Firm Investment with Shareholder Heterogeneity*

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October 2, 2022
Most recent draft available here

Abstract

Does rising household income risk change aggregate investment and output by corporate firms? To answer this question, I study a stochastic, general equilibrium model featuring aggregate risk, incomplete markets, households who are heterogeneous in productivity and wealth, and shareholder-owned firms that own capital. The firm's objective is typically not well defined in models like this, so I introduce a pair of financial intermediaries that jointly pin down market prices and the investment choices of production firms. In equilibrium, firms maximize their net market value (or cum-dividend share price), which is determined by the post-trade share-weighted marginal rate of substitution. This method nests both the standard representative household case and a variety of no-trade models popular in the literature. I then apply this model to the data to see how a change in earnings variance between 1970 and 2010 changes firm dynamics. Increased household idiosyncratic risk causes firms to accumulate more capital, resulting in lower consumption and output volatility over the business cycle. I also find that the observed change in the idiosyncratic productivity process for US households can explain 60% of the decline in dividend yields between 1970 and 2010 and over half of the rise in price-earnings ratio. Additionally, my model generates the procyclical price-earnings ratio and countercyclical equity rate of return observed in the data.

JEL Codes: D52, E13, G12, L21

Keywords: Incomplete markets, Asset prices, Firm objectives

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*I gratefully acknowledge the valuable comments and suggestions from Aubhik Khan, Julia Thomas, Kyle Dempsey, and participants at the OSU workshop and the Midwest Macro conference. A finite horizon version of this paper was presented under the title, "How Does Household Earnings Risk Influence Aggregate Investment?". All errors are my own.

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1 Introduction

Does shareholder wage risk change firm investment behavior and, if so, does an increase in household risk meaningfully shape business cycle dynamics? The incomplete markets literature already documents that earnings risk drives precautionary savings when households directly hold capital. However, households generally are not the primary sources of investment. Households are only responsible for 20% of aggregate investment, while corporate firms perform 60-65% of investment.[1] Understanding the investment choices of shareholder-owned firms plays a crucial role in understanding the evolution of aggregate capital.

If household risk and firm dynamics are both important to understanding macroeconomic fluctuations, why are models with these features not more common? The problem of the firm is generally not well defined when owned by heterogeneous shareholders. Each shareholder's valuation of payoffs over different aggregate states will depend on their current idiosyncratic income and wealth levels. When this is the case, an investment choices that maximizes value for one shareholder may not maximize value for another. To determine a consistent objective for the firm, I construct a model with uninsurable aggregate and household risk featuring a set of financial intermediaries who discipline a consistent valuation of payoffs across future states. In equilibrium, firms will value future payoffs using a share-weighted average of their shareholders' expected marginal rates of substitution across states. I find an increase in household earnings risk increases investment by firms, lowers the price-earnings ratio, and lowers aggregate consumption volatility over the business cycle.

In a complete markets setting, the value of a payoff in any given state is given by the price of a state-contingent claim for that state. Under incomplete markets, payoffs in each state are not uniquely priced. This uncertainty about the value of payoffs causes two problems. First, it becomes more difficult to price assets. An asset can generally be priced as $P = \mathbb{E}[M'X]$ where X is a vector of payoffs and M is the stochastic discount factor (SDF). With complete markets, this SDF vector is the vector of state-contingent claim prices. When markets are incomplete, there are trivially many SDF vectors that satisfy $P = \mathbb{E}M'X$. Second, firms no longer have a well-defined goal. Firms generally want to maximize shareholder value. When markets are complete, maximizing shareholder value means maximizing payoffs in future states valued by the SDF. Under incomplete markets, each shareholder might have slightly different valuations of payoffs in each future state.

[1]Households perform approximately 20% of aggregate investment, while partnerships, sole proprietor businesses, and nonprofits make up the remainder.

A choice that maximizes value for one shareholder might not maximize value for another.

To resolve the joint problems of asset pricing and firm discounting, I introduce a pair of financial agents that find the stochastic discount factor (or pricing kernel) and discipline the production sector's choices. The mutual fund finds the market price for payoffs across states, while the private equity firm pins down the discounting regime for production firms.

Households save in aggregate equity through a mutual fund[2] which bundles shares of all production firms into a single investment instrument. The mutual fund's bundling of production shares plays the dual role of simplifying the household's problem to a single continuous choice variable and preventing production firms from becoming financial innovators.[3] The private equity firm disciplines the choices of the production firm, similar to the one proposed by Grossman and Hart (1979). The private equity firm takes over the production firm if it can find an alternate investment scheme supported by the mutual fund. If the private equity firm cannot find an alternate capital investment choice that is strictly preferred by the mutual fund, it does not act in the period.

In equilibrium, each household buys or sells shares of aggregate equity at a market-clearing price. Each household's shareholding choice depends on the aggregate price, future payoffs, and their current idiosyncratic state. The mutual fund measures a pricing kernel as the post-trade share-weighted marginal rates of substitution of all marginal shareholders. This pricing kernel determines the price the mutual fund is willing to pay for a production firm given its future returns (which depend on its current capital level and investment decision). To prevent the private equity firm from finding a deviation, the production firms will value future payoffs with the pricing kernel found by the mutual fund. Discounting future payoffs with the same pricing kernel as the mutual fund results in firms maximizing shareholder value by maximizing their net market value (or cum-dividend share price).

The equilibrium of this model will generally not be constrained efficient. Production firms are atomistic, so they do not consider their investment's impact on wages and other prices. A social planner will be able to find an improvement by changing aggregate capital and shifting wages.

The primary contribution of this paper is the aggregate stochastic discount factor and its

[2]The mutual fund I propose is similar to a capital mutual fund described in Carlstrom and Fuerst (1997). The key difference is that the mutual fund in my model holds equity rather than capital.

[3]Financial innovation can happen when a firm creates a new set of payoffs that were not spanned by the previous set of possible investment choices. If a firm promises a tiny deviation in one future aggregate state, households could trade this firm purely as a financial asset, even if it does not meaningfully change output.

relationship to firm choices in an incomplete market setting. I find a discount factor that both prices assets and pins down shareholder-owned firm behavior. The discount factor is a post-trade share-weighted sum of marginal rates of substitution among marginal shareholders.[4] The discount factor I describe nests several special cases. It nests the representative household case and the environment with zero aggregate risk. Further, this method can be applied to the case with exogenously-imposed zero trade as proposed by Krusell, Mukoyama and Smith (2011). Nesting this last case allows me to evaluate the role of trade in an incomplete markets environment. I also demonstrate that alternate discounting schemes proposed in the literature are inconsistent with observed shareholder and intermediary behavior. This theory contribution enables researchers to build models featuring rich heterogeneity among both firms and households.

This paper also contributes to the literature studying the relationship between household risk and macroeconomic aggregates. From 1970 to 2010, the variance in earnings has increased in the United States. I find this higher level of risk results in a lower expected rate of return on capital, higher aggregate investment, less volatile output, and less volatile aggregate consumption. Additionally, the model can explain 60% of the observed fall in dividend yield and 55% of the rise in the price-earnings ratio for S&P 500 stocks.

Related Literature An extensive literature studies the role of household risk in incomplete markets where households own capital. Aiyagari (1994) develops this in a setting without aggregate risk, which Krusell and Smith (1998) extends with aggregate risk. Challe and Ragot (2016) documents the role of household precautionary savings with unemployment risk over the business cycle. I contribute to this literature by including a production sector that makes a meaningful choice between profit and rental payments. This distinction is crucial when considering that corporate firms perform three times more investment than households.

The closest link between household risk and firm behavior generally comes from the entrepreneurship literature. Cagetti and De Nardi (2006) describe a setting where household wealth generates a distribution of entrepreneur firms. In their setting, however, small businesses take their discounting directly from their owners. My approach focuses on larger corporate firms who are responsible for 60% of total investment, while small businesses only account for about 10% of investment.

My paper is most closely related to the firm discounting and price perception litera-

[4] Marginal shareholders are those who endogenously participate in the market by choosing to hold more than the minimum required level of shares.

ture. Early work by Drèze (1974) describes the problem of uncertainty in the firm's valuation of future payoffs. Grossman and Hart (1979) proposed aggregating discount factors weighed by current shareholding to try to discipline the production sector's problem. Carceles-Poveda and Coen-Pirani (2009) used this aggregation method to study how firm behavior under the proposed discount valuation. These models share two common shortcomings. First, the compensations to shareholders rely on off-equilibrium perceptions of price changes. Each household has a different belief about how prices will change after a deviation in investment. Second, the discount factor used by firms cannot be used to price the asset, except in the special case of constant returns. In contrast, I construct a model where price perceptions are consistent with equilibrium. Additionally, the stochastic discount factor in my model both disciplines the firm's choices and prices assets. Firms that weigh the future by the methods proposed by Grossman and Hart (1979) and Carceles-Poveda and Coen-Pirani (2009) will be worth less than firms that weigh the future using my methodology because my approach takes into account post-trade optimal conditions. However, the difference between these models is relatively small, which is consistent with the comparative analysis performed in Carceles-Poveda and Coen-Pirani (2009).

Instead of using price perceptions, Krusell, Mukoyama and Smith (2011) exogenously imposes zero trade, which results in a setting where a single household pins down the aggregate discount factor. My model nests their approach and demonstrates that it results in excessive investment. Empirically, Gormsen and Huber (2022) documents that firms have higher discount rates than what is implied by the cost of equity. My model replicates this as payoffs are more valued in low states than high states, which results in firms who look like they are risk averse. Bejan (2020) find that financial innovation can break the standard link between firm value and discounted returns. Their model focuses on the case of a group of stakeholders called the *control group*. The production firm operates to maximize the preferences of this subgroup. This setting would be particularly useful when studying the problem of a firm with a block of a few, distinct shareholders. My setting is more general and assumes that control of a firm is pinned down by the external threat of a shareholder challenge. However, the pricing kernel described in this paper can be used to value any asset, including those governed by a control group.

The asset pricing literature also relates closely to my work. Constantinides and Duffie (1996), Braun and Nakajima (2012), and Constantinides and Ghosh (2017) combine household marginal rates of substitution to create an aggregate stochastic discount factor. The discount factors calculated in their settings differs from the standard result from representative household models, which partly explains the . However, these papers construct an income process that results in zero trade while my model allows for shareholders to change over time. Marcet and Singleton (1999) finds asset prices are higher with higher income risk, but they only focus on the price and not the discount factor required to find that price. Krueger and Lustig (2010) documents that a lack of insurance for idiosyncratic risk only shifts the price of aggregate risk if household risk is uncorrelated with aggregate risk. Household wage income risk is correlated with aggregate risk in my setting, so their result reinforces my findings. Paron (2021) documents a similar phenomenon in continuous time models.

To tie my model's assumptions back to observed behavior, I rely on a number of works examining the interaction between shareholders, intermediaries, and firms. Carlstrom and Fuerst (1997) sets up an environment where risky firms rent capital from a single financial intermediary. I follow the literature by extending their approach to an equity market. The private equity challenge is based on theory from Grossman and Hart (1979) but supported in data by Fos (2017). Fos finds that shareholder challenges are most common and are more likely to succeed when they attempt to increase firm value. Fichtner, Heemskerk and Garcia-Bernardo (2017) and Edelman, Thomas and Thompson (2014) document that large financial intermediaries control a majority of voting shares and that they are required to vote in their shareholders' best interests. In equilibrium, these shareholder challenges are never observed in my model. However, the off-equilibrium behavior is consistent with observed shareholder challenges to management.

The paper proceeds as follows. Section 2 describes the model environment in detail, with particular focus given to the problem of the private equity firm. Section 3 describes the conditions required for equilibrium and shows how I derive the aggregate stochastic discount factor. Section 4 discusses the algorithm I use to solve the model. Section 5 discusses business cycles moments and impulse responses with varying levels of realistic idiosyncratic risk. Section 6 concludes.

2 Model Environment

In this model economy, production firms own capital and make meaningful intertemporal decisions on behalf of their shareholders. Households face idiosyncratic labor productivity risk and can only save in aggregate equity. I begin the description of this economy with details about the maximization problem facing each household, the production firms, and the mutual fund. Once the dynamic agents are introduced, I describe the off-equilibrium private equity firm.

2.1 Households

There is a single production good which is used for both consumption and investment. This good is the model's numeraire. There are a unit measure of households in this economy, identified by their start of period assets a and idiosyncratic labor productivity η. Each household has identical, time-separable, concave, strictly increasing preferences over consumption. Each supplies labor inelastically and saves in aggregate equity a. I assume η is a Markov chain; $\eta \in \mathbf{N} \equiv \{\eta_1, \ldots, \eta_{N_\eta}\}$, where $\Pr(\eta' = \eta_j | \eta = \eta_i) = \pi_{ij} \geq 0$ and $\sum_{j=1}^{N_\eta} \pi_{ij} = 1$ for each $i = 1, \ldots, N_\eta$. For simplicity and without loss of generality, I assume higher indexed values of η denote higher productivity levels: $\eta_1 < \eta_2 < \cdots < \eta_{N_\eta}$.

The household's asset holding in the mutual fund is given by $a \in \mathbf{A} \subset \mathbf{R}$. The set $\mathbf{A}$ is bounded above by 1 and below by $\underline{a}$. This lower bound $\underline{a}$ is a parameter in the model. The lower bound must fall in the range $[\underline{A}, 1]$, where $\underline{A}$ is the natural borrowing limit. I define this limit as the smallest level of debt a household could service conditional on entering the period with that debt and holding the lowest productivity draw. I derive an expression for the natural borrowing limit in Appendix B. If $\underline{a} = 0$, this constraint would prohibit short sales. If $\underline{a} = 1$, the economy is in exogenously-imposed autarky, similar to the no-trade scenario described in Krusell, Mukoyama and Smith (2011). I discuss similarities between my method and theirs further in Section 3.4.1.

I summarize the distribution of households over (a, η) using the probability measure μ_H defined on the Borel algebra $\mathcal{S}$ generated by the open subsets of the product space, $\mathbf{S}_H = \mathbf{A} \times \mathbf{N}$.

I require two more components to fully define the aggregate state. The first is aggregate exogenous TFP z. I assume z is a Markov chain; $z \in \mathbf{Z} \equiv \{z_1, \ldots, z_{N_z}\}$, where $\Pr(z' = z_n | z = z_m) = \pi_{mn} \geq 0$ and $\sum_{n=1}^{N_z} \pi_{mn} = 1$ for each $m = 1, \ldots, N_z$. As with labor productivity, I assume higher indexed levels of z are more productive: $z_1 < z_2 < \cdots < z_{N_z}$.

The final component of the aggregate state is the distribution of firms over their start of period capital, $k \in \mathcal{K} \subset \mathbf{R}_{++}$. Similar to households, I summarize the distribution of firms over k using the probability measure μ_F. The aggregate state of the economy is then (z, μ_H, μ_F).

The per-productivity-unit wage $w(z, \mu_H, \mu_F)$ is taken as given by the household. In each state, the price of equity (or shares in the mutual fund) is expressed as $P(z, \mu_H, \mu_F)$ which pays dividends $D(z, \mu_H, \mu_F)$. These are equilibrium prices which the household takes as given when making its decisions.

I now describe the recursive problem of each household in the economy. Let $V(a, \eta_i; z_m, \mu_H, \mu_F)$ be the start of period value of a household with assets a, productivity η_i, and the aggregate state given by (z_m, μ_H, μ_F). For convenience of notation, I define a shorthand for the aggregate state as $\mathbb{Z} \equiv (z_m, \mu_H, \mu_F)$. The dynamic problem of each household is given by:

$$V(a, \eta_i; z_m, \mu_H, \mu_F) = \max_{c,a'} u(c) + \beta \sum_{j=1}^{N_\eta} \pi_{ij} \sum_{n=1}^{N_z} \pi_{mn} V(a', \eta_j; z_n, \mu'_H, \mu'_F) \tag{1}$$

$$\text{s.t. } \; c + P(\mathbb{Z})a' \leq (P(\mathbb{Z}) + D(\mathbb{Z}))\, a + w(\mathbb{Z})\eta_i \tag{2}$$

$$\underline{\mathrm{a}} \leq a' \tag{3}$$

$$\mu'_F = \Gamma_F(\mathbb{Z}), \qquad \mu'_H = \Gamma_H(\mathbb{Z})$$

where β is the commmon subjective discount factor. Equation 3 describes the minimum savings policy, where $\underline{\mathrm{a}} \in [\underline{\mathrm{A}}, 1]$ and $\underline{\mathrm{A}}$ is the natural borrowing limit as described above and in Appendix B.

The distribution of households over productivity and shareholding evolves over time according to a mapping Γ_H which depends on the current aggregate state. That is, $\mu'_H = \Gamma_H(z, \mu_H, \mu_F)$. This evolution depends on the asset choices of households in the previous period and the realization of idiosyncratic shocks. The distribution of firms over capital is similar, with $\mu'_F = \Gamma_F(z, \mu_H, \mu_F)$. The household takes both of these laws of motion as given when making its shareholding choice.

Let $c(a, \eta; \mathbb{Z})$ and $a(a, \eta; \mathbb{Z})$ be the decision rules for consumption and future shareholding of a household with current state (a, η) and aggregate state $\mathbb{Z}$.

2.2 Equity Mutual Fund

A risk-neutral mutual fund bundles shares of the production firms and sells the bundle to households as aggregate equity. Each period, the intermediary collects dividends from production firms, chooses how many shares of each production firm it wants to hold for the next period, and pays out aggregate dividends to households. Aggregate dividends are the dividends collected from production firms plus the net revenue from changing its shareholding of production firms.

The mutual fund chooses aggregate dividends $D(z, \mu_H, \mu_F)$ and its portfolio of future shareholding in production firms $\{s'_k\}$ to maximize its net market value. It buys shares $\{s'_k\}$ in each firm indexed by their capital level k at price $p(k; \mathbb{Z})$ and collects dividends $d(k; \mathbb{Z})$. The goal of the intermediary is to maximize net market value, with payoffs in future states valued by the price vector χ, which the intermediary takes as given. As in Makowski (1983), maximizing net market value expands the budget constraint of all households who held shares at the start of the period and is therefore unanimously supported.[5]

The mutual fund's recursive problem is written as:

$$J(\{s_k\};\mathbb{Z}) = \max_{\{s'_k\}} D + \sum_{n=1}^{N_z} \chi(z_n|\mathbb{Z}) J(\{s'_k\}; z_n, \mu_H, \mu_F) \tag{4}$$

$$\text{s.t.} \quad D \leq \int_{\mathcal{K}} \left((p_k + d_k)s_k - p_k s'_k \right) \mu(dk) \tag{5}$$

$$\mu'_F = \Gamma_F(\mathbb{Z}), \qquad \mu'_H = \Gamma_H(\mathbb{Z})$$

where $\chi(z_n|\mathbb{Z})$ is the pricing kernel or stochastic discount factor, which values payments in future state $\{z_n, \mu'_H, \mu'_F\}$ conditional on the current aggregate state $\mathbb{Z}$. This discount factor is taken as given by the mutual fund. In Section 3.3, I describe the equilibrium properties of this discount factor.

2.3 Production Firms

A unit measure of production firms produce a homogeneous output using labor n and start of period capital stock k. They produce using a strictly increasing and concave production function $y = zF(k, n)$. The variable z is the common exogenous stochastic TFP level which was described in the household section.

A firm enters each period with its predetermined stock of capital, $k \in \mathbf{K} \subset \mathbf{R}_{++}$. The goal of each production firm is to maximize dividends plus discounted future value, with payoffs in future states valued by the price vector $\tilde{\chi}$. Each firm chooses labor to maximize period profits, then selects future capital and current dividends. A portion of the firm's capital stock δ depreciates each period. The firm pays a convex adjustment cost $I(k', k)$ that depends on both its current and future capital levels.

Each firm takes the price vector $\tilde{\chi}$ as given. One important distinction here is that the discount factor used by each production firm $\tilde{\chi}$ is not assumed to be the same as the financial intermediary's discount factor χ. The mutual fund values future payoffs at χ while the production firms value future payoffs at $\tilde{\chi}$. These will be the same in equilibrium, but not because it was imposed as a modeling assumption.

As before, I use a shorthand for the aggregate state $\mathbb{Z} \equiv (z_m, \mu_H, \mu_F)$. Each production

[5] I need to flesh this out a bit.

firm's problem can be written recursively as:

$$G(k;z_m,\mu_H,\mu_F) = \max_{n,k'} d + \sum_{n=1}^{N_z} \tilde{\chi}(z_n|\mathbb{Z})G(k';z_n,\mu'_H,\mu'_F) \tag{6}$$

$$\text{s.t.} \quad d + k' + I(k',k) \leq z_m F(k,n) - w(\mathbb{Z})n + (1-\delta)k \tag{7}$$

$$\mu'_F = \Gamma_F(\mathbb{Z}), \qquad \mu'_H = \Gamma_H(\mathbb{Z})$$

where $\tilde{\chi}(z_n|\mathbb{Z})$ is the aggregate valuation for dividends paid in future state $\{z_n,\mu'_H,\mu'_F\}$ conditional on the current aggregate state. This discount factor is taken as given by each production firm and is described in detail in section 3.3.

Let $k(k;\mathbb{Z})$ and $d(k;\mathbb{Z})$ be the decision rules for future capital and dividends of a firm with current capital k and aggregate state $\mathbb{Z}$.

2.4 Private Equity

Finally, I introduce a private equity firm to discipline the choices of the production sector. The private equity firm is inspired by Grossman and Hart (1979) with some meaningful modifications. The private equity firm has access to a technology that allows it to implement an alternate production plan if and only if it can secure support from the firm's owners, the mutual fund.[6] If the private equity firm cannot find a strictly profitable deviation for any production firm, the private equity firm does not act in the period.

To find a profitable deviation, the private equity firm first proposes some alternate investment plan $\hat{k}'$. The private equity firm is only able to alter the plans of a single firm, so it will not shift aggregate prices. The private equity fund then chooses a side payment $\xi(\hat{k})$ to the mutual fund to compensate it for changing the value of its portfolio. If the portfolio becomes more valuable after the change ($\xi(\hat{k}) < 0$), the mutual fund would pay the private equity firm to implement the change. If the portfolio becomes less valuable ($\xi(\hat{k}') > 0$), the private equity firm would have to compensate the mutual fund. If the private equity firm cannot find a strictly profitable deviation for any production firm, the private equity firm does not act in the period.

The private equity firm's problem is written as:

$$\max_{\xi(\hat{k}'),\hat{k}'} -\xi(\hat{k}') \tag{8}$$

$$\text{s.t.} \quad J(\{s_k\};\mathbb{Z}) < \hat{J}(\{s_k\};\mathbb{Z}|\hat{k}') \tag{9}$$

[6] I consider a case where the private equity firm instead directly compensates marginal shareholders in Appendix A

where hat variable $(\hat{J})$ denotes the change in the mutual fund's value conditional on one of the production firms changing its capital investment to $\hat{k}'$ and after receiving the side payment $\xi(\hat{k}')$. Equation 9 is the mutual fund's participation constraint. It will vote unanimously in favor of the new plan only if it is made better off than it was before the deviation.

When the private equity firm chooses an alternate scheme $\hat{k}'$, it changes prices and dividend payments today and in the future. To make this problem tractable, I first assume that the firm takes over a tiny, identical mass of firms with size ε. I then take $\varepsilon \to 0$ to find the atomistic limit. I write the new level of dividends as $\hat{D} \equiv (1-\varepsilon)D + \varepsilon\hat{d}$ and the new price level as $\hat{P} \equiv (1-\varepsilon)P + \varepsilon\hat{p}$. There is no need to include changes in shareholding because the mutual fund holds all shares and, in equilibrium, it will continue to hold shares at the new price level.

2.5 Discussion of Assumptions

Before continuing the analysis, I would like to pause briefly to discuss the rationality of the modeling choices presented above. Specifically, I want to discuss the mutual fund and the private equity firm.

2.5.1 Benefits of a Mutual Fund

Is it reasonable to impose a mutual fund as the sole savings instrument for household savings? Retirement fund managers typically suggest shifting a portfolio toward bonds while approaching retirement age, so there is some empirical evidence for portfolio management beyond simple market savings. However, common financial advice suggests that saving in aggregate equity will generally outperform stock picking. Warren Buffett famously (Perry, 2017) wagered in 2008 that a low-cost S&P500 index fund would outperform a portfolio of actively managed hedge funds. Further, the SCF documents that safe assets only make up about 10% of wealth for the top 90% of households.

Is it reasonable to expect that a mutual fund will support net market value maximization by the firms it owns? As discussed in the introduction, shareholder challenges tend to be more successful when they target low market value firms. Additionally, mutual funds are legally required to act in their shareholders' best interest. Pursuing net market value maximization expands the budget constraint of households with long positions in equity.

The mutual fund plays three key roles in this economy. It prevents production firms from becoming financial innovators, it simplifies the problem of price discovery, and it makes this problem tractable.

First and most importantly, it prevents atomistic production firms from becoming financial innovators. DeAngelo (1981), Makowski (1983), and Krouse (1985) argue that shareholders will be unanimous in supporting the firm's decision to maximize net market value if firms are sufficiently small. That is, shareholders have to believe that a firm's deviation will not change the set of available prices or future outcomes. By imposing a financial intermediary between the household and the production sector, a production firm will not be able to change the available choice set for households when it produces differently than its peers. Preventing financial innovation keepts the problem more tractable.

Second, it simplifies the potential problem of price discovery. It could be difficult for every production firm to ask its shareholders how they value payoffs across time, aggregate those answers, and predict future shareholding. A financial intermediary sector could much more realistically study markets and make prices available to production firms.[7] In a full information model, this isn't particularly necessary, but it is a helpful feature for future work.

Finally, the mutual fund reduces the size of the problem to something tractable. As written, the model now features a distribution of households over shareholding and productivity and a distribution of firms over capital. If shareholders were allowed to own individual production firms, shareholding would become a portfolio choice and the distribution of households would increase by the number of firms in the economy. Firms would also need to know who specifically holds their shares and how their decisions impacts those specific households, including the portfolio balancing effects of a change in capital.

The tractability result also has important implications for equilibrium. Imagine a setting where households directly invested in firms. Two production firms might start the period identically, but for some reason attract different types of shareholders. If one firm attracts poor shareholders, it will likely invest less than the firm that attracts rich shareholders. If they are ex-ante identical, which firm will attract which type of shareholders? The answer isn't immediately clear. Rather than spending effort to track shareholder-to-firm combinations, it is much more straightforward to impose a financial intermediary.

This is not to say the mutual fund is an assumption without flaws. Clearly, households do differentiate their portfolios for a variety of reasons. There is also evidence from Brockman et al. (2022) that firm risk earns a return premium over the market rate. This

[7] An example of this behavior comes from Investor Relations departments at large firms. These groups frequently interact with institutional investors, who might provide feedback about how a proposed capital investment plan will change share prices.

would not be the case with a single mutual fund holding all production firms. Private or closely-held corporate firms may also be inconsistent with the mutual fund presented here. These cases leave room for future research.

2.5.2 Private Equity Firms

The problem of the private equity firm is modeled from observed shareholder proxy battles. Fos (2017) documents that a majority of shareholder challenges state their goal as increasing market value. Further, shareholder challenges that target market value tend to be more likely to succeed.

This modeling choice is the most important one in this paper because it ultimately pins down the objective of the production firm. Other authors approach this question differently. Grossman and Hart (1979) and Carceles-Poveda and Coen-Pirani (2009) assume that the production firm tries to maximize start-of-period shareholder value and that those shareholders have control over the firm. These authors also consider cases where the date-zero shareholders control the firm. These assumptions each lead to slightly different capital choices by production firms (as documented in Carceles-Poveda and Coen-Pirani (2009)). A micro-founded private equity firm eliminates the need for assumptions about firm control.

One weakness of this modeling choice is its assumption that all production firms care about a shareholder challenge. Closely-held private firms, entrepreneur firms, partnerships, sole proprietorships, nonprofits, and corporate firms with large blocks of insider shareholding[8] are all types of firms who might reasonably ignore a shareholder challenge. The research on the distribution of wealth (De Nardi and Fella, 2017) documents that entrepreneurs can also generate higher rates of return than the market. Because corporate firms are responsible for nearly two thirds of aggregate investment, adding these types of firms to the model is best left as an exercise for future work.

3 Equilibrium Definition and Properties of the Discount Factor

In this section, I describe the conditions for a recursive competitive stock market equilibrium. Then, I discuss properties of the equilibrium stochastic discount factor.

[8] Berkshire Hathaway is an example of such a firm. 43.3% of Berkshire Hathaway stock is held by directors and executive officers of the company as of March 2, 2022. A firm like this is likely immune to all but the most united shareholder challenge. `https://www.berkshirehathaway.com/meet01/2022proxy.pdf`

3.1 Recursive Competitive Stock Market Equilibrium

A Recursive Competitive Stock Market Equilibrium is a set of functions,

$$\{w, G, \chi, \tilde{\chi}, d, n, k, p, d, J, P, D, V, s, c\}$$

that jointly solve the household, firm, and mutual fund's problems, and clear the markets for goods, labor, production firm shares, and aggregate equity, as described by the following:

i. V solves Eq. 1 with policy functions $\{c, a\}$

ii. J solves Eq. 4 with policy function D

iii. G solves Eq. 6 with policy functions $\{d, n, k\}$

iv. The market for aggregate equity clears in each date and state:

$$1 = \int_{\mathcal{S}} a(a, \eta, \mathbb{Z}) \mu(d[a \times \eta])$$

v. The intermediary holds all shares of the production firms $1 = s'_k \ \forall\, k \in \mathcal{K}$

vi. The labor market clears:

$$\int_{\mathcal{K}} n(k; \mathbb{Z}) \mu(dk) = \int_{\mathcal{S}} \eta \mu(d[a \times \eta])$$

vii. The goods market clears:

$$\int_{\mathcal{K}} \left(k(k; \mathbb{Z}) - (1 - \delta)k\right) \mu(dk) + \int_{\mathcal{S}} c(a, \eta; \mathbb{Z}) \mu(d[a \times \eta]) = \int_{\mathcal{K}} zF(k, n(k; \mathbb{Z})) \mu(dk)$$

viii. Γ_H is defined by:

$$\mu'(A, \eta_j) = \int_{\{a, \eta_i | (a(a, \eta_i; \mathbb{Z})) \in A\}} \pi_{ij} \mu(d[a \times \eta_i]) \quad \forall\, (A, \eta_j) \in \mathcal{S}$$

ix. Γ_H is defined by:

$$\mu'(k) = \int_{\{k | (k(k; \mathbb{Z})) \in K\}} \mu(dk) \quad \forall\, k \in \mathcal{K}$$

x. The private equity firm cannot find a profitable deviation for any production firm, which is satisfied when the maximum of Equation 8 is zero.

While not explicitly listed, χ and $\tilde{\chi}$ are determined by the conditions above. The production firm's discount factor $\tilde{\chi}$ is determined by the last equilibrium condition and will be derived in further detail in Section 3.3. The financial intermediary's discount factor χ is determined by market clearing for aggregate equity.

There are many values of $\tilde{\chi}$ that can survive a shareholder challenge when $N_z \geq 2$. However, the discount factor I derive in Section 3.3 pins down a family of discount factors that jointly lead to the same prices and allocations.

3.2 Optimal Choices

I begin by describing the conditions that pin down optimal choices for each type of agent. With those optimal choices in mind, I will construct a pair of discount factors that are consistent with both clearing the aggregate equity market (χ) and surviving a proxy battle ($\tilde{\chi}$).

Each household's optimal choice of a' satisfies:

$$Pu'(c) = \beta \mathbb{E}_{\eta',z'}(P' + D')u'(c') + \lambda_a \tag{10}$$

where $u'(c)$ is the marginal utility of consumption in the current period and $\mathbb{E}$ reflects the expectation of transitioning over both idiosyncratic state η' and aggregate productivity state z'. Future outcomes $P',\ D',\ c'$ are each optimal choices of each type of agent in each realized future state. The λ_a term reflects the fact that some households may want to save less than is allowed by the minimum savings constraint described in Equation 3. This term will be equal to zero for households who choose $a' > \underline{\text{a}}$.

The financial intermediary chooses shareholding of each production firm, which has optimality conditions:

$$p(k;\mathbb{Z}) = \sum_{n=1}^{N_z} \chi(z_n|\mathbb{Z})[p'(k(k;\mathbb{Z});z_n,\mu'_H,\mu'_F) + d'(k(k;\mathbb{Z});z_n,\mu'_H,\mu'_F)] \quad \forall\, k \in \mathcal{K} \tag{11}$$

The price the mutual fund is willing to pay for a firm with capital level k and a vector of future payoffs $p',\ d'$ depends on the future price and dividend it can expect to receive, weighed by some discounting χ for each future state. Because the financial intermediary is not bound by a short sales constraint, the *law of one price* will hold.[9] The mutual fund will price future payoffs at the same rate for all of the production firms that it owns.

[9] The law of one price is common in the finance literature. See Chapter 4 of Campbell (2018).

Each production firm's optimal choices are given by:

$$1 + \frac{\partial I(k', k)}{\partial k'} = \sum_{n=1}^{N_z} \tilde{\chi}(z_n | \mathbb{Z}) D_1 G(k'; z_n, \mu'_H, \mu'_F) \tag{12}$$

Because each firm is atomistic, it takes as given future prices and its shareholders' valuation of payoffs in future states. Because the firm is owned by the mutual fund, the firm cannot become a financial innovator. This is similar to the case in Makowski (1983) but with a different mechanism. The firm only weighs the lower dividends against the change in future valued payments. In equilibrium, the distribution of firms will be degenerate with all of the mass at a single capital level.

Finally, I describe the choices of the private equity firm, which will discipline the production firms' discount factor $\tilde{\chi}$. When deriving the private equity firm's optimal choices, I assume it takes over a mass of identical production firms of size ε. This is assumed to be small enough that the private equity firm does not have market power to change other firms' behavior in the current or future date. Then I take the limit as $\varepsilon \to 0$ to find the specific results at the atomistic firm limit.

The private equity firm chooses a capital deviation $\hat{k}'$ for a mass ε of production firms with current capital level k_i. A capital deviation changes the firm's value to the mutual fund across three channels. It changes dividends d, equity price p, and the vector of future returns $\{p'_n + d'_n\}$.

These production firm changes pass through directly to the mutual fund's balance sheet. The mutual fund will not change its shareholding level. Rather, the price the mutual fund is willing to pay changes as described in Equation 11, depending on the vector of future returns $\hat{p}'$ & $\hat{d}'$.

The firm chooses $\hat{k}'$ and side payment $\xi(\hat{k}')$ satisfying:

$$\begin{aligned} [\xi(\hat{k}')] : \quad & 1 = \Omega \\ [\hat{k}'] : \quad & \varepsilon \Omega \frac{\partial p}{\partial \hat{k}'} = \varepsilon \Omega \left(\frac{\partial p + \partial d}{\partial \hat{k}'} + \sum_{n=1}^{N_z} \chi(z_n | \mathbb{Z}) \frac{\partial p'_n + \partial d'_n}{\partial \hat{k}'} + \underbrace{o(\varepsilon)}_{\to 0} \right) \end{aligned} \tag{13}$$

where Ω is the multiplier on the participation constraint described in Equation 9 and ε are the mass of firms controlled. The term $o(\varepsilon)$ accounts for changes to the firm's value through channels other than direct price and dividend, like the change on other firms' value in the next period if some mass ε of competitors behaved differently than expected.

This term goes to zero in the atomistic limit. Rearranging yields:

$$-\frac{\partial d}{\partial \hat{k}'} = \sum_{n=1}^{N_z} \chi(z_n|\mathbb{Z}) \frac{\partial p' + \partial d'}{\partial \hat{k}'} \tag{14}$$

The capital choice of the private equity firm $\hat{k}'$ determined by Equation 14 is the same as the capital choice of the production firm k' determined by Equation 12 if the production firm's value is equal to its net market value: $G(k;\mathbb{Z}) = p(k;\mathbb{Z}) + d(k;\mathbb{Z})$.

Lemma 1. *If the production firm discounts future states with the mutual fund's discount factor χ, its choice of k' is the same as the private equity firm's optimal deviation $\hat{k}'$ and will therefore survive a proxy challenge.*

The proof is by construction. Suppose $G(k;\mathbb{Z}) = p(k;\mathbb{Z}) + d(k;\mathbb{Z})$. I rewrite the Benveniste-Scheinkman condition and the production firm's change in dividends with respect to capital as:

$$D_1 G(k;\mathbb{Z}) = \frac{\partial p + \partial d}{\partial k} \tag{15}$$

$$\frac{\partial d}{\partial k'} = -(1 + \frac{\partial I(k',k)}{\partial k'}) \tag{16}$$

I substitute these expressions back into the production firm's optimal choices and combine with the private equity firm's optimality condition. Finally, I evaluate this at $k' = \hat{k}'$, which will be the case in equilibrium. Together, these substitutions yield:

$$1 + \frac{\partial I(k',k)}{\partial k'} = \sum_{n=1}^{N_z} \tilde{\chi}(z_n|\mathbb{Z}) D_1 G(k'; z_n, \mu_H', \mu_F') \tag{12 repeated}$$

$$-\frac{\partial d}{\partial k'} = \sum_{n=1}^{N_z} \tilde{\chi}(z_n|\mathbb{Z}) \frac{\partial p' + \partial d'}{\partial k'} \tag{substituting 15 \& 16}$$

$$\sum_{n=1}^{N_z} \chi(z_n|\mathbb{Z}) \frac{\partial p' + \partial d'}{\partial \hat{k}'} = \sum_{n=1}^{N_z} \tilde{\chi}(z_n|\mathbb{Z}) \frac{\partial p' + \partial d'}{\partial k'} \tag{17}$$

Lemma 2. *The production firm survives a proxy challenge if its discount factor $\tilde{\chi}$ is the same as the financial intermediary's discount factor χ.*

While there are trivially many firm discounting regimes $\tilde{\chi}$ that satisfy Equation 17,

This expression can also be derived by directly compensating marginal shareholders. I discuss that alternate setting (with identical results) in Appendix A.

3.3 Analytic Form of the Intermediary's Discount Factor

I construct an aggregate discount factor from each household's optimality conditions as described in Equation 10. To simplify notation, I suppress the current aggregate state $\mathbb{Z} \equiv \{z_m, \mu_H, \mu_F\}$ and the transition of future distributions. I first rewrite each household's Euler equation for shareholding in terms of the aggregate equity price. I then multiply through by future shareholding choice a' (Equation 18) and aggregate over all households (19):

$$Pu'(c) = \beta \sum_{n=1}^{N_z} \pi_{mn}(P'_n + D'_n) \sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn}) + \lambda_a \tag{10 rewritten}$$

$$P = \frac{\beta \sum_{n=1}^{N_z} \pi_{mn}(P'_n + D'_n) \sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} + \frac{\lambda_a}{u'(c)}$$

$$Pa' = \left(\frac{\beta \sum_{n=1}^{N_z} \pi_{mn}(P'_n + D'_n) \sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} \right) a' + \frac{\lambda_a}{u'(c)} a' \tag{18}$$

$$\int_{\mathcal{S}} Pa' \mu(d[a \times \eta]) = \int_{\mathcal{S}} \left(\left(\frac{\beta \sum_{n=1}^{N_z} \pi_{mn}(P'_n + D'_n) \sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} \right) a' + \frac{\lambda_a}{u'(c)} a' \right) \mu(d[a \times \eta]) \tag{19}$$

where c_{jn} is a shorthand for the household's consumption rule when it transitions to idiosyncratic productivity level $\eta' = \eta_j$, aggregate TFP transitions to level $z' = z_n$, and the shareholding in the next period is the solution to the household's maximization problem: $c_{jn} \equiv c(a(a, \eta; \mathbb{Z}), \eta_j; z'_n, \mu'_H, \mu'_F)$.

When the stock market clears in equilibrium ($1 = \int_{\mathcal{S}} a(a, \eta, \mathbb{Z}) \mu(d[a \times \eta])$), the left hand side of Equation 19 is equal to the aggregate equity price P. I can express aggregate equity price as a function of weighted marginal rates of substitution and future payoffs:

$$P = \sum_{n=1}^{N_z} (P'_n + D'_n) \left(\beta \pi_{mn} \int_{\mathcal{S}} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} a' \mu(d[a \times \eta]) \right) + \int_{\mathcal{S}} \frac{\lambda_a}{u'(c)} a' \mu(d[a \times \eta]) \tag{20}$$

A stochastic discount factor directly values an asset conditional on a vector of future returns without an additive wedge. Therefore, I want to eliminate Lagrange multiplier λ_a in Equation 20. I start by rearranging the household's optimal choice for a' to be in terms of $\lambda_a/u'(c)$, which will allow me to simplify equation 20.

$$\frac{\lambda_a}{u'(c)} = P - \frac{\beta \sum_{n=1}^{N_z} \pi_{mn}(P'_n + D'_n) \sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} \tag{10 rearranged}$$

For households who choose $a' > \underline{\text{a}}$, the expression above is equal to zero. I introduce an indicator function that will let me separate out households who are at the savings limit against those who save more than the minimum.

$$\mathbb{I} = \begin{cases} 1 & a' > \underline{\text{a}} \\ 0 & a' = \underline{\text{a}} \end{cases}$$

With this indicator I separate out Equation 20 into households who are and aren't bound by the savings condition.

$$\begin{aligned} P = &\sum_{n=1}^{N_z} (P'_n + D'_n) \left(\beta \pi_{mn} \int_{\mathcal{S}} \mathbb{I} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} a' \mu(d[a \times \eta]) \right) + \\ &\sum_{n=1}^{N_z} (P'_n + D'_n) \left(\beta \pi_{mn} \int_{\mathcal{S}} (1 - \mathbb{I}) \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} \underline{\text{a}} \mu(d[a \times \eta]) \right) + \\ &\int_{\mathcal{S}} (1 - \mathbb{I}) \left(P - \frac{\beta \sum_{n=1}^{N_z} \pi_{mn}(P'_n + D'_n) \sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} \right) \underline{\text{a}} \mu(d[a \times \eta]) \end{aligned}$$

With some additional algebra, this leads to the equation:

$$P = \sum_{n=1}^{N_z} (P'_n + D'_n) \frac{\left(\beta \pi_{mn} \int_{\mathcal{S}} \mathbb{I} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} a' \mu(d[a \times \eta]) \right)}{\int_{\mathcal{S}} \mathbb{I} a' \mu(d[a \times \eta])} \tag{21}$$

This equation says that the value of aggregate equity is determined by discounted future payoffs. The discounting comes from shareholders' expected marginal rates of substitution across aggregate states, weighted by their end of period shareholding. I define this discount factor as:

$$\chi(z_n|\mathbb{Z}) \equiv \beta \pi_{mn} \frac{\int_{\mathcal{S}} \mathbb{I} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} a' \mu(d[a \times \eta])}{\int_{\mathcal{S}} \mathbb{I} a' \mu(d[a \times \eta])} \tag{22}$$

Before moving on, I would like to discuss a special case of the discount factor if $\underline{a} = 0$. In that case, the discount factor simplifies to:

$$\chi(z_n|\mathbb{Z}) = \pi_{mn}\beta \int_{\mathcal{S}} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} a' \mu(d[a \times \eta]) \tag{23}$$

This form of the discount factor is identical to the one proposed in Drèze (1974). However, the discount factor in that paper is assumed rather than being derived as an equilibrium expression. Additionally, the discount factor proposed by Drèze (1974) cannot handle cases where the minimum savings constraint is binding while the discount factor described in Equation 22 can.

3.4 Analysis

As discussed in Lemma 1, the production firm will survive a shareholder challenge if its discount factor is the same as the mutual fund's. Therefore, I will simplify notation in the remainder of this paper and use χ to describe the aggregate stochastic discount factor.

In the sections below, I discuss the equilibrium discount factor. I describe properties of the discount factor, describe how it compares to discount factors proposed in the literature, and briefly discuss uniqueness and unanimity with the discount factor above.

3.4.1 Properties of the Discount Factor

The discount factor described in Equation 22 features a number of useful properties. I discuss below how it nests a number of standard models, including the representative household case, the exogenous no-trade case, constant returns to production environments, and the Makowski (1983) Criterion. Additionally, this discount factor results in net market value maximization, which is consistent with observed firm behavior.

First, it neatly nests the representative household discount factor. With a representative household, the future shareholding choice is always $a' = 1$. And because there is no idiosyncratic risk, the distribution is degenerate. That discount factor can be written as:

$$\chi_{\text{rep}}(z_n|\mathbb{Z}) = \pi_{mn}\beta \frac{u'(c'_n)}{u'(c)}$$

which is a special case of the discount factor I derive in Equation 22. This discount factor is standard in the literature featuring dynamic firms, such as Khan and Thomas (2013).

Another useful feature is that this discount factor nests the exogenous no-trade approach proposed by Krusell, Mukoyama and Smith (2011). In their model, the minimum savings rule for equity is $\underline{a} = 1$, which requires all households to save the median number of shares. In that setting, only a single shareholder (or type of shareholders) would not want to choose $a' < 1$, meaning $\mathbb{I} = 0$ for all shareholders except one. Their discount factor is then:

$$\chi_{\text{KMS}}(z_n|\mathbb{Z}) = \pi_{mn}\beta\frac{\sum_{j=1}^{N_\eta}\pi_{ij}u'(c'_{jn})}{u'(c)} \qquad \text{for } \eta_i = \max \mathbf{N} \tag{24}$$

If there is only a single household (or single type of household) who would not want to choose $a' < \underline{a}$, then my model exactly replicates this result. There is only one household for whom the indicator function $\mathbb{I}$ in Equation 22 is nonzero, so that household's marginal rate of substitution is the stochastic discount factor.

In a constant returns environment without adjustment costs, the price of a production firm p is equal to its next-period capital stock k'. This is a standard result in the literature, as in Carceles-Poveda and Coen-Pirani (2009).

The proposed discount factor also meets a criterion as set out by Makowski (1983). That criterion requires the discount factor used by the firm satisfy $P = \max[\sum_n SDF_{a,\eta_i}(P'_n + D'_n)]$, where SDF_{a,η_i} is the stocahastic discount factor across aggregate states for the household indexed by $\{a, \eta_i\}$. However, every household that chooses $a' > \underline{a}$ will satisfy this condition as shown by the optimality conditions in Equation 10.[10]

One downside of the Murkowski criterion is that it does not identify a unique discount factor. Every household that chooses to hold more than the minimum level of assets will satisfy the criterion, and each of these discount factors may lead to the firm making a slightly different choice in capital. Because my discount factor uses information from all shareholders in equilibrium, it is distinct.

Another useful property of this discount factor is that it implicitly maximizes value weighed by current shareholding. As in DeAngelo (1981), maximizing the firm's net value expands each household's budget constraint by the size of their current shareholdings. This wealth effect allows households to choose more consumption or savings. The price effect and the change to future payoffs can be ignored because the shareholder can freely adjust their future shareholding choice.

[10]Households who choose $a' = \underline{a}$ will have $\lambda_a > 0$, which means only their SDF will not satisfy the Murkowski criterion.

3.4.2 My Approach Relative to Alternatives

The two most common alternate discount factors stem from Grossman and Hart (1979) and Carceles-Poveda and Coen-Pirani (2009). For each, they set the discount factor to:

$$\chi_{\text{alt}}(z_n|\mathbb{Z}) = \pi_{mn}\beta \int_S \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} \mathbf{W}_{a,\eta}\mu(d[a \times \eta]) \tag{25}$$

where $\mathbf{W}_{a,\eta}$ is the model-specific weighting for shareholder indexed by $\{a, \eta\}$. Grossman and Hart (1979) consider weights equal to either date zero holdings ($\mathbf{W}_{a,\eta} = a_{0_{,a,\eta}}$) or beginning of period holdings ($\mathbf{W}_{a,\eta} = a_{a,\eta}$). In comparison, my model uses $\mathbf{W}_{a,\eta} = a'_{a,\eta}$ in the special case $\underline{a} = 0$, as shown in Equation 23.

How does my model differ from these expressions? First, I explicitly allow for cases where $\underline{a} \neq 0$ by only considering households who choose to be shareholders. Second, the discount factor in my model is built up explicitly from equilibrium conditions. The alternate methods simply assume that the firm weighs future payoffs by these weighting factors. The most important difference is that the discount factor used by firms is also the pricing kernel.

From the mutual fund's problem, any asset in my problem is priced as $p = \sum_{n=1}^{N_z}(p'_n + d'_n)$. However, this relationship does not necessarily hold with these alternate weights.[11] This is the case because the alternate discount factors do not utilize the asset market clearing condition in their calculation.

How inaccurate would the guess of aggregate equity price be if one were to incorrectly guess that aggregate price is given by $P = \sum_{n=1}^{N_z} \chi_{\text{alt},n}(P'_n + D'_n)$? For this analysis, I assume $\underline{a} = 0$ so the numerator of χ from Equation 22 goes to 1. With that assumption in place, the price error is given by:

$$P_{err} = \sum_{n=1}^{N_z}(P'_n + D'_n)\left(\beta\pi_{mn} \int_S \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c'_{jn})}{u'(c)} (a - a')\mu(d[a \times \eta])\right) \tag{26}$$

In a complete markets setting where all households have the same marginal rate of substitution across states, the error will be zero. Similarly, the error would be zero in cases where all households endogenously choose not to trade. In initial testing, this term is tiny but positive ($< 0.0001\%$ of P). The error is small because the spread in marginal rates of substitution are not very large.

I discuss the numerical difference between my model and the one proposed by Gross-

[11] Except in the special case of constant returns to production or if households don't trade shares.

man and Hart (1979) in Section 5.4. A firm that discounts using the Grossman discounting will not survive a proxy challenge in my model. However, production firms that follow the initial-shareholding discounting regime will not maximize net market value, so they would be taken over by the private equity firm.

3.4.3 Unanimity

A common question in this literature is around the concept of unanimity. In a setting that requires unanimity, the goal is for all shareholders to want the firm to pursue exactly the same capital investment plan. Carceles-Poveda and Coen-Pirani (2009) shows that this is the case in models with constant returns and no short selling. However, DeAngelo (1981) and Makowski (1983) instead propose that ex-ante shareholders unanimously prefer for the firm to engage in net value (or cum-dividend price) maximization. The best a firm can do for its shareholders is to maximally expand their budget sets, which is done by maximizing the firm's net value.

In my model, households are unanimous in their preference for firm value maximization, as is the case in DeAngelo (1981). They are not unanimous, however, in the firm's exact choice of future capital. Some households would be better off if the firm increased its capital choice and vice-versa. However, a lack of unanimity about specific capital plans is not a shortcoming of the model. In observed equity markets, shareholder votes about capital choices (or firm management in general) are relatively rare. Fos (2017) documents that the most frequent cause for shareholder challenges are poor stock performance, while proxy challenges that target capital structure tend to be less successful. The empirical data support ignoring unanimity in capital choice as long as firms are maximizing their net market value.

4 Algorithm

The solution algorithm utilizes a modified version of the backward induction developed by Reiter (2009). While I still use a proxy distribution to find consistent behavior, I also require the representative household to behave consistently with perceived aggregates. Details are described below.

I begin by discretizing the aggregate state z into 7 states using the Tauchen algorithm. I similarly discretize idisoyncratic productivity levels η into 7 states. I then need to choose a proxy aggregate state. I use the total level of log capital, which serves as a good measure of aggregate total wealth. I linearly space log capital M into 9 grid points. Finally, I discretize the choice values for a' and k' on a grid with 99 points for shareholding and 299 points for capital.

I begin with a guess for the proxy distribution of households over shareholding and productivity at each point on the aggregate grid (z, M). A naive guess of the distribution could be to assume that all households start with shareholding $a = 1$ and productivity is distributed at the steady state level.

I also start with a guess for the aggregate law of motion, equity prices, and dividends at each state. I guess that $K' = K$, which also pins down aggregate dividends. The guess of share price is trivial, but I start by assuming $P = \beta D/(1-\beta)$. This guess of the price is consistent with an asset priced in a riskless Lucas economy. Finally, I also guess starting levels for the firm's value G, the household's value V, and the household's period marginal utility of consumption MUC.

In each iteration o, the algorithm proceeds as follows:

1 **Outer Loop:** In each aggregate state indexed by (z, M):

(a) **Solve LOM:** Guess a future aggregate state M'_g, which implies dividends D_g

i. **Clear the Equity Market:** Guess a price for aggregate equity P.

ii. Solve each household's optimal choice of a' given the previous V^o, law of motion M'_g, dividends D_g, and future prices and dividends P^o & D^o.

iii. Measure total shareholding $A(P) = \int_S a' \mu(d[a \times \eta])$.

A. If $A(P) - 1$ >precision, there is too much demand for shares, so the price needs to rise. Return to step 1(a)i

B. If $1 - A(P)$ >precision, there is insufficient demand for shares, so the price is too high and needs to fall. Return to step 1(a)i.

iv. Once the share price has cleared the equity market, measure the implied stochastic discount factor (Equation 22) using the MUC^o array.

v. **Consistency with aggregates:** Solve the firm's problem given the discount factor and firm values G^o. This yields k'.

A. If $k' - K'$ >precision, the guess of K' was too low. Guess a higher K' and return to step 1a

B. If $K' - k'$ >precision, the guess of K' was too high. Guess a lower K' and return to step 1a

(b) Once capital choice is consistent, update the guess of $P^{o+1}, D^{o+1}, K'^{o+1}$.

2 With a consistent guess of aggregates, solve the household's and the firm's problem conditional on the updated guesses of $P^{o+1}, D^{o+1}, K'^{o+1}$

(a) $V^{o+1} = u(c) + \beta V^o$

(b) $MUC^{o+1} = u'(c)$

(c) $G^{o+1} = d + \beta G^o$

3 If the norm is sufficiently small $|V^{o+1} - V^o| <$precision, proceed to the simulation. Otherwise, return to step 1.

In the algorithm above, I search for price via bisection. Conditional on a guess of M', demand for shares is weakly decreasing in price, so there is a single price that clears the equity market. I solve the household's problem and with the endogenous grid method and I solve the firm's problem via golden section search.

Once I've solved the value functions and aggregate laws of motion with the proxy distribution, I simulate the economy to calculate a new reference distribution. In the simulation, I draw a TFP shock on the grid, then solve it at the previously determined level of aggregate capital using the process described in step 1 above. However, instead of using the proxy distribution, I use the distribution of households from the previous simulation step. I run this simulation for 750 periods to "pre-heat" before tracking the distribution of households in each date and with each realization of shocks on the z grid. I simulate 750 periods, then update the reference distribution as described in Reiter (2009). After updating the reference distribution, I solve the household's value function again. I typically run this process four times in total, though results generally don't change after the second update of the reference distribution.

5 Business Cycle Moments and Impulse Responses

I now apply my method to a simple business cycle example to see how aggregate behavior varies with idiosyncratic risk. To begin, I specify explicit forms for the utility and production functions. Households value consumption with CRRA utility of the form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

where σ is the relative risk aversion parameter. I assume a Cobb-Douglas production function: $F(k, n) = zk^\alpha n^\gamma$ with $\alpha + \gamma \in (0, 1]$. I further assume a quadratic adjustment cost function: $I(k', k) = \frac{\psi}{2k}(k' - k)^2$. If $\psi = 0$, this nests a model without adjustment costs.

5.1 Model Parameters

Table 1 summarizes the model parameters that are constant across iterations. All results are from a parameterized model which captures rough trends.

σ	δ	β	α	γ	σ_z	ρ_z	$\underline{a}$	ψ
3.0	0.070	0.96	0.30	0.55	0.0120	0.7898	0	1.0

Table 1: Summary of economic parameters. σ is the coefficient of relative risk aversion. δ is depreciation. β is the household's subjective discount factor. α is capital's share of production and γ is labor's share. σ_z is standard deviation of TFP and ρ_z is the persistence of the TFP shock. $\underline{a}$ is the borrowing constraint, with 0 indicating no short sales are allowed. ψ is the scale of the quadratic adjustment cost.

The period of the model is annual. The coefficient of relative risk aversion σ is set to 3.0, which is on the high end of levels used in the real business cycle literature. Depreciation is set to 7%, which approximately matches the investment to capital ratio in the United States in the post-war period. The subjective discount factor β is set to 0.96, which would imply a real interest rate of 4% in a riskless representative agent economy. The adjustment parameter ψ is in the range of values in the literature, which typically vary from 0.15-150.

Labor's share of production γ is estimated from NIPA tables as total payments to labor divided by GDP, which is approximately 55%. I choose α at the low end of the literature's range at 0.3, which implies a total return to scale of 0.85. For TFP, I calculate Solow residuals for 1956-2019. I estimate TFP's AR(1) process with a standard deviation of 1.204% and a persistence of 78.98%. This level of persistence is a bit lower than the literature, but it leads to a standard deviation of TFP of 2.1%, which is consistent with the literature.

Idiosyncratic Risk

		Variance of Log Earnings	
Setting	σ_η	Model	Data
Rep HH	0.0	0.0	-
1970	0.1154	0.25	0.25
2010	0.1548	0.45	0.45

Table 2: Comparison of parameter values across settings. σ_η is the standard deviation of shocks to the persistent earnings process. The persistence ρ_η is 0.973 from Heathcote, Storesletten and Violante (2010).

For idiosyncratic risk, I consider three cases. I start with a baseline model with zero idiosyncratic risk, which will represent the representative agent case. For the cases with idiosyncratic risk, I consider risk levels matching 1970 and 2010. For both, I use estimates from Heathcote, Storesletten and Violante (2010) for the size and persistence of the productivity shock. I target the 1970's and 2010's with shock sizes of 0.1154 and 0.1548, respectively. For each, the persistence of the shock is set to 0.973. These shock sizes target the variance of log earnings documented in Heathcote, Storesletten and Violante (2010). I do not include the transitory shocks[12]featured in their models and instead only focus on the persistent shocks.

While Heathcote, Storesletten and Violante (2010) documents a number of causes for the increase in variance, I model it as a change to a technology parameter. If the variance in earnings is driven by endogenous choices, my model will overstate results.

5.2 Business Cycle Moments

Table 1 describes statistics about business cycles in this economy with three different types of productivity risk. Aggregate risk and aggregate labor supply are held constant across settings.

An increase in income risk reduces the standard deviation of output, consumption, investment, and share price. Production firms hold higher capital stocks, which delivers more market value to shareholders. Households use this higher average market value to smooth consumption across riskier idiosyncratic states.

As idiosyncratic risk increases, the average return on equity falls from 3.9% to 3.5%. This reflects two driving factors. First, the demand for shares rises as idiosyncratic risk increases. Households want to ensure themselves against a low idiosyncratic productivity shock, so they save in equity, which drives up the price of the asset and lowers average returns. Second, idiosyncratic risk increases the household's desire for dividend payments in low productivity states. This means firms will invest more, which lowers the rate of return on capital.

When households directly own capital, higher investment would be considered precautionary savings. However, the firm directly owns capital, so the concept of precautionary savings can't exactly be considered the same way as it is used in the literature. The firm has no utility function, is not risk averse, and doesn't face any change in productivity risk across these scenarios, so why would a firm engage in precautionary savings? This entirely driven by the household's risk aversion and insurance against earnings risk. Wage is perfectly correlated with output, so every household has a higher marginal utility of consumption in low productivity states. With higher productivity risk, each household has a higher expected marginal utility of consumption in each aggregate state (due to Jensen's Inequality). As risk increases, households value payoffs in low productivity states more than they did in a riskless environment. Firms see that households value

[12]Including transitory shocks will increase the precautionary savings motive. This would amplify any results I find when moving away from the representative agent case.

Business Cycle Moments

	Representative Household Economy					
	Y	C	I	D	P	r^e
Average	0.6498	0.5255	0.1243	0.1681	4.0605	4.06%
σ/μ	2.78%	1.75%	7.83%	1.82%	3.84%	4.23%
SDX/SDY	*1.000*	0.630	2.813	0.652	1.381	1.520
CORR(X,Y)	*1.000*	0.952	0.957	-0.386	0.942	-0.995
AutoCorr	0.839	0.929	0.752	0.721	0.937	0.810

	1970's wage risk ($\sigma_\eta = 0.10$)					
	Y	C	I	D	P	r^e
Average	0.6856	0.5369	0.1487	0.1599	5.5834	2.83%
σ/μ	2.74%	1.63%	7.33%	2.24%	3.61%	4.61%
SDX/SDY	*1.000*	0.597	2.679	0.817	1.317	1.683
CORR(X,Y)	*1.000*	0.943	0.963	-0.575	0.936	-0.992
AutoCorr	0.833	0.932	0.753	0.690	0.937	0.796

	2010's wage risk ($\sigma_\eta = 0.1414$)					
	Y	C	I	D	P	r^e
Average	0.7128	0.5436	0.1692	0.1515	7.1339	2.11%
σ/μ	2.69%	1.55%	6.94%	2.71%	3.52%	4.99%
SDX/SDY	*1.000*	0.577	2.579	1.007	1.307	1.856
CORR(X,Y)	*1.000*	0.930	0.965	-0.657	0.923	-0.990
AutoCorr	0.827	0.936	0.748	0.679	0.941	0.784

Table 3: Columns are output, consumption, investment, dividends, equity price, and realized return on equity, respectively. σ/μ is the standard deviation divided by the average. SDX/SDY is the relative standard deviation of the variable divided by the relative standard deviation of output. CORR(X,Y) describes the variable X's correlation with output. AutoCorr is the variable's correlation with itself over time.

payoffs more in low states, so they save more in good states to ensure a higher stream of dividend payments.

5.2.1 Model Fit of Financial Moments

With a simple shift in earnings risk, my model is able to replicate additional secular trends in equity markets. The first is a trend toward lower dividend yields. In the 1970's, dividends were roughly 3.5% of equity price, while they are now closer to 2%. My model features dividend yields of 3% on average with a 1970's level of risk which falls to 2.1% with the 2010's level of idiosyncratic risk. Dividend yields fell by 45% in the data and 27% in my model, so I explain over 60% of the fall in dividend yields with only a shift in idiosyncratic risk.

My model also replicates the rise in the price earnings ratio. Shiller (2016) documents

Pricing Ratios Over Time

	Data		Model	
Year	Dividend Yield	PE Ratio	Dividend Yield	PE Ratio
1970	3.5	15.1	2.9%	18.1
2010	1.9	21.2	2.1%	22.2
Change	-45.7%	40.3%	-27.6%	22.6%

Table 4: Dividend yields and price earnings ratios evaluated at 1970 and 2010. Dividend yield and PE ratio are from Shiller (2016). Model dividend yield is evaluated as average dividend divided by average price and PE ratio is price divided by the sum of investment and dividends. The only change in the model is from idiosyncratic earnings risk which rises from 0.10 in 1970 to 0.1414 in 2010.

the PE ratio as rising 40% between 1970 and 2010. My model generates an increase in PE ratio of 22%. An increase in household income risk explains 55% of the rise in the price earnings ratio.

5.3 Impulse Responses

How does idiosyncratic risk change recovery from recessions? I start by simulating the economy for 900 periods with TFP at the median level. This produces a steady state distribution of households and a constant level of capital. I then hit the economy with a negative TFP shock that is 1 standard deviation in size (2.14% below average). I simulate for 100 periods with the TFP shock decaying naturally at the rate of $\rho_z = 0.7337$. In each simulated date, I solve the equity price and law of motion using the process I described in Section 4.

In Figure 1, I show that the economy without aggregate risk goes into a deeper recession than the economies with some level of idiosyncratic risk. During a recession, dividend payments increase to help smooth consumption. In a representative household economy, dividends are higher than the steady state for 6 years. The economies with 1970's (2010's) household earnings risk feature dividends higher than the steady state for 7 (8) years. While not pictured, the price of equity falls by less than output but stays low longer due to the firm's lower capital level over the recession.

The increased smoothing in output and dividends comes from a higher initial capital stock which can be safely drawn down during a recession. Firms can then pay more in dividends without cutting into the capital stock as severely.

5.4 Household Distributions

I now briefly discuss the distribution of wealth in my model as I change levels of idiosyncratic risk. In the representative economy, there is clearly a degenerate distribution

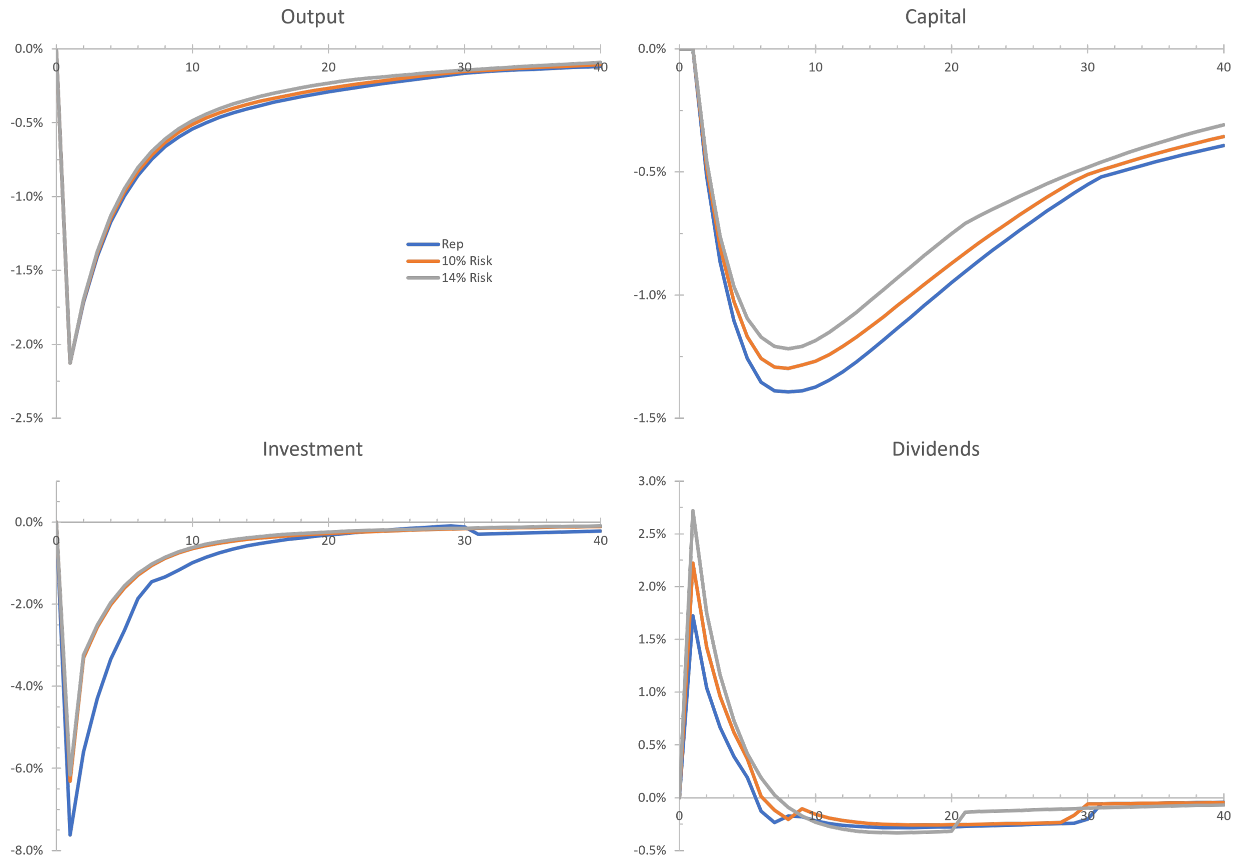

Figure 1: Response of dividends to a 2.14% negative TFP shock. The baseline representative household case is shown in blue. The 10% income risk representing 1970 is shown in gold and the 14% income risk representing 2010 is shown in grey. Note that the lumps in the dividend impulse response are approximately 1.4E-4 in size, which is only 0.02% of steady state output.

of wealth. Table 5 shows how the wealth distribution changes with increasing levels of household earnings risk.

As the variance of income risk increases, the wealthiest households accumulate more wealth. This finding is standard in the literature, so this result primarily demonstrates that this model generates expected results.

The wealth distribution is not remotely close to the data, but that is a common feature in the household heterogeneity literature without certain features. There are a few common approaches to rectify this issue. They include stochastic preferences (β heterogeneity), rates of return that increase with wealth, and alternate income schemes. Any of these extensions would fit well into my model and could help better target the wealth distribution. However, I leave that as an exercise for later work.

Additionally, allowing negative wealth or short sales would both concentrate more wealth at the top of the distribution as it creates some households with negative wealth.

Wealth Distribution

Model	Percentage of wealth held by top X of households				
	1%	5%	10%	20%	30%
10% earnings risk	4.3%	16.9%	29.3%	48.4%	63.1%
14% earnings risk	4.6%	17.8%	30.4%	49.5%	63.6%
Data[13]	30%	51%	64%	79%	88%

Table 5: Distribution of household wealth by top percentiles of households. Evaluated at the average TFP level and the average aggregate capital level.

This paper does not explicitly target the distribution of wealth, so I do not pursue these modeling changes at this time.

5.5 Comparison to Other Discounting Regimes

I now compare some key business cycle results from my model to results generated by alternate discounting regimes. I set the persistence of earnings to $\rho_\eta = 0.8$ and the standard deviation of the earnings process to $\sigma_\eta = 0.2$.[14] I consider three alternatives - discounting using current shareholders' discount factors, discounting at the implied safe rate, and discounting with a limited subset of interior shareholders. Discounting using current shareholders' valuation is the method proposed by Grossman and Hart (1979) and evaluated by Carceles-Poveda and Coen-Pirani (2009). The second approach, discounting each future state at one rate dependent on the current state, is more common in the New Keynesian literature. The final approach, discounting with a limited subset of shareholders, is described in Krusell, Mukoyama and Smith (2011) as a method to get around the question of aggregation altogether.

When constructing these alternate discount factors, I make a few small changes to the model environment. When discounting using current shareholder valuations, I calculate an alternate discount factor $\tilde{\chi}(z_n|\mathbb{Z})$ using the same formula as Equation 22, but I replace future shareholding weights a' with current shareholding a.

When I evaluate safe rate discounting, I use the standard expression to find the stochastic discount factor described in Equation 22, but I assume the firm discounts the future using the sum of these weights: $\bar{\chi}(\mathbb{Z}) \equiv \sum \chi(z_n|\mathbb{Z})$.

To find the business cycle moments in the environment with limited participation, I set $\underline{a} = 0.975$. This minimum savings rule is close to the zero trade setting proposed by

[14]These parameter values are chosen for run time speed. They result in less income risk than the examples discussed in the business cycle setting. This means well-calibrated estimates of earnings risk will increase the size of these differences.

Krusell, Mukoyama and Smith (2011). I plan on evaluating $\underline{a} \to 1$, but my algorithm needs to be refined to handle truly zero-trade cases.

To compare settings, I run the algorithm described in Section 4 with each variant discount factor. I then run a simulation with the same TFP shocks and measure the result from the last 750 periods. Table 6 describes results.

Discounting Comparison - Average Percentage Difference

Discounting Method	Y	C	I	$\frac{P}{K}$	$\frac{P}{E}$
Current Shareholders	0.002	0.001	0.007	-0.008	-0.003
Safe Rate	0.081	0.033	0.268	-0.307	-0.119
Limited Participation	7.969	2.591	29.11	20.23	43.79

Table 6: Comparison of three alternate discounting proposals. The table above compares averages for each model relative to the baseline across a 750 period simulation. Columns are output, consumption, investment, price to capital (or price to book) ratio, and price to earnings ratio. For example, discounting the future using current shareholders' marginal rates of substitution results in an economy with 0.002% higher average output relative to the baseline scenario.

The first two alternate models generate similar behavior. They both result in higher average output, consumption, and investment. This happens because firms generally undervalue payouts in low states when using alternate discount factors. Because firms undervalue payouts in low states, households know that they will have less insurance in low states, so demand for shares will rise. As demand for shares increases, the interest rate falls (or the discount factor rises), which increases firm investment. The lower rate of return results in lower price to book and price to earnings ratios than are generated by my model. Firms that discount using either of these alternate discount rates would not survive a proxy challenge in the benchmark model.

While these gaps are small, the key problem with discounting using the current shareholder methodology is that the SDF that prices assets is different than the discount factor that governs firm behavior. With a representative firm, this difference isn't critical because the aggregate equity price is the same as firm prices. But in a model with firm heterogeneity, an analysis of firm price would not be feasible. With the SDF described in Equation 22, any asset can be priced given a set of future payments.

The limited participation case results in much higher average output, consumption, and investment. Unlike the other alternates considered, price to book and price to earnings ratios are significantly higher under the limited participation setting. With limited participation, only the highest productivity shareholders price the aggregate equity asset. They have a lot to lose if the economy shifts to a low aggregate state and their productivity falls to the low level. And because they cannot dissave when they receive a low labor productivity shock, their marginal valuation of payoffs in future states is much higher than it is in the standard model. This results in much higher price to book and price to earnings ratios in the limited participation setting.

5.6 Robustness

When returns to scale are constant, my model replicates the standard finding that a firm's value is equal to its future capital choice. In this special case, my model and the approach suggested by Grossman and Hart (1979) or Carceles-Poveda and Coen-Pirani (2009) are identical.

Higher adjustment costs ψ will typically reduce the volatility of investment. This parameter can be used to ensure that the dividend process becomes procyclical as is the case in the data. And as adjustment costs increases, the average price of equity does not change significantly, but it becomes much more volatile. In the baseline case $\psi = 1$, price is 1.3 times as volatile as output. If adjustment costs become very high ($\psi = 150$), the standard deviation of equity price becomes 3.4 times as large as the standard deviation of output.

With log utility ($\sigma = 1$), an increase in earnings risk causes similar changes as the baseline case, though with lower magnitude. This result is intuitive. As risk aversion falls, an increase in risk has less of an impact on outcomes.

6 Conclusion

In this paper, I derive an equilibrium discounting mechanism that firms can use to maximize their value when they are owned by heterogeneous shareholders. I show that an increase in idiosyncratic household productivity risk increases capital investment levels, lowers equity rate of return, and results in less volatile aggregate consumption sequences. Additionally, a rough calibration of the increase in idiosyncratic risk explains half of the fall in dividend yields observed between the 1970's and the 2010's.

The model presented here has numerous avenues for refinement. First, I plan on better matching the wealth distribution by implementing a β heterogeneity plan as is the case in Krusell and Smith (1998). I expect the increase in wealth inequality will drive additional precautionary savings and will increase share price. Second, I plan on modeling the income process more seriously. Skewness of earnings is procyclical in the data with a long right tail. I anticipate this would also lead to more precautionary savings and higher equity price. Temporary, one period earnings shocks are also a feature of the data that would likely shift firm behavior in my model.

There are a number of possible extensions of this framework, but I focus here on two. In future work, I plan on extending my model to examine the interaction between firm and household heterogeneity over the last 40 years. My discounting method can be immediately extended to a setting with firm heterogeneity with only a small change to the solution algorithm. There is evidence that household earnings are becoming more dispersed over time (Hubmer, Krusell and Smith., 2021) alongside evidence that firm dynamism is falling (Akcigit and Ates, 2022). These phenomena have not been studied in tandem in part because there is not a consensus method for linking heterogeneous shareholders to firm choices.

The next natural extension is in the realm of common ownership. The current frontier of common ownership literature (Azar and Vives, 2021) does not feature endogenous valuation of future states. My approach would extend that literature by providing it with a consistent method for valuing future payoffs when owned by heterogeneous shareholders.

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Appendices

A Alternate Problem of the Private Equity Firm

In this section, I derive the private equity firm's problem if it compensates marginal equity shareholders directly.

Assumption 1. *Shareholders only have voting rights if they hold more than the minimum number of shares.*

Assumption 2. *A private equity firm needs 100% of voting shares to approve a capital deviation for it to be approved.*

Assumption 3. *A successful deviation will be small enough that households do not change their shareholding choice.*

Assumptions 1 and 2 require that the private equity firm make side payments to all marginal[15] investors. Households who are made better off by the alternate investment plan $\hat{k}'$ will vote for the plan and pay the private equity firm to implement it. Households who are made worse off by $\hat{k}'$ will be compensated for their votes so that they are at least indifferent to the alternate plan. In this case, the side payment is the price of their vote.

I define the set of shareholder who choose $a' > \underline{a}$ as $\hat{\mathcal{S}} \subset \mathcal{S}$. The private equity firm's problem is written as:

$$\max_{\xi(a,\eta|\hat{k}'),\hat{k}'} \int_{\hat{\mathcal{S}}} -\xi(a,\eta|\hat{k}')\mu(d[a\times\eta]) \tag{27}$$

$$\text{s.t.} \quad u(c) + \beta \mathbb{E} V(a'; z', \mu'_H, \mu'_F) \leq u(\hat{c}) + \beta \mathbb{E} V(a'; z', \mu'_H, \hat{\mu}'_F) \quad \forall (a,\eta) \in \hat{\mathcal{S}} \tag{28}$$

$$\hat{c} \equiv w\eta + \xi + (\hat{P} + \hat{D})a - \hat{P}a'$$

where hat variables $(\hat{c}, \hat{P}, \hat{D}, \hat{\mu}'_F)$ denote the components that shift when $\hat{k}'$ changes. When the private equity firm chooses an alternate scheme $\hat{k}'$, it changes prices and dividend payments today and in the future.

Just as in the baseline case, the private equity firm targets a mass $\varepsilon_{\to 0}$ of identical production firms. Unlike the baseline case, the firm needs to consider how its choice of alternate capital $\hat{k}'$ changes current prices.

[15]Marginal investors, as described earlier, are households who choose to hold more than the minimum number of shares.

The private equity firm's optimal choices are given as:

$$
\begin{aligned}
[\xi(\hat{k}')]: &\qquad \mu_i = \Omega_i u'(c) \\
[\hat{k}']: &\quad \varepsilon\Omega_i \frac{\partial p}{\partial \hat{k}'} a' u'(c) = \varepsilon\Omega_i \left(\frac{\partial p + \partial d}{\partial \hat{k}'} a u'(c) + \sum_{n=1}^{N_z} \beta\pi_{,m} \frac{\partial p' + \partial d'}{\partial \hat{k}'} a' \sum_{j=1}^{N_\eta} \pi_{ij} u'(c') + \underbrace{o(\varepsilon)}_{\to 0} \right)
\end{aligned} \tag{29}
$$

where Ω_i is the Lagrange multiplier on each household's participation constraint. If capital rises, households lose out by having to pay a higher price today for future shareholding and they lose out on some amount of dividends. They then benefit from higher price and dividend payments in the future.

If the production firm was already maximizing its net market value (or cum-dividend share price), then $\frac{\partial p' + \partial d'}{\partial k'} = 0$. In equilibrium, $k' = \hat{k}'$. Integrating over households and substituting in the equilibrium result that production firms are net value maximizers yields:

$$
-\frac{\partial d}{\partial k'} = \sum_{n=1}^{N_z} \frac{\partial p_n' + \partial d_n'}{\partial k'} \frac{\left(\beta\pi_{mn} \int_{\mathcal{S}} \mathbb{I} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c_{jn}')}{u'(c)} a' \mu(d[a \times \eta]) \right)}{\int_{\mathcal{S}} \mathbb{I} a' \mu(d[a \times \eta])} \tag{30}
$$

Finally, I substitute in the production firm's optiamlity conditions:

$$
\sum_{n=1}^{N_z} \frac{\partial p_n' + \partial d_n'}{\partial k'} \tilde{\chi}(z_n) = \sum_{n=1}^{N_z} \frac{\partial p_n' + \partial d_n'}{\partial k'} \frac{\left(\beta\pi_{mn} \int_{\mathcal{S}} \mathbb{I} \frac{\sum_{j=1}^{N_\eta} \pi_{ij} u'(c_{jn}')}{u'(c)} a' \mu(d[a \times \eta]) \right)}{\int_{\mathcal{S}} \mathbb{I} a' \mu(d[a \times \eta])} \tag{31}
$$

With the assumptions laid out at the beginning of this section, the firm's discounting regime $\tilde{\chi}$ is consistent with the the mutual fund's pricing kernel χ.

B Endogenous Borrowing Limit

The natural borrowing limit is set such that the household will always be able to service its debt in aggregate state $\mathbb{Z}$. Additionally, it must be able to service this debt if it starts with the lowest level of savings $a = \underline{\mathrm{A}}$ and with the lowest productivity draw $\eta = \eta_1$.

That is:

$$\underbrace{0}_{c} + P(\mathbb{Z})\underline{\text{A}} = (P(\mathbb{Z}) + D(\mathbb{Z}))\underline{\text{A}} + w(\mathbb{Z})\eta_1 \tag{32}$$

$$\Rightarrow \underline{\text{A}} = \max \frac{-w(\mathbb{Z})\eta_1}{D(\mathbb{Z})} \tag{33}$$

If the utility function satisfies the standard Inada condition,[16] then households will never choose $a' = \underline{\text{A}}$.

[16] $\lim_{c \to 0} u'(c) = +\infty$