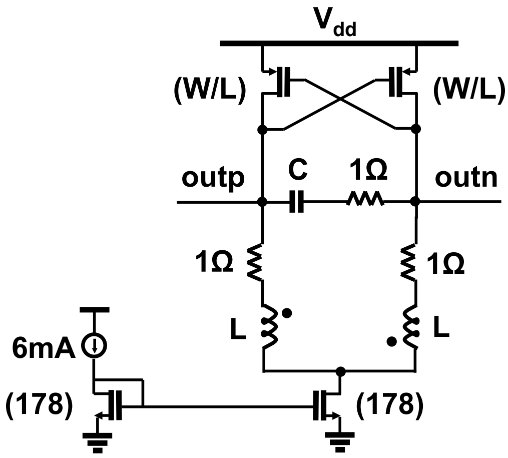
RF Circuit Design **Prof. Salvatore Levantino**

Available time: 2 hours 23 Luglio 2014

Problem #1



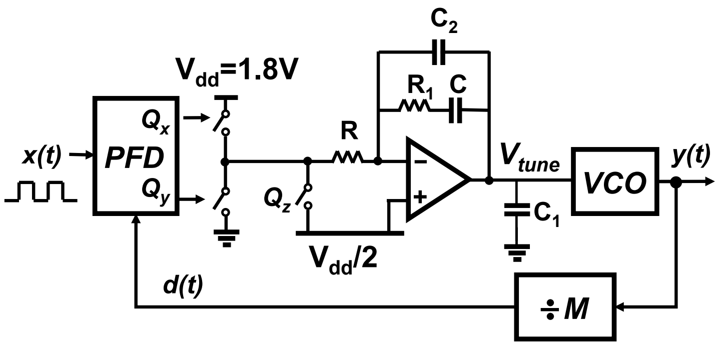
Assume the FETs with V_t=0.4V, constant $1/2\mu_nC_{ox}$=0.2mA/V^2 (for the nMOSFETs), $1/2\mu_pC_{ox}$=0.1mA/V^2 (for the pMOSFETs).

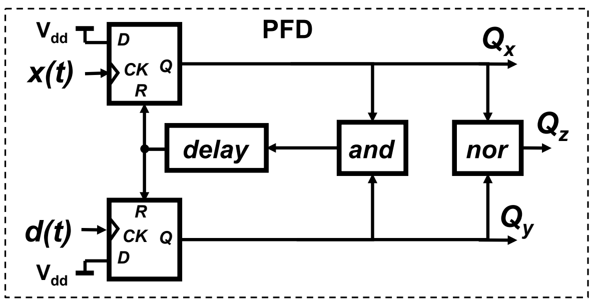
a) Neglecting the mutual inductance between the inductors, set **the value of the capacitance *C* and the inductance *L*** so that the zero-peak amplitude of the differential voltage (V_{outp}-V_{outn}) be equal to 1V at 5GHz.

b) **Set the values of the voltage V_{dd} and the (W/L) of the pMOSFETs** to get a gain margin factor on the oscillation startup equal to 2 and to guarantee proper behavior of the transistors in DC biasing.

c) Taking into account the **magnetic coupling** between the inductors L with coupling factor k = 0.5, derive the **new values of V_{dd}, (W/L), L and C,** which fullfil the conditions in questions a) and b).

Problem #2





The PLL in the figure embeds the PFD in the inset, where the block “delay” introduces a delay of 0.5ns. The swtiches have infinite resistance (when off) and 10Ω (when on). The reference clock x(t) has 50MHz frequency. The frequency-division factor is M=55 and the VCO frequency varies in the range 1900-2100MHz, sweeping the V_{tune} from 0 to V_{dd}. Let the capacitors be C_1=100pF and the resistor R_1=100Ω.

a) Assuming an ideal opamp (with infinite gain and bandwidth), set **the value of R and C** to get two complex dominant (closed-loop) poles at 100kHz located at 45 degree on the Gauss plane.

b) Assuming that the resistance of the switch driven by Q_X is 15Ω (when on), set the minimum **value of C_2**, to get the level of the spur at 50 MHz in the spectrum of y(t) lower than -80dBc.

c) Assuming all the switches with resistance 10Ω (when on), but an offset voltage of 100mV for the opamp, **can the loop lock? If yes,** what is the value of the **output frequency**, the **delay between x(t) and d(t) at steady state,** the **reference-spur level**?

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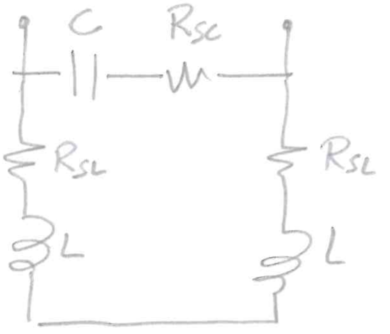
Esercizio n. 1 Traccia di soluzione

a) $V_{out_p} - V_{out_n} = A \cos \omega_0 t$

$A = \frac{2}{\pi} I_{bias} R_{tank}$; $R_{tank} = \frac{\pi}{2} \frac{A}{I_{bias}} = \frac{\pi}{2} \frac{1}{6 \cdot 10^{-3}} = 262\ \Omega$



Ip. $Q^2 = \left(\frac{2\omega_0 L}{R_{SC} + 2R_{SL}}\right)^2 \gg 1$

$R_{tank} = \frac{(\omega_0 \cdot 2L)^2}{R_{SC} + 2R_{SL}}$; $2\omega_0 L = \sqrt{R_{tank}(R_{SC} + 2R_{SL})}$

$L = \frac{1}{2\omega_0} \cdot \sqrt{R_{tank}(R_{SC} + 2R_{SL})} = \frac{1}{4\pi \cdot 5 \cdot 10^9} \sqrt{262 \cdot 3} = \underline{0.45\ nH}$

$C = \frac{1}{\omega_0^2 \cdot 2L} = \underline{1,12\ pF}$ $Q = 9.42$ ip. verificata

b) Startup $\frac{g_m R_{tank}}{2} = 2$; $g_m = \frac{4}{R_{tank}}$

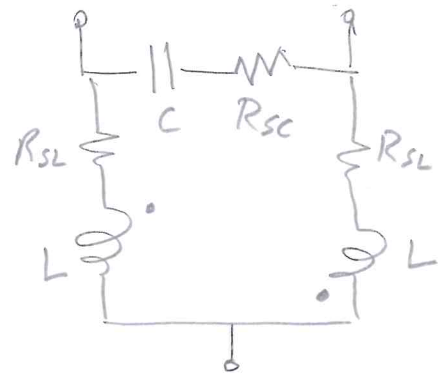
$g_m = 2\sqrt{K_p \frac{W}{L} \frac{I_{bias}}{2}} \Rightarrow \left(\frac{W}{L}\right) = \left(\frac{g_m}{2}\right)^2 \frac{1}{K_p} \cdot \frac{2}{I_{bias}} =$

$= \frac{g_m^2}{4} \frac{1}{K_p} \frac{2}{I_{bias}} = \frac{1}{2} \cdot \frac{16}{(262)^2} \frac{1}{0.1 \cdot 10^{-3}} \frac{1}{6 \cdot 10^{-3}} = \underline{194} \Rightarrow V_{ov} = 0.39\ V$

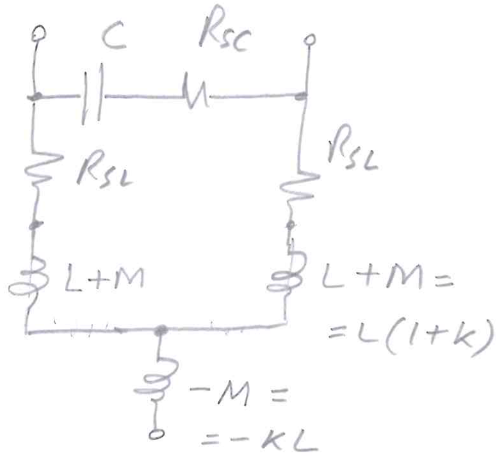
$V_{ov\,nmos} = \sqrt{\frac{I_{bias}}{K_n (W/L)_n}} = \sqrt{\frac{6 \cdot 10^{-3}}{0.2 \cdot 10^{-3} \cdot 178}} = 0.41\ V$

$\underbrace{V_{dd} - V_{ov\,pmos} - V_T}_{\text{DC voltage}} \geq V_{ov\,nmos}$; $V_{dd} \geq \underline{1,2\ V}$

c)



$\Rightarrow$



$$K = \frac{M}{L} \quad ; \quad M = K \cdot L$$

$$R_{tank} = \frac{[\omega_0 \cdot 2L(1+K)]^2}{R_{SC} + 2R_{SL}} \quad ; \quad L = \frac{1}{2\omega_0(1+K)} \sqrt{R_{tank}(R_{SC}+2R_{SL})} =$$

$$= 0{,}3\ nH$$

$$C = \frac{1}{\omega_0^2 2L(1+K)} = 1{,}12\ pF$$

Esercizio n. 2

$$K_{VCO} = 2\pi \cdot \frac{200 \cdot 10^6}{1{,}8} = 697 \frac{Mrad}{Vs}, \quad M = 55, \quad R_1 = 100\,\Omega$$

a) $$H_{loop} = \frac{Vdd/2}{2\pi} \cdot \frac{1}{R'} \cdot \frac{1}{s(C+C_2)} \cdot \frac{1 + sR_1C}{1 + sR_1\frac{CC_2}{C+C_2}} \cdot \frac{K_{VCO}}{sM} \simeq$$

$$\simeq \underbrace{\frac{Vdd}{4\pi} \cdot \frac{1}{R'C} \cdot \frac{K_{VCO}}{s^2 M}}_{K} \cdot (1 + s\underbrace{(R_1C)}_{\tau_z}) \qquad R' = R + r_{on}$$

$C_2 \ll C$

$$= \frac{K}{s^2}(1 + s\tau_z) = -1 \quad ;$$

$$s^2 + s\tau_z \cdot K + K = 0$$

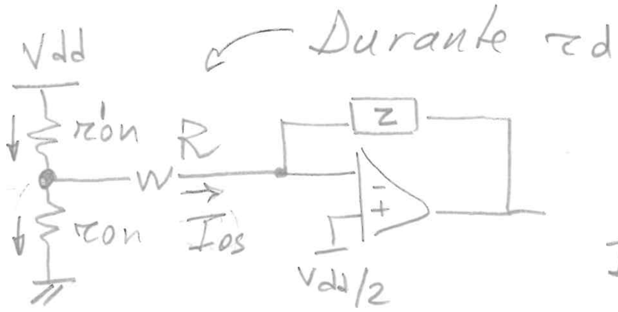
$$\Rightarrow \begin{cases} \omega_n^2 = K \\ 2\xi\omega_n = K \cdot \tau_z \end{cases} \Rightarrow \quad K = \omega_n^2 = (2\pi \cdot 10^5)^2$$

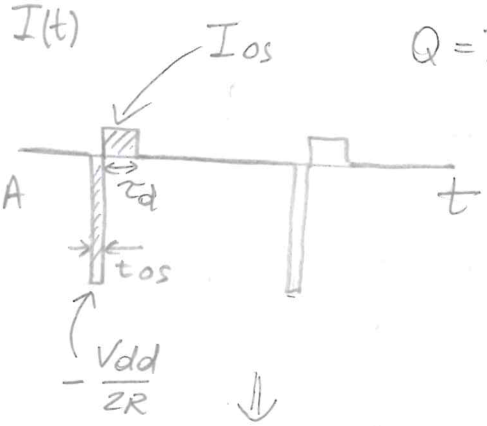
$$\xi = \frac{K \cdot \tau_z}{2\omega_n} = \frac{\omega_n^{\cancel{2}} \cdot \tau_z}{2\cancel{\omega_n}}$$

$$\xi = \frac{\sqrt{2}}{2} \Rightarrow \omega_n \cdot \tau_z = \sqrt{2} \quad ; \quad \tau_z = \frac{\sqrt{2}}{\omega_n} = 2{,}25\ \mu s$$

$\Rightarrow C = 22{,}5\ nF$ $\quad R' = 205\,\Omega$ $\quad R = R' - r_{on} = 195\,\Omega$

b)





$Q = I_{os} \cdot \tau_d$

$$I_{os} \overset{(*)}{=} \frac{V_{dd}}{2} \cdot \frac{r_{on} - r'_{on}}{r_{on} r'_{on} + R(r_{on} + r'_{on})} = \cancel{-1.1}\ \mathbf{-0.92}\ mA$$

$$t_{os} = \frac{I_{os} \cdot \tau_d}{V_{dd}/2R} = \cancel{0.12}\ \mathbf{0.105}\ ns$$

$$\Downarrow$$

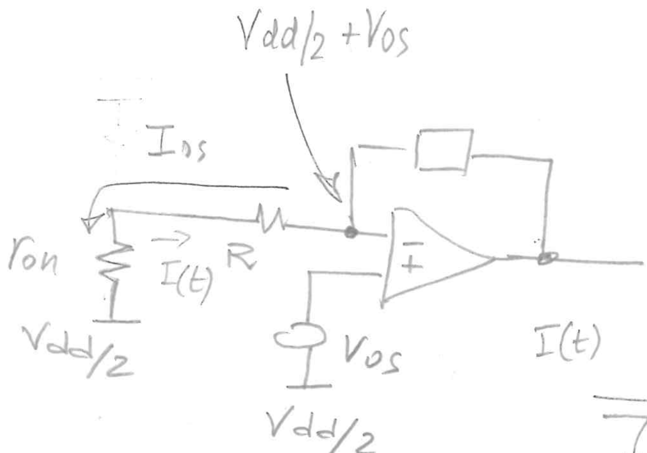
$$I(t) = \frac{2Q}{T_{ref}} \cdot \omega_{ref} \cdot \frac{\tau_d + t_{os}}{2} \cdot \cos \omega_{ref} t + \dots$$

$$\mathcal{L} = \frac{S_\varphi}{2} = \left(\frac{1}{2} \cdot \frac{K_{vco}}{\cancel{\omega_{ref}}/2\pi} \cdot \frac{1}{\cancel{\omega_{ref}} C_2} \cdot \frac{Q}{\cancel{T_{ref}}} \cdot \cancel{\omega_{ref}} \cdot (\tau_d + t_{os}) \right)^2$$

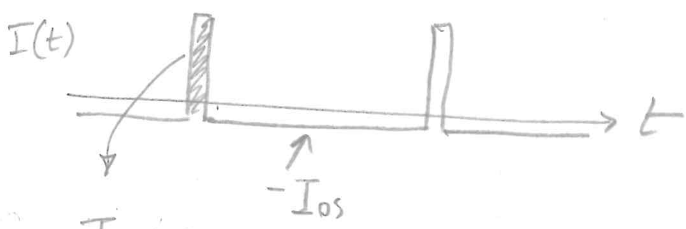
$$\Rightarrow \quad \frac{1}{4\pi} \cdot \frac{K_{vco}}{C_2} \cdot I_{os} \cdot \tau_d (\tau_d + t_{os}) = 10^{-4}$$

$$C_2 = \cancel{189}\ \mathbf{154}\ pF$$

c)



$$I_{os} = \frac{V_{os}}{R + r_{on}}$$

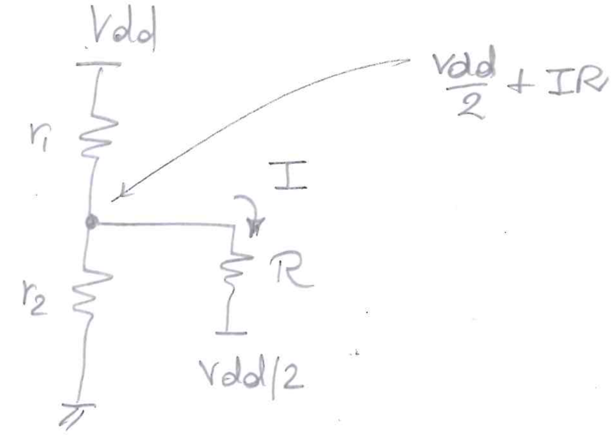


$$Q = I_{os} \cdot T_{ref} = \frac{V_{os}}{R + r_{on}} \cdot T_{ref}$$

$$I(t) = \frac{2Q}{T_{ref}} \cos \omega_{ref} t = 2 \frac{V_{os}}{R + r_{on}} \cdot \frac{\cancel{T_{ref}}}{\cancel{T_{ref}}} \cos \omega_{ref} t$$

$$\mathcal{L} = \left(\frac{1}{2} \frac{K_{vco}}{\omega_{ref}} \cdot \frac{1}{\omega_{ref} C_2} \cdot \frac{2 V_{os}}{R + r_{on}} \right)^2 = -32{,}8\ dBc$$

(*)



$$\frac{\frac{Vdd}{2} - IR}{r_1} = \frac{\frac{Vdd}{2} + IR}{r_2} + I$$

$$\frac{Vdd}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = I\left(1 + \frac{R}{r_2} + \frac{R}{r_1}\right)$$

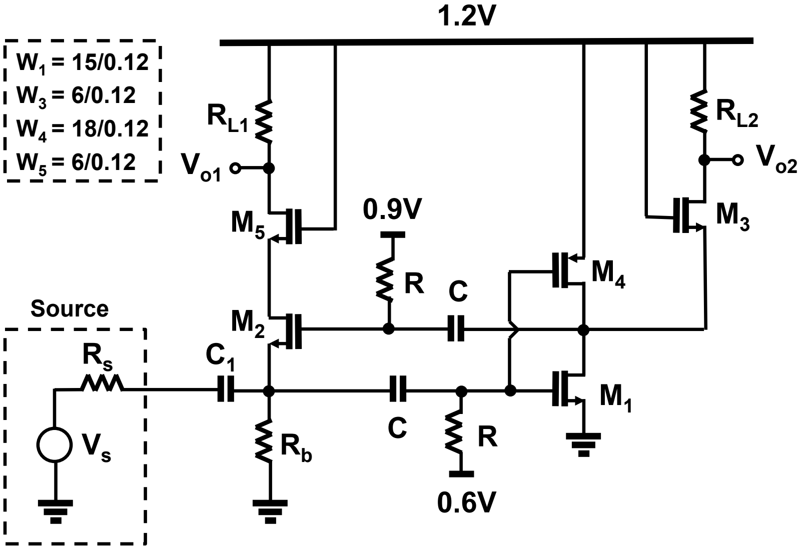
$$I = \frac{Vdd}{2} \frac{\frac{r_2 - r_1}{r_1 r_2}}{1 + R\frac{r_1 + r_2}{r_1 r_2}} = \frac{Vdd}{2} \frac{r_2 - r_1}{r_1 r_2 + R(r_1 + r_2)}$$

RF Circuit Design **Prof. Salvatore Levantino**

Available time: 2 hours 8 Luglio 2014

Problem #1

Assume R_b=750Ω, R=100kΩ, C_1=1nF, C=1pF. The FETs have V_t=0.4V, $1/2\mu_nC_{ox}$=0.2mA/V^2 (for the nMOS), $1/2\mu_pC_{ox}$=0.1mA/V^2 (for the pMOS), γ=2/3 e α=1. The (W/L) of the FETs are: (15/0.12) for M1, (6/0.12) for M3 and M5, and (18/0.12) for M4.



a) **Set the value of (W/L) of M2** to get source matching (R_s=50Ω) at 1GHz, and derive **the circuit DC biasing** (Please provide the values of DC currents/voltages at all branches/nodes, and verify the hypotheses).

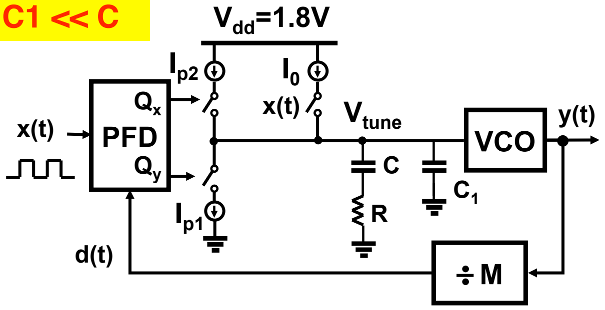
b) **Set the values of the resistors R_{L1} and R_{L2}** and verify the **DC biasing of the circuit**, so that: (i) the voltage gain $(V_{o1}-V_{o2})/V_s$ is equal to 18dB at 1GHz and (ii) the thermal noise of M2 does not contribute to the amplifier noise figure at 1GHz.

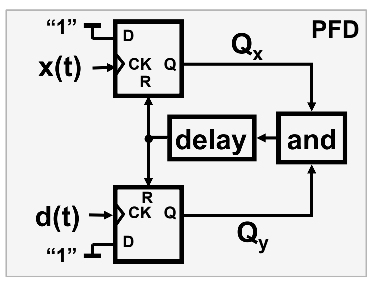
c) Calculate the **noise figure** at 1GHz, taking into account the noise of **R_s, R_b, R_{L1}, R_{L2}, M5**.

Problem #2

Note: Take into account that C1 << C

The PLL in the figure embeds the PFD in the inset, where the block "delay" introduces a delay of 3ns. The current generators in the PLL are off when the switches in series are off. The reference clock x(t) has 50% duty cycle and 50MHz frequency. The frequency-division factor is M=55. The charge-pump currents: $I_{P1}=I_{P2}$=1.8mA and I_0=10μA. Let the capacitors be C=1μF and C_1=1nF and the resistor R=100Ω.





a) **Set the value of the VCO gain K_{vco}** to get the maximum loop phase-margin. What is the value of the **phase margin (in degree)** and the **closed-loop bandwidth**?

b) Calculate the **time delay** between x(t) and d(t) at steady state, plot the **total current** injected into the loop filter versus time, and calculate **the level of the spur** in the spectrum of y(t).

c) Assuming x(t) to be affected by a white phase noise with SSCR = -155dBc/Hz and the free-running VCO to have 1/f2 phase noise with SSCR = -135dBc/Hz at 10MHz offset from the carrier, evaluate the **phase noise level of y(t) at 100Hz and at 10MHz** offset from the carrier.

Esame del 8/7/'14 — Elettronica RF

Esercizio n. 1 — Traccia di soluzione

a)

$$Z_{in} = R \,//\, R_b \,//\, \left(\frac{1/g_{m2}}{1 + \frac{g_{m1}+g_{m4}}{1/g_{m3} + 1/R}} \right)$$

IP. M1, M4, M3 SAT.

$I_{m4} = 0.6\ mA$ — $I_{m3} = 0.4\ mA$

$g_{m4} = 6\ mA/V$ — $g_{m3} = 4\ mA/V$

$I_{m1} = 1\ mA$

$g_{m1} = 10\ mA/V$

$V_{GDm1} = 0$, $V_{GDm4} = 0$ $\Rightarrow$ M1,4 sat.

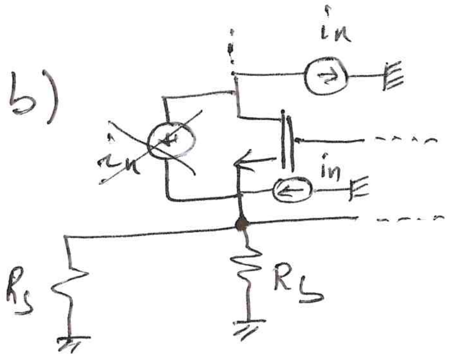
$$Z_{in} \simeq R_b \,//\, \frac{1/g_{m2}}{1 + \frac{g_{m1}+g_{m4}}{g_{m3}}} = R_S = 50\,\Omega$$

$$\Rightarrow g_{m2} = 3.7\ mA/V$$

IP. M2 SAT. $\quad V_{OVm2} = 0.209\,V \quad I_{m2} = 0{,}39\ mA$

$$(W/L)_{m2} = 5{,}3/0{,}12$$

b)



$$\frac{V_{o1} - V_{o2}}{i_n} = -\frac{R_{L1}}{2}\left(1 + \frac{R_S}{R_L}\right) + \frac{R_S}{2}(g_{m1}+g_{m4})\,\cdot$$

$$= 0$$

$$\Rightarrow R_{L1}\left(1 + \frac{R_S}{R_b}\right) = R_S\,(g_{m1}+g_{m4})\,R_{L2}$$

Gain: $\frac{V_{01}-V_{02}}{V_{in}} = \frac{R_{L1}}{2R_S}\left(1-\frac{R_S}{R_b}\right) + \frac{(g_{m1}+g_{m4})R_{L2}}{2} =$

$$= \frac{R_{L1}}{R_S} \Rightarrow R_{L1} = 397\ \Omega$$

$$R_{L2} = \frac{R_{L1}}{R_S}\frac{1+\frac{R_S}{R_b}}{(g_{m1}+g_{m4})} = 1{,}33 \cdot R_{L1} = 529\ \Omega$$

c) $NF = 1 + \frac{R_S}{R_b} + \frac{R_{L1}+R_{L2}}{G^2 \cdot R_S} = 1{,}36 \to 1{,}34\,dB$

M2, M5 non contribuiscono a NF

Esercizio n. 2

a) $\omega_u = \frac{K_{VCO}}{M} \cdot \frac{I_P}{2\pi} \cdot R$

$\omega_u = \sqrt{\omega_z \cdot \omega_p} = \frac{1}{R\sqrt{CC_1}}$

$\Rightarrow K_{VCO} = \frac{M}{R}\frac{2\pi}{I_P}\frac{1}{R\sqrt{CC_1}} = 2\pi \cdot 96{,}6\ \frac{Mrad}{V \cdot s}$

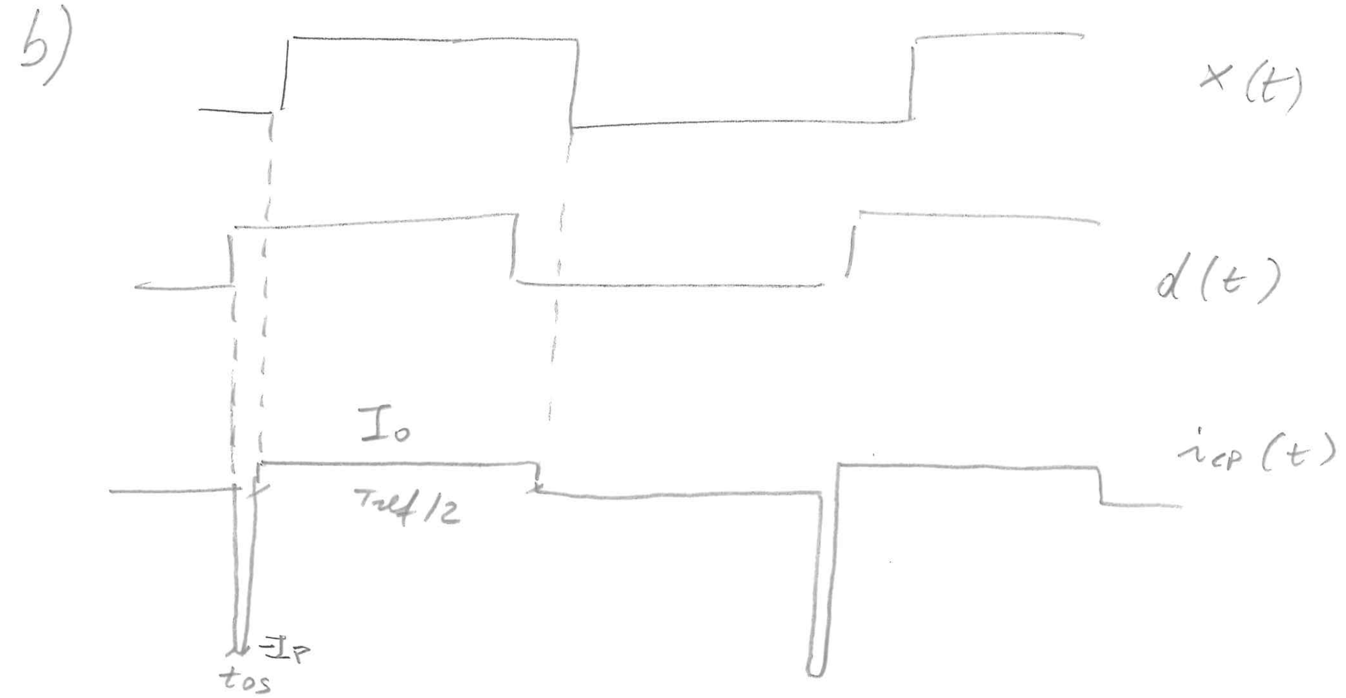
$$\varphi_M = \text{atan}\left(\frac{\omega_u}{\omega_z}\right) - \text{atan}\left(\frac{\omega_u}{\omega_p}\right) =$$

$$= \text{atan}\left(\sqrt{\frac{\omega_p}{\omega_z}}\right) - \text{atan}\left(\sqrt{\frac{\omega_z}{\omega_p}}\right) =$$

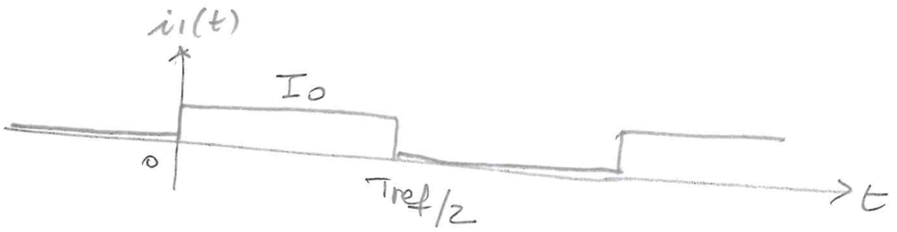
$$= \frac{\pi}{2} - 2\,\text{atan}\sqrt{\frac{\omega_z}{\omega_p}} \Rightarrow 86.4^\circ$$

$\omega_u = 2\pi \cdot 50{,}3\ \frac{Krad}{s}$

b)

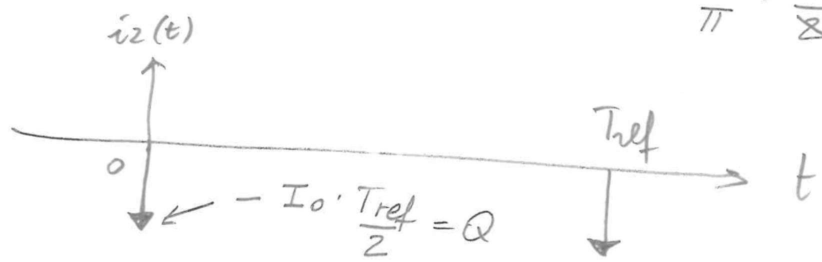


$$I_P t_{os} = I_0 T_{ref}/2 \quad \Rightarrow \quad t_{os} = \frac{I_0}{I_P} \frac{T_{ref}}{2} = 55.5\,\text{ps}$$



$$i_{cp}(t) = i_1(t) + i_2(t)$$

$\rightarrow$ First harmonic : $\frac{\cancel{4}^{\,2}}{\pi} \cdot \frac{I_0}{\cancel{2}} \cdot \sin(\omega_{ref} \cdot t)$



$\rightarrow$ First harmonic : $\frac{2Q}{T_{ref}} \cdot \cos(\omega_{ref} t) =$

$$= -\cancel{2} \frac{I_0 \cancel{T_{ref}}}{\cancel{2} \cancel{T_{ref}}} \cos(\omega_{ref} t)$$

$\Rightarrow$ First harmonic of $i_{cp}(t)$ has amplitude :

$$\sqrt{\left(\frac{2}{\pi} I_0\right)^2 + I_0^2} = I_0 \cdot \sqrt{\frac{4}{\pi^2} + 1}$$

$$\text{Ref. spur} = \frac{S_\varphi}{2} = \frac{1}{2} \cdot \frac{1}{2} \cdot \left(\frac{K_{VCO}}{\omega_{ref}} \cdot I_0 \sqrt{\frac{4}{\pi^2} + 1} \cdot \frac{1}{\omega_{ref} C_1} \right)^2 \rightarrow -89\,\text{dBc}$$

c)

$$S_{\phi_y} = S_{\phi_x} \cdot M^2 \cdot \left| \frac{G_{LOOP}}{1+G_{LOOP}} \right|^2 +$$

$$+ S_{\phi_{VCO}} \cdot \left| \frac{1}{1+G_{LOOP}} \right|^2$$

$$K = \frac{K_{VCO}\, I_P}{M\; 2\pi} \cdot \frac{1}{C+C_1} = (2\pi \cdot 10\,\mathrm{Krad/s}$$

$$G_{LOOP} = \frac{K}{s^2} \cdot \frac{1+s\tau_z}{1+s\tau_p}$$

$\omega_z = 2\pi \cdot 1.5\,\mathrm{K\,rad/s}$

$\omega_p = 2\pi \cdot 1.5\,\mathrm{M\,rad/s}$

At 100 Hz

$$\cdot \begin{cases} s\tau_z \ll 1 \\ s\tau_p \ll 1 \end{cases} : \quad G_{LOOP}(s) \simeq \frac{K}{s^2} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{G_{LOOP}}{1+G_{LOOP}} = \frac{K}{K+s^2} \simeq 1 \qquad \boxed{\frac{s}{\sqrt{K}} \ll 1}$$

$$\frac{1}{1+G_{LOOP}} = \frac{s^2}{K+s^2} \simeq 0$$

At 10 MHz

$$K\frac{\omega_p}{\omega_z} = \left(2\pi \cdot 320\,\frac{\mathrm{Krad}}{s}\right)^2$$

$$\cdot \begin{cases} s\tau_z \gg 1 \\ s\tau_p \gg 1 \end{cases} \quad G_{LOOP} \simeq \frac{K}{s^2} \cdot \frac{\tau_z}{\tau_p} = \frac{K}{s^2} \cdot \frac{\omega_p}{\omega_z}$$

$$\Rightarrow \quad \frac{G_{LOOP}}{1+G_{LOOP}} = \frac{K \cdot \omega_p/\omega_z}{K\,\omega_p/\omega_z + s^2} \simeq 0 \qquad \boxed{\frac{s}{\sqrt{K\frac{\omega_p}{\omega_z}}} \gg 1}$$

$$\frac{1}{1+G_{LOOP}} = \frac{s^2}{K\frac{\omega_p}{\omega_z} + s^2} \simeq 1$$

$$SSCR_y(100\,\mathrm{Hz}) = SSCR_x + 20 \log m =$$

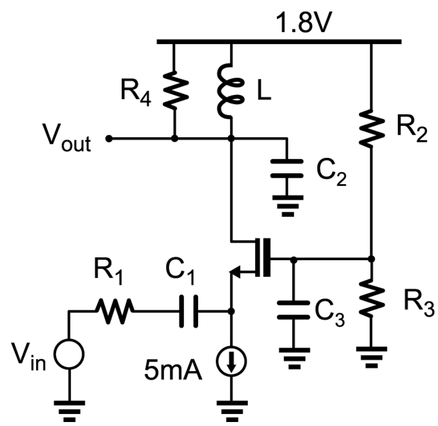
$$= -155 + 34{,}8 = -120{,}2\ \mathrm{dBc/Hz}$$

$$SSCR_x(10\,\mathrm{MHz}) = SSCR_{VCO}(10\,\mathrm{MHz}) = -135\ \mathrm{dBc/Hz}$$

RF Circuit Design **Prof. Salvatore Levantino**

Maximum available time: 2 hours July 24, 2013

Problem #1



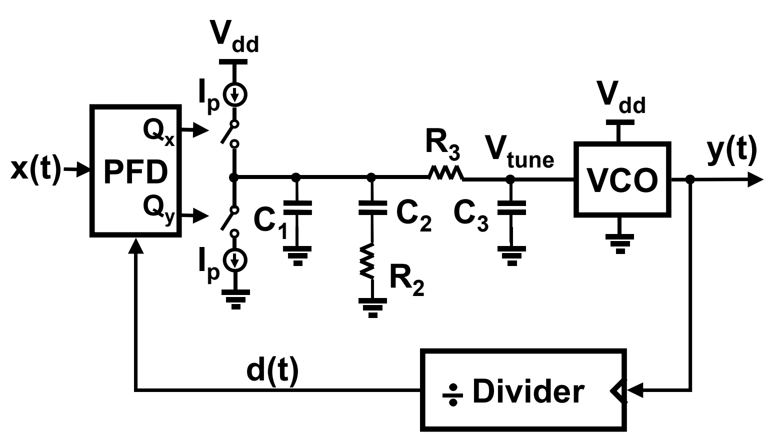
Let the source impedance be R_1=50Ω and the capacitor C_1 equal to 1nF. Let the MOSFET device have threshold voltage V_T=0.5V, coefficient $(1/2)\mu C_{ox}$=200μA/V^2 and noise parameter $(\gamma/\alpha) = 2/3$. Let the inductor L be equal to 1nH, the resistors R_2=2kΩ ed R_3=8kΩ.

a) After having derived the **bias point** (the voltages at all nodes, the currents at all branches and the working condition of the FET), compute the values of **R_4, C_2** and aspect ratio **(W/L)** of the FET that guarantee (i) input matching, (ii) maximum gain at 3.3GHz and (iii) noise figure equal to 2.7dB, assuming C_3=30pF and accounting for the thermal noise of ALL the resistors in the circuit and of the FET channel.

b) Let modify the circuit in figure, tying the terminal of C_2 to the GATE of the FET (instead of tying it at ground). On the modified circuit, keeping the same values of the other devices, update the values of **C_2, C_3** and **(W/L)** of the FET, so to achieve input matching at 3.3GHz, but noise figure equal to 1.2dB.

c) What is the value of the **transducer power gain in decibels**?

Problem #2



Let V_{dd} = 3.3V, x(t) a periodic signal whose frequency is 1MHz and the divider modulus equal to 1350. Let us assume a linear tuning characteristic of the VCO between 1200 and 1500MHz, when V_{tune} is varied from 0 to V_{dd}. Let R_2=1kΩ and C_2=50nF.

a) Neglecting R_3 and C_3 (i.e. R_3=0 and C_3=0), compute the values of the current $\mathbf{I_p}$ and the capacitor $\mathbf{C_1}$ so to have (i) phase margin equal to 60 degrees and (ii) bandwidth (approximated as the unity-gain frequency of the loop gain) equal to 10kHz.

b) Assuming R_3=1kΩ and C_3=350pF and under proper approximations (to be verified), update the values of **phase margin and unit-gain frequency**, after the insertion of R_3 and C_3?

c) Accounting for R_3, C_3 and for a constant leakage current, which is drained by the charge pump, equal to 10nA, compute the **level of the spurious tone** in the spectrum of y(t).

Written Test of RF Circuit Design
Solutions

Prof. Salvatore Levantino
Politecnico di Milano

May 21^{st}, 2014

Problem #1

a)

Input matching requires

$$\frac{1}{g_m} = R_1 = 50\ \Omega \tag{1}$$

Thus, the aspect ratio W/L has to be 100 and the bias point of the FET is given by $V_D = 1.80$V, $V_G = 1.44$V, $V_S = 0.44$V.

Gain is maximum when the capacitor C_2 resonates with the inductor L. So, we impose

$$C_2 = \frac{1}{w_0^2 L} = 2.33 \text{ pF}. \tag{2}$$

After some computations, we get the following expression of the noise figure

$$NF = 1 + \frac{\gamma}{\alpha} + \frac{4R_1}{R_4} + \frac{R_{23}}{R_1} \cdot \frac{1}{1 + \omega^2 R_{23}^2 C_3^2} \tag{3}$$

where $R_{23} = R_2 || R_3 = 1.6\ k\Omega$.

Finally, imposing $NF = 10^{-0.27} = 1.86$, we obtain

$$\frac{4R_1}{R_4} \approx 0.2 \tag{4}$$

So, $R_4 = 1\ k\Omega$.

b)

Let us assume that the equivalent resistance R_{23} in parallel to C_3 gives negligible variation of the parallel resistance of the LC resonator. In other words, let assume that (I) $Q_3 = \omega R_{23} C_3 \gg 1$. This means that R_{23} can be equivalently placed in parallel to R_4 after multiplication by $1/n^2$, where

$$n = \frac{C_2}{C_2 + C_3} \tag{5}$$

Thus, the equivalent resistance in parallel to the load resonator is $R_L = R_4 || \frac{R_{23}}{n^2}$.

After some computations, we get the following expression of the noise figure

$$NF = 1 + \frac{\frac{\gamma}{\alpha}}{g_m R_1} + \frac{R_1}{R_L}\left(1 + \frac{1}{g_m R_1}\right)^2 \tag{6}$$

Imposing $NF = 10^{-0.12} = 1.32$, (6) gives an equation in the unknowns g_m and n. To simplify the solution of (6), we make the assumption (II) $R_L \approx R_4$. Thus, being $R_4 = 1\ k\Omega$ and $R_1 = 50\ k\Omega$, the last term in (6) can be neglected. Doing so, we get

$$g_m R_1 \approx 2.1 \tag{7}$$

$$\frac{1}{g_m} = 24\ \Omega \tag{8}$$

Thus, the aspect ratio W/L has to be increased to 434 and the bias point of the FET is slightly modified to $V_D = 1.80$V, $V_G = 1.44$V, $V_S = 0.7$V.

In the modified circuit, input matching requires

$$\frac{1}{g_m} + n \cdot R_4 = R_1 = 50\ \Omega \tag{9}$$

It follows that n, given in (5), has to be approximately equal to $1/38$.

Gain is maximum when the capacitor $C_{23} = n \cdot C_3$ resonates with the inductor L. So, we impose

$$C_3 = \frac{1}{n w_0^2 L} \approx 89.6 \text{ pF} \tag{10}$$

and

$$C_2 = \frac{C_{23}}{1 - n} \approx 2.39 \text{ pF}. \tag{11}$$

Finally, we verify our initial hypotheses. (I) $Q_3 = \omega R_{23} C_3 = 2970$, which is much greater than one, and (II) $R_{23}/n^2 \approx 2.37\ M\Omega$, which is much greater than R_L.

c)

The transducer power gain is defined as the ratio between the output power and the available input power. It is given by

$$G = 10 \log_{10} \left(\frac{\frac{V_{out}^2}{2R_L}}{\frac{V_{in}^2}{8R_1}} \right) \tag{12}$$

Substituting the expression of the voltage gain

$$\frac{V_{out}}{V_{in}} = \frac{R_L}{2R_1} \tag{13}$$

we obtain

$$G = 10 \log_{10} \left(\frac{R_L}{R_1} \right) = 13 \text{ dB}. \tag{14}$$

Problem #2

a)

From the VCO tuning characteristic, we get

$$K_{vco} = 2\pi \cdot \frac{f_H - f_L}{V_{dd}} = 2\pi \frac{1500 - 1200}{3.3} \text{ Mrad/(Vs)} = 570.9 \text{ Mrad/(Vs)}. \tag{15}$$

The loop gain of the PLL can be written as

$$H(s) = \frac{I_p}{2\pi} \frac{K_{vco}}{sN} Z(s) = \frac{I_p}{2\pi} \frac{K_{vco}}{sN} \frac{1}{s(C_1 + C_2)} \cdot \frac{1 + s/\omega_z}{1 + s/\omega_p} \tag{16}$$

where

$$\omega_z = \frac{1}{R_2 C_2} \tag{17}$$

$$\omega_p = \frac{1}{R_2 C_{12}} \tag{18}$$

being

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2}. \tag{19}$$

The stability constraint imposes that the loop gain crosses the 0-dB axis with -20 dB/decade slope. Under this condition, the *crossover frequency* is given by

$$\omega_u = \frac{K_{vco} I_p R_2}{2\pi N} \cdot \frac{C_2}{C_1 + C_2} \tag{20}$$

If we assume that $C_1 \ll C_2$ and we impose $\omega_u = 2\pi \cdot 10$ krad/s (being $R_2 = 1\ k\Omega$ and $N = 1350$), it follows that $I_p = 0.932$ mA.

The *phase margin* is given by

$$\phi_m \approx \arctan\left(\frac{\omega_u}{\omega_z}\right) - \arctan\left(\frac{\omega_u}{\omega_p}\right) \tag{21}$$

Substituting the value of ω_z

$$\omega_z = \frac{1}{R_2 C_2} = 2\pi \cdot 3.2 \text{ krad/s} \tag{22}$$

and imposing $\phi_m = 60$ degrees, we get

$$\omega_p \approx 4.51 \cdot \omega_u = 2\pi \cdot 45.1 \text{ krad/s} \tag{23}$$

Thus,

$$C_{12} \approx 3.53 \text{ nF} \tag{24}$$

and

$$C_1 = \frac{C_2 C_{12}}{C_2 - C_{12}} = 3.80 \text{ nF} \tag{25}$$

We can iterate this computation by substituting the value of C_1 into (20). Doing so, we obtain a new, more accurate value of I_p, that is 1.002 mA.

b)

Taking into account R_3 and C_3, another pole is added into the loop gain

$$H_1(s) = \frac{1}{2\pi}\frac{K_{vco}}{sN}\frac{I_p}{s(C_1+C_2)} \cdot \frac{1+s/\omega_z}{(1+s/\omega_p)(1+sR_3C_3) + \frac{C_3}{C_1+C_2}\cdot(1+s/\omega_z)} \tag{26}$$

The last equation shows that, as long as

$$C_3 \ll C_1 + C_2 \tag{27}$$

and

$$C_3 \ll C_1 + \frac{R_3}{R_2}\cdot\left(1+\frac{C_1}{C_2}\right)\cdot C_3, \tag{28}$$

the two poles does not interact and the new pole is placed at $1/(R_3C_3)$ and the loop gain is approximately given by

$$H_1(s) \approx \frac{1}{2\pi}\frac{K_{vco}}{sN}\frac{I_p}{s(C_1+C_2)} \cdot \frac{1+s/\omega_z}{(1+s/\omega_p)(1+sR_3C_3)} \tag{29}$$

Being $C_3 \ll C_1$, both (27) and (28) hold true. Thus, looking at (29), we can conclude that the *crossover frequency* is unchanged, whereas the *phase margin* is reduced by

$$\Delta = \arctan\left(\omega_u R_3 C_3\right) \approx 1.3 \text{ degrees} \tag{30}$$

c)

The constant leakage current causes a ripple on $V_{tune}(t)$ at the reference frequency. At this frequency the filter trans-impedance can be approximated as:

$$Z_m(s) \approx \frac{1}{s(C_1 + C_3)} \cdot \frac{1}{1 + sR_3 \frac{C_1 C_3}{C_1 + C_3}} \tag{31}$$

The spur level can be calculated from the following expression

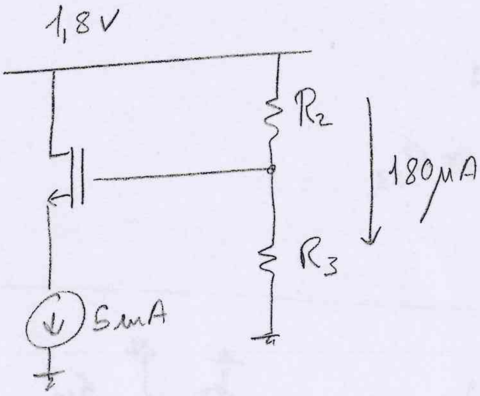
$$\text{SFDR} \approx 10 \log \left[\frac{K_{vco}^2 I_1^2}{4\omega_{ref}^6 C_1^2 R_3^2 C_3^2} \right] \tag{32}$$

where I_1 is the first harmonic of the steady-state current drained from the filter. Being $I_1 = 2 \cdot Q_{leakage}/T_{ref} =$ 20 nA, SFDR is about -95.2 dBc.

ESERCIZIO 1 (LNA)

FATTO IL 26/5/2014

(a) Polarizzazione



$$V_G = 1{,}8V \cdot \frac{R_3}{R_3+R_2} = \boxed{1{,}44V}$$

$$g_m = \underbrace{\frac{1}{50\Omega}}_{\text{input matching}} = \frac{2 I_D}{V_{GS}-V_T}$$

$$\Rightarrow V_{GS}-V_T = 0{,}5V \Rightarrow V_S = \boxed{0{,}44V}$$

$$I_D = \frac{1}{2}\mu C_{ox} \frac{W}{L}(V_{GS}-V_T)^2 \Longrightarrow \frac{W}{L} = \frac{5m}{200\mu} \cdot \frac{1}{(0{,}5)^2} = \boxed{100 = \frac{W}{L}}$$

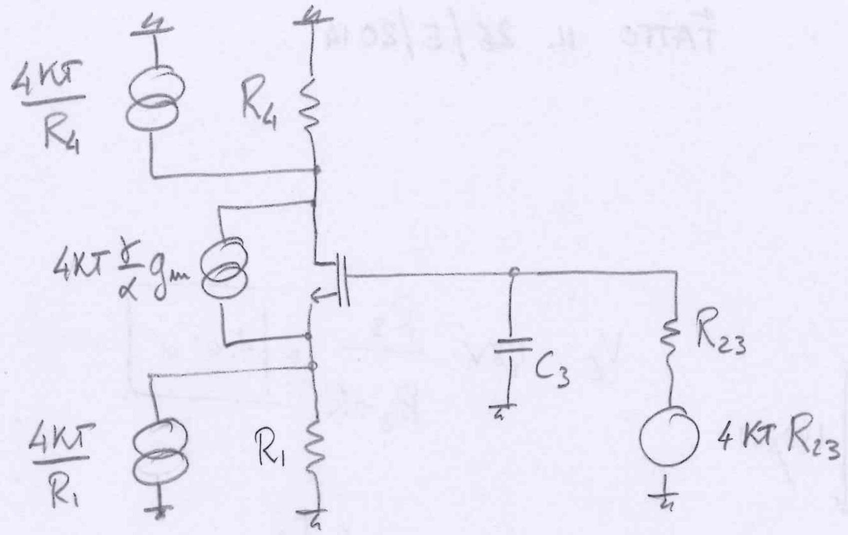
$$\underbrace{W_0^2 = \frac{1}{LC_2}}_{\text{guadagno massimo}} \longrightarrow C_2 = \frac{1}{W_0^2 L} = \boxed{2{,}33\,pF}$$

Noise figure

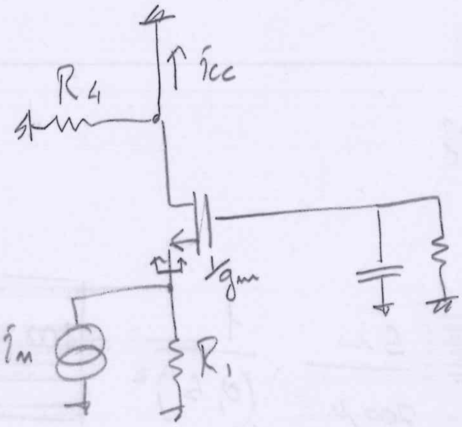
Hp: C_1 corto circuito @ 3,3GHz

$$\Rightarrow \tau_{C_1} = \Big(R_1 + \underset{\uparrow\,R_{in}}{\frac{1}{g_m}}\Big) C_1 = 100\,ns \Rightarrow \text{passa alto con } f_{pb} \text{ a } 1{,}6\,MHz \ll 3{,}3\,GHz$$

$R_{23} \triangleq R_2 // R_3 = 1{,}6\,K\Omega$



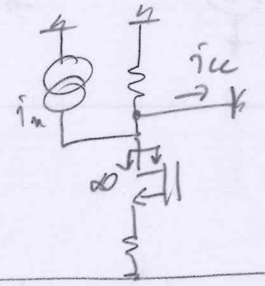
$\boxed{R_1}$



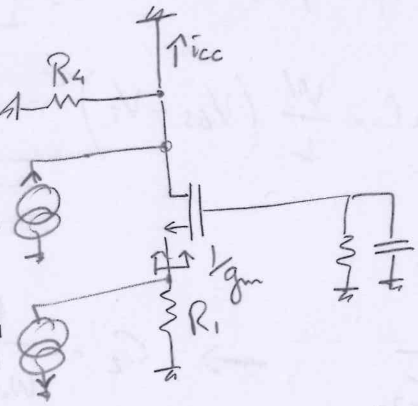
$$\frac{i_{cc}}{i_n} = \frac{R_1}{R_1 + 1/g_m}$$

$\boxed{R_4}$

$$\frac{i_{cc}}{i_n} = 1$$

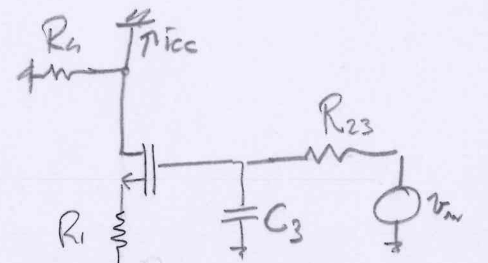


$\boxed{MOS}$



$$i_{cc} = i_n' - i_n'' \frac{R_1}{R_1 + 1/g_m} \Rightarrow \frac{i_{cc}}{i_n} = 1 - \frac{R_1}{R_1 + 1/g_m} = \frac{1/g_m}{R_1 + 1/g_m}$$

$\boxed{R_{23}}$



$$\frac{i_{cc}}{v_n} = \sqrt{\left|\frac{1/sC_3}{1/sC_3 + R_{23}}\right|^2_{s=j\omega_0}} \cdot \frac{1}{R_1 + 1/g_m} = \frac{1}{R_1 + 1/g_m} \cdot \frac{1}{\sqrt{1+\omega_0^2 C_3 R_{23}}}$$

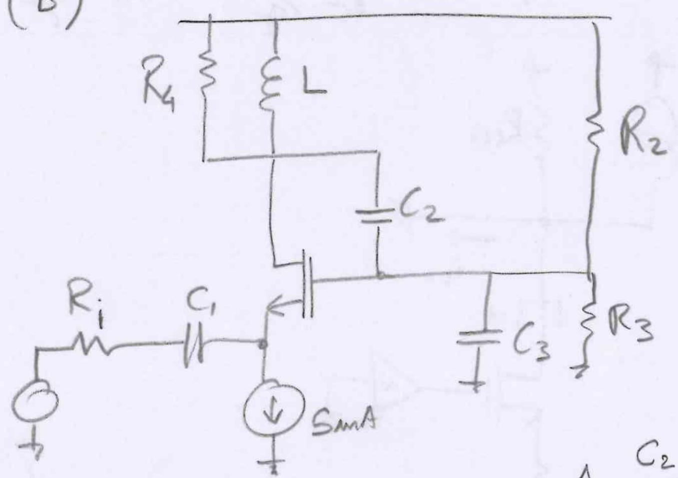
$$NF = 1 + \frac{\frac{4KT}{R_4}}{\frac{4KT}{R_1}\cdot\left(\frac{R_1}{R_1+1/g_m}\right)^2} + \frac{4KT\frac{\gamma}{\alpha}g_m\left(\frac{1/g_m}{1/g_m+R_1}\right)^2}{\frac{4KT}{R_1}\left(\frac{R_1}{R_1+1/g_m}\right)^2} + \frac{4KT R_{23}\left(\frac{1}{R_1+1/g_m}\right)^2\cdot\frac{1}{1+\omega_0^2 C_3 R_{23}}}{\frac{4KT}{R_1}\cdot\left(\frac{R_1}{R_1+1/g_m}\right)^2}$$

$$NF = 1 + \frac{\cancel{R_1}}{R_4}\cdot\frac{\left(R_1+\frac{1}{g_m}\right)^2}{R_1^{\cancel{2}}} + \frac{\gamma}{\alpha}\cdot\cancel{g_m}\cdot\frac{1}{g_m^{\cancel{2}}}\cdot\frac{\cancel{R_1}}{R_1^{\cancel{2}}} + \frac{R_{23}}{R_1^{\cancel{2}}}\cdot\cancel{R_1}\cdot\frac{1}{1+\omega_0^2 C_3 R_{23}}$$

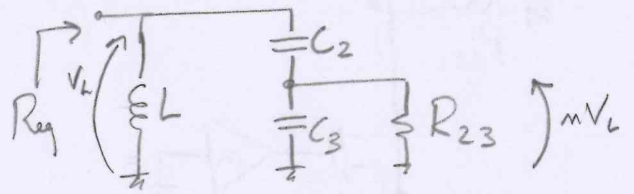
Per $R_1 = \frac{1}{g_m}$ $\quad NF = 1 + \frac{4R_1}{R_4} + \frac{\gamma}{\alpha} + \frac{R_{23}}{R_1}\cdot\frac{1}{1+\omega_0^2 C_3 R_{23}} = 10^{\frac{-2,7}{10}} = 1,86$

$$\Rightarrow \frac{4R_1}{R_4} \simeq 0,19 \Rightarrow \boxed{R_4 = 1K\Omega}$$

(b)



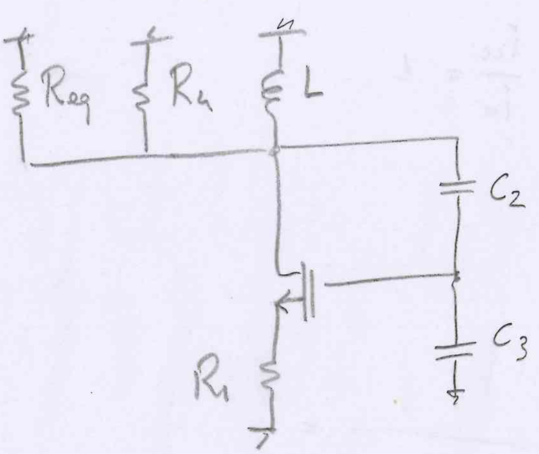
$\underline{Hp\,1}$ $\quad R_{23} \gg \frac{1}{\omega_0 C_3}$

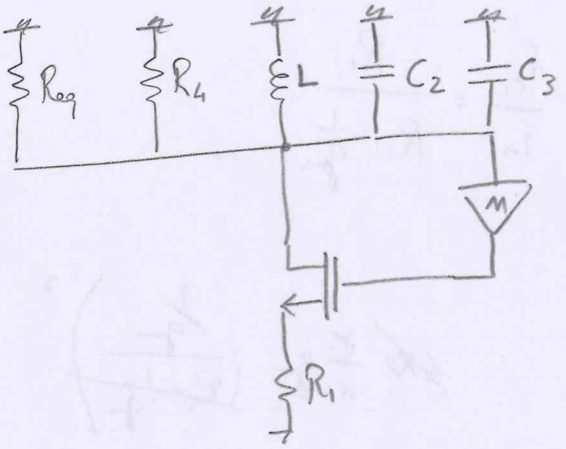


$$\frac{V_L^2}{R_{eq}} = \frac{(nV_L)^2}{R_{23}} \Rightarrow R_{eq} = \frac{R_{23}}{n^2}$$

$$n \triangleq \frac{C_2}{C_2+C_3}$$

@ 3,3 GHz

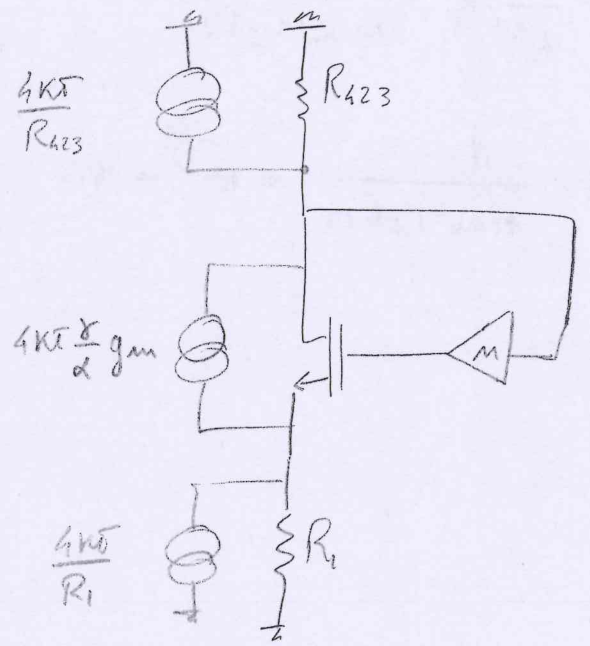




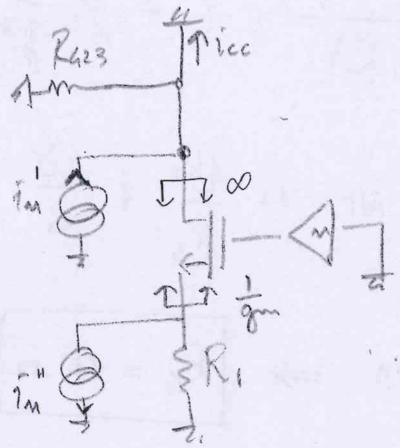
$R_{423} \triangleq R_{eq} \,//\, R_4$

Guadagno massimo $\quad \omega_0^2 = \frac{1}{L\,\frac{C_2 C_3}{C_2+C_3}}$

Noise figure



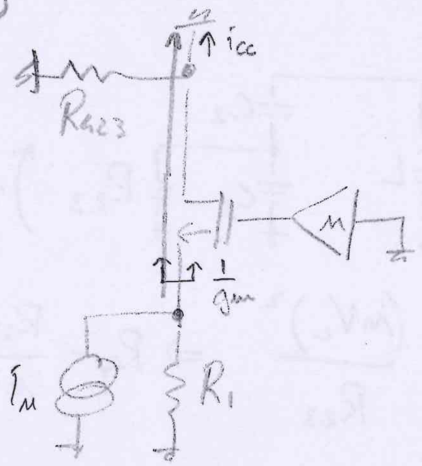
$\boxed{MOS}$



$$i_{cc} = i_n' - i_n'' \cdot \frac{R_1}{R_1 + \frac{1}{g_m}}$$

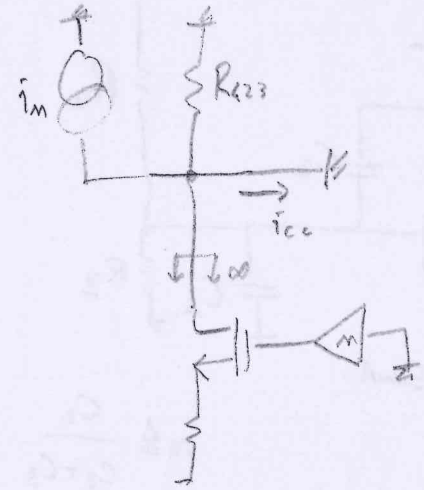
$$\frac{i_{cc}}{i_n} = 1 - \frac{R_1}{R_1 + 1/g_m} = \frac{R_1 + \frac{1}{g_m} - R_1}{R_1 + 1/g_m} = \frac{1/g_m}{R_1 + 1/g_m}$$

$\boxed{R_1}$



$$\frac{i_{cc}}{i_n} = \frac{R_1}{R_1 + \frac{1}{g_m}}$$

$\boxed{R_{423}}$

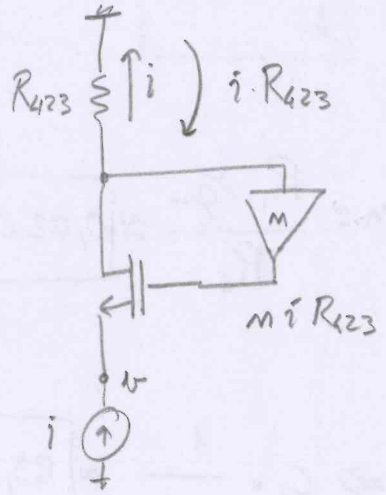


$$\frac{i_{cc}}{i_n} = 1$$

$$NF = 1 + \frac{\cancel{4kT}\frac{\gamma}{\alpha}g_m\left(\cancel{\frac{1/g_m}{R_1 + 1/g_m}}\right)^2}{\frac{\cancel{4kT}}{\cancel{R_1}}\cdot\left(\cancel{\frac{R_1}{R_1 + 1/g_m}}\right)^2} + \frac{\frac{\cancel{4kT}}{R_{423}}}{\frac{\cancel{4kT}}{R_1}\left(\frac{R_1}{R_1 + 1/g_m}\right)^2} =$$

$$= 1 + \frac{\gamma}{\alpha}\cdot\frac{1}{g_m R_1} + \frac{\cancel{R_1}}{R_{423}}\cdot\frac{\left(R_1 + 1/g_m\right)^2}{R_1^{\cancel{2}}} = 1 + \frac{\gamma/\alpha}{g_m R_1} + \frac{R_1}{R_{423}}\cdot\left(1 + \frac{1}{g_m R_1}\right)^2$$

input matching



$$i = -g_m\left(n R_{423}\, i - v\right)$$

$$i\left(1 + g_m n R_{423}\right) = g_m v$$

$$\frac{v}{i} = \boxed{\frac{1}{g_m} + n R_{423} = R_1}$$

$$\Longrightarrow NF = 1 + \frac{\gamma/\alpha}{g_m R_1} + \frac{R_1}{R_{423}}\left(1 + \frac{1}{g_m R_1}\right)^2 = 10^{-12/10} = 1{,}32$$

$$R_{423} \triangleq R_4 \,/\!/\, \frac{R_{23}}{n^2}$$

<u>Hp 2</u> $\quad R_4 \ll \frac{R_{23}}{n^2} \quad \Longrightarrow \quad R_{423} \simeq R_4$

$$\Longrightarrow NF \simeq 1 + \frac{\gamma/\alpha}{g_m R_1} + \frac{R_1}{R_4}\left(1 + \frac{1}{g_m R_1}\right)^2 = 1{,}32$$

<u>Hp 3</u>: $\quad \frac{R_1}{R_4}\left(1 + \frac{1}{g_m R_1}\right)^2 \ll 1$

$$\Longrightarrow NF \simeq 1 + \frac{\gamma/\alpha}{g_m R_1} = 1{,}32 \quad \Longrightarrow \quad \frac{1}{g_m} = 24\,\Omega \qquad (g_m R_1 = 2{,}1)$$

$$g_m = \frac{2 I_D}{V_{GS} - V_T} \quad \Longrightarrow \quad \frac{1}{24} = \frac{2 \cdot 5m}{V_{GS} - V_T} \quad \Longrightarrow \quad V_{GS} - V_T = 0{,}24\,V$$

• con $V_G = 1{,}44\,V \implies V_S = 0{,}7\,V$

$$I_D = \frac{1}{2}\mu C_{ox} \frac{W}{L}\left(V_{GS} - V_T\right)^2 = 5\,mA \implies \boxed{\frac{W}{L} = 434}$$

input matching: $\frac{1}{g_m} + n\underbrace{R_{423}}_{\simeq R_4} = R_1 \implies n \simeq \frac{R_1 - 1/g_m}{R_4} \simeq \boxed{0{,}026}$

guadagno max: $\omega_0^2 = \frac{1}{L\underbrace{\frac{C_2 C_3}{C_2+C_3}}_{n}} = \frac{1}{LnC_3} \implies C_3 = \frac{1}{\omega_0^2 n L} = \boxed{89{,}6\,pF}$

$$\frac{C_2}{C_2+C_3} = n \implies \frac{1}{1+\frac{C_3}{C_2}} = n \implies \frac{C_3}{C_2} = \frac{1-n}{n} \implies C_2 = \frac{nC_3}{1-n} = \boxed{2{,}4\,pF}$$

<u>Verifica delle ipotesi</u>

<u>Hp1</u> $R_{23} >> \frac{1}{\omega_0 C_3}$ $\qquad 1600 >> 0{,}54$ OK!

<u>Hp2</u> $R_4 << \frac{R_{23}}{n^2}$ $\qquad 1000 << 2{,}37 \cdot 10^6$ OK!

<u>Hp3</u> $\frac{R_1}{R_4}\left(1 + \frac{1}{g_m R_1}\right)^2 << 1$ $\qquad 0{,}1 << 1$ OK!

(c) guadagno di potenza di trasduzione

$$G \triangleq 10 \log_{10} \left(\frac{V_{out}^2 / 2R_4}{V_{in}^2 / 8R_1} \right) = 10 \log_{10} \left(\left(\frac{V_{out}}{V_{in}} \right)^2 \cdot 4 \frac{R_1}{R_4} \right)$$

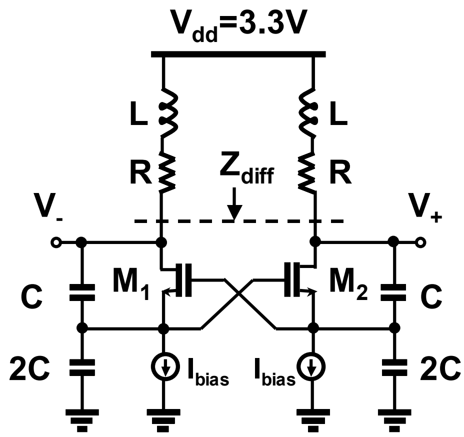
guadagno di tensione $\dfrac{V_{out}}{V_{in}} = \dfrac{R_4}{2R_1}$

$$\Longrightarrow \quad G \triangleq 10 \log_{10} \left(\frac{R_4^{\cancel{2}}}{\cancel{4} R_1^{\cancel{2}}} \cdot \cancel{4} \frac{\cancel{R_1}}{\cancel{R_4}} \right) = 10 \log_{10} \left(\frac{R_4}{R_1} \right) \simeq \boxed{13\,dB}$$

Elettronica a Radiofrequenza **Prof. Salvatore Levantino**

Tempo massimo a disposizione: 3 ore 26/09/2011

Problema 1



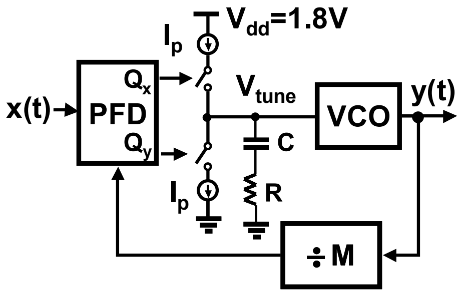
Siano L = 8nH, R = 8Ω, V_t = 0.5V, K_n = 4mA/V^2.

a) Ricavare l'espressione dell'impedenza differenziale $Z_{diff}(\omega)$.

b) Dimensionare I_{bias} e C per garantire un margine di un fattore 2 sull'innesco dell'oscillazione e frequenza di oscillazione di 3GHz.

c) Ricavato il guadagno d'anello del circuito, calcolare l'ampiezza d'oscillazione di V_+ e V_-, approssimando la corrente in M1/M2 con un treno di impulsi.

Problema 2

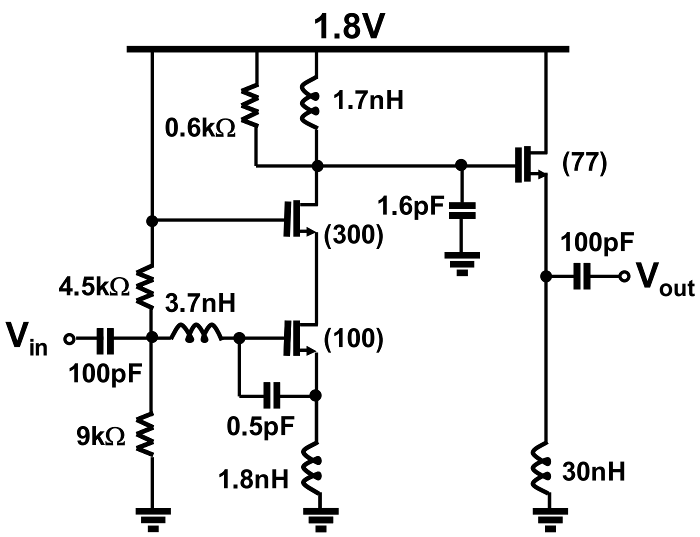


a) Nel sistema in figura, il VCO ha frequenza di free-running di 3GHz e sensitività di 300MHz/V, M=100, I_p=0.1mA. Dopo aver ricavato il modello equivalente lineare, dimensionare R e C, per garantire poli ad anello chiuso a frequenza di 10kHz e a 45° sul piano di Gauss.

b) Quanto vale il contributo del rumore termico della resistenza R sullo spettro di y(t) a 1MHz dalla portante?

c) Tenuto conto che il segnale periodico x(t) ha spettro di fase bianco pari a -140dBc/Hz e considerato il rumore termico di R, disegnare su grafico quotato lo spettro di fase totale di y(t).

Problema 3



Assumere FET con (W/L) in figura, soglia V_t=0.5V, costante $1/2\mu C_{ox}$=0.1mA/V^2 e rumore termico con α=1 e γ=2/3 (trascurare gate-induced noise).

a) Dopo aver polarizzato il circuito, ricavare alla frequenza di risonanza il guadagno di potenza disponibile a 50Ω.

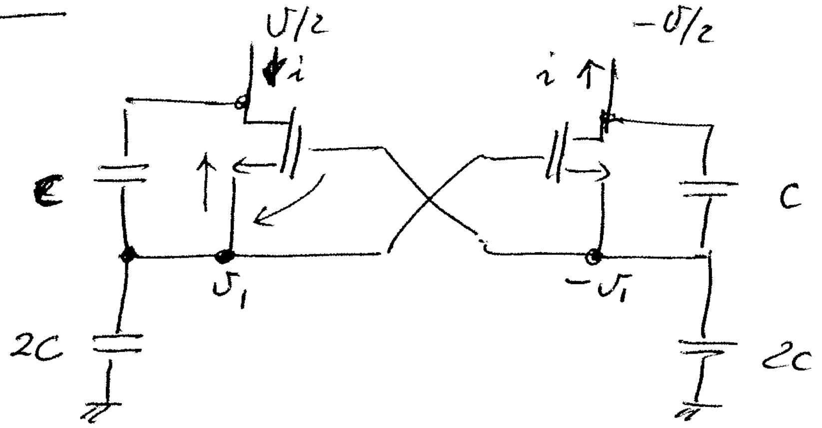
b) Ricavare la figura di rumore del circuito.

c) Assumendo di connettere due filtri passabanda, rispettivamente all'ingresso e all'uscita del circuito, e che tali filtri abbiano perdita di potenza di 3dB (in banda passante) e avendo un segnale all'ingresso della catena di potenza disponibile -100dBm, quanto vale in uscita il rapporto segnale/rumore nella banda di 1MHz.

Tema d'esame 26/9/2011

Problema 1

a)



$$\begin{cases} i = (v/2 - v_1)\cdot sC - g_m \cdot 2v_1 \\ (v/2 - v_1)\cdot sC - g_m\, 2v_1 = s2C \cdot v_1 \end{cases}$$

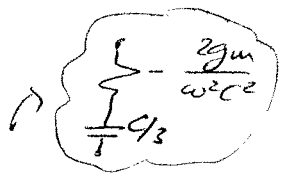
$$sC\,v/2 = (s2C + sC + 2g_m)\cdot v_1$$

$$v_1 = \frac{sC/2}{s3C + 2g_m}\cdot v$$

$$i = v/2 \cdot sC - (2g_m + sC)\cdot \frac{sC/2}{s3C + 2g_m}\cdot v$$

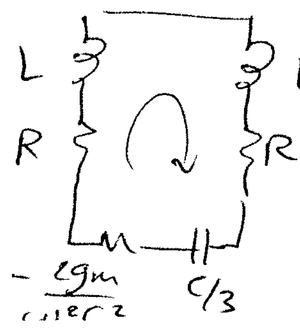
$$i/v = s\frac{C}{2}\,\frac{s3C + \cancel{2g_m} - \cancel{2g_m} - sC}{s3C + 2g_m} = \frac{s\frac{C}{\cancel{2}}\cdot s\cancel{2}C}{s3C + 2g_m} =$$

$$= \frac{s^2C^2}{s3C + 2g_m}$$



$$Z_{diff} = \frac{v}{i} = \frac{2g_m}{s^2C^2} + \frac{\cancel{s}3C}{s^{\cancel{2}}C^{\cancel{2}}}\Bigg|_{s=j\omega} = -\frac{2g_m}{\omega^2C^2} + \frac{3}{j\omega C}$$

b)



Condizioni di oscillazione $Z_{tot} = 0$:

1) $Re(Z_{tot}) = 0$: $\quad \cancel{2}R - \frac{\cancel{2}g_m}{\omega^2C^2} = 0$

2) $Im(Z_{tot}) = 0$: $\quad \omega = \frac{1}{\sqrt{2L\cdot C/3}}$

1) $g_{m_{min}} = \omega_0^2 C^2 \cdot R = 0.79\ mA/V$

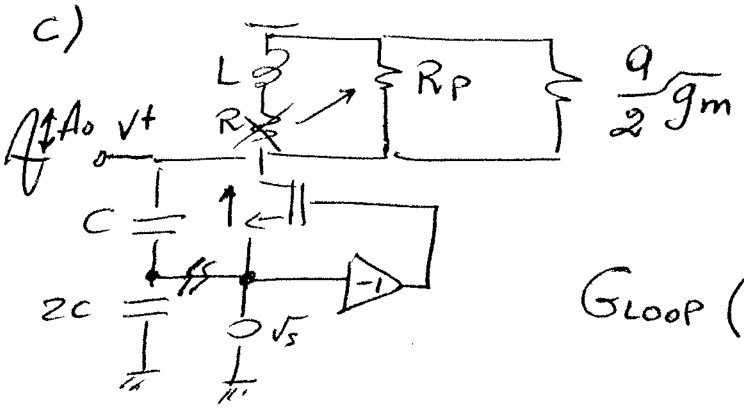
2) $\omega = \dfrac{1}{\sqrt{2L \cdot 4/3}} = \omega_0 \rightarrow C = \dfrac{1}{\omega_0^2} \cdot \dfrac{1}{L} \cdot \dfrac{3}{2} = 528\ fF$

Per avere margine di fattore 2 su startup:

$g_m = 2 g_{m\,min} = 1.58\ mA/V$

$g_m = 2\sqrt{k \cdot I_{bias}} \Rightarrow I_{bias} = \dfrac{g_m^2}{4} \cdot \dfrac{1}{K} = 157\ \mu A$

c)



$\dfrac{1}{2g_m} \cdot \dfrac{1}{n^2}$

$$G_{LOOP}(\omega_0) = 2g_m \cdot \left(R_p \parallel \frac{9}{2g_m}\right) \cdot \frac{1}{3}$$

$$= \frac{2}{3} g_m \cdot \frac{9 R_p}{9 + 2 g_m R_p} = 1$$

$$\Rightarrow g_m R_p = 9/4 \Rightarrow g_m = (\omega_0 C)^2 R$$

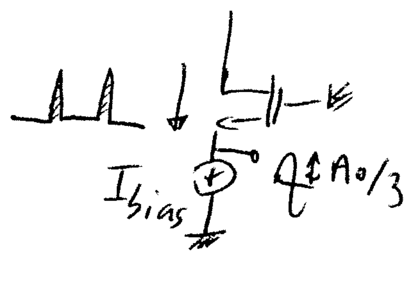
COME OTTENUTO IN a)

$$R_p = \frac{1}{(\omega_0 \frac{2}{3} C)^2 R} = \frac{9}{4} \frac{1}{(\omega_0 C)^2 \cdot R}$$

Per grande segnale: $G_{meff} = (\omega_0 C)^2 R$

d'altra parte $G_{meff} \simeq \dfrac{2 \cdot I_{bias}}{A_0/3} \Rightarrow$

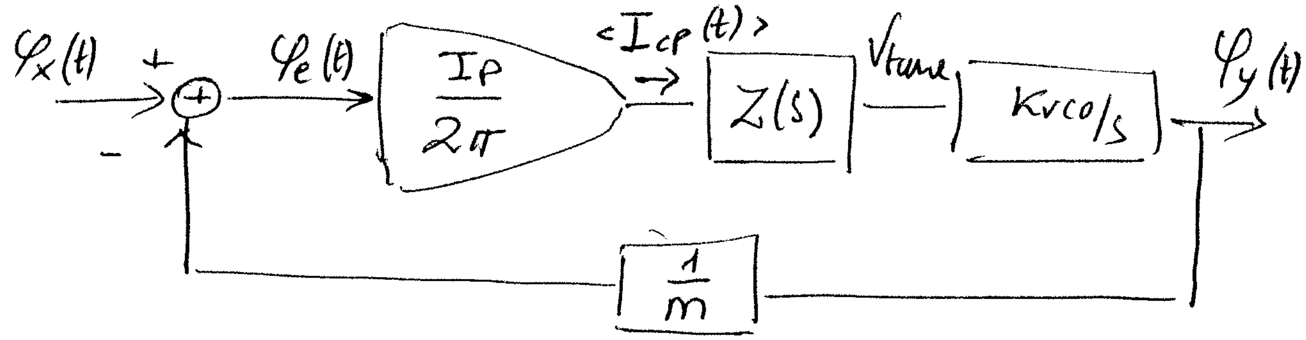
$\Rightarrow A_0 = \dfrac{6\ I_{bias}}{(\omega_0 C)^2 \cdot R} \simeq 1.2\ V$

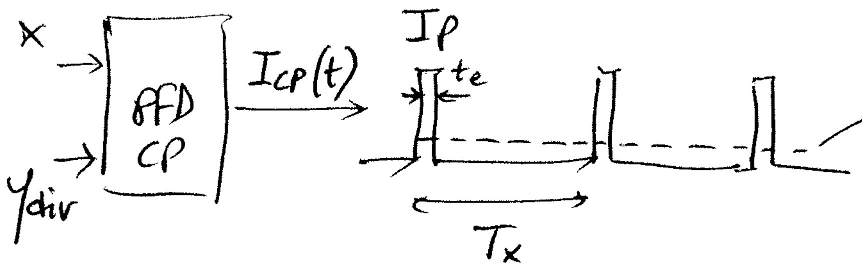


Problema 2

$$\begin{cases} M = 100 \\ K_{VCO} = 2\pi \cdot 300 \ \text{Mrad/sV} \\ I_P = 0.1 \ \text{mA} \end{cases}$$

a)





$$\langle I_{CP}(t)\rangle = I_P \cdot \frac{t_e}{T_x} = I_P \frac{\varphi_e}{2\pi}$$

Guadagno equivalente di PFD/CP

Ipotizzando che $Z(s)$ sia filtro a banda più stretta di $1/T_x$ possiamo considerare la sola componente DC di I_{CP}

$$G_{LOOP}(s) = \frac{I_P}{2\pi}\left(R + \frac{1}{sC}\right)\frac{K_{VCO}}{sM}$$

$$1 - G_{LOOP}(s) = 0 \quad : \quad s^2 + \underbrace{\frac{I_P R K_{VCO}}{2\pi M}}_{2\xi\omega_n} s + \underbrace{\frac{I_P K_{VCO}}{2\pi M C}}_{\omega_n^2} = 0$$

$$\Rightarrow \quad \omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi M C}}$$

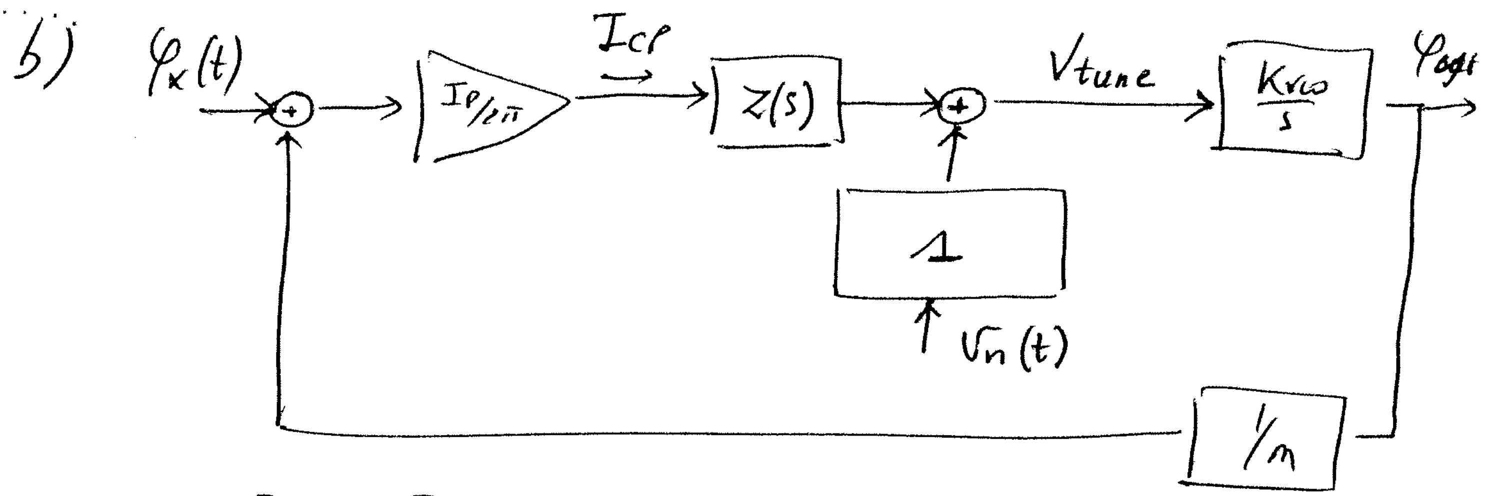
$$\xi = \frac{R}{2}\sqrt{\frac{I_P K_{VCO} C}{2\pi M}} \quad \Rightarrow \quad C = \left(\frac{I_P \cdot K_{VCO}}{2\pi \cdot M}\right) \cdot \frac{1}{\omega_n^2} = 76 \ nF$$

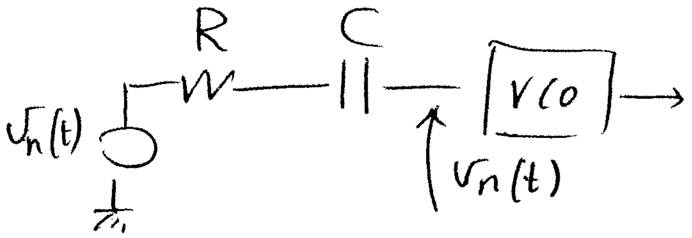
$$\omega_n = 2\pi \cdot 10k$$

$$\xi = \sqrt{2}/2$$

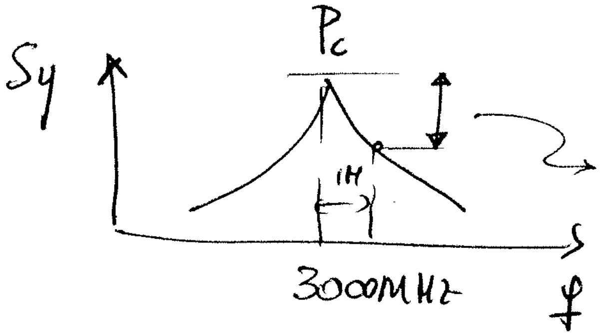
$$R = \frac{2\xi}{\sqrt{\frac{I_P K_{VCO} C}{2\pi M}}} = 296 \ \Omega$$

b)





guadagno ad anello aperto da $v_n(t)$ a v_{tune} è 1



$$\mathcal{L} = \frac{S_{\varphi_y}}{2} = \frac{1}{2} \cdot \frac{K_{VCO}^2}{\omega^2} \cdot \underbrace{\frac{1}{|1 - G_{loop}(j\omega)|^2}}_{\to 1 \text{ a } 1MHz} \cdot S_{vn}$$

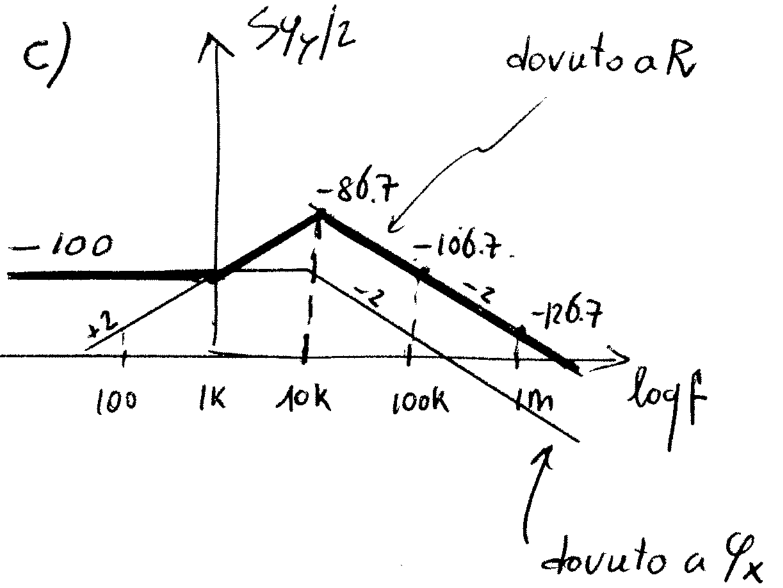
dove $S_{vn} = 4KTR$

1 a 1MHz (essendo banda a 10kHz)

$$\mathcal{L}(\omega = 2\pi \cdot 1M) \approx \frac{1}{2} \frac{K_{VCO}^2}{\omega^2} \cdot 4KTR =$$

$$= \frac{1}{2} \cdot (300)^2 \cdot 16 \cdot 10^{-21} \cdot 296 \rightarrow -126.7 \ \frac{dBc}{Hz}$$

c)



$$\frac{\varphi_y}{\varphi_x} = \frac{M(1 + sRC)}{1 + \frac{2\xi s}{\omega_n} + \frac{s^2}{\omega_n^2}}$$

zero a 7kHz

2 poli c.c. a 10kHz

$M = 100 \rightarrow 40\,dB$

$S_{\varphi_x}/2 = -140\ dBc/Hz$

⇓

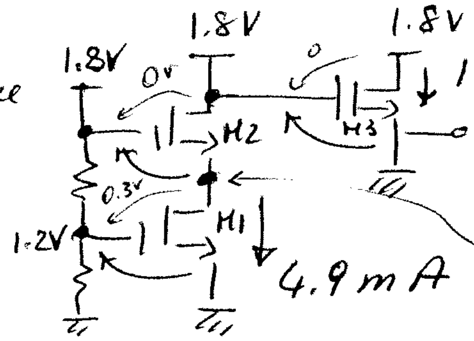
In banda $\frac{S_{\varphi_y}}{2} = -100 \frac{dBc}{Hz}$

Problema 3

- $g_{m_1} = 14\ mA/V$
- $g_{m_2} = 24.5\ mA/V$
- $g_{m_3} = 20\ mA/V$

a) Polarizzazione

Transistori in z. di sat.



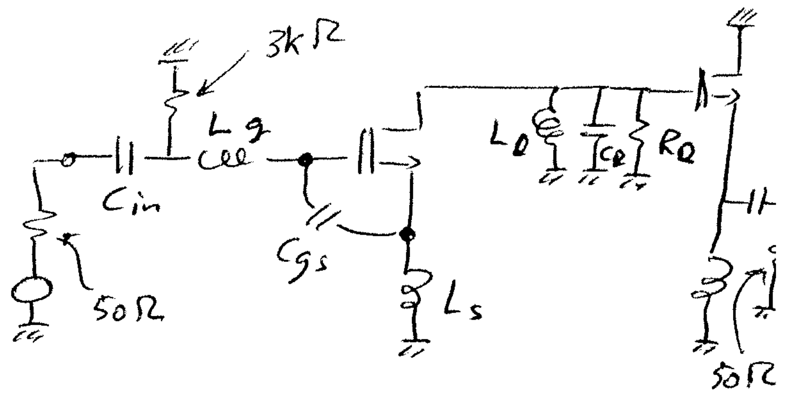
$0.5 + \sqrt{\frac{4.9}{30}} = 0.9V$

1) • $f = f_{RES} =$

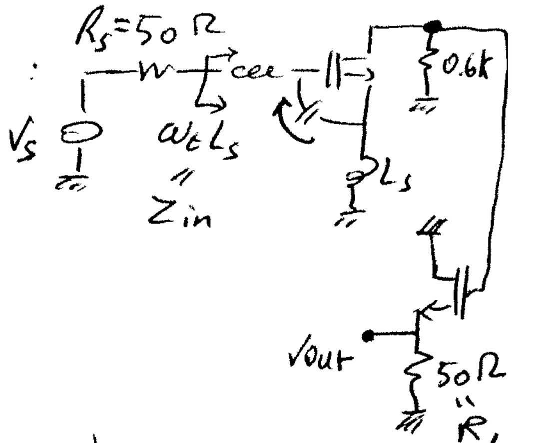
$$= \frac{1}{2\pi} \frac{1}{\sqrt{(L_g + L_s) \frac{C_{in} C_{gs}}{C_{in} + C_{gs}}}}$$

$$\simeq \frac{1}{2\pi} \frac{1}{\sqrt{5.5n \cdot 0.5p}} =$$

$$= 3.04\ GHz$$



A $f = f_{res}$:



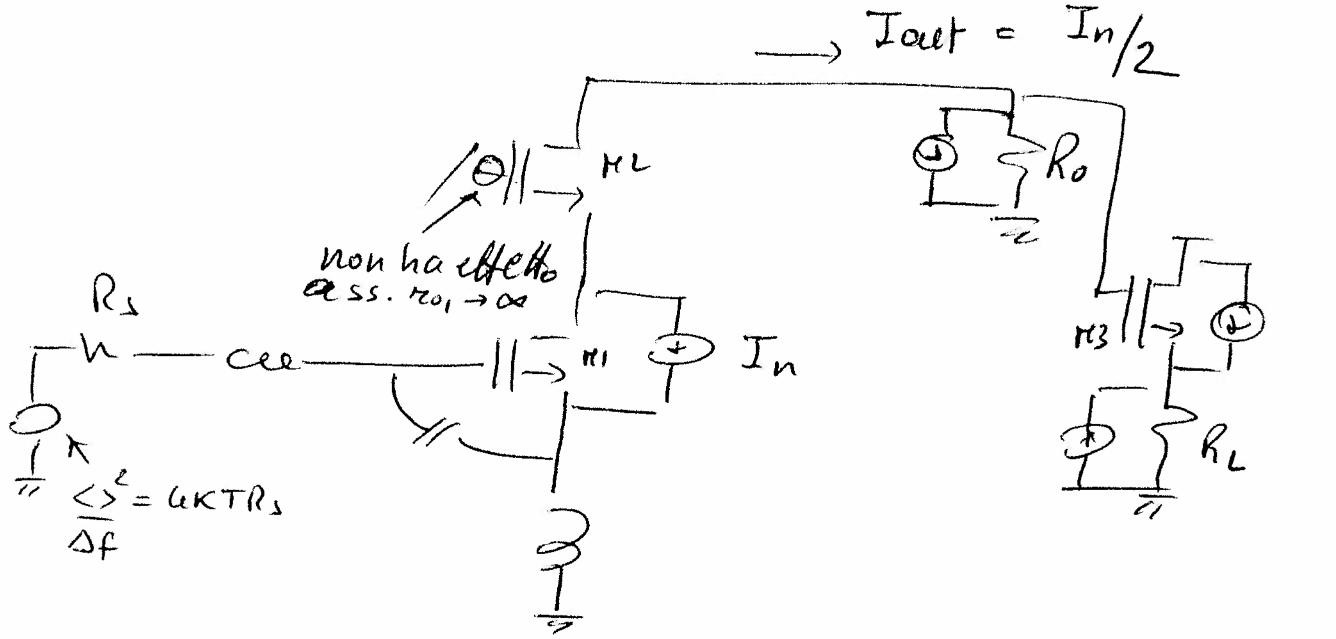
• $Z_{in} = \omega_T L_s =$

$$= \frac{g_{m_1}}{C_{gs}} \cdot L_s = 50\Omega$$

• $$\frac{V_{out}}{V_s} = g_{m_1} \cdot Q \cdot R_0 \cdot \underbrace{\left(\frac{R_L}{R_L + 1/g_{m_2}}\right)}_{1/2} \cdot 50\Omega = 4.45 \Rightarrow 13\ dB$$

$$Q = \frac{1}{\omega_{RES} C_{gs} \cdot \underbrace{2R_s}_{\text{ossia } R_s + Z_{in}}} = \frac{1}{2\pi \cdot 3G \cdot 0.5p \cdot 100} = 1.06$$

$$\Rightarrow G_P = \frac{P_{out,av}}{P_{in,av}} = \frac{V_{out}^2/8R_L}{V_s^2/8R_s} = \left(\frac{V_{out}}{V_s}\right)^2 \Rightarrow 13\ dB$$



Si può dimostrare che $I_{out} = I_n/2$ (...)

$$\Rightarrow \quad NF = 1 + \frac{\cancel{4KT}\frac{8}{3}g_{m1} \bullet \frac{1}{4^2}}{\cancel{4KT}R_s \bullet (g_{m1}Q)^2} +$$

$$+ \frac{\cancel{4KT}/R_0}{\cancel{4KT}R_s(g_{m1}Q)^2} + \frac{\left(4KT\,\frac{2}{3}g_{m3} + 4KT/R_L\right)R_L^2}{4KTR_s \cdot G_P} =$$

$//R_L$

$$= 1 + \frac{1}{6}\,\frac{1}{g_{m1}R_s}\,\frac{1}{Q^2} + \frac{1}{R_oR_s\,g_{m1}^2Q^2} + \frac{\cancel{4KTR_L}\left(\frac{2}{3}+1\right)}{\cancel{4KTR_s}\cdot G_P} =$$

$R_L = R_s$

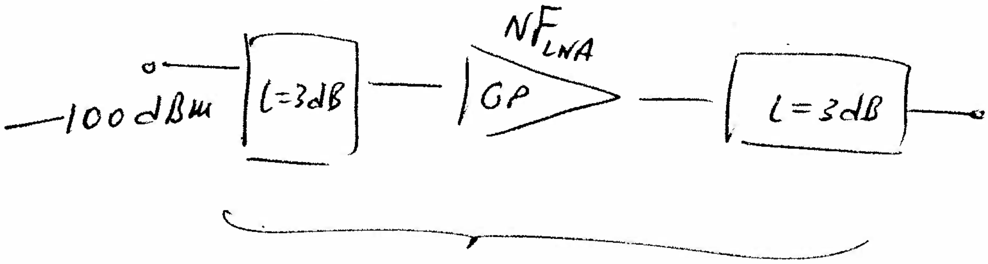
$$= 1 + \frac{1}{6}\,\frac{1}{g_{m1}R_s}\,\frac{1}{Q^2} + \frac{1}{R_oR_s\,g_{m1}^2Q^2} + \frac{5}{3}\cdot\frac{1}{G_P} =$$

$$= 1 + 0.21 + 0.15 + 0.08 = 1.44$$

$\downarrow$

1.59 dB

c)



$$NF = L + \frac{NF_{LNA}-1}{1/L} + \frac{L-1}{\frac{G_P}{L}} =$$

$$= L \cdot NF_{LNA} + \frac{L(L-1)}{G_P} =$$

$L = 3\,dB$
$\downarrow$
2

$$= 2 \cdot 1.44 + \frac{2 \cdot 1}{20} =$$

$$= 2.88 + 0.1 = 2.98$$

$\downarrow$
$4.6\,dB$

$$SNR = \frac{P_S}{P_N} = \frac{P_S}{\frac{\cancel{4}KT\cancel{R_S} \cdot NF \cdot BW}{\cancel{4R_S}}} = \frac{P_S}{KT \cdot NF \cdot BW} =$$

BW: 1MHz

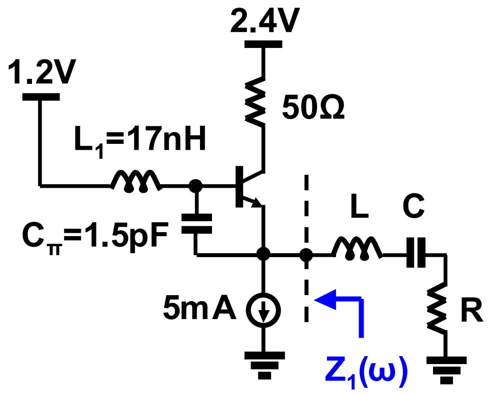
$$\Rightarrow \frac{P_S}{P_N} = -100\,dBm + 174\,\frac{dBm}{Hz} - 4.6\,dB - 60 =$$

$$= 9.4\,dB$$

Elettronica a Radiofrequenza | **Prof. Salvatore Levantino**

Tempo massimo a disposizione: 3 ore | 30/06/2011

Problema 1



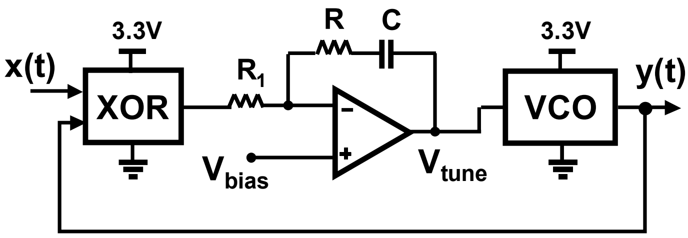
Si trascuri la capacità C_μ del BJT e si consideri $\beta >> 1$.

a) Ricavare l'espressione dell'impedenza $Z_1(\omega)$, trascurando la corrente di base del BJT.

b) Dimensionare L, C e la R limite per garantire 1) innesco dell'oscillazione a 2GHz (f_0) e 2) fattore Q della rete RLC (isolata) pari a 5 (**Nota:** Imporre le condizioni di oscillazione e approssimare per $\mathbf{f_0} << \mathbf{f_T}$, con f_T frequenza di taglio del BJT).

c) Con i valori ottenuti, ma con corrente ridotta a 50µA, ricalcolare frequenza di oscillazione e condizione d'innesco (approssimando per $\mathbf{f_0} >> \mathbf{f_T}$).

Problema 2



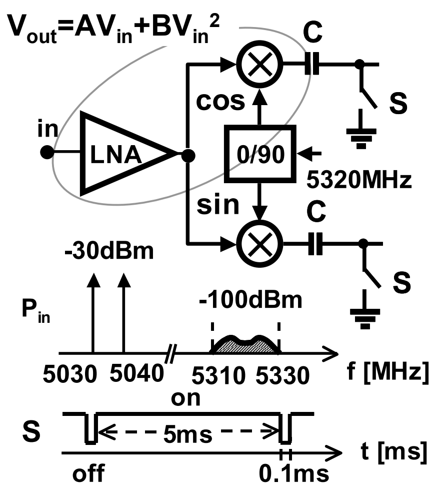
Sia la frequenza di x(t) pari a 200 kHz. Assumere nulla l'impedenza d'uscita della porta XOR, amplificatore operazione ideale e VCO con caratteristica di tuning lineare sull'intervallo 0-3.3V.

a) Ipotizzando V_{bias} pari a 1.65V, ricavare e disegnare il modello equivalente di fase, e dimensionare i parametri R_1, R, C, K_{VCO} per avere 1) massima corrente in uscita alla XOR pari a 1mA, 2) tuning range pari a 50kHz, 3) margine di fase 60°, 4) frequenza di crossover del guadagno d'anello pari a 1kHz.

b) Ricavare lo sfasamento in gradi tra x(t) e y(t) in funzione della tensione V_{bias} tra 0 e 3.3V.

c) Fissando V_{bias} a 1.65V, valutare l'effetto sul funzionamento del sistema di un offset di tensione di 10mV e di una corrente di bias di 10µA dell'amplificatore operazionale: l'anello rimane agganciato e perché? Se sì, con che relazione di fase tra x(t) e y(t)?

Problema 3



Assumere che il segnale d'ingresso durante un "burst" abbia spettro di potenza (riferito a 50Ω) riportato in figura, che il guadagno di LNA/mixer sia 20dB e che gli interruttori S siano chiusi/aperti quando il ricevitore è inattivo/attivo.

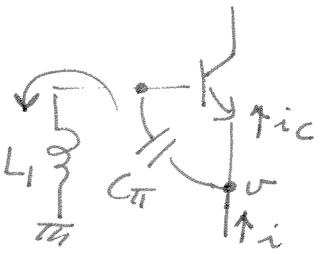
a) Considerando i due blocker all'ingresso di -30dBm ciascuno, discutere l'impatto della nonlinearità del secondo ordine e ricavare la massima IIP2 tollerabile del ricevitore per avere un rapporto segnale/interferenti di almeno 20dB.

b) Dopo aver discusso la funzione degli switch S, ricavare il valore di C che garantisca un SNR di almeno 20dB, tenuto conto del rumore termico degli interruttori S con r_{on}=1kΩ.

c) Spiegare perché il ricevitore in figura utilizza due mixer e non solamente uno.

ES. 1

a)



$$\begin{cases} i = i_c + \dfrac{v - i_c/g_m}{sL_1} \\ \dfrac{v - i_c/g_m}{sL_1}\,\dfrac{1}{sC_\pi} = \dfrac{i_c}{g_m} \end{cases}$$

$L_1 = 17\,nH$
$C_\pi = 1.5\,pF$
$f_0 = 2\,GHz$
$f_T = \dfrac{10^{12}}{2\pi\cdot 5\cdot 1.5} = 21.26$

$$\Longrightarrow Z(s) = \frac{v}{i} = \frac{1 + s^2 L_1 C_\pi}{g_m + s C_\pi}$$

$$\frac{1}{g_m} = \frac{25m}{5m} = 5\,\Omega$$

$$Z(\omega) = \frac{1 - \omega^2 L_1 C_\pi}{g_m(1 + j\,\omega/\omega_T)} = \frac{1}{g_m}\,\frac{(1-\omega^2 L_1 C_\pi)}{1+(\omega/\omega_T)^2}(1 - j\,\omega/\omega_T)$$

b) $\omega_0 \ll \omega_T$: $Z(\omega_0) \simeq \dfrac{1}{g_m}(1 - \omega_0^2 L_1 C_\pi)(1 - j\,\omega_0/\omega_T)$

I) $-Re(Z(\omega_0)) = R$; $R_{max} = \dfrac{1}{g_m}(\omega_0^2 L_1 C_\pi - 1) \simeq 15.1\,\Omega$

II) $-Im(Z(\omega_0)) = j\left(\omega_0 L - \dfrac{1}{\omega_0 C}\right)$; $\dfrac{1}{g_m}(\omega_0^2 L_1 C_\pi - 1)\,\dfrac{\omega_0}{\omega_T} = \dfrac{1}{\omega_0 C} - \omega_0 L$,

$R\cdot\dfrac{\omega_0}{\omega_T} = \dfrac{1}{\omega_0 C} - \omega_0 L$; $\omega_0 = \dfrac{1}{\sqrt{C(L + R/\omega_T)}} \Rightarrow C = \dfrac{1}{\omega_0^2\left(L + \frac{R}{\omega_T}\right)} =$

$Q = \dfrac{\omega_0 L}{R} = 5 \Rightarrow L = \dfrac{Q\cdot R}{\omega_0} = \dfrac{5\cdot 15.1}{2\pi\cdot 2G} = 6\,nH$ $\qquad = 1.037\,pF$

c) $\omega_0 \gg \omega_T$: $Z(\omega_0) \simeq \dfrac{1}{g_m}\cdot\dfrac{1-\omega_0^2 L_1 C_\pi}{\omega_0^2/\omega_T^2}\cdot(1 - j\,\omega_0/\omega_T) =$

$f_T = \dfrac{10^{12}}{2\pi\cdot 500\cdot 1.5} = 212\,MHz$ $\qquad = \dfrac{1}{g_m}\left(\dfrac{\omega_T}{\omega_0}\right)^2 - \omega_T L_1 + j\,\omega_0 L_1 + \dfrac{1}{j\omega_0 C_\pi}$

I) $\omega_T L_1 - \dfrac{1}{g_m}\left(\dfrac{\omega_T}{\omega_0}\right)^2 \geq R$ for startup ; $22.6 - 12.2\,\Omega = 10.4\,\Omega \leq 15.1\,\Omega = R$

$\Rightarrow$ no startup

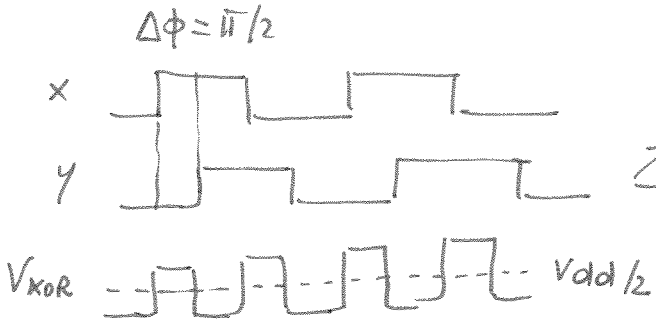
II) $\omega_0 = \dfrac{1}{\sqrt{(L+L_1)\cdot\frac{C C_\pi}{C + C_\pi}}} = \dfrac{1}{\sqrt{23\cdot 10^{-9}\cdot 0.6\cdot 10^{-12}}} = 8.5\,\dfrac{Grad}{s}$

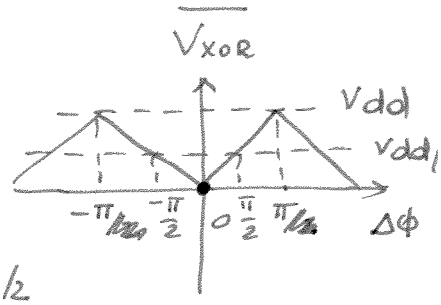
$\Rightarrow f_0 = 1.355\,GHz$

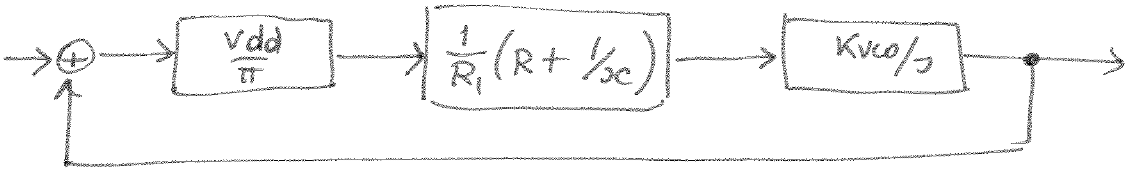
ES. 2

a)

x\y	0	1
0	0	1
1	1	0







1) $R_1 = \dfrac{V_{dd}/2}{I_{max}} = \dfrac{1.65V}{1mA} = 1.65\,k\Omega$

2) $K_{vco} = 2\pi \cdot \dfrac{50k}{3.3} = 95.2\ krad/Vs$

3) $G_{LOOP}(s) = \underbrace{\left(\dfrac{V_{dd}}{\pi} \cdot \dfrac{K_{vco}}{s} \dfrac{R}{R_1}\right)}_{K} \left(1 + \dfrac{1}{sRC}\right)$

** $|G_{LOOP}(\omega_u)| = \dfrac{K}{\omega_u}\sqrt{1 + \dfrac{\omega_z^2}{\omega_u^2}} =$

↓

ess. $\omega_u/\omega_z = \sqrt{3}$:

$\omega_u = K \cdot \sqrt{1 + 1/3} = 1.15 \cdot K$

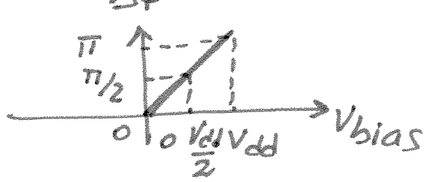
$|G_{LOOP}(\omega_u)| \cong \dfrac{V_{dd}}{\pi} \dfrac{K_{vco}}{\omega_u} \dfrac{R}{R_1} = 1 \Rightarrow \omega_u = \dfrac{V_{dd}}{\pi} \cdot K_{vco} \cdot \dfrac{R}{R_1} = 2\pi \cdot 1K$

$\Rightarrow \dfrac{R}{R_1} = \dfrac{2\pi \cdot 1K \cdot \pi}{V_{dd} \cdot \pi_{vco}} = \dfrac{\cancel{2\pi} \cdot 1K \cdot \pi}{\cancel{2\pi} \cdot 50K} = 0.0628 \Rightarrow R = 103.62\,\Omega$ $(90\Omega)^*$

$\measuredangle G_{LOOP}(\omega_u) = \arctan\left(\dfrac{\omega_u}{\omega_z}\right) = \dfrac{\pi}{3} \Rightarrow \dfrac{\omega_u}{\omega_z} = \sqrt{3} \Rightarrow$

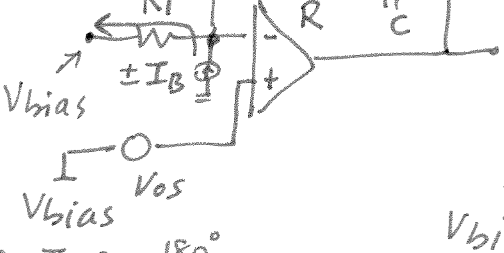
$\omega_z = \dfrac{\omega_u}{\sqrt{3}} \Rightarrow RC = \dfrac{\sqrt{3}}{\omega_u} = 276\,\mu s \Rightarrow C = 2.66\,\mu F$ $(3.06\,\mu F)^{**}$

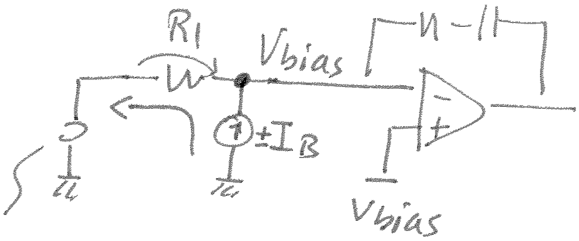
b) $\Delta\varphi = 90°$ per $V_{bias} = 1.65V$ →



Assumendo che VCO abbia guadagno negativo (ossia frequenza aumenta al diminuire della tensione di controllo)

c) $V_{bias} = 1.65V$





$V_{bias} + V_{os} + I_B R_1$

↓

$V_{bias} \mp I_B R_1$

$\Delta\varphi = 90° + V_{os} \cdot \dfrac{180°}{V_{dd}} + I_B R_1 \cdot \dfrac{180°}{V_{dd}} =$

$= 90° + \dfrac{0.01}{3.3} \cdot 180° + \dfrac{0.0165}{3.3} \cdot 180° = 90° + 0.54° + 0.95° = 91.49°$

ES. 3

a) $IIP_2 = \Delta P + P_{in} = 90 - 30 = 60 \text{ dBm}$

$-30-(-120) = +90$

b) $P_n = -100 \text{ dBm} \Rightarrow C = \frac{KT}{P_n} = \frac{\frac{4 \cdot 10^{-21} V^2}{50 \Omega}}{10^{-13} W} =$

$= 800 \text{ pF}$