Rapid #: -9283484

FROM HARVESTING TO NONHARVESTING UTILITY: AN OPTIMAL CONTROL APPROACH TO SPECIES CONSERVATION

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ABSTRACT. The purpose of this paper is to retrace the evolution of mathematical models focused on relation and interaction between economic growth, sustainable development, and natural environment conservation. First, generic defensive expenditures are introduced into a common-property harvesting model in order to favor the species growth. Second, a transition model comprising both harvesting and nonharvesting values of wildlife biological species emerges. The latter gives rise to a group of purely nonharvesting models where anthropic activities and economic growth may have positive or negative impact on the natural evolution of wildlife species. Several scholars have proved that optimal strategies that are relatively good for harvesting purposes are not simply "transferrable" to the context of conservation of wildlife biological species with no harvesting value. In addition, the existence of optimal policies for long-term conservation of all biological species (with or without harvesting value) cannot be guaranteed without having relatively large species populations at the initial time. Therefore, all such strategies are incapable of enhancing the scarce populations of endangered species and, therefore, cannot save these species from eventual (local) extinction. As an alternative, policymakers may soon be compelled to design and implement short-term defensive actions aimed at recovery and enhancement of endangered wildlife species.

KEY WORDS: Nonharvesting utility, optimal control, species conservation, bioeconomic model.

1. Introduction. Among many factors contributing to biodiversity loss,[1] the majority of scientists usually identify overexploitation, deforestation, invasive species, air and water pollution, soil contamination, and climate change. Despite significant efforts of human society (such as ecosystem stewardship, new common-property ecological policies, legislative initiatives aimed at habitat restoration, increase of protected terrestrial, coastal and maritime areas, etc.), biodiversity continues to decline worldwide (see a thorough analysis by Hill et al. [2013] and numerous references therein). Therefore, it is fair to say that global commitments made in 2002, through the Convention on Biological Diversity (CBD), have not been met yet.

Eppink and Van Den Bergh [2007] had summarized the key features of four basic categories of models that integrate the strategies aimed at species conservation with economic theories. These models help to define strategies for optimal and/or

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sustainable *harvesting*. Such strategies contribute to the economic development by ensuring the preservation of natural stock and stability of forthcoming profits for future generations.

A pioneering work of Gordon [1954] provided the framework for harvest modeling of common-property renewable resources (such as fisheries, for example), and the fundamental principles of sustainable bioeconomics were further developed and refined by Clark [1976] using mathematical modeling. During the last decades, there has been a notable upsurge in research focusing on the interaction between the conservation of natural environment, sustainable development, and economic growth. However, besides direct consumption of renewable resources (i.e., harvesting), there have been identified other factors that may provoke drastic reductions and, possibly, extinction of wild species populations.[2]

Several authors tried to predict the long-run consequences of pollution and natural resource scarcity using macroeconomic growth models. Such models consider man-made capital, natural stock and emissions as necessary factors of production, where pollution directly affects both the growth of renewable resources and social welfare (see, e.g., Tahvonen and Kuuluvainen [1991, 1993], Comolli [2006]).

Alternatively, Swanson [1994] had proposed a constructive adjustment to classical harvesting models of Gordon–Schaefer type (thoroughly described by Clark [1976], Anderson and Seijo [2011], among other similar texts) where harvesting effort is traditionally modeled by a control variable. He had explicitly included another control variable that expresses the allocation of resources required for a species' survival. The latter can be regarded as an initial endeavor to amend the species biological dynamics with defensive actions aimed at species protection.

Swanson's idea was then explored in a broader sense by Alexander [2000] who suggested a new form of the objective function by including both consumptive (i.e., harvesting) and *nonconsumptive*[3] (i.e., nonharvesting) values of the biological species. This skillful approach helped to delineate and to comprehend the population dynamics of endangered species with both consumptive and nonconsumptive values, as well as to propose suitable conservation policies aimed at maintaining sustainability and bioeconomic viability of these species.

On the other hand, many wildlife species that are endangered by negative side-effects of human activity (such as urbanization, pollution, habitat loss, etc.) do not have any harvest value. Several scholars (see, e.g., Madan and Madan [2009, p. 133] and Aggarwal [2010]) assert that most of the species that are currently threatened with extinction, are not regarded as "food species," that is, they are not directly consumed by people. However, their biomass is (indirectly) turned into human food when their habitat is converted into pasture, cropland, and orchards. Dumont [2012] points out that the humankind is facing a considerable reduction of the surface area of wild biodiverse land by the year 2050 because of steadily growing global human population. One may argue that the presence

of wild species is not crucially essential for human survivorship. However, ongoing loss of wilderness may irrevocably reduce an important source of human wellbeing.

To study the evolution of wild endangered species, Alexander and Shields [2003] had proposed a single-species *nonharvesting* dynamical model where the control variable expresses an index of the quantity of land resources which are considered vital for the species survival and reproduction. The latter reflects a defensive expenditure undertaken by society (or resource owner) in order to endorse the conservation of the species natural habitat. This nonharvesting model does not contemplate that human activity and aggregated production may have negative impact on the natural evolution of species population.

On the other hand, studies conducted by Antoci et al. [2005a,b] have clearly demonstrated that stability properties of the natural dynamics of biological species can be altered by human actions (both negatively and positively). Although based on linear dynamics of two interacting species, these studies had revealed an interesting fact. Namely, when the equilibrium levels of the species are sufficiently high, all properties of local stability will be preserved if the natural biological dynamics (that is, without human intervention) becomes amended with additional economic and ecological features (such as, negative impact of aggregated production and positive impact of defensive expenditures). Moreover, other authors (see Campo-Duarte and Vasilieva [2011], Cruz-Rivera and Vasilieva [2013, 2015]) have arrived to the same outcome while analyzing a single-species nonlinear model with Gompertz-type and logistic population growth, respectively.

All previously mentioned models have been formulated for *infinite time horizon* and are aimed at design of *long-term* policies for species conservation that must prevent future collapses of species populations, which are still relatively abundant. Their meticulous analysis[4] had disclosed that the initial abundance of the species population is crucial for existence of sustainable policies for long-term conservation planning. In other words, such long-term policies *can maintain* the species population at some desirable level and thus prevent its local extinction. However, the same long-term policies *cannot enhance* the scarce populations of endangered species. Therefore, a different short-term policy will be needed to move up the species population to some desirably high target level. When such target level is achieved, the policymakers may safely apply the long-term policy aimed at maintaining the species abundance.

Regarding this argument, an emergent study conducted by Cruz-Rivera et al. [2013] has revealed the existence of *short-term* decision policies capable of enhancing the population of single endangered species within a *finite* period time. However, such policies do require a relatively high level of initial capital to be quickly spent for gradual regeneration of the species. Thus, an aggregated production (that generates an additional capital surplus) may also have a positive effect on species conservation.

2. A brief survey of harvesting models. A customary framework of bio-economic models with harvesting of so-called Schaefer–Gordon type can be found in many textbooks (see, e.g., Clark [1976], Anderson and Seijo [2011], among others). Such type of models fits into the following mathematical formulation:

$$(1) \qquad \frac{d\boldsymbol{x}}{dt} = \boldsymbol{F}(\mathbf{x}) - H(\boldsymbol{x}, h), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0,$$

where $\boldsymbol{x}$ can be scalar ($\boldsymbol{x} = x$) or vector ($\boldsymbol{x} = (x_1, x_2, \ldots, x_n)'$) quantity that stands for the biomass of renewable natural stock consisting of one or more biological species with harvesting value, and $\boldsymbol{x}_0$ denotes an initial stock level (constant). Function $\boldsymbol{F} = (F_1, F_2, \ldots, F_n)$ describes the biological recruitment or growth of the species (and possibly an interaction between species in the ecosystem), and $H(\boldsymbol{x}, h)$ is a function of harvest with $h = h(t)$ expressing a stock removal (or harvesting effort) at time t. Variable $h(t)$ is usually exogenous and should be chosen by the social planner in order to maximize the present values of its future net revenues (or utility) $U(\boldsymbol{x}, h)$:

$$(2) \qquad \max_{h(t) \geq 0} \int_0^\infty e^{-\rho t} U\left(\boldsymbol{x}(t), h(t)\right) dt \quad \text{s.t. (1)},$$

where $\rho > 0$ denotes the instantaneous discount rate. The above utility is understood as "consumptive" since it expresses the net revenue obtained from direct consumption (or harvesting) of the biological resource:

$$U(\boldsymbol{x}, h) = [\text{total income}] - [\text{total cost}].$$

In case of single species, $\boldsymbol{x} = x$, scalar function F is usually supposed to be a hump-shaped function satisfying the following conditions:

$$\begin{aligned} F(x) &> 0 \quad \text{for} \quad 0 < x < K, \\ F(0) = F(K) &= \quad 0, \\ F''(x) &< 0 \quad \text{for} \quad 0 < x < K, \end{aligned}$$

where $K > 0$ represents the maximal stock level sustainable by the environment which is usually referred to as "carrying capacity." This term reflects the fact that without harvesting ($H \equiv 0$) the natural stock x is bounded from above:

$$\lim_{t \to \infty} x(t) = K.$$

A classical example of F is Verhulst's logistic function:

$$F(x) = rx \left(1 - \frac{x}{K} \right)$$

(as well as its numerous modifications), where $r > 0$ stands for intrinsic growth rate of the stock. Other useful examples are Gompertz growth function

$$F(x) = rx \ln \left(\frac{K}{x} \right)$$

(see, e.g., Nobile et al. [1982]) and

$$F(x) = \frac{rx(K - x)}{K + (r/a)x}$$

attributed to Smith [1963].

Optimal solutions to the problem (2) are usually related to biological or economical equilibria of the system (1) (in the sense that $\frac{d\boldsymbol{x}}{dt} = 0$) and can be obtained using an appropriate variant of so-called "golden rule."[5] In fishery models of the type (1)–(2), the "golden rule" accounts for unit harvesting cost (which is a decreasing function of the stock size x) and provides a relationship that specifies an "optimal" equilibrium biomass level x^*. Therefore, the control variable of fishing effort, $h^*(t)$, is chosen in order to move the initial stock x_0 toward x^*. For more detail, the reader may refer to the book by Clark [2010].

From the biological standpoint, it is optimal to maintain the stock size at the point of its maximum growth rate x_{MSY} where $F'(x_{MSY}) = 0$ by choosing an appropriate harvesting effort h_{MSY} and to harvest only the individuals which otherwise would be added to the population, thus allowing the population to be productive indefinitely. This approach is known as *maximum sustainable yield* (MSY, cf. the subscript notation) and it usually neglects the harvesting costs.

When harvesting costs are essential, it is optimal to choose the harvesting effort h_{MEY} and the corresponding level of stock x_{MEY} for which the total utility $U(x_{MEY}, h_{MEY})$ (i.e., the difference between the gross revenue and the cost of harvesting) is maximized. This approach is known as *maximum economic yield* (MEY, cf. the subscript notation) and proffers the largest economically profitable harvesting rate that can be sustained indefinitely. Both x_{MSY} and x_{MEY} equilibria are rather sensitive with respect to initial stock level, the population growth rates, and the discount rates. Therefore, the underlying stationary strategies h_{MSY} and h_{MEY} cannot be sustained indefinitely in practice and may provoke the stock overexploitation and further extermination.

In particular, Clark [1976, 2010] had pointed out that low growth rate of biological stock with respect to discount rate (that is, $\rho > r > 0$) and/or gradual decay of the carrying capacity K will inevitably lead to stock extermination in common-property models with harvesting. In this situation, there are two possible scenarios, namely:

(i) *Stock extermination or resource mining.* If a biological resource is unable to generate a competitive return, a social planner may choose so-called "optimal extinction" (see more details provided by Clark [1973]). This option is considered *economically* viable when
    (a) the discount rate sufficiently exceeds the maximum reproductive potential of the population, and
    (b) an immediate profit can be made from harvesting the last remaining stock. Swanson [1994] provides a valid example concerned with deforestation of tropical hardwood forests. He points out: "These trees represent substantial amount of standing value, but they have very low growth potential. Thus, it is economically rational to 'cash in' the hardwoods and invest the return in other, more productive assets."

(ii) *Avoiding extermination.* If the natural growth rate of biological stock is decelerated by limited base resources required for species survival (such as land, water, etc.) then social planner should provide them even by incurring an additional cost. This option (also suggested by Swanson [1994] in his attempt to avoid species extinction) calls for adjustments to the traditional harvesting model (1)–(2), namely:

$$(3) \qquad \max_{h(t), R(t) \geq 0} \int_0^\infty e^{-\rho t} \left[U\left(x(t), h(t)\right) - \rho C_R R(t) \right] dt$$

$$(4) \qquad \text{s.t.} \quad \frac{dx}{dt} = F(x, R) - H(x, h), \quad x(0) = x_0,$$

where C_R is the price of a unit of base resource $R = R(t)$ and the latter stand for an additional control variable. These alterations in the model indicate that, given sufficient resources for the species survival and reproduction, the biological stock should give a competitive return even after discounting the total cost of allocation of additional base resource (expressed by the term $\rho C_R R(t)$ in (3)). Swanson [1994] introduces an additional first-order optimality condition for R to be satisfied in the target stationary stock level. This condition is conceptually similar to "golden rule" equilibrium and requires that base resources be allocated to a particular species only in proportion to its ability to generate a competitive return and to provide an additional surplus which compensates the investments in its sustenance.

It is worthwhile to note that common-property models with harvesting described by (1)–(2) are derived under the implicit assumption that biological resources are naturally "free goods" and do not require (external) investments. However, Swanson's model (3)–(4) does require societal investment R in allocation of the ancillary resources to potentially competitive biological assets. If the species is of high or medium value and has a significant rate of growth, then an increase in its value will make the investment in its base resources economically viable. On the other hand, the biological species with high value and low growth rates should be subject to disinvestment programs (such as resource mining, asset replacement, and conversion) and will be driven toward local extinction under common-property regimes.

3. Harvesting and nonharvesting utility.   Effectively, the idea of Swanson [1994] can be regarded as an initial endeavor to amend the species biological dynamics with defensive actions aimed at species protection. This idea was then explored in a broader sense by Alexander [2000] who suggested a new form of the objective function by including both consumptive (i.e., harvesting) and nonconsumptive (i.e., nonharvesting) values of the biological species. Alexander's original term "nonconsumptive utility" can be perceived as "nonconsumptive use values of endangered species" (in the sense of Boyle and Bishop [1987]); thus, it principally refers to species existence values.

Traditional harvesting models fail to account for existence values of endangered biological species even though some particular species may have more significant existence values than their consumptive (or harvesting) valuation. Using the African elephant (*Loxodonta africanus*) as an example, Alexander had proposed another bioeconomic model that accounts for both harvesting and nonharvesting utility of this particular species and highlights the incentives faced by economic agents who make decisions affecting endangered species:

$$(5) \qquad \max_{h(t) \geq 0} \int_0^\infty e^{-\rho t} \left[U\left(x(t), h(t)\right) - C_R R x + U_{nh}(x) \right] dt$$

$$(6) \qquad \text{s.t.} \quad \frac{dx}{dt} = F(x) - H(x, h), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0.$$

In the above model, $U(x, h)$ expresses the utility of harvesting (ivory and nonivory products, revenues from safari hunting minus the total cost of harvesting), C_R is

the unit value of land resources used by elephants, Rx is quantity of land resources used by elephants as a constant proportion of their population, and $U_{nh}(x)$ stands for *nonharvesting* utility. The latter can be displayed as

(7) $$U_{nh}(x) = P_T T(x) + N(x),$$

where P_T is the unit price of one tourist day, $T(x)$ is tourist days as function of population, and $N(x)$ is the nonmarket existence value of elephants as function of population.[6] The nonharvesting utility (7) displays revenues obtained from tourism and contributes to people's awareness that this species exists (for both harvest and tourism purposes).

It is worthwhile to note that in contrast to Swanson's model (3)–(4), land resources allocated to elephants are not expressed as a control variable in the model (5)–(6). Instead, Alexander suggests that if the correct incentives are put in place in society, further transfers of land resources from alternative uses may naturally arise through the market.

Standard application of the optimal control techniques to the model (5)–(6) results in a relationship to be fulfilled by the stationary stock and harvest levels that equates the discount rate with the marginal productivity of the biological asset in both harvesting and nonharvesting sense. Essentially, this relationship can be regarded as a hybrid "harvesting/nonharvesting golden rule" variant since it states that both harvest and tourism returns should cover the costs of base resources allocated to the species for its sustenance.

Inclusion of nonharvesting values of a wildlife resource illustrates social values and benefits of maintaining that resource. Such benefits become particularly important for studying endangered species with no harvest value.

Many scholars agree that the principal cause of (local) extinction of wildlife species with no harvesting value is merely incidental. In other words, local extinction is frequently provoked by habitat reduction due to urbanization, farming, and other anthropic activities that are not directly intended to harm the species. The obvious questions that arise are:

— Do such wildlife species have a chance to avoid eventual extinction?

— Can human external intervention contribute to conservation of such species?

In the attempt to answer these questions, Alexander and Shields [2003] had proposed a purely *nonharvesting* bioeconomic model for conservation of the New Zealand yellow-eyed penguin (*Megadyptes antipodes*). This model is significantly different from those revised above, and its objective functional represents the net

returns to society derived from tourism, in other word, there is no harvesting value involved. This model is formulated as follows:

$$(8) \qquad \max_{L(t) \geq 0} \int_0^\infty e^{-\rho t} U_{tour}\left(x(t), L(t)\right) dt$$

$$(9) \qquad \text{s.t.} \quad \frac{dx}{dt} = F(x, L), \quad x(0) = x_0,$$

where $U_{tour}(x, L)$ expresses the *nonharvesting* utility obtained from tourism business[7], x is the population of yellow-eyed penguins, and L represents an index of the quality of land resources used by the penguins (control variable). The resource owner may reallocate his land resources $L(t)$ available for penguins, and the latter will be reflected in the population growth, for example,

$$F(x, L) = rx\left(1 - \frac{x}{K \cdot L}\right)$$

assuming logistic growth of the species. Within the mathematical framework of the optimal control theory, Alexander and Shields [2003] had derived a purely *nonharvesting* variant of "golden rule." Their further argument, based on this relationship, results in a rather logical conclusion. Namely, if the population is relatively small (less than a half of current carrying capacity) and the intrinsic growth rate of the species is below the discount rate (i.e., $r < \rho$), then it will be optimal to allocate more land resources to the species. On the other hand, if the population is relatively small and the intrinsic growth rate of the species exceeds the discount rate (i.e., $r > \rho$), then it will be optimal to decrease the land allocated to the species up to some "sustainable" level. This implies that the initial population density was too low on the land initially available, and that by increasing that density (that is, by reducing the carrying capacity) one may guarantee an economically viable income from tourism. Impartially speaking, a tourist may choose not to pay for a visit when the probability of sighting is small, but once that probability becomes close to certainty, additional population is unlikely to generate additional visits. Therefore, there must be an "economically sustainable" level of population and an "economically viable" amount of land resources should be allocated to the species.

4. Conservation policies under economic growth. In a broader sense, the control variable L in the model (8)–(9) can be viewed as a defensive expenditure of the society aimed at the conservation of the natural habitat of the species. On the other hand, there are many wild species with rather low intrinsic growth rate

that may require not only the land but other resources (such as additional water, food, special conditions for breeding, etc.) for their survival and sustainable growth. In addition, the model (8)–(9) does not explicitly include any impact (positive or negative) that anthropic activity and aggregated production may have on the natural evolution of species population.

Another study conducted by Antoci et al. [2005a,b] claims that defensive actions may deeply alter the natural ecological dynamics and modify its equilibria. In support of this argument, the authors had proposed another variant of bioeconomic model with two interacting species $\boldsymbol{x} = (x_1, x_2)$ and nonharvesting utility $\hat{U}_{nh}$ which is stated as follows:

$$(10) \qquad \max_{\substack{c(t) > 0 \\ d(t) \geq 0}} \int_0^\infty e^{-\rho t} \hat{U}_{nh}\left(\boldsymbol{x}(t), c(t)\right) dt$$

$$(11) \qquad \text{s.t.} \quad \begin{cases} \frac{d\boldsymbol{x}}{dt} = \boldsymbol{F}(\boldsymbol{x}) - k^\alpha(t)\boldsymbol{\epsilon} + d^\mu(t)\boldsymbol{\sigma}, \ \ \boldsymbol{x}(0) = \boldsymbol{x}_0 > 0, \\[2mm] \frac{dk}{dt} = pk^\alpha(t) - c(t) - d(t), \qquad k(0) = k_0 > 0, \end{cases}$$

where $\boldsymbol{F}(\boldsymbol{x}) = (F_1(x_1, x_2), F_2(x_1, x_2))\prime$ is a linear affine vector function of $(x_1, x_2)\prime$ while the positive components of $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2)\prime$ and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2)\prime$ measure, respectively, the negative impact of aggregated production k and the positive impact of generic defensive expenditure d on the target populations x_1 and x_2. The second equation in (11) describes the accumulation of capital $k(t)$ which is considered as a sole product of global economy. The capital output can be used for reinvestment (expressed by the production function $pk^\alpha(t)$, $0 < \alpha < 1$ of Cobb–Douglas type), consumption $c(t)$, and defensive expenditures $d(t)$.

Economic activity modeled by the second equation in (11) induces obvious changes in the natural biological dynamics of the species given by the first equation in (11). Namely, the pollution, habitat reduction, and other consequences attributed to aggregated production $k^\alpha(t)$, will have *negative* effect for both populations $x_1(t)$ and $x_2(t)$. On the other hand, defensive expenditures $d(t)$ will constitute *positive* effect for $x_1(t)$ and $x_2(t)$. Thus, the natural biological dynamics becomes amended with both negative and positive effects of anthropic activities.

It is worthwhile to note that parameter $\mu \in (0, 1)$ has a specific role in formalization of the model. It indicates that the positive effect of defensive investment on the specie evolution is not directly proportional to the population growth. The latter implies that eventual extra spending on species conservation (i.e., an additional

increase in $d(t)$) may reduce the positiveness of such action since the population growth is subject to carrying capacity limitations of the environment.

It is assumed that there is a representative agent in the economy whose welfare $\hat{U}_{nh}(\boldsymbol{x}(t), c(t))$ depends, in each instant of time t, on consumption $c(t)$ and on the present amounts of both species $\boldsymbol{x} = (x_1, x_2)$. It should be emphasized that these species have no harvest value. In the paper by Antoci et al. [2005a], two alternative forms of $\hat{U}_{nh}(\boldsymbol{x}(t), c(t))$ are proposed:

(12) $$U_1(x_1, x_2, c) = q_1 x_1 + q_2 x_2 + q \ln c,$$

(13) $$U_2(x_1, x_2, c) = q_1 \ln x_1 + q_2 \ln x_2 + q \ln c,$$

where q_1, q_2, and q are strictly positive weight parameters. These utility functions U_1 and U_2 clearly reflect the priorities of decision-making. Namely, function U_1 expresses that the *nonharvesting utility* of the species is directly proportional to the species abundance and has constant utility gain, while the utility of consumption (logarithmic term) has decreasing utility gain. Thus, the maximization problem (10)–(11) with function (12) may yield an optimal policy $(c^*(t), d^*(t))$ which is not adverse to species' extinction since $x_1 = 0$ and/or $x_2 = 0$ can be paid off by an aggregated consumption level.

Alternatively, the maximization problem (10)–(11) with function (13) requires that $x_1 > 0, x_2 > 0$ since otherwise the representative agent may suffer an infinite loss of utility when either $x_1 \to 0^+$ or $x_2 \to 0^+$. Therefore, this definition of utility function should yield an optimal policy $(c^*(t), d^*(t))$ that favors the conservation of the species.

The analysis of the model (10)–(11) performed by Antoci et al. [2005a] suggested that if the agents get constant marginal utility from x_1 and x_2 (i.e., function (12) is applied), then the stability features of natural equilibrium (that is, of the ecosystem $d\boldsymbol{x}/dt = \boldsymbol{F}(\boldsymbol{x})$ with no human intervention) will be preserved in the integrated bioeconomic system regardless of both populations' initial or fixed-point levels. In other words, if the fixed point $\bar{\boldsymbol{x}}$ of the natural system ($\boldsymbol{F}(\bar{\boldsymbol{x}}) = 0$) is a repeller (or saddle point, or attractor) then under optimal policy $(c^*(t), d^*(t))$ it will continue being a repeller (or saddle point, or attractor) of the bioeconomic system (11). The latter implies that there is no policy capable of converting an "initial" repeller into a "future" attractor when the agents do not care for the species protection.

On the contrary, if the agents suffer an infinite loss of utility from species extinction (i.e., function (13) is applied), the defensive expenditures may alter the natural evolutionary dynamics of the interacting species. A study conducted by Antoci et al. [2005b] revealed the existence of the optimal policy $(c^*(t), d^*(t))$ under which the stability can be achieved in the bioeconomic system (11) even for "naturally unstable" interacting populations.

The results of Antoci et al. [2005a,b] comply with a study focused on single-species model with Gompertz population growth under human intervention (see Campo-Duarte and Vasilieva [2011]). This bioeconomic model fits into the formal framework of (10)–(11) when instead of two interacting species x_1, x_2 a single one $(0 \leq x \leq K$) is considered, and its biological growth is described by the so-called Gompertz function $F(x) = rx \ln(K/x)$ where $K > 0$ stands for carrying capacity of the environment. Since $x = K$ is an attractor of natural dynamics with Gompertz population growth, there exists an optimal policy $(c^*(t), d^*(t))$ capable of preserving stability features in the bioeconomic system (11) with either $U_1(x, c) = q_1 x + q \ln c$ or $U_2(x, c) = q_1 \ln x + q \ln c$ substituted in (10). However, the latter is possible only if the population value in the fixed point $\bar{x}$ of bioeconomic system (11) is relatively high, namely, $(K/e) < \bar{x} \leq K$, and if the initial population $x(0) = x_0$ is sufficiently close to $\bar{x}$.

Another pertinent contribution to this strand of research is a single-species bioeconomic model with logistic population growth and nonharvesting utility of the form (12) which was thoroughly analyzed by Cruz-Rivera and Vasilieva [2013, 2015]. This bioeconomic model also fits into the formal framework of (10)–(11) when instead of two interacting species x_1, x_2 a single one $(0 \leq x \leq K)$ is considered, and its biological growth is described by logistic function $F(x) = rx(1 - x/K)$ where $K > 0$ denotes the carrying capacity, while the nonharvesting utility $\hat{U}_{nh}$ is expresses by (12) and (13), respectively.

Besides confirming the general trend on preservation of stability features (claimed in preceding works by Antoci et al. [2005a,b], Campo-Duarte and Vasilieva [2011]), this study also addressed an important question: *Can the defensive expenditures mitigate the negative impact of aggregated production on the species population and to what extent?*

To answer this question, the underlying features of logistic dynamics (such as its quadratic nature and symmetry of the fixed points with respect to the axis $x = K/2$) were rather beneficial, and helped to disclose and visualize some essential changes in stability properties of the "amended" ecological dynamics, that is, natural population dynamics modified by human intervention. It was established that such changes principally depend on the model's parameters, including two control variables c and d.

The range of damage possibly caused by the aggregated production (term $-\epsilon k^\alpha$ in the first equation of (11)) to the species population can be estimated by varying $\epsilon > 0$ while holding other parameters of the model constant. This procedure yields two ϵ-depended fixed points $x_-^*(\epsilon)$ and $x_+^*(\epsilon)$ satisfying the following relationships:

$$0 \leq x_-^*(\epsilon) \leq \frac{K}{2} \leq x_+^*(\epsilon) \leq K, \quad x_-^*(\epsilon) + x_+^*(\epsilon) = K.$$

In addition, it was possible to define particular values of the parameter $\epsilon > 0$ such that

$$(14) \qquad \epsilon_* : x_-^*(\epsilon_*) = 0, \quad x_+^*(\epsilon_*) = K \quad \text{(logistic case)},$$

$$(15) \qquad \epsilon_0 : x_-^*(\epsilon_0) = x_+^*(\epsilon_0) = \frac{K}{2} \quad \text{(degenerate case)}.$$

From the ecological standpoint, ϵ_* should be viewed as *opportune reference value* since it describes an "ideal situation," when both positive and negative effects of aggregated production equiponderate each other and thus make no alteration in the natural equilibrium of the dynamic system. Alternatively, ϵ_0 should be referred to as *critical reference value* since it depicts an "unhealthy situation" when the equilibrium of the system becomes unreachable in infinite time t. Both *reference values* ϵ_* and ϵ_0 can be determined outside of the model in accordance with other constant parameters, and then provide the decision-maker with some useful insights regarding to actual ecological situation described by the model (10)–(11). Logically, if ϵ from the first equation of (10) stands close to ϵ_*, then the ecological situation is characterized as "good"; otherwise, if ϵ stands close to ϵ_0, the situation is characterized as "bad."

Moreover, if $\epsilon \neq \epsilon_0$ and if the initial population $x(0) = x_0$ is situated closer to $x_+^*(\epsilon)$ than to $x_-^*(\epsilon)K$ (that is, $x_0 > K/2$) there exists an optimal policy $(c^*(t), d^*(t))$ capable of guaranteeing stability of the bioeconomic system (11) even when the ecological situation is regarded as "bad" (that is, if $\epsilon > 0$ given in the model is closer to ϵ_0 than to ϵ_*). Under such policy, the species population trajectory will eventually reach the fixed point of maximal possible abundance of the species ($K/2 < x_+^*(\epsilon) \leq K$).

It is worthwhile to note that all results described so far deal with design of *long-term optimal policies* capable of providing stability to bioeconomic systems when $t \to \infty$ and when the initial populations are proximal enough to their stationary (or fixed-point) values. Such policies are rather useful if the primary goal is *to maintain* the species population(s) at some "desired" level. However, these policies are incapable of enhancing (in finite or even infinite time) the initially scarce populations and, therefore, they cannot save the species from eventual extinction.

5. Conservation policies aimed at enhancement of scarce populations.
In the previous section, it was shown that initial abundance of the species population is indispensable for existence of long-term conservation policies. How crucial is it in the case of short-term planning? To answer this question, a finite-time variant

of bioeconomic model (10)–(11) was proposed by Cruz-Rivera et al. [2013] for a single-species population with logistic biological growth, namely:

$$(16) \qquad \max_{\substack{c(t) > 0 \\ d(t) \geq 0}} \int_0^T U_1\left(x(t), c(t)\right) dt + x(T) + k(T)$$

$$(17) \qquad \text{s.t.} \quad \begin{cases} \frac{dx}{dt} = F(x) - \epsilon k^\alpha(t) + \epsilon d^\mu(t), & x(0) = x_0 > 0, \\ \\ \frac{dk}{dt} = pk^\alpha(t) - c(t) - d(t), & k(0) = k_0 > 0, \end{cases}$$

where $F(x)$ has logistic form, other entries of (17) are the same as in the system (11), and $U_1(x, c) = q_1 x + q \ln c$ expresses the preferences of decision-making that are not adverse to local species extinction (rather cruel but more realistic case). Here, the objective functional (16) is different from the previous one (10) and targets not only to maximize the overall utility within the finite period of time $[0, T]$ but also to enhance the terminal values of both state variables—species population $x(T)$ and capital $k(T)$. In addition, the discount factor $e^{-\rho t}$ is suppressed in (16) in order to highlight the short-term planning and patience of decision-making.[8]

Among several scenarios meticulously considered by Cruz-Rivera et al. [2013], there is one of particular interest and it deserves to be mentioned here. This scenario deals with the design of optimal policies aimed at the enhancement of initially scarce populations (i.e., $x(0) = x_0$ is rather low) when the initial level of production $k(0) = k_0$ is reasonably high.

Under this scenario, there are sufficient capital resources k_0 to be spent immediately for defensive expenditures $d(t)$ in order to obtain a significant increase in the species population $x(t)$ by the final time T. Given the scarcity of biological species at initial time, the social planner must implement the defensive policy from the very beginning and continue to spend increasingly up to the midpoint $(T/2)$ of the time lapse. At the same time, the optimal consumption $c(t)$ and capital accumulation $k(t)$ should be maintained strictly increasing within $[0, T]$ in order to guarantee maximization of the integral part and second terminal-value term of the objective (16).[9] The optimal decision policy $(c^*(t), d^*(t))$ derived by Cruz-Rivera et al. [2013] complies with the maximization criterion (16) in the sense that:

- it guarantees a significant increase of final states $x^*(T), k^*(T)$ and thus impedes the species extinction;
- it ensures the growth of utility (expressed by the integral part) since $U(x, c)$ is an increasing function of x and c.

Thus, high initial level of production k_0 plays a significant and *notably positive* role for further enhancement of initially scarce populations. However, initial abundance of capital resources is only the necessary (but not sufficient) condition for existence of the optimal policy $(c^*(t), d^*(t))$. The length of planning horizon $T > 0$ also plays an essential role. It is worthwhile to note that the optimal policy $(c^*(t), d^*(t))$ designed for $t \in [0, T]$ cannot be adjusted to shorter or longer time intervals (such as $[0, T/2]$ or $[0, 2T]$, for example) by mere "truncation" or "extension."

Numerical experiments held by Cruz-Rivera et al. [2013] disclosed that the variation of the length of the planning period $[0, T]$ has significant impact on decision policies and the underlying value of the objective (16). Namely, shorter periods are more "expensive" since they result in the decline of capital accumulation during the whole period while ensuring steady increase in consumption and providing moderate rise of the species population. Alternatively, longer planning periods guarantee strictly increasing capital accumulation together with significant enhancement of the species population and without neglecting the consumption. Thus, there is coherence between the planning horizon T and the preferences of decision-making.

Effectively, to achieve faster the highest level of consumption while trying to enhance the species population, one may apply a shorter time decision policy that disregards the capital accumulation. Conversely, a longer time decision policy should assure significant increase in both state variables (species population x and capital k) on account of lesser overall consumption. Given the above argument, it seems reasonable to implement consecutively a series of decision policies for $[T_0, T_1] \cup [T_1, T_2] \cup \cdots \cup [T_{N-1}, T_N]$ where $T_0 = 0$ and $(x^*_{j+1}(t), k^*_{j+1}(t), c^*_{j+1}(t), d^*_{j+1}(t))$ are defined as solutions of the optimization problem (16) subject to (17) for $t \in [T_j, T_{j+1}]$, $j = 0, \ldots, N-1$ with aptly assigned initial conditions:

$$(x_1(T_0), k_1(T_0)) = (x_0, k_0) \text{ when } t \in [T_0, T_1] \text{ and } j = 0,$$

$$(x_{j+1}(T_j), k_{j+1}(T_j)) = \big(x^*_j(T_j), k^*_j(T_j)\big) \text{ when } t \in [T_j, T_{j+1}] \text{ and } j = 1, \ldots, N-1.$$

Such "parceled" planning may come up rather advantageous due to its flexibility and adjustment to different priorities of decision-making within each subinterval $[T_j, T_{j+1}]$.

Finally, it would be fair to assert that economic growth has "twofold" effect on the species evolution. On the one hand, an aggregated production may reduce the species population to dangerous levels (leading to local extinction) if the social planner does not spend at all (or spends too little) on defensive measures, preferring to disburse the whole capital surplus (or a great part of it) solely for consumption.

On the other hand, an aggregated production may provide additional surplus which, being destined for defensive expenditures, may significantly enhance the species population. In this case, the social planner should find and implement an optimal decision policy $(c^*(t), d^*(t))$ which depends on three factors:

– the initial species population x_0,

– the initial capital k_0, and

– the length of policy implementation (that is, final time T),

while other parameters of the model (16)–(17) are kept unchanged.

In mathematical terminology, if either one or both initial values x_0, k_0 are too small for a chosen final time T, the *optimality system*[10] may simply fail to have *feasible* solution in bioeconomic sense, i.e., such that

$$(x^*(t), k^*(t)) \in \mathbb{R}_+^2, \quad \forall t \in [0, T].$$

To reclaim the situation, the social planner will be compelled to prolong the period of policy implementation; in other words, to raise the value of T.

In addition, policymakers should be aware of the following fact. In case of short-term planning (that is, when planning horizon T is finite), the current value of x_0 has lesser concern than the value of initial capital resources k_0. From this standpoint, the prospects for near future can be quite reassuring and optimistic even for initially scarce populations of endangered species. Thereby, it is not enough to have time for skillful planning, it is also crucial to have relatively high level of initial capital resources and be able to spend them quickly for the benefit of conservation.

6. Further perspectives and conclusions. The purpose of this paper was to bring out a fresh outlook on the evolution of bioeconomic modeling from its commercial fisheries roots to its application for protection of noncommercial wildlife species. In order to disclose the intrinsic logics of academic thought that has been accompanying this evolution for the last five decades, several elucidative models has been described and briefly revised. The controversy between the inevitable proliferation of multiple economic activities and the preservation of existing biological capital has brought forward numerous mathematical models that account for interaction between economic growth, sustainable development, and natural environment conservation. However, the main stream of underlying research has been focused on the design of long-term conservation policies seeking to maintain so-called "bioeconomic equilibrium" that permits a "sustainable use" of natural resources. Under this posture, the conservation of wildlife species with no harvesting value has been nearly ignored by both policymakers and natural resource modelers.

Many wildlife species that inhabit the surroundings of metropolitan areas worldwide are threatened by local extinction not because of malevolence and animal slaughter but because of poor breeding conditions, lack of food, water or land resources that are crucial for the species survival and reproduction. Unfortunately, gradual shrinking of wildlife species habitats is being disregarded by local authorities since the conservation of noncommercial wildlife species has, in practice, little priority among other societal deeds. As time goes by, the populations of wildlife

species may fall down to some dangerously low levels where the long-term or "preventive" conservation policies would be of no practical use. This argument is plainly supported by preceding studies (see, e.g., Antoci et al. [2005a,b], Campo-Duarte and Vasilieva [2011], Cruz-Rivera and Vasilieva [2013, 2015], and references therein), which have also disclosed that the role of initial abundance of the species population x_0 is really crucial for successful implementation of long-term decision policies aimed at sustainable conservation.

Subliminally thinking, one might imagine that "prevention is better than cure" and, therefore, the "preventive" (or long-term) conservation planning should be given priority over the "curative" (or short-term) policies and measures aimed at fast recovery and enhancement of wildlife species. In addition, a timely and provident prevention usually comes cheaper than an urgent cure, but prevention alone cannot be regarded as 100% panacea from any eventual malady. If the damage is already done, curative treatment should be imperative.

A recent study accomplished by Possingham et al. [2015] offers some interesting insights and provides solid arguments to support this strand of thinking. Using as a basis another type of conservation model,[11] its authors come to an elegant conclusion that both *preventive* and *curative* actions (that is, habitat protection and habitat restoration) are crucial components of a conservation strategy that seeks to optimize either biodiversity conservation or ecosystem services provision.

Nowadays, saving the wildlife species from eventual (local) extinction is not among the priorities of social planning. However, this attitude may change when human society acquires more consciousness of ubiquitous solid decay in wildlife diversity. In such a case, policymakers would be compelled to design and implement "curative" timely actions aimed at recovery and enhancement of wildlife species populations. As natural resource modelers we still have little guidance to offer for this type of short-term planning. Therefore, one of the open avenues in our future research could be to examine more closely this underlying issue, to design and analyze new models, and to offer viable policies for recovery and protection of wildlife diversity.

ENDNOTES

1. Madan and Madan [2009, pp. 133–135] offers a comprehensive review on imminent threats to biodiversity.

2. Ehrlich [1988] pointed out that: "The primary cause of the decay of organic diversity is not direct human exploitation or malevolence, but the habitat destruction that inevitably results from the expansion of human populations and human activities."

3. Alexander also cites several other studies where 'nonconsumptive" values (principally, tourism values) of particular biological species were addressed; however, their authors had never considered such "nonconsumptive" values in the frameworks of mathematical modeling.

4. This analysis was focused on the $2n$-dimensional "optimality system" that stems from the application of Pontryagin maximum principle and consists of n state equations with matching initial conditions and n equations for adjoint variables with limiting transversality conditions. Stability of this system is usually understood only in local sense as the existence of n-dimensional stable manifold that attracts the state trajectories (see more details in Dockner [1985], Dockner and Feichtinger [1991]).

5. This rule originates from the capital investment theory and basically states that the resource must be maintained at a stock level x^* such that its marginal productivity equates to the discount rate, that is $F'(x^*) = \rho$ when harvesting costs are neglected.

6. Alexander does not provide explicit forms for $T(x)$ and $N(x)$ and merely states that they are strictly increasing and concave. The latter makes sense since the nonharvesting utility (7) augments as x increases while its marginal return decreases due to saturation or overpopulation.

7. According to Alexander and Shields [2003], $U_{tour}(x, L) = [P_T T(x) - C_L - C_O]L$ where $P_T T(x)$ represents the income generated by tourist visits, C_L is the cost per unit of land resources used by the penguins, and C_O is the cost of operation of tourism enterprise.

8. Properly speaking, lower values of ρ describe rather patient social planning ($U_1(x, c)e^{-\rho t} \approx U_1(x, c)$ when $\rho \approx 0$) when it comes to obtaining the maximum overall utility.

9. The term $k(T)$ is included in the objective in order to ensure sufficient capital resources for next planning period (e.g., $[T, 2T]$) if a desired level of species population is not achieved during $[0, T]$.

10. That is, the boundary value problem that results from the application of Pontryagin maximum principle to the model (16)–(17).

11. Their model is focused on optimal allocation of monetary resources available for conservation of natural habitats; therefore, this model does not fit into the modeling frameworks considered in this paper.

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