

1 Write a proof of Lemma 4.2 without looking at the proof given in the paper.

First, note that $h_{U \setminus \{i\}}^{d+1}$ contains all polynomials of degree $d + 1$ in U that do not contain x_i . We may then subtract this polynomial from h_U^{d+1} , leaving only the polynomials of degree $d + 1$ that contain x_i . By dividing out x_i , this can be written

$$x_i h_U^d = h_U^{d+1} - h_{U \setminus \{i\}}^{d+1}.$$

By replacing the index i with j , we also obtain

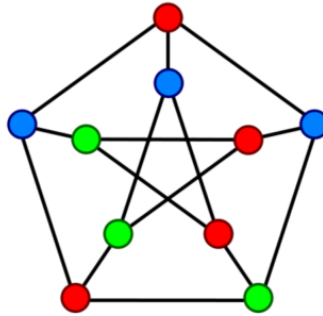
$$x_j h_U^d = h_U^{d+1} - h_{U \setminus \{j\}}^{d+1}.$$

Thus

$$\begin{aligned} (x_i - x_j) h_U^d &= x_i h_U^d - x_j h_U^d \\ (x_i - x_j) h_U^d &= h_U^{d+1} - h_{U \setminus \{i\}}^{d+1} - (h_U^{d+1} - h_{U \setminus \{j\}}^{d+1}) \\ (x_i - x_j) h_U^d &= h_{U \setminus \{j\}}^{d+1} - h_{U \setminus \{i\}}^{d+1}. \end{aligned}$$

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2 Verify Lemma 4.3 on the graph below (label the vertices however you like):



This has been verified.

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3 Verify Lemma 4.4 on the graph in Problem 2. You will want to read Section 2 of the paper to know what a radical ideal is.

This has been verified using Macaulay 2.

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4 Read and understand the proof of Lemma 4.4.