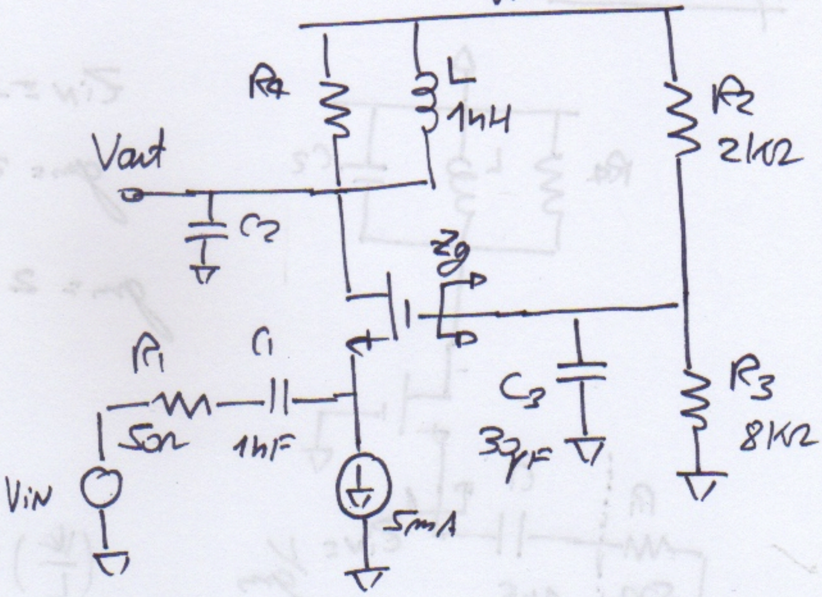
EXERCISE 7.1

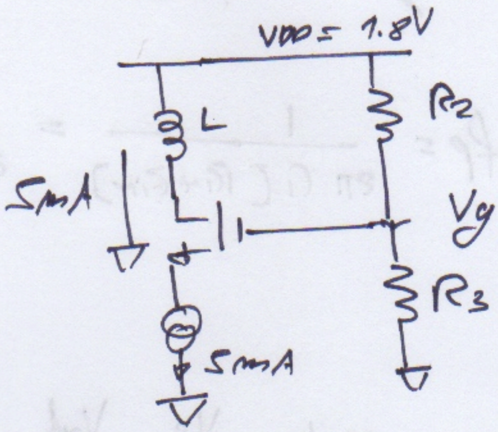
$V_T = 0.5V$

$\frac{1}{2}\mu C'_{ox} = 0.2 \frac{mA}{V^2}$

$\frac{\gamma}{\alpha} = \frac{2}{3}$



a) DERIVE THE BIAS point.



$V_g = V_{DD} \frac{R_3}{(R_2+R_3)} = 1.44V$

$f_0 = 3.3GHz$

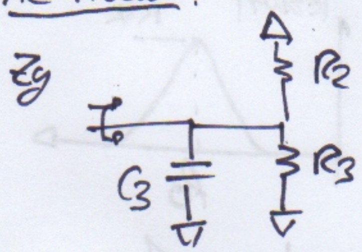
• Size R_a, C_2, $\left(\frac{W}{L}\right)$

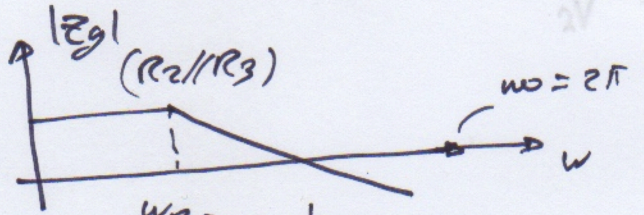
(i) input matching

(ii) MAX GAIN

(iii) NF = 2.7dB

• AC Model:



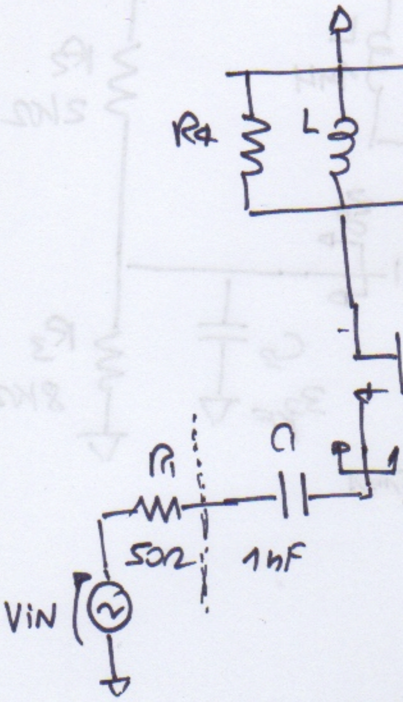


$w_p = \frac{1}{C_3[R_3//R_2]} = 2\pi\ 3.32MHz$

$Z_g \simeq \emptyset$ since $w_0 >> w_p$

(i) input

(i) <u>input MATCHING:</u>



$Z_{IN} = 50\Omega = R_1 = 1/g_m$

$g_m = 20\,mA/V$

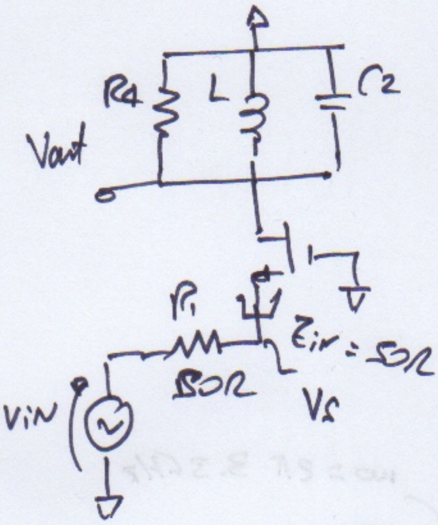
$g_m = 2\sqrt{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$ — $5mA$, $0.2\,\frac{mA}{V^2}$

$\left(\frac{W}{L}\right) = 100$

$f_P = \frac{1}{2\pi\, C_1 [R_1 + Z_{IN}]} = \frac{1}{2\pi}\,\frac{1}{2 C_1 R_1} = 1.59\,MHz$

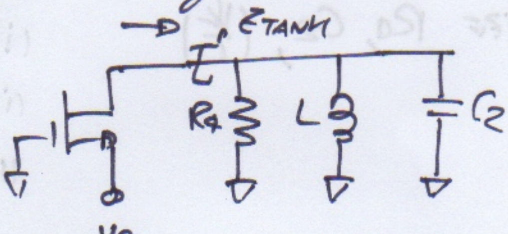
$f_P \ll 3.3GHz = f_0$

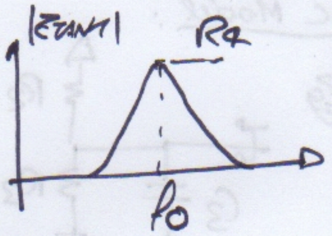
(ii) <u>MAX GAIN:</u>



$\frac{V_{out}}{V_{IN}} = \frac{V_S}{V_{IN}}\,\frac{V_{out}}{V_S}$

$= \left(\frac{1}{2}\right)$

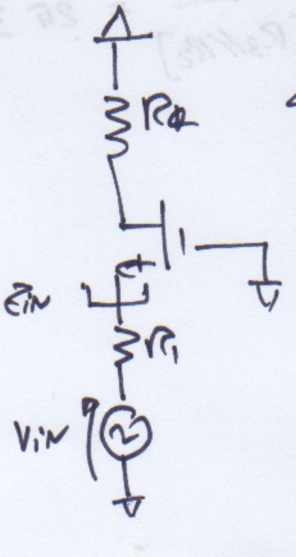




$f_0 = \frac{1}{2\pi}\,\frac{1}{\sqrt{L C_2}}$

$C_2 = 2.33pF$

@f_0 =

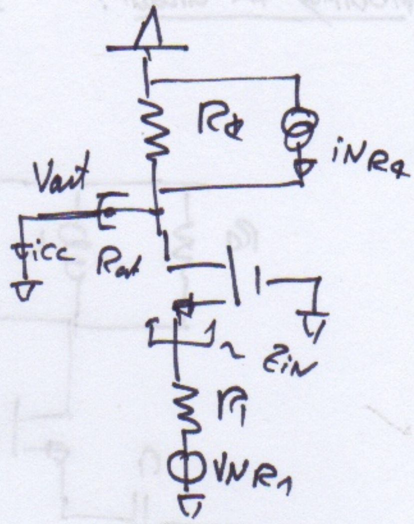


$\frac{V_{out}}{V_{IN}} = \frac{1}{2}\, g_m R_4 = \frac{1}{2}\,\frac{R_4}{R_1}$

(iii) NF = 2.7dB

$$NF = \frac{S_{V\,ALL} \text{ (at one Node)}}{S_V \text{ only input resistor (at same Node)}}$$

$$= 1 + \frac{S_V \text{ others without } R_{IN}}{S_V \text{ only input resistor}}$$

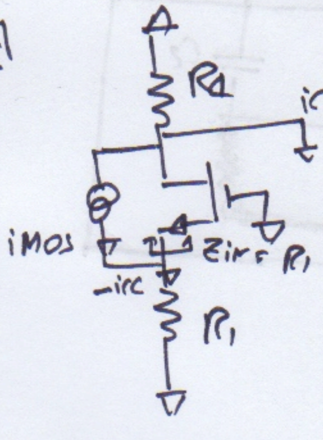


$$NF = 1 + \frac{S_{Vout\ other}}{S_{Vout}\ by\ R_1} = 1 + \frac{S_{icc\,out\ other}\, R_{out}^2}{S_{icc\ by\ R_1}\, R_{out}^2}$$

$\boxed{R_4}$: $S_{icc}^{R_4} = \frac{4KT}{R_4}$

$\boxed{R_1}$: $S_{icc}^{R_1} = 4KTR_1 \left|\frac{1}{(R_1 + Z_{IN})}\right|^2 = \frac{4KTR_1}{(2R_1)^2} = \frac{4KT}{R_1}\left(\frac{1}{2}\right)^2$

$\boxed{MOS}$:



$$S_{icc}^{MOS} = 4KT\frac{\gamma}{\alpha} g_m \left(\frac{1}{2}\right)^2$$

$$NF = 1 + \frac{S_{icc}^{R_4}}{S_{icc}^{R_1}} + \frac{S_{icc}^{MOS}}{S_{icc}^{R_1}} = 1 + \frac{\frac{\cancel{4KT}}{R_4}}{\frac{\cancel{4KT}}{R_1}\left(\frac{1}{2}\right)^2} + \frac{\cancel{4KT}\frac{\gamma}{\alpha} g_m \cancel{\left(\frac{1}{2}\right)^2}}{\frac{\cancel{4KT}}{R_1}\cancel{\left(\frac{1}{2}\right)^2}}$$

$$= 1 + \frac{4R_1}{R_4} + \frac{\gamma}{\alpha}\overbrace{R_1 g_m}^{=1} = 10^{\frac{2.7dB}{10}}$$

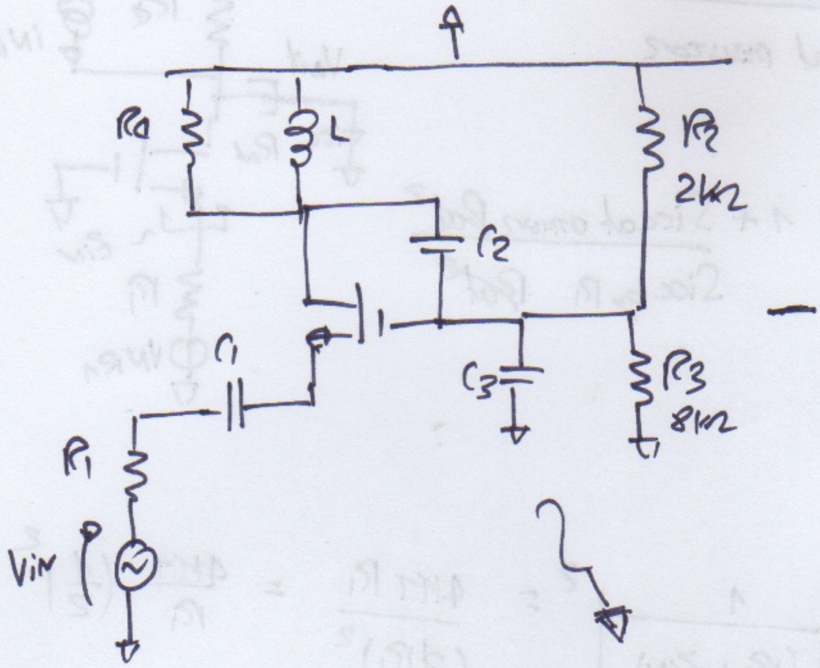
$$R_4 = \frac{4R_1}{\left[10^{\frac{2.7dB}{10}} - 1 - \frac{\gamma}{\alpha}\right]} = 1023{,}4\Omega$$

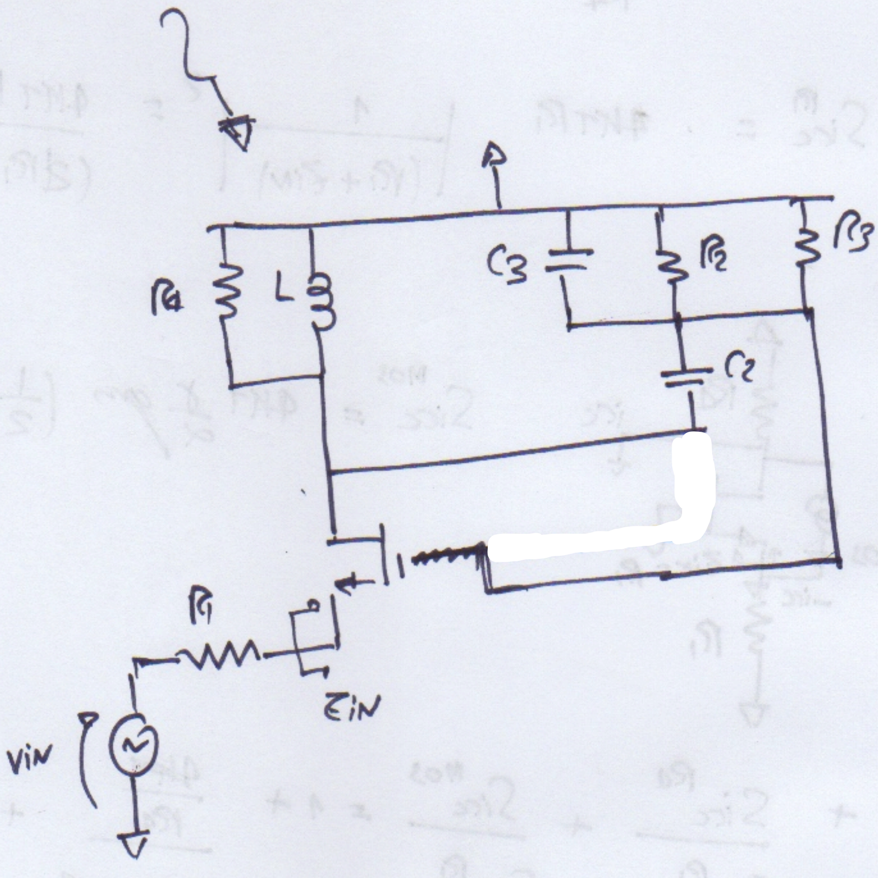
$$\frac{V_{out}}{V_{IN}} = \frac{R_4}{2R_1} = \frac{1023,4\Omega}{100\Omega} = 20.2\,dB$$

b) Modify the circuit: Size: C_2, C_3, $\left(\frac{W}{L}\right)$

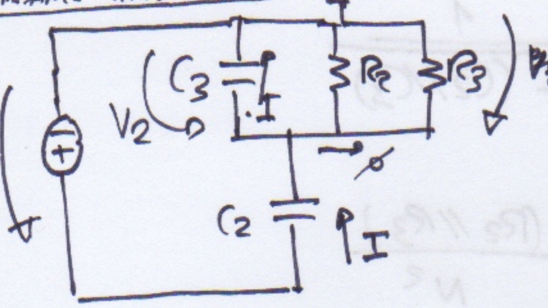
(i) = input matching

(ii) $NF = 1.2\,dB$





• IMPEDANCE TRANSFORMATION



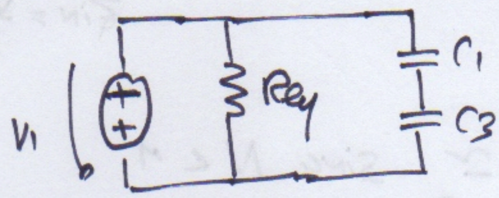
$$\frac{V_2}{V_1} \simeq \frac{1/sC_3}{\frac{1}{sC_2}+\frac{1}{sC_3}} = \frac{C_2}{C_3+C_2} = N$$

LOW LOSSES APPROXIMATION

$$\left|\frac{1}{\omega_0 C_3}\right| \ll (R_2 // R_3)$$

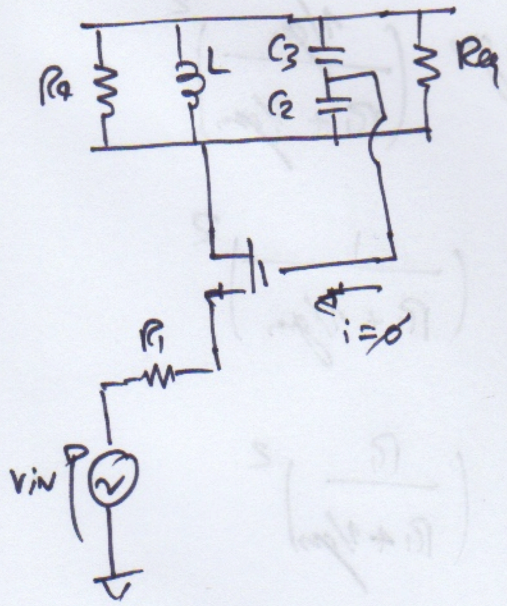
$$P_{\text{delivered by } V_1} = \frac{1}{2}\frac{V_2^2}{(R_2//R_3)}$$

$$P = \frac{1}{2}\frac{(V_1)^2}{R_{eq}} \Longrightarrow$$

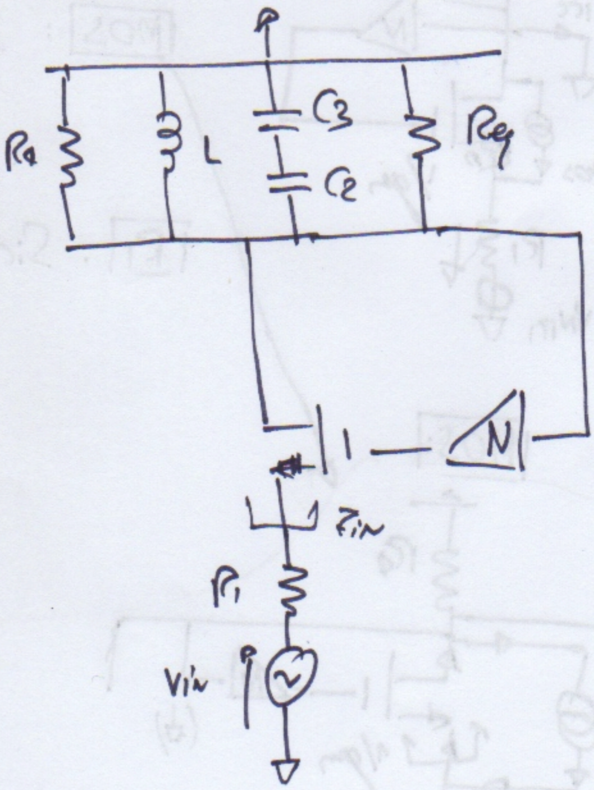


$$\frac{1}{2}\frac{V_1^2}{R_{eq}} = \frac{1}{2}\frac{V_2^2}{(R_2//R_3)} \Longrightarrow R_{eq} = \frac{(R_2//R_3)}{\left(\frac{V_2}{V_1}\right)^2} = \frac{(R_2//R_3)}{N^2}$$

• AC Model: @ f_0



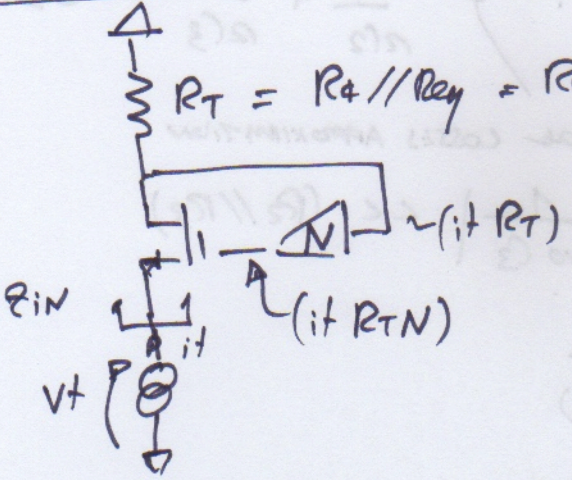
$\Longrightarrow$



- For MAX gain condition

$$f_0 = \frac{1}{2\pi} \frac{1}{\sqrt{L\,(C_2 // C_3)}}$$

(i) <u>input MATCHING</u>:



$$R_T = R_4 // R_{eq} = R_4 // \frac{(R_2 // R_3)}{N^2}$$

$$-i_t = g_m [i_t R_T N - V_t]$$

$$\frac{-i_t}{g_m} = i_t R_T N - V_t$$

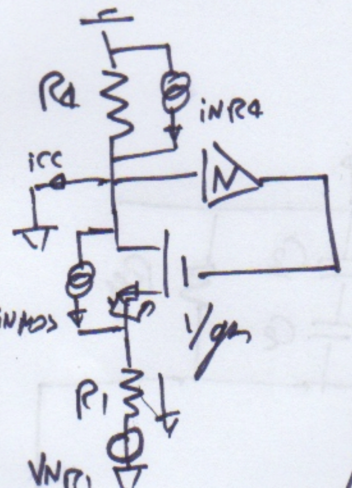
$$Z_{IN} = \frac{V_t}{i_t} = \frac{1}{g_m}\left[1 + g_m R_T N\right] = 50\Omega$$

- Hyp:

$$R_T = R_4 // \frac{(R_2 // R_3)}{N^2} \simeq \text{ since } N < 1$$

$$R_T = R_4$$

- NOISE FIGURE = 1.2 dB @ f_0.



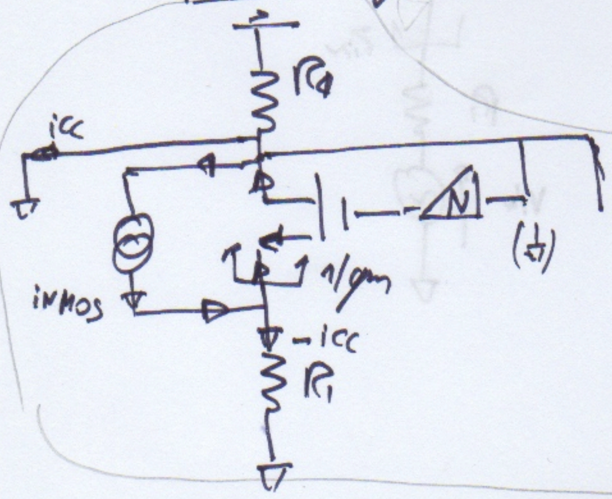
$\boxed{R_4}$ $\quad S_{icc}^{R4} = \frac{4KT}{R_4}$

$\boxed{MOS}$: $\quad S_{icc}^{MOS} = 4KT \frac{\gamma}{\alpha} g_m \left(\frac{1/g_m}{R_1 + 1/g_m} \right)^2$

$\boxed{R_1}$: $\quad S_{icc}^{R1} = 4KTR_1 \left(\frac{1}{R_1 + 1/g_m} \right)^2$

$$= \frac{4KT}{R_1} \left(\frac{R_1}{R_1 + 1/g_m} \right)^2$$

$\boxed{MOS:}$



$$i_{cc} = -i_{NMOS} \frac{1/g_m}{(1/g_m + R_1)}$$

$$S_{icc}^{MOS} = 4KT \frac{\gamma}{\alpha} g_m \left(\frac{1/g_m}{1/g_m + R_1} \right)^2$$

$$NF = 1 + \frac{\dfrac{4KT}{R_4}}{\dfrac{4KT}{R_1}\left(\dfrac{R_1}{(R_1 + 1/g_m)}\right)^2} + \frac{4KT\dfrac{\gamma}{\alpha}\,g_m\left(\dfrac{1/g_m}{1/g_m + R_1}\right)^2}{\dfrac{4KT}{R_1}\left(\dfrac{R_1}{R_1 + 1/g_m}\right)^2}$$

$$= \left[1 + \frac{(R_1 + 1/g_m)^2}{R_1 R_4} + \frac{\gamma}{\alpha}\frac{1}{g_m R_1} = 10^{\frac{1.2\,dB}{10}}\right]$$

$R_1 = 50\Omega$

$R_4 = 1023,4\,\Omega$

$\frac{\gamma}{\alpha} = \frac{2}{3}$

$\rightarrow g_m = 58\,\frac{mA}{V}$

$$g_m = 2\sqrt{\frac{1}{2}\underbrace{\mu_n C'_{ox}}_{0.2\frac{mA}{V^2}}\left(\frac{W}{L}\right)\underbrace{I_d}_{5mA}} \rightarrow \left(\frac{W}{L}\right) = 841$$

FROM $Z_{IN} = \frac{1}{g_m}\left[1 + g_m R_4 N\right] = 50\Omega$

$\rightarrow N = \frac{(g_m Z_{IN} - 1)}{g_m R_4} = 0.032$

FROM MAX GAIN $f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{L\,(C_2 // C_3)}} = 3.3\,GHz$

$N = \frac{C_2}{C_2 + C_3}$

$\rightarrow C_3 = C_2\frac{[1-N]}{N}$

$$(2\pi f_0)^2 = \frac{1}{L\,\frac{C_2 C_3}{C_2 + C_3}} = \frac{1}{L\,C_2\,(1-N)}$$

$\rightarrow C_2 = 2,4\,pF$

$$C_3 = C_2\frac{(1-N)}{N} = 72,63\,pF$$

- check the assumption:

1) $R_T = R_4 // \left(\frac{(R_2//R_3)}{N^2}\right) \simeq R_4 = 1023,4\,\Omega$

$\searrow 1,56\,M\Omega$

2) LOW-LOSSES

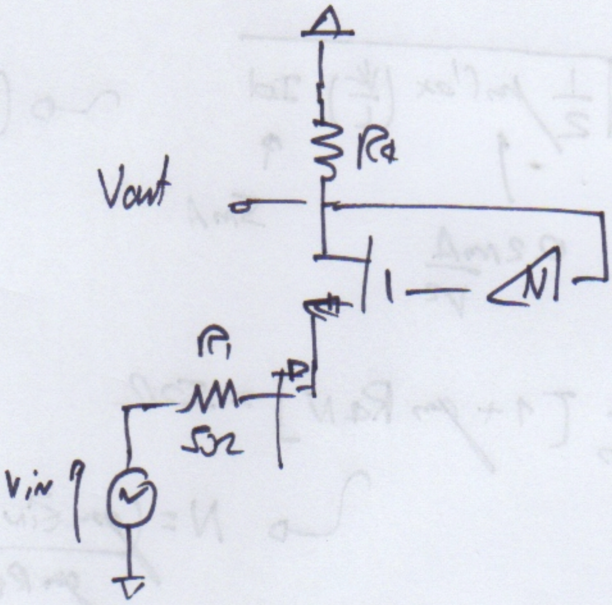
$\left|\frac{1}{\omega_0 C_3}\right| << (R_2//R_3)$

$\nearrow 1.6\,k\Omega$

$\searrow 0.66\,\Omega$

— - - .

c) TRANSDUCER POWER GAIN



$G_T = \frac{P_{out}}{P_{in,\,AV\ MATCHING}}$

$= \dfrac{\frac{1}{2}\frac{V_{out}^2}{R_4}}{\frac{1}{2}\frac{(V_{in}/2)^2}{Z_{in}}}$

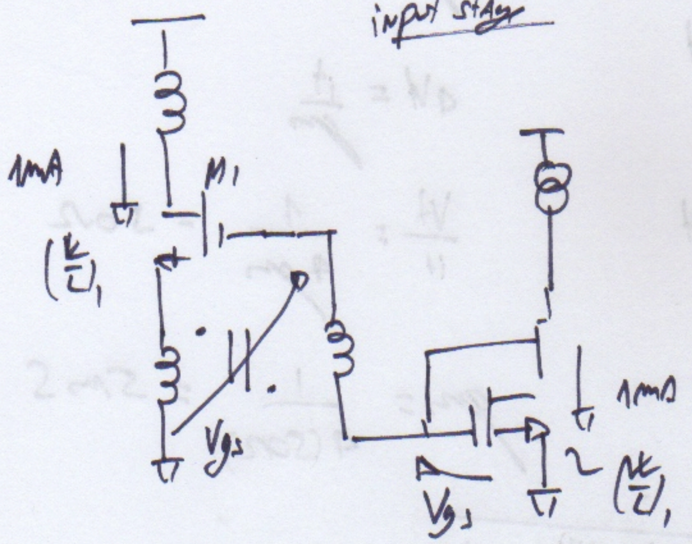
$= \left(\frac{V_{out}}{V_{in}}\right)^2 \frac{4R_1}{Z_{in}} = \left(\frac{R_4}{2R_1}\right)^2 \frac{4R_1}{R_1} = \frac{R_4}{R_1} = \frac{1023,4\,\Omega}{50\,\Omega} = 13.11\,dB$

$Z_{in} = R_1 = 50\,\Omega$

EXERCISE 7.2

- BIAS point:

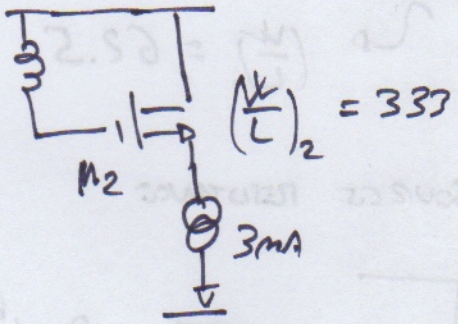
input stage



$$g_{m1} = 2\sqrt{\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_1 I_{d1}}$$

1mA

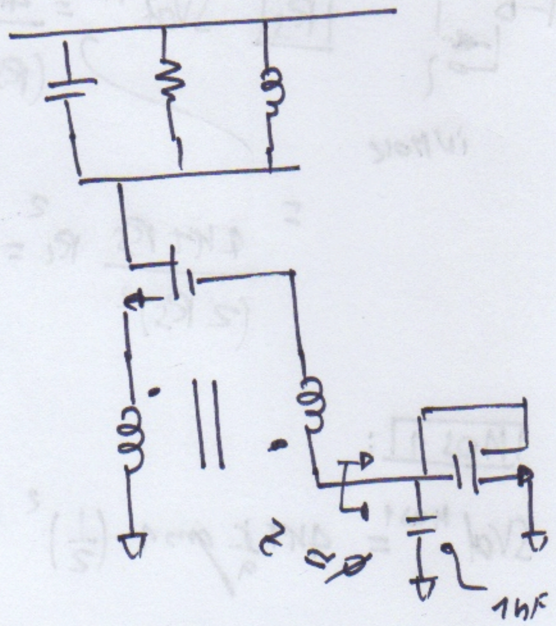
output stage:



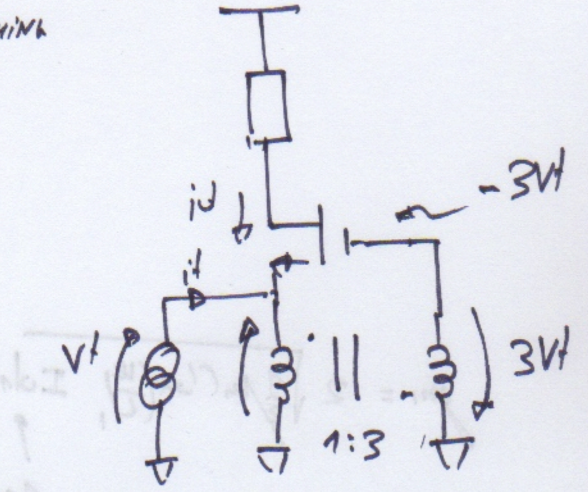
$$g_{m2} = 2\sqrt{\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_2 I_{d2}}$$

$= 20\,mA/V = \frac{1}{53\Omega}$

- input impedance @ fo.



- input matching



$$id = gm[-3Vt - Vt] = -it$$

$$4Vt = \frac{it}{gm}$$

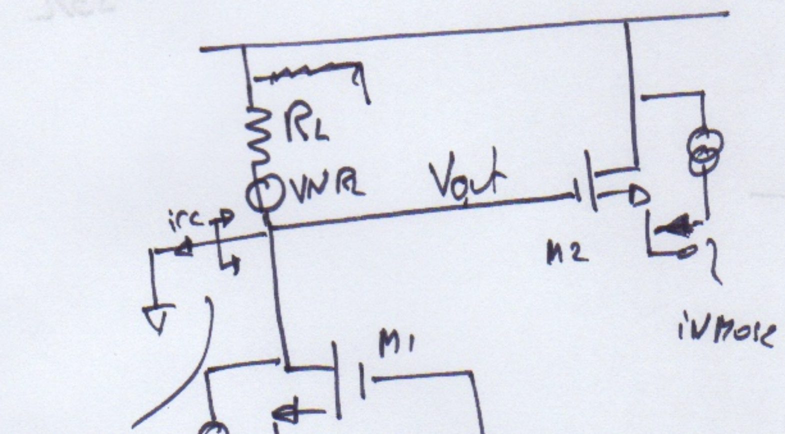
$$\frac{Vt}{it} = \frac{1}{4gm} = 50\Omega$$

$$gm = \frac{1}{4(50\Omega)} = 5\,mS$$

$$gm = 2\sqrt{\frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_1 Id_1}$$

$$\rightarrow \left(\frac{W}{L}\right)_1 = 62.5$$

b) EVALUATE NF FOR 50Ω SOURCE RESISTANCE



$\boxed{RL}$ $$Svout^{RL} = 4kT\,RL$$

$\boxed{Rs}$ $$Svout^{Rs} = \frac{4kTRs}{(Rs + Zin)^2} \cdot RL^2$$

$Zin = Rs$

$$= \frac{4kTRs}{(2Rs)^2} RL^2 = \frac{4kT}{Rs}\left(\frac{RL}{2}\right)^2$$

$\boxed{MOS\,1}$:

$$Svout^{MOS1} = 4kT\frac{\gamma}{\alpha} gm_1 \left(\frac{1}{2}\right)^2 RL^2$$

$\boxed{MOS2}$ $$Svout^{MOS2} = 4kT\frac{\gamma}{\alpha} gm_2 \left(\frac{1}{gm_2}\right)^2$$

$$NF = 1 + \frac{4kTR_L}{\frac{4kT}{R_S}\left(\frac{R_L}{2}\right)^2} + \frac{4kT\frac{\gamma}{\alpha}g_{m1}\frac{R_L^2}{4}}{\frac{4kT}{R_S}\frac{R_L^2}{4}} + \frac{4kT\frac{\gamma}{\alpha}\frac{1}{g_{m2}}}{\frac{4kT}{R_S}\frac{R_L^2}{4}}$$

$$= 1 + \frac{4R_S}{R_L} + \frac{\gamma}{\alpha}g_{m1}R_S + \frac{\gamma}{\alpha}\frac{4R_S}{g_{m2}R_L^2}$$

$$\simeq \;= 2.355\,dB$$

$R_L = 1k\Omega$

$R_S = 50\Omega$

$g_{m1} = 5mS$

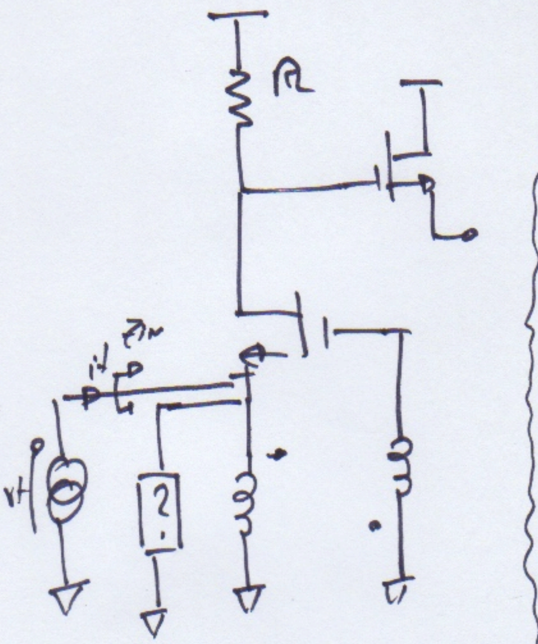
$\frac{\gamma}{\alpha} = 2$

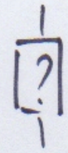
$g_{m2} = 20mS$

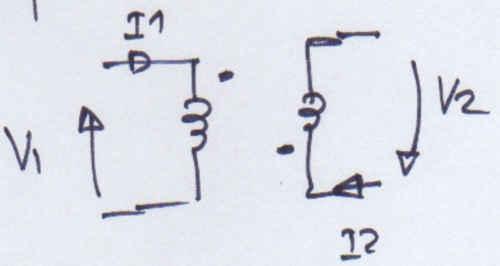
c) $L_{11} = 1nH$

$L_{22} = 9nH$

$K = 1$



Size  to have $Z_{in} = 50\Omega$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = s\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

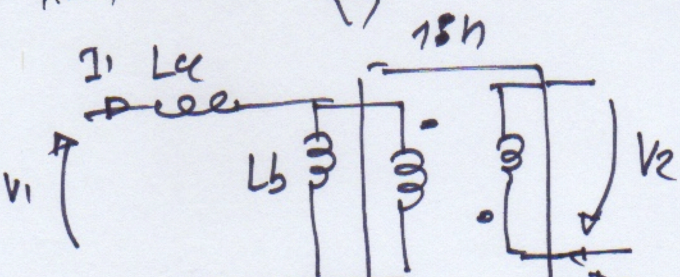
$$\begin{cases} V_1 = sL_{11}I_1 + sMI_2 \\ V_2 = sMI_1 + sL_2I_2 \end{cases} \qquad L_{12} = L_{21} = M$$

$L_1 = L_{11} = 1nH$

$L_2 = L_{22} = 9nH$

$K = 1$

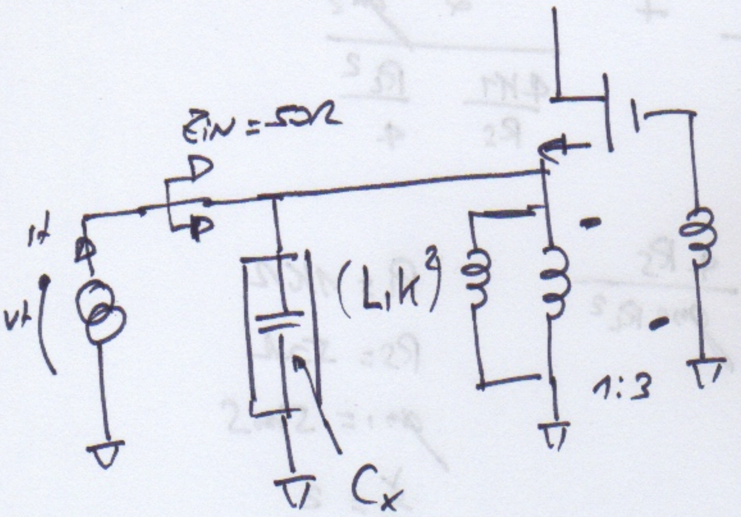
$\Downarrow$ Eq Model for CPL inductor.



$$n = \frac{1}{K}\sqrt{\frac{L_2}{L_1}} = \frac{1}{1}\sqrt{\frac{9nH}{1nH}} = 3$$

$$L_a = L_1(1-K^2) = \emptyset$$

$$L_b = L_1K^2 = L_1$$



$$f_0 = \frac{1}{2\pi} \frac{1}{\sqrt{C_x L_{11}}}$$

$$\rightarrow C_x = 4{,}053\,pF$$