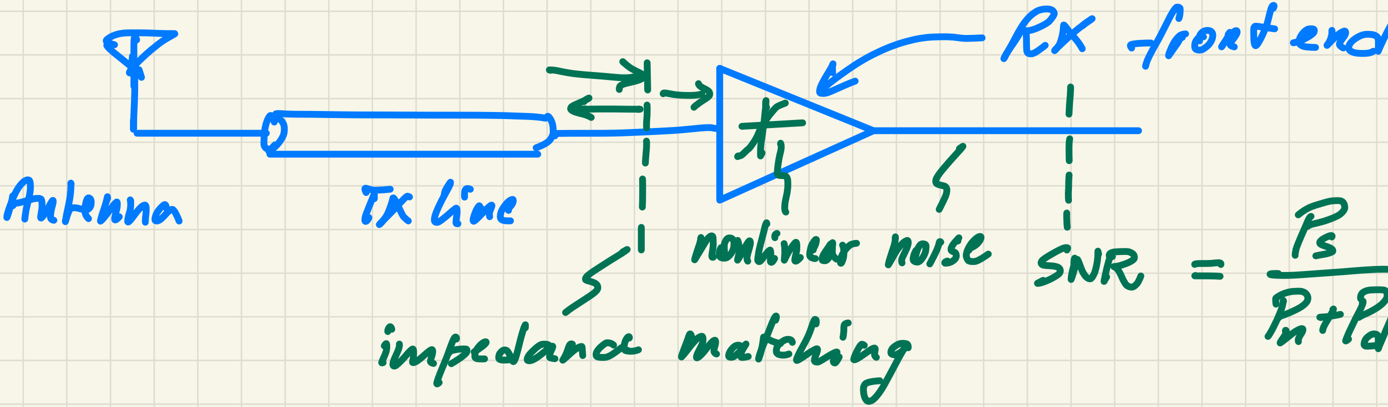
RF Circuit Design

L15

Basics of RF Systems

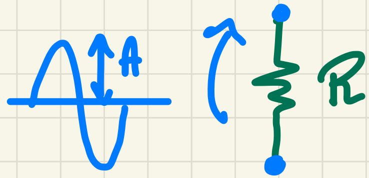


$$SNR = \frac{P_s}{P_n + P_d}$$

Sensitivity : min. detectable signal ($SNR > SNR_{min}$)

↳ limited by :
1. impedance matching
2. noise
3. nonlinearity

Note : Power of a signal



$v(t) = A \sin \omega t$

$$P = \frac{A^2}{2R} \; [W] = \frac{A_{rms}^2}{R}$$

e.g. $P = 1\,W \longrightarrow 10 \log_{10} P_{mW} =$

$R = 50\,\Omega$

$= 10 \log_{10} 1000 =$

$= \underline{30\ dBm}$

$$A = \sqrt{P \cdot 2R} = \sqrt{1 \cdot 2 \cdot 50} = 10\ V_p$$

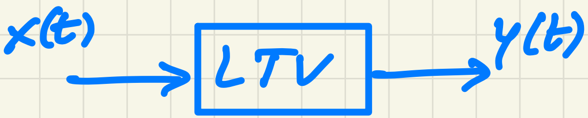
$$\longrightarrow 20 \log_{10}(A) = \underline{20\ dBV}$$

Effects of Nonlinearity

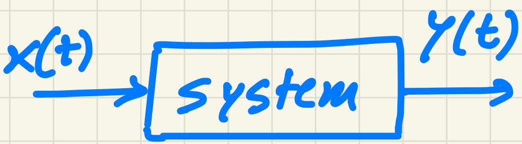
① Single tone at input

impulse response

$$y(t) = x(t) * h(t)$$

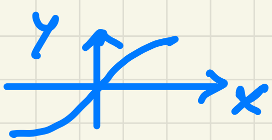


$$y(t) = x(t) * h(t,\tau)$$



Nonlinear system

- Memoryless or static model

e.g. 

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \dots$$

- Nonlinear dynamic system

$$y(t) = \text{Volterra series}$$

$$x(t) \xrightarrow{A} \boxed{\text{System}} \longrightarrow y(t)$$

HD , THD

a. Harmonic generation

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\cos^3 x = \cos x \cdot \cos^2 x$$

$$x(t) = A\cos\omega t$$

$$y(t) \simeq \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

memoryless nonlinear system

$$x^2(t) = A^2\cos^2\omega t = \frac{A^2}{2} + \frac{A^2}{2}\cos 2\omega t$$

$$x^3(t) = A^3\cos^3\omega t = A^3 \cdot \cos\omega t \cdot \frac{1+\cos 2\omega t}{2} =$$

$$= \frac{A^3}{2}\cos\omega t + \frac{A^3}{4}\cos\omega t + \frac{A^3}{4}\cos 3\omega t$$

$$x^2(t) = \underset{\substack{\uparrow \\ \text{DC} \\ \text{"rectification"}}}{\frac{A^2}{2}} + \underset{\substack{\uparrow \\ \text{2nd harmonic}}}{\frac{A^2}{2}\cos 2\omega t} \qquad x^3(t) = \underset{\substack{\uparrow \\ \text{fundemental}}}{\frac{3}{4}A^3\cos\omega t} + \underset{\substack{\uparrow \\ \text{3rd harmonic}}}{\frac{A^3}{4}\cos 3\omega t}$$

$$y(t) = \underbrace{\alpha_1 \cdot A}\cos\omega t + \underbrace{\alpha_2 \cdot \frac{A^2}{2}}_{B_0} + \alpha_2 \frac{A^2}{2}\cos 2\omega t +$$

$$+ \underbrace{\alpha_3 \cdot \frac{3}{4} A^3}\cos\omega t + \alpha_3 \frac{A^3}{4}\cos 3\omega t + \cdots =$$

Desired

$$= B_0 + B_1 \cos\omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + \cdots$$

$$B_0 = \alpha_2 \frac{A^2}{2} \qquad B_2 = \alpha_2 \frac{A^2}{2}$$

$$B_1 = \underset{\substack{\uparrow \\ \text{small-signal gain}}}{\alpha_1}\underline{A} + \frac{3}{4}\alpha_3 A^3 \qquad B_3 = \frac{1}{4}\alpha_3 A^3$$

$\Rightarrow$ • Generated harmonic amplitude :

$$B_n \doteq \sim A^n \qquad n \geq 1$$

n-th harmonic has amplitude $\doteq A^n$

• $B_{2n} = 0$ if $\alpha_{2n} = 0$ (fully-different.)

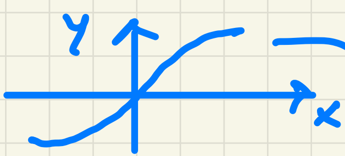
even-order harmonics come from even-order nonlinearity

2. Gain compression : $B_1 = \alpha_1 A + \frac{3}{4}\alpha_3 A^3$

$$\text{Gain of the system} = \frac{B_1}{A} = \alpha_1 + \underbrace{\frac{3}{4}\alpha_3 A^2}_{\text{gain compression}}$$

$\alpha_1 > 0$

$\alpha_3 < 0$

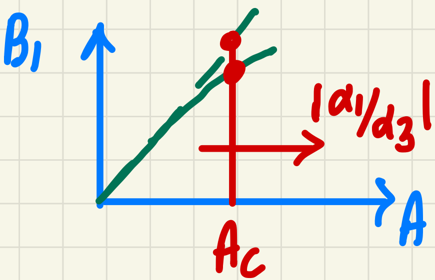


for COMPRESSIVE system : $\alpha_1\alpha_3 < 0$

Def. 1-dB compression point:

input amplitude (power) "A_c" such that the system gain is reduced by 1dB

$$\frac{\overbrace{\alpha_1 \cancel{A_c} + \frac{3}{4}\alpha_3 A_c^{\cancel{3}\,2}}^{B_1 \text{ compress. output amplit.}}}{\underbrace{\alpha_1 \cancel{A_c}}_{\text{ideal (linear) output amplit.}}} = \underbrace{10^{-\frac{1}{20}}}_{-1\,\text{dB}}$$



$$\Downarrow$$

$$1 + \frac{3}{4}\frac{\alpha_3}{\alpha_1} A_c^2 = 0.89$$

$$\Downarrow$$

$$\underline{A_{c,dB} = 20\log A_c = -9.6\,\text{dB} + 10\log_{10}\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

② Two tones at the input

$$x(t) = \underbrace{A_1 \cos \omega_1 t}_{a} + \underbrace{A_2 \cos \omega_2 t}_{b}$$

For simplicity : $y(t) \simeq \alpha_1 x(t) + \alpha_3 x^3(t)$

$$(a+b)^3 = a^3 + b^3 + \underbrace{3a^2 b} + \underbrace{3ab^2}$$

$$y(t) = B_1 \cos \omega_1 t + B_2 \cos \omega_2 t + B_{221} \cos(2\omega_2 - \omega_1)t + B_{112} \cos(2\omega_1 - \omega_2)t + \dots$$

$$3\alpha_3 \cdot A_1 \cos \omega_1 t \cdot \frac{A_2^2 + A_2^2 \cos 2\omega_2 t}{2}$$

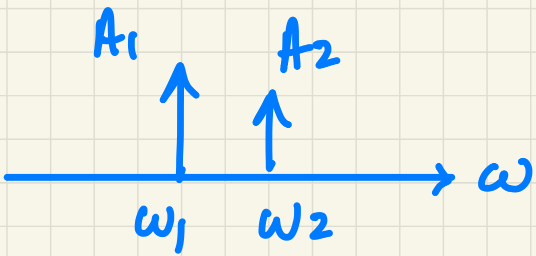
- $B_1 = \alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2$
- $B_2 = \alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_1^2 A_2$

$$\frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\omega_2 - \omega_1)t + \frac{3}{4}\alpha_3 A_1 A_2^2 \cos(2\omega_2 + \omega_1)t$$

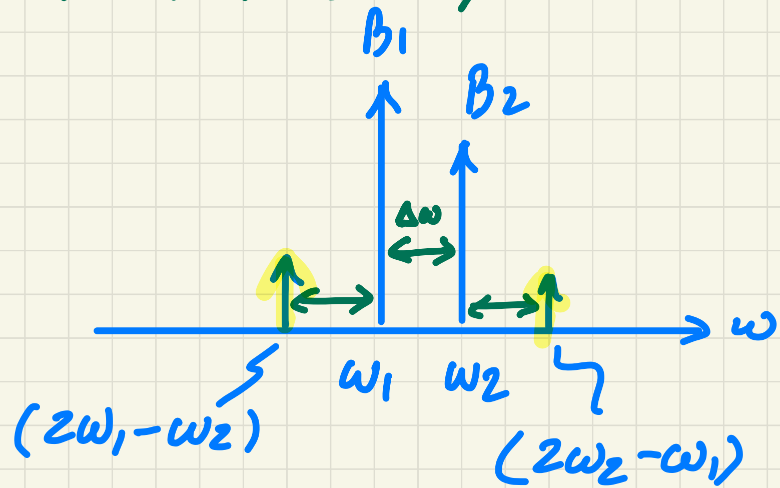
$$\bullet\ B_{221} = \frac{3}{4}\,\underline{\underline{\alpha_3}}\,A_1 A_2^2$$

$$\bullet\ B_{112} = \frac{3}{4}\,\underline{\underline{\alpha_3}}\,A_1^2 A_2$$

IM3: third-order intermodulation products



$\Longrightarrow$

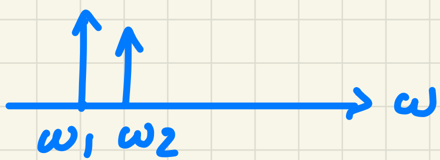


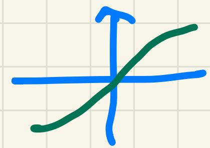
$$\omega_2 = \omega_1 + \Delta\omega \Rightarrow 2\omega_1 - \omega_2 = 2\omega_1 - \omega_1 - \Delta\omega =$$

$$= \omega_1 - \Delta\omega$$

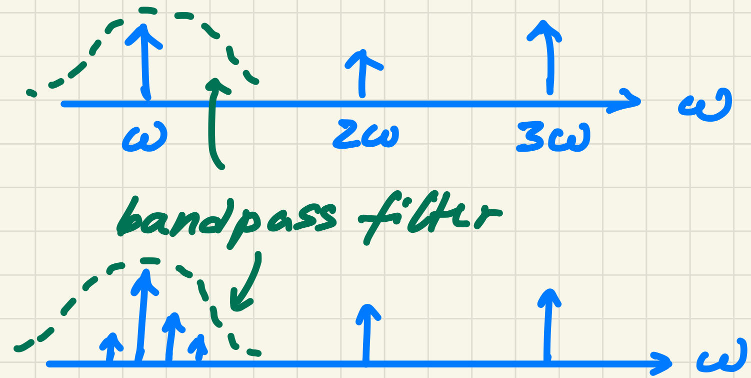
$$2\omega_2 - \omega_1 = 2\omega_1 + 2\Delta\omega - \omega_1 =$$

$$= 2\omega_1 + 2\Delta\omega$$

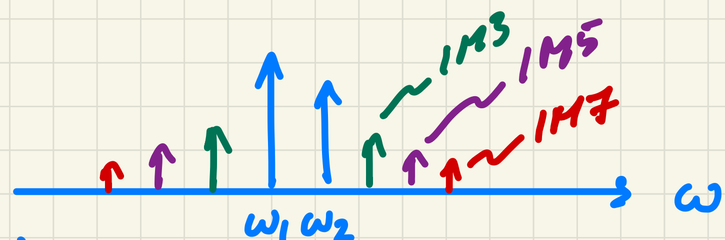




nonlinearity



IM3 products are typically not filtered as instead the HD products



- 3rd order :

$$\cos\omega_1 t \times \cos^2\omega_2 t$$

$$\Rightarrow (2\omega_2 - \omega_1), \ (2\omega_1 - \omega_2)$$

- 5th order : $\cos^2\omega_1 t \times \cos^3\omega_2 t$

$$\Rightarrow 3\omega_2 - 2\omega_1$$

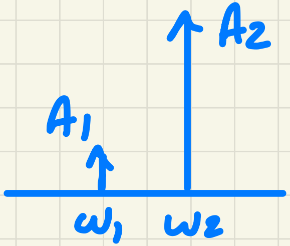
same offset f.

$$\underbrace{(3\omega_2 - 2\omega_1)}_{\text{IM5}} - \underbrace{(2\omega_2 - \omega_1)}_{\text{IM3}} = \omega_2 - \omega_1$$

1. Blocking :

small wanted A_1
large unwanted A_2

$A_1 \ll A_2$



output component at ω_1

$$B_1 = \alpha_1 A_1 + \underbrace{\frac{3}{4}\alpha_3 A_1^3}_{\text{negligible}} + \frac{3}{2}\alpha_3 A_1 A_2^2 =$$

if $A_1^3 \ll A_1 A_2^2$

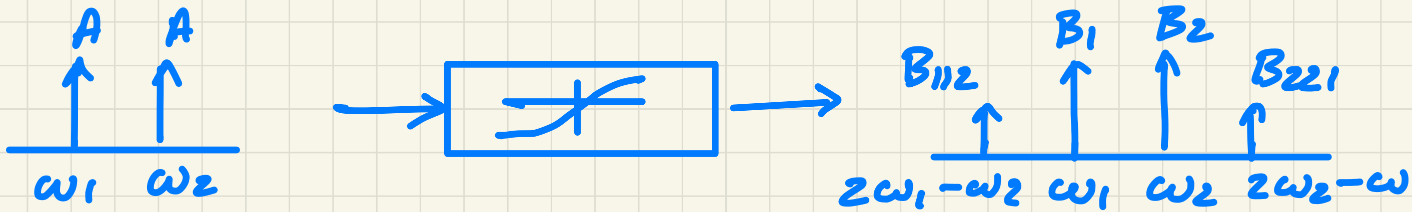
RF Interferens
a.k.a. "BLOCKERS"

$$= \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right)\cdot A_1$$

$\Rightarrow$ Harm. gain of the system $= \dfrac{B_1}{A_1} = \alpha_1 + \frac{3}{2}\alpha_3 A_2^2$

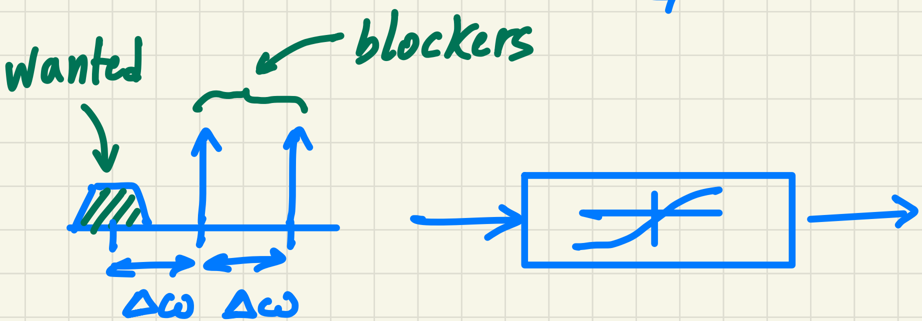
Harmonic gain $\longrightarrow 0$ at large A_2

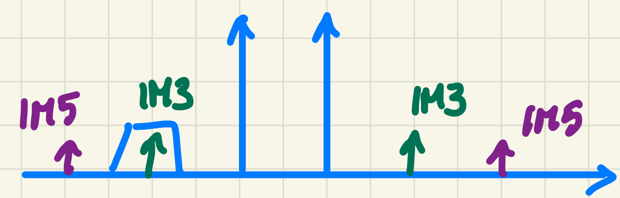
2. Intermodulation : $A_1 = A_2 = A$



$$B_{221} = B_{112} = \frac{3}{4}\alpha_3 A^3$$

IM3 terms (in the signal band)





IM3 degrades $SNDR = \dfrac{P_s}{P_n + P_d}$

Second-order nonlinearity

$$x(t) = \overbrace{A\cos\omega_1 t}^{a} + \underbrace{A\cos\omega_2 t}_{b}$$

$$\alpha_2 x^2(t) =$$

$$= \alpha_2 A^2 (\cos\omega_1 t + \cos\omega_2 t)^2$$

$$\Downarrow$$

$$B_0 = \alpha_2 A^2$$

$$B_{11} = \alpha_2 A^2$$

$$B_{12} = \alpha_2 A^2$$

$$B_{22} = \alpha_2 A^2$$

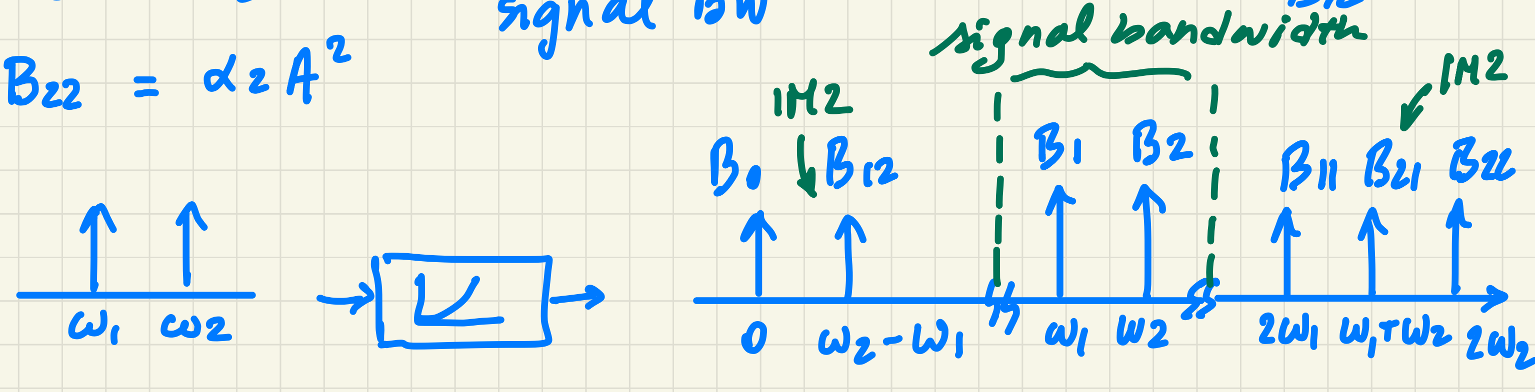
$$(a+b)^2 = \underline{\underline{a}}^2 + \underline{\underline{b}}^2 + 2ab$$

$\cos^2\omega_1 t \Rightarrow B_0, B_{11}$

$\cos^2\omega_2 t \Rightarrow B_0, B_{22}$

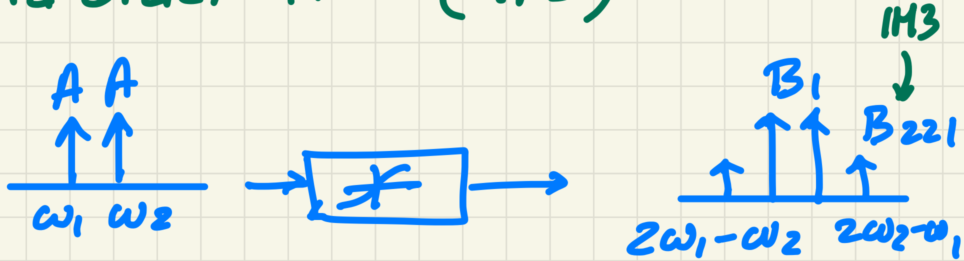
$2\cos\omega_1 t\cos\omega_2 t \Rightarrow B_{12}$

IM2 products fall outside signal BW



Intercept Point

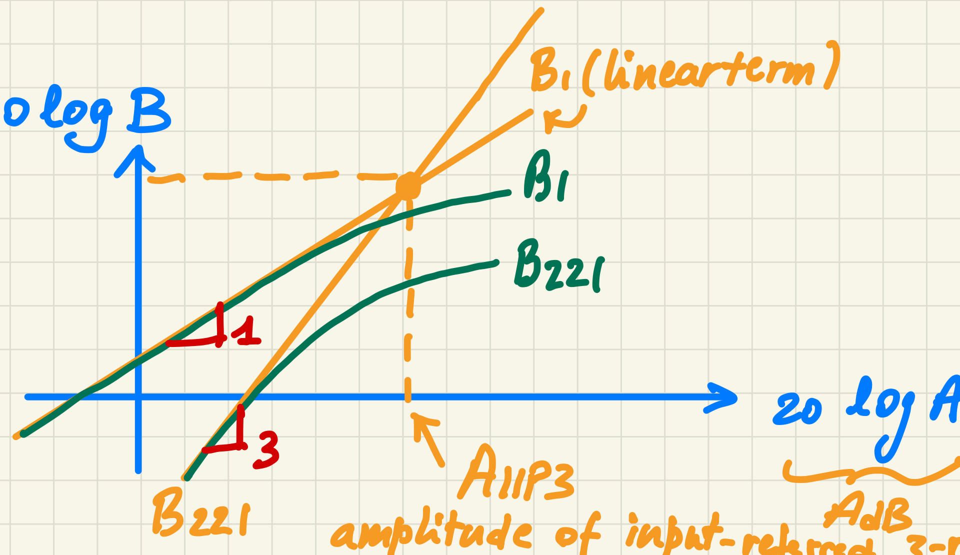
3rd order IP (IP3)



$$B_1 = \alpha_1 A + \frac{3}{4}\alpha_3 A^3 + \frac{3}{2}\alpha_3 A^3$$

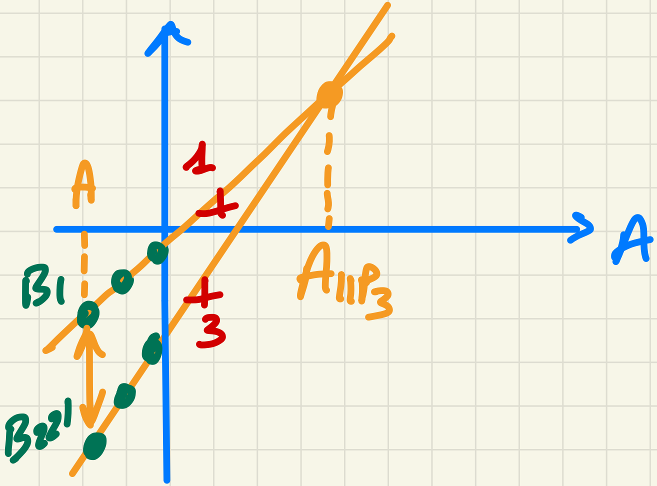
$$= \underbrace{\alpha_1 A} + \underbrace{\frac{9}{4}\alpha_3 A^3}$$

$$B_{221} = \underbrace{\frac{3}{4}\alpha_3 A^3}$$



$$20\log_{10}(\alpha_1 A) = \alpha_{1,dB} + A_{dB}$$

$$20\log_{10}(B_{221}) = 20\log\left(\frac{3}{4}\alpha_3\right) + 3\cdot A_{dB}$$



"Two-tone test"

- From analysis:

$$\underbrace{\alpha_1 A_{IIP_3}}_{B_1 \text{ (extrapol.)}} = \underbrace{\frac{3}{4}\alpha_3 A_{IIP_3}^3}_{B_{221} \text{ (extrap.)}}$$

$$\Downarrow$$

$$A_{IIP_3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$\downarrow$$

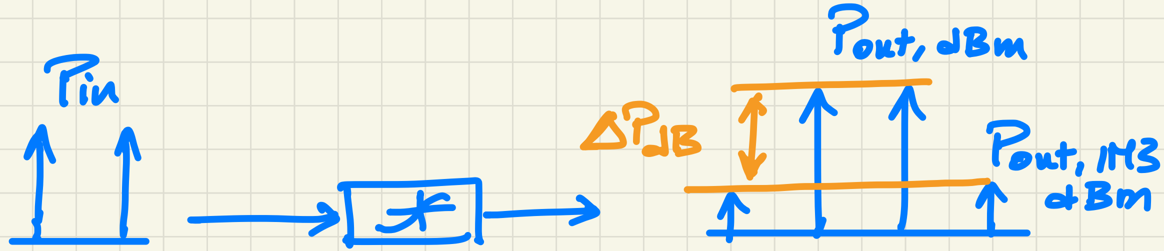
$$A_{IIP_3,dB} = 20\log_{10} A_{IIP_3} = 10\log_{10}\left(\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|\right)$$

1-dB compression point

$$A_{C,dB} = -9.6\,dB + A_{IIP_3,dB} \quad \Leftarrow$$

$\Rightarrow$ 1 dB compression point is typically about 9.6 dB lower than the IIP3

- Operative
  Practical measur. of IIP_3



$$\frac{B_1}{B_{221}} = \frac{V_{out}}{V_{out,IM3}} = \frac{\alpha_1 \cancel{A}}{\frac{3}{4}\alpha_3 A^{\cancel{3}2}} = \frac{A_{IIP_3}^2}{A^2};$$

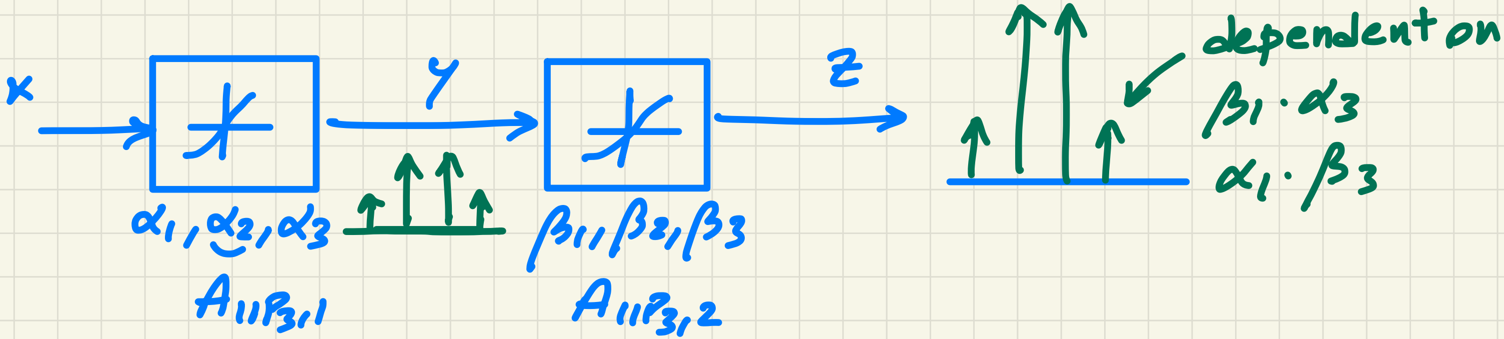
$$A_{IIP_3} = A \cdot \sqrt{\frac{V_{out}}{V_{out,IM3}}}$$

$$\Rightarrow \underset{(dBm)}{IIP_3} = P_{in} + \frac{1}{2}\cdot \Delta P$$

dBm

$$\Rightarrow IIP_3 = P_{in} + \frac{1}{2}\cdot [\underbrace{P_{out} - P_{out,IM3}}_{\Delta P}]$$

IIP_3 of cascaded stages



$$y = \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 \qquad x = A\cos\omega_1 t + A\cos\omega_2 t$$

$$z = \beta_1 y + \beta_2 y^2 + \beta_3 y^3$$

$$\underbrace{\frac{1}{A_{IIP_3}^2} \overset{(*)}{\cong} \frac{1}{A_{IIP_3,1}^2} + \frac{\alpha_1^2}{A_{IIP_3,2}^2}}_{\text{Nonlinearity of latter stages dominates}}$$

(*) typ. bandpass filtering between the two blocks

$\alpha_2 x^2 \Rightarrow \underbrace{(\omega_2 - \omega_1)}$

$\beta_2 y^2 \Rightarrow (2\omega_2 - \omega_1) \longrightarrow$ IM3 term

$(2\omega_1 - \omega_2)$

$(\omega_2 - \omega_1) \otimes \omega_2$

$(\omega_2 - \omega_1) \otimes \omega_1$

$\Rightarrow$ cascade of two 2nd order nonlinearity terms produces the same effect of a 3rd ord. nonlin. (IM3)