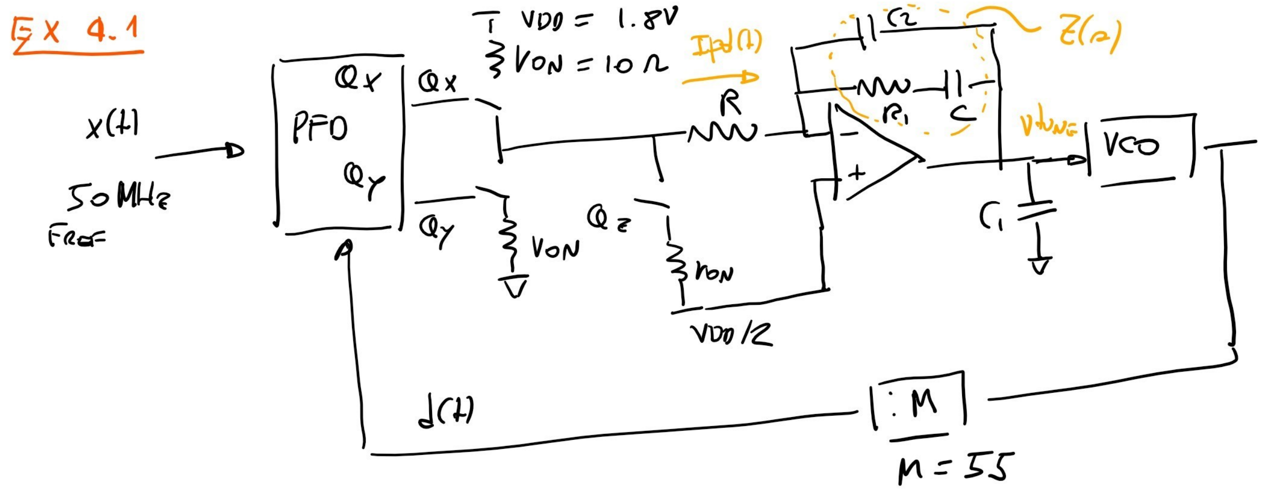
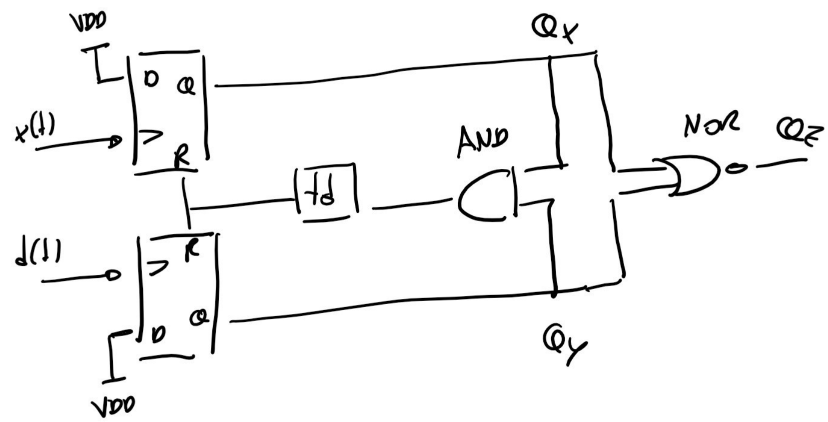
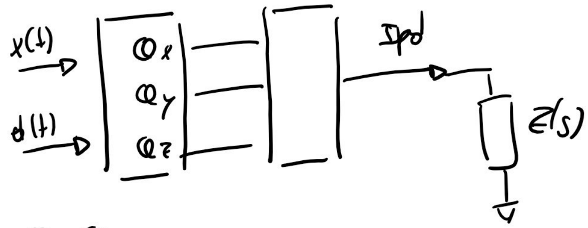
EX 4.1







- POSSIBLE STATES OF Q_X, Q_Y, Q_Z

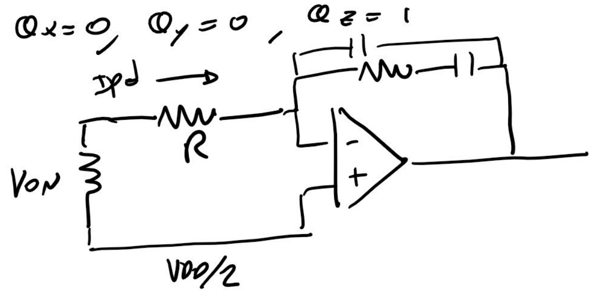
Q_X	Q_Y	R	Q_Z
0	0	0	1
0	1	0	0
1	0	0	0
1	1	1	0

$R = AND(Q_X, Q_Y)$

$Q_Z = NOR(Q_X, Q_Y)$

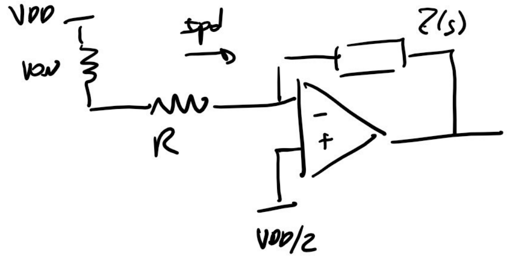
- RELATION BETWEEN Q_X, Q_Y, Q_Z AND I_{pd}

- STATE 1 $\quad Q_X = 0, \; Q_Y = 0, \; Q_Z = 1$



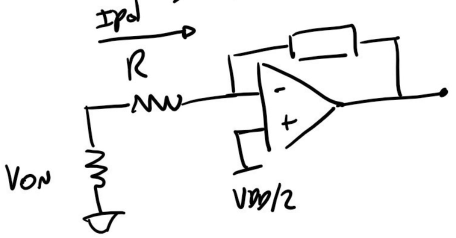
$I_{pd} = \emptyset$

- STATE 2 $\quad Q_x = 1,\ Q_y = 0,\ Q_z = 0$



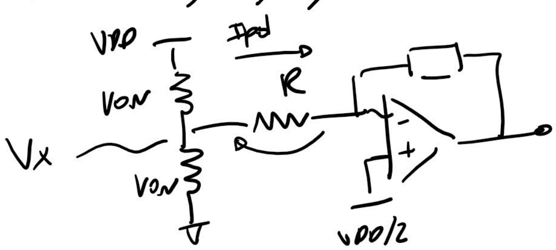
$$I_{pd} = \frac{(V_{DD} - V_{DD}/2)}{V_{ON} + R} = I_{po}$$

- STATE 3 $\quad Q_x = 0,\ Q_y = 1,\ Q_z = 0$



$$I_{pd} = \frac{(0 - V_{DD}/2)}{V_{ON} + R} = -I_{po}$$

- STATE 4 $\quad Q_x = 1,\ Q_y = 1,\ Q_z = 0$



$$I_{pd} = \frac{(V_x - V_{DD}/2)}{R}$$

$$V_x = \frac{(V_{ON} /\!/ R)}{(V_{ON} /\!/ R) + V_{ON}} V_{DD} + \frac{(V_{ON}/2)\frac{V_{DD}}{2}}{V_{ON}/2 + R}$$

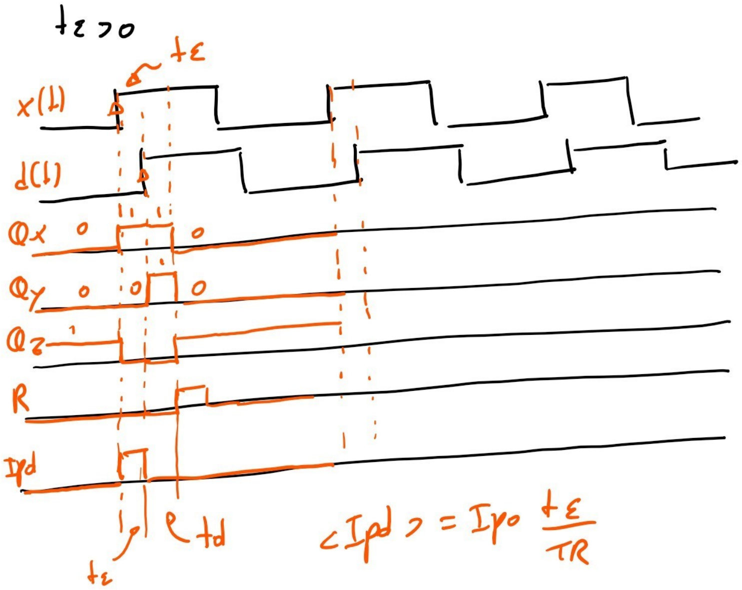
$R >> V_{ON}$

$$\simeq \frac{V_{ON}}{V_{ON} + V_{ON}} V_{DD} \simeq \frac{V_{DD}}{2}$$

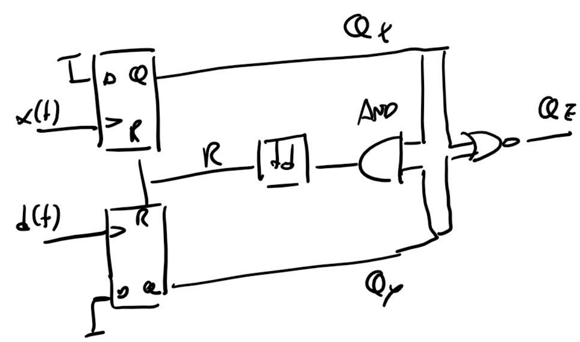
$$I_{pd} \simeq \emptyset$$

- DERIVE THE PHASE DETECTOR EQUIVALENT MODEL

$t_\varepsilon > 0$

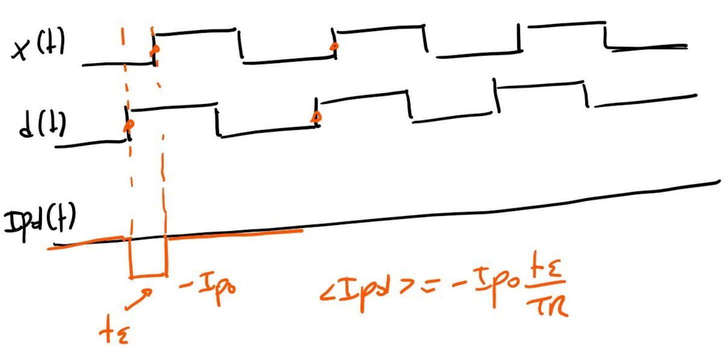


$$\langle I_{pd} \rangle = I_{po} \frac{t_\varepsilon}{T_R}$$



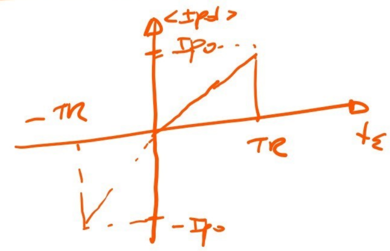
Q_x	Q_y	Q_z	I_{pd}
0	0	1	0
1	0	0	I_{po}
0	1	0	$-I_{po}$
1	1	0	0

$t_\varepsilon < 0$

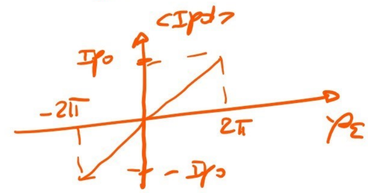


$$\langle I_{pd} \rangle = -I_{po} \frac{t_\varepsilon}{T_R}$$

PD CHARACTERISTIC

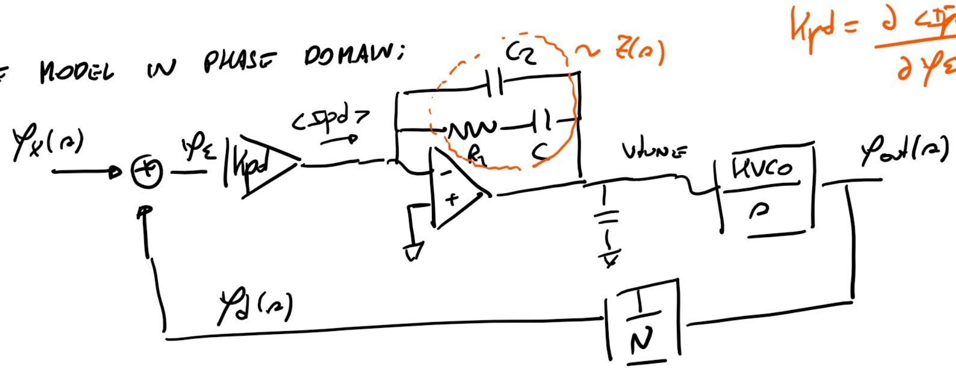


$$\varphi_\varepsilon = \frac{2\pi}{T_R} t_\varepsilon$$



$$K_{pd} = \frac{\partial \langle I_{pd} \rangle}{\partial \varphi_\varepsilon} = \frac{I_{po}}{2\pi}$$

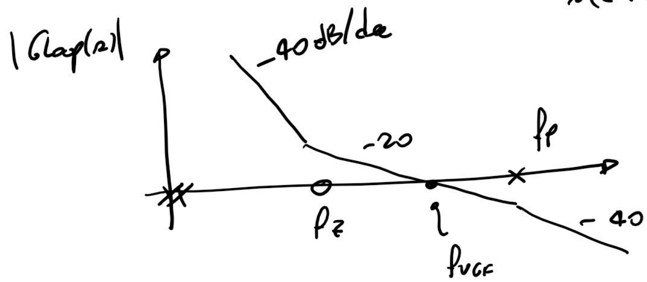
- COMPLETE MODEL IN PHASE DOMAIN:



$$G_{loop}(s) = K_{pd}\, Z(s) \frac{K_{vco}}{s} \frac{1}{N} = K_{pd} \left[\left(R_1 + \frac{1}{sC}\right) // \frac{1}{sC_2} \right] \frac{K_{vco}}{s} \frac{1}{N}$$

$$= K_{pd} \frac{(1 + sCR_1)}{s(C+C_2)\left[1 + s(C//C_2)R_1\right]} \frac{K_{vco}}{s} \frac{1}{N} = K \frac{(1 + s\tau_z)}{s^2 (1 + s\tau_p)}$$

$$\begin{cases} K = \dfrac{K_{pd} K_{vco}}{(C+C_2)N} \\ \tau_z = C R_1 \\ \tau_p = R_1 (C // C_2) \end{cases}$$



a) Set R, C

CLOSED LOOP POLES @ 100 KHz

LOCATED @ 45° GAUSS PLANE

$$1 + G_{loop}(s) = 0 \qquad 1 + \frac{K(1 + s\tau_z)}{s^2 (1 + s\tau_p)} = 0$$

SINCE $f_p \gg f_{UGF}$ FOR STABILITY

$$(1 + s\tau_p)\left[1 + \frac{K(1 + s\tau_z)}{s^2}\right] = 0 \implies (1 + s\tau_p)(s^2 + sK\tau_z + K) = 0$$

HF POLE NOT AFFECTED BY Gloop

$$\begin{cases} s^2 + s\,\tau_z K + K = 0 \\ s^2 + s\,\dfrac{\omega_N}{Q} + \omega_N^2 = 0 \end{cases}$$

COMPARE WITH CANONICAL FORM

$\omega_N = 2\pi\ 100\,KHz$

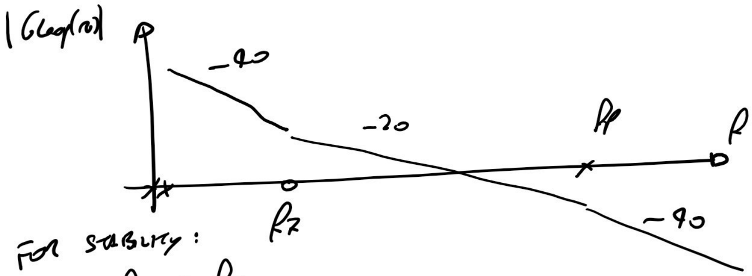
$Q = \dfrac{\sqrt{2}}{2}$

$K = \omega_N^2$

$\tau_z = \dfrac{1}{\omega_N Q} = R_1 C \rightsquigarrow$ $R_1 = 100\,\Omega$ (FROM DATA)

$$\boxed{C = 22.51\ nF}$$

$$K = \frac{K_{pd}\,K_{VCO}}{N\,(C + C_2)} = \omega_N^2$$



FOR STABILITY:

$f_p >> f_z$

$$\frac{1}{2\pi R_1 (C // C_2)} >> \frac{1}{2\pi R_1 C} \Rightarrow (C//C_2) << C$$

$$\Downarrow$$

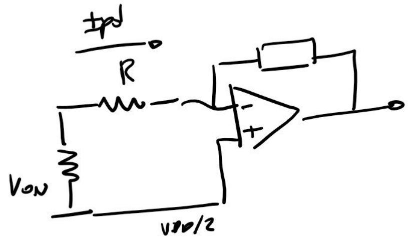
$$C_2 << C$$

$$\begin{cases} K = \dfrac{K_{pd}\,K_{VCO}}{N\,(C + C_2)} \simeq \dfrac{K_{pd}\,K_{VCO}}{N\,C} = \omega_N^2 \\ K_{pd} = \dfrac{I_{pd}}{2\pi} \end{cases}$$

$\rightsquigarrow I_{pd} = 4.4\,mA = \dfrac{(V_{DD}/2)}{R} \rightsquigarrow \boxed{R = 194{,}5\,\Omega}$

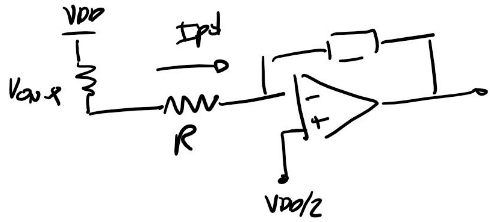
b) Von $Q_x = 15\Omega$ Set C_2 $\mathcal{L}(\Delta f = 50\,MHz) = -80\,dBc$

- STATE 1 $Q_x = 0$, $Q_y = 0$, $Q_z = 1$



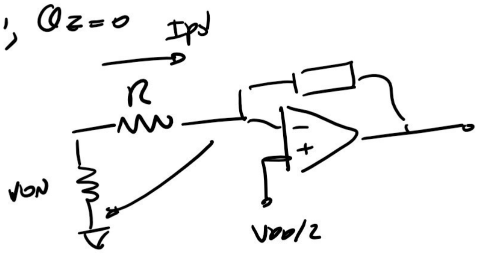
$I_{pd} = 0$

- STATE 2 $Q_x = 1$, $Q_y = 0$, $Q_z = 0$



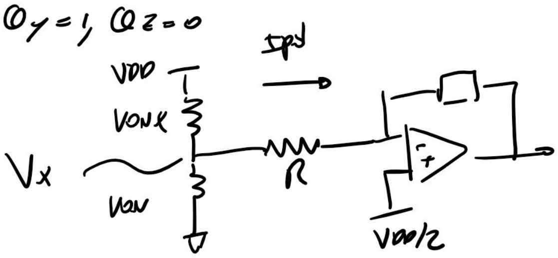
$$I_{pd} = \frac{V_{DD}/2}{Von\,X + R} = 4{,}3\,mA = I_{pd}^{+}$$

- STATE 3 $Q_x = 0$, $Q_y = 1$, $Q_z = 0$



$$I_{pd} = \frac{-(V_{DD}/2)}{(Von + R)} = -4{,}4\,mA = -I_{pd}^{-}$$

- STATE $Q_x = 1$, $Q_y = 1$, $Q_z = 0$



$$I_{pd} = \frac{(V_x - V_{DD}/2)}{R}$$

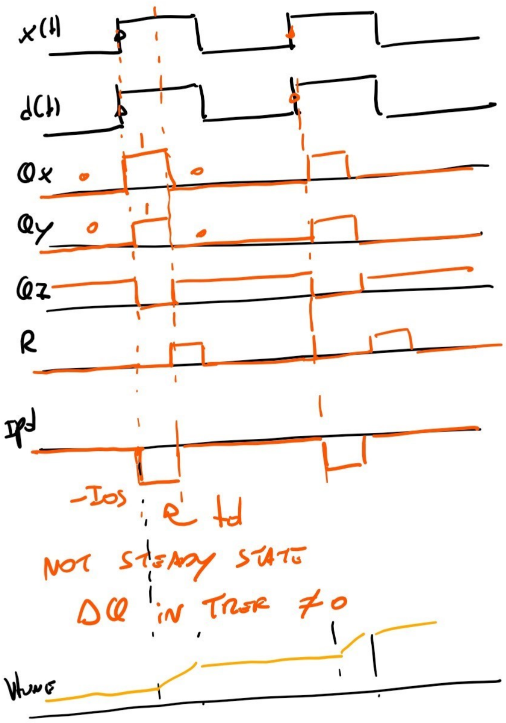
$R >> Von, Von\,x$

$$V_x \simeq \frac{Von}{Von + Von\,x} V_{DD}$$

$I_{pd} \simeq -0{,}925\,mA = I_{os}$

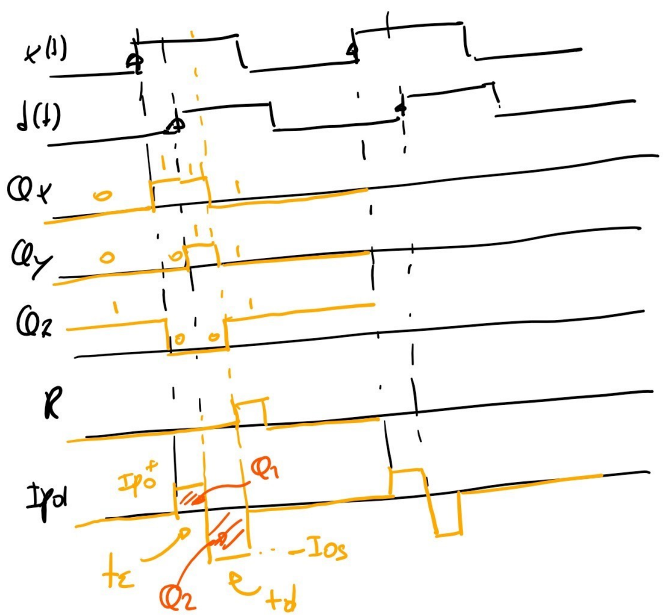
- ANALYZE STEADY STATE OPERATION OF PLL

- ideal STEADY STATE $t_\varepsilon = 0$



Q_x	Q_y	Q_E	I_{pd}
0	0	1	0
1	0	0	I_{po}^+
0	1	0	$-I_{po}^-$
1	1	0	$-I_{os}$

REAL STEADY STATE
Vtune = COSTANT in AVERAGE
$t_\varepsilon > 0$

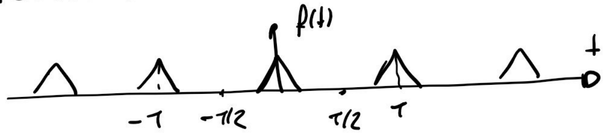


@ STEADY STATE → CHARGE BALANCE in TREF → Vtune CONSTANT in AVERAGE

$|Q_2| = |Q_1|$

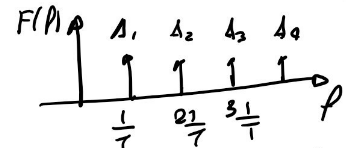
- WE HAVE TO DERIVE $\mathcal{L}(\Delta f = 50\,MHz)$ AS FUNCTION OF THE 1ST HARMONIC OF $I_{pd}(t)$

FOURIER SERIES OF PERIODIC FUNCTIONS

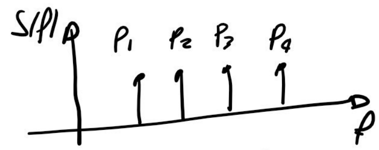


$$f(t) = \gamma_0 + \sum_{n=1}^{+\infty} 2|\gamma_n| \cos\left(\frac{2\pi n t}{T} + \angle \gamma_n\right)$$ POLAR FORM

$$\gamma_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-j\frac{2\pi}{T}nt}\, dt$$

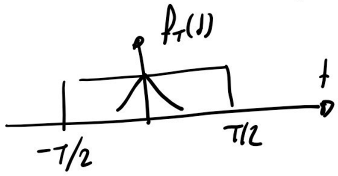


$A_i = 2|\gamma_i|$



$P_i = \frac{1}{2}A_i^2 = \frac{1}{2}(2|\gamma_i|)^2$

RELATION BETWEEN γ_N AND $F_T\{f_T(t)\}$

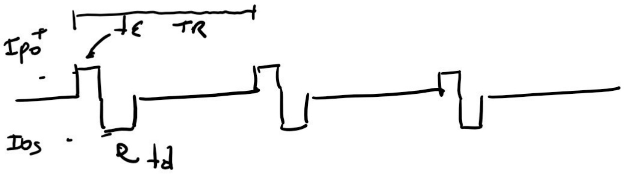


$f_T(t) = f(t)\, rect\left(\frac{t}{T}\right)$

$$F_T\{f_T(t)\} = \int_{-\infty}^{+\infty} f_T(t) e^{-j2\pi f t} dt = \int_{-\infty}^{+\infty} f(t)\, rect\left(\frac{t}{T}\right) e^{-j2\pi f t} = \int_{-T/2}^{T/2} f(t) e^{-j2\pi f t} dt$$

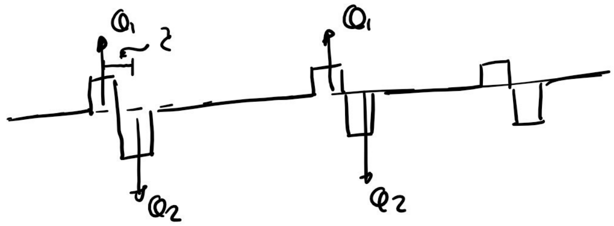
$$\boxed{\gamma_N = \frac{1}{T} F_T\{f_T(t)\}_{f = n/T}}$$

$I_{pd}(t)$



SINCE $\frac{t_\varepsilon}{T_R} \to 0$ AND $\frac{t_d}{T_R} \to 0$

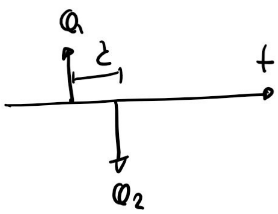
$I_{pd}(t)$



$$\tau = \left(\frac{t_\varepsilon}{2} + \frac{t_d}{2}\right)$$

TRUNCATED VERSION OF I_{pd}

$I_{pdT}(t)$



$$I_{pdT}(t) = |Q_1|\delta(t) - |Q_2|\delta(t-\tau) = Q[\delta(t) - \delta(t-\tau)]$$

$$|Q_1| = |Q_2|$$

WE WANT to derive the FIRST HARMONIC

$$A_1 = 2|\gamma_1| \qquad P_1 = \frac{1}{2}A_1^2 = \frac{1}{2}(2|\gamma_1|)^2$$

$$F_T\{I_{pdT}(t)\} = Q\left[1 - e^{-j2\pi f\tau}\right]$$

since $\tau \to 0$ $\quad e^{x} \simeq 1 - x$ FOR $x \to 0$

$$\simeq Q[\cancel{1} - \cancel{1} - j2\pi f\tau]$$

$$A_1 = 2|\gamma_1| = 2\left|\frac{1}{T_R} F_T\{I_{pdT}(t)\}_{f=\frac{n}{T_R}}\right| = 2\left|\frac{1}{T_R} Q\, j2\pi \frac{n}{T_R}\tau\right| \quad n=1$$

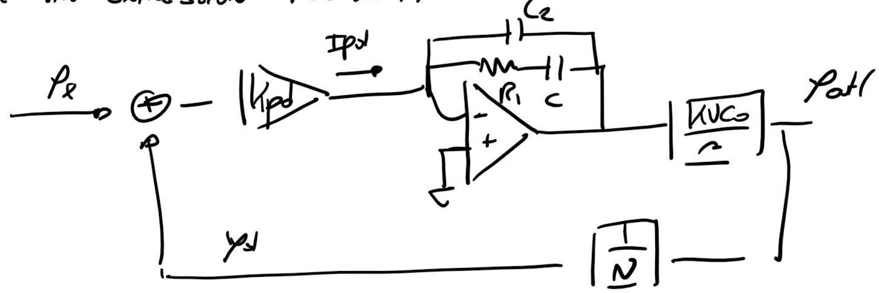
$$\tau = \frac{t_\varepsilon}{2} + \frac{t_d}{2}$$

$$Q = t_\varepsilon I_{po}^{+}$$

P_1 = POWER OF 1ST COMPONENT OF $I_{pd}(t)$

$$= \frac{1}{2}(2|\gamma_1|)^2 = \frac{1}{2}\left(2\,(F_{REF})^2\left(\frac{t_\varepsilon + t_d}{2}\right)2\pi\,(t_\varepsilon I_{po}^{+})\right)^2$$

• DERIVE THE EXPRESSION FOR $\mathcal{L}(\Delta f)$

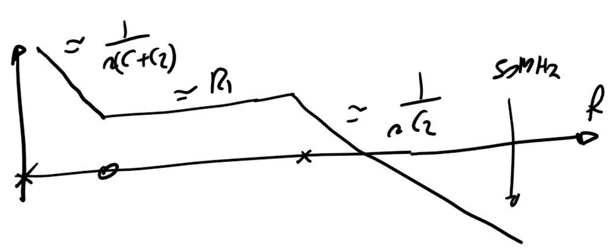


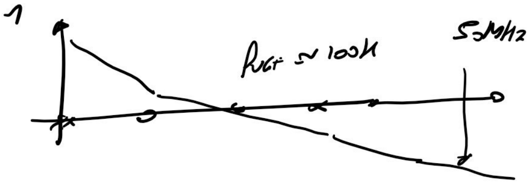
$$\mathcal{L}(\Delta f) = \frac{1}{2} S_{\phi}^{SSB}(f)$$

$$\mathcal{L}(\Delta f) = \frac{1}{2} P_1 \left| \frac{(1+sR_1C)}{s(C+C_2)(1+sR_1(C\|C_2))} \right|^2 \left| \frac{K_{VCO}}{s} \right|^2 \left| \frac{1}{1+G_{loop}(s)} \right|^2$$

$\simeq 1$

$\Delta f = 50MHz$



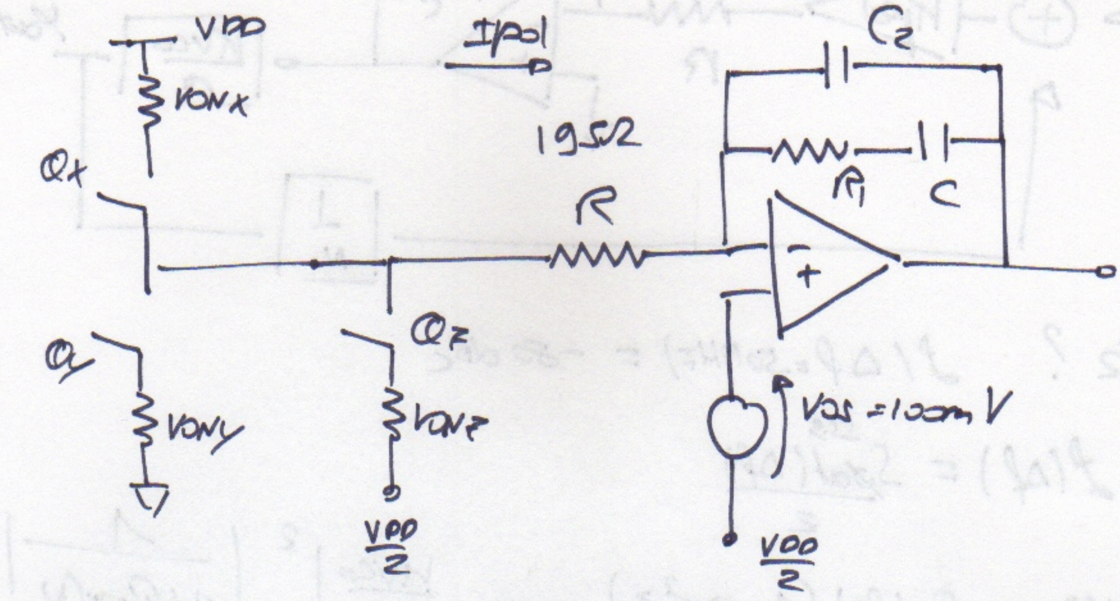


$$\simeq \frac{1}{2} P_1 \left| \frac{1}{sC_2} \right| \left| \frac{K_{VCO}}{s} \right|^2 \quad s = j2\pi F_{REF}$$

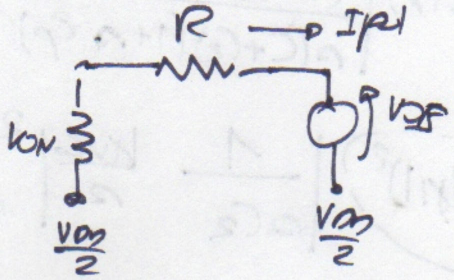
$$= \frac{1}{2} \left(2 F_{REF}^2 \, 2\pi \, t_c I_{pd}^2 \tau\right)^2 \left| \frac{K_{VCO}}{(2\pi F_{REF})^2 C_2} \right|^2 = -80dBc = 10^{-80/10}$$

$$\boxed{C_2 = 154\,pF}$$

c) $V_{ONX} = V_{ONY} = 10\Omega$, $V_{OS\ OPAMP} = 100mV$



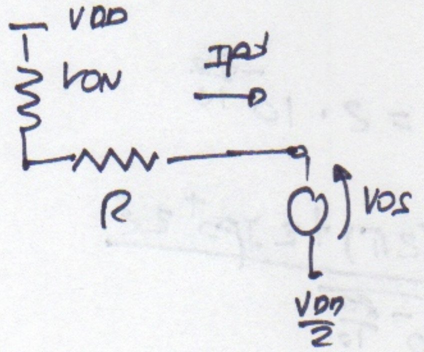
- $Q_x = 0$, $Q_y = 0$, $Q_z = 1$



$$I_{pol} = -\frac{V_{OS}}{(V_{ON}+R)}$$

$$\simeq -0{,}5\,mA$$

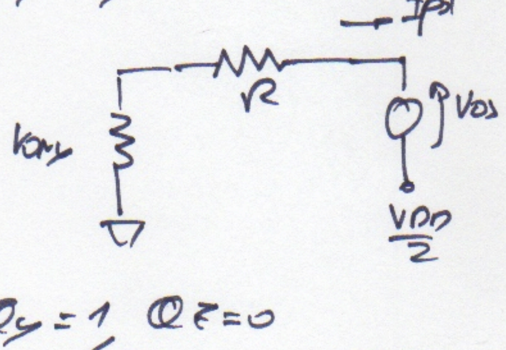
- $Q_x = 1$, $Q_y = 0$, $Q_z = 0$



$$I_{pol} = \frac{V_{DD} - \left[V_{OS} + \frac{V_{DD}}{2}\right]}{(V_{ON}+R)}$$

$$\simeq 3{,}92\,mA = I_p^+$$

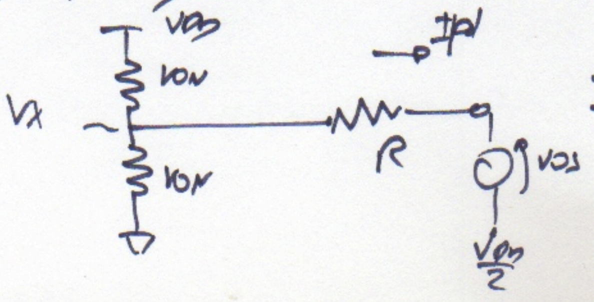
- $Q_x = 0$, $Q_y = 1$, $Q_z = 0$



$$I_{pol} = -\frac{\left(\frac{V_{DD}}{2} + V_{OS}\right)}{(R + V_{ON})}$$

$$\simeq -4{,}9\,mA$$

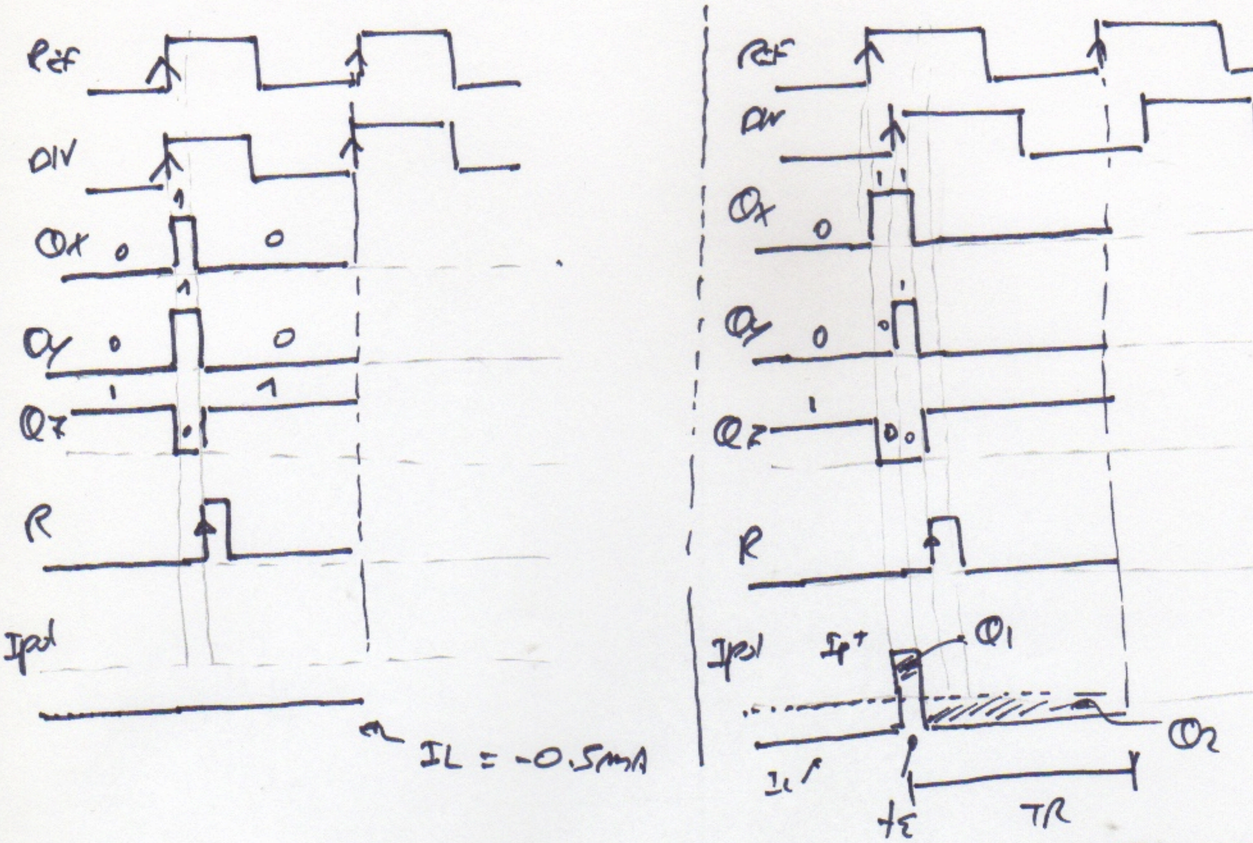
- $Q_x = 1$, $Q_y = 1$, $Q_z = 0$



$$I_{pol} = \frac{\frac{V_{DD}}{2} - \left[\frac{V_{DD}}{2} + V_{OS}\right]}{R}$$

$$= -\frac{V_{OS}}{R} = -0.5\,mA$$

$t_\varepsilon = 0$ (OPEN-LOOP) $\longrightarrow$ $t_\varepsilon > 0$ STEADY-STATE CLOSED-LOOP.



$I_p^+ = 3{,}92\,mA$

$Q_1 = Q_2$

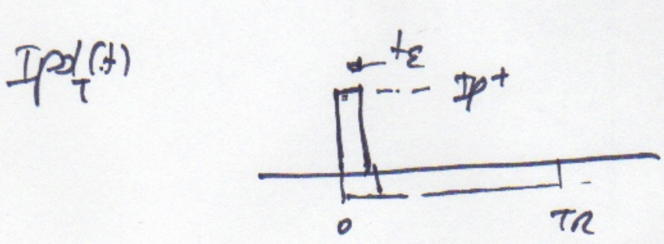
$I_p^+ t_\varepsilon = I_L T_R$

$t_\varepsilon = \dfrac{I_L}{I_p^+} T_R$

$= 2.55\,nsec.$

$$\mathcal{L}(\Delta f = 50MHz) = \frac{S\phi_{out}^{SSB}(f = 50MHz)}{2}$$

$$= \frac{1}{2}\left(2|\gamma_1|\right)^2 \left|\frac{1}{2\pi F_{REF} C_2}\right|^2 \left|\frac{K_{VCO}}{2\pi F_{REF}}\right|^2 \cdot \frac{1}{2}$$



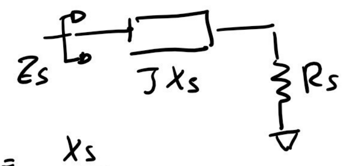
$$\gamma_1 = I_p^+ \frac{t_\varepsilon}{T_R}$$

$$\mathcal{L}(\Delta f = 50MHz) = \frac{1}{2}\left(2\, I_p^+ t_\varepsilon F_{REF}\right)^2 \frac{(K_{VCO})^2}{(2\pi F_R)^2 C_2^{\,2}} \frac{1}{2} = -32{,}8\,dBc$$

T6

IMPEDANCE TRANSFORMATION:

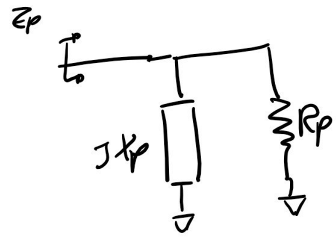
- SERIES



$Q_S = \dfrac{X_S}{R_S}$

SERIES TO PARALLEL TRANSFORMATION @ ω_0

- PARALLEL NETWORK



$Q_P = \dfrac{R_P}{X_P}$

@ ω_0

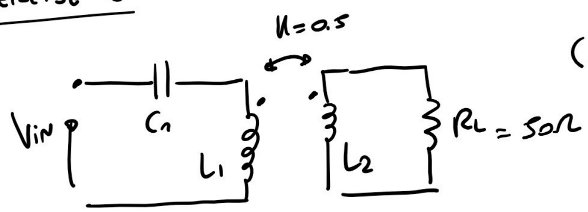
$$Z_S(\omega_0) = Z_P(\omega_0)$$

$$Q_T(\omega_0) = Q_S(\omega_0) = Q_P(\omega_0) = \frac{X_S}{R_S} = \frac{R_P}{X_P}$$

$$R_P = R_S(1 + Q_T^2)$$

$$X_P = X_S\left(1 + \frac{1}{Q_T^2}\right)$$

EXERCISE 6



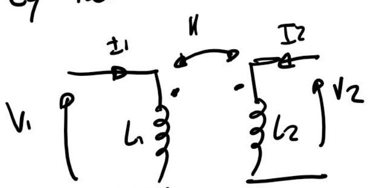
$C_1 = ?$

$L_1 = ?$

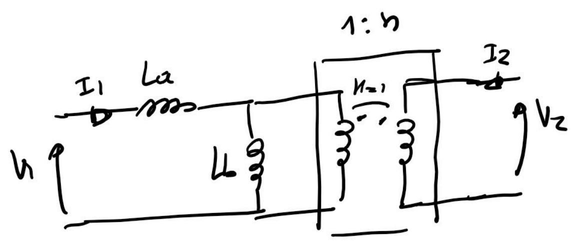
$L_2 = 1,59\,nH$

- $Z_{IN} = 5\Omega$
- Q OF THE NETWORK?

EQ MODEL OF THE TRANSFORMER:



$\Longleftrightarrow$ EQ MODEL



$$\begin{cases} V_1 = sL_1I_1 + sMI_2 \\ V_2 = sMI_1 + sL_2I_2 \end{cases}$$

$$M = K\sqrt{L_1L_2}$$

$M > 0$
if both I_1 and I_2
ENTERS / LEAVES
the dot

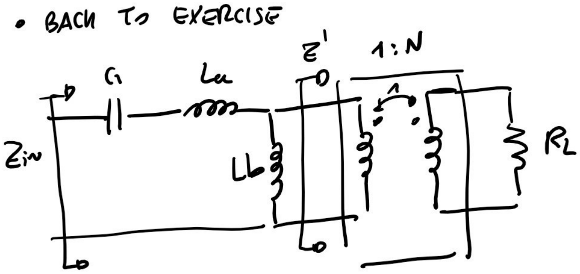
$$L_a = L_1(1-k^2)$$

$$L_b = L_1k^2$$

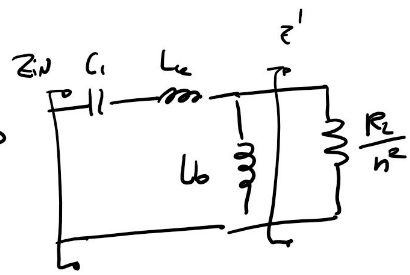
$$L_1 = L_a + L_b$$

$$n = \frac{1}{K}\sqrt{\frac{L_2}{L_1}}$$

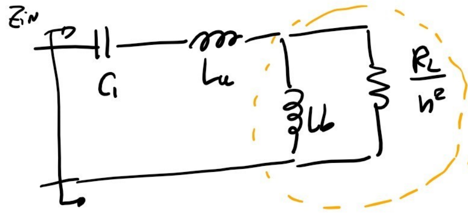
- BACK TO EXERCISE

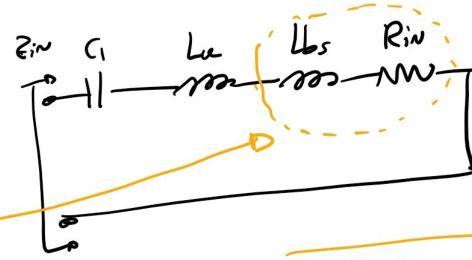


$\Longrightarrow$



- PARALLEL TO SERIES TRANSFORMATION

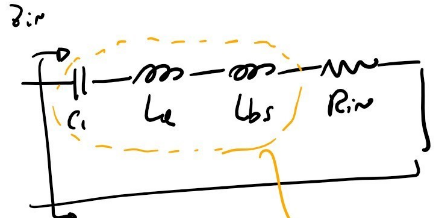




PAR – TO – SER

$$\begin{cases} \dfrac{R_L}{n^2} = R_{in}\left(1+Q_T^2\right) \\ \omega_0 L_b = \omega_0 L_{bs}\left(1+\dfrac{1}{Q_T^2}\right) \\ Q_T = \dfrac{\omega_0 L_{bs}}{R_s} = \dfrac{R_L/n^2}{\omega_0 L_b} \end{cases}$$

$$\boxed{\begin{aligned} R_p &= R_s\left(1+Q_T^2\right) \\ X_p &= X_s\left(1+\frac{1}{Q_T^2}\right) \\ Q_T &= \frac{X_s}{R_s} = \frac{R_p}{X_p} \end{aligned}}$$

- 

$$\omega_0 = 2\pi\,5\,GHz = \frac{1}{\sqrt{C_1\left(L_e+L_{bs}\right)}}$$

$R_{in} = 5\Omega$

$$Q_T = \frac{\omega_0 L_{bs}}{R_{in}} = \frac{(R_L/n^2)}{\omega_0 L_b} \qquad n^2 = \frac{1}{K^2}\frac{L_2}{L_1}$$

$$L_b = L_1 K^2$$

$$= \frac{R_L}{\omega_0 \cancel{L_1 K^2}}\frac{\cancel{K^2 L_1}}{L_2} = \frac{\overset{50\Omega}{R_L}}{\underset{2\pi\,5GHz}{\omega_0}\underset{1{,}59\,nH}{L_2}} = 1$$

$$L_{bs} = \frac{R_{in} Q_T}{\omega_0} = 159{,}15\,pH$$

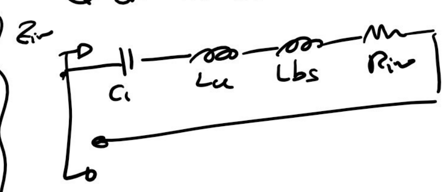
$$L_b = L_{bs}\left(1+\frac{1}{Q_T^2}\right) = 318{,}3\,pH$$

$$\boxed{L_1 = \frac{L_b}{K^2} = 1{,}273\,nH}$$

$$L_e = L_1 - L_b = 957{,}75\,pH$$

$$\boxed{C_1 = \frac{1}{\omega_0^2\left(L_e+L_{bs}\right)} = 909{,}61\,fF}$$

Q OF THE NETWORK @ ω_0



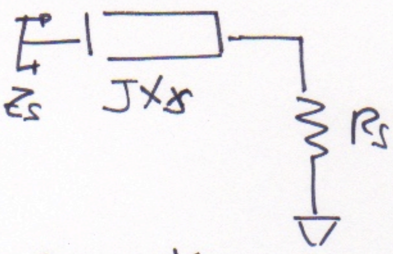
$$Q_{TOT} = \frac{1}{R_{in}\,\omega_0 C_1} = 7$$

$$= \frac{\omega_0\left(L_e+L_{bs}\right)}{R_{in}}$$

REMEMBER $Q_T \neq Q_{TOT}$

- IMPEDANCE TRANSFORMATION

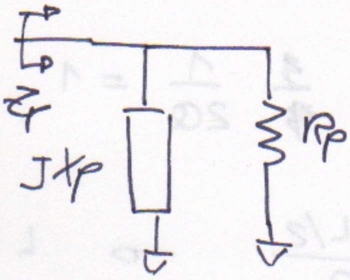
- SERIES



SERIES TO PARALLEL TRANSFORMATION

@ w_0

PARALLEL



$$Q_s = \frac{X_s}{R_s} \qquad Q_p = \frac{R_p}{X_p}$$

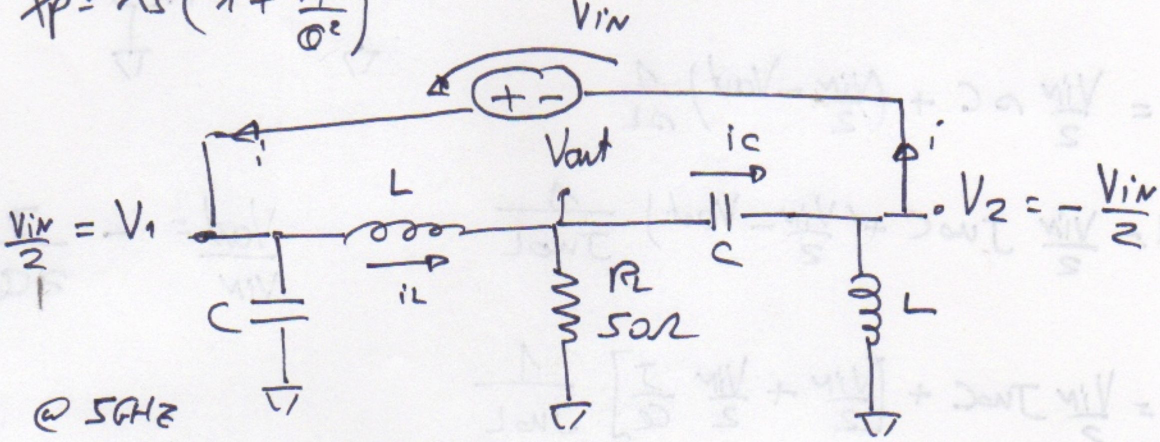
$$Z_s(w_0) = Z_p(w_0)$$

$$Q \; @ \; w_0 = Q_s(w_0) = Q_p(w_0)$$

$$R_p = R_s\left(1 + Q^2\right)$$

$$X_p = X_s\left(1 + \frac{1}{Q^2}\right)$$

EXERCISE 4.4



a) L, C ?

$$\left|\frac{V_{out}}{V_1 - V_2}\right| = 1 \quad @ \; 5GHz$$

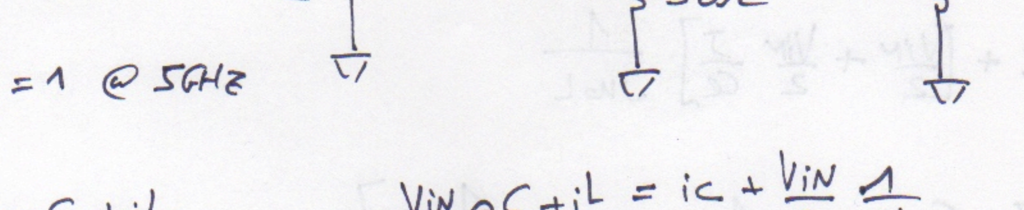
$$\begin{cases} i = \frac{Vin}{2} sC + i_L \\ i = i_C - \left(-\frac{Vin}{2}\right)\frac{1}{sL} \\ V_{out} = R_L(i_L - i_C) \end{cases}$$

$$\frac{Vin}{2} sC + i_L = i_C + \frac{Vin}{2}\frac{1}{sL}$$

$$(i_L - i_C) = \frac{V_{out}}{R_L} = \frac{Vin}{2}\left[\frac{1}{sL} - sC\right]$$

$$s = Jw_0$$

$$\frac{V_{out}}{Vin} = \frac{R_L}{2}\left[\frac{1}{Jw_0L} - Jw_0C\right]$$



$$Q = \frac{w_0 L/2}{R_L} = \frac{1}{w_0 2C R_L}$$

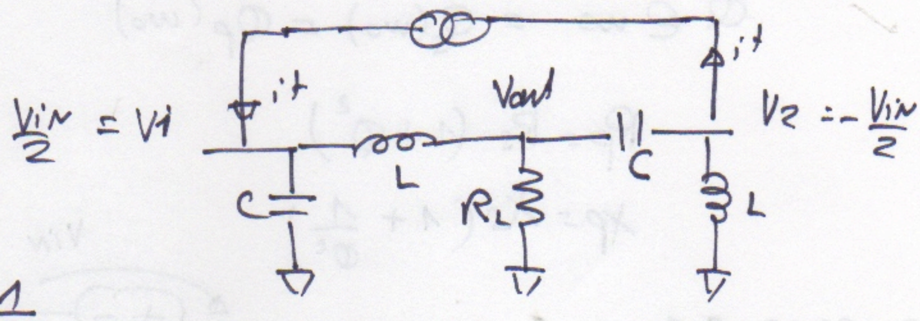
$$\frac{V_{out}}{V_{IN}} = \frac{1}{4}\left[\frac{2R_L}{j\omega_0 L} - j\omega_0 C R_L 2\right] = \frac{1}{4}\left[\frac{1}{jQ} - \frac{j}{Q}\right] = \frac{1}{j2Q}$$

$$\left|\frac{V_{out}}{V_{IN}}\right| = \cancel{\frac{4}{4}}\ \frac{1}{2Q} = 1 \Rightarrow Q = \frac{1}{2}$$

$$Q = \frac{\omega_0 L/2}{R_L} \Rightarrow L = 1{,}591\ nH$$

$$Q = \frac{1}{\omega_0 2 C R_L} \Rightarrow C = 636{,}62\ pF$$

b) DIFFERENTIAL IMPEDANCE:
@ 5GHz



$$i_t = \frac{V_{IN}}{2} sC + \left(\frac{V_{IN}}{2} - V_{out}\right)\frac{1}{sL}$$

$$i_t = \frac{V_{IN}}{2} j\omega_0 C + \left(\frac{V_{IN}}{2} - V_{out}\right)\frac{1}{j\omega_0 L} \qquad \frac{V_{out}}{V_{IN}} = -\frac{j}{2Q}$$

$$i_t = \frac{V_{IN}}{2} j\omega_0 C + \left[\frac{V_{IN}}{2} + \frac{V_{IN}}{2}\frac{j}{Q}\right]\frac{1}{j\omega_0 L}$$

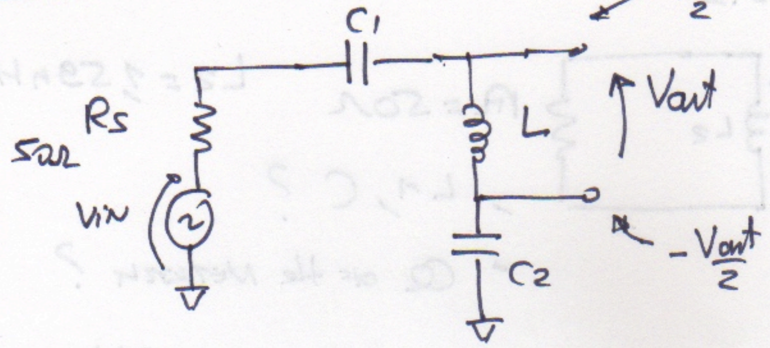
$$= V_{IN}\left[\frac{j\omega_0 C}{2} + \frac{1}{2j\omega_0 L} + \frac{1}{2Q\omega_0 L}\right]$$

$$\frac{V_{IN}}{i_t} = \frac{1}{\left[j\frac{\omega_0 C}{2} + \frac{1}{2j\omega_0 L} + \frac{1}{2Q\omega_0 L}\right]\left(\frac{2}{2}\right)\frac{R_L}{R_L}}$$

$$= \frac{1}{\frac{j}{R_L 4Q} + \frac{1}{jR_L 4Q} + \frac{1}{Q^2 4 R_L}} = \frac{1}{\frac{j}{R_L 4Q} - \frac{j}{R_L 4Q} + \frac{1}{Q^2 4 R_L}}$$

$$\frac{V_{IN}}{i_t} = \underbrace{Q}_{\frac{1}{2}}{}^2\, 4R_L = R_L = 50\,\Omega$$

EXERCISE 4.5



C_1, C_2, L

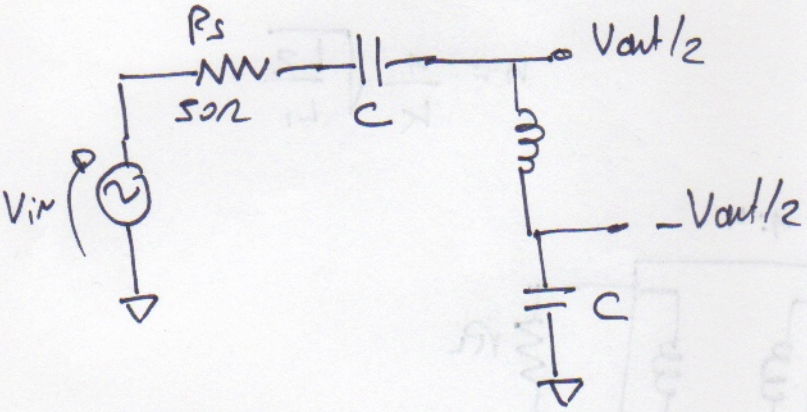
$\frac{V_{out}}{V_{in}} = 3$ @ 5GHz.

• Resonance freq of the network: $\omega_0 = \frac{1}{\sqrt{L \; C_1 // C_2}}$

• $\frac{V_{out}}{sL} = -\frac{V_{out}}{2} s C_2 \qquad s = j\omega \Rightarrow \frac{1}{j\omega L} = -\frac{j\omega C_2}{2}$

$$\omega_0 = \frac{1}{\sqrt{L \frac{C_2}{2}}}$$

$$C = C_1 = C_2$$



$$\frac{\left[V_{in} - \frac{V_{out}}{2}\right]}{R_s + \frac{1}{sC}} = -\frac{V_{out}}{2} sC$$

$$\frac{\left(V_{in} - \frac{V_{out}}{2}\right) \cancel{sC}}{(1 + sCR_s)} = -\frac{V_{out}}{2} \cancel{sC}$$

$$V_{in} - \cancel{\frac{V_{out}}{2}} = -\cancel{\frac{V_{out}}{2}} - \frac{V_{out}}{2} sCR_s$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{2}{\omega_0 C R_s} = 3 \Rightarrow C = \frac{2}{3} \frac{1}{\omega_0 R_s} = 424,41 \text{ fF}$$

$$L = \frac{1}{\frac{C}{2} \omega_0^2} = 4,775 \text{ nH}$$