Solving the Berger Equation

$$\frac{\partial \phi}{\partial t} + \phi \frac{\partial \phi}{\partial x} = d \frac{\partial^2 \phi}{\partial x^2}$$

Equation 1 - Berger Equation

The discretization of the Berger Equation gives us:

$$\phi_{n+1}^{k} = \phi_{n}^{k} + \Delta t \left(d \frac{\phi_{n}^{k+1} - 2\phi_{n}^{k} + \phi_{n}^{k-1}}{\Delta x^2} - \phi_{n}^{k} \frac{\phi_{n}^{k+1} - \phi_{n}^{k-1}}{2\Delta x} \right)$$

Equation 2 - Discretized Berger Equation

Characteristics of the simulated Equation:

$$d = 0{,}01 \qquad x \in [0,1]$$

Boundary conditions:

$$\frac{\partial \phi}{\partial t}(1,t) = e^{-2t} \qquad \phi(0,t) = 0$$

Initial conditions:

$$\phi(x,0) = e^{-16(t-0.5)^2} - e^{-4}$$