Discussion paper

Stochastic Optimization for Multispecies Fisheries in the Barents Sea

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STOCHASTIC OPTIMIZATION FOR MULTISPECIES FISHERIES IN THE BARENTS SEA

Short title: Optimization for Multispecies Fisheries

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ABSTRACT

This study presents a multispecies stochastic model. The model suggests optimal fishing policy for two species in a three species predator prey ecosystem in the Barents Sea. We have employed stochastic dynamic programing to solve a three dimensional model, where catch is optimized by a multispecies feedback strategy. Application of the model in cod, capelin and herring ecosystem in the Barents Sea shows that the optimal catch for stochastic interaction model is more conservative compared to deterministic policy. Furthermore, we found that stochasticity has strong effect on optimal exploitation policy in the prey (capelin) compared to the predator (cod) species.

Key words Barents Sea, Ecosystem based management, Multispecies, Stochastic optimization

JEL Classification Codes   C61, Q22, Q57

1. Introduction

Barents Sea is one of the most productive ocean areas in the world (Aanestad, Sandal, & Eide, 2009; O'Brien, Tompkins, Eriksen, & Prestrud, 2004) and represents a highly diverse arctic ecosystem (Larsen, Nagoda, & Anderson, 2001). The major commercial fish species in the Barents Sea include capelin (*Mallotus villosus*), herring (*Clupea harengus*) and their main predator, Northeast Arctic cod (*Gadus morhua*). Of the three species, only capelin and cod are harvested in the Barents Sea. However, the influx and out flux of the herring species affect the growth of these main commercial species through their predator-prey relationships. The relationships between these species are highly dynamic (Bogstad, Hauge, & Ulltang, 1997). There are biological, physical and economic interactions among these species in the ecosystem. The optimal catch of the two commercial species in the Barents Sea is affected due to the interactions of the species. There exist management problems for the species that involve interactions between species at different trophic levels (May, Beddington, Clark, Holt, & Laws, 1979). The most common approach, single species management in multispecies fisheries, ignores the ecological relationships among species as well as the technological and economic relationships (Kasperski, 2010). This may lead to misleading results and incorrect policy decisions causing an over or under exploitation of the stocks (Fleming & Alexander, 2003; Hoff et al., 2010). Therefore, an ecosystem based fishery management (EBFM) approach is required and it is gaining importance in recent years (Scandol, Holloway, Gibbs, & Astles, 2005; Zhou et al., 2010). EBFM is a holistic approach for maintaining ecosystem quality and sustaining associated benefits (Brodziak & Link, 2002; Zhou et al., 2010). Fisheries may affect the marine ecosystem in several ways such as fish populations and their growth habits, interaction among fish species, fish migrations, food preferences and more importantly its management. Among many factors of fisheries management, fish harvesting is one of the key determinants that affects the marine ecosystem. An ecosystem based management strategy for marine fisheries would be to minimize the potential impacts on ecosystem together with sustainable exploitation of fish resources (Witherell, Pautzke, & Fluharty, 2000). EBFM strategy includes conservative and precautionary harvest limits, comprehensive monitoring and enforcements, by-catch controls, gear restrictions, temporal and spatial distribution of fisheries and marine protected areas (Witherell et al., 2000). EBFM requires a multispecies ecosystem model for sustainable harvest of fisheries that considers beyond the sustainable yield for individual species. Multispecies management is to examine the big picture to address the long term consequences of present decisions. Therefore, it is a key approach for sustainable management (Hollowed et al., 2000) and consideration of multispecies interaction in fishery model is increasing in recent years (for example, see Agnarsson et al., 2008; Bogstad et al., 1997; Clark, 1990; Fleming & Alexander, 2003; Iversen, 2006; Kar & Chaudhuri, 2004; May et al., 1979; Poudel, Sandal, Steinshamn, & Kvamsdal, 2012; Sandal & Steinshamn, 2010; Woodward & Bishop, 1999; Yodzis, 1994). However, most of the studies focused on predators prey interactions between two species while three species continuous time interaction

models are very few, although there is increasing interest in multispecies model in recent years (for example, Aanestad et al., 2009; Das, Srinivas, Srinivas, & Gazi, 2012; Nieminen, Lindroos, & Heikinheimo, 2012; Samanta, Manna, & Maiti, 2003; Sharma & Samanta, 2013).

Apart from the interactions among species, growth of the individual species is also affected by physical uncertainties and random shocks (Sarkar, 2009). Random shocks such as environmental disturbances and variability, zooplankton abundance, and predator distribution and migration affects recruitment and growth of fish stocks (Misund et al., 1998). Migration of herring stock in the Barents Sea is observed to be highly variable (Lindstrøm, Haug, & Røttingen, 2002), which creates uncertainty in growth of the stock, its predators and preys in the ecosystem. To our knowledge, three species continuous time model with stochastic stock dynamics are very rare. The reasons for the lack of multispecies stochastic bioeconomic models are due to unavailability of analytical solutions (Kar & Chaudhuri, 2004; Posch & Trimborn, 2010) and computational difficulties particularly in solving non-linear dynamic models in higher dimensions (Munos & Zidani, 2005; Singh, Weninger, & Doyle, 2006). In this study, we develop a multispecies model that not only considers the interactions among these species but also takes into account of the physical and biological or environmental system uncertainties by including stochasticity in the stock growth dynamics. The inclusion of stochasticity in the model makes the model more realistic because the Barents Sea ecosystem is highly volatile partly due to the variability in the abundance of immature herring year classes (Lindstrøm et al., 2002). Bogstad et al. (1997) found that herring is most likely to be affected by changes in the abundance of its top predators such as minke whales, which further influences the growth of the targeted species. Capelin and herring also compete for food during food scarcity in the Barents Sea (Huse & Toresen, 1996). In addition to cod and herring predation, capelin are also predated in large quantities by whales, seals, and others such as haddock (Ushakov & Prozorkevich, 2002). Similarly, the cod recruitment and survivability is directly affected by climatic and food availability. Higher temperature at spawning time and high capelin biomass has a positive effect on cod recruitment. However, high temperature results in decrease in capelin biomass through high herring recruitment which affects the cod recruitment negatively through cannibalism (Hjermann et al., 2007). Thus, the fluctuations in the distribution and abundance of herring have an impact on growth of its prey and predators, the commercially exploitable species.

We apply stochastic dynamic programing to derive the feedback optimal control rules, which is a central tool for tackling stochastic optimization problems (Sydsæter, Hammond, Seierstad, & Strom, 2005). Dynamic programming is a mathematical technique for solving certain types of sequential decision problems in which a sequence of decisions must be made with each decision affecting the future decisions (Howard, 1966). The dynamic programming technique has tremendous methodological appeal in solving inter-temporal economic problems but it has not been as widely used

as anticipated for empirical analysis of natural resource problems (Howitt, Msangi, Reynaud, & Knapp, 2002). One reason could be the computational difficulties for these types of problems. We solved stochastic dynamic problems through a numerical approximation scheme labeled "the probabilistic approach" to finite difference approximation of the associated Hamilton-Jackobi-Belmann equation.

The main contribution of the work is a development of multispecies bioeconomic model with stochastic dynamics, an essential component of the ecosystem based fisheries management. The contribution of the study is two-fold. This study shows that dynamic programming (DP) approach can be used for empirical model in higher dimensions and solving a stochastic dynamic programing problem in such higher dimension can be a very useful resource in the field of bioeconomic modeling. Secondly, it provides empirical results on optimal management in a complex bio ecosystem that harbors three species.

2. The Bioeconomic Model

Our model is a surplus production model (Schaefer, 1957) in continuous time. The general biological interdependent deterministic growth functions for three interacting species are specified by adding one species in the model from Sandal and Steinshamn (2010). Let x be capelin species, a prey for both cod and herring, y be the cod species, a predator for both capelin and herring, and z be the herring that is preyed by cod and predator of capelin species. The deterministic growth increments of these species can be given as:

$$
\begin{aligned}
dx &= [f(x,y,z) - u_x]dt \\
dy &= [g(x,y,z) - u_y]dt \\
dz &= [j(x,y,z)\,]dt
\end{aligned}
\tag{1}
$$

The functions $f(x,y,z)$, $g(x,y,z)$ and $j(x,y,z)$ are the biological growth functions of three fish species capelin, cod and herring respectively. u_x and u_y represent the harvest rate of species x and y while there is no herring harvest in the Barents Sea.

Furthermore, a three-species interaction model with stochastic dynamic can be formulated by adding stochastic terms in equation (1) as:

$$
\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} f(x,y,z) - u_x \\ g(x,y,z) - u_y \\ j(x,y,z) \end{pmatrix} dt + \begin{pmatrix} \sigma_{11}(x,y,z) & \sigma_{12}(x,y,z) & \sigma_{13}(x,y,z) \\ \sigma_{21}(x,y,z) & \sigma_{22}(x,y,z) & \sigma_{23}(x,y,z) \\ \sigma_{31}(x,y,z) & \sigma_{32}(x,y,z) & \sigma_{33}(x,y,z) \end{pmatrix} \begin{pmatrix} dB_1 \\ dB_2 \\ dB_3 \end{pmatrix}
\tag{2}
$$

Although it is likely that the stochastic events are correlated among the species, it is not necessary to make excessively complex models to observe the effects of stochasticity (Stefansson, 2001).

Therefore, we assume it to be small enough to be neglected, i.e. $\sigma_{ij}(x, y, z) = 0 \ for \ all \ i \neq j$.

We further simplify by setting the volatility as a linear function of its own stock level, $\sigma_{11}(x, y, z) = \sigma_1 x$; $\sigma_{22}(x, y, z) = \sigma_2 y$; and $\sigma_{33}(x, y, z) = \sigma_3 z$. This assumption is made for two reasons; firstly, for the simplicity in the numerical approximation, and secondly, complicated functional volatility dependence is not much known. Since the species are prey and predator species, the stochastic processes that affect prey directly may not affect the predators in a direct sense or vice versa. The other species is still indirectly affected by such a process through the interaction in the drift (deterministic) part. Epidemics are typically of this kind. Equation (2) can now be written as:

$$dx = [f(x, y, z) - u_x]dt \ + \sigma_1 x \, dB_1$$
$$dy = [g(x, y, z) - u_y]dt \ + \sigma_2 y \, dB_2 \qquad (3)$$
$$dz = [j(x, y, z)]dt \ + \sigma_3 z \, dB_3$$

The biological growth functions and their coefficients obtained from (Aanestad et al., 2009) are given in equation (4):

$$f(x, y, z) = r_1 x(1 - \frac{x}{k_1} - b_1(y + z)) \qquad (4)$$

$$g(x, y, z) = r_2 y(1 - \frac{y}{k_2} + b_2(1 + x)\sqrt{z})$$

$$j(x, y, z) = \widetilde{r_3}(x, y)z\left(1 - \frac{z}{\widetilde{k_3}(x, y)} + b_3 x \frac{(100 + x + y)}{1 + z}\right) + C$$

Where $\widetilde{r_3}(x, y) = \frac{r_3}{100 + x + y}$, $\widetilde{k_3}(x, y) = k_3\widetilde{r_3}(x, y)$ and $r_i, k_i, b_i, C > 0$ are parameters.

In equation (4), the growth function for each species is concave with respect to its own species. The coefficients r_i and k_i represent the intrinsic growth rates and carrying capacity in the logistic functions. The additional coefficient $b_i > 0$ is the interaction coefficient that adjusts the actual biomass in the multispecies model. Capelin biomass decreases due to predation of cod and herring while increases in cod due to predation on capelin and herring. The term $(1 + x)\sqrt{z}$ implies that the biomass of capelin is more important than the biomass of the herring species. The growth function of the herring is somewhat different from the straightforward logistic growth function. The term $\widetilde{r_3}(.)$ and $\widetilde{k_3}(.)$ have dependency on the biomass of cod and capelin. The coefficients $C > 0$ implies that herring stock can never extinct in the Barents Sea ecosystem due to predation or natural mortality because of its nature of migration or influx in the Barents Sea. In equation (3), the term $\sigma_{()}(.)dB_{()}$ represents the stochastic part of the stock growth relationship. $\sigma_{()}(.)$ is the diffusion term and represents volatility in the growth models. The term dt is the time increment and the term $dB_{()}$ is Brownian motion which is identically and independently distributed with mean zero and variance dt. We assume stocks and harvests rates to be non-negative.

By substituting the growth functions from equation (4) into equation (3), our basic biological model is now given as:

$$dx = \left[r_1 x (1 - \frac{x}{k_1} - b_1(y+z)) - u_x \right] dt + \sigma_1 x \, dB_1 \tag{5}$$

$$dy = \left[r_2 y (1 - \frac{y}{k_2} + b_2(1+x)\sqrt{z}) - u_y \right] dt + \sigma_2 y \, dB_2$$

$$dz = \left[\frac{r_3}{100 + x + y} z \left(1 - \frac{z}{k_3 \frac{r_3}{100 + x + y}} + b_3 x \frac{(100 + x + y)}{1 + z} \right) \right] dt + \sigma_3 z \, dB_3$$

Equation (5) states that all of the species have stochastic growth and interactions among them. But what if only one of the species has stochastic growth? To understand the effect of stochastic growth of one species on optimal exploitation in multispecies ecosystem, equation (5) can be simplified by setting other two stochastic parameters to zero. For example by setting $\sigma_2 = \sigma_3 = 0$ in equation (5), the stochasticity in capelin growth can be analyzed. Similarly by setting $\sigma_1 = \sigma_2 = 0$, the stochastic effect of herring in multispecies ecosystem can be modeled. However, it is noteworthy that an assumption of stochasticity in one of the species makes all the other species stochastic due to the existence of predator prey interaction terms (xy, yz, xz) in the growth functions.

The second part of the **bioeconomic model consists of economic component**, which is the net revenue from the harvest of capelin and cod species.

$$\pi \left(x, y, u_x, u_y \right) = \pi_x(x, u_x) + \pi_y(y, u_{y)} \tag{6}$$

$\pi_x(x, u_x)$ and $\pi_y(y, u_{y)}$ are the net revenue from capelin species (x) and cod species (y) respectively. The revenue functions are obtained from (Sandal & Steinshamn, 2010) expressed as:

$$\pi_x(x, u_x) = p_x(u_x)u_x - c_x(x, u_x) \tag{7}$$
$$\pi_y(x, u_y) = p_y(u_y)u_y - c_y(y, u_y)$$

The functions $p(.)$ and $c(.)$ are inverse demand functions and cost functions respectively. The demand functions and cost functions are specified as:

$$p_x(u_x) = p_1 \tag{8}$$
$$c_x(x, u_x) = q_1 u_x^{\alpha_1}$$
$$p_y(u_y) = p_2 - p_3 u_y$$
$$c_y(y, u_y) = \frac{q_2 u_y^{\alpha_2}}{y}$$

After substituting the expression from equation (7) and (8) into equation (6), the profit function can be specified as:

$$\pi \left(x, y, u_x, u_y \right) = p_1 u_x - q_1 u_x^{\alpha_1} + p_2 u_y - p_3 u_y^2 - \frac{q_2 u_y^{\alpha_2}}{y} \tag{9}$$

Where $p_1, p_2, q_1, q_2, \alpha_1, \alpha_2$ are economic parameters.

We assume that prey is a schooling species and, therefore, the unit cost of harvest is independent of stock size. Our revenue function depends only on the predator stock and harvest level of prey and predators. The revenue function equation (9) can simply be written as a function of three arguments $\pi(y, u_x, u_y)$.

Given the growth functions and profit function, the management objective is to maximize expected net present value of the return from the harvest schedule over an infinite time horizon. This can be achieved by maximizing following function:

$$J(u_x, u_y) = E\left[\int_0^\infty e^{-\delta t} \pi(y, u_x, u_y) dt\right] \qquad (10)$$

The non-negative parameter δ is the discount rate and E is the expectation operator. Along with dynamic constraints and appropriate boundary conditions, the dynamic optimization problem can be written as:

$$V(x_0, y_0, z_0) = \max_{h_x, h_y \geq 0} J(y, u_x, u_y) \qquad (11)$$
$$x(t = 0) = x_0$$
$$y(t = 0) = y_0$$
$$z(t = 0) = z_0$$

The optimal solution in predator-prey model can be obtained by solving following Hamilton-Jacobi-Bellman (HJB) equation (Kushner & Dupuis, 2001) along with appropriate boundary conditions. This is an equation for any feasible initial condition and hence we replace (x_0, y_0, z_0) with (x, y, z).

$$\delta V(x, y, z) = \max_{u_x, u_y \geq 0} \{\pi(y, u_x, u_y) + V_a^T(x, y, z) F(x, y, z, u_x, u_y) \qquad (12)$$

$$+ \frac{1}{2} tr[\sigma(x, y, z) \sigma^T(x, y, z) V_{aa}(x, y, z)]\}$$

where, $V_a(x, y, z) = \begin{pmatrix} V_x((x,y,z)) \\ V_y((x,y,z)) \\ V_z((x,y,z)) \end{pmatrix}$, $F(x, y, z, u_x, u_y) = \begin{pmatrix} f(x,y,z) - u_x \\ g(x,y,z) - u_y \\ j(x,y,z) \end{pmatrix}$,

$\sigma(x, y, z) = \begin{pmatrix} \sigma_1 x & 0 & 0 \\ 0 & \sigma_2 y & 0 \\ 0 & 0 & \sigma_3 z \end{pmatrix}$ and $V_{aa}(x, y, z) = \begin{pmatrix} V_{xx}(x,y,z) & V_{xy}(x,y,z) & V_{xz}(x,y,z) \\ V_{yx}(x,y,z) & V_{yy}(x,y,z) & V_{yz}(x,y,z) \\ V_{zx}(x,y,z) & V_{zy}(x,y,z) & V_{zz}(x,y,z) \end{pmatrix}$ are

matrices.

The HJB equation (12) can be rearranged and simplified as:

$$\delta V = \max_{u_x, u_y \geq 0} \{\pi(y, u_x, u_y) + (f(x,y,z) - u_x)V_x \tag{13}$$

$$+ (g(x,y,z) - u_y)V_x + j(x,y,z)V_z + \frac{1}{2}\sigma_1{}^2 x^2 Vxx$$

$$+ \frac{1}{2}\sigma_2{}^2 y^2 Vyy + \frac{1}{2}\sigma_3{}^2 z^2 Vzz\}$$

The subscripts of V denote partial derivatives with respect to the index $(i = x, y, z)$. Optimal solution can be derived by solving the HJB equation (13). While it is difficult or impossible to solve analytically the HJB equation together with boundary conditions, we solve it using numerical approximation methods by applying stochastic dynamic programing (SDP) technique. Among a number of techniques available, the Markov chain approximation approach which is based on probability theory, is one of the most effective methods (Song, 2008).

3. Approximation Procedure for Numerical Solution

We have employed the Markov chain approximation approach of (Kushner & Dupuis, 2001). The basic idea of the Markov chain approximation approach is to discretize the HJB control problem. Let the HJB control problem (13) for multidimensional space be rewritten as:

$$\delta V(x) = \max_{u \geq 0} \left\{\pi(x,u) + V_x(x)^T F(x,u) + \frac{1}{2}tr[a(x)V_{xx}(x)]\right\} \tag{14}$$

where $V_x(x)$ and $F(x,u)$ are matrices of $1 \times n$ dimensions and $a(x) = \sigma(x)\sigma(x)^T$ is the covariance matrix with $\sigma(x)$ and $V_{xx}(x)$ are the matrices of $n \times n$ dimensions.

The numerical method entails a direct discretization of HJB equation (4). Let e_i be the unit basis of $\mathbb{R}^n$ for $i = 1, \ldots, n$ and $h > 0$, is a uniform step size or the approximation parameter. Let $V(x)$ be the numerical solution to the value function. Then by following standard finite difference approximations (Kushner & Dupuis, 2001), the first derivative and second derivative for $V(x)$ are given as equations (15) and (16) respectively.

$$V_{x_i}^{\pm}(x) = \frac{V(x \pm e_i h) - V(x)}{h} \tag{15}$$

$$V_{x_i x_j}(x) = -\frac{[2V(x) + V(x + e_i h - e_j h) + V(x - e_i h + e_j h)]}{2h^2} \tag{16}$$

$$+ \frac{[V(x + e_i h) + V(x - e_i h) + V(x + e_j h) + V(x - e_j h)]}{2h^2}$$

The next step is to define the positive and negative parts of the drift terms as in (17):

$$F_i^+(x, u) = \max(0, F_i(x, u)) \qquad (17)$$

$$F_i^-(x, u) = \max(0, -F_i(x, u))$$

where, $F_i^+(x, u)$ is the positive and $F_i^-(x, u)$ is flow in positive or negative direction.

Next, we define the transition probabilities (18) and interpolation interval (19) as:

$$P(x, x \pm e_i h | u) = \left[\frac{1}{2} a_{ii}(x) - \frac{\sum_{j: j \neq i} |a_{ij}(x)|}{2} + F_i^+(x, u)h\right]/Q^h(x, u) \qquad (18)$$

$$P(x, x + e_i h + e_j h | u) = (x, x - e_i h - e_j h | u) = [a_{ij}^+]/2Q^h(x, u)$$

$$P(x, x - e_i h + e_j h | u) = (x, x + e_i h - e_j h | u) = [a_{ij}^-]/2Q^h(x, u)$$

$$P(x, x | u) = 1 - \sum_{y \neq x} P(x, y | u)$$

where, $P(x, x|u)$ denotes the probability of remaining at the same point and $\sum_{y \neq x} P(x, y|u)$ is the sum of above listed values in (18). The implicit given stochastic interpolation time is then

$$\Delta t = \left[\frac{h^2}{1 + \delta h^2}\right]/Q^h(x, u). \qquad (19)$$

where $Q^h(x, u) = \sum_i a_{ii}(x) - \frac{\sum_{i,j: j \neq i} |a_{ij}(x)|}{2} + h \sum_i F_i(x, u)$ is the normalization coefficient. Defining $\beta(x, y) = 1 - \delta \Delta t$ with δ being the discount rate, the proper discrete form (14) (Markov chain) is given by

$$V(x) = max_u \left\{ \pi(x, u)\Delta t(x, u) + \beta(x, y) \sum_y P(x, y|u)V(y) \right\}. \qquad (20)$$

Given the transition probabilities and interpolation intervals, the optimal value function in the policy space is obtained and updated by value iteration and policy iteration on the HJB equation. We carried out value function iteration with initial guess, $V_0(x)$ for a given policy for example $u = u_0$ using the value iteration and policy iteration $V_{n+1}^k(x) = L(u_k)[V_n^k(x)]$ and $u_{k+1,n} = argmax\{L(u)[V_n^k(x)]\}$ respectively. Here, $L(u)$ is representing the Markov chain operator. The policy iterations (given $u_k(x)$) converges to the true value $\tilde{V}^k(x)$ for the given policy and the sequence $\{\tilde{V}^k(x)\}$ converges to the value function $V(x) = \tilde{V}(x)$ of our problem.

The value function iteration is carried out with initial guess, $V_0(x) = 0$ for a given policy $u = u_k$ using value iteration:

$$V_{n+1}^k(x) = L(x, u_k)V_n^k(x) \qquad (21)$$

where, the Markov chain operator L is defined by $L(x,u)V(x) = [\sum_i P(x, x \pm e_i h|u) \cdot V(x, x \pm e_i h|u) + \pi(x,u) \cdot \Delta t]$

V^* is obtained when n and k are large enough i.e. $V_n^k \xrightarrow[n \to \infty]{} (V^k)^* \xrightarrow[k \to \infty]{} V^*$. Similarly, the policy iteration is given as:

$$u_{k+1,n} = \arg \max_u [L(x,u), V_n^k(x)] \qquad (22)$$

The new policy $u_{k+1,n}$ is then employed in the value function iteration.

A mixture of value and policy iterations was carried out until the value function converged to the optimal value function. The function to which this procedure converges is the unique value function for the infinite horizon problem and the policy associated with the optimal value function is the optimal policy. The uniqueness is due to the contraction operator nature of our approach (Kushner & Dupuis, 2001). The iteration is stopped by employing stopping criteria (22) for policy and value functions.

$$|| u_{k+1,n+1}(x) - u_{k,n+1}(x) || < \varepsilon_1 \qquad (23)$$

$$|| V_{n+1}^{k+1}(x) - V_n^{k+1}(x) || < \varepsilon_2$$

where ε_i are a predefined threshold values. We carried out the policy and value iteration until the largest absolute difference in the value function was smaller than $\varepsilon_1 = \varepsilon_2 = 10^{-6}$ from one iteration to the next to ensure the convergence of the value function.

4. Specification of Parameters in the Model

The biological parameters were obtained from Aanestad et al. (2009) and the economic parameters were used from Sandal and Steinshamn (2010) and Aanestad et al. (2009). These parameters were estimated from International Council for the Exploration of the Seas (ICES) Barents Sea stock and harvest data during 1973 to 2005. The value of biological and economic parameter is given in the table 1 and table 2 respectively.

Table 1. The value of coefficients in the biological growth functions

Parameter	value	Parameter	value	Parameter	value
r_1	1.8515	k_1	7890.1	b_1	$1.1142e^{-4}$
r_2	0.5490	k_2	3191.3	b_2	$2.3e^{-6}$
r_3	1380.7	k_3	$1.0572e^7$	b_3	$1.87e^{-6}$

Sources: obtained from Aanestad et al. (2009)

Table 2. The value of economic parameters

Parameter	Value	Parameter	Value
p_1	1.0	p_2	12.65
q_1	0.67	p_3	0.00839
α_1	1.4	q_2	5848.1
δ	0.05	α_2	1.0

Sources: adopted from Sandal and Steinshamn (2010) and Aanestad et al. (2009)

5. Results and Discussions

In this section, the feedback exploitation policy obtained by employing numerical result is presented in multidimensional graphical surface plots. Exploitation policy in deterministic and stochastic model is discussed and compared. Although, there are four dimensions in the numerical output, only three dimensional surface plots are presented by fixing one of the species at some stock level due to difficulty in presenting in four dimensions.

5.1 Results in a Deterministic Model

The deterministic exploitation policy is obtained by employing dynamic equation (1). The feedback optimal exploitation policy for cod and capelin species is presented in figures 1 and 2 respectively.

The fishing moratorium for cod in the deterministic growth model is found to be nearly one million ton, when the capelin and herring stock is high in the ecosystem. As soon as the cod stock biomass exceeds one million ton, its exploitation should be increased. For example an increase of cod biomass from two million ton to six million ton, its optimal exploitation can be increased from 0.36 million ton to 0.62 million ton. We also observe that the abundance of biomass of other stocks such as capelin and herring in the ecosystem has no strong influence on the cod exploitation policy (figure 1).The main reason could be that there are two prey species for cod that serve as a food for its growth.
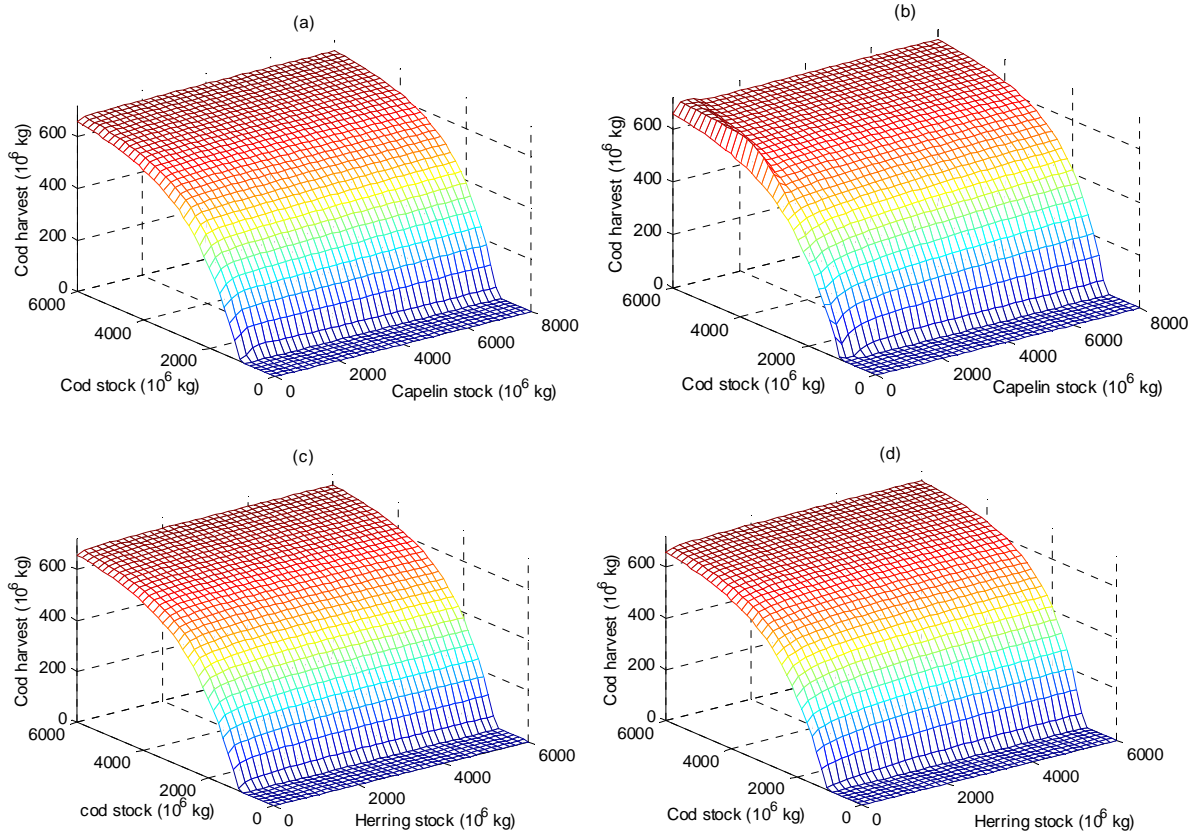
10

Figure 1: Optimal exploitation policy for cod in three species interaction ecosystem (a) 1.5 million tons of herring (b) 6 million tons of herrings (c) 2 million tons of capelin (d) 6 million tons of capelin.

The optimal exploitation policy for capelin is influenced by the presence of its predators in the ecosystem. The higher the influx of the juvenile herring, the more it is affected. We observed that exploitation of capelin should be more conservative at high herring biomass compared to low herring biomass. The main reason is that high herring biomass could consume more capelin and therefore one should be conservative in capelin exploitation to avoid its (capelin) extinction due to herring and cod predation.

Similarly, when there is 1.5 million ton of herring and no cod, one should be conservative compared to high cod at the same level of herring biomass. The reason might be that when there is high cod stock, the cod predates on herring and therefore capelin can be harvested without being conservative. On the other hand, the exploitation of capelin should be conservative especially when the cod stock is low. The reason is that immature cod need capelin for their growth (Hamre, 2003) and if harvested capelin could extinct due to cod and herring predation. As soon as the cod stock becomes large capelin exploitation could be done earlier because the cod may utilize herring and may not depend solely on the capelin biomass.
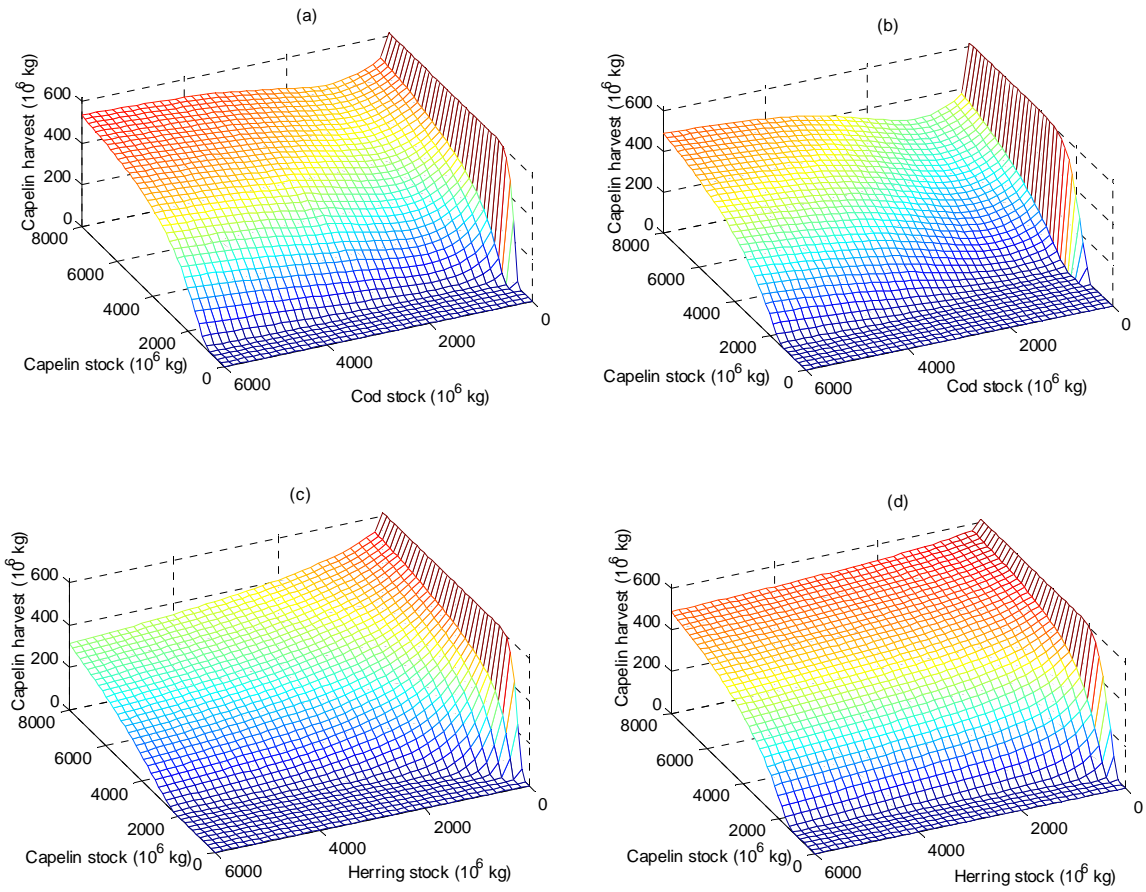
Figure 2: Optimal exploitation policy for capelin in three species interaction ecosystem (a) 1.5 million tons of herring (b) 6 million tons of herrings (c) 1.5 million tons of cod (d) 6 million tons of cod.

When there is low herring and very high cod in the ecosystem one should exploit the capelin as in a single species model (figure 2d), because the conservation of capelin to such a very large cod stock would not add value. With increasing herring biomass one should be conservative in capelin exploitation to avoid its collapse due to herring predation.

In case of low cod with increasing herring biomass, one should be conservative to avoid the risk of capelin collapse because a high herring influx causes a mass death of capelin fry (Hamre, 2003; Mehl, 1991) to extinction if harvested without considering the multispecies interaction in the ecosystem. There might be several reasons: (i) the herring predation on it, (ii) due to low cod in the ecosystem, there is high chance that herring is not affected by cod predation (iii) there is a food competition in the ecosystem (Kar & Chaudhuri, 2004).

5.2 Results in a Stochastic Model

In this section, we discuss the results from the dynamic equation (3), where we choose different level of stochasticity to compare the results with the deterministic model. The effect of stochasticity on optimal exploitation policy for cod and capelin species is presented following subsection. We analyze the result at different level of stochasticity in each species.

5.2.1 Effect of Stochasticity in the Cod Exploitation Policy

To analyze the effect of stochasticity in cod exploitation policy, we consider stochasticity only in one of the species at a time for all combination and also include stochasticity in all species. We observe that the stochastic effect is not very different whether one of the species is stochastic or all of the species are stochastic to cod harvest policy. The reason may be that if one of the species is stochastic, all are affected due to the interaction terms in equation (5). Due to similarity in results, we only present the results with stochasticity in all species.

When the level stochasticity is low ($\sigma_1 = \sigma_2 = \sigma_3 = 0.1$), there is no pronounced effect on optimal policy (results not shown). With increasing stochasticity in all of the species, the exploitation level should be decreased. For example, when the level of stochasticity increased to 0.5 i.e. ($\sigma_1 = \sigma_2 = \sigma_3 = 0.5$), at zero capelin and high level of herring stock in the ecosystem, it is optimal to delay the exploitation to higher stock because of the lack of capelin could cause slow growth of cod species (could be because the young cod prefer capelin to herring). At an increased level of stochasticity ($\sigma_1, \sigma_2, \sigma_3 > 0.9$) the moratorium moves towards a small stock level. This suggests that myopic exploitation is optimal for a high level of stochasticity. As in the deterministic model, the size of the other stocks (the prey species) has less influence in the optimal exploitation policy of cod (figure 3).
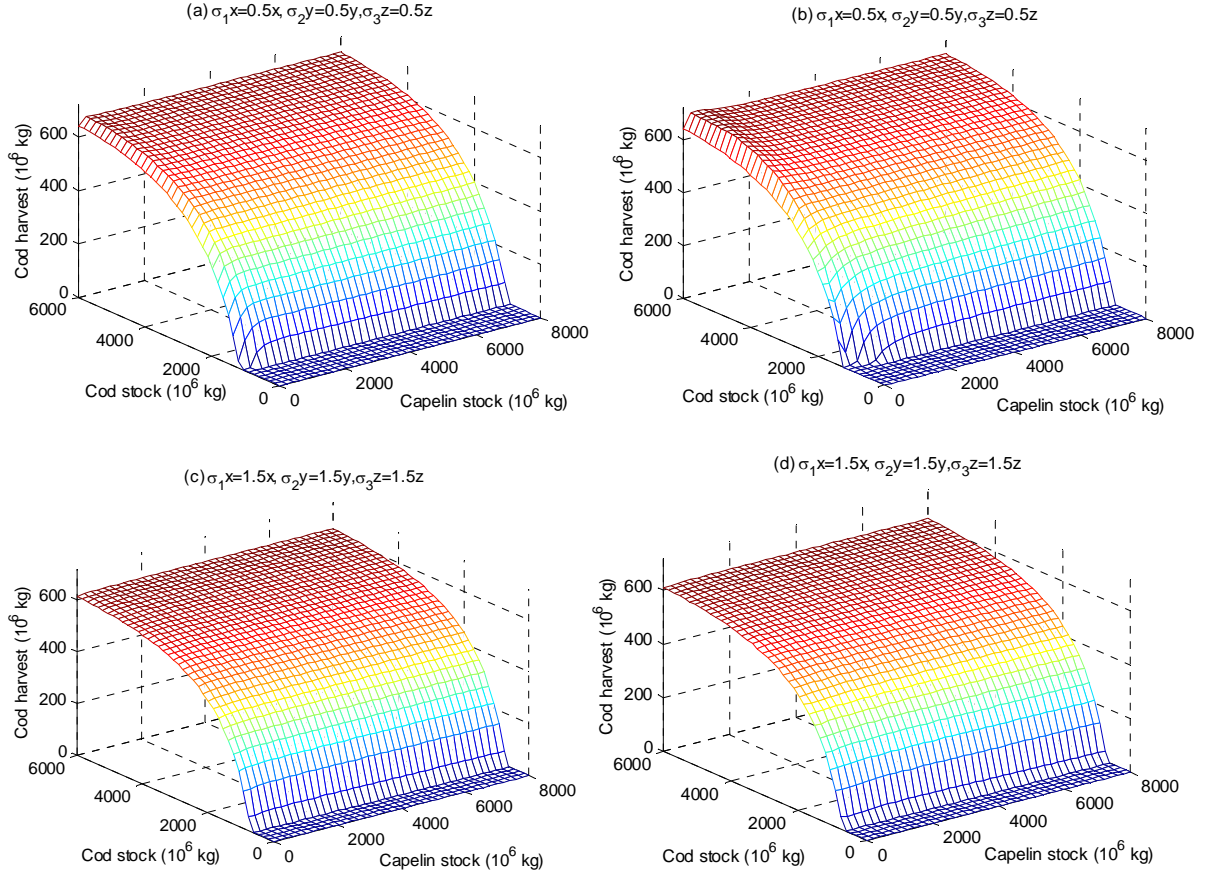
13

Figure 3: The effect of stochasticity in optimal exploitation policy for cod in three species interaction model (a) 1.5 million tons of herring (b) 6 million tons of herring (c)1.5 million tons of herring (d) 6 million tons of herring.

5.2.2 Effect of stochasticity in capelin exploitation policy

To analyze the effect of stochasticity in capelin exploitation policy, we consider stochasticity in all species, and again only in one of the species. Here we observe that capelin is more sensitive to stochasticity in different species, different biomass levels and also at different level of stochasticity. The effect due to cod species and herring species is not much pronounced. We present the results with stochasticity in all species, in capelin species and in herring species in flowing three different cases.

5.2.2.1 Case 1. All of the Species are Stochastic

Stochasticity has a large effect on the exploitation policy for capelin compared to cod species. If all three species are stochastic the exploitation policy should be conservative at low level of stochasticity (figure 4). The reason is that when these species are stochastic, there is high risk of stock collapse (Poudel, Sandal, & Kvamsdal, 2012) and therefore one should be more conservative to avoid the risk of extinction of the stock. When the herring stock is high, there is higher risk of stock collapse due to

herring predation; therefore, one should be very conservative compared to the low herring stock in the ecosystem.
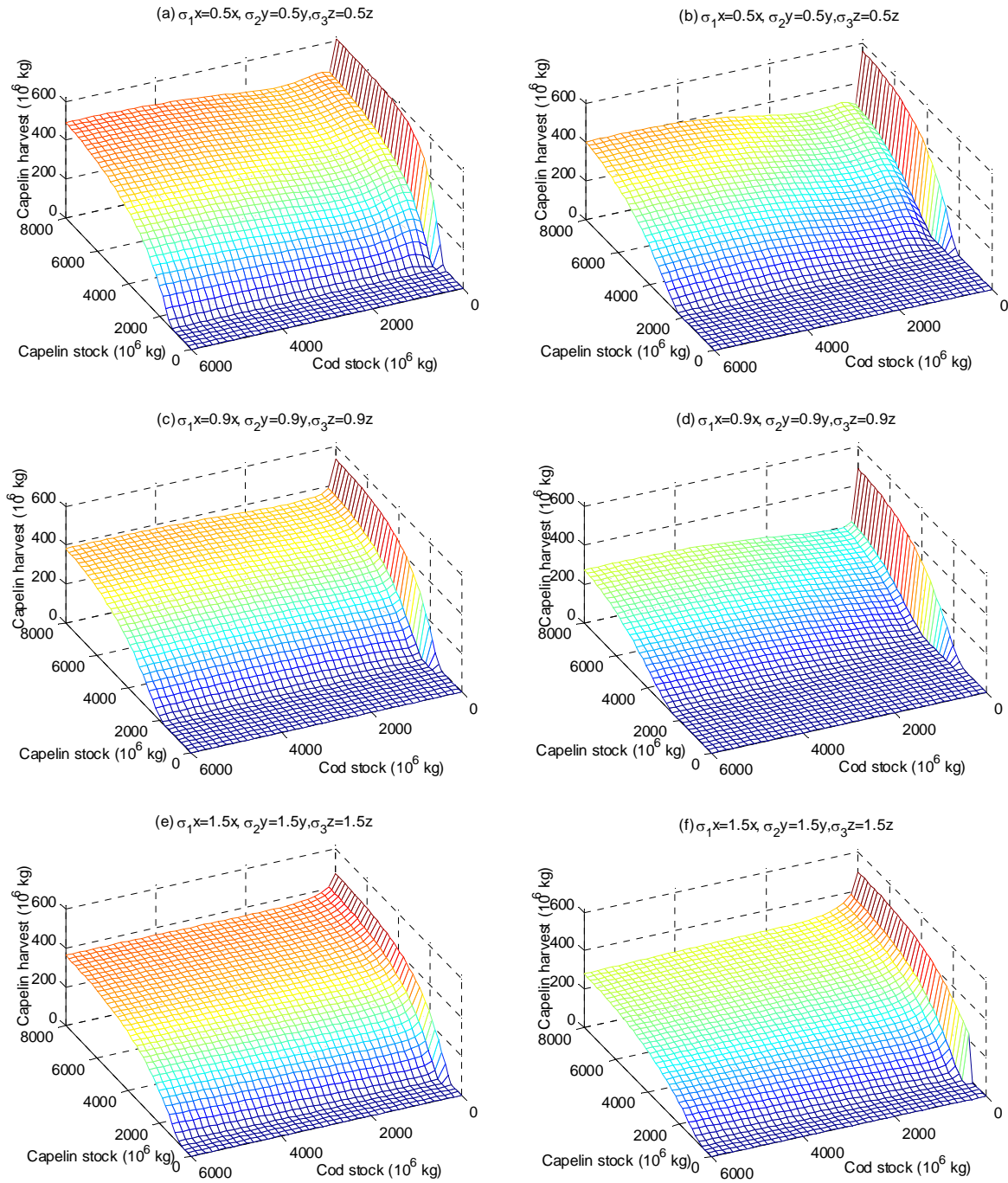

Figure 4: The feedback optimal exploitation policy for capelin in three species interaction model (a) 1.5 million tons of herring (b) 6 million tons of herring (c) 1.5 million tons of herring (d) 6 million tons of herring (e)1.5 million tons of herring (f) with 6 million tons of herring.

For substantially high level of stochasticity ($\sigma_1, \sigma_2, \sigma_3 > 0.9$), it is optimal to exploit capelin species myopically when the stock is low but should be conservative when the stock gets larger. At a very small stock, it is almost impossible to avoid extinction due to predation and the stochasticity, therefore, it is profitable to exploit to collapse. While at high stock level the risk of stock collapse can be avoided by being conservative. One should be more conservative when the herring stock is very high for high stochastic growth model compared to the low herring present in the ecosystem (figure 4e 4f).

5.2.2.2 Case 2. Only Capelin Species is Stochastic

In this section, we study the optimality in the capelin if the capelin stock is stochastic while other stocks have deterministic growth. The numerical results are obtained by setting the stochastic parameters for herring and cod to zero and choosing different level of stochasticity in the capelin stock.

We observe that one should be further conservative in capelin exploitation if it stochastic and the other species are deterministic (figure 5). The reasons are at least two-folds. First the stochastic stock has a high risk of extinction and therefore conservative exploitation is required. Second, by saving the stochastic capelin for cod food, we can increase the revenue from the cod harvest especially when the cod stock is small and the herring is very high.

Furthermore, we observe that substantially high level stochasticity ($\sigma_1 x = 1.5x$), one should be conservative except at a very low capelin stock. The exploitation, however, should be myopic at this stochasticity level when the cod is zero because in the absence of cod, the stock could extinction due to the stochasticity and the herring predation.
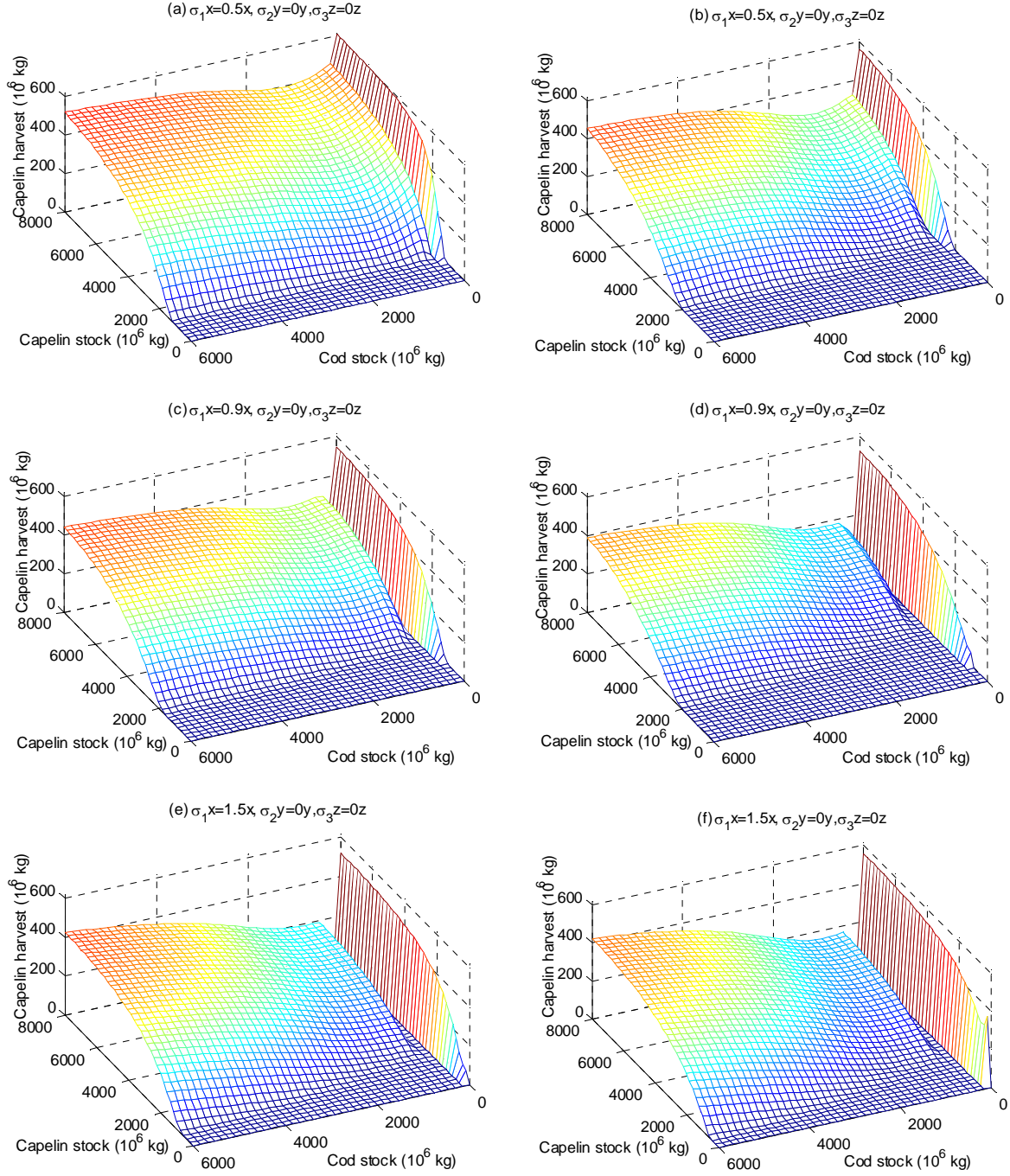
Figure 5: Effect of capelin stochasticity in exploitation policy of capelin (a) Low herring with $\sigma_1 x = 0.5x$ (b) High herring with $\sigma_1 x = 0.5x$ (c) Low herring with $\sigma_1 x = 0.9x$ (d) High herring level with $\sigma_1 x = 0.9x$) (e) Low herring with $\sigma_1 x = 1.5x$ (f) High herring with $\sigma_1 x = 1.5x$

5.2.2.3 Case 3. Only Herring Species is Stochastic

In this section, we study the optimality in the capelin if the herring stock is stochastic while other stocks, cod and capelin, have deterministic growth. The numerical results are obtained by setting the stochastic parameters for capelin and cod to zero and choosing different level of stochasticity in the

herring stock. The effect of herring stochasticity in the exploitation of capelin stock is presented in figure 6.

It is interesting to observe that there is no strong influence of the herring stochasticity in the capelin exploitation policy for relatively high degree of stochasticity ($\sigma_3 z < 0.9z$). For substantially high degree of stochasticity ($\sigma_3 z > 0.9z$), one should be myopic when the stock is small because the stochastic herring can deplete capelin to extinction when capelin is very low. As long as the capelin stock becomes large, there is no effect of herring stochasticity in the optimal exploitation policy.
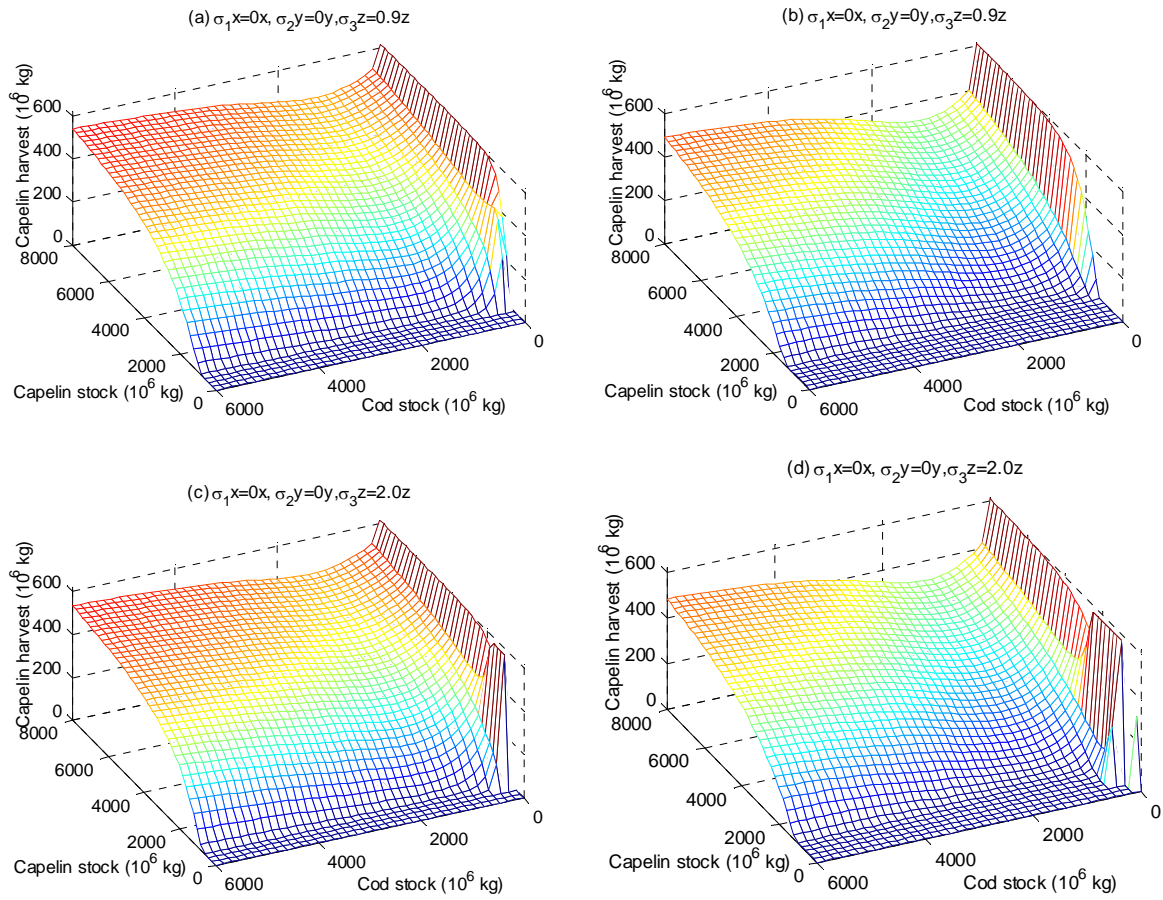

Figure 6: Effect of herring stochasticity on optimal exploitation of capelin species (a) Low herring with $\sigma_3 z = 0.9z$ (b) High herring with $\sigma_3 z = 0.9z$ (c) Low herring with $\sigma_3 z = 2z$ (d) High herring with $\sigma_3 z = 2z$.

6. Long-term sustainable optimal State (LSOS)

To sustain resources over a long time horizon, it is important to manage them optimally. Therefore, it is important to determine sustainable levels for the resource and their harvests. To determine the optimal sustainable levels or to derive LSOS, we conduct Monte Carlo simulations with implementation of optimal solutions. We simulate the system forward in time for a range of initial stock levels with all the species presents in the system[1]. The LSOS for the deterministic and stochastic models are defined as follows.

Deterministic LSOS: In the deterministic setting, the simulated paths approach a certain level over time, which is defined as the LSOS or steady state or the equilibrium level that can be achieved after a certain period of time if the stocks are managed optimally.

Stochastic LSOS: In the stochastic setting, there is no equilibrium but most paths[2] become confined to the same level after some period. This is a region around a zero drift level which is defined as the stochastic LSOS (Poudel, Sandal, Kvamsdal, & Steinsham, 2013). In other words, it can be defined as the optimal stochastic stationary state (Smith, 1986).

The LSOS and biological equilibrium (BE - equilibrium that can be achieved without taking into consideration of the harvest, a natural equilibrium level) are characterized and compared in table 3-5, both in the deterministic and stochastic models. Table 3 shows results on biological equilibrium (BE) for stock and long-term sustainable optimal states (LSOS) for stocks and their harvest in deterministic model.

Table 3: The BE and LSOS in deterministic model

Species and harvest	Type of Equilibrium	
	Bio-Equilibrium (BE)	LSO States
Capelin (10^6 kg)	2711	3025
Capelin harvest (10^6 kg)	-	228
Cod (10^6 kg)	4046	3156
Cod harvest (10^6 kg)	-	560
Herring (10^6 kg)	1844	2012

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[1]For example, LSOS for capelin is obtained with optimal feedback solution for capelin along with cod and herring present in the model.

[2] In this study, each paths are obtained by taking an average of 1000 stochastic realizations.

Biological equilibrium (BE) is lower than the LSOS in capelin and herring but higher in cod. This is an interesting but contrasting result. This is contrasting because in general, LSOS is expected to be lower compared to the biological equilibrium because of the harvest. But this can be explained as follows. Cod LSOS is smaller than the biological equilibrium i.e. when we harvest cod there is low predator for capelin and herring in the ecosystem, thus allowing it to grow to a higher equilibrium level.

We also derive biological equilibrium levels at different degree of stochasticity (table 4). It is interesting to note that for an increased stochasticity, the biological equilibrium in the capelin is increases until some level of stochasticity (for example $\sigma = 0.5$) and then decreases when the stochasticity is very high ($\sigma \geq 0.7$). The main reason is that due to stochastic downward drag, both of the predators (cod and herring) have smaller equilibrium at increased stochasticity. This means that there is less predator stock in the ecosystem to prey upon capelin, thus allowing the capelin species to grow to a higher level until it is not affected by its own stochastic drag. But at very high stochasticity the biological equilibrium decreases as a result of the stochastic downward drag of itself.

Table 4: The biological equilibrium levels under different degree of stochasticity

Species	Degree of stochasticity				
	$\sigma = 0.1$	$\sigma = 0.3$	$\sigma = 0.5$	$\sigma = 0.7$	$\sigma = 0.9$
Capelin (10^6 kg)	2744	3059	3485	2824	1988
	(354)∫	(1061)	(1776)	(1780)	(1629)
Cod (10^6 kg)	4011	3727	3102	2131	1046
	(321)	(938)	(1405)	(1641)	(1274)
Herring (10^6 kg)	1812	1545	1294	827	645
	(277)	(734)	(1060)	(1268)	(1189)

∫Figures in the parentheses represent standard deviation

The stochastic LSO levels of stocks and their exploitations and the stochastic biological equilibrium stock levels in our model are presented and compared in table 5. This is generally expected that LSOS is lower compared to the biological equilibrium due to harvest. This can be observed in the cod species, where the LSOS is smaller compared to the BE. However, we found that biological equilibrium is smaller compared to the LSOS in capelin species, which is contrasting result. But the interesting point behind this result is that due to decreased LSOS in cod species, there is less prey in the ecosystem thus allowing capelin to increase at LSOS.

Table 5: The BE and LSOS under different degree of stochasticity

Species and Harvest	Degree of stochasticity			
	σ=0.1		σ=0.3	
	Biological Equilibriums	LSO States	Biological Equilibriums	LSO States
Capelin (10^6 kg)	2744 (354)∫	3084 (351)	3059 (1061)	3357 (1036)
Capelin harvest (10^6 kg)	-	221 (50)	-	209 (138)
Cod (10^6 kg)	4011 (321)	3136 (292)	3727 (938)	3000 (815)
Cod harvest (10^6 kg)	-	547 (32)	-	443 (130)
Herring (10^6 kg)	1812 (277)	1938 (290)	1545 (734)	1662 (783)

∫Figures in the parentheses represent standard deviation

7. Evolution of stock over time

We have illustrated how different stocks evolve over time as they approach the LSOS in the prey-predator ecosystem. Several different optimal paths for various combinations of initial biomass levels of capelin, cod and herring are shown in figures 7 and 8 for deterministic and stochastic stock overtime. The paths in the stochastic setting were obtained by taking an average of thousand realizations from the same point.

7.1 Capelin evolution over time

Capelin evolution to LSOS is shown in figure 7. It can be seen that for any combination of initial biomass of capelin, cod and herring, the stock reaches to LSOS about 40 years in a deterministic stock (figure 7a) but it takes more than 60 years in the stochastic stock. However, an un-harvested stock approaches to LSOS earlier when there are low predators in the ecosystem. During the evolution process, we can observe overshooting and undershooting phenomena. Overshooting occurs when there is a fast growth of the stock and in the absence of harvest or predation, while undershooting takes place if a large stock is heavily harvested because a large stock is less costly to harvest due to harvest cost.

Overshooting is observed in a high initial capelin stock (5 million ton) with low cod and low herring in the ecosystem because of the low predation. Furthermore, it takes longer duration to reach to LSOS because it is suboptimal to exploit heavily due to market condition. But an undershooting is observed

in a high initial capelin stock (6 million ton) with high cod and high herring in the ecosystem due to both exploitation and predation. Overshooting can also be observed if there is only one predator in the ecosystem because of the low predation. A small capelin stock (0.1 million ton) with high cod and high herring leads to a slow growth towards LSOS compared to medium but un-harvested stock.

The trend of stock evolution in the stochastic and in deterministic model is similar but the stochastic stock evolution seems to take longer duration to reach to the LSOS compared to the deterministic evolution.
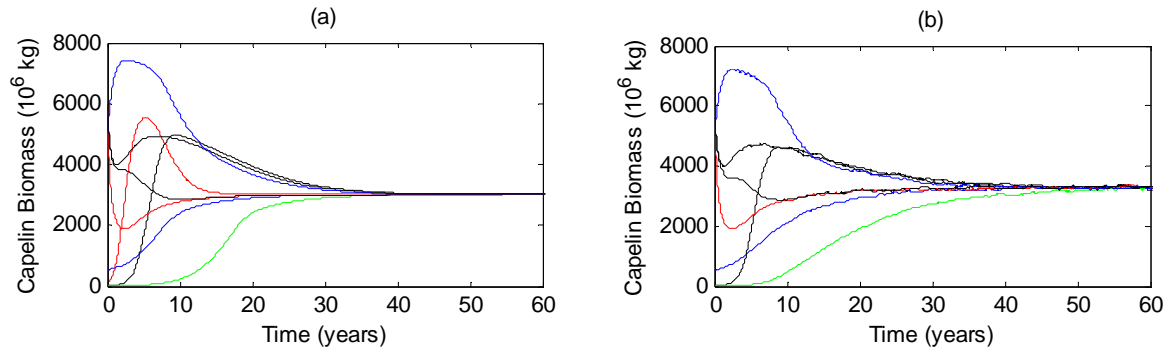

Figure7: Evolution of capelin stock over time towards LSOS from a combination of several initial conditions of capelin, cod and herring (a) Deterministic model (b) Stochastic model

7.2 Cod evolution of over time

Cod evolution is shown in figure 8. A large cod reaches to LSOS earlier than the small initial stock. No overshooting is observed in the cod evolution but an undershooting occurs when high cod, large capelin but low herring in the ecosystem. At large stock levels, both cod and capelin can be harvested extensively but due to low of herring in the ecosystem, cod might lack food and resulting competition and cannibalism until the herring and capelin increases in the ecosystem. Furthermore, with high capelin and herring stock in the ecosystem, even a small cod reaches to LSOS fast compared to only a large capelin in the ecosystem because capelin is harvested thus leading to food deficiency for cod growth. However, its own initial stock level influences the evolution directly. A small initial stock takes longer compared to the higher initial stock level. In the stochastic stock, the LSOS is lower but having other features similar to the deterministic growth model. In the stochastic stock, same initial stock take more time to reach to LSO compared to the deterministic model (figure 8b). However compared to the capelin and herring evolution, the cod stochastic evolution takes shorter period. This could be because prey growth is negatively affected by the cod stock as the stochastic cod needs more food.
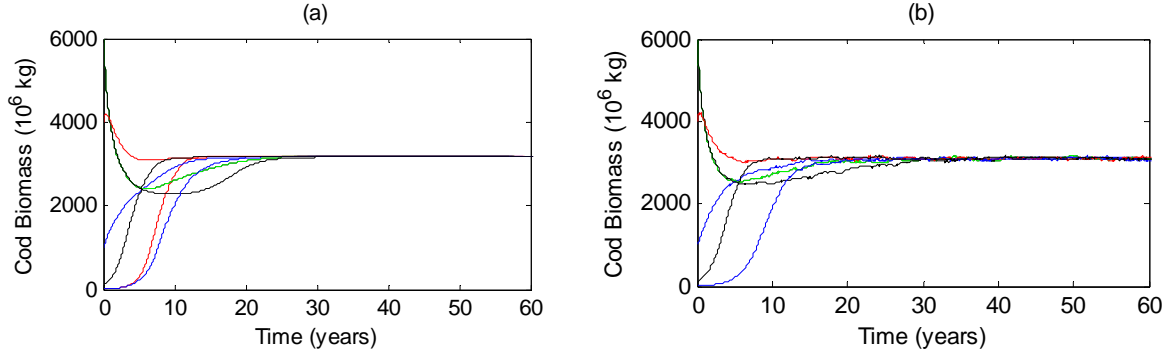
Figure 8: Evolution of cod stock over time towards LSOS from a combination of several initial conditions of capelin cod and herring (a) Deterministic model (b) Stochastic model

7.3 Paths towards LSOS

The dynamics paths from different initial combination of cod capelin and herring

Species are also shown in figure 9. Generally the tendency towards LSOS appears alike in both the deterministic model except the LSOS lower in stochastic case. The initial biomass level determines how the paths move towards to LSOS. Figure 9 shows seven different combination of initial biomass levels how they reach to LSOS in the long-term.
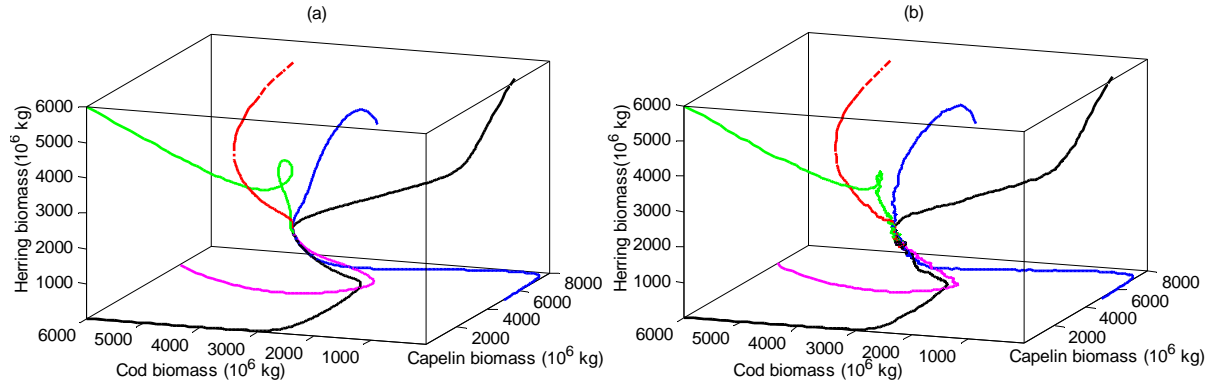

Figure 9: Dynamic paths towards LSOS in capelin, cod and herring space (a) deterministic paths (b) stochastic paths (mean of 1000 realizations)

8. Concluding Remarks

We solve a three dimensional model to suggest feedback optimal strategy for Barents Sea species. This study employed stochastic dynamic programing (SDP) technique for the numerical approximation. Since, such types of multispecies models are rare in the economic literatures due to the complexity in solving model in higher dimensions; our work is a break-through in dealing with solution to complex three dimensional model. This work is an important step towards the ultimate goal of Ecosystem based fishery management (EBFM) under uncertainty.

Since we have employed the parameters from previous work, our findings in the deterministic growth model confirms the results of (Aanestad et al., 2009). That is the dynamic programming (DP) approach can be used for empirical model in higher dimensions.

Secondly, our study added stochasticity in the model. Solving a stochastic dynamic programing problem in such higher dimension can be a very useful resource in the field of bioeconomic modeling.

Although the empirical results may not be truly representing the reality, we argue that incorporating rather than avoiding uncertainty will increase the chances of successfully achieving conservation and management goals as suggested by Halpern, Regan, Possingham, and McCarthy (2006). Our model suggests that in multispecies ecosystem the effect of the stochasticity is less pronounced when the stochasticity is relatively low. And the effect of stochasticity is higher in prey species compared to the predators. Furthermore, a very interesting result is that juvenile herring in the Barents Sea has to be considered when exploiting the capelin species, if not a higher herring may lead to capelin extinction. However, the stochasticity is not very important for the capelin exploitation. The reason may be that its influx is stochastic and the deterministic feedback model takes into account of the influx itself.

Acknowledgments

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