

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

$$Ax = b \quad \Downarrow$$



$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ \hline a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \hline \dots \quad \quad \quad \uparrow a_{nn}x_n = b_n \end{array}$$

$$\begin{array}{l} n \rightarrow 1 \\ \hline (n-1) \rightarrow 0 \end{array}$$

$$x_n = \frac{b_n}{a_{nn}}$$

$$a_{n-1,n-1}x_{n-1} + a_{n-1,n}x_n = b_{n-1}$$

$$x_{n-1} = \frac{1}{a_{n-1,n-1}} (b_{n-1} - a_{n-1,n}x_n)$$

$$x_{n-2} = \frac{1}{a_{n-2,n-2}} \left(b_{n-2} - a_{n-2,n-1} x_{n-1} - a_{n-2,n} x_n \right)$$

~~$$0 \quad 0 \quad 0 \quad a_{ii} \quad a_{i,i+1} \quad \dots \quad a_{in} \quad b_i$$~~

$$x_i = \frac{1}{a_{ii}} \left(b_i - \sum_{j=i+1}^n a_{ij} x_j \right)$$

Sum

$$R_1: \boxed{a_{11}}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$R_2: \textcircled{a_{21}}x_1 + \boxed{a_{22}}x_2 + a_{23}x_3 = b_2$$

$$R_3: \textcircled{a_{31}}x_1 + a_{32}x_2 + \boxed{a_{33}}x_3 = b_3$$

$$\begin{cases} R_2 - \left(\frac{a_{21}}{a_{11}} \right) R_1 = R_2 \\ R_3 - \left(\frac{a_{31}}{a_{11}} \right) R_1 = R_3 \end{cases}$$

$$R_1: a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$R_2: \quad \quad \quad a_{22}x_2 + a_{23}x_3 = b_2$$

$$R_3: \quad \quad \quad a_{33}x_3 = b_3$$

$$\textcircled{2} \left\{ \underline{R_3} - \left(\frac{a_{32}}{a_{22}} \right) R_2 = R_3 \right.$$

$$\begin{array}{l} i \rightarrow \quad \textcircled{a_{i1}}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i \\ R_2 \quad \textcircled{a_{21}}x_1 + \textcircled{a_{22}}x_2 + \dots + a_{2n}x_n = b_2 \\ n-1 \rightarrow R_n \quad \textcircled{a_{n1}}x_1 + \textcircled{a_{n2}}x_2 + \dots + \textcircled{a_{nn}}x_n = b_n \end{array}$$

$$A_{n \times n} x_{n \times 1} = B_{n \times 1}$$

$$\underline{i = 1 : (n-1)}$$

$$j = \underline{i+1}, \underline{n}$$

$$\begin{aligned}
 (1) & \rightarrow a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1 \\
 (2) & \rightarrow a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_2 \\
 & \quad a_{33}x_3 + \dots + a_{3n}x_n = b_3 \\
 & \quad \vdots \\
 (n-1) & \rightarrow a_{n-1,n-1}x_{n-1} + \dots + a_{n-1,n}x_n = b_{n-1} \\
 (n) & \rightarrow a_{nn}x_n = b_n
 \end{aligned}$$

Diagram illustrating the elimination process for the first column. The first column of coefficients $a_{11}, a_{21}, \dots, a_{n1}$ is shown being zeroed out in rows 2 through n. The resulting system of equations is shown to the right.

i.

$$(1) \Rightarrow R_j = R_j - \left(\frac{a_{j1}}{a_{11}} \right) R_1, \quad j=2, \dots, n$$

$$(2) \Rightarrow R_j = R_j - \left(\frac{a_{j2}}{a_{22}} \right) R_2, \quad j=3, \dots, n$$

$$(i) \Rightarrow R_j = R_j - \left(\frac{a_{ji}}{a_{ii}} \right) R_i, \quad j=i+1, \dots, n$$

$$a_{jk} = a_{jk} - \frac{a_{ji}}{a_{ii}} a_{ik}$$

Diagram illustrating the row operation $R_j = R_j - \left(\frac{a_{ji}}{a_{ii}} \right) R_i$. The diagram shows the original row i and row j of the augmented matrix. The element a_{ji} in row j is circled, and an arrow points to the element a_{ii} in row i . The element a_{ik} in row i is circled, and an arrow points to the element a_{jk} in row j . The resulting row j is shown below, with the element a_{jk} updated to $a_{jk} - \frac{a_{ji}}{a_{ii}} a_{ik}$.

$$x_{n+1} = f(x_n)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= g(x_n)$$

$$x^2 - 4x + 1$$

$$g(x) = x - \frac{x^2 - 4x + 1}{2x - 4}$$

$$= \frac{2x^2 - 4x - x^2 + 4x - 1}{2x - 4}$$

$$= \frac{x^2 - 1}{2(x - 2)}$$

$$x_{n+1} = g(x_n)$$

$$= x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x = g(x)$$

$$f(x) = 0$$

