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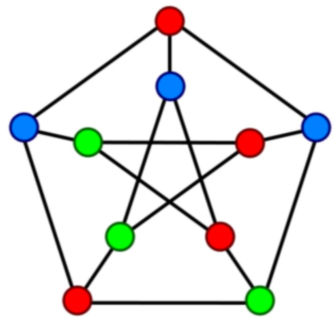
Summer 2014

1 Let G be the following 3-coloring of the following graph:



Using Definition 1.4, write down a ν-basis for G, and hence find a generating set for A_ν. (Label the vertices with the labels $1, 2, \ldots, 10$ in any way you like.)

The ν-basis is

$$\begin{aligned}
g_1 &= x_1 - x_8 \\
g_2 &= x_2 - x_6 \\
g_3 &= x_3 - x_{10} \\
g_4 &= x_4 - x_8 \\
g_5 &= x_5 - x_6 \\
g_6 &= x_6 + x_8 + x_{10} \\
g_7 &= x_7 - x_8 \\
g_8 &= x_8^2 + x_8x_{10} + x_{10}^2 \\
g_9 &= x_9 - x_{10} \\
g_{10} &= x_{10}^3 - 1
\end{aligned}$$

∎

2 Read example 1.8 and verify the computations in this example using Macaulay 2.

This has been verified using Macaulay 2. ∎

3 Using the definition before Theorem 1.11, write down the reduced ν-basis for the ideal A_ν from Problem 1 in this problem set. Use Macaulay 2 to verify that G from Problem 1 violates the conclusion in Theorem 1.11.

The reduced ν-basis is

$$\begin{aligned}
\tilde{g}_1 &= x_1 - x_8 \\
\tilde{g}_2 &= x_2 + x_8 + x_{10} \\
\tilde{g}_3 &= x_3 - x_{10} \\
\tilde{g}_4 &= x_4 - x_8 \\
\tilde{g}_5 &= x_5 + x_8 + x_{10} \\
\tilde{g}_6 &= x_6 + x_8 + x_{10} \\
\tilde{g}_7 &= x_7 - x_8 \\
\tilde{g}_8 &= x_8^2 + x_8 x_{10} + x_{10}^2 \\
\tilde{g}_9 &= x_9 - x_{10} \\
\tilde{g}_{10} &= x_{10}^3 - 1
\end{aligned}$$

■

4 Use Macaulay 2 to verify Example 1.13.

This has been verified using Macaulay 2. ■