RF Circuits Design

L11

Oscillators

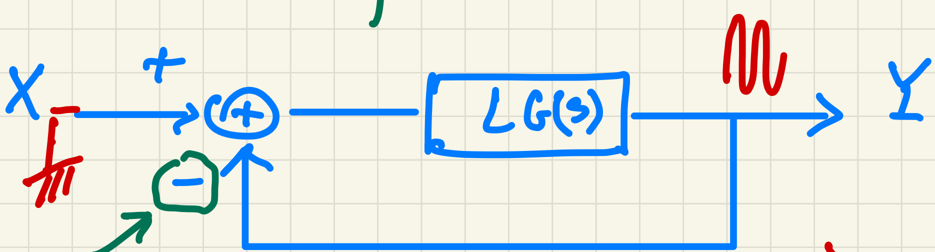
ex. VCO or CCO electrically-tuned oscillators

XO crystal oscillators

Mathematical models : 1) Feedback system

2) Negative resistance

Feedback system :



$$\frac{Y}{X} = \frac{LG(s)}{1 + LG(s)}$$

→ autonous system

a) negative feedback

$$\frac{Y}{X} = \frac{LG(s)}{1+LG(s)}$$

$y(t)$  ω_0

$\Downarrow$

Oscillation condition: $Y(j\omega_0) \neq 0$ with $X(j\omega_0) = 0$

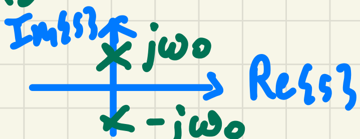
$$0 \neq \frac{Y(j\omega_0)}{\underbrace{X(j\omega_0)}_{0}} = \frac{LG(j\omega_0)}{1+LG(j\omega_0)} \to \infty$$

$$\iff \boxed{LG(j\omega_0) = -1}$$

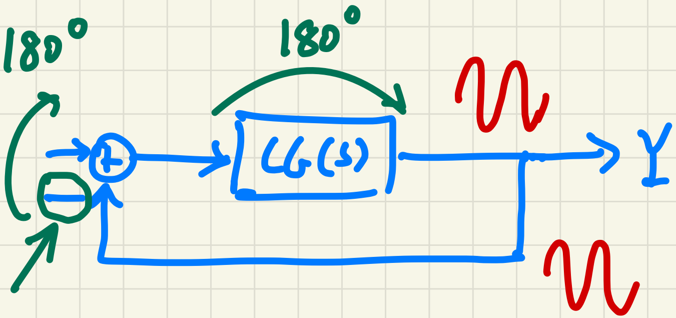
ω_0 is such that the loop at that freq. is equal to -1

$\iff$ $s = j\omega_0$ is a solution of $LG(s) = -1$

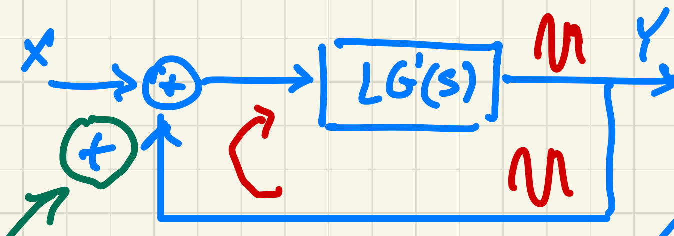
($j\omega_0$ is a pole of the closed-loop system)



$$LG(j\omega_0) = -1 \iff \underbrace{\begin{cases} |LG(j\omega_0)| = 1 \\ \angle LG(j\omega_0) = 180^\circ \end{cases}}_{\text{Barkhausen's conditions}}$$



b) positive feedback



$$Y/X = \frac{LG'}{1 - LG'} \rightarrow \infty$$

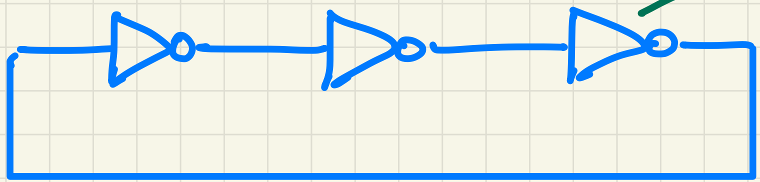
Oscillation condition

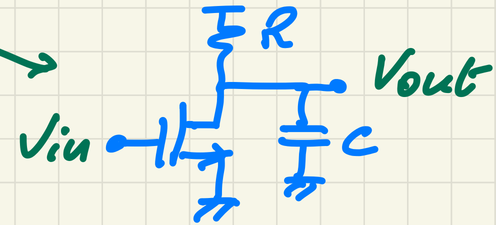
$$\underline{LG'(j\omega_0) = 1}$$

$$\underbrace{\begin{cases} |LG'(j\omega_0)| = 1 \\ \angle LG'(j\omega_0) = 0^\circ \end{cases}}$$

EXAMPLES

- RC oscillator (e.g. ringoscillator)

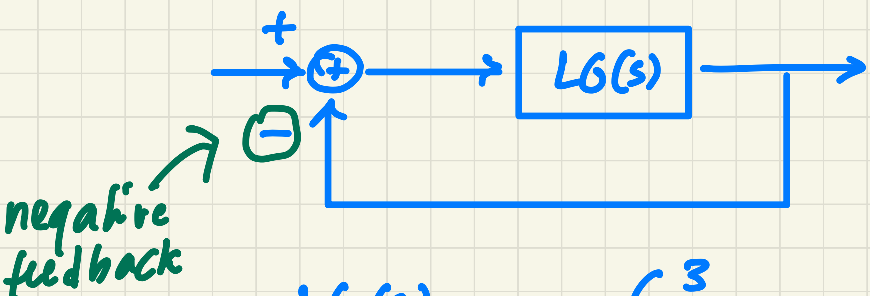




$$\frac{V_{out}}{V_{in}} = -\frac{G}{1+s\tau}$$

$$G > 0$$

(simple linear model)



$$LG(s) = \frac{G^3}{(1+s\tau)^3}$$

Oscillation condition : 1. $\measuredangle LG(j\omega_0) = \pi$

$$\measuredangle \frac{G^3}{(1+j\omega_0\tau)^3} = \pi \quad ;$$

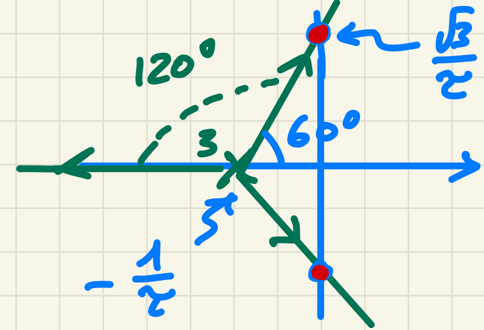
$$\underbrace{\measuredangle G^3}_{0} - 3\cdot\arctan(\omega_0\tau) = \pi \quad ; \qquad \arctan\omega_0\tau = -\frac{\pi}{3}$$

$$\omega_0\tau = \sqrt{3} \quad ; \quad \underline{\underline{\omega_0}} = \frac{\sqrt{3}}{\tau} \qquad \tau = RC$$

2. $|LG(j\omega_0)| = 1 \;;\quad \dfrac{G^3}{\left[1+\underbrace{(\omega_0\tau)^2}_{\sqrt{3}}\right]^{3/2}} = 1$

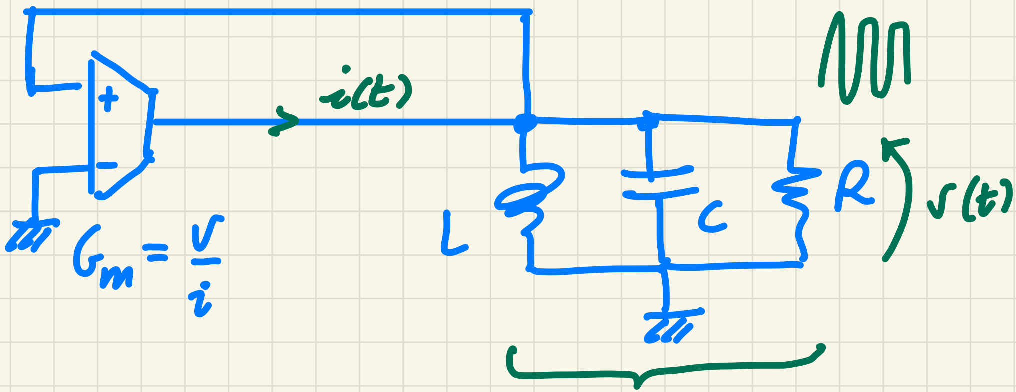
$$\Rightarrow \frac{G^3}{(1+3)^{3/2}} = 1 \quad ; \quad G^3 = 2^3 \quad ; \quad \underline{\underline{G}} = 2$$

Root locus $\quad LG(s) = \dfrac{G^3}{(1+s\tau)^3}$



$$s = \pm j\omega_0 = \pm j\frac{\sqrt{3}}{\tau} \quad \Leftarrow$$

- LC oscillator

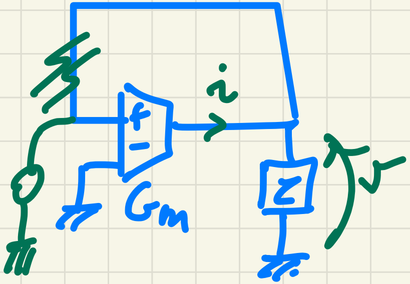


$$G_m = \frac{v}{i}$$

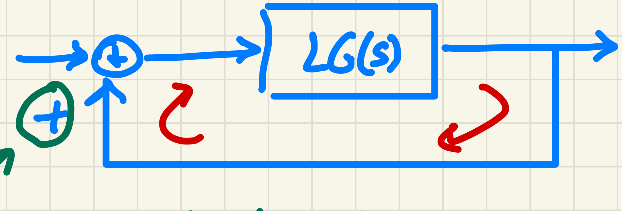
$$Z(s) = R \cdot \frac{s\,\omega_r/Q}{s^2 + s\,\omega_r/Q + \omega_r^2}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_r RC$$



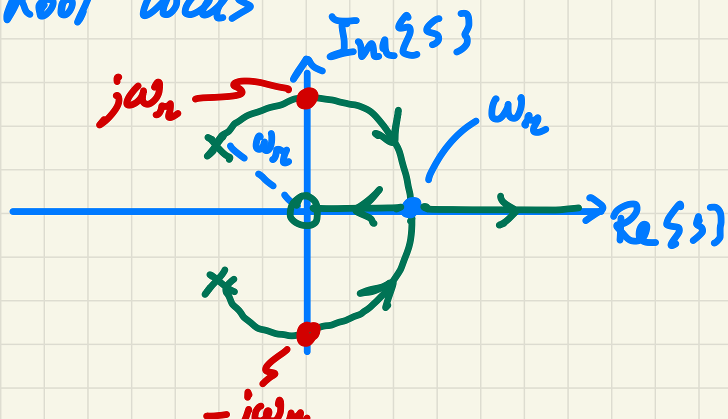
$\Rightarrow$



to highlight, it's positive feedback

$$LG(s) = G_m \cdot Z(s) = G_m R \cdot \frac{s\,\omega_r/Q}{\underbrace{s^2 + s\,\omega_r/Q + \omega_r^2}_{\text{2 complex poles}}}$$

Root locus



$\Rightarrow \quad s = \pm j\omega_r = \pm j\omega_0$

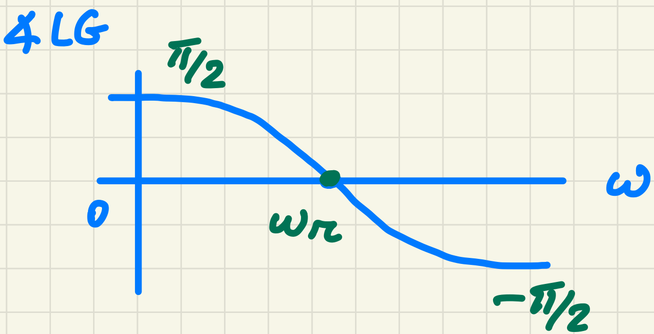
frequency of oscillation is identical to the resonance frequency

1. $\measuredangle LG(j\omega_0) = 0$;

$$\measuredangle \frac{j\omega_0 \omega_r/Q}{-\omega_0^2 + j\omega_0\omega_r/Q + \omega_r^2} = \frac{\pi}{2} - \arctan \frac{\omega_0\omega_r/Q}{\omega_r^2 - \omega_0^2} = 0$$

$$\Downarrow$$

$$\underline{\omega_0 = \omega_r}$$

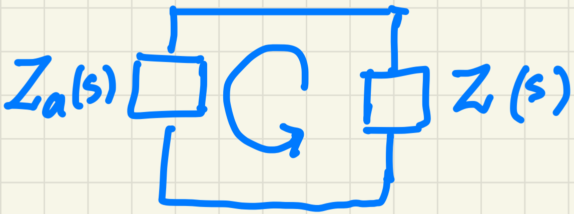


2. $|LG(j\omega_0)| = 1$;

$$\frac{G_m R \cdot \cancel{\omega_0 \omega_r / Q}}{\sqrt{\underbrace{(\omega_r^2 - \omega_0^2)^2}_{0} + \cancel{\left(\frac{\omega_0\omega_r}{Q}\right)^2}}} = 1$$

$$\Rightarrow \underline{G_m R = 1}$$

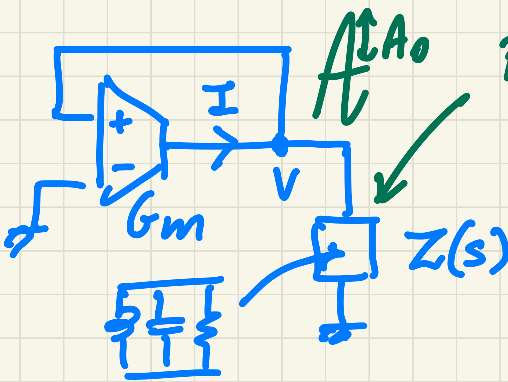
Negative Resistance Model



oscillation condition :
balance between dissipated power and the active power

$$\underline{Z_a(j\omega_0) + Z(j\omega_0) = 0}$$

example :



Power dissipated | Active power

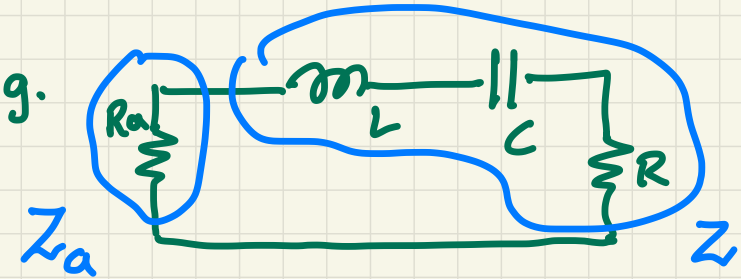
$$\frac{1}{2} \cdot \frac{A_0^2}{R} = \frac{1}{2} G_m \cdot A_0^2$$

$$G_m = \frac{1}{R}\,;\; G_m R = 1$$

$$\underbrace{Z_a(j\omega_0) = -Z(j\omega_0)}$$

$$\begin{cases} \mathrm{Re}\{Z_a(j\omega_0)\} = -\mathrm{Re}\{Z(j\omega_0)\} \\ \mathrm{Im}\{Z_a(j\omega_0)\} = -\mathrm{Im}\{Z(j\omega_0)\} \end{cases}$$

e.g.



$$\begin{cases} R_a = -R \\ \omega_0 L + \dfrac{1}{\omega_0 C} = 0 \end{cases} ; \quad \omega_0 = \frac{1}{\sqrt{LC}}$$