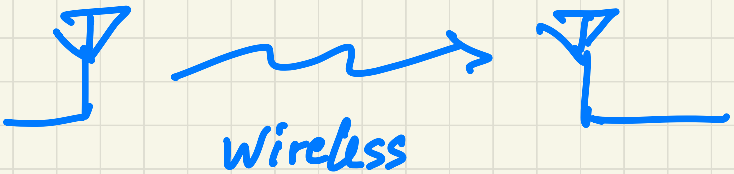
RF Circuit Design

L1

2020/21

Communication Theory

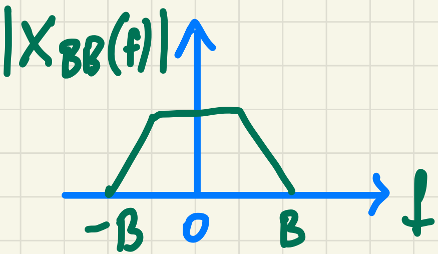


Carrier modulation

sinusoid

$$\underbrace{A_c}_{\text{amplitude}} \cdot \cos[\underbrace{\omega_c t}_{\text{phase}}]$$

"carries" informat.



Baseband signal (information)

$c = 3 \cdot 10^8$ m/s

$\lambda/2 \approx 15$ cm

$\lambda \approx 30$ cm

$\Downarrow$

$f_c = 1$ GHz

$\lambda/2$ physical dimension

Hertz dipole (antenna)

$$\lambda = \frac{c}{f_c}$$

AM modulation (amplitude modulation)

$$x(t) = A_c \left[1 + m \cdot \underbrace{x_{BB}(t)}_{\text{baseband signal}} \right] \cdot \underbrace{\cos \omega_c t}$$

Spectrum : Fourier transform of $x(t)$

$$e^{\pm j\omega_c t} \longleftrightarrow \delta(f \mp f_c)$$

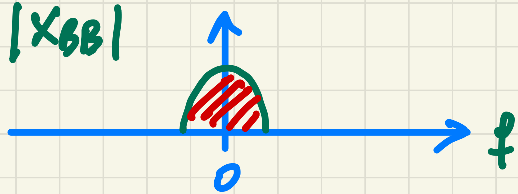
$$X(f) \triangleq \int_{-\infty}^{+\infty} x(t)\, e^{-j 2\pi f t}\, dt$$

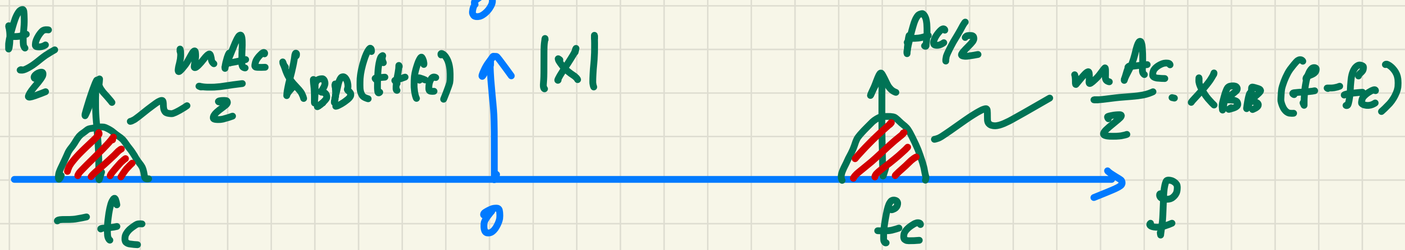
$$x(t) = A_c \left[1 + m \cdot x_{BB}(t) \right] \cdot \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$$

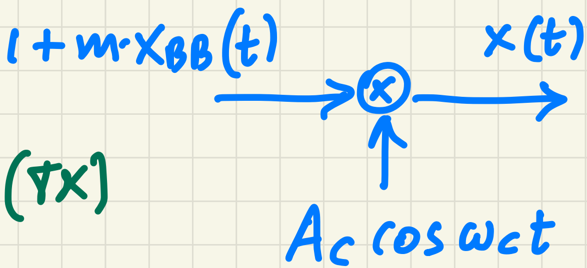
$$X(f) = \frac{A_c}{2} \delta(f - f_c) + \frac{A_c}{2} \delta(f + f_c) +$$

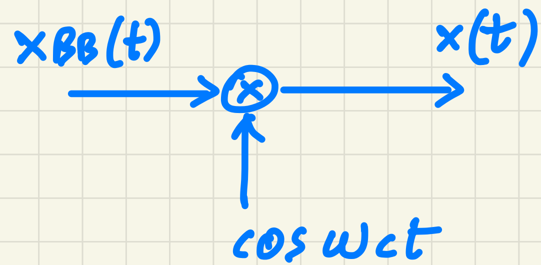
$$+ \frac{m A_c}{2} \cdot X_{BB}(f) * \delta(f + f_c) + \frac{m A_c}{2} X_{BB}(f) * \delta(f - f_c)$$

$$X(f) = \frac{A_c}{2}\delta(f-f_c) + \frac{A_c}{2}\delta(f+f_c) + \frac{mA_c}{2}X_{BB}(f-f_c) + \frac{mA_c}{2}X_{BB}(f+f_c)$$



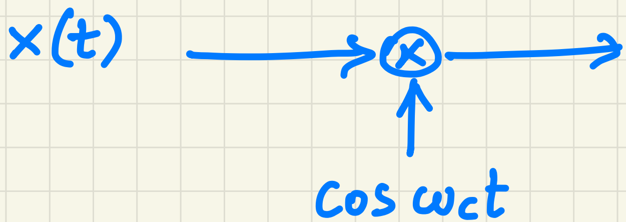






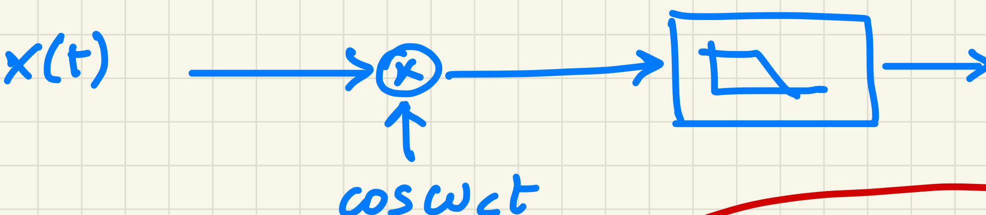
- AM modulation with transmitted carrier
- AM without transmitted c.

Coherent demodulation (RX)

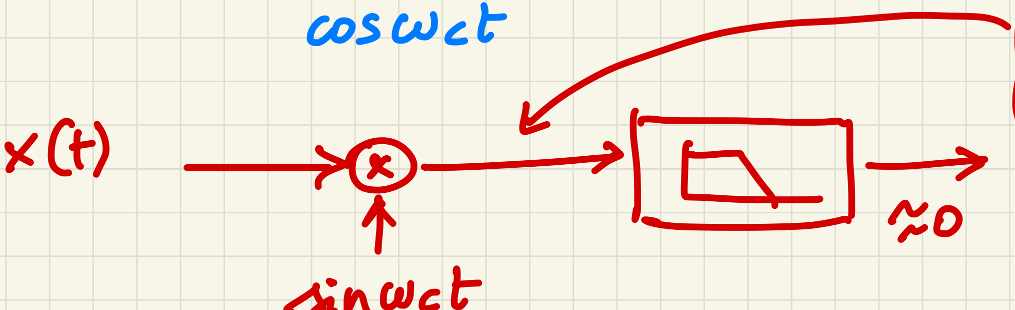


$$x(t)\cdot\cos\omega_c t =$$
$$= x_{BB}(t)\cdot\cos^2\omega_c t =$$
$$= x_{BB}(t)\cdot\frac{1+\cos 2\omega_c t}{2}$$

$$x(t) = x_{BB}(t)\cdot\cos\omega_c t$$



$$\approx \frac{1}{2}\cdot x_{BB}(t)$$

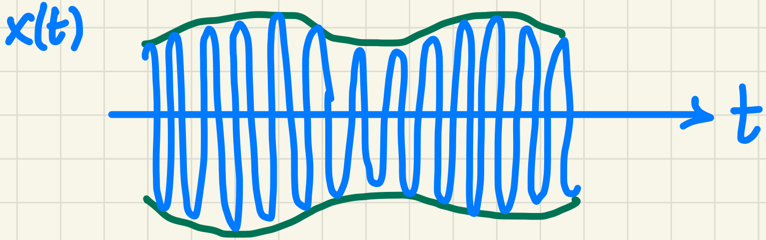


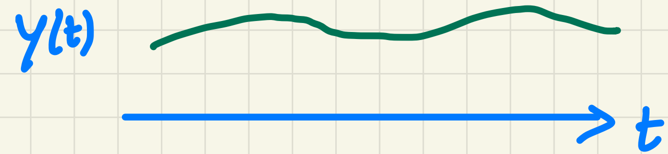
$$x(t)\cdot\sin\omega_c t =$$
$$= x_{BB}(t)\sin\omega_c t\cos\omega_c t$$
$$= x_{BB}(t)\cdot\frac{1}{2}\sin 2\omega_c t$$

Non-coherent demodulation

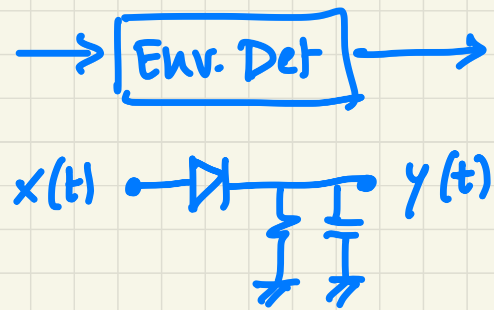
AM with TX carrier

$$x(t) = A_c\,[1 + m\,x_{BB}(t)]\cos\omega_c t \qquad m\,x_{BB} < 1$$

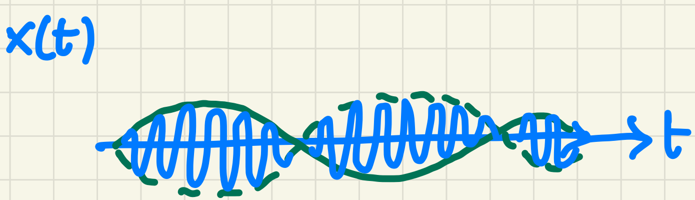




AM without TX'ed carrier



$$x(t) = x_{BB}(t)\cdot\cos\omega_c t$$



Phasor representation of a sinusoidal AM

$$\cos x \cdot \cos y = \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y)$$

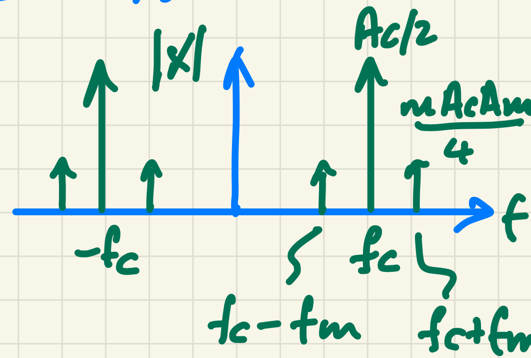
$$x_{BB}(t) = A_m \cos \omega_m t$$

$$x(t) = A_c\,[\,1 + m\,x_{BB}(t)\,]\cos \omega_c t =$$

$$= A_c \cdot \cos\omega_c t + m A_m A_c \cdot \cos\omega_c t \cdot \cos\omega_m t =$$

$$= A_c \cdot \cos\omega_c t + \frac{m A_m A_c}{2}\cos(\omega_c - \omega_m)t +$$

$$+ \frac{m A_m A_c}{2}\cos(\omega_c + \omega_m)\cdot t$$



$$x(t) = \mathrm{Re}\{\,\bar{X}(t)\cdot e^{j\omega_c t}\,\}$$

Phasor

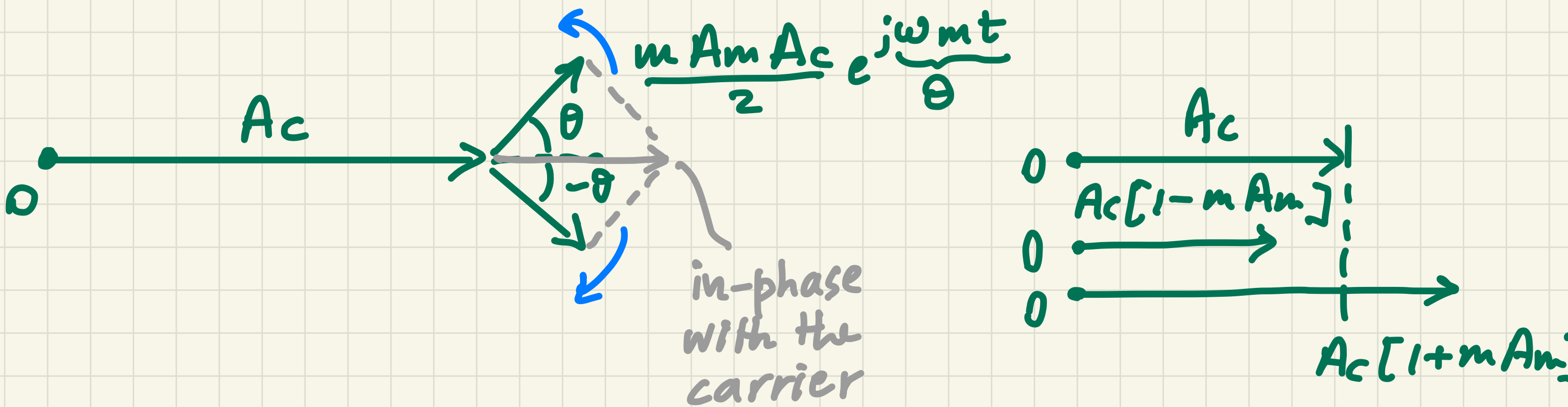
$$e^{j\omega_c t} = \cos\omega_c t + j\sin\omega_c t$$

$$x(t) = A_c \cdot \cos\omega_c t + \frac{m A_m A_c}{2} \cos(\omega_c - \omega_m)t + $$
$$+ \frac{m A_m A_c}{2} \cos(\omega_c + \omega_m)\cdot t$$

$$x(t) = \mathrm{Re}\left\{ \bar{X}(t) \cdot e^{j\omega_c t} \right\}$$

Phasor

$$\bar{X}(t) = A_c + \frac{m A_m A_c}{2} \cdot e^{-j\omega_m t} + \frac{m A_m A_c}{2} e^{j\omega_m t}$$



Frequency Modulation (FM)

$$x(t) = A_c \cos\left[\omega_c t + \underbrace{m \int_{-\infty}^{t} x_{BB}(t')\,dt'}_{\varphi(t)}\right]$$

$$\begin{cases} \omega(t) = \dfrac{d\Phi}{dt} \\ \Phi(t) = \displaystyle\int_{-\infty}^{t} \omega(t')\,dt' \end{cases}$$

Relationship between angular frequency ω and phase Φ of a periodic signal

Narrowband FM approximation (NBFM):

$$\varphi(t) = m \int_{-\infty}^{t} x_{BB}(t')\,dt' \ll 1 \text{ rad}$$

$$x(t) = A_c \cos\left[\omega_c t + \varphi(t)\right] =$$

$$= A_c \cos \omega_c t \cdot \cos\left[\varphi(t)\right] - A_c \sin \omega_c t \cdot \sin\left[\varphi(t)\right] =$$

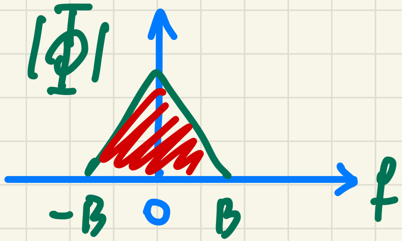
$$\simeq A_c \cdot \cos \omega_c t \cdot 1 - A_c \cdot \sin \omega_c t \cdot \varphi(t) =$$

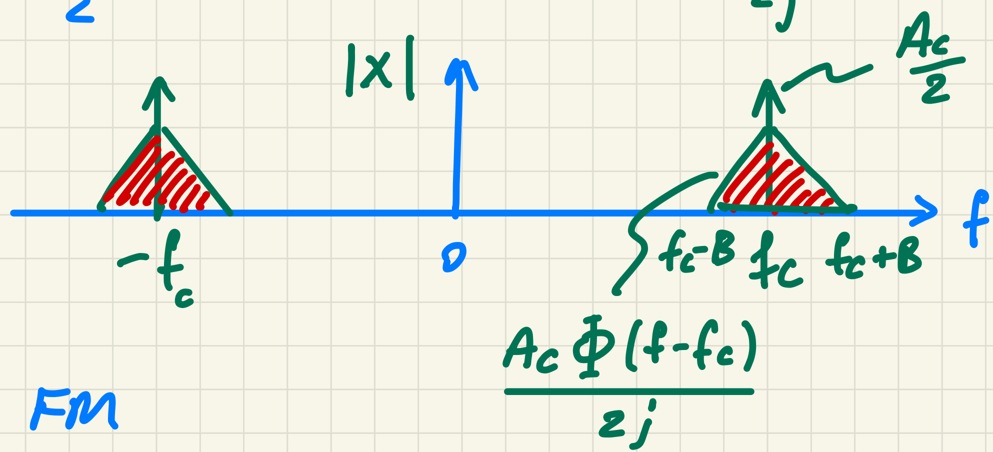
NBFM

$$= \underbrace{A_c \cos \omega_c t}_{\text{carrier}} - \underbrace{A_c \cdot \varphi(t) \cdot \sin \omega_c t}_{\text{AM modulation of the quadrature component of the carrier}} =$$

$$= A_c \cdot \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} - A_c \varphi(t) \cdot \frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}$$

$$x(t) = A_c \cdot \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} - A_c \varphi(t) \cdot \frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}$$





Case : Sinusoidal FM

$$x_{BB}(t) = A_m \cos \omega_m t$$

$$x(t) = A_c \cos\left[\omega_c t + m\int_{-\infty}^{t} A_m \cos \omega_m t' \, dt'\right] =$$

$$= A_c \cos\left[\omega_c t - \overbrace{\underbrace{\frac{m A_m}{\omega_m}}_{\beta} \cdot \sin \omega_m t}^{\varphi(t)}\right] =$$

$$x(t) = A_c \cos\Big[\omega_c t - \underbrace{\beta \cdot \sin \omega_m t}_{\varphi(t)}\Big] =$$

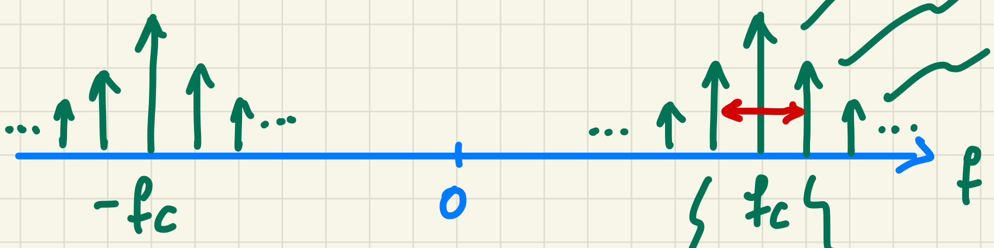
modulation depth

$$= A_c \sum_{n=-\infty}^{+\infty} \underbrace{J_n(\beta)}_{\text{First Kind Bessel Function}} \cdot \cos\left[\omega_c + n\,\omega_m t\right]$$

n = 0

n = ±1

n = ±2



$f_c - f_m$ $f_c + f_m$

BW of the FM carrier

CARSON'S BW

$$BW_{98\%} = 2 \cdot (\beta + 1) \cdot f_m$$

BW of the baseband signal

$$BW_{98\%} = 2(\beta + 1)\cdot f_m \approx 2\cdot f_m$$

NBFM

$\varphi \ll 1$ rad

$\Rightarrow \beta \ll 1$ rad

Phasor Representation

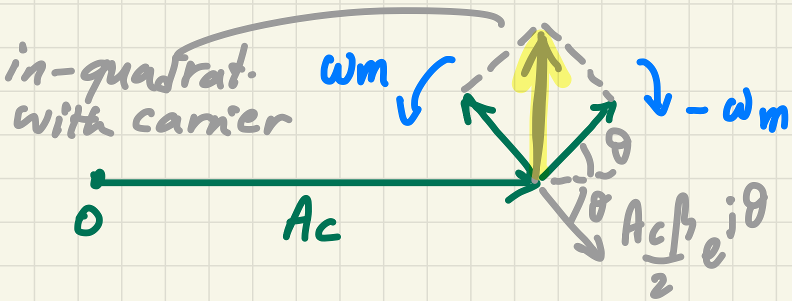
$$x(t) = A_c \cos[\omega_c t + \varphi(t)] \approx A_c \cos\omega_c t - A_c\cdot\varphi(t)\cdot\sin\omega_c t$$

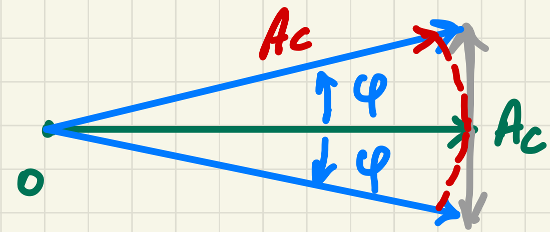
$$= A_c \cos\omega_c t - A_c\cdot\sin\omega_c t\cdot[-\beta \sin\omega_m t] =$$

$$= A_c \cos\omega_c t + \frac{A_c\beta}{2}\cos(\omega_c - \omega_m)t - \frac{A_c\beta}{2}\cos(\omega_c + \omega_m)t$$

$$\Rightarrow \bar{X}(t) = A_c + A_c\beta/2 \cdot e^{-j\underbrace{\omega_m t}_{\theta}} - A_c\beta/2 \cdot e^{j\underbrace{\omega_m t}_{\theta}}$$

$$\bar{X}(t) = A_c + \frac{A_c\beta}{2} \cdot e^{-j\underbrace{\omega_m t}_{\theta}} - \frac{A_c\beta}{2} \cdot e^{j\underbrace{\omega_m t}_{\theta}}$$





PM (or FM) is equivalent to amplitude modulat. of the quadrature component of the carrier

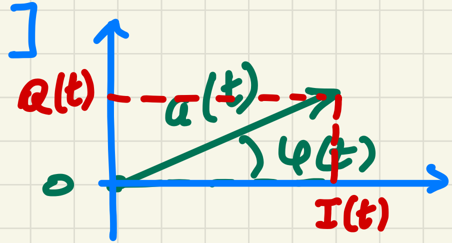
Why is not a pure PM modulation ?

The equivalence holds only under NBFM approximation

$\beta \ll 1$ rad (or $\varphi \ll 1$ rad)

AM and PM modulation (Quadrature modulation)

- $x(t) = a(t)\cdot\cos[\omega_c t + \varphi(t)]$



Phasor $\bar{x}(t) = a(t)\cdot e^{j\varphi(t)}$

$$\mathrm{Re}\{x(t)\cdot e^{j\omega_c t}\} = \mathrm{Re}\{a(t)\cos[\omega_c t+\varphi] + j\,a(t)\sin[\omega_c t+\varphi]\}$$

- $x(t) = I(t)\cdot\cos\omega_c t - Q(t)\sin\omega_c t =$

$$= I(t)\frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2} + jQ(t)\cdot\frac{e^{+j\omega_c t}-e^{-j\omega_c t}}{2\cancel{j}} =$$

$$= \frac{1}{2}[\underbrace{I(t)+jQ(t)}_{\bar{x}}]\cdot e^{j\omega_c t} + \frac{1}{2}[I(t)-jQ(t)]\,e^{-j\omega_c t}$$

$$= \frac{1}{2}\bar{x}\,e^{j\omega_c t} + \frac{1}{2}\bar{x}^{*}e^{-j\omega_c t} = \cancel{2}\,\frac{1}{\cancel{2}}\,\mathrm{Re}\{\bar{x}\,e^{j\omega_c t}\}$$