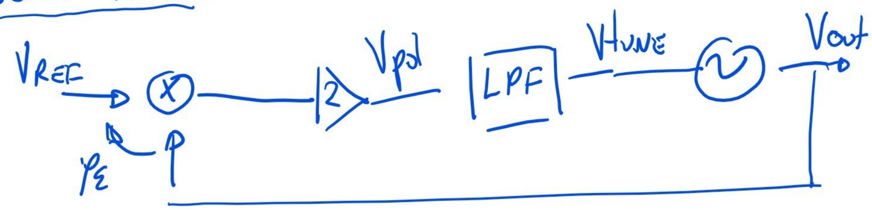
- LOCK RANGE



$$\begin{cases} \omega_{out} = \omega_{REF} \overset{!}{=} \omega_{FR} + K_{VCO} V_{tune} \\ \langle V_{pd} \rangle = V_{tune} = K_{pd} \sin(\varphi_E) \end{cases}$$

$\langle V_{pd} \rangle^{MAX} = \pm 1$

$\omega_{out}^{MAX} = \omega_{FR} + K_{VCO}(+1)$

$\omega_{out}^{MIN} = \omega_{FR} + K_{VCO}(-1)$

IN OUR CASE: $\omega_{FR} = \omega_0$

$\omega_{REF} = \omega_0 + \underbrace{\Delta\omega}_{\text{INPUT STEP}}$

MAXIMUM INPUT STEP: $\Delta\omega^{MAX} = \underbrace{K_{VCO}}_{2\ \frac{rad/s}{V}}(\pm 1V) = 2\ rad/s$

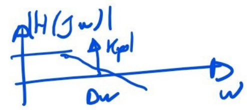
- CAPTURE RANGE
  - APPLY AN input FREQUENCY step $\Delta\omega$

$\sin(\omega_0 t + \Delta\omega t)$

$V_{out} = \cos(\omega_0 t + \cancel{\varphi_{out}})$ negligible

$V_{pd} = 2\left[\frac{1}{2}\sin(\Delta\omega t) + \cancel{\frac{1}{2}\sin([2\omega_0 + \Delta\omega]t)}\right]$ FILTERED BY LPF

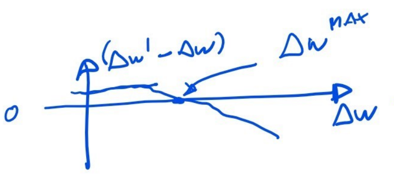
This signal is @ $\Delta\omega$ FREQ with AMPLITUDE K_{pd} $\rightsquigarrow$ it is partially filtered by LPF.



- the PLL can REACH LOCK if

$\Delta\omega' = K_{pd}\,|H(j\Delta\omega)|\,K_{VCO} \geq \Delta\omega$

GRAPHICAL SOLUTION:

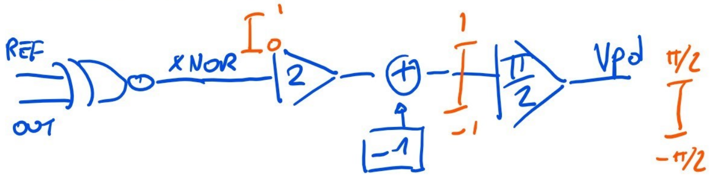


input step

SEE MATLAB SCRIPT

$\Delta\omega^{MAX}$ = CAPTURE RANGE

NXOR PHASE DETECTOR

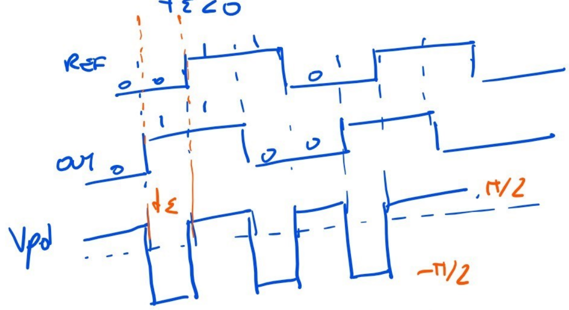


NXOR

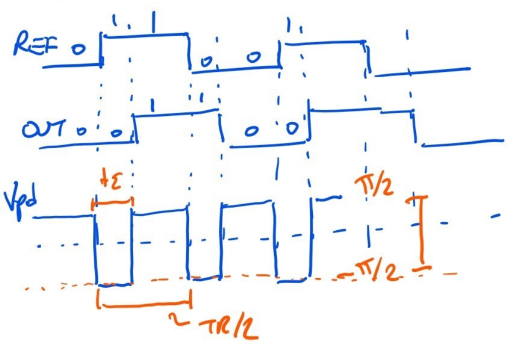
REF	OUT	XNOR
0	0	1
0	1	0
1	0	0
1	1	1

- DERIVE THE LINEARIZED AVERAGED MODEL IN PHASE DOMAIN

$t_\varepsilon < 0$



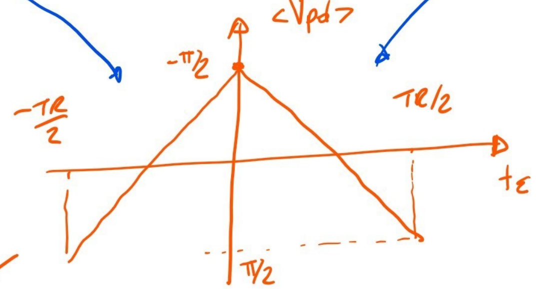
$t_\varepsilon > 0$



$$\langle V_{pd} \rangle = -\frac{\pi}{2} + 2\pi \frac{|t_\varepsilon|}{T_R}$$

$$\langle V_{pd} \rangle = -\frac{\pi}{2} + \pi \frac{(T_R/2 - t_\varepsilon)}{T_R/2} = -\frac{\pi}{2} + \pi - 2\pi \frac{t_\varepsilon}{T_R} = \frac{\pi}{2} - 2\pi \frac{t_\varepsilon}{T_R}$$

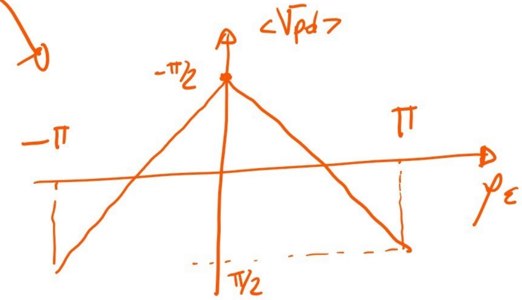
$t_\varepsilon < 0$ $\quad$ $t_\varepsilon > 0$



in PHASE DOMAIN

$$\varphi_\varepsilon = \frac{2\pi}{T_R} t_\varepsilon \rightsquigarrow t_\varepsilon = \varphi_\varepsilon \frac{T_R}{2\pi}$$

$$\langle V_{pd} \rangle = \frac{\pi}{2} - (\varphi_\varepsilon \ 1V/rad)$$



$\varphi_\varepsilon < 0$:

$$K_{pd} = \frac{\partial \langle V_{pd} \rangle}{\partial \varphi_\varepsilon} = 1$$

$\varphi_\varepsilon > 0$:

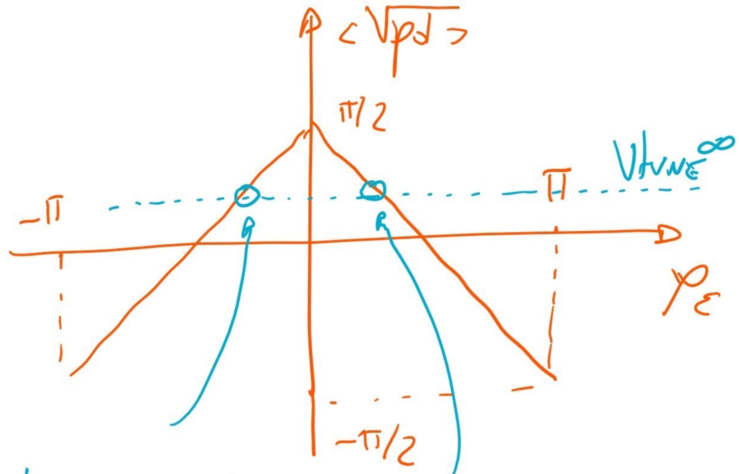
$$K_{pd} = \frac{\partial \langle V_{pd} \rangle}{\partial \varphi_\varepsilon} = -1$$

PLL EQ POINT:

$$\begin{cases} \omega_{OUT} = \omega_{REF} \stackrel{!}{=} \omega_{FR} + K_{VCO} V_{TUNE} \\ V_{TUNE} = \langle V_{pd} \rangle = K_{pd} \varphi_\varepsilon^\infty \end{cases}$$

$$V_{TUNE}^\infty = \frac{\omega_{out} - \omega_{FR}}{K_{VCO}} = \frac{d\omega}{K_{VCO}} = 0.1V$$

$$\varphi_\varepsilon^\infty = \frac{V_{TUNE}^\infty}{K_{pd}} = \pm 0.1 \text{ rad}$$



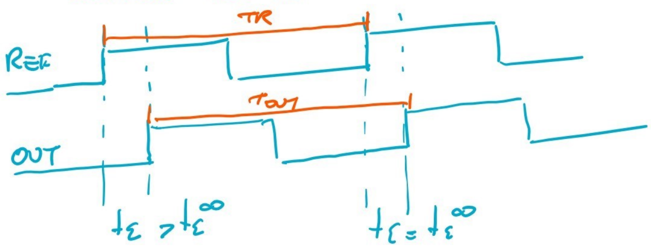
two possible eq. points

- WHICH IS THE STABLE EQ POINT?

- METHOD 1: APPLY PERTURBATION

- CASE $t_\varepsilon > 0$

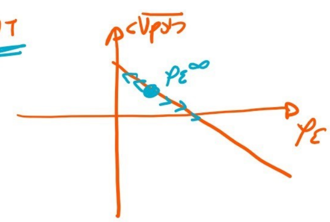
SUPPOSE $t_\varepsilon > t_\varepsilon^\infty$



CASE $t_\varepsilon > 0$

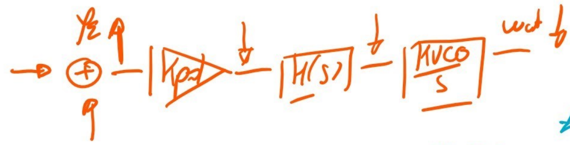
IF I APPLY A PERTURBATION $t_\varepsilon > t_\varepsilon^\infty$ to REACH $t_\varepsilon = t_\varepsilon^\infty$ the PLL needs to set $T_{out} < T_R$ (INCREASE FREQUENCY)

BUT



For $\varphi_\varepsilon > 0$

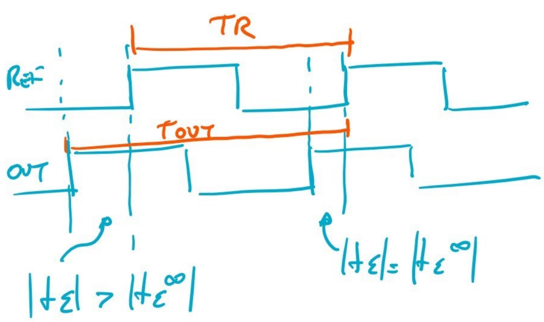
IF I MOVE $\varphi_\varepsilon > \varphi_\varepsilon^\infty$ the PLL DECREASE THE OUTPUT FREQUENCY



UNSTABLE eq. POINT.

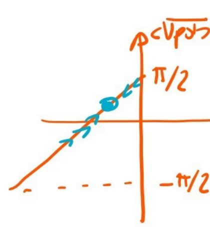
- CASE $t_\varepsilon < 0$

SUPPOSE $|t_\varepsilon| > t_\varepsilon^\infty$



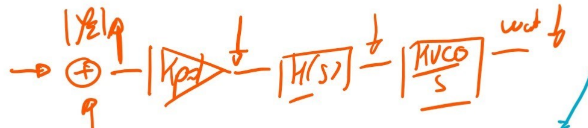
CASE $t_\varepsilon < 0$

IF I APPLY A PERTURBATION $|t_\varepsilon| > |t_\varepsilon^\infty|$ to REACH $|t_\varepsilon| = |t_\varepsilon^\infty|$ the PLL needs to set $T_{out} > T_R$ (DECREASE FREQUENCY)



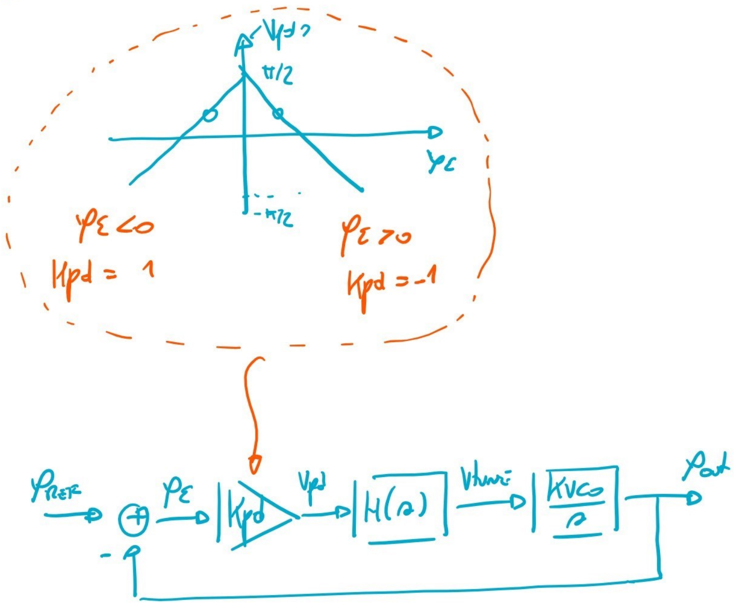
For $\varphi_\varepsilon > 0$

IF I MOVE $|\varphi_\varepsilon| > |\varphi_\varepsilon^\infty|$ the PLL DECREASE THE OUTPUT FREQUENCY



STABLE eq. POINT.

• METHOD 2: LOOP GAIN SIGN



to have $G_{loop} < 0 \rightarrow K_{pd} > 0 \rightsquigarrow \varphi_\varepsilon^\infty$ in $\varphi_\varepsilon < 0$ stable eq point.