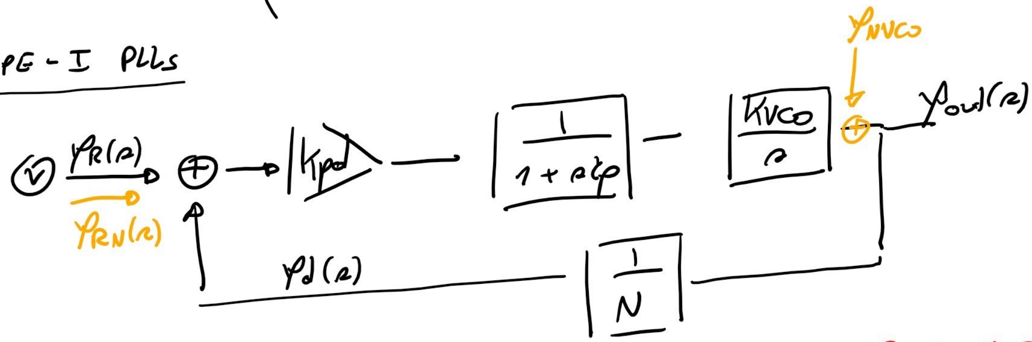
TYPE - II PLLs

- ISSUES OF TYPE-I PLLs
- TRI-STATE PHASE DETECTOR (PFD)
- CP - PLL

- **ISSUES OF TYPE-I PLLs**



$$G_{loop}(s) = K_{pd}\,\frac{1}{(1+s\tau_p)}\,\frac{K_{VCO}}{s}\,\frac{1}{N}$$

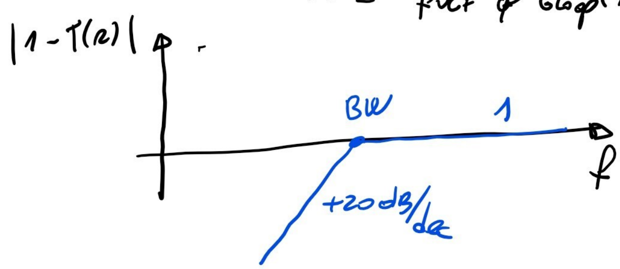
\# TYPE-I SYSTEM = 1 POLE IN ORIGIN

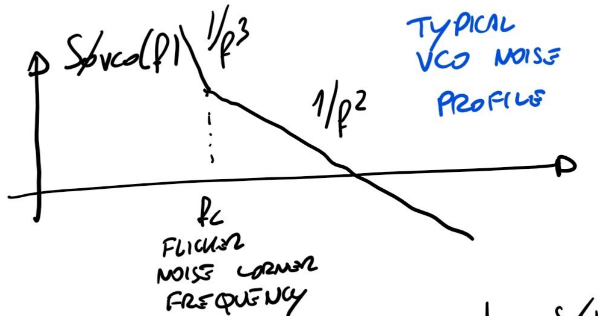
1) LIMITED VCO-NOISE FILTERING

- $\dfrac{\varphi_{out}(s)}{\varphi_{RN}(s)} = N\,T(s) = N\,\dfrac{G_{loop}(s)}{(1+G_{loop}(s))}$

BW ≃ f_{UGF} of $G_{loop}(s)$

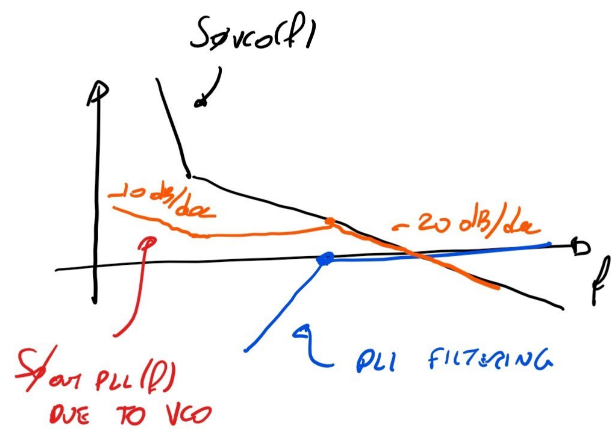
- $\dfrac{\varphi_{out}(s)}{\varphi_{nVCO}(s)} = 1 - T(s) = \dfrac{1}{1+G_{loop}(s)}$



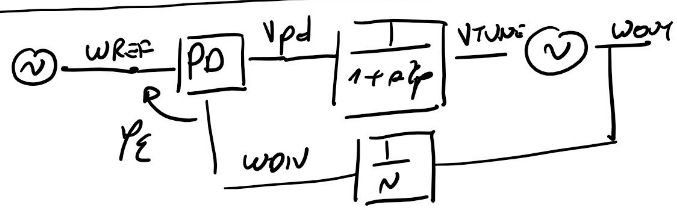


TYPICAL VCO NOISE PROFILE

if we apply PLL FILTERING to $S_{\varphi VCO}(f)$



2) STATIC PHASE ERROR @ PHASE DETECTOR INPUT @ STEADY STATE



@ STEADY STATE

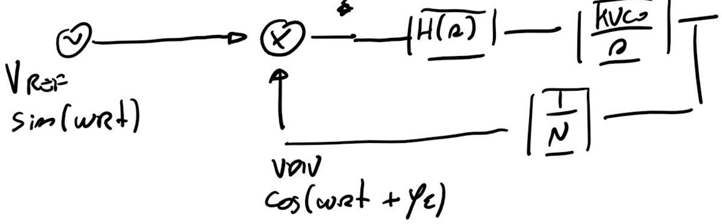
$$\begin{cases} \omega_{OUT} = N\,\omega_{REF} \overset{!}{=} \omega_{FR} + K_{VCO}\cdot V_{TUNE} \\ \langle V_{pd}\rangle = V_{TUNE} = K_{pd}\,\varphi_E \end{cases}$$

$$\boxed{\varphi_E = \frac{(N\omega_{REF} - \omega_{FR})}{K_{VCO}\cdot K_{pd}}}$$

φ_E is PARAMETER DEPENDENT

3) REFERENCE SPURS

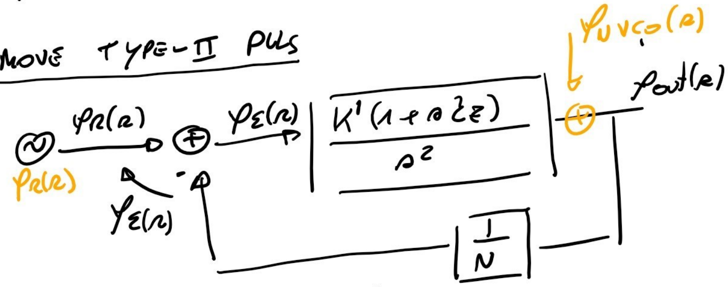
$$V_{pd} = \frac{1}{2}\sin(\varphi_\varepsilon) + \cancel{\frac{1}{2}\sin(2\omega_R t)}$$



We want to remove the HF component from the PD output.

- TO SOLVE THESE ISSUES WE MOVE TYPE-II PLLs

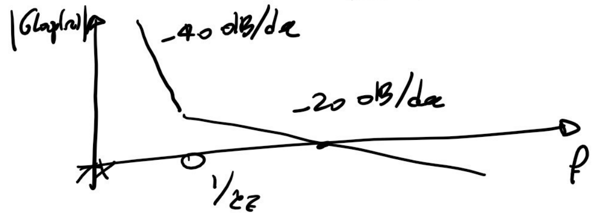
BASIC STRUCTURE:



$$G_{loop}(s) = \frac{K'(1+s\tau_z)}{s^2}\frac{1}{N}$$

#2 POLES ON THE ORIGIN
TYPE-II SYSTEM

$$= \frac{K(1+s\tau_z)}{s^2}$$

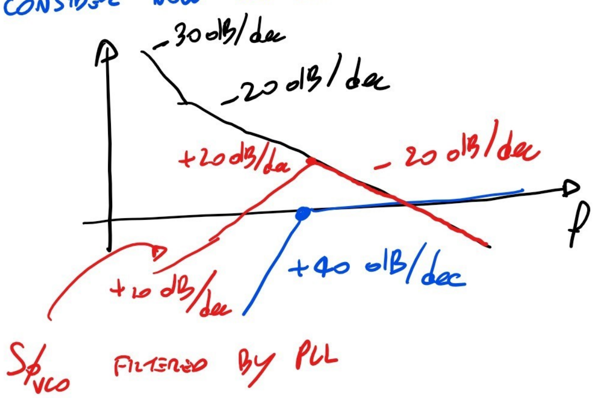


FEATURES:

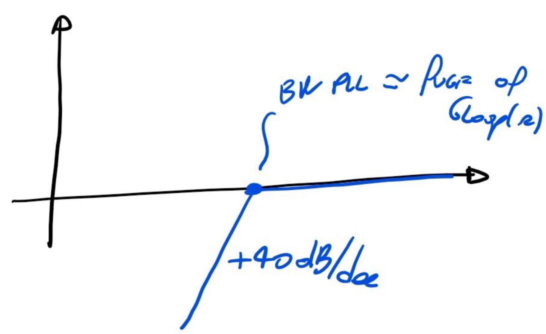
1) BETTER VCO NOISE FILTERING

$$\frac{\varphi_{out}(s)}{\varphi_{N\,VCO}(s)} = 1 - T(s) = \frac{1}{1+G_{loop}(s)} = \frac{1}{1+\frac{K(1+s\tau_z)}{s^2}} = \frac{s^2}{s^2+sK\tau_z+K}$$

CONSIDER NOW the FILTERING OF VCO NOISE: $|1-T(s)|$



$S_{\varphi_{VCO}}$ FILTERED BY PLL



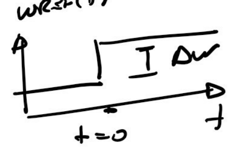
2) ZERO STEADY STATE ERROR @ PHASE DETECTOR input

$$\frac{\varphi_\varepsilon(s)}{\varphi_R(s)} = \frac{1}{1+G_{loop}(s)} = 1 - T(s)$$

LET'S APPLY an input FREQUENCY STEP:

$$\omega_{REF}(s) = \frac{\Delta\omega}{s}$$

$$\varphi_{REF}(s) = \frac{\Delta\omega}{s^2}$$



STEADY STATE $\varphi_\varepsilon^\infty$

$$\varphi_\varepsilon^\infty = \lim_{s\to 0} s\left(\frac{\Delta\omega}{s^2}\right)\cdot\left(\frac{\varphi_\varepsilon(s)}{\varphi_R(s)}\right) = \lim_{s\to 0} s\left(\frac{\Delta\omega}{s^2}\right)\frac{s^2}{s^2+s\tau_z K+K} = 0$$

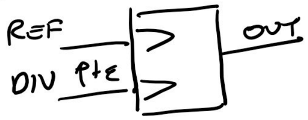
3) REFERENCE SPURS

@ STEADY STATE $\varphi_\varepsilon = 0$

WE CAN BUILD A PHASE DETECTOR $\quad OUT = 0$ when $\varphi_\varepsilon = 0$

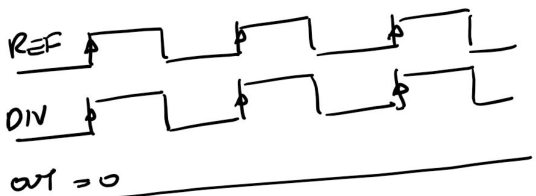
AND NOT JUST $\langle OUT \rangle = 0$

- TRI-STATE PHASE DETECTOR (PFD)



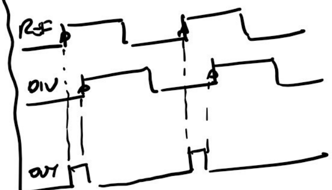
RISING-EDGE SENSITIVE

$t_\varepsilon = 0$ STATE 0



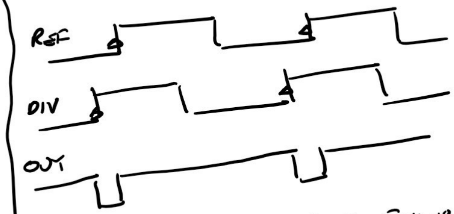
NOT ONLY $\langle OUT \rangle = 0$

$t_\varepsilon > 0$ STATE 1



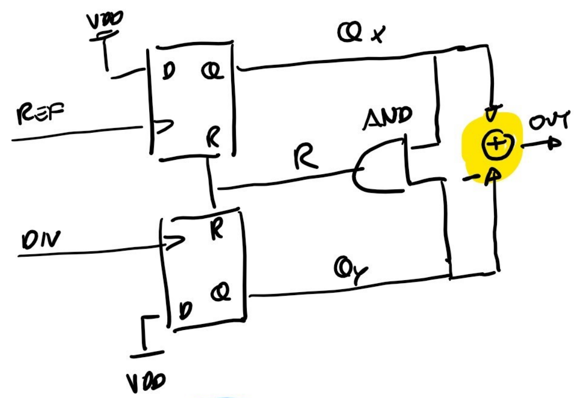
$\langle OUT \rangle = \frac{t_\varepsilon}{T_R}$ REF LEADS THE DIV

$t_\varepsilon < 0$ STATE 2

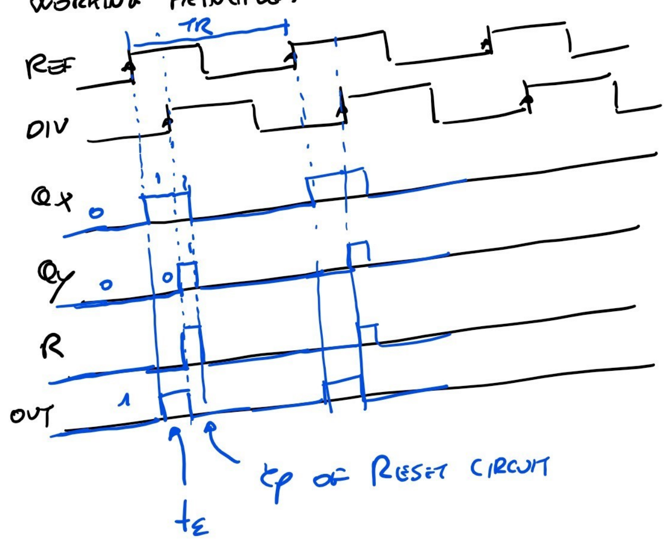


$\langle OUT \rangle = -\frac{t_\varepsilon}{T_R}$ REF FOLLOWS THE DIV

- IMPLEMENTATION



WORKING PRINCIPLE: $t_\varepsilon > 0$

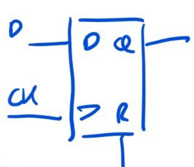


$\langle OUT \rangle = \frac{t_\varepsilon}{T_R}$

- the SAME HAPPENS FOR $t_\varepsilon < 0$

$\langle OUT \rangle = -\frac{t_\varepsilon}{T_R}$

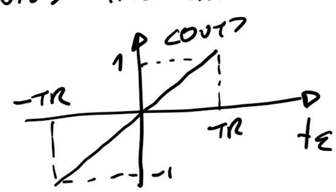
D - FLIP - FLOP



D	CK	R	Q
0	⨍	X	0
1	⨍	X	1
X	X	⨍	0

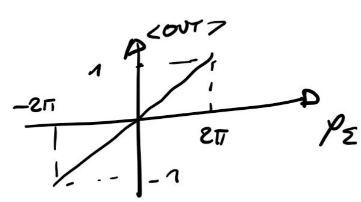
$X = 0/1$

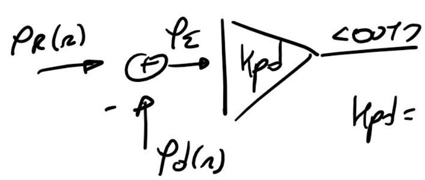
- BUILD THE CHARACTERISTIC:



$$\varphi_\varepsilon = \frac{2\pi}{T_R} t_\varepsilon$$

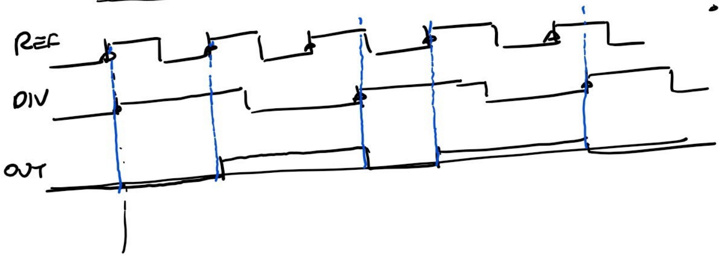
$$\langle OUT \rangle = \frac{t_\varepsilon}{T_R} = \frac{\varphi_\varepsilon}{2\pi}$$





$K_{pd} = \frac{1}{2\pi}$

- WHY IS IT CALLED PHASE/FREQUENCY DETECTOR (PFD)
  - DURING STARTUP: $F_{REF} \gg F_{DIV}$

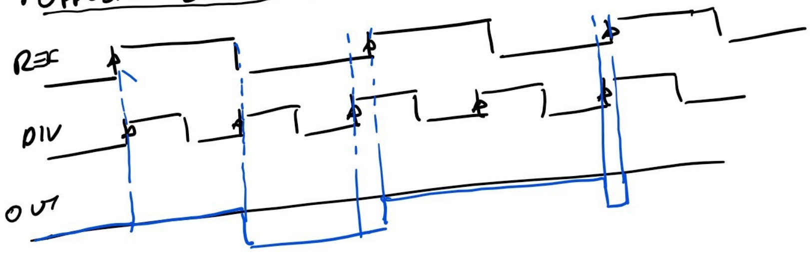


- PFD PROVIDES ONLY POSITIVE OUTPUT

  ↓

  the LOOP is FORCED ONLY TO INCREASE FREQUENCY UNTIL $F_{DIV} = F_{REF}$.

- OPPOSITE SITUATION: $F_{REF} \ll F_{DIV}$



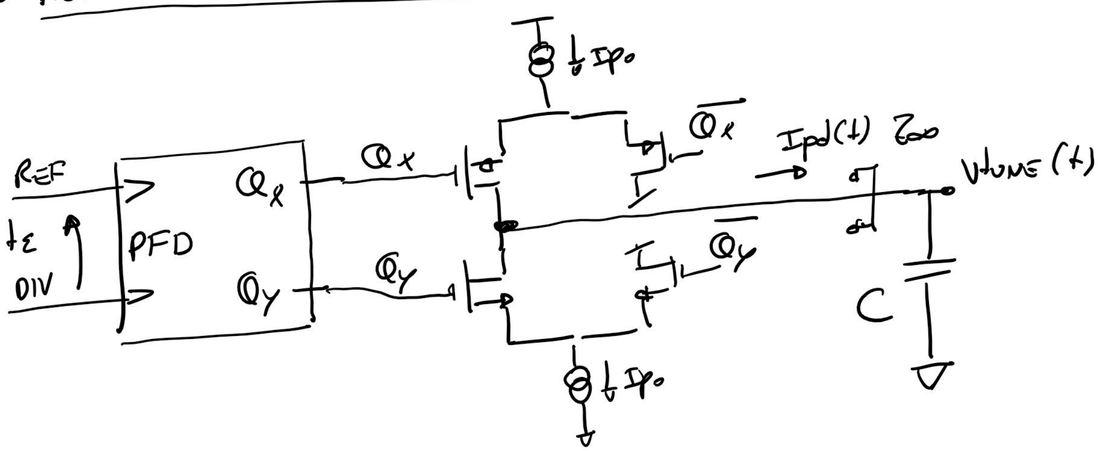
- FOR $F_{DIV} > F_{REF}$ the PFD output is NEGATIVE

  ↓

  THE LOOP DECREASE THE VCO FREQUENCY UNTIL $F_{DIV} = F_{REF}$

- PFD SENSE ALSO THE FREQUENCY DIFFERENCE BETWEEN FREF AND FDIV

- HOW TO IMPLEMENT THE SUM IN PFD : CHARGE-PUMP PLLs



- SUM OF PFD OUT WITH CURRENT
- HIGH OUTPUT IMPEDANCE OF CP

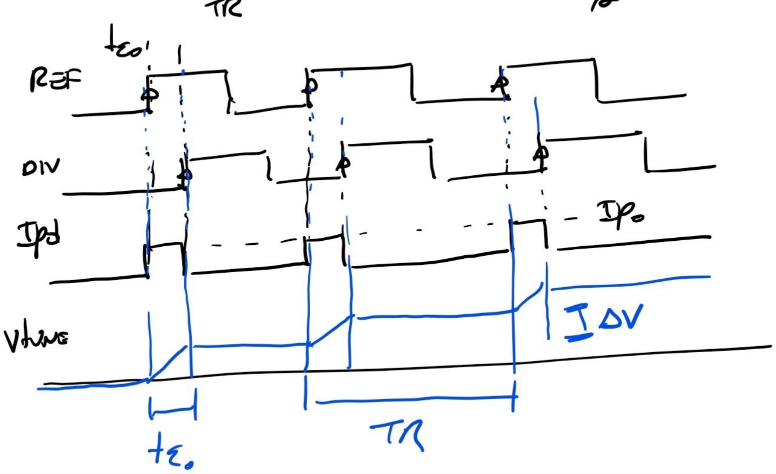
  ↓

  CONNECT C TO PERFORM INTEGRATION WITHOUT THE NEED OF OPAMP

- DERIVE THE EQ. MODEL
  - CONSIDER to apply a t_{ε_0} STEP @ $t=0$

$$\varphi_{\varepsilon_0} = \frac{2\pi}{T_R} t_{\varepsilon_0} \qquad \varphi_\varepsilon(s) = \frac{\varphi_{\varepsilon_0}}{s}$$

$$\Delta V = \frac{t_{\varepsilon_0} \cdot I_{P_0}}{C} \qquad C = \frac{dV}{dQ} \qquad i = \frac{dQ}{dt}$$



- GARDNER'S LIMIT FOR STABILITY:

$$BW_{PLL} < \frac{F_{REF}}{20}$$

$$V_{tune}(t) \simeq \frac{\Delta V}{T_R} \cdot t = \frac{t_{\varepsilon_0} I_{P_0}}{C \, T_R} \cdot t$$

$V_{TUNE}(t)$ CAN BE APPROXIMATED AS A RAMP DUE TO BW LIMITATION

$$V_{tune}(t) = \frac{t_{\varepsilon_0} I_{p0}}{C\, T_R}\, t = \frac{I_{p0}}{C} \frac{\varphi_{\varepsilon_0}}{2\pi}\, t \qquad \left(\varphi_{\varepsilon_0} = \frac{2\pi}{T_R} t_{\varepsilon_0}\right)$$

LAPLACE TRANSFORM:

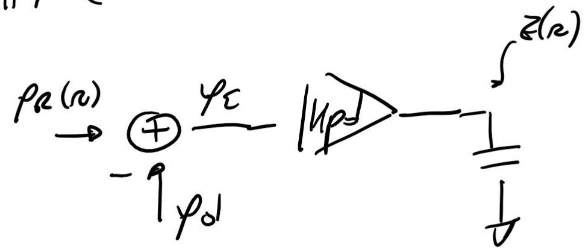
$$V_{tune}(s) = \frac{I_{p0}}{C} \frac{\varphi_{\varepsilon_0}}{2\pi} \frac{1}{s^2} = \underbrace{\left(\frac{I_{p0}}{2\pi}\right)}_{K_{pd}} \underbrace{\left(\frac{1}{sC}\right)}_{Z(s)} \underbrace{\left(\frac{\varphi_{\varepsilon_0}}{s}\right)}_{\text{input step}}$$

$Z(s)$ connected to the CP

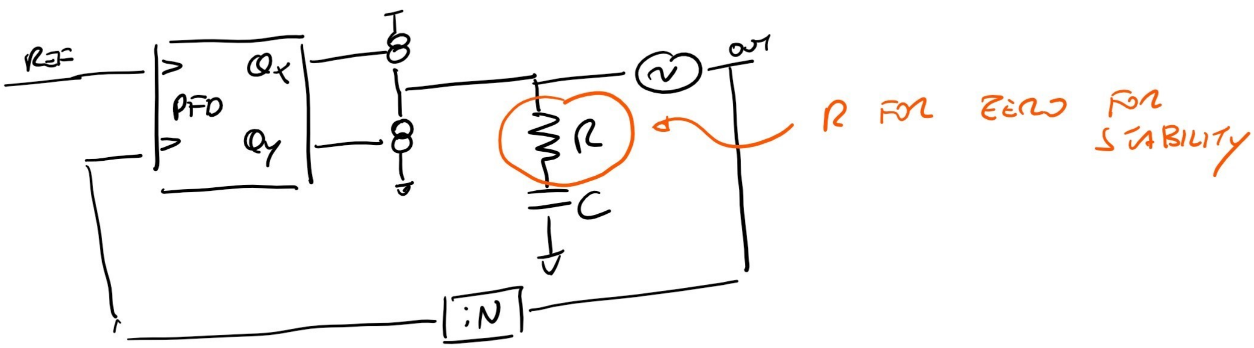
input step $\varphi_\varepsilon(s) = \frac{\varphi_{\varepsilon_0}}{s}$

THE TRANSFER FUNCTION:

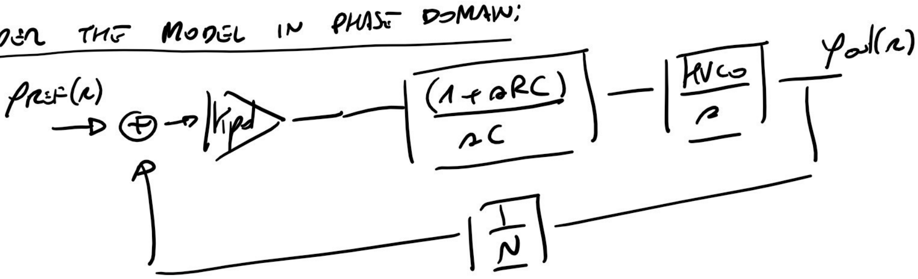
$$\frac{V_{tune}(s)}{\varphi_\varepsilon(s)} = \left(\frac{I_{p0}}{2\pi}\right)\left(\frac{1}{sC}\right) = K_{pd}\, Z(s)$$



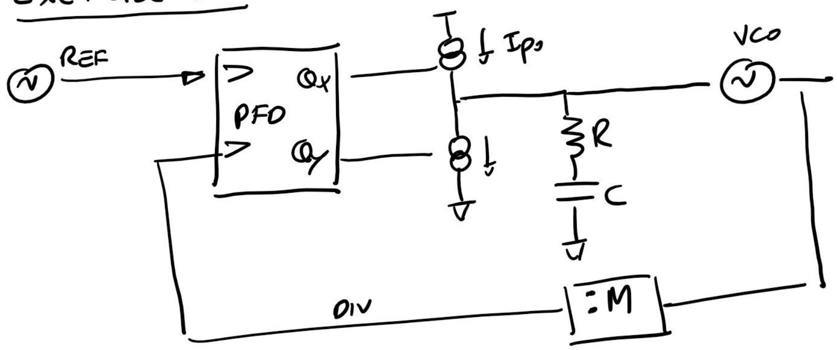
- COMPLETE SYSTEM: CP-PLL WITH PFD



- CONSIDER THE MODEL IN PHASE DOMAIN:



• EXERCISE 3.1



$F_{FR} = 3\,GHz$

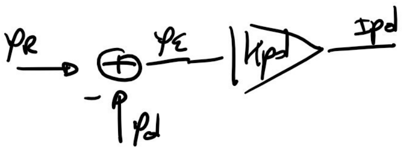
$K_{VCO} = 2\pi\ 300\,MHz/V$

$I_{P0} = 100\,\mu A$

$M = 100$

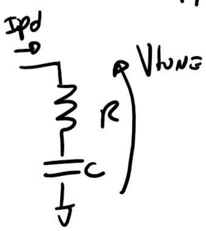
a) DERIVE THE EQUIVALENT LINEAR MODEL IN PHASE DOMAIN:

PFD + CP:



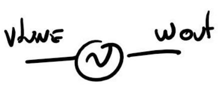
$$K_{pd} = \frac{I_{pd}}{\varphi_E} = \frac{I_{P0}}{2\pi}$$

FILTER:



$$\frac{V_{tune}(s)}{I_{pd}(s)} = Z(s) = \frac{(1 + sRC)}{sC}$$

VCO:



$$w_{out} = w_{FR} + K_{VCO} V_{tune}$$

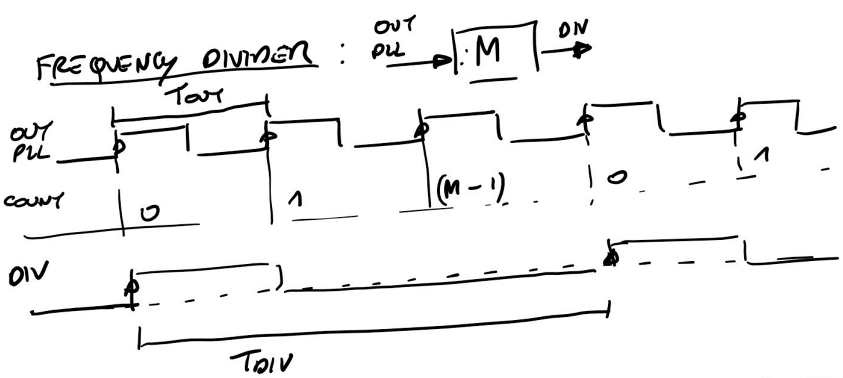
EXCESS PHASE

$$\phi_{out}(t) = \int_{-\infty}^{t} w_{out}(t)\,dt = w_{FR} t + \underbrace{K_{VCO}\int_{-\infty}^{t} V_{tune}(t)\,dt}$$

LAPLACE TRANSFORM OF EXCESS PHASE:

$$\varphi_{out}(s) = \frac{K_{VCO}}{s} V_{tune}(s) \qquad \frac{\varphi_{out}(s)}{V_{tune}(s)} = \frac{K_{VCO}}{s}$$

FREQUENCY DIVIDER:



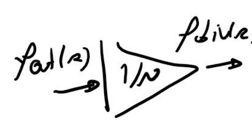
$$T_{OUT} = \frac{T_{DIV}}{N}$$

$$F_{OUT} = N F_{DIV}$$

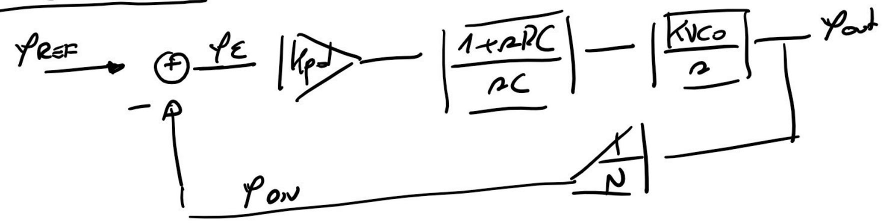
$$\varphi(s) = \frac{1}{s} w(s)$$

$$\frac{\varphi_{DIV}(s)}{\varphi_{OUT}(s)} = \frac{\cancel{\phi} F_{DIV}}{\cancel{\phi} F_{OUT}} = \frac{1}{N}$$

MODEL OF DIVIDER: LINEAR GAIN



• COMPLETE MODEL:



SIZE R, C to have CLOSED LOOP POLES: • $\omega_0 = 10kHz \cdot 2\pi$

• @ $45°$ IN GAUSS-PLANE $\rightarrow$ $\xi = \frac{\sqrt{2}}{2} = \frac{1}{2Q}$

$$G_{loop}(s) = K_{pd} \frac{(1+sRC)}{sC} \frac{K_{VCO}}{s} \frac{1}{N} = K \frac{(1+s\tau_z)}{s^2}$$

$\tau_z = RC$

$K = \frac{K_{pd} K_{VCO}}{C N}$

TO FIND CLOSED-LOOP POLES:

$1 + G_{loop}(s) = 0$

$1 + K \frac{(1+s\tau_z)}{s^2} = 0$ $\qquad s^2 + sK\tau_z + K = 0$

COMPARE with the FORM $\frac{s^2}{\omega_N^2} + \frac{s}{\omega_N Q} + 1 = 0$

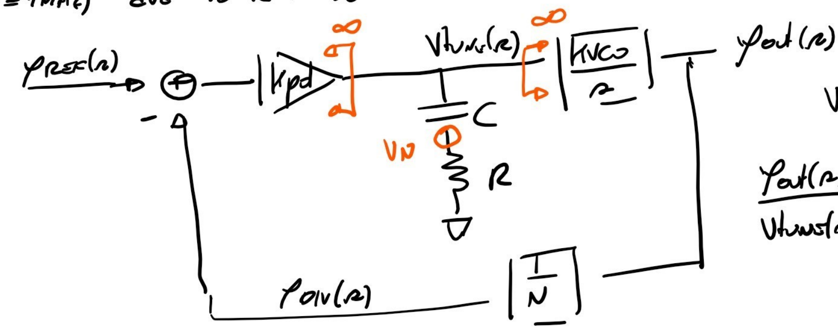
$\frac{s^2}{K} + s\tau_z + 1 = 0$

$K = \frac{K_{pd} K_{VCO}}{C N} = (2\pi \, 10kHz)^2$

$\rightarrow C = 76\,nF$

$\tau_z = RC = \frac{2\xi}{\omega_N} \rightarrow R = \frac{2\xi}{\omega_N C} = 296\,\Omega$

b) $\mathcal{L}(\Delta f = 1MHz)$ due to R NOISE

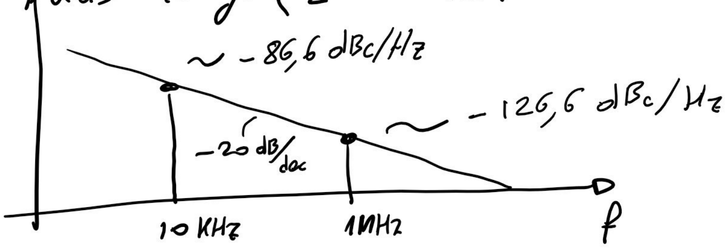


$$V_{TUNE}(s) \simeq V_n(s)$$

$$\frac{\varphi_{out}(s)}{V_{tune}(s)} = \left(\frac{K_{VCO}}{s}\right) \frac{1}{1 + G_{loop}(s)}$$

$$\mathcal{L}(\Delta f) = \frac{1}{2} S_{\varphi}^{SSB}(f) = \underbrace{\frac{1}{2} (4KTR) \left|\frac{K_{VCO}}{2\pi f}\right|^2}_{} \underbrace{\left|\frac{1}{1 + G_{loop}(j2\pi f)}\right|^2}_{}$$

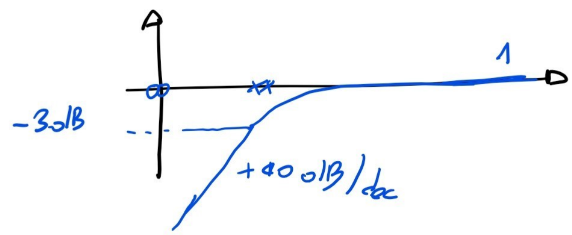
$$\mathcal{L}_{dB} = 10 \log_{10}\left(\frac{1}{2} 4KTR \left|\frac{K_{VCO}}{2\pi f}\right|^2\right)$$



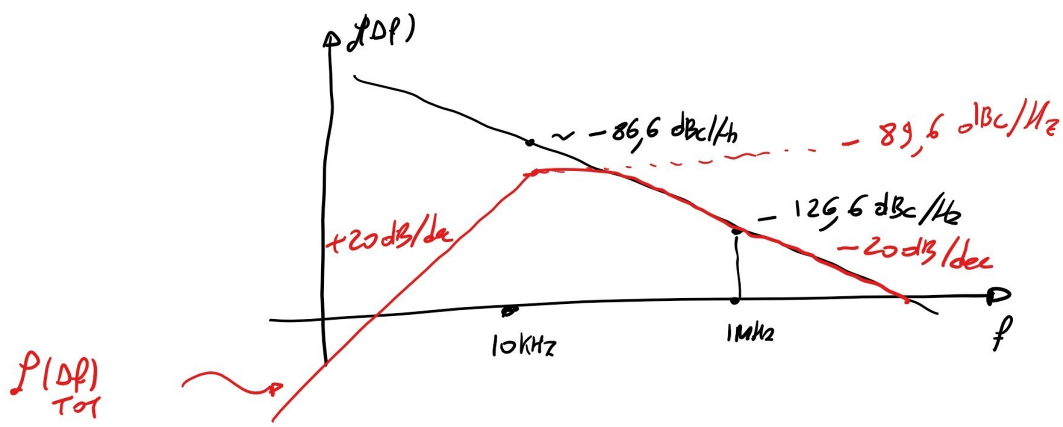
$$\frac{1}{1 + G_{loop}(s)} = \frac{s^2}{s^2 + sK\tau_z + K} = \frac{s^2}{s^2 + s\omega_N 2\xi + \omega_N^2}$$

$$\left|\frac{1}{1 + G_{loop}(j2\pi f)}\right|$$

$$\left|\frac{1}{1 + G_{loop}(j2\pi\omega_N)}\right| = \frac{1}{2\xi} = \frac{\sqrt{2}}{2} = -3dB$$



- $\mathcal{L}(\Delta f) = \frac{1}{2} 4KTR \left|\frac{K_{VCO}}{2\pi f}\right|^2 \left|\frac{1}{1 + G_{loop}(j2\pi f)}\right|^2$



$\mathcal{L}(\Delta f = 1MHz) = -126$ dBc/Hz due to R.

c) DERIVE $\mathcal{L}_{out}$ DUE TO R AND TO REFERENCE NOISE

$\mathcal{L}_{REF}$ = WHITE = $-140\ dBc/Hz$

$$\frac{\varphi_{out}(s)}{\varphi_{REF}(s)} = \frac{N\, G_{loop}(s)}{1+G_{loop}(s)} = \frac{N\,(1+s\tau_z)}{\frac{s^2}{K}+s\tau_z+1}$$

$$= \frac{N\left(1+\frac{s}{\omega_N Q}\right)}{\frac{s^2}{\omega_N^2}+\frac{s}{\omega_N Q}+1}$$

REMEMBER:

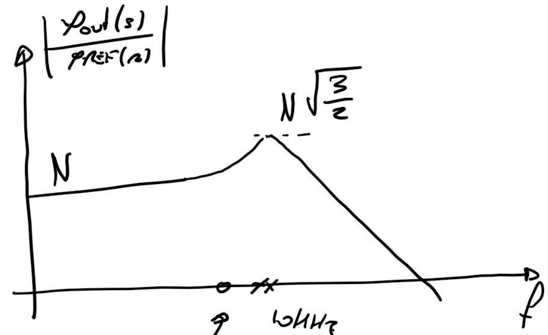
$$\frac{s^2}{\omega_N^2}+\frac{s}{\omega_N Q}+1=0$$

$$\tau_z = \frac{1}{2Q} \qquad K=\omega_N^2$$

$$\left|\frac{\varphi_{out}(j\omega_N)}{\varphi_{REF}(j\omega_N)}\right| = \left|N\,\frac{\left(1+\frac{j}{Q}\right)}{-1+\frac{j}{Q}+1}\right| = N\sqrt{1+Q^2}$$

$$Q=\frac{1}{2\zeta}=\frac{\sqrt{2}}{2}$$

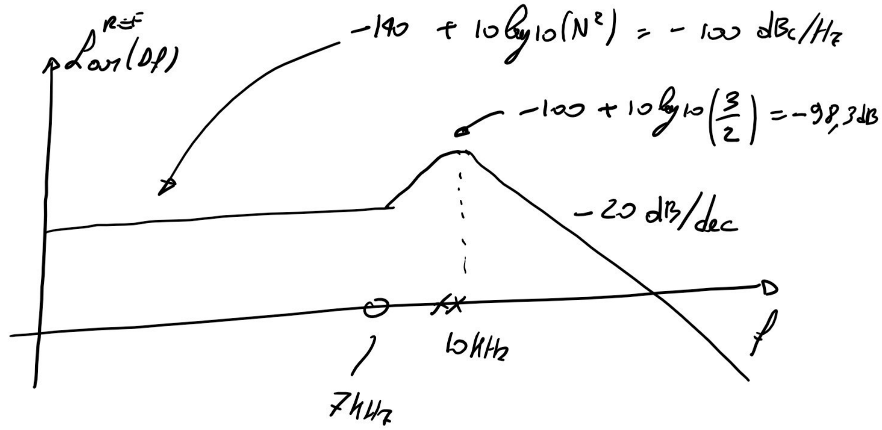
$$= N\sqrt{1+\left(\frac{\sqrt{2}}{2}\right)^2} = N\sqrt{\frac{3}{2}}$$



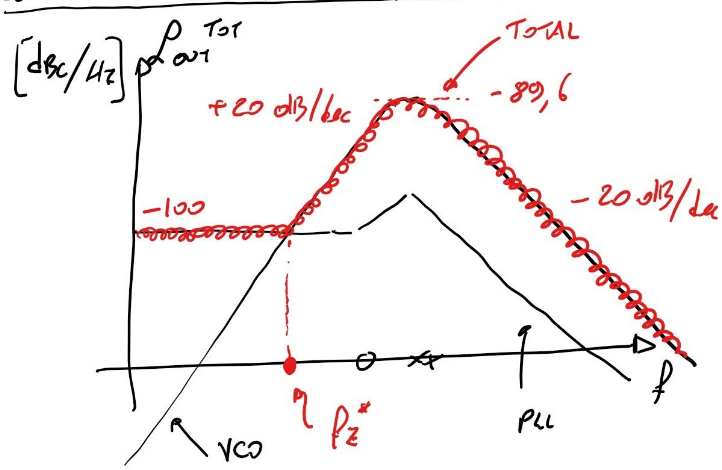
$$f_z = \frac{1}{2\pi RC} = 7\,kHz$$

- REFERENCE OUTPUT SPECTRUM:

$$\mathcal{L}_{out}^{REF} = \mathcal{L}_{REF}\cdot\left|\frac{\varphi_{out}(f)}{\varphi_{REF}(f)}\right|^2$$



- COMPOSITION OF R NOISE WITH REF NOISE:



f_z^* = ZERO OF THE COMPOSITION

$$-100+20\log_{10}\left(\frac{10kHz}{f_z^*}\right) = -89{,}6$$

$$f_z^* = \frac{10kHz}{10^{\frac{(-89{,}6+100)}{20}}} = 3kHz$$