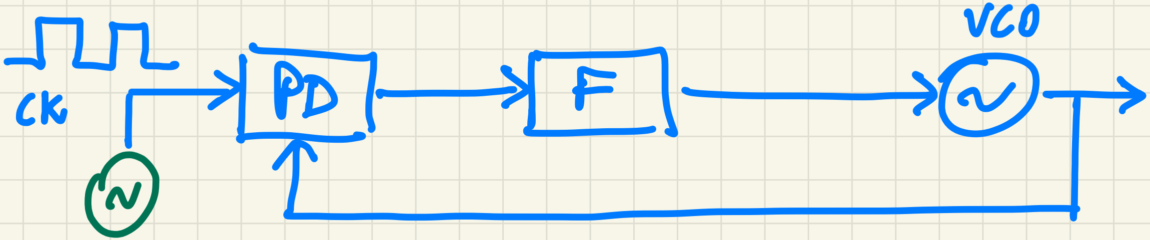
RF Circuit Design

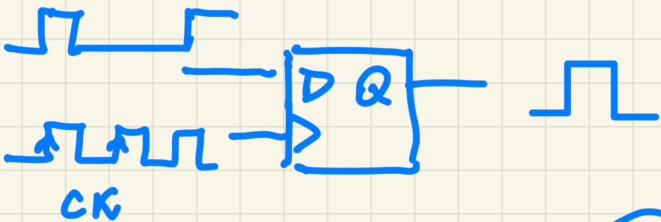
L9



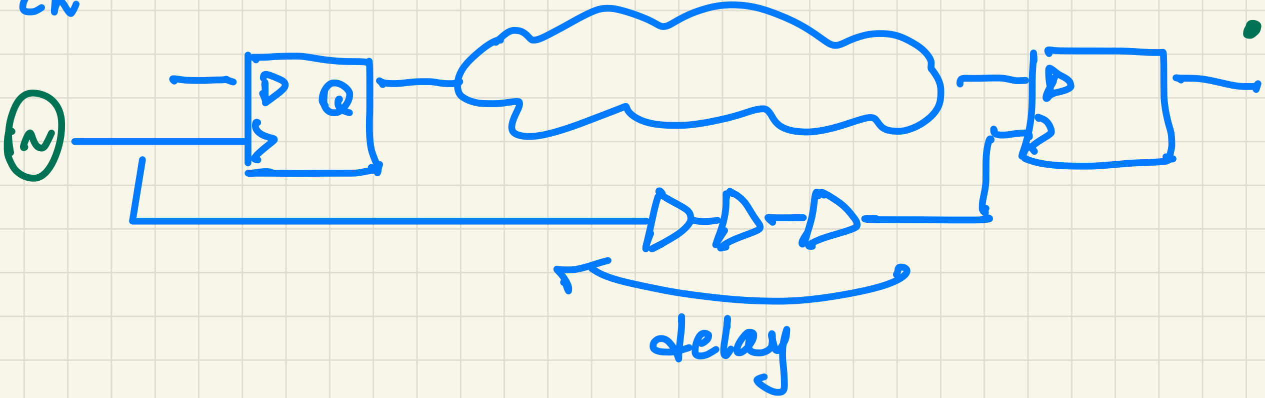
CK replica

PLL Applications

- Frequ. synth.
- Bandpass filter (clock recov.)
- Synchronizes two signals (CK replica)

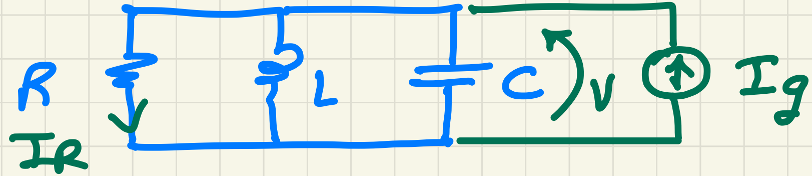


Clock skews



Passive Networks

Resonant circuits



Impedance

$$Z = \frac{V}{I_g} = \frac{I_R \cdot R}{I_g} = R \cdot \underbrace{\frac{I_R}{I_g}}_{H(s)}$$

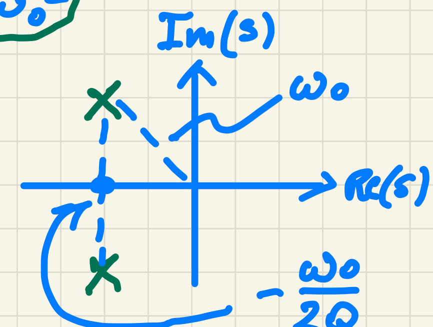
$$H(s) = \frac{1/R}{1/R + 1/sL + sC} = \frac{s\,\omega_0/Q}{s^2 + s\,\underbrace{\omega_0/Q}_{2\xi\omega_0} + \omega_0^2}$$

Define : $Q \triangleq \omega_0 RC$

$\omega_0 \triangleq \frac{1}{\sqrt{LC}}$

$p_1 p_2 = \omega_0^2$

$-(p_1 + p_2) = \omega_0/Q$



$$p_{1,2} = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - 1/4Q^2}$$ pole location

$$\xi = \frac{1}{2Q} = \frac{1}{2\omega_0 RC}$$ damping factor

Meaning of Q factor :

1. inversely prop. to the damping factor ξ

   ξ small $\Leftrightarrow$ Q large $\Leftrightarrow$ underdamped poles

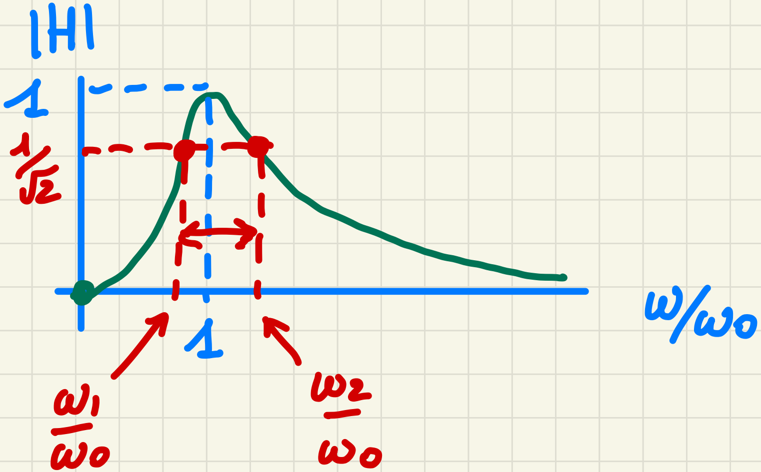
   Q related to pole location

2. -3dB BW :

$$H(j\omega) = \frac{j\omega\omega_0/Q}{-\omega^2 + j\omega\,\omega_0/Q + \omega_0^2} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$|H(j\omega)|^2 = \frac{1}{\left|1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right|^2} = \frac{1}{1 + \underbrace{Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}} = \frac{1}{2}$$

−3dB bandwidth $(\omega_2 - \omega_1)$



$$\Downarrow$$

$$Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2 = 1$$

$$\Downarrow$$

$$Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \pm 1$$

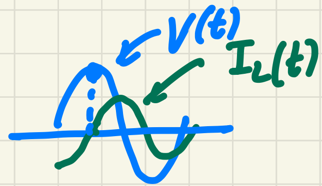
$$\omega^2 \mp Q\,\omega_0\omega - \omega_0^2 = 0$$

$$\omega_{1,2} = \omega_0\left(\mp\frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}\right) \Leftarrow$$

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega_2 - \omega_1}{\omega_0} = \frac{\omega_0/2Q + \omega_0/2Q}{\omega_0} = \frac{\omega_0/Q}{\omega_0} = \frac{1}{Q}$$

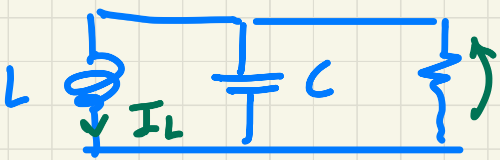
Q is the ratio between the center frequency and the −3dB BW of the frequency response

$$Q = \frac{\omega_0}{\Delta\omega}$$



3. Energy meaning

$$Q = \omega_0 RC = \omega_0 \frac{\overbrace{\tfrac{1}{2}C|\bar{V}|^2}^{E_{stored}}}{\underbrace{\tfrac{1}{2}\frac{|\bar{V}|^2}{R}}_{P_{DISS}}}$$



$$V(t) = \mathrm{Re}\{\bar{V} \cdot e^{j\omega_0 t}\}$$

$$\Downarrow$$

$$E_{stored} = \frac{1}{2} L I_L^2(t) + \frac{1}{2} C V^2(t) = \frac{1}{2} C |\bar{V}|^2$$

$$Q = \omega_0 \cdot \frac{E_{stored}}{P_{diss}}$$

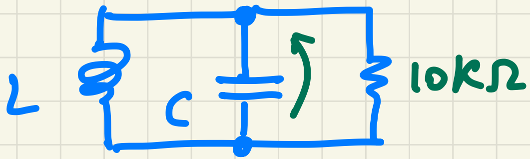
Q is ω_0 times the ratio between stored energy and diss. power in a resonator

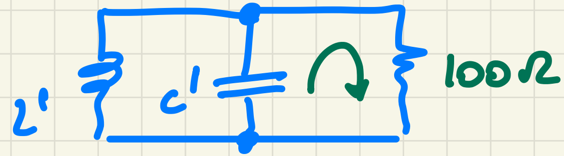
$$Q = 2\pi \cancel{f_0} \cdot \frac{E_{stored}}{E_{diss\ in\ 1\ cycle} \cdot \cancel{f_0}} \Rightarrow$$

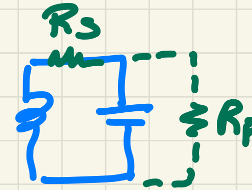
$$\Rightarrow Q = 2\pi \cdot \frac{E_{stored}}{E_{diss.\ in\ 1\ cycle}}$$

Q is 2π times the ratio between stored and dissipated energy

Quality factor is high if the stored energy is much than the dissipated energy per cycle e.g.







ideal resonator (lossless)

$R_p \to \infty$

$R_s \to 0$

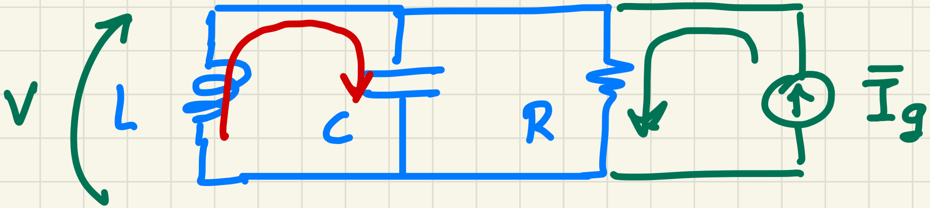
$LC = L'C' \Rightarrow$ same resonance frequency

Which resonator has the highest Q ?

$$Q = \omega_0 R C = \frac{R}{\omega_0 L} = \sqrt{\frac{C}{L}} \cdot R$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad ; \quad L = \frac{1}{\omega_0^2 C} \quad ; \quad \omega_0 L = \frac{1}{\omega_0 C}$$

4. Amplification at resonance



$I_g(t)$ sinusoid at resonance $\left(\omega_0 = \frac{1}{\sqrt{LC}}\right)$

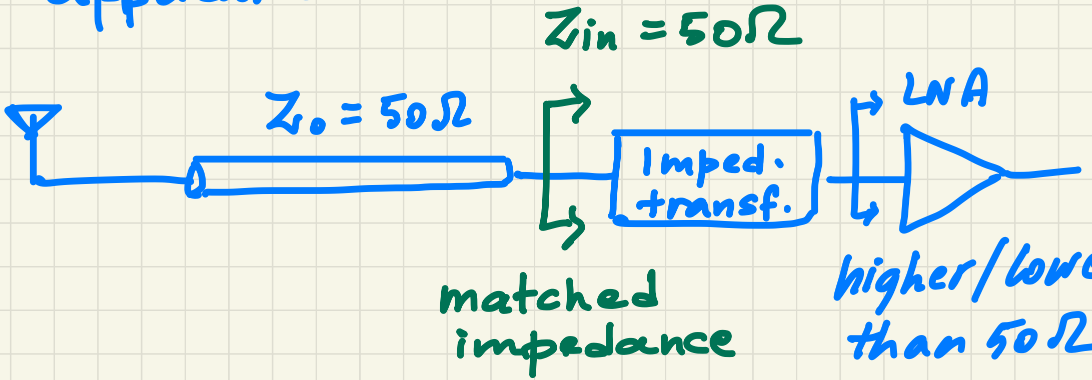
$$|\bar{I}_c| = \omega_0 C \cdot |\bar{V}| = \omega_0 C \cdot \underbrace{|\bar{I}_g| \cdot R}_{\text{ohm's law}} =$$

$$= Q \cdot |\bar{I}_g|$$

Q is the current gain between input current and capacitor / inductor current

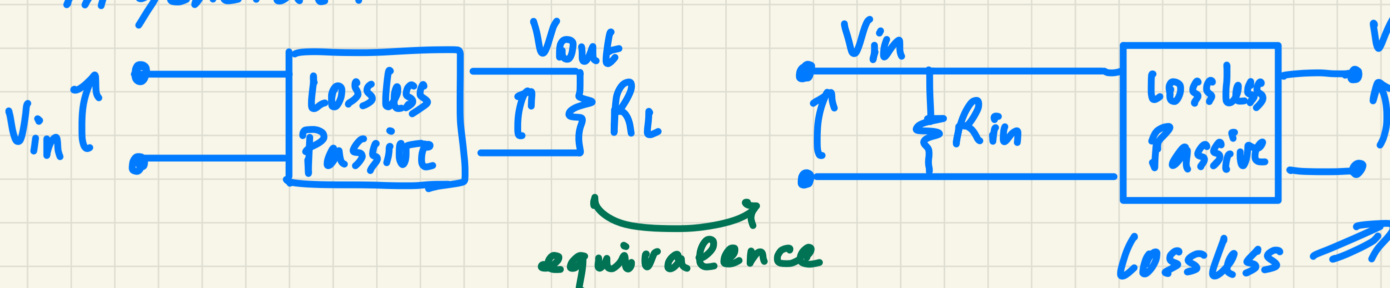
Impedance transformation (Matching networks)

e.g. application



impedance transformation: DOWNWARD / UPWARD

In general:



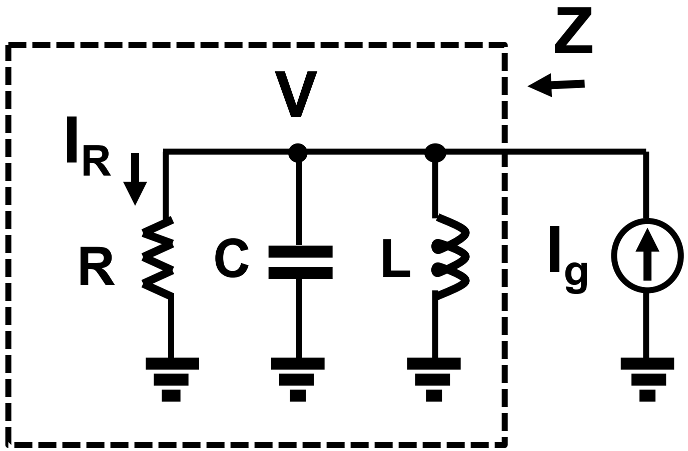
$$\frac{1}{2}\frac{|\bar{V}_{in}|^2}{R_{in}} = \frac{1}{2}\frac{|\bar{V}_{out}|^2}{R_L}$$

$$\Rightarrow R_{in} = \frac{R_L}{\frac{|\overline{V_{out}}|^2}{|\overline{V_{in}}|^2}} = \frac{R_L}{G^2}$$

$$G = \left|\frac{V_{out}}{V_{in}}\right|$$

- $G > 1$: amplification $\Rightarrow$ DOWNWARD imp. tran
- $G < 1$: attennation $\Rightarrow$ UPWARD imp. trans.

Resonant Circuits



Impedance in Laplace Transform:

$$Z(s) = \frac{V}{I_g} = R \cdot \frac{I_R}{I_g} = R \cdot H(s)$$

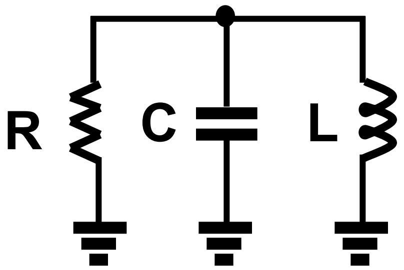
Definition

$$Q = \omega_0 RC$$

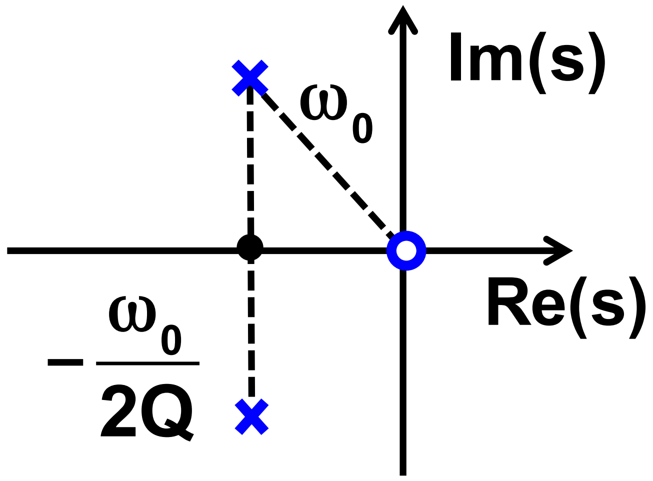
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$H(s) = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}} = \frac{s\frac{\omega_0}{Q}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Resonant Circuits: Complex Singularities



$$H(s)=\frac{\frac{s}{RC}}{s^2+\frac{s}{RC}+\frac{1}{LC}}=\frac{s\frac{\omega_0}{Q}}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

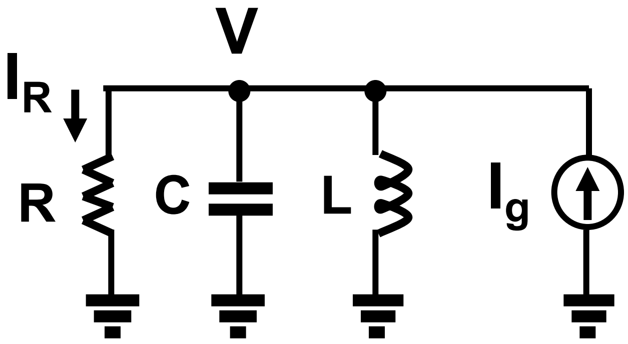


Damping Factor

$$\Rightarrow \zeta=\frac{-Re(\omega_p)}{|\omega_p|}=\frac{1}{2Q}$$

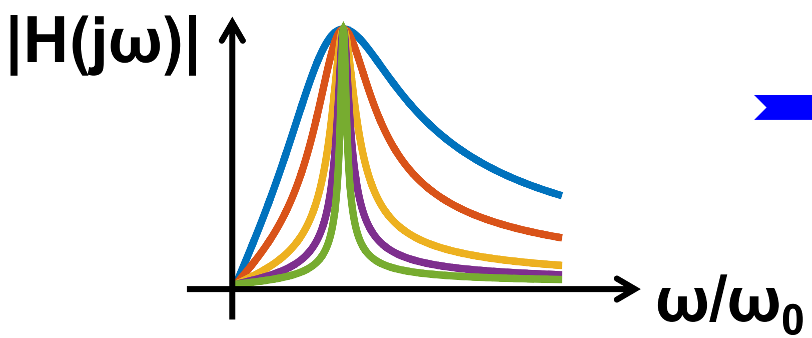
- **Damping factor is inversely proportional to Q**

Resonant Circuits: Network Functions



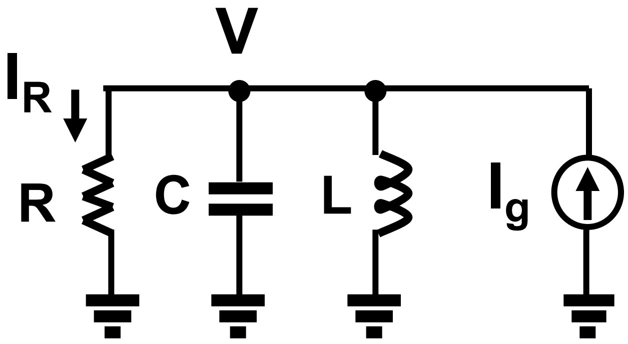
$$Z(j\omega) = \frac{V}{I_g} = R \cdot \frac{I_R}{I_g} = R \cdot H(j\omega)$$

$$H(j\omega) = \frac{1}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$



- **Band-pass frequency response dependent on Q**

Resonant Circuits: -3dB Bandwidth



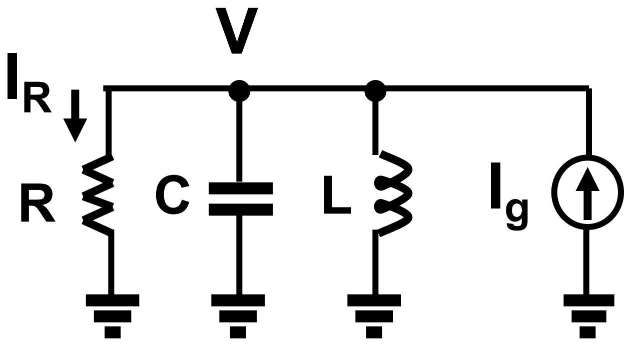
$$\left|H(j\omega)\right|^2 = \frac{1}{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{\omega}{\omega_0}-\frac{\omega_0}{\omega} = \pm\frac{1}{Q} \quad \Rightarrow \quad \frac{\omega_{1,2}}{\omega_0} = \mp\frac{1}{2Q}+\sqrt{\frac{1}{4Q^2}+1}$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega_0} = \frac{\omega_2-\omega_1}{\omega_0} = \frac{1}{Q}$$

- **Q factor is the ratio of the center frequency over the -3dB BW of the network function**

Resonant Circuits: Energy Relationship



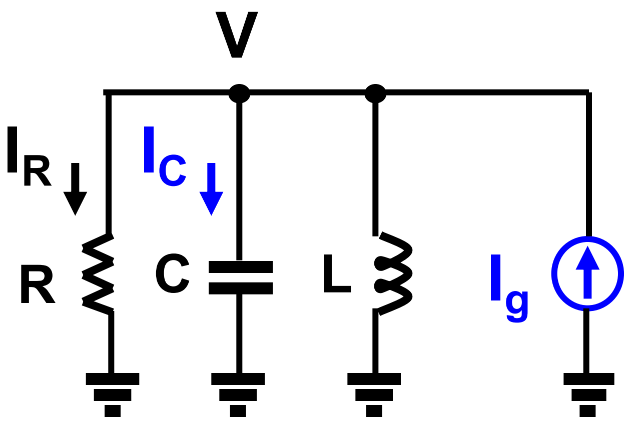
$$V(t) = \mathrm{Re}\left\{\overline{V} \cdot e^{j\omega_0 t}\right\}$$

Phasor

$$Q = \omega_0 RC = \omega_0 \frac{\frac{1}{2} C \left|\overline{V}\right|^2}{\frac{1}{2} \frac{\left|\overline{V}\right|^2}{R}} = \omega_0 \frac{E_{stored}}{P_{diss.}} = 2\pi \cdot \frac{E_{stored}}{E_{diss.per\,cycle}}$$

- Q factor is proportional to the ratio of the energy stored over the energy dissipated in one oscillation cycle

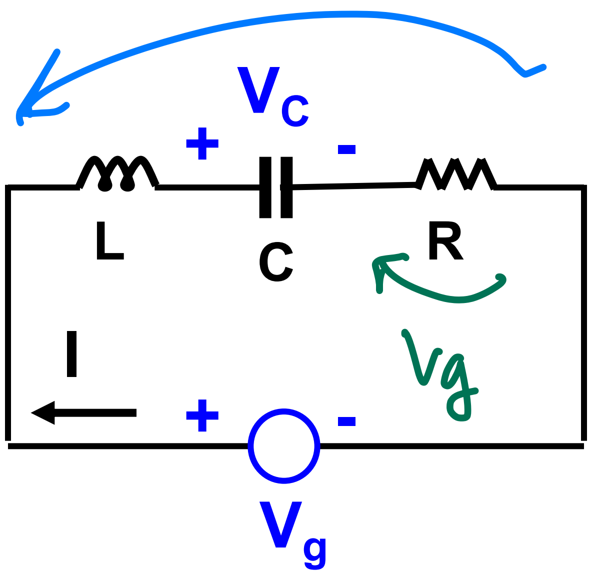
Current/Voltage Amplification at Resonance



Current Amplification <u>at Resonance</u>

$$\left|\overline{I_C}\right| = \omega_0 C \cdot \left|\overline{V}\right| = \omega_0 C \cdot \left|\overline{I_g}\right| R = Q \cdot \left|\overline{I_g}\right|$$

$$Q \triangleq \omega_0 RC$$

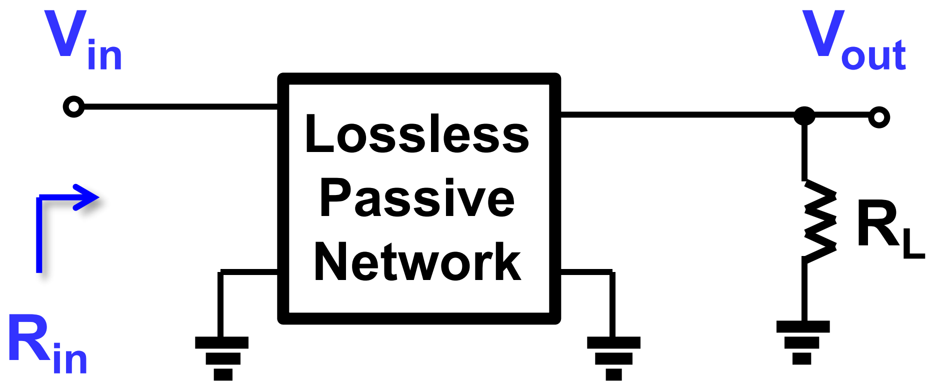


Voltage Amplification <u>at Resonance</u>

$$\left|\overline{V_C}\right| = \frac{\left|\overline{I}\right|}{\omega_0 C} = \frac{\left|\overline{V_g}\right|}{\omega_0 RC} = Q \cdot \left|\overline{V_g}\right|$$

$$Q \triangleq \frac{1}{\omega_0 RC}$$

Impedance Transformations: General Result



$$G = \frac{V_{out}}{V_{in}}$$

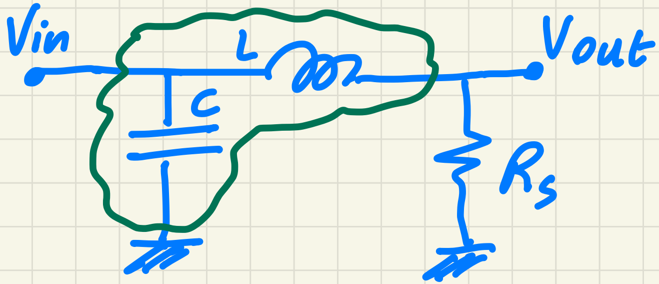
Voltage gain

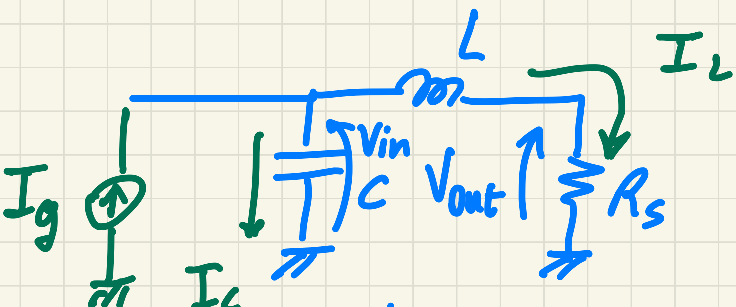
$$\frac{1}{2}\frac{V_{in}^2}{R_{in}} = \frac{1}{2}\frac{V_{out}^2}{R_L} \quad \Rightarrow \quad R_{in} = R_L \cdot \frac{V_{in}^2}{V_{out}^2} = \frac{R_L}{G^2}$$

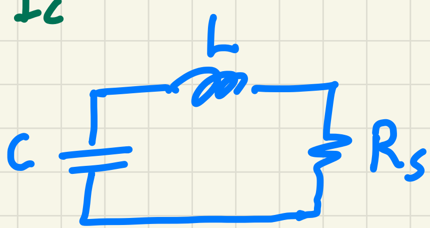
Impedance transformation

- ***Upward*** **transformation if G < 1**
- ***Downward*** **transformation if G > 1**

L - match network (simplest network)







- lossless approximation :

$R_s^{**} \simeq 0$ at resonance

$$|\bar{I}_L| \simeq Q \cdot |\bar{I}_g|$$

where $Q = \dfrac{1}{\omega_0 R_s C}$

(quality of a series LC network)

** $Q \gg 1$

$$|\overline{V_{out}}| = |\overline{I_L}| \cdot R_s \underset{\text{at resonance}}{=} |\overline{I_C}| \cdot R_s = \omega_0 C \cdot |\overline{V_{in}}| \cdot R_s =$$

$$= \frac{|\overline{V_{in}}|}{Q}$$

voltage attenuation

$$|Z_{in}| = \frac{|\overline{V_{in}}|}{|\overline{I_g}|} = \frac{|\overline{V_{out}}| \cdot Q}{|\overline{I_L}| / Q} = Q^2 \cdot R_s$$

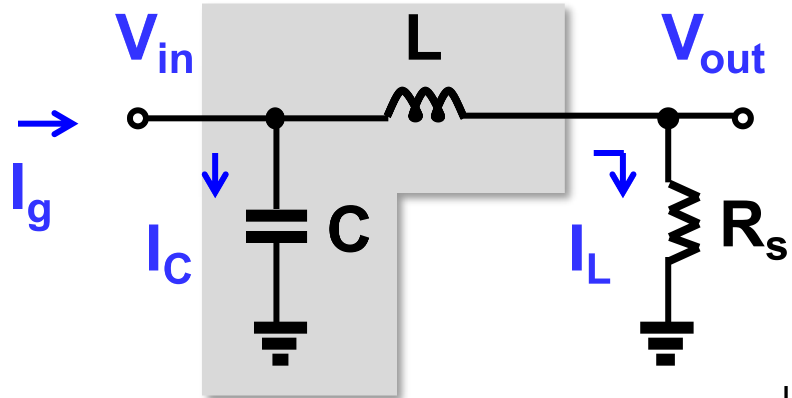
current in R_s

$$\boxed{Z_{in} = Q^2 \cdot R_s}$$

at resonance (with $Q \gg 1$)

upward impedance transformation

L-match Networks (Small Losses)



For small losses, at resonance:

$$\left|\overline{I_C}\right| = Q_L \cdot \left|\overline{I_g}\right| = \left|\overline{I_L}\right|$$

$$Q_L = \frac{\omega_0 L}{R_s} >> 1$$

$$\left|\overline{V_{out}}\right| = \left|\overline{I_L}\right| R_s = \left|\overline{V_{in}}\right| \omega_0 C R_s = \left|\overline{V_{in}}\right| / Q_L$$

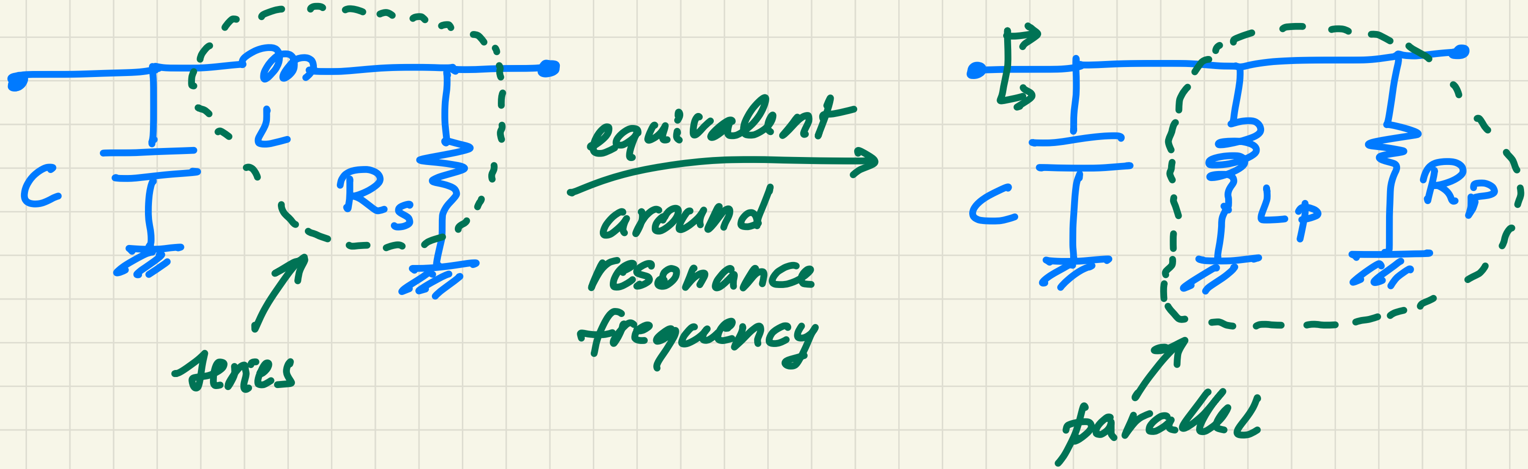
Voltage attenuation

↓

$$\left|Z_{in}\right| = \frac{\left|\overline{V_{in}}\right|}{\left|\overline{I_g}\right|} \approx \frac{Q_L \left|\overline{V_{out}}\right|}{\left|\overline{I_L}\right| / Q_L} = \boxed{R_s \cdot Q_L^2}$$

Upward impedance transformation

- General case (no lossless approx.)



series - to - parallel transformation :

$$j\omega L + R_s = \frac{R_p \cdot j\omega L_p}{R_p + j\omega L_p} \qquad Q_L = \frac{\omega L}{R_s} = \frac{1}{\omega C R_s}$$

$$R_s(1 + jQ_L) = \frac{R_p \cdot j\omega L_p}{R_p + j\omega L_p}$$

$$R_S(1+jQ_L) = \frac{R_P \cdot j\omega L_P}{R_P + j\omega L_P}$$

$$R_S(1+jQ_L)(R_P + j\omega L_P) = j\omega L_P R_P$$

Re — Im

Re:

$$\cancel{R_S} R_P - \cancel{R_S} Q_L \cdot \omega L_P = 0$$

$$\Downarrow$$

$$Q_L = \frac{R_P}{\omega L_P}$$

Im:

$$R_S \cdot Q_L R_P + R_S \overbrace{\omega L_P} = \overbrace{\omega L_P} R_P$$

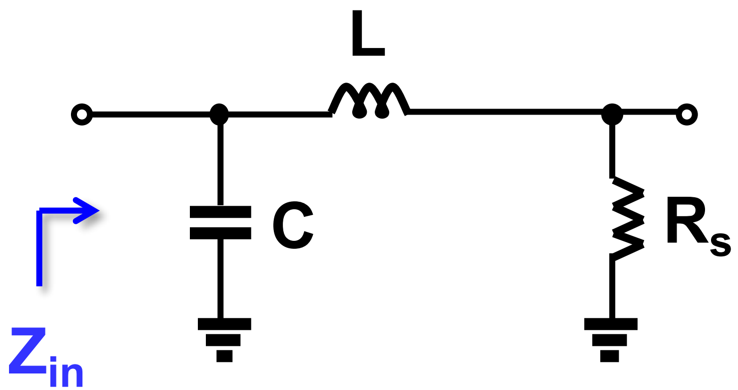
$$\Downarrow$$

$$R_S \cancel{R_P} Q_L + R_S \cdot \frac{\cancel{R_P}}{Q_L} = R_P \cdot \frac{\cancel{R_P}}{Q_L}$$

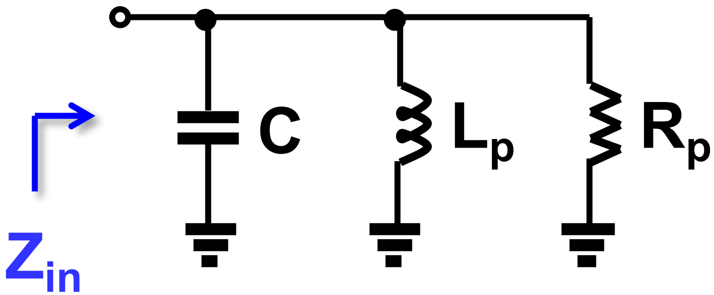
$$\Downarrow$$

$$R_P = R_S \cdot (1 + Q_L^2)$$

L-match Networks (General Case)



Equivalent Network



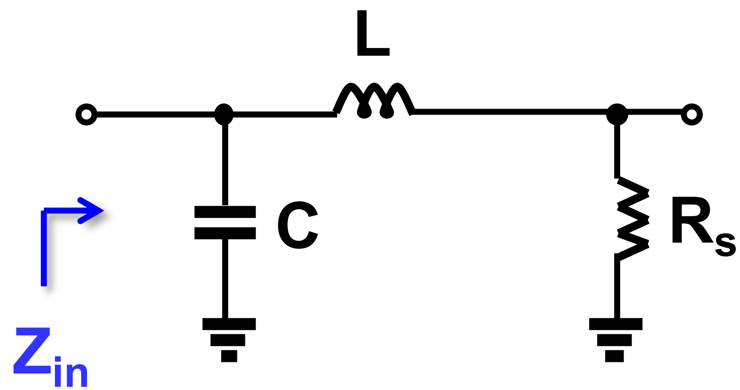
$$\frac{j\omega R_p L_p}{R_p + j\omega L_p} = R_s + j\omega L$$

$$\boxed{Q_L = \frac{\omega L}{R_s}}$$

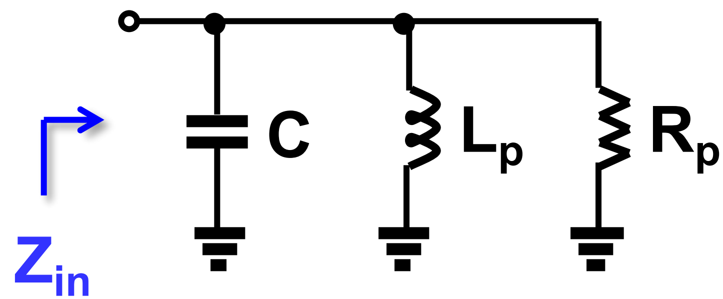
$$\downarrow$$

$$\frac{j\omega R_p L_p}{R_p + j\omega L_p} = R_s\left(1 + jQ_L\right) \rightarrow \begin{cases} \omega L_p R_p = \omega L_p R_s + R_s R_p Q_L \\ 0 = R_s R_p - R_s Q_L \omega L_p \end{cases}$$

L-match Networks (Continued)



Equivalent Network



$$\begin{cases} \omega L_p R_p = \omega L_p R_s + R_s R_p Q_L \\ 0 = R_s R_p - R_s Q_L \omega L_p \end{cases} \rightarrow \begin{cases} \dfrac{R_s}{Q_L} + R_s Q_L = \dfrac{R_p}{Q_L} \\ L_p = \dfrac{R_p}{\omega Q_L} \end{cases}$$

$$\rightarrow \begin{cases} R_p = R_s \cdot \left(1 + Q_L^2\right) \\ L_p = L \cdot \dfrac{1 + Q_L^2}{Q_L^2} \end{cases}$$

Note that R_p and L_p in general depends on frequency