SURPLUS PRODUCTION (continued)

Transition to a New Equilibrium

The following materials are adapted from Fletcher (1978), on the *Recommended Reading* list.

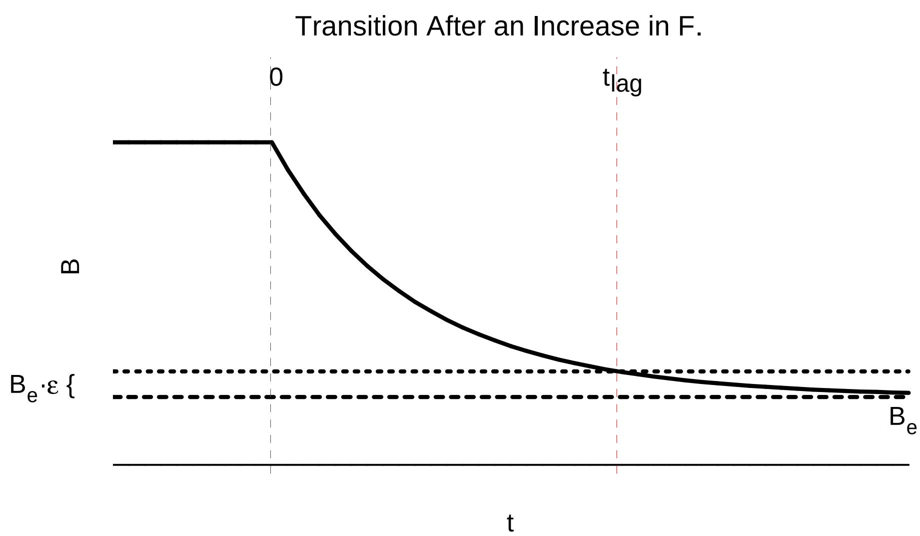
Because B(t) approaches the new equilibrium value asymptotically, it takes an infinite amount of time to actually reach the new equilibrium. However, we can determine how long it will take to get to within any fixed proportion of the new equilibrium following a sudden change in the rate of fishing mortality F. If the initial rate of fishing mortality F_0 is less than the new rate F_1, then the old equilibrium biomass was greater than the new B_e and we want to determine how long it takes for B(t) to reach the value $B_e + \varepsilon \cdot B_e$. We want to find t_{lag} such that

$$B(t_{lag}) = B_e + \varepsilon \cdot B_e = B_e \cdot (1+\varepsilon) \qquad ==> \qquad \frac{B(t_{lag})}{B_e} = 1 + \varepsilon$$

$$==> \qquad \frac{1}{1 + C \cdot \exp\left[-(r - F_1)\cdot t_{lag}\right]} = 1 + \varepsilon$$

where $\quad C = \dfrac{new\_B_e - old\_B_e}{old\_B_e} \quad$ and $\quad r = \dfrac{4 \cdot MSY}{K}$

Here is a picture of what we are doing.



Solve the following equation for t_{lag}

$$1 = (1+\varepsilon)\cdot\left[1 + C\cdot\exp\left[-(r - F_1)\cdot t_{lag}\right]\right]$$

$$\frac{1}{1+\varepsilon} - 1 = C \cdot \exp\left[-(r - F_1)\cdot t_{lag}\right] \qquad ==> \qquad \frac{1 - (1+\varepsilon)}{1+\varepsilon}\cdot\frac{1}{C} = \exp\left[-(r - F_1)\cdot t_{lag}\right]$$

$$\ln\left(\frac{-\varepsilon}{1+\varepsilon}\cdot\frac{1}{C}\right) = -(r - F_1)\cdot t_{lag} \qquad ==> \qquad t_{lag} = \frac{\ln\left[\left(\dfrac{1+\varepsilon}{-\varepsilon}\right)\cdot\left(\dfrac{new\_B_e - old\_B_e}{old\_B_e}\right)\right]}{r - F_1}$$

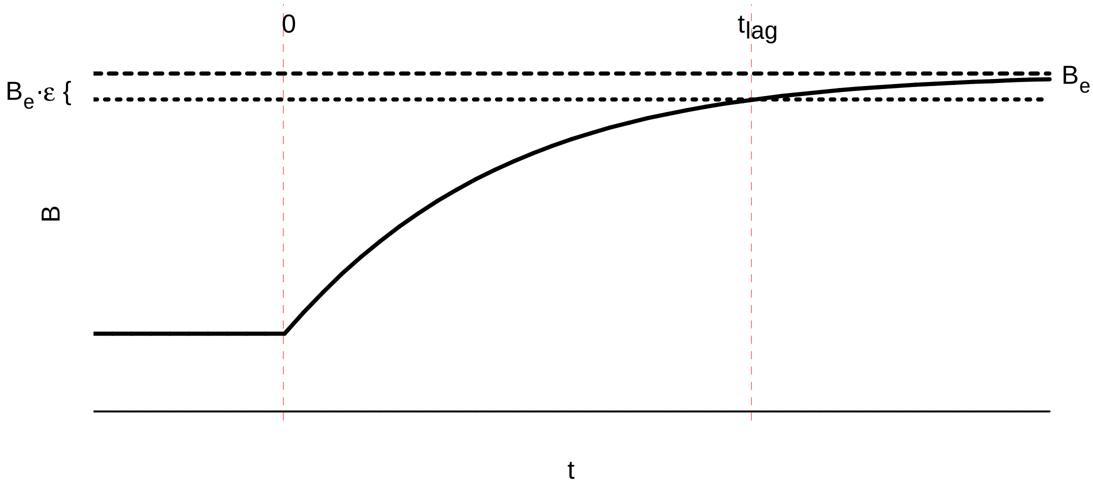
Suppose F_1 is less than F_0 (i.e., the new B_e is greater than the old B_e). In this case the approach to the new equilibrium is from below and

$$B(t_{lag}) = B_e - \varepsilon \cdot B_e = B_e \cdot (1 - \varepsilon) \qquad ==> \qquad \frac{B(t_{lag})}{B_e} = 1 - \varepsilon$$

The solution for t_{lag} is

$$t_{lag} = \frac{\ln\left[\left(\frac{1-\varepsilon}{\varepsilon}\right)\cdot\left(\frac{new\_B_e - old\_B_e}{old\_B_e}\right)\right]}{r - F_1}$$

Transition After a Decrease in F.



In either case, the yield that accumulates during the transition period $(0, t_{lag})$ is

$$Y = \int_0^{t_{lag}} F \cdot B(t)\, dt = \int_0^{t_{lag}} F \cdot \left[\frac{B_e}{1 + C \cdot \exp[-(r-F)\cdot t]}\right] dt$$

$$Y = F \cdot B_e \cdot \int_0^{t_{lag}} \frac{1}{1 + C \cdot \exp[-(r-F)\cdot t]}\, dt$$

The integral is of the form

$$\int \frac{1}{1 + a \cdot \exp(-b \cdot u)}\, du = \int \frac{\exp(b \cdot u)}{\exp(b \cdot u) + a}\, du = \frac{\ln(\exp(b \cdot u) + a)}{b} + Arb$$

$$\int_0^X \frac{1}{1 + a \cdot \exp(-b \cdot u)}\, du = \frac{\ln(\exp(b \cdot X) + a)}{b} - \frac{\ln(1 + a)}{b} = \frac{1}{b} \cdot \ln\left(\frac{\exp(b \cdot X) + a}{1 + a}\right)$$

We can use this result with b = r - F and a = C and write the equation for Y as

$$Y = \frac{F \cdot B_e}{r - F} \cdot \ln\left[\frac{\exp\left[(r - F) \cdot t_{lag}\right] + C}{1 + C}\right] \qquad \text{with} \qquad C = \frac{B_e - B_0}{B_0}$$

Now substitute for C to get $Y = \frac{F \cdot B_e}{r - F} \cdot \ln\left[\frac{\exp\left[(r - F) \cdot t_{lag}\right] + \frac{B_e - B_0}{B_0}}{1 + \frac{B_e - B_0}{B_0}}\right]$.

$$Y = \frac{F \cdot B_e}{r - F} \cdot \ln\left[\frac{B_0 \cdot \exp\left[(r - F) \cdot t_{lag}\right] + \left(B_e - B_0\right)}{B_e}\right]$$

$$Y = \frac{F \cdot B_e}{r - F} \cdot \ln\left[\frac{B_0}{B_e} \cdot \exp\left[(r - F) \cdot t_{lag}\right] + 1 - \frac{B_0}{B_e}\right]$$

The first term can be written as

$$\frac{F}{r - F} \cdot B_e = \frac{F}{r - F} \cdot K \cdot \left(1 - \frac{F}{r}\right) = \frac{F}{r - F} \cdot K \cdot \left(\frac{r - F}{r}\right) = K \cdot \frac{F}{r}$$

$$= K - K + K \cdot \frac{F}{r} = K - K \cdot \left(1 - \frac{F}{r}\right)$$

So, Y can be written as $Y = \left(K - B_e\right) \cdot \ln\left[1 + \frac{B_0}{B_e} \cdot \left[\exp\left[(r - F) \cdot t_{lag}\right] - 1\right]\right]$.

Pella and Tomlinson's Generalized Surplus-Production Model

One problem with the Graham-Schaefer model is that the maximum sustainable yield MSY always occurs when the biomass is half the carrying capacity K. This is a direct consequence of the parabolic relationship between dB/dt and B, which in turn follows from the linear relationship between per capita productivity and population size. Pella and Tomlinson (1969), on the *Supplemental Reading* list, proposed an alteration to the model for latent productivity, which uncouples B_{msy} from K.

$$\frac{dB}{dt} = \begin{cases} a \cdot B^n - b \cdot B & \text{... for } 0 < n < 1 \\ b \cdot B - a \cdot B^n & \text{... for } 1 < n \end{cases}$$

This is not a convenient formulation for this model. The values for MSY, K, and B_{msy} all depend on parameter n, and you have to reverse the signs of parameters a and b depending on whether n is greater or less than one.

Fletcher's Parameterization of the Pella & Tomlinson Model

Fletcher (1978), on the *Supplemental Reading* list, proposed the following alternative parameterization to avoid the problems mentioned above.

$$\frac{dB}{dt} = \gamma \cdot MSY \cdot \left(\frac{B}{K}\right) - \gamma \cdot MSY \cdot \left(\frac{B}{K}\right)^{n} \qquad \text{with} \qquad \gamma = \frac{n^{\frac{n}{n-1}}}{n-1}$$

Parameter γ (gamma), which is a pure number, automatically changes sign as n increases through one. Parameter n controls the location of B_{msy}. To see this, differentiate the equation for dB/dt with respect to B.

$$\frac{d}{dB}\frac{dB}{dt} = \gamma \cdot \frac{MSY}{K} - n \cdot \gamma \cdot \frac{MSY}{K} \cdot \left(\frac{B}{K}\right)^{n-1}$$

If we set this to zero and solve for B, we can determine B_{msy}.

$$\gamma \cdot \frac{MSY}{K} = n \cdot \gamma \cdot \frac{MSY}{K} \cdot \left(\frac{B_{msy}}{K}\right)^{n-1} \qquad ==> \qquad \frac{1}{n} = \left(\frac{B_{msy}}{K}\right)^{n-1}$$

$$\left(\frac{1}{n}\right)^{\frac{1}{n-1}} = \frac{B_{msy}}{K} \qquad ==> \qquad \frac{B_{msy}}{K} = n^{\frac{-1}{n-1}} \qquad ==> \qquad B_{msy} = K \cdot n^{\frac{-1}{n-1}}$$

When n = 2, the model is equivalent to the Graham-Schaefer model.

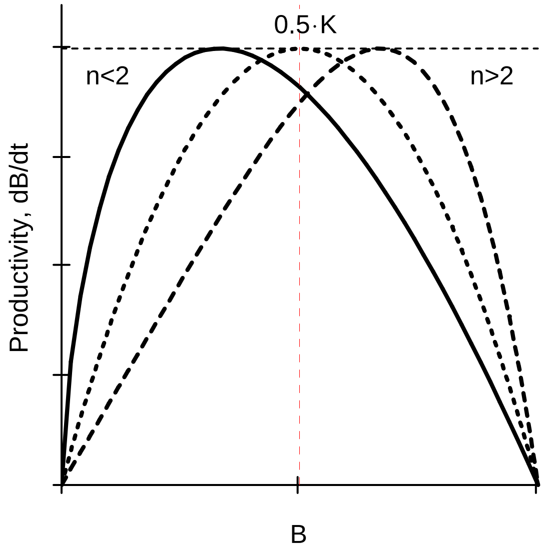
$$B_{msy} = K \cdot 2^{-1} = \frac{K}{2}$$

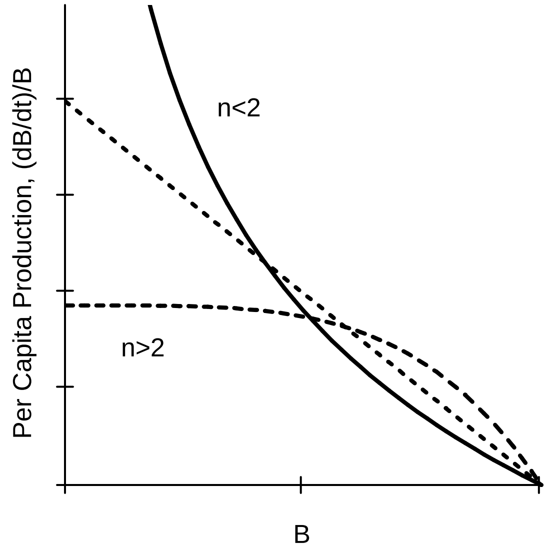
When n > 2, $B_{msy} > \frac{K}{2}$ and maximum productivity is closer to K.

For example, $\quad n = 5 \quad ==> \quad B_{msy} = K \cdot 5^{\frac{-1}{5-1}} = K \cdot 0.66874$

When n < 2, $B_{msy} < \frac{K}{2}$ and maximum productivity is closer to zero.

For example, $\quad n = 0.5 \quad ==> \quad B_{msy} = K \cdot 0.5^{\frac{-1}{0.5-1}} = K \cdot 0.25$





When n = 1, the system reduces to the so called "exponential" surplus-production model of Fox (1970), on the *Supplemental Reading* list.

$$\frac{dB}{dt} = -e \cdot MSY \cdot \frac{B}{K} \cdot \ln\left(\frac{B}{K}\right)$$

This is described as an exponential model because the graph of equilibrium CPUE versus effort declines exponentially. Here is a derivation of Fox's model from Fletcher's version of the Pella and Tomlinson model.

$$\frac{dB}{dt} = \gamma \cdot MSY \cdot \left(\frac{B}{K}\right) - \gamma \cdot MSY \cdot \left(\frac{B}{K}\right)^n$$

Factor out B/K and write out γ *in full.*

$$\frac{dB}{dt} = \left(\frac{n^{\frac{n}{n-1}}}{n-1}\right) \cdot MSY \cdot \left(\frac{B}{K}\right) \cdot \left[1 - \left(\frac{B}{K}\right)^{n-1}\right] = \left(n^{\frac{n}{n-1}}\right) \cdot MSY \cdot \left(\frac{B}{K}\right) \cdot \left[\frac{1 - \left(\frac{B}{K}\right)^{n-1}}{n-1}\right]$$

Now take the limit as n goes to one.

$$\lim_{n \to 1} \left(n^{\frac{n}{n-1}}\right) = e \qquad \lim_{n \to 1} \left[\left(\frac{1 - X^{n-1}}{n-1}\right) = -\ln(X)\right]$$

$$\Longrightarrow \qquad \lim_{n \to 1} \left(\frac{dB}{dt}\right) = -e \cdot MSY \cdot \frac{B}{K} \cdot \ln\left(\frac{B}{K}\right)$$

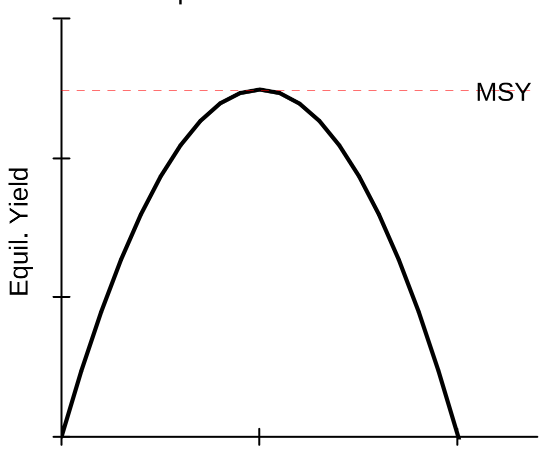
With the Fox surplus production model $B_{msy} = \frac{K}{e} = K \cdot 0.367879$.

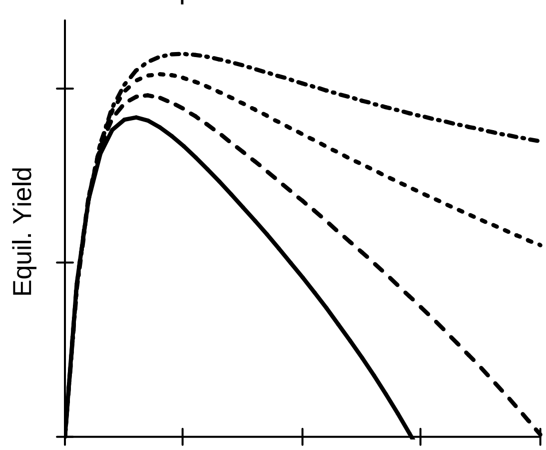
Explore the influence of parameters MSY, K, and n
(n<2, n>2, and n≅1) using the Excel demonstration..

In practice researchers have often had difficulty in fitting the Pella-Tomlinson model to real data. Because of the inherent relationship between the curvature in the model and the value of n, this parameter is often difficult to determine. Very different values for n can give almost the same fit to many data sets. One approach to this problem is to choose n on the basis of other analyses or ecological arguments and then determine the remaining parameters.

Fletcher (1978), on the *Supplemental Reading* list, discusses the problem of applying the Pella-Tomlinson model. Fletcher (1974), on the *Supplemental Reading* list, develops an alternative method for controlling the location of B_{msy}, by rotating the main axis of the parabolic curve.

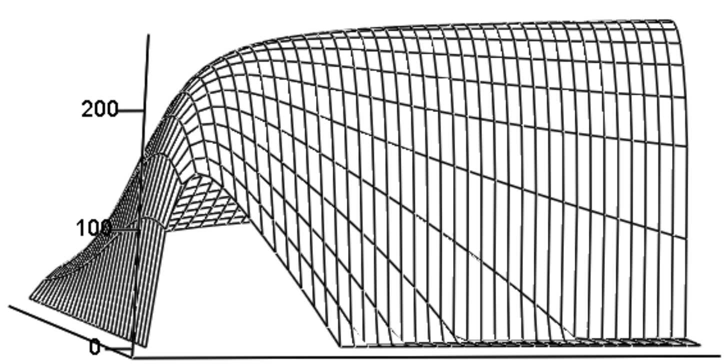
These surplus production models do not specify the biological processes responsible for the curvature in the graph of dB/dt versus B, but we have already studied a model that does include the biological details; the model for equilibrium yield, which we developed by combining a yield-per-recruit model with a stock-recruit relationship.





The Equilibrium Yield Surface

The horizontal axis is F, the axis going into the page is t_e, and the vertical axis is Y.



Y

With any of the surplus production models the population is driven to extinction if the rate of fishing mortality F is greater than or equal to the slope of the graph of dB/dt versus B at the origin (equivalent to parameter r in the Graham-Schaefer model). With the equilibrium yield model, extinction only occurs if the age-at-entry is too small. This discrepancy arises because the two models make different assumptions about whether all the reproductive animals are susceptible to capture. In the equilibrium yield model mature animals younger than the age-at-entry provide a buffer against the effects of fishing. In the surplus production models all the productive animals are vulnerable.

Surplus Production Models and Fisheries Management

Although surplus-production models are intuitively appealing and involve reasonably simple mathematics, using them as guides for fisheries management can lead to serious problems. During the '60s and '70s the research focus of many fisheries agencies was to determine MSY for all the exploited stocks, and the main management objective was to maintain the stocks at B_{msy}. Some researchers cautioned against such a narrow view. Larkin (1977), on the *Recommended Reading* list, describes some of the problems with using MSY as the objective for managing a fishery. Here are some of the problems:

- In multispecies fisheries, the stocks with lower productivity may end up being eliminated if the rate of fishing mortality is maintained at F_{msy}.
- The MSY values for different stocks may not be independent. Species are not ecologically isolated, and as a consequence changes in the biomass of one may affect the values of r and K for other species.
- Fishing at the F_{msy} rate, as opposed to any lower rate, implies a younger average age and a reduced number of age classes, which may lead to greater variability in recruitment.
- Fishing may reduce the genetic variability of a fish stock and lead to reduced productivity in the long run.
- Fishing at MSY is not cost effective. The economically optimum yield is generally less than MSY.

