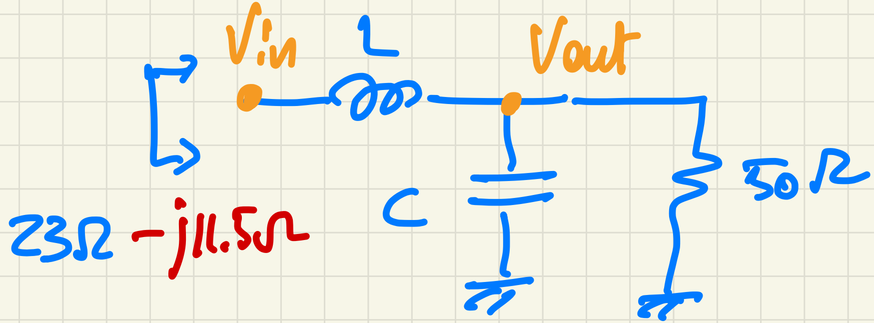
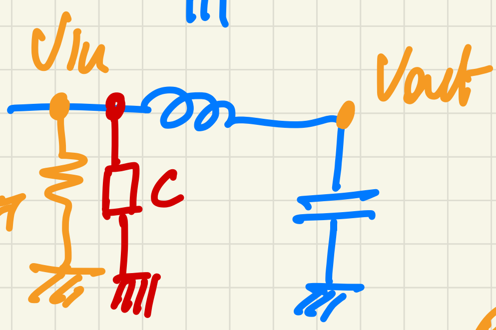
RF Circuit Design

L16



$f_0 = 2.5GHz$

$|V_{out}/V_{in}| = 1.32$



$$\frac{23^2 + 11.5^2}{23} = 29\Omega$$

$$\frac{|V_{out}|^2}{50\Omega} = \frac{|V_{in}|^2}{\cancel{23\Omega}\ 29\Omega}$$

Re Z_L

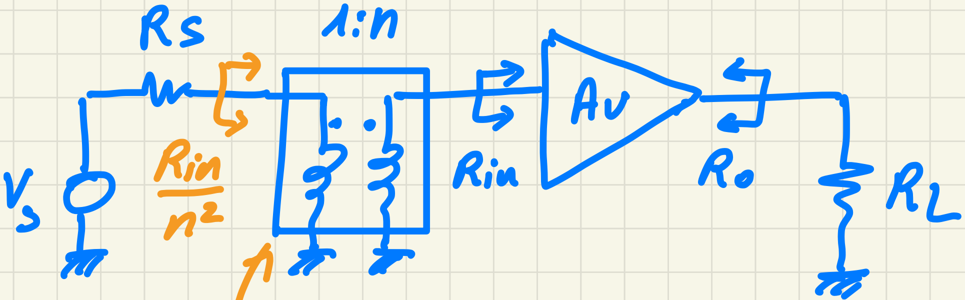
Re Z_{in}

$$\Downarrow$$

$$|V_{out}/V_{in}| = \sqrt{\frac{50}{\cancel{23}\ 29\Omega}}$$

$$= \cancel{1.47}\ 1.32$$

Impedance Matching

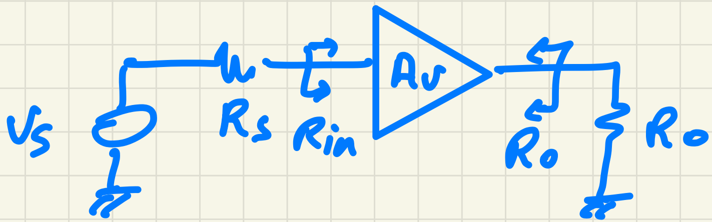


impedance transf. network (model)

unloaded voltage gain

$$\frac{V_{out}}{V_s} = \underbrace{\frac{\cancel{n^2}\,\frac{R_{in}}{\cancel{n^2}}}{\cancel{n^2}\,\frac{R_{in}}{\cancel{n^2}} + R_s n^2}\cdot n}_{\alpha \text{ (input voltage division)}} \cdot A_v \cdot \frac{R_L}{R_L + R_o}$$

** Voltage amplifier design



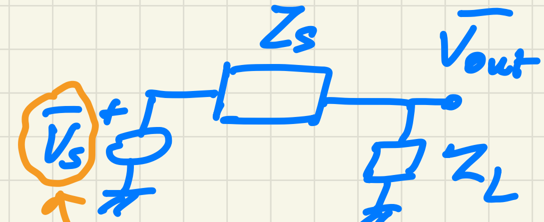
$$\frac{V_{out}}{V_S} = \underbrace{\frac{R_{in}}{R_{in}+R_S}} \cdot A_v \cdot \underbrace{\frac{R_L}{R_o+R_L}} = \alpha \cdot A_v \cdot \frac{R_L}{R_o+R_L}$$

to maximize gain : $R_{in} \gg R_S$

$R_L \gg R_o$

$$\left.\frac{V_{out}}{V_S}\right|_{max} \rightarrow A_v \quad \text{(unloaded gain)}$$

** Max. power transfer theorem

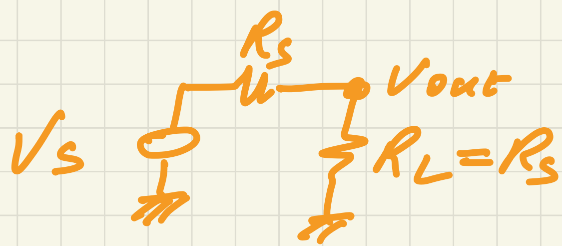


phasor of $v_s(t)$

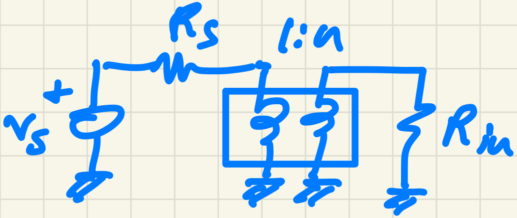
For a given source impedance Z_s
max. power at load is obtained
with $Z_L = Z_s^*$ (conjugate matching)

Available power: $P_{L,av} = \dfrac{|\bar{V}_{out}|^2}{2R_L} = \dfrac{|\bar{V}_s|^2}{8R_L}$

$Z_L = R_L + jX_L$



$\dfrac{V_{out}}{V_s} = \dfrac{1}{2}$



$$\alpha = \frac{n R_{in}}{R_{in} + n^2 R_s}$$

maximize α :

$$\frac{\partial \alpha}{\partial n} = \frac{\overbrace{R_{in}(R_{in}+n^2R_s)}^{R_{in}^2 + n^2 R_{in} R_s} - \overbrace{nR_{in}\cdot 2nR_s}^{2n^2 R_{in} R_s}}{(R_{in}+n^2R_s)^2}$$

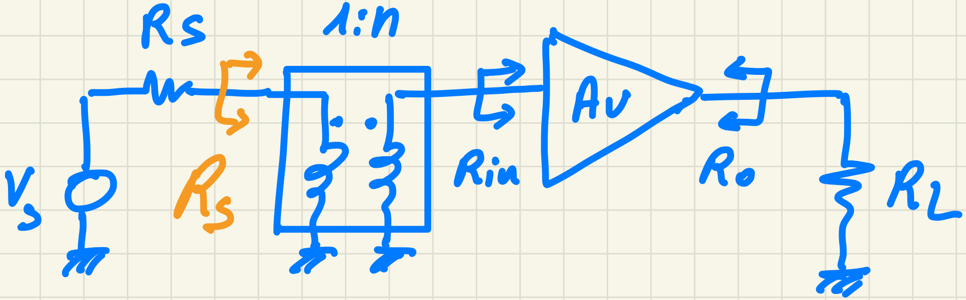
$$\frac{\partial \alpha}{\partial n} = 0 \iff R_{in}^{\cancel{2}} = n^2 \cancel{R_{in}} R_s \iff n_{opt} = \sqrt{\frac{R_{in}}{R_s}}$$

(hyp. $R_{in} \neq 0$)

maximum of α : $\alpha_{max} = \frac{n \cancel{R_{in}}}{2 \cancel{R_{in}}} = \frac{1}{2}\sqrt{\frac{R_{in}}{R_s}}$

$$\boxed{\frac{R_{in}}{n^2} = R_s}$$

Transformed imped. MATCHED to the source impedance



input matching maximizes voltage gain

$$\left.\frac{V_{out}}{V_s}\right|_{max} = \underbrace{\frac{1}{2}\cdot n_{opt}}_{\alpha_{max}} \cdot A_v \cdot \frac{R_L}{R_L+R_o}$$

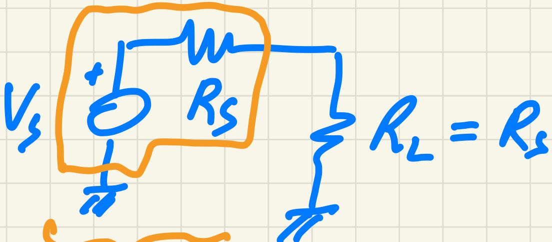
e.g. $R_s = 50\Omega$

$R_{in} = 1k\Omega$

$n_{opt} = \sqrt{\frac{1k}{50}} = 4.47 \Rightarrow \alpha_{max} = \frac{1}{2}\cdot 4.47 =$

$= 2.24$

Power Gain

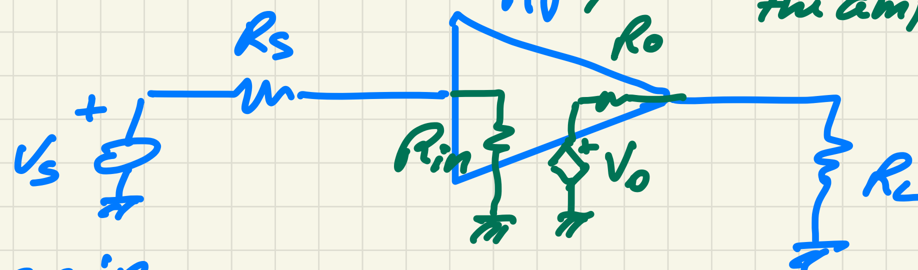


model of antenna

$$P_{s,av} = \frac{V_s^2}{8R_s}$$

$$\frac{V_o}{V_s} = \alpha \cdot A_v = $$
$$= A_o$$

voltage gain of the amplif.



Available power gain
(max. achievable power gain)

$$G_A = \frac{P_{out,av}}{P_{in,av}} = \frac{V_o^2/8R_o}{V_s^2/8R_s} =$$

$$= \left(\frac{V_o}{V_s}\right)^2 \cdot \frac{R_s}{R_o}$$

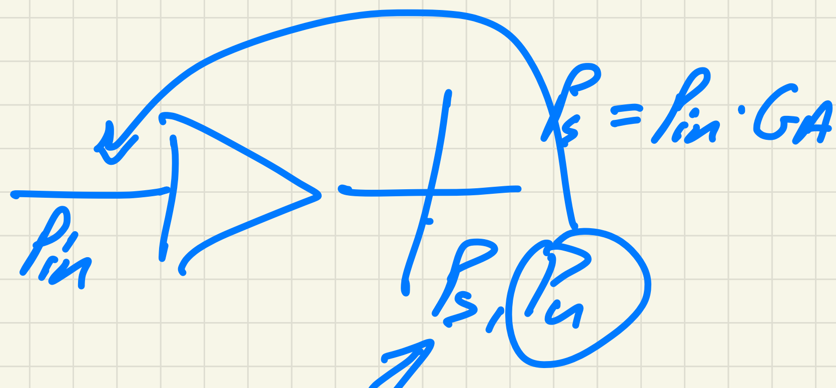
$\Rightarrow$ av. power gain $G_A = (\alpha \cdot A_V)^2 \cdot \frac{R_S}{R_O} =$

$$= A_O^2 \cdot \frac{R_S}{R_O}$$

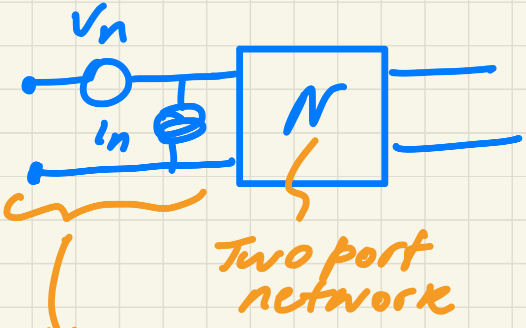
In general: a. power gain $\neq$ the square of voltage gain. This is true ONLY when $R_S = R_O$:

$$G_A = A_O^2$$

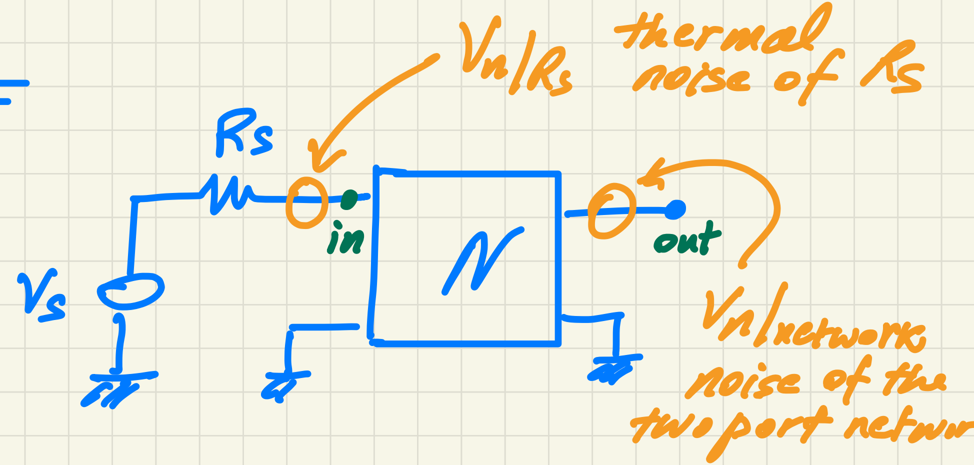
$$SNR = \frac{P_{in}}{(P_n / G_A)}$$



Effects of Noise



2 equivalent generators modelling noise of the network



Noise Figure

$$NF \triangleq \frac{SNR_{in}}{SNR_{out}} =$$

$$= \frac{\overline{V^2_{s\,in}}}{\overline{V^2_{n\,in}}} \cdot \frac{\overline{V^2_{n\,out}}}{\overline{V^2_{s\,out}}} = \frac{1}{A_o^2} \cdot \frac{\overline{V^2_{n|network}} + A_o^2\,\overline{V^2_{n|R_s}}}{\overline{V^2_{n|R_s}}}$$

$$A_o = V_{s\,out} / V_{s\,in}$$

$$NF = \frac{1}{A_0^2} \cdot \left(\frac{\overline{V_{n\,network}^2}}{\overline{V_{nR_S}^2}} + A_0^2 \right) =$$

(1) $$= 1 + \frac{\boxed{\overline{V_{n\,network}^2} / A_0^2}}{\overline{V_{nR_S}^2}} =$$

noise of the network (input-referred)

noise of source resistance

(2) $$= \frac{\boxed{\overline{V_{n\,out,total}^2} / A_0^2}}{\overline{V_{nR_S}^2}}$$

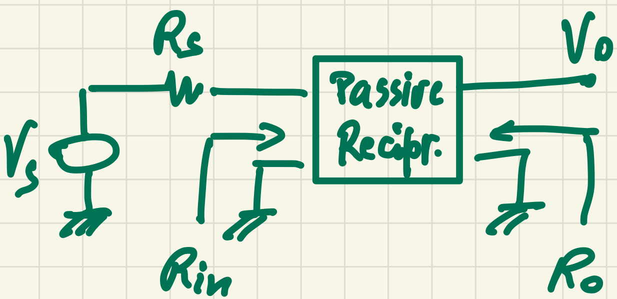
noise of network + source resistance (input-referred)

$$\frac{\overline{V^2_{n\,out\,total}}}{A_0^2} = NF \cdot \overline{V^2_{n R_s}}$$

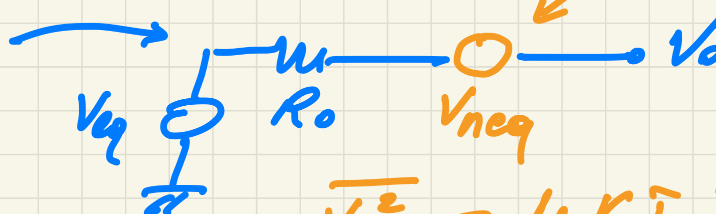
factor that multiplies source noise to get total noise (input referred)

$$\Rightarrow PSD_{Rx(input\ ref.)} = 4KTR_s \cdot NF_{Rx}$$

NOISE FIGURE of a lossy circuit (eg passive filter)



Thevenin theorem



Nyquist theorem

$$\overline{V^2_{neq}} = 4KT \cdot Re\{Z_o\} = $$
$$= 4KT \cdot R_o$$

$$A_o = \frac{V_{eq}}{V_s}$$

$$NF = \frac{\overline{V^2_{n\,out\,total}} / A_o^2}{\overline{V^2_{nRs}}} = \frac{\underbrace{\overline{V^2_{neq}}}}{A_o^2} \cdot \frac{1}{\underbrace{\overline{V^2_{nRs}}}} =$$

$$= \frac{\cancel{4kT}R_o}{A_o^2} \cdot \frac{1}{\cancel{4kT}R_s} =$$

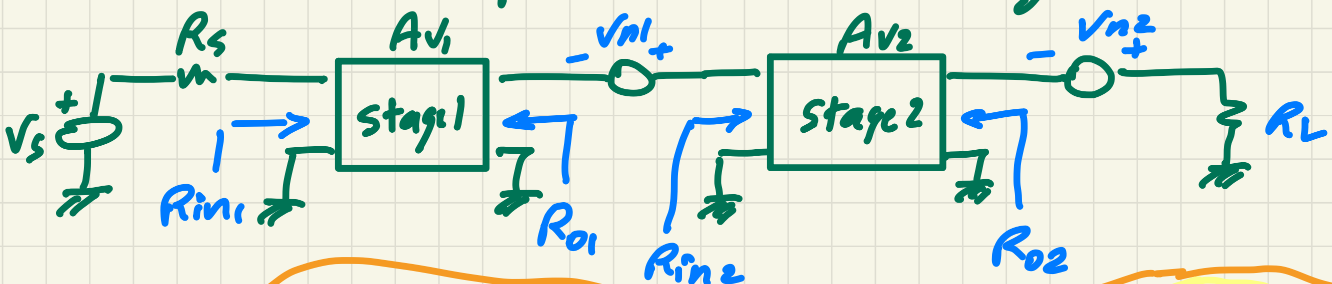
$$= \frac{1}{\underbrace{A_o^2 \cdot \frac{R_s}{R_o}}_{\text{available power gain } G_A}} = \frac{1}{G_A} = L_A$$

available power loss

⇒ Noise figure of a lossy circuit is given by its available power loss

e.g. filter with 2 dB power loss → NF = 2 dB

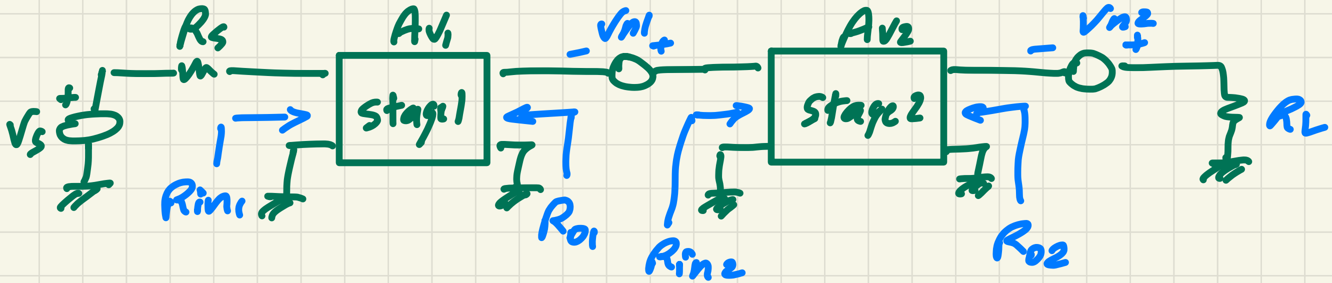
NOISE FIGURE of cascaded stages



?

$$NF = \underbrace{1 + \frac{\overline{V_{n1}^2}}{\underbrace{\alpha_1^2 A_{v1}^2}_{A_{o1}^2}} \cdot \frac{1}{4KTR_s}}_{NF_1} + \frac{\overline{V_{n2}^2}}{\alpha_1^2 A_{v1}^2 \cdot \alpha_2^2 A_{v2}^2} \cdot \frac{1}{4KTR_s}$$

$$\alpha = \frac{R_{in1}}{R_{in1} + R_s} \qquad \alpha_2 = \frac{R_{in2}}{R_{in2} + R_{o1}}$$



$$NF_2\Big|_{R_{o1}} = 1 + \frac{\overline{v_{n2}^2}}{\alpha_2^2 \cdot A_{v2}^2} \cdot \frac{1}{4kTR_{o1}}$$

$$\Downarrow$$

$$\Rightarrow NF = NF_1 + \frac{(NF_2|_{R_{o1}} - 1) \cdot \cancel{4kT}R_{o1}}{\alpha_1^2 A_{v1}^2 \cdot \cancel{4kT}R_S}$$

$$= NF_1 + \frac{NF_2|_{R_{o1}} - 1}{G_{A1}}$$

$$\alpha_1^2 A_{v1}^2 \cdot \frac{R_S}{R_{o1}} = G_{A1}$$

available power gain of stage 1

$$\Rightarrow NF = 1 + \underbrace{(NF_1 - 1)}_{\text{stage 1}} + \underbrace{\frac{NF_2\big|_{R_{01}} - 1}{G_{A1}}}_{\text{stage 2}} + \underbrace{\frac{NF_3\big|_{R_{02}} - 1}{G_{A1}\, G_{A2}}}_{\text{stage 3}} + \ldots$$

$\Rightarrow$ First stages most critical for noise figure