RF Circuit Design

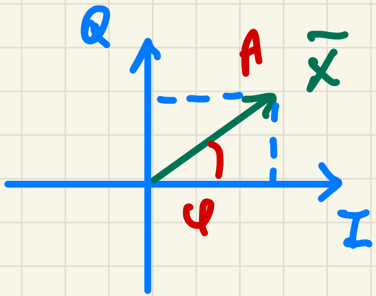
L4

Modulated signal (quadrature mod.) Cartesian

$$x(t) = I(t)\cos\omega_c t - Q(t)\sin\omega_c t =$$

$$= A(t)\cos[\omega_c t + \varphi(t)]$$

Polar



$$A(t) = \sqrt{I^2(t) + Q^2(t)}$$

$$\varphi(t) = \arctan\frac{Q(t)}{I(t)}$$

e.g. QPSK modulation $a_n = \pm 1$; $b_n = \pm 1$

$$x(t) = \underbrace{\sum a_n\, p(t - nT_b)}_{I(t)} \cdot \cos\omega_c t \; +$$

$$- \underbrace{\sum b_n\, p(t - nT_b)}_{Q(t)} \cdot \sin\omega_c t \; =$$

$$= \mathrm{Re}\left\{ \underbrace{\sum (a_n + j b_n)\, p(t - nT_b)}_{\tilde{x}(t)} \, e^{j\omega_c t} \right\}$$

$\tilde{x}(t)$ phasor of $x(t)$

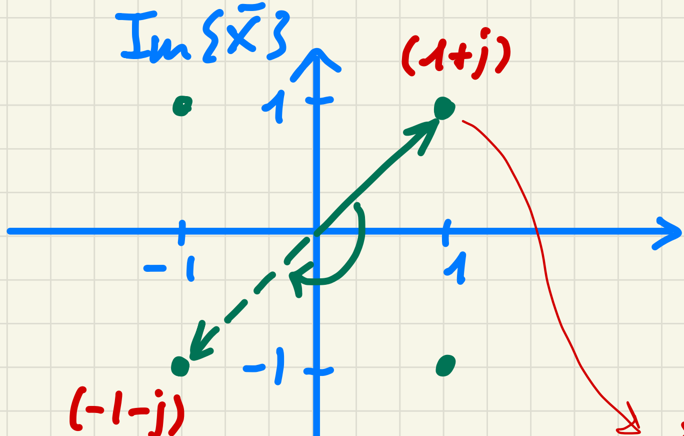
$$x(t) = \mathrm{Re}\left\{ \bar{x}(t)\, e^{j\omega_c t} \right\}$$

$$a_n = \pm 1$$

$$\bar{x}(t) = \sum (a_n + j b_n) \cdot p(t - nT_b)$$

$$b_n = \pm 1$$

QPSK



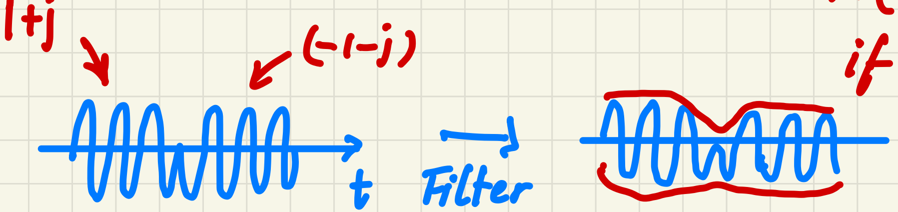
$t = KT_b$

$p(0) = 1$

$$\bar{x}(KT_b) = a_K + j b_K$$

$$x(t) = \cos\omega_c t + \sin\omega_c t$$

if $p(t)$ is rectangle in time

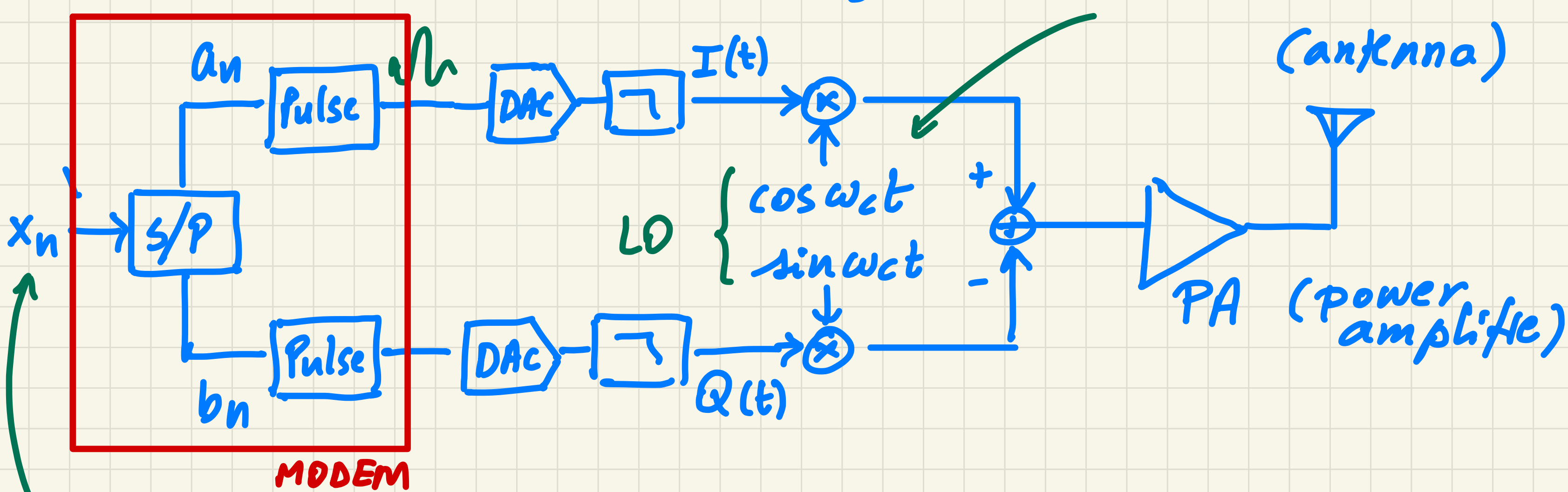


if $p(t)$ rectangle

↳ non-constant envelope

QPSK Tx block diagram

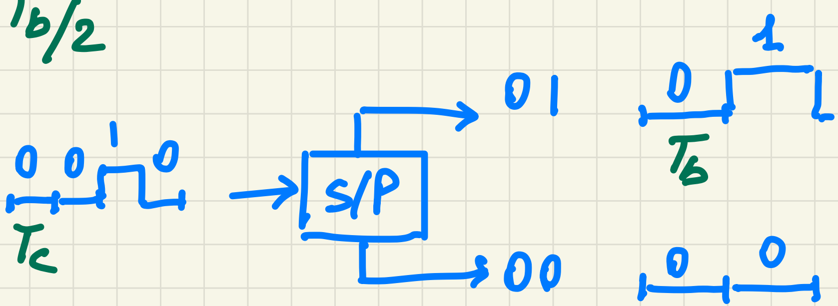
ideal LO



T_c chip period $\qquad$ $1/T_c$ chip rate

$T_c = T_b/2$

e.g.



series-to-parallel converter

RF bandwidth

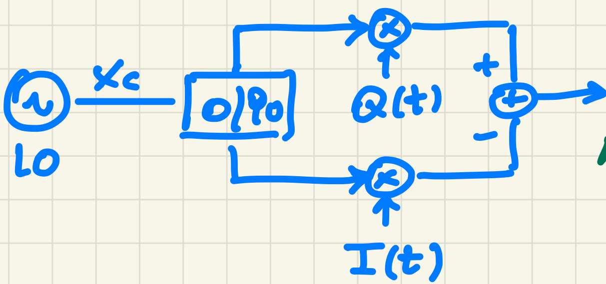
For $\alpha = 0$ (Roll-off) : $\mathrm{sinc}(t/T_b)$ shape

$$BW_{bb} = \frac{1}{2T_b} \Rightarrow \underset{2 \times BW_{bb}}{BW} = \frac{1}{T_b} = \frac{1}{2T_c}$$

RF bandwith of QPSK is given by the CHIP rate divided by 2

Impact of LO PHASE NOISE on the quality of our modulation

$$x_c(t) = \cos[\omega_c t + \varphi_n(t)] \qquad \xrightarrow{\text{Phasor}}$$



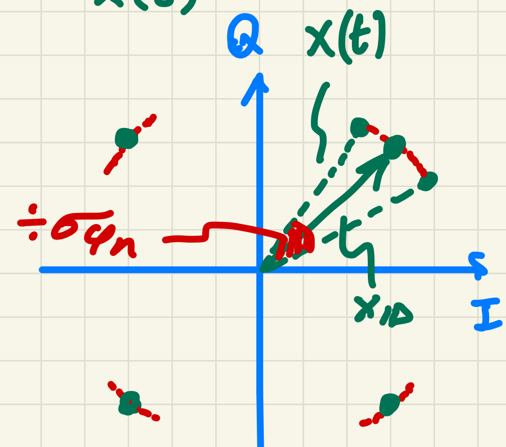
$$x(t) = \mathrm{Re}\left\{ \underbrace{\underbrace{\bar{X}_{ID}(t)} \cdot e^{j\omega_c t} \cdot \underbrace{e^{j\varphi_n(t)}}}_{X(t)} \right\}$$

modulated carrier

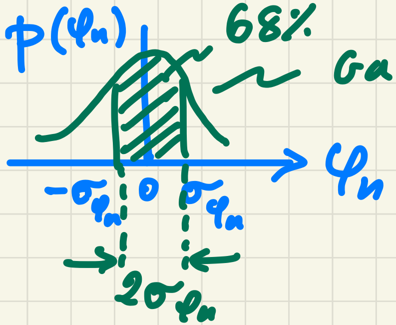
$$\bar{X}_{ID}(t) = I(t) + jQ(t)$$

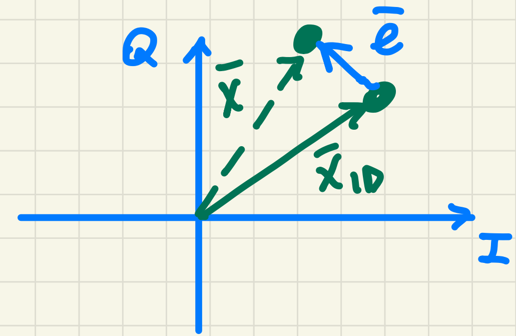
Phasor affected by LO phase noise :

$$\bar{X}(t) = \bar{X}_{ID}(t) \cdot e^{j\varphi_n(t)}$$

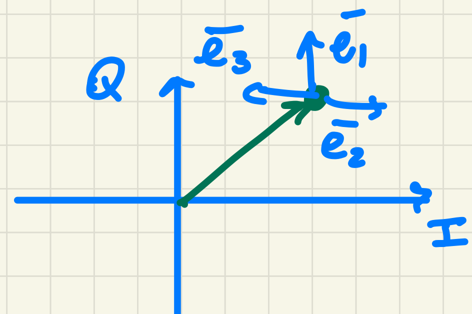


$$\sigma_{\varphi_n}^2 = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \varphi_n^2(t)\, dt = \int_0^\infty S_{\varphi_n}^{SSB}(f)\, df$$





Error-Vector Magnitude

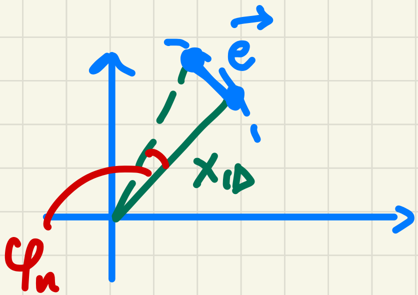


$$EVM \triangleq \frac{\frac{1}{N}\sum_{j=1}^{N} |e_j|^2}{P_{avg}} = \frac{P_e}{P_{avg}}$$

It is a Noise-to-signal ratio

$$EVM = \frac{1}{SNR}$$

- EVM induced by Phase Noise



$$EVM = \frac{|\bar{e}|^2}{|x_{ID}|^2} \simeq \frac{\cancel{|\bar{x}_{ID}|^2} \cdot \sigma^2_{\varphi_n}}{\cancel{|\bar{x}_{ID}|^2}} = \sigma^2_{\varphi_n}$$

For small φ_n : $|\bar{e}| \simeq \overbrace{|\bar{x}_{ID}| \cdot \varphi_n}^{\text{arc}} \Rightarrow |\bar{e}|^2 \simeq |\bar{x}_{ID}|^2 \cdot \sigma^2_{\varphi_n}$

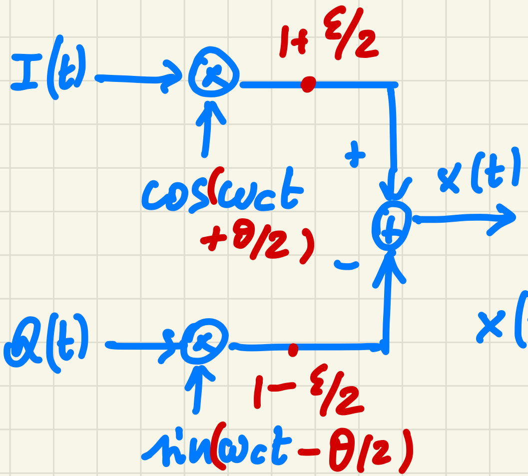
$$EVM \simeq \sigma^2_{\varphi_n}$$

Regardless of P_{avg} (Tx power) SNR at TX output is limited by phase noise

Rx phase noise (LO) → degrades SNR at Rx.

$$SNR = 1/\sigma^2_{\varphi_n}$$

- EVM induced by amplitude / phase errors



ε amplitude imbalance

ϑ phase imbalance

$$x(t) = \left(1 + \varepsilon/2\right) \cdot \cos\left[\omega_c t + \vartheta/2\right] \cdot I(t) + $$
$$- \left(1 - \varepsilon/2\right) \cdot \sin\left[\omega_c t - \vartheta/2\right] \cdot Q(t)$$

$$EVM = \frac{P_e}{P_{avg}} = \frac{|\bar{e}|^2}{|\bar{x}_{ID}|^2}$$

$$\bar{e} = \bar{x} - \bar{x}_{ID}$$

$I + jQ$

$$\bar{x} = I \cdot e^{j\vartheta/2} \cdot \left(1 + \varepsilon/2\right) + jQ \cdot e^{-j\vartheta/2} \cdot \left(1 - \varepsilon/2\right)$$

$$-\bar{e} = \bar{x}_{1D} - \bar{x} = I\left[1 - e^{j\vartheta/2}(1+\varepsilon/2)\right] + jQ\left[1 - e^{-j\vartheta/2}\left(1-\frac{\varepsilon}{2}\right)\right]$$

$$= I\cdot\Big[\underbrace{1-e^{j\vartheta/2}}_{\simeq -j\vartheta/2} - \underbrace{e^{j\vartheta/2}}_{(1+j\vartheta/2)}\cdot\varepsilon/2\Big] + jQ\Big[\underbrace{1-e^{-j\vartheta/2}}_{\simeq +j\vartheta/2} + \underbrace{e^{-j\vartheta/2}}_{(1-j\vartheta/2)}\cdot\frac{\varepsilon}{2}\Big]$$

small ϑ

$e^x \simeq 1+x$

$x \simeq 0$

$1-e^x \simeq -x$

$$\simeq I\cdot\left[-j\vartheta/2 - (1+j\vartheta/2)\varepsilon/2\right] + jQ\left[j\vartheta/2 + (1-j\vartheta/2)\cdot\frac{\varepsilon}{2}\right]$$

$$\simeq I\cdot\left[-j\vartheta/2 - \varepsilon/2\right] + jQ\left[j\vartheta/2 + \varepsilon/2\right] =$$

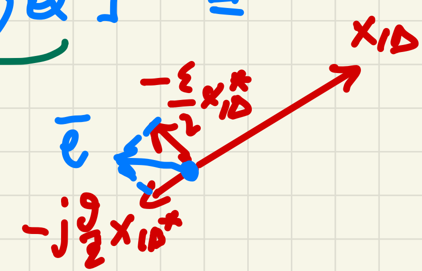
$\theta \ll 1\,\text{rad}$

$\varepsilon \ll 1$

$\varepsilon\cdot\vartheta$ negligible

$$= -(j\vartheta/2 + \varepsilon/2)\left[\underbrace{I - jQ}\right] =$$

$$= -(j\vartheta/2 + \varepsilon/2)\cdot x_{1D}^*$$



$$EVM = \frac{|\bar{e}|^2}{|\bar{x}_{1D}|^2} = \frac{|\underbrace{(\varepsilon/2 + j\theta/2)}\,\bar{x}^*_{1D}|^2}{|\bar{x}_{1D}|^2} =$$

$$= \left(\varepsilon^2/4 + \theta^2/4\right) \cdot \frac{\cancel{|\bar{x}^*_{1D}|^2}}{\cancel{|\bar{x}_{1D}|^2}}$$

$$EVM = \varepsilon^2/4 + \theta^2/4$$

e.g. $\varepsilon = 1\%$

$\theta = 1\,\text{deg}$

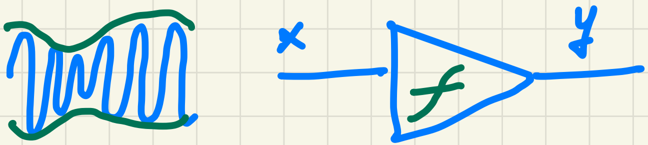
$\downarrow$

$1\,\text{deg} \cdot \frac{\pi}{180} = 0.0174\ \text{rad}$

$$EVM = \frac{(0.01)^2}{4} + \frac{(0.0174)^2}{4} = \frac{0.00040}{4}$$

$$EVM_{dB} = 10\log_{10} EVM = -40\,\text{dB}$$

- Impact of Non-linearity on the modulated signal



spectral regrowth

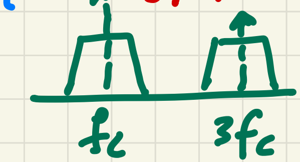
$$y = \alpha_1 \cdot x(t) + \underbrace{\alpha_3 \cdot x^3(t)}_{\text{cubic nonlinearity}} + \dots$$

static nonlinear model

- Constant envelope : $x(t)$ is PM modulation

no distors. of φ

$$x(t) = \underbrace{A_c}_{\text{constant}} \cos\left[\omega_c t + \underbrace{\varphi(t)}_{\text{information signal}}\right]$$



$$\alpha_3 x^3(t) = \alpha_3 A_c^3 \cos^3\left[\omega_c t + \varphi(t)\right] = \alpha_3 A_c^3 \frac{3}{4} \cos\left[\omega_c t + \varphi(t)\right] +$$
$$+ \alpha_3 A_c^3 \frac{1}{4} \cos\left[3\omega_c t + 3\varphi(t)\right]$$

$$\bullet\ \cos^3 x = \cos x \cdot \cos^2 x = \cos x \cdot \frac{1+\cos 2x}{2} =$$

$$= \frac{1}{2}\cos x + \frac{1}{2}\left[\frac{1}{2}\cos x + \frac{1}{2}\cos 3x\right] =$$

$$= \underbrace{\left(\frac{1}{2} + \frac{1}{4}\right)}_{3/4} \cdot \cos x + \frac{1}{4} \cdot \cos 3x$$

$$\bullet\ \sin^3 x = $$

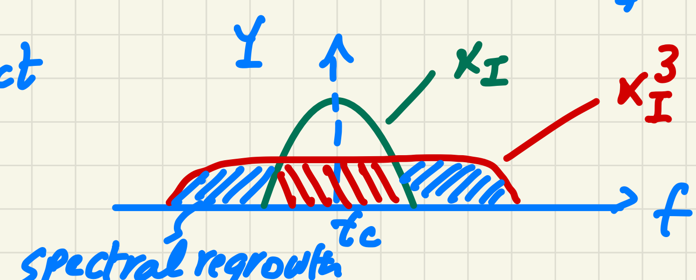
$$= \frac{3}{4}\sin x - \frac{1}{4}\sin 3x$$

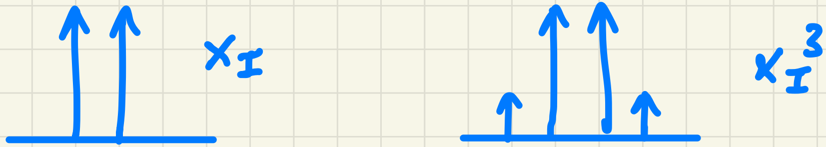
- Non-constant envelope modulation :

$$\alpha_3 x^3(t) = \alpha_3 \cdot x_I^3 \cdot \left(\frac{1}{4}\cos 3\omega_c t + \frac{3}{4}\cos\omega_c t\right) - \alpha_3 x_Q^3 \left(\frac{3}{4}\sin\omega_c t - \frac{1}{4}\sin 3\omega_c t\right)$$

$$x(t) = x_I(t)\cos\omega_c t - x_Q(t)\sin\omega_c t$$

$$y = \alpha_1 x(t) + \alpha_3 x^3(t)$$



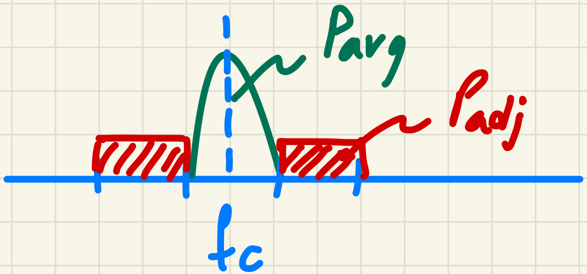


Nonlinearity degrades → EVM — inband disturbance

Nonlinearity degrades → ACPR

$$\text{ACPR} \triangleq \frac{\text{Power leaking in adj. channel}}{\text{Power of the signal}} \quad \begin{matrix} \leftarrow P_{adj} \\ \leftarrow P_{avg} \end{matrix}$$

(Adjacent channel power ratio)



Amplifiers trade-off

- linearity
- Power efficiency $\eta = P_{out}/P_{dc}$