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***Electricity Trading Between Smart Nano-Grids:  
Matching Supply and Demand in the Face of Unpredictable  
Supply***

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# Declaration

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### **Abstract**

As fossil fuels across the world are steadily depleted, the majority of the world's energy production will shift towards renewable sources. However, renewable energy sources do not always guarantee the same reliability in rates of production. Therefore new strategies must be developed to match supply and demand in the face of unpredictable supply. The growing popularity and proliferation of smart grid technology is one proposed method of solving this problem.

In this paper, a network implementation of a game theoretic approach to this problem is implemented in a smart nanogrid to prove whether or not such an approach is first feasible in a network, and secondly whether or not it would provide a better approach to the problem. A number of optimisation techniques, such as convex optimisation and hyperplane projection optimisation, are also employed to find a better solution.

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## **Part I**

# **Introduction**



Currently in the field of smart grid technologies, there is an abundance of theoretical approaches to the problems of managing the transactions between consumers and suppliers of energy [1]. While there are multiple facets to this problem, this project examines the problem of matching supply and demand in the face of unpredictable supply. The world is slowly moving towards a future in which renewable energy sources will become our primary method of energy production, particularly with the inevitable depletion of fossil fuels [2]. This future, which will unfortunately be accompanied by a more unpredictable supply of energy, will require a smarter approach to the accumulation and distribution of energy. Due to the proliferation of decision making computers, the smart grid is a very popular solution. As mentioned previously, research has been conducted by hundreds of papers on theoretical solutions to this problem. However very few have real implementations and do not take into account the limitations and overheads of a real network.

The goal of this project is to implement a network solution to the problem of matching supply and demand in the face of unpredictable supply. The paper authored by Wayes Tushar [3] formed the basis of the project and therefore this report. The system will then be compared against a simple scenario of the current REFIT scheme [4] and an analysis will be provided to see whether or not this particular smart grid solution would be effective in a real world situation.

First a discussion of the technologies used and an explanation of the associated terms will be presented, including topics such as the smart grid, the current implementation of managing supply (REFIT), definitions of auctions and game theory, as well as a brief summary of the mathematical optimisation techniques employed in this project. Next, the design of the project will be examined and finally the actual implementation itself will be presented. To conclude there will be an assessment of the system designed and a number of potential continuations of the project will be discussed.

## **Part II**

# **Background**

# Chapter 1

## Decentralised Grid

At present, in Ireland and in many other countries, the national electric grid infrastructure is controlled by a central body, namely the ESB. While there are several electricity providers in Ireland, such as Bord Gáis Energy, SSE Airtricity and Energy Ireland, each of them use the same distribution network. Fundamentally, the power is provided from each of the different providers and then routed into the same centralised hub belonging to the ESB. From there, each consumer (a household) receives the energy that they pay for accordingly at a fixed rate through that same infrastructure belonging to the ESB. This is similar to many other countries where there is a centralised grid.

This system has been in place for decades and lends itself very well to large companies can providing a steady supply of energy by way of electricity plants that use both renewable and nonrenewable energy sources. Non-renewable energy sources, also known as fossil fuels, include resources such as coal, gas and oil. While these are finite resources, at present they can be burned at a steady rate in order to meet the demands of customers. Electricity from renewable sources can also be produced at a reasonably steady rate by placing large farms in areas that are particularly well suited to the type of renewable energy being produced. For example, large wind farms are set up in windy regions far removed from residential or urban areas and solar panels can be placed in regions that typically enjoy clearer skies than other areas.

However in the future, with the ongoing depletion of nonrenewable resources, increasing numbers of customers will turn to erecting solar panels and local wind farms on their own properties, regardless of whether or not they are living in a particularly sunny or windy area. Presently there are a few houses that use a solar panel to heat their water or other smaller tasks but soon more and more people will become more and more dependent on what they can produce either within their own home, or in a more collective sense in their own neighbourhood to power their houses [5].

The issue that then arises in these areas that aren't as sunny or windy is that supply of electricity is no longer steady. The current system could not be maintained as the energy produced on a local level would be

small enough that it would not be worth passing this energy upstream to the central grid. The energy would instead be used at a local level to try to cover the demand for electricity of the house or business with which that particular device is associated.

The model of infrastructure that would then be required is that of a decentralised grid. This model would require a massive infrastructure overhaul in order to implement, so it would not exist until it is absolutely necessary and has been accepted by the major companies who would then go about implementing it. The rough idea of a distributed grid is described in figure 1.1. Throughout the rest of this report the phrases “distributed grid” and “decentralised grid” are used interchangeably.

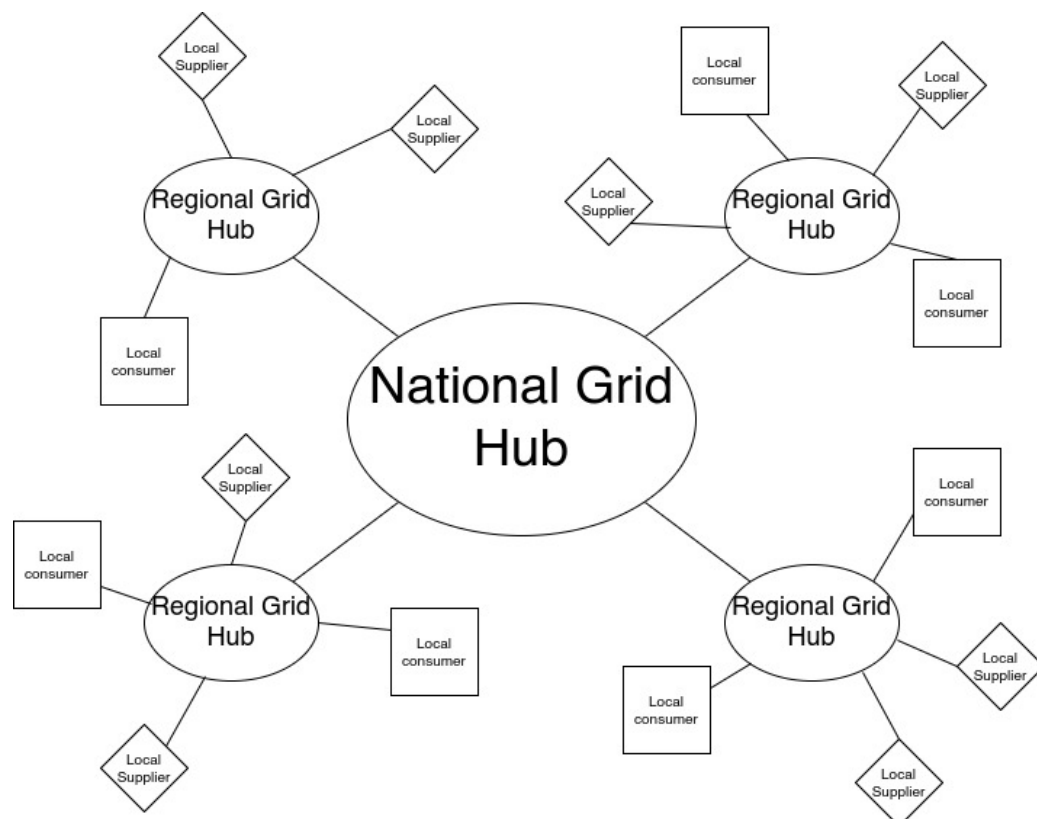


Figure 1.1: Each local consumer and supplier is attached to a regional grid hub which manages the allocation of electricity between suppliers and consumers. This is just a simple overview of the idea but conceivably a consumer or a supplier could be connected to two or more different regional grid hubs.

## Chapter 2

# Smart Grid

### 2.1 Overview

Due to advancements in networking technologies, and the field of sophisticated decision making technologies, the idea of a smart grid has become increasingly popular. Actors within a smart grid, be they individual consumers or suppliers, or groups thereof, can be fitted with small computers that perceive changes in the grid and then these actors can react accordingly. Several different types of management systems have been constructed in order to successfully, fairly and efficiently allocate resources for each of these different types of actors. The two primary types of management systems that were examined as part of this final year project were Auctions and Game Theory which will both be discussed in detail later on.

The smart grid is not only used in this manner, but has many other potential applications, some of which have been implemented already in several cities and regions throughout the world. These applications include energy consumption or production prediction, scheduling the use of consumers in order to reduce costs of operation, and smart reaction to disruptions or blackouts within the grid to reduce the damage that occurs as a result.

In this project it is assumed that the consumers within the system are outfitted with some kind of prediction technology. An example of such a system has been proposed by Garcia et al [6] where a device tries to time its own operation within a certain time-frame in accordance with when the price of energy is cheapest. It also attempts to predict how much energy the system will consume based off its own knowledge of previous experiences in buying power at that particular time of day, allowing the system to learn over time and make smarter decisions as time goes on.

## 2.2 Microgrids and Nanogrids

At present smart grids have generally been implemented at the level of microgrids. Microgrids are generally thought of having a consumer be a single house, or perhaps a group of houses, and a supplier being a small wind farm or solar farm, or perhaps a group of these together. In the case of a microgrid, actors within the system are defined in similar terms to those involved in a centralised grid system, meaning that the transition from a centralised grid to the microgrid scheme is a relatively easy one.

An example of a real world implementation is that of the system in place in Japan. The majority of the work completed on this system was conducted following the disaster of Fukushima, where it became clear that a reliance on a single power source and a centralised power distribution network left the country vulnerable following the disaster [7]. Several regions were cut off from power as a result of the disaster which hampered relief efforts as well as making the lives of ordinary Japanese citizens more difficult. Had a microgrid system been in place then fewer hospitals and homes would have been left without power following the disaster.

The company ENEL has also introduced a smart grid system in the region of Apulia in southern Italy [8] with great success. The system there allows customers to produce and network their own electricity as well as making them more aware of their consumption and any potential savings.

The nanogrid system is very similar to that of the microgrid system conceptually but is concerned with a much smaller scale. A nanogrid is one that operates within the confines of a single building, generally where each consumer is a single appliance such as a washing machine or an electronic vehicles (EV). Suppliers would also be very small scale perhaps a set of solar panels or a small wind turbine. A nanogrid system could also be adapted to aggregate a number of devices to act as one as a single actor within the nanogrid system, for example all the lights on one floor of a house could act as a single consumer and draw on a shared reserve of power.

Another extension would be to incorporate a nanogrid system as a sub-node of a microgrid. This would create a hierarchy of distributed grids. This tree could also be adapted into a graph where a parent node in the tree could have multiple children and a child could have multiple parents. This will be discussed more in the conclusion.

## Chapter 3

# REFIT Scheme

The REFIT scheme (Renewable Energy Feed In Tariff) is one of the most common ways in which countries around the world, including Germany, Spain and the state of Hawaii [9], try to incentivise renewable energy sources and suppliers to sell energy into the main grid for consumption by consumers. The primary tenet of the REFIT scheme is to guarantee a fixed price for energy provided by suppliers at particular times of the day. These prices are offered in a non-discriminatory fashion for every kWh produced by the supplier. The prices can be lower or higher based on the type of energy being produced. For example in Germany the price is higher for suppliers of solar energy than for suppliers of wind energy, according to the EU at the time of the writing of this report [10].

The main advantage of this type of a scheme is that firstly it incentivises companies to invest in renewable energy because they know they'll receive a good return on their investment. It also incentivises landowners and homeowners to invest, thereby creating a large infrastructure of renewable energy resources in a relatively small space of time and this has worked effectively in Germany [4]. The payment also easily covers the cost of creating the solar panels or wind turbines for regular consumers.

The main downside to the REFIT scheme however is that because it provides a fixed amount based primarily on the type of energy produced and for how long it is being provided, which means that it is not worth it for a supplier to sell if it has a poor supply in reserve for example. In this case, the incentive to sell energy is quite low as selling any energy would drain the supplier of most of its power. Therefore a scheme involving a dynamic price model that incentivises all suppliers at all times to contribute to the demand and maximise their own utility in the system might be better.

## Chapter 4

# Auctions

### 4.1 Overview

The first type of node management systems considered as part of this project was that of auctions. Auctions generally have a number of different types of properties and as such, can be classified into different groupings.

- Single- or multi-dimensional
- One- or two-sided
- Open-cry or sealed-bid
- First- or k<sup>th</sup>-price
- Single- or multi-unit
- Single- or multi-item

While all of these are examined in detail in the book by Simon Parsons [11], only one of these will be discussed here as it the only type of auction that was considered. The reasoning for the decision as to why this was the only type considered is also explained. The type of auction investigated was a continuous double auction.

### 4.2 Continuous Double Auction

A continuous double auction was discussed in the paper by Ramachandran [12] among others and was therefore a popular candidate by several potential energy management systems. The idea of a double auction is a simple one. Instead of trying to match multiple bidders to a single seller or multiple sellers to a single buyer,



a double auction is where there are multiple sellers and multiple bidders. Through combining the buy-side and the sell-side of an auction into a single process, we then have a two-sided or double action.

A continuous double auction is an extension and a refinement of a double auction where multiple rounds are conducted until as many bidders and sellers have been satisfied as is possible. The first stage attempts to match up as many bidders and sellers as possible who have compatible bids. After that both the sellers and the bidders attempt to adjust their respective ask and bid prices and then another round begins. This process continues iteratively until either all actors involved in the auction are satisfied or until all remaining actors have reached their respective buying or selling thresholds.

The reason why this particular style of auction was chosen to be investigated was that it matches the real world scenario of having multiple consumers within a nanogrid environment as well as multiple suppliers. It is also reasonable to assume that some kind of memory might be built into the consumers and suppliers so that they might remember what each other offered on previous occasions and submit bids in order to be accepted quicker. The iterative style of the continuous auction was appealing and realistic due to the nature of managing the bids and sales of so many different actors within one given system. Most of the auctions investigated as part of this project required the central controller having access to all the private information of all the other nodes. This, among other reasons, led to auctions not being implemented for this project and this will be discussed in further detail later.

## Chapter 5

# Game Theory

### 5.1 Overview

The field of game theory has been one that has many different facets and versions depending on the type of situation required. In this section of the report the nomenclature and jargon of game theory will be discussed, as will a short explanation about the decision of selection of the type of game implemented as part of this final year project. First the two primary types of interactions between players in a game will be discussed and after that the two primary types of playing styles will be discussed. However first of all there are certain traits that are universal for any type of game that must first be explained in order to grasp the concept of game theory enough to understand some of the implementation decisions later in this report as well as to grasp the general concept of game theory itself.

In game theory, players within a game compete for a finite resource with the objective of maximising their own utility within the scope of that game. Each player within the game has an associated utility function that is generally the same for all players within that game. The utility function generally results in some scalar value that is trying to reach some max value, whether on an individual or collective level. There is also generally some kind of manager node that helps to conduct the game between all of the players involved. Within any particular game, the players are all trying to maximise their own utility, however in different types of games they may also be conscious of the utilities of all the other players involved and try to react accordingly, whether to further their own goal or to further the goals of the collective group.

A well defined game also has some form of state of equilibrium. This state of equilibrium is when the sum of utilities of all the players within the game reaches a maximum. The central managing node, if there is one, generally decides whether or not this state has been reached. This state is the success state of the game. In a well-designed game the utility function must be designed such that the state of equilibrium not only can be reached but also that reaching that state is appealing to all players within the game.

## 5.2 Non-Cooperative Game Theory

Non-Cooperative games are the simplest types of games to both understand and design. As previously stated, each player is trying to maximise its own utility but the core component of a non-cooperative game is that all of the players are operating purely independently. Each player within the game knows the best strategy to take in order to maximise its own utility. Because each player in a game has the same moves open for them to take and therefore the same strategy that each other player will take to maximise their own respective utilities.

This is where the concept of Nash Equilibrium comes into play. Nash Equilibrium is the state in which there is the least disparity between the best player and the worst player, that is that each player performs the best that it can with the knowledge that all other players are similarly going to try to maximise their own utilities. With this knowledge, each player is then able to pick the strategy that maximises its own utility, taking into consideration that all other players are trying to do the exact same thing and therefore it picks an appropriate strategy. In a well designed game, there should also be no incentive for a player to change their strategy to try to undercut other players. If made correctly, such an action would have an adverse effect on the player in the game. In this case all other players would then be aware that this players strategy had changed and would then react accordingly in order to maximise their own utility and decrease that player's utility.

## 5.3 Cooperative Game Theory

Cooperative game theory shares many similar traits with that of non-cooperative game theory as outlined in the overview section of this part on game theory in this report. However the key aspect of cooperative game theory is that players within the game will form coalitions based on threats and incentives that occur between each other. The key component of cooperative game theory is the analysis of which coalitions are likely to form within any given game and what the projected outcomes are based upon these permutations of coalitions. In this way the study of cooperative games have two main facets. First of all they are concerned with what might cause different groups of players to act together in unison. Secondly they are concerned with the outcomes from the most likely of each of these games that happen when different groups form.

In this project, the nodes involved in the game are all energy suppliers who are each trying to maximise their own profit based on the amount of energy that they are able to sell. The utility functions of the nodes and other such details will be discussed later in the Implementation section of this report. The desired outcome of each player is therefore entirely selfish and because they are all trying to compete for a finite price, they each want to obtain as much of that money as possible. Therefore it does not make sense to design this game in such a way that these players should be able to form coalitions, as any coalition would involve compromising and receiving less money which doesn't make sense in this game. Similarly due to the lack of communication

between the players in the game, they can also never know if other players could change their strategies so are unable to even realise that cooperation is even possible at any given stage.

## 5.4 Cournot and Stackelberg Games

Cournot and Stackelberg games are two manners in which players participate in the game, in other words they constitute the structure of the game as opposed to how players react to one another and strategise within the game. Both of these are relatively easy concepts to understand so this section should be quite short. Because these different structures of games effect the way in which a player interacts with the other players in the game, different strategies can be better or worse based on whether the game is a Cournot game or a Stackelberg game and in some cases some strategies may not even be possible within different game structures.

A Cournot game is simply where all the players make their moves at the same time. For example, all players may submit their moves separately to a central manager node who then reveals all of the different moves at the same time and tries to work out and resolve all the different collisions and determine what exactly the outcome of the game was on that particular turn. In a Cournot game, the players all have to predict what the most likely turn of all the other players are and react accordingly for every round of the game.

A Stackelberg game is where there is a leader within the game who plays first, attempting to maximise its own utility first and then all other players in the game play in turn after that and are able to see the moves of all other players before them. Obviously in this kind of a game, where players are competing over a finite resource, whoever plays first has an immediate advantage over the over players in the game. This trickles down through the game, so that while any given player has a disadvantage compared the whoever had the preceding turn, they have a distinct advantage over all players who come afterwards.

The reasoning behind choosing a Stackelberg game over a Cournot game for this project will be discussed later in the Implementation section of this report.

## Chapter 6

# Optimisation Techniques

### 6.1 Overview

Optimisation techniques are an important part of the field of mathematics and are reasonably simple to understand, but can be extremely difficult to formulate. Optimisation problems concern themselves with a key problem that is relevant to many different fields of engineering and computer programming.

For a function  $f: A \rightarrow \mathbb{R}^n$  for a particular set  $A$ , an optimisation problem is concerned with finding an element  $x_o$  of  $A$  where  $f(x_o) \leq f(x)$  for a minimisation problem or  $f(x_o) \geq f(x)$  for a maximisation problem,  $\forall x \in A$ . These optimisation problems manifest themselves in countless fields from economics [13], civil engineering [14] and of course as part of the smart grid [15]. The optimisation techniques involved in this particular project are used on each of the two utility functions involved in the process namely that of each of the game players and then the moderator actor process involved in the system. This will of course be discussed in more detail later on.

One of the main benefits of an optimisation technique is that it is often obtainable using linear algebraic methods which means that a computer can figure out the solution to the optimisation problem in polynomial time. Another benefit of this is that an optimisation technique can be used in tandem with any other problem solving technique in order to find a better solution much faster. If any problem fits the parameters of the optimisation as defined above then different optimisation techniques can be applied or at least the same one in multiple places.

While the basic premise and motivation behind every optimisation technique is the same, different types of sets of values can be used for the set  $A$  and as a result. Fortunately, different types of optimisation techniques have been developed in order to more efficiently solve problems in each of these areas. In some cases, the type of values in the set such as in a convex set, actually make other optimisation methods useless. In this project, two main optimisation methods were used, namely Convex Optimisation and Hyperplane Projection

Optimisation. Both techniques are involved with quickly and accurately solving for a maximum in the case of two different utility functions but operate with different types of sets, each one being suitable for the relevant type of problem.

## 6.2 Convex Optimisation

Convex optimisation is defined as the solving of minimisation problems that involve convex functions being applied to convex sets [16]. Due to the nature of the convexity of the sets involved in these sorts of problems, a term that I will discuss momentarily, the local minimum that is discovered is actually a global minimum. Basically this means that the curve of the graphed outputs from mapping the values of a convex set through a convex function, only has a single minimum as opposed to a situation where the curve could have multiple minimums or values that can be converged on which are not the true minimum of the curve. This property of a convex optimisation problem as well as the property of general optimisation problems of being able to solve the problem in polynomial time means that the true solution can be discovered relatively quickly.

A convex set is simply a region in which, if you draw a line between any two arbitrary points in the region, then all points on the line are also inside the region as outlined in the left side of Fig 6.1. The right side shows a non-convex set where there is a hollow section to the region.

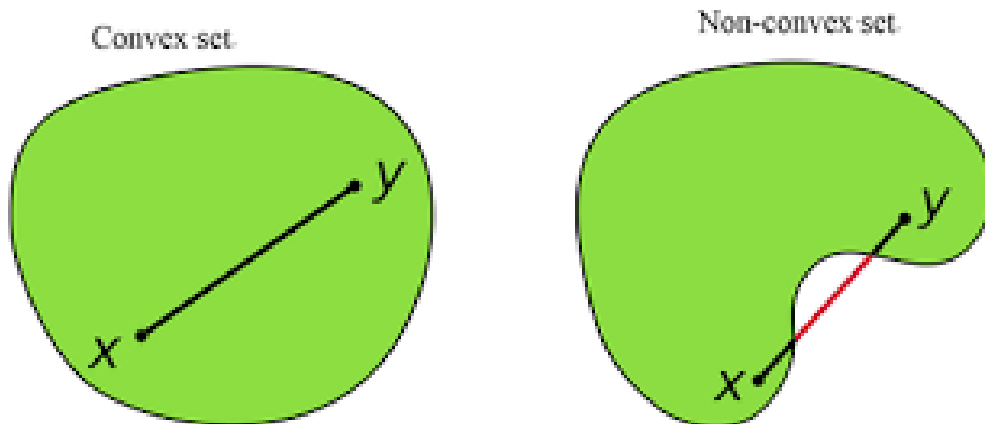


Figure 6.1: A convex set (gtMath March 2016) [17]

A convex function on the other hand is simply a function where the entire line segment between any two points on the graph is above the or on the graph. This is the part of convex optimisation that determines the fact that the local minimum is a global minimum. Convex functions are extremely common in the field of mathematics such as the quadratic function  $x^2$  and the exponential function  $e^x$ .

Convex optimisation is therefore a relatively simple concept to understand and is clearly seen to be a very useful and efficient method of accurately and quickly finding solutions to minimisation problems.

## 6.3 Hyperplane Projection

### 6.3.1 Variational Inequality Problem

The hyperplane projection method is a tool for solving problems that suit the criteria of a variational inequality problem so first that must be explained before moving onto the concept of the solution to such a problem.

A variational inequality is an inequality that involves a functional that must be solved for all variables in a set, usually a convex set. As a side note, although this problem also involves a convex set like the convex optimisation problem, the functional is not a convex function and therefore convex optimisation does not apply in this instance. A functional is a function that maps a vector space onto its underlying field of scalars. Often this vector space can be a series of functions, meaning that the functional takes a function as an argument and can be interpreted as a function of functions. This is similar to the Haskell idea of higher order functions, where a single higher order function can be used to operate on multiple functions and perhaps capture some other important piece of data for a given system.

The origin of, and primary application of, variational inequality problems is in the field of finding solutions of equilibrium in a given system. As we'll see later on in the implementation section of this report, finding the state of Nash Equilibrium between the different suppliers that take part in the game requires a state of equilibrium. Therefore it can be easily inferred that the variational inequality problem is applicable and the problem can be solved as such using a method appropriate for such a problem.

The hyperplane projection method defined here also stipulates that the underlying functional involved in the problem must meet a certain monotonicity criteria. Monotonicity is a property of a function that says that the function must either be non-decreasing or non-increasing. The function does not have to be constantly increasing or decreasing but for example if it is increasing then it cannot decrease or vice versa in order to be deemed monotonic. This can be represented mathematically as  $f(x) \leq f(y) \forall x \leq y$  or  $f(x) \geq f(y) \forall x \leq y$ . Functions that cleave to this mould are called monotonically increasing and monotonically decreasing respectively.

### 6.3.2 Hyperplane Projection Method

Having covered a number of the prerequisites for using a hyperplane projection method, the method itself can be explained. The version I looked at was developed by Solodov and Svaiter and is called the Solodov and Svaiter Hyperplane Projection Method (SSHPPM) [18]. Figure 6.2 will be referred to as a part of the explanation.

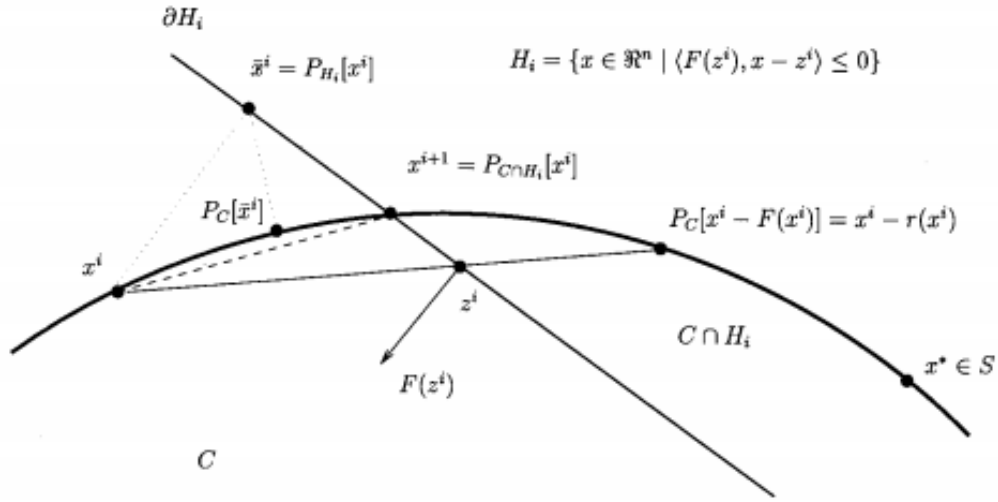


Figure 6.2: Solodov and Svaiter Hyperplane Projection Method

The curve in the figure describes the functional in the variational inequality (VE) problem. This method uses the projection operator  $P_C[x] := \operatorname{argmin}_{y \in C} \|y - x\|$  where  $y \in C$ . Suppose we have a point  $x^i$  which is the current approximation of the solution to the VE problem involving the set  $C$  and the functional  $F$ . First we calculate a projection point  $P_C[x^i - F(x^i)]$ . The segment between  $x^i$  and  $P_C[x^i - F(x^i)]$  is searched for a point  $z^i$ , using a linesearch method like the Armijo linesearch method [19], such that a hyperplane  $\delta H_i$  (using the definition of  $H_i$  as defined in figure 6.2) strictly separates  $x^i$  from any solution  $x^*$  of the problem. The next approximation to the solution  $x^{i+1}$  is calculated by projecting  $x^i$  onto the intersection of the set  $C$  and the halfspace  $H^i$  that contains the solution set using  $P_{C \cap H_i}$ .

The benefit of this solution is that each iteration of the method only requires two projections which makes it computationally efficient, the first to calculate the hyperplane  $H_i$  and another onto the intersection  $C \cap H^i$  to find the next iterate in finding the solution. Later on in the Implementation section, the application of this method will be discussed in further detail.



# **Part III**

## **Implementation**

# Chapter 7

## Design

### 7.1 Games vs Auctions

In the background section of this report both the concepts of Auctions and Game Theory as both were considered as potential candidates for the management system to match supply and demand in a nanogrid system. Ultimately however, a non-cooperative game was chosen as the prime candidate for the smart grid in this project. It is important to first consider the reasons as to why this choice was made before explaining how the game was designed.

In the process of investigation of auctions and game theory, certain similarities stood out between the two management systems. Ultimately all actors within either of these systems are trying to maximise their utility, a scalar value that is determined based on a number of key variables that each actor considers pertinent to their operation. In the case of a model such as this one, where a price value is involved, the utility of any given actor is usually modelled as a balance between any profit that the unit could make versus some kind of risk factor of selling too much at any one given time. In this regard, the modelling of any actors within the grid would end up being the same on a conceptual level and only the interactions between them would change based on what kind of system was chosen.

As has been outlined in previous sections, one of the main criteria for the nanogrid system, was that of minimal sharing of information between actors in the grid. This was to decrease the size of packets exchanged between nodes in the network as well as to hopefully decrease the number of packets sent between each other in order to improve the efficiency of such a system such that it might be practical for a real world scenario. Therefore the focus was on a system that would fit this design. Every auction that was investigated as part of this report had a crucial element of either all nodes being aware of the each others' private information or at the very least the central node needed to have all this information to hand. Therefore a non-cooperative game seemed more appropriate based off this particular design.

## 7.2 System Design

In this section I will discuss a brief overview of the operation of the system implemented in this project. Below in Figure 7.2 is a basic flowchart of a single iteration of the operation of the system, followed by a brief summary of each step. The summary below assumes that all the nodes within the network have connected with one another already, although in my code submission there is an extra step to ensure that the system process doesn't start until the user decides that it should so that the system can be monitored on a step by step basis. Figure 7.1 is a simple diagram of the connections between different actors within the system.

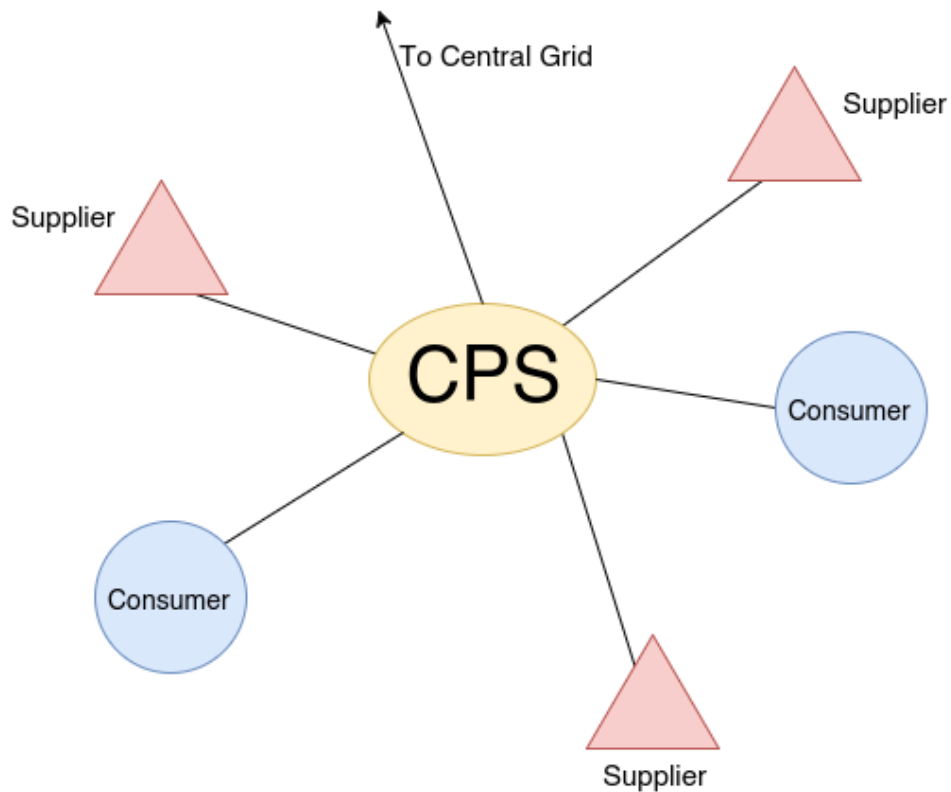


Figure 7.1: Simple diagram to understand the connections between the different actors involved in the system in a given iteration

An iteration of the system is conducted to match supply and demand for in a nanogrid situation for a given upcoming timeslot. Some kind of system where a consumer can predict their energy usage for the next timeslot is presumed to be in place. The suppliers of course know what their own supply of energy is as well as having a caution variable  $c \in (0, 1)$ . The caution value determines how willing they are to sell larger amounts of energy, a low caution value representing a willingness to sell more energy and a high value standing for a more conservative supplier.

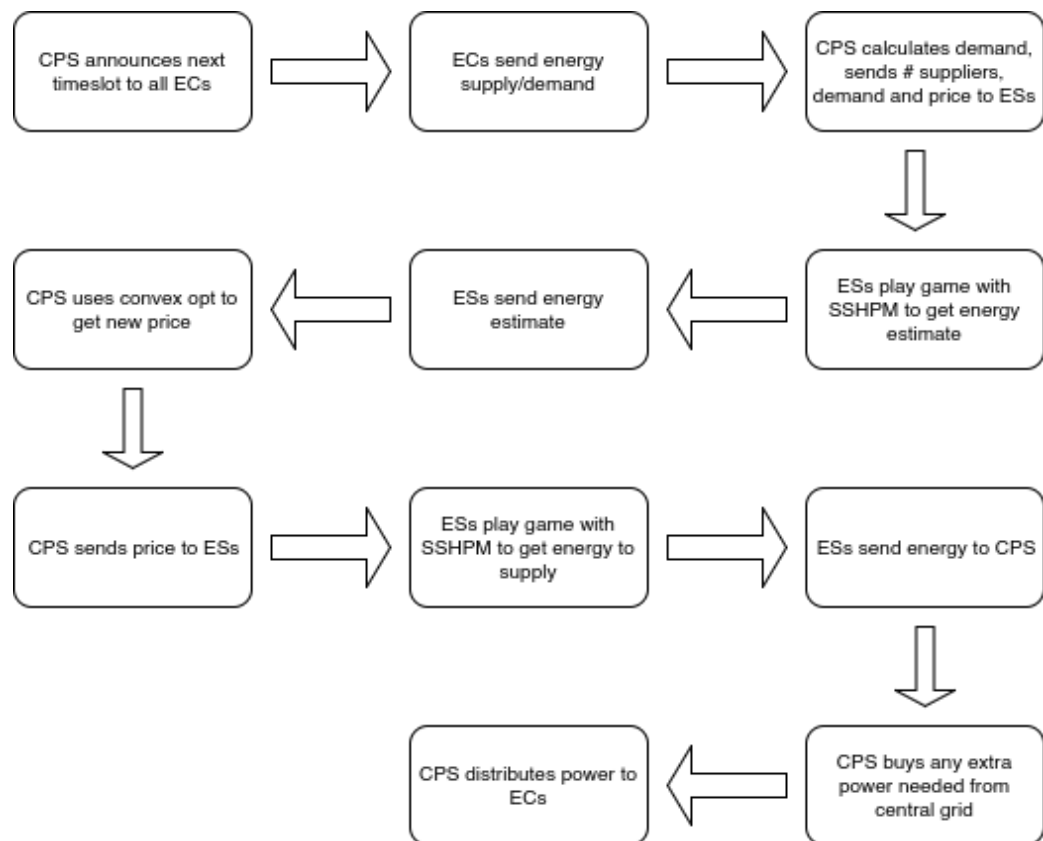


Figure 7.2: Flow chart depicting the operation of the system in terms of the Central Power Station (CPS), Energy Consumers (ECs) and Energy Suppliers (ESs) in a single iteration

The operation begins with the Central Power Station (CPS) announcing a timeslot to all consumers and all suppliers within the network. At the beginning the CPS doesn't know who is a consumer and who is a supplier in order to accommodate the situation where a consumer has proactively bought too much energy in anticipation of needing it or has been instructed by some logic to sell excess energy into the grid. Each Energy Consumer (EC) then notifies the CPS as to whether it is in need of energy or whether it has energy to sell and if it's the case of the former then it also sends how much energy it requires. Figure 7.1 shows the situation where ECs within the grid have already made it clear as to whether they are a supplier or a consumer for this particular timeslot.

The CPS then simply sums the total demand and can begin the game. It sends to the total demand, the total amount of money it has available to give and the number of suppliers within the system to each Energy Supplier (ES). The total price is calculated naively by multiplying the current price per unit that is offered by the central grid by the number of units of energy required by the consumers within the nanogrid. A standard unit would be kWh. Each ES first calculates how much energy it can be offered by dividing the total price by the number of players. Each one then uses the SSHPM optimisation method to determine an estimate for the energy it is willing to give to the CPS at that price and sends that estimate to the CPS. The functional used as part of the SSHPM is the utility function of each EC and the set of values being mapped over is a one dimensional vector space that goes from zero to whatever the total energy of each EC is.

The CPS then receives each ES's energy estimate. From this it is able to estimate how willing each ES is to giving more or less energy. It cannot work out the private store or the caution of each ES but rather understands the ratio that exists between all the different players involved. The CPS then uses its own utility function and the vector of energy estimates from each ES as the inputs to a convex optimisation problem. A disciplined convex optimisation method is employed [20] as any standard convex optimisation technique is all that is required and the Python solver CVXPY [21] was readily available. A new vector of prices per ES is generated and each one is sent to each ES. This is the actual price that each ES receives.

The ESs then play another game using their utility functions and the new price that they have been offered by the CPS and try to find the actual amount of energy that they are willing to give away using SSHPM. This energy is then sent to the CPS. The CPS then sums the total of energy that has been provided at that time. If this energy matches the total demand of the consumers in the nanogrid, then the energy is simply supplied to those who need it, on a first come first serve basis. However, if the supply does not reach the demand then the CPS buys the extra power that is needed from the central power grid as seen in Figure 7.1. This system accepts the fact that it may not be able to supply all consumers within the nanogrid using solely local sources that exist within its own grid. Once the supply matches the demand, the power is then distributed as before. The process then starts again ahead of the next timeslot to ensure that everyone that needs power during that time is supplied.

## 7.3 Game Design

First some of the key components of the game as well as a brief overview of how it is conducted will be explained and then after that, the game will be discussed ins and outs will be discussed in further detail. The game played between all of the ESs that are trying to receive remuneration for the energy they are willing to offer is played across two steps. First of all the ESs use their utility functions along with a number of other important variables such as their energy capacity  $E_n$ , caution  $c_n$  and the current price offer  $p_n$  in order to determine their new estimate for how much energy they are willing to offer to the CPS  $e_n$ , where  $n \in N$ ,  $N$  being the set of all ESs taking part in the nanogrid. Next they use that energy estimate to calculate a slack variable  $\varepsilon_n$  which is a variable indicating the amount of energy it is willing to offer without giving up any private information. These slack variables are derived from the ES's utility functions which will be discussed in the next section. The slack variables are used by the CPS to determine Nash Equilibrium within the game, namely this is when all of the slack variables are equal. Once this state of equilibrium is reached, then the CPS asks for the energy offer from each of the ESs.

When the hyperplane projection is initially calculated there is a small piece of logic that determines what slack variable is sent to the CPS as well as what energy should be offered. If the projection is equal to zero then  $\varepsilon_n = E_n - 2c_n e_n + p_n$ . Otherwise the second part of the hyperplane projection method is run, where the halfspace is determined and from that a new projection is worked out. In this case the slack variable sent back to the CPS is  $\varepsilon_n = E_n - e_n + p_n$ . These slack variables are then sent to the CPS. If the slack variables are all equal, as previously mentioned, then the game has reached the state of Nash Equilibrium and the ESs are informed to end their iterations and they instead send back the amount of energy they are offering. If the slack variables are not equal then the CPS instructs the ESs to perform another iteration of the SSHPM.

## 7.4 Utility Functions

### 7.4.1 EC Utility Function

Each EC has a utility function that is used as the functional in the the hyperplane projection optimisation. The utility function in question takes into account the energy that EC  $n$  has stored  $E_n$ , the price being offered to it  $p_n$ , the caution value of that EC  $c_n$  and the energy that it is offering  $e_n$ .

$$U(e_n, E_n, p_n) = p_n e_n + (E_n - c_n e_n) e_n$$

This utility function is based on the profit that the EC could get when it is supplying energy, that is  $p_n e_n$ .  $(E_n - c_n e_n) e_n$  represents the loss that the EC incurs by giving away a certain amount of power. Ultimately the system is trying to maximise the utilities of all ECs in the nanogrid, where the sum of all offered energies

is less than or equal to the energy deficiency (demand) of the system for a given timeslot  $E_{def}$ , that is

$$\sum_n e_n \leq E_{def}$$

The utility function defined for the EC is the the crux of this project in order to both structure the game itself and to determine the efficacy of the system. The utility function is defined such that each EC is better utilised for each timeslot but also does not expend too much electricity at one time unless the incentive, namely the price, for it to do so is very high. This means that at a later stage when there is perhaps a higher deficit, it can make more money in the future as opposed to potentially being depleted of energy for the times of high profit.

### 7.4.2 CPS Utility Function

The CPS has its own utility function that serves as the convex function for the convex optimisation problem in trying to find appropriate prices for each of the ESs that have submitted energy estimates for how much they are willing to offer. The function is represented as a minimisation problem in terms of the energy that each ES is offering  $e_n$ , the price that the CPS would offer for that energy  $p_n$  and two scalar values  $a_n$  and  $b_n$  that account for the costs associated with storing and transmitting the energy.

$$\min_p L(p, e) = \min_{p_n} \sum_n (e_n p_n^r + a_n p_n + b_n), \text{ subject to } \sum_n p_n = P, p_{min} \leq p_n \leq p_{max}$$

For each ES, the CPS is trying to find the value of  $p_n$  that will give the smallest value. However all values of  $p_n$  must sum to be equal to the value of  $P$ , the total price that the CPS is willing to pay. As can be seen in this model, the system doesn't pay any less for power overall, but rather incentivises all suppliers of electricity to try to match the demand in question. Another caveat of the minimisation problem is that  $p_n$  must be between the values of  $p_{min}$  and  $p_{max}$ . This simply means that there is a minimum and a maximum value that the CPS is willing to pay for energy.

## Chapter 8

# Application

### 8.1 Python Twisted Framework

In the course of this project the Python Twisted Framework [22] was used in order to implement a network between the CPS and the ECs. The Twisted Framework has a number of layers in order to abstract out the problem for the user so that they only need care about their own application [23]. It also has a number of inbuilt functions so that the programmer does not have to care about things like sockets that are very tricky and are far removed from the problem trying to be solved in this project.

Both the Client and the Server have two main layers where all the Twisted magic happens, the Factory layer and the Protocol layer. Essentially the Factory layer contains all of the persistent information of any given network actor and the Protocol layer contains actions and information for every connection made by that actor. In the code produced as part of this project, the factories of the CPS and the ECs were mostly used to store the values of variables pertinent to each of them such as the price vectors for the CPS and the energy storage for each of the ECs.

The majority of the logic that controls both the CPS and ECs was contained within separate Finite State Machine (FSM) files which were connected to their respective Protocol files the FSM only changes state based on the inputs it receives from any given connection. It was easiest to abstract the problem out in this fashion for ease of reading and understanding of the code for both the programmer and any potential readers. Each EC and the CPS is finally wrapped by a simple run script that just sets up the factory from which everything else is run. In each section below, the FSM of both the ECs and the CPS will be examined as the system is conducted in the same fashion for both, where the protocol calls a different function in the FSM depending on the state of the actor at that time.



## 8.2 Client (EC)

### 8.2.1 Idle State

The idle state is merely a state for in between operations of the system, where no game is being played. In this project it was also used as an initial state before the user decides to start the game. When a new game is started by the CPS, it notifies each EC to move into the start state.

### 8.2.2 Start State

The start state is where the EC sends a message to the CPS to inform it as to whether it is a supplier or a consumer for the upcoming timeslot. If it is a consumer then it also sends the amount of energy that it requires at that time. Also if an EC in the nanogrid requires no energy for the next timeslot, then it simply puts itself back into the idle state, awaiting the next timeslot when it might need or be able to supply energy. An EC moves to the Estimate 1 State if it is a supplier and to the receive state if it is a consumer.

### 8.2.3 Estimate 1 State

This state is used for when the ESs are playing the game and making their first estimate of how much energy they are willing to offer to the CPS. In this state, if an ES is told to "End" its iterations then it moves to the second estimation state (Estimate 2 State). Otherwise it uses the hyperplane projection method solver (SSHPPM Implementation) that was developed as part of this project and sends a slack variable to the CPS, used in determining Nash Equilibrium for the game.

### 8.2.4 Estimate 2 State

The Estimate 2 State is more or less the exact same as Estimate 1 State except that when it receives the "End" message, it instead moves back to the idle state, having successfully supplied energy to the CPS and having been remunerated for that energy. If it doesn't receive the end message then it uses SSHPPM to calculate a new slack variable and continues playing the game.

### 8.2.5 Receive State

The Receive State is the state for any consumers for the current timeslot. An EC stays in this state until the operation of the system has been completed and the energy is distributed to it accordingly. Once it receives this energy, it returns to the Idle state in order to wait for the next timeslot.

### 8.2.6 SSHPM Implementation

The SSHPM implementation caused the greatest amount of difficulty as part of this final year project. The paper [18] details a complex and dense mathematical algorithm that was difficult to grasp and to implement. The functions with the SSHPM.py file follow the steps in the algorithm defined by Solodov and Svaiter.

## 8.3 Server (CPS)

### 8.3.1 Idle State

The Idle State is used while there is no game happening and the CPS is idle. The CPS is then able to move into the start state when it wants to begin a game immediately preceding a new timeslot. In this implementation it's also used on start-up of the system so that the operation of the system doesn't begin until every EC is connected to the CPS. Each connected EC is stored in a Python dictionary with its relevant connection.

### 8.3.2 Start State

The Start state accepts incoming messages from each of the ECs about whether they will be a consumer or a supplier for the current timeslot or if they will be abstaining from the current round of operation. It stores each EC's role in the upcoming game and doesn't move to the next state until every single EC in the network has given an answer as to what their role shall be. The CPS then naively calculates the demand by simply summing the values given by each of the consumers and then moves to the Init State,

### 8.3.3 Init State

The Init State is used to allow the suppliers who will be taking part in the game to start the game by providing them with the values that they need. Each EC is sent the energy deficiency ( $E_{def}$ ) for the current timeslot and price that it is willing to offer to each EC which it calculates by multiplying  $E_{def}$  by the current price per unit of energy and dividing that by the number of ESs in the system. The CPS then moves to the Game 1 State.

### 8.3.4 Game 1 State

The Game 1 State is used for the first game that is played by the ECs that are supplying energy for the current timeslot. When all connected ESs have responded with their slack variables, the CPS runs a quick check as to whether or not the slack variables are equal. If they are then it tells the ESs to finish their iterations of SSHPM and to send the energy estimate that they used to calculate the last slack variable that they sent and

then the CPS moves to the Optimisation State. If the slack variables are not all equal then the CPS tells the ESs to continue playing the game.

### 8.3.5 Optimisation State

The Optimisation state first waits to receive the energy estimates from each ES in the game of the current timeslot before beginning the convex optimisation. It formulates the problem and then solves it using the CVXPY [21] solver library. The new prices are then extracted from the solver and the CPS sends the relevant price to each ES before moving to the Game 2 State.

### 8.3.6 Game 2 State

The Game 2 State is more or less identical to the Game 1 State except that it is concerned with managing the ESs who are trying to calculate the actual amount of energy that they will give to the CPS as opposed to an estimate. In this state, when the slack variables are equal, it similarly tells the ECs to stop the iterations of their game and then it moves the CPS to the Distribute State.

### 8.3.7 Distribute State

The Distribute State first makes sure that all ESs have submitted the amount of energy that they are going to provide to the CPS. It then sums these values and compares that to  $E_{def}$  and if it the supplied energy is insufficient, then it buys the extra energy needed from the central grid. It then disperses the energy needed to each of the consumers within the current timeslot and moves into the idle state, ready for the next game before the next timeslot.

## **Part IV**

# **Conclusion**

## Chapter 9

# Results

Ultimately the success of the system was unable to be determined due to difficulties with the implementation of the hyperplane projection method. Instead of converging on a value for the energy estimate, it instead approaches zero for every instance but never reaches it, and therefore runs infinitely. In order to solve this obstacle, a number of lecturers within the school of computer science were contacted for assistance and the author of the paper, Wayes Tushar, was approached. However Mr Tushar was unavailable and the assistance provided was not enough to reach a proper understanding of the mathematics involved in SSHPM in order to work out a correct code implementation.

However, a pen and paper version of a comparison between the REFIT scheme and the proposed scheme was worked out to demonstrate a potential scenario of operation in order to compare the two systems. The slack variables were calculated on paper and are hard-coded into the current system in order that the code can be run as opposed to running forever. The actual code is still there but merely commented out. This is commented on in the code. This scenario was run between a single consumer and two suppliers. The consumer sets  $E_{def} = 700$  and the two suppliers have  $E_1 = 1100$  and  $E_2 = 1000$ . The price per unit (kWh) is set at 1.85. When the proposed system is run with these values the sum of utilities  $\sum_n U_n = 1069618.928$ . Compared to the REFIT scheme where the ratio between the ratio of capacities is used as a naive method of deciding how much energy should be given by each EC, in a system where they share private information. In this scenario, the sum of utilities  $\sum_n U_n = 1068167.8$ .

Unfortunately, due to the lack of conversion on correct values by the system, no further scenarios could be calculated as it is unclear as to what the values that the optimisation methods would reach in instances where the supply does not meet the demand. Optimisation techniques can often require several iterations before converging on a correct value and therefore are not feasible to be worked out on paper.

## Chapter 10

# Assessment

From the numerical results it can be easily seen that the proposed method in this instance is superior to the REFIT scheme as the sum of utilities is greater in the former approach. However the difference between the two utilities is minimal so this system may not be the worth implementation in the real world. A working solution, however, would verify whether this statement is true or not.

Regardless, an important question to answer as part of this project, is about the likeliness of such an system being implemented in the real world. First of all it must be noted that such a system could only exist in a few years' time when suitable prediction methods have been developed for predicting the amount of energy that a consumer will require within the next timeslot. Secondly, the installation of such a smart grid would require a huge overhaul of the current network if it were to jump straight to this system. Rather a more realistic approach would be to first introduce smart meters into homes in conjunction with the current grid, like is in place in Italy as discussed previously and to follow that with a REFIT scheme to incentivise the installation and construction of local renewable energy sources as is in place in Germany. At this point it would then be possible and feasible to introduce a system such as the one discussed in this report. The reason why a REFIT scheme would need to precede a game theoretic solution is that on a human level, people would need a tangible and static amount of money to ensure that they would see a return on their investment whereas the game theoretic approach does not yield a concrete and easy to grasp amount of money.

## Chapter 11

# Future Work and Continuations

There are a number of interesting ways in which this project could be continued, that were realised during the course of the investigation of the field of smart grids and in the particular area around which this project is based. These are in two distinct categories: the former being ways in which this project could have been extended had there been more time and the latter being ways in which investigations could be made into pairing this project with other proposed smart grid technologies.

Originally there was the intention to eventually implement a prototype for the supply and demand matching using a number of Raspberry Pis connected over a WiFi network in order to analyse network latencies as well as to create a real, tangible network and prove whether or not this system could be implemented on a larger scale.

Another potential extension would be to create a hierarchy of CPSs in a larger smart grid, where the CPS of a nanogrid in a house would then act as an EC within say the community or neighbourhood. The CPS controlling that region would then be an EC for a larger region, say a county or even an area code. However a major change would need to take place in order to implement such a system. The current system that was examined in this project, only ever tried to supply energy less than or equal to the upcoming energy deficiency. This extension however would require the game to be able to produce a surplus of electricity for a single house at a given time period. If this was possible then a house could then act as a supplier in the game within its own neighbourhood and actually generate a further profit for that house.

As mentioned previously in this report, as well as supply side management systems, there are also plenty of papers concerned with demand side management. Most of these papers were primarily concerned with driving down the price of energy at any one given timeslot so it would be interesting to investigate pairing such a system with the system in this paper in order to see whether the low price generated is still enough to incentivise suppliers to give energy to the CPS. If successful, such an amalgamation would help citizens who had such a grid system set up in two different ways, first by driving down their costs and then by allowing

them to sell energy to further reduce their costs.

The final potential continuation of this project would be to pair any devices that would solely be suppliers in a nanogrid such as wind turbines and solar panels, with some form of prediction software relating to weather patterns [24]. This data could then be used to inform an EC's caution value. For example if the EC was nearing capacity and it knew that it was going to be generating enough electricity that it would be unable to store it, then it could have a very low caution value. Conversely, low energy capacity and low production in the future could be used to inform a higher caution value.



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