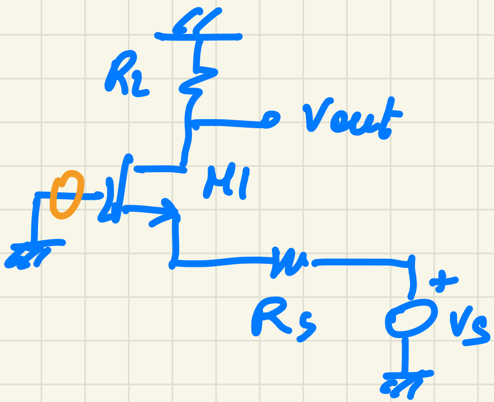
RF Circuit Design

L18

Common gate config.



- matching : $1/g_{m1} = R_s$

- Gain : $\dfrac{V_{out}}{V_s} = \dfrac{R_L}{2R_s} = A_0$ (matched)

- noise figure :

$$NF = 1 + \frac{\gamma}{\alpha}\underbrace{\frac{1}{g_{m1}R_s}}_{1} + \underbrace{\frac{4R_s}{R_L}}_{\text{matched}}$$

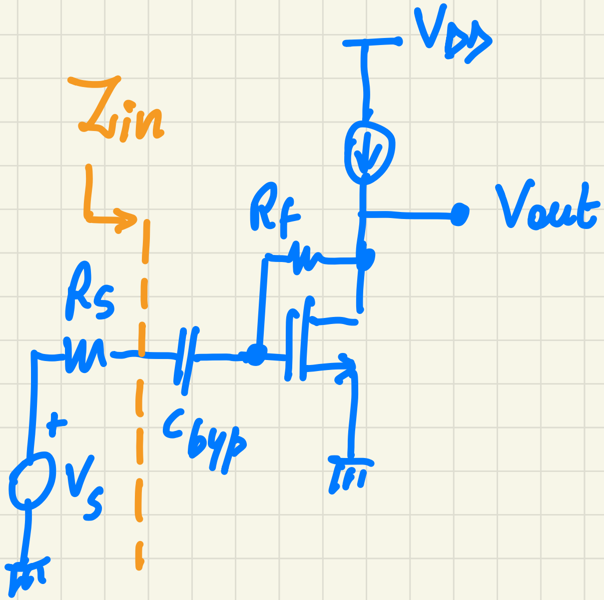
$$= 1 + \frac{\gamma}{\alpha} + \frac{2}{A_0}$$

to overcome NF limit

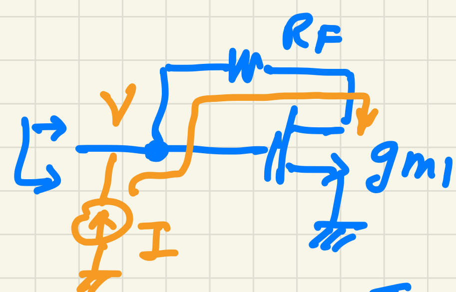
- Feedback : to decouple $1/g_m$ from R_s resistance
- Noise cancelling
- Impedance transformation

Noise cancelling

Shunt feedback topology :



- input impedance :

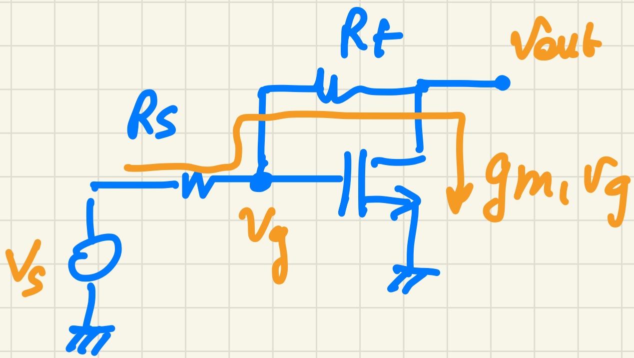


$$V = \frac{I}{g_{m1}} \Rightarrow Z_{in} = \frac{1}{g_{m1}}$$

matching : $\frac{1}{g_{m1}} = R_S$

(same as CG

- Voltage gain:

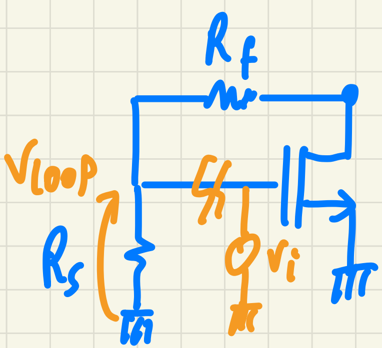


$$\begin{cases} V_{out} = V_s - g_{m1} V_g \cdot (R_s + R_f) & \text{KVL} \\ \dfrac{V_s - V_g}{R_s} = \dfrac{V_g - V_{out}}{R_f} & \text{KCL} \end{cases}$$

$$\Downarrow$$

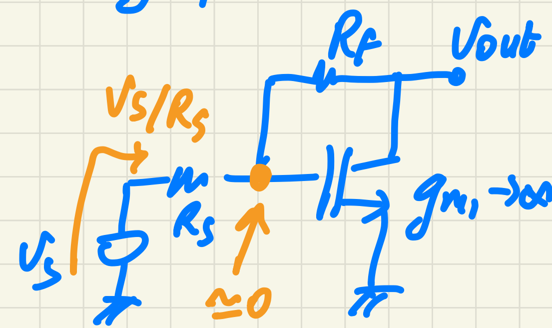
$$A_o = \frac{V_o}{V_s} = \frac{1 - g_{m1} R_f}{1 + g_{m1} R_s}$$

Feedback theory:



$$G_{LOOP} = -g_{m1} R_s$$

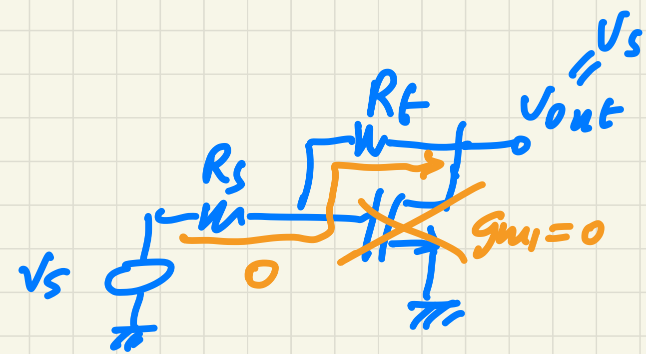
$$T_{ID} = -\frac{R_f}{R_s}$$



$$A_o = \frac{T_{ID}}{1 - \frac{1}{G_{LOOP}}} + \frac{T_{DIR}}{1 - G_{LOOP}}$$

$$= \frac{-R_f/R_s}{1 + \frac{1}{g_{m1} R_s}} + \frac{1}{1 + g_{m1} R_s}$$

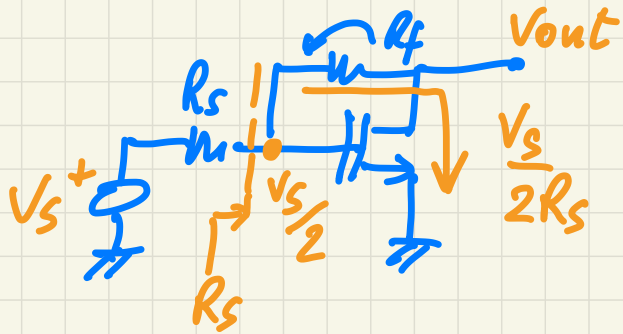
$$= \frac{1 - g_{m1} R_f}{1 + g_{m1} R_s}$$



matching condition: $1/g_{m1} = R_S \Rightarrow G_{loop} = -1$

$$\Rightarrow A_0\Big|_{matched} = \frac{1 - R_f/R_S}{1+1} = \frac{1}{2}\left(1 - \frac{R_f}{R_S}\right)$$

Alternative calculation:



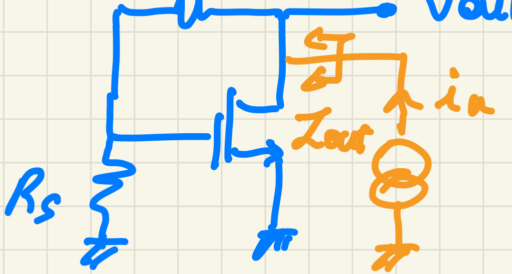
$$A_0\Big|_{matched} = \frac{V_{out}}{V_S} = \frac{V_S/2 - \frac{V_S}{2R_S}\cdot R_f}{V_S}$$

$$A_0\Big|_{matched} \longrightarrow -\frac{R_f}{2R_S} \qquad R_f \gg R_S$$

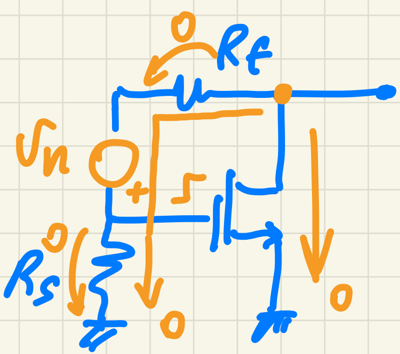
i.i same as CG

- Noise figure :

$$NF = 1 + \frac{\cancel{4kT}\,\frac{\gamma}{\alpha}\cdot \overbrace{g_{m1}}^{/\!/\ 1/R_s}\cdot \overbrace{\left(\frac{R_f+R_s}{\cancel{2}}\right)^2}^{\to R_f^2}}{\cancel{4kT}\,R_s\cdot \cancel{\frac{1}{4}}\cdot \underbrace{\left(1-\frac{R_f}{R_s}\right)^2}_{R_f^2/R_s^2}} + \text{0.f.}^{(*)}$$



$$\frac{V_{out}}{i_n} = Z_{out} = \frac{R_f+R_s}{1+G_{loop}} = \frac{R_f+R_s}{2}$$

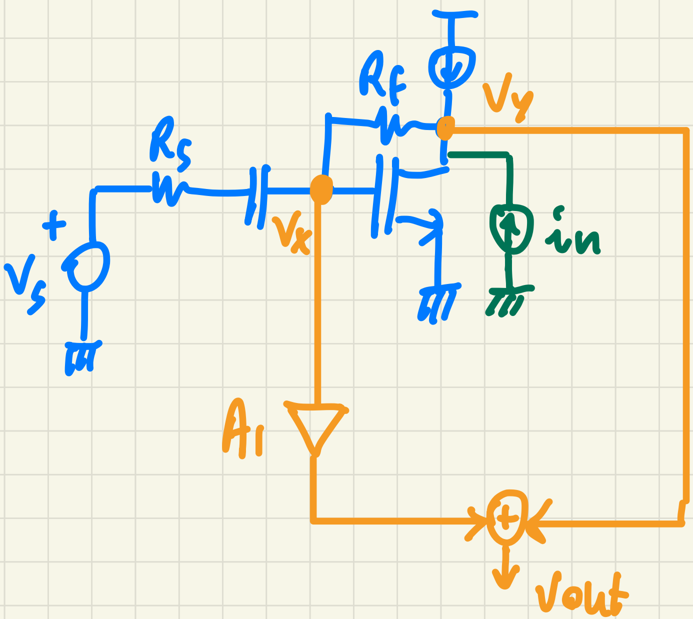


$$v_{out} = v_n$$

$$(*)\quad \frac{\cancel{4kT}\,R_f}{\cancel{4kT}\,R_s\,\frac{1}{4}\left(1-\frac{R_f}{R_s}\right)^2}$$

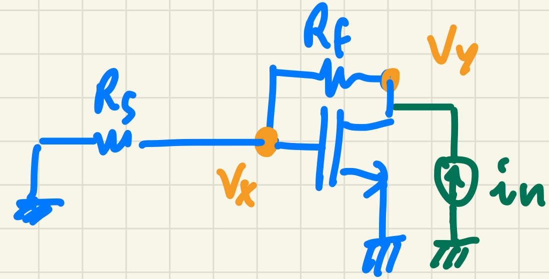
$$NF \xrightarrow[R_f \gg R_s]{} 1 + \frac{\gamma}{\alpha} + 4\frac{R_s}{R_f}$$ (same as CG)

□ Noise canalling technique (to shunt-feedback)



2 nodes to be combined such that noise source (in) is cancelled out, while signal (V_s) is not cancelled

- **Noise transfer :**



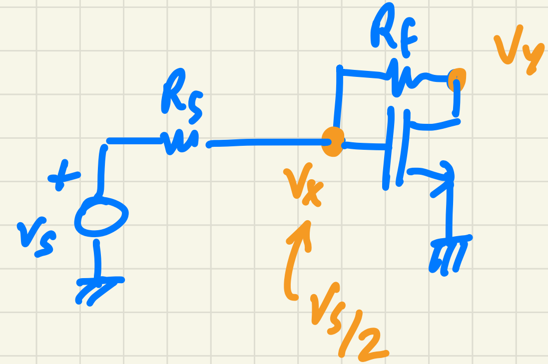
$$\frac{V_y}{i_n} = \frac{R_s + R_f}{2} > 0$$

$$\frac{V_x}{i_n} = \frac{V_y}{i_n} \cdot \frac{R_s}{R_s + R_f} = \frac{\cancel{R_s + R_f}}{2} \cdot \frac{R_s}{\cancel{R_s + R_f}} > 0$$

$$V_{out} = A_1 \cdot V_x + V_y = A_1 \cdot \frac{R_s}{2} + \frac{R_s + R_f}{2}$$

$$V_{out} = 0 \iff A_1 = -\underbrace{\left(1 + \frac{R_f}{R_s}\right)}$$

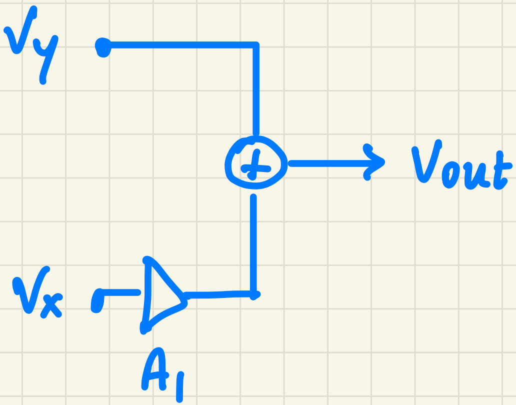
- Signal transfer :

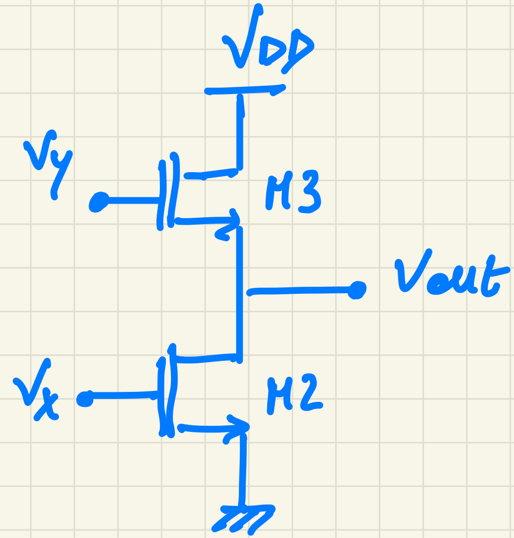


$$\frac{v_{out}}{v_s} = \frac{v_y}{v_s} + A_1 \frac{v_x}{v_s} =$$

$$= \underbrace{\frac{1}{2}\left(\cancel{1} - \frac{R_f}{R_s}\right)}_{A_0|\text{matched}} - \left(\cancel{1} + \frac{R_f}{R_s}\right) \cdot \underbrace{\frac{1}{2}}_{\text{matched condition}} =$$

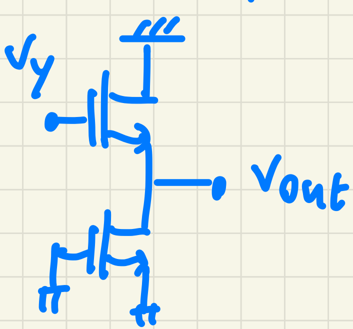
$$= -\frac{R_f}{R_s}$$

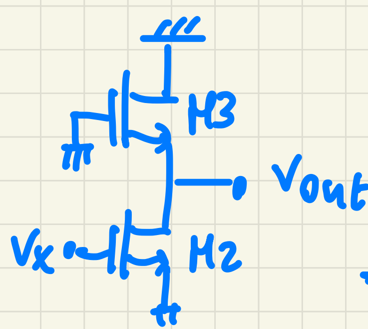




Superposition principle :

$$\frac{V_{out}}{V_y} \approx 1 \qquad r_o \to \infty$$

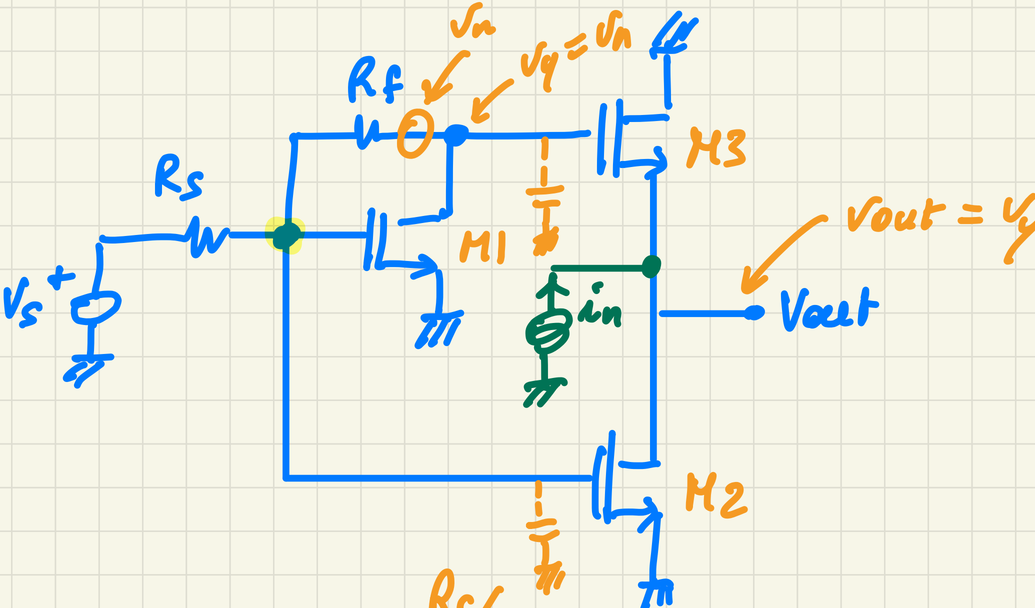




$$\frac{V_{out}}{V_x} = -\frac{g_{m2}}{g_{m3}} = A_1$$

$$\Rightarrow V_{out} = V_y - \frac{g_{m2}}{g_{m3}} V_x$$

Noise canc. topology : noise figure



$$\frac{g_{m2}}{g_{m3}} = 1 + \frac{R_f}{R_s}$$

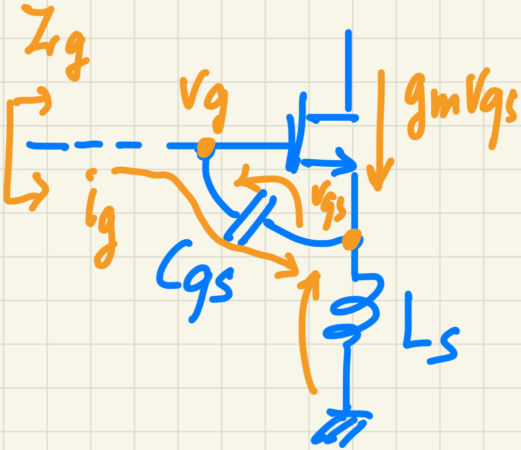
$$NF = 1 + \frac{\overbrace{\cancel{4kT}\cancel{R_f}}^{R_s/R_f}}{\cancel{4kT}\cancel{R_s}\cdot\left(\frac{R_f}{R_s}\right)^{\cancel{2}}} + \frac{\overbrace{\cancel{4kT}\frac{\gamma}{\alpha}\cdot(g_{m2}+g_{m3})\cdot\frac{1}{g_{m3}^2}}^{\left(\overbrace{\frac{g_{m2}}{g_{m3}}+1}\right)\cdot\frac{1}{g_{m3}}}}{\cancel{4kT}R_s\cdot\left(\frac{R_f}{R_s}\right)^2}$$

$$NF \xrightarrow[R_f \gg R_s]{} 1 + \underbrace{\frac{\gamma}{\alpha} \cdot \frac{\cancel{\frac{R_f}{R_s}} \cdot \frac{1}{g_{m3}}}{\cancel{R_s} \cdot \left(\frac{R_f}{\cancel{R_s}}\right)^{\cancel{2}}}}_{\gamma/\alpha \,\cdot\, 1/g_{m3}R_f}$$

If g_{m3} is larger than $1/R_f$: NF of this stage is lower than the NF of the shunt feedback topology

- M1 : $g_{m1} = 1/R_s$ for input matching
- M3 : $g_{m3} > \dfrac{1}{R_f}$ for low NF

Impedance transformation : CS Inductive degeneration



$$\begin{cases} v_g = v_{gs} + sL_s\left(g_m v_{gs} + i_g\right) \\ i_g = sC_{gs} \cdot v_{gs} \end{cases}$$

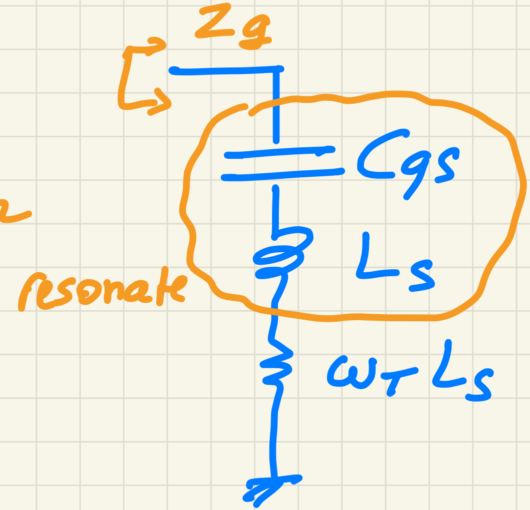
$$\Downarrow$$

$$v_g = v_{gs} + \left(sL_s g_m + s^2 C_{gs} L_s\right) v_{gs}$$

$$Z_g = \frac{v_g}{i_g} = \frac{\cancel{v_{gs}}\left(1 + sL_s g_m + s^2 C_{gs} L_s\right)}{sC_{gs} \cdot \cancel{v_{gs}}} =$$

$$= \frac{1}{sC_{gs}} + g_m \cdot \frac{L_s}{C_{gs}} + sL_s$$

$$Z_g = \underbrace{\frac{1}{sC_{gs}}}_{\text{capacitor}} + \underbrace{\overbrace{g_m}^{\approx \omega_T} \cdot \frac{L_s}{C_{gs}}}_{\text{real positive impedance}} + \underbrace{sL_s}_{\text{inductor}}$$



$$\omega_T \simeq \frac{g_m}{C_{gs}} \quad (\text{neglecting } C_{gd})$$

Matching condition :

- $\omega_0 = \dfrac{1}{\sqrt{L_s C_{gs}}}$
- $\omega_T L_s = R_s$