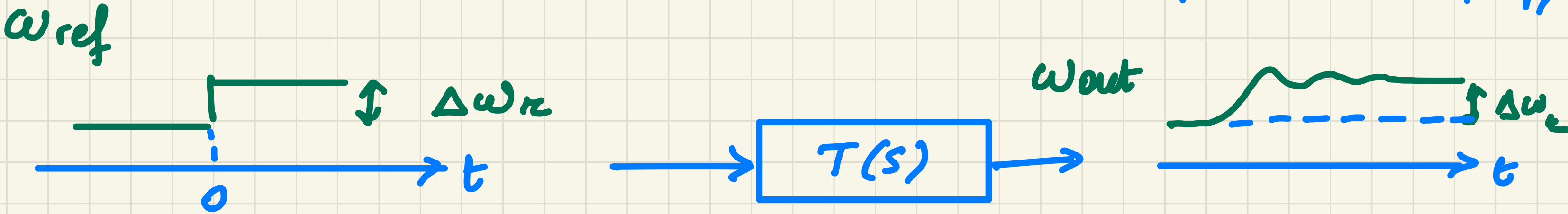
RF Circuit Design

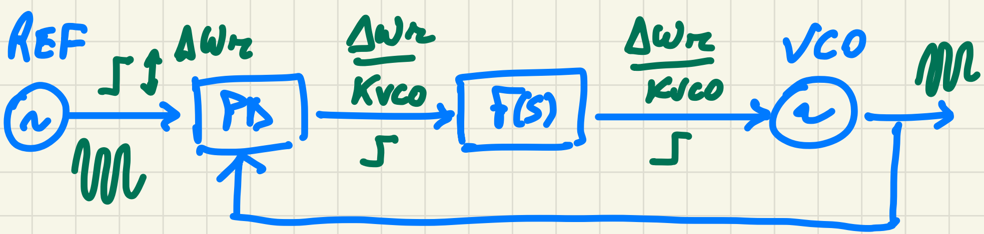
L8

Static Phase Error

(residual error at steady state between ϕ_{out} and ϕ_{ref})

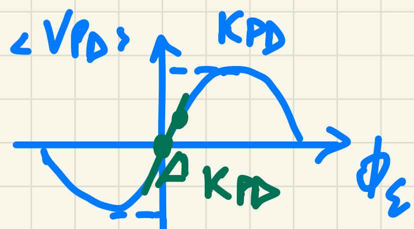


- What is the value of ϕ_ε at steady state?



$\Delta\omega_r$ (variations at steady state)

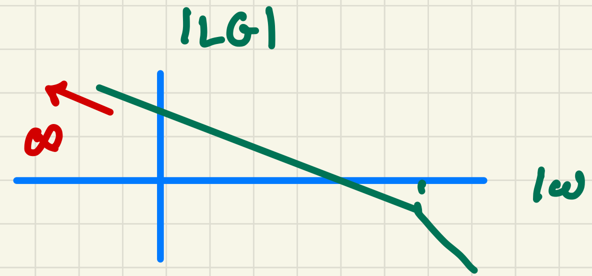
$$F(f=0) = 1$$



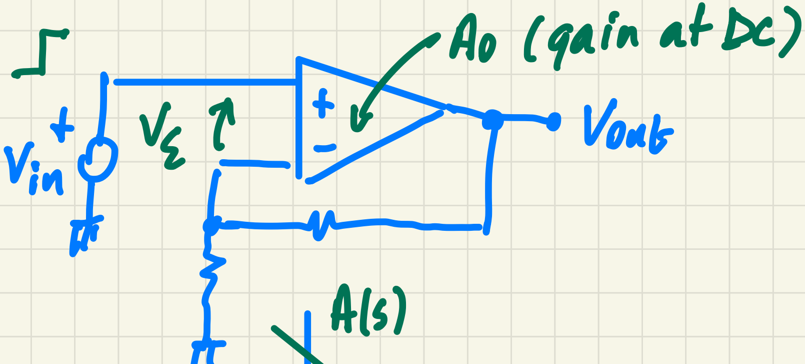
$$\phi_\varepsilon = \frac{\Delta\omega_r / K_{VCO}}{K_{PD}} = \frac{\Delta\omega_r}{K}$$

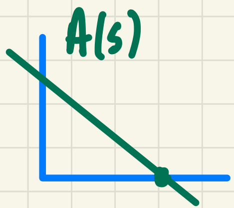
- Why the static ϕ_ε is not null, although $|LG| \to \infty$ at DC?

$$LG = \frac{1}{s} \cdot \frac{1}{1+s\tau}$$

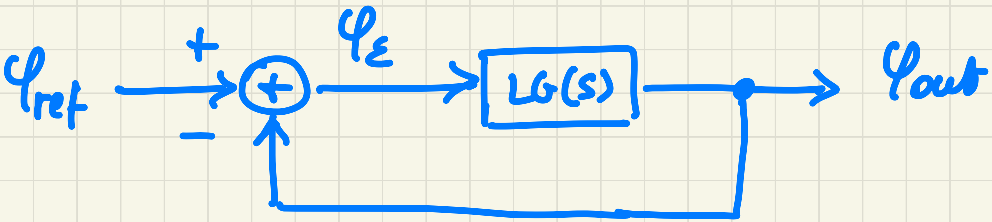


Voltage amplifier analogy





$$V_\varepsilon = \frac{\overbrace{V_{out}}^{\text{finite}}}{\underbrace{A_0}_{\infty}} \to 0$$



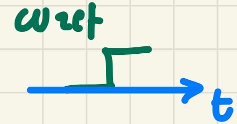
$$\underbrace{\lim_{t\to\infty} \varphi_\varepsilon(t)}_{\text{Static phase error}} = \lim_{s\to 0} s\cdot \Phi_\varepsilon(s)$$

$$\frac{\Phi_\varepsilon(s)}{\Phi_{ref}(s)} = \frac{1}{1+LG(s)} = 1 - T(s)$$

$$LG(s) = \frac{K}{s}\cdot\frac{1}{1+s\tau} \Rightarrow \quad = \frac{s(1+s\tau)}{s(1+s\tau)+K}$$

- case of FREQUENCY step

$$\omega_{ref}(t) = \Delta\omega_r \cdot u(t)$$



$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

$$\Rightarrow \quad \Omega_{ref}(s) = \frac{\Delta\omega_r}{s}$$

$$\Rightarrow \quad \Phi_{ref}(s) = \frac{\Omega_{ref}(s)}{s} = \frac{\Delta\omega_r}{s^2}$$

$$\Phi_\varepsilon(s) = \frac{\Delta\omega_r}{s^2} \cdot \frac{s(1+s\tau)}{s(1+s\tau)+K}$$

$$\lim_{s\to 0} s\cdot\Phi_\varepsilon(s) = \lim_{s\to 0} \cancel{s}\cdot \frac{\Delta\omega_r}{\cancel{s^2}} \cdot \frac{\cancel{s}(1+\overbrace{s\tau}^{0})}{\underbrace{s(1+s\tau)}_{\to 0}+K} = \frac{\Delta\omega_r}{K}$$

$$\lim_{t\to\infty} \varphi_\varepsilon(t) = \frac{\Delta\omega_r}{K}$$ static phase error is not NULL

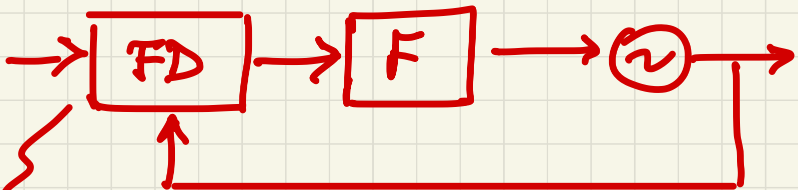
• case of phase step $\varphi_{ref} = \frac{\Delta\Phi}{s}$

$$\lim_{t\to\infty} \varphi_\varepsilon(t) = \lim_{s\to 0} s\,\Phi_\varepsilon(s) = \lim_{s\to 0} s \cdot \frac{\Delta\Phi}{\cancel{s}} \cdot \frac{\cancel{s}(1+s\tau)}{s(1+s\tau)+K} = 0$$

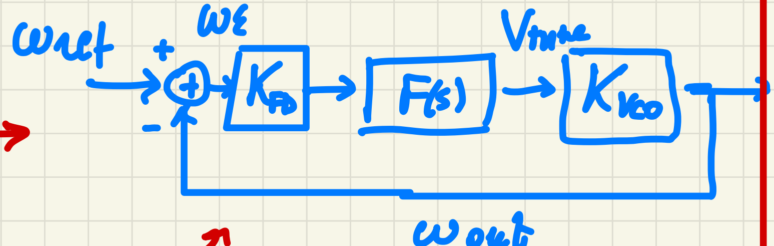
static phase error is NULL

How can we build a PLL with zero static φ_ε after an input frequency step?

? FLL (freq. locked loop)



frequency detector



FLL model

no integrator!

→ no good

Ingeneral : LG has n integrators and Φ_{ref} is of m-th order

$$\lim_{t\to\infty} \varphi_\varepsilon(t) = \lim_{s\to 0} s \underbrace{\frac{\Delta}{s^m}}_{\Phi_{ref}} \cdot \underbrace{\frac{s^n H(s)}{s^n \cdot H(s) + K}}_{\frac{1}{1+LG}} =$$

$$LG(s) = \frac{K}{s^n} \cdot \frac{1}{H(s)}$$

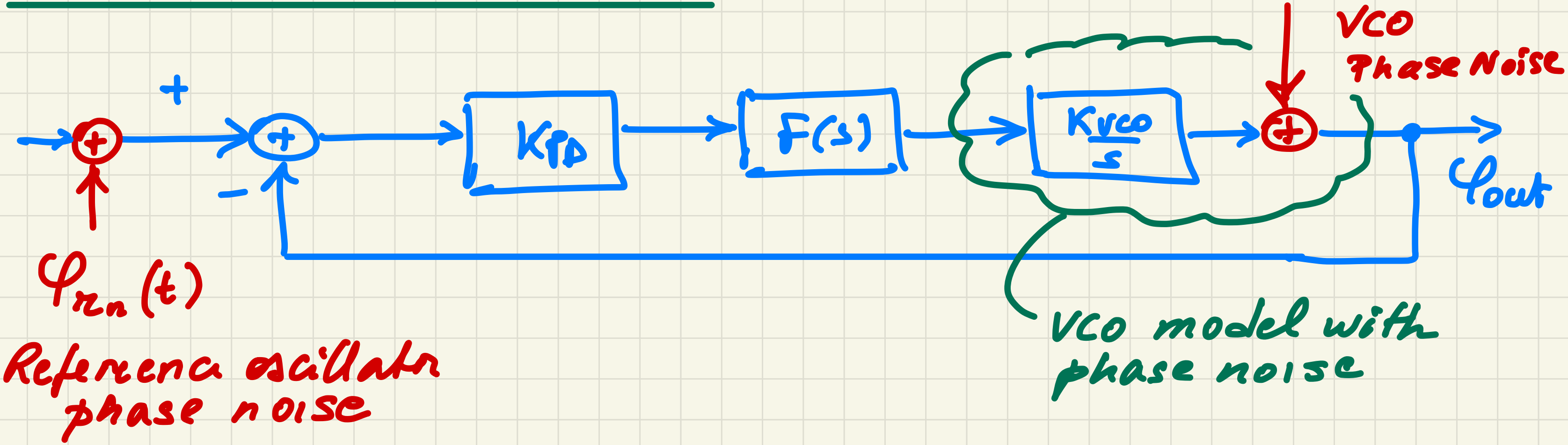
$H(s) \not\to 0 \ (s \to 0)$

$$= \lim_{s\to 0} \frac{\Delta}{K} \cdot s^{n-m+1} = \begin{cases} \frac{\Delta}{K} & n = m-1 \\ 0 & \underline{n > m-1} \\ & n \geq m \end{cases}$$

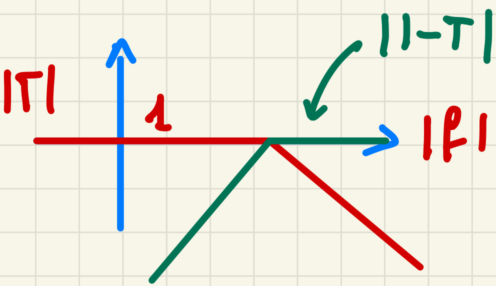
static (phase) error is zero IF the # of integrators in $LG(s)$ is at least equal to the order of the input perturbation

TYPE of a FEEDBACK SYSTEM is # of INTEGRATORS in LG

Phase Noise in PLLs



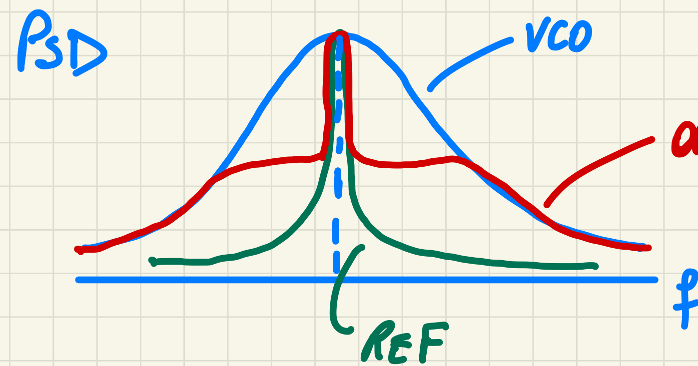
$$S_{\varphi_{out}}(f) = S_{\varphi,rn}\cdot\underbrace{|T(f)|^2}_{\text{low-pass}} + S_{\varphi,vn}\cdot\underbrace{|1-T(f)|^2}_{\text{high pass}}$$



$$\varphi_{vn} \rightarrow \varphi_{out} : \frac{1}{1+LG} = 1-T(s)$$

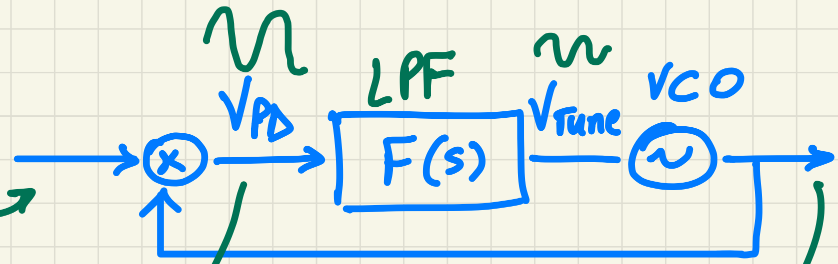
Intuitive meaning: $S_{\varphi_{out}}(f) = S_{\varphi,rn} \cdot \underbrace{|T(f)|^2}_{\text{low-pass}} + S_{\varphi,vn} \cdot \underbrace{|1-T(f)|^2}_{\text{high pass}}$

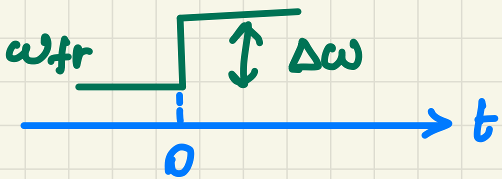
- within PLL BW : the VCO follows the phase noise of the reference clock
- out-of-PLL-BW : the vco follows its own phase noise



Capture Range

$$\cos\overbrace{(\underbrace{\omega_{fr}+\Delta\omega}_{\omega_{ref}})\,t}^{\phi_{ref}}$$





$$\underbrace{\sin\left(\omega_{fr}\,t + \underbrace{\int_{-\infty}^{t} K_{vco} V_{tune}(t')\,dt'}_{\text{Let's neglect this term}}\right)}_{\phi_{out}}$$

$$V_{PD} \simeq K_{PD} \cdot \sin\left[\Delta\omega t - 0.t.\right]$$

$$V_{tune} \simeq K_{PD} \cdot |F(\Delta\omega)| \cdot \underbrace{\sin\left[\Delta\omega \cdot t + \measuredangle F(\Delta\omega)\right]}_{|\cdot| \leq 1} \Rightarrow$$

$$|V_{tune}| \le K_{PD} \cdot |F(\Delta\omega)|$$

$$\Downarrow$$

$$\left|\frac{\Delta\omega}{K_{VCO}}\right| \le K_{PD} \cdot |F(\Delta\omega)|$$

$$\Downarrow$$

$$\Delta\omega_C = \underbrace{K_{VCO} \cdot K_{PD}}_{K_v} \cdot |F(\Delta\omega_C)|$$

CAPTURE or HOLD RANGE

→ capture range in general ≤ lock range