Number of intrinsic white dwarf binaries, N_{MW}, and number detectable by ZTF, N_{ZTF}

The ELM DWDB production rate, and expected number calculation

From Brown et al. 2016 the merger rate is 0.003 yr^{-1} from detached double white dwarf binaries. Kilic et al. 2016 has 0.00017 yr^{-1} from AM CVn binaries. These set a range for the rate of DWDB production rate.

In[1]:=
```
RMWlow = 0.00017;
RMWhigh = 0.003;
```

For the frequency bounds in a "steady state" we require a merger time < Gyr timescale

In[3]:=
$$\texttt{dfdt[fGW\_, mchirp\_]} = \frac{96\,\pi^{8/3}\left(6.67\times10^{-8}\right)^{5/3}(\texttt{fGW})^{11/3}\left(\texttt{mchirp}\,2\times10^{33}\right)^{5/3}}{5\left(3\times10^{10}\right)^{5}}$$

Out[3]= $5.82578\times10^{-7}\,\texttt{fGW}^{11/3}\,\texttt{mchirp}^{5/3}$

In[4]:= $\texttt{Tsteady} = 1\times10^{9}\,\pi\,10^{7};$

In[5]:= $\texttt{dfdt[fGW, 0.3]}^{-1}$

Out[5]= $\dfrac{1.27676\times10^{7}}{\texttt{fGW}^{11/3}}$

In[6]:= $\texttt{Integrate}\left[\dfrac{1.276762868707655\grave{}*\wedge 7}{\texttt{fGW}^{11/3}}, \{\texttt{fGW}, \texttt{fmin}, \infty\}\right]$

Out[6]= $\dfrac{4.78786\times10^{6}}{\texttt{fmin}^{8/3}}$ if Im[fmin] $\neq$ 0 || Re[fmin] > 0

In[7]:= $\texttt{fmin /. Solve}\left[\dfrac{4.787860757653706\grave{}*\wedge 6}{\texttt{fmin}^{8/3}} == \texttt{Tsteady}, \texttt{fmin}\right][[1]]$

Out[7]= 0.000208268

So for the minimum frequency a 0.3 solar mass chirp mass binary needs at formation, to reach mass transfer in 1 Gyr (so that the steady state assumption is justified) is 0.2mHz.

We note here that 1.3 mHz from ZTF J1749 is the longest period binary in the table.

We limit it to a minimum frequency of 1.3 mHz, and take the maximum to be 3.8 mHz (ZTF J2243), but note that the higher frequency is not so sensitive.

In[8]:=
```
fGWlow = 0.0013;
fGWhigh = 0.0038;
```

For the detectable volume, the furthest detached binary is at 2kpc (table 4 in Burdge et al. 2020) . More recent estimates in Kupfer & Korol et al. 2024 suggests up to 2 kpc. Considering that the scale height of the galactic thick disc is ~ 1kpc we can construct a cylinder volume

In[10]:=
```
vZTF = π 1.76^2 × 1
```
Out[10]=
```
9.7314
```

While the volume of the milky way will have a diameter of 30 kpc and scale height 1 kpc:

In[11]:=
```
vMW = π 15^2 × 1 // N
```
Out[11]=
```
706.858
```

So that the observable volume is

In[12]:=
$$\text{vobs} = \frac{\text{vZTF}}{\text{vMW}} \ // \ \text{N}$$
Out[12]=
```
0.0137671
```

In[13]:=
```
vZTF / vMW // N
```
Out[13]=
```
0.0137671
```

Then, for the eclipsing probability, we consider that if the components are ~ equal size, it scales like $\frac{(R_1+R_2)}{a}$
which for this sample is on average ~0.2. For ZTF J1901 , which we argue in the paper is an outlier in that it is hot because it must have formed recently, rather than be tidally heated, it is 0.15.

In[14]:=
```
eclipseprob = 0.2
```
Out[14]=
```
0.2
```

So finally, we can write

In[15]:=
$$\text{NZTF} = \text{eclipseprob}\ \frac{\text{RMWhigh}}{\pi\, 10^7}\ \text{vobs Integrate}\left[\text{dfdt[fGW, 0.3]}^{-1}, \{\text{fGW, fGWlow, fGWhigh}\}\right]$$
Out[15]=
```
58.9569
```

NZTF is a factor 7 larger than the number of binaries in the Table. But if we used Brown+2011's R_{MW}, then NZTF = number in the Table..

In[16]:=
```
Integrate[fGW^(-11/3), fGW]
```
Out[16]=

$$-\frac{3}{8\ \mathrm{fGW}^{8/3}}$$

We see that NZTF ∝ $\mathrm{fGW}_{\mathrm{low}}{}^{-8/3}$. Therefore this estimate is highly sensitive to our (dubious?) choice of $\mathrm{fGW}_{\mathrm{low}}$.

In[17]:=

$$\mathrm{NZTF2} = \mathrm{eclipseprob}\ \frac{\mathrm{RMWlow}}{\pi\ 10^{7}}\ \mathrm{vobs}\ \mathrm{Integrate}\left[\mathrm{dfdt}[\mathrm{fGW}, 0.3]^{-1}, \{\mathrm{fGW}, \mathrm{fGWlow}, \mathrm{fGWhigh}\}\right]$$

Out[17]=

3.34089

In[18]:=

$$\mathrm{eclipseprob}\ \frac{\mathrm{RMWlow}}{\pi\ 10^{7}}\ \mathrm{vobs}\ \mathrm{Integrate}\left[\mathrm{dfdt}[\mathrm{fGW}, 0.3]^{-1}, \{\mathrm{fGW}, 0.001, 0.1\}\right]$$

Out[18]=

7.13364

In[19]:=

$$\mathrm{eclipseprob}\ \frac{\mathrm{RMWlow}}{\pi\ 10^{7}}\ \mathrm{vobs}\ \mathrm{Integrate}\left[\mathrm{dfdt}[\mathrm{fGW}, 0.3]^{-1}, \{\mathrm{fGW}, 0.003, 0.1\}\right]$$

Out[19]=

0.381024

In[20]:=

$$\mathrm{Integrate}\left[\mathrm{dfdt}[\mathrm{fGW}, 0.3]^{-1}, \{\mathrm{fGW}, \mathrm{fgwstart}, 0.1\}\right]$$

Out[20]=

$$1.27676\times 10^{7}\left(-174.06+\frac{0.375}{\mathrm{fgwstart}^{8/3}}\right)\ \text{if}\ \mathit{condition}$$

In[21]:=

$$\frac{(303+329+275+270+310+290+313+286+311)}{9}\ //\ \mathrm{N}$$

Out[21]=

298.556

In[22]:=

$$\frac{(0.3-0.27)}{0.3}$$

Out[22]=

0.1

In[23]:=

$$\mathrm{Integrate}\left[\left(\frac{96\ \pi^{8/3}\ \left(6.67\times 10^{-8}\right)^{5/3}\ (\mathrm{mHz}\ \mathrm{fGW})^{11/3}\ \left(0.3\ \mathrm{mchirp}\ 2\times 10^{33}\right)^{5/3}}{5\ \left(3\times 10^{10}\right)^{5}}\right)^{-1}, \mathrm{fGW}\right]$$

Out[23]=

$$-\frac{4.78786\times 10^{6}\ \mathrm{fGW}}{\mathrm{mchirp}^{5/3}\ (\mathrm{fGW}\ \mathrm{mHz})^{11/3}}$$

In[24]:= $\text{Integrate}\left[\left(\frac{96\,\pi^{8/3}\,(6.67\times 10^{-8})^{5/3}\,(\text{mHz fGW})^{11/3}\,(0.3\,\text{mchirp}\,2\times 10^{33})^{5/3}}{5\,(3\times 10^{10})^{5}}\right)^{-1},\ \text{fGW}\right]$

$(\text{mHz}^{-11/3})\,(0.3\times 2\times 10^{33})^{-5/3}\,\frac{0.001}{3.15\times 10^{7}}\,\text{eclipseprob vobs}$

Out[24]=

$-\frac{9.80509\times 10^{-62}\,\text{fGW}}{\text{mchirp}^{5/3}\,\text{mHz}^{11/3}\,(\text{fGW mHz})^{11/3}}$

In[25]:= $\frac{41850.80065723604\grave{}}{1^{8/3}}-\frac{41850.80065723604\grave{}}{4^{8/3}}$

Out[25]=

40 812.8

In[26]:= $\text{Integrate}\left[\left(\frac{96\,\pi^{8/3}\,(6.67\times 10^{-8})^{5/3}\,(\text{fGW})^{11/3}\,(\text{mchirp})^{5/3}}{5\,(3\times 10^{10})^{5}}\right)^{-1},\ \text{fGW}\right]$

Out[26]=

$-\frac{2.04359\times 10^{61}}{\text{fGW}^{8/3}\,\text{mchirp}^{5/3}}$

In[27]:= $-\frac{2.043589184607502\grave{}*\hat{}61}{\text{fGW}^{8/3}\,\text{mchirp}^{5/3}}\,\left(\text{mHz}^{-8/3}\,(0.3\,\text{Msol})^{-5/3}\right)\,\frac{0.001}{3.15\times 10^{7}}\,\text{eclipseprob vobs}$

Out[27]=

$-\frac{1.32868\times 10^{49}}{\text{fGW}^{8/3}\,\text{mchirp}^{5/3}\,\text{mHz}^{8/3}\,\text{Msol}^{5/3}}$

In[28]:= $\frac{41.85080065723604\grave{}}{1.3^{8/3}\,\text{mchirp}^{5/3}}\,3-\frac{41.85080065723604\grave{}}{3.8^{8/3}\,\text{mchirp}^{5/3}}\,3$

Out[28]=

$\frac{58.7995}{\text{mchirp}^{5/3}}$

In[29]:= $\text{eclipseprob}\,\frac{\text{RMWlow}}{\pi\,10^{7}}\,\text{vobs Integrate}\left[\text{dfdt}[\text{fGW},\,0.3]^{-1},\ \{\text{fGW},\,\text{fGWlow},\,\text{fGWhigh}\}\right]$

Out[29]=

3.34089

In[30]:= $\text{Integrate}\left[\text{dfdt}[\text{fGW},\,0.3]^{-1},\ \text{fGW}\right]$

Out[30]=

$-\frac{4.78786\times 10^{6}}{\text{fGW}^{8/3}}$

In[31]:= $\text{eclipseprob}\,\frac{\text{RMWlow}}{\pi\,10^{7}}\,\text{vobs}\left(-\frac{4.787860757653706\grave{}*\hat{}6}{\text{fGW}^{8/3}}\right)$

Out[31]=

$-\frac{7.13368\times 10^{-8}}{\text{fGW}^{8/3}}$

In[32]:= $\text{eclipseprob} \dfrac{\text{RMWlow}}{\pi\, 10^7} \text{vobs dfdt[fGW, 0.3]}^{-1}$

Out[32]=

$\dfrac{1.90231 \times 10^{-7}}{\text{fGW}^{11/3}}$

In[33]:= $-\dfrac{7.133675884536345\text{`*}\wedge\text{-8}}{(0.001\ \text{fGW})^{8/3}}$

Out[33]=

$-\dfrac{7.13368}{\text{fGW}^{8/3}}$

In[34]:= $\text{eclipseprob} \dfrac{\text{RMWhigh}}{\pi\, 10^7} \text{vobs} \left(-\dfrac{4.787860757653706\text{`*}\wedge 6}{(0.001\ \text{fGW})^{8/3}}\right)$

Out[34]=

$-\dfrac{125.888}{\text{fGW}^{8/3}}$

In[35]:= $\text{eclipseprob} \dfrac{\text{RMWlow}}{\pi\, 10^7} \text{vobs} \left(-\dfrac{4.787860757653706\text{`*}\wedge 6}{(0.001\ \text{fGW})^{8/3}}\right)$

Out[35]=

$-\dfrac{7.13368}{\text{fGW}^{8/3}}$

In[36]:= eclipseprob vobs

Out[36]=

0.00275342

Visualizing the frequency distribution and computation of N_{MW} and N_{ZTF}

In[37]:=
```
Dist = ProbabilityDistribution[7.133675884536338 z^(-8/3), {z, 0.0001, Infinity}];
Dist3 = ProbabilityDistribution[125.88839796240595 z^(-8/3), {z, 0.0001, Infinity}];
pdfH[z_] := PDF[Dist, z];
pdfH3[z_] := PDF[Dist3, z];
```

In[41]:=
```
Nint = {{100 (eclipseprob vobs), "100"},
   {1000 (eclipseprob vobs), "1000"}, {10 000 (eclipseprob vobs), "10^4"},
   {100 000 (eclipseprob vobs), "10^5"}, {(eclipseprob vobs) 1 000 000, "10^6"}};
```

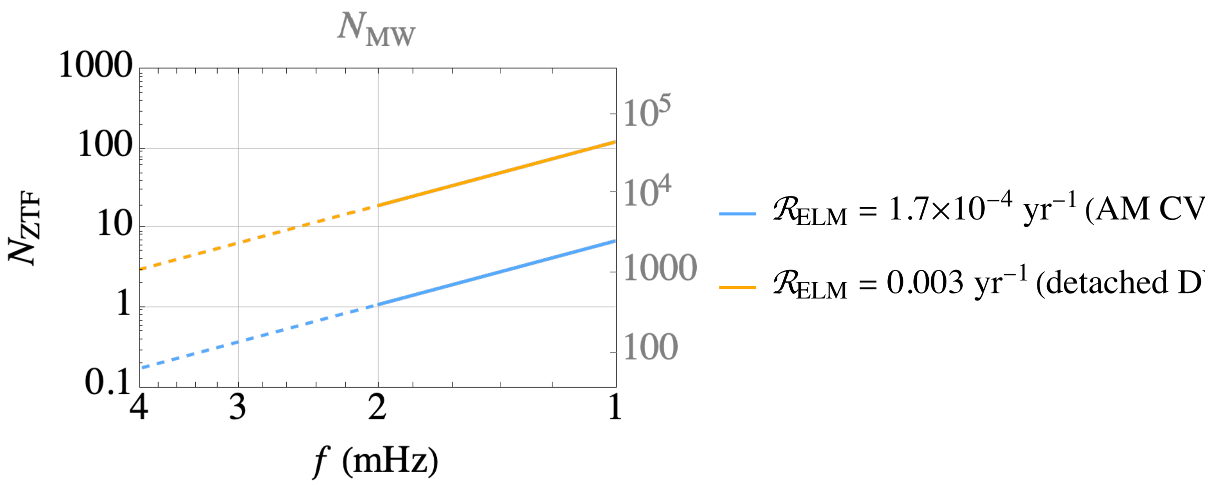
In[42]:=
```
p1 = LogLogPlot[{pdfH[z], pdfH3[z]}, {z, 5, 2}, AspectRatio → 2 / 3,
   Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
   FrameLabel → {Style["f (mHz)"], Style["N_ZTF"], Style["N_MW", Gray]},
   BaseStyle → {FontSize → 20}, GridLines → Automatic,
   PlotStyle → { {Blend[{Cyan, Blue, White}], Dashed},
     { Blend[{Orange, Orange, Yellow}], Dashed}},
   PlotRange → {{4, 1}, {0.1, 1000}}, FrameTicks →
    {{Automatic, Nint}, {Automatic, Automatic}}, FrameTicksStyle →
    {{Black, Gray}, {Black, Black}}, ScalingFunctions → {"Reverse", Identity}];
```

In[43]:=
```
p2 = LogLogPlot[{pdfH[z], pdfH3[z]}, {z, 2, 0}, AspectRatio → 1,
    Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
    FrameLabel → {Style["f (mHz)"], Style["NZTF"]},
    BaseStyle → {FontSize → 20}, GridLines → Automatic, PlotStyle →
     {{Blend[{Cyan, Blue, White}]}, {Blend[{Orange, Orange, Yellow}]}},
    PlotRange → {{4, 1}, {0.1, 1000}}, ScalingFunctions → {"Reverse", Identity},
    PlotLegends → {Style["ℛELM = 1.7×10^-4 yr^-1 (AM CVn binaries)", 18, Black]}];
```

In[44]:=
```
p3 = LogLogPlot[{pdfH3[z], pdfH[z]}, {z, 2, 0}, AspectRatio → 1,
    Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
    FrameLabel → {Style["f (mHz)"], Style["NZTF"]}, BaseStyle → {FontSize → 20},
    GridLines → Automatic, PlotStyle → {{Blend[{Orange, Orange, Yellow}]},
      {Blend[{Cyan, Blue, White}]}}, PlotRange → {{4, 1}, {0.1, 1000}},
    ScalingFunctions → {"Reverse", Identity}, PlotLegends →
     {Style["ℛELM = 0.003 yr^-1 (detached DWDBs, RCrB stars)", 18, Black]}];
```

In[45]:=
```
Show[p1, p2, p3]
```

Out[45]=



References

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Brown, W. R., Kilic, M., Hermes, J. J., et al. 2011, ApJL, 737, L23

Brown, W. R., Kilic, M., Kenyon, S. J., & Gianninas, A. 2016, ApJ, 824, 46

Kilic, M., Brown, W. R., Heinke, C. O., et al. 2016, MNRAS, 460, 4176

Kupfer & Korol, Littenberg, T. B., Shah, S., et al. 2024, ApJ, 963, 100,