

Homework Set 3, CPSC 8420, Fall 2024

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Due 11/11/2024, 11:59PM EST

Problem 1

Please download the image from [https://en.wikipedia.org/wiki/Lenna#/media/File:Lenna_\(test_image\).png](https://en.wikipedia.org/wiki/Lenna#/media/File:Lenna_(test_image).png) with dimension $512 \times 512 \times 3$. Assume for each RGB channel data X , we have $[U, \Sigma, V] = svd(X)$. Please show each compression ratio and reconstruction image if we choose first 2, 5, 20, 50, 80, 100 components respectively. Also please determine the best component number to obtain a good trade-off between data compression ratio and reconstruction image quality. (Open question, that is your solution will be accepted as long as it's reasonable.)

Answer:

The code for this problem is as follows:

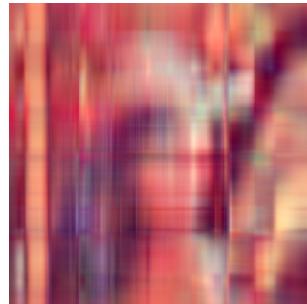
```
1 clear all; close all;
2
3 ks = [2, 5, 20, 50, 80, 100];
4 img = imread("Lenna.png");
5 image = im2double(img);
6
7 [m, n, channel] = size(img);
8
9 for k = 1:length(ks)
10    rec = reconstruction(image, channel, ks(k));
11    rec = im2uint8(rec);
12    compression_ratio = (m*k+k*k+k*n) / (m*n);
13    % plot
14    ax = subplot(2,3,k);
15    imshow(rec, 'Parent', ax);
16    title(ax, sprintf("first %d components,\n compression ratio=%.3f", ks(k), compression_ratio))
17 end
18
19 function image_rec = reconstruction(image, channel, k)
20    image_rec = zeros(size(image));
21    for ch = 1:channel
22        X = image(:, :, ch);
23        [U, S, V] = svd(X);
24        X_rec = U(:, 1:k) * S(1:k, 1:k) * V(:, 1:k)';
25        image_rec(:, :, ch) = X_rec;
26    end
27 end
```

The reconstruction images are shown in Fig. 1. I'd select the first 50 components to reconstruct the image, as 50 is the smallest number among these six options that achieves good image quality.

**first 2 components,
compression ratio=0.004**



**first 5 components,
compression ratio=0.008**



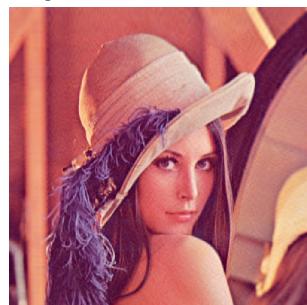
**first 20 components,
compression ratio=0.012**



**first 50 components,
compression ratio=0.016**



**first 80 components,
compression ratio=0.020**



**first 100 components,
compression ratio=0.024**

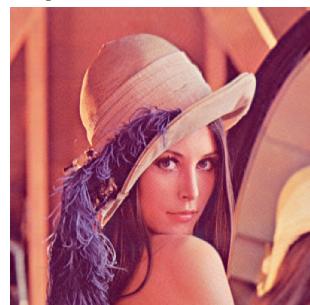


Figure 1: Reconstruction images