



## Assessing power of large river fish monitoring programs to detect population changes: the Missouri river sturgeon example

By M. L. Wildhaber<sup>1</sup>, S. H. Holan<sup>2</sup>, J. L. Bryan<sup>1</sup>, D. W. Gladish<sup>2</sup> and M. Ellersieck<sup>2</sup>

<sup>1</sup>U.S. Geological Survey, Columbia Environmental Research Center, Columbia, MO, USA; <sup>2</sup>University of Missouri – Columbia, Department of Statistics, Columbia, MO, USA

### Summary

In 2003, the US Army Corps of Engineers initiated the Pallid Sturgeon Population Assessment Program (PSPAP) to monitor pallid sturgeon and the fish community of the Missouri River. The power analysis of PSPAP presented here was conducted to guide sampling design and effort decisions. The PSPAP sampling design has a nested structure with multiple gear subsamples within a river bend. Power analyses were based on a normal linear mixed model, using a mixed cell means approach, with variance estimates from the original data. It was found that, at current effort levels, at least 20 years for pallid and 10 years for shovelnose sturgeon is needed to detect a 5% annual decline. Modified bootstrap simulations suggest power estimates from the original data are conservative due to excessive zero fish counts. In general, the approach presented is applicable to a wide array of animal monitoring programs.

### Introduction

Monitoring programs are used for long-term assessment of populations and communities that often include long-lived, rare species. Such programs can be a critical component of adaptive management (Williams et al., 2007). The Comprehensive Everglades Restoration Plan (1999) and Colorado River Recovery Implementation Program (2001) are two examples of long-term monitoring applied to adaptive management. The Pallid Sturgeon Population Assessment Program (PSPAP), initiated to monitor Missouri River fishes (Drobish, 2005a), is used here to demonstrate a general approach for such monitoring program power analyses.

The Missouri River is 4180 km long (Galat et al., 2005) and drains one-sixth of the contiguous United States (1 396 117 km<sup>2</sup>). Intensive management for purposes of navigation, flood control, and power generation has resulted in dramatic physical changes in the river (Galat and Lipkin, 2000 and references therein). These changes have been implicated in the dramatic declines of native river fishes and their resource base, in particular the endangered pallid sturgeon (*Scaphirhynchus albus*) (Dryer and Sandvol, 1993; Hesse et al., 1989; Galat et al., 2005).

The pallid sturgeon is endemic to the turbid waters of the Missouri River and Lower Mississippi River, from the Missouri River mouth to New Orleans (Mayden and Kuhajda, 1997). In the Lower Missouri River, no reliable pallid sturgeon population estimates exist; however, based on recent capture rates and incidence of occurrence, only several thousand wild individuals are estimated to remain in the Lower Missouri

River (Duffy et al., 1996; U.S. Fish and Wildlife Service, 2000). The sympatric shovelnose sturgeon (*Scaphirhynchus platyrhynchus*) is more common and widespread but is also declining (Moos, 1978; Keenlyne, 1997).

In 2000 the US Fish and Wildlife Service issued US Army Corps of Engineers a Biological Opinion that included the Missouri River pallid sturgeon (U.S. Fish and Wildlife Service, 2000). In response to the Biological Opinion, in 2003, US Army Corps of Engineers initiated the Pallid Sturgeon Population Assessment Program to monitor the fish community of the Missouri River based on the design of the Missouri River Benthic Fishes Study (Berry et al., 2005). The Missouri River Benthic Fishes Study was the first system wide cooperative study, initiated in 1995, to produce a baseline for 26 Missouri River benthic fishes including shovelnose and pallid sturgeon (Table 1). The PSPAP goal is to detect changes in populations and habitat preferences over time for pallid sturgeon and other native fish species in the Missouri River Basin (Table 1) (Drobish, 2005a). The need for data analysis support was identified by independent science reviews of PSPAP and the US Army Corps of Engineers Habitat Assessment and Monitoring Program initiated in 2006 to assess the effects of habitat modifications on the lower Missouri River fish community (Sustainable Ecosystem Institute 2004, 2005). In 2006, the concern over data analysis and statistical design had become a cornerstone issue. To help address this critical need, three reports were provided to US Army Corps of Engineers (Wildhaber unpubl. data) that articulate the philosophy and rationale for meeting short-term and long-term analytical needs for the US Army Corps of Engineers Missouri River Recovery Program and assessments of PSPAP and Habitat Assessment and Monitoring Program (both part of the Missouri River Recovery Program) power to detect population changes of pallid sturgeon and targeted fish species (Table 1). Here we focus on the PSPAP results.

We present the approach developed to conduct PSPAP power analyses using the shovelnose and pallid sturgeon examples. The purpose of the power analysis was to determine the most efficient sampling (number of bends) and subsampling (gear deployments within a bend) efforts in combination. Typically, testing for trends proceeds using a *t*-test on the fixed effect for trend with the proper variance term obtained under a mixed model (Sims et al., 2007; and the references therein). To take advantage of the PSPAP sampling design, the power analysis that was conducted uses a standard ANOVA formulation with multiple fixed effects (year, season, and segment) and nested random effects (gear deployment within bend) that results in a *F*-test for the main effect due to year. However,

Table 1

Common and scientific names of benthic fishes studied in Missouri River Benthic Fishes Study (Berry et al., 2005) and monitored as part of the PSPAP (Drobish, 2008) with PSPAP only species in bold and species in both Missouri River Benthic Fishes Study and PSPAP underlined

Bigmouth buffalo	Emerald shiner	River carpsucker	Smallmouth buffalo
<i>Ictiobus cyprinellus</i>	<i>Notropis atherinoides</i>	<i>Carpionodes carpio</i>	<i>Ictiobus bubalus</i>
Blue catfish	Fathead minnow	<u>Sand shiner</u>	<b>Speckled chub</b>
<i>Ictalurus furcatus</i>	<i>Pimephales promelas</i>	<i>Notropis stramineus</i>	<i>Macrhybopsis asetivalis</i>
<u>Blue sucker</u>	Flathead chub	<u>Sauger</u>	Stonecat
<i>Cycoreptus elongatus</i>	<i>Platygobio gracilis</i>	<i>Sander canadensis</i>	<i>Noturus flavus</i>
Brassy minnow	Flathead catfish	Shorthead redhorse	<u>Sturgeon chub</u>
<i>Hybognathus hankinsoni</i>	<i>Ptyodictis olivaris</i>	<i>Moxostoma macrolepidotum</i>	<i>Macrhybopsis gelida</i>
Burbot	Freshwater drum	<u>Shovelnose sturgeon</u>	Walleye
<i>Lota lota</i>	<i>Aplodinotus grunniens</i>	<i>Scaphirhynchus platyrhynchus</i>	<i>Sander vitreus</i>
Channel catfish	<u>Pallid sturgeon</u>	<u>Sicklefin chub</u>	<u>Western silvery minnow</u>
<i>Ictalurus punctatus</i>	<i>Scaphirhynchus albus</i>	<i>Macrhybopsis meeki</i>	<i>Hybognathus argyritis</i>
Common carp	<u>Plains minnow</u>		White sucker
<i>Cyprinus carpio</i>	<i>Hybognathus placitus</i>		<i>Catostomus commersonii</i>

because the species is rare, the data contains excess zeros and thus some of the ANOVA assumptions are violated. Therefore, the effects of these violations are investigated through simulation.

## Materials and methods

### Study area

The PSPAP encompasses the Missouri River from Fort Peck Dam, Montana at River kilometer (RK) 2851 downstream to the confluence of the Missouri and Mississippi Rivers near St. Louis, Missouri (KM 0) and the lower reach of the Kansas River (Fig. 1). For PSPAP, the Missouri River was stratified into 13 sampling segments. Segments were uniquely identified based on differences between them relative to water temperature, turbidity, tributary influences, degrading or aggrading stream bed, stream gradient, hydrograph, spillway releases, and flow fluctuations. This study focuses on the last five segments in the Lower Missouri River or Zone 4.

### Sampling time frame

Sampling year (year from this point) was dependent on temperature restrictions to minimize stress on pallid sturgeon (see below). Sampling year began on October 31 of the preceding calendar year and ended on October 30 of the

current calendar year. Each year was divided into two sampling seasons, the Sturgeon Season (focused on sturgeons) and the Fish Community seasons (focused on native fishes). Sturgeon Season began in fall when water temperature was  $\leq 12.8^{\circ}\text{C}$  ( $55^{\circ}\text{F}$ ) and continued through June 30. Fish Community Season ran from July 1 through October 31 and overlapped the Sturgeon Season when temperatures fell below  $12.8^{\circ}\text{C}$  prior to October 31. The data for this study include 2003–2005.

### Habitat sampling

Unlike the Missouri River Benthic Fishes Study that used a stratified random sampling design based on macrohabitats and river bends (Berry et al., 2005), the PSPAP sampling design is based on bends in which all macrohabitats available were subsampled (Fig. 2). A bend included three continuous macrohabitats (i.e., outside bend, inside bend, and channel crossover) and, potentially, 12 discrete macrohabitats (i.e., large and small tributary mouths, confluences, large and small secondary connected channels, non-connected secondary channels, deranged, braided, and dendritic).

Standard gears were selected in PSPAP to target specific habitats. For the Lower Missouri River that meant: (i) drifting trammel nets (TN) and otter trawls (OT) for channel borders (a mesohabitat within the macrohabitat) of all flowing

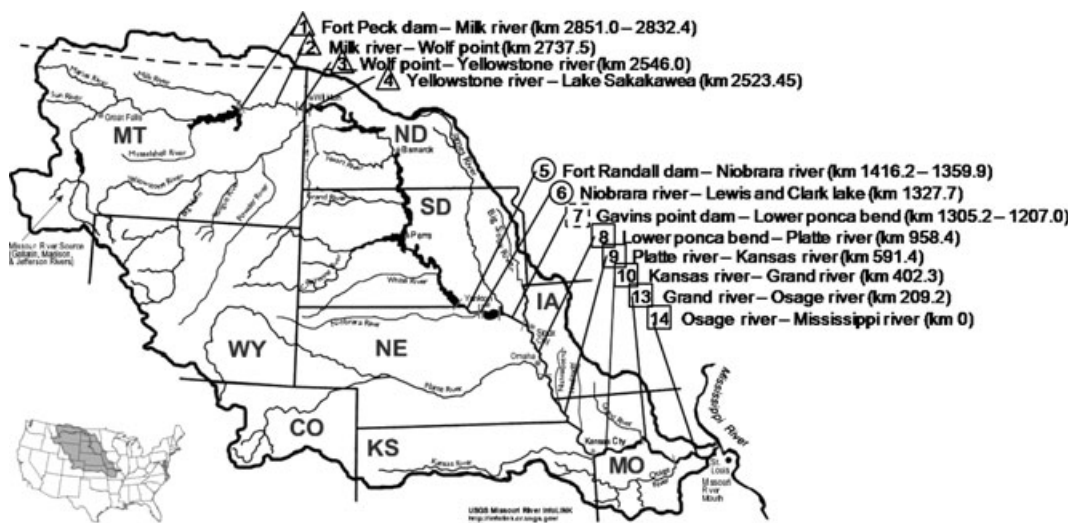


Fig. 1. The Zones (triangles = Zone 1, circles = Zone 2, all squares = Zone 3, only solid lined squares = Zone 4) and Segments designated in the Pallid Sturgeon Population Assessment Program on the Missouri River. This manuscript focuses on Zone 4

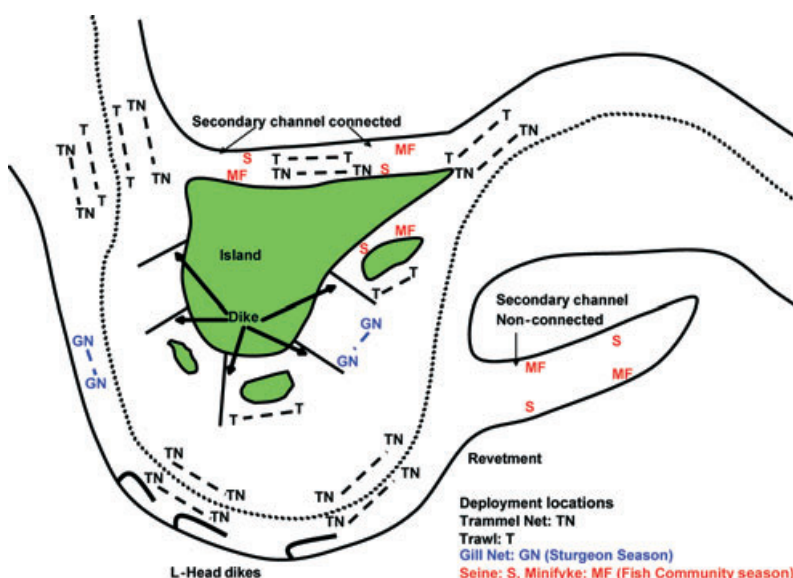


Fig. 2. Gear deployment locations for PSPAP (based on Drobish, 2008)

macrohabitat (i.e., not for non-connected secondary channels) in Sturgeon and Fish Community seasons, (ii) stationary gill nets (GN) only in the Sturgeon Season where TN and OT were fished and in pool and island tip mesohabitats, and (iii) beach seines (BS) and mini-fyke nets (MF) only in Fish Community Season in connected and non-connected secondary channel bar mesohabitats. For each bend sampled, a minimum of two subsamples (i.e., gear deployments) were required for each standard gear for the macrohabitats and mesohabitats identified within a bend with a minimum of eight sub-samples of each standard gear per bend (see details in Drobish, 2005a,b).

Following Berry et al. (2005) and Arab et al. (2008), the unit of measure for active gears (i.e., actively moved across the bottom) was catch per unit area (CPUA) based on minimum deployment area. The minimum deployment area for OT was  $360 \text{ m}^2 = 4.9 \text{ m wide} \times 75 \text{ m minimum deployment distance}$ , TN was  $2858 \text{ m}^2 = 38.1 \text{ m wide} \times 75 \text{ m minimum distance}$ , and a BS was  $10 \text{ m}^2 = \text{the minimum area sampled using any BS technique}$ . Unit of measure for passive gears (i.e., stationary fish entangle gears) was catch per unit effort (CPUE) based on each gear's minimum deployment time (i.e., 12 h). If a gear deployment did not meet the minimum deployment requirement, it was not used in the analyses. This occurred only for two gears: four out of 1758 OT and 12 out of 1908 TN deployments.

### Analysis

Each analysis was performed separately for all fish collected in each species for each gear and separately for juveniles ( $< 750 \text{ mm}$  for pallids and  $< 550 \text{ mm}$  for shovelnose; Bajer and Wildhaber (2007) and Moos (1978), respectively) and adults. Prior to analyses, the same set of criteria used by Berry et al. (2005) was used to filter the data to provide improved distributional characteristics and thus, decrease assumption violations. The basis of these criteria is that if no or nearly no fish were collected in a study design component it could be excluded from analyses since it could be considered essentially zero. These criteria were: (i) year, season, or gear was included if at least one fish was caught. This resulted in elimination of BS and MF for pallid sturgeon since 0 of 66 were collected by these gears. It also eliminated adult pallid sturgeon in the Fish Community Season since 0 were collected, (ii) a gear was not

analyzed if catch was  $< 5\%$  of the total catch. For shovelnose sturgeon, BS and MF were not analyzed since only 2 and 5, respectively, of 19 678 were caught. For pallid sturgeon adults, OT and TN were not analyzed since only 1 and 2, respectively, were caught, (iii) a gear for a species was not analyzed if presence of the species did not exceed 5% of the total number of bends sampled. This never occurred, (iv) a year for a species in a gear was excluded if the catch in a year by the gear was  $< 10\%$  of the total fish collected by that gear for that species. This excluded from analyses: GN in 2003 for pallid sturgeon as a whole since it represented 2 of 24; GN for juvenile pallid sturgeon since it represented 1 of 15; OT in 2004 and 2005 for adult pallid sturgeon because 0 were collected; and TN in 2004 for adult pallid sturgeon since 0 were collected, and (v) a season for a species in a gear was excluded if the catch in a season by the gear was  $< 5\%$  of the total collected by that gear for that species. This never occurred. When application of these criteria resulted in a reduction of data, our inference was necessarily restricted to the reduced dataset.

### Model description and power calculation

The PSPAP sampling design has a nested structure with multiple subsamples within each bend. For each species, the model consisted of three fixed effects (year, season, and segment). Within each year, season, and segment combination, a random sample of river bends is selected. Within each river bend, multiple gear deployments (subsamples) were distributed randomly across macrohabitats present.

To calculate power for lower and higher levels of effort than observed in the original data requires knowledge of population variances associated with random effects due to bend and subsample. These values are inferred based on estimates obtained from the data, and are subsequently considered known. Specifically, the power analysis is based on the normal linear mixed model

$$Y_{ijklm} = \mu + \alpha_i + \beta_j + \gamma_k + \alpha\beta_{ij} + \alpha\gamma_{ik} + \beta\gamma_{jk} + \alpha\beta\gamma_{ijk} + D_{ijkl} + S_{ijklm}$$

where  $Y_{ijklm}$  is catch per unit area/effort for year  $i$ , season  $j$ , segment  $k$ , bend  $l$ , and subsample  $m$ ;  $\mu$  is the overall mean over all years, seasons, segments, bends, and subsamples;  $\alpha_i$  is the



fixed effect due to year  $i$ , ( $i = 1, \dots, a$ );  $\beta_j$  is the fixed effect due to season  $j$ , ( $j = 1, \dots, b$ );  $\gamma_k$  is the fixed effect due to segment  $k$ , ( $k = 1, \dots, c$ );  $D_{ijkl}$  is the random effect due to bend  $l$ , ( $l = 1, \dots, d$ ); and  $S_{ijklm}$  is the random effect due to subsample (gear deployment)  $m$ , ( $m = 1, \dots, s$ ).

Further, we assume  $D_{ijkl} \sim iidN(0, \sigma_D^2)$  and  $S_{ijklm} \sim iidN(0, \sigma_S^2)$ , with  $D_{ijkl}$  and  $S_{ijklm}$  mutually independent (Kuehl, 2000). This model can be written in cell means notation (Moser, 1996; Littell et al., 2006)

$$Y_{ijklm} = \mu_{ijk} + D_{ijkl} + S_{ijklm}$$

where  $\mu_{ijk}$  denotes the fixed portion of the model and  $D_{ijkl}$  and  $S_{ijklm}$  are defined as before. Let

$$\mathbf{Y} = (Y_{11111}, \dots, Y_{abc ds})'$$

then, using matrix notation, the model can be written as

$$\mathbf{Y} = \mathbf{W}\mu + \mathbf{U}\mathbf{D} + \mathbf{S}$$

where  $\mathbf{Y} = (Y_{11111}, \dots, Y_{abc ds})'$ ,  $\mu = (\mu_{111}, \dots, \mu_{abc})'$ ,  $\mathbf{D} = (D_{11111}, \dots, D_{abc d})'$ ,  $\mathbf{S} = (S_{111111}, \dots, S_{abc ds})'$ ,  $\mathbf{W} = (\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{I}_c \otimes \mathbf{I}_d \otimes \mathbf{1}_s)$ ,  $\mathbf{U} = (\mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{I}_c \otimes \mathbf{I}_d \otimes \mathbf{1}_s)$ ,  $\mathbf{I}_k$  is the  $k \times k$  identity matrix,  $\otimes$  denotes the Kronecker product and  $\mathbf{1}_k$  is the vector of ones of length  $k$ . Further, the covariance matrix of  $\mathbf{Y}$ , denoted  $\Sigma$ , is given by

$$\Sigma = \sigma_S^2 \mathbf{I}_{abc ds} + \sigma_D^2 (\mathbf{I}_{abcd} \otimes \mathbf{J}_s),$$

where  $\mathbf{J}_k$  is the  $k \times k$  matrix of ones.

The goal of our analysis is to determine if there exists a significant difference among any of the years sampled. The appropriate hypothesis test in cell means notation is

$$H_0: \bar{\mu}_{1..} = \dots = \bar{\mu}_{a..}$$

$H_a$ : At least one inequality,

where

$$\bar{\mu}_{i..} = \frac{1}{bc} \sum_{j=1}^b \sum_{k=1}^c \mu_{ijk}.$$

Let  $E[\text{MSD}]$  and  $E[\text{MS}\alpha]$  denote the expected mean square for bend and year respectively. It follows that (see Appendix for details)

$$E[\text{MSD}] = \sigma_S^2 + s\sigma_D^2,$$

$$E[\text{MS}\alpha] = \sigma_S^2 + s\sigma_D^2 + bc ds \frac{\sum_{i=1}^a (\bar{\mu}_{i..} - \bar{\mu}_{...})^2}{(a-1)}.$$

Under the hypothesis of no main effect due to year, it follows that

$$\sum_{i=1}^a (\bar{\mu}_{i..} - \bar{\mu}_{...})^2 \equiv 0.$$

Thus, under the null hypothesis,  $E[\text{MSD}] = E[\text{MS}\alpha]$ . Therefore, the appropriate test statistic for testing no main effect due to year is

$$F = \frac{\text{MS}\alpha}{\text{MSD}}$$

where  $\text{MS}\alpha$  and  $\text{MSD}$  denote the mean square for the main effect due to year and for the main effect due to bend (in this case the error term in the model), respectively.

Under the null hypothesis, the test statistic  $F$  follows a central  $F$  distribution with numerator degrees of freedom  $a-1$  and denominator degrees of freedom  $abc(d-1)$ . However, under the alternative hypothesis, the test statistic follows a non-central  $F$  distribution with numerator degrees of freedom  $a-1$  and denominator degrees of freedom  $abc(d-1)$ , and non-centrality parameter  $\phi$ , defined:

$$\phi = \frac{bc ds}{\sigma_S^2 + s\sigma_D^2} \sum_{i=1}^a (\bar{\mu}_{i..} - \bar{\mu}_{...})^2.$$

To determine power, we calculate the probability of rejecting the null hypothesis given that the alternative is true. This is the probability that our test statistic is greater than the critical value of the test under significance level  $\alpha$ , under the non-central  $F$  distribution with the appropriate degrees of freedom.

To calculate this power, the number of years  $a$  and the value of  $\bar{\mu}_{i..}$  for each  $i$  need to be specified. Since the goal is to obtain power estimates when there is a decrease in the population from 1 year to the next, we let

$$\bar{\mu}_{i..} = (1 - \Delta)^{i-1} \bar{\mu}_{1..}$$

for  $i = 2, \dots, a$  and  $0 < \Delta < 1$ . Hence for each year  $i$ , the mean  $\bar{\mu}_{i..}$  was decreased over the previous year's mean  $\bar{\mu}_{i-1..}$  by a constant percentage. The initial mean  $\bar{\mu}_{1..}$  is estimated based on the current data. This estimate is slightly problematic since, although we base power off of a balanced design, the original data is unbalanced with respect to the number of bends sampled per year, season, segment, and number of subsamples per bend. To estimate the first year allowing for each bend to have the same influence on the mean (as was done by Berry et al., 2005), we average by

$$\hat{\mu}_{1..} = \frac{1}{3bc} \sum_{i=1}^3 \sum_{j=1}^b \sum_{k=1}^c \left\{ \frac{1}{d_{ijk}} \sum_{l=1}^d \left( \frac{1}{s_{ijkl}} \sum_{m=1}^s Y_{ijklm} \right) \right\}$$

where  $d_{ijk}$  is the number of bends in year  $i$ , season  $j$ , and segment  $k$ , and  $s_{ijkl}$  is the number of subsamples in year  $i$ , season  $j$ , segment  $k$ , and bend  $l$ . Note that this is not necessarily the best linear unbiased predictor for the mean of the first year, but it does provide a biologically meaningful initial value under the alternative hypothesis. The variance components  $\sigma_D^2$  and  $\sigma_S^2$  were estimated using Restricted Maximum Likelihood Estimation (REML) and thus are restricted to be non-negative.

To calculate power, we first define  $\Delta$  and the number of years  $a$ . Then we calculate  $\bar{\mu}_{i..}$  for  $i = 1, \dots, a$ . We also specify the overall mean  $\bar{\mu}_{...} = (1/a) \sum_{i=1}^a \bar{\mu}_{i..}$ . Then, specifying  $s$ ,  $d$ ,  $c$ , and  $b$ , we calculate  $\phi$ . We repeat this procedure for various combinations of  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $s$ ,  $\Delta$ , and  $\alpha$ . The results give us the desired power for a desired level of significance and difference under the most reasonable conditions for sampling the number of subsamples, bends, and years.

Current target levels of effort for the PSPAP are 12 bends with eight subsamples per bend for each year, season, and segment combination [subsequently referenced as 12/8 or number of bends/number of subsamples per bend, (Drobish, 2005a)]. However, in the data used for the power analysis, these levels fluctuated slightly between years and segments (Table 2). Here levels of bend effort and subsample tested were 6, 12, 18, 24, and 4, 8, 12, and 16, respectively. Different monitoring lengths of 5, 10, and 20 years also were examined, as was the power to detect different population rates of decline over the

Table 2

The total count of fish (proportion of non-occurrence in subsamples) and the number of bends the species occupied (proportion of non-occurrence in bends) in the power analysis for each gear type and model statistics. The number of years was 3 for all models except for pallid sturgeon as a whole and juveniles for gillnet where it was 2 (i.e., without year 2003). The number of Seasons was 2 and Segments was 5 for all models except for gillnets for all shovelnose and pallid sturgeon models where Seasons was 1 (i.e., without Fish Community Season and Segment 10)

Gear	Species	Count	Bend	Yearly catch per unit	Bend	Subsamples	Bend variance component estimate $10^{-2}$ (initial $10^{-3}$ )	Subsample variance component estimate
Gill net	Pallid-all	22 (0.97)	20 (0.74)	0.0093	76	704	0 (-0.0258)	0.0028
	Pallid-adult	9 (0.99)	8 (0.92)	0.0030	105	895	0.0014	0.0009
	Pallid-juvenile	14 (0.98)	14 (0.82)	0.0054	76	704	0 (-0.0337)	0.0018
	Shovelnose-all	12248 (0.15)	103 (0.02)	4.3567	105	895	343.4610	21.7928
	Shovelnose-adult	8397 (0.21)	102 (0.03)	2.8998	105	895	181.1319	10.8579
	Shovelnose-juvenile	3851 (0.3)	99 (0.06)	1.4569	105	895	27.3304	2.7839
Otter trawl	Pallid-all	20 (0.99)	18 (0.92)	0.0034	239	1894	0 (-0.0011)	0.0022
	Pallid-juvenile	19 (0.99)	17 (0.93)	0.0032	239	1894	0 (-0.0012)	0.0022
	Shovelnose-all	2854 (0.53)	231 (0.03)	0.5329	239	1894	4.0924	1.0838
	Shovelnose-adult	1338 (0.72)	204 (0.15)	0.2426	239	1894	1.1845	0.4303
	Shovelnose-juvenile	1516 (0.63)	217 (0.09)	0.2904	239	1894	0.7508	0.2994
	Pallid-all	22 (0.99)	22 (0.91)	0.0057	242	1754	0 (-0.0311)	0.0046
Trammel net	Pallid-juvenile	20 (0.99)	20 (0.92)	0.0053	242	1754	0 (-0.0219)	0.0040
	Shovelnose-all	4569 (0.5)	226 (0.07)	1.2393	242	1754	42.0978	10.5805
	Shovelnose-adult	2588 (0.62)	212 (0.12)	0.6841	242	1754	17.5500	3.6586
	Shovelnose-juvenile	1981 (0.62)	210 (0.13)	0.5553	242	1754	8.2349	2.4628

years. The compounding rates of decline were 1, 3, and 5% annually; for 10 years, this meant 8.6, 24, and 37% decline, respectively (20 years = 17.4, 43.9, and 62.3%). For all tests the  $\alpha$ -level was 0.05 and power of 0.80 was considered adequate.

#### Effect of ANOVA assumption violations

Due to the high percentage of zeros in our data, the assumptions of normality and homoscedasticity are violated. Therefore, appropriateness of the linear mixed model is questionable. To help validate the results, a bootstrap procedure was performed (Davidson and Hinkley, 1997) to investigate the effect of the percentage of zeros in the data on the estimated power. This bootstrap was used to determine how power behaves as the number of zeros increases or decreases while non-zero values of the data still resemble the characteristics of the original data.

The nonparametric bootstrap we propose resamples the data with replacement and assigns a subsample either a zero or a non-zero observed value from the original data with a given probability. That is, we initially select a desired percentage of zeros for the new data, which then becomes our probability  $P$  of selecting a zero. Each value in the bootstrap sample is assigned value of zero with probability  $P$  or some non-zero value from the original data with probability  $1 - P$ . For observations that are not zero in the bootstrap sample, the desire was to maintain equal influence on the inference for each bend. Thus, non-zero bootstrap observations are assigned a value from the original data by first randomly choosing a bend and then randomly choosing a non-zero value (originally observed) from that bend. That value gets assigned to the chosen non-zero observation in the bootstrap sample. This process is repeated for each observation chosen as a non-zero and thus gives us a modified bootstrap sample. With this new data, the power analysis was performed as described previously. This process is repeated multiple times to obtain several estimates of power based on the bootstrap for a given probability of zero.

The bootstrap analysis was performed on the shovelnose sturgeon trammel net model. We used 1000 bootstrap samples for six different zero percentage probabilities, using 0, 5, 25, 50, 75, and 95%. We performed a power analysis on each sample using the obtained variance components from the implied mixed model and the estimated initial mean using the same methodology applied to the data. For this analysis, we only used a 5% annual decline in population.

#### Results

Each individual species and size category was susceptible to certain gear types with high percentages of non-occurrence (Table 2), therefore, 50 out of 126 species/gear/year/season categories were removed from analyses due to the deletion criteria. In most cases, a 5% annual decline was necessary to demonstrate adequate power; therefore, results discussed will be limited to the 5% case.

#### Gill net

Only for pallid sturgeon as a whole was there enough power to detect a 5% annual decline using gill nets. This occurred for 20 years with 18 bends within a year, season, segment combination and 16 subsamples within each of those bends (Table 3). It would require 35–40 years to detect a decline with 12 bends and eight subsamples, the current PSPAP level. Conversely, a >45% (maximum tested) annual decline was required to detect a change within 5 years under current PSPAP effort levels or more than 60 (maximum tested) bends sampled at the current subsampling level.

In all cases, shovelnose sturgeon had enough power to detect a 5% annual decline either after 10 years with 12 bends and eight subsamples or six bends and four subsamples over 20 years (Table 3). Conversely, a 10–15% annual decline was required to detect a change within 5 years under the current PSPAP effort levels or sampling of 45–50 bends at the current subsampling level.

Table 3

Summary of the range of bends and percent annual decrease needed to reach a power of 0.8 for each gear of the PSPAP

Species	Gill net	Otter trawl	Trammel net
Bend/subsample level when detecting a 5% change over 10 years			
Pallid-all	> 24/16 <sup>a</sup>	> 24/16 <sup>a</sup>	> 24/16 <sup>a</sup>
Pallid-adult	> 24/16	—	—
Pallid-juvenile	> 24/16 <sup>a</sup>	> 24/16 <sup>a</sup>	> 24/16 <sup>a</sup>
Shovelnose-all	12/8	6/12	12/12
Shovelnose-adult	12/8	12/12	12/12
Shovelnose-juvenile	12/8	6/12	12/12
Bend/subsample level when detecting a 5% change over 20 years			
Pallid-all	18/16 <sup>a</sup>	> 24/16 <sup>a</sup>	> 24/16 <sup>a</sup>
Pallid-adult	> 24/16	—	—
Pallid-juvenile	> 24/16 <sup>a</sup>	> 24/16 <sup>a</sup>	> 24/16 <sup>a</sup>
Shovelnose-all	6/4	6/4	6/4
Shovelnose-adult	6/4	6/4	6/8
Shovelnose-juvenile	6/4	6/4	6/8
Range of bends needed in 5 years for each season and segment at a 8 subsample level of effort with 5% annual decrease			
Pallid-all	> 60 <sup>a</sup>	> 60 <sup>a</sup>	> 60 <sup>a</sup>
Pallid-adult	> 60	—	—
Pallid-juvenile	> 60 <sup>a</sup>	> 60 <sup>a</sup>	> 60 <sup>a</sup>
Shovelnose-all	45–50	30–35	> 60
Shovelnose-adult	20–25	30–35	25–30
Shovelnose-juvenile	40–45	25–30	> 60
Range of percent annual decrease needed in 5 years at a 12 bend/8 subsample level of effort			
Pallid-all	> 45 <sup>a</sup>	> 45 <sup>a</sup>	> 45 <sup>a</sup>
Pallid-adult	> 45	—	—
Pallid-juvenile	> 45 <sup>a</sup>	> 45 <sup>a</sup>	> 45 <sup>a</sup>
Shovelnose-all	10–15	5–10	10–15
Shovelnose-adult	10–15	10–15	10–15
Shovelnose-juvenile	10–15	5–10	10–15
Range of years at a 12 bend/8 subsample level of effort and five percent annual decrease			
Pallid-all	35–40 <sup>a</sup>	> 100 <sup>a</sup>	75–100 <sup>a</sup>
Pallid-adult	> 100	—	—
Pallid-juvenile	> 100 <sup>a</sup>	> 100 <sup>a</sup>	75–100 <sup>a</sup>
Shovelnose-all	5–10	5–10	10–15
Shovelnose-adult	5–10	10–15	10–15
Shovelnose-juvenile	5–10	5–10	10–15

A ‘—’ means a species did not meet minimum catch criteria and power was not calculated.

<sup>a</sup>Models with an initial negative bend variance component estimate were subsequently re-estimated using standard REML estimation methods.**Otter trawl**

No pallid sturgeon groupings had enough power to detect a decline (Table 3) using otter trawls. It would require more than 100 years before a change would be detected with 12 bends and eight subsamples. Conversely, a greater than 45% annual decline was required to detect a change over 5 years under the current PSPAP effort levels or sampling more than 60 bends at the current sampling levels.

In all cases, shovelnose sturgeon had enough power to detect a decline after 10 years with 6 or 12 bends (the latter for adults) and 12 subsamples or six bends and four subsamples over 20 years (Table 3). Conversely, a 5–10% annual decline (10–15 for adults) was required to detect a change over 5 years under the current PSPAP effort levels or sampling of 30–35 bends (25–30 for juveniles) at the current subsampling level. The same groups would require 5–10 years (10–15 for adults) before a decline would be detected with 12 bends and eight subsamples.

**Trammel net**

No pallid sturgeon groups had enough power to detect a decline (Table 3) with trammel nets. The same groups would require 75–100 years before a change would be detected with 12 bends and eight subsamples. Conversely, a >45% annual decline was required to detect a change over 5 years under the current PSPAP levels of effort or sampling of more than 60 bends at current subsampling levels.

In all cases, shovelnose sturgeon had enough power to detect a decline after 10 years with 12 bends and 12 subsamples and six bends and four subsamples (eight for juveniles and adults) over 20 years (Table 3). Conversely, a 10–15% annual decline was required to detect a change over 5 years under the current PSPAP levels of effort or sampling of 25–30 bends (>60 for shovelnose sturgeon as a whole and juveniles) at current subsampling levels. The same groups would require 10–15 years before a change would be detected with 12 bends and eight subsamples.

**Effects of ANOVA assumption violations**

Due to the high frequency of zeros in the data, the distributions were not normal. However, we found that power increased as the proportion of zeros decreased (Fig. 3). Simultaneously, the mean and estimated variance components decreased as the number of zeros increased (Table 4). The former results suggest that the power estimates presented are conservative with respect to excessive zero observations.

**Discussion**

What has been presented here is a method for assessing power for large-river fish monitoring programs that allows for use of the fully-nested design of the study to maximize information gain from the data. The PSPAP on the Lower Missouri River

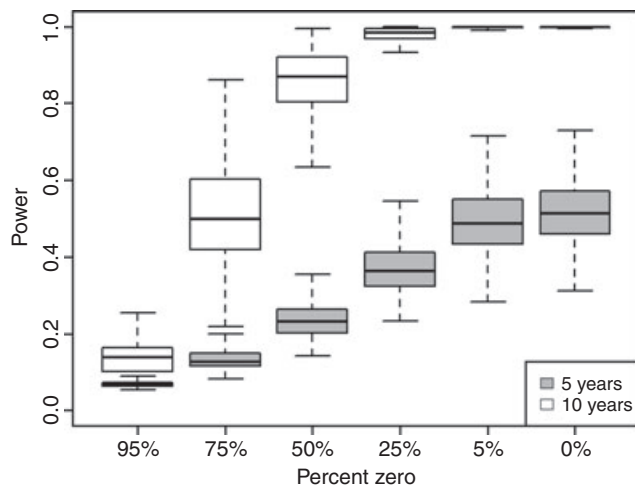


Fig. 3. Box plots of the relationship between power and percent zeros based on bootstrap analyses

Table 4  
Distribution of the parameter values used in the bootstrap analyses

Parameter	Percent zero	5% quantile	Median	95% quantile
$\bar{\mu}_{...}$	0	2.121324	2.266981	2.443082
	5	2.012912	2.165104	2.324002
	25	1.570303	1.701894	1.863781
	50	1.024305	1.140408	1.27192
	75	0.4886834	0.5650625	0.6671629
	95	0.08010349	0.10931761	0.15979772
$\sigma_D^2$	0	1.10E-07	1.44E-06	4.00E-01
	5	1.14E-07	1.46E-06	3.79E-01
	25	9.84E-08	3.27E-06	3.23E-01
	50	5.81E-08	9.67E-07	2.14E-01
	75	3.02E-08	4.28E-07	1.07E-01
	95	3.06E-09	8.13E-08	1.75E-02
$\sigma_S^2$	0	10.25721	15.45153	21.86305
	5	9.807816	14.86784	21.312463
	25	8.100546	12.452031	18.615939
	50	5.178645	8.950755	13.991404
	75	2.303079	4.613186	8.479802
	95	0.2969518	0.629764	2.7203495

provided a comprehensive data set for method development. The approach taken is general enough to be applicable to a broad array of animal monitoring programs. The method could be applied to the other factors (e.g., season, segment) to address various temporal and spatial scales. The results support the use of multiple gear types to monitor fish communities to assure all targeted species are adequately sampled, especially since each species has different gear susceptibilities. Further, multiple gears provide the ability to combine active gears or passive gears in multi-gear models (Arab et al., 2008). It was found that with additional data, higher catch rates, and the use of multi-gear models, the power of PSPAP to detect trends in fish populations should improve.

The PSPAP is a good example of community population monitoring using multiple gear types to target short-lived, small fish species along with long-lived, rare fish species of concern (Drobish, 2008). It was found that at least 20 years for pallid sturgeon and 10 years for shovelnose sturgeon at the current level of sampling effort are required to detect a 5% compounded annual decline. This is similar to other monitoring efforts such as described by Gray and Burlews (2007) who

estimate that sampling would need to occur for 20–30 years to detect an annual 5% trend in fingernail clams (Family: Sphaeriidae). Purcell et al. (2005) determined it would take 10–20 years to detect a decline in abundance of 30–56% in bird communities in oak-woodland sites in California. For shovelnose sturgeon, as few as 25 bends would need to be sampled at the current level of subsampling effort to detect a 5% annual decline after 5 years; not enough bends could be sampled to get a similar outcome for pallid sturgeon.

Since the modified bootstrap simulation essentially only modified the number of zeros and not the distribution of the data for the nonzero values, simulation results show that a decrease in zeros can increase power. Furthermore, decreasing the population mean as we did, while keeping the variance fixed results in a lower non-centrality parameter and thus, a conservative estimate of power results as compared to an increasing population mean. Similarly, a decrease in variance with a constant mean will result in an increase in power. If the observations were spread out more uniformly (e.g., less zeros) with the mean held constant the variance would decrease. One caution with application of the results is when an estimated variance component is 0, under REML, this indicates a different model may be more effective (Kuehl, 2000), in this case, due to the large number of zeros.

Often times, in studies of abundance, a logistic or Poisson regression is used. However, in our context these approaches would not provide a viable solution. First, logistic regression would change the problem to one of presence/absence and therefore limit the inferential scope. Additionally, similar to the ANOVA situation, the observed data would fail to meet the assumptions of a Poisson regression due to excessive zeros. One alternative might be to use a zero-inflated Poisson (ZIP) regression. However, the design imposed for the Pallid Sturgeon Population Assessment Program is such that random effects arise due to subsamples. Therefore, in order to respect the sampling design, the model would need to be a ZIP mixed model. Calculating power under this model could, in principal, be conducted using a simulation-based approach. However, we are not aware of any place in the literature where such an approach has been developed.

Joint monitoring and parallel power analyses for a more common species (i.e., shovelnose sturgeon) and a related, rare species of concern (i.e., pallid sturgeon) provide guidance otherwise not obtainable for the latter. It is important to note that the shovelnose and pallid sturgeon are different species with different demographics (Bajer and Wildhaber, 2007). Shovelnose sturgeon persistence and resiliency relative to the pallid sturgeon is likely the result of its earlier maturity (Keenlyne and Jenkins, 1993), lower trophic status (Keenlyne, 1997), and adaptability to a wider range of environmental conditions (Mayden and Kuhajda, 1997). Despite these differences, morphological, physiological, and genetic similarity clearly indicates these two sympatric species are closely related (Bailey and Cross, 1954; Campton et al., 2000). The data collected as a result of the sampling design and gear used by PSPAP provide valuable population information to assess these differences. Additionally, the PSPAP addresses many of the critical factors identified in the conceptual life-history model developed by Wildhaber et al. (2007) and the population viability analysis conducted by Bajer and Wildhaber (2007) that affect reproduction, growth, and survival of Scaphirhynchus sturgeons. The Bajer and Wildhaber (2007) model was most sensitive to age-0, juvenile, and young-adult survival. This suggests that PSPAP could be improved by



changing the sampling design to increase power to detect changes in juveniles and young adults.

Continuous updating of information and models is central to the practice of adaptive management of natural resources (Williams et al., 2007). The power analyses presented here provide a foundation of information on the performance of the PSPAP, and can help guide future sampling designs and application of effort. Importantly, the results may change substantially over time as more data are available and variances estimates are consequently reduced. Periodic updates of power analyses would serve to facilitate optimal performance of monitoring efforts.

### Acknowledgements

The authors thank all the people that have been involved in the PSPAP from which the data came and all the coordinators and data managers. We also acknowledge the US Army Corps of Engineers as the funding agency. Lastly, thanks to all who helped from the US Geological Survey, Columbia Environmental Research Center.

### References

- Arab, A.; Wildhaber, M. L.; Wikle, C. K.; Gentry, C. N., 2008: Zero-inflated modeling of catch per unit area resulting from multiple gears: application to channel catfish and shovelnose sturgeon in the Missouri River. *N. Am. J. Fish. Manage.* **28**, 1044–1058.
- Bailey, R. M.; Cross, F. B., 1954: River sturgeon of the American genus *Scaphirhynchus*: characters, distribution and synonymy. *Pap. Mich. Acad. Sci. Arts Let.* **39**, 169–208.
- Bajer, P. G.; Wildhaber, M. L., 2007: Population viability analysis of lower Missouri River shovelnose sturgeon with initial application to the pallid sturgeon. *J. Appl. Ichthyol.* **23**, 457–464.
- Berry, C. R., Jr; Wildhaber, M. L.; Galat, D. L., 2005: Population structure and habitat use of benthic fishes along the Missouri and lower Yellowstone Rivers. Volume 3: fish distribution and abundance. Final Report of Missouri River Benthic Fish Study PD-95-5832 to USACE and U.S. Bureau of Reclamation.
- Campton, D. E.; Bass, A. L.; Chapman, F. A.; Bowen, B. W., 2000: Genetic distinction of pallid, shovelnose, and Alabama sturgeon—emerging species and the U.S. Endangered Species Act. *Conserv. Genet.* **1**, 17–32.
- Colorado River Recovery Implementation Program, 2001: Program highlights 2000–2001. <http://www.fws.gov/coloradoriverrecovery/documents.htm>.
- Comprehensive Everglades Restoration Plan, 1999: Central and Southern Florida Project, Comprehensive Review Study, Final Integrated Feasibility Report and Programmatic Environmental Impact Statement. [http://www.evergladesplan.org/about/about\\_cerp\\_brief.aspx](http://www.evergladesplan.org/about/about_cerp_brief.aspx).
- Davidson, A. C.; Hinkley, D. V., 1997: Bootstrap methods and their application. Cambridge University Press, New York, NY, pp. 582.
- Drobish, M. R. (Ed.), 2005a: Pallid sturgeon population assessment program: vol. 1, June 6, 2005. USACE, Omaha District, Yankton, SD, pp. 48.
- Drobish, M. R. (Ed.), 2005b: Missouri river standard operating procedures for sampling and data collection: June 17, 2005. USACE, Omaha District, Yankton, SD, pp. 125.
- Drobish, M. R. (Ed.), 2008: Pallid sturgeon population assessment project, Volume 1.3. USACE, Omaha District, Yankton, SD, pp. 207.
- Dryer, M. P.; Sandvol, A. J., 1993: Recovery plan for the pallid sturgeon (*Scaphirhynchus albus*). U.S. Fish and Wildlife Service, Bismarck, ND, pp. 55.
- Duffy, W. G.; Berry, C. R.; Keenlyne, K. D., 1996: Biology of the pallid sturgeon with an annotated bibliography through 1994. South Dakota State University, Cooperative Fish and Wildlife Research Unit, Technical Bulletin 5, Brookings, SD, pp. 32.
- Galat, D. L.; Lipkin, R., 2000: Restoring ecological integrity of great rivers: historical hydrographs aid in defining reference conditions for the Missouri River. *Hydrobiologia* **422**, 29–48.
- Galat, D. L.; Berry, C. R.; Gardner, W. M.; Hendrickson, J. C.; Mestl, G. E.; Power, G. J.; Stone, C.; Winston, M. R., 2005: Spatio-temporal patterns and changes in Missouri River fishes. *Am. Fish. S. S.* **45**, 249–291.
- Gray, B. R.; Burlews, M. M., 2007: Estimating trend precision and power to detect trends across grouped count data. *Ecology* **88**, 2364–2372.
- Hesse, L. W.; Schmulback, J. C.; Carr, J. M.; Keenlyne, K. D.; Unkenholz, D. G.; Robinson, J. W.; Mestl, G. E., 1989: Missouri River fishery resources in relation to past, present, and future stresses. In: Proceedings of the International Large River Symposium. D. P. Dodge (Ed.). *Can. Spec. Publ. Fish. Aquat. Sci.* **106**, 352–371.
- Keenlyne, K. D., 1997: Life history and status of the shovelnose sturgeon, *Scaphirhynchus platyrhynchus*. *Environ. Biol. Fish.* **48**, 291–298.
- Keenlyne, K. D.; Jenkins, L. G., 1993: Age at sexual maturity of the pallid sturgeon. *Trans. Am. Fish. Soc.* **122**, 393–396.
- Kuehl, R., 2000: Design of experiments: statistical principles of research design and analysis, 2nd edn. Duxbury Press, Pacific Grove, CA, pp. 666.
- Littell, R. C.; Milliken, G. A.; Stroup, W. W.; Wolfinger, R. D.; Schabenberger, O., 2006: SAS for mixed models, 2nd edn. SAS Institute Inc., Cary, NC, pp. 814.
- Mayden, R. L.; Kuhajda, B. R., 1997: Threatened fishes of the world: *Scaphirhynchus albus* (Forbes & Richardson, 1905) (Acipenseridae). *Environ. Biol. Fish.* **48**, 420–421.
- Moos, R. E., 1978: Movement and reproduction of shovelnose sturgeon *Scaphirhynchus platyrhynchus* (Rafinesque) in the Missouri River, South Dakota. PhD Dissertation, University of South Dakota, Vermillion, SD, pp. 213.
- Moser, B., 1996: Linear models: a mean model approach. Academic Press, New York, NY, pp. 228.
- Purcell, K. L.; Mori, S. R.; Chase, M. K., 2005: Design considerations for examining trends in avian abundance using point counts: examples from oak woodlands. *Condor* **107**, 305–320.
- Searle, S. R., 1997: Linear models. Wiley, New York, NY, pp. 57–59, 101. ISBN: 0471184993.
- Sims, M.; Elston, D. A.; Harris, M. P.; Wanless, S., 2007: Incorporating variance uncertainty into a power analysis of monitoring designs. *J. Agric. Biol. Environ. Stat.* **12**, 236–249.
- Sustainable Ecosystem Institute, 2004: Independent science review of the Pallid Sturgeon Assessment Program: final report. Sustainable Ecosystems Institute, Portland, OR.
- Sustainable Ecosystem Institute, 2005: Independent science review of the Habitat Assessment and Monitoring Program study plan for the Missouri River. Sustainable Ecosystems Institute, Portland, OR.
- U.S. Fish and Wildlife Service, 2000: Biological opinion on the operation of the Missouri River main stem reservoir system, operation and maintenance of the Missouri River bank stabilization and navigation project, and operation of the Kansas River reservoir system. U.S. Fish and Wildlife Service, Bismarck, ND, pp. 286.
- Wildhaber, M. L.; DeLonay, A. J.; Papoulias, D. M.; Galat, D. L.; Jacobson, R. B.; Simpkins, D. G.; Braaten, P. J.; Korschegen, C. E.; Mac, M. J., 2007: A conceptual life-history model for pallid and shovelnose sturgeon. *USGS Circ.* **1315**, 18.
- Williams, B. K.; Szaro, R. C.; Shapiro, C. D., 2007: Adaptive management: The U.S. Department of Interior technical guide. Adaptive Management Working Group, U.S. Department of Interior, Washington, D.C.

**Author's address:** Mark Wildhaber, U.S. Geological Survey, Columbia Environmental Research Center, 4200 New Haven Road, Columbia, MO 65201, USA.  
E-mail: mwildhaber@usgs.gov



## Appendix

To test a hypothesis of no year effect, the null hypothesis for the cell means model is

$$H_0 : \mathbf{H}\mu = \mathbf{0}$$

where

$$\mathbf{H} = \Delta_a \otimes \frac{1}{b} \mathbf{1}'_b \otimes \frac{1}{c} \mathbf{1}'_c,$$

$$\Delta_a = \begin{bmatrix} 1 & 0 & \cdots & 0 & -1 \\ 0 & 1 & & \vdots & -1 \\ \vdots & & \ddots & 0 & -1 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix}_{a-1 \times a},$$

and  $\mathbf{0}$  is a vector of zeros of length  $abc$ . Thus,  $\mathbf{H}\mu = \mathbf{0}$  can be written

$$\begin{aligned} \frac{1}{bc} \left( \sum_{j=1}^b \sum_{k=1}^c \mu_{1jk} - \sum_{j=1}^b \sum_{k=1}^c \mu_{ajk} \right) &= 0 \\ \vdots \\ \frac{1}{bc} \left( \sum_{j=1}^b \sum_{k=1}^c \mu_{a-1jk} - \sum_{j=1}^b \sum_{k=1}^c \mu_{ajk} \right) &= 0 \end{aligned}$$

which simplifies to

$$H_0 : \bar{\mu}_{1\bullet\bullet} = \cdots = \bar{\mu}_{a\bullet\bullet}.$$

The expected mean square for the year effect,  $E(\text{MS}\alpha)$ , is

$$E(\text{MS}\alpha) = \sigma_S^2 + s\sigma_D^2 + bc ds \frac{\sum_{i=1}^a (\bar{\mu}_{i\bullet\bullet} - \bar{\mu}_{\bullet\bullet\bullet})^2}{a-1}.$$

Under the null hypothesis it follows that  $E(\text{MS}\alpha) = E(\text{MSD})$ , which implies that the appropriate test statistic, for testing no main effect due to year is

$$F = \frac{\text{MS}\alpha}{\text{MSD}}.$$

The sum of squares for year can be written as  $\text{SS}\alpha = \mathbf{Y}'\mathbf{M}_\alpha\mathbf{Y}$  where

$$\mathbf{M}_\alpha = \left( \mathbf{I}_a - \frac{1}{a} \mathbf{J}_a \right) \otimes \frac{1}{b} \mathbf{J}_b \otimes \frac{1}{c} \mathbf{J}_c \otimes \frac{1}{d} \mathbf{J}_d \otimes \frac{1}{s} \mathbf{J}_s.$$

Therefore,  $\mathbf{M}_\alpha$  is idempotent and symmetric. Similarly, the sum of squares for bend is  $\text{SSD} = \mathbf{Y}'\mathbf{M}_D\mathbf{Y}$  where,

$$\mathbf{M}_D = \mathbf{I}_a \otimes \mathbf{I}_b \otimes \mathbf{I}_c \otimes \left( \mathbf{I}_d - \frac{1}{d} \mathbf{J}_d \right) \otimes \frac{1}{s} \mathbf{J}_s,$$

is idempotent and symmetric. As a result, it follows that  $\mathbf{M}_\alpha\mathbf{\Sigma}/(\sigma_S^2 + s\sigma_D^2)$  is idempotent and symmetric with rank equal to  $a-1$ . Finally, let  $\mathbf{W}$  and  $\mu$  be defined as before then  $\phi = \mu'\mathbf{W}'\mathbf{M}_\alpha\mathbf{W}\mu/(\sigma_S^2 + s\sigma_D^2)$ ; i.e.,

$$\phi = \frac{bc ds}{\sigma_S^2 + s\sigma_D^2} \sum_{i=1}^a (\bar{\mu}_{i\bullet\bullet} - \bar{\mu}_{\bullet\bullet\bullet})^2.$$

Collecting these results, it follows that (Theorem 2, Page 57, Searle, 1997),  $\mathbf{Y}'\mathbf{M}_\alpha\mathbf{Y}/(\sigma_S^2 + s\sigma_D^2)$  has a chi-squared distribution with  $a-1$  degrees of freedom and non-centrality parameter  $\phi$ . Analogously,  $\mathbf{Y}'\mathbf{M}_D\mathbf{Y}/(\sigma_S^2 + s\sigma_D^2)$  has a central chi-squared distribution (i.e.,  $\phi_D = 0$ ) with  $abc(d-1)$  degrees of freedom.

With the distributions of the sum of squares for year and bend established, we can combine the two to form the  $F$  statistic. However, the quadratic forms need to be independent. Now, since  $\mathbf{M}_\alpha\mathbf{\Sigma}\mathbf{M}_D = 0$ ,  $\mathbf{Y}'\mathbf{M}_\alpha\mathbf{Y}/(\sigma_S^2 + s\sigma_D^2)$  and  $\mathbf{Y}'\mathbf{M}_D\mathbf{Y}/(\sigma_S^2 + s\sigma_D^2)$  are independent (Theorem 3 – Searle, 1997, Page 59). Additionally, note that

$$\begin{aligned} F &= \frac{\text{MS}\alpha}{\text{MSD}} = \frac{((\mathbf{Y}'\mathbf{M}_\alpha\mathbf{Y})/(\sigma_S^2 + s\sigma_D^2))/(a-1)}{((\mathbf{Y}'\mathbf{M}_D\mathbf{Y})/(\sigma_S^2 + s\sigma_D^2))/abc(d-1)} \\ &\sim \frac{\chi_{a-1,\phi}^2/(a-1)}{\chi_{abc(d-1)}^2/abc(d-1)}. \end{aligned}$$

Hence,  $F$  follows a non-central  $F$  distribution with  $a-1$  and  $abc(d-1)$  numerator and denominator degrees of freedom respectively and non-centrality parameter  $\phi$  (Searle, 1997, Page 101) with  $\phi = 0$  under the null hypothesis.

Copyright of Journal of Applied Ichthyology is the property of Wiley-Blackwell and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.