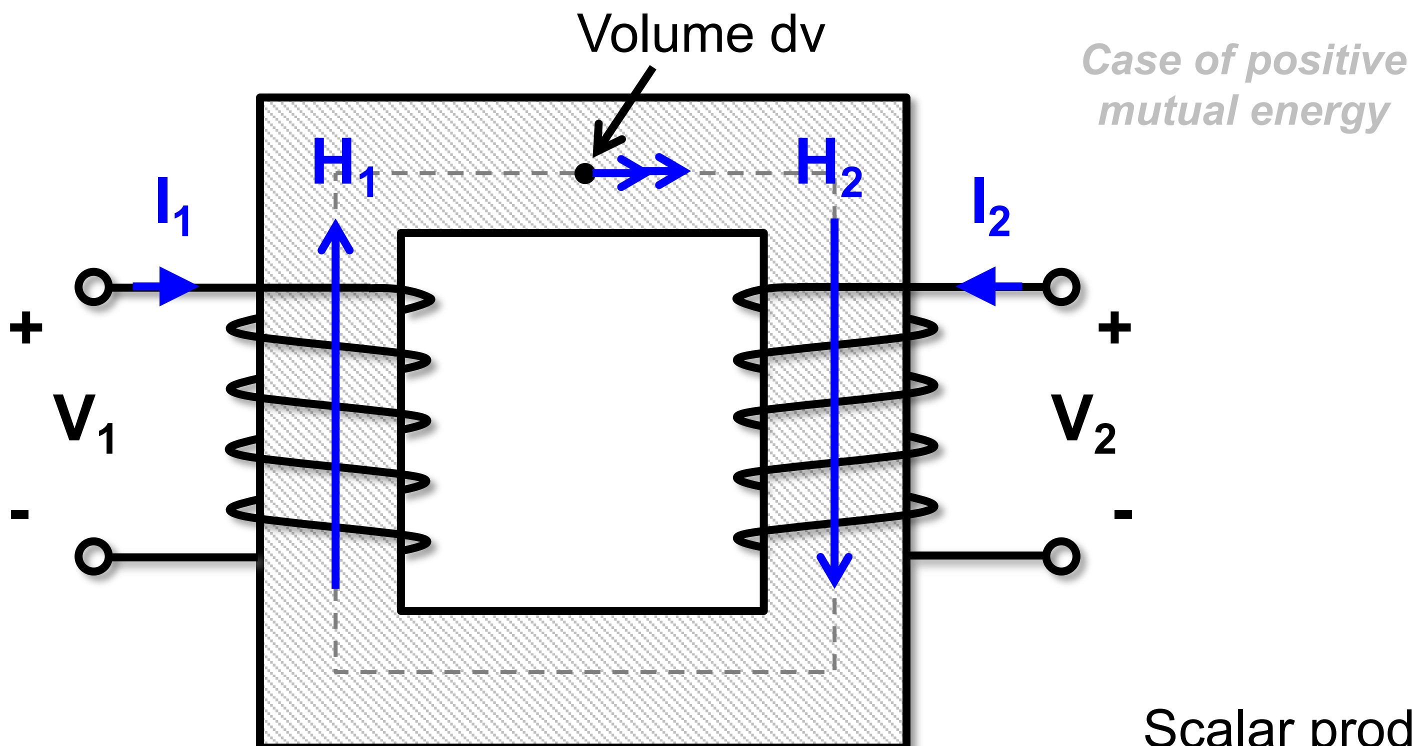
RF Circuit Design

TG

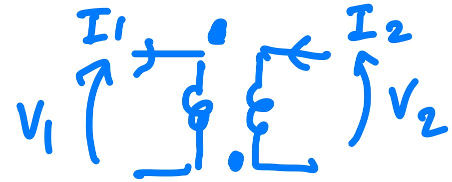
Transformers: Physical View



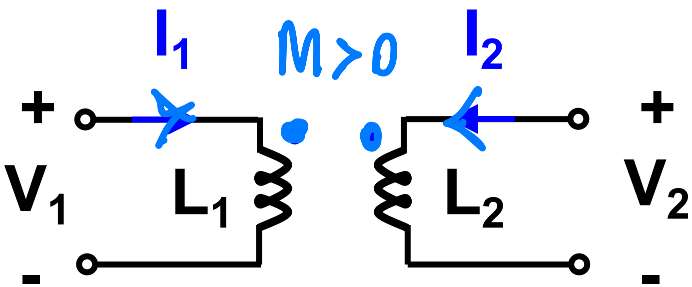
Scalar product

$$\text{Energy} = \frac{\mu}{2}\left|\vec{H}_1 + \vec{H}_2\right|^2 dv = \underbrace{\frac{\mu}{2}\left|\vec{H}_1\right|^2 dv}_{\text{Energy of coil 1}} + \underbrace{\frac{\mu}{2}\left|\vec{H}_2\right|^2 dv}_{\text{Energy of coil 2}} + \underbrace{\mu \vec{H}_1 \cdot \vec{H}_2 dv}_{\text{Mutual Energy}}$$

Transformers: Circuit View



M<0
Mutual en.<0



Case of positive mutual energy

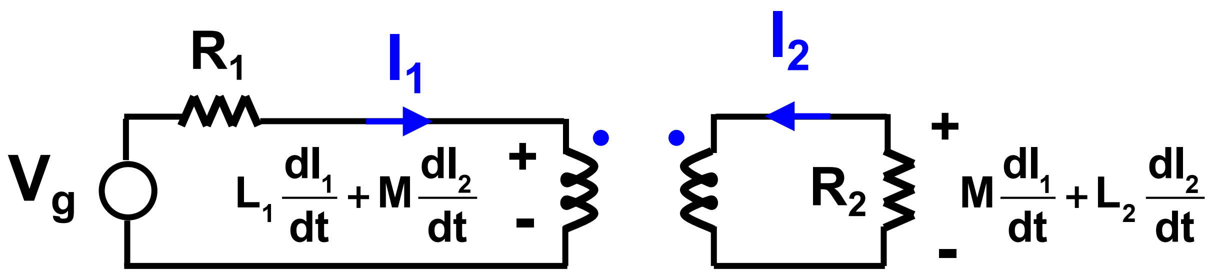
$$\begin{cases} \phi_1 = L_1 I_1 + M I_2 \\ \phi_2 = M I_1 + L_2 I_2 \end{cases} \qquad \begin{cases} V_1 = \dot{\phi}_1 \\ V_2 = \dot{\phi}_2 \end{cases}$$

Passive sign convention

$$\text{Energy} = \int_0^t \left(V_1 I_1 + V_2 I_2\right) dt' = \underbrace{\frac{1}{2} L_1 I_1^2(t)}_{\text{Energy of coil 1}} + \underbrace{\frac{1}{2} L_2 I_2^2(t)}_{\text{Energy of coil 2}} + \underbrace{\overset{M>0}{M} \cdot \overset{>0}{I_1(t)}\,\overset{>0}{I_2(t)}}_{\text{Mutual Energy}}$$

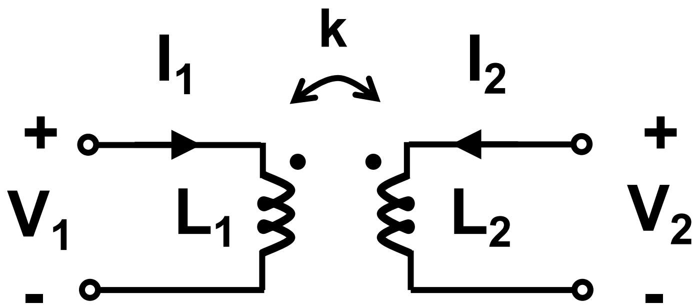
- Sign of mutual energy depends on the sign of **M** and on the sign of $I_1 I_2$
- Dot convention: **M** is positive if both currents enter or leave the dotted terminal

Transformers: More on dot convention



- **If a current enters the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is positive at its dotted terminal.**
- **If a current leaves the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is negative at its dotted terminal.**

Coupling Coefficient



Mutual Inductance

$$\begin{cases} \phi_1 = L_1 I_1 + M I_2 \\ \phi_2 = M I_1 + L_2 I_2 \end{cases}$$

Self-Inductance

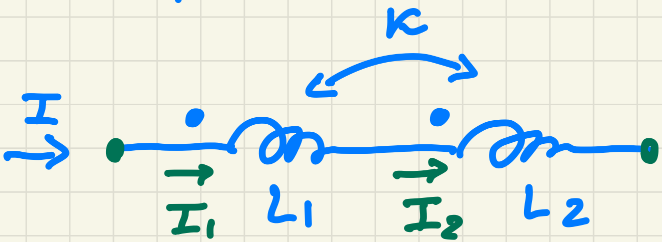
- Definition of coupling coefficient

$$k = \frac{|M|}{\sqrt{L_1 L_2}}$$

- Conservation of energy implies that

$$0 \le k \le 1$$

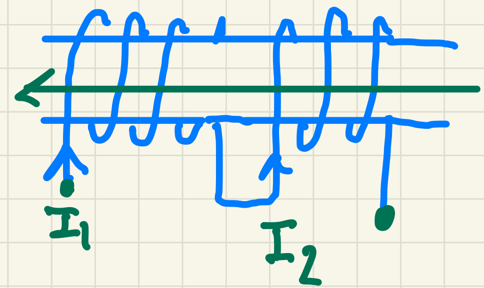
Examples : Series of coupled inductors



$I = I_1 = I_2$

- This the case of <u>POSITIVE M</u> (mutual energy) because I_1 and I_2 enter the dotted terminal

- Total inductance L_{TOT} :

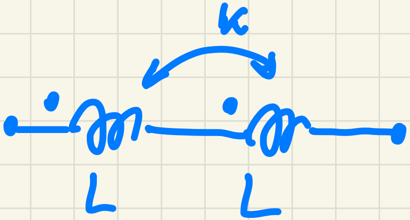


flux

$$\phi = \phi_1 + \phi_2 = \underbrace{L_1 I_1 + M I_2}_{\phi_1} +$$

$$+ \underbrace{M I_1 + L_2 I_2}_{\phi_2} = (\underbrace{L_1 + L_2 + 2M}_{L_{TOT}}) \cdot I$$

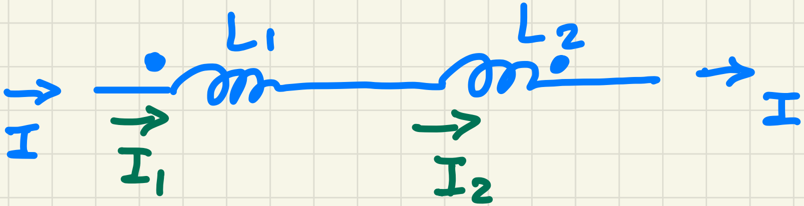
$$L_{TOT} = L_1 + L_2 + 2M = L + L + 2\cdot K \cdot L = 2(1+K)\cdot L$$



- case: $L_1 = L_2 = L$

$$|M| = K\cdot\sqrt{L_1 L_2} = K\cdot L$$

$$2(1+K)\cdot L \xrightarrow{K=0} 2L \qquad 2(1+K)\cdot L \xrightarrow{K=1} 4L$$



NEGATIVE M ($M<0$)

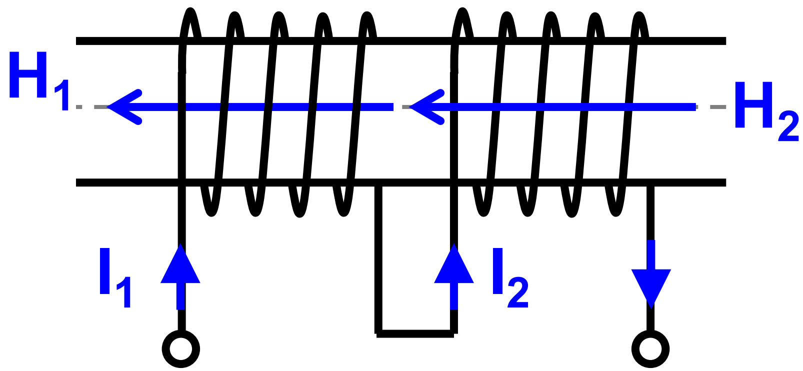
$$\Phi = \Phi_1 + \Phi_2 = L_1 I_1 - |M| I_2 - |M| I_1 + L_2 I_2 =$$

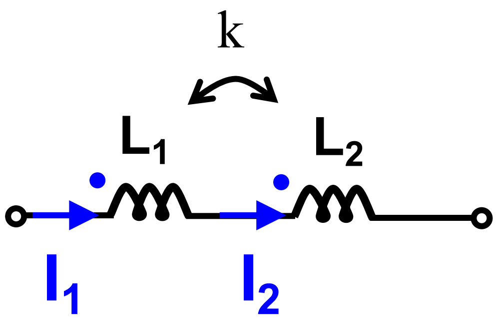
$$= \underbrace{(L_1 + L_2 - 2|M|)}_{\text{total inductance } L_{tot}} I$$

- case $L_1 = L_2 = L$: $L_{TOT} = 2\cdot(1-K)\cdot L$

$$2\cdot(1-K)\cdot L \xrightarrow{K=0} 2L \qquad 2\cdot(1-K)\cdot L \xrightarrow{K=1} 0$$

Example: Series of Two Coupled Inductors (I)





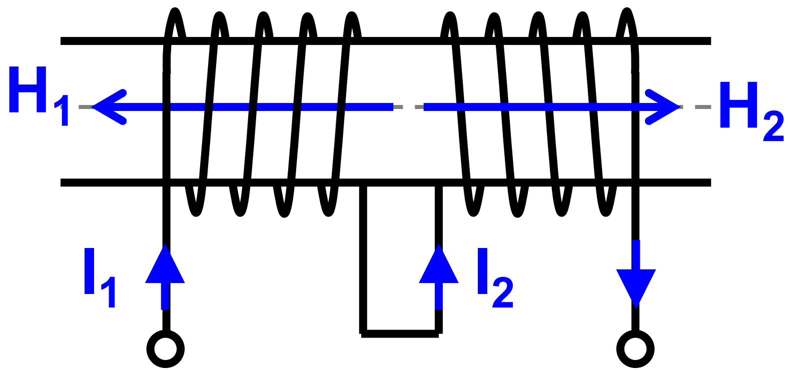
Case of positive mutual energy (M is positive)

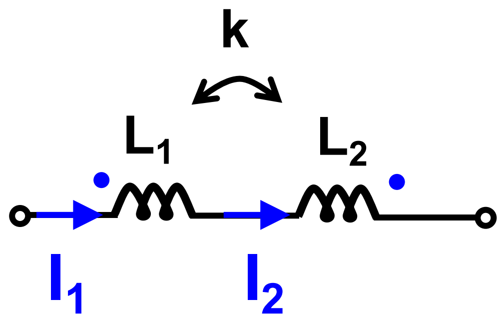
$$\phi = \phi_1 + \phi_2 = L_1 I_1 + M I_2 + M I_1 + L_2 I_2 = \underbrace{\left(L_1 + L_2 + 2M\right)}_{\text{Total Inductance } L_{tot}} I$$

- **If $L_1 = L_2 = L$:**

$$L_{tot} = 2\left(1 + k\right)L$$

Example: Series of Two Coupled Inductors (II)





Case of negative mutual energy (M is negative)

$$\phi = \phi_1 + \phi_2 = L_1 I_1 - |M| I_2 - |M| I_1 + L_2 I_2 = \underbrace{\left(L_1 + L_2 - 2|M|\right)}_{\text{Total Inductance } L_{tot}} I$$

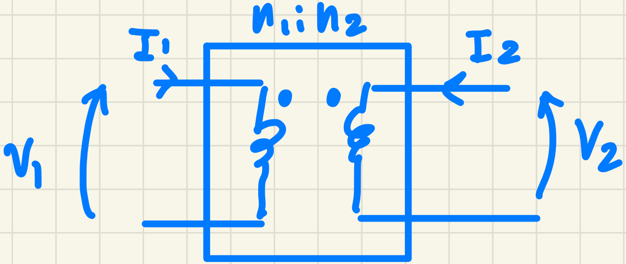
- **If $L_1 = L_2 = L$:**

$$L_{tot} = 2\left(1 - k\right)L$$

Model of coupled inductors :

- model based on ideal transformer
- T-circuit model

Ideal transformer :



1) No flux dispersion
$(k = 1)$

$\Downarrow$

$\Phi_1 = n_1 \, \phi$ ← flux of a single turn

$\Phi_2 = n_2 \, \phi$

$\Rightarrow$ $\boxed{\dfrac{V_1}{V_2} = \dfrac{n_1}{n_2}}$

e.g. $1:n$ ideal transf.

$V_2 = V_1 \cdot n$

voltage amplification

2) Infinite self-inductance ($L_1, L_2 \to \infty$)

HOPKINSON LAW

$$\underbrace{m.m.f.}_{\text{magnetomotive force}} = \underbrace{\Phi}_{\text{flux}} \cdot \underbrace{\mathcal{R}}_{\text{Reluctance } (H^{-1}) \to 0} = \frac{\Phi}{\Lambda}$$

Λ ← Permeance → ∞

$$m.m.f. \to 0$$

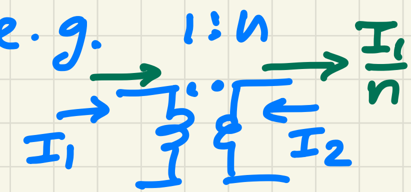
AMPERE'S LAW

$$\underbrace{m.m.f.}_{0} = n_1 I_1 + n_2 I_2$$

$$\Downarrow$$

$$\boxed{\frac{I_1}{I_2} = -\frac{n_2}{n_1}}$$

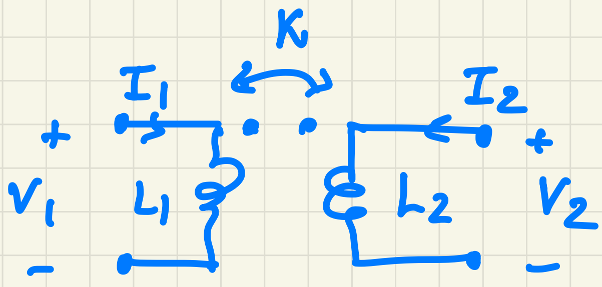
e.g.



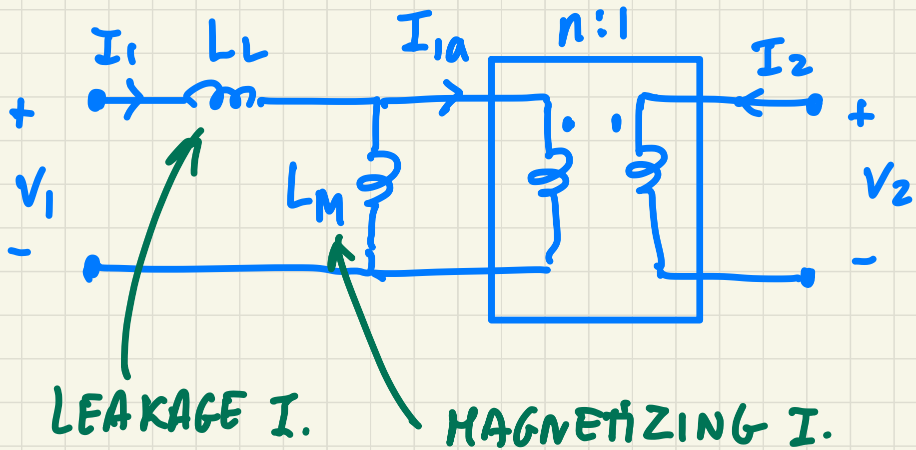
$$I_2 = -\frac{I_1}{n}$$

$$\frac{V_1}{V_2} \cdot \frac{I_1}{I_2} = \frac{n_1}{n_2} \cdot \left(-\frac{n_2}{n_1}\right) = -1 \Rightarrow V_1 I_1 + V_2 I_2 = 0 \Rightarrow \text{LOSSLESS}$$

Equivalent model of coupled inductors



$$
\boxed{\begin{cases} L_L = (1-k^2)\,L_1 \\ L_M = k^2 \cdot L_1 \\ n = k \cdot \sqrt{\dfrac{L_1}{L_2}} \end{cases}}
$$



verification:

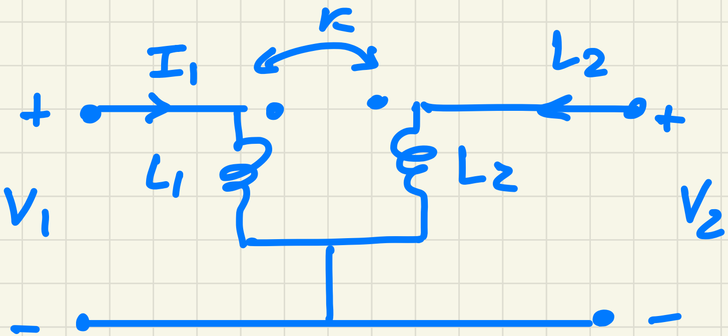
$$\Phi_1 = L_1 I_1 + M I_2$$

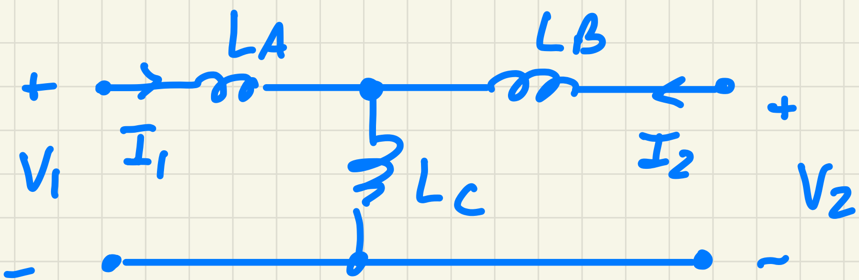
$$\Downarrow$$

$$L_1 = \left.\frac{\Phi_1}{I_1}\right|_{I_2=0} = L_L + L_M$$

$$I_2 = 0 \Rightarrow I_{1a} = 0 \Rightarrow \Phi_1 = (L_L + L_M) \cdot I_1$$

T- circuit model of coupled inductors





$$\begin{cases} L_A = L_1 - M \\ L_B = L_2 - M \\ L_C = M \end{cases}$$

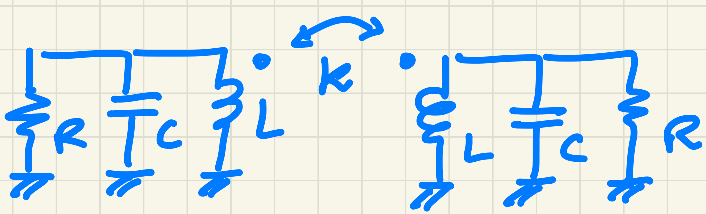
$$L_1 = \left.\frac{\Phi_1}{I_1}\right|_{I_2=0} =$$

$$= L_A + L_C$$

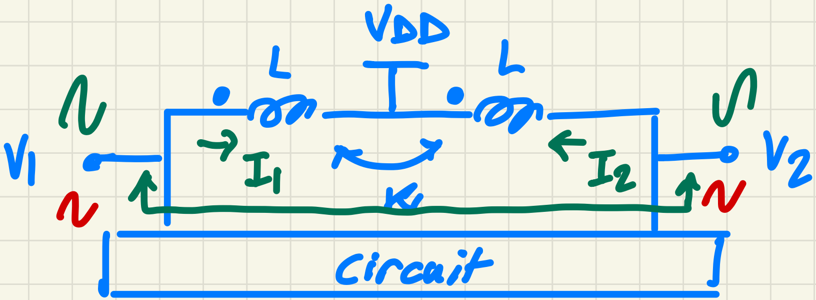
$$L_2 = \left.\frac{\Phi_2}{I_2}\right|_{I_1=0} =$$

$$= L_B + L_C$$

$\vdots$



- How many resonant frequencies has this circ?

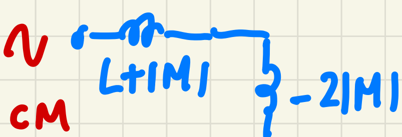


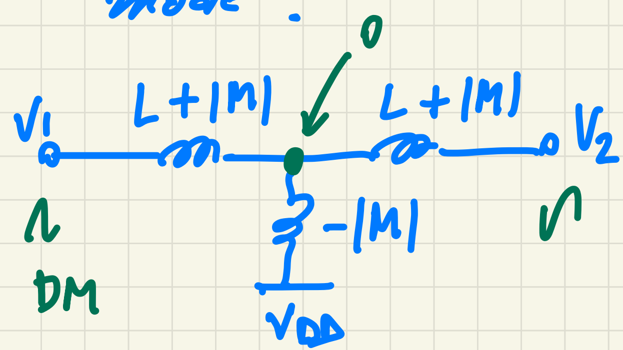
- What is the Load imped. in common mode and in differential mode?

"common-mode killer"

$K = 1$: $Z_{cm} \to 0$

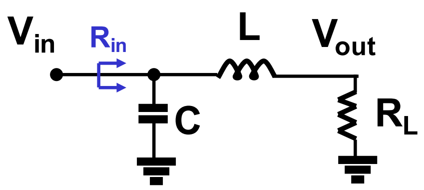
$Z_d \to j\omega\, 4L$





Tutorial T6

T6.1 Let us consider the L-match network in figure, where R_L=50Ω.

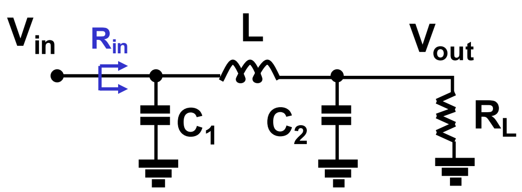


L - match

a) Size L and C in order to obtain R_{in}=100Ω at 5 GHz. What is the Q of the network?
b) Driving the input port of the network with a current source, evaluate the complex transimpedance V_{out}/I_{in} at 5GHz.

[Solution: a. C = 318fF, L = 1.59nH; b. $V_{out}/|I_{in}$ = 50Ω - j50Ω]

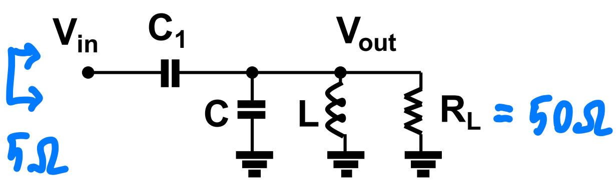
T6.2 Let us consider the π-match network in figure, where R_L=50Ω. Size L, C_1 and C_2 in order to obtain R_{in}=100Ω at 5GHz, and a quality factor of the resulting network equal to Q=5.



π- match

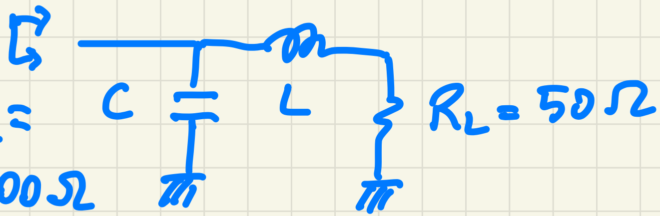
[Solution: a. C_1 = 955fF, C_2 = 1.274fF, L = 1.59nH]

T6.3 Let us consider the impedance transformation network in figure. Assuming R_L=50Ω and C=2pF, size L and C_1 to obtain an equivalent input impedance of 5Ω at 5GHz. What is the quality factor of the network?



[Solution: L = 258pH, C_1 = 2.12pH, Q = 3]

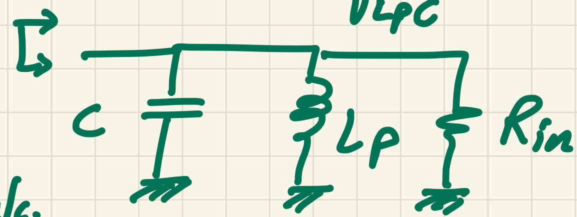
TG.1 $f_0 = 5\,GHz$



$R_{in} = 100\,\Omega$

$Z_{in} = R_{in}$ at $\omega_0 = \dfrac{1}{\sqrt{L_p C}}$

$\longrightarrow$ Equival. for Z_{in} calc.



a.

- $R_{in} = R_L \cdot (1 + Q_L^2)$ where $Q_L = \dfrac{\omega_0 L}{R_L}$

$$\omega_0 = \frac{1}{\sqrt{L_p C}}$$ where

- $L_p = L \cdot \left(1 + \dfrac{1}{Q_L^2}\right)$

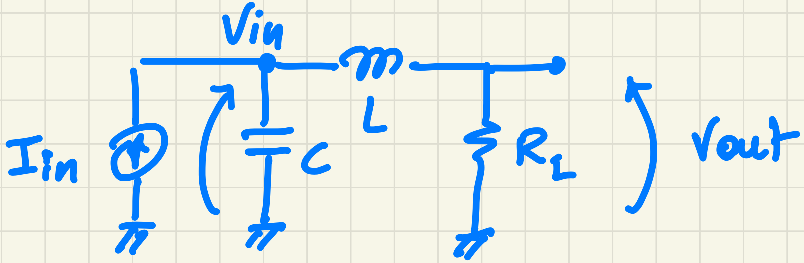
$$\Rightarrow Q_L = \sqrt{\frac{R_{in}}{R_L} - 1} = \sqrt{\frac{100}{50} - 1} = 1$$

$$L_p = L \cdot (1+1) = 2L$$

$$L = \frac{Q_L \cdot R_L}{\omega_0} = \frac{1 \cdot 50}{2\pi \cdot 5 \cdot G} = 1.59\,nH$$

$$\Rightarrow C = \frac{1}{\omega_0^2 L_p} = 318.6\,fF$$

b.



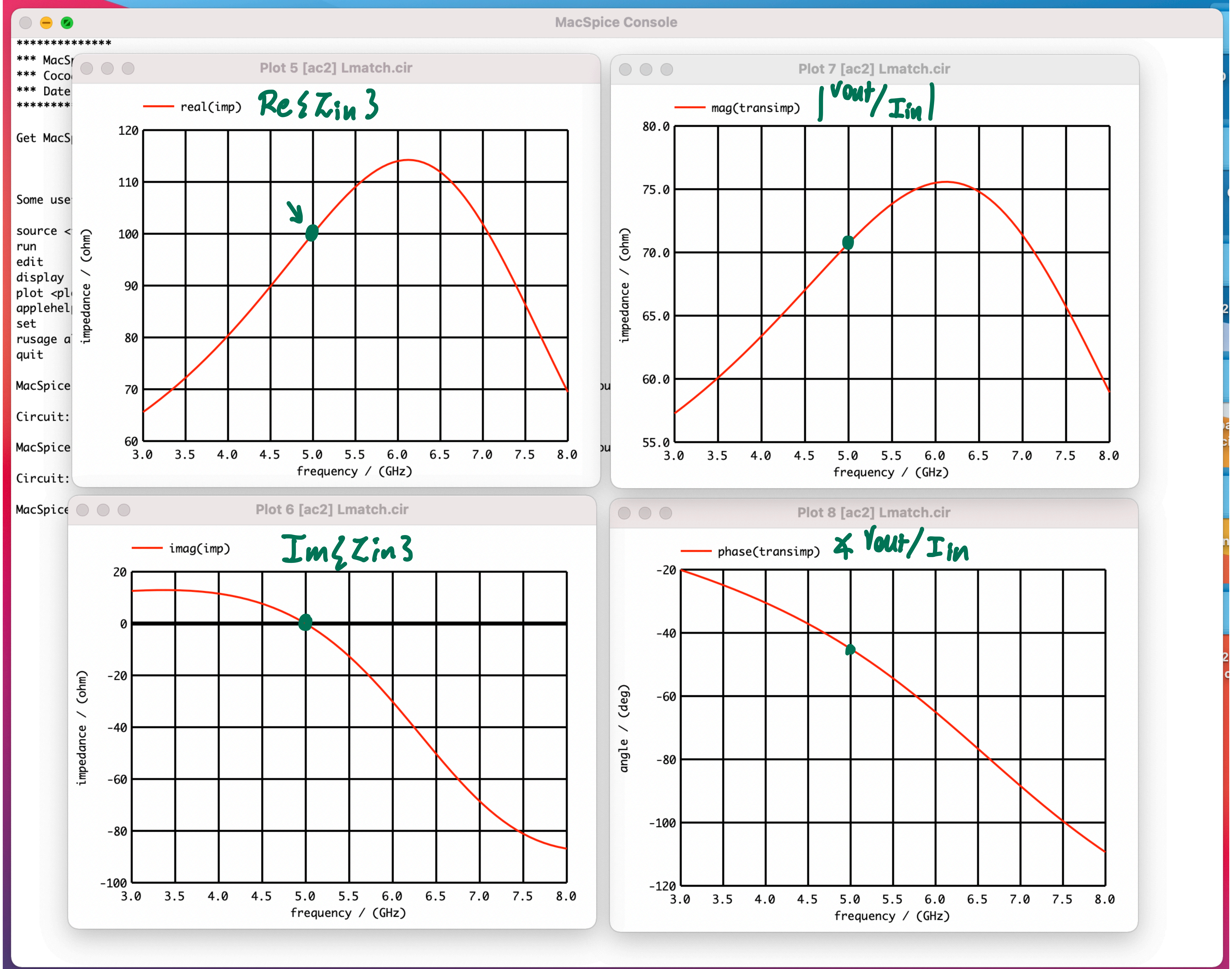
$\frac{V_{out}}{I_{in}}$ at $f_0 = 5GHz$

$$\frac{V_{out}}{I_{in}} = \underbrace{\frac{V_{in}}{I_{in}}}_{Z_{in}} \cdot \frac{V_{out}}{V_{in}} = R_{in} \cdot \frac{\cancel{R_L}\ 1}{\cancel{R_L}\ 1 + \underbrace{\frac{j\omega_0 L}{R_L}}_{Q_L}} =$$

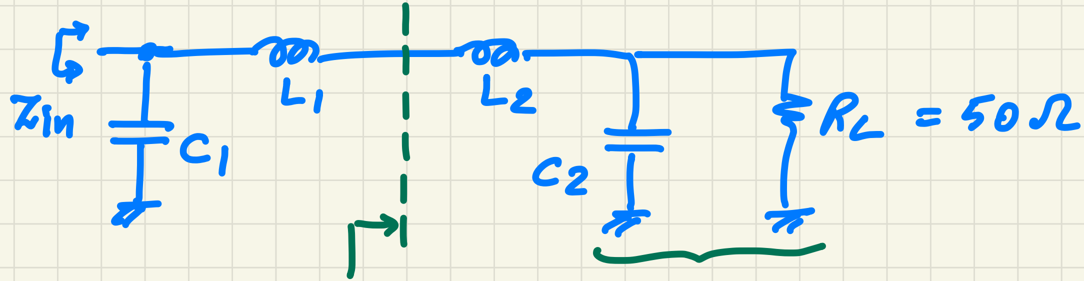
$$= R_{in} \cdot \frac{1}{1 + jQ_L} =$$

$$= 100 \cdot \frac{1}{1+j} =$$

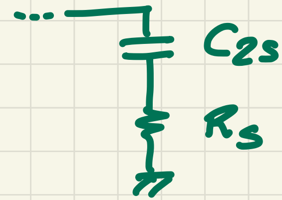
$$= 100 \cdot \frac{1-j}{1+1} = 50\Omega - j50\Omega$$


MacSpice Console
Plot 5 [ac2] Lmatch.cir
real(imp)
Re{Zin}
impedance / (ohm)
frequency / (GHz)
Plot 7 [ac2] Lmatch.cir
mag(transimp)
|Vout/Iin|
impedance / (ohm)
frequency / (GHz)
Plot 6 [ac2] Lmatch.cir
imag(imp)
Im{Zin}
impedance / (ohm)
frequency / (GHz)
Plot 8 [ac2] Lmatch.cir
phase(transimp)
∠ Vout/Iin
angle / (deg)
frequency / (GHz)

T6.2



$$R_s = \frac{R_L}{1+Q_2^2}$$



$$Q_2 = \frac{\omega_0 L_2}{R_s}$$

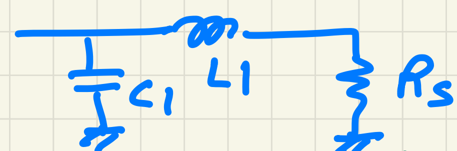
$$\omega_0 = \frac{1}{\sqrt{L_2 C_{2s}}}$$

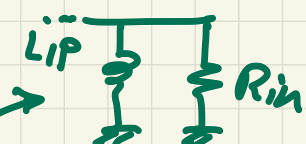
$$C_{2s} = C_2 \cdot \left(1 + \frac{1}{Q_2^2}\right)$$

$$R_{in} = R_s \cdot (1 + Q_1^2)$$

$$Q_1 = \frac{\omega_0 L_1}{R_s}$$

$$\omega_0 = \frac{1}{\sqrt{L_{1P} C_1}}$$





$$L_{1P} = L_1 \cdot \left(1 + \frac{1}{Q_1^2}\right)$$

$$\Rightarrow \begin{cases} R_{in} = R_L \cdot \dfrac{1+Q_1^2}{1+Q_2^2} = R_L \cdot \dfrac{1+Q_1^2}{1+(Q-Q_1)^2} \\ Q = Q_1 + Q_2 \; ; \quad Q_2 = Q - Q_1 \end{cases}$$

Known

$$\frac{100}{50} = 2 = \frac{1+9}{1+4} = \frac{10}{5}$$

$$\begin{cases} Q_1 = 3 \\ Q_2 = 2 \end{cases}$$

$$R_s = \frac{R_{in}}{1+Q_1^2} = \frac{100}{1+9} = 10\,\Omega$$

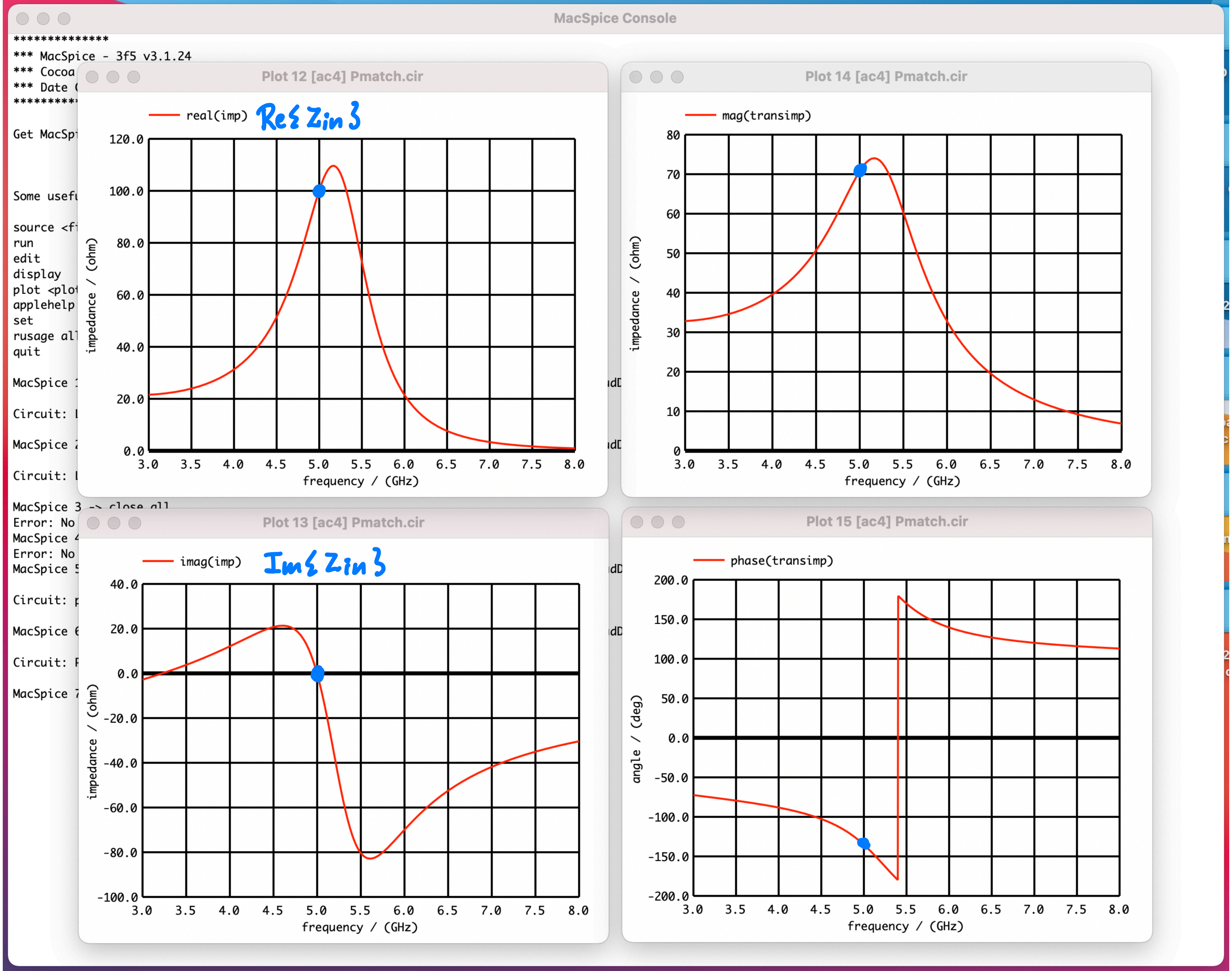
$$L_1 = \frac{Q_1 R_s}{\omega_0} = \frac{3 \cdot 50}{2\pi \cdot 5G} = 955\ pH$$

$$L_2 = \frac{Q_2 R_s}{\omega_0} = 637\ pH$$

$$L = L_1 + L_2 = 1.59\ nH$$

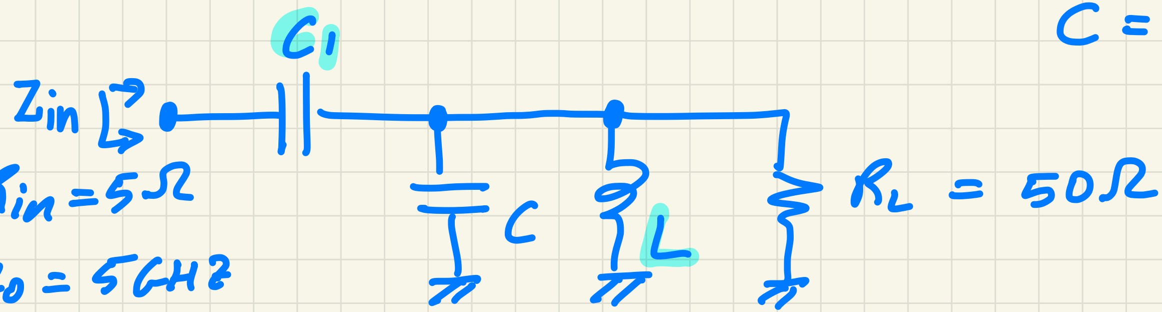
$$C_1 = \frac{1}{\omega_0^2 L_1 \left(1 + \frac{1}{Q_1^2}\right)} = 955\ fF$$

$$C_2 = \frac{1}{\omega_0^2 L_2 \cdot \left(1 + \frac{1}{Q_2^2}\right)} = 1.273\ pF$$


MacSpice Console
****
*** MacSpice - 3f5 v3.1.24
*** Cocoa
*** Date
Get MacSpi
Some usefu
source <f
run
edit
display
plot <plo
applehelp
set
rusage all
quit
MacSpice
Circuit:
MacSpice
Circuit:
MacSpice 3 -> close all
Error: No
MacSpice
Error: No
MacSpice
Circuit:
MacSpice
Circuit:
MacSpice
Plot 12 [ac4] Pmatch.cir
real(imp)
Re{Zin}
impedance / (ohm)
frequency / (GHz)
Plot 14 [ac4] Pmatch.cir
mag(transimp)
impedance / (ohm)
frequency / (GHz)
Plot 13 [ac4] Pmatch.cir
imag(imp)
Im{Zin}
impedance / (ohm)
frequency / (GHz)
Plot 15 [ac4] Pmatch.cir
phase(transimp)
angle / (deg)
frequency / (GHz)

T6.3

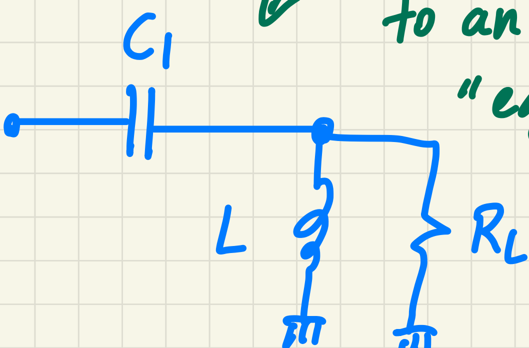
$C = 2\,fF$



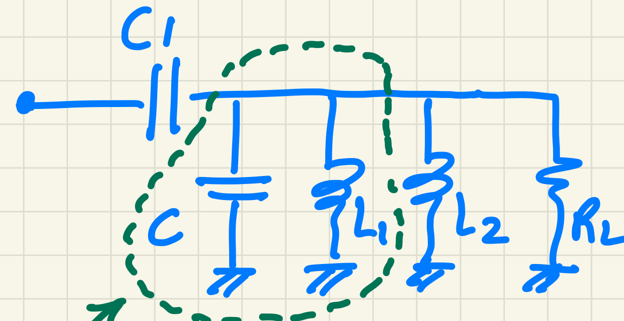
$R_{in} = 5\,\Omega$

$f_0 = 5\,GHz$

How can we get to an L-match "equivalent" case?



L-match

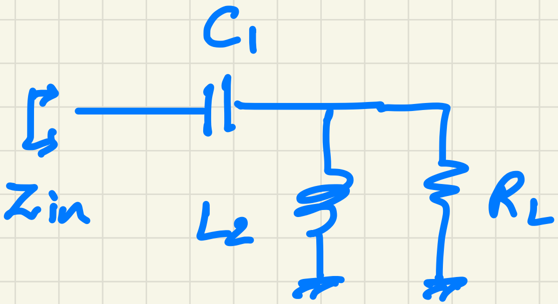


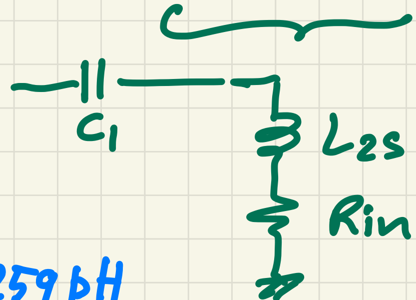
$L = L_1 \parallel L_2$

Resonance:

$$L_1 = \frac{1}{\omega_0^2 \cdot C} = 507\,pH$$

L-match





$L = L_1 // L_2 = 259\,pH$

(1) $$Z_{in} = \frac{R_L}{1 + Q_{L2}^2} = R_{in}$$

(2) $$Q_{L2} = \frac{R_L}{\omega_0 L_2}$$

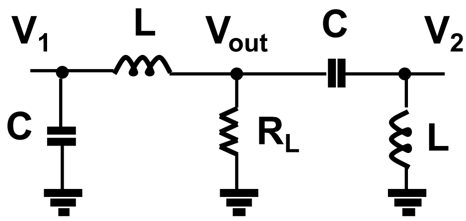
(3) $$\omega_0 = \frac{1}{\sqrt{C_1 L_{2s}}} = \frac{1}{\sqrt{C_1 \dfrac{L_2}{1 + 1/Q_{L2}^2}}}$$

(1)
$$Q_{L2} = \sqrt{\frac{R_L}{R_{in}} - 1} = \sqrt{\frac{50}{5} - 1} = 3$$

(2)
$$L_2 = \frac{\omega_0 Q_{L2}}{R_L} = 531\,pH$$

(3) $C_1 = 2{,}12\,pF$

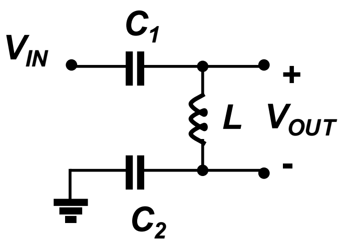
T6.4 We want to design a differential to single-ended signal converter Let us consider the circuit in figure, where R_L=50Ω.



a) Find the values of L and C to have a gain $|V_{out}|/|V_1-V_2|$ = 1 at 5GHz.
b) Evaluate the differential impedance between V_1 and V_2 at 5GHz.

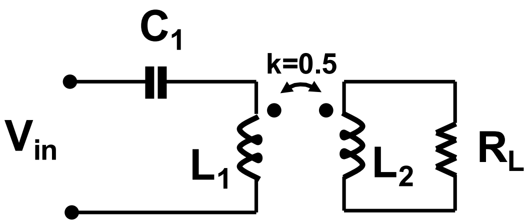
[Solution: a. L=1.59nH, C=637fF; b. R_{diff}=50Ω]

T6.5 We want to design a single-ended-to-differential converter of a sinusoidal signal from V_{IN} to V_{OUT}, using the circuit in figure. Assuming a 50Ω source resistance for V_{IN}, set C_1, C_2, and L to achieve differential output and V_{OUT}/V_{IN} = 3 at 5GHz.



[Solution: $C_1 = C_2$ = 424fF, L = 4.78nH]

T6.6 Let us consider the impedance-matching network in figure, based on a real transformer. Given a coupling factor k=0.5 between primary and secondary windings, L_2 = 1.59nH and R_L=50Ω, size L_1 and C_1 to obtain an equivalent input impedance of 5Ω at 5GHz. What is the Q of the resulting network?



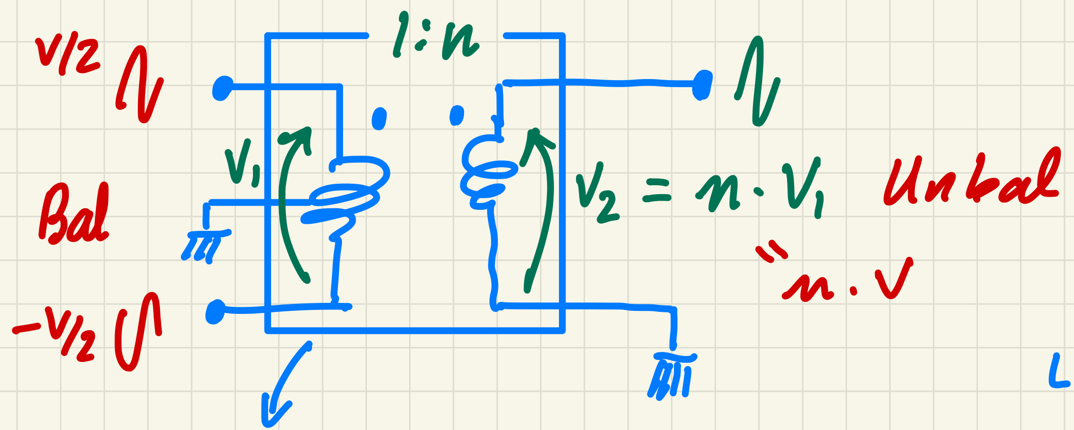
Coupled inductors

[Solution: L_1 = 1.237nH, C_1= 909.5fF, Q=7]

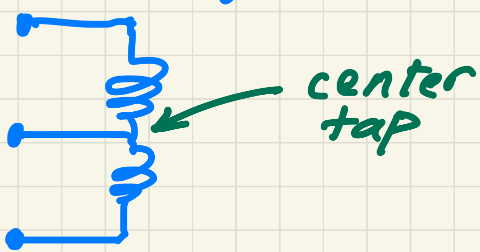
Differential-to-single-ended converter

is a "BalUn": balanced-to-unbalanced converter

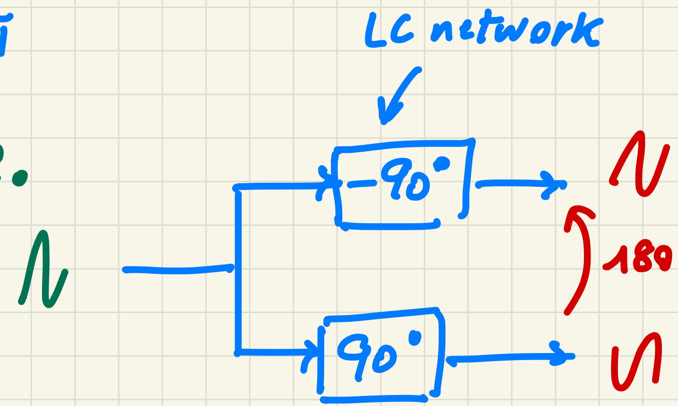
1.



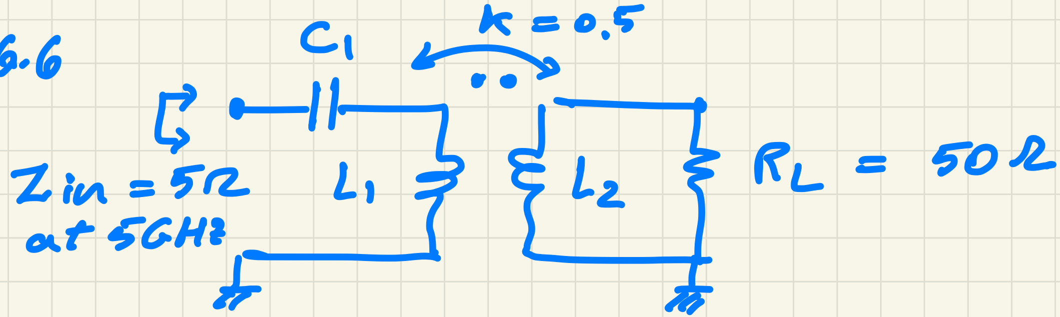
possible implementations of Balun based on transformers



2.



T6.6



use equiv. model of coupled inductors