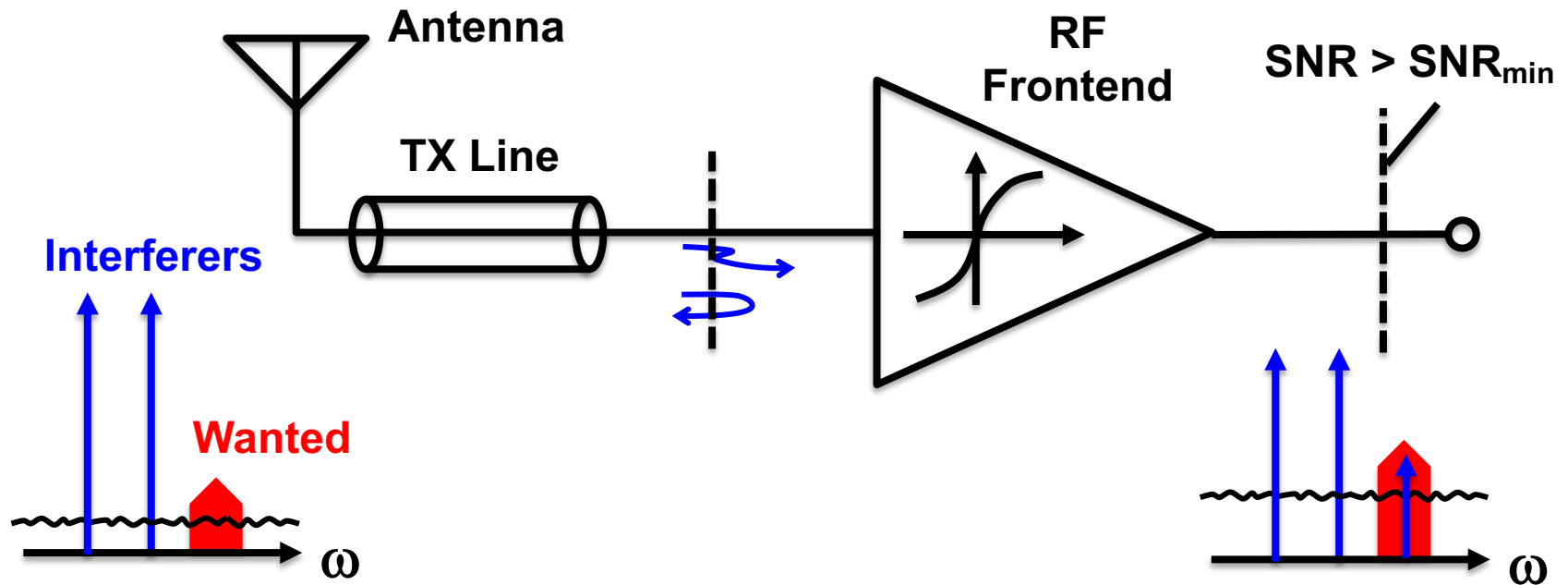

Basics of RF Systems

Prof. Salvatore Levantino
Politecnico di Milano

RF Receivers

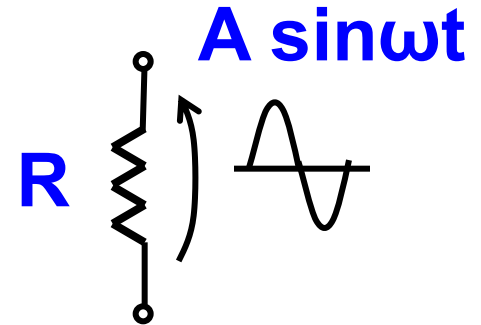


- **Max. tolerable signal** limited by nonlinearity
- **Min. detectable signal** limited by impedance matching, noise nonlinearity

Amplitude and Power

- Amplitude A [V]: $20 \log_{10}(A)$

- 1V (0dBV)



- Power P [W] : $10 \log_{10}(P)$

- 1W (30dBm) \longrightarrow 10Vp (20dBV)

- 10μW (-20dBm) \longrightarrow 31.6mVp (-30dBV)

if $R = 50\Omega$

Sinusoid with zero-peak amplitude A: $P = \frac{A^2}{2R}$

Effects of Non-linearity

Memoryless and Dynamic Systems



- **Memoryless** nonlinear systems

$$y = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \dots$$

with coefficients depending on time if time-variant

- **Dynamic** systems

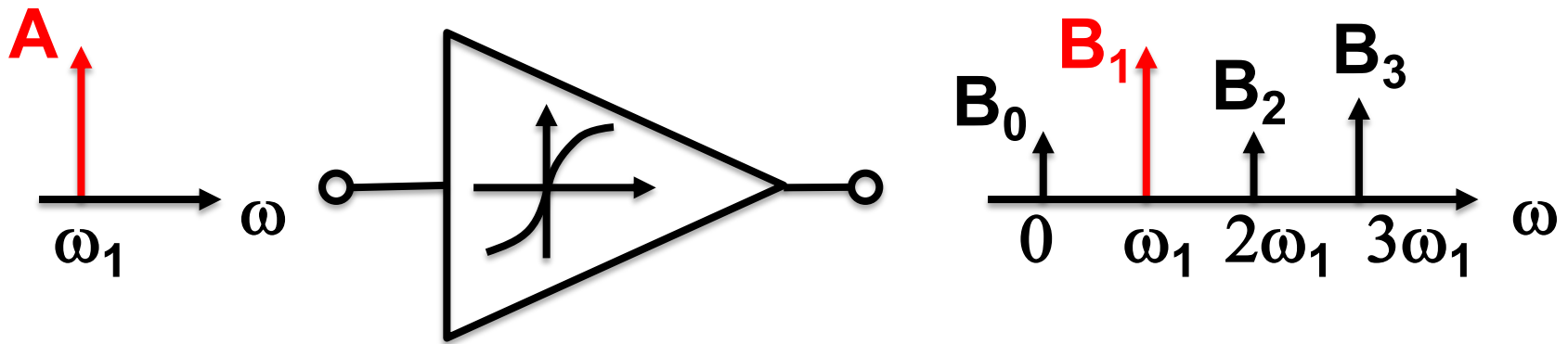
- **LTI**: $y(t) = h(t) * x(t)$
- **LTV**: $y(t) = h(t, \tau) * x(t)$
- **Nonlinear**: $h(t)$ approximated with Volterra series

Effects of Nonlinearity

- **Case 1: Single tone at the input**
 - Harmonic generation
 - Gain compression
- Case 2: Two tones at the input
 - Blocking
 - Third-order intermodulation

Harmonic Generation

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$



$$B_0 = \alpha_2 A^2 / 2$$

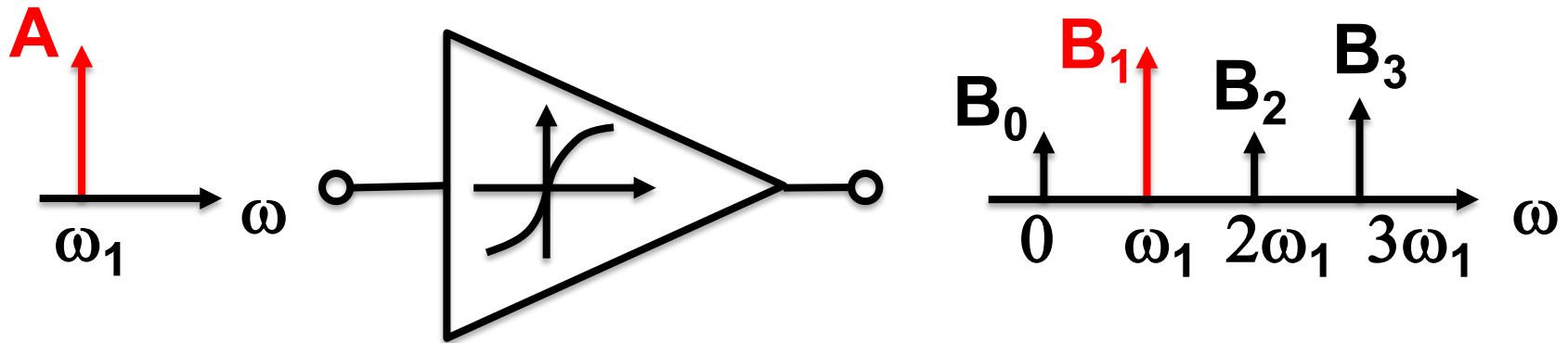
$$B_2 = \alpha_2 A^2 / 2$$

$$B_1 = \alpha_1 A + 3\alpha_3 A^3 / 4$$

$$B_3 = \alpha_3 A^3 / 4$$

- $B_{2n} = 0$ for fully-differential. Mismatches...
- B_n approx. prop. to A^n

Gain Compression

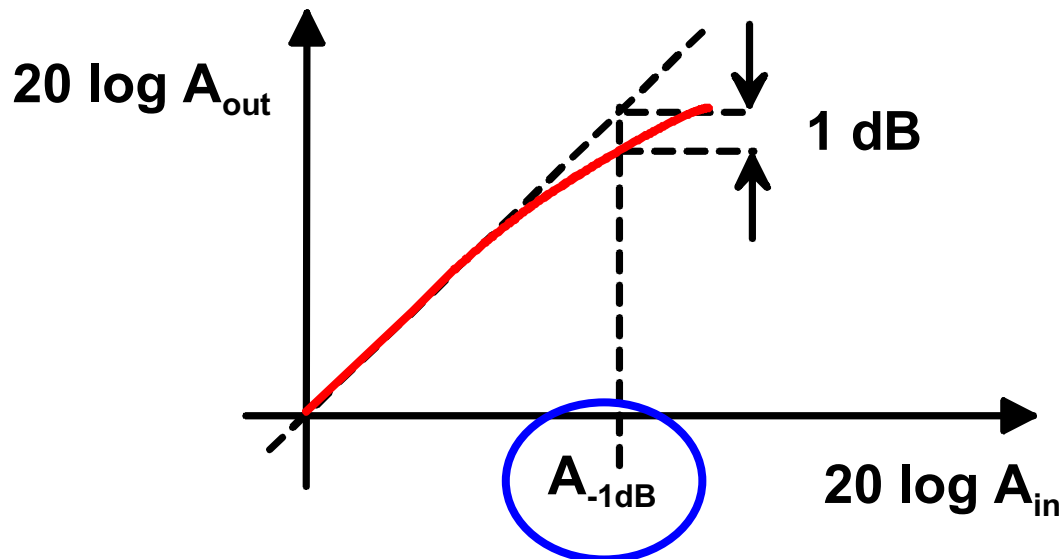


$$B_1 = \left(\alpha_1 + \underbrace{3\alpha_3 A^2 / 4}_{\downarrow} \right) A$$

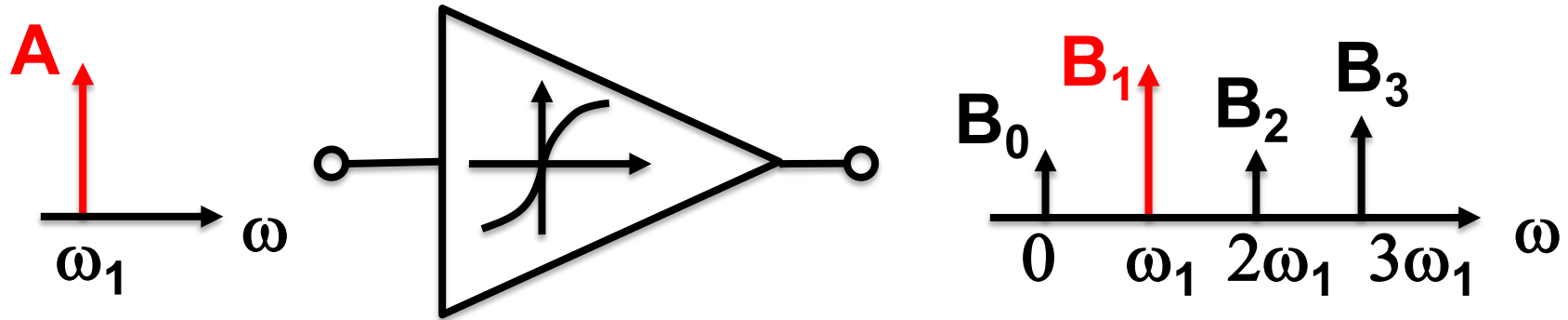
For $\alpha_3 < 0$, decreasing function of A

Definition of “1-dB Compression Point”

- $A_{-1\text{dB}}$ is the input amplitude at which the output is 1 dB less than the ideal one
- Quantifies the gain compression
- Measures the input full-scale range



1-dB Compression Point



$$B_1 = \left(a_1 + \frac{3a_3 A^2}{4} \right) \cdot A \rightarrow \frac{a_1 A_{-1\text{dB}}}{a_1 A_{-1\text{dB}} + \frac{3}{4} a_3 A_{-1\text{dB}}^3} = 10^{\frac{1}{20}}$$

$$\rightarrow A_{-1\text{dB}} = \sqrt{\underbrace{\left(1 - 10^{-\frac{1}{20}} \right)}_{\sim 0.11} \cdot \frac{4}{3} \frac{|a_1|}{|a_3|}}$$

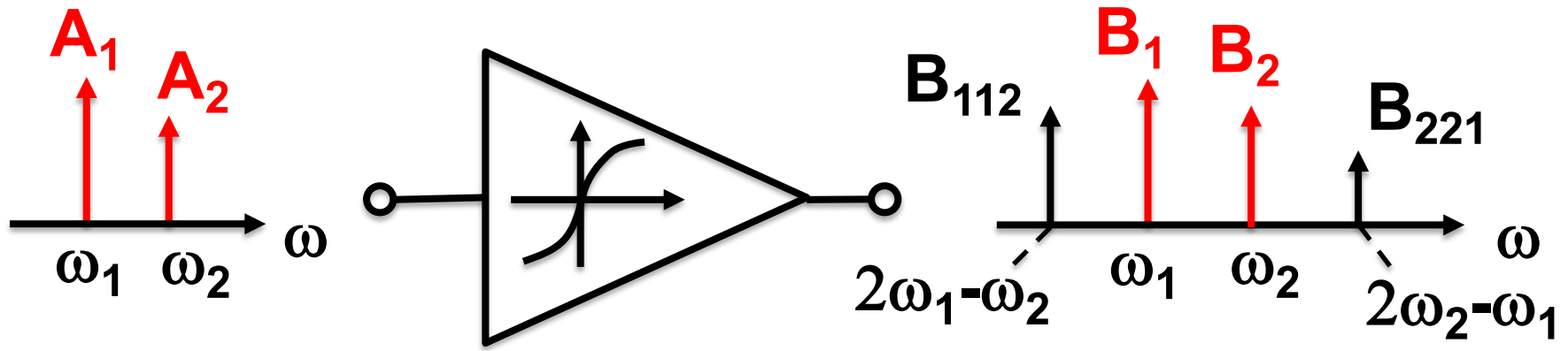
$$\rightarrow 20 \log_{10}(A_{-1\text{dB}}) = -9.6 \text{ dB} + 10 \log_{10}\left(\frac{4}{3} \frac{|a_1|}{|a_3|} \right)$$

Effects of Nonlinearity

- Case 1: Single tone at the input
 - Harmonic generation
 - Gain compression

- **Case 2: Two tones at the input**
 - **Blocking**
 - **Third-order intermodulation**

Intermodulation



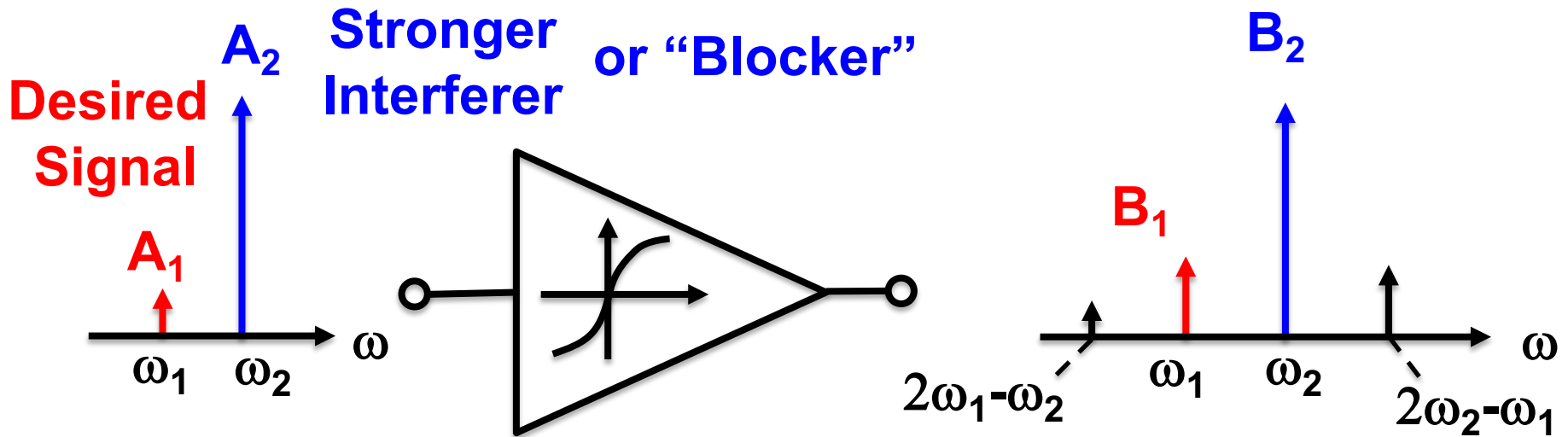
$$B_1 = \alpha_1 A_1 + 3\alpha_3 A_1^3 / 4 + 3\alpha_3 A_1 A_2^2 / 2$$

$$B_{112} = 3\alpha_3 A_1^2 A_2 / 4$$

$$B_2 = \alpha_1 A_2 + 3\alpha_3 A_2^3 / 4 + 3\alpha_3 A_2 A_1^2 / 2$$

$$B_{221} = 3\alpha_3 A_2^2 A_1 / 4$$

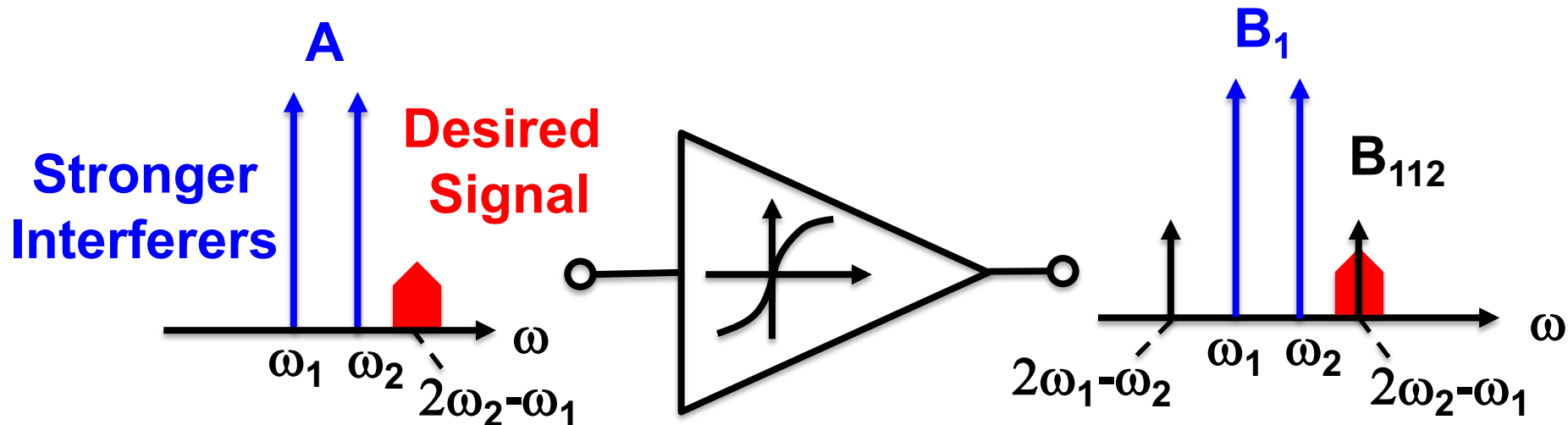
Desensitization and Blocking



$$B_1 = \alpha_1 A_1 + 3\alpha_3 A_1^3 / 4 + 3\alpha_3 A_1 A_2^2 / 2 \approx A_1 \left(\alpha_1 + \underbrace{3\alpha_3 A_2^2 / 2}_{\text{blocking term}} \right)$$

- For $\alpha_3 < 0$, decreasing function of A_2
- Signal is "blocked" when gain is zero

Intermodulation (IM)

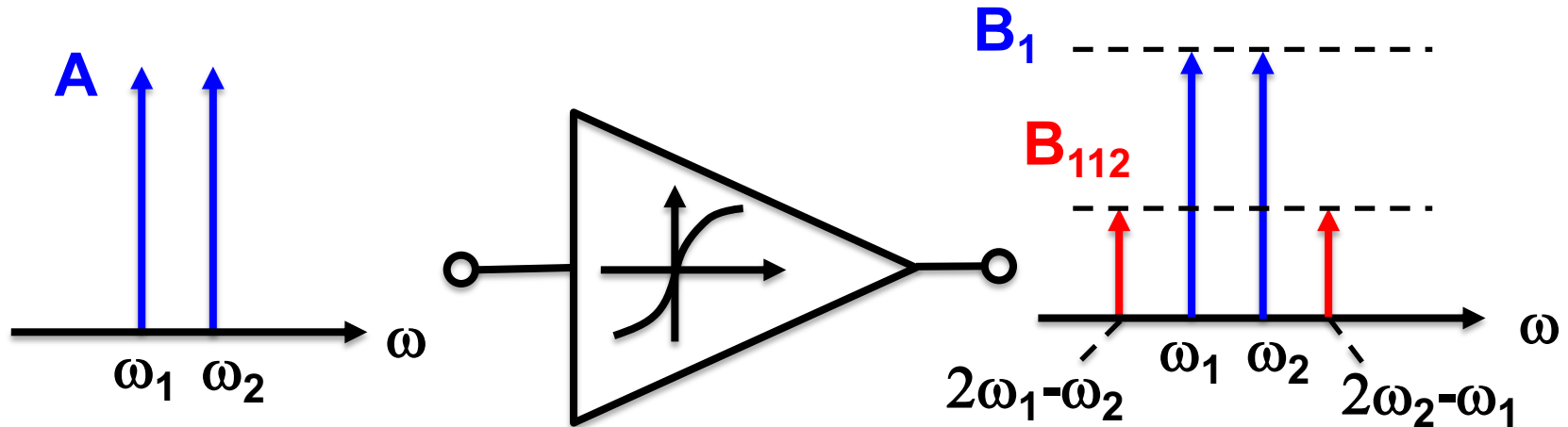


$$B_1 = \alpha_1 A + 3\alpha_3 A^3/4 + 3\alpha_3 A^3/2 \approx \alpha_1 A$$

$$B_{112} = 3\alpha_3 A^3/4$$

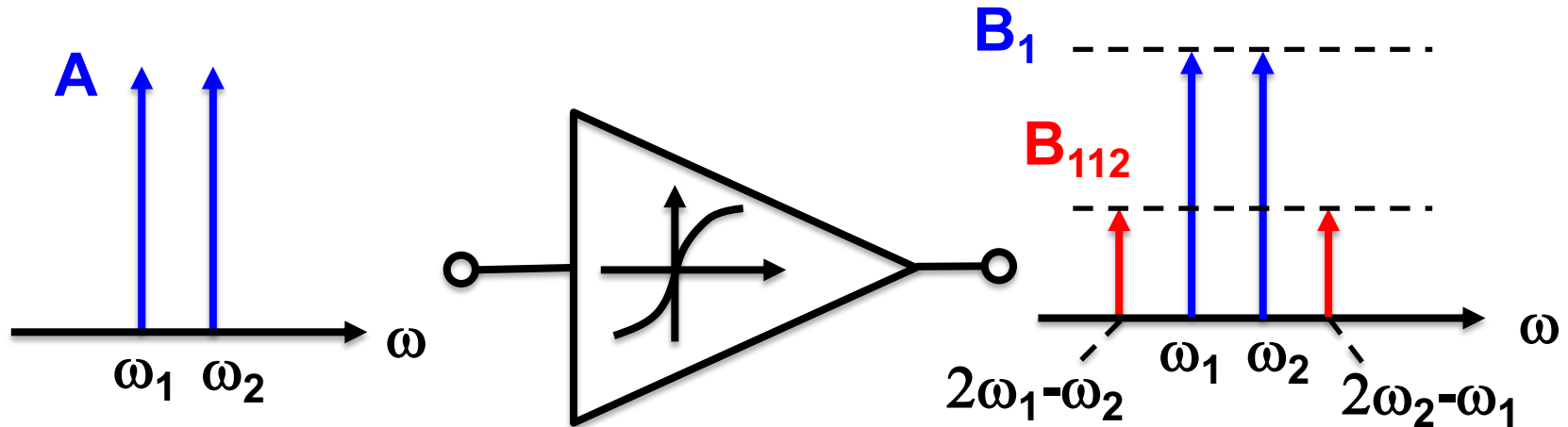
- The third-order inter-modulation (IM3) product corrupts the desired signal

Third-Order Intercept Point (IP3)



- **The input-referred intercept point (A_{IP3}) is the input amplitude at which the IM3 products have the same amplitude as the fundamental tones**

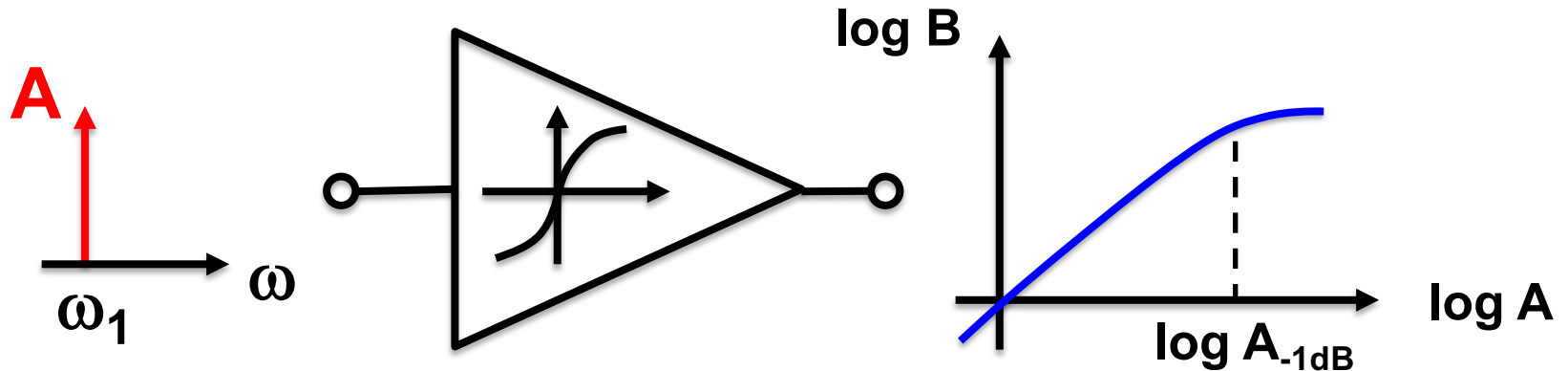
IIP3 and Memory-less Model



$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$\begin{cases} B_1 = \left(\alpha_1 + 9\alpha_3 A^2 / 4 \right) A \\ B_{112} = 3\alpha_3 A^3 / 4 \end{cases} \Rightarrow A_{\text{IIP}_3} \approx \sqrt{\frac{4}{3} \cdot \frac{|\alpha_1|}{|\alpha_3|}}$$

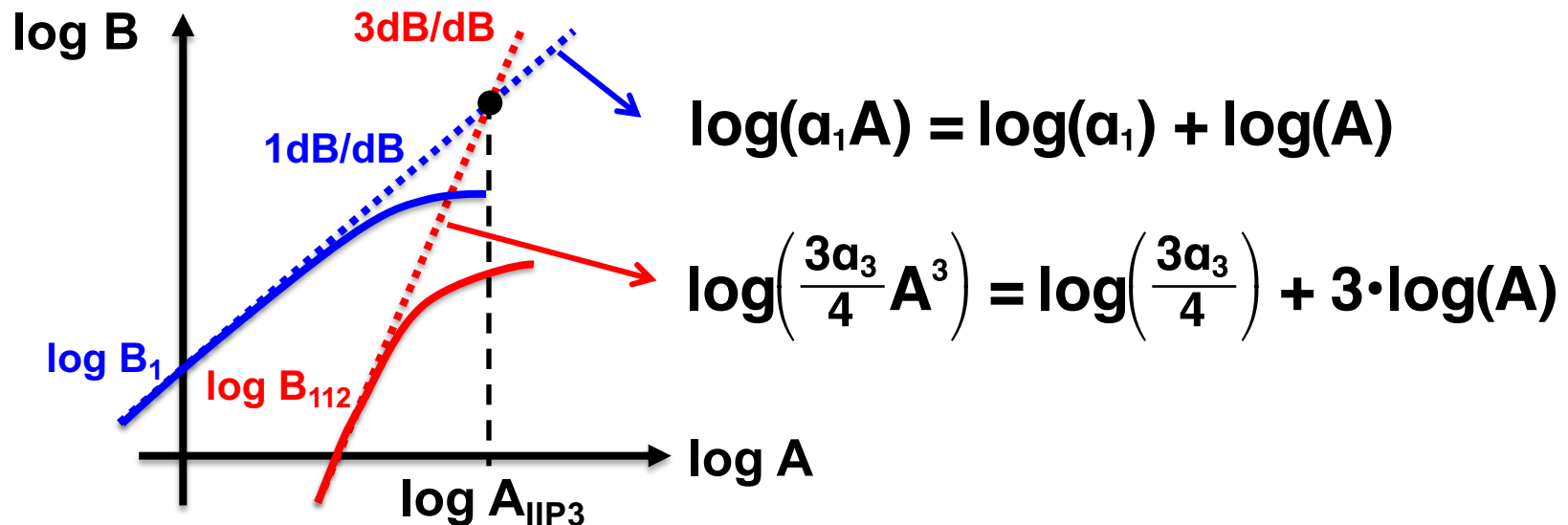
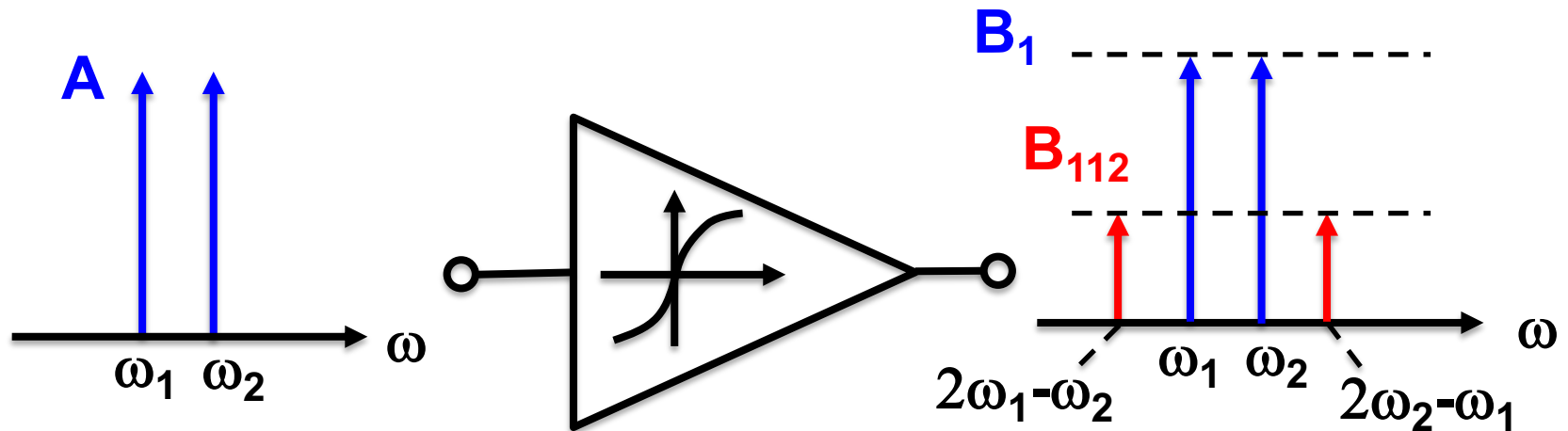
1-dB Compression Point and IIP3



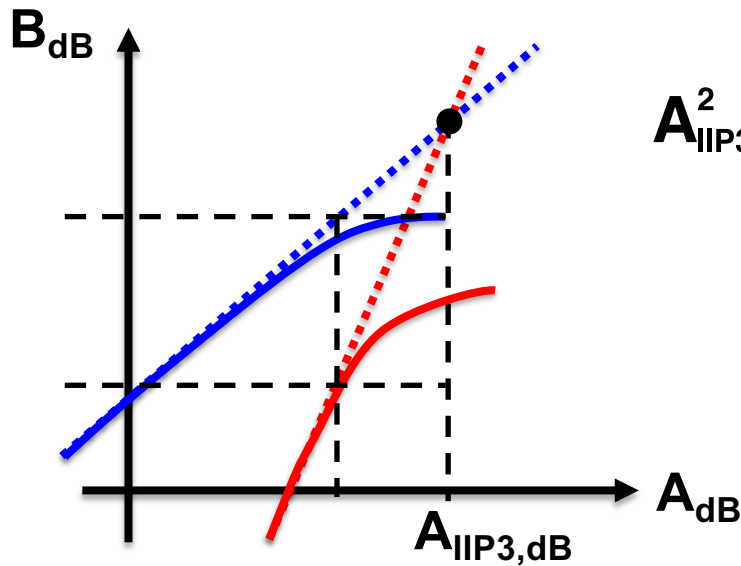
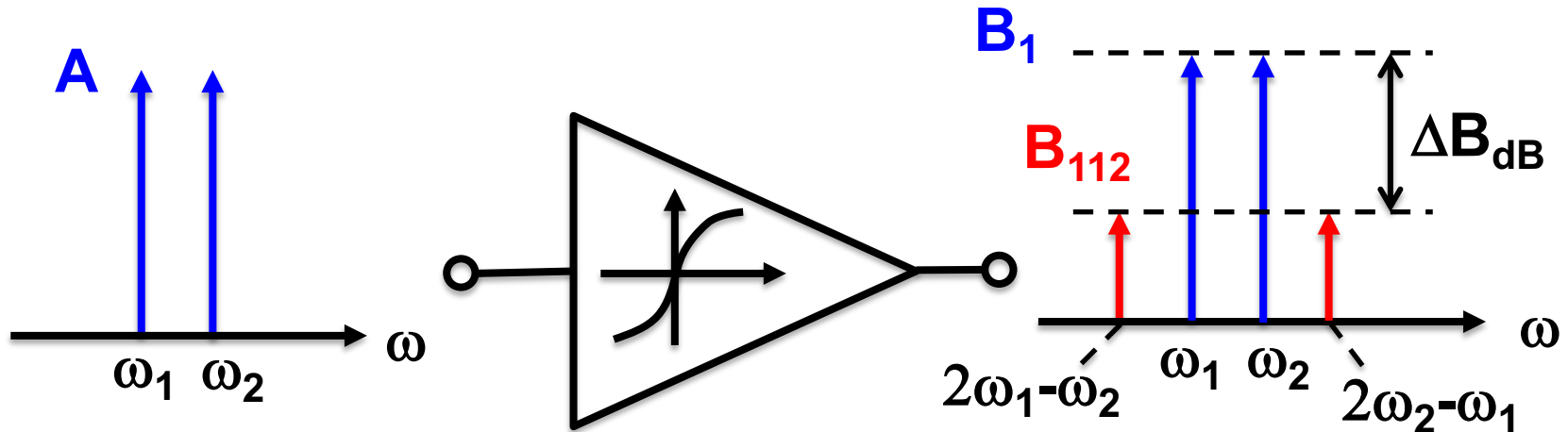
$$20 \log_{10}(A_{-1\text{dB}}) = -9.6 \text{ dB} + \underbrace{10 \log_{10}\left(\frac{4 |a_1|}{3 |a_3|}\right)}_{20 \log_{10}(A_{\text{IIP3}})}$$

$$\rightarrow A_{\text{IIP3}} = A_{-1\text{dB}} + 9.6 \text{ dB}$$

Two-Tone Test and IIP3



Two-Tone Test

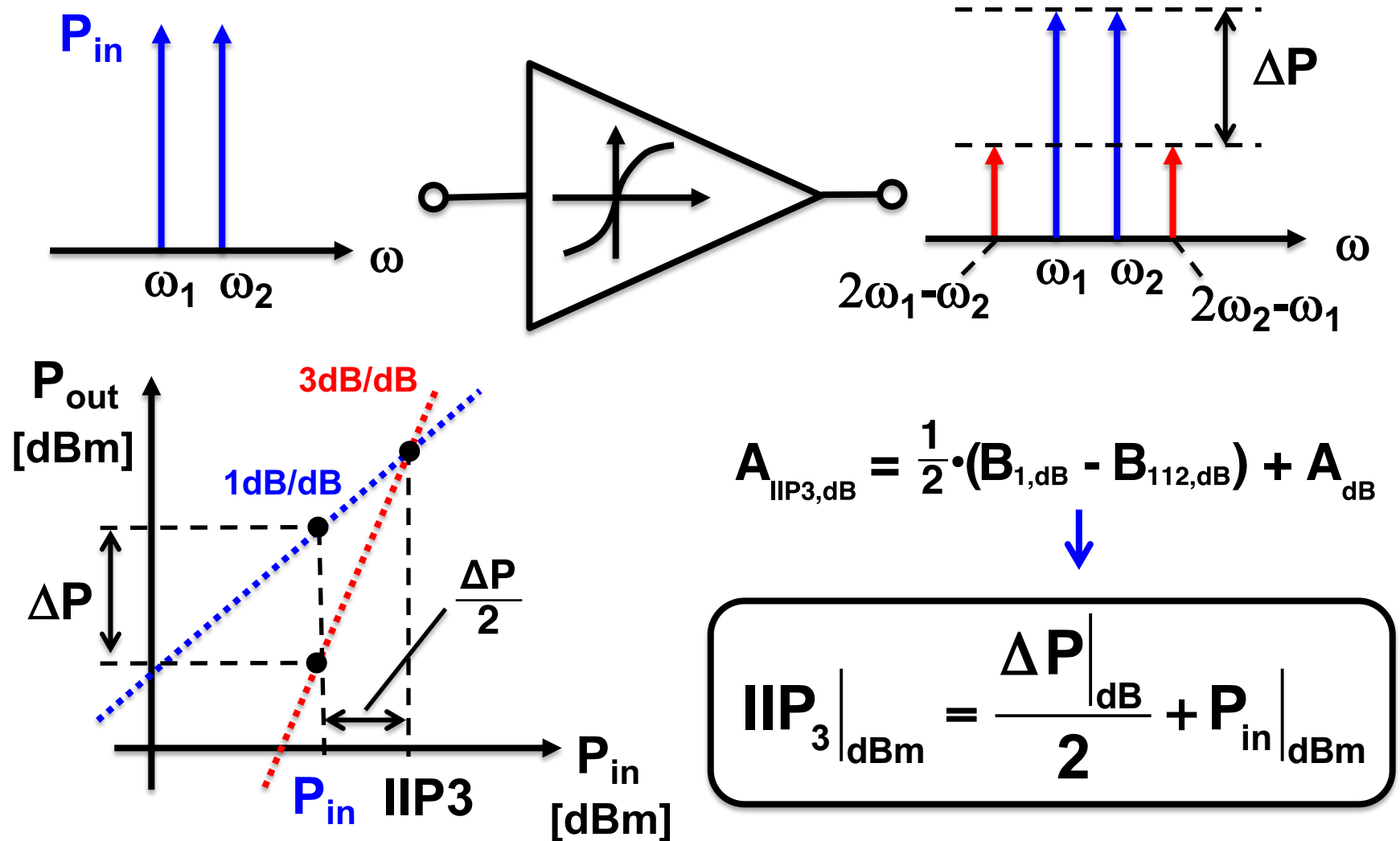


$$A_{IP3}^2 = \frac{4}{3} \cdot \left| \frac{a_1}{a_3} \right| = \left| \frac{a_1 A}{\frac{3}{4} a_3 A^3} \right| \cdot A^2 = \frac{B_1}{B_{112}} \cdot A^2$$

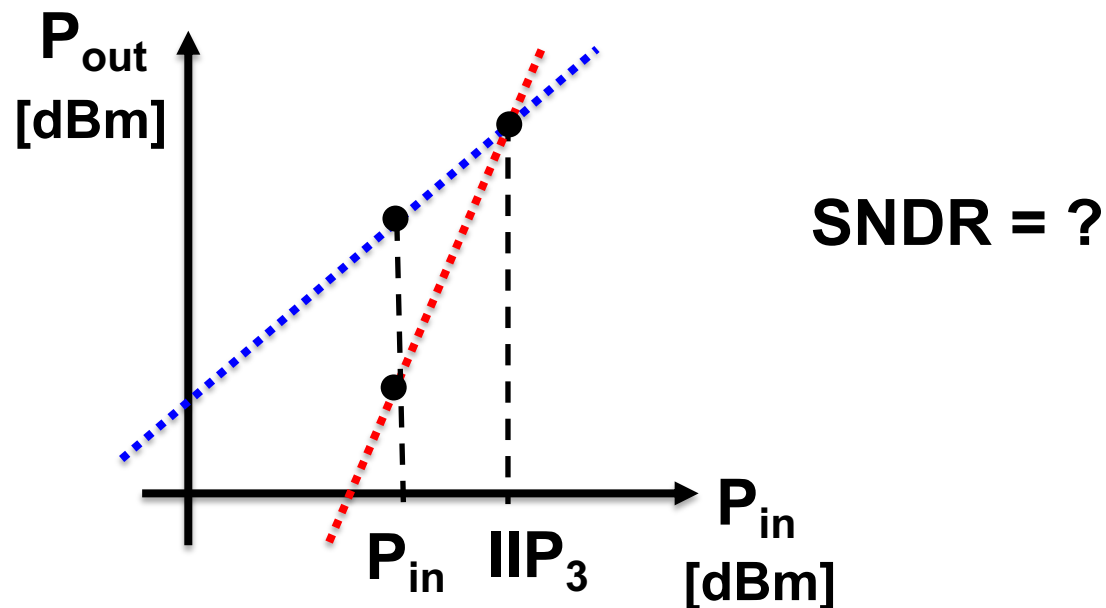
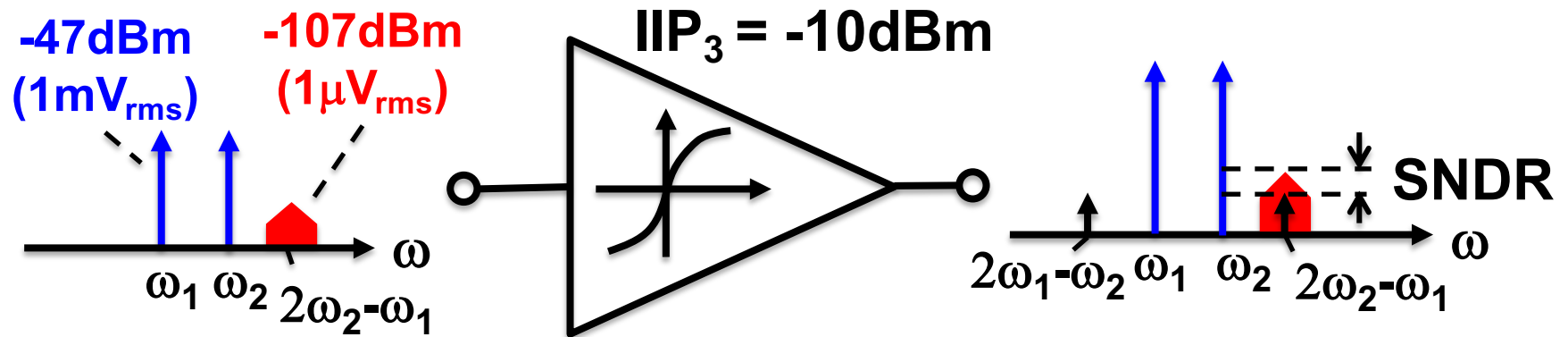


$$A_{IP3,dB} = \frac{1}{2} \cdot \underbrace{(B_{1,dB} - B_{112,dB})}_{\Delta B_{dB}} + A_{dB}$$

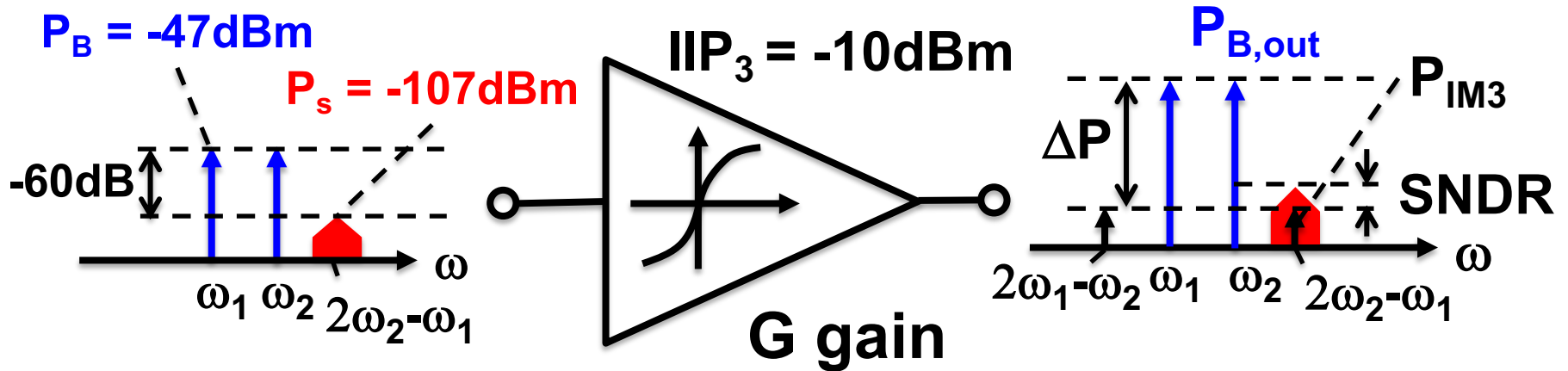
How to measure IIP3



Exercise: Compute SNDR at the Output



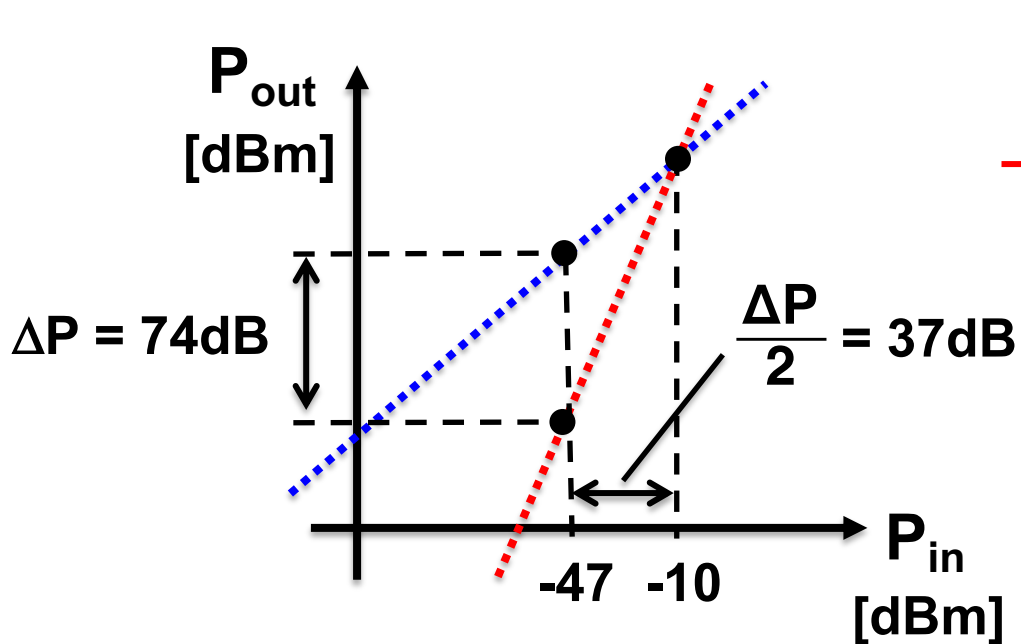
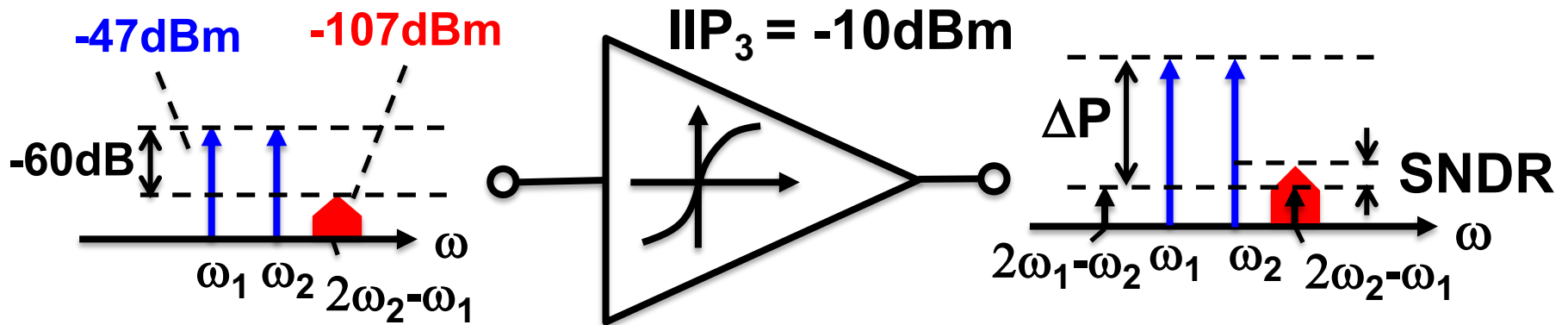
Exercise: Solution in Formulas



$$IIP3 = P_B + \Delta P/2 = P_B + P_{B,out}/2 - P_{IM3}/2$$

$$\begin{aligned} SNDR &= P_{S,out} - P_{IM3} = P_S + G - P_{IM3} = \\ &= P_S + G - (2P_B + P_{B,out} - 2IIP3) = \\ &= P_S + G - 2P_B - P_B - G + 2IIP3 = \\ &= P_S - 3P_B + 2IIP3 = -107 + 3 \cdot 47 - 20 = 14 \text{ dB} \end{aligned}$$

Exercise: Quicker Solution

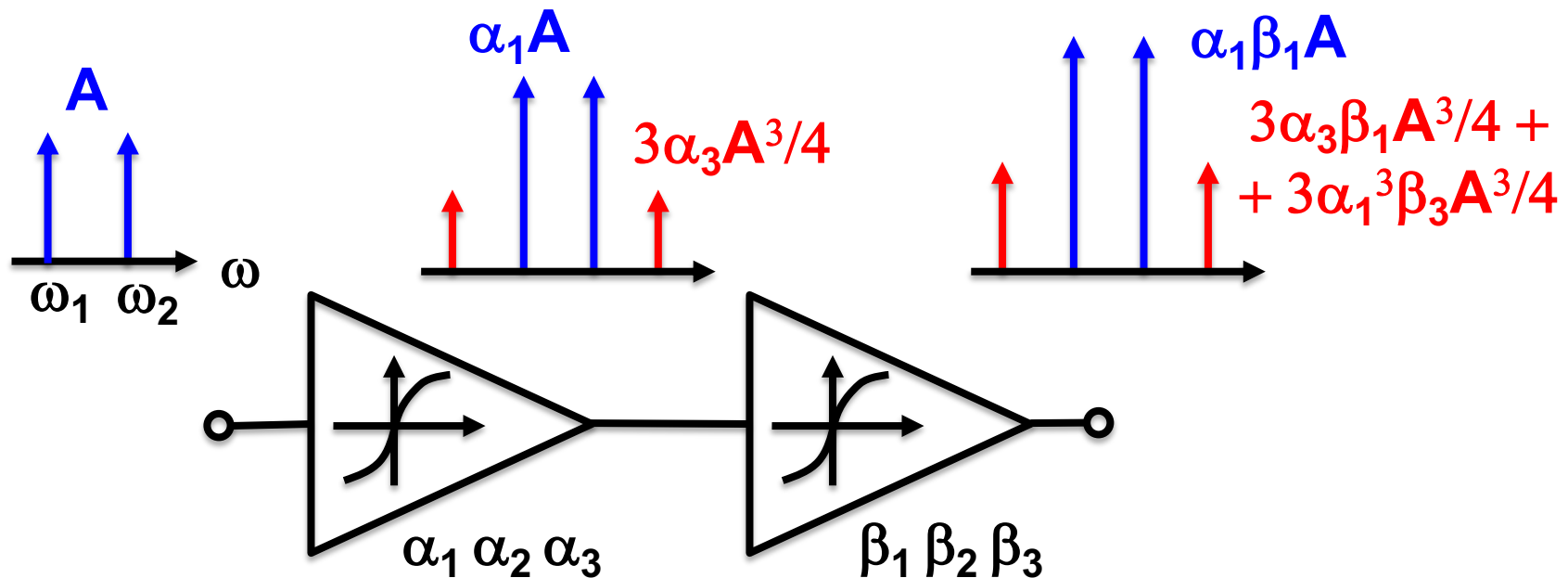


$$\rightarrow \text{SNDR} = \underbrace{P_S}_{-107} - \underbrace{(P_B - \Delta P)}_{(-47 - 74)} = 14 \text{ dB}$$

IIP3 of Cascaded Amplifiers

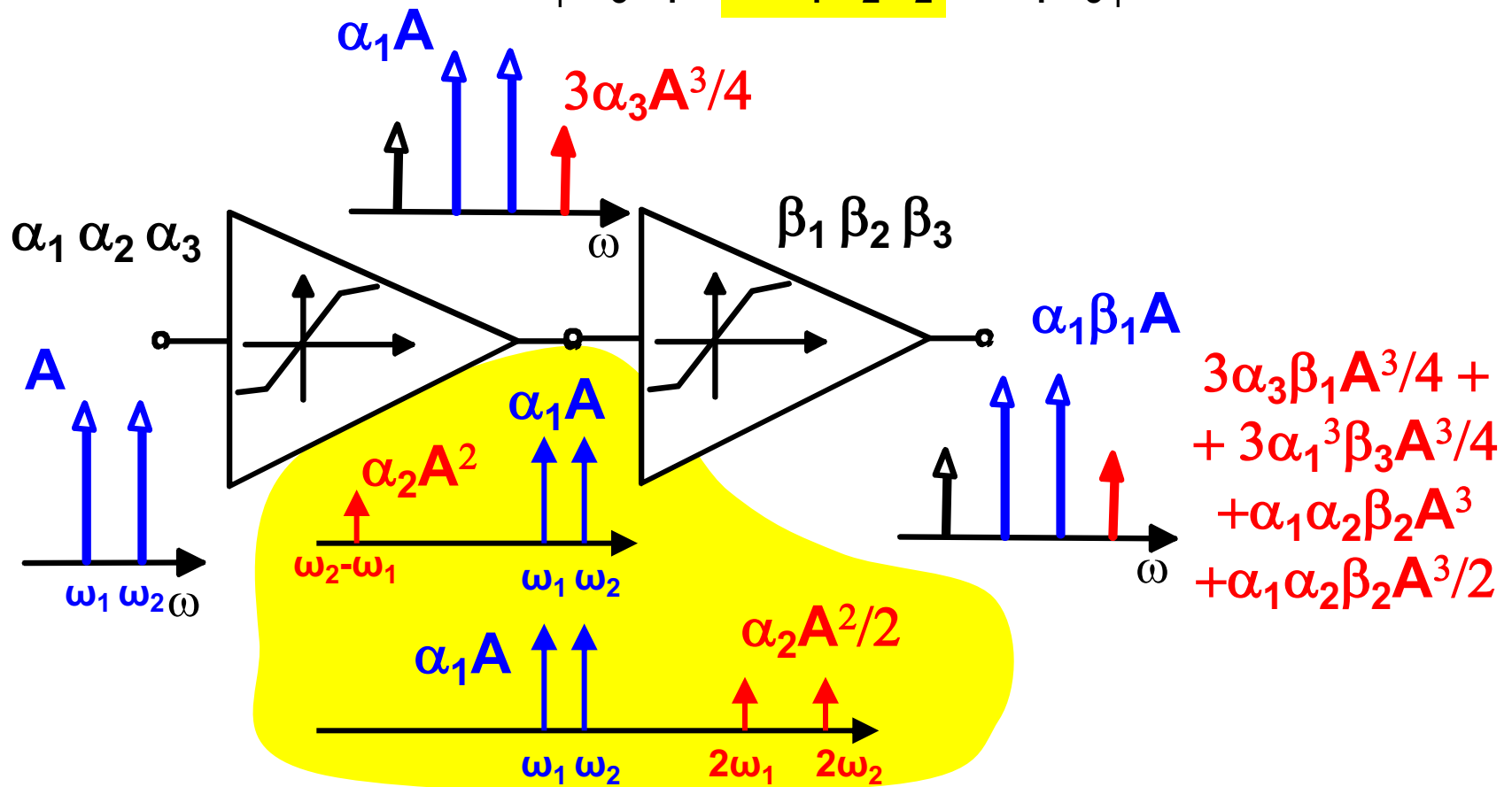
$$y_2 = \beta_1 [\alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3] + \beta_2 [\alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3]^2 + \beta_3 [\alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3]^3$$

$$A_{\text{IIP3}}^2 = \frac{4}{3} \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|$$



IIP3 of Cascaded Amplifiers: Insight

$$A_{\text{IIP3}}^2 = \frac{4}{3} \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|$$



Formula of IIP3 of Cascaded Amplifiers

$$A_{\text{IIP3}}^2 = \frac{4}{3} \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|$$



$$\frac{1}{A_{\text{IIP3}}^2} = \left| \frac{1}{A_{\text{IIP3},1}^2} + \frac{3\alpha_2 \beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{\text{IIP3},2}^2} \right|$$



$$\frac{1}{A_{\text{IIP3}}^2} \approx \frac{1}{A_{\text{IIP3},1}^2} + \frac{\alpha_1^2}{A_{\text{IIP3},2}^2} + \frac{\alpha_1^2 \beta_1^2}{A_{\text{IIP3},3}^2} + \dots$$

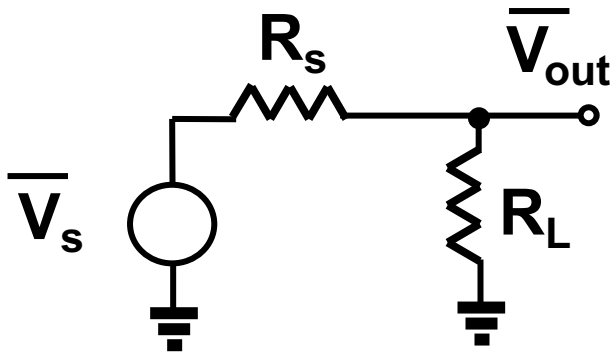
Highlights

- **Nonlinearity** causes:
 - Harmonic generation
 - Gain Compression ($P_{1\text{dB}}$)
 - Desensitization and blocking
 - Intermodulation (IP)
- **Nonlinear cascaded stages**
 - Nonlinearity of **latter stages** is more critical

Basic Concepts in RF:
Impedance Matching

Impedance Matching: Resistive Case

- **Maximum power transfer** to the load is obtained matching the load resistance to the source resistance



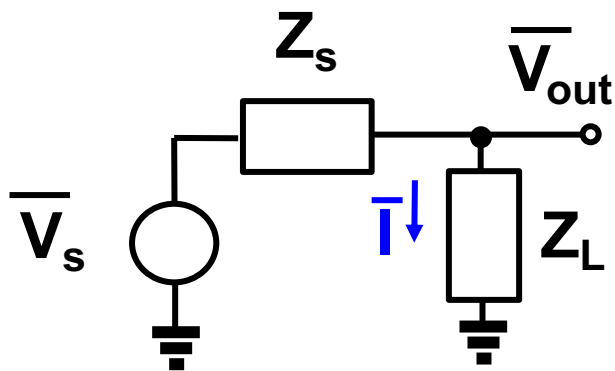
$$P_L = \frac{|\overline{V}_{out}|^2}{2R_L}$$

$$P_{L,max} = \frac{|\overline{V}_s|^2}{8R_L} \quad \text{for } R_L = R_s$$

Available Power

Impedance Matching: Reactive Case

- Maximum power transfer to the load is obtained with **conjugate matching** of load impedance to the source impedance



$$Z_s = R_s + jX_s$$

$$Z_L = R_L + jX_L$$

$$P_L = \frac{|\bar{I}|^2 R_L}{2} = \frac{1}{2} \frac{|\bar{V}_s|^2}{|Z_s + Z_L|^2} R_L$$

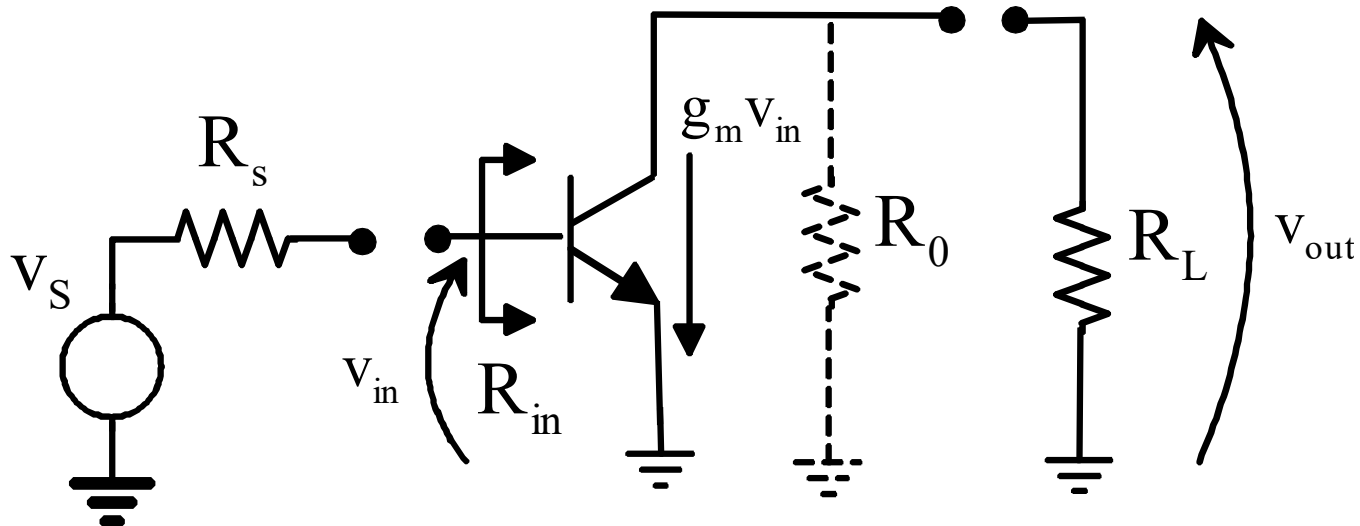
$$P_{L,\max} = \frac{|\bar{V}_s|^2}{8R_L} \quad \text{for } Z_L = Z_s^*$$

$$R_L = R_s$$

$$X_L = -X_s$$

resonance

Voltage Amplifier

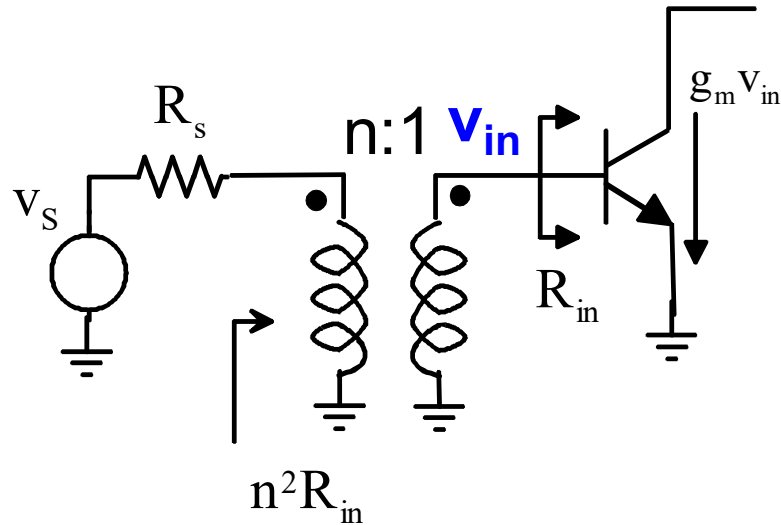


Unloaded Voltage Gain (V_{out}/V_s): $A_0 = \alpha A_v$

$$V_{out} = V_s \cdot \frac{R_{in}}{(R_s + R_{in})} \cdot \boxed{g_m R_0} \cdot \frac{R_L}{(R_0 + R_L)}$$

Unloaded Voltage Gain (V_{out}/V_{in}): A_v

Impedance Matching



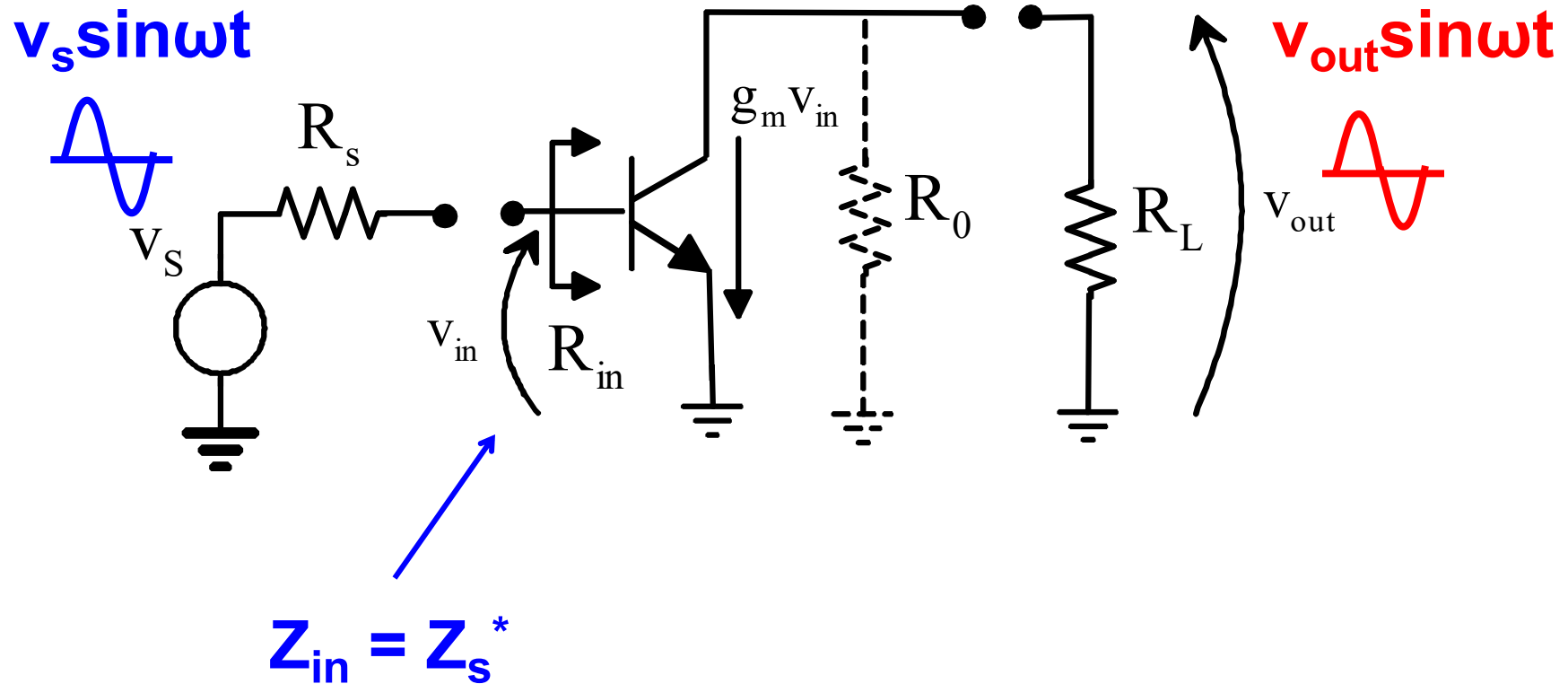
$$\begin{cases} v_1 = n v_2 \\ i_1 = \frac{i_2}{n} \end{cases} \Leftrightarrow \begin{cases} i_1 v_1 = i_2 v_2 \\ \frac{v_1}{i_1} = n^2 \frac{v_2}{i_2} \end{cases}$$

$$V_{in} = V_s \cdot \frac{n^2 R_{in}}{(R_s + n^2 R_{in})} \cdot \frac{1}{n}$$

Maximum voltage gain if

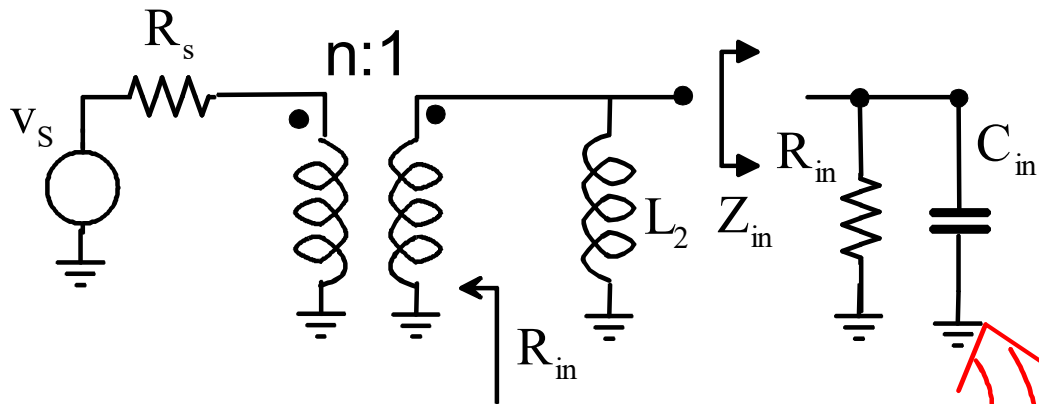
$$n = \sqrt{\frac{R_s}{R_{in}}} \Leftrightarrow n^2 R_{in} = R_s$$

Theorem of Maximum Power Transfer



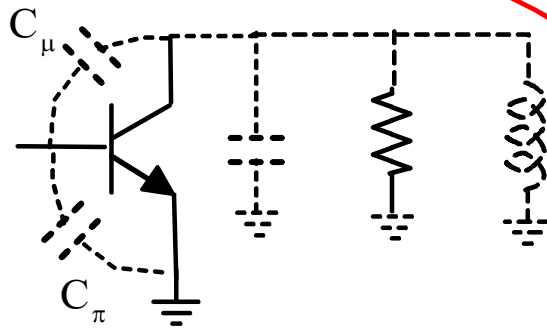
Conjugate Matching
for given source
impedance

Resonant Matching



$$\omega_0 = \frac{1}{\sqrt{L_2 C_{in}}}$$

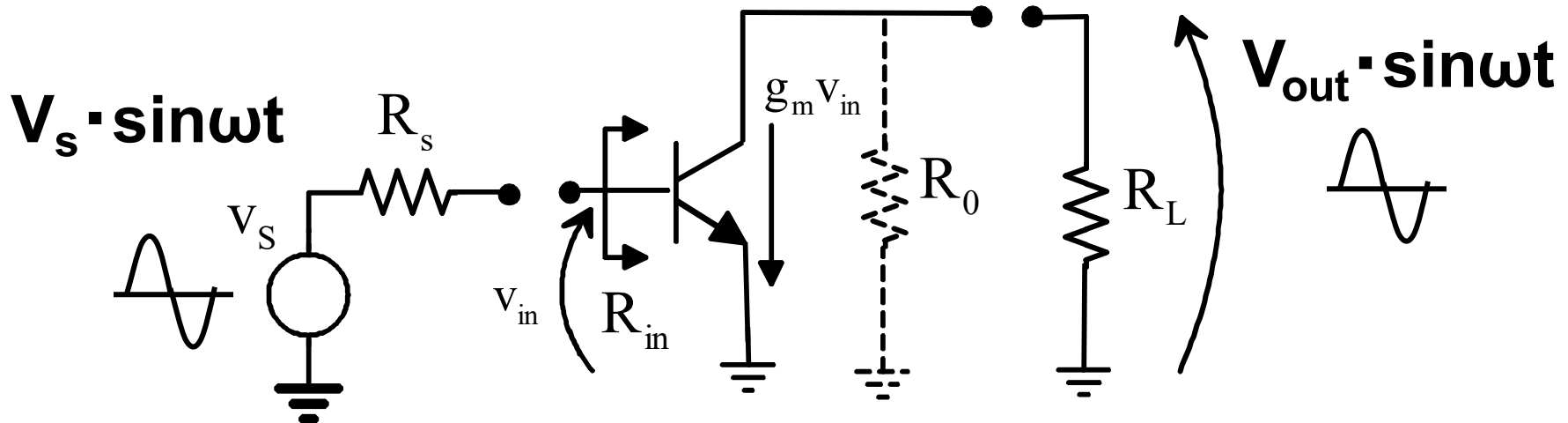
$$Q_{in} = \omega_0 \frac{R_{in}}{2} C_{in}$$



Definition of Power Gains

- **Operating Power Gain:** $G_P = \frac{P_{out}}{P_{in}}$
- **Transducer Power Gain:** $G_T = \frac{P_{out}}{P_{in,av}}$
- **Available Power Gain:** $G_A = \frac{P_{out,av}}{P_{in,av}}$

Power Transfer



$$P_{out} = \frac{V_s^2}{8R_s} \cdot \frac{4R_s R_{in}}{(R_s + R_{in})^2} \cdot \frac{R_{in} g_m^2 R_0}{4} \cdot \frac{4R_0 R_L}{(R_0 + R_L)^2}$$

Maximum or available power gain G_A (points to $\frac{R_{in} g_m^2 R_0}{4}$)

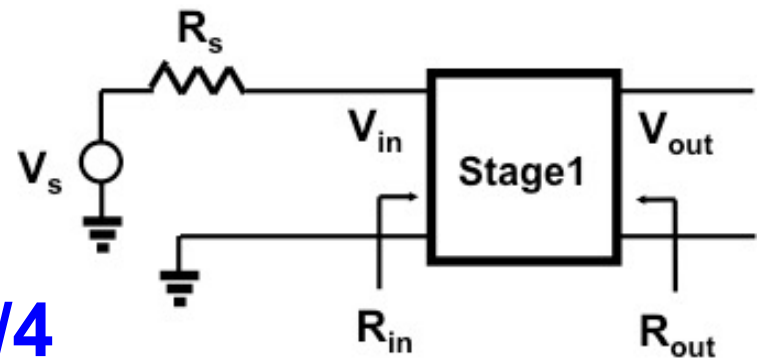
Transducer Power Gain G_T (points to the entire fraction)

Power Gains as a Function of Voltage Gain A_0

$$G_T = \frac{P_{out}}{P_{in,av}} = \frac{\frac{V_s^2 A_0^2}{2R_L} \left(\frac{R_L}{R_L + R_{out}} \right)^2}{\frac{V_s^2}{8R_s}} = A_0^2 \frac{4R_s R_L}{(R_L + R_{out})^2}$$

$$G_A = \frac{P_{out,av}}{P_{in,av}} = \frac{\frac{V_s^2 A_0^2}{8R_s}}{\frac{V_s^2}{8R_s}} = A_0^2 \frac{R_s}{R_{out}}$$

Unloaded Voltage Gain
(V_{out}/V_s): $A_0 = \alpha A_v$



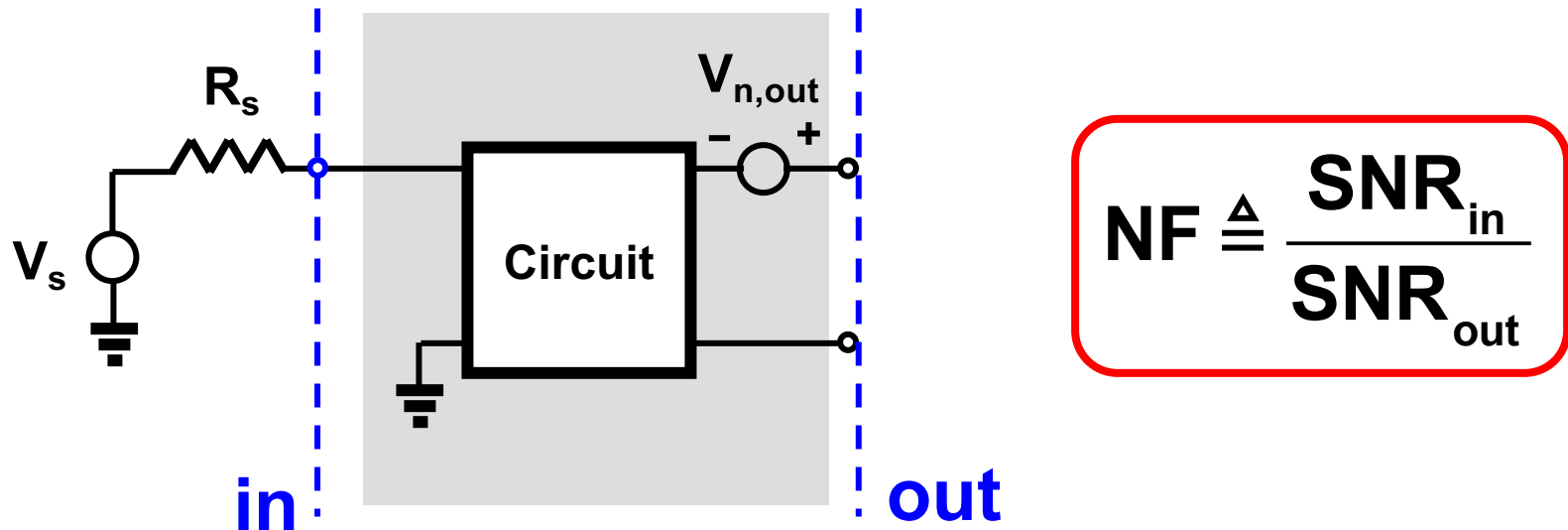
If $R_s = R_{out}$, then $G_A = A_0^2 = A_v^2/4$

Highlights

- **Impedance matching** improves voltage/power gain
- **Power gain** is not in general the square of voltage gain
- **Available power gain G_A** is the square of the unloaded voltage gain A_0 only if $R_s = R_{out}$
- **Reactive matching** is narrowband

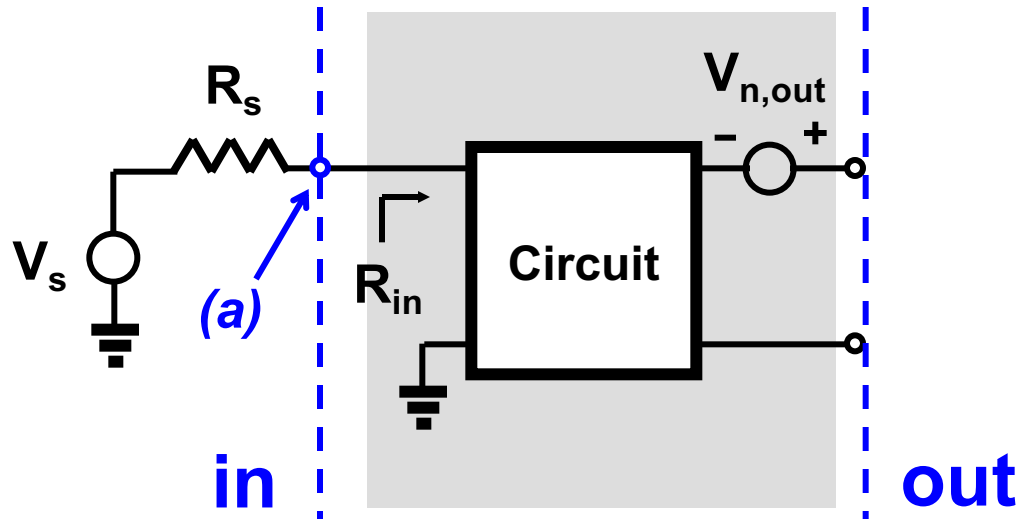
Basic Concepts in RF:
Noise Figure

Impact of Thermal Noise



- In RF receivers, the input signal is corrupted by the thermal noise of the radiation resistance of the antenna ($4kTR_s$)
- The receiver adds other noise ($S_{V_{n,out}}$)
- Definition of Noise Figure (NF)

Noise Figure from Output Noise

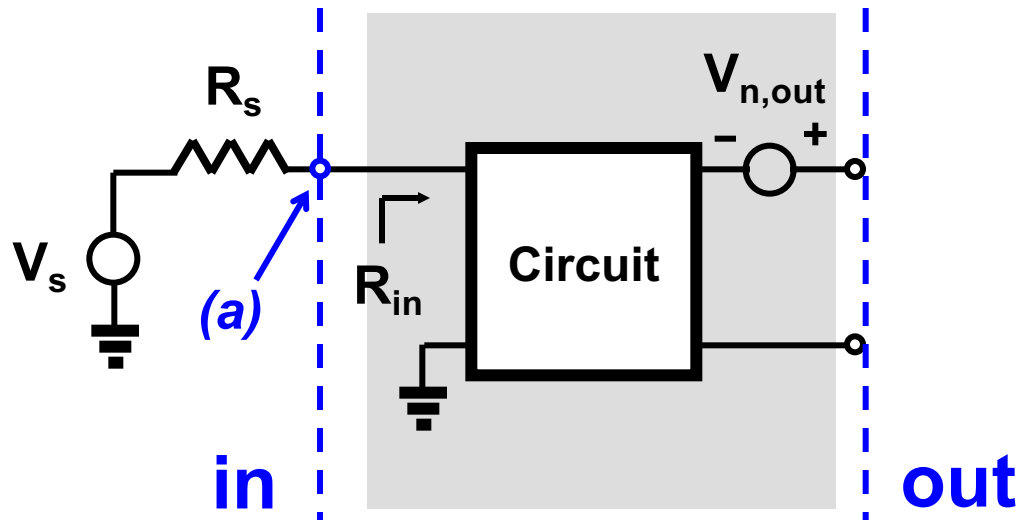


$$NF \triangleq \frac{SNR_{in}}{SNR_{out}}$$

$$NF = \frac{\frac{\alpha^2 \cdot V_{s,eff}^2}{\alpha^2 \cdot 4kTR_s}}{\frac{\alpha^2 A_v^2 \cdot V_{s,eff}^2}{\alpha^2 A_v^2 \cdot 4kTR_s + \overline{V_{n,out}^2}}} = \frac{\alpha^2 \cdot 4kTR_s + \frac{\overline{V_{n,out}^2}}{A_v^2}}{\alpha^2 \cdot 4kTR_s}$$

$\frac{R_{in}}{R_s + R_{in}}$

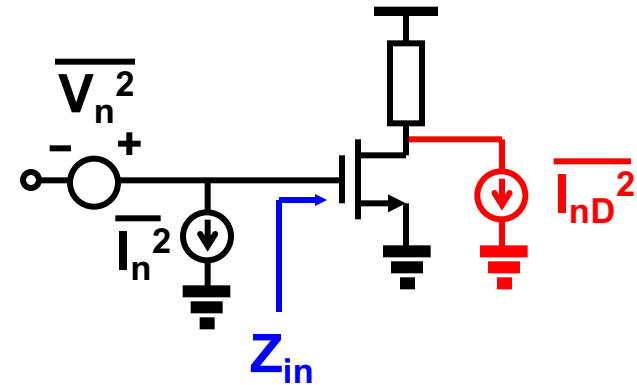
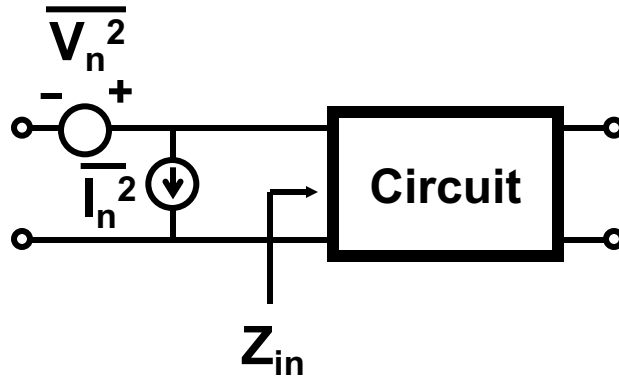
Meaning of Noise Figure



$$NF \triangleq \frac{SNR_{in}}{SNR_{out}}$$

$$NF = \frac{\text{Total Voltage Noise at (a)}}{\text{Voltage Noise at (a) from } R_s}$$

Input-Referred Noise of a Two-port Network



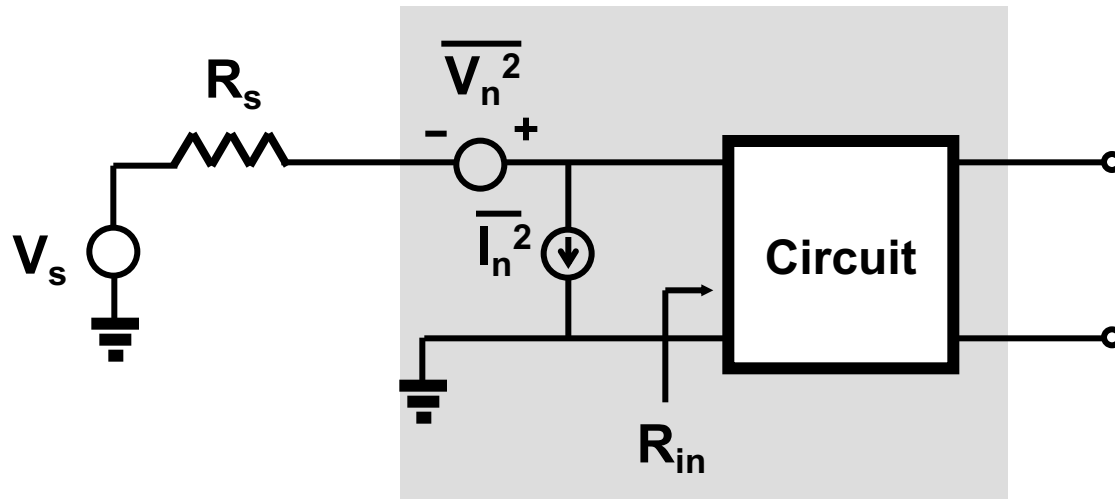
- **Two noise generators** for a two-port network
- Each generator must produce the same output noise of the original noise source. E.g.

$$\overline{V_n^2} = \frac{\overline{I_{nD}^2}}{g_m^2} = \frac{8kT}{3g_m} \quad \overline{I_n^2} = \frac{\overline{I_{nD}^2}}{g_m^2 |Z_{in}|^2} = \frac{8kT}{3g_m |Z_{in}|}$$

$\Delta f = 1\text{Hz}$

correlation

Noise Figure from Input-Referred Noise

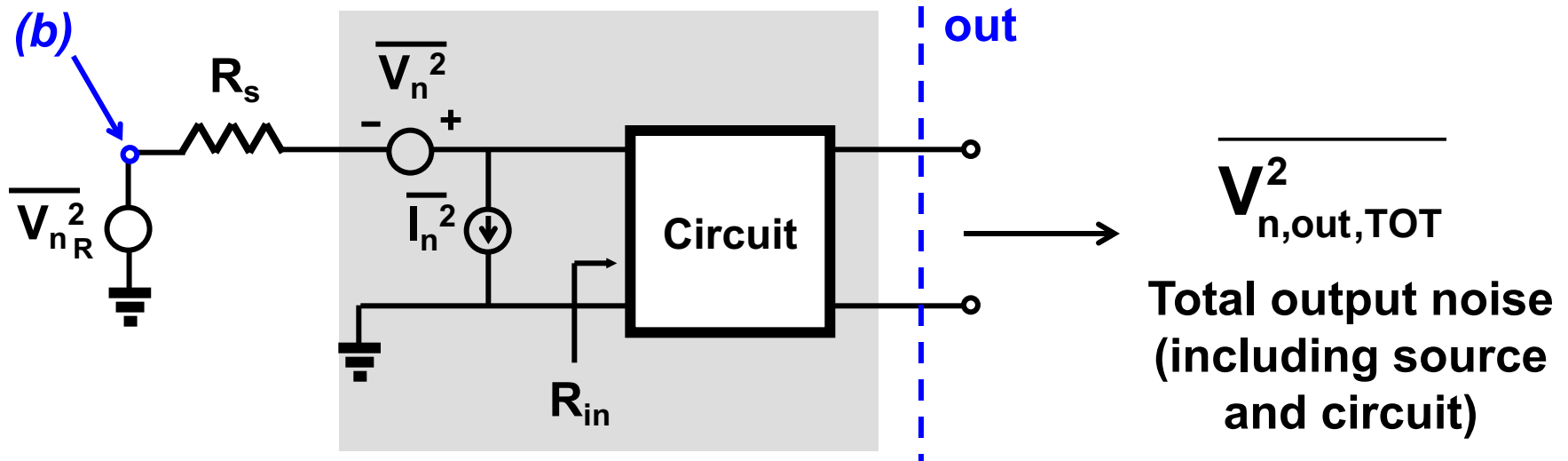


$$NF = \frac{4kTR_s + \overline{(V_n + I_n R_s)^2}}{4kTR_s} = 1 + \frac{\overline{(V_n + I_n R_s)^2}}{4kTR_s}$$

If uncorrelated:

$$NF = 1 + \frac{\overline{V_n^2}}{4kTR_s} + \frac{\overline{I_n^2}}{4kT/R_s}$$

Practical Calculation of Noise Figure



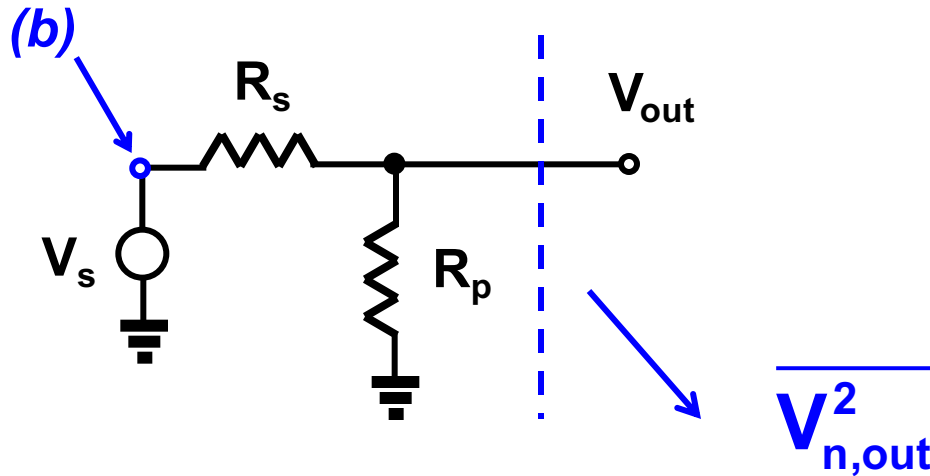
Useful for calculation/simulation of NF:

$$NF = \frac{\overline{V_{n,out,TOT}^2}}{A_0^2} \cdot \frac{1}{4kTR_s}$$

$$A_0 = \frac{V_{out}}{V_s} = \alpha \cdot A_v$$

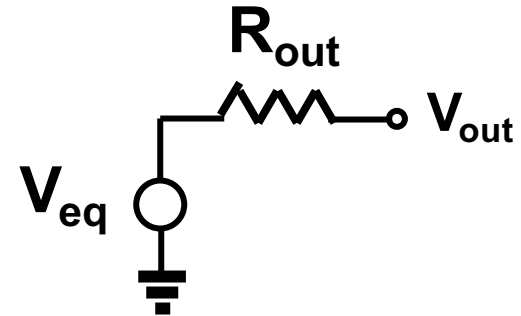
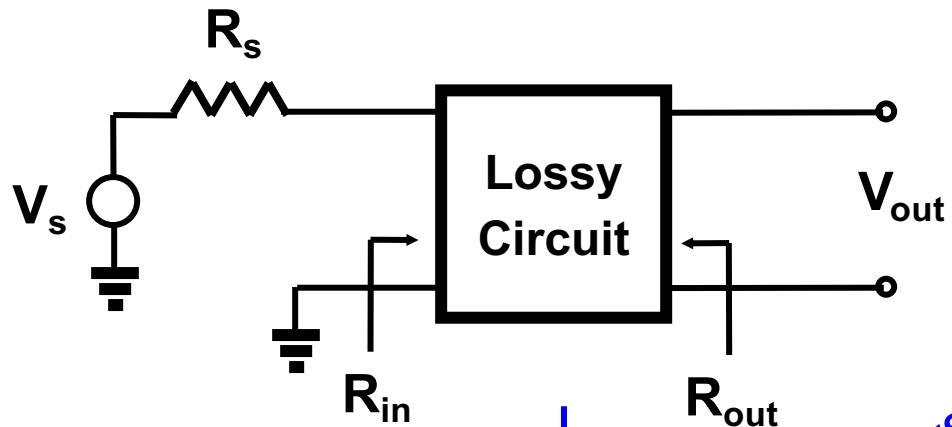
Unloaded voltage gain

Example: NF of Voltage Divider



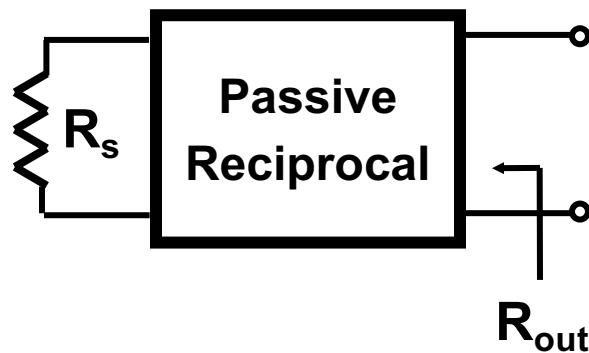
$$NF = \frac{4kT(R_s // R_p)}{\underbrace{\left(\frac{R_p}{R_p + R_s} \right)^2}_{A_0}} \cdot \frac{1}{4kTR_s} = 1 + \frac{R_s}{R_p}$$

NF of Lossy Circuits

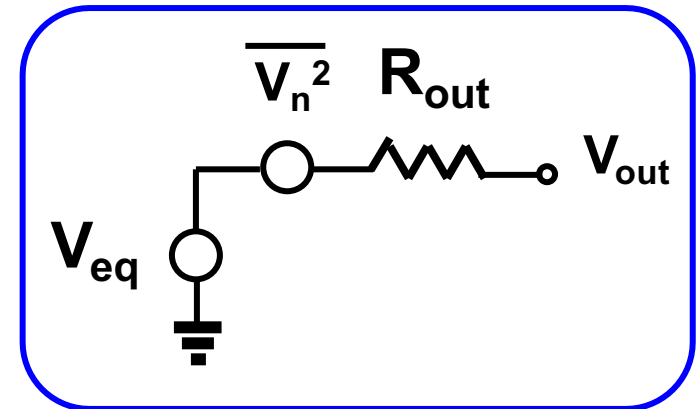


Thevenin equivalent

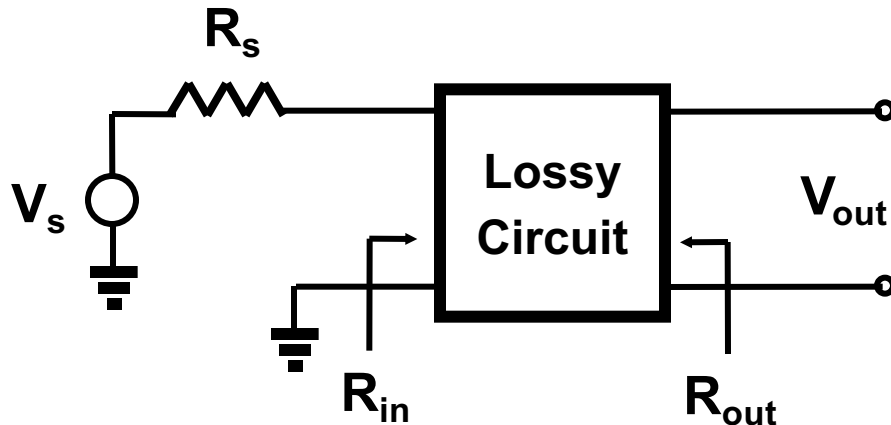
$$\overline{V_n^2} = 4kTR_{out}$$



Nyquist's Theorem



NF of Lossy Circuits



Available Power Loss

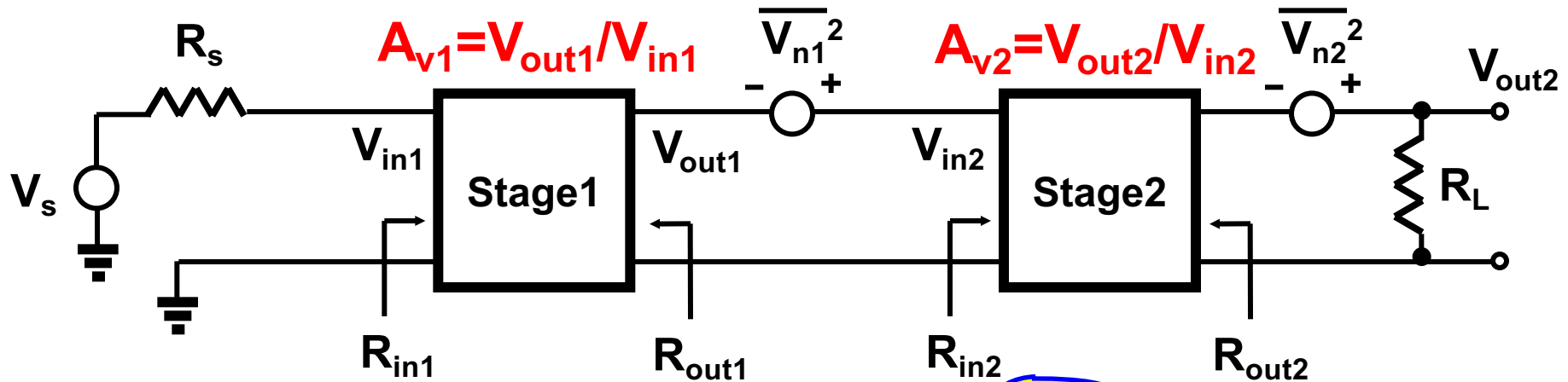
$$L = \frac{P_{in,av}}{P_{out,av}} = \frac{V_s^2 / 8R_s}{V_{eq}^2 / 8R_{out}}$$

$$NF = \frac{\overbrace{4kTR_{out}}^{V_{n,out}^2}}{\underbrace{V_{eq}^2}_{V_s}} \frac{1}{4kTR_s} = \frac{V_s^2}{V_{eq}^2} \frac{R_{out}}{R_s} = L$$

$\underbrace{V_s}_{\text{Gain } A = V_{out}/V_s}$

Cascading NF

Unloaded Gain



$$NF = 1 + \frac{\overline{V_{n1}}^2}{\left(\frac{R_{in1}}{R_{in1} + R_s} \right)^2 A_{v1}^2} + \frac{1}{4kTR_s} + \frac{\overline{V_{n2}}^2}{\left(\frac{R_{in1}}{R_{in1} + R_s} \right)^2 A_{v1}^2 \left(\frac{R_{in2}}{R_{in2} + R_{out1}} \right)^2 A_{v2}^2} + \frac{1}{4kTR_s}$$

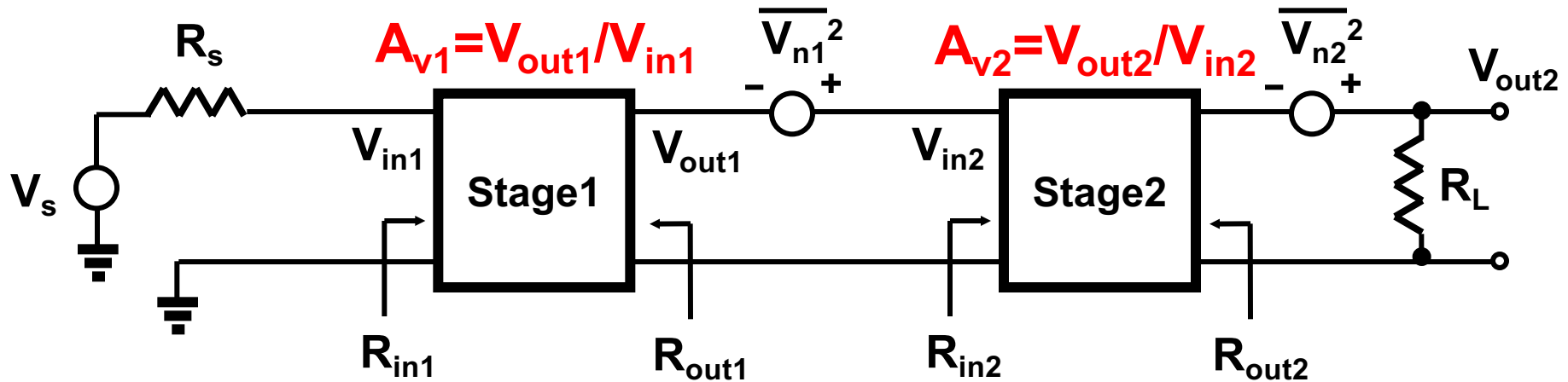
The first term is highlighted with a blue dashed line and labeled $(A_{01})^2$. The second term is highlighted with a yellow box. A blue arrow points from the yellow box to the simplified equation below.

$$NF_1 = 1 + \frac{\overline{V_{n1}}^2}{\left(\frac{R_{in1}}{R_{in1} + R_s} \right)^2 A_{v1}^2} + \frac{1}{4kTR_s}$$

$$NF_2|_{R_{out1}} = 1 + \frac{\overline{V_{n2}}^2}{\left(\frac{R_{in2}}{R_{in2} + R_{out1}} \right)^2 A_{v2}^2} + \frac{1}{4kTR_{out1}}$$

Cascading NF

Unloaded Gain



$$NF = NF_1 + \frac{NF_2|_{R_{out1}} - 1}{\left(\frac{R_{in1}}{R_{in1} + R_s} \right)^2 A_{v1}^2 \frac{R_s}{R_{out1}}}$$

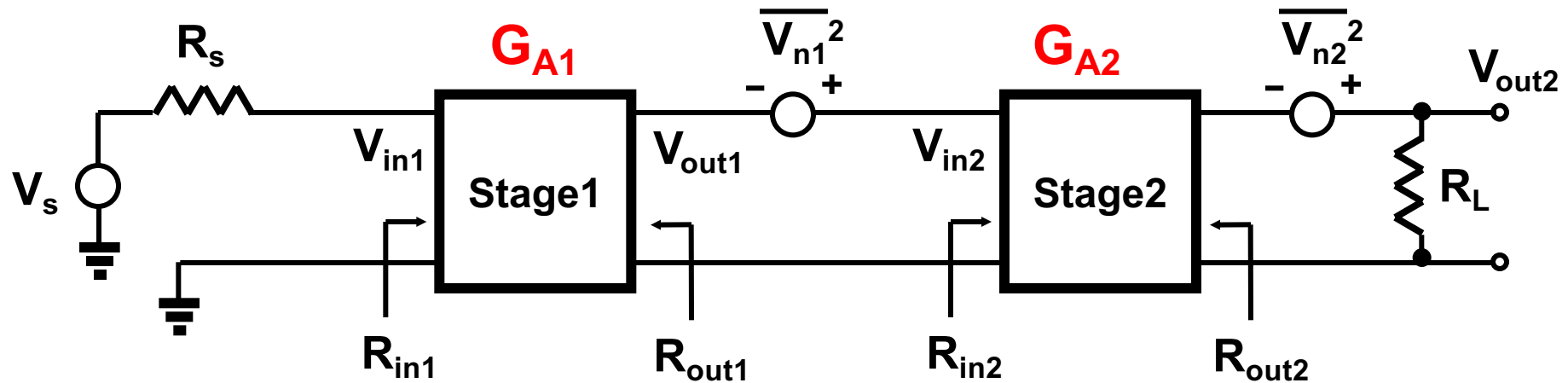
$(A_{01})^2$

Voltage Gain

$V_{out1}/V_s: A_{01}$

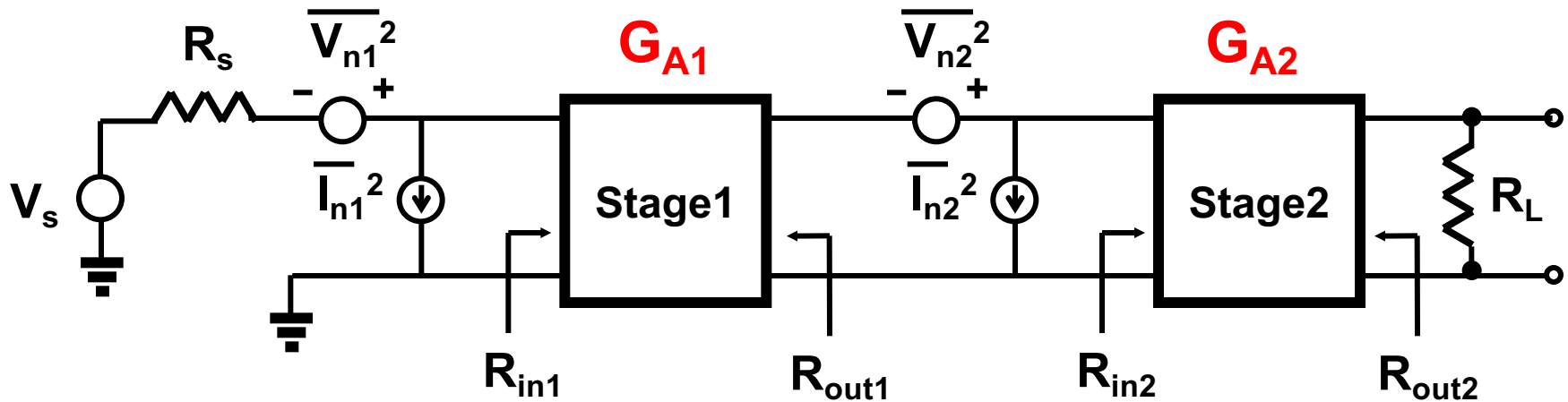
Available Power Gains

Available Power Gain



$$G_{A1} = \frac{P_{out,av}}{P_{in,av}} = \frac{V_s^2 \left(\frac{R_{in1}}{R_{in1} + R_s} \right)^2 A_{v1}^2 \cdot \frac{1}{8R_{out1}}}{\frac{V_s^2}{8R_s}} = A_{01}^2 \frac{R_s}{R_{out1}}$$

Friis Equation



Referred to R_{out1}

Referred to R_{out2}

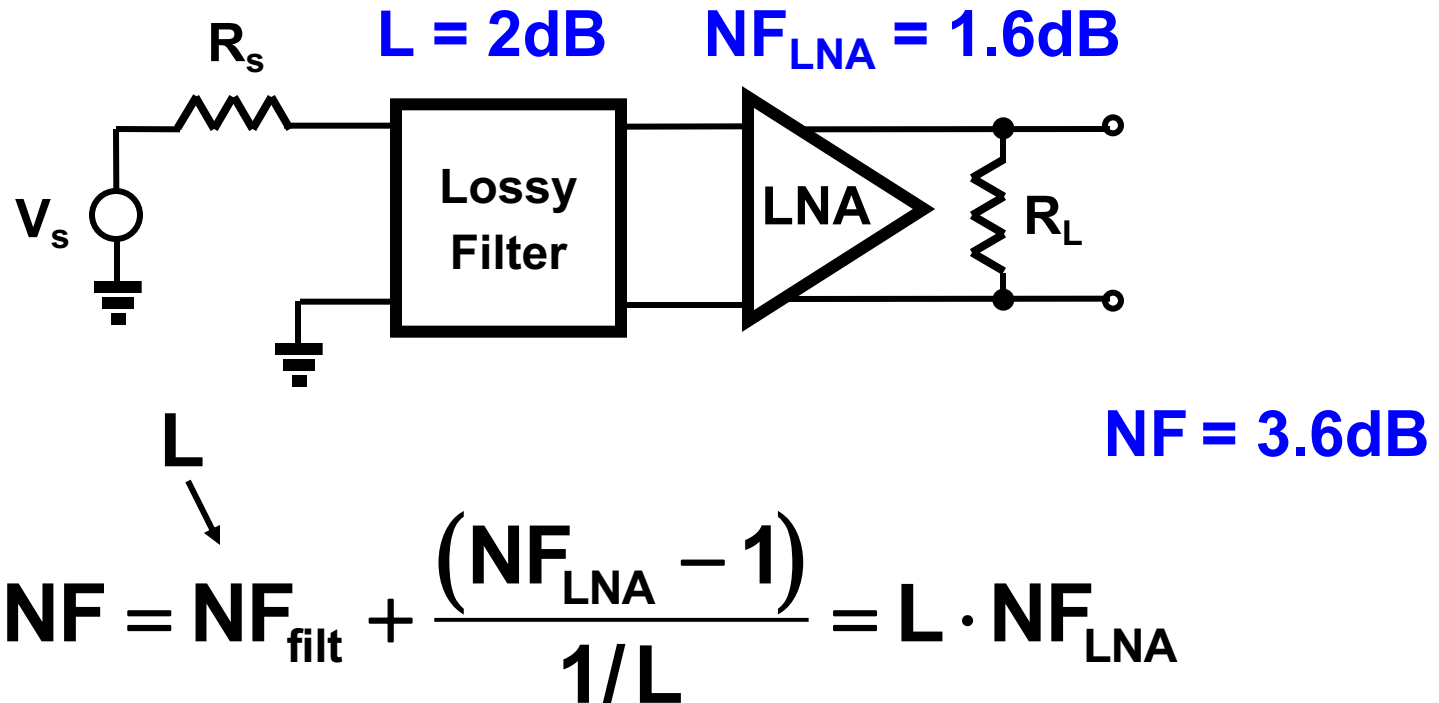
$$NF = NF_{1,Rs} + \frac{(NF_{2,Rout1} - 1)}{G_{A1}} + \frac{(NF_{3,Rout2} - 1)}{G_{A1}G_{A2}} + \dots$$

Referred to R_s

Available Power Gains

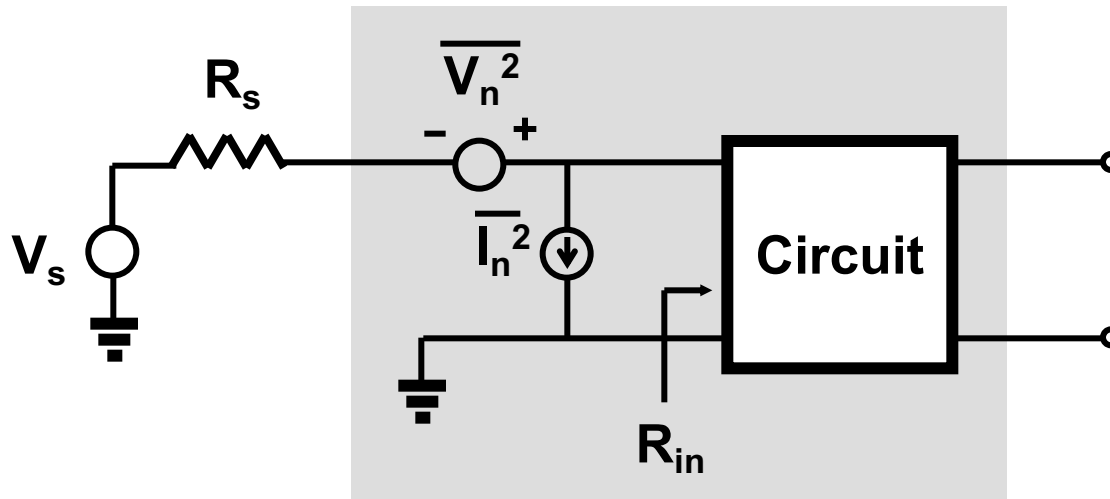
This sum is not in dB!

Effect of Lossy Filter and Example



- First stage are most critical for noise
- The noise figure is amplified by losses

Matching for Minimum Noise (Noise Matching)



$$NF = 1 + \frac{\overline{(V_n + I_n R_s)^2}}{4kTR_s} \approx 1 + \frac{\overline{V_n^2}}{4kTR_s} + \frac{\overline{I_n^2}}{4kT/R_s}$$

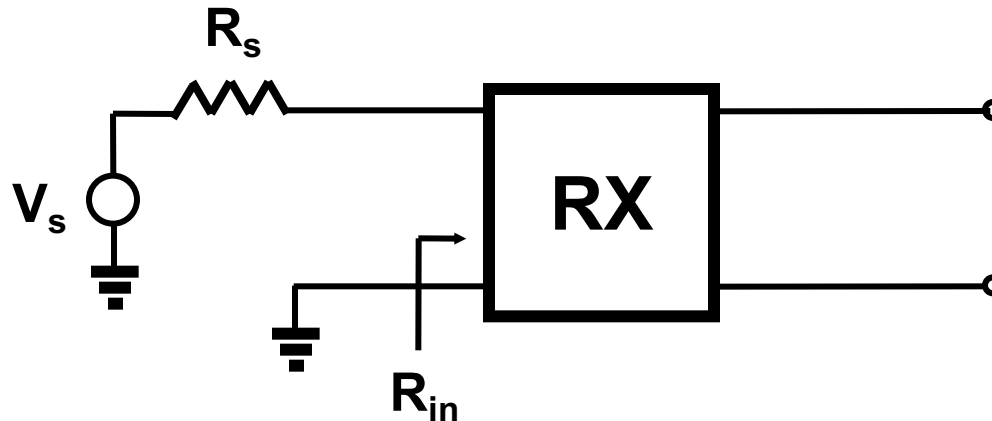
$$\frac{\partial NF}{\partial R_s} = 0 \quad \longrightarrow \quad R_{s,opt} = \sqrt{\frac{\overline{V_n^2}}{\overline{I_n^2}}}$$

Highlights

- Two input-referred generators needed to model noise of a two-port network
- **Noise Figure:**
 - is the ratio between total noise and source noise
 - is a single parameter, referred to a specific source resistance
- **Cascaded stage: first stages** more critical
- **Lossy filters** degrade the noise figure
- Minimum noise figure is obtained with optimum source resistance (**Noise matching**)

Basic Concepts in RF:
Sensitivity and Dynamic Range

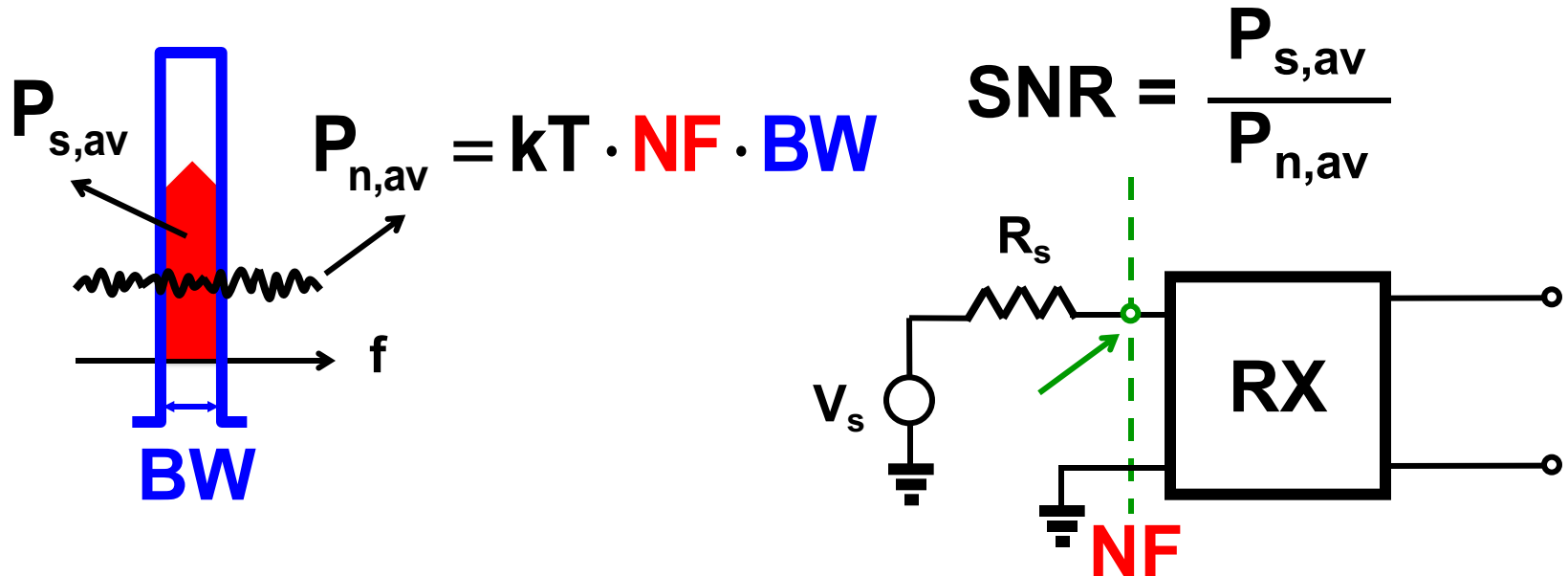
Available Noise Power



$$\begin{array}{ccc} \text{[V}^2\text{/Hz]} & & \text{[W/Hz]} \\ \overline{V_{n,s}^2} & \rightarrow & P_{n,av} \\ \frac{\overline{V_{n,s}^2}}{\Delta f} = 4kTR_s & & \frac{P_{n,av}}{\Delta f} = kT \\ & & \downarrow \end{array}$$

-174 dBm/Hz at room temperature (25 degrees Celsius)

Sensitivity

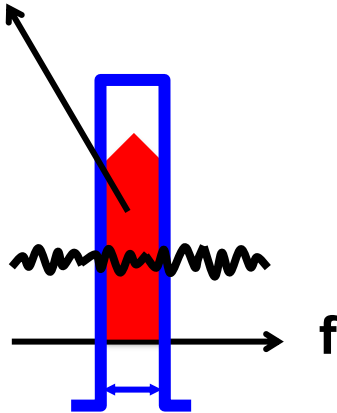


- **Sensitivity** is the minimum detectable signal

$$P_{s,min}|_{dBm} = \overbrace{kT|_{dBm} + NF|_{dB} + 10\log(BW)}^{P_{n,av}} + \overbrace{SNR|_{dB}}^{\text{Minimum SNR}}$$

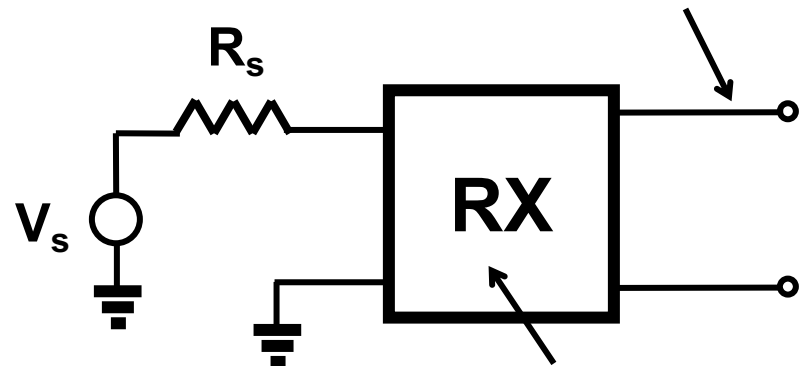
Example: NF of a GSM Receiver

Sensitivity
 $P_{s,min} = -100 \text{ dBm}$



Noise BW
200 kHz

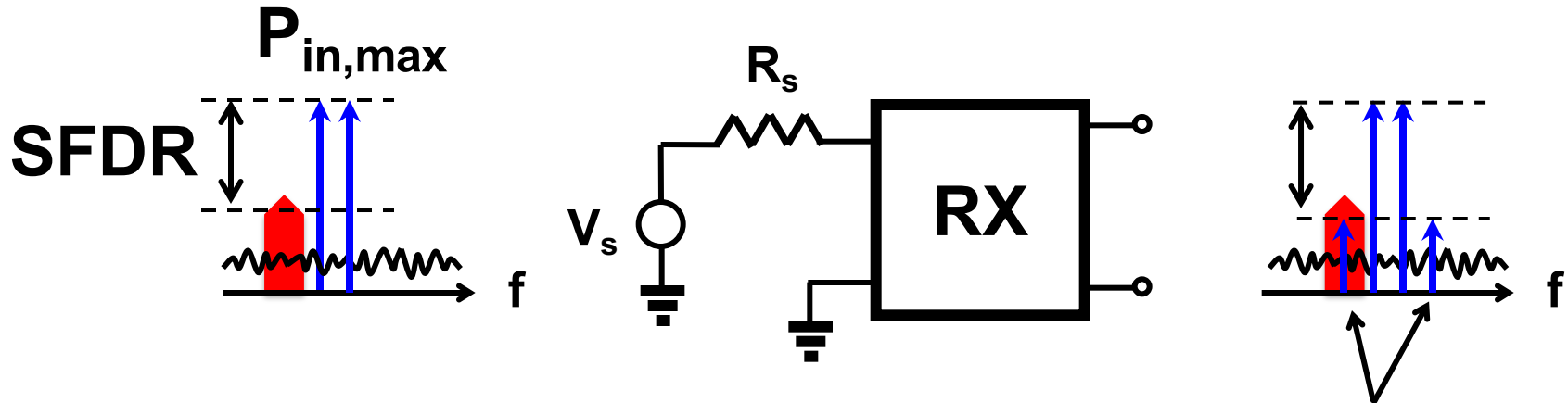
Signal-to-Noise Ratio
SNR = 12 dB



Noise Figure NF

$$\begin{aligned} \text{NF}_{\text{dB}} &= -kT - 10 \log(\text{BW}) - \text{SNR}_{\text{min}} + P_{s,min} \\ &= 174 - 53 - 12 - 100 = 9 \text{ dB} \end{aligned}$$

Dynamic Range

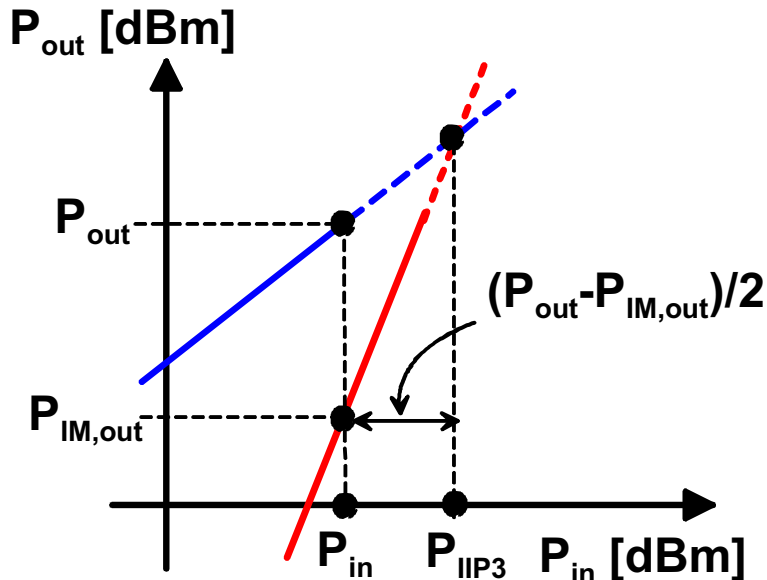
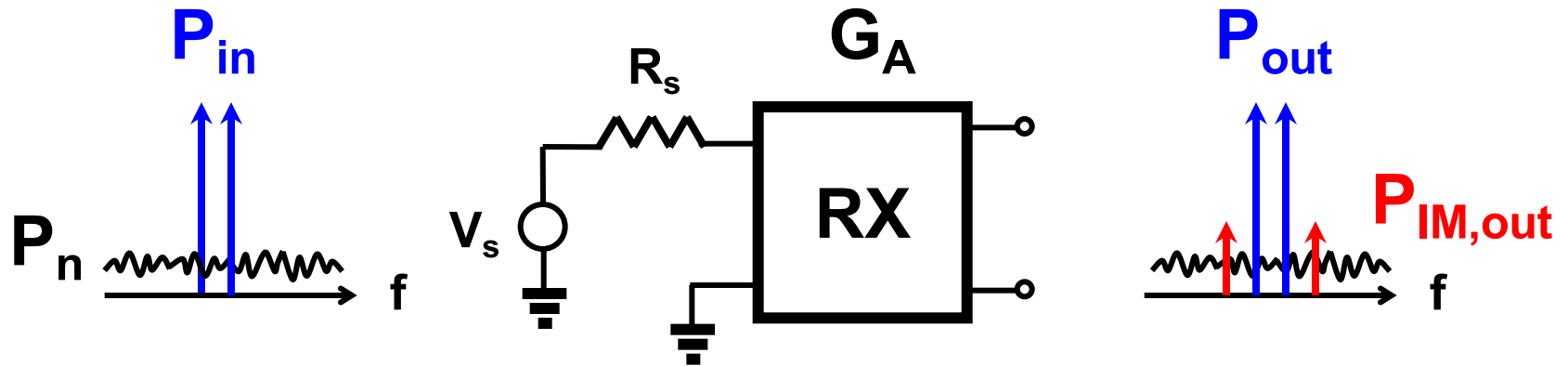


Third-order nonlinearity gives rise to **IM3 products**

- **Upper end:** IM3 products equal to noise
- **Lower end:** sensitivity level

→ The available full-scale range is the **Spurious-Free Dynamic Range (SFDR)**

Dynamic Range (cont'd)



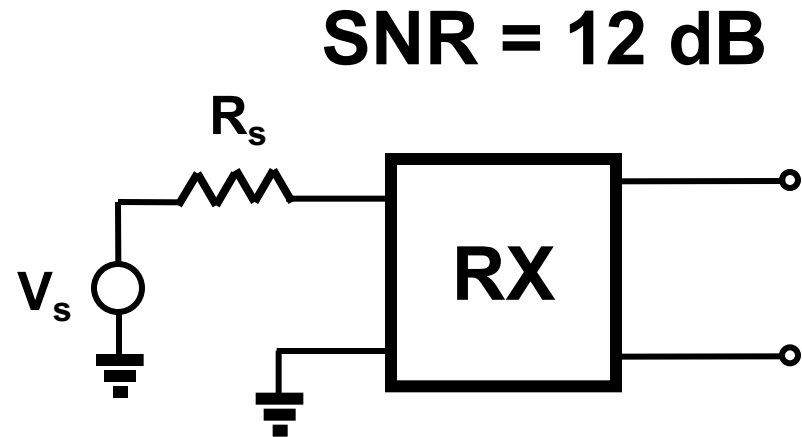
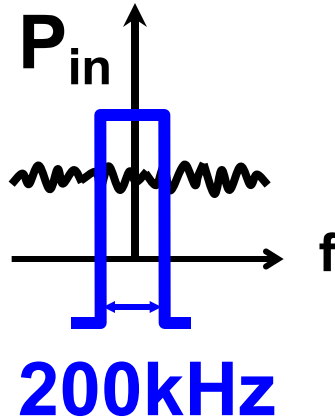
$$P_{IIp3} = P_{in} + \frac{(P_{out} - P_{IM,out})}{2} =$$

$$= P_{in} + \frac{(P_{in} + \cancel{G_A} - P_{IM,in} - \cancel{G_A})}{2}$$

$$P_{in,max} = \frac{2P_{IIp3} + P_n}{3}$$

in dBm!

Example: SFDR of a GSM Receiver



$$NF = 9 \text{ dB} \rightarrow P_n = -112 \text{ dBm} \rightarrow P_{in,min} = -100 \text{ dBm}$$

$$IIP3 = -15 \text{ dBm} \rightarrow P_{in,max} = \frac{2IIP_3 + P_n}{3} = -47.3 \text{ dBm}$$

$$\rightarrow SFDR = P_{in,max} - P_{in,min} = 52.7 \text{ dB}$$

Highlights

- **RX Sensitivity** is the power of the minimum detectable signal (with minimum SNR)
- **RX Spurious-Free Dynamic Range** is the ratio between maximum signal producing IM products equal to noise and sensitivity