

Application of the DBSCAN clustering algorithm in High Energy Physics

Unsupervised neural networks



Istituto Nazionale di Fisica Nucleare

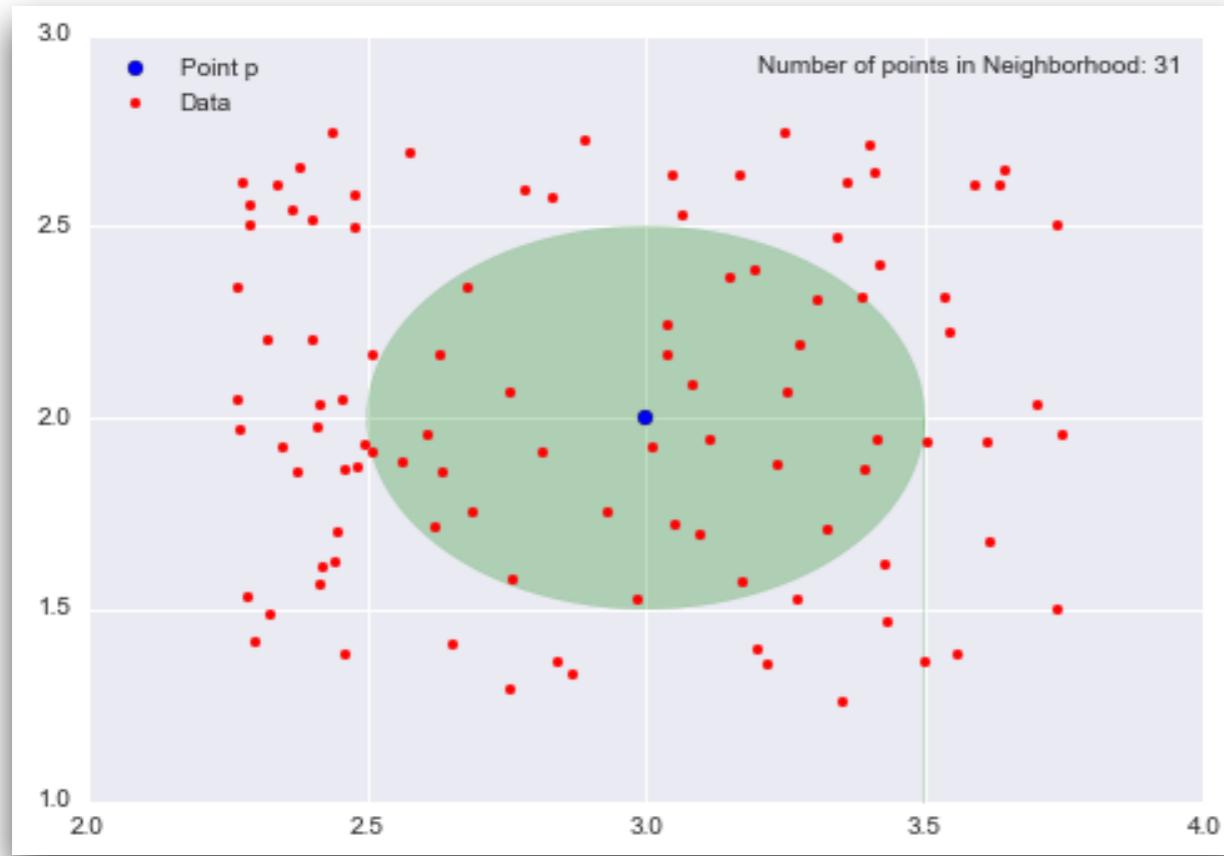


Antonio Bianchi, Luca Barioglio,
Matteo Concas, Fabrizio Gerosa

Neighbourhood density

- For some real value $\varepsilon > 0$ and some point p , the ε -neighbourhood of p is defined as the set of points that are at most distance ε away from p

$$Nbhd(p, \varepsilon) = \{q : d(q, p) < \varepsilon\}$$



- The neighbourhood of point p with $\varepsilon = 0.5$ is formed by 31 points
- The density of the neighbourhood is defined as the number of points contained within the neighbourhood (mass) divided by the volume of the neighbourhood:

$$\rho = 31/(\pi\varepsilon^2) \approx 39.5$$

📚 <https://blog.dominodatalab.com/topology-and-density-based-clustering/>

Density-Based Spatial Clustering of Applications with Noise

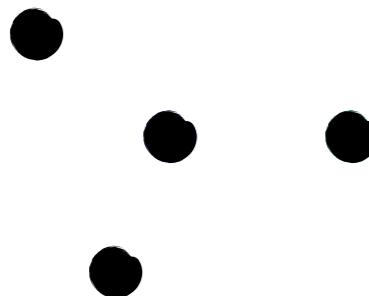
- DBSCAN is a density-based clustering algorithm, introduced in 1996 by Ester et. al. (<http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.71.1980>)
- It does not require the number of clusters as a parameter
- It infers the number of clusters based on the data, and it can discover clusters of arbitrary shape
- To approximate the local density, two parameters are needed:
 - the radius ϵ of the neighbourhoods around a data point p
 - the minimum number of data points ($minPts$) we want in a neighbourhood to define a cluster
- Using these two parameters, DBSCAN categories the data points into three categories:
 - Core points
 - Border points
 - Outliers

Core points

- A data point p that belongs to a cluster is defined *core point* if the neighbourhood of p contains at least $minPts$

$$Nbhd(p, \epsilon) \geq minPts$$

- Example: let's consider a cluster obtained with $minPts = 3$ and radius ϵ

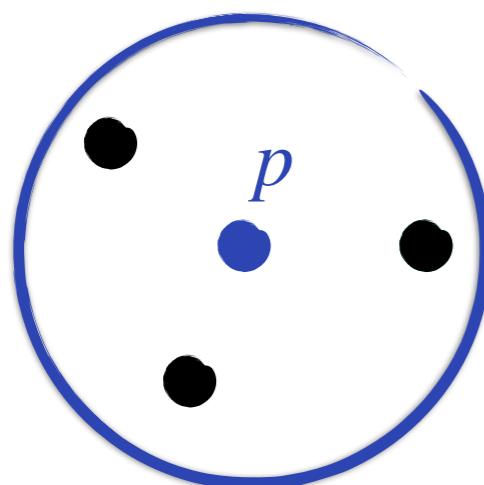


Core points

- A data point p that belongs to a cluster is defined *core point* if the neighbourhood of p contains at least $minPts$

$$Nbhd(p, \epsilon) \geq minPts$$

- Example: let's consider a cluster obtained with $minPts = 3$ and radius ϵ



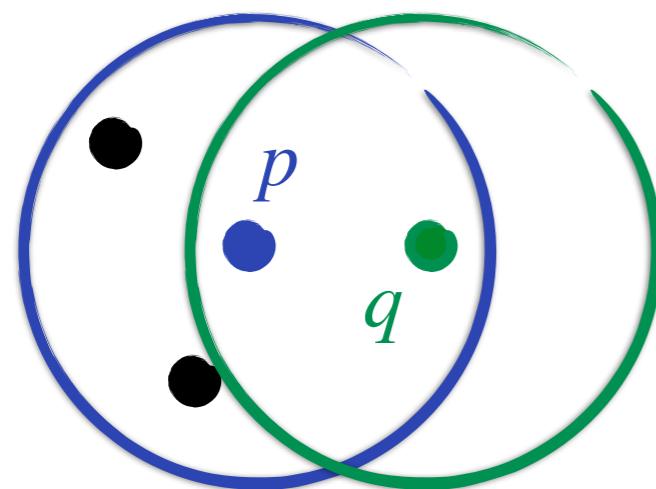
→ p is a core point, since there are 3 points in its neighbourhood

Core points

- A data point p that belongs to a cluster is defined *core point* if the neighbourhood of p contains at least $minPts$

$$Nbhd(p, \epsilon) \geq minPts$$

- Example: let's consider a cluster obtained with $minPts = 3$ and radius ϵ



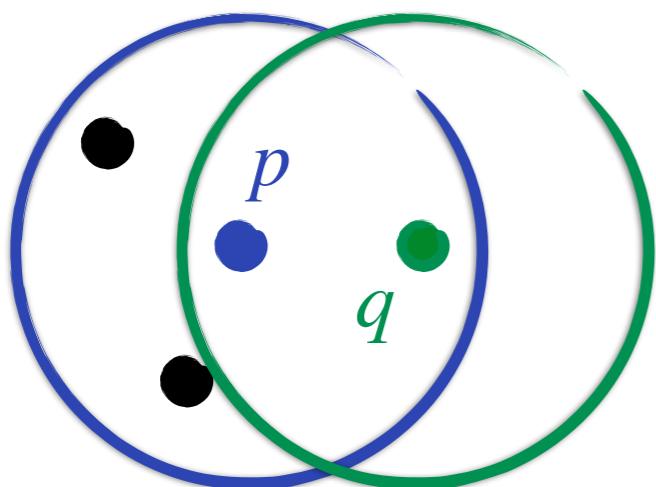
- p is a core point, since there are 3 points in its neighbourhood
- q is not a core point, since there is only 1 point in its neighbourhood

- Clusters are build around core points

Border points

- A data point q which belongs to a cluster is defined *border point* if the neighbourhood of p contains at less points than minPts , but q is *reachable* from some *core point* p .

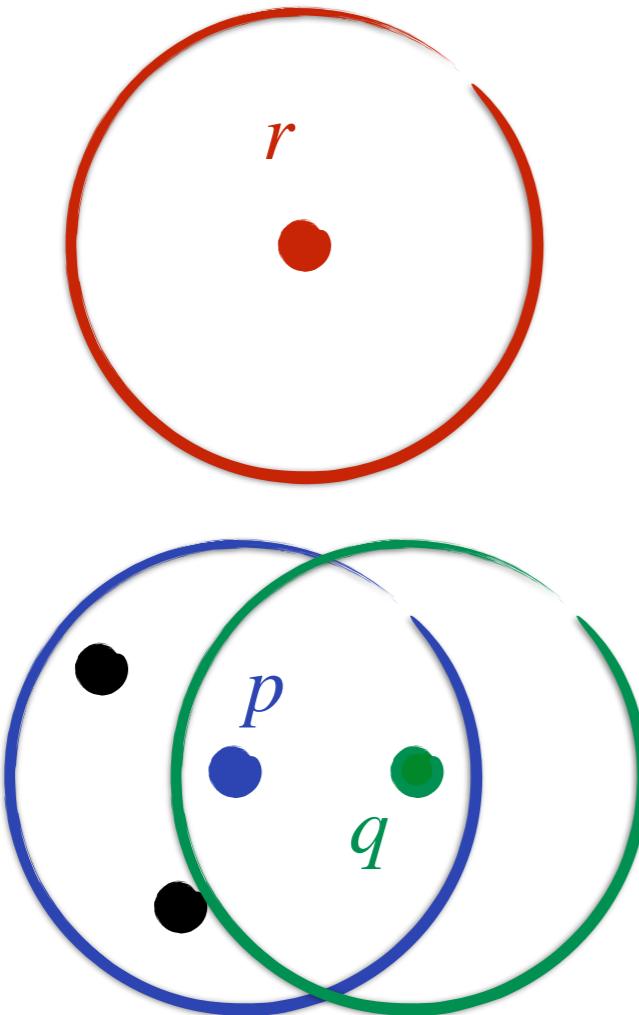
$$Nbhd(q, \epsilon) < \text{minPts} \wedge \exists p : q \in Nbhd(p, \epsilon) \geq \text{minPts}$$



- p is a core point, since there are 3 points in its neighbourhood
- q is a border point, since there is only 1 points in its neighbourhood, but it belongs to the cluster of the core point p

Outliers

- A data point r which is neither a core or a border point, is defined as an *outlier*

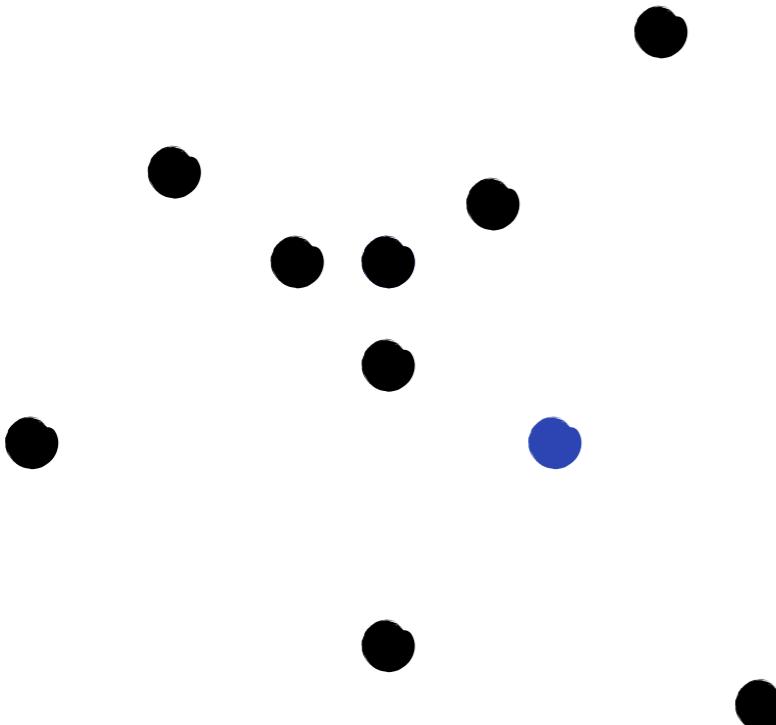


- r is an outlier, since there are no other points in its neighbourhood and it does not belong to a cluster
- p is a core point, since there are 3 points in its neighbourhood
- q is a border point, since there is only 1 points in its neighbourhood, but it belongs to the cluster of the core point p

DBSCAN algorithm

→ The steps of the DBSCAN algorithm are:

- (i) Pick a random point that has not been assigned to a cluster or designated as an *outlier* and compute its neighbourhood to determine whether it's a *core point*

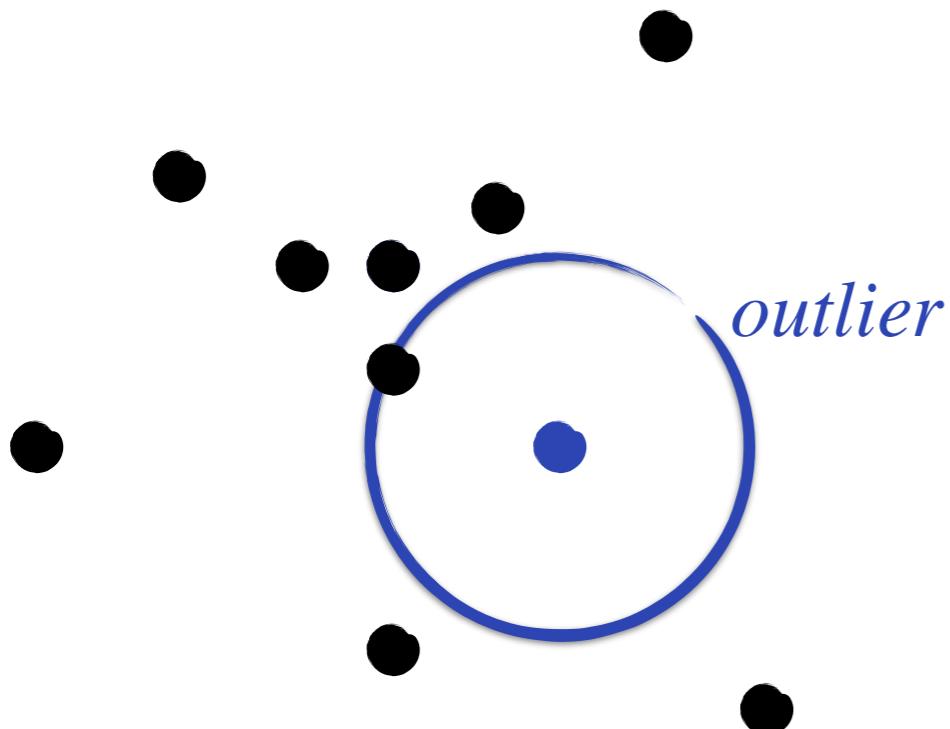


DBSCAN algorithm

→ The steps of the DBSCAN algorithm are:

- (i) Pick a random point that has not been assigned to a cluster or designated as an *outlier* and compute its neighbourhood to determine whether it's a *core point*

if not, label the point as an *outlier*

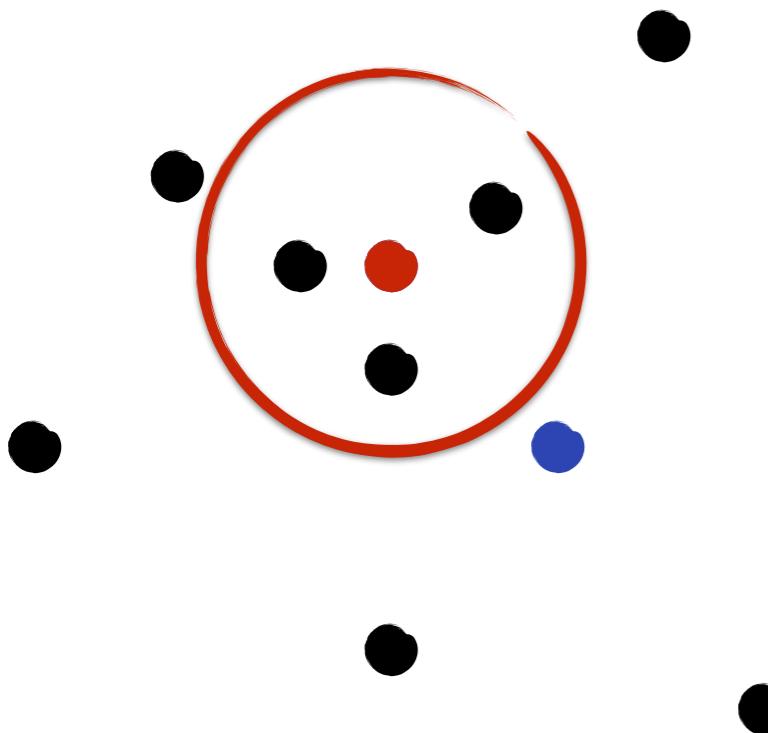


DBSCAN algorithm

→ The steps of the DBSCAN algorithm are:

- (i) Pick a random point that has not been assigned to a cluster or designated as an *outlier* and compute its neighbourhood to determine whether it's a *core point*

if yes, start a cluster around this point

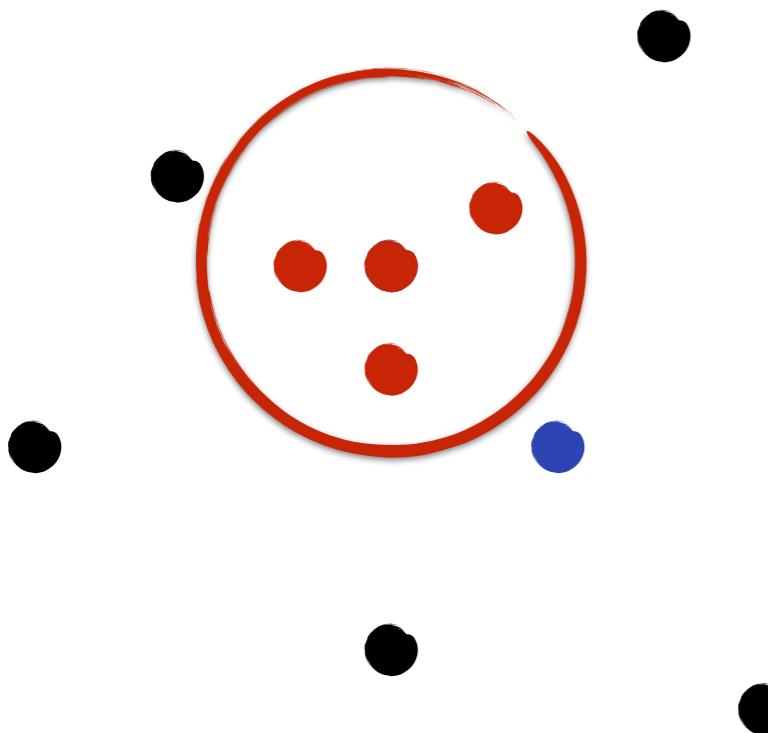


DBSCAN algorithm

→ The steps of the DBSCAN algorithm are:

- (i) Pick a random point that has not been assigned to a cluster or designated as an *outlier* and compute its neighbourhood to determine whether it's a *core point*

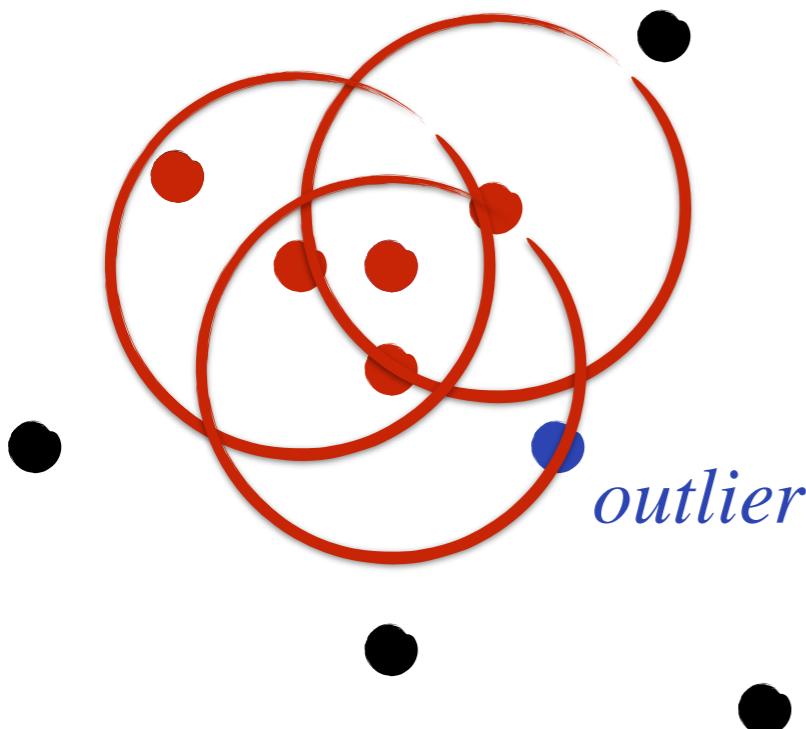
if yes, start a cluster around this point



DBSCAN algorithm

→ The steps of the DBSCAN algorithm are:

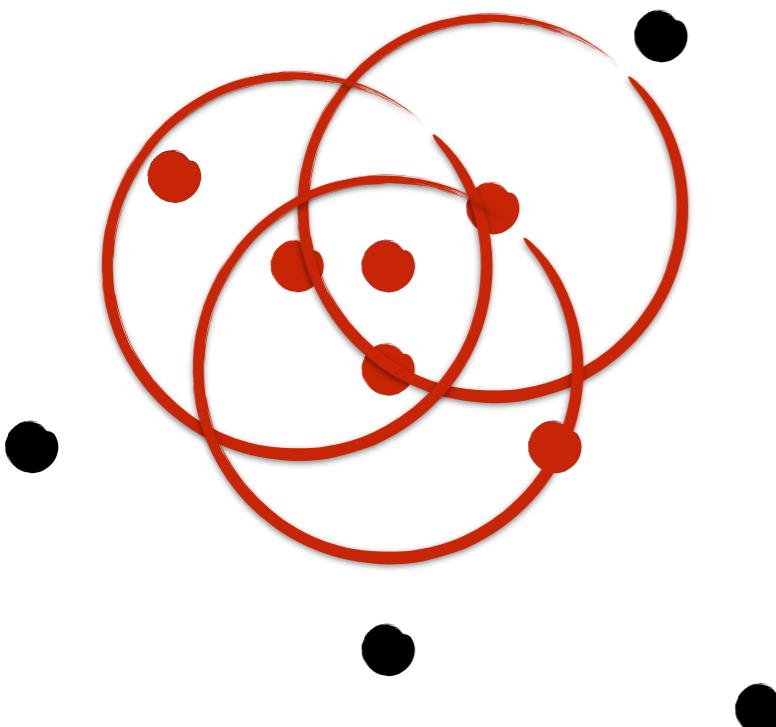
- (i) Pick a random point that has not been assigned to a cluster or designated as an *outlier* and compute its neighbourhood to determine whether it's a *core point*
- (ii) Once we find a *core point* and thus a cluster, expand the cluster by adding all *directly-reachable* points to the cluster. Perform "neighbourhood jumps" to find all *density-reachable* points and add them to the cluster.



DBSCAN algorithm

→ The steps of the DBSCAN algorithm are:

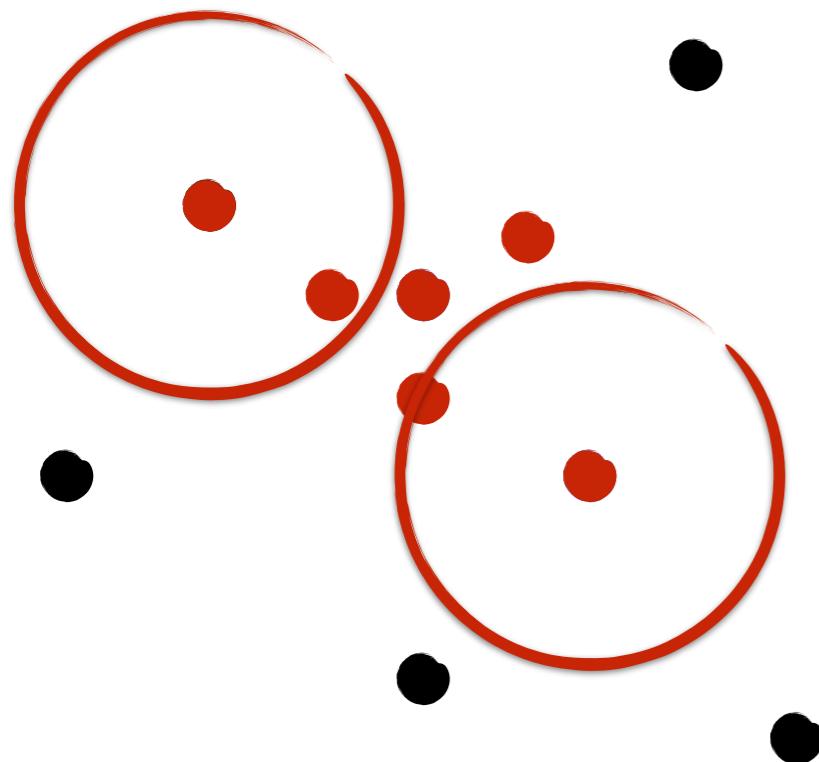
- (i) Pick a random point that has not been assigned to a cluster or designated as an *outlier* and compute its neighbourhood to determine whether it's a *core point*
- (ii) Once we find a *core point* and thus a cluster, expand the cluster by adding all *directly-reachable* points to the cluster. Perform "neighbourhood jumps" to find all *density-reachable* points and add them to the cluster. If an *outlier* is added, change that point's status from *outlier* to *border point*



DBSCAN algorithm

→ The steps of the DBSCAN algorithm are:

- (i) Pick a random point that has not been assigned to a cluster or designated as an *outlier* and compute its neighbourhood to determine whether it's a *core point*
- (ii) Once we find a *core point* and thus a cluster, expand the cluster by adding all *directly-reachable* points to the cluster. Perform "neighbourhood jumps" to find all *density-reachable* points and add them to the cluster. If an *outlier* is added, change that point's status from *outlier* to *border point*
- (iii) Repeat these two steps until all points are either assigned to a cluster or designated as an *outlier*

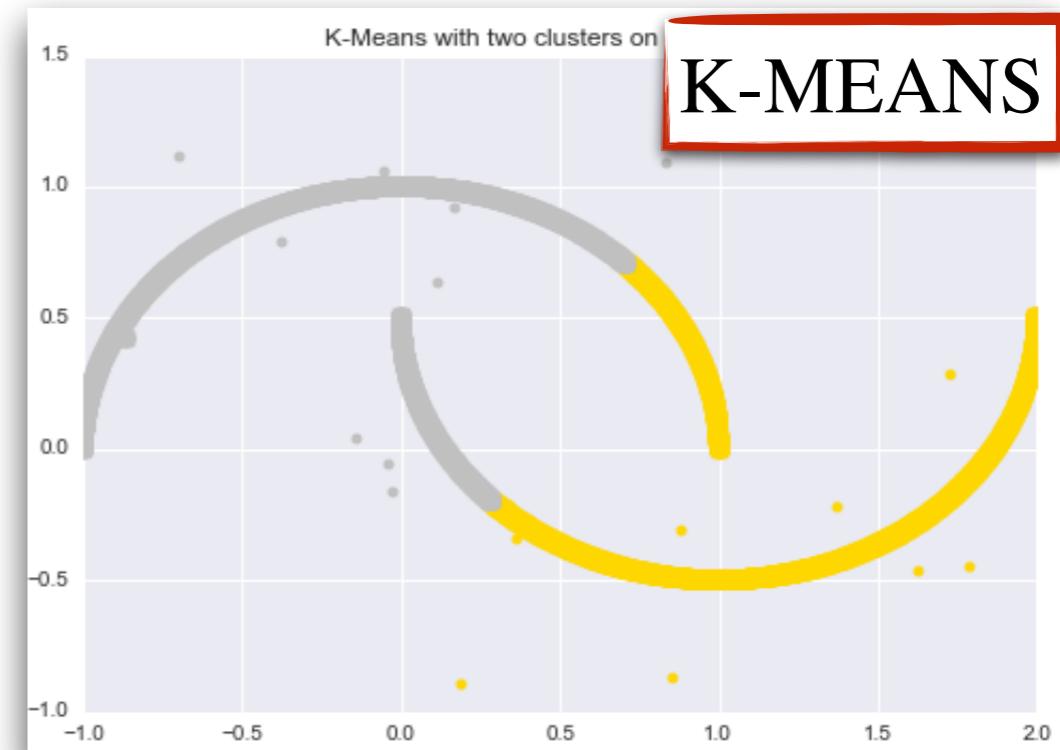
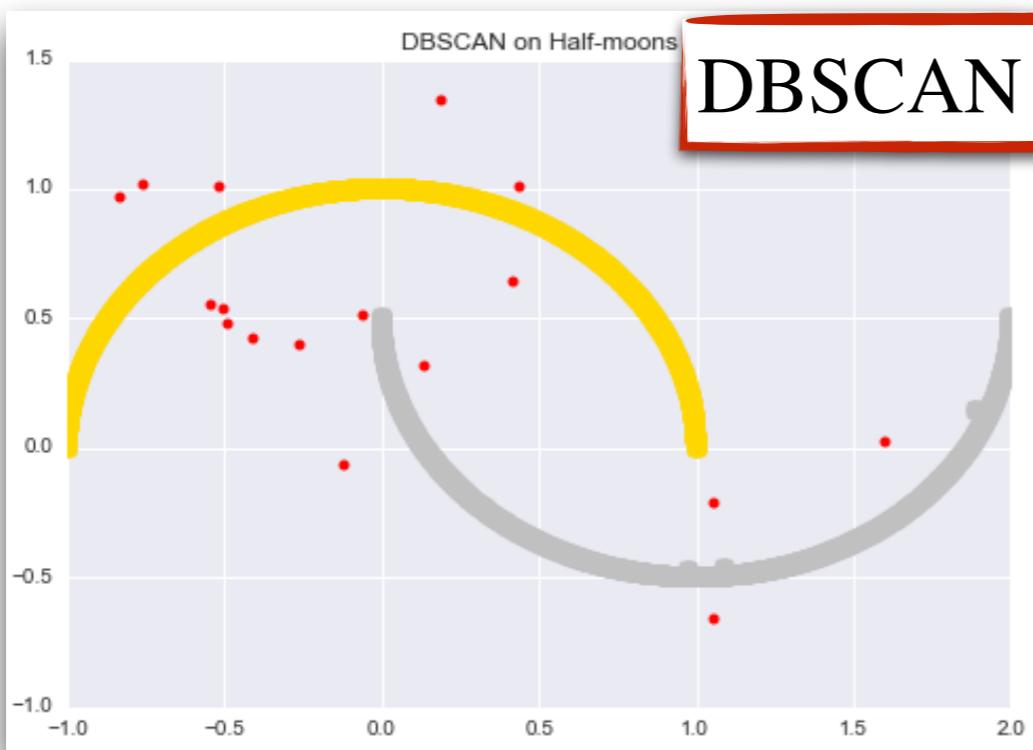
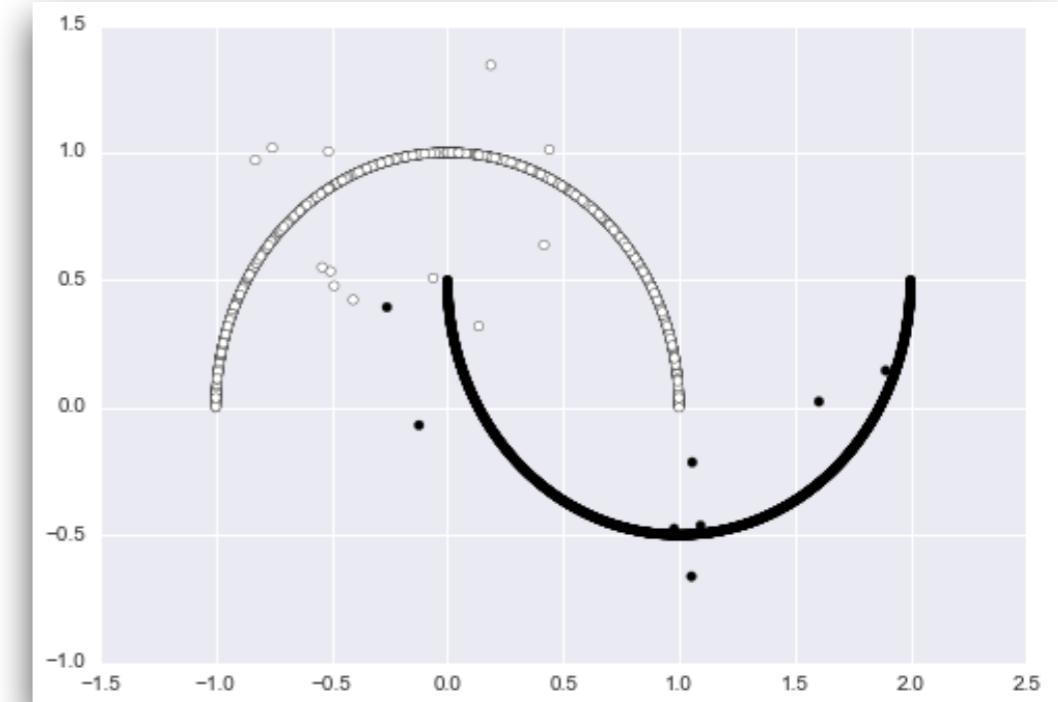


DBSCAN vs. k-MEANS

→ Example: consider the following dataset:

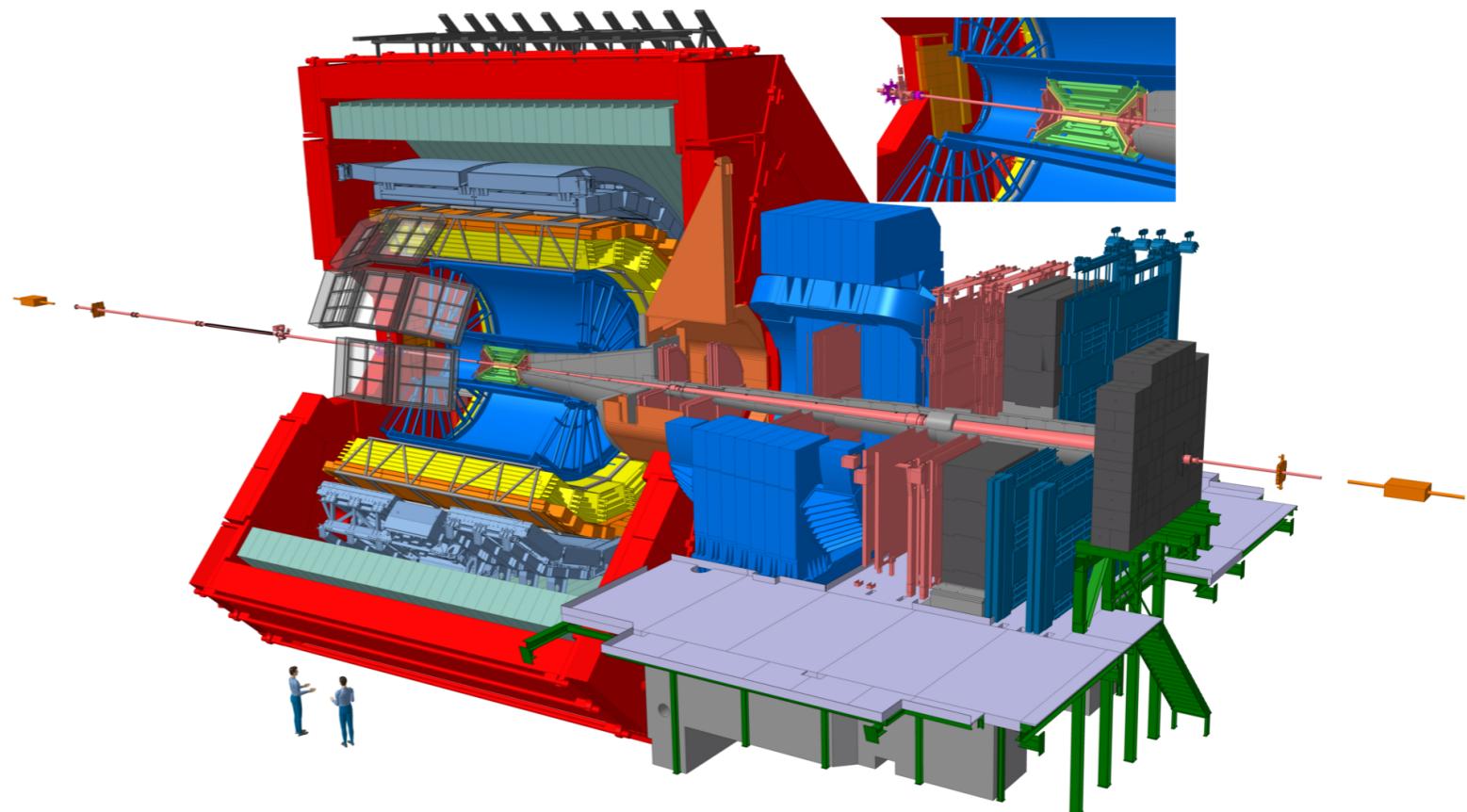
- (i) two “half-moon” clusters
- (ii) some noise points

→ Using DBSCAN the two clusters are recognised and most of the noise points tagged as “outliers”

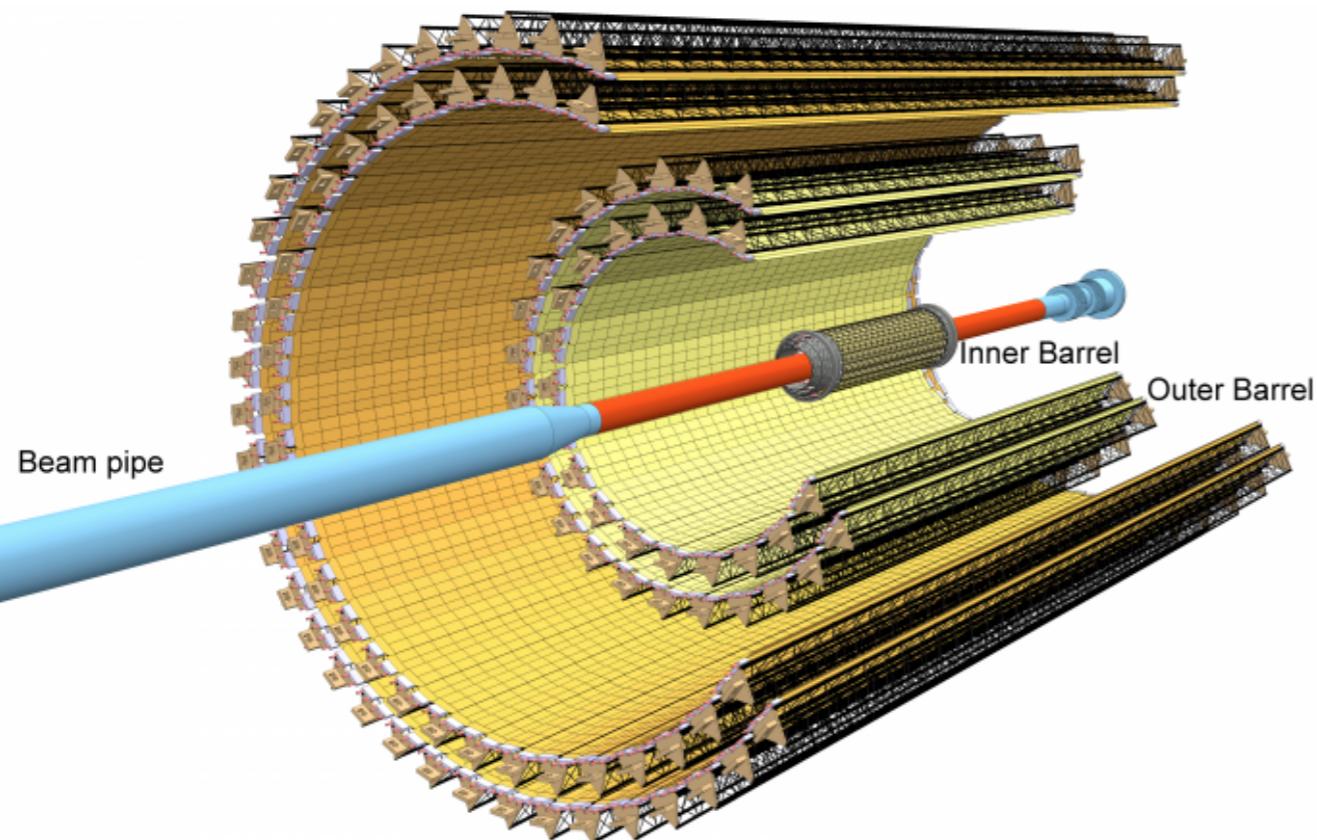


A Large Ion Collider Experiment

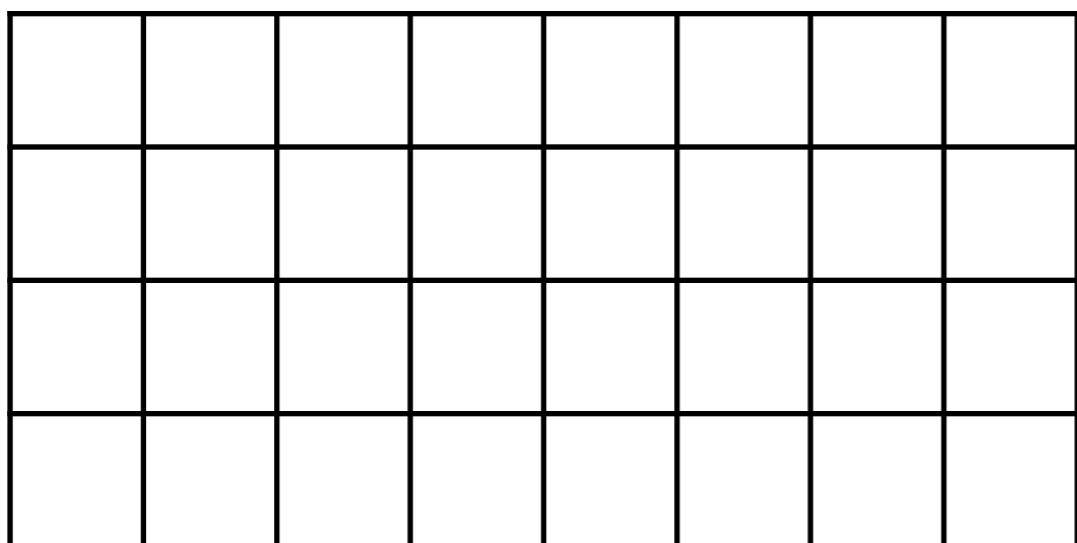
- ALICE is one of the four LHC experiments at CERN
- It is designed to study the matter created in ultra-relativistic heavy-ion collisions
- In each collision many particles are created
- Trajectories of charged particles are reconstructed using various detectors
- The closest detector to the interaction point is the Inner Tracking System (ITS), which consists of several cylindrical layers of silicon sub-detectors



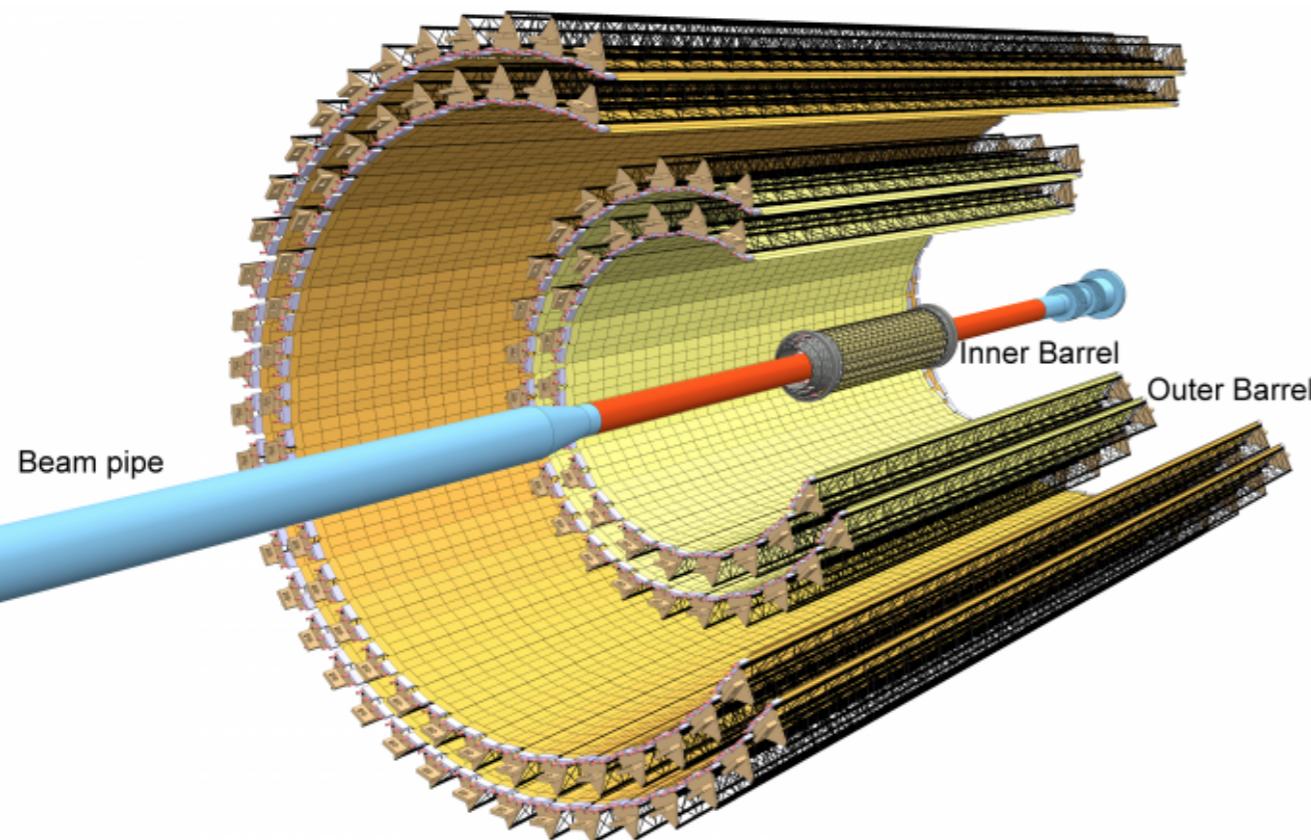
Detection of charged particles with the ITS



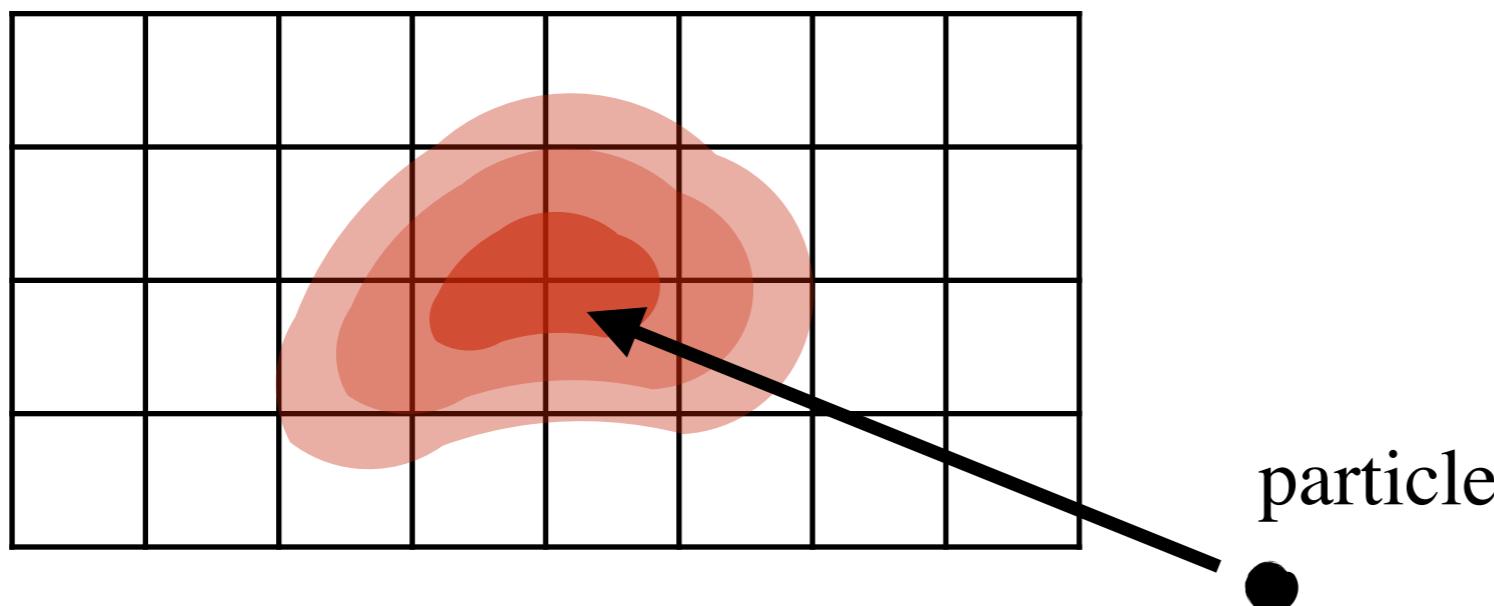
When the charged particles pass through each layer, they fire a certain amount of pixels, depending on their energy loss



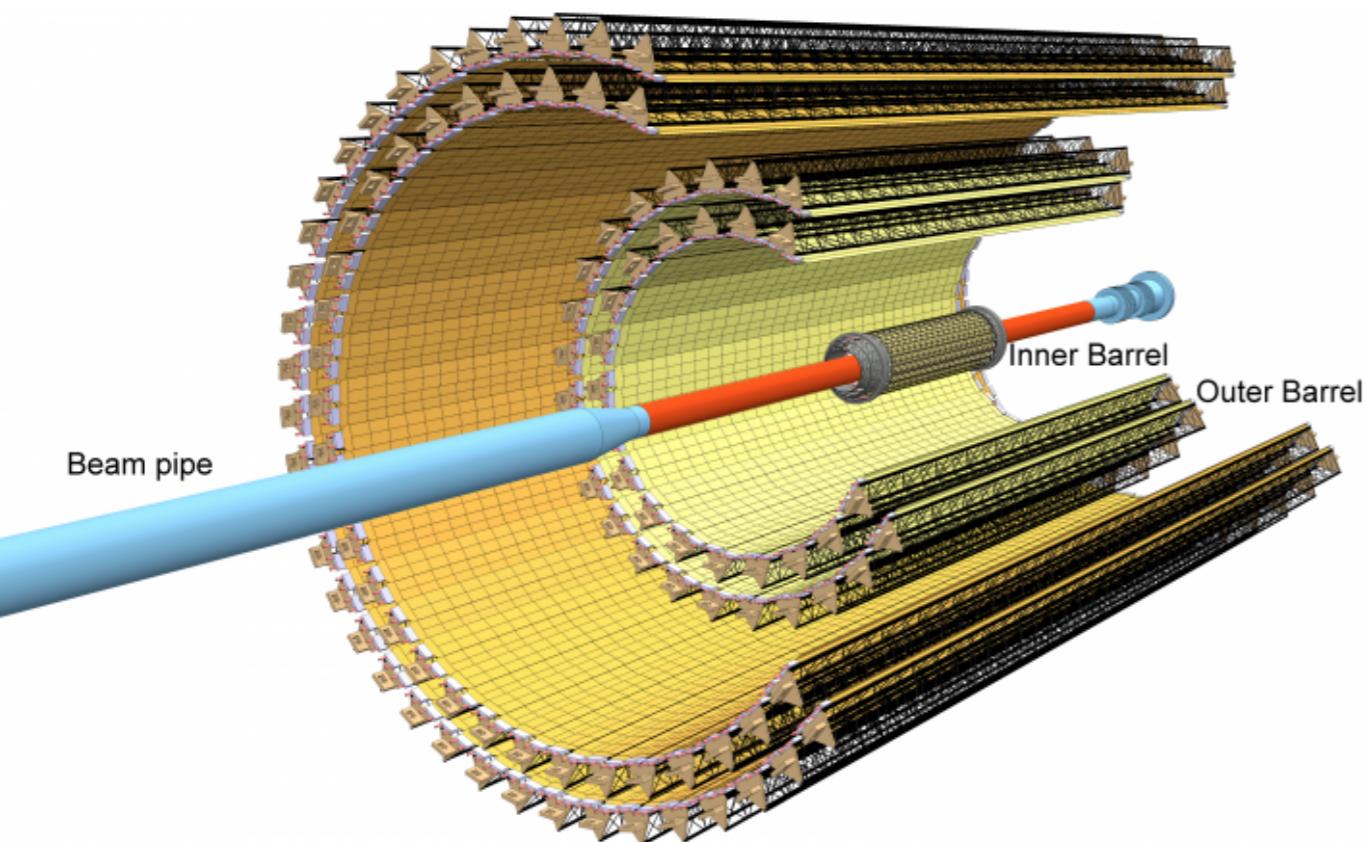
Detection of charged particles with the ITS



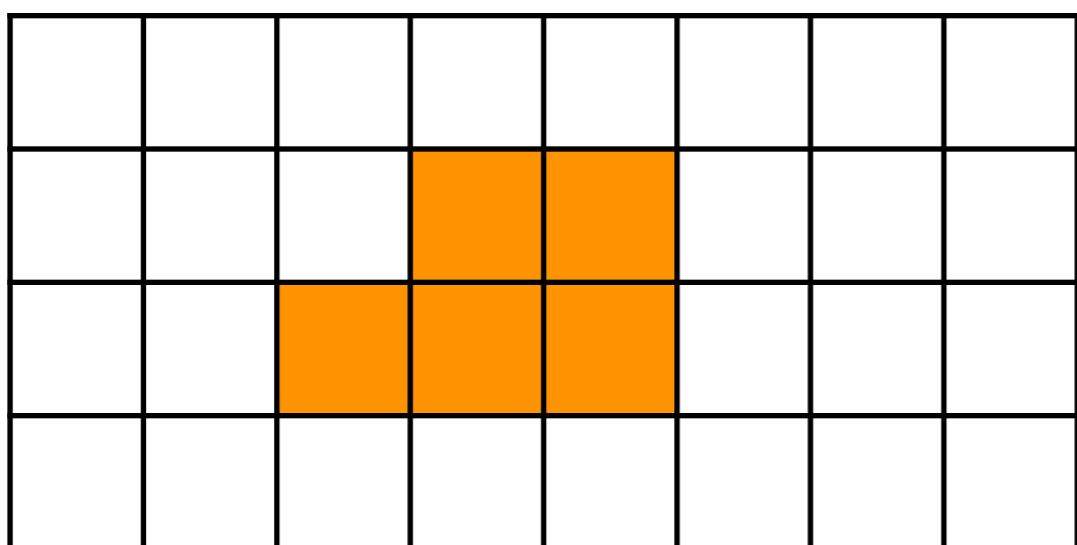
When the charged particles pass through each layer, they fire a certain amount of pixels, depending on their energy loss



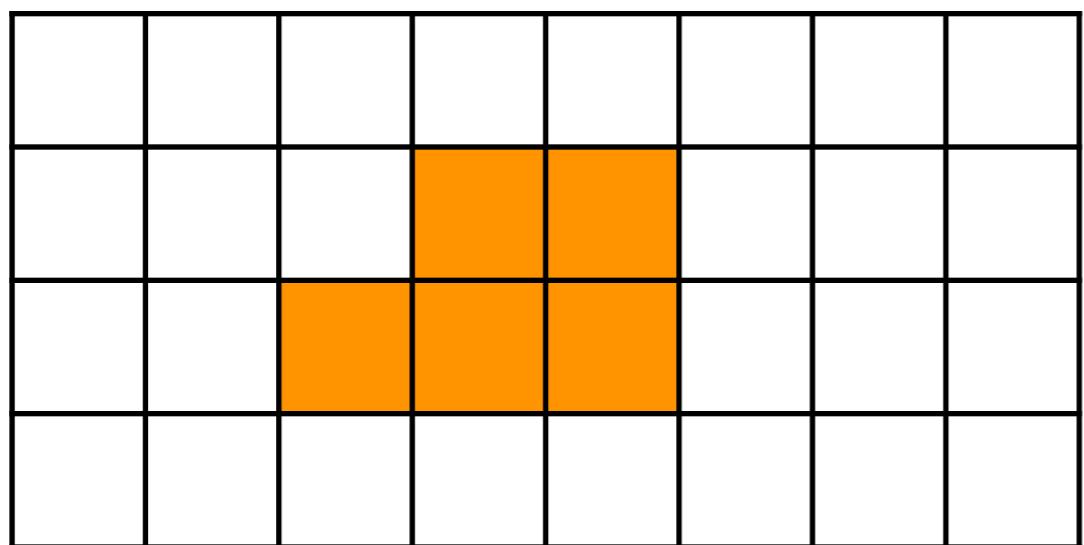
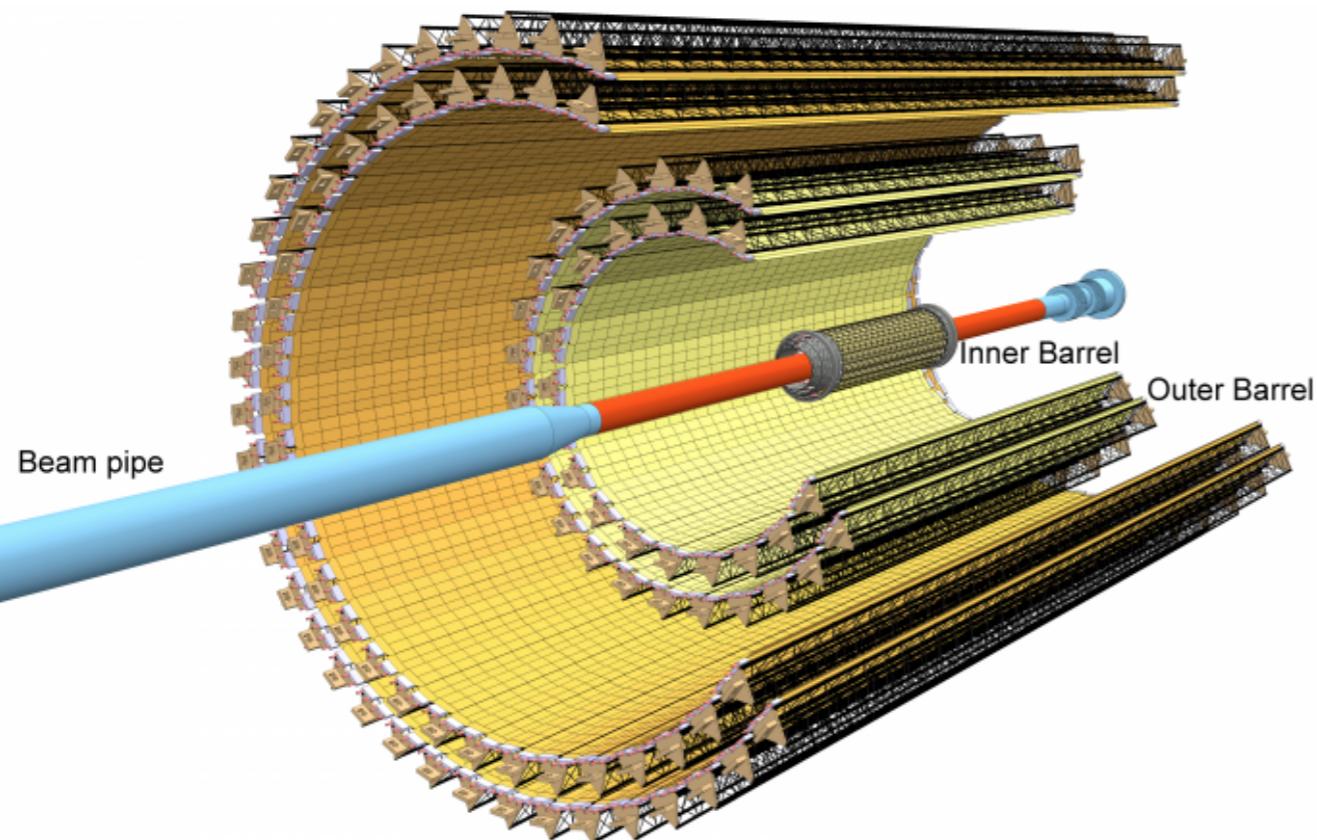
Detection of charged particles with the ITS



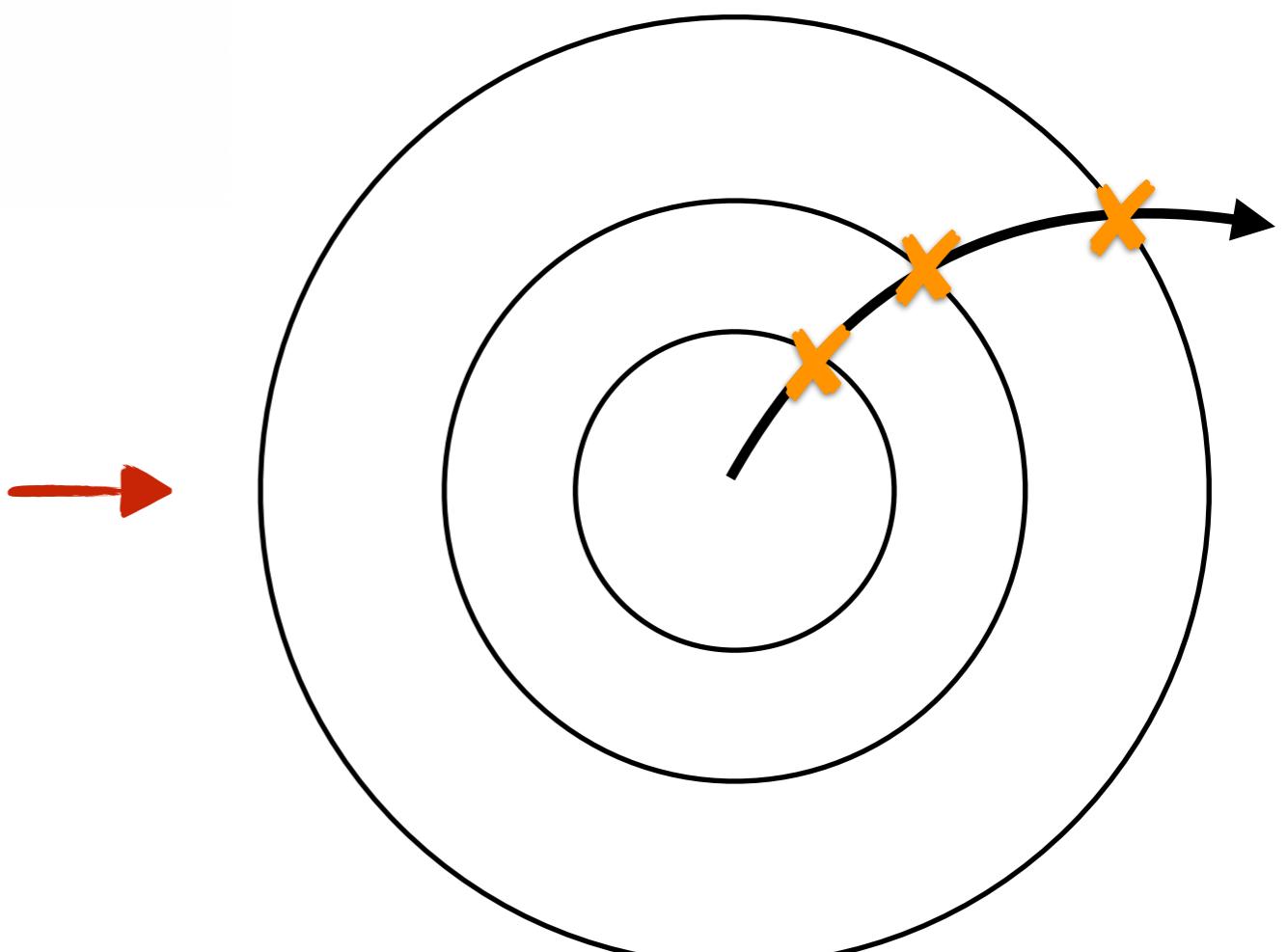
→ When the charged particles pass through each layer, they fire a certain amount of pixels, depending on their energy loss



Detection of charged particles with the ITS



When the charged particles pass through each layer, they fire a certain amount of pixels, depending on their energy loss



Data simulation

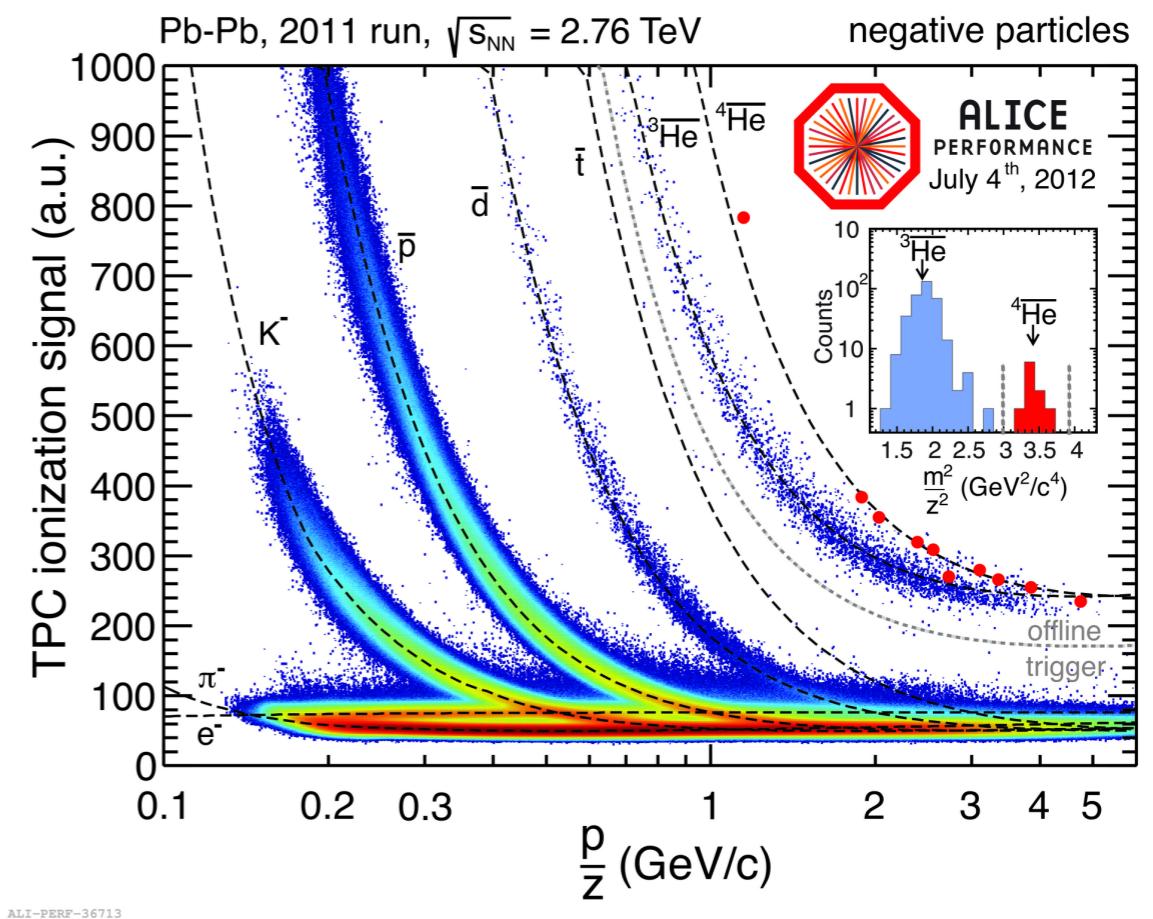
→ For the simulation of the data considered two different particle species were considered:

- (i) pions (π^\pm)
- (ii) helium nuclei (${}^3\text{He}$)

→ The cluster size of pions and helium nuclei is expected to be different because of their energy loss

→ Simulated transport of 1000 pions and 250 helium nuclei only through the innermost layer of the detector with a Monte Carlo (no noise included)

→ Coordinates of the fired pixels are the input data for the DBSCAN algorithm



Application of the DBSCAN algorithm

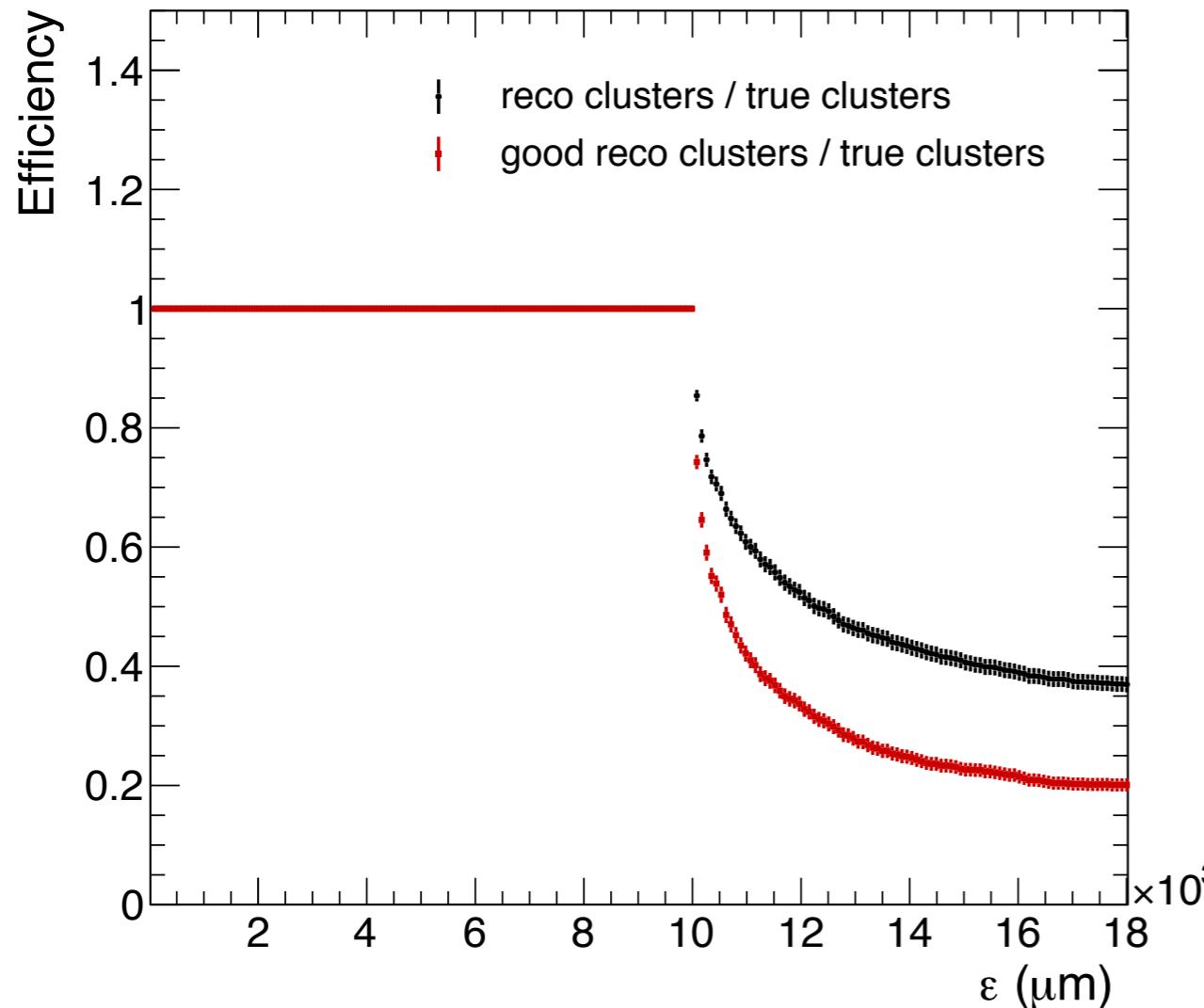
- Python implementation with `sklearn` module (<https://pypi.python.org/pypi/scikit-learn/>)
- Analysis with ROOT framework (<https://root.cern.ch/>)
- Study of the cluster reconstruction efficiency as a function of the parameters ε and $minPts$
- Quantities of interest:
 - (i) number of reconstructed clusters
 - (ii) number of good reconstructed clusters → clusters formed by pixels fired by the same particle

Results - $minPts$ fixed

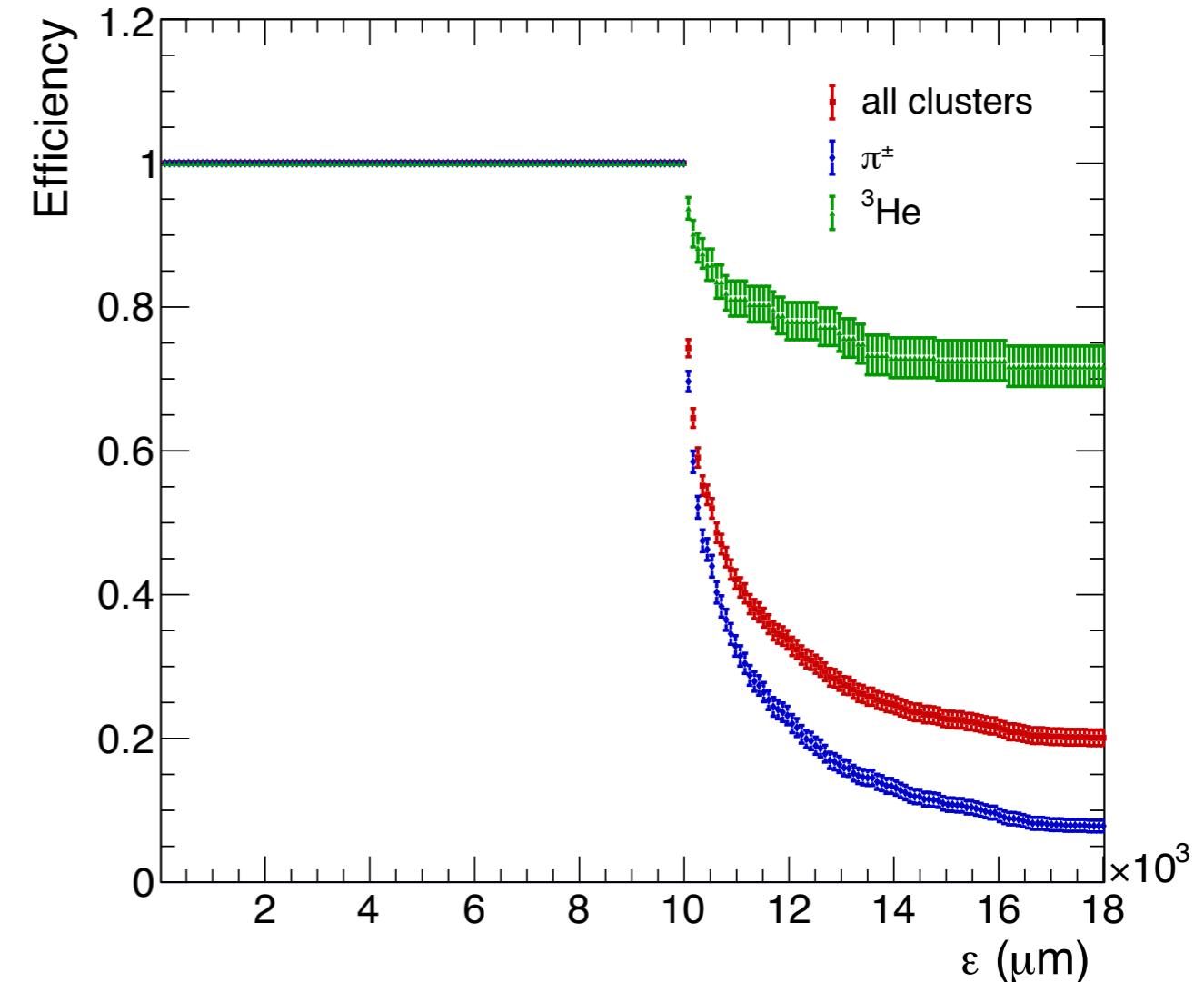
→ The efficiency of the algorithm was tested varying ε , keeping $minPts$ fixed

$minPts = 1$

Cluster reconstruction efficiency



Good reco clusters / true clusters

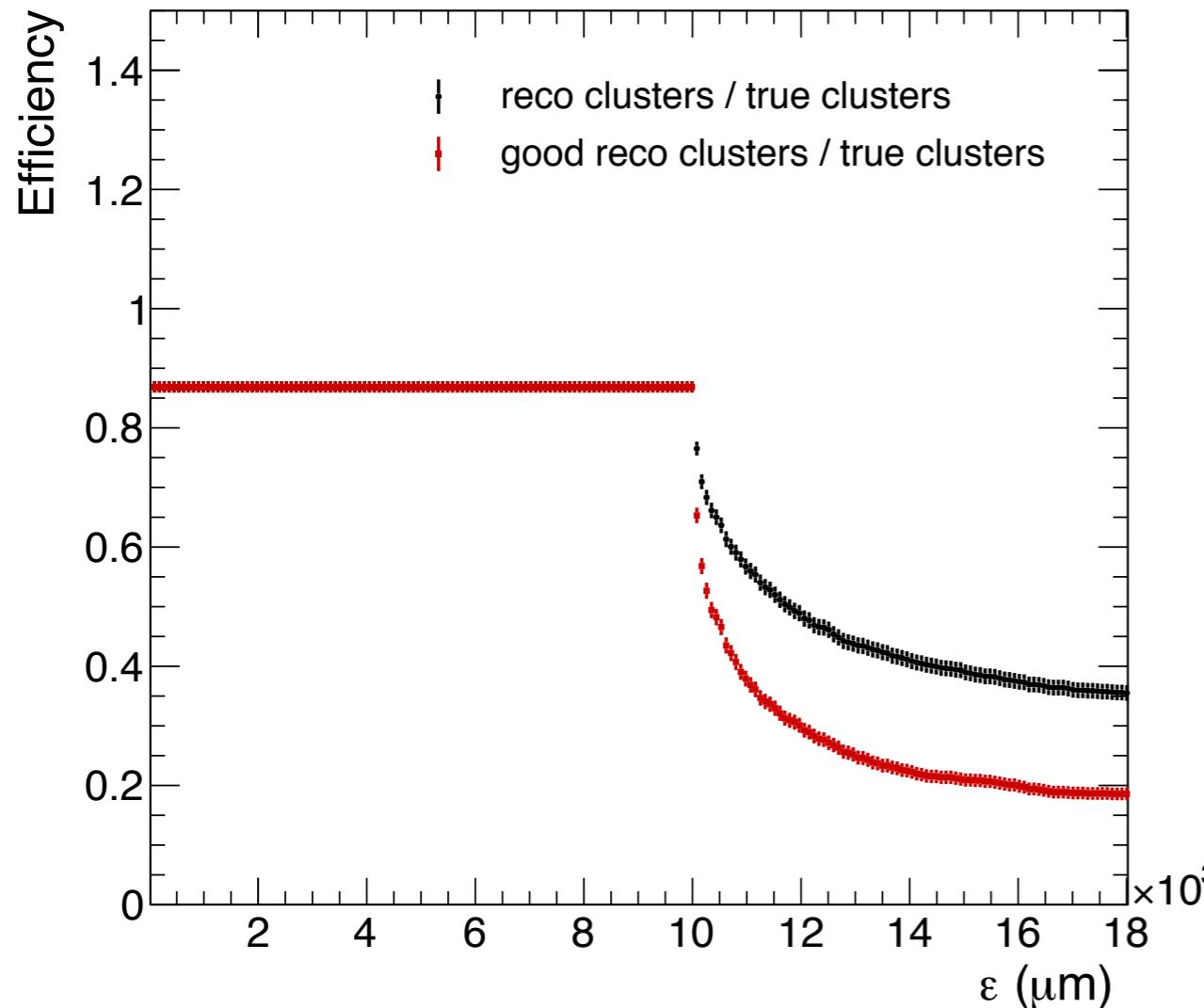


Results - $minPts$ fixed

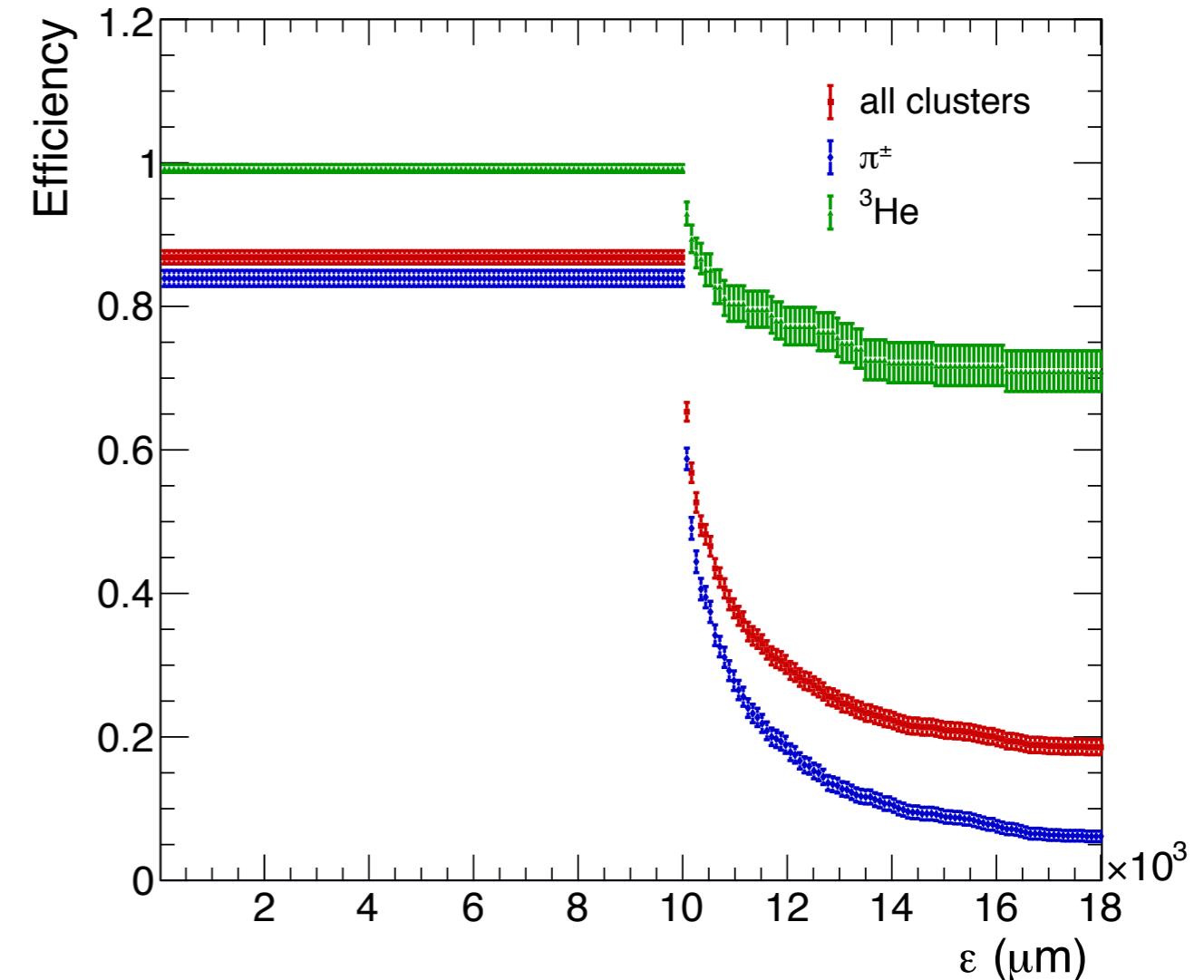
→ The efficiency of the algorithm was tested varying ε , keeping $minPts$ fixed

$minPts = 2$

Cluster reconstruction efficiency



Good reco clusters / true clusters

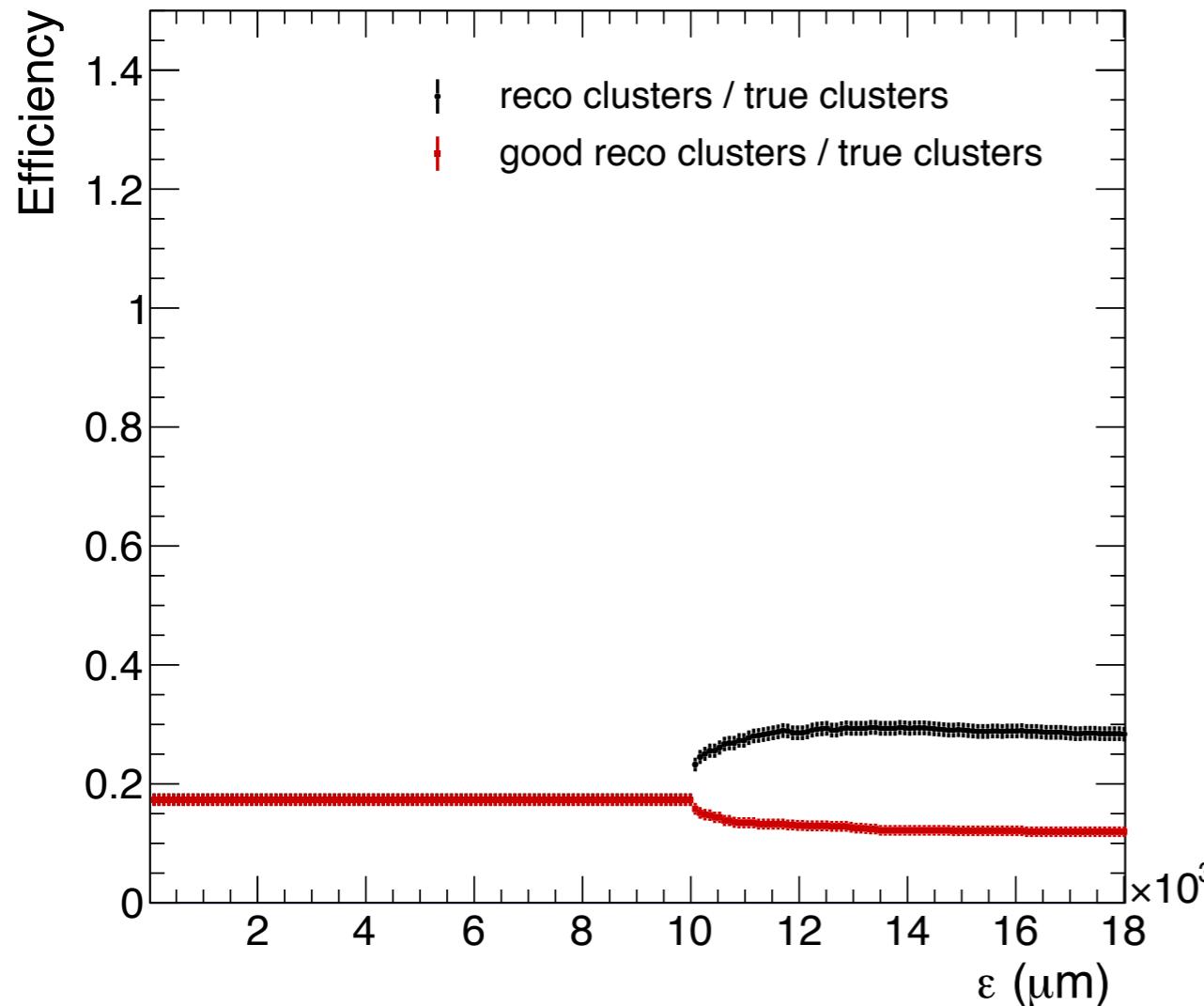


Results - $minPts$ fixed

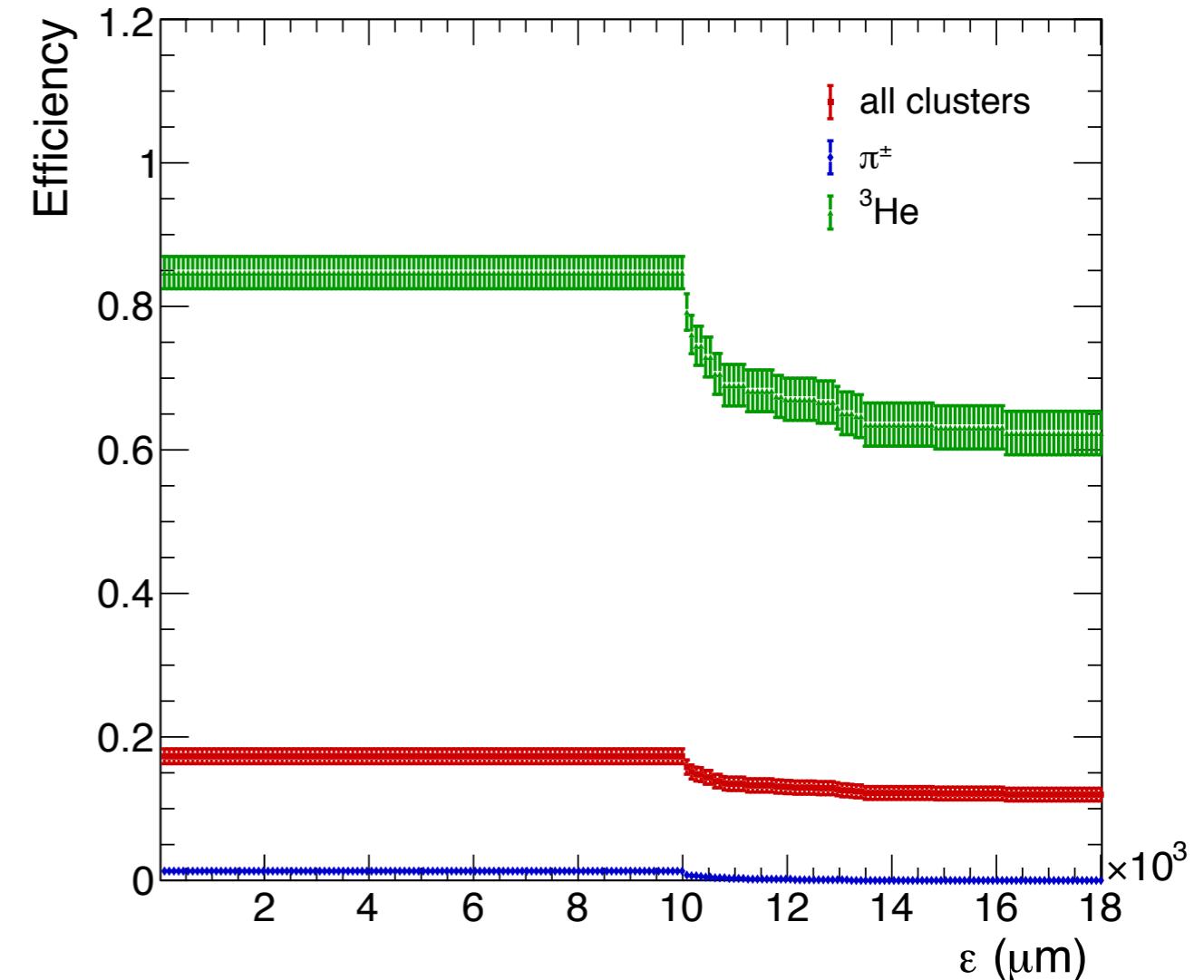
→ The efficiency of the algorithm was tested varying ε , keeping $minPts$ fixed

$minPts = 5$

Cluster reconstruction efficiency



Good reco clusters / true clusters

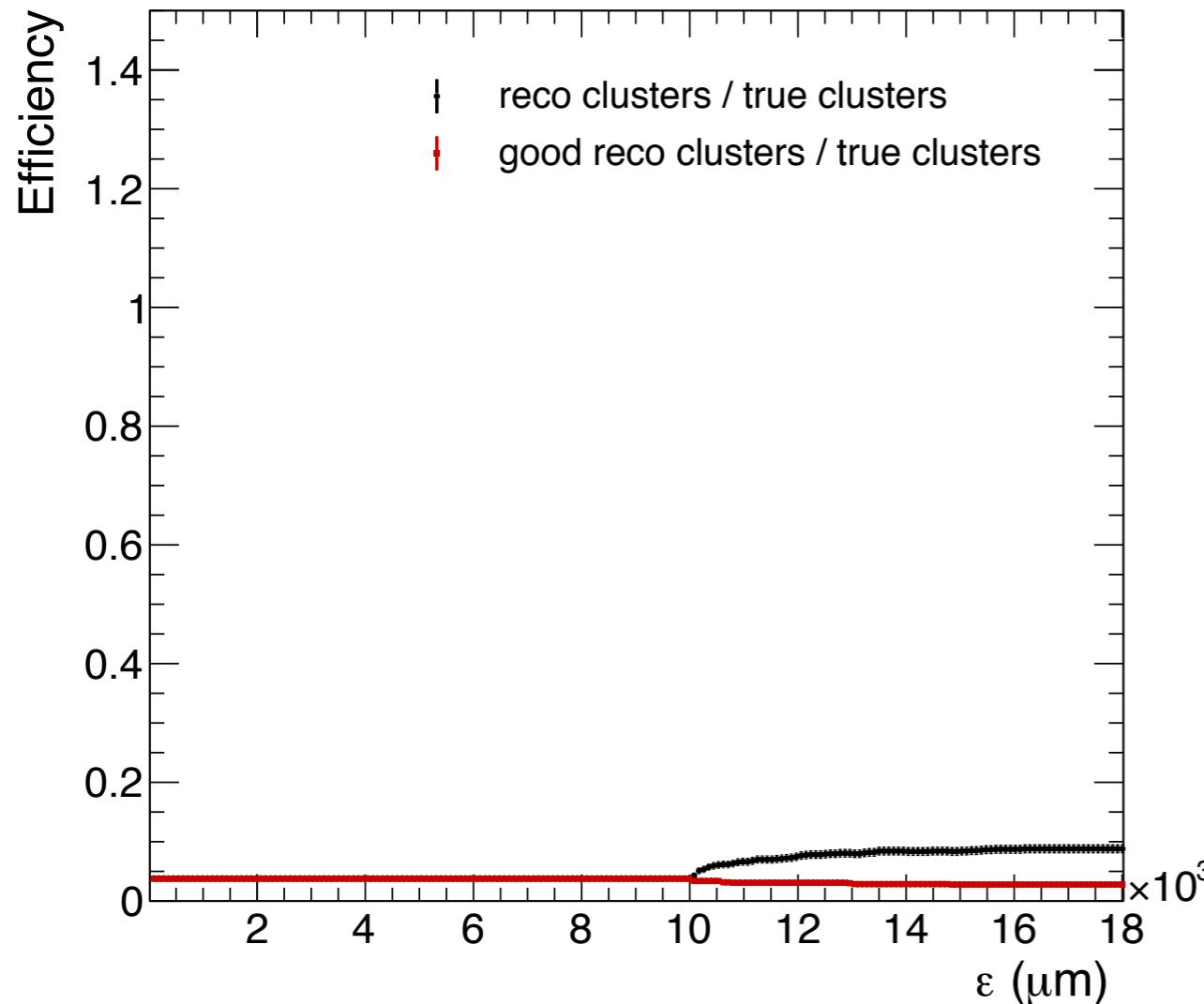


Results - $minPts$ fixed

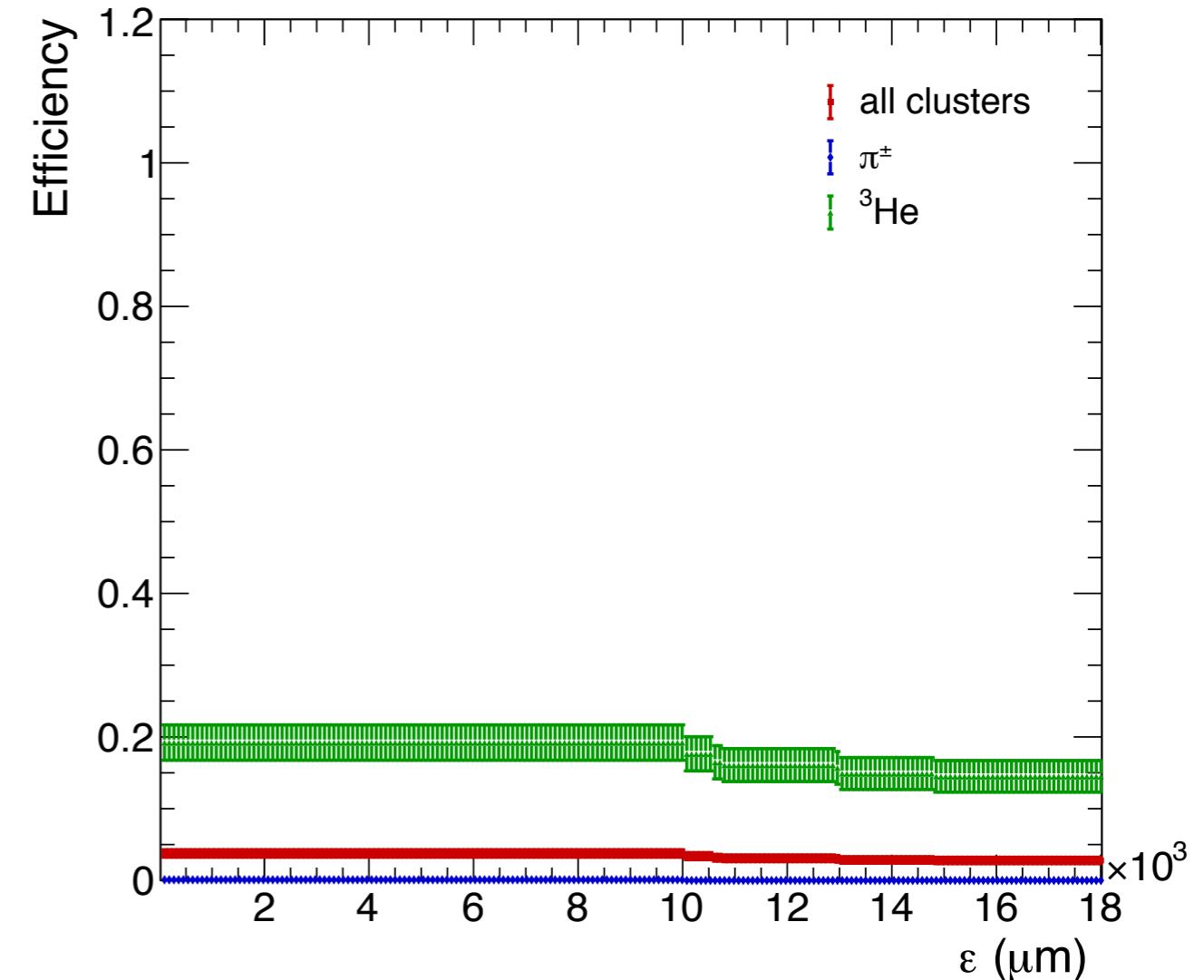
→ The efficiency of the algorithm was tested varying ε , keeping $minPts$ fixed

$minPts = 10$

Cluster reconstruction efficiency



Good reco clusters / true clusters

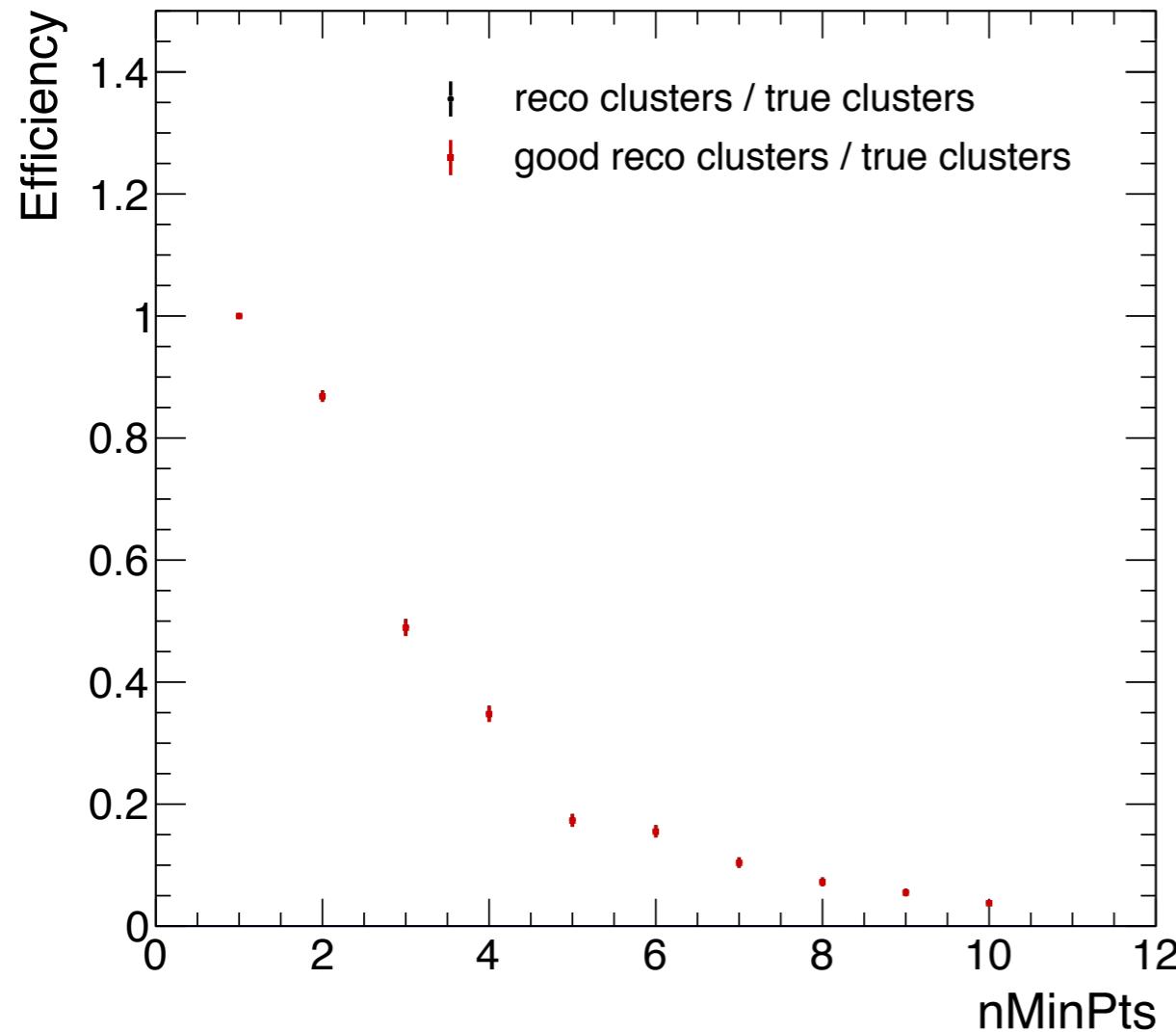


Results - ε fixed

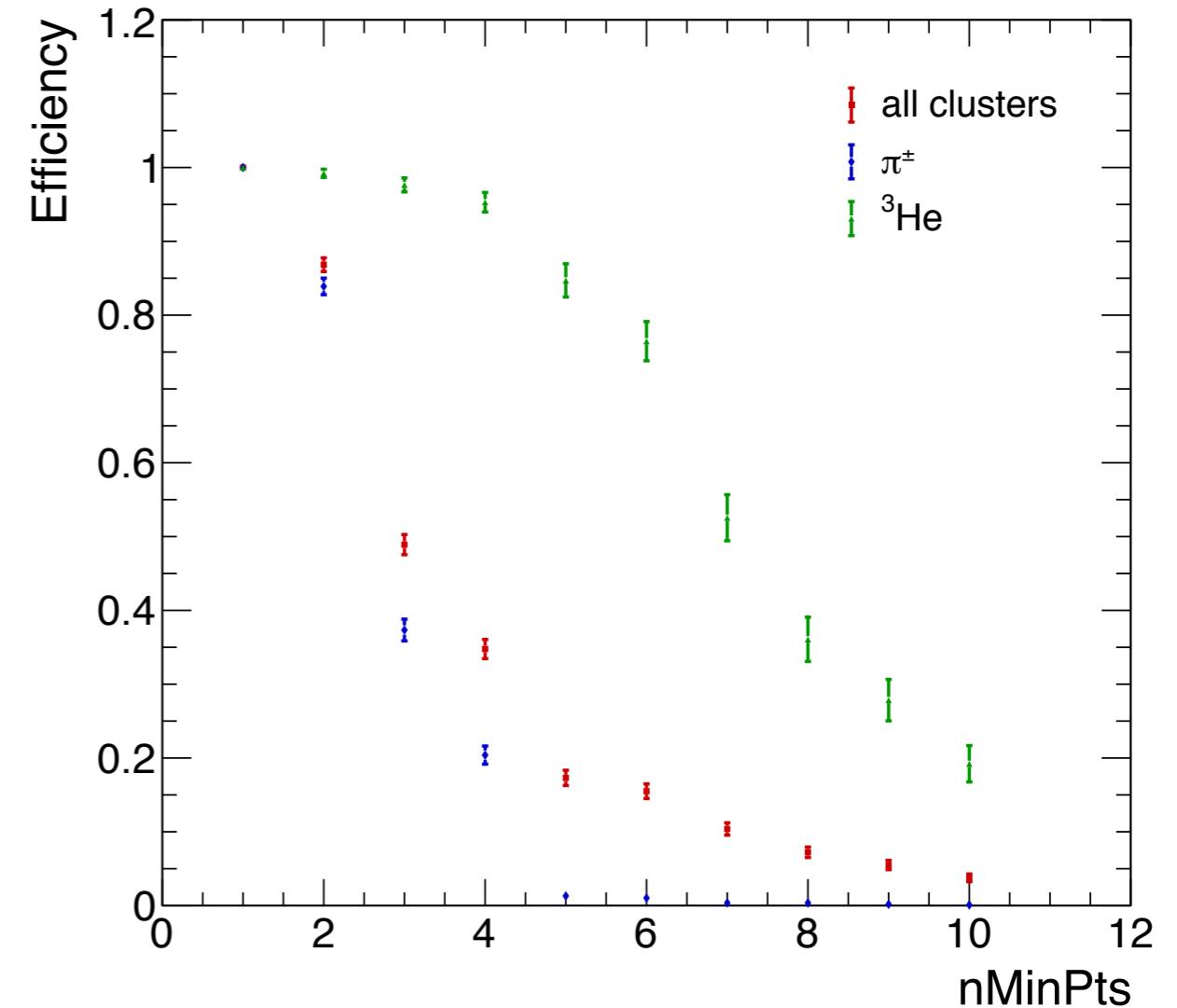
→ The efficiency of the algorithm was tested varying $minPts$, keeping ε fixed

$$\varepsilon = 90 \mu\text{m}$$

Cluster reconstruction efficiency



Good reco clusters / true clusters



Summary

- DBSCAN is a valuable algorithm both to find clusters and to separate clusters generated by different particle species
- Optimisation of the parameters depends on pixel density (particle multiplicity) and kind of particles searched (energy deposition
 - cluster size)
- For real life use cases, the procedure is much more complex because the information has to be matched with other sub-detectors, and there are many particle species, dead pixels and noise, that should be rejected (not included in the example presented)
 - The noise suppression is usually performed by combining the results from different layers (reconstruction of full trajectories)