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Errors in Estimating Mean Weight and Other Statistics from Mean Length

LARRY A. NIELSEN

Department of Fisheries and Wildlife Sciences
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

WILLIAM F. SCHOCH

Department of Ecology, Fisheries, and Wildlife
Oklahoma State University
Stillwater, Oklahoma 74074

Abstract

Computer simulations of length-weight data indicated that the error incurred by estimating mean weight from mean length is a function of the exponent in the length-weight relationship and the coefficient of variation in the length-frequency distribution. Errors were less than 10% for exponents up to 4.0 if the coefficient of variation were less than 0.1, but error increased to near 90% of actual mean weight for large exponents and coefficients of variation. Similar errors arise for other exponential relationships, such as length-fecundity, that commonly are developed in fisheries work.

Morphometric comparisons such as length-weight and length-fecundity relationships appear in almost every study of life history or population dynamics of fishes. Such comparisons are useful because measurements which are imprecise in the field (for example, weight) or require killing fish (for example, fecundity) can be estimated from an easily measured feature such as length (Tesch 1971). In the construction of population models, morphometric relationships may allow extrapolation of data to provide new information. For example, the egg production by a cohort is often calculated as the number of mature females times the fecundity for a female of average length as determined from a length-fecundity relationship.

The form of the relationship, however, and the method of computation can affect the accuracy of estimates (Pienaar and Ricker 1968). Many morphometric relationships are nonlinear, as in the typical length-weight relationship,

$$W = aL^b,$$

where L is length, W is weight, and a and b are constants. Mean weight, $\bar{W}$, of a group of N fish can be estimated in two ways from this formula. Mean length, $\bar{L}$, can be calculated and entered directly into the formula,

$$\bar{W}_e = a(\bar{L})^b. \quad (1)$$

A more tedious procedure involves estimating the weight of each fish and averaging these, as in the formula

$$\bar{W} = \left(\sum_{i=1}^{N} aL_i^b\right)/N. \quad (2)$$

Equation (2) gives an accurate estimate of mean weight. If the exponent (b) in the relationship is greater than 1.0, however, Equation (1) will always underestimate the actual mean weight (Fig. 1).

Equation (1) is used frequently in fisheries studies, but more often than not the computational method is not specified. Tesch (1971) suggested that the error involved in Equation (1) is about 5–10%, but a direct analysis of this conclusion has not been performed. This study was conducted to describe the effects which the magnitudes of the exponent and the variation in sample lengths have on the accuracy of weights estimated by Equation (1).

Methods

The analysis was performed by 104 computer simulations of fish lengths and weights. The program was written in FORTRAN and is available from the authors. For each simulation, the program generated a set of 1,000 positive values from a normal distribution with a chosen

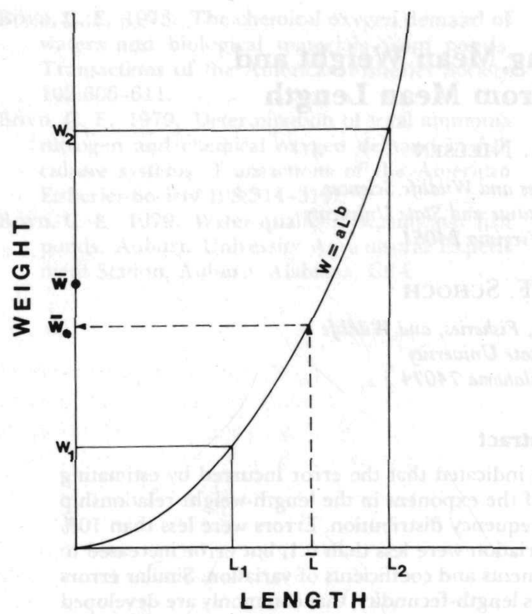

FIGURE 1.—*The hypothetical difference between actual mean weight ($\bar{W}$) and mean weight ($\bar{W}_e$) estimated from mean length ($\bar{L}$) for a pair of fish.*

mean and standard deviation. This set of numbers represented fish lengths.

Preliminary investigations indicated that the joint effect of varying the mean and standard deviation of the length-frequency distribution could be described by the coefficient of variation, CV = SD/mean. In all simulations, therefore, mean length was held constant at 10, and standard deviation was varied. The program was written to produce CV values from 0.05 to 1.0 at intervals of 0.05. Restricting the simulated lengths to positive numbers, however, deformed the length distribution such that CV did not exceed 0.62. This is of minor concern because length distributions with CV values that large are unlikely.

Mean weights were estimated from length-weight relationships according to Equations (1) and (2). The percent error (E) involved in Equation (1) was

$$E = 100(\bar{W} - \bar{W}_e)\bar{W}.$$

In the length-weight relation, a was held constant at 1.0, and b was varied from 2.00 to 5.00 at 0.25 intervals. Although the exponents for most length-weight relationships fall between

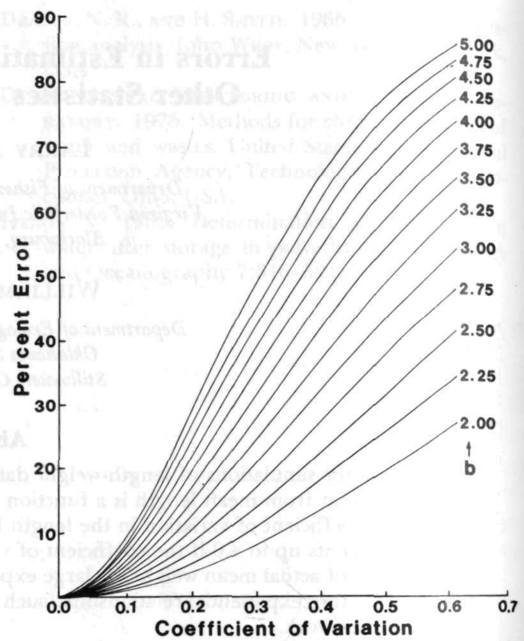

FIGURE 2.—*Percent error in estimated mean weights as a function of the coefficient of variation in the length-frequency distribution. Each curve represents a different exponent (b) in the length-weight relationship.*

2.0 and 4.0, exponents up to 5.0 were included to represent other relationships, such as length-fecundity, which may involve higher exponents.

Results

For a given exponent, the magnitude of error increases in a logistic pattern as the coefficient of variation increases (Fig. 2). All curves pass through zero because with no variation in lengths, the individual weight and mean weight are identical. Calculations of coefficients of variation for samples of yellow perch (*Perca flavescens,* Nielsen 1978), largemouth bass (*Micropterus salmoides,* Savitz 1978), and Atlantic menhaden (*Brevoortia tyrannus,* Nicholson 1975) suggest that typical CV values range between 0.04 and 0.20 for large samples or small fish. Error in predicting weights (exponent near 3.0) would not exceed approximately 10% with coefficients of variation in this range. When small sample size requires pooling data for several year classes, CV would be expected to increase and larger errors would result.

For a given coefficient of variation, magni-

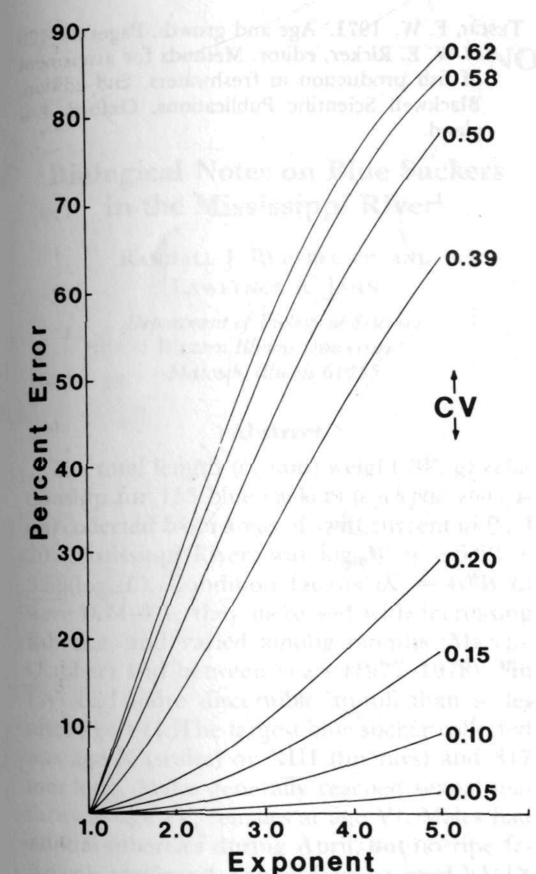

FIGURE 3.—*Percent error in estimated mean weights as a function of the exponent in the length-weight relationship. Each curve represents a different coefficient of variation (CV) in the length-frequency distribution.*

tude of error also increases with an increase in exponent (Fig. 3). All curves pass through zero error when the exponent is 1.0 because the relationship is linear. With exponents less than 1.0, as might occur for length-scale radius relationships, the error would be negative. Exponents in the range of 2.5–3.5 are typical for length-weight relationships, and relatively small errors would occur at normal CV. Large exponents, however, characteristic of length-fecundity equations, may produce large errors, especially when combined with high coefficients of variation. Recent literature contains exponents for length-fecundity equations as high as 4.20 for bloater (*Coregonus hoyi,* Emery and Brown 1978) and 5.68 for white bass (*Morone chrysops,* Ruelle 1977). With exponents greater than 5.0, errors will exceed 10% with coefficients of variation as low as 0.11.

Discussion

Tesch's (1971) generalization that a 5–10% error results from using Equation (1) holds for a limited set of conditions. When the coefficient of variation for the independent variable is less than 0.1, error is low regardless of the exponent; beyond that level, the exponent and the coefficient of variation both greatly affect the magnitude of error.

Errors can be very large under unusual conditions, and individuals should determine what level of error is acceptable before choosing a method of computation. Figure 2 can be used to estimate an error for given coefficients of variation and exponents. These errors are approximate only because the normal distribution and specific values used in the simulations will not match empirical data exactly.

Pienaar and Ricker (1968) presented an alternative method for computing mean weights when mean length, standard deviation, and the length-weight relationship are known. The computational method, however, is rather complicated, and it has not been used routinely in fisheries. The availability of programmable calculators and computer programs should relieve the tedious calculations involved in Equation (2), justifying its use if larger errors are liable to occur.

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