

## ME 470 Final Project: Detailed Drawing with Kinematic and Dynamic Analysis

### 4 bar linkage

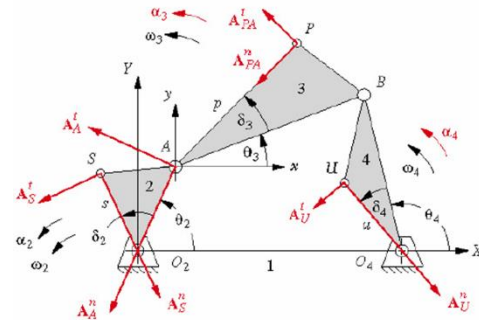
a, b, c, d, $\omega_2$ , $\alpha_2$	Choose Values
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Ideally you will determine the following values at all possible values of  $\theta_2$  so that you can visualize where the forces will be the highest.

$K_1 = \frac{d}{a}$ $K_2 = \frac{d}{c}$ $K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$ $K_4 = \frac{d}{b}$ $K_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab}$	$A = \cos\theta_2 - K_1 - K_2\cos\theta_2 + K_3$ $B = -2\sin\theta_2$ $C = K_1 - (K_2 + 1)\cos\theta_2 + K_3$ $D = \cos\theta_2 - K_1 + K_4\cos\theta_2 + K_5$ $E = -2\sin\theta_2$ $F = K_1 + (K_4 - 1)\cos\theta_2 + K_5$	$\theta_{4,1,2} = 2 \tan^{-1} \left( \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \right)$ $\theta_{3,1,2} = 2 \tan^{-1} \left( \frac{-E \pm \sqrt{E^2 - 4DF}}{2D} \right)$
$\mu$	$\mu = \theta_4 - \theta_3$	

### Velocity

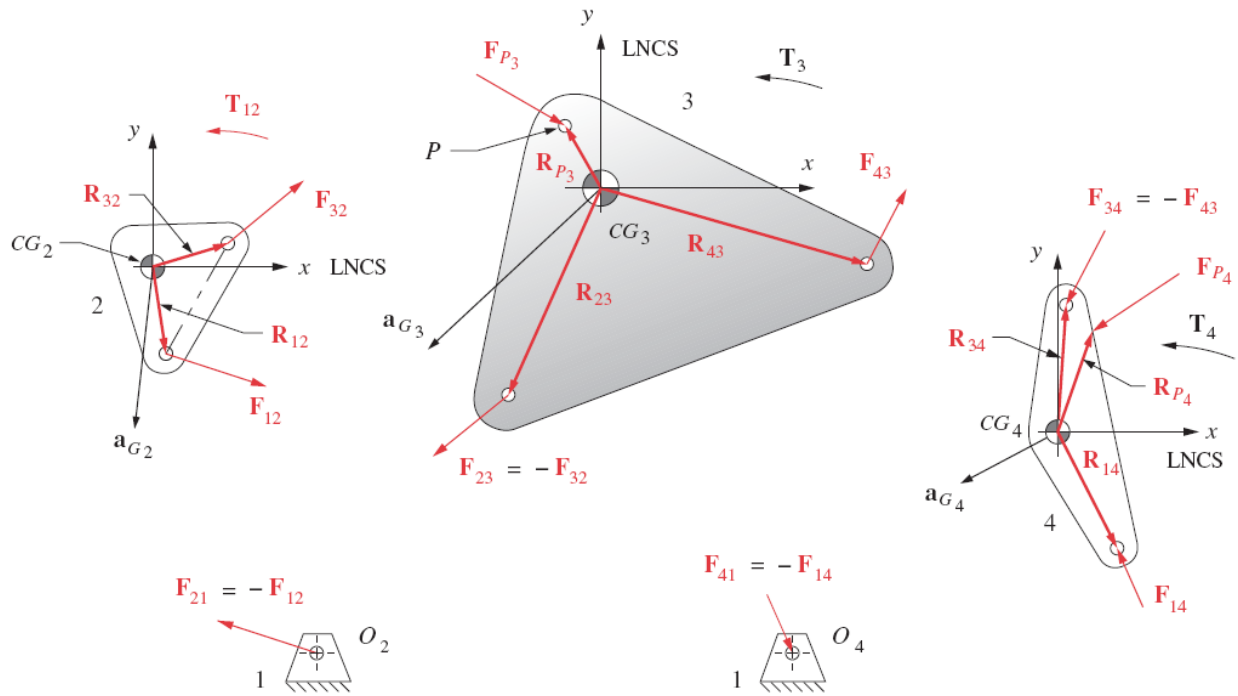
$\omega_3$	$\omega_3 = \frac{a\omega_2 \sin(\theta_4 - \theta_2)}{b \sin(\theta_3 - \theta_4)}$
$\omega_4$	$\omega_4 = \frac{a\omega_2 \sin(\theta_2 - \theta_3)}{c \sin(\theta_4 - \theta_3)}$



### Acceleration

	$A = c \sin \theta_4 \quad B = b \sin \theta_3$ $C = a\alpha_2 \sin \theta_2 + a\omega_2^2 \cos \theta_2 + b\omega_3^2 \cos \theta_3 - c\omega_4^2 \cos \theta_4$ $D = c \cos \theta_4 \quad E = b \cos \theta_3$ $F = a\alpha_2 \cos \theta_2 - a\omega_2^2 \sin \theta_2 - b\omega_3^2 \sin \theta_3 + c\omega_4^2 \sin \theta_4$
$\alpha_3$ $\alpha_4$	$\alpha_3 = \frac{CD - AF}{AE - BD} \quad \alpha_4 = \frac{CE - BF}{AE - BD}$
	Determine the location of the center of mass for link 2, link 3, link 4: s, p, u, $\delta_2$ , $\delta_3$ , $\delta_4$
$A_{G2}$ $A_{G3}$ $A_{G4}$	$A_{G2} = A_S = -[s\alpha_2 \sin(\theta_2 + \delta_2) + s\omega_2^2 \cos(\theta_2 + \delta_2)] + j[s\alpha_2 \cos(\theta_2 + \delta_2) - s\omega_2^2 \sin(\theta_2 + \delta_2)]$ $A_{G3} = A_P = A_A + A_{PA} = -[a\alpha_2 \sin \theta_2 + a\omega_2^2 \cos \theta_2] + j[a\alpha_2 \cos \theta_2 - a\omega_2^2 \sin \theta_2] + -[p\alpha_3 \sin(\theta_3 + \delta_3) + p\omega_3^2 \cos(\theta_3 + \delta_3)] + j[p\alpha_3 \cos(\theta_3 + \delta_3) - p\omega_3^2 \sin(\theta_3 + \delta_3)]$ $A_{G4} = A_U = -[u\alpha_4 \sin(\theta_4 + \delta_4) + u\omega_4^2 \cos(\theta_4 + \delta_4)] + j[u\alpha_4 \cos(\theta_4 + \delta_4) - u\omega_4^2 \sin(\theta_4 + \delta_4)]$

	Determine mass of link 2, link 3, and link 4: $m_2, m_3, m_4$ in slugs, blobs, or kg Determine mass moment of inertia of link 2, link 3, link 4: $I_{G2}, I_{G3}, I_{G4}$ Bar Link: $I_G = \frac{1}{12} mass(width^2 + length^2)$ Circle Link: $I_G = \frac{1}{2} mass(radius^2)$ Utilizing the value of $\theta_3, \theta_4, \mu, \omega_3, \omega_4, \alpha_3, \alpha_4, a_{G2}, a_{G3}, a_{G4}, s, p, u, \delta_2, \delta_3, \delta_4, m_2, m_3, m_4, I_{G2}, I_{G3}, I_{G4}$ Determine $R_{12x}, R_{12y}, R_{32x}, R_{32y}, R_{23}, R_{23y}, R_{43x}, R_{43y}, R_{14x}, R_{14y}, R_{34x}, R_{34y}$ position vectors. Estimate position and magnitude of any externally applied forces and torques. Determine $F_{12x}, F_{12y}, F_{32x}, F_{32y}, F_{43x}, F_{43y}, F_{14x}, F_{14y}, T_{12}$ based on any given input values.																		
	<table><tr><th colspan="3">R-values</th></tr><tr><th>Link 2</th><th>Link 3</th><th>Link 4</th></tr><tr><td><math>R_{12x} = s \cos(\theta_2 + \delta_2 + 180^\circ)</math></td><td><math>R_{23x} = p \cos(\theta_3 + \delta_3 + 180^\circ)</math></td><td><math>R_{14x} = u \cos(\theta_4 + \delta_4 + 180^\circ)</math></td></tr><tr><td><math>R_{12y} = s \sin(\theta_2 + \delta_2 + 180^\circ)</math></td><td><math>R_{23y} = p \sin(\theta_3 + \delta_3 + 180^\circ)</math></td><td><math>R_{14y} = u \sin(\theta_4 + \delta_4 + 180^\circ)</math></td></tr><tr><td><math>R_{32x} = a \cos(\theta_2) + R_{12x}</math></td><td><math>R_{43x} = b \cos(\theta_3) + R_{23x}</math></td><td><math>R_{34x} = c \cos(\theta_4) + R_{14x}</math></td></tr><tr><td><math>R_{32y} = a \sin(\theta_2) + R_{12y}</math></td><td><math>R_{43y} = b \sin(\theta_3) + R_{23y}</math></td><td><math>R_{34y} = c \sin(\theta_4) + R_{14y}</math></td></tr></table>	R-values			Link 2	Link 3	Link 4	$R_{12x} = s \cos(\theta_2 + \delta_2 + 180^\circ)$	$R_{23x} = p \cos(\theta_3 + \delta_3 + 180^\circ)$	$R_{14x} = u \cos(\theta_4 + \delta_4 + 180^\circ)$	$R_{12y} = s \sin(\theta_2 + \delta_2 + 180^\circ)$	$R_{23y} = p \sin(\theta_3 + \delta_3 + 180^\circ)$	$R_{14y} = u \sin(\theta_4 + \delta_4 + 180^\circ)$	$R_{32x} = a \cos(\theta_2) + R_{12x}$	$R_{43x} = b \cos(\theta_3) + R_{23x}$	$R_{34x} = c \cos(\theta_4) + R_{14x}$	$R_{32y} = a \sin(\theta_2) + R_{12y}$	$R_{43y} = b \sin(\theta_3) + R_{23y}$	$R_{34y} = c \sin(\theta_4) + R_{14y}$
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$$\begin{bmatrix}
 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 -R_{12y} & R_{12x} & -R_{32y} & R_{32x} & 0 & 0 & 0 & 0 & 1 \\
 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & R_{23y} & -R_{23x} & -R_{43y} & R_{43x} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & R_{34y} & -R_{34x} & -R_{14y} & R_{14x} & 0
 \end{bmatrix}
 \begin{bmatrix}
 F_{12x} \\
 F_{12y} \\
 F_{32x} \\
 F_{32y} \\
 F_{43x} \\
 F_{43y} \\
 F_{14x} \\
 F_{14y} \\
 T_{12}
 \end{bmatrix}
 =
 \begin{bmatrix}
 m_2 a_{G2x} \\
 m_2 a_{G2y} \\
 I_{G2} \alpha_2 \\
 m_3 a_{G3x} - F_{P3x} \\
 m_3 a_{G3y} - F_{P3y} \\
 I_{G3} \alpha_3 - (R_{P3x} F_{P3y} - R_{P3y} F_{P3x}) - T_3 \\
 m_4 a_{G4x} - F_{P4x} \\
 m_4 a_{G4y} - F_{P4y} \\
 I_{G4} \alpha_4 - (R_{P4x} F_{P4y} - R_{P4y} F_{P4x}) - T_4
 \end{bmatrix}$$