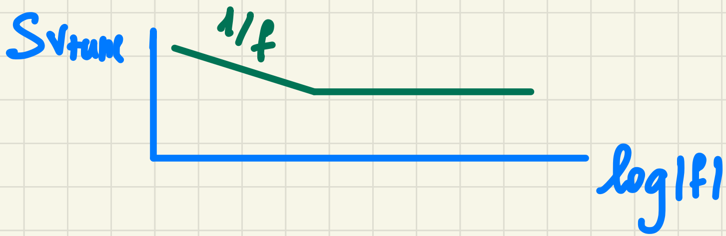
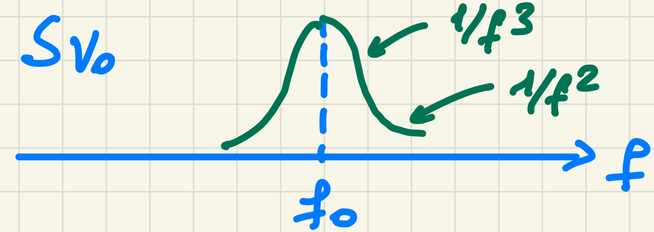
RF Circuit Design

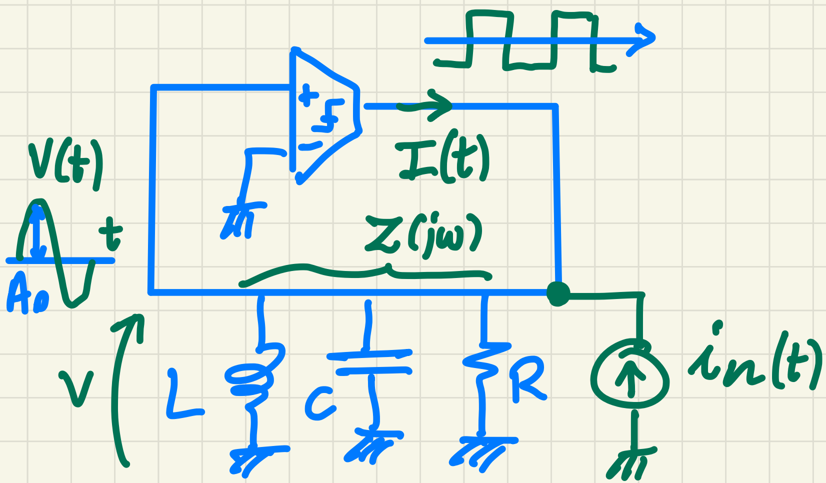
L14





$$\mathcal{L}(f) \simeq \frac{1}{2} S_\varphi(\omega) = \frac{1}{2} K_{vco}^2 \cdot \frac{S_{v_{tune}}(\omega)}{\omega^2}$$

- Direct mechanism

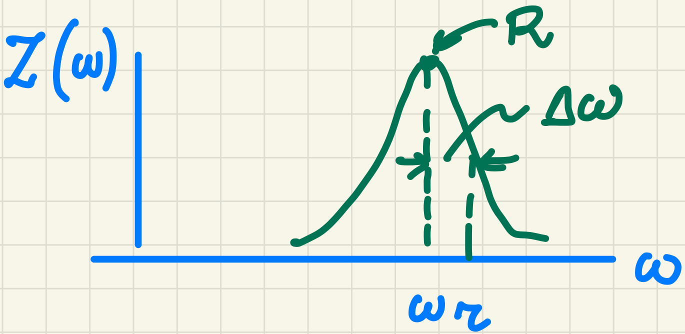


$i_n(t)$ is noise associated to tank losses (resistor R)

$$S_{i_n}(f) = \frac{4KT}{R}$$

$$Z(j\omega) = R \cdot \frac{j\omega \cdot \omega_r / Q}{\omega_r^2 - \omega^2 + j\omega \frac{\omega_r}{Q}}$$

$$Q = \omega_r R C$$



$$\omega = \omega_r \pm \Delta\omega$$

$$Z(j\omega_r \pm j\Delta\omega) = R \cdot \frac{j(\omega_r \pm \Delta\omega)\,\omega_r/Q}{\cancel{\omega_r^2} - \underbrace{(\omega_r \pm \Delta\omega)^2}_{} + j(\omega_r \pm \Delta\omega)\frac{\omega_r}{Q}} =$$

$$-(\cancel{\omega_r^2} + \Delta\omega^2 \pm 2\omega_r\Delta\omega) =$$

$$= -\Delta\omega \cdot (\Delta\omega \pm 2\omega_r)$$

$$= R \cdot \frac{1}{1 + j\,\frac{\Delta\omega}{\frac{\omega_r}{Q}} \cdot \underbrace{\frac{\Delta\omega \pm 2\omega_r}{\omega_r + \Delta\omega}}_{\approx \pm 2}}$$

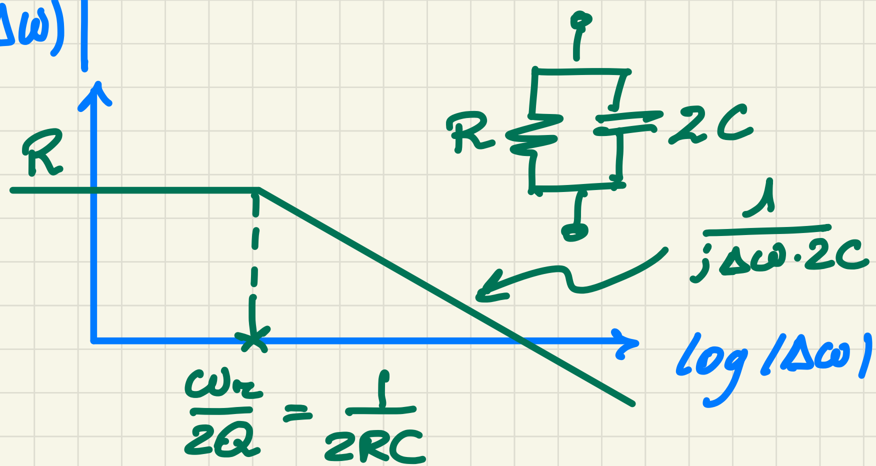
Hyp: $\Delta\omega \ll \omega_r$

$$\Rightarrow Z(j\omega_r \pm j\Delta\omega) \simeq \frac{R}{1 \pm j\cdot\dfrac{\Delta\omega}{\frac{\omega_r}{2Q}}} = \frac{R}{1 \pm j2RC\cdot\Delta\omega} =$$

$$\triangleq Z'(\pm j\Delta\omega)$$

a. frequency offset from the carrier

$|Z'(j\Delta\omega)|$

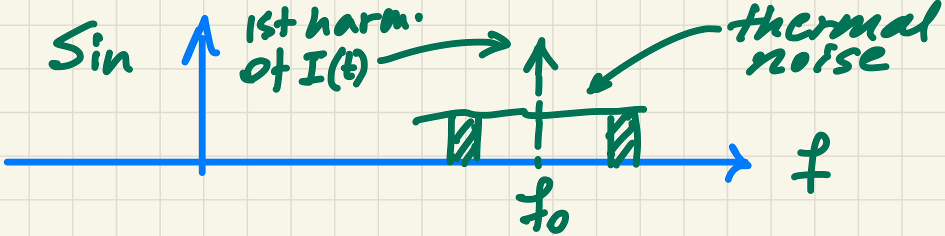


$$Q = \omega_r RC$$

$$\Downarrow$$

$$\frac{\omega_r}{2Q} = \frac{1}{2RC}$$

Baseband equivalent of $Z(j\omega)$ of a RLC resonator around resonance



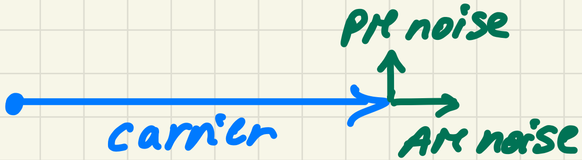
$S_{in} = \frac{4kT}{R}$ white

in phase with the carrier

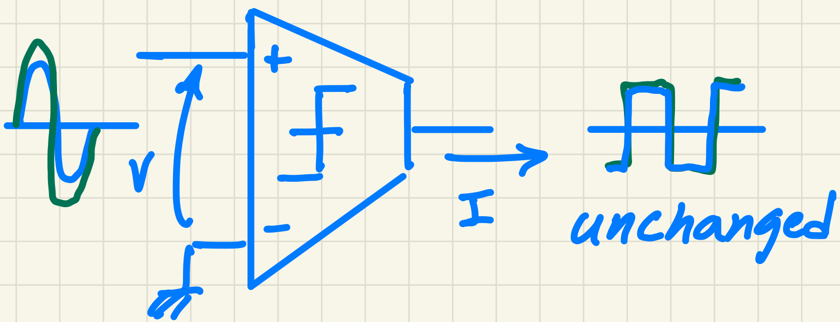
in quadrature with the carr.

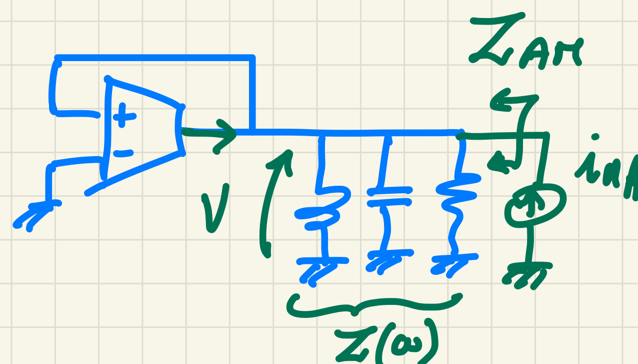
$S_{inAM} = \frac{2kT}{R}$

$S_{inPM} = \frac{2kT}{R}$



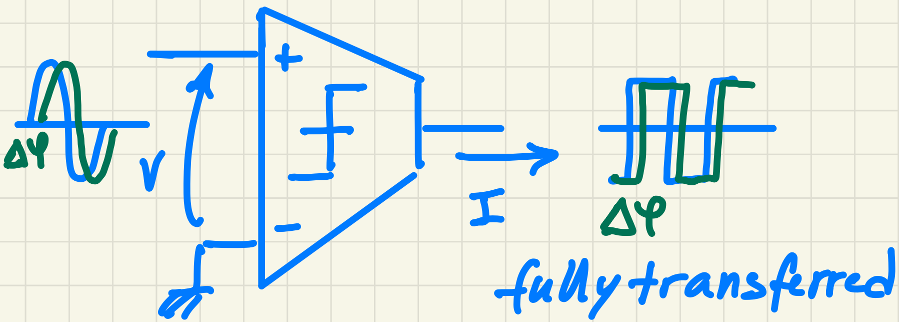
• AM component:

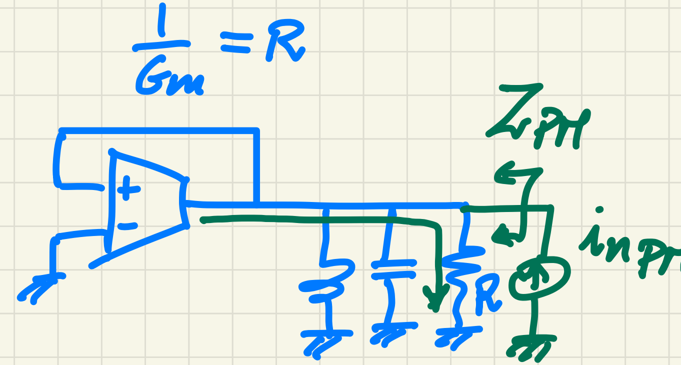




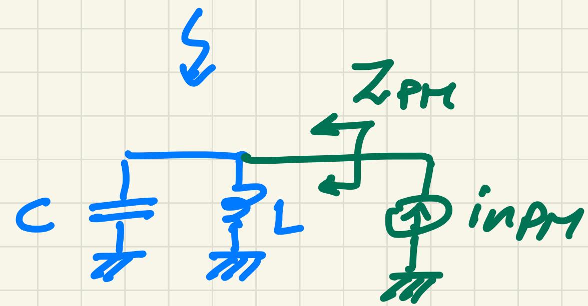
$Z_{AM}(\omega) = Z(\omega)$

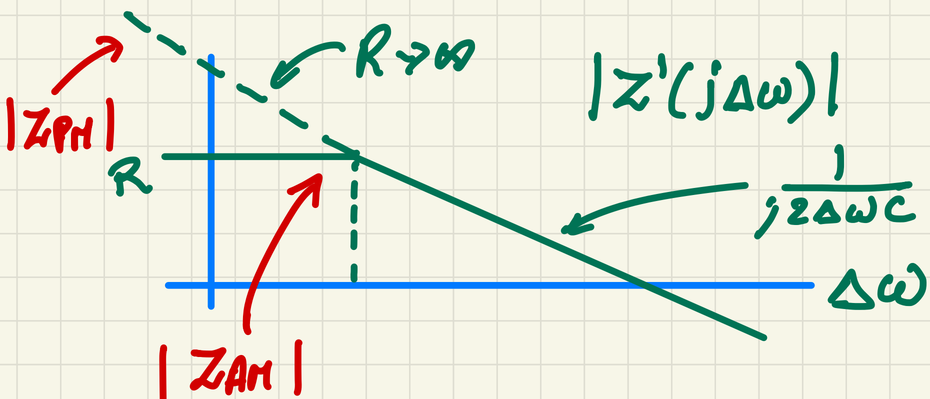
- PM component of noise

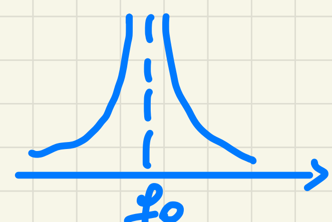




$$Z_{PM}(\omega) = Z(j\omega)\Big|_{R\to\infty}$$







$$\rightsquigarrow S_v(\omega) = \frac{2KT}{R} \cdot |Z_{AM}(\omega)|^2 + \frac{2KT}{R} \cdot |Z_{PM}(\omega)|^2$$

$\Delta\omega \ll \frac{\omega_r}{2Q}$:

$$Z_{AM}(\omega_r \pm \Delta\omega) \simeq R$$

$$Z_{PM}(\omega_r \pm \Delta\omega) \simeq \frac{1}{\pm j\, 2 \Delta\omega C}$$

$$F_a = \frac{S_{Va}}{S_{Vr}}$$

$\Downarrow$

$$S_v(\omega_r \pm \Delta\omega) \simeq \frac{2KT}{\cancel{R}} \cdot R^{\cancel{2}} + \frac{\cancel{2}KT}{R} \cdot \frac{1}{_{2}\cancel{4}\,\Delta\omega^2 C^2} =$$

$$= 2KTR + \frac{KT \cdot R \cdot \omega_r^2}{\underbrace{2R\Delta\omega^2 C^2 \cdot R \cdot \omega_r^2}_{Q^2}} = \qquad Q = \omega_r R C$$

$$= \underbrace{\underbrace{2KTR}_{\text{negligible}}}_{\text{AM noise}} + \underbrace{\frac{1}{2} \cdot KT \cdot R \cdot \left(\frac{\omega_r}{Q}\right)^2 \cdot \frac{1}{\Delta\omega^2}}_{\text{PM noise}} \cdot F_a$$

accounts for active elem. noise

$$\Rightarrow \mathcal{L}(\Delta\omega) \triangleq \frac{S_V(\omega_r + \Delta\omega)}{P_{carrier}} \simeq \quad [dBc/Hz]$$

Phase noise of oscillator

$$\simeq \frac{\frac{1}{\cancel{2}} KT \cdot \left(\frac{\omega_r}{Q}\right)^2 \cdot R \, \frac{1}{\Delta\omega^2} \cdot F_a}{\frac{A_0^2}{\cancel{2}}} =$$

A_0 amplitude of $V(t)$

$$= \frac{KT}{2P_R} \cdot \left(\frac{\omega_r}{Q}\right)^2 \cdot \frac{1}{\Delta\omega^2} \cdot F_a$$

$P_R = \frac{A_0^2}{2R}$ [W]

power diss. in the reson.

$$P_R = \eta \cdot P_{DC}$$

DC power from supply

$\eta \triangleq P_R / P_{DC}$

$0 \leq \eta \leq 1$

Efficiency

$$\mathcal{L}(\Delta\omega) = \frac{kT}{2\eta P_{DC}} \cdot \left(\frac{\omega_r}{Q}\right)^2 \cdot \frac{1}{\Delta\omega^2} \cdot F_a$$

expresses the trade-off between phase noise and dissipated power

Define : Figure of merit of oscillators

$$FoM_{dB} \triangleq 10 \log_{10} \left\{ \frac{1}{\mathcal{L}(\Delta\omega) \cdot P_{DC,mW}} \cdot \left(\frac{\omega_{osc}}{\Delta\omega}\right)^2 \right\}$$

$$\Rightarrow FoM_{dB} = 10 \log_{10} \left\{ \frac{\left(\cancel{\omega_{osc}} / \cancel{\Delta\omega}\right)^2}{\frac{kT}{2\eta} \frac{1}{\cancel{P_{DC}}} \cdot \left(\frac{\cancel{\omega_{osc}}}{Q}\right)^2 \cdot \frac{1}{\cancel{\Delta\omega^2}} \cdot F_a \cdot \underbrace{\cancel{P_{DC}},_{mW}}_{10^3}} \right\} =$$

$$FOM_{dB} = 10 \log_{10} \left\{ 10^{-3} \cdot \frac{2\eta}{KT} \cdot Q^2 \cdot \frac{1}{F_a} \right\}$$

thermodynamic limit of FoM of oscillators :

- ideally : $\eta = 1$
- ideally : $F_a = 1$ (only thermal noise of the resonator)
- assume : $Q = 10$

$$\Rightarrow FoM_{max} = 10 \log_{10} \left\{ \frac{2}{KT} \cdot Q^2 \right\} - 30 = 197 \text{ dB/Hz}$$

e.g. $f_{osc} = 1\,GHz$

$\Delta f = 1\,MHz$

$P_{DC} = 1\,mW$

$Q = 10$

$$\mathcal{L}_{min}(\Delta f) = \frac{1}{FOM_{max}} \cdot \frac{1}{P_{DC,mW}} \cdot \left(\frac{f_{osc}}{\Delta f}\right)^2$$

$$\Downarrow$$

$$-FOM_{max} - 10 \log_{10} P_{DC,mW} + 20 \log \frac{f_{osc}}{\Delta f}$$

$$= -197\,dB - 0\,dBm + 60 = -137\,dBc/Hz$$

Circuit Simulators (e.g. Cadence Spectre
Mentor Eldo
...)

• DC	DC analysis	bias point (non linear)	
• AC	AC analysis	transfer functions (linear*)	
• $\boxed{\text{NOISE}}$	NOISE analysis	based on AC (linear*)	

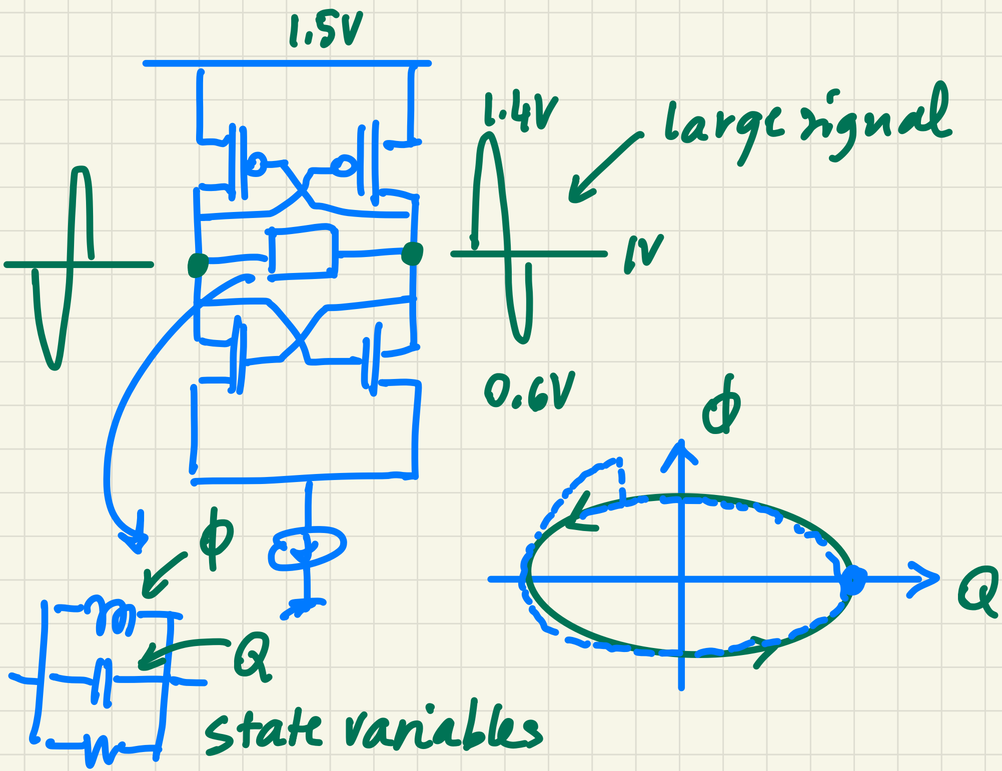
LTI approximation

* nonlinear devices are replaced by equivalent linear circuits

• TRAN	TRANSIENT analysis	transient behavior (nonlinear)	
• NOISETRAN	noise sources are modelled as random sequences needs many runs to get statistics	does not account for noise sources	time consuming

RF circuit simulators (e.g. Spectre RF
Eldo RF
...)

. PSS periodic steady state analysis nonlinear



$V_1(t+T_0) = V_1(t)$

$V_2(t+T_0) = V_2(t)$

$\vdots$

For every voltage and current

$\rightarrow$ find T_0

- $\boxed{\text{PAC}}$ periodic AC analysis — transfer functions of small-signal perturbations

LTV approximation (linear time-variant)

- PNOISE periodic noise analysis

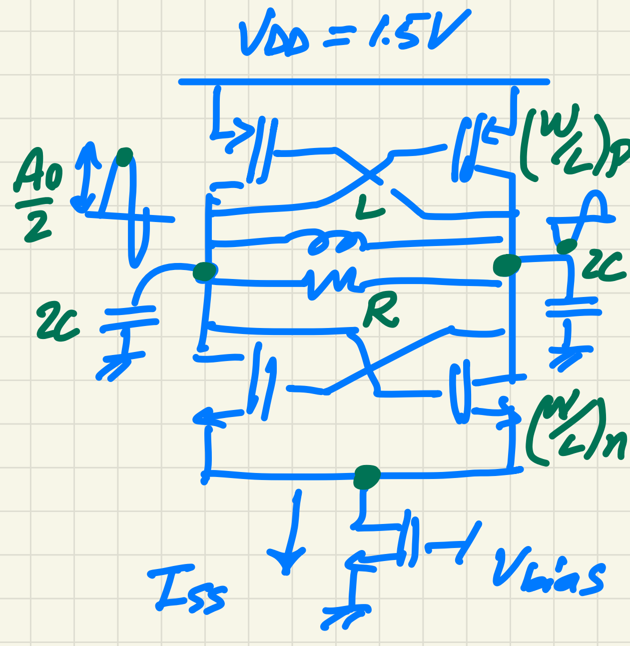
Design of an LC oscillator

MOS devices: $V_T = 0.35V$

$\mu_n C_{ox} = 120\,\mu A/V^2$

$\mu_p C_{ox} = 60\,\mu A/V^2$

Specifications:
- $f_0 = 1.5\,GHz$
- $Q = 20$
- $I_{SS} = 3\,mA$
- max. FoM



Unknowns : $(W/L)_n$, $(W/L)_p$, R, L, C, A_0, $\mathcal{L}$

Design ① Startup : $\angle LG(j\omega_0) = EG$

$$G_m R = EG > 1$$

we choose Excess gain $EG = 5$

② max FoM $\div \frac{2\eta}{kT} \cdot Q^2 \iff$ max. η

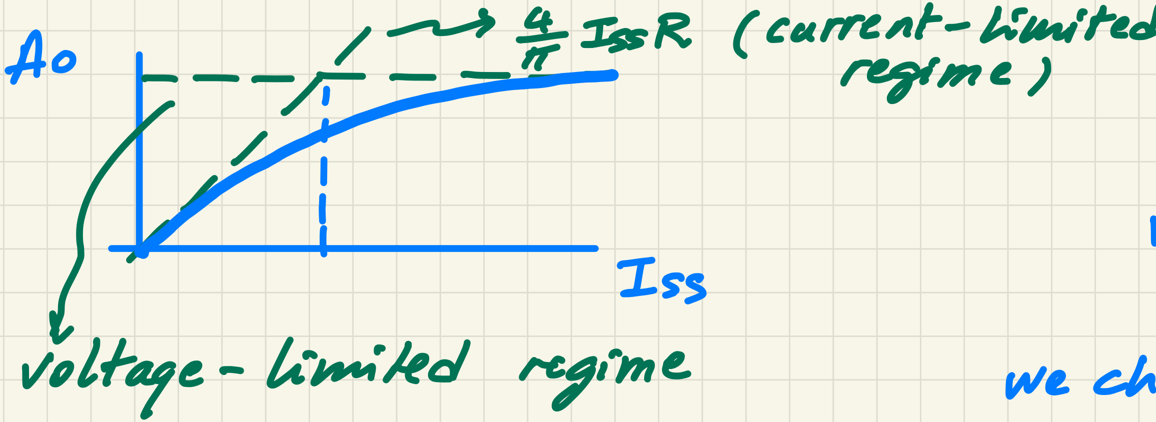
$$\eta = \frac{P_R}{P_{DC}} = \frac{A_0^2/2R}{I_{SS} \cdot V_{DD}} \iff \text{max. } A_0$$

Oscillation amplitude : $\frac{4}{\pi} \cdot \frac{I_{SS}}{A_0} R = 1$;

$$G_{m_n} \cdot R = 1$$

$$A_0 = \frac{4}{\pi} I_{SS} \cdot R$$

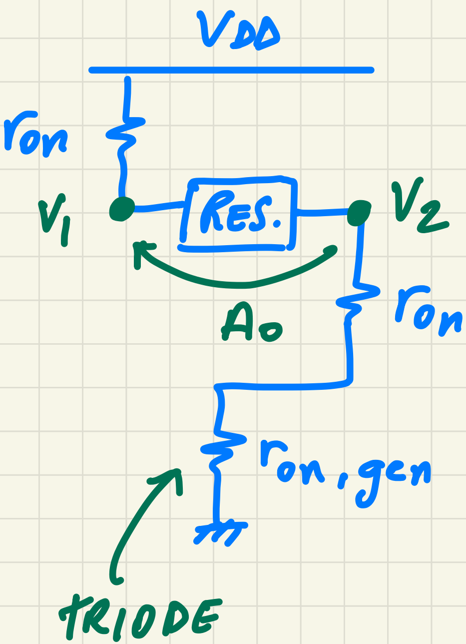
** only in current-limited



$V_{DD} = 1.5V$

$\downarrow$

we choose $A_0 = 0.9V$



$$A_0 \simeq V_{DD} \cdot \frac{R}{R + 2r_{on} + r_{on,gen}}$$

In a Spreadsheet :

Data

f_o	1.5 GHz
Q	10
V_{DD}	1.5 V
$\mu C_{ox}'$	$\left\{ \begin{matrix} 120 \\ 60 \end{matrix} \right.$ $\mu A/V^2$
V_T	0.35 V
I_{SS}	3 mA
• EG	5
• A_o	0.9 V

Equations

$$R = \frac{\pi}{4} \frac{A_o}{I_{SS}} \qquad 236\,\Omega$$

$$L = \frac{R}{\omega_o Q} \qquad 1.25\,nH$$

$$C = \frac{1}{\omega_o^2 L} \qquad 9\,pF$$

$$G_m = EG \cdot \frac{1}{R} \qquad 2.12\,\frac{mA}{V}$$

$$Q = \frac{R}{\omega_o L}, \quad \omega_o = \frac{1}{\sqrt{LC}}$$

$$G_m = \frac{g_{mn} + g_{mp}}{2}$$

Hyp. $g_{mn} = g_{mp} = g_m \Rightarrow G_m = g_m = \sqrt{2\mu C_{ox}' \frac{W}{L} \underbrace{I_{ss}/2}_{\text{DC curr.}}}$

$$\left(\frac{W}{L}\right)_n = \frac{g_m^2}{\cancel{2}\mu_n C_{ox}' \, I_{ss}/\cancel{2}} \qquad 1250$$

$$\left(\frac{W}{L}\right)_p = \frac{g_m^2}{\cancel{2}\mu_p C_{ox}' \, I_{ss}/\cancel{2}} \qquad 2500$$

MOSFET length
$L = 60\,nm$

W_n $\qquad$ $75\,\mu m$

W_p $\qquad$ $150\,\mu m$

$$\mathcal{L}(\underset{2\pi \cdot 1M\,rad/s}{\Delta\omega}) = 10 \log_{10} \left\{ \frac{\overbrace{KT}^{4\cdot 10^{-21} J} \cdot R}{A_0^2} \cdot \frac{\omega_0^2}{Q^2} \cdot \frac{1}{\Delta\omega^2} \cdot \underset{1}{F_a} \right\}$$

$-142\,dBc/Hz$

Exercise :

- calculate the power efficiency η
- calculate the FoM