Tidal heating model for detached double white dwarf binaries

In this notebook, we formulate the tidal heating model presented in McNeill and Hirai 2025 (submitted).
Figure 4, and Figure 5 are produced here.

physical constants

In[1]:=
$$\text{afun} = \frac{\text{GG}^{1/3}\,((\text{m1}+\text{m2}))^{1/3}}{(\text{f})^{2/3}\,\pi^{2/3}};$$

In[2]:=
$$\text{Rsol} = 6.995\times 10^{10};$$
$$\text{Msol} = 2\times 10^{33};$$
$$\text{Mchirpf[m11\_, m22\_]} = \frac{(\text{m11 m22})^{3/5}}{(\text{m11}+\text{m22})^{1/5}};$$
$$\text{G} = 6.67\times 10^{-8};$$
$$\text{c} = 3\times 10^{10};$$
$$\text{Msol} = 2\times 10^{33};$$
$$\text{mHz} = 0.001;$$
$$\text{kK4} = 10^{4};$$
$$\text{rg2} = 0.1;$$
$$\sigma = 5.67\times 10^{-5};$$

Figure 4: Future temperature and frequency evolution for three binaries

In[12]:=
```
Clear[f]
```

Relations

In[13]:=
$$\text{Rscale[m1a\_, T1a\_]} := 10^{-0.02792426461145596\grave{}+0.7641778013995925\grave{}\,\sqrt{\text{T1a}}}\,\text{m1a}^{0.14797691065884058\grave{}-0.940895504247887 3\grave{}\,\sqrt{\text{T1a}}}$$

In[14]:=

```
dRscaledt[m1a_, T1a_] :=
```

$$\frac{1}{\sqrt{T1a}}\ 0.8797922069498331\text{`} \times 10^{-0.02792426461145596\text{`}+0.7641778013995925\text{`}\ \sqrt{T1a}}\ m1a^{0.14797691065884058\text{`}-0.9408955042478873\text{`}\ \sqrt{T1a}} - \frac{1}{\sqrt{T1a}}\ 0.47044775212394363\text{`} \times 10^{-0.02792426461145596\text{`}+0.7641778013995925\text{`}\ \sqrt{T1a}}\ m1a^{0.14797691065884058\text{`}-0.9408955042478873\text{`}\ \sqrt{T1a}}\ Log[m1a]$$

In[15]:=

$$Mchirpf[m11\_, m22\_] = \frac{(m11\ m22)^{3/5}}{(m11 + m22)^{1/5}};$$

J2029

In[16]:=

```
mp = 3.2;
ms = 3.0;
fbin = 1.6;
Tp = 1.825;
```

In[20]:=

```
kQratioa = 8 × 10^-12;
```

Here we determine prefactors in Equations (3) and (17) for f , Equation (18) for Ω_1 and Equation (21) for T_1 of McNeill and Hirai 2025 to simplify later numerical integrations.

In[21]:=

$$preGW = D\left[\frac{fGW}{mHz}, fGW\right] \frac{96\ G^{5/3}\ \pi^{8/3}\ (Msol / 10)^{5/3}}{5\ c^5}\ (mHz)^{11/3}\ D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[21]=

```
0.0394863
```

In[22]:=

$$preTDa = D\left[\frac{fGW}{mHz}, fGW\right] \frac{18\ (mHz)^{13/3}\ (Msol / 10)\ \pi^{13/3}\ (Rsol / 100)^5\ (mHz)}{G^{5/3}\ (Msol / 10)\ ((Msol / 10))^{5/3}}\ kQratioa\ D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

$$pre\Omega a = D\left[\frac{\Omega}{mHz}, \Omega\right] \left(\frac{3\ mHz^3\ (Msol / 10)^2\ \pi^3\ (Rsol / 100)^3\ (mHz)}{G\ (Msol / 10)\ rg2\ ((Msol / 10))^2}\right) kQratioa\ D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[22]=

```
0.00014425
```

Out[23]=

```
0.0600658
```

For the prefactors we must solve for the initial $\Omega_1(0)$ according to Equation 23. This finally determines the $\mathcal{F}$ listed in Table 4 (called “kQratiob” here).

In[24]:=
$$\text{d}\Omega\text{df} = \left(\left(\text{preGW}\ (\text{mp ms})\ \left(\frac{1}{(\text{mp} + \text{ms})^{1/3}}\right)\ \text{fbin}^{11/3}\right) + \frac{\text{ms}}{\text{mp}\ (\text{mp}+\text{ms})^{5/3}}\ \text{preTDa fbin}^{13/3}\ (\text{Rscale}[\text{mp},\ \text{Tp}])^{5}\ (\text{fbin}/2-\Omega)\ \frac{1}{\left(1-2\ \frac{\Omega}{\text{fbin}}\right)}\right)^{-1}$$
$$\left(\text{pre}\Omega\text{a}\ \frac{1}{\left(1-2\ \frac{\Omega}{\text{fbin}}\right)}\ \frac{\text{ms}^2}{\text{mp}\ (\text{mp}+\text{ms})^2}\ \text{fbin}^3\ (\text{Rscale}[\text{mp},\ \text{Tp}])^3\ (\text{fbin}/2-\Omega)\right);$$

$$\text{d2}\Omega\text{df2}[\text{f\_}] = \text{D}\left[\left(\left(\text{preGW}\ (\text{mp ms})\ \left(\frac{1}{(\text{mp} + \text{ms})^{1/3}}\right)\ \text{f}^{11/3}\right) + \frac{\text{ms}}{\text{mp}\ (\text{mp}+\text{ms})^{5/3}}\ \text{preTDa}\ \frac{1}{\left(1-2\ \frac{\Omega}{\text{f}}\right)}\ \text{f}^{13/3}\ (\text{Rscale}[\text{mp},\ \text{Tp}])^{5}\ (\text{f}/2-\Omega)\right)^{-1}\right.$$
$$\left.\left(\text{pre}\Omega\text{a}\ \frac{1}{\left(1-2\ \frac{\Omega}{\text{f}}\right)}\ \frac{\text{ms}^2}{\text{mp}\ (\text{mp}+\text{ms})^2}\ \text{f}^3\ (\text{Rscale}[\text{mp},\ \text{Tp}])^3\ (\text{f}/2-\Omega)\right),\ \text{f}\right];$$

$$\Omega\text{start} = \Omega\ /.\ \text{FindRoot}\left[\frac{2\ \Omega}{\text{fbin}^3} - \frac{2\ \text{d}\Omega\text{df}}{\text{fbin}^2} + \frac{\text{d2}\Omega\text{df2}[\text{fbin}]}{\text{fbin}} == 0,\ \{\Omega,\ 0.9\}\right][[1]];$$

$$\text{factorsyn} = 2\ \Omega\text{start}\ /\ \text{fbin}$$

$$\text{kQratiob} = \frac{1}{(1-\text{factorsyn})}\ \text{kQratioa}$$

Out[27]=

0.425069

Out[28]=

1.39147×10^{-11}

In[29]:=
$$\text{preTDb} = \text{D}\left[\frac{\text{fGW}}{\text{mHz}},\ \text{fGW}\right]\ \frac{18\ (\text{mHz})^{13/3}\ (\text{Msol}/10)\ \pi^{13/3}\ (\text{Rsol}/100)^5\ (\text{mHz})}{\text{G}^{5/3}\ (\text{Msol}/10)\ ((\text{Msol}/10))^{5/3}}\ \text{kQratiob D}\left[\frac{\text{t}}{31.46 \times 10^{13}},\ \text{t}\right]^{-1}$$

Out[29]=

0.0002509

In[30]:=
$$\text{preTb} = \text{D}\left[\frac{\text{T}}{\text{kK4}},\ \text{T}\right]\ \left(\frac{135\ \text{mHz}^{19/3}\ (\text{Msol}/10)^3\ \pi^{25/3}\ (\text{Rsol}/100)^9}{\text{G}^{8/3}\ (\text{Msol}/10)\ \sigma\ ((\text{Msol}/10))^{11/3}\ \text{kK4}^3\ ((\text{Rsol}/100))}\ (\text{mHz})^3\right)\ \text{kQratiob}^2\ \text{D}\left[\frac{\text{t}}{31.46 \times 10^{13}},\ \text{t}\right]^{-1}$$

Out[30]=

0.0000230828

In[31]:=
$$\text{preJb} = \text{D}\left[\frac{\text{J}}{(\text{Msol}/10)\ (\text{Rsol}/100)^2\ \text{mHz}},\ \text{J}\right]\ \left(2\ \pi\ \frac{3\ \text{mHz}^3\ (\text{Msol}/10)^2\ \pi^3\ (\text{Rsol}/100)^5\ (\text{mHz})}{\text{G}\ ((\text{Msol}/10))^2}\right)\ \text{kQratiob D}\left[\frac{\text{t}}{31.46 \times 10^{13}},\ \text{t}\right]^{-1}$$

Out[31]=

0.0656433

In[32]:=
```
preΩb =
  D[Ω/mHz, Ω] ((3 mHz^3 (Msol / 10)^2 π^3 (Rsol / 100)^3 (mHz))/(G (Msol / 10) rg2 ((Msol / 10))^2)) kQratiob D[t/(31.46 × 10^13), t]^-1
```

Out[32]=
```
0.104475
```

In[33]:=
```
soltestgenb = NDSolve[{f'[t] == (preGW (mp ms) (1/(mp + ms)^(1/3)) f[t]^(11/3)) +
     ms/(mp (mp + ms)^(5/3)) preTDb f[t]^(13/3) (Rscale[mp, T[t]])^5 (f[t] / 2 - Ω[t]),
    T'[t] ==
     preTb (((ms)^3 f[t]^(19/3) Rscale[mp, T[t]]^9 (f[t] / 2 - Ω[t])^2 (f[t] / 2 - 3/5 Ω[t]))/
        ((mp (mp + ms)^(11/3))
          (T[t]^3 (2 Rscale[mp, T[t]] + T[t] × dRscaledt[mp, T[t]])))),
    Ω'[t] == preΩb ms^2/(mp (mp + ms)^2) f[t]^3 (Rscale[mp, T[t]])^3 (f[t] / 2 - Ω[t]),
    f[0] == fbin, T[0] == Tp, Ω[0] == factorsyn fbin / 2 },
   {f, T, Ω}, {t, 0, 0.51}, Method → "StiffnessSwitching"]
```

Power: Infinite expression $\frac{1}{0.}$ encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

NDSolve: The function value $\{2.00672\times10^{20}$, Indeterminate, 0.$\}$ is not a list of numbers with dimensions {3} at {t, f[t], T[t], Ω[t]} = $\{0.51, 529630., 3.77064\times10^{7}, 3912.11\}$.

Out[33]=
```
{{f → InterpolatingFunction[ Domain: {{0., 0.508}} Output: scalar ],
  T → InterpolatingFunction[ Domain: {{0., 0.508}} Output: scalar ],
  Ω → InterpolatingFunction[ Domain: {{0., 0.508}} Output: scalar ]}}
```

In[34]:=
```
endt = 0.51;
stepsize = 0.0001;
```

```
In[36]:= fvals1b =
   Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp}+\text{ms})\ \text{Msol}/10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\,\pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLvalx} = 3^{-4/3} \times 2\ \text{mp}^{1/3} \Big/ (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\ (\text{mp}/\text{ms})^{2/3}}{0.6\ (\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction: Input value {0.5084} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.5085} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.5086} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {0.5084} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.5085} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.5086} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {0.5084} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.5085} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.5086} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

Out[43]=

```
RRLval
```

Out[45]=

```
0.38452
```

Here we get information about when Roche contact occurs, the temperature there, the time taken since the initial condition, and the fractional temperature increase.

In[52]:=

```
RLposia = x /. Solve[
      Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x]⟦1⟧;
Abs[Ra1cutb / RRLval - 1]⟦RLposia⟧;
fGWend1 = fvals2b⟦RLposia⟧
Tend1 = Tvals2b⟦RLposia⟧ kK4
timevals2b⟦RLposia⟧
(Tvals2b⟦RLposia⟧ - Tp) / Tp
```

Out[54]=

```
7.24804
```

Out[55]=

```
21727.6
```

Out[56]=

```
0.5045
```

Out[57]=

```
0.190555
```

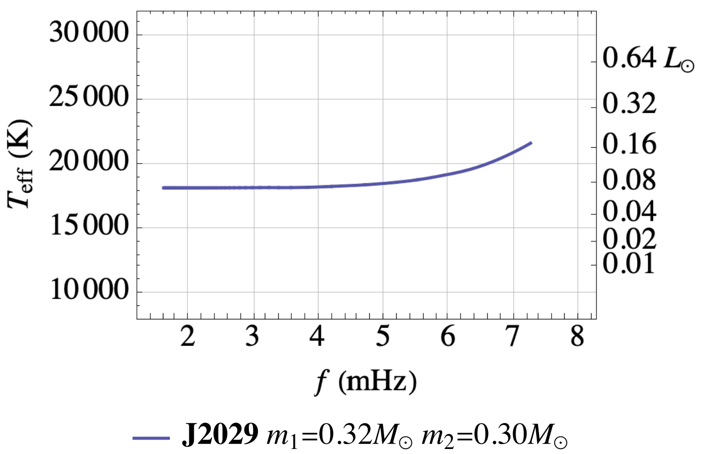
In[58]:=

```
x4 = {{12012, "0.01"}, {13897, "0.02"}, {16049, "0.04"},
    {18502, "0.08"}, {21291, "0.16"}, {24454, "0.32"}, {28031, "0.64 L⊙"}};
```

In[59]:=

```
plota21lin =
 ListPlot[Transpose[{fvals2b⟦ ;; RLposia⟧, 10^4 Tvals2b⟦ ;; RLposia⟧}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Gray, Gray, Blue}],
  PlotRange → {1000 {0.0012, 0.0083}, {8000, 32000}}, AspectRatio → 1 / 1.5,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f (mHz)"], Style["Teff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic,
  PlotLegends → {Style["J2029 m1=0.32M⊙ m2=0.30M⊙", 16]},
  FrameTicks → {{Automatic, x4}, {Automatic, Automatic}}]
```

Out[59]=



Here we obtain the fractional radius increase

In[60]:=
```
Rscale[mp, Tvals2b⟦RLposia⟧]
Rscale[mp, Tvals2b⟦1⟧]
(Rscale[mp, Tvals2b⟦RLposia⟧] - Rscale[mp, Tvals2b⟦1⟧]) / Rscale[mp, Tvals2b⟦1⟧]
```

Out[60]=
```
2.96928
```

Out[61]=
```
2.73582
```

Out[62]=
```
0.0853326
```

Here are the three frequencies that appear in Table 3

In[63]:=

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RRLval}^{-1}\ \text{Rscale[mp, 0.4] Rsol } 0.01\right)^{3/2}},\ \text{fRL}\right][[1]]$$

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RRLval}^{-1}\ \text{Rscale[mp, Tp] Rsol } 0.01\right)^{3/2}},\ \text{fRL}\right][[1]]$$

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RRLval}^{-1}\ \text{Rscale[mp, Tvals2b[[RLposia]]] Rsol } 0.01\right)^{3/2}},\ \text{fRL}\right][[1]]$$

Out[63]=
```
0.0168858
```

Out[64]=
```
0.00824498
```

Out[65]=
```
0.00729198
```

J2243

In[66]:=
```
mp = 3.23;
ms = 3.35;
fbin = 3.8;
Tp = 2.63;
```

In[70]:=

$$\text{preGW} = D\left[\frac{\text{fGW}}{\text{mHz}},\ \text{fGW}\right]\ \frac{96\ G^{5/3}\ \pi^{8/3}\ (\text{Msol}/10)^{5/3}}{5\ c^{5}}\ (\text{mHz})^{11/3}\ D\left[\frac{t}{31.46\times 10^{13}},\ t\right]^{-1}$$

Out[70]=
```
0.0394863
```

In[71]:=

$$\text{preTDa} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{18\ (\text{mHz})^{13/3}\ (\text{Msol}/10)\ \pi^{13/3}\ (\text{Rsol}/100)^5\ (\text{mHz})}{G^{5/3}\ (\text{Msol}/10)\ ((\text{Msol}/10))^{5/3}}\ \text{kQratioa}\ D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

$$\text{pre}\Omega\text{a} = D\left[\frac{\Omega}{\text{mHz}}, \Omega\right] \left(\frac{3\ \text{mHz}^3\ (\text{Msol}/10)^2\ \pi^3\ (\text{Rsol}/100)^3\ (\text{mHz})}{G\ (\text{Msol}/10)\ \text{rg2}\ ((\text{Msol}/10))^2}\right) \text{kQratioa}\ D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[71]=

0.00014425

Out[72]=

0.0600658

In[73]:=

In[74]:=

$$\text{d}\Omega\text{df} = \left(\left(\text{preGW}\ (\text{mp ms})\ \left(\frac{1}{(\text{mp} + \text{ms})^{1/3}}\right) \text{fbin}^{11/3}\right) + \frac{\text{ms}}{\text{mp}\ (\text{mp} + \text{ms})^{5/3}}\ \text{preTDa}\ \text{fbin}^{13/3}\ (\text{Rscale}[\text{mp}, \text{Tp}])^5\ (\text{fbin}/2 - \Omega)\ \frac{1}{\left(1 - 2\ \frac{\Omega}{\text{fbin}}\right)}\right)^{-1}$$
$$\left(\text{pre}\Omega\text{a}\ \frac{1}{\left(1 - 2\ \frac{\Omega}{\text{fbin}}\right)}\ \frac{\text{ms}^2}{\text{mp}\ (\text{mp} + \text{ms})^2}\ \text{fbin}^3\ (\text{Rscale}[\text{mp}, \text{Tp}])^3\ (\text{fbin}/2 - \Omega)\right);$$

$$\text{d2}\Omega\text{df2}[\text{f\_}] = D\left[\left(\left(\text{preGW}\ (\text{mp ms})\ \left(\frac{1}{(\text{mp} + \text{ms})^{1/3}}\right) f^{11/3}\right) + \frac{\text{ms}}{\text{mp}\ (\text{mp} + \text{ms})^{5/3}}\ \text{preTDa}\ \frac{1}{\left(1 - 2\ \frac{\Omega}{f}\right)}\ f^{13/3}\ (\text{Rscale}[\text{mp}, \text{Tp}])^5\ (f/2 - \Omega)\right)^{-1}\right.$$
$$\left.\left(\text{pre}\Omega\text{a}\ \frac{1}{\left(1 - 2\ \frac{\Omega}{f}\right)}\ \frac{\text{ms}^2}{\text{mp}\ (\text{mp} + \text{ms})^2}\ f^3\ (\text{Rscale}[\text{mp}, \text{Tp}])^3\ (f/2 - \Omega)\right), f\right];$$

$$\Omega\text{start} = \Omega\ /.\ \text{FindRoot}\left[\frac{2\ \Omega}{\text{fbin}^3} - \frac{2\ \text{d}\Omega\text{df}}{\text{fbin}^2} + \frac{\text{d2}\Omega\text{df2}[\text{fbin}]}{\text{fbin}} == 0, \{\Omega, 0.9\}\right][[1]]$$

$$\text{factorsyn} = 2\ \Omega\text{start}/\text{fbin}$$

$$\text{kQratiob} = \frac{1}{(1 - \text{factorsyn})}\ 8\times 10^{-12};$$

Out[76]=

1.76315

Out[77]=

0.927973

In[79]:=

$$\text{preTDb} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{18\ (\text{mHz})^{13/3}\ (\text{Msol}/10)\ \pi^{13/3}\ (\text{Rsol}/100)^5\ (\text{mHz})}{G^{5/3}\ (\text{Msol}/10)\ ((\text{Msol}/10))^{5/3}}\ \text{kQratiob}\ D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[79]=

0.00200272

In[80]:=

$$\text{preTb} = D\left[\frac{T}{\text{kK4}}, T\right] \left(\frac{135\,\text{mHz}^{19/3}\ (\text{Msol}/10)^3\,\pi^{25/3}\ (\text{Rsol}/100)^9}{G^{8/3}\ (\text{Msol}/10)\ \sigma\ ((\text{Msol}/10))^{11/3}\ \text{kK4}^3\ ((\text{Rsol}/100))}\ (\text{mHz})^3\right) \text{kQratiob}^2\, D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[80]=

0.00147072

In[81]:=

$$\text{preJb} = D\left[\frac{J}{(\text{Msol}/10)\ (\text{Rsol}/100)^2\ \text{mHz}}, J\right] \left(2\,\pi\,\frac{3\,\text{mHz}^3\ (\text{Msol}/10)^2\,\pi^3\ (\text{Rsol}/100)^5\ (\text{mHz})}{G\ ((\text{Msol}/10))^2}\right) \text{kQratiob}\, D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[81]=

0.523975

In[82]:=

$$\text{pre}\Omega\text{b} = D\left[\frac{\Omega}{\text{mHz}}, \Omega\right] \left(\frac{3\,\text{mHz}^3\ (\text{Msol}/10)^2\,\pi^3\ (\text{Rsol}/100)^3\ (\text{mHz})}{G\ (\text{Msol}/10)\ \text{rg2}\ ((\text{Msol}/10))^2}\right) \text{kQratiob}\, D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[82]=

0.833933

In[83]:=

```
soltestgenb = NDSolve[{f'[t] == (preGW (mp ms) (1/(mp + ms)^(1/3)) f[t]^(11/3)) +
      ms/(mp (mp + ms)^(5/3)) preTDb f[t]^(13/3) (Rscale[mp, T[t]])^5 (f[t] / 2 - Ω[t]),
    T'[t] ==
     preTb (((ms)^3 f[t]^(19/3) Rscale[mp, T[t]]^9 (f[t] / 2 - Ω[t])^2 (f[t] / 2 - 3/5 Ω[t])) /
         ((mp (mp + ms)^(11/3))
           (T[t]^3 (2 Rscale[mp, T[t]] + T[t] × dRscaledt[mp, T[t]])))),
    Ω'[t] == preΩb ms^2/(mp (mp + ms)^2) f[t]^3 (Rscale[mp, T[t]])^3 (f[t] / 2 - Ω[t]),
    f[0] == fbin, T[0] == Tp, Ω[0] == 0.9 fbin / 2 },
   {f, T, Ω}, {t, 0, 0.042}, Method → "StiffnessSwitching"]
```

Power: Infinite expression $\frac{1}{0.}$ encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

NDSolve: The function value $\{3.08435\times10^{67}$, Indeterminate, 0.$\}$ is not a list of numbers with dimensions {3} at {t, f[t], T[t], Ω[t]} = $\{0.0412615, 3.81276\times10^{18}, 6.15852\times10^{26}, 5.61058\times10^{11}\}$.

Out[83]=

```
{{f → InterpolatingFunction[ Domain: {{0., 0.0408}}  Output: scalar ],
  T → InterpolatingFunction[ Domain: {{0., 0.0408}}  Output: scalar ],
  Ω → InterpolatingFunction[ Domain: {{0., 0.0408}}  Output: scalar ]}}
```

In[84]:=

```
endt = 0.042;
stepsize = 0.00001;
```

In[86]:=
```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp}+\text{ms})\,\text{Msol}/10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\,\pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLvalx} = 3^{-4/3} \times 2\,\text{mp}^{1/3} / (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\,(\text{mp}/\text{ms})^{2/3}}{0.6\,(\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction: Input value {0.0408} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.04081} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.04082} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {0.0408} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.04081} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.04082} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {0.0408} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.04081} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.04082} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

Out[93]=

```
0.38452
```

Out[95]=

```
0.375767
```

In[102]:=

```
RLposia = x /. Solve[
      Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]];
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fGWend2 = fvals2b[[RLposia]]
Tend2 = Tvals2b[[RLposia]] kK4
Tend2 = Tvals2b[[RLposia]] kK4
timevals2b[[RLposia]]
(Tvals2b[[RLposia]] - Tp) / Tp
```

Out[104]=

```
6.20219
```

Out[105]=

```
27 261.6
```

Out[106]=

```
27 261.6
```

Out[107]=

```
0.03214
```

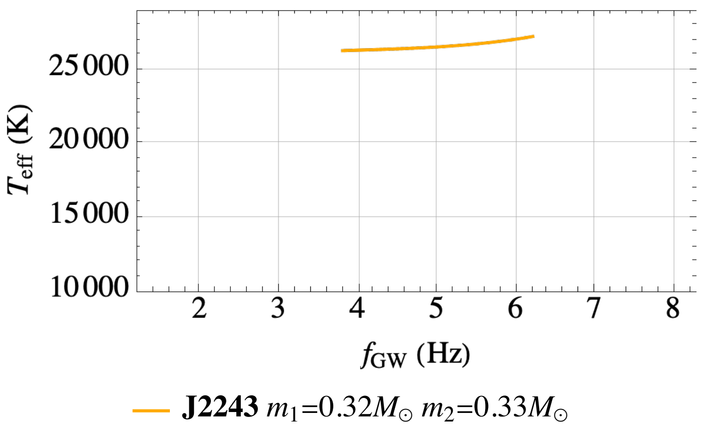
Out[108]=

```
0.0365644
```

In[109]:=

```
plota23lin =
 ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Orange, Orange, Yellow}],
  PlotRange → {1000 {0.0012, 0.0083}, {10 000, 29 000}},
  AspectRatio → 1 / 2, Frame → True, LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["f_GW (Hz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic,
  PlotLegends → {Style["J2243 m₁=0.32M⊙ m₂=0.33M⊙", 16]}]
```

Out[109]=



In[110]:=

```
Rscale[mp, Tvals2b[[RLposia]]]
Rscale[mp, Tvals2b[[1]]]
(Rscale[mp, Tvals2b[[RLposia]]] - Rscale[mp, Tvals2b[[1]]]) / Rscale[mp, Tvals2b[[1]]]
```

Out[110]=

3.29693

Out[111]=

3.23396

Out[112]=

0.0194739

In[113]:=

In[114]:=

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RLval}^{-1}\ \text{Rscale}[\text{mp}, 0.4]\ \text{Rsol}\ 0.01\right)^{3/2}}, \text{fRL}\right][[1]]$$

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RLval}^{-1}\ \text{Rscale}[\text{mp}, \text{Tp}]\ \text{Rsol}\ 0.01\right)^{3/2}}, \text{fRL}\right][[1]]$$

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RLval}^{-1}\ \text{Rscale}[\text{mp}, \text{Tvals2b}[[\text{RLposia}]]]\ \text{Rsol}\ 0.01\right)^{3/2}}, \text{fRL}\right][[1]]$$

Out[114]=

0.0169105

Out[115]=

0.00638463

Out[116]=

0.00620257

J0538

In[117]:=

```
mp = 3.2;
ms = 4.5;
fbin = 2.3;
Tp = 1.28;
```

In[121]:=

$$\text{preGW} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{96\ G^{5/3}\ \pi^{8/3}\ (\text{Msol}/10)^{5/3}}{5\ c^5}\ (\text{mHz})^{11/3}\ D\left[\frac{t}{31.46\times10^{13}}, t\right]^{-1}$$

Out[121]=

0.0394863

In[122]:=

$$\text{preTDa} = \text{D}\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right]\frac{18\,(\text{mHz})^{13/3}\,(\text{Msol}/10)\,\pi^{13/3}\,(\text{Rsol}/100)^5\,(\text{mHz})}{\text{G}^{5/3}\,(\text{Msol}/10)\,((\text{Msol}/10))^{5/3}}\,\text{kQratioa}\,\text{D}\left[\frac{\text{t}}{31.46\times 10^{13}}, \text{t}\right]^{-1}$$

$$\text{pre}\Omega\text{a} = \text{D}\left[\frac{\Omega}{\text{mHz}}, \Omega\right]\left(\frac{3\,\text{mHz}^3\,(\text{Msol}/10)^2\,\pi^3\,(\text{Rsol}/100)^3\,(\text{mHz})}{\text{G}\,(\text{Msol}/10)\,\text{rg2}\,((\text{Msol}/10))^2}\right)\text{kQratioa}\,\text{D}\left[\frac{\text{t}}{31.46\times 10^{13}}, \text{t}\right]^{-1}$$

Out[122]=

0.00014425

Out[123]=

0.0600658

In[124]:=

In[125]:=

$$\text{d}\Omega\text{df} = \left(\left(\text{preGW}\,(\text{mp}\,\text{ms})\left(\frac{1}{(\text{mp}+\text{ms})^{1/3}}\right)\text{fbin}^{11/3}\right) + \frac{\text{ms}}{\text{mp}\,(\text{mp}+\text{ms})^{5/3}}\,\text{preTDa}\,\text{fbin}^{13/3}\,(\text{Rscale}[\text{mp}, \text{Tp}])^5\,(\text{fbin}/2-\Omega)\,\frac{1}{\left(1-2\,\frac{\Omega}{\text{fbin}}\right)}\right)^{-1}\left(\text{pre}\Omega\text{a}\,\frac{1}{\left(1-2\,\frac{\Omega}{\text{fbin}}\right)}\,\frac{\text{ms}^2}{\text{mp}\,(\text{mp}+\text{ms})^2}\,\text{fbin}^3\,(\text{Rscale}[\text{mp}, \text{Tp}])^3\,(\text{fbin}/2-\Omega)\right);$$

$$\text{d2}\Omega\text{df2}[\text{f\_}] = \text{D}\left[\left(\left(\text{preGW}\,(\text{mp}\,\text{ms})\left(\frac{1}{(\text{mp}+\text{ms})^{1/3}}\right)\text{f}^{11/3}\right) + \frac{\text{ms}}{\text{mp}\,(\text{mp}+\text{ms})^{5/3}}\,\text{preTDa}\,\frac{1}{\left(1-2\,\frac{\Omega}{\text{f}}\right)}\,\text{f}^{13/3}\,(\text{Rscale}[\text{mp}, \text{Tp}])^5\,(\text{f}/2-\Omega)\right)^{-1}\left(\text{pre}\Omega\text{a}\,\frac{1}{\left(1-2\,\frac{\Omega}{\text{f}}\right)}\,\frac{\text{ms}^2}{\text{mp}\,(\text{mp}+\text{ms})^2}\,\text{f}^3\,(\text{Rscale}[\text{mp}, \text{Tp}])^3\,(\text{f}/2-\Omega)\right), \text{f}\right];$$

$$\Omega\text{start} = \Omega\ /.\ \text{FindRoot}\left[\frac{2\,\Omega}{\text{fbin}^3} - \frac{2\,\text{d}\Omega\text{df}}{\text{fbin}^2} + \frac{\text{d2}\Omega\text{df2}[\text{fbin}]}{\text{fbin}} == 0, \{\Omega, 0.9\}\right][[1]]$$

$$\text{factorsyn} = 2\,\Omega\text{start}/\text{fbin}$$

$$\text{kQratiob} = \frac{1}{(1-\text{factorsyn})}\;8\times 10^{-12};$$

Out[127]=

0.372118

Out[128]=

0.323581

In[130]:=

$$\text{preTDb} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{18\ (\text{mHz})^{13/3}\ (\text{Msol}/10)\ \pi^{13/3}\ (\text{Rsol}/100)^5\ (\text{mHz})}{G^{5/3}\ (\text{Msol}/10)\ ((\text{Msol}/10))^{5/3}}\ \text{kQratiob}\ D\left[\frac{t}{31.46\times10^{13}}, t\right]^{-1}$$

Out[130]=

0.000213256

In[131]:=

$$\text{preTb} = D\left[\frac{T}{\text{kK4}}, T\right] \left(\frac{135\ \text{mHz}^{19/3}\ (\text{Msol}/10)^3\ \pi^{25/3}\ (\text{Rsol}/100)^9}{G^{8/3}\ (\text{Msol}/10)\ \sigma\ ((\text{Msol}/10))^{11/3}\ \text{kK4}^3\ ((\text{Rsol}/100))}\ (\text{mHz})^3\right) \text{kQratiob}^2\ D\left[\frac{t}{31.46\times10^{13}}, t\right]^{-1}$$

Out[131]=

0.0000166759

In[132]:=

$$\text{preJb} = D\left[\frac{J}{(\text{Msol}/10)\ (\text{Rsol}/100)^2\ \text{mHz}}, J\right] \left(2\pi\ \frac{3\ \text{mHz}^3\ (\text{Msol}/10)^2\ \pi^3\ (\text{Rsol}/100)^5\ (\text{mHz})}{G\ ((\text{Msol}/10))^2}\right) \text{kQratiob}\ D\left[\frac{t}{31.46\times10^{13}}, t\right]^{-1}$$

Out[132]=

0.0557945

In[133]:=

$$\text{pre}\Omega\text{b} = D\left[\frac{\Omega}{\text{mHz}}, \Omega\right] \left(\frac{3\ \text{mHz}^3\ (\text{Msol}/10)^2\ \pi^3\ (\text{Rsol}/100)^3\ (\text{mHz})}{G\ (\text{Msol}/10)\ \text{rg2}\ ((\text{Msol}/10))^2}\right) \text{kQratiob}\ D\left[\frac{t}{31.46\times10^{13}}, t\right]^{-1}$$

Out[133]=

0.0887996

In[134]:=

```
soltestgenb = NDSolve[{f'[t] == (preGW (mp ms) (1/(mp + ms)^(1/3)) f[t]^(11/3) +
     ms/(mp (mp + ms)^(5/3)) preTDb f[t]^(13/3) (Rscale[mp, T[t]])^5 (f[t] / 2 - Ω[t]),
   T'[t] ==
    preTb (((ms)^3 f[t]^(19/3) Rscale[mp, T[t]]^9 (f[t] / 2 - Ω[t])^2 (f[t] / 2 - 3/5 Ω[t])) /
       ((mp (mp + ms)^(11/3))
        (T[t]^3 (2 Rscale[mp, T[t]] + T[t] × dRscaledt[mp, T[t]])))),
   Ω'[t] == preΩb ms^2/(mp (mp + ms)^2) f[t]^3 (Rscale[mp, T[t]])^3 (f[t] / 2 - Ω[t]),
   f[0] == fbin, T[0] == Tp, Ω[0] == factorsyn fbin / 2 },
  {f, T, Ω}, {t, 0, 0.15}, Method → "StiffnessSwitching"]
```

Power: Infinite expression $\frac{1}{0.}$ encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

NDSolve: The function value $\{1.1007\times10^{27}$, Indeterminate, 0.$\}$ is not a list of numbers with dimensions {3} at {t, f[t], T[t], Ω[t]} = $\{0.137343, 3.33013\times10^{7}, 5.4506\times10^{11}, 858.759\}$.

Out[134]=

```
{{f → InterpolatingFunction[ Domain: {{0., 0.137}} Output: scalar ],
  T → InterpolatingFunction[ Domain: {{0., 0.137}} Output: scalar ],
  Ω → InterpolatingFunction[ Domain: {{0., 0.137}} Output: scalar ]}}
```

In[135]:=

```
endt = 0.15;
stepsize = 0.0001;
```

In[137]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\ ((\text{mp} + \text{ms})\ \text{Msol} / 10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\ \pi^{2/3}} \Big/ (\text{Rsol} / 100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLvalx} = 3^{-4/3} \times 2\ \text{mp}^{1/3} \Big/ (\text{mp} + \text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\ (\text{mp} / \text{ms})^{2/3}}{0.6\ (\text{mp} / \text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp} / \text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction: Input value {0.1374} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.1375} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.1376} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {0.1374} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.1375} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.1376} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {0.1374} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.1375} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {0.1376} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

Out[144]=

```
0.375767
```

Out[146]=

```
0.349817
```

In[153]:=

```
RLposia = x /. Solve[
     Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]];
Abs[Ra1cutb / RRLval - 1][[RLposia]]
fGWend3 = fvals2b[[RLposia]]
Tend3 = Tvals2b[[RLposia]] kK4
timevals2b[[RLposia]]
(Tvals2b[[RLposia]] - Tp) / Tp
```

Out[154]=

```
0.0105195
```

Out[155]=

```
7.58384
```

Out[156]=

```
19187.2
```

Out[157]=

```
0.1345
```

Out[158]=

```
0.499002
```

In[159]:=

```
Tvals2b // Length
```

Out[159]=

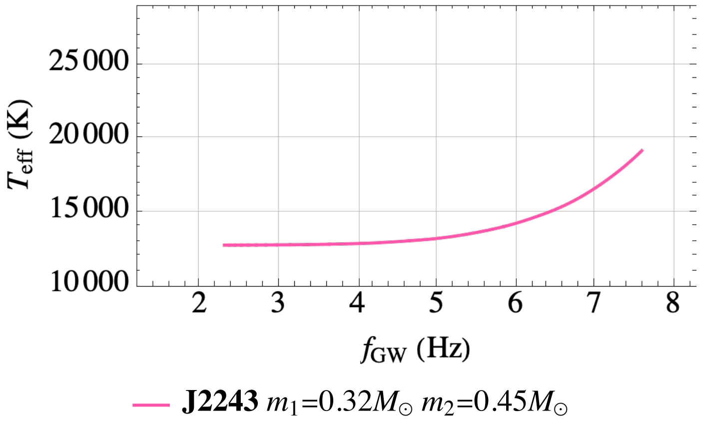
```
1473
```

In[160]:=

In[161]:=

```
plota24lin =
 ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Magenta, Red, White}],
  PlotRange → {1000 {0.0012, 0.0083}, {10 000, 29 000}},
  AspectRatio → 1 / 2, Frame → True, LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["f_GW (Hz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic,
  PlotLegends → {Style["J2243 m_1=0.32M_⊙ m_2=0.45M_⊙", 16]}]
```

Out[161]=



In[162]:=

```
Rscale[mp, Tvals1b[[RLposia]]]
```

Out[162]=

2.79887

In[163]:=

```
Rscale[mp, Tvals1b[[1]]]
```

Out[163]=

2.36408

In[164]:=

```
Rscale[mp, Tvals2b[[RLposia]]]
Rscale[mp, Tvals2b[[1]]]
(Rscale[mp, Tvals2b[[RLposia]]] - Rscale[mp, Tvals2b[[1]]]) / Rscale[mp, Tvals2b[[1]]]
```

Out[164]=

2.79887

Out[165]=

2.36408

Out[166]=

0.183917

In[167]:=

$$\frac{2^{3/2}}{9\,\pi}\left(\frac{\text{G mp 0.1 Msol}}{(\text{Rscale[mp, Tp] Rsol / 100})^{3}}\right)^{1/2}$$

Out[167]=

0.00971922

In[168]:=

$$\frac{2^{3/2}}{9\,\pi}\left(\frac{\text{G mp 0.1 Msol}}{(\text{Rscale[mp, Tvals2b[[RLposia]]] Rsol / 100})^{3}}\right)^{1/2}$$

Out[168]=

0.00754483

In[169]:=

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RRLval}^{-1}\ \text{Rscale[mp, 0.4] Rsol 0.01}\right)^{3/2}},\ \text{fRL}\right][[1]]$$

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RRLval}^{-1}\ \text{Rscale[mp, Tp] Rsol 0.01}\right)^{3/2}},\ \text{fRL}\right][[1]]$$

$$\text{fRL /. Solve}\left[\text{fRL} == \frac{G^{1/2}\ ((\text{mp}+\text{ms})\ 0.1\ \text{Msol})^{1/2}}{\pi\ \left(\text{RRLval}^{-1}\ \text{Rscale[mp, Tvals2b[[RLposia]]] Rsol 0.01}\right)^{3/2}},\ \text{fRL}\right][[1]]$$

Out[169]=

0.0163288

Out[170]=

0.00992568

Out[171]=

0.0077051

In[172]:=

```
RRLval Rsol 0.01
```

Out[172]=

2.44697×10^{8}

In[173]:=

```
RRLval
```

Out[173]=

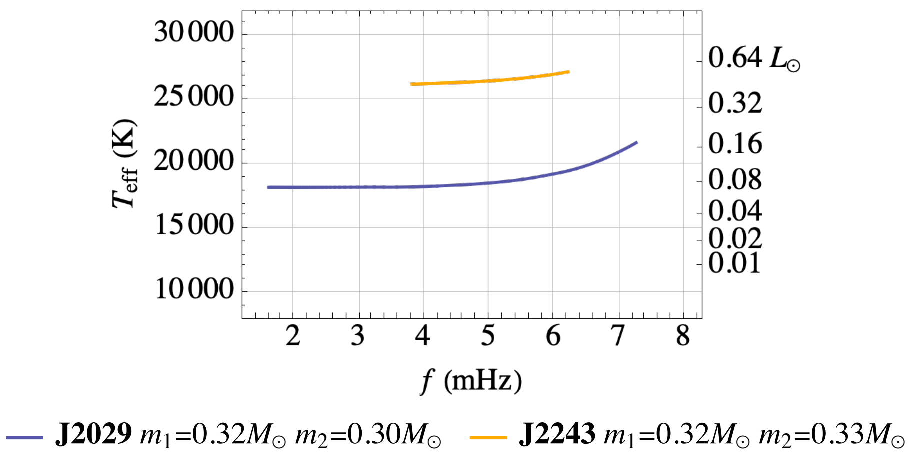
0.349817

together

In[174]:=

```
Show[plota21lin, plota23lin]
```

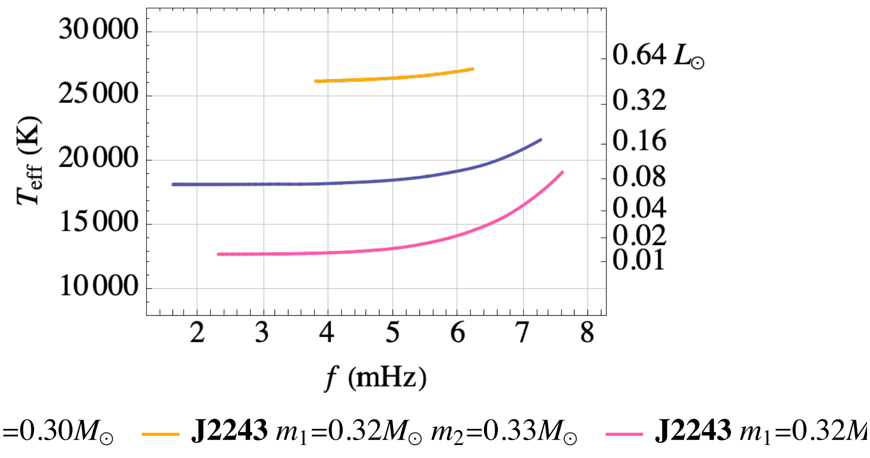
Out[174]=



In[175]:=

```
Show[plota21lin, plota23lin, plota24lin]
```

Out[175]=



In[176]:=

```
fGWs3 = {fGWend1, fGWend2, fGWend3}
T1prims3 = {Tend1, Tend2, Tend3}
```

Out[176]=

```
{7.24804, 6.20219, 7.58384}
```

Out[177]=

```
{21 727.6, 27 261.6, 19 187.2}
```

In[178]:=

```
df4 = Transpose[{fGWs3, 1.002 T1prims3, T1prims3}]
pts4 = df4
stylesTemp4 = ColorData["Pastel"] /@ Rescale[pts4⟦All, 3⟧]
Pltfuntemp4[ii_] := ListPlot[{pts4⟦All, {1, 2}⟧⟦ii⟧},
  PlotRange → {{0.1, 10}, {6000, 40 000}}, AspectRatio → 1, PlotMarkers → {"∞", 25},
  PlotStyle → {{stylesTemp4⟦ii⟧}}, LabelStyle → (FontFamily → "Times"),
  PlotLegends → {Style["R (m,Teff) of detached WD", 16]}]
```

Out[178]=

```
{{7.24804, 21 771.1, 21 727.6},
 {6.20219, 27 316.2, 27 261.6}, {7.58384, 19 225.6, 19 187.2}}
```

Out[179]=

```
{{7.24804, 21 771.1, 21 727.6},
 {6.20219, 27 316.2, 27 261.6}, {7.58384, 19 225.6, 19 187.2}}
```

Out[180]=

```
{■, ■, ■}
```

In[182]:=

```
RL1 = ListPlot[{Transpose[{fGWs3[[1]], 1.013 T1prims3[[1]]}]},
   PlotRange → {{0.1, 10}, {6000, 40 000}}, AspectRatio → 1,
   PlotMarkers → {"∞", 25}, PlotStyle → {Blend[{Gray, Gray, Blue}]},
   LabelStyle → (FontFamily → "Times"),
   PlotLegends → {Style["R(m,Teff) of detached WD", 16]}];
RL2 = ListPlot[{Transpose[{fGWs3[[2]], 1.01 T1prims3[[2]]}]},
   PlotRange → {{0.1, 10}, {6000, 40 000}}, AspectRatio → 1,
   PlotMarkers → {"∞", 25}, PlotStyle → {Blend[{Orange, Orange, Yellow}]},
   LabelStyle → (FontFamily → "Times"),
   PlotLegends → {Style["R(m,Teff) of detached WD", 16]}];
RL3 = ListPlot[{Transpose[{fGWs3[[3]], 1.01 T1prims3[[3]]}]},
   PlotRange → {{0.1, 10}, {6000, 40 000}}, AspectRatio → 1,
   PlotMarkers → {"∞", 25}, PlotStyle → {Blend[{Magenta, Red, White}]},
   LabelStyle → (FontFamily → "Times"),
   PlotLegends → {Style["R(m,Teff) of detached WD", 16]}];
```

In[185]:=

```
Show[plota21lin, plota23lin, plota24lin, RL1, RL2, RL3]
```

Out[185]=

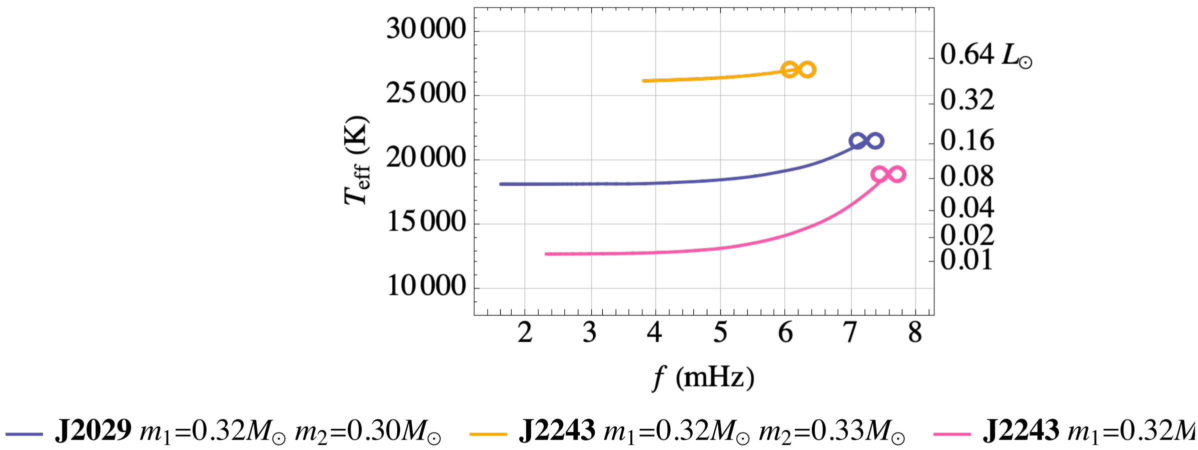


Figure 5: Evolution of J1539

Relations

In[186]:=

```
Rscale[m1a_, T1a_] :=
 10^(-0.02792426461145596`+0.7641778013995925` √T1a) m1a^(0.14797691065884058`-0.9408955042478873` √T1a)
```

In[187]:=

```
dRscaledt[m1a_, T1a_] :=
```

$$\frac{1}{\sqrt{T1a}}\,0.8797922069498331\grave{}\times 10^{-0.02792426461145596\grave{}+0.7641778013995925\grave{}\sqrt{T1a}}\ m1a^{0.14797691065884058\grave{}-0.9408955042478873\grave{}\sqrt{T1a}} - \frac{1}{\sqrt{T1a}}\,0.47044775212394363\grave{}\times 10^{-0.02792426461145596\grave{}+0.7641778013995925\grave{}\sqrt{T1a}}\ m1a^{0.14797691065884058\grave{}-0.9408955042478873\grave{}\sqrt{T1a}}\ \mathrm{Log}[m1a]$$

In[188]:=

$$\mathrm{Mchirpf}[m11\_, m22\_] = \frac{(m11\ m22)^{3/5}}{(m11 + m22)^{1/5}};$$

In[189]:=

$$\mathrm{fdotGW}[m1\_, m2\_, fGW\_] = \frac{96\,\pi^{8/3}\,G^{5/3}\,(fGW)^{11/3}\,(\mathrm{Mchirpf}[m1, m2])^{5/3}}{5\,c^5}\ //\ \mathrm{FullSimplify}$$

Out[189]=

$$1.83501\times 10^{-62}\ fGW^{11/3}\left(\frac{(m1\ m2)^{3/5}}{(m1+m2)^{1/5}}\right)^{5/3}$$

non rotating frequency derivative :

In[190]:=

$$\mathrm{fdotTD1}[m1\_, m2\_, fGW\_, R1\_] = \frac{18\ fGW^{13/3}\ m2\ \pi^{13/3}\ R1^5\left(\frac{fGW}{2}\right)}{G^{5/3}\ m1\ (m1+m2)^{5/3}}\ konQ$$

Out[190]=

$$\frac{1.17044\times 10^{15}\ fGW^{16/3}\ konQ\ m2\ R1^5}{m1\ (m1+m2)^{5/3}}$$

rotating at 90 percent synchronous

In[191]:=

$$\mathrm{fdotTD2}[m1\_, m2\_, fGW\_, R1\_] = \frac{18\ fGW^{13/3}\ m2\ \pi^{13/3}\ R1^5\left(\frac{fGW}{2}\ \frac{1}{10}\right)}{G^{5/3}\ m1\ (m1+m2)^{5/3}}\ konQ$$

Out[191]=

$$\frac{1.17044\times 10^{14}\ fGW^{16/3}\ konQ\ m2\ R1^5}{m1\ (m1+m2)^{5/3}}$$

4000K case

In[192]:=

```
mp = 2.1;
ms = 6.1;
fbin = 0.8;
Tp = 0.4;
fform = 2.0 ;
fstep = (fform / fbin)^(10/3)
```

Out[197]=

```
21.2064
```

structure constant

In[198]:=

```
factorsyn = 0.9999
```

Out[198]=

0.9999

In[199]:=

$$\text{kQratiob} = \frac{1}{(1 - \text{factorsyn})}\ 8 \times 10^{-12}$$

Out[199]=

$8. \times 10^{-8}$

In[200]:=

differential equation prefactors

In[201]:=

$$\text{preTDb} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right]$$

$$\frac{18\ (\text{mHz})^{13/3}\ (\text{Msol} / 10)\ \pi^{13/3}\ (\text{Rsol} / 100)^{5}\ (\text{mHz})}{G^{5/3}\ (\text{Msol} / 10)\ ((\text{Msol} / 10))^{5/3}}\ \text{kQratiob}\ D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[201]=

1.4425

In[202]:=

$$\text{preTb} = D\left[\frac{T}{\text{kK4}}, T\right] \left(\frac{135\ \text{mHz}^{19/3}\ (\text{Msol} / 10)^{3}\ \pi^{25/3}\ (\text{Rsol} / 100)^{9}}{G^{8/3}\ (\text{Msol} / 10)\ \sigma\ ((\text{Msol} / 10))^{11/3}\ \text{kK4}^{3}\ ((\text{Rsol} / 100))}\ (\text{mHz})^{3}\right)$$

$$\text{kQratiob}^{2}\ D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[202]=

762.994

In[203]:=

$$\text{preJb} = D\left[\frac{J}{(\text{Msol} / 10)\ (\text{Rsol} / 100)^{2}\ \text{mHz}}, J\right]$$

$$\left(2\,\pi\ \frac{3\ \text{mHz}^{3}\ (\text{Msol} / 10)^{2}\ \pi^{3}\ (\text{Rsol} / 100)^{5}\ (\text{mHz})}{G\ ((\text{Msol} / 10))^{2}}\right)\ \text{kQratiob}\ D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[203]=

377.404

In[204]:=

$$\text{pre}\Omega\text{b} =$$

$$D\left[\frac{\Omega}{\text{mHz}}, \Omega\right] \left(\frac{3\ \text{mHz}^{3}\ (\text{Msol} / 10)^{2}\ \pi^{3}\ (\text{Rsol} / 100)^{3}\ (\text{mHz})}{G\ \ (\text{Msol} / 10)\ \text{rg2}\ ((\text{Msol} / 10))^{2}}\right)\ \text{kQratiob}\ D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[204]=

600.658

In[205]:=

$$\text{soltestgenb} = \text{NDSolve}\Big[\Big\{f'[t] == \Big(\text{preGW}\,(\text{mp}\,\text{ms})\left(\frac{1}{(\text{mp}+\text{ms})^{1/3}}\right) f[t]^{11/3}\Big) + \frac{\text{ms}}{\text{mp}\,(\text{mp}+\text{ms})^{5/3}}\,\text{preTDb}\,f[t]^{13/3}\,(\text{Rscale}[\text{mp}, T[t]])^5\,(f[t]/2 - \Omega[t]),$$

$$T'[t] == \text{preTb}\,\Big(\Big((\text{ms})^3 f[t]^{19/3}\,\text{Rscale}[\text{mp}, T[t]]^9\,(f[t]/2-\Omega[t])^2\,\Big(f[t]/2 - \frac{3}{5}\Omega[t]\Big)\Big)\Big/ \Big(\Big(\text{mp}\,(\text{mp}+\text{ms})^{11/3}\Big)\,\big(T[t]^3\,(2\,\text{Rscale}[\text{mp}, T[t]] + T[t]\times\text{dRscaledt}[\text{mp}, T[t]])\big)\Big)\Big),$$

$$\Omega'[t] == \text{pre}\Omega\text{b}\,\frac{\text{ms}^2}{\text{mp}\,(\text{mp}+\text{ms})^2}\,f[t]^3\,(\text{Rscale}[\text{mp}, T[t]])^3\,(f[t]/2-\Omega[t]),$$

$$f[0] == \text{fbin},\ T[0] == \text{Tp},\ \Omega[0] == \text{factorsyn}\,\text{fbin}/2\Big\},$$

$$\{f, T, \Omega\},\ \{t, 0, 0.214\,\text{fstep}/1.68\},\ \text{Method} \to \text{"StiffnessSwitching"}\Big]$$

General: Overflow occurred in computation.

General: Overflow occurred in computation.

General: Overflow occurred in computation.

General: Further output of General::ovfl will be suppressed during this calculation.

NDSolve: The function value {Overflow[], Overflow[], Overflow[]} is not a list of numbers with dimensions {3} at {t, f[t], T[t], Ω[t]} = $\{2.69941, 1.35671\times10^{123}, 1.21701\times10^{193}, 2.14015\times10^{73}\}$.

Out[205]=

{{f → InterpolatingFunction[Domain: {{0., 2.7}} Output: scalar],

T → InterpolatingFunction[Domain: {{0., 2.7}} Output: scalar],

Ω → InterpolatingFunction[Domain: {{0., 2.7}} Output: scalar]}}

In[206]:=

```
endt = 0.214 fstep / 1.68;
stepsize = 0.0001 / fstep;
```

In[208]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp}+\text{ms})\,\text{Msol}/10)^{1/3}}{(\text{fvals1b}\,\text{mHz})^{2/3}\,\pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3} \times 2\,\text{mp}^{1/3} \Big/ (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\,(\text{mp}/\text{ms})^{2/3}}{0.6\,(\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1+(\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction: Input value {2.69943} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69943} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69944} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {2.69943} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69943} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69944} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {2.69943} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69943} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69944} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

Out[215]=

```
0.349817
```

Out[217]=

```
0.291442
```

In[224]:=

```
RLposia = x /.
  Solve[Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]]
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[224]=

```
569 967
```

Out[226]=

```
5.42597
```

Out[227]=

```
9894.4
```

In[228]:=

```
Tvals2b // Length
```

Out[228]=

```
570 365
```

In[229]:=

```
timevals2b[[RLposia]]
fvals2b[[RLposia]]
```

Out[229]=

```
2.68771
```

Out[230]=

```
5.42597
```

In[231]:=

```
formfreq = 1.5
presfreq = 4.8
```

Out[231]=

```
1.5
```

Out[232]=

```
4.8
```

In[233]:=

```
fposiform = x /. Solve[
    Position[Abs[fvals2b / formfreq - 1], Min[Abs[fvals2b / formfreq - 1]]] == x][[1]]
Abs[fvals2b / formfreq - 1][[fposiform]];
2 Ωvals2b[[fposiform]]
fvals2b[[fposiform]];
Tvals2b[[fposiform]] kK4;
timevals2b[[fposiform]];
timevals2b[[RLposia]] - timevals2b[[fposiform]]

fpres = x /. Solve[Position[
      Abs[fvals2b / presfreq - 1], Min[Abs[fvals2b / presfreq - 1]]] == x][[1]];
Abs[fvals2b / presfreq - 1][[fpres]];
2 Ωvals2b[[fpres]]
fvals2b[[fpres]]
Tvals2b[[fpres]] kK4
timevals2b[[fpres]];
timevals2b[[RLposia]] - timevals2b[[fpres]]
```

Out[233]=

466 493

Out[235]=

1.49976

Out[239]=

0.487938

Out[242]=

4.79994

Out[243]=

4.80017

Out[244]=

7985.93

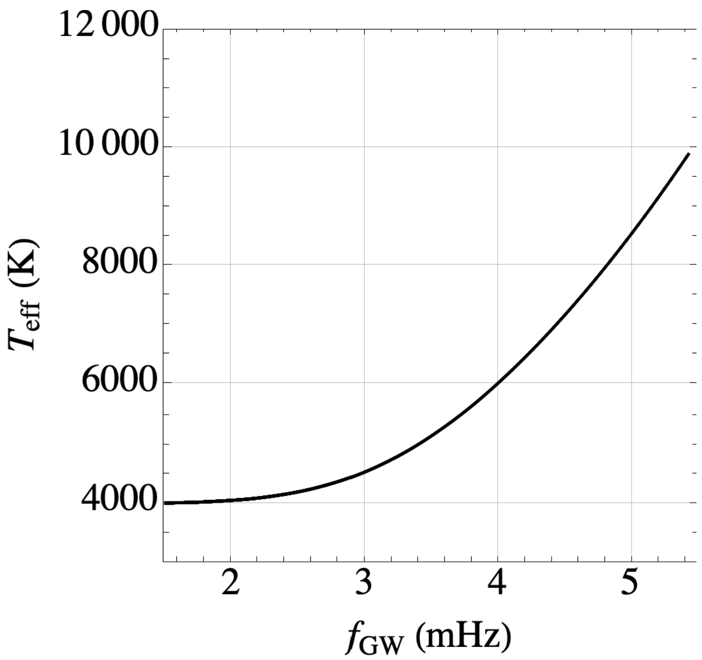
Out[246]=

0.00617738

In[247]:=

```
plotaT42 = ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}],
  Mesh → All, PlotMarkers → None, Joined → True, PlotStyle → Black,
  PlotRange → {1000 {0.0015, 0.0055}, {3000, 12 000}}, AspectRatio → 1,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f_GW (mHz)"], Style["T_eff (K)"]},
  BaseStyle → {FontSize → 20}, GridLines → Automatic]
```

Out[247]=



5000K case

In[248]:=

```
Tp = 0.5;
```

In[249]:=

$$\text{soltestgenb} = \text{NDSolve}\Big[\Big\{f'[t] == \left(\text{preGW}\,(\text{mp}\,\text{ms})\left(\frac{1}{(\text{mp}+\text{ms})^{1/3}}\right) f[t]^{11/3}\right) + \frac{\text{ms}}{\text{mp}\,(\text{mp}+\text{ms})^{5/3}}\,\text{preTDb}\, f[t]^{13/3}\,(\text{Rscale}[\text{mp}, T[t]])^5\,(f[t]/2 - \Omega[t]),$$

$$T'[t] == \text{preTb}\left(\left((\text{ms})^3 f[t]^{19/3}\,\text{Rscale}[\text{mp}, T[t]]^9\,(f[t]/2-\Omega[t])^2\left(f[t]/2 - \frac{3}{5}\Omega[t]\right)\right)\Big/\left(\left(\text{mp}\,(\text{mp}+\text{ms})^{11/3}\right)\left(T[t]^3\,(2\,\text{Rscale}[\text{mp}, T[t]] + T[t]\times \text{dRscaledt}[\text{mp}, T[t]])\right)\right)\right),$$

$$\Omega'[t] == \text{pre}\Omega\text{b}\,\frac{\text{ms}^2}{\text{mp}\,(\text{mp}+\text{ms})^2}\, f[t]^3\,(\text{Rscale}[\text{mp}, T[t]])^3\,(f[t]/2-\Omega[t]),$$

$$f[0] == \text{fbin},\ T[0] == \text{Tp},\ \Omega[0] == \text{factorsyn}\,\text{fbin}/2\Big\},$$

$$\{f, T, \Omega\},\ \{t, 0, 0.214\,\text{fstep}/1.68\},\ \text{Method} \to \text{"StiffnessSwitching"}\Big]$$

General: Overflow occurred in computation.

General: Overflow occurred in computation.

General: Overflow occurred in computation.

General: Further output of General::ovfl will be suppressed during this calculation.

NDSolve: The function value {Overflow[], Overflow[], Overflow[]} is not a list of numbers with dimensions {3} at {t, f[t], T[t], Ω[t]} = $\{2.6984, 1.76419\times10^{163}, 3.50867\times10^{256}, 3.0941\times10^{97}\}$.

Out[249]=

{{f → InterpolatingFunction[Domain: {{0., 2.7}} Output: scalar],

T → InterpolatingFunction[Domain: {{0., 2.7}} Output: scalar],

Ω → InterpolatingFunction[Domain: {{0., 2.7}} Output: scalar]}}

In[250]:=

```
endt = 0.214 fstep / 1.68;
stepsize = 0.0001 / fstep;
```

In[252]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp}+\text{ms})\,\text{Msol}/10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\,\pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3} \times 2\,\text{mp}^{1/3} \big/ (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\,(\text{mp}/\text{ms})^{2/3}}{0.6\,(\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1+(\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction: Input value {2.69841} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69842} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69842} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {2.69841} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69842} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69842} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {2.69841} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69842} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69842} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

Out[259]=

```
0.291442
```

Out[261]=

```
0.291442
```

In[268]:=

```
RLposia = x /.
  Solve[Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]]
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[268]=

```
569 701
```

Out[270]=

```
5.394
```

Out[271]=

```
9967.11
```

In[272]:=

```
Tvals2b // Length
```

Out[272]=

```
570 313
```

In[273]:=

```
timevals2b[[RLposia]]
fvals2b[[RLposia]]
```

Out[273]=

```
2.68645
```

Out[274]=

```
5.394
```

In[275]:=

```
fposiform = x /. Solve[Position[
        Abs[fvals2b / formfreq - 1], Min[Abs[fvals2b / formfreq - 1]]] == x][[1]];
Abs[fvals2b / formfreq - 1][[fposiform]];
2 Ωvals2b[[fposiform]]
fvals2b[[fposiform]];
Tvals2b[[fposiform]] kK4;
timevals2b[[fposiform]];
timevals2b[[RLposia]] - timevals2b[[fposiform]]

fpres = x /. Solve[Position[
        Abs[fvals2b / presfreq - 1], Min[Abs[fvals2b / presfreq - 1]]] == x][[1]];
Abs[fvals2b / presfreq - 1][[fpres]];
2 Ωvals2b[[fpres]]
fvals2b[[fpres]];
Tvals2b[[fpres]] kK4
timevals2b[[fpres]];
timevals2b[[RLposia]] - timevals2b[[fpres]]
```

Out[277]=

```
1.49981
```

Out[281]=

```
0.487363
```

Out[284]=

```
4.79982
```

Out[286]=

```
8252.88
```

Out[288]=

```
0.00592274
```

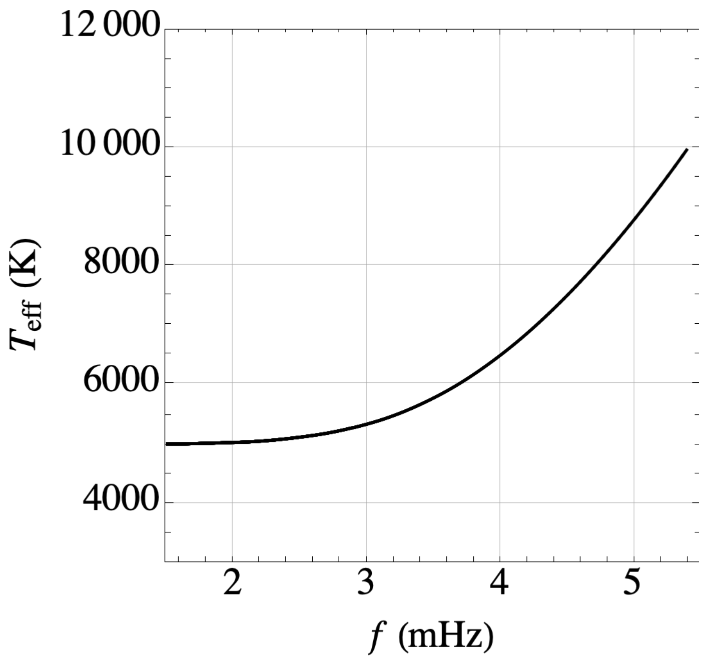
In[289]:=

```
list1b =  Transpose[{fvals2b mHz, Tvals2b kK4}];
list2b =  Transpose[{fvals2b mHz, 0 Tvals2b kK4}];
list3b =  Transpose[{Ra1cutb, Tvals2b kK4}];
list4b =  Transpose[{Ra1cutb, 0 Tvals2b kK4}];
```

In[293]:=

```
plotaT52 = ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}],
  Mesh → All, PlotMarkers → None, Joined → True, PlotStyle → Black,
  PlotRange → {1000 {0.0015, 0.0055}, {3000, 12 000}}, AspectRatio → 1,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f (mHz)"], Style["Teff (K)"]},
  BaseStyle → {FontSize → 20}, GridLines → Automatic]
```

Out[293]=



6000K case

In[294]:=

```
Tp = 0.6;
```

In[295]:=

```
soltestgenb = NDSolve[{f'[t] == (preGW (mp ms) (1/(mp + ms)^(1/3)) f[t]^(11/3)) +
    ms/(mp (mp + ms)^(5/3)) preTDb f[t]^(13/3) (Rscale[mp, T[t]])^5 (f[t] / 2 - Ω[t]),
   T'[t] ==
    preTb (((ms)^3 f[t]^(19/3) Rscale[mp, T[t]]^9 (f[t] / 2 - Ω[t])^2 (f[t] / 2 - 3/5 Ω[t]))/
       ((mp (mp + ms)^(11/3))
         (T[t]^3 (2 Rscale[mp, T[t]] + T[t] × dRscaledt[mp, T[t]])))),
   Ω'[t] == preΩb ms^2/(mp (mp + ms)^2) f[t]^3 (Rscale[mp, T[t]])^3 (f[t] / 2 - Ω[t]),
   f[0] == fbin, T[0] == Tp, Ω[0] == factorsyn fbin / 2 },
  {f, T, Ω}, {t, 0, 0.214 fstep / 1.69}, Method → "StiffnessSwitching"]
```

Out[295]=

```
{{f → InterpolatingFunction[ Domain: {{0., 2.69}} Output: scalar ],
  T → InterpolatingFunction[ Domain: {{0., 2.69}} Output: scalar ],
  Ω → InterpolatingFunction[ Domain: {{0., 2.69}} Output: scalar ]}}
```

In[296]:=

```
0.214 fstep
```

Out[296]=

```
4.53817
```

In[297]:=

```
endt = 0.214 fstep / 1.69;
stepsize = 0.0001 / fstep;
```

In[299]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp}+\text{ms})\ \text{Msol}/10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\,\pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3} \times 2\ \text{mp}^{1/3} \Big/ (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\ (\text{mp}/\text{ms})^{2/3}}{0.6\ (\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
```

Out[306]=

0.291442

Out[308]=

0.291442

In[315]:=

```
RLposia = x /.
  Solve[Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]]
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[315]=

569 386

Out[317]=

5.34332

Out[318]=

10 087.2

In[319]:=

```
Tvals2b // Length
```

Out[319]=

569 386

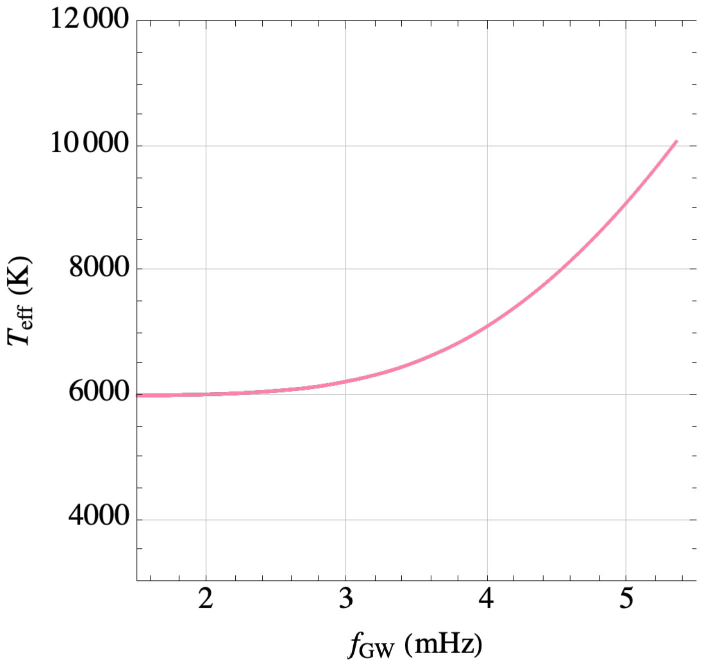
In[320]:=
```
timevals2b⟦RLposia⟧
fvals2b⟦RLposia⟧
```
Out[320]=
```
2.68497
```
Out[321]=
```
5.34332
```
In[322]:=
```
fposiform = x /. Solve[Position[
        Abs[fvals2b / formfreq - 1], Min[Abs[fvals2b / formfreq - 1]]] == x]⟦1⟧;
Abs[fvals2b / formfreq - 1]⟦fposiform⟧;
2 Ωvals2b⟦fposiform⟧
fvals2b⟦fposiform⟧;
Tvals2b⟦fposiform⟧ kK4;
timevals2b⟦fposiform⟧;
timevals2b⟦RLposia⟧ - timevals2b⟦fposiform⟧

fpres = x /. Solve[Position[
        Abs[fvals2b / presfreq - 1], Min[Abs[fvals2b / presfreq - 1]]] == x]⟦1⟧;
Abs[fvals2b / presfreq - 1]⟦fpres⟧;
2 Ωvals2b⟦fpres⟧
fvals2b⟦fpres⟧;
Tvals2b⟦fpres⟧ kK4
timevals2b⟦fpres⟧;
timevals2b⟦RLposia⟧ - timevals2b⟦fpres⟧
```
Out[324]=
```
1.49985
```
Out[328]=
```
0.486226
```
Out[331]=
```
4.79976
```
Out[333]=
```
8642.74
```
Out[335]=
```
0.00550306
```
In[336]:=
```
list1b = Transpose[{fvals2b mHz, Tvals2b kK4}];
list2b = Transpose[{fvals2b mHz, 0 Tvals2b kK4}];
list3b = Transpose[{Ra1cutb, Tvals2b kK4}];
list4b = Transpose[{Ra1cutb, 0 Tvals2b kK4}];
```

In[340]:=

```
plotaT6 =
 ListPlot[Transpose[{fvals2b⟦ ;; RLposia⟧, 10^4 Tvals2b⟦ ;; RLposia⟧}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle →  Blend[{Magenta, White, Orange}],
  PlotRange → {1000 {0.0015, 0.0055}, {3000, 12 000}}, AspectRatio → 1,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f_GW (mHz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic]
```

Out[340]=



7000K case

In[341]:=

```
Tp = 0.7;
```

In[342]:=

```
soltestgenb = NDSolve[{f'[t] == (preGW (mp ms) (1/(mp + ms)^(1/3)) f[t]^(11/3)) +
    ms/(mp (mp + ms)^(5/3)) preTDb f[t]^(13/3) (Rscale[mp, T[t]])^5 (f[t] / 2 - Ω[t]),
  T'[t] ==
   preTb (((ms)^3 f[t]^(19/3) Rscale[mp, T[t]]^9 (f[t] / 2 - Ω[t])^2 (f[t] / 2 - 3/5 Ω[t])) /
      ((mp (mp + ms)^(11/3))
        (T[t]^3 (2 Rscale[mp, T[t]] + T[t] × dRscaledt[mp, T[t]])))),
  Ω'[t] == preΩb ms^2/(mp (mp + ms)^2) f[t]^3 (Rscale[mp, T[t]])^3 (f[t] / 2 - Ω[t]),
  f[0] == fbin, T[0] == Tp, Ω[0] == factorsyn fbin / 2 },
 {f, T, Ω}, {t, 0, 0.214 fstep / 1.68}, Method → "StiffnessSwitching"]
```

General: Overflow occurred in computation.

General: Overflow occurred in computation.

General: Overflow occurred in computation.

General: Further output of General::ovfl will be suppressed during this calculation.

NDSolve: The function value {Overflow[], Overflow[], Overflow[]} is not a list of numbers with dimensions {3} at {t, f[t], T[t], Ω[t]} = $\{2.69615, 4.5998\times10^{94}, 1.87808\times10^{148}, 1.242\times10^{56}\}$.

Out[342]=

```
{{f → InterpolatingFunction[ Domain: {{0., 2.7}} Output: scalar ],
  T → InterpolatingFunction[ Domain: {{0., 2.7}} Output: scalar ],
  Ω → InterpolatingFunction[ Domain: {{0., 2.7}} Output: scalar ]}}
```

In[343]:=

```
endt = 0.214 fstep / 1.68;
stepsize = 0.0001 / fstep;
```

In[345]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\ ((\text{mp}+\text{ms})\ \text{Msol}/10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\ \pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3} \times 2\ \text{mp}^{1/3} \Big/ (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\ (\text{mp}/\text{ms})^{2/3}}{0.6\ (\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction: Input value {2.69615} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69615} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69616} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {2.69615} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69615} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69616} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction: Input value {2.69615} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69615} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {2.69616} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

General: $2.1^{-7569.77+0.\,i}$ is too small to represent as a normalized machine number; precision may be lost.

General: $2.1^{-38553.7+0.\,i}$ is too small to represent as a normalized machine number; precision may be lost.

General: $2.1^{-82620.+0.\,i}$ is too small to represent as a normalized machine number; precision may be lost.

General: Further output of General::munfl will be suppressed during this calculation.

Out[352]=

0.291442

Out[354]=

0.291442

In[361]:=

```
RLposia = x /.
  Solve[Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]]
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[361]=

569 009

Out[363]=

5.26863

Out[364]=

10 266.

In[365]:=

```
Tvals2b // Length
```

Out[365]=

570 099

In[366]:=

```
timevals2b[[RLposia]]
fvals2b[[RLposia]]
```

Out[366]=

2.68319

Out[367]=

5.26863

In[368]:=

```
Max[fvals2b]
```

Out[368]=

2.02152×10^{15}

In[369]:=

```
fposiform = x /. Solve[Position[
        Abs[fvals2b / formfreq - 1], Min[Abs[fvals2b / formfreq - 1]]] == x][[1]];
Abs[fvals2b / formfreq - 1][[fposiform]];
2 Ωvals2b[[fposiform]]
fvals2b[[fposiform]];
Tvals2b[[fposiform]] kK4;
timevals2b[[fposiform]];
timevals2b[[RLposia]] - timevals2b[[fposiform]]

fpres = x /. Solve[Position[
        Abs[fvals2b / presfreq - 1], Min[Abs[fvals2b / presfreq - 1]]] == x][[1]];
Abs[fvals2b / presfreq - 1][[fpres]];
2 Ωvals2b[[fpres]]
fvals2b[[fpres]];
Tvals2b[[fpres]] kK4
timevals2b[[fpres]];
timevals2b[[RLposia]] - timevals2b[[fpres]]
```

Out[371]=

```
1.49987
```

Out[375]=

```
0.485434
```

Out[378]=

```
4.79969
```

Out[380]=

```
9153.11
```

Out[382]=

```
0.00486646
```

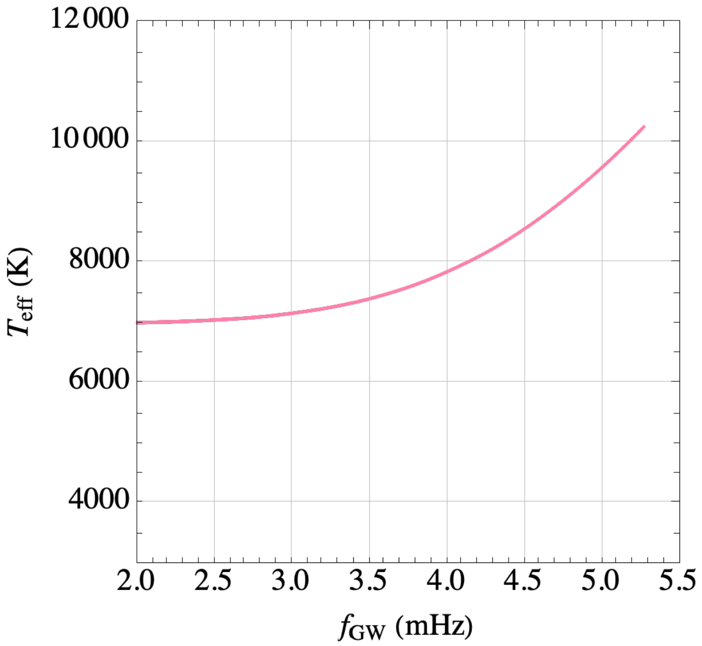
In[383]:=

```
list1b = Transpose[{fvals2b mHz, Tvals2b kK4}];
list2b = Transpose[{fvals2b mHz, 0 Tvals2b kK4}];
list3b = Transpose[{Ra1cutb, Tvals2b kK4}];
list4b = Transpose[{Ra1cutb, 0 Tvals2b kK4}];
```

In[387]:=

```
plotaT7 =
 ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Magenta, White, Orange}],
  PlotRange → {1000 {0.002, 0.0055}, {3000, 12 000}}, AspectRatio → 1,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f_GW (mHz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic]
```

Out[387]=



8000K case

In[388]:=

```
Tp = 0.8;
```

In[389]:=

```
soltestgenb = NDSolve[{f'[t] == (preGW (mp ms) (1/(mp + ms)^(1/3)) f[t]^(11/3) +
      ms/(mp (mp + ms)^(5/3)) preTDb f[t]^(13/3) (Rscale[mp, T[t]])^5 (f[t] / 2 - Ω[t]),
    T'[t] ==
     preTb (((ms)^3 f[t]^(19/3) Rscale[mp, T[t]]^9 (f[t] / 2 - Ω[t])^2 (f[t] / 2 - 3/5 Ω[t]))/
        ((mp (mp + ms)^(11/3))
          (T[t]^3 (2 Rscale[mp, T[t]] + T[t] × dRscaledt[mp, T[t]])))),
    Ω'[t] == preΩb ms^2/(mp (mp + ms)^2) f[t]^3 (Rscale[mp, T[t]])^3 (f[t] / 2 - Ω[t]),
    f[0] == fbin, T[0] == Tp, Ω[0] == factorsyn fbin / 2 },
   {f, T, Ω}, {t, 0, 0.214 fstep / 1.69}, Method → "StiffnessSwitching"]
```

Out[389]=

```
{{f → InterpolatingFunction[ Domain: {{0., 2.69}} Output: scalar ],
  T → InterpolatingFunction[ Domain: {{0., 2.69}} Output: scalar ],
  Ω → InterpolatingFunction[ Domain: {{0., 2.69}} Output: scalar ]}}
```

In[390]:=

```
endt = 0.214 fstep / 1.69;
stepsize = 0.0001 / fstep;
```

In[392]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp}+\text{ms})\ \text{Msol}/10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\,\pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3} \times 2\ \text{mp}^{1/3} \Big/ (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\ (\text{mp}/\text{ms})^{2/3}}{0.6\ (\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
```

Out[399]=

0.291442

Out[401]=

0.291442

In[408]:=

```
Max[timevals2b]
```

Out[408]=

2.68105

In[409]:=

In[410]:=

```
RLposia = x /.
  Solve[Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]]
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[410]=

568 555

Out[412]=

5.16576

Out[413]=

10 518.7

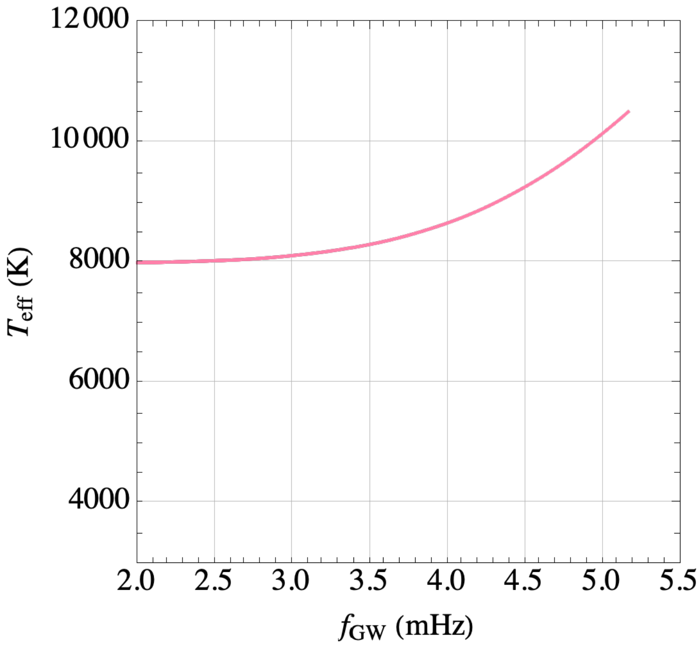
In[414]:=
```
timevals2b⟦RLposia⟧
fvals2b⟦RLposia⟧
```
Out[414]=
```
2.68105
```
Out[415]=
```
5.16576
```
In[416]:=
```
Tvals2b // Length
```
Out[416]=
```
568 555
```
In[417]:=
```
fposiform = x /. Solve[Position[
        Abs[fvals2b / formfreq - 1], Min[Abs[fvals2b / formfreq - 1]]] == x]⟦1⟧;
Abs[fvals2b / formfreq - 1]⟦fposiform⟧;
2 Ωvals2b⟦fposiform⟧
fvals2b⟦fposiform⟧;
Tvals2b⟦fposiform⟧ kK4;
timevals2b⟦fposiform⟧;
timevals2b⟦RLposia⟧ - timevals2b⟦fposiform⟧

fpres = x /. Solve[Position[
        Abs[fvals2b / presfreq - 1], Min[Abs[fvals2b / presfreq - 1]]] == x]⟦1⟧;
Abs[fvals2b / presfreq - 1]⟦fpres⟧;
2 Ωvals2b⟦fpres⟧
fvals2b⟦fpres⟧;
Tvals2b⟦fpres⟧ kK4
timevals2b⟦fpres⟧;
timevals2b⟦RLposia⟧ - timevals2b⟦fpres⟧
```
Out[419]=
```
1.49989
```
Out[423]=
```
0.483576
```
Out[426]=
```
4.79996
```
Out[428]=
```
9766.53
```
Out[430]=
```
0.00392806
```
In[431]:=
```
list1b =  Transpose[{fvals2b mHz, Tvals2b kK4}];
list2b =  Transpose[{fvals2b mHz, 0 Tvals2b kK4}];
list3b =  Transpose[{Ra1cutb,  Tvals2b kK4}];
list4b =  Transpose[{Ra1cutb, 0 Tvals2b kK4}];
```

In[435]:=

```
plotaT8 =
 ListPlot[Transpose[{fvals2b⟦ ;; RLposia⟧, 10^4 Tvals2b⟦ ;; RLposia⟧}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Magenta, White, Orange}],
  PlotRange → {1000 {0.002, 0.0055}, {3000, 12 000}}, AspectRatio → 1,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f_GW (mHz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic]
```

Out[435]=



9000K case

In[436]:=

```
Tp = 0.9;
```

In[437]:=

```
soltestgenb = NDSolve[{f'[t] == (preGW (mp ms) (1/(mp + ms)^(1/3)) f[t]^(11/3)) +
      ms/(mp (mp + ms)^(5/3)) preTDb f[t]^(13/3) (Rscale[mp, T[t]])^5 (f[t] / 2 - Ω[t]),
    T'[t] ==
     preTb (((ms)^3 f[t]^(19/3) Rscale[mp, T[t]]^9 (f[t] / 2 - Ω[t])^2 (f[t] / 2 - 3/5 Ω[t])) /
        ((mp (mp + ms)^(11/3))
          (T[t]^3 (2 Rscale[mp, T[t]] + T[t] × dRscaledt[mp, T[t]])))),
    Ω'[t] == preΩb ms^2/(mp (mp + ms)^2) f[t]^3 (Rscale[mp, T[t]])^3 (f[t] / 2 - Ω[t]),
    f[0] == fbin, T[0] == Tp, Ω[0] == factorsyn fbin / 2 },
   {f, T, Ω}, {t, 0, 0.214 fstep / 1.692}, Method → "StiffnessSwitching"]
```

Out[437]=

```
{{f → InterpolatingFunction[ Domain: {{0., 2.68}} Output: scalar ],
  T → InterpolatingFunction[ Domain: {{0., 2.68}} Output: scalar ],
  Ω → InterpolatingFunction[ Domain: {{0., 2.68}} Output: scalar ]}}
```

In[438]:=

```
endt = 0.214 fstep / 1.692;
stepsize = 0.0001 / fstep;
```

In[440]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\ ((\text{mp} + \text{ms})\ \text{Msol} / 10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\ \pi^{2/3}} \Big/ (\text{Rsol} / 100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3} \times 2\ \text{mp}^{1/3} \Big/ (\text{mp} + \text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\ (\text{mp} / \text{ms})^{2/3}}{0.6\ (\text{mp} / \text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp} / \text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
```

Out[447]=

0.291442

Out[449]=

0.291442

In[456]:=

```
RLposia = x /.
  Solve[Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]]
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[456]=

567 996

Out[458]=

5.02935

Out[459]=

10 864.3

In[460]:=

```
Tvals2b // Length
```

Out[460]=

567 996

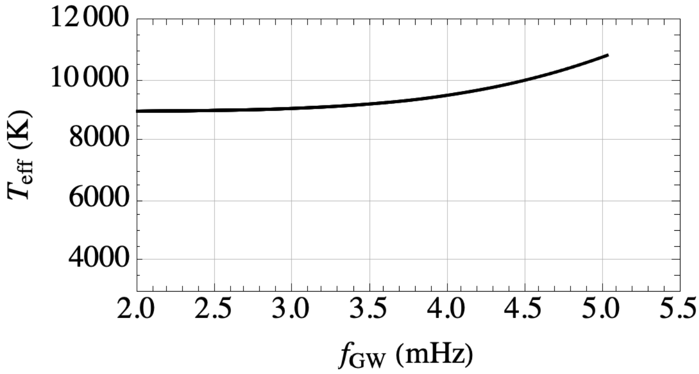
In[461]:=
```
timevals2b⟦RLposia⟧
fvals2b⟦RLposia⟧
```
Out[461]=
```
2.67841
```
Out[462]=
```
5.02935
```
In[463]:=
```
fposiform = x /. Solve[Position[
        Abs[fvals2b / formfreq - 1], Min[Abs[fvals2b / formfreq - 1]]] == x]⟦1⟧;
Abs[fvals2b / formfreq - 1]⟦fposiform⟧;
2 Ωvals2b⟦fposiform⟧
fvals2b⟦fposiform⟧;
Tvals2b⟦fposiform⟧ kK4;
timevals2b⟦fposiform⟧
timevals2b⟦RLposia⟧ - timevals2b⟦fposiform⟧

fpres = x /. Solve[Position[
        Abs[fvals2b / presfreq - 1], Min[Abs[fvals2b / presfreq - 1]]] == x]⟦1⟧;
Abs[fvals2b / presfreq - 1]⟦fpres⟧;
2 Ωvals2b⟦fpres⟧
fvals2b⟦fpres⟧;
Tvals2b⟦fpres⟧ kK4
timevals2b⟦fpres⟧;
timevals2b⟦RLposia⟧ - timevals2b⟦fpres⟧
```
Out[465]=
```
1.49991
```
Out[468]=
```
2.19617
```
Out[469]=
```
0.482246
```
Out[472]=
```
4.79973
```
Out[474]=
```
10 466.6
```
Out[476]=
```
0.00258413
```
In[477]:=
```
list1b = Transpose[{fvals2b mHz, Tvals2b kK4}];
list2b = Transpose[{fvals2b mHz, 0 Tvals2b kK4}];
list3b = Transpose[{Ra1cutb, Tvals2b kK4}];
list4b = Transpose[{Ra1cutb, 0 Tvals2b kK4}];
```

In[481]:=

```
plotaT9 = ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}],
  Mesh → All, PlotMarkers → None, Joined → True, PlotStyle → {Black},
  PlotRange → {1000 {0.002, 0.0055}, {3000, 12 000}}, AspectRatio → 1 / 2,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f_GW (mHz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic]
```

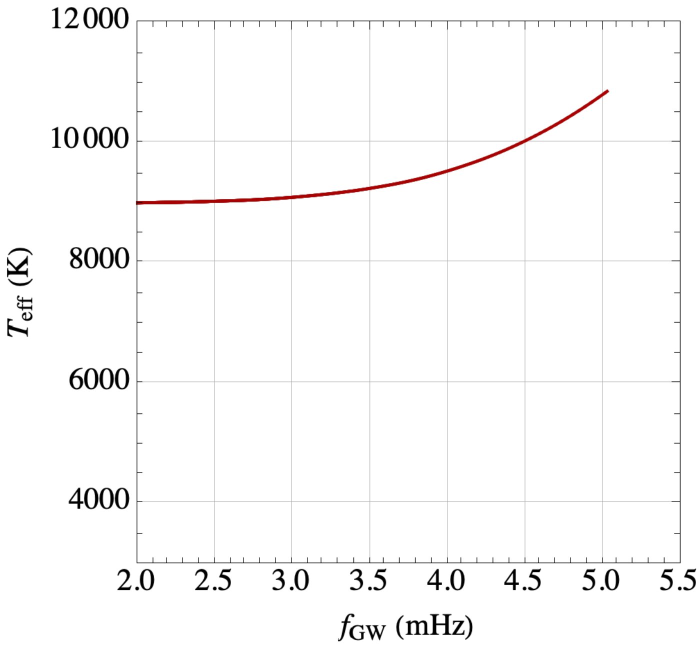
Out[481]=



In[482]:=

```
plotaT92 =
 ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Black, Red, Red}],
  PlotRange → {1000 {0.002, 0.0055}, {3000, 12 000}}, AspectRatio → 1,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f_GW (mHz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic]
```

Out[482]=



Contour plot

In[483]:=

```
m1prims = {0.32, 0.167, 0.32, 0.33, 0.28, 0.36, 0.323, 0.26, 0.21};
m2secs = {0.45, 0.652, 0.3, 0.38, 0.4, 0.36, 0.335, 0.5, 0.61};
```

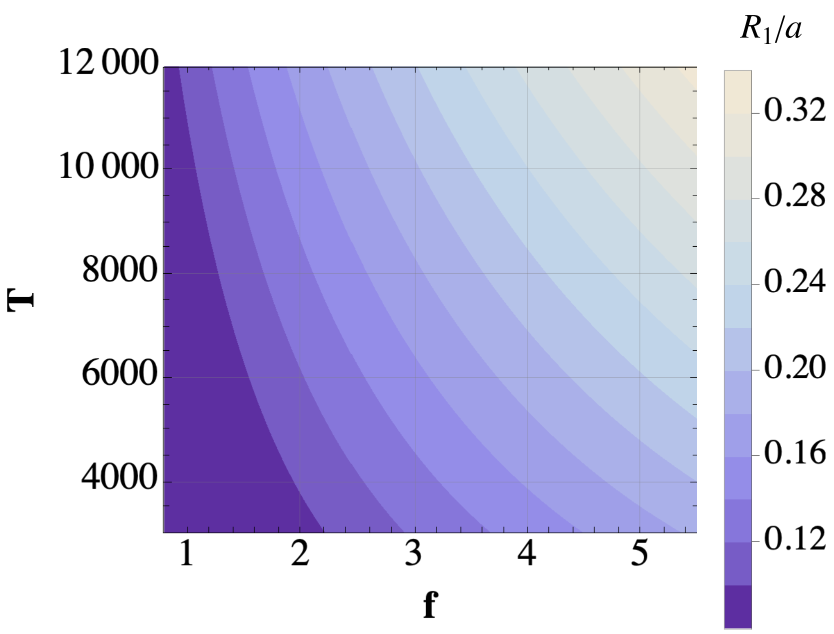
In[485]:=

```
labels = Directive[FontSize → 18, FontFamily → "Times", Black];
```

In[486]:=

```
Rrlcontour2 = ContourPlot[(Rscale[m1prims[[9]] 10, temp / 10 000] Rsol / 100) /
    (G^(1/3) ((m1prims[[9]] + m2secs[[9]]) Msol)^(1/3) / ((fgw mHz)^(2/3) π^(2/3))), {fgw, fbin, 5.5},
   {temp, 10 000 × 0.3, 10 000 × 1.2}, Contours → {0.05, 0.10, 0.12`, 0.14`,
     0.16`, 0.18`, 0.2`, 0.22`, 0.24`, 0.26`, 0.28`, 0.3`, 0.32`},
   ImageSize → Medium, ColorFunction → {"LakeColors"}, Axes → True,
   FrameLabel → {Style["f", Bold, 20, Black], Style["T", Bold, 20]},
   FrameTicksStyle → Directive[FontSize → 20], ContourStyle → None,
   ScalingFunctions → {None, None}, BaseStyle → {FontSize → 20},
   PlotLegends → Placed[BarLegend[Automatic, LegendLabel → Style["R1/a", Black],
       LabelStyle → {labels, Black}], {After, Top}],
   PlotRange → {{fbin, 5.5}, 10 000 {0.3, 1.2}}, AspectRatio → 7 / 8,
   LabelStyle → (FontFamily → "Times"), GridLines → Automatic
  ]
```

Out[486]=



J1539 upper bound

In[487]:=

```
T1prims3 = {12.8, 20, 18.25, 16.8, 12, 26, 26.3, 16.53, 28, 27.689, 27.26,
    21.288918, 21.93, 18.6938, 19.4547, 10.9988, 10.3693, 6, 6, 9.7} 1000;
Porb3 = {866.6, 1233.97, 1252.06, 1422.55, 1586.03, 2436.11, 528,
   765, 10000, 2 / (0.006168791304123794`), 2000 / (6.255216513999142`),
   2000/7.4654682414395825, 2000/7.267013989096072, 2000/7.7853741570106845,
   2000/7.588645085136682, 2000/4.99993, 2000/5.25661, 10000, 2000/2, 414.8};
fGWs3 = 2 / Porb3 × 1000;
```

In[490]:=

```
df3 = Transpose[{fGWs3, T1prims3, T1prims3}];
pts3 = df3;
```

In[492]:=

```
stylesTemp = ColorData["Pastel"] /@ Rescale[pts3[[All, 3]]]
```

Out[492]=

In[493]:=

```
Pltfuntri[ii_] := ListPlot[{pts3[[All, {1, 2}]][[ii]]},
  PlotRange → {{0.1, 10}, {6000, 40000}}, AspectRatio → 1, PlotMarkers → {"▼", 18},
  PlotStyle → {{stylesTemp[[ii]]}}, LabelStyle → (FontFamily → "Times"),
  PlotLegends → {Style["present day upper bound", 16]}, GridLines → Automatic]
```

In[494]:=

```
PltfuntriLarge[ii_] := ListPlot[{pts3[[All, {1, 2}]][[ii]]},
  PlotRange → {{0.1, 10}, {6000, 40000}}, AspectRatio → 1, PlotMarkers → {"▼", 25},
  PlotStyle → {Black}, LabelStyle → (FontFamily → "Times"),
  PlotLegends → {Style["present day upper bound", 16]}, GridLines → Automatic]
```

In[495]:=

```
tempcont = temp /.
  Solve[Rscale[mp, temp / 10000] / ((G^(1/3) ((mp + ms) Msol / 10)^(1/3)) / ((fgw mHz)^(2/3) π^(2/3)) / (Rsol / 100)) ==
    RRLval, temp][[1]]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

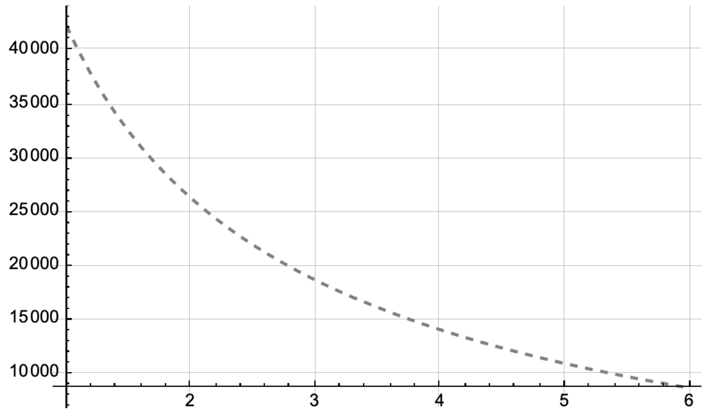
Out[495]=

$$8874.85\,\mathrm{Log}\left[\frac{8.87629139077674}{\mathrm{fgw}^{2/3}}\right]^2$$

In[496]:=

```
Rochebound2 =
 Plot[tempcont, {fgw, 1, 6}, PlotStyle → {Gray, Dashed}, GridLines → Automatic]
```

Out[496]=



all together

In[497]:=

```
Show[plotaT52, Rrlcontour2, plotaT52, plotaT42, plotaT6, plotaT7,
 plotaT8, plotaT92, PltfuntriLarge[20], Pltfuntri[20], Rochebound2]
```

Out[497]=