The rate / expected number calculation

From Brown + 2016 (https://arxiv.org/abs/1604.04269) the merger rate is 0.003 yr^{-1}. Note that in earlier works it was smaller in Brown+2011 (https://arxiv.org/abs/1011.3047) of 0.00004 yr^{-1}. Also this paper (https://academic.oup.com/mnras/article/460/4/4176/2609184) has 0.00014 yr^{-1}?

In[1]:=
```
RMWlow = 0.00004;
RMWlow = 0.00014;
RMWlow = 0.00017;
RMWmid = 0.003;
RMWhigh = 0.1;
```

For the frequency bounds in a "steady state" we require a merger time < Gyr timescale

In[6]:=
$$\text{dfdt[fGW\_, mchirp\_]} = \frac{96\,\pi^{8/3}\left(6.67\times10^{-8}\right)^{5/3}(\text{fGW})^{11/3}\left(\text{mchirp}\,2\times10^{33}\right)^{5/3}}{5\left(3\times10^{10}\right)^{5}}$$

Out[6]=
$$5.82578\times10^{-7}\,\text{fGW}^{11/3}\,\text{mchirp}^{5/3}$$

In[7]:=
$$\text{Tsteady} = 1\times10^{9}\,\pi\,10^{7};$$

In[8]:=
$$\text{dfdt[fGW, 0.3]}^{-1}$$

Out[8]=
$$\frac{1.27676\times10^{7}}{\text{fGW}^{11/3}}$$

In[9]:=
$$\text{Integrate}\left[\frac{1.276762868707655\grave{}*\hat{}7}{\text{fGW}^{11/3}}, \{\text{fGW}, \text{fmin}, \infty\}\right]$$

Out[9]=
$$\frac{4.78786\times10^{6}}{\text{fmin}^{8/3}}\quad \text{if}\quad \text{Im[fmin]} \neq 0 \;||\; \text{Re[fmin]} > 0$$

In[10]:=
$$\text{fmin /. Solve}\left[\frac{4.787860757653706\grave{}*\hat{}6}{\text{fmin}^{8/3}} == \text{Tsteady, fmin}\right][[1]]$$

Out[10]=
```
0.000208268
```

So the minimum frequency a 0.3 solar mass chirp mass binary needs to reach mass transfer in 1 Gyr (so that the steady state assumption is justified), it’s 0.2mHz.
We note here that 1.3 mHz from ZTF J1749 is the longest period binary in the table (except for ZTF J1901 which we will exclude for outlier-based arguments later).
Therefore we limit it to a minimum frequency of 1.3 mHz, and take the maximum to be 3.8 mHz (ZTF J2243), but note that the higher frequency is not so sensitive.

In[11]:=
```
fGWlow = 0.0013;
fGWhigh = 0.0038;
```

In terms of the detectable volume, the furthest detached binary is at 2kpc (table 4 here https://iopscience.iop.org/article/10.3847/1538-4357/abc261/pdf) . This is much more recent https://arxiv.org/pdf/2302.12719.pdf but basically suggests up to 2 kpc. Considering that the scale height of the galactic thick disc is ~ 1kpc we can construct a cylinder volume

In[13]:=
```
vZTF = π 1.76^2 × 1
```
Out[13]=
```
9.7314
```

While the volume of the milky way will have a diameter of 30 kpc and scale height 1 kpc:

In[14]:=
```
vMW = π 15^2 × 1 // N
```
Out[14]=
```
706.858
```

So that the observable volume is

In[15]:=
```
vobs = vZTF/vMW // N
```
Out[15]=
```
0.0137671
```

In[16]:=
```
vZTF / vMW // N
```
Out[16]=
```
0.0137671
```

Then, for the eclipsing probability, we consider that if the components are ~ equal size, it scales like $\frac{(R_1+R_2)}{a}$ (https://iopscience.iop.org/article/10.3847/1538-4357/aa9ce7/pdf)
which for this sample is on average ~0.2. For ZTF J1901 , which we argue in the paper is an outlier in that it is hot because it must have formed recently, rather than be tidally heated, it is 0.15.

In[17]:=
```
eclipseprob = 0.2
```
Out[17]=
```
0.2
```

So finally, we can write

In[18]:=
```
NZTF = eclipseprob RMWmid/(π 10^7) vobs Integrate[dfdt[fGW, 0.3]^-1, {fGW, fGWlow, fGWhigh}]
```
Out[18]=
```
58.9569
```

NZTF is a factor 10 larger than the number of binaries in the Table. But if we used Brown+2011's R_{MW}, then NZTF = number in the Table..

In[19]:=
```
Integrate[fGW^(-11/3), fGW]
```
Out[19]=

$$-\frac{3}{8\, fGW^{8/3}}$$

We see that NZTF ∝ $fGW_{low}^{-8/3}$. Therefore this estimate is highly sensitive to our (dubious?) choice of fGW_{low}.

In[20]:=
```
NZTF2 =
  eclipseprob RMWlow/(π 10^7) vobs Integrate[dfdt[fGW, 0.3]^-1, {fGW, fGWlow, fGWhigh}]
```
Out[20]=
```
3.34089
```