And So it Grows: Using a Computer-Based Simulation of a Population Growth Model to Integrate Biology & Mathematics

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Source: The American Biology Teacher, 75(4):274-279.
Published By: National Association of Biology Teachers
URL: http://www.bioone.org/doi/full/10.1525/abt.2013.75.4.9

And So It Grows: Using a Computer-Based Simulation of a Population Growth Model to Integrate Biology & Mathematics

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Abstract

We provide a 5E structured-inquiry lesson so that students can learn more of the mathematics behind the logistic model of population biology. By using models and mathematics, students understand how population dynamics can be influenced by relatively simple changes in the environment.

Key Words: *Simulation; logistic growth; model; integration; biology; mathematics.*

In the August 2010 issue of the *Notices of the American Mathematical Society*, Friedman wrote that he believed "the coming decade will demonstrate very clearly that mathematics is the future frontier of biology and biology is the future frontier of mathematics" (p. 857). The interdependency between these disciplines is supported by *A Framework for K–12 Science Education: Practices, Crosscutting Concepts, and Core Ideas* (National Research Council [NRC], 2012), in that "developing and using models" and "using mathematics and computational thinking" are two of eight practices that have been identified as representative of professional scientists and engineers. There are numerous fields of biology that have advanced by the utilization of modeling and mathematics, such as computational neuroscience, epidemiology, and population ecology. The latter field and its mathematical application in high school biology will be our focus here.

> *In population dynamics, a useful tool to help predict and explain growth patterns is the mathematical model.*

○ Population Biology

Most high school biology textbooks (e.g., Biggs et al., 2005; Miller & Levine, 2006; Postlethwait & Hopson, 2006) include a chapter devoted to population biology. A closer examination of this chapter will reveal two central ideas: population dynamics and human population. In population dynamics, a useful tool to help predict and explain growth patterns is the mathematical model, the simplest being exponential growth, as depicted by a J-shaped population growth curve that increases indefinitely. In nature, exponential models do not apply to most populations because uninhibited growth cannot occur over an extended period, owing to various limiting factors in the environment. These factors are typically distinguished in biology textbooks as density dependent (e.g., competition, predation, disease) or density independent (e.g., weather, natural disasters, seasonal cycles; Miller & Levine, 2006).

To accommodate these real-world growth limitations, a more complex model is needed. Density-dependent (logistic) growth builds on the exponential model but accounts for these limiting factors via environmental carrying capacity (the largest sustainable population size). The logistic model is characterized by an S-shaped curve that indicates initially slow growth, subsequent rapid growth, and, finally, decelerating growth. Most high school biology textbooks will present a corresponding graph to represent the logistic model. Some textbooks will even provide students the opportunity to create a logistic graph based on presented data. In both cases, an explanation of the underlying mathematical function is nonexistent. Consequently, many students do not develop an appreciation of the mathematics that influenced the development of the logistic model, nor do they understand the complexities of how certain populations respond to internal and external environmental pressures.

To this end, we provide an inquiry-based lesson that guides students through the mathematics of population growth. The lesson is a means by which high school biology teachers can effectively integrate mathematics by introducing their students to population growth dynamics via combining modeling techniques with research-based instructional strategies (Bybee, 2004).

○ Lesson Background

The logistic growth model (Figure 1) is an extension of the exponential growth model. It represents the simplest form of density-dependent growth and is frequently utilized in population studies (Thornley & Johnson, 2000). The logistic growth model creates a

The American Biology Teacher, Vol. 75, No. 4, pages 274–279. ISSN 0002-7685, electronic ISSN 1938-4211.
DOI: 10.1525/abt.2013.75.4.9

$$N(t+1) = \underbrace{N(t) + rN(t)}_{\text{Exponential Growth}} * \underbrace{[1 - \tfrac{N(t)}{K}]}_{\text{Limiting Factor}}$$

Figure 1. The logistic growth model.

sigmoidal (or "S") curve by multiplying the exponential growth component by a mathematical "limiting factor."

As students manipulate parameters in the logistic growth equation, they realize how population dynamics can be influenced by relatively simple changes in the environment – for example, the way in which carrying capacity (K) and growth rate (r) serve to control net population growth.

Once the model is ready, we implement the subsequent population biology lesson, which follows the 5E lesson design (Bybee, 2004). In addition to this lesson addressing the scientific practices of "developing and using models" and "using mathematics and computational thinking," it addresses two Core Ideas in the Life Sciences: (1) Core Idea LS2.A: Interdependent Relationships in Ecosystems:

> Ecosystems have carrying capacities, which are limits to the numbers of organisms and populations they can support. These limits result from such factors as the availability of living and nonliving resources and from such challenges as predation, competition, and disease. Organisms would have the capacity to produce populations of great size were it not for the fact that environments and resources are finite. This fundamental tension affects the abundance (number of individuals) of species in any given ecosystem. (NRC, 2012, p. 152)

and (2) Core Idea LS2C: Ecosystem Dynamics, Functioning and Resilience:

> A complex set of interactions within an ecosystem can keep its numbers and types of organisms relatively constant over long periods of time under stable conditions. If a modest biological or physical disturbance to an ecosystem occurs, it may return to its more or less original status (i.e., the ecosystem is resilient), as opposed to becoming a very different ecosystem. Extreme fluctuations in conditions or the size of any population, however, can challenge the functioning of ecosystems in terms of resources and habitat availability. Moreover, anthropogenic changes (induced by human activity) in the environment – including habitat destruction, pollution, introduction of invasive species, overexploitation, and climate change – can disrupt an ecosystem and threaten the survival of some species. (NRC, 2012, pp. 155–156)

Specifically, the concepts associated with this lesson are that (1) increasing the carrying capacity allows the population to grow to a larger size, but the population always stops growing at the value established by the carrying capacity; and (2) changing the growth rate results in different curves that may "max out" at the predetermined carrying capacity and vary in the time required to achieve it.

○ Creating the Simulation Model

We begin with a step-by-step procedure to create the logistic population growth model using Excel, version 2007 or later. To prepare the model for student use, we suggest preloading the spreadsheet on each student computer; emailing the spreadsheet to each student; or providing the procedures for creating the model for individual entry (which is, incidentally, an opportunity to integrate computer science with this lesson). Lastly, we suggest selecting 'Protect Sheet' in the 'Review' tab for all cells except the three 'Value' cells containing the numeric parameters, to prevent students from modifying the spreadsheet.

1. In Excel 2007, open a new document.
2. In Cell A1, type "Parameter"; in B1, type "Value"; in D1 type "Time (t)" for time interval; in E1 type "Population Size at Time N(t)" for population size at a specific time interval. See Figure 2A.
3. In A2, type "N_0 =" to represent the initial population for the model.
4. In B2, type your desired initial population size. For this example, we are using an initial population size of "2."
5. In A3, type "r =" to represent the growth rate for the model.
6. In B3, type your desired growth rate. For this example, we are using a growth rate of "0.5."
7. In A4, type "K =" to represent the carrying capacity for the model.
8. In B4, type your desired carrying capacity. For this example, we are using a carrying capacity of "300." See Figure 2B.
9. In D2, type "0" for initial time; in D3, type "1" for first time interval (t).
10. Select Cells D2 and D3 and then drag the fill handle down Column D to your desired end time. For this example, we filled 51 time intervals from D2 to D52. See Figure 2C.
11. In E2, type "= B2" for the initial population size and press "Enter." For this example, since we are using an initial population size of "2," E2 should now read "2."
12. Because population size is realistically a whole number, format column E so that there are no decimal places for these values. Right click column E and select "Format Cells." In the "Number" tab under "Category:" select "Number" and change "Decimal places" to "0."
13. In E3 for population at t = 1, type "=E2+($B$3*E2)*(1-(E2/$B$4))" to represent the logistic growth model and press "Enter." The population at t = 1 in E3 should now read "3." See Figure 2D.

    NOTE: The "$" is used in Excel to represent cells that do not change. In effect, "$B$3" tells Excel that cell B3 (r = growth rate) is an unchanging, constant value. This prevents us from

A

	A	B	C	D	E
1	Parameter	Value		Time (*t*)	Population Size at Time *N* (*t*)

B

	A	B	C	D	E
1	Parameter	Value		Time (*t*)	Population Size at Time *N* (*t*)
2	*No* =	2			
3	*r* =	0.5			
4	*K* =	300			

C

	A	B	C	D	E
1	Parameter	Value		Time (*t*)	Population Size at Time *N* (*t*)
2	*No* =	2		0	
3	*r* =	0.5		1	
4	*K* =	300		2	
5				3	
6				4	
7				5	
8				6	
9				7	
10				8	
11				9	
12				10	
13				11	
14				12	
15				13	
16				14	
17				15	
18				16	
19				17	
20				18	
21				19	
22				20	
23				21	
24				22	
25				23	
26				24	
27				25	
28				26	

D

E3 | *fx* =E2+(E2*$B$3)*(1-(E2/$B$4))

	A	B	C	D	E	F
1	Parameter	Value		Time (*t*)	Population Size at Time (*Nt*)	
2	*No* =	2		0	2	
3	*R* =	0.5		1	3	
4	*K* =	300		2		

E

	A	B	C	D	E
1	Parameter	Value		Time (*t*)	Population Size at Time *N* (*t*)
2	*No* =	2		0	2
3	*r* =	0.5		1	3
4	*K* =	300		2	4
5				3	7
6				4	10
7				5	15
8				6	22
9				7	32
10				8	46
11				9	66
12				10	91
13				11	123
14				12	159
15				13	197
16				14	230
17				15	257
18				16	276
19				17	287
20				18	293
21				19	296
22				20	298
23				21	299
24				22	300
25				23	300
26				24	300
27				25	300
28				26	300
29				27	300

Figures 2. Screenshots illustrating the model creation procedure.

having to manually type the equation for every time step in the model.

14. In E4 for population size at t = 2, the logistic growth equation would be modified to =E3+($B$3*E3)*(1-(E3/$B$4)). Excel handles this change automatically thanks to the "$" notation we used in step 13. Therefore, to fill Column E, select Cell E3 and then drag the fill handle down Column E to your desired end time. See Figure 2E.
15. For this example, the population reached the carrying capacity of K = 300 in 22 time intervals.
16. To obtain the corresponding graph in Excel 2007, highlight the values of t (cells D2 to D52) and the values of N(t) (cells

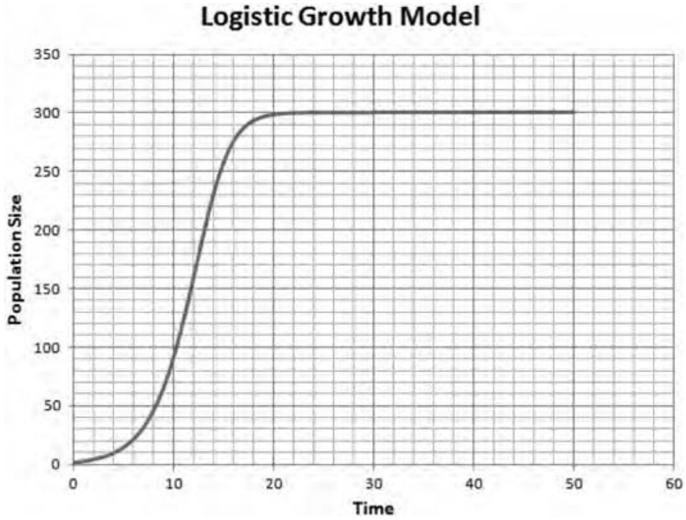

Figure 3. Graphic output assessing population size (N) versus time (t) as generated by the completed logistic growth model.

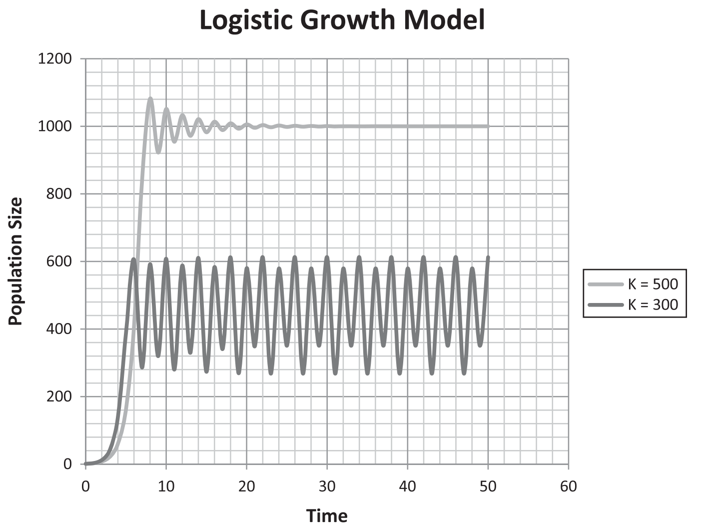

Figure 4. Population size (N) versus time (t) with varying carrying capacity (K) values. Growth rate (r) is set at its default value of 0.5.

E2 to E52). Click the "Insert" tab and choose the "Scatter" chart type followed by "Scatter with Smooth Lines." Complete the chart by selecting "Layout" in the "Chart Tools" tab and including appropriate axis titles and chart title and major and minor horizontal and vertical gridlines. See Figure 3.

Note: If you prefer students to analyze only the computed data or to graph by hand the data that are produced by the model, omit the final step of the procedures.

○ 5E Lesson

Engage

We watch fields turn from a sickly brown to a vibrant green every spring season, and then return to this brown hue in subsequent seasons. Consider how this change occurs: plants first sprout new leaves and shoots, grow very rapidly, and expand across an area until the whole location is covered in new plant life. Then these plants slow down until they stop growing at all.

The preceding scenario is used to interest students in the underlying concepts of the population dynamics lesson as well as to generate student curiosity in ecological phenomena. We use two probing questions related to the scenario as an opportunity to assess students' conceptions before launching into the lesson: (1) Why do these plants slow down? (2) What mechanisms control how fast the population grows? After an informal class discussion, we review previous terminology of population growth, such as birth rate and death rate, and the more familiar exponential growth curve, which demonstrates the relationship between growth rate and population size.

Explore

Students access computers on which the logistic growth model is available. We encourage students in each group to take turns manipulating the parameters (N, K, and r) and recording observations from each set of parameters. As parameters are entered into the model, the population size is computed while the corresponding size-versus-time graph is produced.

Part 1: How Does Carrying Capacity (K) Affect Population Size?

Students first run the model to identify carrying capacity (K). We use a starting population size (N_0) of 1 and a growth rate (r) of 0.5 (r will be manipulated in the subsequent activity). For students who are curious as to what a growth rate of 0.5 represents, we equate the decimal to a percent; ergo, a growth rate of 0.5, or 50%, results in one-half of the current population size being added at the next time step. Students notice that varying K while maintaining the other parameters at constant values produces logistic growth curves that vary in terms of their maximum population size (Figure 4), and that the generated graphs depart from the J-shaped curves of exponential growth. A formal discussion takes place after students have investigated the questions in the Explore section.

Part 2: How Does Growth Rate (r) Affect Population Size?

Next, students identify the model parameter r. Students are asked to set the population size (N) to 1, carrying capacity (K) to 1000, and growth rate (r) to a value greater than 0.5 (or 50%) but less than 1 (or 100%) (Figure 5, dark gray line). Students repeat the process but change r to a value greater than 0 but less than 0.5 (Figure 5, light gray line). (More radical changes to r are covered in later sections.) Students realize that changing r results in different curves that vary in the time required to achieve carrying capacity, with larger values of r growing the population more quickly. Students then predict and justify the shape of the population graph if r is changed to 0 (answer: there is no net population growth; therefore, the population will never change).

Explain

After students see that population dynamics are influenced by simple changes in the two parameters under investigation, we provide them the opportunity to discuss their results in small groups and present their graphs based on their selected parameter values. At this point, students should tell the "story" of logistic growth, that when the population is small, birth rates are high and death rates are low, which

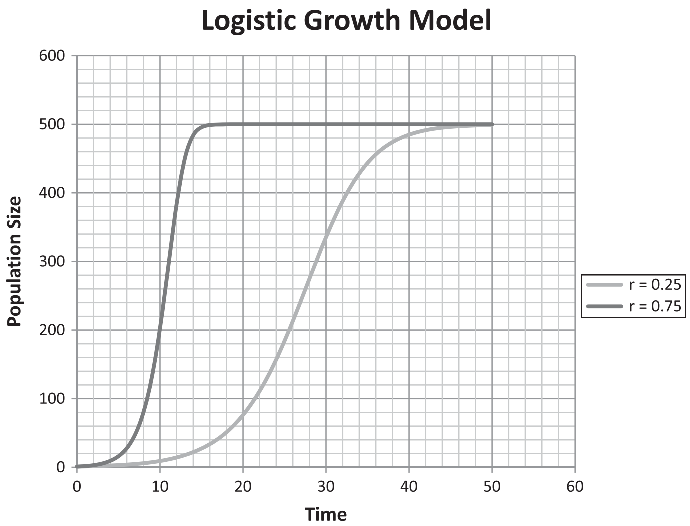

Figure 5. Population size (N) versus time (t) with varying growth rate (r) values. Carrying capacity (K) is set at its default value of 500.

is similar to the exponential model. However, as the population approaches the carrying capacity, the growth rate slows because of falling birth rate and/or increasing death rate. When birth rate equals death rate, the population growth ceases. (Be aware that students may develop a misconception that birth rate equals zero when the population growth stops.)

With this conceptual foundation in place, discussion of the logistic equation and its value as a biological model leads to a deeper understanding of how mathematics is used in biology. As demonstrated, the logistic model creates an S-shaped curve by including a "limiting factor" in the mathematical equation. We suggest providing the equation for the logistic model at this point in the lesson (Figure 1). The equation is divided into its constituent parts where N(t) is the population size at time t, r is the population growth rate, and K is the environmental carrying capacity (Thornley & Johnson, 2000). Teachers can also refer students to the logistic equation built into the logistic model as an example of how mathematical equations can be recreated in spreadsheets (see Creating the Simulation Model, above).

Although students manipulated these parameters in the simulation, they may be intimidated by the mathematical computation involved here. Students may comment that the parenthetical is confusing to them. Teachers can address student apprehension by stating that understanding the logistic growth model requires nothing more than basic algebra and order of operations. The teacher explains that logistic growth differs from the more familiar exponential, or "unlimited," growth via the addition of a limiting factor, which is the parenthetical of the equation, then encourages the students to perform the parenthetical calculation by hand using predetermined values for N and K. Using this approach, previous students quickly realized that, as the population size approaches the environmental carrying capacity, the value of the parenthetical reduces to zero, which scales the number of new individuals added to the population (Thornley & Johnson, 2000). These calculations reinforce the notion that net growth is dependent on resource availability and other factors typically attributed to carrying capacity.

Elaborate

Now that the students have developed an understanding of the components of logistic growth, they extend their knowledge to unexpected circumstances. Students have seen that logistic growth is defined by the S-shaped curve, and they may simply depend on knowing that curve to explain the phenomenon. Therefore, changing the shape of the curve allows students to develop greater comprehension of population growth dynamics.

Students should now run several simulations in which r is increased to values greater than 1. They will find that when populations more than double in size with each time step, new growth curves are created (Figure 6), including oscillations ($1 < r < 2.5$) and even chaos ($r > 2.5$). These atypical curves challenge the notion that density-dependent growth is always stable and characterized by a smooth curve. This realization allows students to develop a working knowledge of how growth rate interacts with carrying capacity to create a maintainable population size that is not possible with traditional lecture-based scientific teaching. This is also an excellent opportunity to discuss equilibrium and stability of a model system, as many of the atypical curves will eventually achieve stable oscillation. These examples could serve as a natural transition toward later lessons covering ecological succession, climax communities, and predator–prey interactions, as suggested by the NRC (2012).

Evaluate

To determine students' understanding of the inherent concepts and their facility with the logistic model and computer simulation, we give them an opportunity to solve a real-world problem according to the following scenario: *Imagine 20 rats of a new species have just arrived in San Francisco on a boat from Asia. The sewers of San Francisco have enough food to support a population of 100,000 rats at most. Use the model to determine the approximate number of years it will take the population to reach carrying capacity if the population doubles in size every year* [Solution: Using K = 100,000; N = 20; r = 1; it will take ~16 years.] Students are expected to support their

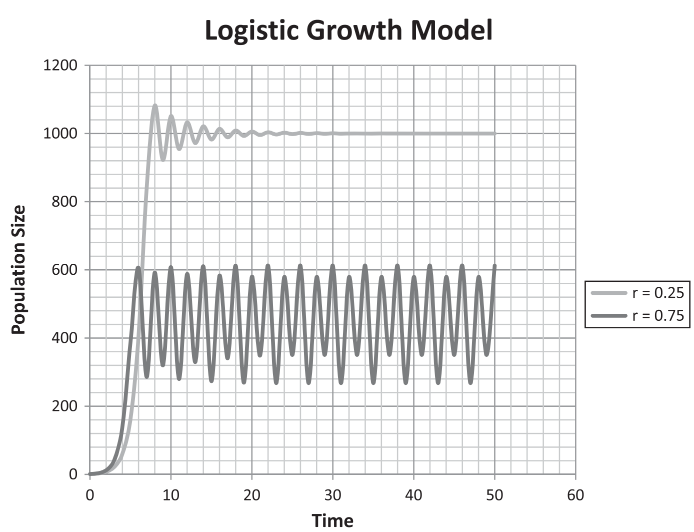

Figure 6. Population size (N) versus time (t) with growth rate (r) values exceeding 1. Carrying capacity values differ between lines for legibility (light gray = 1000, dark gray = 500).

mathematical findings with a coherent explanation as to why the model produced the result.

Lastly, we return to the scenario provided in the Engage section. Allow students the opportunity to revisit their initial ideas and compare them to their current understanding of population dynamics by revisiting the following questions: (1) Why do these plants slow down? (2) What mechanisms control how fast the population grows? Students utilize their new knowledge to explain that net population growth declines as the plant population approaches its maximum sustainable size (carrying capacity), and carrying capacity is based on space and resource limitations.

○ Conclusion

If Friedman is correct in stating that mathematics is the future frontier of biology, it is essential that students be exposed to biological concept models encompassing mathematical models that drive our current understanding of the biological sciences. This lesson takes a traditionally taught model of population dynamics and introduces students to the mechanisms that control the model. As students experience the logistic model, they understand how population dynamics can be influenced by simple changes to the environment. They learn through the manipulation of a computer-based simulation how carrying capacity and rate of growth serve to control population abundance beyond simple definitions and shallow understandings.

By becoming actively involved in their education through this and other inquiry-based lessons that require mathematical applications in learning biology, students may discover a greater appreciation for related ecological concerns such as overpopulation and wildlife management. Encouraging students to be responsible for their education gives them the tools they need to become active participants in science who ask and independently explore questions to improve their understanding of the world around them, which is an important goal of scientific education.

○ Acknowledgments

We thank Lori Pettijohn, Daneice Foster, Teresa Christensen, and Michael Dean for allowing this lesson to be tested in their classrooms. Additional comments and suggestions were provided by Mark Nanny and Chris Dalton. This work was funded through the NSF GK-12 fellowship, "Engineering in Practice for a Sustainable Future" (DGE-0538655).

References

Biggs, A., Hagins, W.C., Kapicka, C. & Lundgren, L. (2005). *Biology: The Dynamics of Life*. Columbus, OH: Glencoe/McGraw Hill.

Bybee, R.W. (2004). *Scientific Inquiry and Science Teaching*. Dordrecht, The Netherlands: Springer.

Friedman, A. (2010). What is mathematical biology and how useful is it? *Notices of the AMS*, *57*, 851–857.

Miller, K.R. & Levine, J.S. (2006). *Prentice Hall Biology*. Boston, MA: Pearson Prentice Hall.

National Research Council. (2012). *A Framework for K–12 Science Education: Practices, Crosscutting Concepts, and Core Ideas.* Washington, D.C.: National Academies Press.

Postlethwait, J.H. & Hopson, J.L. (2006). *Modern Biology*. Austin, TX: Holt, Rinehart, and Winston.

Thornley, J.H.M. & Johnson, I.R. (2000). *Plant and Crop Modelling: A Mathematical Approach to Plant and Crop Physiology*. Caldwell, NJ: Blackburn Press.

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