In[1]:= $afun = \frac{GG^{1/3}\ ((m1 + m2))^{1/3}}{(f)^{2/3}\ \pi^{2/3}};$

In[2]:= $Rsol = 6.995 \times 10^{10};$

$Msol = 2 \times 10^{33};$

$Mchirpf[m11\_, m22\_] = \frac{(m11\ m22)^{3/5}}{(m11 + m22)^{1/5}};$

$G = 6.67 \times 10^{-8};$

$c = 3 \times 10^{10}$

$k1 = 0.143;$

$Q = 5 \times 10^{8};$

$c = 3 \times 10^{10};$

$Msol = 2 \times 10^{33};$

$mHz = 0.001;$

$kK4 = 10^{4};$

$\sigma = 5.67 \times 10^{-5};$

$rg2 = 0.1;$

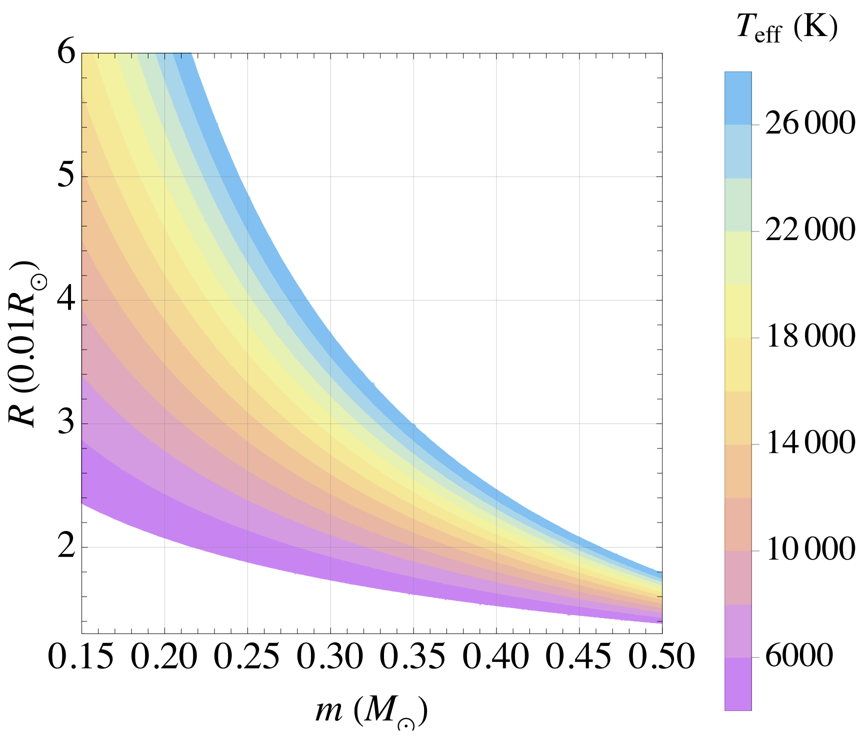
Out[6]= 30 000 000 000

Figure 1 : mass radius relation

In[15]:= labels = Directive[FontSize → 18, FontFamily → "Times", Black];

In[16]:=
```
contourprim = ContourPlot[
  (1.1798232975286564`*^47 Log[mmm]^2 - 1.4023785637137418`*^48 Log[mmm]
      Log[0.74269870382108113407122043463241581874`15.954589770191005 rr] +
     4.167288525500679`*^48
      Log[0.74269870382108113407122043463241581874`15.954589770191005 rr]^2) /
   (7.70388920852663`*^43 + 3.4482434674930335`*^44 Log[mmm] +
     3.858565034251842`*^44 Log[mmm]^2) , {mmm, 0.15, 0.5},
  {rr, 1.3, 6}, Contours → {1, 4000, 6000, 8000, 10 000, 12 000, 14 000,
    16 000, 18 000, 20 000, 22 000, 24 000, 26 000, 28 000},
  ImageSize → Medium, ColorFunction → "Pastel", Axes → True,
  FrameLabel → {Style["m  (M⊙)", 20, Black], Style["R  (0.01R⊙)", 20, Black]},
  FrameTicksStyle → Directive[FontSize → 20, Black],
  ContourStyle → None, ScalingFunctions → {None, None},
  BaseStyle → {FontSize → 20},
  PlotLegends → Placed[BarLegend[Automatic,
     LegendLabel → Style["Teff  (K)", Black], LabelStyle → labels], {After, Top}],
  PlotRange → {{0.15, 0.5}, {1.3, 6}, {4000, 28 000}},
  LabelStyle → (FontFamily → "Times"), GridLines → Automatic
 ]
```

Out[16]=

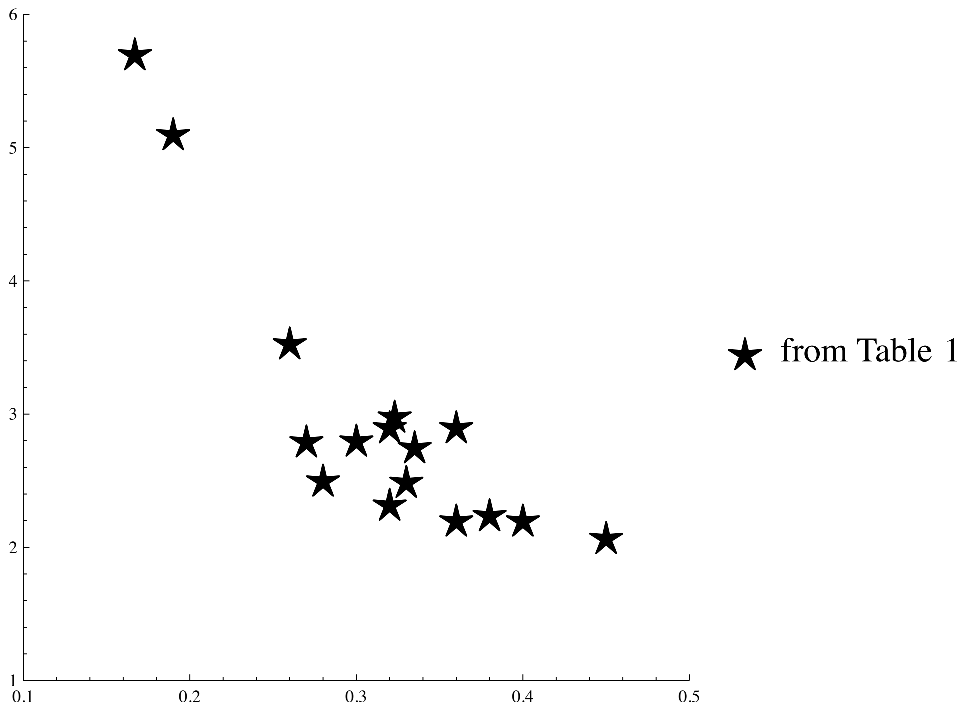


In[17]:=
```
cplot1 = ContourPlot[
   (1.1798232975286564`*^47 Log[mmm]^2 - 1.4023785637137418`*^48 Log[mmm]
       Log[0.74269870382108113407122043463241581874`15.954589770191005 rr] +
      4.167288525500679`*^48
       Log[0.74269870382108113407122043463241581874`15.954589770191005 rr]^2) /
    (7.70388920852663`*^43 + 3.4482434674930335`*^44 Log[mmm] +
      3.858565034251842`*^44 Log[mmm]^2) , {mmm, 0.15, 0.5},
   {rr, 1.3, 6}, Contours → {1, 4000, 6000, 8000, 10 000, 12 000, 14 000,
     16 000, 18 000, 20 000, 22 000, 24 000, 26 000, 28 000},
   ImageSize → Medium, ColorFunction → "Pastel", Axes → True,
   FrameLabel → {Style["m (M⊙)", Bold, 20], Style["R (0.01R⊙)", Bold, 20]},
   FrameTicksStyle → Directive[FontSize → 20],
   ContourStyle → None, ScalingFunctions → {None, None},
   LabelStyle → (FontFamily → "Times"), BaseStyle → {FontSize → 20},
   PlotLegends → Placed[BarLegend[Automatic,
      LegendLabel → Style["Teff (K)", 18], LabelStyle → labels], {After, Top}],
   PlotRange → {{0.15, 0.5}, {1.3, 3.25}, {4000, 20 000}},
   BaseStyle → Directive[Opacity[1]]];
```

In[18]:=
```
mtest = {0.32, 0.45, 0.167, 0.32, 0.3, 0.33, 0.38,
   0.28, 0.4, 0.36, 0.36, 0.323, 0.335, 0.26, 1, 1, 0.27, 0.19};
rtest = {2.319, 2.069, 5.70, 2.90, 2.80, 2.49,
   2.24, 2.5, 2.2, 2.9, 2.2, 2.98, 2.75, 3.53, 1, 1, 2.794, 5.1};
Ttest = 1000 {12.8, 26.45, 20, 18.25, 15.3, 16.8,
    19.9, 12, 20.4, 26, 16.5, 26, 19, 16.4, 28, 4, 13.4, 16.4};
df2 = Transpose[{mtest, rtest, Ttest}];
pts2 = df2;
Graphics[{AbsoluteThickness[3], Point[pts2[[All, {1, 2}]],
    VertexColors → ColorData["Pastel"] /@ Rescale[pts2[[All, 3]]]]},
  AspectRatio → 1, Frame → True];
stylesTemp = ColorData["Pastel"] /@ Rescale[pts2[[All, 3]]];
Pltfun[ii_] := ListPlot[{pts2[[All, {1, 2}]][[ii]]},
   PlotRange → {{0.1, 1}, {1, 6}}, AspectRatio → 1, PlotMarkers → {"★", 18},
   PlotStyle → {{stylesTemp[[ii]]}}, LabelStyle → (FontFamily → "Times"),
   PlotLegends → {Style["R (m,Teff) of detached WD", 16]}];
```

In[26]:=
```
outline = ListPlot[pts2⟦All, {1, 2}⟧, PlotRange → {{0.1, 0.5}, {1, 6}},
  AspectRatio → 1, PlotMarkers → {"★", 24}, PlotStyle → {{Black}},
  PlotLegends → {Style["from Table 1", 18]}, LabelStyle → (FontFamily → "Times")]
```

Out[26]=



In[27]:=
```
outline = ListPlot[pts2⟦All, {1, 2}⟧, PlotRange → {{0.1, 0.5}, {1, 6}},
   AspectRatio → 1, PlotMarkers → {"★", 24}, PlotStyle → {{Black}},
   PlotLegends → {Style["from Table 1", 18, Bold]},
   LabelStyle → (FontFamily → "Times")];
```

In[28]:=
```
Show[outline, Pltfun[1], Pltfun[2], Pltfun[3], Pltfun[4], Pltfun[5], Pltfun[6],
  Pltfun[7], Pltfun[8], Pltfun[9], Pltfun[10], Pltfun[11], Pltfun[12],
  Pltfun[13], Pltfun[14], Pltfun[15], Pltfun[16], Pltfun[17], Pltfun[18]];
```

In[29]:=
```
Show[ListPlot[pts2⟦All, {1, 2}⟧, PlotRange → {{0.9, 1.2}, {0.5, 6}},
   AspectRatio → 1, PlotMarkers → {"★", 24}, PlotStyle → {{Black}}, PlotLegends →
    {Style["from Table 1", 18]}, LabelStyle → (FontFamily → "Times")],
  Pltfun[1], Pltfun[2], Pltfun[3], Pltfun[4], Pltfun[5], Pltfun[6],
  Pltfun[7], Pltfun[8], Pltfun[9], Pltfun[10], Pltfun[11], Pltfun[12],
  Pltfun[13], Pltfun[14], Pltfun[15], Pltfun[16], Pltfun[17], Pltfun[18]];
```

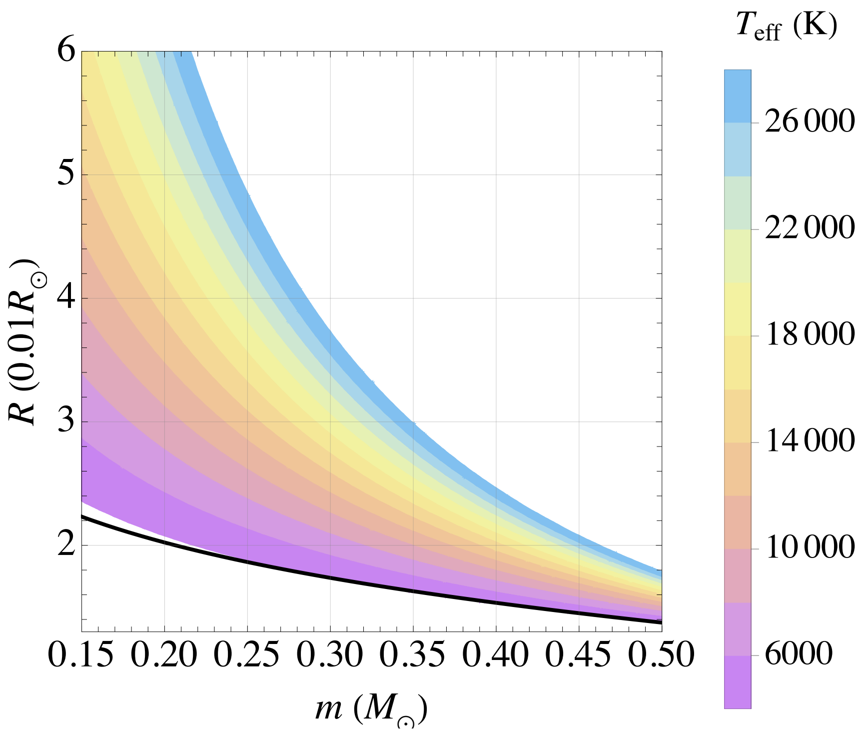
In[30]:=
```
Regg[m_] := 0.0114 ((m / 1.44)^(-2/3) - (m / 1.44)^(2/3))^(1/2)
  (1 + 3.5 (m / (5.7 × 10^-4))^(-2/3) + ((5.7 × 10^-4) / m))^(-2/3)
```

In[31]:=
```
plotegg = Plot[100 Regg[m], {m, 0.15, 0.5}, AspectRatio → 1,
   AxesLabel → {Style["mass (solar)", 16], Style["radius (solar)", 16]},
   BaseStyle → {FontSize → 15}, LabelStyle → (FontFamily → "Times"),
   PlotStyle → {Black, Thick}, PlotRange → {{0.15, 0.5}, {100 × 0.013, 100 × 0.06}},
   PlotLegends → {Style["R(m)", 16, Italic]},
   FrameLabel → {Style["mᵢ (M⊙)", 16], Style["Rᵢ (R⊙)", 16],
     Style["White dwarf masses and radii", Bold, 16], Style["Rᵢ (10^8cm)", 16]},
   BaseStyle → {FontSize → 15}, LabelStyle → (FontFamily → "Times"),
   Frame → True, FrameTicks → {{{.1, .2, .3, .4, .5}, {Automatic},
      ChartingScaledTicks[{# / (Rsol / 10^8) &, Rsol / 10^8 # &}]}}];
```

In[32]:=
```
Plot[100 Regg[m], {m, 0.15, 0.5}, AspectRatio → 1,
  AxesLabel → {Style["mass (solar)", 16], Style["radius (solar)", 16]},
  BaseStyle → {FontSize → 15}, LabelStyle → (FontFamily → "Times"),
  PlotStyle → {Black, Thick}, PlotRange → {{0.15, 0.5}, {100 × 0.013, 100 × 0.06}},
  PlotLegends → {Style[" \"cold\" ", 16]},
  FrameLabel → {Style["mᵢ (M⊙)", 16], Style["Rᵢ (R⊙)", 16],
    Style["White dwarf masses and radii", Bold, 16], Style["Rᵢ (10^8cm)", 16]},
  BaseStyle → {FontSize → 15}, LabelStyle → (FontFamily → "Times"),
  Frame → True, FrameTicks → {{{.1, .2, .3, .4, .5}, {Automatic},
     ChartingScaledTicks[{# / (Rsol / 10^8) &, Rsol / 10^8 # &}]}}];
```

In[33]:=
```
Show[contourprim, plotegg]
```

Out[33]=



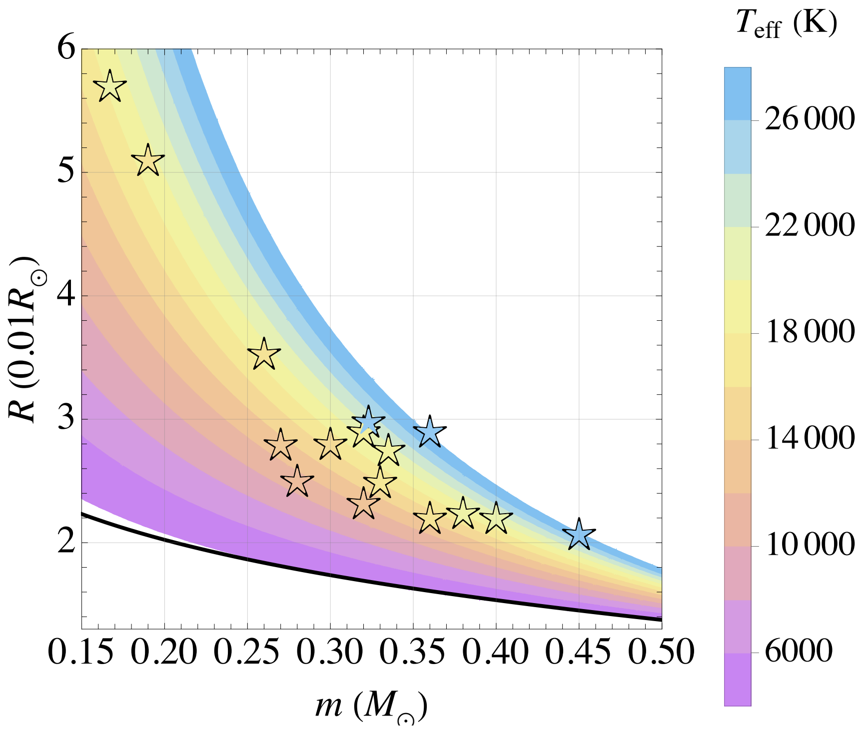
In[34]:=
```
Show[outline, Pltfun[1], Pltfun[2], Pltfun[3], Pltfun[4], Pltfun[5], Pltfun[6],
  Pltfun[7], Pltfun[8], Pltfun[9], Pltfun[10], Pltfun[11], Pltfun[12],
  Pltfun[13], Pltfun[14], Pltfun[15], Pltfun[16], Pltfun[17], Pltfun[18]];
```

In[35]:=
```
contourprimempty = ContourPlot[
   0 (1.1798232975286564`*^47 Log[mmm]^2 - 1.4023785637137418`*^48 Log[mmm]
        Log[0.74269870382108113407122043463241581874`15.954589770191005 rr] +
       4.167288525500679`*^48
        Log[0.74269870382108113407122043463241581874`15.954589770191005 rr]^2) /
    (7.70388920852663`*^43 + 3.4482434674930335`*^44 Log[mmm] +
       3.858565034251842`*^44 Log[mmm]^2) , {mmm, 0.15, 0.5},
   {rr, 1.3, 6}, Contours → {1, 4000, 6000, 8000, 10 000, 12 000, 14 000,
     16 000, 18 000, 20 000, 22 000, 24 000, 26 000, 28 000},
   ImageSize → Medium, ColorFunction → "Pastel", Axes → True,
   FrameLabel → {Style["m (M⊙)", Bold, 20], Style["R (0.01R⊙)", Bold, 20]},
   FrameTicksStyle → Directive[FontSize → 20],
   ContourStyle → None, ScalingFunctions → {None, None},
   BaseStyle → {FontSize → 20},
   PlotLegends → Placed[BarLegend[Automatic,
      LegendLabel → Style["Teff (K)", Bold], LabelStyle → labels], {After, Top}],
   PlotRange → {{0.15, 0.5}, {1.3, 6}, {4000, 28 000}},
   LabelStyle → (FontFamily → "Times")
  ];
```

In[36]:=
```
Show[contourprimempty, plotegg, outline, Pltfun[1], Pltfun[2],
  Pltfun[3], Pltfun[4], Pltfun[5], Pltfun[6], Pltfun[7], Pltfun[8],
  Pltfun[9], Pltfun[10], Pltfun[11], Pltfun[12], Pltfun[13],
  Pltfun[14], Pltfun[15], Pltfun[16], Pltfun[17], Pltfun[18]];
```

In[37]:=
```
Show[contourprim, plotegg, outline, Pltfun[1], Pltfun[2],
 Pltfun[3], Pltfun[4], Pltfun[5], Pltfun[6], Pltfun[7], Pltfun[8],
 Pltfun[9], Pltfun[10], Pltfun[11], Pltfun[12], Pltfun[13],
 Pltfun[14], Pltfun[15], Pltfun[16], Pltfun[17], Pltfun[18]]
```

Out[37]=



In[38]:=
```
Show[ListPlot[pts2[[All, {1, 2}]], PlotRange → {{0.9, 1.2}, {0.5, 6}},
  AspectRatio → 1, PlotMarkers → {"★", 24}, PlotStyle → {{Black}},
  PlotLegends → {Style["Panei+2000: R(m,T_eff) ∝ m^(T_eff^1/2)", 18]},
  LabelStyle → (FontFamily → "Times")]]
```

Out[38]=

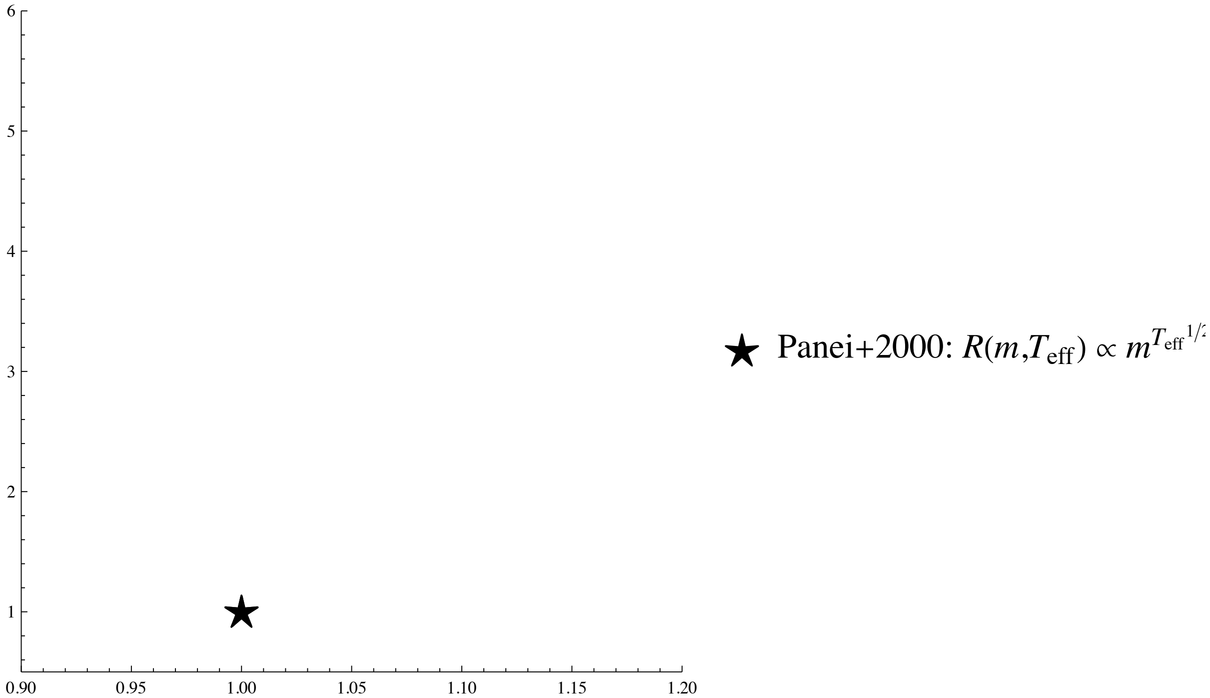


Figure 2 : tidal heating vs cooling regime

In[39]:=
$$\text{fdotTD1}[m1\_, m2\_, fGW\_, R1\_] = \frac{18\, fGW^{13/3}\ m2\ \pi^{13/3}\, R1^5 \left(\frac{fGW}{2}\right)}{G^{5/3}\, m1\, (m1+m2)^{5/3}}\ konQ1$$

Out[39]=
$$\frac{1.17044\times 10^{15}\, fGW^{16/3}\, konQ1\, m2\, R1^5}{m1\, (m1+m2)^{5/3}}$$

In[40]:=
$$\text{fdotGW}[m1\_, m2\_, fGW\_] = \frac{96\,\pi^{8/3}\, G^{5/3}\, (fGW)^{11/3}\, (\text{Mchirpf}[m1, m2])^{5/3}}{5\, c^5}\ //\ \text{FullSimplify}$$

Out[40]=
$$1.83501\times 10^{-62}\, fGW^{11/3}\left(\frac{(m1\, m2)^{3/5}}{(m1+m2)^{1/5}}\right)^{5/3}$$

In[41]:=
```
ratiosol = konQ1 /. Solve[fdotTD1[0.21 Msol, 0.61 Msol, 0.0048, 0.0314 Rsol] ==
     0.1 fdotGW[0.21 Msol, 0.61 Msol, 0.0048] , konQ1][[1]]
```

Out[41]=
$$7.6605\times 10^{-12}$$

In[42]:=
$$\text{RPanei}[m\_, T\_] := 0.0132\times 10^{-0.00177\, T^{1/2}}\, m^{0.148-0.00941\, T^{1/2}}$$

```
In[43]:= Rscale[m1a_, T1a_] :=
          10^(-0.02792426461145596`+0.7641778013995925` Sqrt[T1a]) m1a^(0.14797691065884058`-0.9408955042478873` Sqrt[T1a])

In[44]:= kQratio = 8 × 10^-12;

In[45]:= WDid = {J0538, J0533, J2029, J0722, J1749, J1901, J2243, J0651, J1539};
         m1prims = {0.32, 0.167, 0.32, 0.33, 0.28, 0.36, 0.323, 0.26, 0.21};
         T1prims = {12.8, 20, 18.25, 16.8, 12, 26, 26.3, 16.53, 10} 1000;
         m2secs = {0.45, 0.652, 0.3, 0.38, 0.4, 0.36, 0.335, 0.5, 0.61};
         Porb = {866.6, 1233.97, 1252.06, 1422.55, 1586.03, 2436.11, 528, 765, 414.8};
         fGWs = 2 / Porb
Out[50]=
         {0.00230787, 0.00162078, 0.00159737,
          0.00140593, 0.00126101, 0.000820981, 1/264, 2/765, 0.0048216}

In[51]:= Rscale[m1prims 10, T1prims / 10 000]
Out[51]=
         {2.36408, 6.1573, 2.73582, 2.55237, 2.59701, 2.77078, 3.23396, 3.26576, 3.02524}

In[52]:= fGWRL = (2^(3/2) / (9 π)) (G m1prims Msol / (Rscale[m1prims 10, T1prims / 10 000] Rsol / 100)^3)^(1/2)
Out[52]=
         {0.00971922, 0.0016704, 0.00780715, 0.00879815,
          0.00789618, 0.00812451, 0.00610308, 0.00539583, 0.005439}

In[53]:= Rscale[m1prims 10, T1prims / 10 000]
Out[53]=
         {2.36408, 6.1573, 2.73582, 2.55237, 2.59701, 2.77078, 3.23396, 3.26576, 3.02524}

In[54]:= RPanei
Out[54]=
         RPanei

In[55]:= RPanei[m1prims, T1prims]
Out[55]=
         {0.0236543, 0.0616041, 0.0273707, 0.025536,
          0.0259858, 0.0277161, 0.0323496, 0.0326745, 0.0302729}

In[56]:= Rscale[m1prims 10, T1prims / 10 000]
Out[56]=
         {2.36408, 6.1573, 2.73582, 2.55237, 2.59701, 2.77078, 3.23396, 3.26576, 3.02524}

In[57]:= testZ = 0.02;
         Atest = 4;
         Lsol = 3.826 × 10^33;
         Tcold = 4000;

In[61]:= L2a[m1_, t_, Z_] := 300 m1 Z^0.4 / (Atest (t + 0.1))^1.18;
```

In[62]:=
```
L2b[m1_, t_, Z_] := (300 (9000 Atest)^5.3 m1 Z^0.4)/(Atest (t + 0.1))^6.48;
```

In[63]:=
```
τcools2 =
  (t /. Table[NSolve[7.1 × 10^-4 (Rsol RPanei[m1prims[[i]], Tcold])^2 Tcold^4 == Piecewise[
        {{L2a[m1prims[[i]], t, testZ], t < 9000}, {L2b[m1prims[[i]], t, testZ],
          t > 9000}}] Lsol, t], {i, 1, 9}]) // Flatten
```

NSolve : NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

NSolve : NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

NSolve : NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

General : Further output of NSolve::ratnz will be suppressed during this calculation.

Out[63]=
```
{9325.86, 3851.99, 9325.86, 9410.13, 8830.64, 9652.56, 9351.34, 7840.16, 5564.45}
```

In[64]:=
```
RPanei[m1prims[[2]], T1prims[[2]]]
```

Out[64]=
```
0.0616041
```

In[65]:=
```
fGWs
```

Out[65]=
```
{0.00230787, 0.00162078, 0.00159737,
 0.00140593, 0.00126101, 0.000820981, 1/264, 2/765, 0.0048216}
```

In[66]:=
```
τmergeTDfixMyr2 =
  Table[(2/3 × 2/18 ((G^(5/3) m1prims[[i]] (m1prims[[i]] Msol + m2secs[[i]] Msol)^(5/3) kQratio^-1) /
        (fGWs[[i]]^(13/3) m2secs[[i]] π^(13/3) (Rsol / 100 Rscale[m1prims[[i]] 10,
              T1prims[[i]] / 10 000])^5))) / (3.15 × 10^7 × 10^6), {i, 1, 9}]
```

Out[66]=
```
{711.981, 10.9677, 1766.42, 4434.85, 4887.26, 35 675.7, 18.1169, 58.9895, 4.58024}
```

In[67]:=
```
τmergers = Table[
   Integrate[((96 π^(8/3) G^(5/3))/(5 c^5) (Mchirpf[m1prims[[i]] Msol, m2secs[[i]] Msol])^(5/3) f^(11/3))^-1,
     {f, fGWs[[i]], 100}], {i, 1, 9}] / (3.146 × 10^7 × 10^6)
```

Out[67]=
```
{1.40009, 4.85044, 5.21232, 5.86778, 8.6554, 23.9443, 0.471726, 1.10729, 0.225319}
```

In[68]:=
$$\tau RL = \text{Table}\left[\text{Integrate}\left[\left(\frac{96\,\pi^{8/3}\,G^{5/3}}{5\,c^5}\,(\text{Mchirpf}[\text{m1prims}[\![i]\!]\ \text{Msol},\ \text{m2secs}[\![i]\!]\ \text{Msol}])^{5/3}\,f^{11/3}\right)^{-1},\ \{f,\ \text{fGWs}[\![i]\!],\ \text{fGWRL}[\![i]\!]\}\right],\ \{i,1,9\}\right] \Big/ \left(3.146\times 10^{7}\times 10^{3}\right)$$

Out[68]=

```
{1369.82, 374.761, 5136.56, 5823.65, 8590.42, 23891.3, 339.511, 946.937, 61.9168}
```

In[69]:=
$$\text{Table}\left[\text{Integrate}\left[\left(\frac{96\,\pi^{8/3}\,G^{5/3}}{5\,c^5}\,(\text{Mchirpf}[\text{m1prims}[\![i]\!]\ \text{Msol},\ \text{m2secs}[\![i]\!]\ \text{Msol}])^{5/3}\,f^{11/3}\right)^{-1},\ \{f,\ \text{fGWs}[\![i]\!],\ \text{fGWRL}[\![i]\!]\}\right],\ \{i,1,9\}\right] \Big/ \left(3.146\times 10^{7}\times 10^{6}\right)$$

Out[69]=

```
{1.36982, 0.374761, 5.13656, 5.82365,
 8.59042, 23.8913, 0.339511, 0.946937, 0.0619168}
```

In[70]:=
$$\text{Table}\left[\text{Integrate}\left[\left(\frac{96\,\pi^{8/3}\,G^{5/3}}{5\,c^5}\,(\text{Mchirpf}[\text{m1prims}[\![i]\!]\ \text{Msol},\ \text{m2secs}[\![i]\!]\ \text{Msol}])^{5/3}\,f^{11/3}\right)^{-1},\ \{f,\ \text{fGWs}[\![i]\!],\ 100\}\right],\ \{i,1,9\}\right] \Big/ \left(3.146\times 10^{7}\times 10^{6}\right)$$

Out[70]=

```
{1.40009, 4.85044, 5.21232, 5.86778, 8.6554, 23.9443, 0.471726, 1.10729, 0.225319}
```

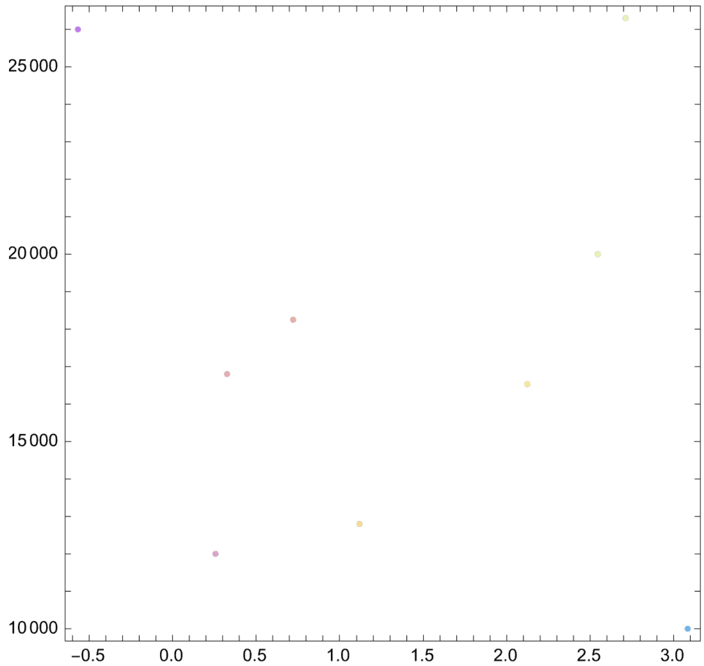
In[71]:=

```
df1 = Transpose[{-Log10[τmergeTDfixMyr2 / τcools2], T1prims, -Log[τRL]}]
```

Out[71]=

```
{{1.11722, 12800., -7.22243},
 {2.54557, 20000, -5.92629}, {0.722596, 18250., -8.54414},
 {0.326716, 16800., -8.66968}, {0.256927, 12000, -9.0584},
 {-0.56773, 26000, -10.0813}, {2.71279, 26300., -5.82751},
 {2.12355, 16530., -6.85323}, {3.08453, 10000, -4.12579}}
```

In[72]:=
```
pts2 = df1
Graphics[{AbsoluteThickness[3], Point[pts2[[All, {1, 2}]],
   VertexColors → ColorData["Pastel"] /@ Rescale[pts2[[All, 3]]]]},
 AspectRatio → 1, Frame → True]
```
Out[72]=
```
{{1.11722, 12 800., -7.22243},
 {2.54557, 20 000, -5.92629}, {0.722596, 18 250., -8.54414},
 {0.326716, 16 800., -8.66968}, {0.256927, 12 000, -9.0584},
 {-0.56773, 26 000, -10.0813}, {2.71279, 26 300., -5.82751},
 {2.12355, 16 530., -6.85323}, {3.08453, 10 000, -4.12579}}
```
Out[73]=



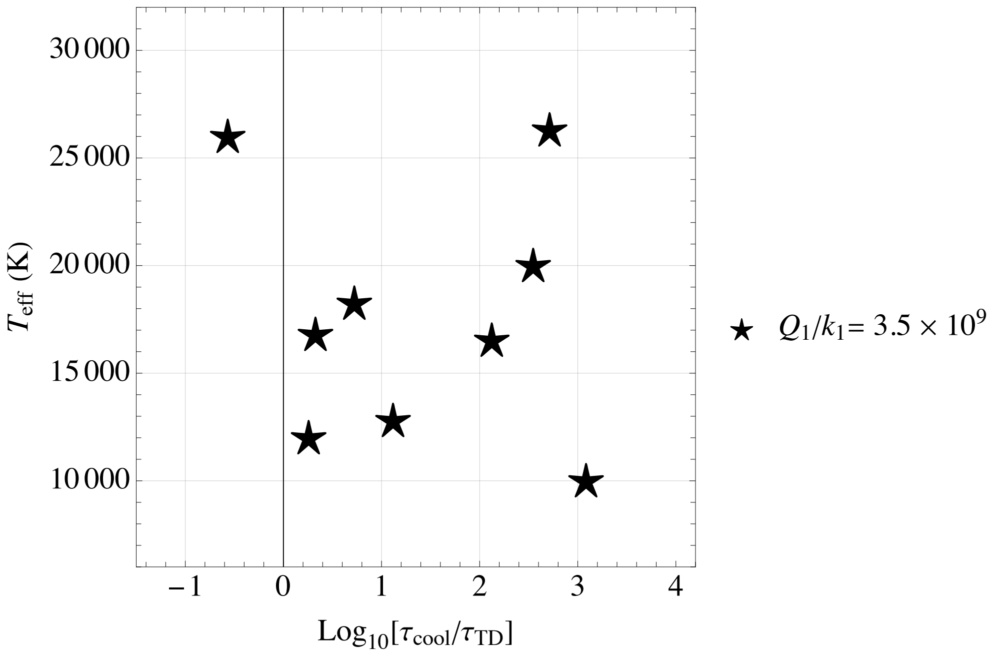
In[74]:=
```
stylesTemp = ColorData["CMYKColors"] /@ Rescale[pts2[[All, 3]]]
```
Out[74]=

In[75]:=
```
stylesTemp = ColorData["AvocadoColors"] /@ Rescale[pts2[[All, 3]]]
```
Out[75]=

In[76]:=
```
Pltfun[ii_] :=
 ListPlot[{pts2[[All, {1, 2}]][[ii]]}, PlotRange → {{-1.5, 4.2}, {6000, 32 000}},
  AspectRatio → 1, PlotMarkers → {"★", 18},
  PlotStyle → {{stylesTemp[[ii]]}}, LabelStyle → (FontFamily → "Times")]
```

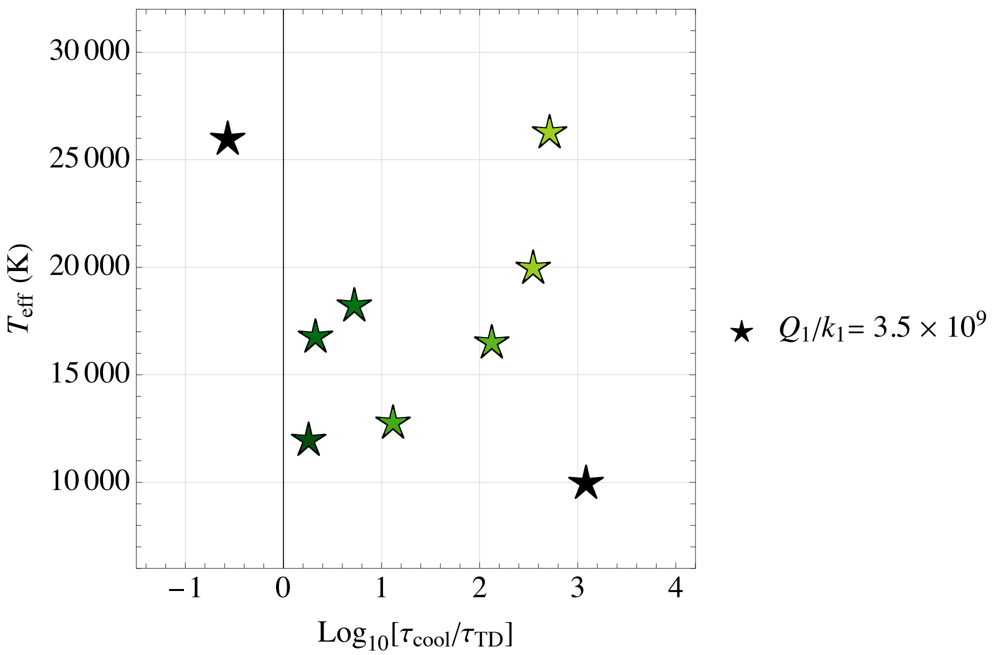
In[77]:=
```
br2 = ListPlot[Transpose[{-Log10[τmergeTDfixMyr2 / τcools2], T1prims}],
  AspectRatio → 1, PlotMarkers → {"★", 25}, PlotStyle → {{Black}, "★"},
  Frame → True, LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["Log10[τcool/τTD]"], Style["Teff (K)", 16]},
  BaseStyle → {FontSize → 16}, PlotRange → {{-1.5, 4.2}, {6000, 32 000}},
  PlotLegends → {Style["Q1/k1= 3.5 × 10^9", 16]}, GridLines → Automatic]
```

Out[77]=



In[78]:=
```
Show[br2, Pltfun[1], Pltfun[2], Pltfun[3],
 Pltfun[4], Pltfun[5], Pltfun[6], Pltfun[7], Pltfun[8]]
```

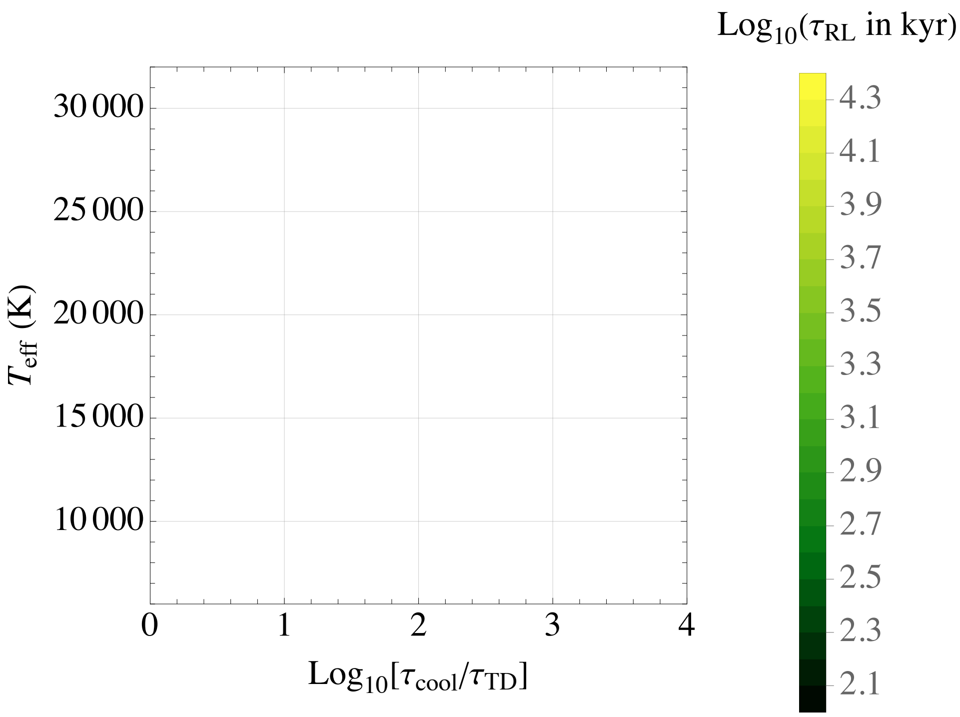
Out[78]=



```
In[79]:= WDid
Out[79]=
{J0538, J0533, J2029, J0722, J1749, J1901, J2243, J0651, J1539}

In[80]:= (τmergeTDfixMyr2 / τcools2)^-1
Out[80]=
{13.0985, 351.211, 5.27954, 2.12186, 1.80687, 0.270564, 516.166, 132.908, 1214.88}

In[81]:= Log10[Min[τRL]]
Out[81]=
1.79181

In[82]:= Log10[Max[τRL]]
Out[82]=
4.37824

In[83]:= τRL
Out[83]=
{1369.82, 374.761, 5136.56, 5823.65, 8590.42, 23 891.3, 339.511, 946.937, 61.9168}

In[84]:= labels = Directive[FontSize → 18, FontFamily → "Times"];
```

In[85]:=
```
cptrack =
 ContourPlot[ tscale, {ratio, 0, 4}, {tscale, Min[Log10[τRL]], Max[Log10[τRL]]},
  Contours → Table[(i + 1) 0.1, {i, 0, 43}], ImageSize → Medium,
  ColorFunction → (ColorData["AvocadoColors"]),
  Axes → True, FrameTicksStyle → Directive[FontSize → 18],
  ContourStyle → None, ScalingFunctions → {None, None, None},
  PlotLegends → Placed[BarLegend[Automatic, LegendLabel →
      Style["Log10(τRL in kyr)", 18], LabelStyle → labels], {After, Top}],
  PlotRange → {{0, 4}, {6000, 32 000}, {2, 4.5}}, Frame → True,
  LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["Log10[τcool/τTD]", 18], Style["Teff (K)", 18]},
  GridLines → Automatic, FrameStyle → Automatic]
```

Out[85]=

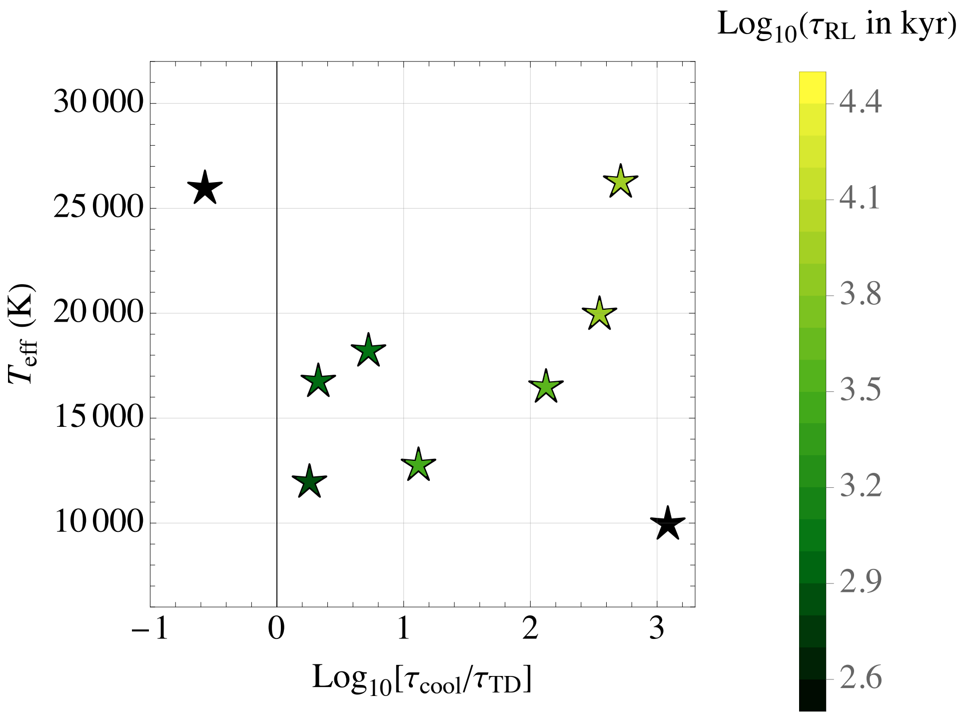


In[86]:=
```
cptrack = ContourPlot[ tscale, {ratio, 0, 4},
   {tscale, 2, 5}, Contours → Table[(i + 1) 0.1, {i, 0, 43}],
   ImageSize → Medium, ColorFunction → (ColorData["AvocadoColors"]),
   Axes → True, FrameTicksStyle → Directive[FontSize → 18],
   ContourStyle → None, ScalingFunctions → {None, None, None},
   PlotLegends → Placed[BarLegend[Automatic, LegendLabel →
       Style["Log10(τRL in kyr)", 18], LabelStyle → labels], {After, Top}],
   PlotRange → {{0, 4}, {6000, 32 000}, {2.5, 4.5}}, Frame → True,
   LabelStyle → (FontFamily → "Times"),
   FrameLabel → {Style["Log10[τcool/τTD]", 18], Style["Teff (K)", 18]},
   GridLines → Automatic, FrameStyle → Automatic];
```

In[87]:=
```
cptrack = ContourPlot[ tscale, {ratio, 0, 4},
    {tscale, 2.5, 4.5}, Contours → Table[(i + 1) 0.1, {i, 0, 43}],
    ImageSize → Medium, ColorFunction → (ColorData["AvocadoColors"]),
    Axes → True, FrameTicksStyle → Directive[FontSize → 18, Black],
    ContourStyle → None, ScalingFunctions → {None, None, None},
    PlotLegends → Placed[BarLegend[Automatic, LegendLabel →
        Style["Log10(τRL in kyr)", 18], LabelStyle → labels], {After, Top}],
    PlotRange → {{0 - 1, 3.3}, {6000, 32 000}, {2.5, 4.5}}, Frame → True,
    LabelStyle → (FontFamily → "Times"), FrameLabel →
     {Style["Log10[τcool/τTD]", 18, Black], Style["Teff (K)", 18, Black]},
    GridLines → Automatic, FrameStyle → Automatic];
```

In[88]:=
```
Show[cptrack, br2, Pltfun[1], Pltfun[2], Pltfun[3],
 Pltfun[4], Pltfun[5], Pltfun[6], Pltfun[7], Pltfun[8]]
```

Out[88]=



In[89]:=
```
fGWs
```

Out[89]=

$\{0.00230787, 0.00162078, 0.00159737, 0.00140593, 0.00126101, 0.000820981, \frac{1}{264}, \frac{2}{765}, 0.0048216\}$

In[90]:=
```
τfricTD =
 Table[(1/9 ((G^(5/3) m1prims[[i]] (m1prims[[i]] Msol + m2secs[[i]] Msol)^(5/3) kQratio^-1) /
      (fGWs[[i]]^(13/3) m2secs[[i]] π^(13/3) (Rsol / 100 Rscale[m1prims[[i]] 10,
            T1prims[[i]] / 10 000])^5))) / (3.15 × 10^7 × 10^6), {i, 1, 9}]
```

Out[90]=

{1067.97, 16.4516, 2649.62, 6652.28, 7330.89, 53 513.5, 27.1754, 88.4843, 6.87036}

In[91]:=
```
τTempTD =
 Table[(1/9 ((G^(5/3) m1prims[[i]] (m1prims[[i]] Msol + m2secs[[i]] Msol)^(5/3) kQratio^-1) /
      (fGWs[[i]]^(13/3) m2secs[[i]] π^(13/3) (Rsol / 100 Rscale[m1prims[[i]] 10,
            T1prims[[i]] / 10 000])^5))) / (3.15 × 10^7 × 10^6), {i, 1, 9}]
```

Out[91]=

{1067.97, 16.4516, 2649.62, 6652.28, 7330.89, 53 513.5, 27.1754, 88.4843, 6.87036}

In[92]:=
```
τmergeTDfixMyr2
```

Out[92]=

{711.981, 10.9677, 1766.42, 4434.85, 4887.26, 35 675.7, 18.1169, 58.9895, 4.58024}

In[93]:=
```
(1/9 ((G^(5/3) 0.21 Msol (0.21 Msol + 0.61 Msol)^(5/3) kQratio^-1) / ((0.0048)^(13/3) (0.61 Msol)
      π^(13/3) (Rsol / 100 Rscale[0.21 × 10, 10 000 / 10 000])^5))) / (3.15 × 10^7 × 10^6)
```

Out[93]=

7.00535

In[94]:=
```
τcool4
```

Out[94]=

τcool4

In[95]:=
```
τcools2
```

Out[95]=

{9325.86, 3851.99, 9325.86, 9410.13, 8830.64, 9652.56, 9351.34, 7840.16, 5564.45}

In[96]:=
```
T1prims
```

Out[96]=

{12 800., 20 000, 18 250., 16 800., 12 000, 26 000, 26 300., 16 530., 10 000}

In[97]:=
```
Tcold
```

Out[97]=

4000

In[98]:=
```
τcools3 = (t /. Table[NSolve[7.1 × 10^-4 (Rsol RPanei[m1prims[[i]], T1prims[[i]] / 2.71])^2
      (T1prims[[i]] / 2.71)^4 == Piecewise[{{L2a[m1prims[[i]], t, testZ], t < 9000},
        {L2b[m1prims[[i]], t, testZ], t > 9000}}] Lsol, t], {i, 1, 9}]) // Flatten
```

NSolve : NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

NSolve : NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

NSolve : NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

General : Further output of NSolve::ratnz will be suppressed during this calculation.

Out[98]=
```
{5855.89, 295.732, 1513.94, 2203.07, 5986., 491.701, 371.863, 1510.42, 7650.54}
```

In[99]:=
```
τcools3 / τfricTD
```

Out[99]=
```
{5.48319, 17.9759, 0.571379, 0.331175,
 0.816544, 0.00918836, 13.6838, 17.0699, 1113.56}
```

In[100]:=
```
Log10[τcools2 / τfricTD]
```

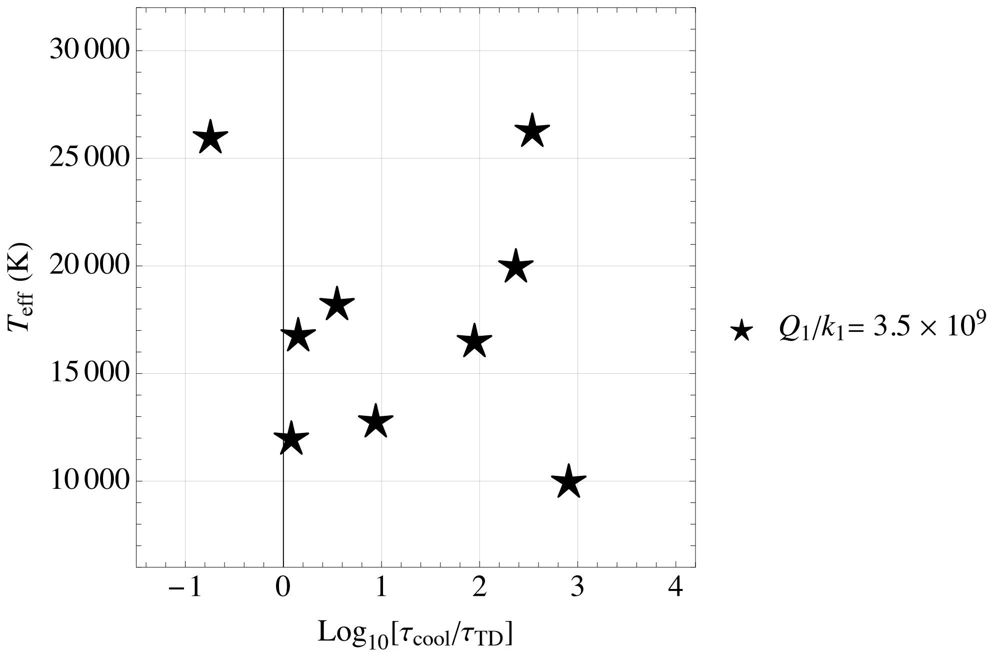
Out[100]=
```
{0.941129, 2.36948, 0.546505, 0.150625,
 0.0808355, -0.743821, 2.5367, 1.94746, 2.90844}
```

In[101]:=
```
br3 = ListPlot[Transpose[{-Log10[τfricTD / τcools3], T1prims}],
   AspectRatio → 1, PlotMarkers → {"★", 25}, PlotStyle → {{Black}, "*"},
   Frame → True, LabelStyle → (FontFamily → "Times"),
   FrameLabel → {Style["Log10[τcool/τTD]"], Style["Teff (K)", 16]},
   BaseStyle → {FontSize → 16}, PlotRange → {{-0.6, 2}, {6000, 32 000}},
   PlotLegends → {Style["Q1/k1= 3.5 × 10^9", 16]}, GridLines → Automatic];
```

In[102]:=

```
br4 = ListPlot[Transpose[{-Log10[τfricTD / τcools2], T1prims}],
  AspectRatio → 1, PlotMarkers → {"★", 25}, PlotStyle → {{Black}, "*"},
  Frame → True, LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["Log10[τcool/τTD]"], Style["Teff (K)", 16]},
  BaseStyle → {FontSize → 16}, PlotRange → {{-1.5, 4.2}, {6000, 32 000}},
  PlotLegends → {Style["Q1/k1= 3.5 × 10^9", 16]}, GridLines → Automatic]
```

Out[102]=



In[103]:=

```
τcools2
```

Out[103]=

```
{9325.86, 3851.99, 9325.86, 9410.13, 8830.64, 9652.56, 9351.34, 7840.16, 5564.45}
```

In[104]:=

```
list1grey = Transpose[{{-2, 0}, {0, 0}}];
list2grey = Transpose[{{-2, 0}, {400 000, 400 000}}];
```

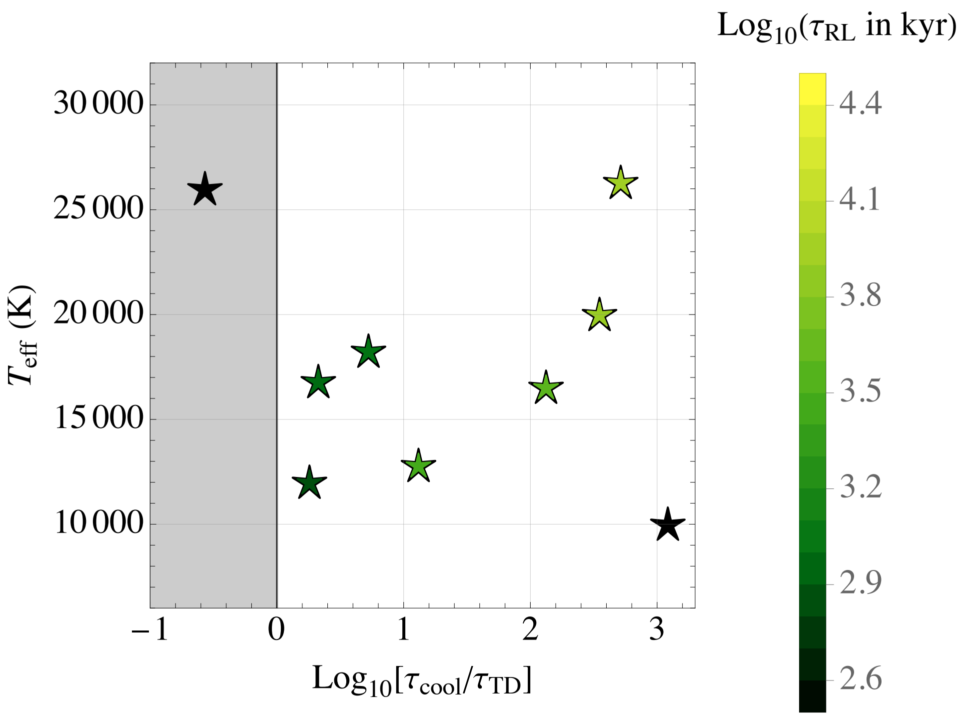
In[106]:=

```
plotgrey =
  ListPlot[{list2grey, list1grey}, Mesh → All, PlotMarkers → None, Joined → True,
   Filling → {1 → {2}}, FillingStyle → {Blend[{Gray, Gray, Black}], Opacity[.3]},
   PlotStyle → {{Blend[{Gray, Gray, Black}], Opacity[0.5]}},
   PlotRange → {{-2, 4}, {0, 222 000}}, AspectRatio → 1, Frame → True,
   LabelStyle → (FontFamily → "Times"), GridLines → Automatic,
   PlotLegends → {Style["J0651 primary (Ω0=0)", 16]}];
```

In[107]:=

```
Show[cptrack, plotgrey, br2, Pltfun[1], Pltfun[2],
 Pltfun[3], Pltfun[4], Pltfun[5], Pltfun[6], Pltfun[7], Pltfun[8]]
```

Out[107]=



In[108]:=

```
Correlation[-Log10[τmergeTDfixMyr2 / τcools2], T1prims]
```

Out[108]=

```
-0.130405
```

In[109]:=

```
-Log10[τmergeTDfixMyr2 / τcools2]
```

Out[109]=

```
{1.11722, 2.54557, 0.722596, 0.326716,
 0.256927, -0.56773, 2.71279, 2.12355, 3.08453}
```

In[110]:=

```
T1prims
```

Out[110]=

```
{12 800., 20 000, 18 250., 16 800., 12 000, 26 000, 26 300., 16 530., 10 000}
```

In[111]:=

```
Correlation[{1.1055364280752897`,
  2.5373372402967753`, 0.7109120374281895`, 0.3150321837678671`,
  0.24869520222317304`, 2.70110493365627`, 2.1153183985579185`},
 {12800.`, 20 000, 18250.`, 16800.`, 12 000, 26300.`, 16530.`}]
```

Out[111]=

```
0.716418
```

In[112]:=

```
J1539τ =
 (2/3 × 2/18 ((G^(5/3) 0.21 (0.21 Msol + 0.61 Msol)^(5/3) kQratio^-1) / (0.0048^(13/3) × 0.61 π^(13/3)
      (Rsol / 100 Rscale[0.21 × 10, 10 000 / 10 000])^5))) / (3.15 × 10^7 × 10^6)
```

Out[112]=

```
4.67023
```

In[113]:=

```
τmergeTDfixMyr2
```

Out[113]=

```
{711.981, 10.9677, 1766.42, 4434.85, 4887.26, 35 675.7, 18.1169, 58.9895, 4.58024}
```

In[114]:=

```
J1539τcool = (t /. NSolve[7.1 × 10^-4 (Rsol RPanei[0.21, Tcold])^2 Tcold^4 ==
      Piecewise[{{L2a[0.21, t, testZ], t < 9000}, {L2b[0.21, t, testZ], t > 9000}}]
       Lsol, t]) // Flatten
```

NSolve : NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[114]=

```
{5564.45}
```

In[115]:=

```
J1539τratio = -Log10[J1539τ / J1539τcool ][[1]]
```

Out[115]=

```
3.07608
```

In[116]:=

```
J1539temp = 10 000
```

Out[116]=

```
10 000
```

In[117]:=

```
(J1539τ / J1539τcool)^-1
```

Out[117]=

```
{1191.47}
```

In[118]:=

```
mtps = ResourceFunction["PolygonMarker"]["Triangle", {Offset[10], π},
  {EdgeForm[Blend[{Cyan, Blue, Cyan}]], FaceForm[stylesTemp[[9]]]}]
```

Out[118]=

In[119]:=

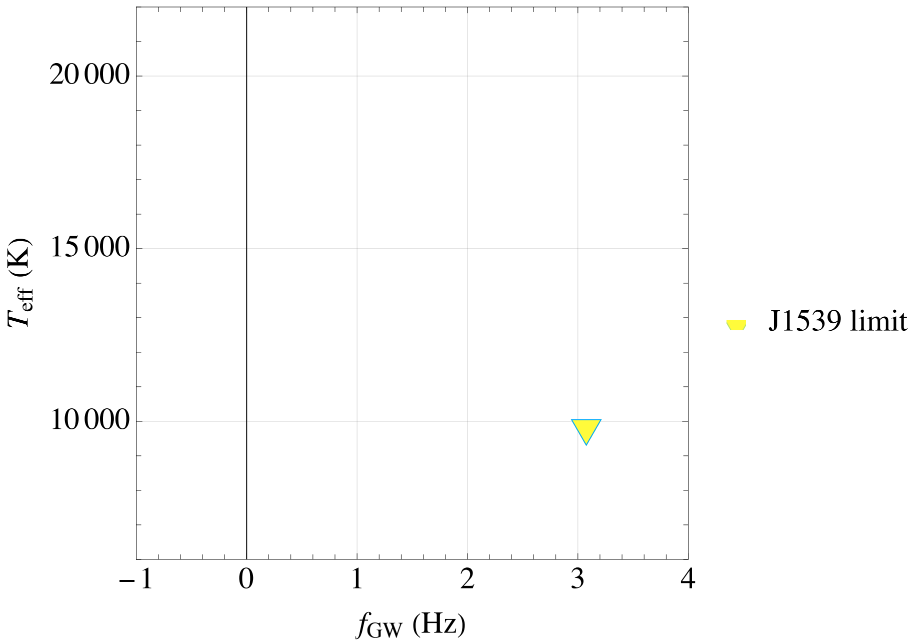
```
stylesTemp[[8]]
```

Out[119]=

In[120]:=

```
J1539plot = ListPlot[{Transpose[{J1539τratio, 0.98 J1539temp}]},
  PlotMarkers → {mtps}, Joined → False, PlotStyle → {{stylesTemp[[9]]}},
  PlotRange → {{-1, 4}, {6000, 22 000}}, AspectRatio → 1,
  Frame → True, LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["f_GW (Hz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic,
  PlotLegends → {Style["J1539 limit", 16]}]
```

Out[120]=



In[121]:=

```
J1539τratio
```

Out[121]=

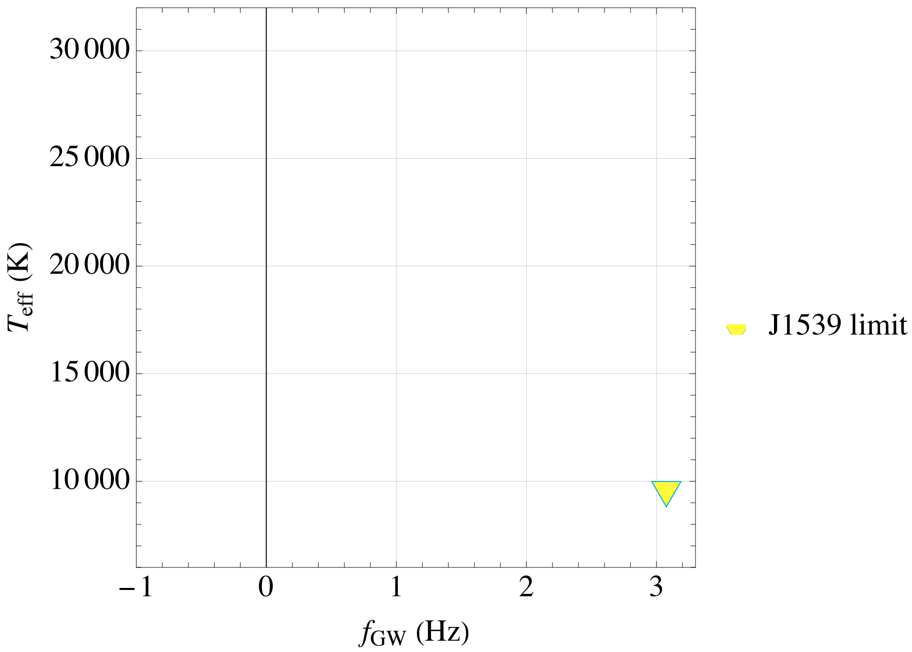
```
3.07608
```

In[122]:=

In[123]:=

```
J1539plot = ListPlot[{Transpose[{J1539τratio, 0.96 J1539temp}]},
  PlotMarkers → {mtps}, Joined → False, PlotStyle → {{stylesTemp⟦9⟧}},
  PlotRange → {{-1, 3.3}, {6000, 32 000}}, AspectRatio → 1,
  Frame → True, LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["fGW (Hz)"], Style["Teff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic,
  PlotLegends → {Style["J1539 limit", 16]}]
```

Out[123]=



In[124]:=

```
Pltfunwhite[ii_] :=
 ListPlot[{pts2⟦All, {1, 2}⟧⟦ii⟧}, PlotRange → {{-1.5, 4.2}, {6000, 32 000}},
  AspectRatio → 1, PlotMarkers → {"★", 40},
  PlotStyle → {White}, LabelStyle → (FontFamily → "Times")]
```

In[125]:=

```
Show[cptrack, plotgrey, br2, J1539plot, Pltfun[1], Pltfun[2], Pltfun[3],
 Pltfun[4], Pltfun[5], Pltfun[6], Pltfun[7], Pltfun[8], Pltfunwhite[9], J1539plot]
```

Out[125]=

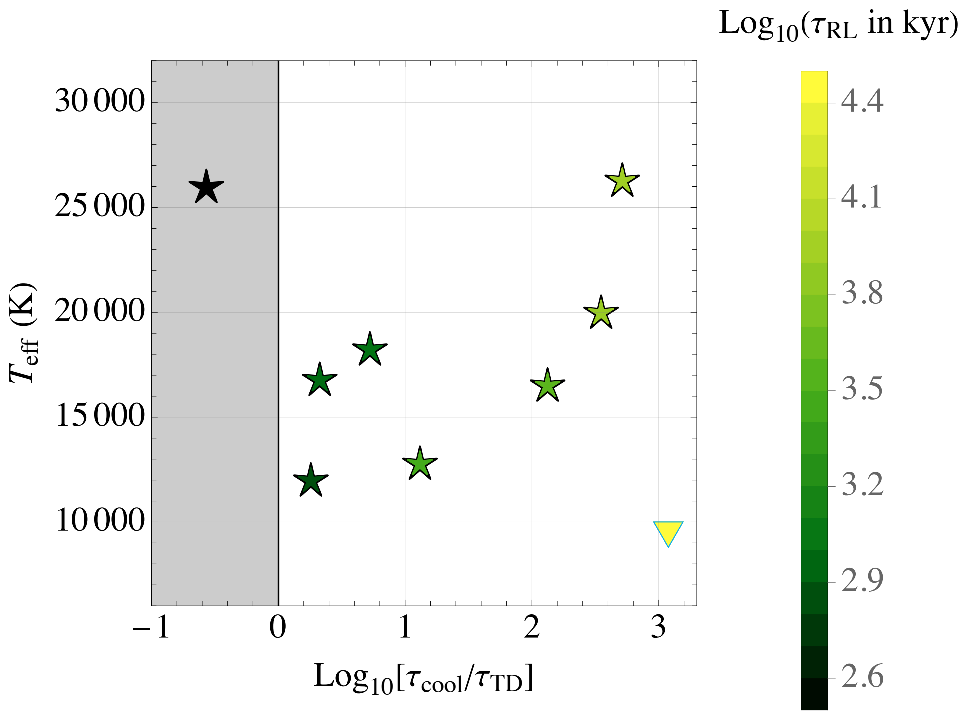


Figure 3 : tracks for three binaries

Relations

In[126]:=

```
Rscale[m1a_, T1a_] :=
 10^(-0.02792426461145596`+0.7641778013995925` Sqrt[T1a]) m1a^(0.14797691065884058`-0.9408955042478873` Sqrt[T1a])
```

In[127]:=

```
dRscaledt[m1a_, T1a_] :=
 1/Sqrt[T1a] 0.8797922069498331` × 10^(-0.02792426461145596`+0.7641778013995925` Sqrt[T1a])
   m1a^(0.14797691065884058`-0.9408955042478873` Sqrt[T1a]) -
  1/Sqrt[T1a] 0.47044775212394363` × 10^(-0.02792426461145596`+0.7641778013995925` Sqrt[T1a])
   m1a^(0.14797691065884058`-0.9408955042478873` Sqrt[T1a]) Log[m1a]
```

In[128]:=

```
Mchirpf[m11_, m22_] = (m11 m22)^(3/5) / (m11 + m22)^(1/5);
```

J2029

In[129]:=

```
mp = 3.2;
ms = 3.0;
fbin = 1.6;
Tp = 1.825;
```

In[133]:=

$$kQratioa = 8 \times 10^{-12};$$

In[134]:=

$$preGW = D\left[\frac{fGW}{mHz}, fGW\right] \frac{96\, G^{5/3}\, \pi^{8/3}\, (Msol / 10)^{5/3}}{5\, c^5}\, (mHz)^{11/3}\, D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

$$preTDa = D\left[\frac{fGW}{mHz}, fGW\right] \frac{18\, (mHz)^{13/3}\, (Msol / 10)\, \pi^{13/3}\, (Rsol / 100)^5\, (mHz)}{G^{5/3}\, (Msol / 10)\, ((Msol / 10))^{5/3}}\, kQratioa\, D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[134]=

0.0394863

Out[135]=

0.00014425

In[136]:=

$$pre\Omega a = D\left[\frac{\Omega}{mHz}, \Omega\right] \left(\frac{3\, mHz^3\, (Msol / 10)^2\, \pi^3\, (Rsol / 100)^3\, (mHz)}{G\, (Msol / 10)\, rg2\, ((Msol / 10))^2}\right) kQratioa\, D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[136]=

0.0600658

In[137]:=

$$\text{d}\Omega\text{df} = \left(\left(\text{preGW}\ (\text{mp ms})\ \left(\frac{1}{(\text{mp} + \text{ms})^{1/3}}\right)\ \text{fbin}^{11/3}\right) + \frac{\text{ms}}{\text{mp}\ (\text{mp}+\text{ms})^{5/3}}\ \text{preTDa fbin}^{13/3}\ (\text{Rscale[mp, Tp]})^{5}\ (\text{fbin}/2-\Omega)\ \frac{1}{\left(1-2\frac{\Omega}{\text{fbin}}\right)}\right)^{-1}$$
$$\left(\text{pre}\Omega\text{a}\ \frac{1}{\left(1-2\frac{\Omega}{\text{fbin}}\right)}\ \frac{\text{ms}^2}{\text{mp}\ (\text{mp}+\text{ms})^{2}}\ \text{fbin}^{3}\ (\text{Rscale[mp, Tp]})^{3}\ (\text{fbin}/2-\Omega)\right)$$

$$\text{d2}\Omega\text{df2[f\_]} = \text{D}\left[\left(\left(\text{preGW}\ (\text{mp ms})\ \left(\frac{1}{(\text{mp} + \text{ms})^{1/3}}\right)\ \text{f}^{11/3}\right) + \frac{\text{ms}}{\text{mp}\ (\text{mp}+\text{ms})^{5/3}}\ \text{preTDa}\ \frac{1}{\left(1-2\frac{\Omega}{\text{f}}\right)}\ \text{f}^{13/3}\ (\text{Rscale[mp, Tp]})^{5}\ (\text{f}/2-\Omega)\right)^{-1}\right.$$
$$\left.\left(\text{pre}\Omega\text{a}\ \frac{1}{\left(1-2\frac{\Omega}{\text{f}}\right)}\ \frac{\text{ms}^2}{\text{mp}\ (\text{mp}+\text{ms})^{2}}\ \text{f}^{3}\ (\text{Rscale[mp, Tp]})^{3}\ (\text{f}/2-\Omega)\right),\ \text{f}\right]$$

$$\Omega\text{start} = \Omega\ /.\ \text{FindRoot}\left[\frac{2\,\Omega}{\text{fbin}^3} - \frac{2\ \text{d}\Omega\text{df}}{\text{fbin}^2} + \frac{\text{d2}\Omega\text{df2[fbin]}}{\text{fbin}} == 0,\ \{\Omega,\ 0.9\}\right][[1]]$$

$$\text{factorsyn} = 2\ \Omega\text{start}\ /\ \text{fbin}$$

$$\text{kQratiob} = \frac{1}{(1-\text{factorsyn})}\ 8\times10^{-12};$$

Out[137]=

$$\frac{0.368603\ (0.8-\Omega)}{\left(1.15618+\frac{0.00759264\ (0.8-\Omega)}{1-1.25\,\Omega}\right)\ (1-1.25\,\Omega)}$$

Out[138]=

$$-\left(\left(0.089991\, f^3 \left(\frac{f}{2}-\Omega\right)\left(0.756586\, f^{8/3}-\frac{0.00198108\, f^{7/3}\left(\frac{f}{2}-\Omega\right)\Omega}{\left(1-\frac{2\,\Omega}{f}\right)^2}+\frac{0.000495271\, f^{13/3}}{1-\frac{2\,\Omega}{f}}+\frac{0.00429235\, f^{10/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)\right)\Big/\left(\left(1-\frac{2\,\Omega}{f}\right)\left(0.206342\, f^{11/3}+\frac{0.000990542\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)^2\right)\right)-\frac{0.179982\, f\left(\frac{f}{2}-\Omega\right)\Omega}{\left(1-\frac{2\,\Omega}{f}\right)^2\left(0.206342\, f^{11/3}+\frac{0.000990542\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)}+\frac{0.0449955\, f^3}{\left(1-\frac{2\,\Omega}{f}\right)\left(0.206342\, f^{11/3}+\frac{0.000990542\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)}+\frac{0.269973\, f^2\left(\frac{f}{2}-\Omega\right)}{\left(1-\frac{2\,\Omega}{f}\right)\left(0.206342\, f^{11/3}+\frac{0.000990542\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)}$$

Out[139]=

0.340055

Out[140]=

0.425069

In[142]:=

$$\text{preTDb} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{18\,(\text{mHz})^{13/3}\,(\text{Msol}/10)\,\pi^{13/3}\,(\text{Rsol}/100)^5\,(\text{mHz})}{G^{5/3}\,(\text{Msol}/10)\,((\text{Msol}/10))^{5/3}}\ \text{kQratiob}\, D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[142]=

0.0002509

In[143]:=

$$\text{preTb} = D\left[\frac{T}{\text{kK4}}, T\right]\left(\frac{135\,\text{mHz}^{19/3}\ (\text{Msol}/10)^3\,\pi^{25/3}\ (\text{Rsol}/100)^9}{G^{8/3}\,(\text{Msol}/10)\,\sigma\,((\text{Msol}/10))^{11/3}\ \text{kK4}^3\,((\text{Rsol}/100))}\ (\text{mHz})^3\right) \text{kQratiob}^2\, D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[143]=

0.0000230828

In[144]:=

$$\text{preJb} = D\left[\frac{J}{(\text{Msol}/10)\ (\text{Rsol}/100)^2\ \text{mHz}}, J\right]$$
$$\left(2\,\pi\,\frac{3\ \text{mHz}^3\ (\text{Msol}/10)^2\ \pi^3\ (\text{Rsol}/100)^5\ (\text{mHz})}{G\ ((\text{Msol}/10))^2}\right)\ \text{kQratiob}\ D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[144]=

0.0656433

In[145]:=

$$\text{pre}\Omega\text{b} =$$
$$D\left[\frac{\Omega}{\text{mHz}}, \Omega\right]\left(\frac{3\ \text{mHz}^3\ (\text{Msol}/10)^2\ \pi^3\ (\text{Rsol}/100)^3\ (\text{mHz})}{G\ (\text{Msol}/10)\ \text{rg2}\ ((\text{Msol}/10))^2}\right)\ \text{kQratiob}\ D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[145]=

0.104475

In[146]:=

$$\text{soltestgenb} = \text{NDSolve}\left[\left\{f'[t] == \left(\text{preGW}\ (\text{mp}\ \text{ms})\ \left(\frac{1}{(\text{mp}\ +\text{ms})^{1/3}}\right)\ f[t]^{11/3}\right) +\right.\right.$$
$$\frac{\text{ms}}{\text{mp}\ (\text{mp}+\text{ms})^{5/3}}\ \text{preTDb}\ f[t]^{13/3}\ (\text{Rscale}[\text{mp}, T[t]])^5\ (f[t]/2-\Omega[t]),$$
$$T'[t] ==$$
$$\text{preTb}\ \left(\left((\text{ms})^3\ f[t]^{19/3}\ \text{Rscale}[\text{mp}, T[t]]^9\ (f[t]/2-\Omega[t])^2\ \left(f[t]/2-\frac{3}{5}\ \Omega[t]\right)\right)\Big/\right.$$
$$\left(\left(\text{mp}\ (\text{mp}+\text{ms})^{11/3}\right)\right.$$
$$\left.\left.\left(T[t]^3\ (2\ \text{Rscale}[\text{mp}, T[t]] + T[t]\times \text{dRscaledt}[\text{mp}, T[t]])\right)\right)\right),$$
$$\Omega'[t] == \text{pre}\Omega\text{b}\ \frac{\text{ms}^2}{\text{mp}\ (\text{mp}+\text{ms})^2}\ f[t]^3\ (\text{Rscale}[\text{mp}, T[t]])^3\ (f[t]/2-\Omega[t]),$$
$$\left. f[0] == \text{fbin},\ T[0] == \text{Tp},\ \Omega[0] == \text{factorsyn}\ \text{fbin}/2\right\},$$
$$\left.\{f, T, \Omega\},\ \{t, 0, 0.51\},\ \text{Method} \to \text{"StiffnessSwitching"}\right]$$

Power : Infinite expression $\frac{1}{0.}$ encountered.

Infinity : Indeterminate expression 0. ComplexInfinity encountered.

NDSolve : The function value $\{2.00672\times 10^{20}, \text{Indeterminate}, 0.\}$ is not a list of numbers with dimensions {3} at {t, f [t], T [t], Ω[t]} = $\{0.51, 529630., 3.77064\times 10^7, 3912.11\}$.

Out[146]=

{{f → InterpolatingFunction[Domain: {{0., 0.508 }} Output: scalar],

T → InterpolatingFunction[Domain: {{0., 0.508 }} Output: scalar],

Ω → InterpolatingFunction[Domain: {{0., 0.508 }} Output: scalar]}}

In[147]:=

```
endt = 0.51;
stepsize = 0.0001;
```

In[149]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp} + \text{ms})\ \text{Msol} / 10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\ \pi^{2/3}} \Big/ (\text{Rsol} / 100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3} \times 2\ \text{mp}^{1/3} \big/ (\text{mp} + \text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\ (\text{mp} / \text{ms})^{2/3}}{0.6\ (\text{mp} / \text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp} / \text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval &/@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction : Input value {0.5084 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.5085 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.5086 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction : Input value {0.5084 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.5085 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.5086 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction : Input value {0.5084 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.5085 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.5086 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

Out[156]=

```
RRLval
```

Out[158]=

```
0.38452
```

In[165]:=

```
RLposia = x /. Solve[
    Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]];
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[167]=

```
7.24804
```

Out[168]=

```
21727.6
```

In[169]:=

```
Tvals2b // Length
```

Out[169]=

```
5046
```

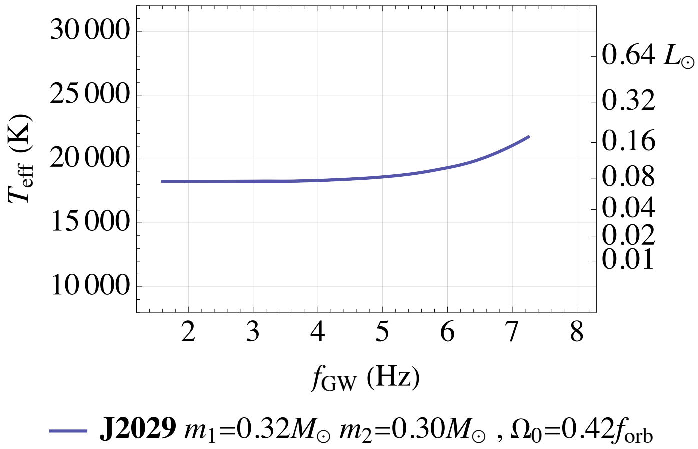
In[170]:=

```
x4 = {{12012, "0.01"}, {13897, "0.02"}, {16049, "0.04"},
   {18502, "0.08"}, {21291, "0.16"}, {24454, "0.32"}, {28031, "0.64 L⊙"}};
```

In[171]:=

```
plota21lin =
 ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Gray, Gray, Blue}],
  PlotRange → {1000 {0.0012, 0.0083}, {8000, 32 000}}, AspectRatio → 1 / 1.5,
  Frame → True, LabelStyle → {(FontFamily → "Times"), Black},
  FrameLabel → {Style["f_GW (Hz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic,
  PlotLegends → {Style["J2029 m_1=0.32M_⊙ m_2=0.30M_⊙ , Ω_0=0.42f_orb", 16]},
  FrameTicks → {{Automatic, x4}, {Automatic, Automatic}}]
```

Out[171]=



J2243

In[172]:=

```
mp = 3.23;
ms = 3.35;
fbin = 3.8;
Tp = 2.63;
```

In[176]:=

$$\text{preGW} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{96\, G^{5/3}\, \pi^{8/3}\, (\text{Msol} / 10)^{5/3}}{5\, c^5} (\text{mHz})^{11/3}\, D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

$$\text{preTDa} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{18\, (\text{mHz})^{13/3}\, (\text{Msol} / 10)\, \pi^{13/3}\, (\text{Rsol} / 100)^5\, (\text{mHz})}{G^{5/3}\, (\text{Msol} / 10)\, ((\text{Msol} / 10))^{5/3}}\, \text{kQratioa}\, D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[176]=

0.0394863

Out[177]=

0.00014425

In[178]:=

```
preΩa =
 D[Ω/mHz, Ω] ((3 mHz^3 (Msol/10)^2 π^3 (Rsol/100)^3 (mHz))/(G (Msol/10) rg2 ((Msol/10))^2)) kQratioa D[t/(31.46×10^13), t]^-1
```

Out[178]=

```
0.0600658
```

In[179]:=

```
kQratioa =  8×10^-12;
```

In[180]:=

```
dΩdf = ((preGW (mp ms) (1/(mp + ms)^(1/3)) fbin^(11/3)) +
     ms/(mp (mp + ms)^(5/3)) preTDa fbin^(13/3) (Rscale[mp, Tp])^5 (fbin/2 - Ω) 1/(1 - 2 Ω/fbin) )^-1
   (preΩa 1/(1 - 2 Ω/fbin) ms^2/(mp (mp + ms)^2) fbin^3 (Rscale[mp, Tp])^3 (fbin/2 - Ω))
d2Ωdf2[f_] = D[((preGW (mp ms) (1/(mp + ms)^(1/3)) f^(11/3)) +
      ms/(mp (mp + ms)^(5/3)) preTDa 1/(1 - 2 Ω/f) f^(13/3) (Rscale[mp, Tp])^5 (f/2 - Ω))^-1
    (preΩa 1/(1 - 2 Ω/f) ms^2/(mp (mp + ms)^2) f^3 (Rscale[mp, Tp])^3 (f/2 - Ω)), f]
Ωstart = Ω /. FindRoot[2 Ω/fbin^3 - 2 dΩdf/fbin^2 + d2Ωdf2[fbin]/fbin == 0, {Ω, 0.9}][[1]]
factorsyn = 2 Ωstart/fbin
kQratiob = 1/(1 - factorsyn) 8×10^-12;
```

Out[180]=

$$\frac{8.94572\ (1.9-\Omega)}{\left(30.4667+\frac{0.745272\ (1.9-\Omega)}{1-0.526316\ \Omega}\right)\ (1-0.526316\ \Omega)}$$

Out[181]=

$$-\left(\left(\left(0.163029\, f^3 \left(\frac{f}{2}-\Omega\right)\left(0.836033\, f^{8/3} - \frac{0.00458088\, f^{7/3}\left(\frac{f}{2}-\Omega\right)\Omega}{\left(1-\frac{2\,\Omega}{f}\right)^2} + \frac{0.00114522\, f^{13/3}}{1-\frac{2\,\Omega}{f}} + \frac{0.00992525\, f^{10/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)\right)\Big/\left(\left(1-\frac{2\,\Omega}{f}\right)\left(0.228009\, f^{11/3} + \frac{0.00229044\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)^2\right)\right) - \frac{0.326058\, f\left(\frac{f}{2}-\Omega\right)\Omega}{\left(1-\frac{2\,\Omega}{f}\right)^2\left(0.228009\, f^{11/3} + \frac{0.00229044\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)} + \frac{0.0815145\, f^3}{\left(1-\frac{2\,\Omega}{f}\right)\left(0.228009\, f^{11/3} + \frac{0.00229044\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)} + \frac{0.489087\, f^2\left(\frac{f}{2}-\Omega\right)}{\left(1-\frac{2\,\Omega}{f}\right)\left(0.228009\, f^{11/3} + \frac{0.00229044\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)}$$

Out[182]=

1.76315

Out[183]=

0.927973

In[185]:=

$$\text{preTDb} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{18\,(\text{mHz})^{13/3}\,(\text{Msol}/10)\,\pi^{13/3}\,(\text{Rsol}/100)^5\,(\text{mHz})}{G^{5/3}\,(\text{Msol}/10)\,((\text{Msol}/10))^{5/3}}\ \text{kQratiob}\ D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[185]=

0.00200272

In[186]:=

$$\text{preTb} = D\left[\frac{T}{\text{kK4}}, T\right]\left(\frac{135\,\text{mHz}^{19/3}\ (\text{Msol}/10)^3\,\pi^{25/3}\ (\text{Rsol}/100)^9}{G^{8/3}\,(\text{Msol}/10)\,\sigma\,((\text{Msol}/10))^{11/3}\ \text{kK4}^3\,((\text{Rsol}/100))}\ (\text{mHz})^3\right) \text{kQratiob}^2\, D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[186]=

0.00147072

In[187]:=

$$\text{preJb} = D\left[\frac{J}{(\text{Msol}/10)\ (\text{Rsol}/100)^2\ \text{mHz}},\ J\right]\left(2\,\pi\,\frac{3\,\text{mHz}^3\ (\text{Msol}/10)^2\ \pi^3\ (\text{Rsol}/100)^5\ (\text{mHz})}{G\ ((\text{Msol}/10))^2}\right)\ \text{kQratiob}\ D\left[\frac{t}{31.46\times 10^{13}},\ t\right]^{-1}$$

Out[187]=

0.523975

In[188]:=

$$\text{pre}\Omega\text{b} = D\left[\frac{\Omega}{\text{mHz}},\ \Omega\right]\left(\frac{3\,\text{mHz}^3\ (\text{Msol}/10)^2\ \pi^3\ (\text{Rsol}/100)^3\ (\text{mHz})}{G\ (\text{Msol}/10)\ \text{rg2}\ ((\text{Msol}/10))^2}\right)\ \text{kQratiob}\ D\left[\frac{t}{31.46\times 10^{13}},\ t\right]^{-1}$$

Out[188]=

0.833933

In[189]:=

$$\begin{aligned}
&\text{soltestgenb} = \text{NDSolve}\Bigg[\Bigg\{f'[t] == \left(\text{preGW}\ (\text{mp ms})\ \left(\frac{1}{(\text{mp} + \text{ms})^{1/3}}\right)\ f[t]^{11/3}\right) + \\
&\quad \frac{\text{ms}}{\text{mp}\ (\text{mp}+\text{ms})^{5/3}}\ \text{preTDb}\ f[t]^{13/3}\ (\text{Rscale}[\text{mp}, T[t]])^5\ (f[t]/2 - \Omega[t]), \\
&\quad T'[t] == \\
&\quad\quad \text{preTb}\ \Bigg(\left((\text{ms})^3\ f[t]^{19/3}\ \text{Rscale}[\text{mp}, T[t]]^9\ (f[t]/2 - \Omega[t])^2\ \left(f[t]/2 - \frac{3}{5}\ \Omega[t]\right)\right) \Big/ \\
&\quad\quad\quad \left((\text{mp}\ (\text{mp}+\text{ms})^{11/3})\right. \\
&\quad\quad\quad\quad \left.(T[t]^3\ (2\,\text{Rscale}[\text{mp}, T[t]] + T[t]\times \text{dRscaledt}[\text{mp}, T[t]]))\right)\Bigg), \\
&\quad \Omega'[t] == \text{pre}\Omega\text{b}\ \frac{\text{ms}^2}{\text{mp}\ (\text{mp}+\text{ms})^2}\ f[t]^3\ (\text{Rscale}[\text{mp}, T[t]])^3\ (f[t]/2 - \Omega[t]), \\
&\quad f[0] == \text{fbin},\ T[0] == \text{Tp},\ \Omega[0] == 0.9\ \text{fbin}/2\ \Bigg\}, \\
&\quad \{f, T, \Omega\},\ \{t, 0, 0.05\},\ \text{Method} \to \text{"StiffnessSwitching"}\Bigg]
\end{aligned}$$

••• Power : Infinite expression $\frac{1}{0.}$ encountered.

••• Infinity : Indeterminate expression 0. ComplexInfinity encountered.

••• NDSolve : The function value $\{4.05494\times 10^{55}$, Indeterminate, 0.$\}$ is not a list of numbers with dimensions {3} at {t, f [t], T [t], Ω[t]} = $\{0.041261, 2.19239\times 10^{15}, 9.31949\times 10^{21}, 5.33244\times 10^{9}\}$.

Out[189]=

{{f → InterpolatingFunction[Domain: {{0., 0.0408 }} Output: scalar],

T → InterpolatingFunction[Domain: {{0., 0.0408 }} Output: scalar],

Ω → InterpolatingFunction[Domain: {{0., 0.0408 }} Output: scalar]}}

In[190]:=

```
endt = 0.05;
stepsize = 0.00001;
```

In[192]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp}+\text{ms})\,\text{Msol}/10)^{1/3}}{(\text{fvals1b}\,\text{mHz})^{2/3}\,\pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3}\times 2\,\text{mp}^{1/3} \big/ (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\,(\text{mp}/\text{ms})^{2/3}}{0.6\,(\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction : Input value {0.0408 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.04081 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.04082 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction : Input value {0.0408 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.04081 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.04082 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction : Input value {0.0408 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.04081 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.04082 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

Out[199]=

0.38452

Out[201]=

0.375767

In[208]:=

```
RLposia = x /. Solve[
     Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]];
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[210]=

6.20388

Out[211]=

27 251.6

In[212]:=

```
Tvals2b // Length
```

Out[212]=

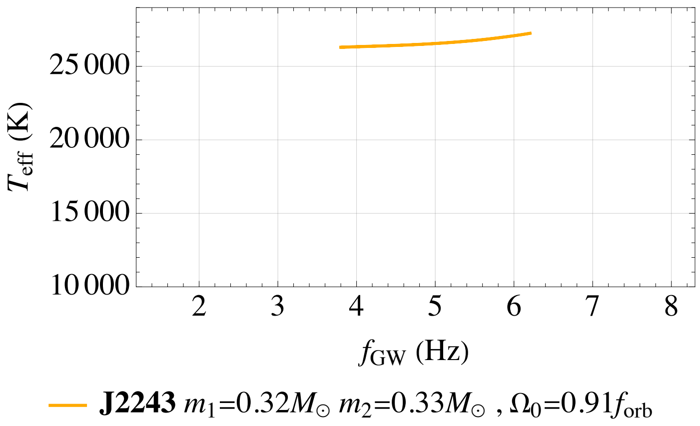
3217

In[213]:=

In[214]:=

```
plota23lin =
 ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Orange, Orange, Yellow}],
  PlotRange → {1000 {0.0012, 0.0083}, {10 000, 29 000}},
  AspectRatio → 1 / 2, Frame → True, LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["f_GW (Hz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic,
  PlotLegends → {Style["J2243 m_1=0.32M_⊙ m_2=0.33M_⊙ , Ω_0=0.91f_orb", 16]}]
```

Out[214]=



In[215]:=

J0538

In[216]:=

```
mp = 3.2;
ms = 4.5;
fbin = 2.3;
Tp = 1.28;
```

In[220]:=

$$\text{preGW} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{96\, G^{5/3}\, \pi^{8/3}\, (\text{Msol}/10)^{5/3}}{5\, c^5} (\text{mHz})^{11/3}\, D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

$$\text{preTDa} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{18\, (\text{mHz})^{13/3}\, (\text{Msol}/10)\; \pi^{13/3}\, (\text{Rsol}/100)^5\, (\text{mHz})}{G^{5/3}\, (\text{Msol}/10)\, ((\text{Msol}/10))^{5/3}}\, \text{kQratioa}\, D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[220]=

0.0394863

Out[221]=

0.00014425

In[222]:=

$$\text{pre}\Omega\text{a} = D\left[\frac{\Omega}{\text{mHz}}, \Omega\right] \left(\frac{3\, \text{mHz}^3\, (\text{Msol}/10)^2\, \pi^3\, (\text{Rsol}/100)^3\, (\text{mHz})}{G\, (\text{Msol}/10)\, \text{rg2}\, ((\text{Msol}/10))^2}\right) \text{kQratioa}\, D\left[\frac{t}{31.46 \times 10^{13}}, t\right]^{-1}$$

Out[222]=

0.0600658

In[223]:=

$$\text{kQratioa} = 8 \times 10^{-12};$$

In[224]:=

```
dΩdf = ((preGW (mp ms) (1/(mp + ms)^(1/3)) fbin^(11/3)) +
    ms/(mp (mp + ms)^(5/3)) preTDa fbin^(13/3) (Rscale[mp, Tp])^5 (fbin / 2 - Ω) 1/(1 - 2 Ω/fbin) )^-1
  (preΩa 1/(1 - 2 Ω/fbin) ms^2/(mp (mp + ms)^2) fbin^3 (Rscale[mp, Tp])^3 (fbin / 2 - Ω))
d2Ωdf2[f_] = D[((preGW (mp ms) (1/(mp + ms)^(1/3)) f^(11/3)) +
     ms/(mp (mp + ms)^(5/3)) preTDa 1/(1 - 2 Ω/f) f^(13/3) (Rscale[mp, Tp])^5 (f / 2 - Ω))^-1
   (preΩa 1/(1 - 2 Ω/f) ms^2/(mp (mp + ms)^2) f^3 (Rscale[mp, Tp])^3 (f / 2 - Ω)), f]
Ωstart = Ω /. FindRoot[2 Ω/fbin^3 - 2 dΩdf/fbin^2 + d2Ωdf2[fbin]/fbin == 0, {Ω, 0.9}][[1]]
factorsyn = 2 Ωstart / fbin
kQratiob = 1/(1 - factorsyn) 8 × 10^-12;
```

Out[224]=

$$\frac{1.0306\ (1.15-\Omega)}{\left(6.10446+\frac{0.0184285\ (1.15-\Omega)}{1-0.869565\ \Omega}\right)\ (1-0.869565\ \Omega)}$$

Out[225]=

$$-\left(\left(0.0847043\, f^3 \left(\frac{f}{2}-\Omega\right)\left(1.0558\, f^{8/3}-\frac{0.000997775\, f^{7/3}\left(\frac{f}{2}-\Omega\right)\Omega}{\left(1-\frac{2\,\Omega}{f}\right)^2}+\frac{0.000249444\, f^{13/3}}{1-\frac{2\,\Omega}{f}}+\frac{0.00216185\, f^{10/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)\right)\Big/\left(\left(1-\frac{2\,\Omega}{f}\right)\left(0.287947\, f^{11/3}+\frac{0.000498888\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)^2\right)\right)-$$
$$\frac{0.169409\, f\left(\frac{f}{2}-\Omega\right)\Omega}{\left(1-\frac{2\,\Omega}{f}\right)^2\left(0.287947\, f^{11/3}+\frac{0.000498888\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)}+$$
$$\frac{0.0423522\, f^3}{\left(1-\frac{2\,\Omega}{f}\right)\left(0.287947\, f^{11/3}+\frac{0.000498888\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)}+$$
$$\frac{0.254113\, f^2\left(\frac{f}{2}-\Omega\right)}{\left(1-\frac{2\,\Omega}{f}\right)\left(0.287947\, f^{11/3}+\frac{0.000498888\, f^{13/3}\left(\frac{f}{2}-\Omega\right)}{1-\frac{2\,\Omega}{f}}\right)}$$

Out[226]=

0.372118

Out[227]=

0.323581

In[229]:=

$$\text{preTDb} = D\left[\frac{\text{fGW}}{\text{mHz}}, \text{fGW}\right] \frac{18\,(\text{mHz})^{13/3}\,(\text{Msol}/10)\;\pi^{13/3}\,(\text{Rsol}/100)^5\,(\text{mHz})}{G^{5/3}\,(\text{Msol}/10)\,((\text{Msol}/10))^{5/3}}\;\text{kQratiob}\;D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[229]=

0.000213256

In[230]:=

$$\text{preTb} = D\left[\frac{T}{\text{kK4}}, T\right]\left(\frac{135\,\text{mHz}^{19/3}\;(\text{Msol}/10)^3\,\pi^{25/3}\;(\text{Rsol}/100)^9}{G^{8/3}\,(\text{Msol}/10)\,\sigma\,((\text{Msol}/10))^{11/3}\;\text{kK4}^3\,((\text{Rsol}/100))}\;(\text{mHz})^3\right)\text{kQratiob}^2\,D\left[\frac{t}{31.46\times 10^{13}}, t\right]^{-1}$$

Out[230]=

0.0000166759

In[231]:=

```
preJb = D[J/((Msol/10) (Rsol/100)^2 mHz), J]
  (2 π (3 mHz^3 (Msol/10)^2 π^3 (Rsol/100)^5 (mHz))/(G ((Msol/10))^2)) kQratiob D[t/(31.46 × 10^13), t]^-1
```

Out[231]=

0.0557945

In[232]:=

```
preΩb =
 D[Ω/mHz, Ω] ((3 mHz^3 (Msol/10)^2 π^3 (Rsol/100)^3 (mHz))/(G (Msol/10) rg2 ((Msol/10))^2)) kQratiob D[t/(31.46 × 10^13), t]^-1
```

Out[232]=

0.0887996

In[233]:=

```
soltestgenb = NDSolve[{f'[t] == (preGW (mp ms) (1/(mp + ms)^(1/3)) f[t]^(11/3)) +
     ms/(mp (mp + ms)^(5/3)) preTDb f[t]^(13/3) (Rscale[mp, T[t]])^5 (f[t]/2 - Ω[t]),
   T'[t] ==
    preTb (((ms)^3 f[t]^(19/3) Rscale[mp, T[t]]^9 (f[t]/2 - Ω[t])^2 (f[t]/2 - 3/5 Ω[t]))/
       ((mp (mp + ms)^(11/3))
        (T[t]^3 (2 Rscale[mp, T[t]] + T[t] × dRscaledt[mp, T[t]])))),
   Ω'[t] == preΩb ms^2/(mp (mp + ms)^2) f[t]^3 (Rscale[mp, T[t]])^3 (f[t]/2 - Ω[t]),
   f[0] == fbin, T[0] == Tp, Ω[0] == factorsyn fbin/2 },
  {f, T, Ω}, {t, 0, 0.15}, Method → "StiffnessSwitching"]
```

Power : Infinite expression $\frac{1}{0.}$ encountered.

Infinity : Indeterminate expression 0. ComplexInfinity encountered.

NDSolve : The function value {1.1007 × 10^{27}, Indeterminate, 0. } is not a list of numbers with dimensions {3} at {t, f[t], T[t], Ω[t]} = {0.137343, 3.33013 × 10^{7}, 5.4506 × 10^{11}, 858.759 }.

Out[233]=

```
{{f → InterpolatingFunction[ Domain: {{0., 0.137 }} Output: scalar ],
  T → InterpolatingFunction[ Domain: {{0., 0.137 }} Output: scalar ],
  Ω → InterpolatingFunction[ Domain: {{0., 0.137 }} Output: scalar ]}}
```

In[234]:=

```
endt = 0.3;
stepsize = 0.0001;
```

In[236]:=

```
fvals1b =
  Table[Evaluate[f[t] /. soltestgenb], {t, 0, endt, stepsize}] // Abs // Flatten;
Tvals1b = Table[Evaluate[T[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
Ωvals1b = Table[Evaluate[Ω[t] /. soltestgenb], {t, 0, endt, stepsize}] // Flatten;
timevals1b = Table[t, {t, 0, endt, stepsize}] // Flatten;
```

$$\text{avals1b} = \frac{G^{1/3}\,((\text{mp}+\text{ms})\,\text{Msol}/10)^{1/3}}{(\text{fvals1b mHz})^{2/3}\,\pi^{2/3}} \Big/ (\text{Rsol}/100);$$

```
Rvals1b = Rscale[mp, Tvals1b];
Ra1b = Rvals1b / avals1b;

RRLval
```

$$\text{RRLval} = 3^{-4/3} \times 2\,\text{mp}^{1/3} \big/ (\text{mp}+\text{ms})^{1/3};$$

$$\text{RRLval} = \frac{0.49\,(\text{mp}/\text{ms})^{2/3}}{0.6\,(\text{mp}/\text{ms})^{2/3} + \text{Log}\left[1 + (\text{mp}/\text{ms})^{1/3}\right]}$$

```
Tvals2b = Pick[Tvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
fvals2b = Pick[fvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ωvals2b = Pick[Ωvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
timevals2b = Pick[timevals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Ra1cutb = Pick[Ra1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
Rvals2b = Pick[Rvals1b, # < RRLval & /@ (Rvals1b / avals1b)] // Abs;
```

InterpolatingFunction : Input value {0.1374 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.1375 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.1376 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction : Input value {0.1374 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.1375 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.1376 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

InterpolatingFunction : Input value {0.1374 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.1375 } lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction : Input value {0.1376 } lies outside the range of data in the interpolating function. Extrapolation will be used.

General : Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

Out[243]=

```
0.375767
```

Out[245]=

```
0.349817
```

In[252]:=

```
RLposia = x /. Solve[
     Position[Abs[Ra1cutb / RRLval - 1], Min[Abs[Ra1cutb / RRLval - 1]]] == x][[1]];
Abs[Ra1cutb / RRLval - 1][[RLposia]];
fvals2b[[RLposia]]
Tvals2b[[RLposia]] kK4
```

Out[254]=

```
7.58384
```

Out[255]=

```
19 187.2
```

In[256]:=

```
Tvals2b // Length
```

Out[256]=

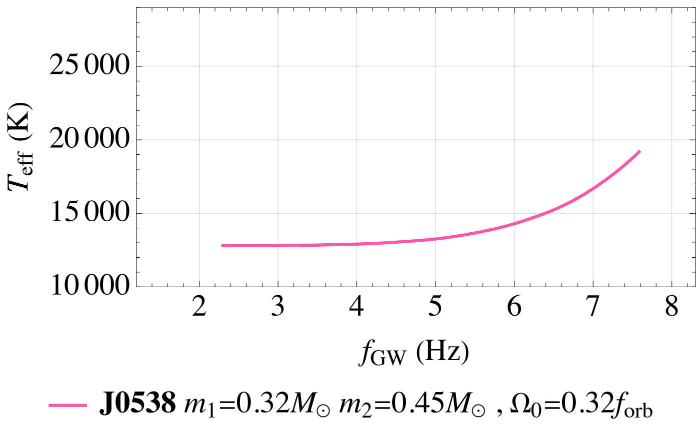
```
2973
```

In[257]:=

In[258]:=

```
plota24lin =
 ListPlot[Transpose[{fvals2b[[ ;; RLposia]], 10^4 Tvals2b[[ ;; RLposia]]}], Mesh → All,
  PlotMarkers → None, Joined → True, PlotStyle → Blend[{Magenta, Red, White}],
  PlotRange → {1000 {0.0012, 0.0083}, {10 000, 29 000}},
  AspectRatio → 1 / 2, Frame → True, LabelStyle → (FontFamily → "Times"),
  FrameLabel → {Style["f_GW (Hz)"], Style["T_eff (K)", 16]},
  BaseStyle → {FontSize → 16}, GridLines → Automatic,
  PlotLegends → {Style["J0538 m_1=0.32M_⊙ m_2=0.45M_⊙ , Ω_0=0.32f_orb", 16]}]
```

Out[258]=



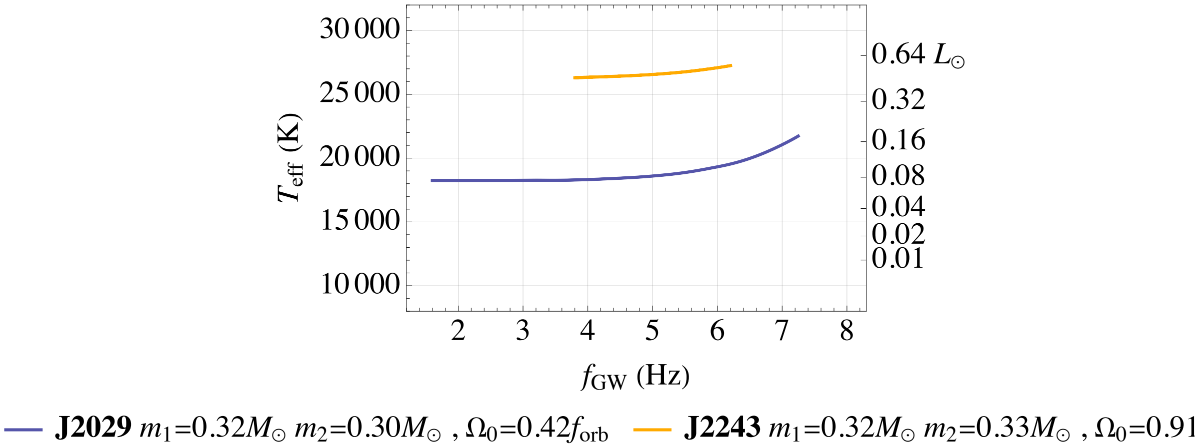
In[259]:=

together

In[260]:=

```
Show[plota21lin, plota23lin]
```

Out[260]=



In[261]:=

```
Show[plota21lin, plota23lin, plota24lin]
```

Out[261]=