1) Create an interesting "completing the square" problem (akin to the one found on page 477 of GGJ) of your own. Write it up in both (i) modern notation and (ii) al-Khwarizmi's descriptive language.

2) Read Sharif al-Tusi's approach to solving a cubic on pages 485 - 486 in *The Crest of the Peacock*. Use modern calculus to confirm what he stated and explain why the three inferences he makes at the end of the argument are mathematically valid.

3) On pages 473 - 474 in *The Crest of the Peacock*, the author details the method for determining whether two positive integers are amicable as described by Thabit ibn Qurra. Apply the criterion for the following values of n. If an amicable pair is permitted, be sure to calculate the values of p, q, r, M and N.

a) $n = 6$ b) $n = 7$ c) $n = 8$

4) You may recall that there was some question as to why triangle DFH in the construction on page 488 in *The Crest of the Peacock* is equilateral. In the supplementary pages linked in this assignment, please find the reference Gheverghese-Joseph mentions at the end of his argument. This excerpt is taken from the text *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook*, edited by Victor J. Katz. Read this argument and write up the explanation with the appropriate details. You may, but do not have to, introduce new pictures or new notation as you see fit.

5) Islamic mathematicians (and Thabit ibn Qurra, in particular) explored "figurate numbers" in nontrivial ways. In this problem, you will examine patterns detailed in several Islamic texts. Begin by recalling some well-known sequences:

Triangular numbers: $1, 3, 6, 10, 15, \ldots$

Square numbers: $1, 4, 9, 16, 25, \ldots$

Pentagonal numbers: $1, 5, 12, 22, 35, \ldots$

Hexagonal numbers: $1, 6, 15, 28, 45 \ldots$

a) For each number type, give a drawing using dots as units that explains how each sequence of numbers can be related to a sequence of polygonal drawings. Your sketches should be clear why, at least, the first four numbers in the sequence fit the descriptive pattern. If you consult external sources, please cite them.

b) For any polygon with s sides (triangles 3, squares 4, pentagons 5, etc.), the n-th s-gonal number is given by:

$$P(s,n) = \frac{n[(s-2)n-(s-4)]}{2}.$$

Write out the formulas this yields for triangular, square, pentagonal, and hexagonal numbers.

c) Find n such that $P(3,n) = 210$.

d) Prove that every hexagonal number is triangular by showing $P(6,n) = P(3,2n-1)$.

e) Prove that the nth square number is the sum of the first n odd numbers, i.e.

$$n^2 = \sum_{k=1}^{n}(2k-1).$$

f) Suppose we want to identify all n values for which $P(3,n) = m^2$ with $n > 0, m > 0$, i.e. the triangular numbers that are perfect squares. Clearly, this holds for $n = m = 1$.

What is the next solution when $n > 1$?

Set up an equation to find the general solution to this question and show it can be written in a form that looks like Pell's equation.