the median productivity state.

Table 3 shows shares and discount factor for each idiosyncratic labor type at the end of period 2 conditional on seeing a low aggregate state. In period 1, all households are ex-ante and ex-post identical, so each employment type starts period 2 owning a third of shares. Low productivity households sell some shares while high productivity households buy shares.

This table also demonstrates that this discounting approach does not feature a dictator, which would be the case in median voter settings like in Anagnostopoulos, Cárceles-Poveda and Mihalache (2021). No single household exactly agrees with the aggregate discount factor. In general, low productivity households will value payoffs in the low aggregate state relatively less than high productivity households. High productivity households have a lot to lose if the economy worsens, while low productivity households are already on a steep part of their utility curve and will see relatively less loss in a low state.

3.1 Wealth and Income Heterogeneity

The evidence so far shows that increasing household labor risk causes firms to increase their precautionary savings behavior. If income heterogeneity causes firms to change their behavior, it is reasonable to guess that economies with different distributions of starting wealth or productivity will also feature different levels of investment.

To examine the role that productivity and wealth distributions play in firm behavior, I run three experiments in the model. In the first experiment, I set productivity to match the distribution of income from the 2019 SCF. In the second, I set shareholding to match the distribution of wealth. In the last experiment, I set both the starting wealth and productivity distributions to match the SCF. Table 4 shows the distribution of income and wealth that I use in these experiments.

Table 5 shows that investment is higher in economies with starting income or wealth

[5] Because the table I am using in the SCF is sorted by income percentile, the top 10% of households hold less wealth than if I used the Federal Reserve's Distribution of Household Wealth tables. That source shows the richest 10% of households own around 76.5% of wealth rather than the 63.9% shown in the SCF.

Percentile of income	0-19.9	20-39.9	40-59.9	60-79.9	80-89.9	90-100
% of initial income	3.0%	6.7%	11.1%	18.2%	14.4%	46.6%
% of initial wealth[5]	3.4%	3.7%	6.0%	11.5%	11.4%	63.9%

Table 4: Distribution of starting income and wealth calculated from the 2019 SCF Tables 1 and 4.

Measure	Starting Inequality			
	None	Income	Wealth	Both
I/Y:	11.91%	11.93%	11.95%	11.91%
$q^{0.95}$	0.1857	0.1849	0.1844	0.1856
$q^{1.00}$	0.1621	0.1622	0.1623	0.1621
$q^{1.05}$	0.1423	0.1430	0.1435	0.1424
$\mathbb{E}\frac{P'+D'}{P}$:	1.0460	1.0454	1.0450	1.0459

Table 5: Firm statistics in the first period in economies with different starting productivity and wealth distributions.

inequality than it is in an economy with starting equality. However, the joint distribution of starting income and wealth also plays a role in investment. An economy with both income and wealth inequality looks nearly identical to an economy with starting equality.

What causes this behavior? When there is income inequality, low productivity households see the future as very risky, so they want to increase their savings. High productivity households are in a relatively safe position, so they can afford to sell some of their equity and enjoy consumption in the current period. These factors put upward pressure on the firm's discount factor, which increases investment. With starting wealth inequality, low asset households save and high asset households dissave. This again puts upward pressure on the firm's valuation of future states.

With both types of inequality, however, these effects cancel out. Poor, low productivity households want to save more, but they are relatively small compared to the initially rich. Rich, productive households are in a safe position along both avenues of risk and therefore don't need the firm to invest as much.

4 Comparison to Alternates

4.1 Households Own Capital

A common approach in incomplete markets settings is to have households own capital (Aiyagari, 1994). I replicate this type of model by building a three period model where households own capital and rent it to firms at a market-determined interest rate. I adjust the capital share of output α to be 0.35, which yields a constant returns production function. In an environment where firms rent capital and labor in competitive factor markets, this constant returns parameterization would lead to a zero profit condition for firms. This also eliminates the need for a stock market and share prices.

With the proxy survival discounting approach described in section 2.5.1, a constant returns firm results in exactly the same outcomes as I would find in an Aiyagari-style model. Capital levels are the same in each state and wealth distributions follow exactly the same process.

5 Conclusion

With a proxy-survival conditions disciplining firm behavior, idiosyncratic household risk plays a meaningful role in determining firm investment and dividend payment choices. The next step in exploring this class of models is to expand it to an infinite horizon setting. This would allow for a fuller exploration of the relationship between precautionary savings by firms when faced with a more realistic TFP process. This would also allow for policy analyses surrounding dividend versus corporate taxation.

References

Aiyagari, S. Rao. 1994. "Uninsured idiosyncratic risk and aggregate saving." *Quarterly Journal of Economics*, 109(3): 659–684.

Anagnostopoulos, Alexis, Eva Cárceles-Poveda, and Gabriel Mihalache. 2021. "Non Linear Dividend Taxation and Shareholder Disagreement."

Anton, Miguel, Mireia Gine, and Martin C. Schmalz. 2016. "Common Ownership, Competition, and Top Management Incentives." *SSRN Electronic Journal*, , (June).

Azar, José, Martin C. Schmalz, and Isabel Tecu. 2018. "Anticompetitive Effects of Common Ownership." *Journal of Finance*, 73(4): 1513–1565.

Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica*, 77(3): 623–685.

Boar, Corina. 2018. "Entrepreneurship , Agency Frictions and Redistributive Capital Taxation." , (1985).

Carceles-Poveda, Eva. 2009. "Asset prices and business cycles under market incompleteness." *Review of Economic Dynamics*, 12(3): 405–422.

Drèze, Jacques H. 1974. *Investment Under Private Ownership: Optimality, Equilibrium and Stability.* London:Palgrave Macmillan UK.

Grossman, Sanford J., and Oliver D. Hart. 1979. "A Theory of Competitive Equilibrium in Stock Market Economies." *Econometrica*, 47(2): 293.

Khan, Aubhik, and Julia K. Thomas. 2008. "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics." *Econometrica*, 76(2): 395–436.

Krusell, Per, Toshihiko Mukoyama, and Anthony A. Smith. 2011. "Asset prices in a Huggett economy." *Journal of Economic Theory*, 146(3): 812–844.

A The firm's problem when share price can change

If I relax **Assumption 2** and allow prices to change, all of my results still hold. Most of the management firm's problem stays the same. The participation constraint is slightly modified. Now, when a firm deviates, it influences both prices and dividends. Dividends still change consistent with Equations 9 and 10.

To find the change in price associated with a deviation, I make two assumptions. First, the capital deviation is relatively small, so a change in price will be the derivative of price with respect to future capital. Second, I guess that the firm's vector of discount factors $\chi(\mathbb{Z}, z_n)$ are consistent with a price change. This is based on Equation 25, which shows that share price is directly related to future value.

When a firm deviates, its new price and the new aggregate price are:

$$\hat{p}(k;\mathbb{Z}) = p(k;\mathbb{Z}) + (\hat{k}' - k') \sum_{n=1}^{N_z} \chi(\mathbb{Z}, z_n) \frac{\partial F(\hat{k}'; z_n, \mu'_f, \mu'_h)}{\partial \hat{k}'} \tag{26}$$

$$\hat{P}(\mathbb{Z}) = P(\mathbb{Z}) + \frac{1}{N_f}(\hat{k}' - k') \sum_{n=1}^{N_z} \chi(\mathbb{Z}, z_n) \frac{\partial F(\hat{k}'; z_n, \mu'_f, \mu'_h)}{\partial \hat{k}'} \tag{27}$$

The management firm's problem is written as:

$$\max_{\{\xi(s,\eta)\},\hat{k}'} \int_{(\underline{s},s_{max}]} \int_{\eta} -\xi(s,\eta)\mu_h(s,\eta)d\eta ds \tag{28}$$

$$\text{s.t. } u(c)+\beta\mathbb{E}V(s',\eta';z',\mu'_f,\mu'_h) \leq u(\hat{c}) + \beta\mathbb{E}V(s',\eta';z',\hat{\mu}'_f,\mu'_h) \tag{29}$$

$$\begin{aligned} \hat{c} \equiv & c + \xi(s,\eta) + \\ & - s\frac{1}{N_f}(\hat{k}' - k) + \\ & s\frac{1}{N_f}(\hat{k}' - k)\sum_{n=1}^{N_z}\chi(\mathbb{Z},z_n)\frac{\partial F(\hat{k}';z_n,\mu'_f,\mu'_h)}{\partial \hat{k}'} \\ & - s'(\hat{k}' - k')\frac{1}{N_f}\sum_{n=1}^{N_z}\chi(\mathbb{Z},z_n)\frac{\partial F(\hat{k}';z_n,\mu'_f,\mu'_h)}{\partial \hat{k}'} \end{aligned}$$

Now, a higher investment deviation changes the household's alternate consumption plan by four components. The first two are the same as before while the last two are new. First, households receive payments ξ as compensation for a deviation (or pay if they benefit). Second, households receive lower dividends for their current shareholding. Next, they receive a higher share price for their current shares. Finally, they have to pay a higher share price for their future shares.

Via the Envelope condition, the second and third terms cancel out. A firm that maximizes its cum-dividend share price already equalizes the benefit of current dividends today against the future value of capital in the future. The alternate consumption path $\hat{c}$ is now simply:

$$\hat{c} \equiv c + \xi(s,\eta) - s'(\hat{k}' - k')\frac{1}{N_f}\sum_{n=1}^{N_z}\chi(\mathbb{Z},z_n)\frac{\partial F(\hat{k}';z_n,\mu'_f,\mu'_h)}{\partial \hat{k}'} \tag{30}$$

Following the same process as in Section 2.4, I find:

$$\sum_{n=1}^{N_z} \chi(\mathbb{Z}, z_n) \frac{\partial F(\hat{k}'; z_n, \mu'_f, \mu'_h)}{\partial \hat{k}'} = \sum_{n=1}^{N_z} \bar{q}^n \left(\frac{\partial P'(\hat{k}')}{\partial \hat{k}'} + \frac{\partial D'(\hat{k}')}{\partial \hat{k}'} \right) \tag{31}$$

This reinforces that in equilibrium, the guesses $\chi = q$ and $F(\cdot) = P(\cdot) + D(\cdot)$ are consistent.

B The discount factor when some households are constrained

When some households are constrained, the equilibrium changes only slightly. If some households choose $s' = \underline{s}$, then the left hand side of Equation 22 is no longer equal to 1. Because I want management firm's problem to be consistent with the firm's choice in equilibrium, I divide the left hand side by the amount of shares that are held internally. Equation 22 becomes:

$$1 = \frac{\int_\eta \int_{(\underline{\mathbf{S}}, s_{max}]} \left(s'_{s,\eta} \mu(s,\eta) \sum_{n=1}^{N_z} q^n_{s,\eta} \left(\frac{\partial P'(\hat{k}')}{\partial \hat{k}'} + \frac{\partial D'(\hat{k}')}{\partial \hat{k}'} \right) \right) ds' d\eta}{\int_\eta \int_{(\underline{\mathbf{S}}, s_{max}]} s_{s,\eta} \mu(s,\eta) ds' d\eta} \tag{32}$$

which yields state-contingent discount factors:

$$\bar{q}^n \equiv \frac{\int_\eta \int_{(\underline{\mathbf{S}}, s_{max}]} s'_{s,\eta} \mu(s,\eta) q^n_{s,\eta} ds' d\eta}{\int_\eta \int_{(\underline{\mathbf{S}}, s_{max}]} s_{s,\eta} \mu(s,\eta) ds d\eta} \tag{33}$$

In the extreme case $\underline{s} = 1$, then there is only one household (or one type of household) that sets the discount factor. In this case, my model exactly nests Krusell, Mukoyama and Smith (2011). In that model, the firm discounts using only the the discount factor of the (typically) wealthiest household.