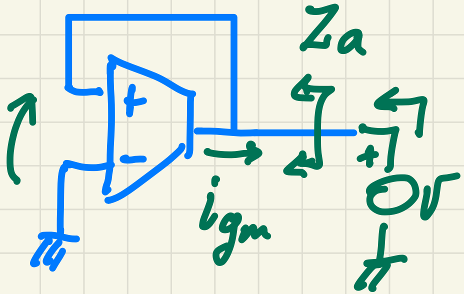
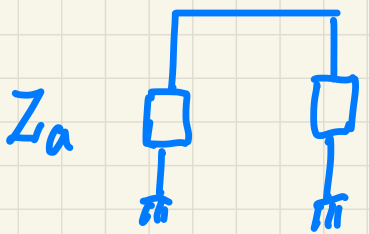
RF Circuit Design

L12



$$i_{gm} = G_m \cdot v \; ; \quad i = -G_m \cdot v$$

$$Z_a = \frac{v}{i} = -\frac{1}{G_m}$$



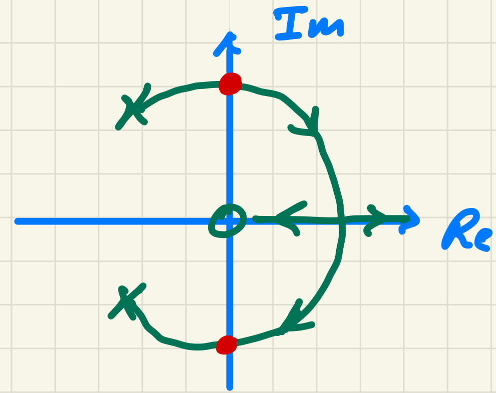
$$Z(s) = R \cdot \frac{s\omega_r/Q}{s^2 + s\frac{\omega_r}{Q} + \omega_r^2}$$

Oscillation condition: $Z_a(j\omega_0) + Z(j\omega_0) = 0$

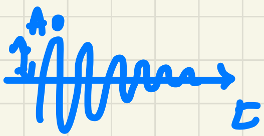
$$-\frac{1}{G_m} + R\,\frac{j\omega_0\omega_r/Q}{-\omega_0^2 + j\omega_0\frac{\omega_r}{Q} + \omega_r^2} = 0 \;\Leftrightarrow\; \begin{cases} -1/G_m + R = 0 \\ \omega_0 = \omega_r \end{cases}$$

Practical Oscillator : amplitude stabilization mechanism

ex. LC oscillator



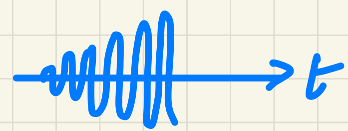
- $G_m R < 1$ poles in LHP

$V(t)$ 

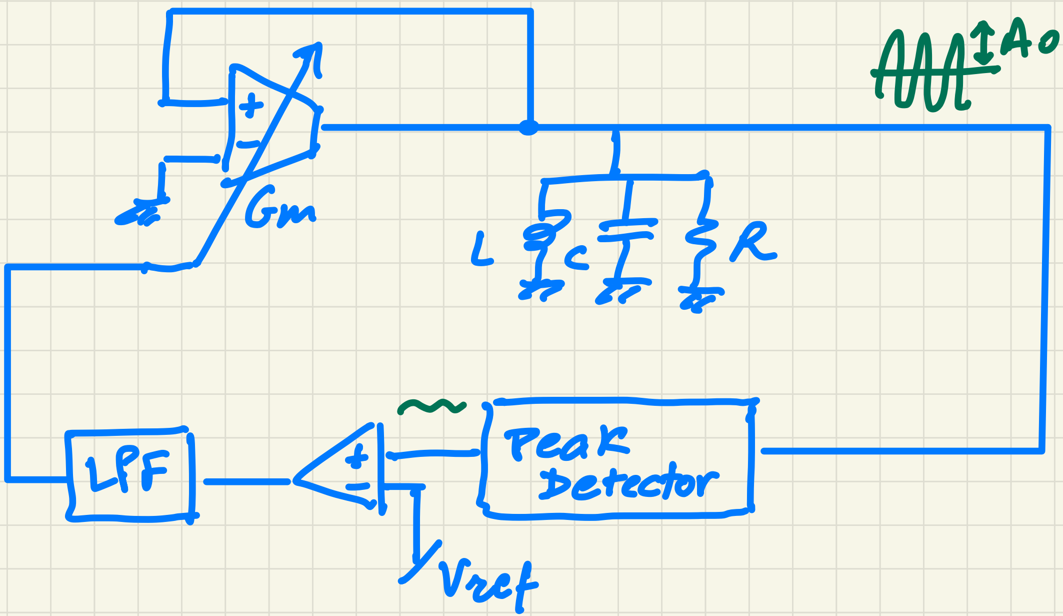
$$G_m \frac{A_0^2}{2} < \frac{A_0^2}{2R}$$

energy provided < dissipated energy

- $G_m R > 1$ poles in RHP

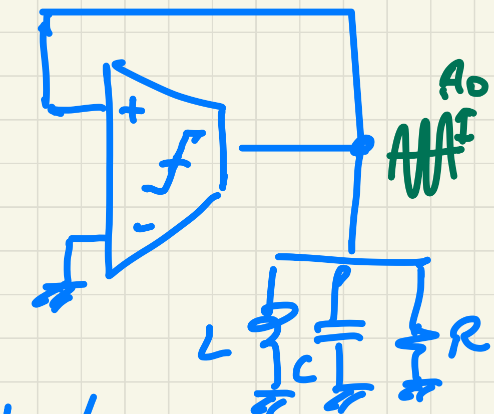


1) Automatic amplitude control (negative feedback)



$G_m = \frac{1}{R}$

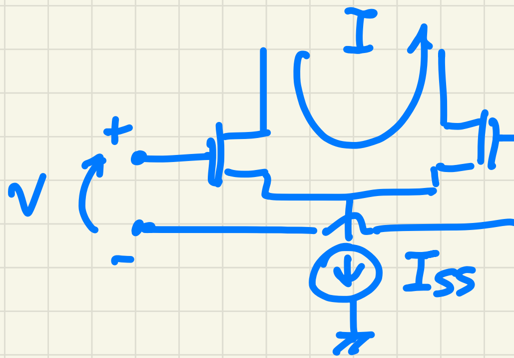
$A_0 \rightarrow V_{ref}$

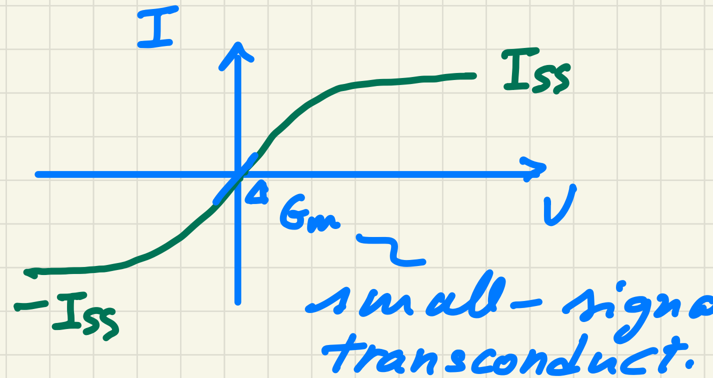


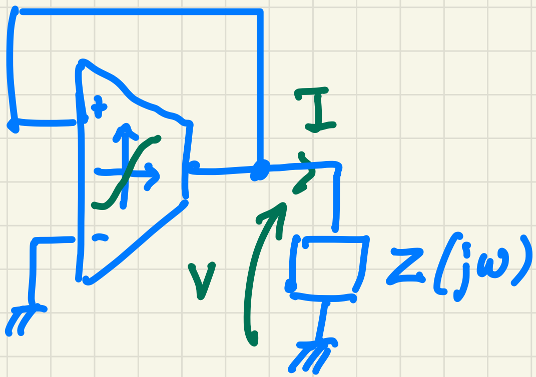
2) Nonlinearity of active devices

small-signal $G_m > 1/R$: oscillator starts up

Oscillation increases until the transconductor saturates example :







a) $I(t) = I[V(t)] =$

$= I\left[\sum_k \bar{V}_k e^{jk\omega_0 t}\right] =$

$= \sum_k \bar{I}_k e^{jk\omega_0 t}$

non linear device

resonator

b) $\bar{V}_k = \bar{I}_k \cdot Z(k\omega_0)$

$$V(t) = \sum_{k=-\infty}^{+\infty} \bar{V}_k e^{jk\omega_0 t}$$

v(t) is periodic with a. freq. ω_0

$$
\begin{cases}
\bar{I}_1 \cdot Z(\omega_0) = \bar{V}_1 \\
\bar{I}_2 \cdot Z(2\omega_0) = \bar{V}_2 \\
\vdots \\
\bar{I}_n \cdot Z(n\omega_0) = \bar{V}_n
\end{cases}
$$

— In analysis, we assume to study HARMONIC OSCILLATORS ($V(t)$ is sinusoidal) i.e. high Q factor*

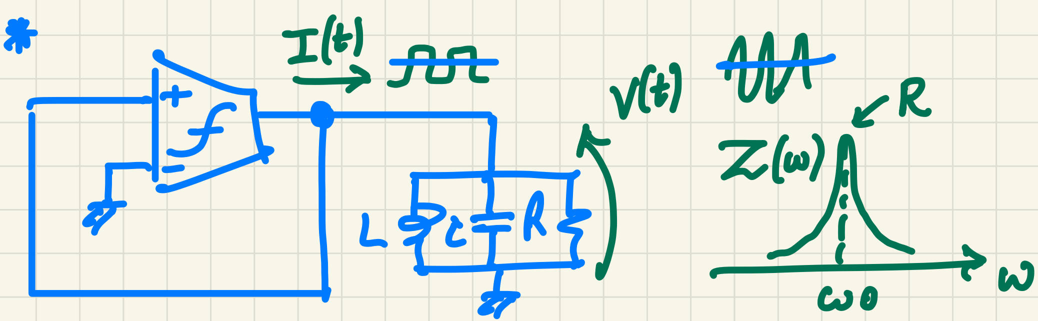
$\downarrow$

$\bar{I}_1 \cdot Z(\omega_0) = \bar{V}_1$;

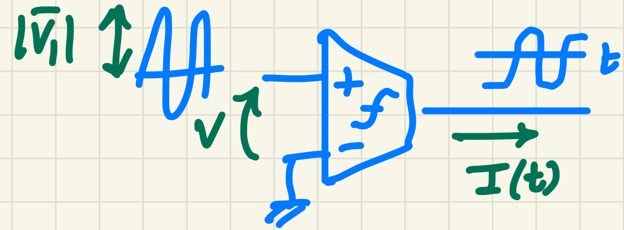
$$\boxed{Z(\omega_0) = \frac{\bar{V}_1}{\bar{I}_1}}$$

harmonic balance with n - harmonics (HB)

*



$$Z(j\omega_0) = \frac{\bar{V}_1}{\bar{I}_1}$$



$$\Downarrow$$

$$Z(j\omega_0) = \frac{1}{G_{mh}}$$

$$\frac{\bar{I}_1}{\bar{V}_1} \triangleq G_{mh}$$

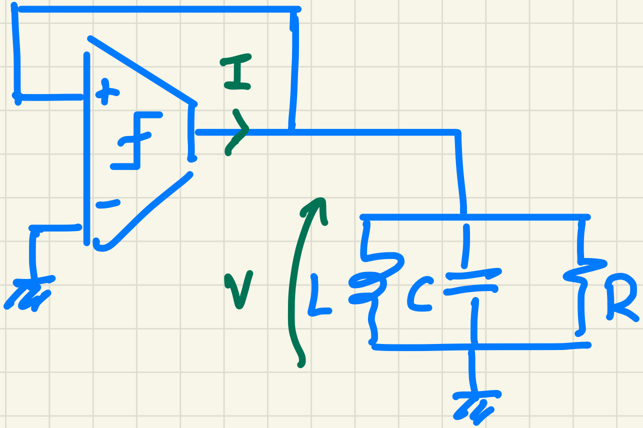
HARMONIC transconduct.
(effective)

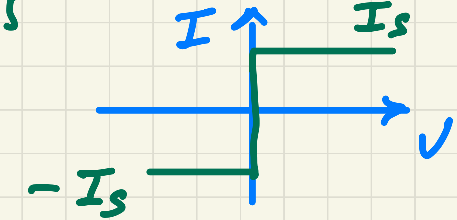
$$\Downarrow$$

$$G_{mh} \cdot Z(j\omega_0) = LG_h(j\omega_0) = 1$$

We replaced the small-signal Gm with a harmonic Gm (method of Descriptive Function)

Case : $I(V) = I_s \cdot \mathrm{sgn}\{V(t)\}$

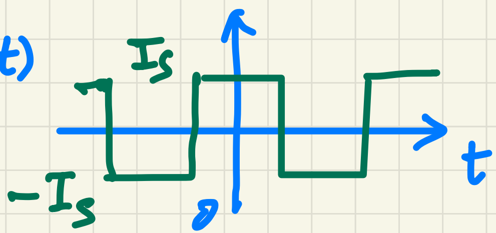




Hyp : $V(t) = A_0 \cos \omega_0 t$

$\Rightarrow \bar{V}_1 = A_0$

$\Rightarrow$

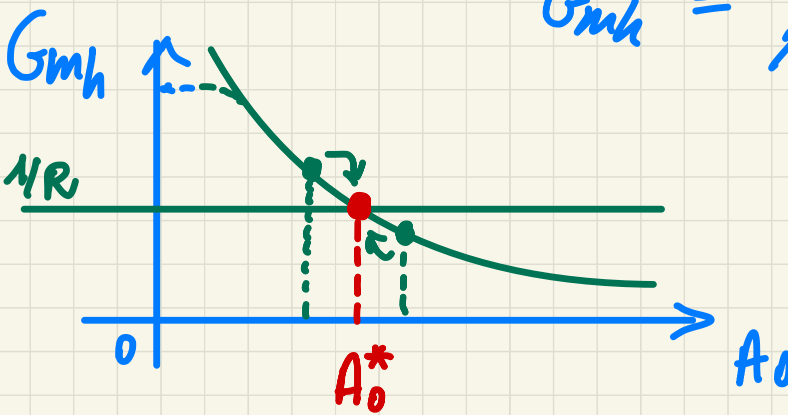


$\Rightarrow \bar{I}_1 = \frac{4}{\pi} \cdot I_s$

$$LG_h(j\omega_0) = 1 \Rightarrow \begin{cases} G_{mh} \cdot R = 1 \quad \text{where } G_{mh} = \dfrac{\frac{4}{\pi} I_s}{A_0} \\ \omega_0 = \dfrac{1}{\sqrt{LC}} \end{cases}$$

$$\underbrace{\frac{4}{\pi}\,\frac{I_s}{A_0}}_{G_{mh}} \cdot R = 1 \quad ; \quad A_0^* = \frac{4}{\pi}\cdot I_s \cdot R$$

$$G_{mh} = \frac{1}{R}$$ oscillation condition



- if $A_0 > A_0^*$:

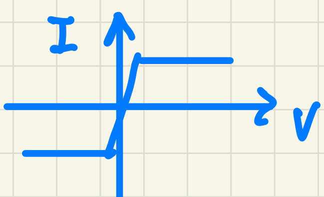
  $G_{mh} \cdot R < 1$

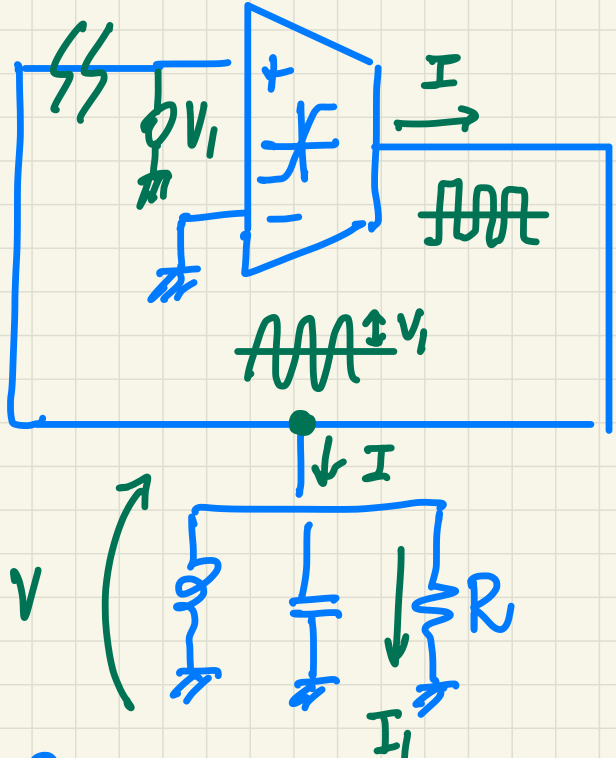
  poles into LHP

  $\to A_0 \searrow$

- if $A_0 < A_0^*$:

  $G_{mh} R > 1 \to A_0 \nearrow$





Hyp. $V(t)$ sinusoidal at ω_0

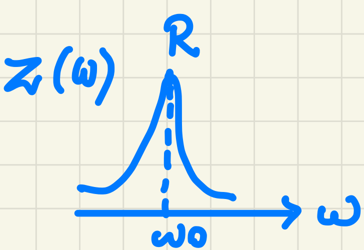
$$\Downarrow$$

$$V_1 = I_1 \cdot Z(j\omega_0)$$

consistency equation

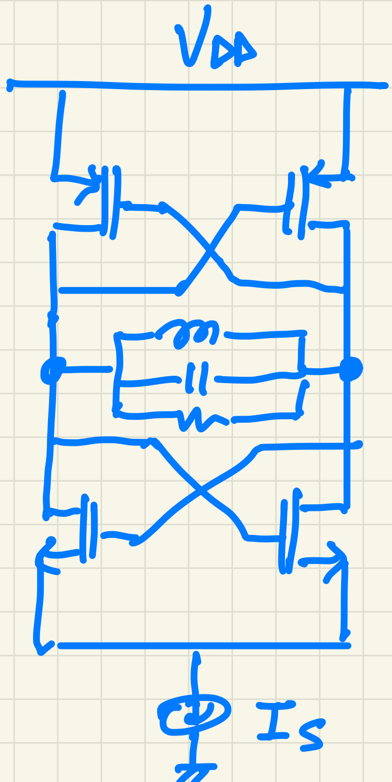
$$LG_h = \underbrace{\frac{I_1}{V_1}}_{} \cdot Z_L(j\omega_0) = 1$$

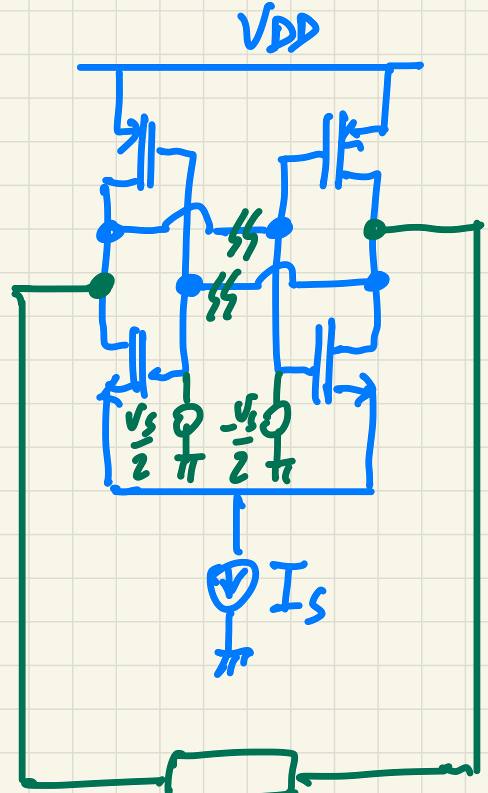
harmonic Gm

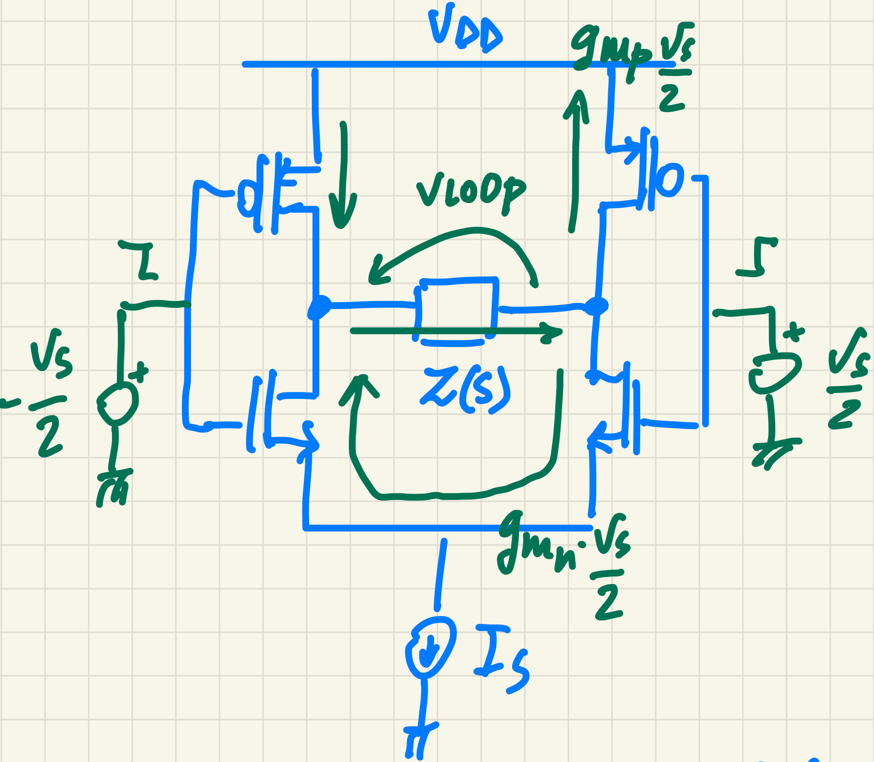


Ex. of real oscillators

- Differential oscillator







Differential mode

$$LG(s) = \frac{V_{LOOP}}{V_S} =$$

$$= Z(s) \cdot \underbrace{\frac{g_{mn}+g_{mp}}{2}}_{\text{small-signal } G_m}$$

Oscillation condition

$$LG(j\omega_0) = 1 \; :$$

- $\measuredangle LG(j\omega_0) = 0 \; ; \; \measuredangle Z(j\omega_0) = 0 \; ;$

$$\omega_0 = \omega_r = \frac{1}{\sqrt{LC}}$$

- $|LG(j\omega_0)| = 1 \; ; \quad \frac{g_{mn}+g_{mp}}{2} \cdot R = 1$

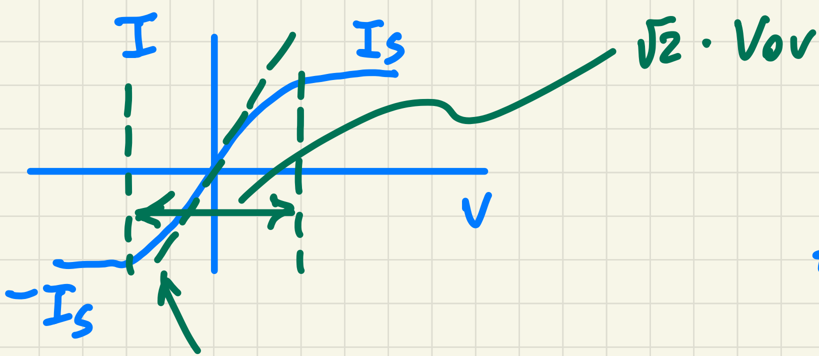
Oscillator design :

small-signal LG

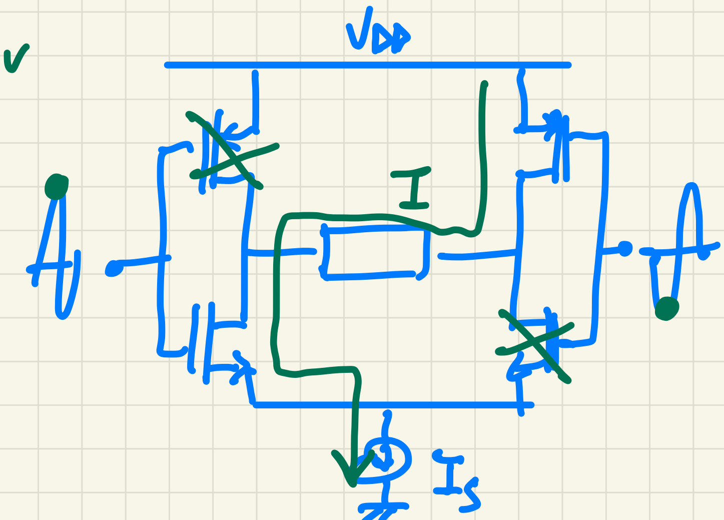
- Startup condition : $\underbrace{LG(j\omega_0) > 1}$

Startup margin $LG(j\omega_0) = EG > 1$
excess gain

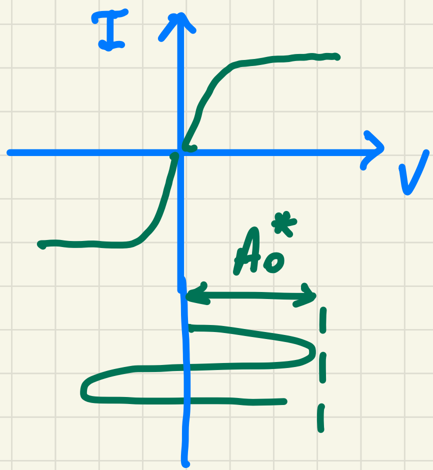
- Oscillation amplitude : $\underbrace{LG_h(j\omega_0) = 1}$

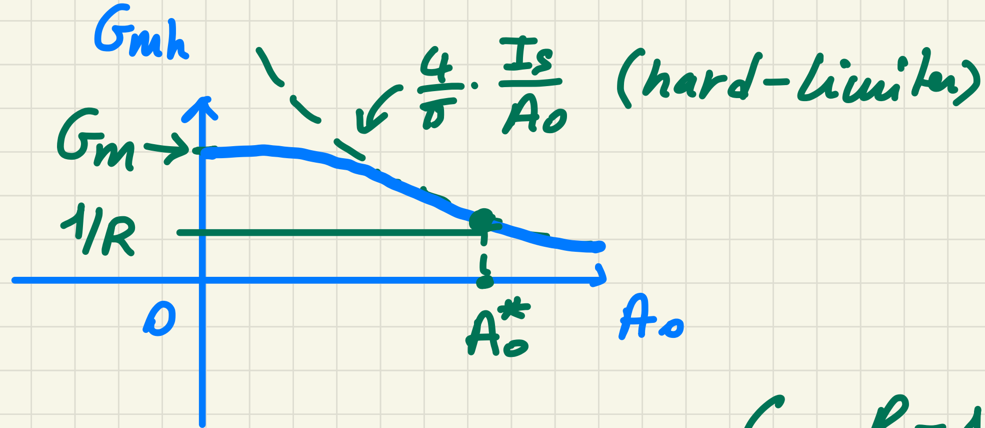


$$\frac{g_{mn} + g_{mp}}{2} = G_m$$



$$A_0^* \gg \sqrt{2} \cdot V_{ov}$$

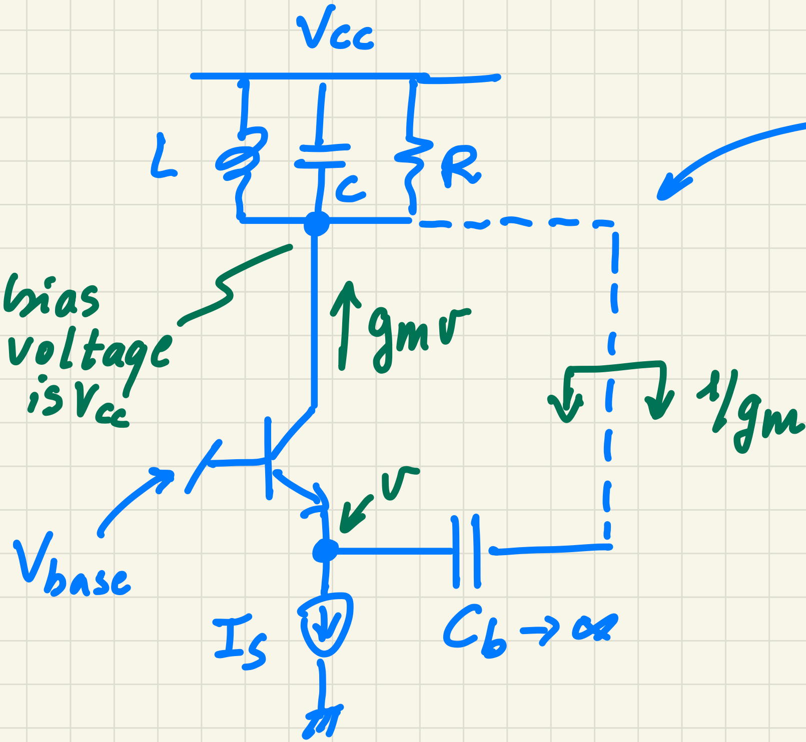


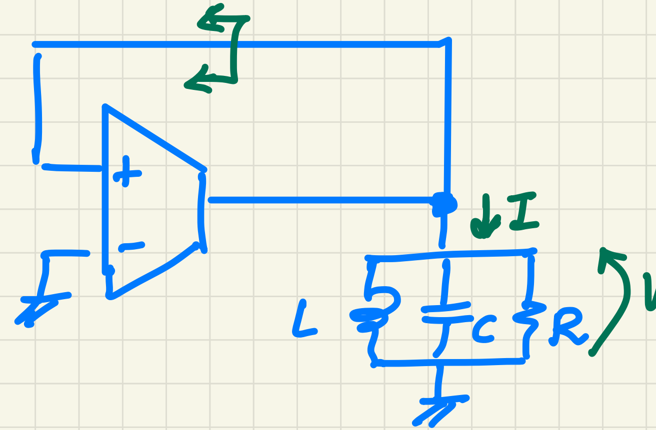


$$G_{mh} \cdot R = 1$$

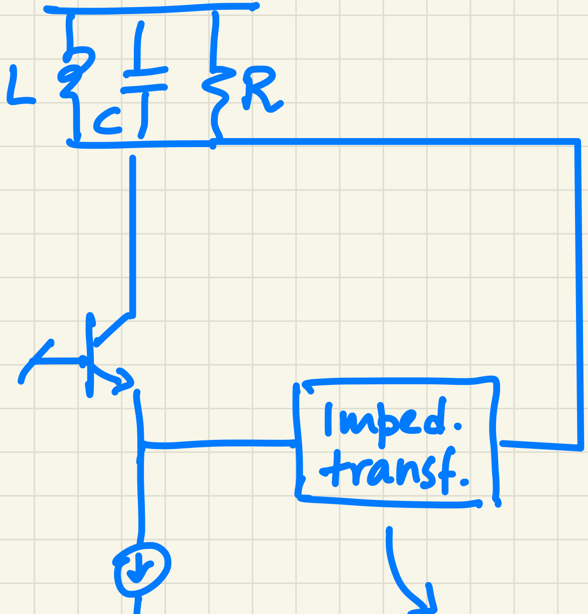
$$A_0^* \simeq \frac{4}{\pi} I_s \cdot R$$

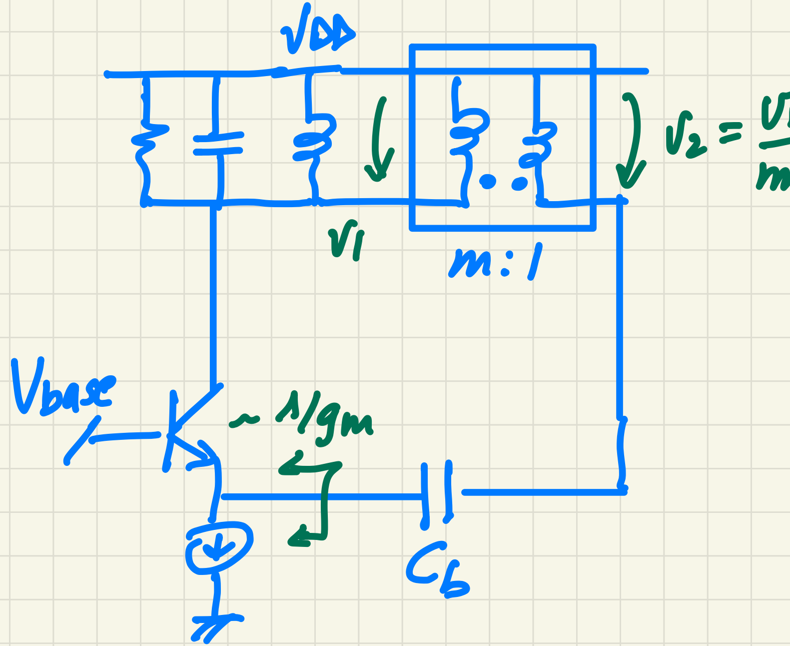
- Single-transistor oscillators





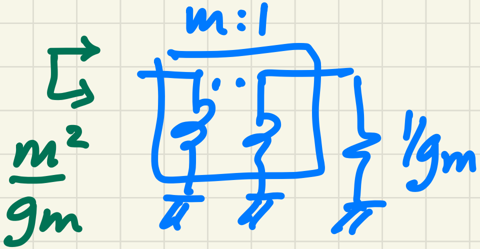
we cannot connect the emitter to the resonator without spoiling the resonator's Q?



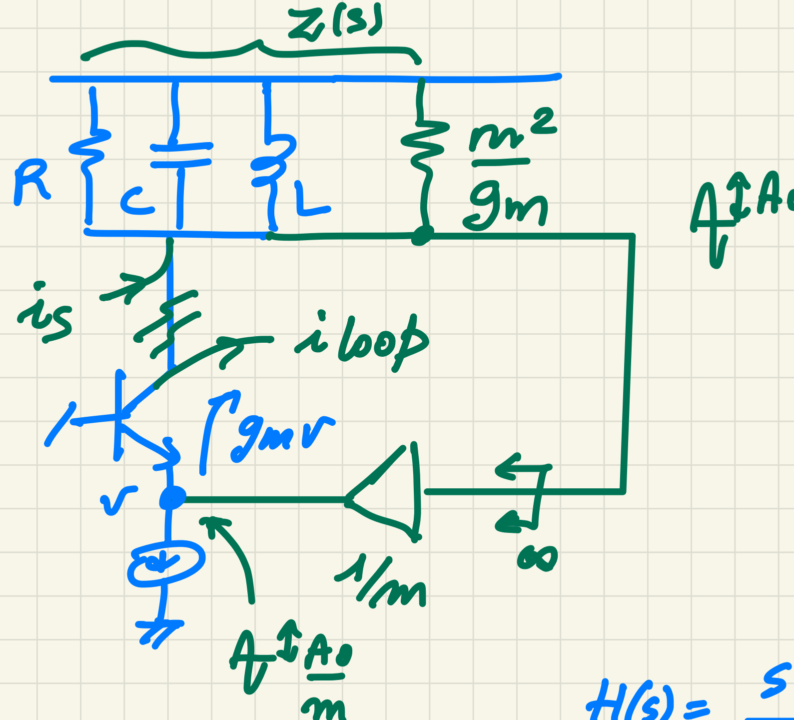


Upward :

voltage attenuation $\Rightarrow$ $m > 1$



$\Rightarrow$



equivalent circuit to compute the loop gain

$$H(s) = \frac{s\,\omega_n/Q}{s^2 + s\frac{\omega_n}{Q} + \omega_n^2}$$

$$LG(s) = \frac{i_{loop}}{i_s} = Z(s)\cdot\frac{g_m}{m}$$

$$Z(s) = R_T \cdot H(s)$$

$$R_T = \frac{R\cdot m^2/g_m}{R + m^2/g_m} = \frac{m^2\cdot R}{m^2 + g_m R}$$

$$LG(s) = \frac{g_m}{m} Z(s) \qquad Z(s) = R_T \cdot H(s)$$

oscillation condition $LG(j\omega_0) = 1$:

$$LG(j\omega_0) = \frac{g_m}{m} \cdot R_T \cdot H(j\omega_0) = 1$$

$$\measuredangle LG(j\omega_0) = 0 \; : \; \underbrace{\omega_0 = \omega_r}$$

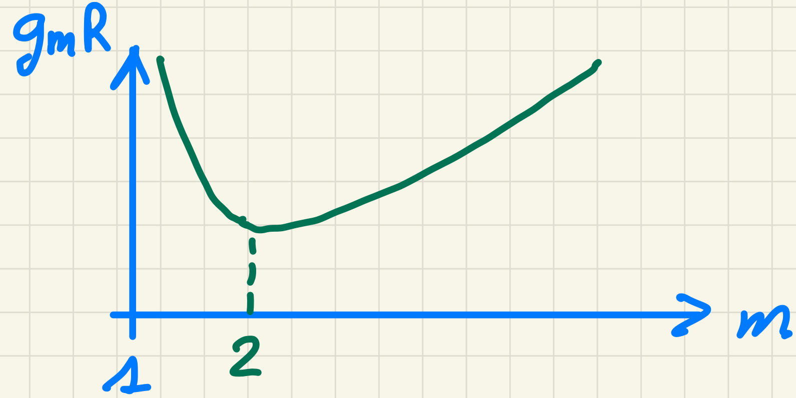
$$|LG(j\omega_0)| = \frac{g_m}{m} \cdot R_T = 1$$

$$\Downarrow$$

$$\frac{g_m}{m} \cdot \frac{m^2 R}{m^2 + g_m R} = 1 \quad \ldots$$

$$\Rightarrow \underbrace{g_m R = \frac{m}{1 - 1/m}}$$

$$g_m R = \frac{m}{1 - 1/m}$$



At large m :

$g_m R \nearrow$

At small m :

$g_m R \nearrow$

min. gain $g_m R = \dfrac{2}{1 - 1/2} = 4$

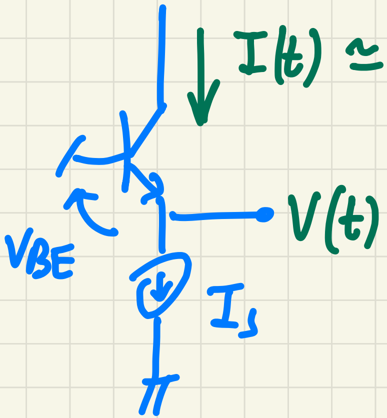
is obtained for $m = 2$

trade - off in the choice of m

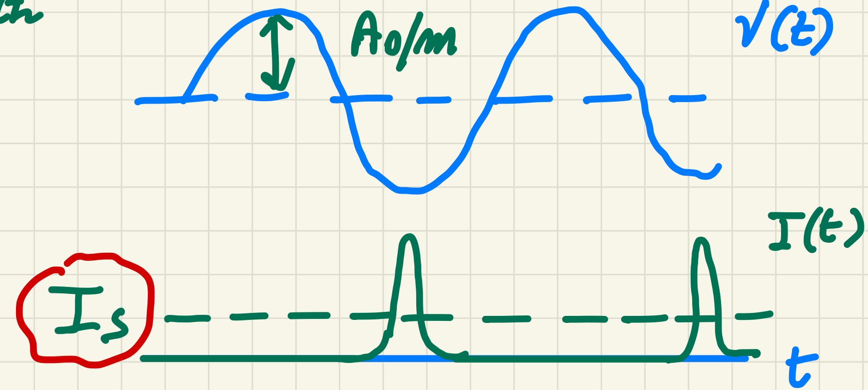
$m = 2$: startup condition $g_m R > 4$

Oscillation amplitude

Large-signal

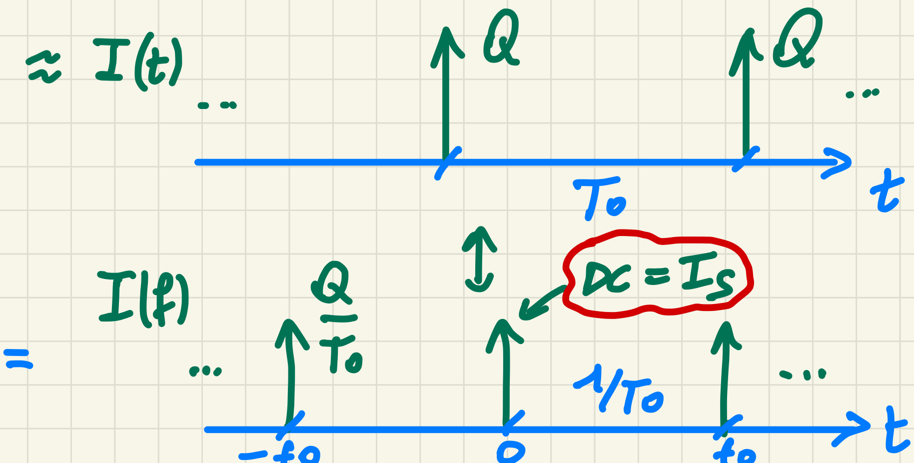


$$I(t) \simeq I_{sat}\, e^{V_{BE}/V_{th}}$$



Approximation

$$\frac{A_0}{m} \gg \frac{kT}{q} = V_{th}$$



$$I(t) \simeq Q \cdot \sum \delta(t - kT_0) =$$
$$= \frac{Q}{T_0} \cdot \sum e^{-jk\omega_0 t}$$

harmonic gm

$$I_1 = 2 \cdot Q/T_0$$

$$g_{mh} = \frac{\bar{I}_1}{\bar{V}_1} = \frac{2Q/T_0}{A_0/m} = \frac{2 I_s}{A_0/m}$$

Oscillation condition (large signal)

$$LG_h(j\omega_0) = 1 \iff \underbrace{g_{mh}}_{\text{we replace small-signal gm}} R = \frac{m}{1-\frac{1}{m}};$$

$$\frac{2I_s}{A_0} \cancel{m} \cdot R = \frac{\cancel{m}}{1-\frac{1}{m}}; \quad A_0 = 2 I_s R \cdot \left(1-\frac{1}{m}\right)$$

$m=1$: 0 $\quad$ $m \to \infty$: $2 I_s R$

$$m = 2: \quad A_0 = 2 I_s R \cdot \frac{1}{2} = I_s R$$