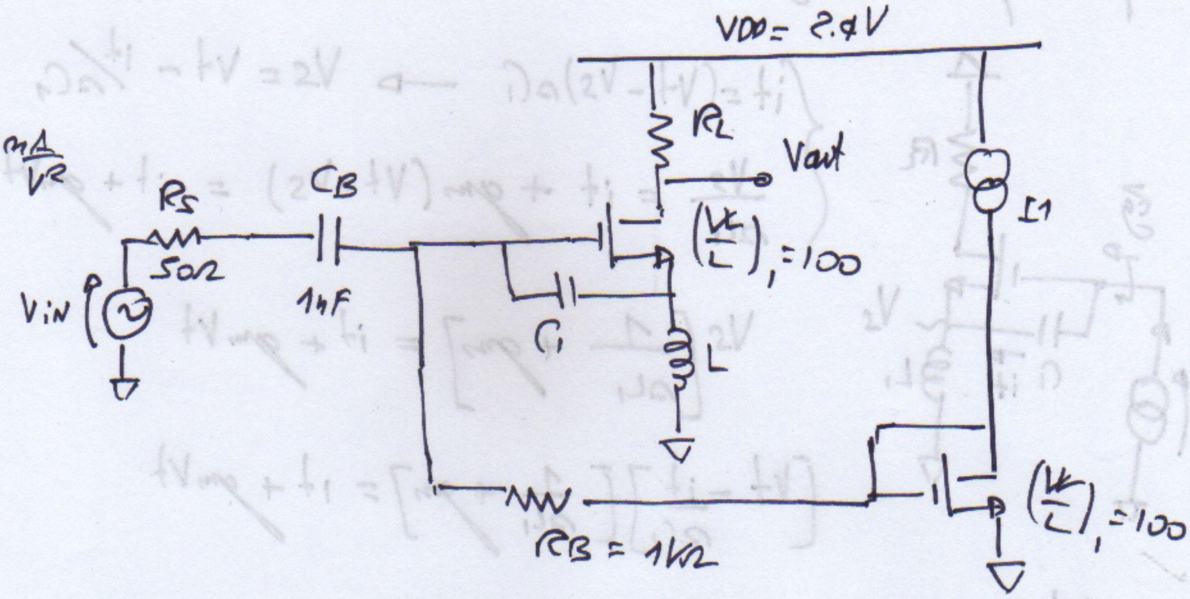
Exercise T11.1

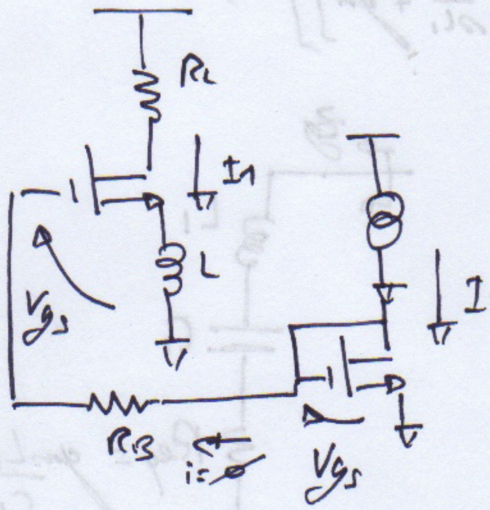
$V_T = 0.5V$

$\frac{1}{2}\mu_n C'_{ox} = 0.1 \frac{mA}{V^2}$

$\frac{\gamma}{\alpha} = \frac{2}{3}$



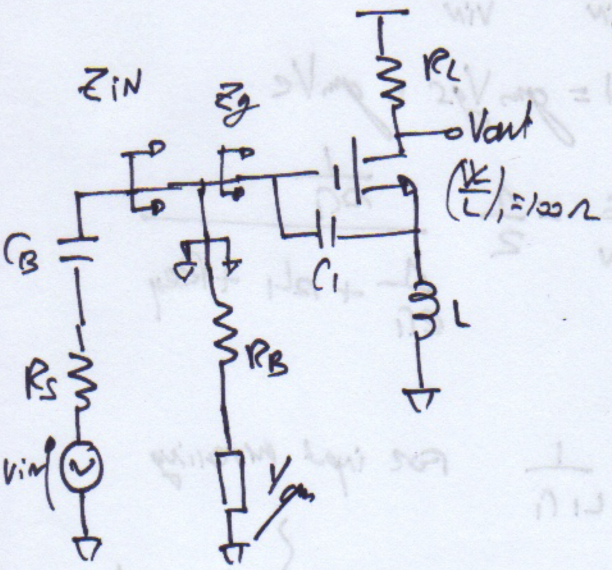
a) - BIAS point:



$$g_m = 2\sqrt{\frac{1}{2}\mu_n C'_{ox} \left(\frac{W}{L}\right)_1 I_1}$$

- DERIVE the ratio $\frac{V_{out}}{V_{in}}$ @ $f_0 = 2.4\,GHz$ assuming input MATCHING. $Z_{IN} = 50\Omega$

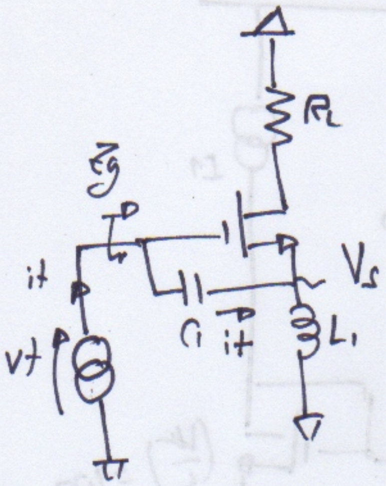
- AC Model:



$$Z_{IN} = Z_g \,//\, [\underbrace{R_B}_{1k\Omega} + 1/g_m] = 50\Omega$$

$\simeq Z_g$

• input impedance Z_g.



$$\begin{cases} i_t = (V_t - V_s)\,sC_1 \longrightarrow V_s = V_t - \dfrac{i_t}{sC_1} \\ \dfrac{V_s}{sL_1} = i_t + g_m(V_t - V_s) = i_t + g_m V_t - g_m V_s \end{cases}$$

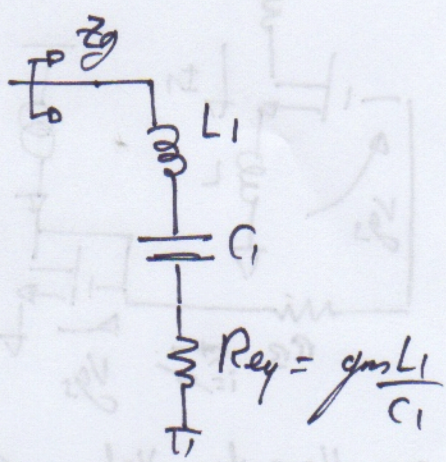
$$V_s\left[\frac{1}{sL_1} + g_m\right] = i_t + g_m V_t$$

$$\left[V_t - \frac{i_t}{sC_1}\right]\left[\frac{1}{sL_1} + g_m\right] = i_t + g_m V_t$$

$$V_t\left[\frac{1}{sL_1} + g_m - g_m\right] = i_t\left[1 + \frac{1}{sC_1}\left[\frac{1}{sL_1} + g_m\right]\right]$$

$$Z_g = \frac{V_t}{i_t} = sL_1\left[1 + \frac{1}{sC_1}\left(\frac{1}{sL_1} + g_m\right)\right]$$

$$= sL_1 + \frac{1}{sC_1} + g_m\frac{L_1}{C_1}$$

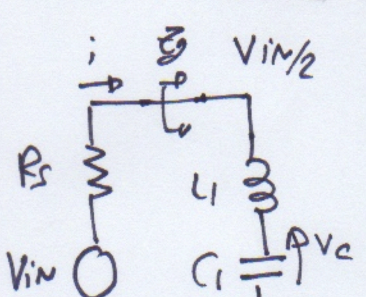


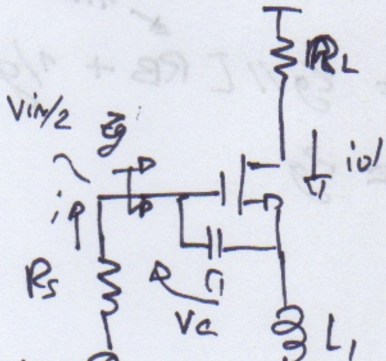
• input MATCHING : $Z_{IN} \simeq Z_g = 50\Omega$

$$w_0 = 2\pi f_0 = \frac{1}{\sqrt{L_1 C_1}}$$

$$R_{eq} = 50\Omega = g_m\frac{L_1}{C_1}$$

• COMPLETE model:





$$\frac{V_{out}}{V_{IN}} = -\frac{i_d}{V_{IN}} \cdot R_L$$

$$i_d = g_m V_{gs} = g_m V_C$$

$$\frac{V_C}{V_{IN}} = \frac{1}{2}\,\frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + sL_1 + R_{eq}}$$

$$\frac{V_C}{V_{IN}} = \frac{1}{2}\,\frac{1}{s^2L_1C_1 + sC_1R_{eq} + 1} \qquad w_0^2 = \frac{1}{L_1C_1} \quad \text{FOR input MATCHING}$$

$s = Jw_0$

$$\frac{V_C}{V_{IN}} = \frac{1}{2}\,\frac{1}{\underbrace{-w_0^2L_1C_1}_{=-1} + Jw_0C_1R_{eq} + 1} = \frac{1}{2J}\,\frac{1}{w_0C_1R_{eq}} = \frac{1}{2}\,\frac{Q_C}{J} \quad \Bigg\} \; Q_C = \frac{1}{w_0C_1R_{eq}}$$

$$\frac{V_{out}}{V_{in}} = -\frac{V_c}{V_{in}} \cancel{g_m} R_L = -\frac{1}{2}\ \frac{1}{\omega_0 C_1 R_{eq}} \cdot \cancel{g_m} R_L = -\frac{1}{2} Q_c \cancel{g_m} R_L$$

b) $R_L = 600\Omega$ $\quad L_1, C_1, I_1$ $\quad$ (i) $Z_{in} = 50\Omega$

(ii) $\frac{V_{out}}{V_{in}} = 12\,dB$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{2}\ \frac{1}{\omega_0 \cancel{C_1} \frac{\cancel{g_m} L_1}{\cancel{C_1}}} \cdot \cancel{g_m} R_L = \frac{1}{2}\ \frac{1}{\omega_0 L_1} R_L = 10^{12/20}$$

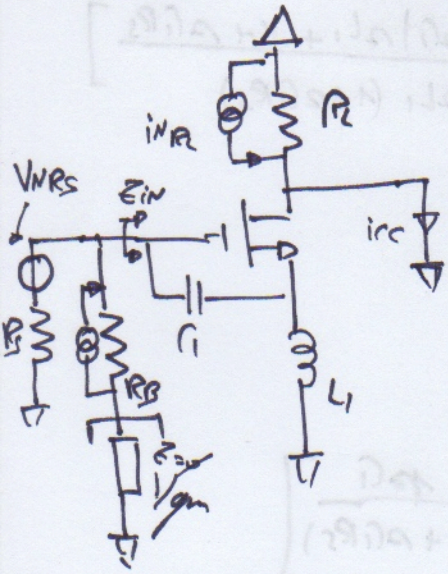
$$\rightarrow L_1 = \frac{1}{2}\ \frac{1}{\omega_0}\ \frac{R_L}{10^{12/20}} = 4,997\,nH \simeq 5nH$$

$$C_1 = \frac{1}{L_1}\ \frac{1}{(2\pi f_0)^2} = 880\,fF$$

$$R_{eq} = \cancel{g_m} \frac{L_1}{C_1} \rightarrow \cancel{g_m} = R_{eq} \frac{C_1}{L_1} = 8,795\ mA/V$$

$$\rightarrow I_d = \frac{(\cancel{g_m}/2)^2}{\frac{1}{2}\mu_n C'_{ox} \left(\frac{W}{L}\right)} = 1,934\ mA$$

c) NF @ f_0. [Neglect the bias transistor contribution]

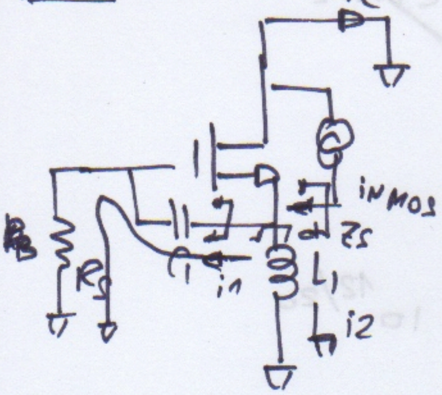


$\boxed{R_L}$ $\quad S_{icc}^{R_L} = \frac{4kT}{R_L}$

$\boxed{R_S}$ $\quad S_{icc}^{R_S} = 4kTR_S \left(\frac{1}{2} Q_c \cancel{g_m}\right)^2$

$\boxed{R_B}$ $\quad S_{icc}^{R_B} = \frac{4kT}{R_B} \left(\frac{R_S}{2}\right)^2 Q_c^2 \cancel{g_m}^2$

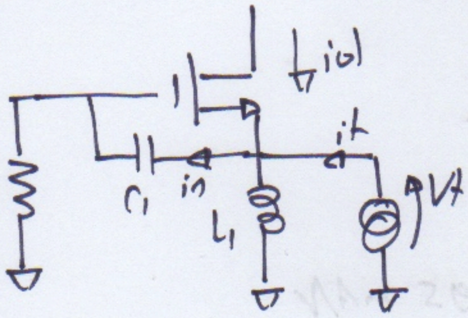
$\boxed{\text{MOS}}$



$$i_{CC} = -i_{NMOS} \cdot \frac{Z_S}{sL_1 \,//\, \left[\frac{1}{sC_1} + R_S\right]}$$

$$i_{CC} = -[i_1 + i_2]$$

• EVALUATE THE IMPEDANCE Z_S:



$$i_t + i_d = i_1 + \frac{V_t}{sL_1}$$

$$i_1 = \frac{V_t}{R_S + \frac{1}{sC_1}} = \frac{V_t sC_1}{(1+sC_1R_S)}$$

$$i_d = g_m V_{gs} = g_m\left[-i_1 \frac{1}{sC_1}\right]$$

$$= -\frac{g_m}{\cancel{sC_1}} \frac{V_t \cancel{sC_1}}{(1+sC_1R_S)} = -g_m \frac{V_t}{(1+sC_1R_S)}$$

$$i_t - \frac{g_m V_t}{(1+sC_1R_S)} = \frac{V_t}{sL_1} + \frac{V_t sC_1}{(1+sC_1R_S)}$$

$$i_t = V_t\left[\frac{g_m}{(1+sC_1R_S)} + \frac{1}{sL_1} + \frac{sC_1}{(1+sC_1R_S)}\right] = V_t\left[\frac{(g_m+sC_1)sL_1 + 1 + sC_1R_S}{sL_1(1+sC_1R_S)}\right]$$

$$Z_S = \frac{V_t}{i_t} = \frac{sL_1(1+sC_1R_S)}{[s^2C_1L_1 + s(C_1R_S + g_mL_1) + 1]}$$

$$\frac{i_{CC}^{MOS}}{i_{NMOS}} = -\frac{Z_S}{\left(\dfrac{1}{\left[\frac{1}{sL_1} + \frac{1}{(R_S + \frac{1}{sC_1})}\right]}\right)} = -Z_S\left[\frac{1}{sL_1} + \frac{sC_1}{(1+sC_1R_S)}\right]$$

$$\frac{i_{cc}}{i_{N\,mos}} = -\frac{sL_1(1+sC_1R_S)}{\left[s^2L_1C_1 + s(C_1R_S + g_mL_1) + 1\right]} \cdot \frac{(1 + sC_1R_S + s^2C_1L_1)}{\cancel{sL_1(1+sC_1R_S)}}$$

$s = j\omega_0$

$$\frac{i_{cc}}{i_N} = -\frac{(1 + j\omega_0C_1R_S - \omega_0^2C_1L_1)}{\left[-\omega_0^2L_1C_1 + j\omega_0(C_1R_S + g_mL_1) + 1\right]} \qquad \omega_0^2 = \frac{1}{L_1C_1}$$

$$= -\frac{\cancel{j\omega_0}C_1R_S}{\cancel{j\omega_0}(C_1R_S + g_mL_1)} = \frac{R_S}{\left(R_S + \frac{g_mL_1}{C_1}\right)} = \frac{1}{2}$$

$R_{eq} = R_S$

$$S_{icc}^{mos} = 4kT\frac{\gamma}{\alpha}g_{m1}\left(\frac{1}{2}\right)^2$$

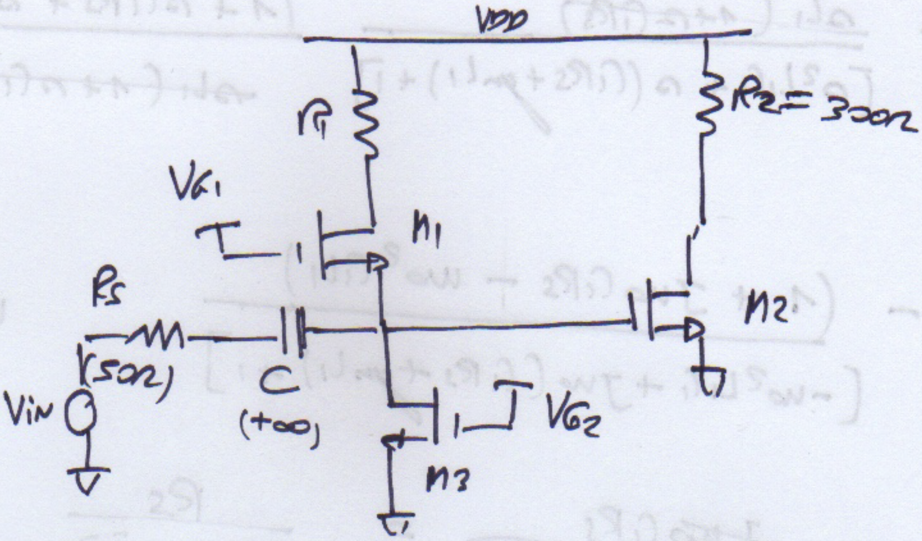
$$NF = 1 + \frac{\frac{4kT}{R_L}}{4kTR_S\left(\frac{1}{2}Q_cg_m\right)^2} + \frac{\frac{4kT}{R_B}\left(\frac{R_S}{2}\right)^2Q_c^2g_m^2}{4kTR_S\left(\frac{1}{2}Q_cg_m\right)^2} + \frac{4kT\frac{\gamma}{\alpha}g_m\left(\frac{1}{2}\right)^2}{4kTR_S\left(\frac{1}{2}Q_cg_m\right)^2}$$

$$= 1 + \frac{1}{R_LR_S\frac{1}{4}Q_c^2g_m^2} + \frac{R_S}{R_B} + \frac{(\gamma/\alpha)}{R_Sg_mQ_c^2} \qquad R_L = 600\Omega \quad R_S = 50\Omega$$

$$Q_c = \frac{1}{\omega_0R_{eq}C_1} = 1.508$$

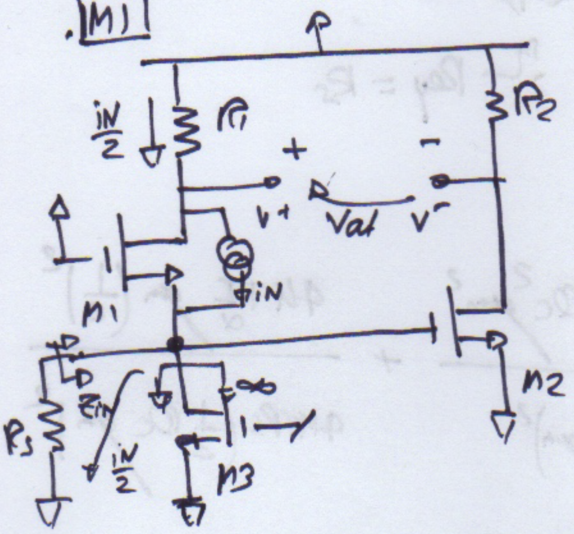
$= 3,962$ dB

Exercise T11.2



a) EXPRESSION FOR THE NF

$\boxed{M1}$



- input matching condition
  $Z_{IN} = R_S = 50\,\Omega = 1/g_{m1}$

$$V_{out} = V^+ - V^-$$

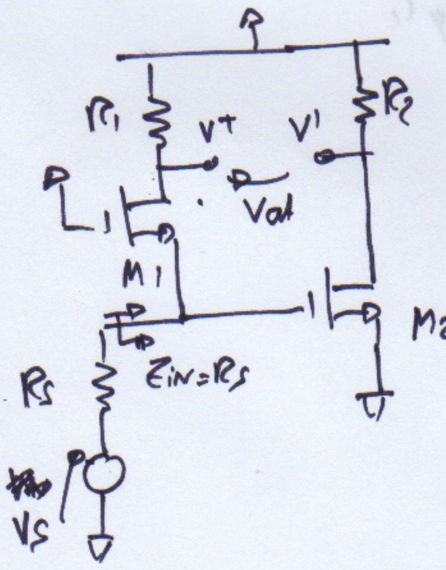
$$V^+ = -\frac{i_N}{2} R_1$$

$$V^- = \frac{i_N}{2}\left(-g_{m2} R_2\right) R_S$$

$$V_{out} = V^+ - V^- = -\frac{i_N}{2} R_1 - \left[-\frac{i_N}{2} g_{m2} R_2 R_S\right]$$

$V_{out} = 0 \rightarrow$ $-\frac{i_N}{2} R_1 + \frac{i_N}{2} g_{m2} R_2 R_S = 0 \Rightarrow \frac{R_1}{R_S} = g_{m2} R_2$

GAIN OF THE STAGE:



$$\frac{V_{out}}{V_S} = \frac{V^+}{V_S} - \frac{V^-}{V_S}$$

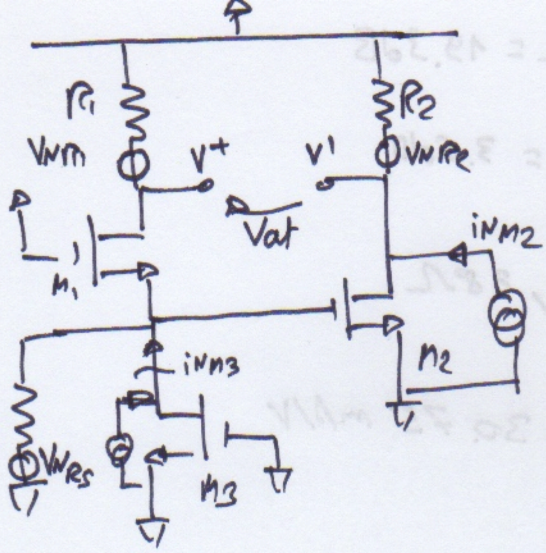
$$\frac{V^+}{V_S} = \frac{1}{(R_S + Z_{IN})} R_1 = \frac{R_1}{2R_S}$$

$$\frac{V^-}{V_S} = \frac{1}{2}\left(-g_{m2} R_2\right)$$

$$\frac{V_{out}}{V_S} = \frac{R_1}{2R_S} + \frac{g_{m2} R_2}{2} = \frac{R_1}{R_S}$$

to have balanced gain

$$\frac{R_1}{R_S} = g_{m2} R_2$$



$\boxed{R_1}$ $V_{out}^{R_1} = V^+ - V^-$

$= V_{NR_1} - 0$

$SV_{out}^{R_1} = 4kT R_1$

$\boxed{R_2}$ $V_{out}^{R_2} = V^+ - V^-$

$= 0 - V_{NR_2}$

$SV_{out}^{R_2} = 4kT R_2$

$\boxed{R_S}$ $V_{out} = V_{NR_S} \cdot \left(\frac{R_1}{R_S}\right) \longrightarrow SV_{out}^{R_S} = \frac{4kT}{R_S} R_1^2$

$\boxed{M3}$: $V_{out} = V^+ - V^-$

$= \left[\frac{i_{NM3}}{2} R_1\right] - \left[i_{NM3}\left(\frac{R_S}{2}\right)\left(-g_{m2} R_2\right)\right]$

FROM NOISE CANCELLING : $\frac{R_1}{R_S} = g_{m2} R_2$

$= \frac{i_{NM3}}{2} R_1 + \frac{i_{NM3}}{2} R_S \left(\frac{R_1}{R_S}\right) = i_{NM3} R_1$

$SV_{out}^{M3} = 4kT \frac{\gamma}{\alpha} g_{m3} (R_1)^2$

$\boxed{M2}$: $V_{out} = V^+ - V^- = 0 - [i_{N2} R_2] \Longrightarrow SV_{out}^{M2} = 4kT \frac{\gamma}{\alpha} g_{m2} R_2^2$

$$NF = 1 + \frac{4kT R_1}{\frac{4kT}{R_S} R_1^2} + \frac{4kT R_2}{\frac{4kT}{R_S} R_1^2} + \frac{4kT \frac{\gamma}{\alpha} g_{m3} R_1^2}{\frac{4kT}{R_S} R_1^2} + \frac{4kT \frac{\gamma}{\alpha} g_{m2} R_2^2}{\frac{4kT}{R_S} R_1^2}$$

$$= 1 + \frac{R_S}{R_1} + \frac{R_2 R_S}{R_1^2} + \frac{\gamma}{\alpha} g_{m3} R_S + \frac{\gamma}{\alpha} g_{m2} R_S \left(\frac{R_2}{R_1}\right)^2 \qquad \boxed{\left\{\frac{R_1}{R_S} = g_{m2} R_2\right\}}$$

$$= 1 + \frac{R_S}{R_1}\left[1 + \frac{R_2}{R_1}\right] + \frac{\gamma}{\alpha} g_{m3} R_S + \frac{\gamma}{\alpha}\left(\frac{R_2}{R_1}\right)$$

b) R_1, $\left(\frac{W}{L}\right)_1$, $\left(\frac{W}{L}\right)_2$, $\left(\frac{W}{L}\right)_3$

(i) $\frac{V_{out}}{V_{in}} = 19.3\,dB$

(ii) $NF = 3.6\,dB$

(i) $\frac{V_{out}}{V_{in}} = \frac{R_1}{R_S} = 10^{\frac{19.3\,dB}{20}} \rightarrow R_1 = 461{,}28\,\Omega$

$\frac{R_1}{R_S} = g_{m2} R_2 \rightarrow g_{m2} = \frac{R_1}{R_2 R_S} = 30.75\ mA/V$

noise cancelling

$R_2 = 300\,\Omega$

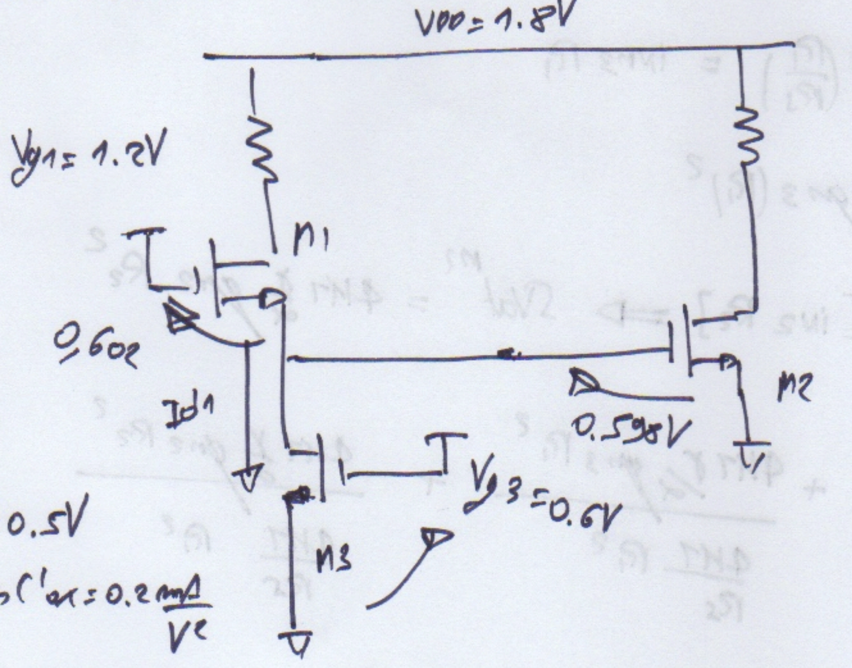
input matching

$\rightarrow g_{m1} = \frac{1}{50\,\Omega} = 20\ mA/V$

(ii) $NF = 1 + \frac{R_S}{R_1}\left[1 + \frac{R_2}{R_1}\right] + \frac{\gamma}{\alpha} g_{m3} R_S + \frac{\gamma}{\alpha}\frac{R_2}{R_1} = 10^{\frac{3.6\,dB}{10}}$

$\rightarrow g_{m3} = 20.35\ mA/V$

• bias point



$V_T = 0.5V$

$\frac{1}{2}\mu_n C'_{ox} = 0.2\ \frac{mA}{V^2}$

• $g_{m3} = \frac{2\,I_{d1}}{V_{ov3}}$

$\rightarrow V_{ov3} = V_{g3} - V_T$

$I_{d1} = 1{,}02\ mA$

$I_{d1} = \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_3 (V_{od3})^2$

$\rightarrow \left(\frac{W}{L}\right)_3 = 509$

• $g_{m1} = 2\sqrt{\frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_1 I_{d1}} \rightarrow \left(\frac{W}{L}\right)_1 = 492$

• $g_{m2} = 2\left[\frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_2 V_{od2}\right]$

$V_{gs2} = V_{g1} - V_{gs1} = 0.598V \rightarrow V_{od2} = 0.098V$

$\left(\frac{W}{L}\right)_2 = 782$

$V_{od1} = \frac{2\,I_{d1}}{g_{m1}} \rightarrow 0{,}102V \rightarrow V_{gs1} = 0{,}602V$

EXERCISE 3

a) CONSIDER CIRCUIT Fig 1.

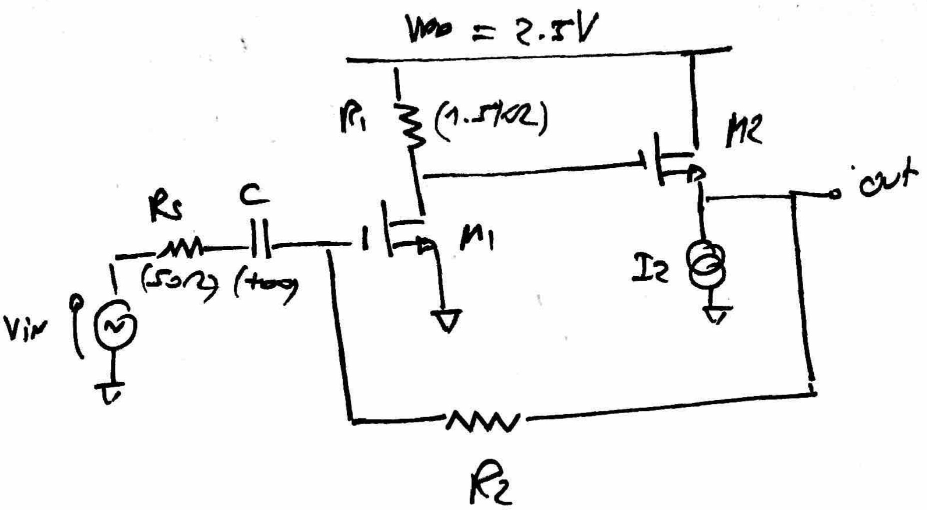
$V_T = 0.5V$

$\frac{1}{2}\mu_n C'_{ox} = 0.2\,\frac{mA}{V^2}$

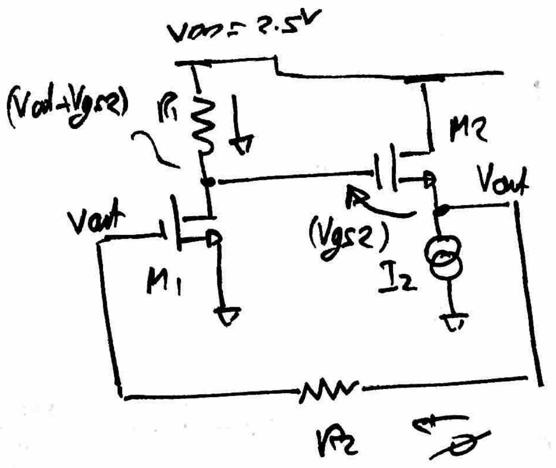
$\left(\frac{W}{L}\right)_1 = 400$

$\left(\frac{W}{L}\right)_2 = 125$

$\frac{\gamma}{\alpha} = \frac{2}{3}$



- BIAS point:



$I_2 = 1mA$

$I_2 = \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_2 (V_{od2})^2$

$\Rightarrow V_{od2} = 200mV$

$\rightarrow V_{gs2} = V_{od2} + V_T = 0.7V$

$g_{m2} = \frac{2 I_2}{V_{od2}} = 10\,mS$

$$\frac{[V_{DD} - (V_{out} + V_{gs2})]}{R_1} = \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_1 \underbrace{(V_{gs1} - V_T)^2}_{(V_{out} - V_T)^2}$$

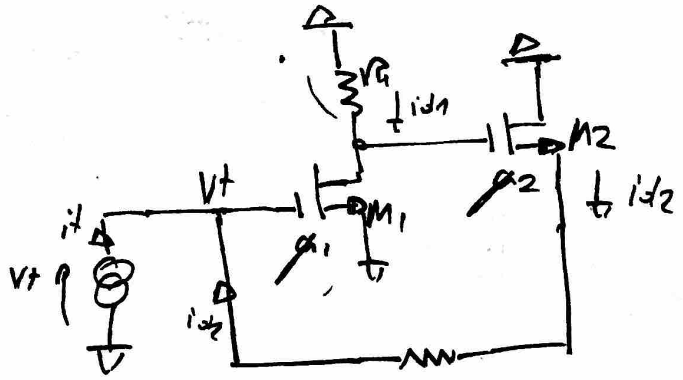
$$\frac{\overbrace{(V_{DD} - \underbrace{V_{gs2}}_{0.7V})}^{(1.8V)} - \underbrace{V_{out}}_{x}}{\underbrace{R_1}_{(1.5k\Omega)}} = \underbrace{\frac{1}{2}\mu_n C'_{ox}}_{0.2\frac{mA}{V^2}}\underbrace{\left(\frac{W}{L}\right)_M}_{400}(\underbrace{V_{out}}_{x} - \underbrace{V_T}_{0.5V})^2$$

$\Rightarrow V_{out} = 0.6V$

$$I_{d1} = \frac{[V_{DD} - (V_{out} + V_{gs2})]}{R_1} = 800\mu A$$

$$g_{m1} = 2\sqrt{\underbrace{\frac{1}{2}\mu_n C'_{ox}}_{0.2\frac{mA}{V^2}}\underbrace{\left(\frac{W}{L}\right)_1}_{400}\underbrace{I_{d1}}_{800\mu A}} = 16\,mS$$

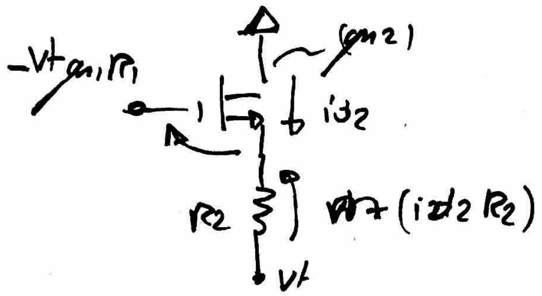
• input ADMITTANCE :



$i_t = -i_{d2}$

$i_{d1} = g_{m1} V_t$

$V_{g2} = -V_t g_{m1} R_1$



$i_{d2} = g_{m2}[-V_t g_{m1} R_1 - [V_t + R_2 i_{d2}]]$

$-i_t = g_{m2} g_{m1} R_1 V_t - V_t g_{m2} + g_{m2} R_2 i_t \Rightarrow Y_{IN} = \frac{i_t}{V_t} = \frac{g_{m2}[1+g_{m1}R_1]}{[1+g_{m2}R_2]}$

to guarantee MATCHING:

$Y_{IN} = \frac{1}{50\Omega} = \frac{g_{m2}[1+g_{m1}R_1]}{(1+g_{m2}R_2)}$

$g_{m1} = 16 mA/V$

$R_1 = 1.5 k\Omega$

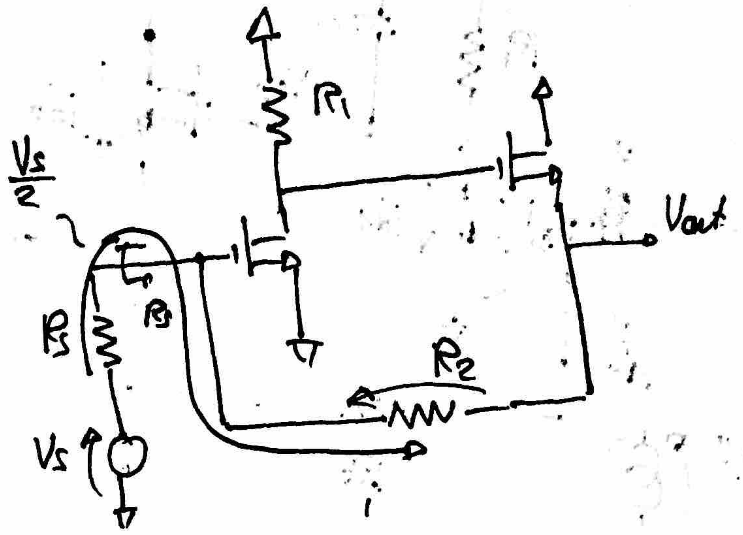
$R_2 = ?$

$g_{m2} = 10 mA/V$

$(R_2 = 1{,}15 k\Omega)$

b) ASSUMING input MATCHING:

• GAIN $\frac{V_{out}}{V_s}$



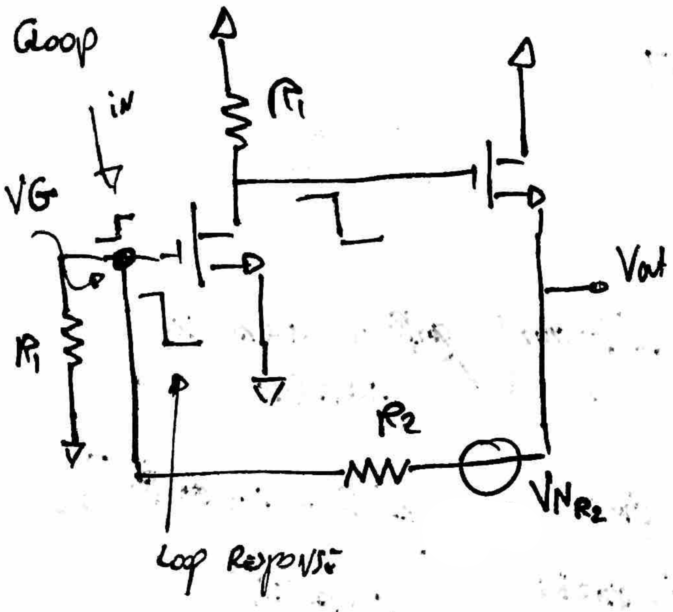
drop across Rs

$$i = \frac{(V_s - V_s/2)}{R_s} = \frac{V_s}{2R_s}$$

$$V_{out} = \frac{V_s}{2} - iR_2$$

$$= V_s\left[\frac{1}{2} - \frac{R_2}{2R_s}\right] \Longrightarrow \frac{V_{out}}{V_s} = \frac{1}{2}\left[1 - \frac{R_2}{R_s}\right]$$

• NF due to R_2 and R_s.



Vg = VIRTUAL GROUND NODE

• $V_{out}^{R_2} = V_{NR_2}$

All the noise is transferred to the output since Vg is a virtual ground node.

$$NF = 1 + \frac{4kTR_2}{4kTR_s\left(\frac{1}{2}\left[1 - \frac{R_2}{R_s}\right]\right)^2}$$

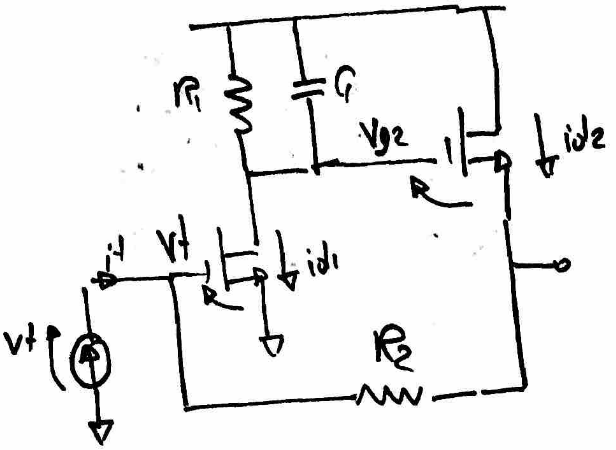
$$= 1 + \frac{R_2}{R_s}\frac{1}{\left[\frac{1}{2}\left(1 - \frac{R_2}{R_s}\right)\right]^2}$$

$$= 0.756\ dB$$

$R_2 = 1.15k\Omega$

$R_s = 50\Omega$

c) EXPRESSION FOR input ADMITTANCE Y_{IN}



$$i_{d2} = g_{m2}\left(V_{g2} - [i_{d2}R_2 + V_t]\right) =$$

$$V_{g2} = V_t\, g_{m1}\, \frac{R_1}{(1+sC_1R_1)} (-1)$$

$$i_{d2} = -i_t$$

$$-i_t = -g_{m2}V_t\, g_{m1}\frac{R_1}{(1+sC_1R_1)} + g_{m2}\, i_t R_2 - V_t\, g_{m2}$$

$$i_t[1+g_{m2}R_2] = V_t\left[g_{m2}\, g_{m1}\frac{R_1}{(1+sC_1R_1)} + g_{m2}\right]$$

$$i_t(1+g_{m2}R_2)\frac{1}{g_{m2}} = V_t\left[\frac{g_{m1}R_1 + (1+sC_1R_1)}{1+sC_1R_1}\right]$$

$$i_t\left(1/g_{m2} + R_2\right) = \frac{V_t}{(1+sC_1R_1)}\left(g_{m1}R_1 + 1 + sC_1R_1\right)$$

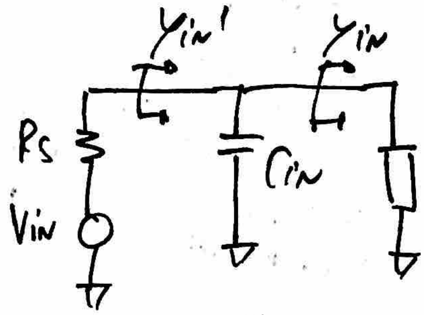
$$s = j\omega_0$$

$$Y_{IN} = \frac{i_t}{V_t} = \frac{(1+g_{m1}R_1 + j\omega_0 C_1R_1)}{(1/g_{m2}+R_2)(1+j\omega_0 C_1R_1)} \frac{(1-j\omega_0C_1R_1)}{(1-j\omega_0C_1R_1)}$$

$$= \frac{1+g_{m1}R_1 \cancel{+ j\omega_0C_1R_1} \cancel{- j\omega_0C_1R_1} - j\omega_0C_1R_1^2 g_{m1} + \omega_0^2(C_1R_1)^2}{(1/g_{m2}+R_2)(1+\omega_0^2C_1^2R_1^2)}$$

$$= \frac{(1+g_{m1}R_1 + \omega_0^2(C_1R_1)^2) - j(\omega_0C_1R_1^2 g_{m1})}{\left[(1/g_{m2}+R_2)(1+\omega_0^2C_1^2R_1^2)\right]}$$

- to guarantee input matching:



$$\frac{1}{R_S} = \mathrm{Re}\{Y_{in}'\}$$

$$\mathrm{Im}\{Y_{IN}'\} = 0$$

$$\frac{1}{R_S} = \frac{1 + g_{m1}R_1 + \omega_0^2 C_1 R_1}{\left(1/g_{m2} + R_2\right)\left(1 + \omega_0^2 C_1^2 R_1^2\right)}$$

$$\underset{\left(\omega_0 \ll \frac{1}{C_1 R_1}\right)}{\simeq} \boxed{\frac{(1 + g_{m1}R_1)}{(1/g_{m2} + R_2)} = \frac{1}{R_S}}$$

(same condition as point a))

$$\mathrm{Im}\{Y_{in}'\} = 0 \rightarrow \omega_0 C_{iN} - \frac{\omega_0 C_1 R_1^2 g_{m1}}{(1/g_{m2} + R_2)(1 + \omega_0^2 C_1^2 R_1^2)} = 0$$

$$C_{IN} = \frac{C_1 R_1^2 g_{m1}}{(1/g_{m2} + R_2)(1 + \omega_0^2 C_1^2 R_1^2)} \underset{\omega_0 \ll \frac{1}{C_1 R_1}}{\simeq} \frac{C_1 R_1^2 g_{m1}}{(1/g_{m2} + R_2)}$$

$$\boxed{C_1 = C_{iN} \frac{(1/g_{m2} + R_2)}{R_1^2 g_{m1}}}$$