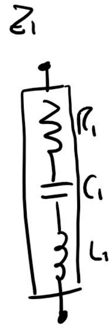
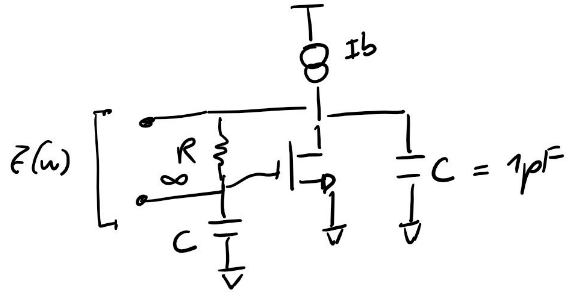
EXERCISE T 7.1

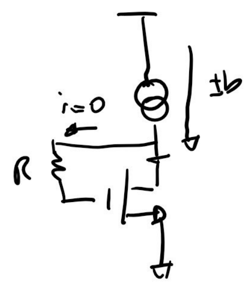
$V_T = 0.5V$

$K = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) = 2\,\mu A / V^2$



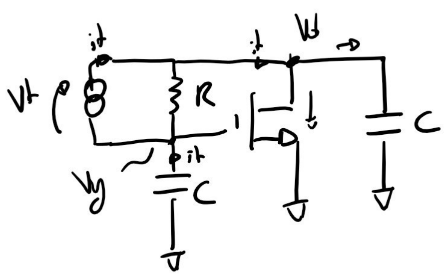


• BIAS POINT OF THE MOS



$g_m = 2\sqrt{K\, I_b}$

a) DERIVE $Z(w)$



$Z(s) = \frac{V_t}{i_t} = \frac{(V_d - V_g)}{i_t} = \frac{V_d}{i_t} - \frac{V_g}{i_t}$

$V_g = -\frac{i_t}{sC}$

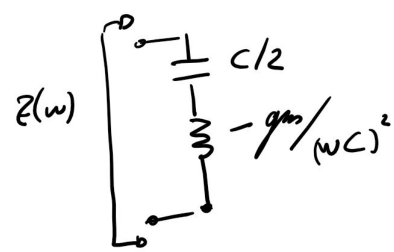
$i_t = V_g g_m + V_d sC = -\frac{i_t}{sC} g_m + V_d sC$

$V_d = \frac{i_t}{sC}\left[1 + \frac{g_m}{sC}\right]$

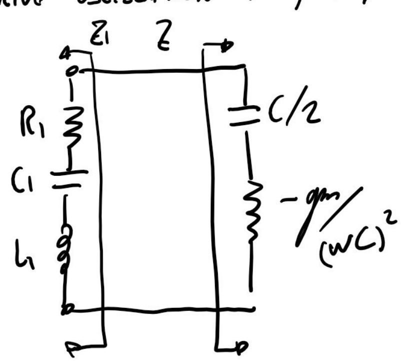
$Z(s) = \frac{1}{sC}\left[1 + \frac{g_m}{sC}\right] + \frac{1}{sC} = \frac{2}{sC} + \frac{g_m}{(sC)^2}$

$s = jw$

$Z(w) = \frac{2}{jwC} + \frac{g_m}{(jwC)^2} = \frac{2}{jwC} - \frac{g_m}{(wC)^2}$



b) DERIVE OSCILLATION FREQUENCY WHEN Z_1 IS CONNECTED TO Z.



• FROM NEGATIVE RESISTOR MODEL TO GUARANTEE POWER BALANCE @ w_0

$Z(jw_0) + Z_1(jw_0) = 0$

① $Im\{Z(jw_0)\} + Im\{Z_1(jw_0)\} = 0$

② $Re\{Z(jw_0)\} + Re\{Z_1(jw_0)\} = 0$

FROM ①: $w_0 L_1 - \frac{1}{w_0 C_1} - \frac{1}{w_0 C/2} = 0 \rightsquigarrow w_0 = \frac{1}{\sqrt{L_1 (C_1 /\!/ C/2)}}$

$= 2\pi\ 40{,}13\ MHz$

c) MINIMUM I_{bias} to GUARANTEE START-UP

@ STEADY STATE $\quad Re\{Z_1(j\omega_0)\} + Re\{Z(j\omega_0)\} = 0$

@ STARTUP $\quad Re\{Z_1(j\omega_0)\} + Re\{Z(j\omega_0)\} < 0$

$$R_1 - \frac{g_m}{\omega_0^2 C^2} < 0$$

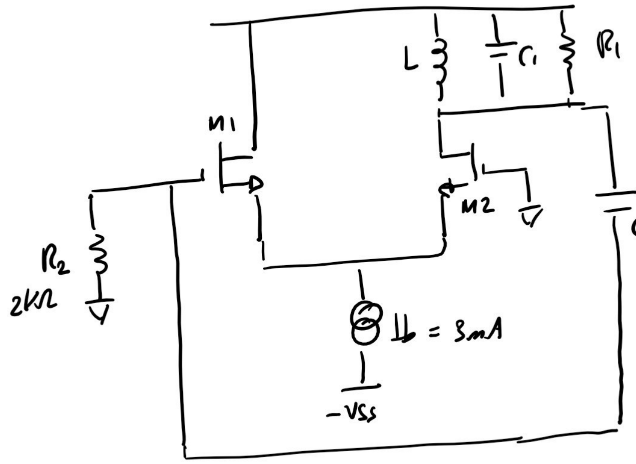
$$\begin{cases} g_m > R_1 \omega_0^2 C^2 \\ g_m = 2\sqrt{K\, I_{bias}} \end{cases}$$

$$I_{bias} \geq \frac{1}{K}\left(\frac{R_1 \omega_0^2 C^2}{2}\right)^2 = 0.449\,\mu A$$

EXERCISE 7.2

$K = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) = 10 \frac{mA}{V^2}$

$V_T = 0.5V$

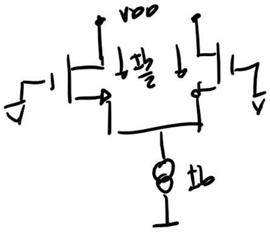


$L = 4nH$

$C_1 = 0.5pF$

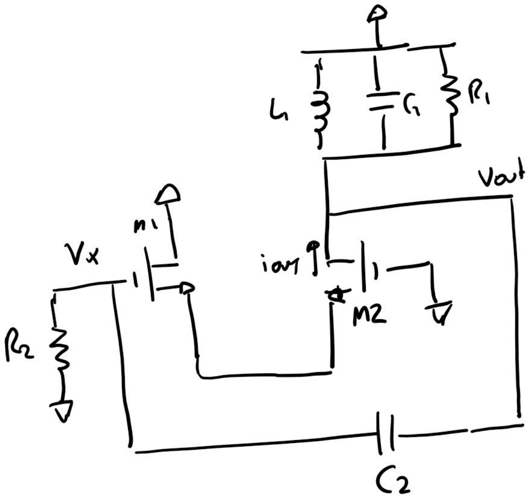
$R_1 = 1K\Omega$

- MOS BIAS POINT

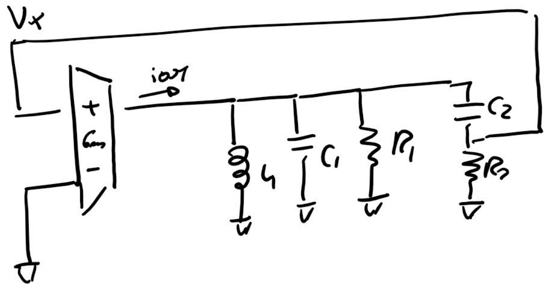


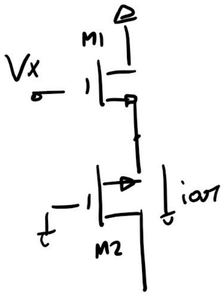
$g_m = 2\sqrt{K \frac{I_b}{2}} = 7{,}75 \frac{mA}{V}$

a) DERIVE ω_0 (WITH SIMPLIFICATIONS)



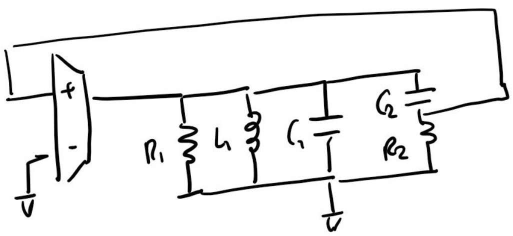
$\Longleftrightarrow$ EQ. MODEL TANK + TRANSCONDUCTOR



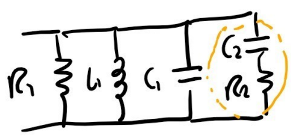


$$i_{OUT} = g_m \left[V_x - \frac{i_{out}}{g_m} \right]$$

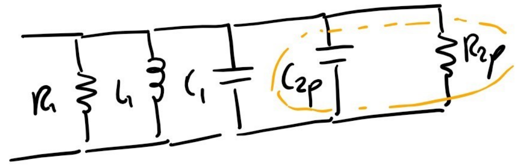
$$\frac{i_{out}}{V_x} = \frac{g_m}{2} = G_m$$



- SER-to-PARALLEL TRANSFORMATION



SER-to-PAR $\Longleftrightarrow$ @ ω_0



SER-TO-PAR @ ω_0

$$\boxed{\begin{aligned} R_p &= R_s\,(1 + Q_T^2) \\ X_p &= X_s\left(1 + \frac{1}{Q_T^2}\right) \\ Q_T &= \frac{X_s}{R_s} = \frac{R_p}{X_p} \end{aligned}}$$

$$R_{2p} = R_2\,(1 + Q_T^2)$$

$$\frac{1}{\omega_0 C_{2p}} = \frac{1}{\omega_0 C_2}\left(1 + \frac{1}{Q_T^2}\right)$$

$$Q_T = \frac{1}{\omega_0 C_2 R_2} = \omega_0 C_{2p} R_{2p}$$

- WE DON'T KNOW ω_0

MAKE HYPOTHESIS OF Q_T

IF $Q_T \ll 1 \rightarrow \omega_0 \gg \frac{1}{C_2 R_2} = 2\pi\ 159\ MHz$

IF $Q_T \gg 1 \rightarrow \omega_0 \ll \frac{1}{C_2 R_2} = 2\pi\ 159\ MHz$

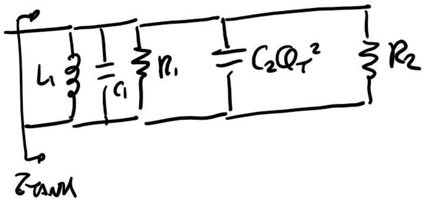
SINCE

$L \sim nH$
$C \sim pF$ $\rightarrow f_0 \sim GHz$

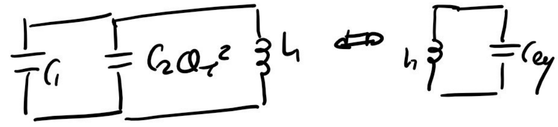
- CONSIDER $Q_T \ll 1$

$$\begin{cases} R_{2p} = R_2\,(1 + Q_T^2) \simeq R_2 \\ \frac{1}{\cancel{\omega_0} C_{2p}} = \frac{1}{\cancel{\omega_0} C_2}\left(1 + \frac{1}{Q_T^2}\right) \simeq \frac{1}{\omega_0 C_2 Q_T^2} \end{cases} \rightsquigarrow C_{2p} = C_2\,Q_T^2$$

• EQ TANK MODEL



$Im\{Z_{TANK}\} = 0$



$$\omega_0 L_1 - \frac{1}{\omega_0 \left[C_1 + C_2 Q_T^2\right]} = 0 \qquad Q_T = \frac{1}{\omega_0 R_2 C_2}$$

$$\omega_0 L_1 - \frac{1}{\omega_0 \left[C_1 + \frac{1}{(\omega_0 R_2)^2 C_2}\right]} = 0$$

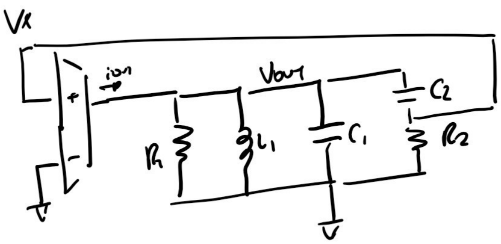
$$\omega_0^2 L_1 C_1 + \frac{\cancel{\omega_0^2} L_1}{\cancel{\omega_0^2} R_2^2 C_2} = 1$$

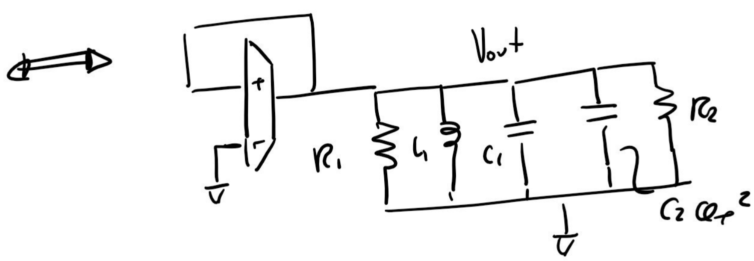
$$\rightsquigarrow \omega_0^2 = \frac{1}{L_1 C_1} - \frac{1}{C_1 C_2 R_2^2}$$

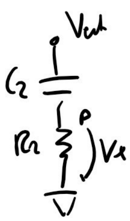
$$= (2\pi \ 3.55 \ GHz)^2$$

• CHECK THE ASSUMPTION

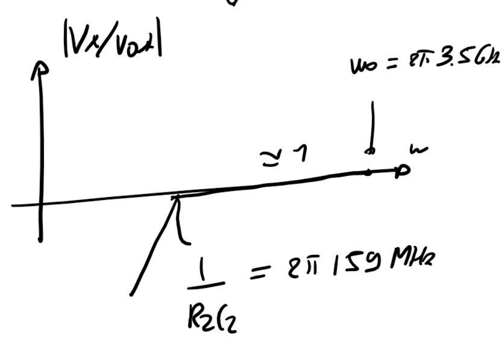
$$Q_T = \frac{1}{\omega_0 C_2 R_2} = 0.045 \ll 1$$



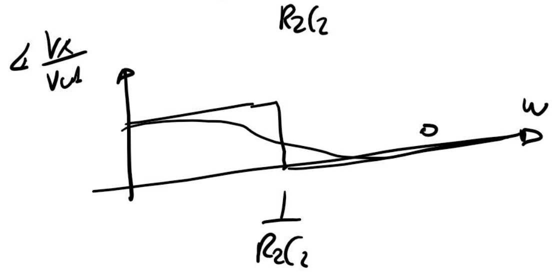




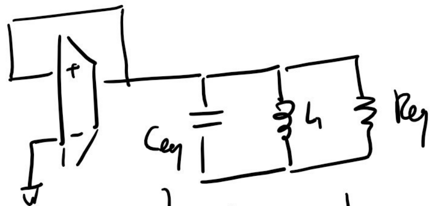
$$\frac{V_x}{V_{out}} = \frac{R_2}{R_2 + \frac{1}{sC_2}} = \frac{sC_2R_2}{1 + sR_2C_2}$$



$V_x \simeq V_{out}$ in AC



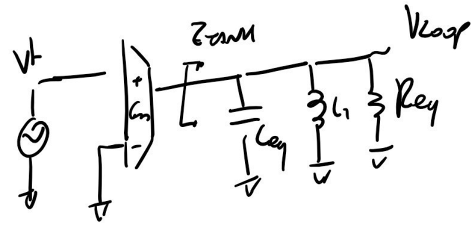
- CHECK THE STARTUP CONDITION



$$\left\{ C_1 + \frac{1}{(\omega_0 R_2)^2 C_2} \right.$$

TO GUARANTEE STARTUP:

$$|G_{loop}(j\omega)| > 1$$

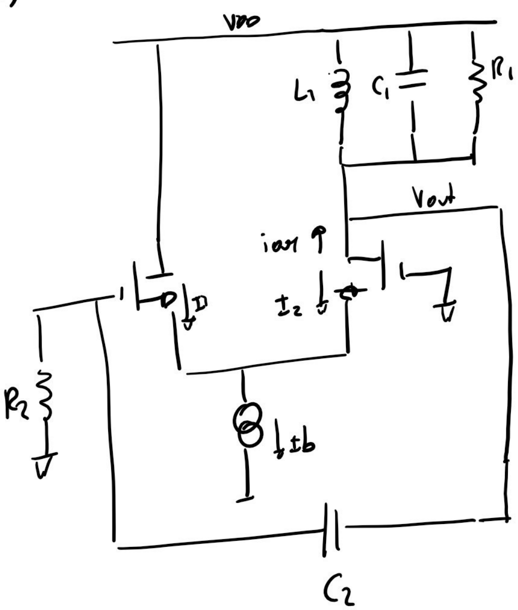


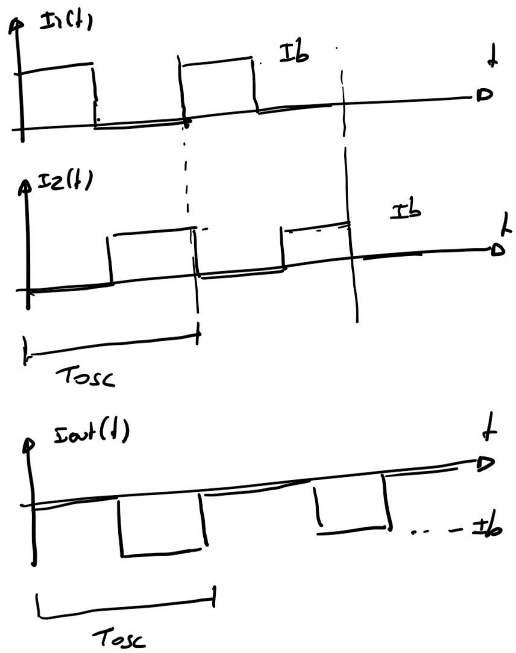
$$G_{loop}(j\omega) = G_m \cdot Z_{tank}(j\omega)$$

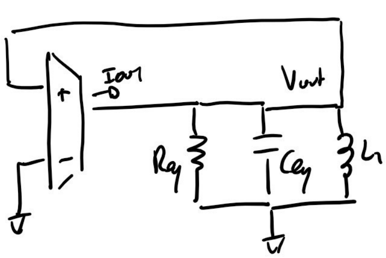
@ ω_0 RESONANCE

$$|G_{loop}(j\omega)| = G_m R_{eq} = \frac{g_m}{2} (R_1 /\!/ R_2) = 2.58 > 1$$

b) OSCILLATION AMPLITUDE CONSIDERING MOS FULLY SWITCHING





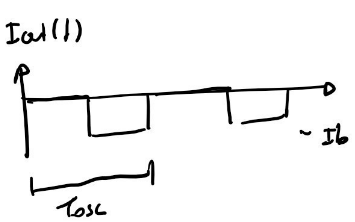


EVALUATE $V_{out}(t)$ WITH HARMONIC BALANCE

$I_{out}(t) \to I_{out}(j\omega)$

$V_{out}(j\omega) = I(j\omega)\, Z_{tank}(j\omega)$

$V_{out}(j\omega) \to V_{out}(t)$



SQUARE WAVE

$D = 0.5$

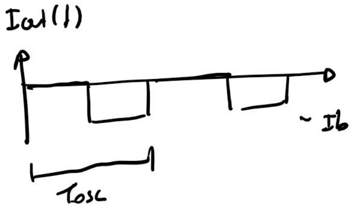
T_{osc}

$A = -I_b$

$$I_{out}(t) = \gamma_0 + \sum_{n=1}^{+\infty} 2|\gamma_n| \cos\left(\frac{2\pi}{T_{osc}} n t + \angle \gamma_n\right)$$

$$I_0 = \gamma_0 = -\frac{I_b}{2}$$

$$\gamma_n = \frac{1}{T_{osc}} F_T \{ I_{outT}(t) \}_{f = \frac{n}{T_{osc}}}$$



SQUARE WAVE

$D = 0.5$

T_{osc}

$A = -I_b$

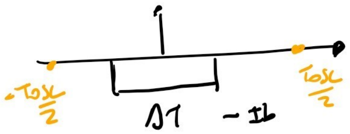
$$I_{out}(t) = \gamma_0 + \sum_{n=1}^{+\infty} 2|\gamma_n| \cos\left(\frac{2\pi}{T_{osc}} nt + \angle \gamma_n\right)$$

$$I_0 = \gamma_0 = -\frac{I_b}{2}$$

$$\gamma_n = \frac{1}{T_{osc}} F_T\{I_{outT}(t)\}_{f=\frac{n}{T_{osc}}}$$

$$I_{out\,TRUNCATED}(t) = I_{out}(t)\, rect\left(\frac{t}{T_{osc}}\right)$$

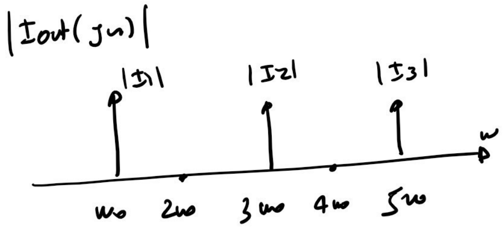
$$I_{OUT\,TRUNCATED}(t) = -I_b\, rect\left(\frac{t}{\Delta t}\right)$$



$$= \frac{1}{T_{osc}} \Delta t\, (-I_b)\, sinc\left(\frac{n}{T_{osc}} \Delta t\right)$$

$$\frac{\Delta t}{T_{osc}} = D$$
= DUTY-CYCLE

$$= -I_b D \frac{\sin(nD\pi)}{nD\pi}$$

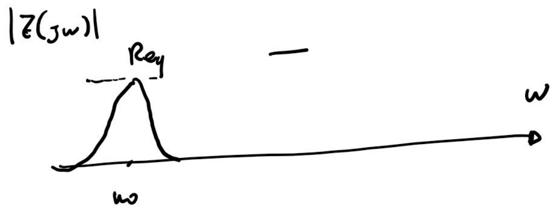


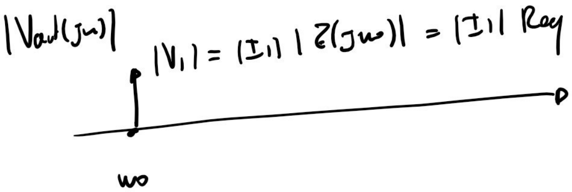
$$|I_1| = 2|\gamma_1| = 2I_b \frac{1}{2} \frac{\sin(\pi/2)}{\pi/2} = \frac{2I_b}{\pi}$$

$$|I_2| = 2|\gamma_2| = 2I_b \frac{1}{2} \frac{\sin(\pi)}{\pi} = 0$$

$$|I_3| = 2|\gamma_3| = \frac{2}{3} \frac{I_b}{\pi}$$

$$|I_4| = 0$$





$$|V_1| = |I_1|\,|Z(j\omega_0)| = |I_1|\, R_{eq}$$

$$V_{out}(t) = V_{out\,DC} + |V_1| \cos(\omega_0 t)$$

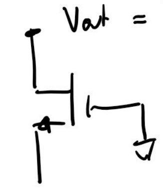
$|V_1|$ = OSCILLATION AMPLITUDE

$$= \frac{2I_b}{\pi} \cdot R_{eq} = \frac{2I_b}{\pi} (R_1 // R_2)$$

$$= 1{,}273\ V$$

c) CHECK TRANSISTOR OPERATION REGION

$\boxed{M2}$



$$V_{out} = V_{DD} + |V_1| \cos(\omega_0 t)$$

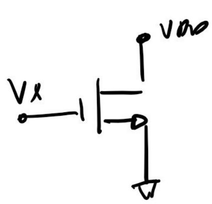
$$V_d^{MIN} > V_g - V_T$$

$$V_{DD} - |V_1| > -V_T$$

$$1{,}25V - 1{,}273V > -0.5V$$

$$-0{,}023\,V > -0.5V \quad \text{OK}$$

$\boxed{M1}$



$$V_d > V_g^{MAX} - V_T$$

$$V_g^{MAX} < V_{DD} + V_T$$

$$1{,}273\,V < 1{,}25V + 0.5V \quad \text{OK}$$

EXERCISE 7.3

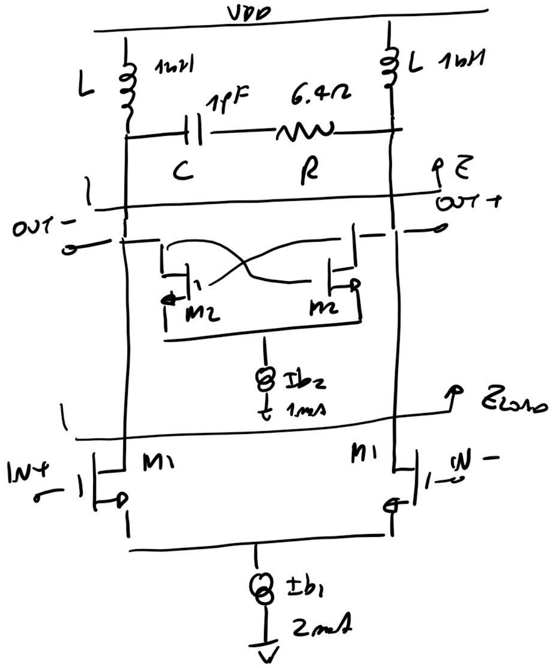
$V_T = 0.5V$

$K = \frac{1}{2}\mu C_{ox} = 0.1 mA/V^2$

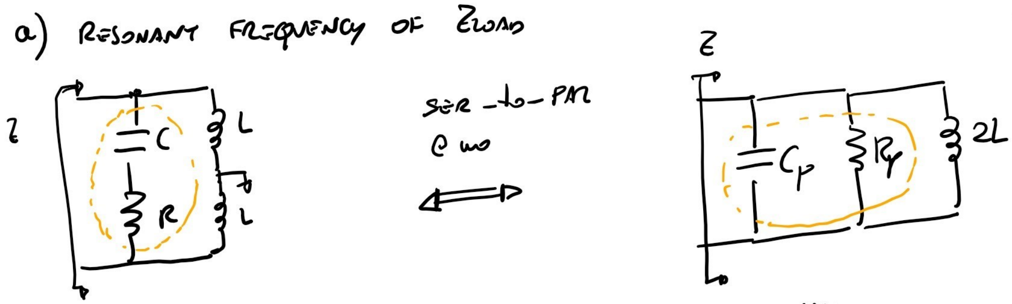
$\gamma/\alpha = 2/3$

$$g_{m1} = 2\sqrt{K \left(\frac{W}{L}\right)_1 \frac{I_{b1}}{2}}$$

$$g_{m2} = 2\sqrt{K \left(\frac{W}{L}\right)_2 \frac{I_{b2}}{2}}$$



a) RESONANT FREQUENCY OF Z_{LOAD}



$$\begin{cases} R_p = R\left(1 + Q_T^2\right) \\ \frac{1}{\omega_0 C_p} = \frac{1}{\omega_0 C}\left(1 + \frac{1}{Q_T^2}\right) \\ Q_T = \frac{1}{\omega_0 CR} = \omega_0 C_p R_p \end{cases}$$

WE DON'T KNOW ω_0

HYPOTHESIS ON Q_T

IF $Q_T \gg 1 \rightarrow \omega_0 \ll \frac{1}{CR} = 2\pi\, 24{,}87\ GHz$

IF $Q_T \ll 1 \rightarrow \omega_0 \gg \frac{1}{CR} = 2\pi\, 24{,}87\ GHz$

SINCE $L \sim nH$ $C \sim pF$

$f_0 \sim GHz$

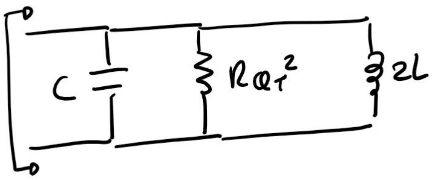
SO $Q_T \gg 1$

if $Q_T >> 1$

$$R_p = R(1+Q_T^2) \simeq R Q_T^2$$

$$\frac{1}{\omega_0 C_{2p}} = \frac{1}{\omega_0 C_2}\left(1+\frac{1}{Q_T^2}\right) \simeq \frac{1}{\omega_0 C_2} \rightsquigarrow C_{2p} \simeq C_2$$

Z



$$\omega_0 = \frac{1}{\sqrt{2LC}} \simeq 2\pi\ 3{,}56\ GHz$$

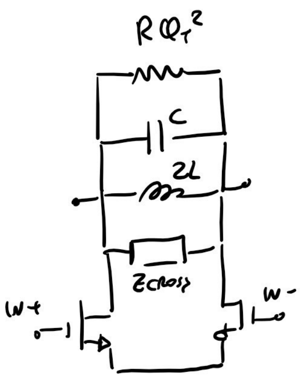
- CHECK THE ASSUMPTION

$$Q_T = \frac{1}{\omega_0 RC} = 7 >> 1$$

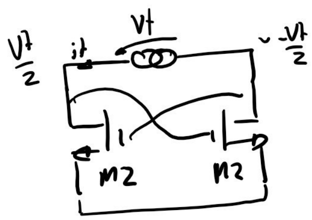
- SIZE $(W/L)_1$ AND $(W/L)_2$

(i) $\frac{V_{outd}}{V_{ind}} = 5\,dB$

(ii) $-3dB$ BW $= 270\ MHz$



IMPEDANCE Zcross

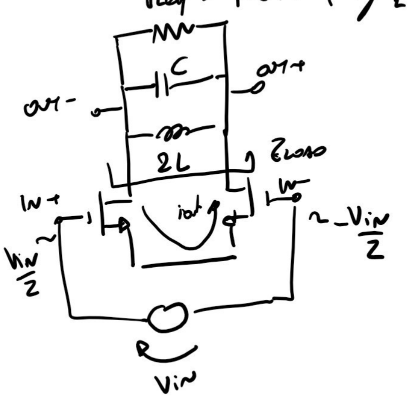


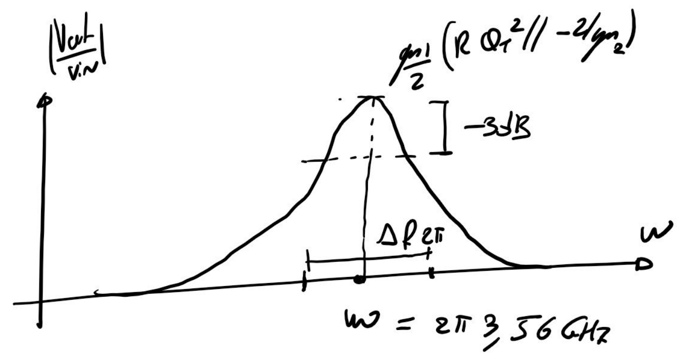
$$Z_{cross} = \frac{V_t}{i_t}$$

$$i_t = g_{m2}\left[-\frac{V_t}{2}\right] \rightsquigarrow \frac{V_t}{i_t} = -2/g_m$$

- DERIVE THE $\frac{V_{outd}}{V_{ind}}$

$$R_{eq} = RQ_T^2 // (-2/g_{m2})$$





$$\omega_0 = 2\pi\ 3{,}56\ GHz$$

$$V_{out} = i_{out} Z_{LOAD} \overset{!}{=} V_{in}\frac{g_{m1}}{2} Z_{LOAD}$$

$$\Delta f = 270\ MHz$$

$$Q_{TANK} = \frac{f_0}{\Delta f} = 13{,}18$$

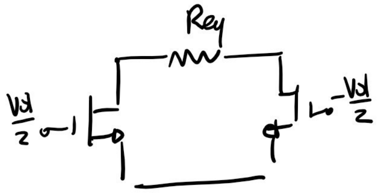
$$Q_{TANK} = \omega_0 C R_{eq} \rightsquigarrow R_{eq} = 589{,}46\,\Omega \overset{!}{=} RQ_T^2 // Z_{cross}$$

$$Z_{cross} = -2/g_{m2}$$

$$g_{m2} = 2\left[\frac{1}{RQ_T^2} - \frac{1}{R_{eq}}\right] = 2{,}98\ mA/V \overset{!}{=} 2\sqrt{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 \frac{I_{b2}}{2}}$$

$$\left(\frac{W}{L}\right)_2 = 45$$

(i) @ w₀ 5dB gain



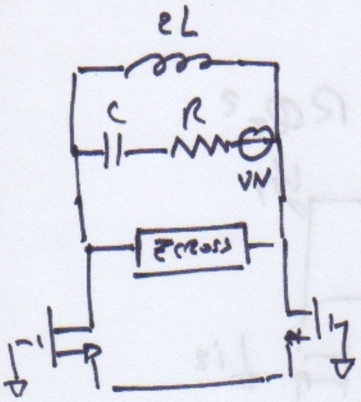
$$\frac{V_{out,d}}{V_{in,d}} = \frac{g_{m}}{2} R_{eq} = 5\,\text{dB} = 10^{5/20}$$

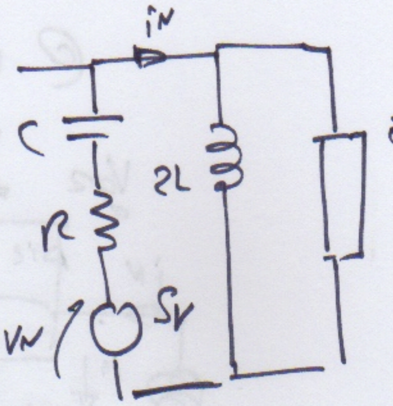
$$\Rightarrow g_{m1} = 6{,}03\ \text{mS}$$

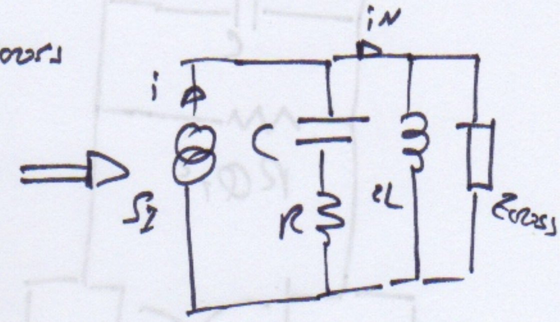
$$= 2\sqrt{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \frac{I_{b1}}{2}}$$

$$\left(\frac{W}{L}\right)_1 = 91$$

b) TRANSISTOR AND RESISTOR NOISE @ ω_0.





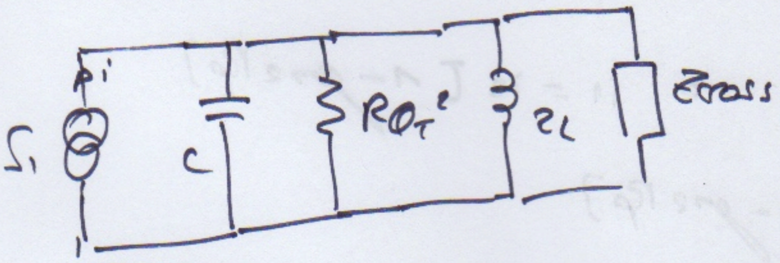


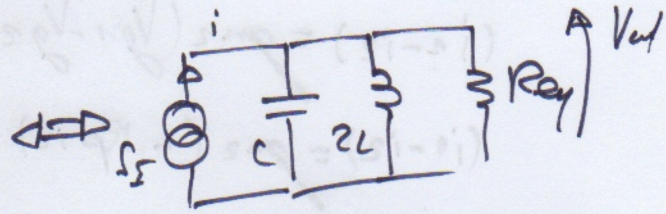
$$i = \frac{V_N}{R + \frac{1}{sC}} = \frac{V_N sC}{(1 + sCR)}$$

$s = j\omega_0$

$$S_I = 4kTR \left| \frac{j\omega_0 C}{1 + j\omega_0 CR} \right|^2 = \frac{4kTR\,R}{R} \frac{\omega_0^2 C^2}{1 + \omega_0^2 R^2 C^2} = \frac{4kT}{R} \frac{\left(\frac{1}{Q_T}\right)^2}{\left(1 + \frac{1}{Q_T^2}\right)}$$

$$S_I = \frac{4kT}{R(1 + Q_T^2)} = \frac{4kT}{R_p}$$

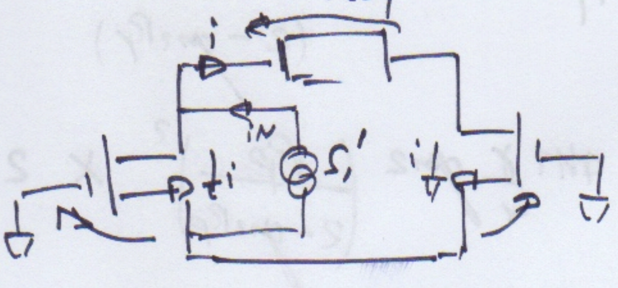




@ ω_0

$$S_{vout} = \frac{4kT}{RQ_T^2} (R_{eq})^2 = \left(4,284 \frac{nV}{\sqrt{Hz}}\right)^2 \qquad T = 300K$$

- CONTRIBUTION OF MAIN DIFFERENTIAL PAIR

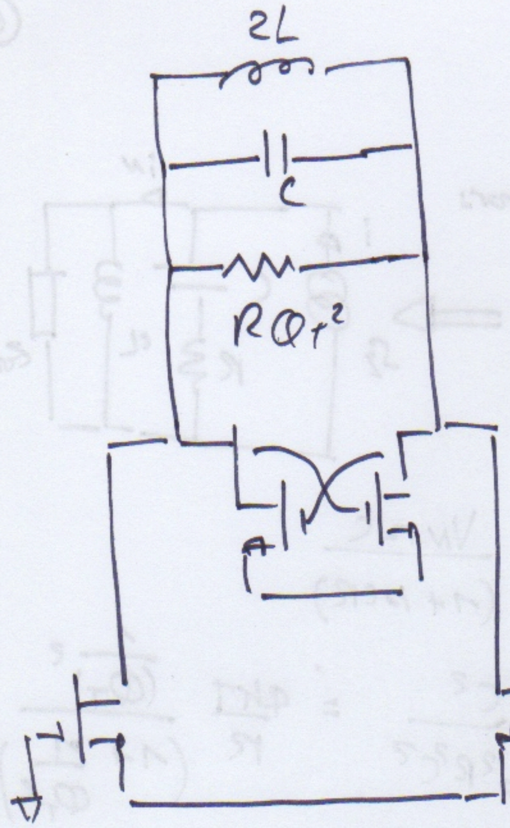


$$\begin{cases} V_{out} = i R_{eq} \\ i_N = 2i \end{cases} \Rightarrow V_{out} = \frac{i_N}{2} R_{eq}$$

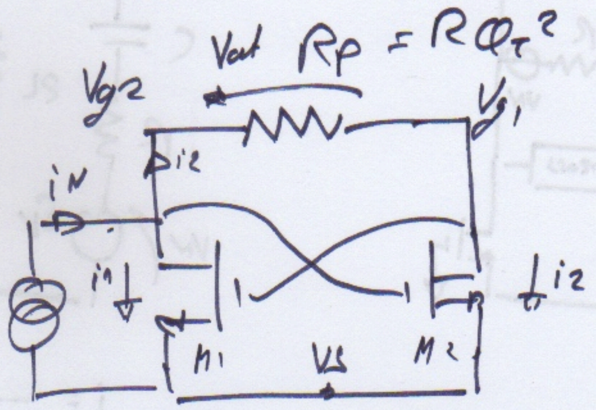
@ ω_0

$$S_{vout} = \frac{1}{4} \underbrace{\left[4kT \frac{\gamma}{\alpha} g_m\right]}_{S_i'} R_{eq}^2 \; X2 \leftarrow \text{BOTH TRANSISTORS}$$

$$= \left(3,4022 \frac{nV}{\sqrt{Hz}}\right)^2$$



@ ω_0



$$i_N = i_1 + i_2$$

$$V_{out} = (V_{g2} - V_{g1}) = R_p i_2$$

$$i_1 = g_{m2}(V_{g1} - V_S)$$

$$i_2 = g_{m2}(V_{g2} - V_S)$$

$$(i_1 - i_2) = g_{m2}(V_{g1} - V_{g2})$$

$$(i_1 - i_2) = g_{m2}(-R_p i_2) \rightarrow i_1 = i_2[1 - g_{m2}R_p]$$

$$i_N = i_2 + i_1 = i_2 + i_2[1 - g_{m2}R_p]$$

$$= i_2[1 + 1 - g_{m2}R_p]$$

$$V_{out} = R_p i_2 = \frac{R_p \, i_N}{(2 - g_{m2}R_p)}$$

@ ω_0

$$S_{Vout} = 4kT\frac{\gamma}{\alpha} g_{m2}\left(\frac{R_p}{2 - g_{m2}R_p}\right)^2 \times 2$$ ← Both TRANSISTORS

$$= \left(2.55\,\frac{nV}{\sqrt{Hz}}\right)^2$$

$$S_{Vout\,TOT} = S_{out}^{RES} + S_{out}^{MAIN} + S_{out}^{CROSS} = \left(6.036\,\frac{nV}{\sqrt{Hz}}\right)^2$$

c) $\left(\frac{W}{L}\right)_2$ LIMIT?

$Req > 0$ to prevent oscillations

$$Req = \frac{2RQ_T^2}{2 - RQ_T^2 g_{m2}} > 0 \qquad 2RQ_T^2 > 0$$

$$2 - RQ_T^2 g_{m2} > 0$$

$$g_{m2} < \frac{2}{RQ_T^2}$$

$$g_m = 2\sqrt{\frac{1}{2} \mu C_{ox} \left(\frac{W}{L}\right)_2 \frac{I_{bc}}{2}}$$

$$\left(\frac{W}{L}\right)_2 < 204$$