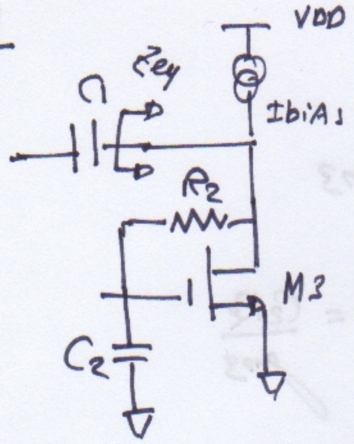
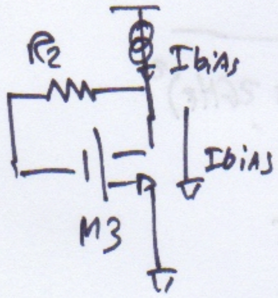
EX 9.1

a)



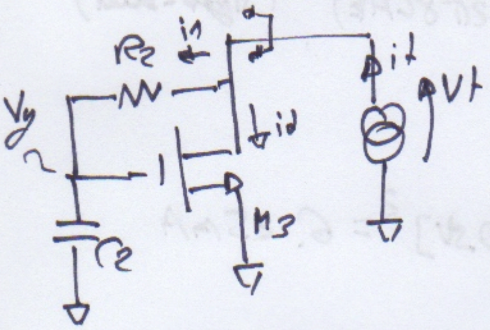
SIZE C_1 to RESONATE at $f_0 = 2GHz$ the output NETWORK R_2, C_1, C_2, M_3

• BIAS OF M3



$$g_{m3} = 2\sqrt{\underbrace{\tfrac{1}{2}\mu C_{ox}}_{0.2mA/V^2}\underbrace{\left(\tfrac{W}{L}\right)_3}_{750}\underbrace{I_{bias}}_{6mA}} = 60\,mA/V$$

• EVALUATE Z_{eq}



$$i_t = i_d + i_1$$

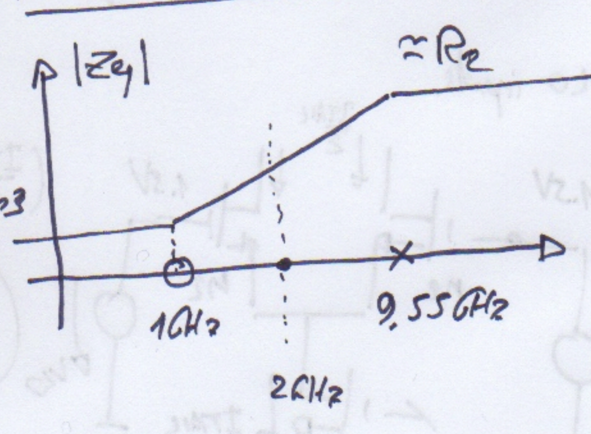
$$i_1 = \frac{V_t}{R_2 + \frac{1}{sC_2}} = \frac{sC_2 V_t}{(1+sC_2R_2)}$$

$$i_d = g_{m3} V_g = g_{m3} \cdot i_1 \cdot \frac{1}{sC_2} = g_{m3}\frac{V_t}{(1+sC_2R_2)}$$

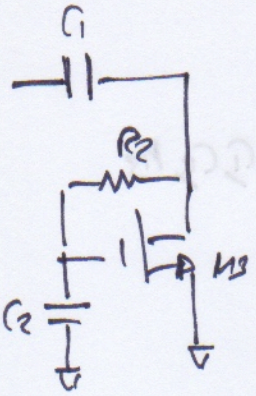
$$i_t = \frac{sC_2}{(1+sC_2R_2)} V_t + g_{m3}\frac{V_t}{(1+sC_2R_2)} \Rightarrow \boxed{Z_{eq} = \frac{V_t}{i_t} = \frac{1}{g_{m3}}\frac{(1+sC_2R_2)}{\left(1+s\frac{C_2}{g_{m3}}\right)}}$$

$$f_z = \frac{1}{2\pi C_2R_2} = 1{,}06\,GHz$$

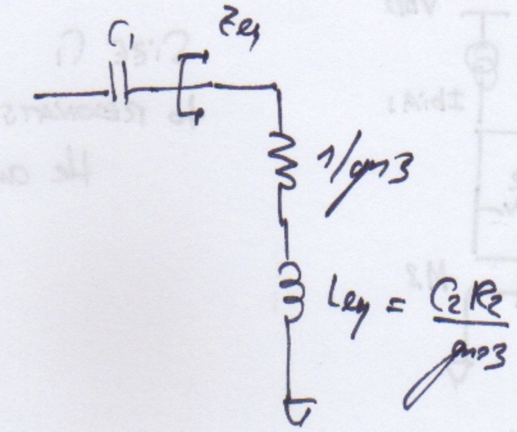
$$f_p = \frac{1}{2\pi\, C_2/g_{m3}} = 9{,}55\,GHz$$



$$Z_{eq}^{\text{AROUND } f_0} \simeq \frac{1}{g_{m3}} + s\underbrace{\frac{C_2R_2}{g_{m3}}}_{\sim L_{eq}}$$



AROUND f_0



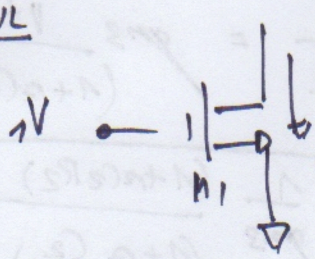
$$2\pi f_0 = \omega_0 = \frac{1}{\sqrt{C_1 \frac{C_2 R_2}{g_{m3}}}} \Rightarrow C_1 = \frac{1}{\frac{C_2 R_2}{g_{m3}}(2\pi\, 2GHz)^2} = 2.53pF$$

b) At RF port $V_{RF} = 1V + A_{RF}\cos(2\pi\, 10GHz)$

$V_{LO} = 1.5V + 300mV$ square-wave $(2\pi\, 8GHz)$ (single-ended)

• Bias point:

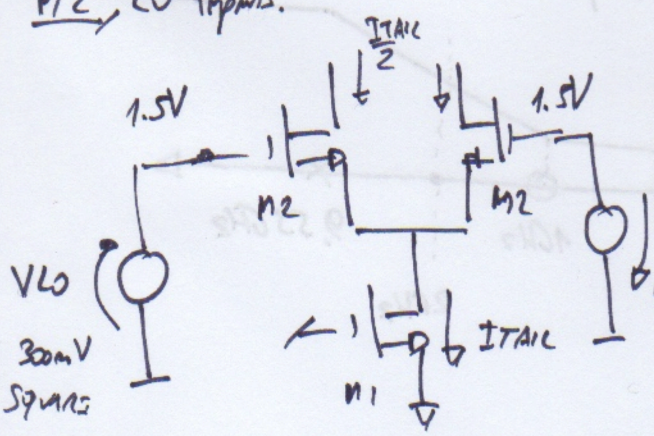
<u>M1, TAIL</u>



$$I_{TAIL} = \underbrace{\frac{1}{2}\mu_n C'_{ox}}_{0.2mA/V^2}\underbrace{\left(\frac{W}{L}\right)_1}_{125}[1V - 0.5V]^2 = 6.25mA$$

$$g_{m1} = \frac{2 I_{TAIL}}{V_{ov1}} = 25mA/V$$

<u>M2, LO inputs.</u>

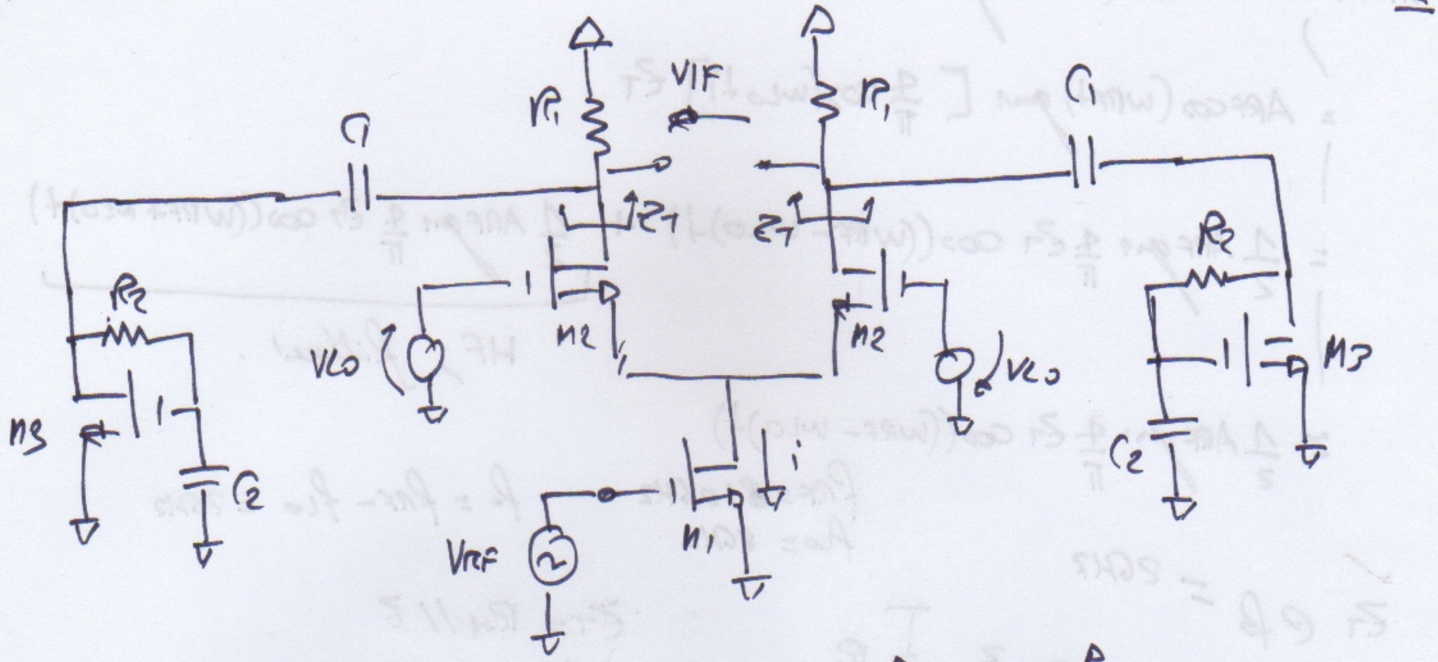


$$\left(\frac{I_{TAIL}}{2}\right) = \underbrace{\frac{1}{2}\mu_n C'_{ox}}_{0.2mA/V^2}\underbrace{\left(\frac{W}{L}\right)_2}_{400}(V_{ov2})^2 \Rightarrow V_{ov2} = 197.6mV$$

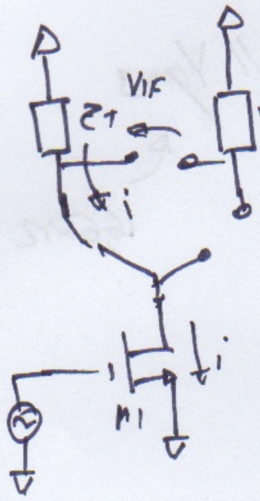
During operation $V_{in\,diff} = 600mV \gg \sqrt{2}\, V_{ov2} = 280mV$

↳ the pair is unbalanced

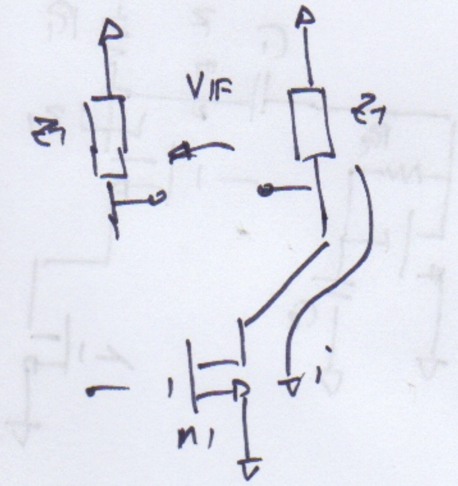
- AC Model



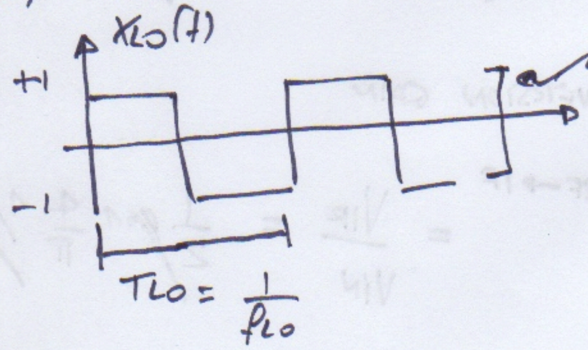
VLO>0



VLO<0



$$V_{IF}(t) = A_{RF}\cos(\omega_{RF} t)\, g_{m1}\, Z_T\, X_{LO}(t)$$



$$x_{LO}(t) = \text{SQUARE-WAVE}$$
$$= \gamma_0 + \sum_{n=1}^{+\infty} 2|\gamma_N| \cos\left(\frac{2\pi}{T_{LO}} n t + \angle \gamma_N\right)$$

$$\gamma_0 = \text{AVERAGE} = 0$$

$$\gamma_N = A_0 D \,\text{sinc}(Dn)$$
$$= A_0 \frac{1}{2} \frac{\sin(\pi \frac{1}{2} n)}{\pi \frac{1}{2} n}$$

$$\gamma_1 = 2 \cdot \frac{1}{2} \frac{1}{\pi/2} = \frac{2}{\pi}$$

$$\gamma_2 = 0$$

$$\gamma_3 = -\frac{2}{3\pi}$$

$$x_{LO}(t) \simeq 0 + \frac{4}{\pi}\cos(\omega_{LO} t) + 0 - \frac{4}{3\pi}\cos(3\omega_{LO} t) + \dots$$

$$V_{IF}(t) = A_{RF}\cos(\omega_{RF}t)\, g_{m1}\, X_{LO}(t)\, Z_T$$

$$= A_{RF}\cos(\omega_{RF}t)\, g_{m1}\left[\frac{4}{\pi}\cos(\omega_{LO}t)\right] Z_T$$

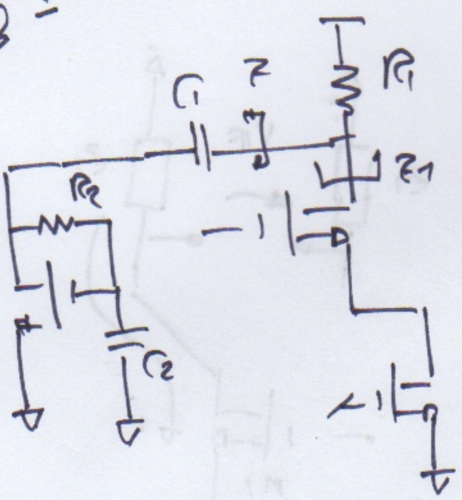
$$= \frac{1}{2}A_{RF}\, g_{m1}\frac{4}{\pi}Z_T\cos((\omega_{RF}-\omega_{LO})t) + \underbrace{\frac{1}{2}A_{RF}\, g_{m1}\frac{4}{\pi}Z_T\cos((\omega_{RF}+\omega_{LO})t)}_{\text{HF, filtered}}$$

$$\simeq \frac{1}{2}A_{RF}\, g_{m1}\frac{4}{\pi}Z_T\cos((\omega_{RF}-\omega_{LO})t)$$

$f_{RF} = 10\,GHz$ $\quad f_0 = f_{RF} - f_{LO} = 2\,GHz$

$f_{LO} = 8\,GHz$

Z_T @ $f_0 = 2\,GHz$



$$Z_T = R_1 // Z$$

$$= R_1 // 1/g_{m3}$$

$200\,\Omega$ $\quad 16{,}67\,\Omega$

$$\simeq 1/g_{m3}$$

$$V_{IF}(t) = \frac{1}{2}A_{RF}\cos(2\pi\, 2GHz\, t)\, g_{m1}\frac{4}{\pi}\left(1/g_{m3}\right)$$

- CONVERSION GAIN

$$A_V^{RF\to IF} = \frac{V_{IF}}{V_{IN}} = \frac{1}{2}g_{m1}\frac{4}{\pi}1/g_{m3} = -11.53\,dB$$

c) NF $(F_{RF} = F_{LO} = 8GHz)$ with $R_S = 50\Omega$

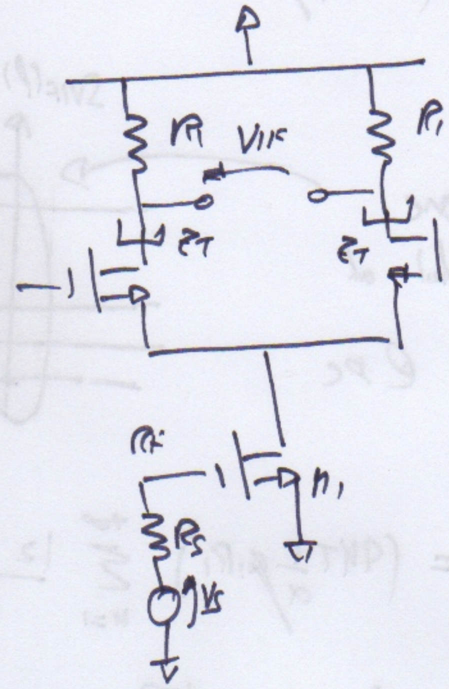
$$V_{IF}(t) = \frac{1}{2} A_{RF}\, g_{m1} \frac{4}{\pi} \cos((\omega_{RF} - \omega_{LO})t)\, Z_T$$

$= 0$

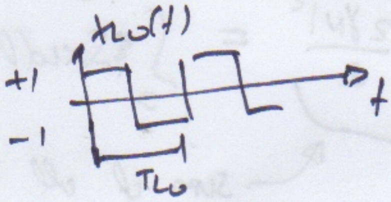
Z is at DC

$$= \frac{1}{2} A_{RF}\, g_{m1} \frac{4}{\pi} R_1$$

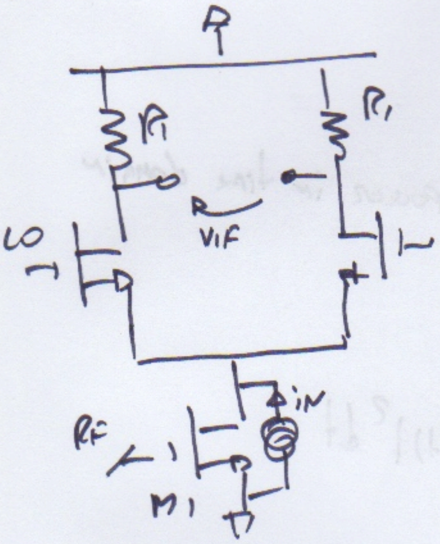
$Z_T = R_1$ because C_1 is open at DC.



$$A_v^{RF-OIF} = \frac{2}{\pi} g_{m1} R_1$$

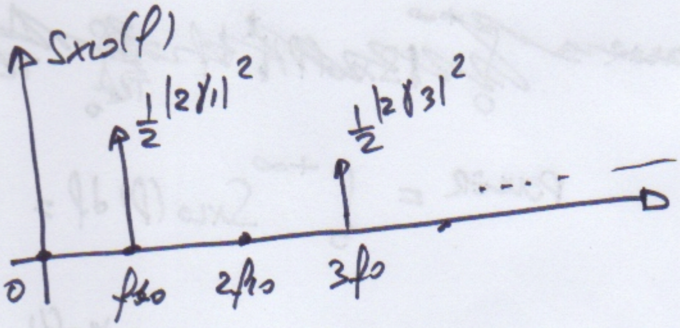


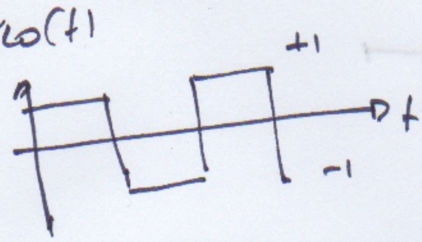
• CONTRIBUTION of M_1 (TAIL)



$$V_{IF} = i_N \cdot x_{LO}\, R_1$$

$$S_{VIF} = 4kT \frac{\gamma}{\alpha} g_{m1} R_1^2 \otimes S_{xLO}$$

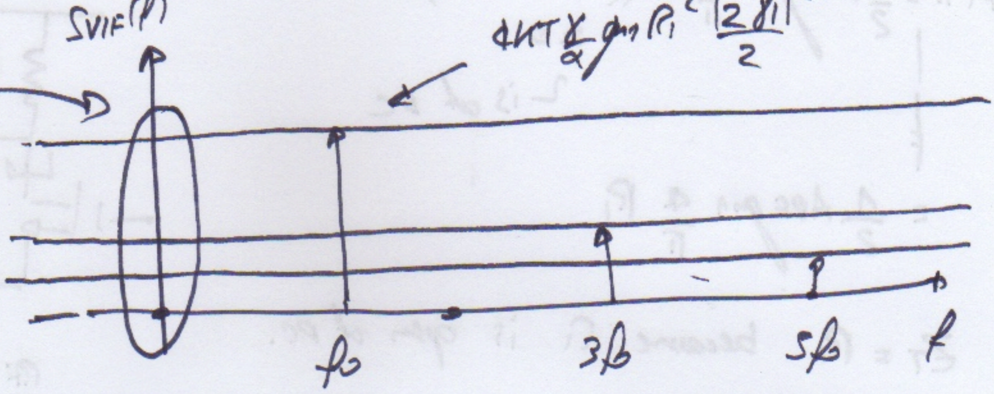




$$x_{LO}(t) = \gamma_0 + \sum_{n=1}^{+\infty} 2|\gamma_n| \cos\left(\frac{2\pi}{T_{LO}} n t + \angle(\gamma_n)\right)$$

$$S_{VIF}(f) = \left(4kT\frac{\gamma}{\alpha} g_{m1} R_1^2\right) \circledast \sum_{n=1}^{+\infty} \frac{|2\gamma_N|^2}{2}\,\delta(f - nf_{LO})$$

we are interested at $S_{VIF}(f)$ @ DC



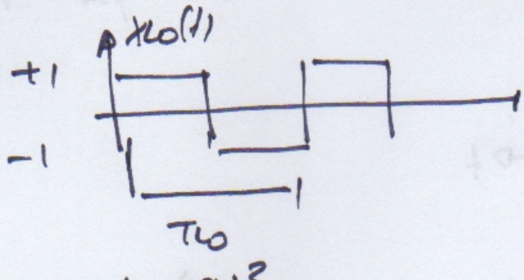
$$S_{VIF}(f) = \left(4kT\frac{\gamma}{\alpha} g_{m1} R_1^2\right) \sum_{n=1}^{+\infty} \frac{|2\gamma_N|^2}{2}$$

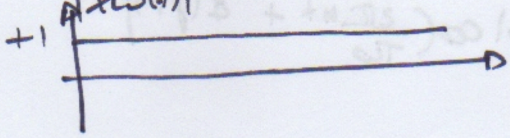
$$\underbrace{\sum_{n=1}^{+\infty} \frac{|2\gamma_N|^2}{2}}_{} = \int_0^{+\infty} S_{xLO}(f)\, df = \text{POWER in frequency domain}$$

sum of all the delta components in the spectrum

PARSEVAL'S THEOREM : POWER in frequency domain = POWER in time domain

$$\text{POWER} = \int_0^{+\infty} S_{xLO}(f)\, df = \frac{1}{T_{LO}} \int_0^{T_{LO}} |x_{LO}(t)|^2\, dt$$





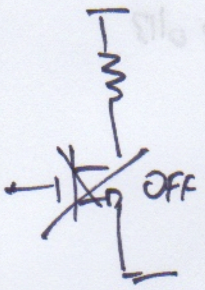
$$\text{POWER} = \frac{1}{T_{LO}} \int_0^{T_{LO}} 1\, dt = 1$$

$$S_{V_{IF}}(f) = 4kT\frac{\gamma}{\alpha} g_{m1} R_1^2 \underbrace{\sum_{n=1}^{+\infty} \frac{|2\gamma_N|^2}{2}}_{=1}$$

$$S_{V_{IF}}^{M1}(f) = 4kT\frac{\gamma}{\alpha} g_{m1} R_1^2$$

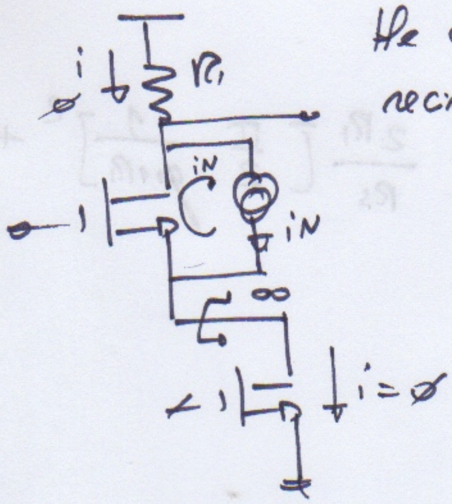
M2, considering the abrupt switching.

OFF



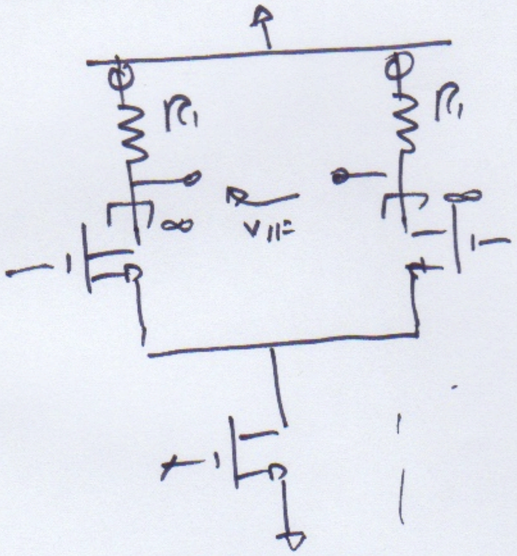
No noise, the transistor is OFF

ON



the current in recirculates.

R_1 contributions:



$$S_{V_{IF}}^{2\times R_1} = 2\times 4kTR_1$$

R_s contribution:

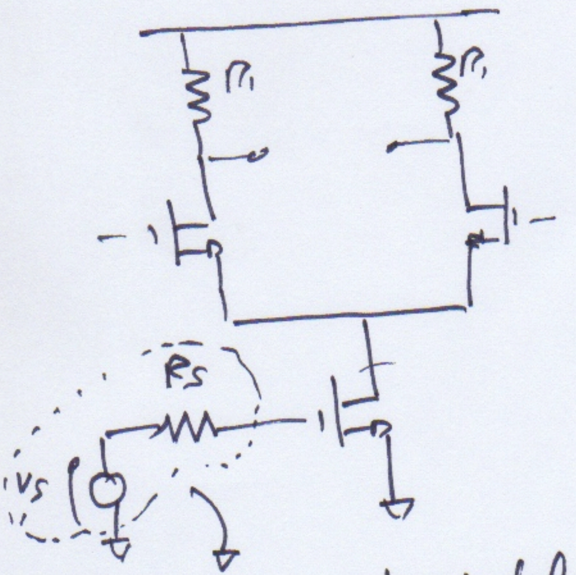
we consider the noise of R_s as a single tone @ f_{RF}

$$V_{IF} = V_N g_{m1} R_1 x_{LO}(t)$$

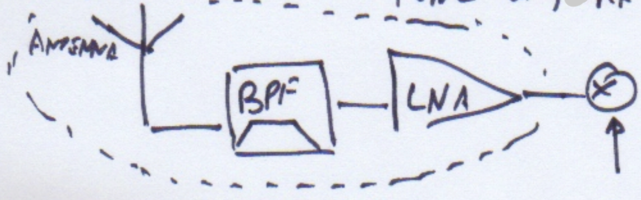
$$= V_N g_{m1} R_1 \frac{1}{2}\frac{4}{\pi}$$

$$S_{V_{IF}}^{R_s} = 4kTR_s A_v^2$$

$$= 4kTR_s \left(g_{m1} R_1 \frac{2}{\pi}\right)^2$$



tuned at f_{RF}

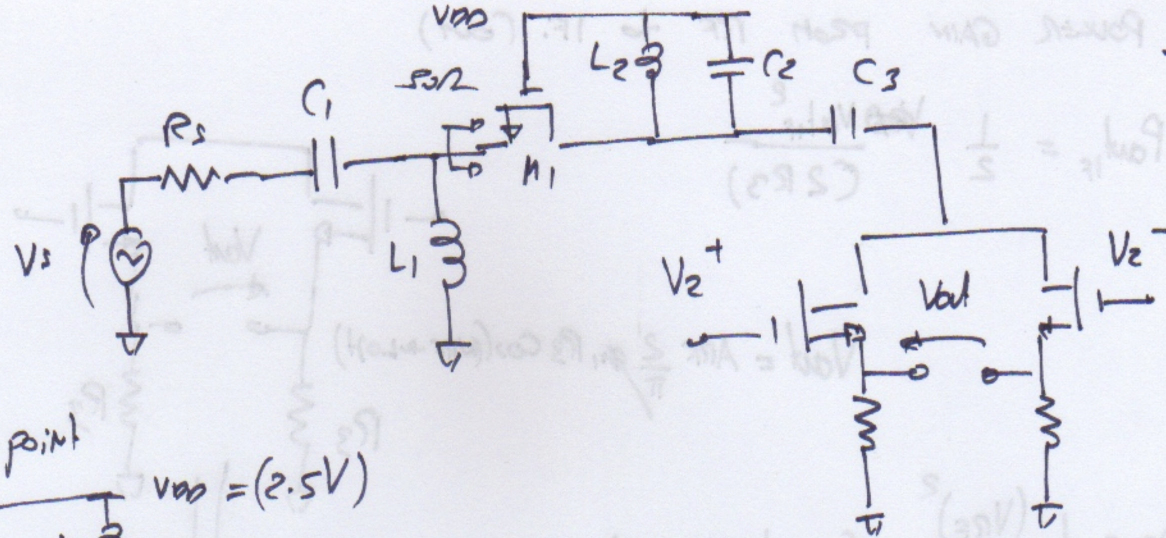


$$NF = 1 + \frac{2 \times 4kT R_1}{4kT R_s A_v^2} + \frac{4kT \frac{\gamma}{\alpha} g_{m1} R_1^2}{4kT R_s A_v^2}$$

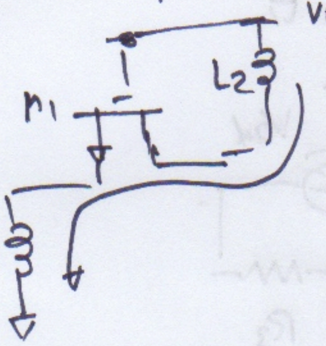
$$= 1 + \frac{2R_1}{R_s g_{m1}^2 R_1^2 \left(\frac{2}{\pi}\right)^2} + \frac{\frac{\gamma}{\alpha} g_{m1} R_1^2}{R_s g_{m1}^2 R_1^2 \left(\frac{2}{\pi}\right)^2}$$

$$= 1 + \frac{2R_1}{R_s}\left[\frac{\pi}{2}\frac{1}{g_{m1}R_1}\right]^2 + \frac{\gamma}{\alpha}\left(\frac{\pi}{2}\right)^2 \frac{1}{g_{m1}R_s} = 4{,}92 \text{ dB}$$

Ex 9.2



a) DERIVE BIAS point

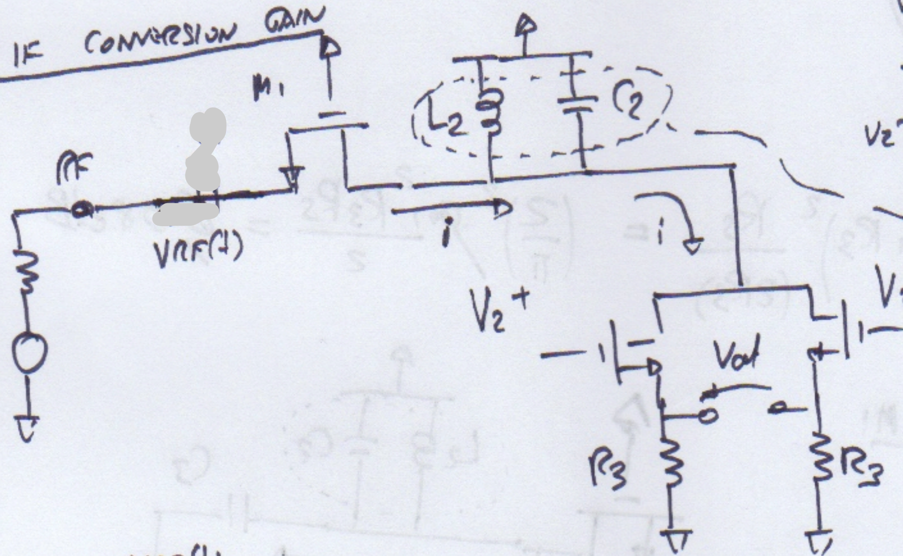


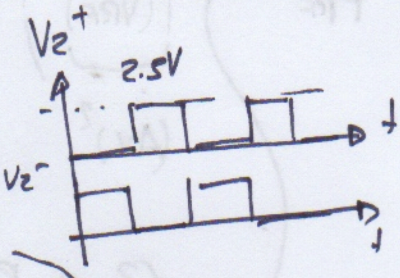
$V_{DD} = (2.5V)$

$$I_{M1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(2.5V - 0.5V)^2 = 20mA$$

$$g_{m1} = \frac{2 I_{M1}}{V_{OV1}} = \frac{2 \cdot 20mA}{2V} = 20mS = \frac{1}{50\Omega}$$

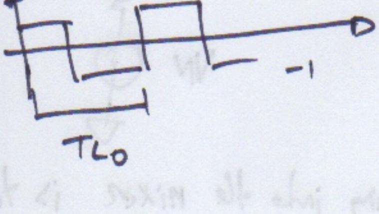
• RF to IF CONVERSION GAIN





$$\omega_0 = \frac{1}{\sqrt{L_2 C_2}} = 2\pi\, 2GHz$$

$$V_{out}(t) = \overbrace{A_{RF}\cos(\omega_{RF}t)}^{V_{RF}(t)}\, g_{m1} R_3\, x_{LO}(t)$$



$$= A_{RF}\cos(\omega_{RF}t)\, g_{m1} R_3 \left[\frac{4}{\pi}\cos(\omega_{LO}t)\right]$$

$$= \frac{1}{2} A_{RF}\cos((\omega_{RF}-\omega_{LO})t)\, g_{m1} R_3 \frac{4}{\pi} + \text{Higher freq. component}$$

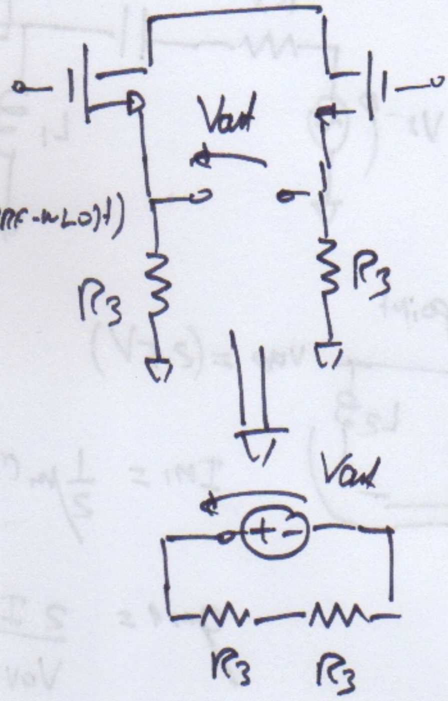
~ Filtered by the system.

$$2\pi(2.1GHz - 2GHz) = 2\pi\, 100MHz$$

$$A_V^{RF \to IF} = \frac{V_{out}}{V_{RF}} = \frac{2}{\pi} g_{m1} R_3 = 22.1\ dB.$$

b) POWER GAIN FROM RF to IF. (OUT)

$$P_{out_{IF}} = \frac{1}{2} \frac{V_{out_{IF}}^2}{(2R_3)}$$



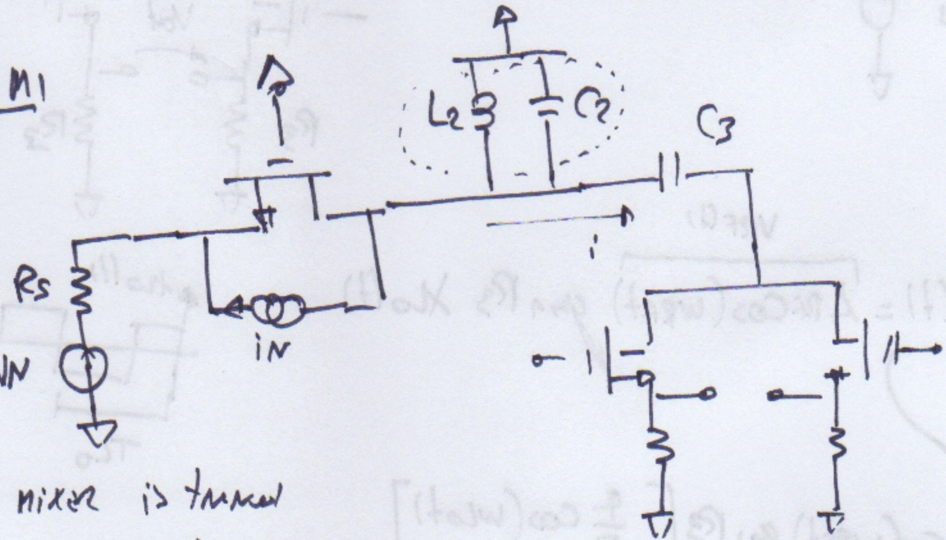
$$V_{out} = A_{RF} \frac{2}{\pi} g_{m1} R_3 \cos((\omega_{RF} - \omega_{LO})t)$$

$$P_{RF} = \frac{1}{2} \frac{(V_{RF})^2}{R_S} \quad \text{(input MATCHING)}$$

$$G_P = \frac{P_{out\,IF}}{P_{RF}} = \underbrace{\left(\frac{V_{out\,IF}}{V_{RF}}\right)^2}_{(A_V)^2} \frac{R_S}{(2R_3)}$$

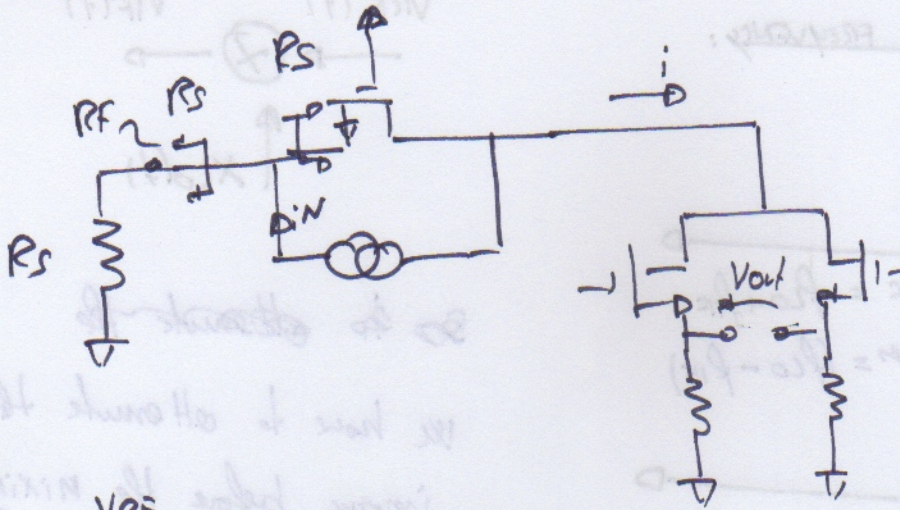
$$= \left(\frac{2}{\pi} g_{m1} R_3\right)^2 \frac{R_S}{(2R_3)} = \left(\frac{2}{\pi}\right)^2 g_{m1}^2 \frac{R_3 R_S}{2} = 6{,}08\,dB$$

c) NF due to R_S and M_1



the noise going into the mixer is tuned at f_{RF} by the two components L_2, C_2.

M1 contribution:

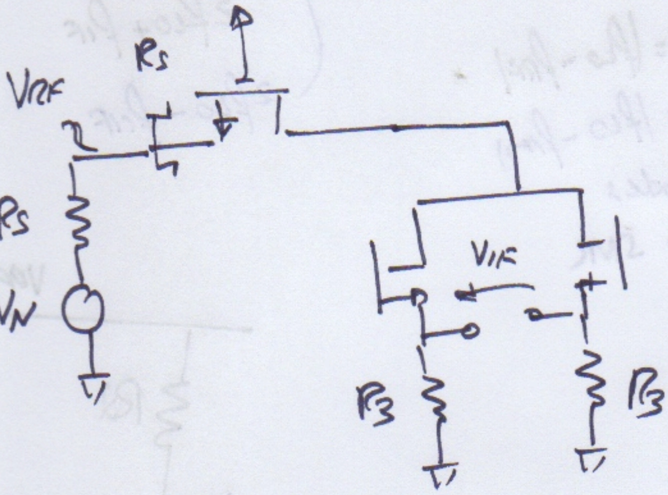


$$V_{out\,IF}^{M1} = \overset{V_{RF}}{\left(i_N \cdot \frac{R_S}{2}\right)} \cdot A_V^{RF \to IF} \implies S_{V_{out\,IF}}^{M1} = 4kT\frac{\gamma}{\alpha} g_{m1} \left(\frac{R_S}{2}\right)^2 (A_V)^2$$

- RS contribution:

$$V_{out\,IF}^{RS} = \overset{V_{RF}}{\left(V_N \frac{1}{2}\right)} \cdot A_V^{RF \to IF}$$

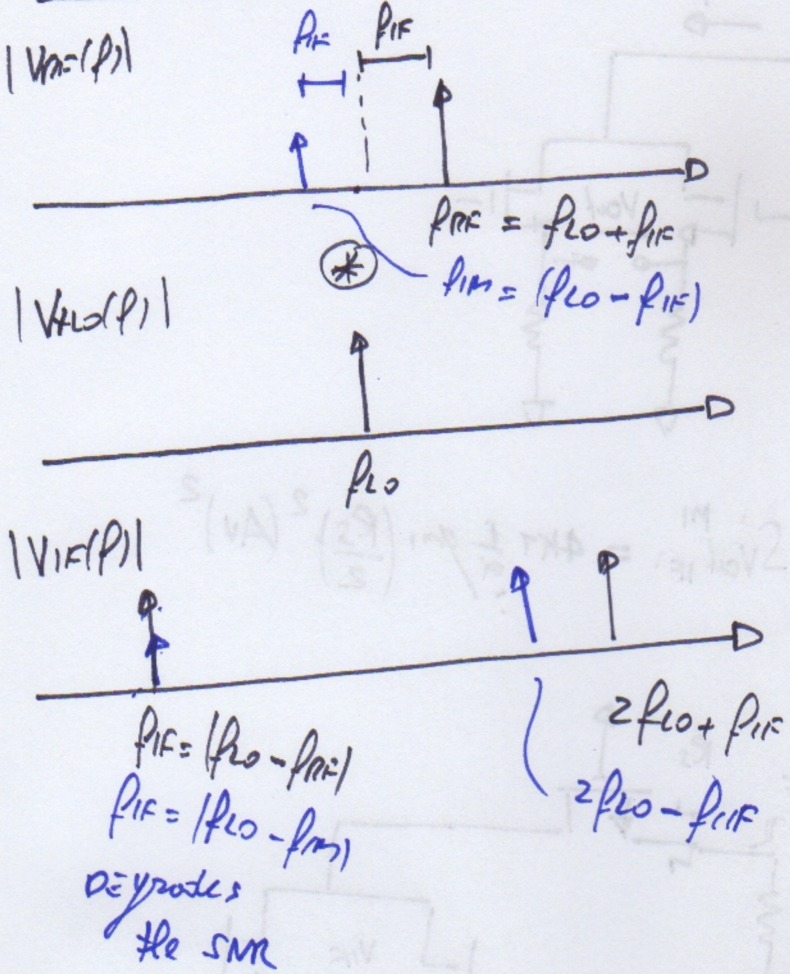
$$S_{V_{out\,IF}}^{RS} = 4kTR_S \left(\frac{1}{2}\right)^2 A_V^2$$

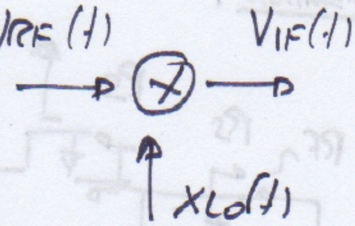


$$NF = 1 + \frac{4kT\frac{\gamma}{\alpha} g_{m1} \left(\frac{R_S}{2}\right)^2 (A_V)^2}{4kT R_S \left(\frac{1}{2}\right)^2 A_V^2} = 1 + \frac{\gamma}{\alpha} g_{m1} R_S = 1 + \frac{\gamma}{\alpha} = 2{,}22\ dB$$

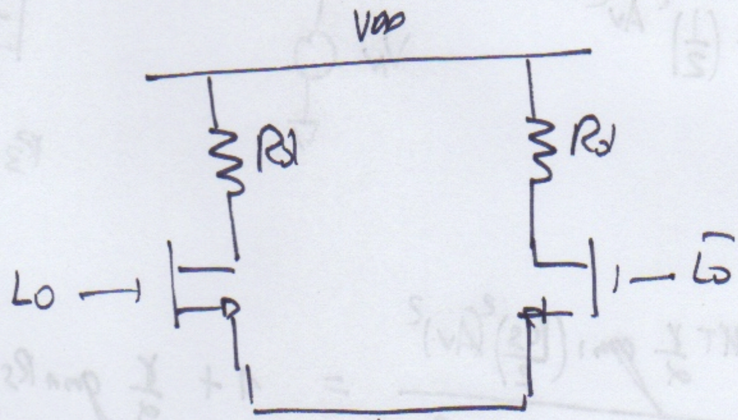
EX 9.31

PROBLEM OF THE IMAGE FREQUENCY:

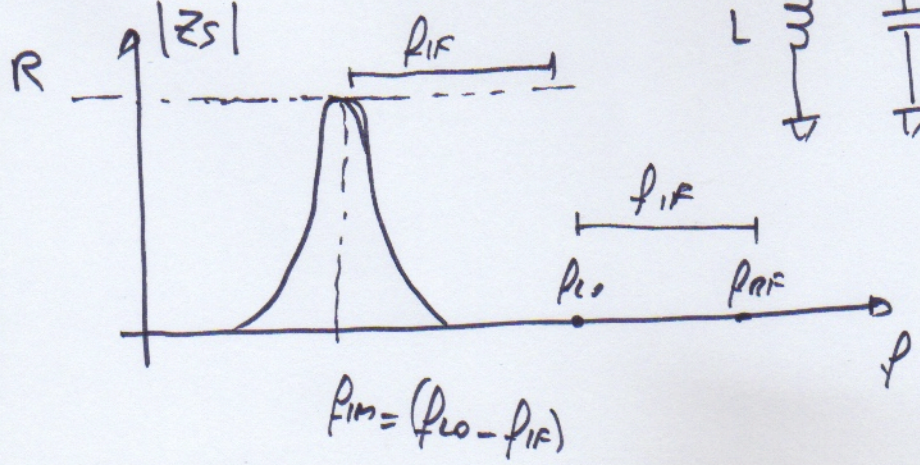




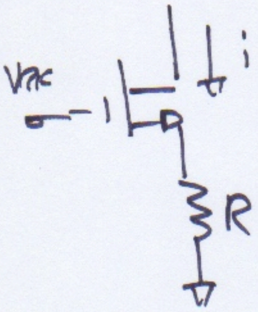
we have to attenuate the image before the mixing.



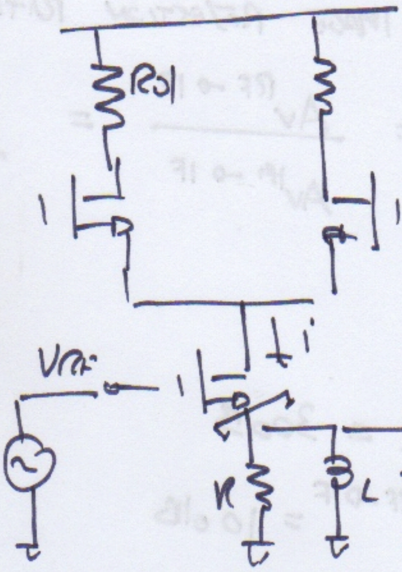
IMPEDANCE Zs



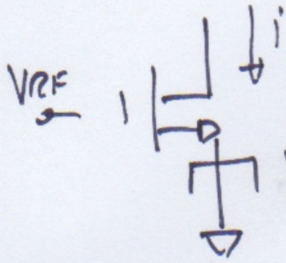
@ $\omega = \omega_{IM}$



$$i^{IM} = V_{RF} \frac{g_{m1}}{(1+g_{m1}R)}$$



@ $\omega = \omega_{RF}$



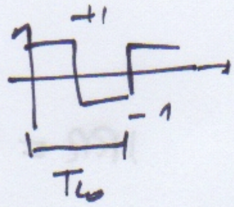
$$i^{RF} = V_{RF}\, g_{m1}$$

very low @ ω_{RF}

- CONVERSION GAIN:

@ ω_{RF}

FIRST HARMONIC OF $x_{LO}(t)$



$$V_{IF}^{RF} = V_{RF}\cos(\omega_{RF}t)\, g_{m1} \left[\frac{4}{\pi}\cos(\omega_{LO}t)\right] R_d$$

$$\simeq \frac{1}{2} V_{RF} \frac{4}{\pi} g_{m1} R_d \cos(\underbrace{(\omega_{RF}-\omega_{LO})}_{\omega_{IF}}t)$$ (just LF component)

$$A_v^{RF \to IF} = \frac{2}{\pi} g_{m1} R_d$$

@ ω_{IM}

$$V_{IF}^{IM} = V_{RF}\cos(\omega_{IM}t) \frac{g_{m1}}{(1+g_{m1}R)} \left[\frac{4}{\pi}\cos(\omega_{LO}t)\right] R_d$$

$$\simeq \frac{1}{2} V_{RF} \frac{g_{m1}}{(1+g_{m1}R)} \frac{4}{\pi} R_d \cos(\underbrace{(\omega_{LO}-\omega_{IM})}_{\omega_{IF}}t)$$

$$A_v^{IM \to IF} = \frac{2}{\pi} \frac{g_{m1}}{(1+g_{m1}R)} R_d$$

IRR = IMAGE REJECTION RATIO

$$= \frac{Av^{RF\to IF}}{Av^{IM\to IF}} = \frac{\frac{2}{\pi}\, \cancel{g_{m1} R_d}}{\frac{2}{\pi}\, \frac{\cancel{g_{m1}}}{(1+g_{m1}R)} \cancel{R_d}} = (1+g_{m1}R)$$

b) $IRR = 30dB$ size $\left(\frac{W}{L}\right)_1$ and R

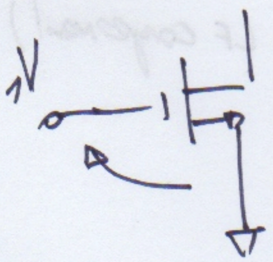
$Av^{RF\to IF} = 10dB$

$(R_d = 200\Omega)$

$$Av^{RF\to IF} = 10dB = \frac{2}{\pi} g_{m1} R_d \longrightarrow g_{m1} = 24,84\ mS$$

$$IRR = (1+g_{m1}R) = 30dB \longrightarrow R = 1,23\ k\Omega$$

• BIAS OF M1



$$I = g_{m1}\frac{V_{ov1}}{2} = 6,21\ mA$$

$$I = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_1 (V_{ov1})^2$$

$$\rightarrow \left(\frac{W}{L}\right)_1 = 125$$