RF Circuit Design

L3

Digital communications

Shannon's channel capacity

AWGN channel

$$C = B \cdot \log_2 (1 + SNR)$$

capacity (bitrate b/s)

Bandwidth (Hz)

$$SNR = \frac{P_s}{P_n}$$

signal power

noise power

Digital modulation

$$x_{BB}(t) = \sum_{n=-\infty}^{+\infty} b_n \cdot p(t - nT_b)$$

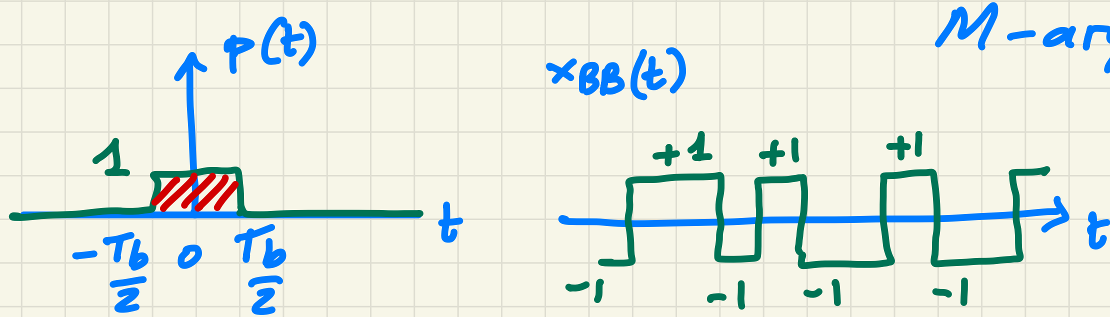
bit period

- $p(t)$: pulse shape
- $\frac{1}{T_b}$: bit rate

$b_n = \pm 1$: binary modulation

$b_n = \pm 1, \pm 2, \ldots, \pm M$: multilevel or M-ary modulation

e.g.



What is the BW of $x_{BB}(t)$?

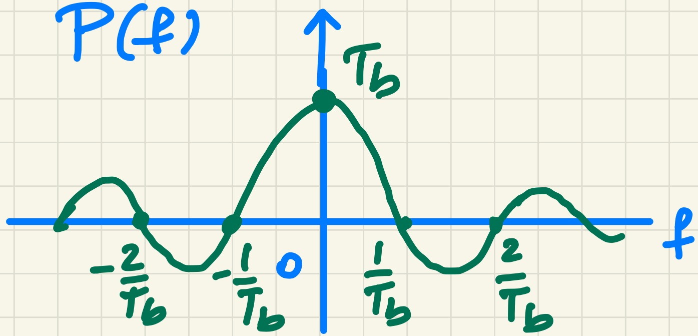
Theorem : $S_{x_{bb}}(f) = \dfrac{|P(f)|^2}{T_b}$

b_n is random

x_{bb} is a stochastic process

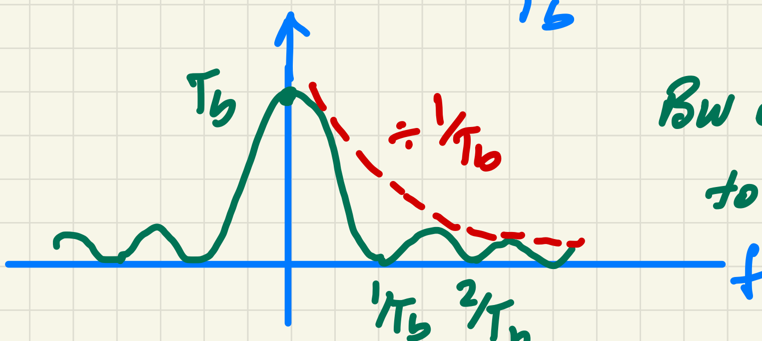
where $P(f) = \mathcal{F}\{p(t)\}$

e.g. $P(f)$



$$P(f) = T_b \cdot \frac{\sin(\pi f T_b)}{\pi f T_b} = T_b \cdot \text{sinc}(f T_b)$$

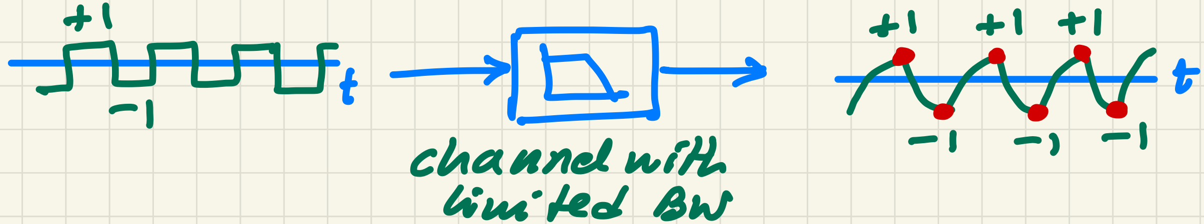
$$\Rightarrow S_{x_{bb}}(f) = \frac{|P(f)|^2}{T_b} = \frac{T_b^{\cancel{2}} \cdot \text{sinc}^2(f T_b)}{\cancel{T_b}}$$

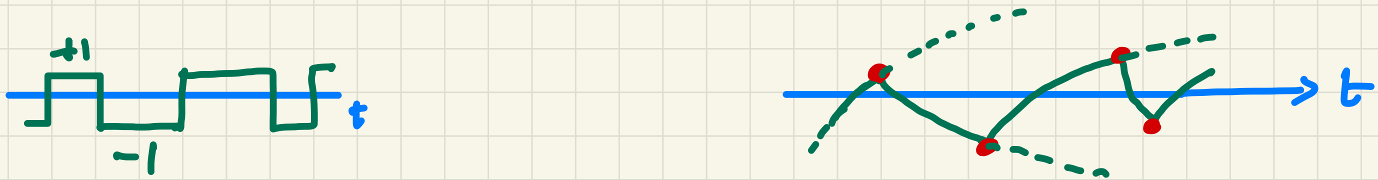


BW of $x_{BB}(t)$ is IDENTICAL to the BW of $p(t)$

Power of $x_{bb}(t) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \overbrace{x_{BB}^2(t)}^{=+1}\, dt = 1$

e.g.



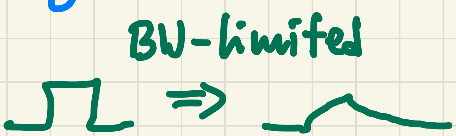


Inter-symbol Interference (ISI)

If a symbol lasts longer than T_b, then it will pile up with the following symbols.

It degrades SNR.

e.g.



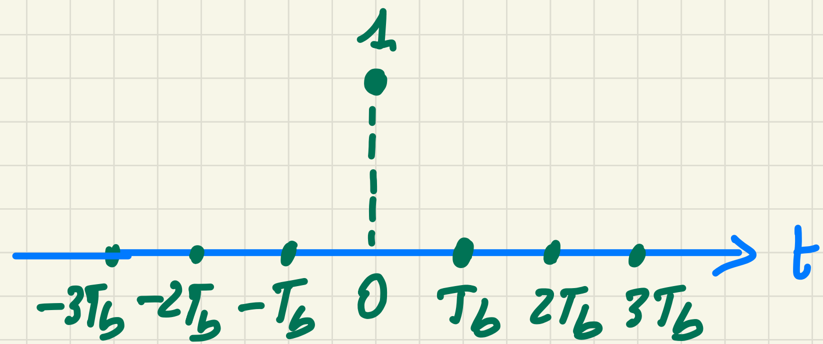
What is the solution to have pulses with limited BW and no ISI?

Solution: Nyquist signalling

$$x_{BB}(t) = \sum_{n=-\infty}^{\infty} b_n \cdot p(t-nT_b)$$

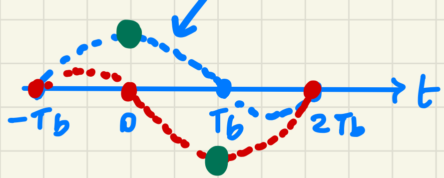
$p(t)$ such that:

$$p(kT_b) = \begin{cases} 1 & k=0 \\ 0 & k \neq 0 \end{cases}$$



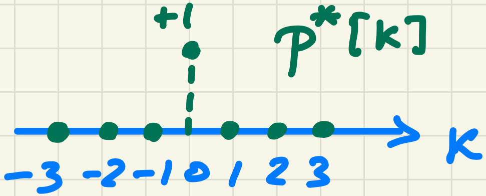
$$x_{BB}(t) = b_0\, p(t) + b_1\, p(t-T_b) + \dots$$

e.g. $b_0 = +1$, $b_1 = -1$



$\Rightarrow$ NO ISI

Spectrum of a Nyquist signal



$$\overset{\mathcal{F}}{\longleftrightarrow} \quad P^*(f) = 1$$

$$p(t) \longleftrightarrow P(f)$$

$$\Rightarrow \quad \sum p(t-kT_b) = p(t) \cdot \sum \delta(t-kT_b)$$

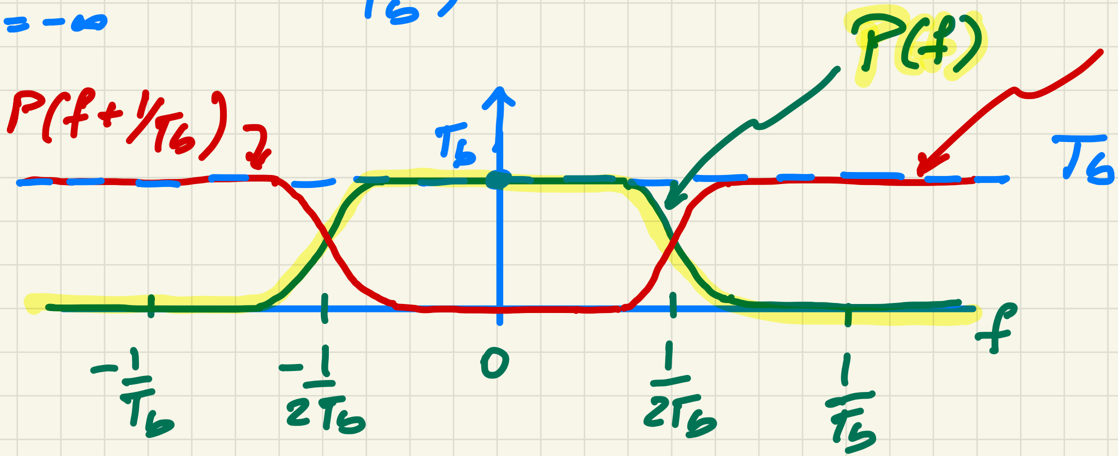
$$\updownarrow \mathcal{F}$$

$$P(f) * \frac{1}{T_b} \sum \delta\left(f - \frac{k}{T_b}\right) =$$

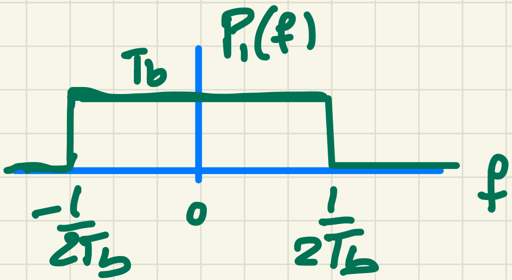
$$= \frac{1}{T_b} \cdot \sum P\left(f - k/T_b\right)$$

$$\boxed{\frac{1}{T_b} \sum P\left(f - \frac{k}{T_b}\right) = 1}$$

$$\sum_{k=-\infty}^{+\infty} P(f - k/T_b) = T_b$$

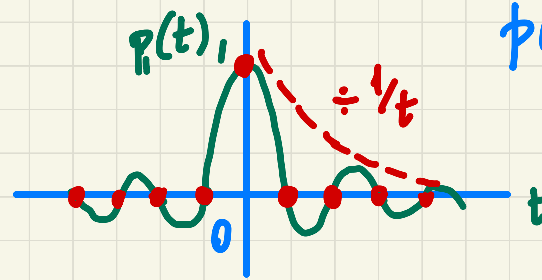


Ex. 1 Rectangular spectrum



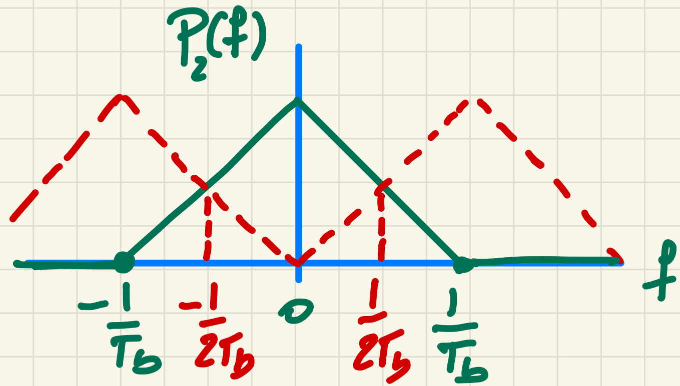
$\mathcal{F}^{-1}$

$\longleftrightarrow$

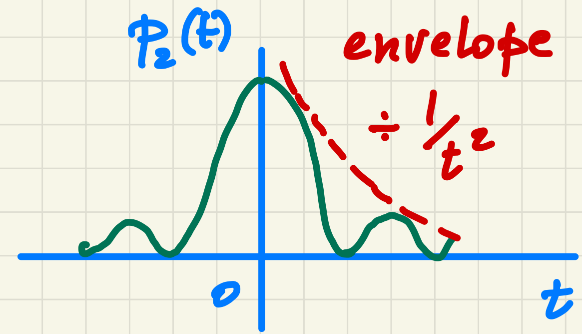


$$p(t) = \operatorname{sinc}\left(\frac{t}{T_b}\right)$$

Ex. 2 Triangular spectrum



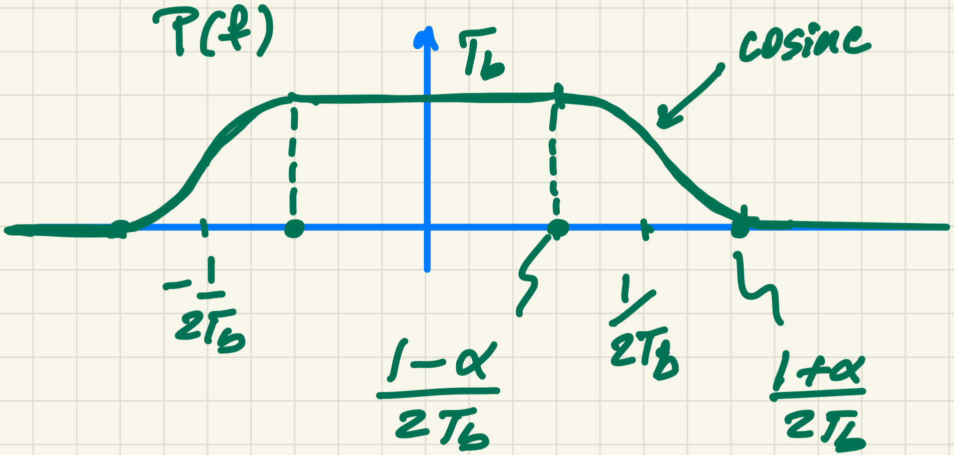
$\mathcal{F}^{-1}$
$\longleftrightarrow$



$$P_2(f) = P_1(f) * P_1(f) \Rightarrow p_2(t) = p_1^2(t) =$$

$$= \mathrm{sinc}^2(t/T_b)$$

Ex. 3 Raised cosine



α Roll-off factor

$0 \le \alpha \le 1$

$\mathcal{F}^{-1}$

$$p(t) = \mathrm{sinc}\left(\frac{t}{T_b}\right) \cdot \frac{\cos\left(\pi\alpha\frac{t}{T_b}\right)}{1-4\alpha^2\left(\frac{t}{T_b}\right)^2}$$

$\alpha = 0$: Narrow spectrum (rect. case) $BW = \frac{1}{2T_b}$

Slow envelope $(\div 1/t)$

BW $\leftrightarrow$ IS

$\alpha = 1$: Wide spectrum $BW = 1/T_b$

Fast envelope → more resilient to sync. errors (ISI)

Synchronization errors $\Rightarrow$ ISI