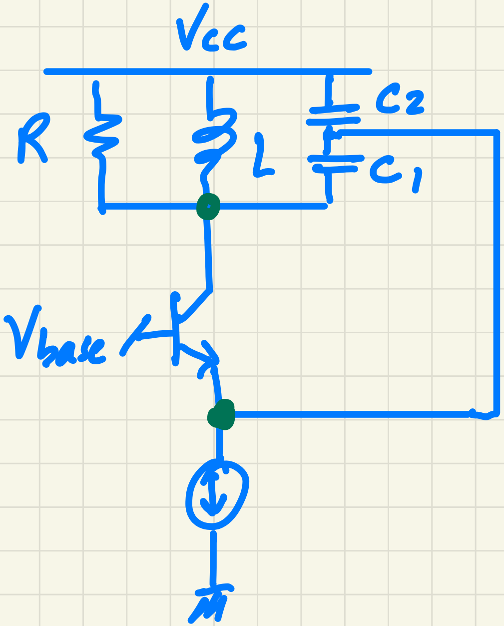
RF Circuit Design

L13

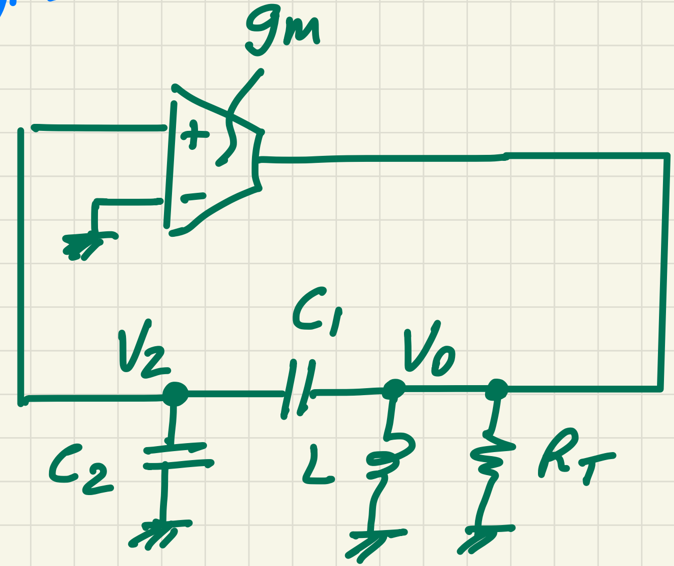
Variants in Oscillator topologies

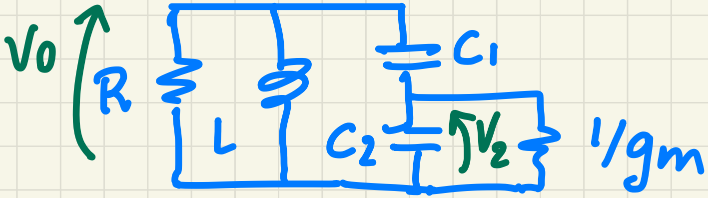
(1)

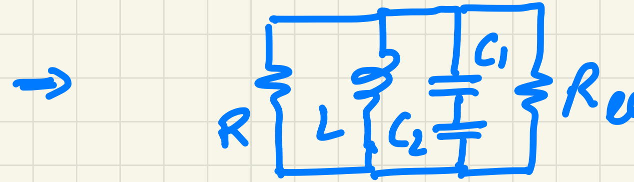


Colpitts Oscillator

small signal

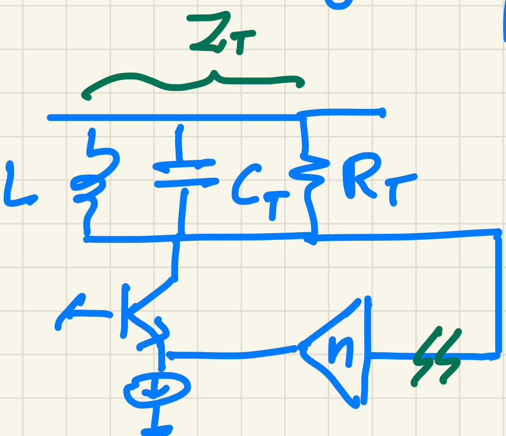






Hyp. low-loss approx. $\overbrace{\frac{1}{g_m} \gg \frac{1}{\omega_0 C_2}}^{Q_c \gg 1}$: $V_2 \simeq \underbrace{\frac{C_1}{C_1 + C_2}}_{n<1} \cdot V_o$

$$\Rightarrow R_{eq} \simeq \frac{1}{g_m} \cdot \frac{1}{\left|\frac{V_2}{V_o}\right|^2} = \frac{1}{n^2 \cdot g_m}$$



$$R_T = R \,//\, R_{eq} = R \,//\, \frac{1}{n^2 g_m}$$

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

$$LG(s) = n \cdot g_m \cdot Z_T(s)$$

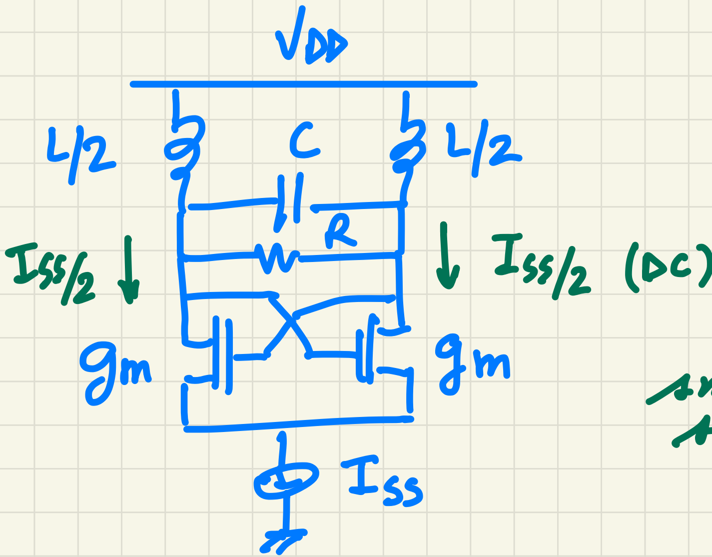
Oscillation condition (small-signal):

$$LG(j\omega_0) = 1 \iff \begin{cases} g_m R_T\, n = 1 \\ \omega_0 = \dfrac{1}{\sqrt{L C_T}} \end{cases} \qquad R_T = \frac{R \cdot \frac{1}{n^2 g_m}}{R + \frac{1}{n^2 g_m}}$$

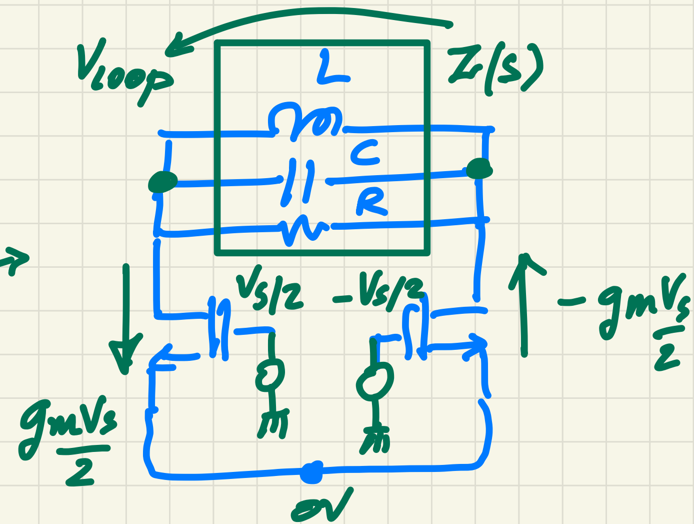
$$\iff \quad g_m R = \frac{1}{n(1-n)}$$

equivalent to the general case where $m = 1/n$

(2)



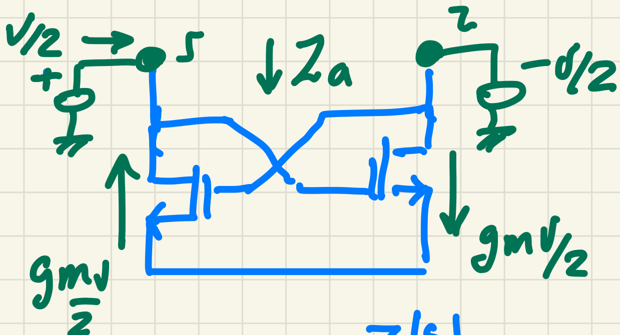
small-signal



$$\rightarrow \quad LG(s) = \frac{V_{loop}}{V_s} = \frac{g_m \frac{V_s}{2} \cdot Z(s)}{V_s} =$$

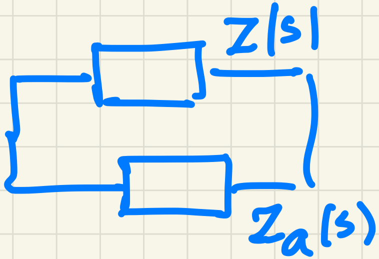
$$= \overbrace{\frac{g_m}{2}}^{G_m} \cdot Z(s)$$

Oscillation condition: $LG(j\omega_0) = 1 \Rightarrow \begin{cases} \omega_0 = \frac{1}{\sqrt{LC}} \\ \frac{g_m}{2} R = 1 \end{cases}$



alternative:

$$Z_a(s) = \frac{v}{-\frac{g_m v}{2}} = -\frac{2}{g_m}$$



Oscillation condition $-Z_a(s) = Z(s)$

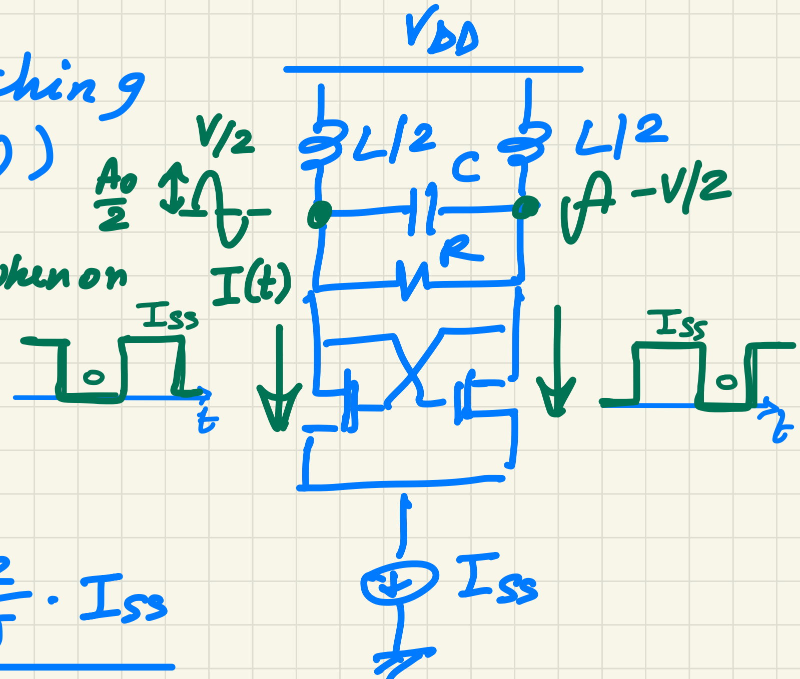
$\Leftrightarrow \quad +\frac{2}{g_m} = R$ and $\omega_0 = \frac{1}{\sqrt{LC}}$

Large signal

Hyp: assuming full switching (hard limiting $I(V)$)

$$\underline{A_0 \gg \sqrt{2} \cdot V_{ov}}$$

assuming MOSFET in sat. when on



Oscillation condition

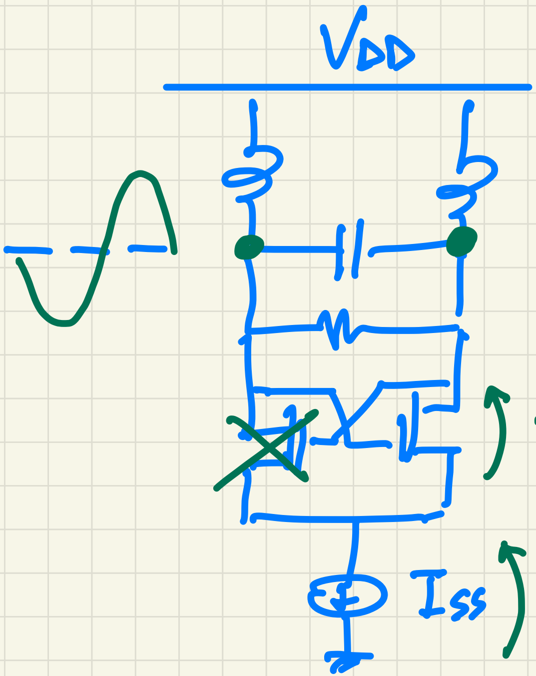
$$|G_{m_h}| \cdot R = 1$$

$$G_{m_h} = \frac{\overline{I}_1}{\overline{V}_1} = \frac{\frac{2}{\pi} \cdot I_{ss}}{A_0}$$

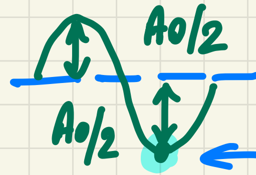
first harmonic of $I(t)$

first harmonic of $V(t)$

$$\Longrightarrow \underbrace{A_0 = \frac{2}{\pi} \cdot I_{ss} \cdot R}_{\text{also valid if MOSFET are in triode when on}}$$



$$I_L = \frac{V_L}{j\omega_0 L}$$

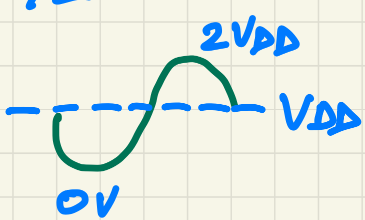


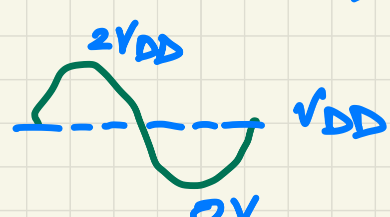
DC voltage = V_{DD}

min. voltage =

$= V_{DSon} + V_{DSgen} \xrightarrow[\text{limit}]{} 0V$

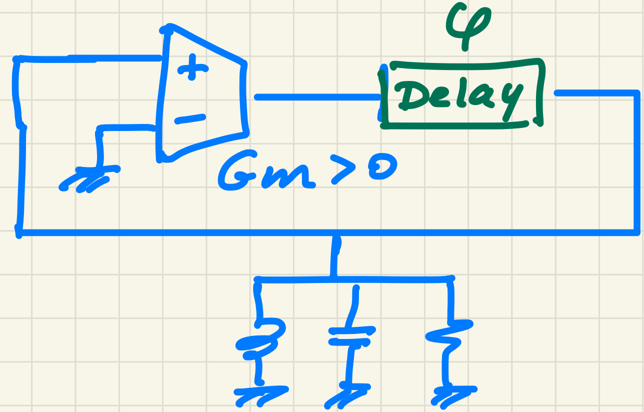
$$A_0/2 \rightarrow V_{DD} - 0V \Rightarrow \underline{A_0 \rightarrow 2V_{DD}}$$





Frequency Stability

φ is an extra delay in the loop



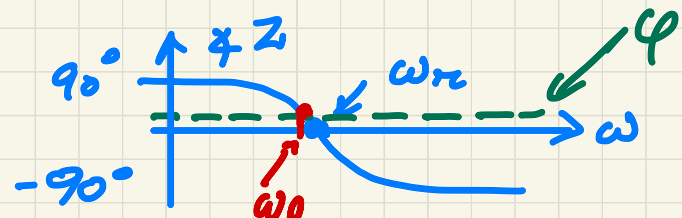
Oscillation condition :

$$\angle LG(j\omega_0) = 0$$

$$\Downarrow$$

$$-\varphi + \angle Z(j\omega_0) = 0$$

$$LG(j\omega) = G_m e^{-j\varphi} \cdot Z(j\omega)$$

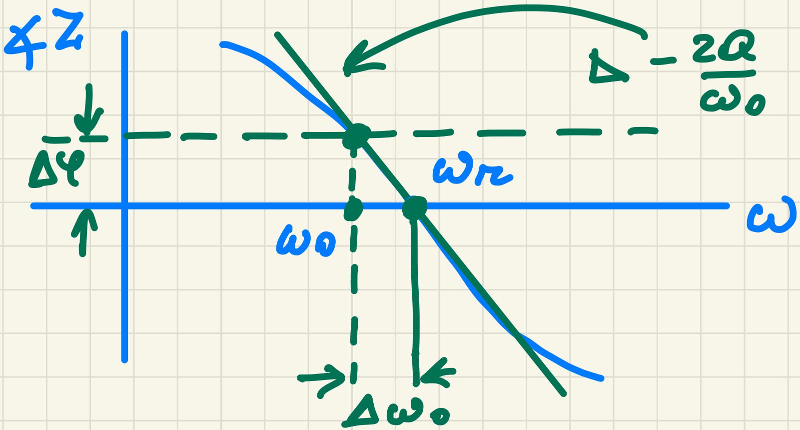


$$-\varphi + \frac{\pi}{2} - \operatorname{atan}\left\{\frac{\omega_0\omega_r/Q}{\omega_r^2 - \omega_0^2}\right\} = 0 \ ;$$

** $$\operatorname{atan}\left\{\frac{\omega_r^2 - \omega_0^2}{\omega_0\omega_r/Q}\right\} = \varphi$$

** $\frac{\pi}{2} - \operatorname{atan}\{x\} = \operatorname{atan}\left\{\frac{1}{x}\right\}$

$$\frac{d}{dx}\operatorname{atan}(x) = \frac{1}{1+x^2}$$



$$\measuredangle Z = \operatorname{atan} \underbrace{\frac{\omega_r^2 - \omega_0^2}{\frac{\omega_0\omega_r}{Q}}}_{x}$$

$$\Delta\omega_0 \simeq \frac{\Delta\varphi}{\left.\frac{d\measuredangle Z}{d\omega_0}\right|_{\omega_0=\omega_r}} = \Delta\varphi \cdot \frac{1}{\left.\frac{1}{1+\Big(\underbrace{Q\cdot\frac{\omega_r^2-\omega_0^2}{\omega_0\omega_r}}_{x}\Big)^2}\cdot\frac{Q}{\omega_r}\cdot\frac{-2\omega_0^2-(\omega_r^2-\omega_0^2)}{\omega_0^2}\right|_{\omega_0=\omega_r}}$$

$$= \frac{\Delta\varphi}{-2Q/\omega_0}$$

$$\Rightarrow \quad \frac{\Delta\omega_0}{\omega_0} = -\frac{\Delta\varphi}{2Q}$$

Relative frequency variation induced by an extra delay φ is INVERSELY PROPORTIONAL TO Q

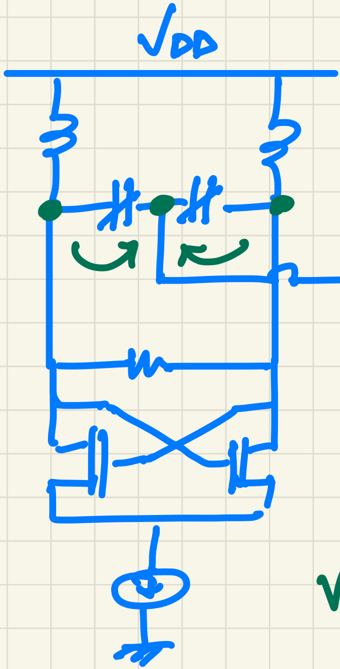
$$\text{Frequency stability Factor} \triangleq \frac{\Delta\omega_0/\omega_0}{\Delta\varphi} = -\frac{1}{2Q}$$

for the basic LC oscillat.

Exercise : Compute frequency stability factor for a ring oscillator ?

Voltage - Controlled Oscillators (VCOs)

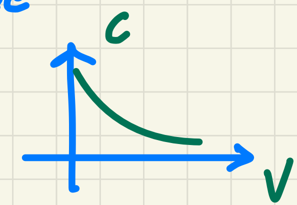
→ use of variable capacitors (VARACTORS)



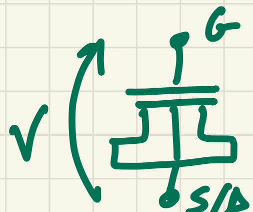
2 main options :

a) pn junction 

in reverse biasing



$$C = \frac{C_{j0}}{\left(1+\frac{V}{V_{j0}}\right)^m}$$

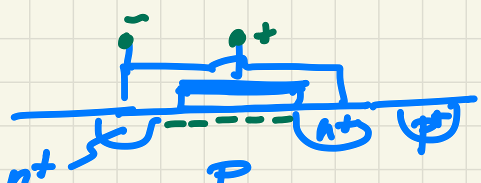


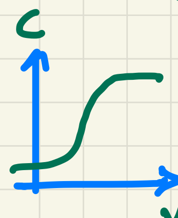
b) MOS junctions

1 - from inversion - to depletion

C_{OX}

C_{DEP}



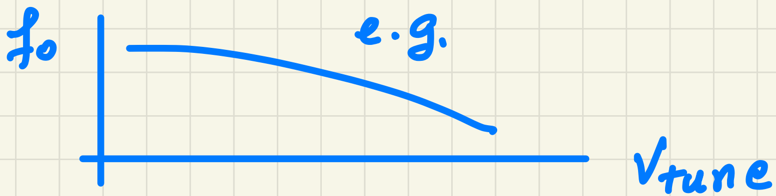


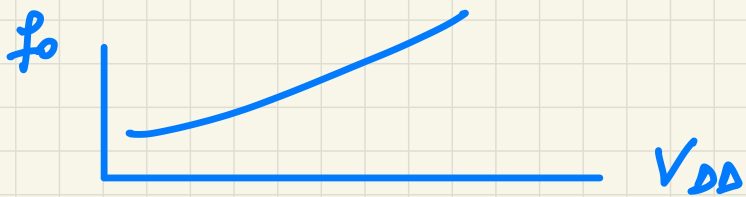
2 - from accumulation to depletion

Phase Noise 2 mechanisms:

- Indirect: AM-to-FM conversion:

upconversion of low-frequency noise





$$K_{VCO} = 2\pi \cdot \frac{\partial f_0}{\partial V_{tune}}$$

VCO sensitivity or gain

$$K_{VDD} = 2\pi \frac{\partial f_0}{\partial V_{DD}}$$

VCO supply pushing

Low frequency noise in V_{tune} PSD $S_{V_{tune}}(\omega)$

$$\underset{FM}{\Longrightarrow} S_{\omega_0}(\omega) = K_{VCO}^2 \cdot S_{V_{tune}}(\omega) \overset{PM}{\Longrightarrow} S_{\varphi}(\omega) = \frac{K_{VCO}^2 S_{V_{tune}}}{\omega^2}$$