Phase-Locked Loop Design
Part 1

RF Circuit Design

Prof. Salvatore Levantino

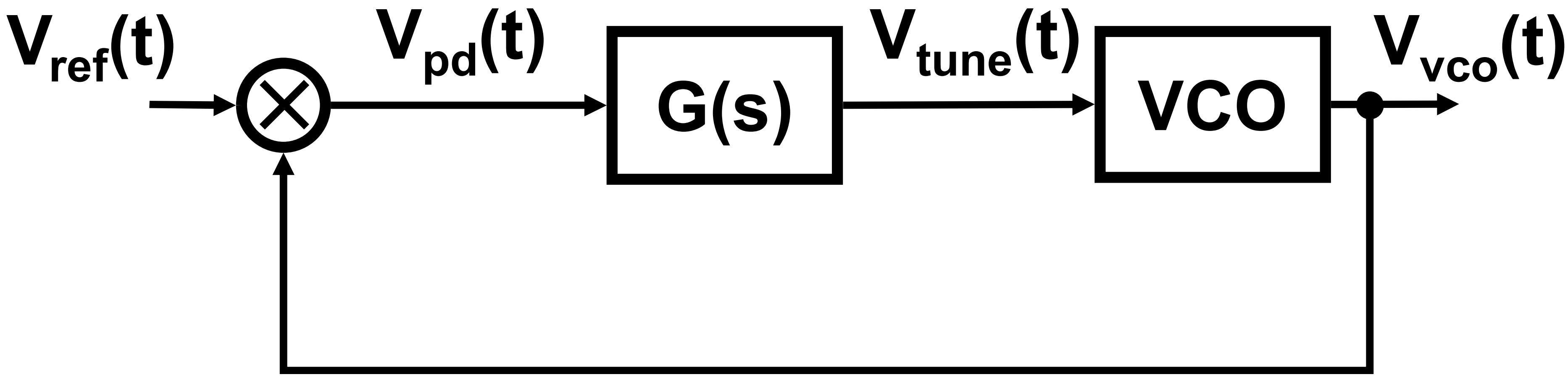
2020/2021

Outline

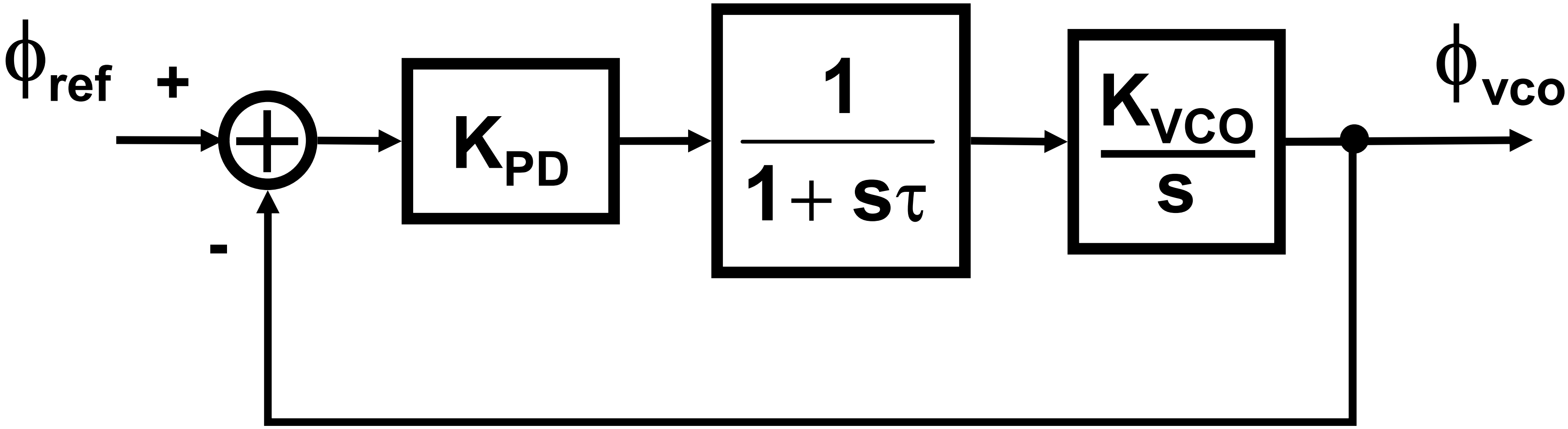
- **Type-I PLL with Analog Multiplier**
- **Type-I PLL with XNOR**

Type-I PLL with Analog Multiplier

PLL with Analog Multiplier



Linear Continuous-Time (CT) Model



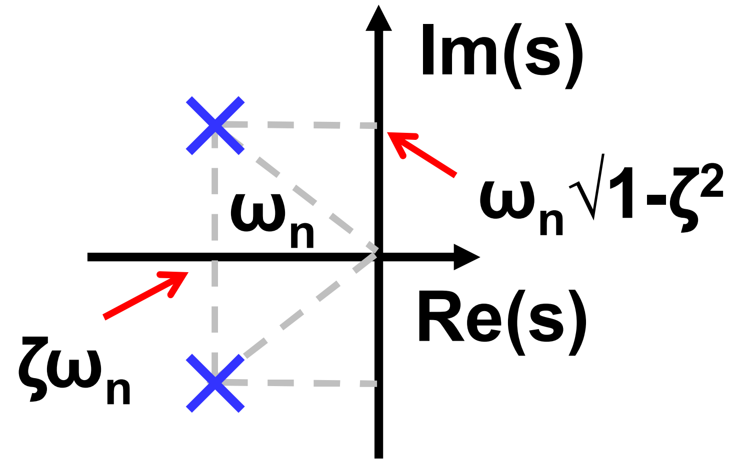
$$\frac{\Phi_{VCO}(s)}{\Phi_{ref}} = \frac{k/\tau}{s^2 + s/\tau + k/\tau}$$

$$\Rightarrow \quad H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Rightarrow \quad \begin{cases} \omega_n = \sqrt{k/\tau} \\ \zeta = \dfrac{1}{2\sqrt{k\tau}} \end{cases}$$

Loop dynamics: Step response of 2nd ord system

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

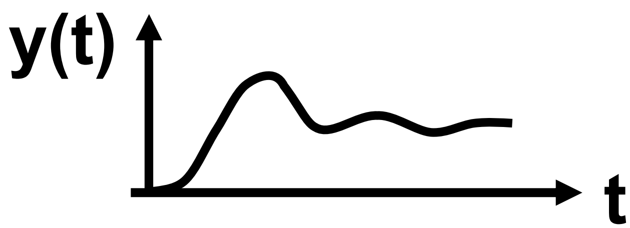


$\zeta < 1$:

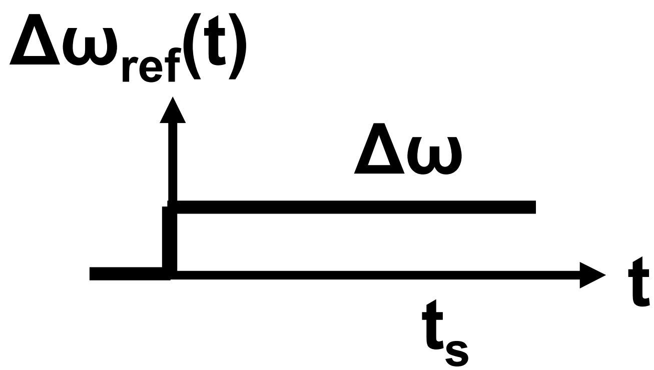
$$y(t) = \left[1 - \overbrace{\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \psi\right)}^{\text{Error w.r.t. final values}}\right] \cdot u(t)$$

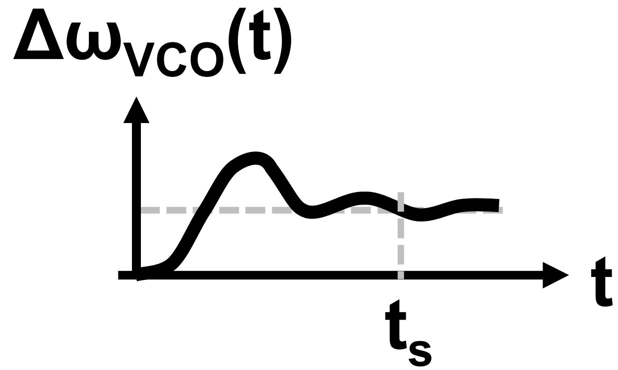
Unit Step Function

$$\psi = \arcsin\sqrt{1-\zeta^2}$$



Loop dynamics: Settling time of 2nd ord. system





$$\omega_{err} \leq \Delta\omega \cdot \frac{1}{\sqrt{1-\zeta^2}} \cdot e^{-\zeta\omega_n t_s}$$

$$\Rightarrow \quad t_s = \frac{1}{\zeta\omega_n} \cdot \ln\left(\frac{\Delta\omega}{\omega_{err}\sqrt{1-\zeta^2}}\right)$$

Parameter Set

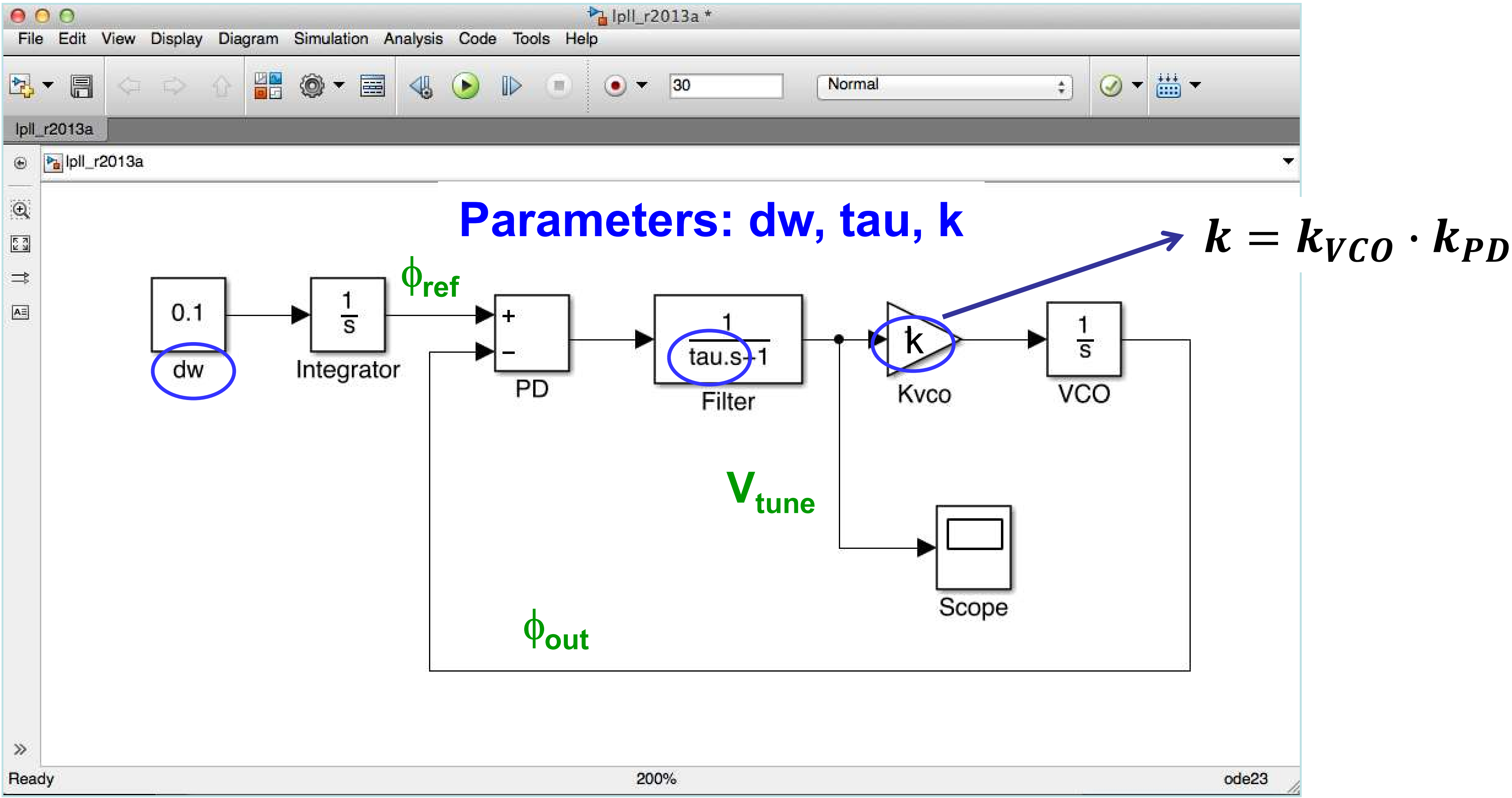
$$\frac{\Phi_{\mathrm{VCO}}(s)}{\Phi_{\mathrm{ref}}} = \frac{k/\tau}{s^2 + s/\tau + k/\tau}$$

$$\begin{cases} \omega_n = \sqrt{k/\tau} \\ \zeta = \dfrac{1}{2\sqrt{k\tau}} \end{cases}$$

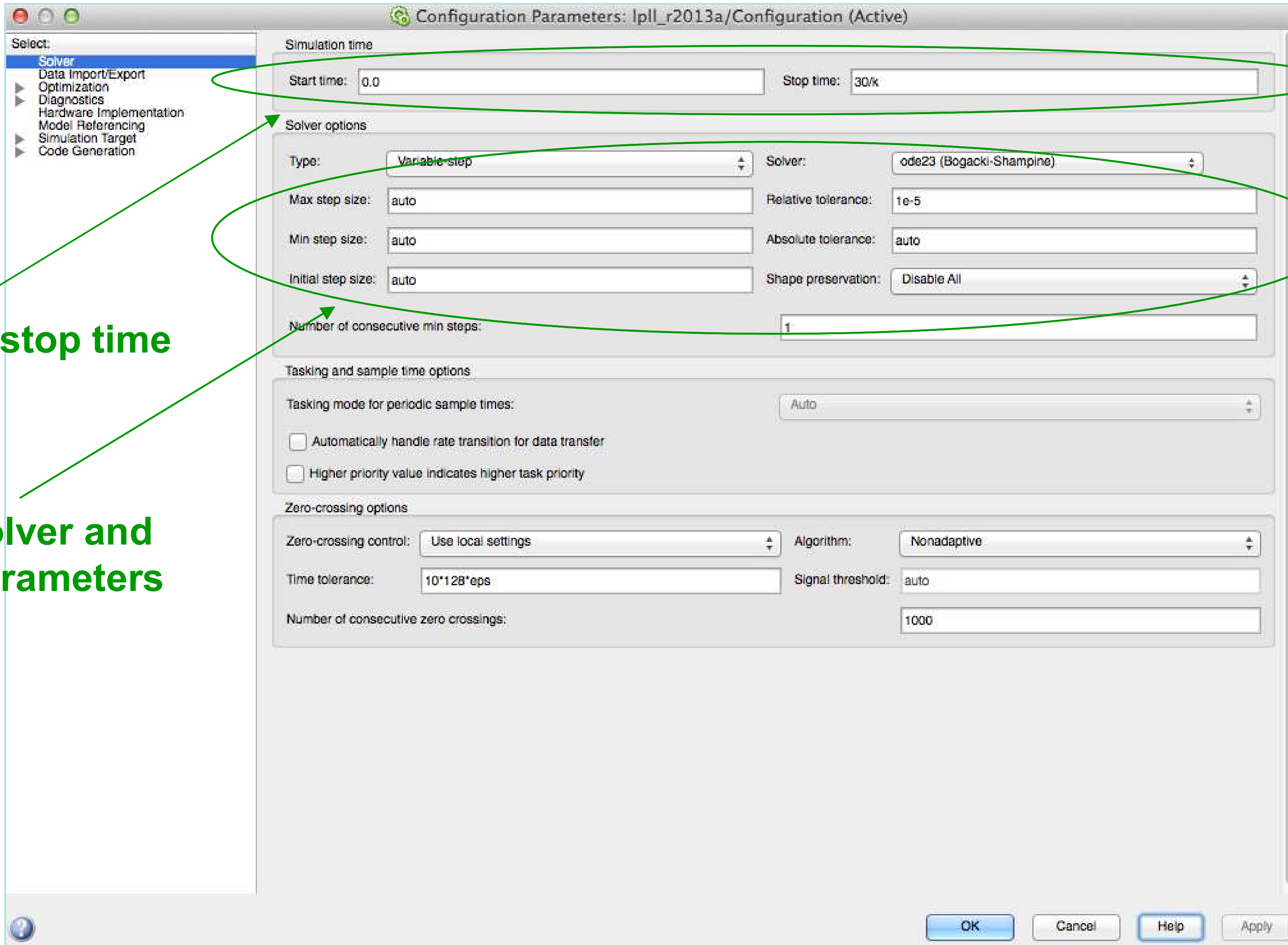
$$\zeta = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} \tau = \dfrac{1}{2k} \\ \omega_n = k\sqrt{2} \end{cases}$$

Critical damping

Linear CT Model (Simulink)



Simulation Parameters (Simulink)



Start/stop time

Solver and parameters

Parameter Set (Simulink)

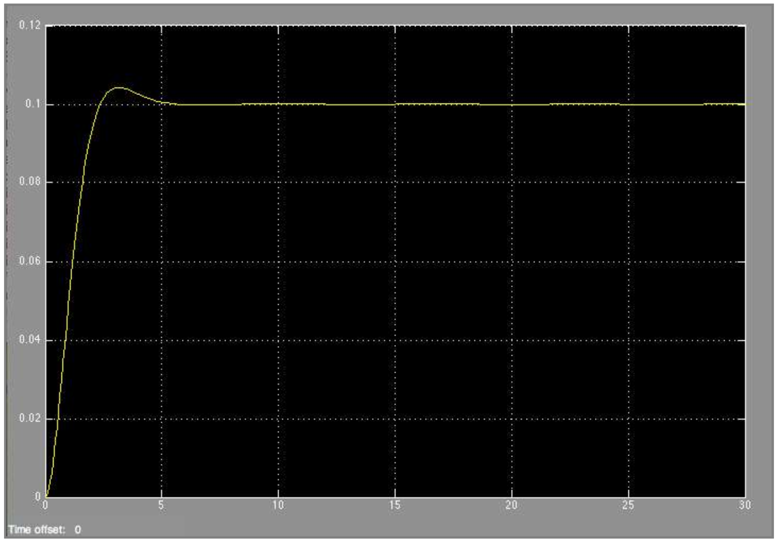
$$\begin{cases} \omega_n = \sqrt{k/\tau} \\ \zeta = \dfrac{1}{2\sqrt{k\tau}} \end{cases}$$

$$\zeta = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} \tau = \dfrac{1}{2k} \\ \omega_n = k\sqrt{2} \end{cases}$$

Critical damping

Parameter set:

- k = 1
- tau = 0.5
- tstop = 30
- dw = 0.1

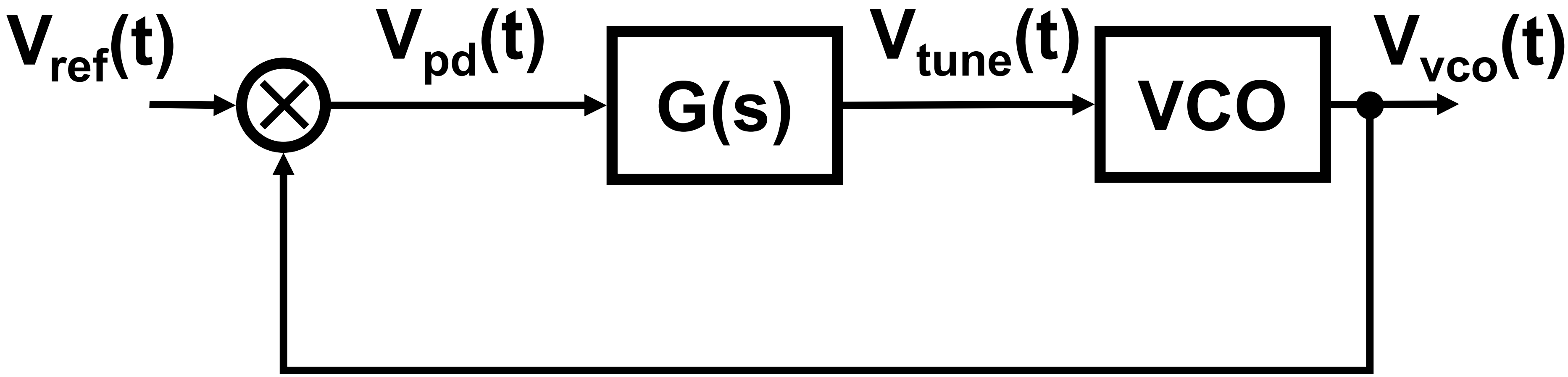


Exercise #1

- **Estimate the settling time of the PLL linear model to make a frequency step $\Delta\omega$ with an error ω_e such that $\omega_e/\Delta\omega$ = 1% from theory and simulation.**

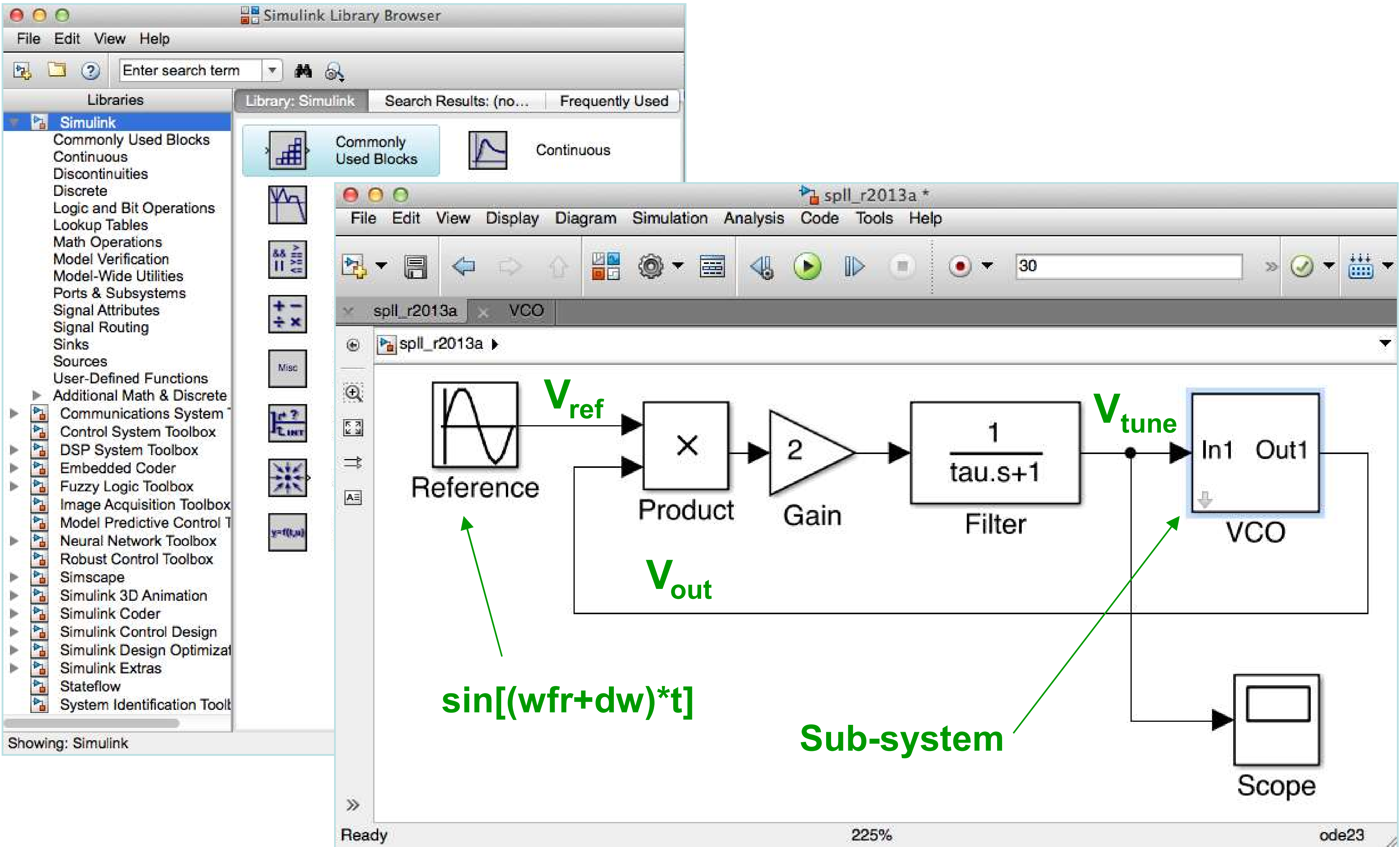
- **Change the parameter settings to halve the damping factor while keeping the same PLL bandwidth value. Plot again the step transient and comment the result.**

PLL with Analog Multiplier: Nonlinear Equations

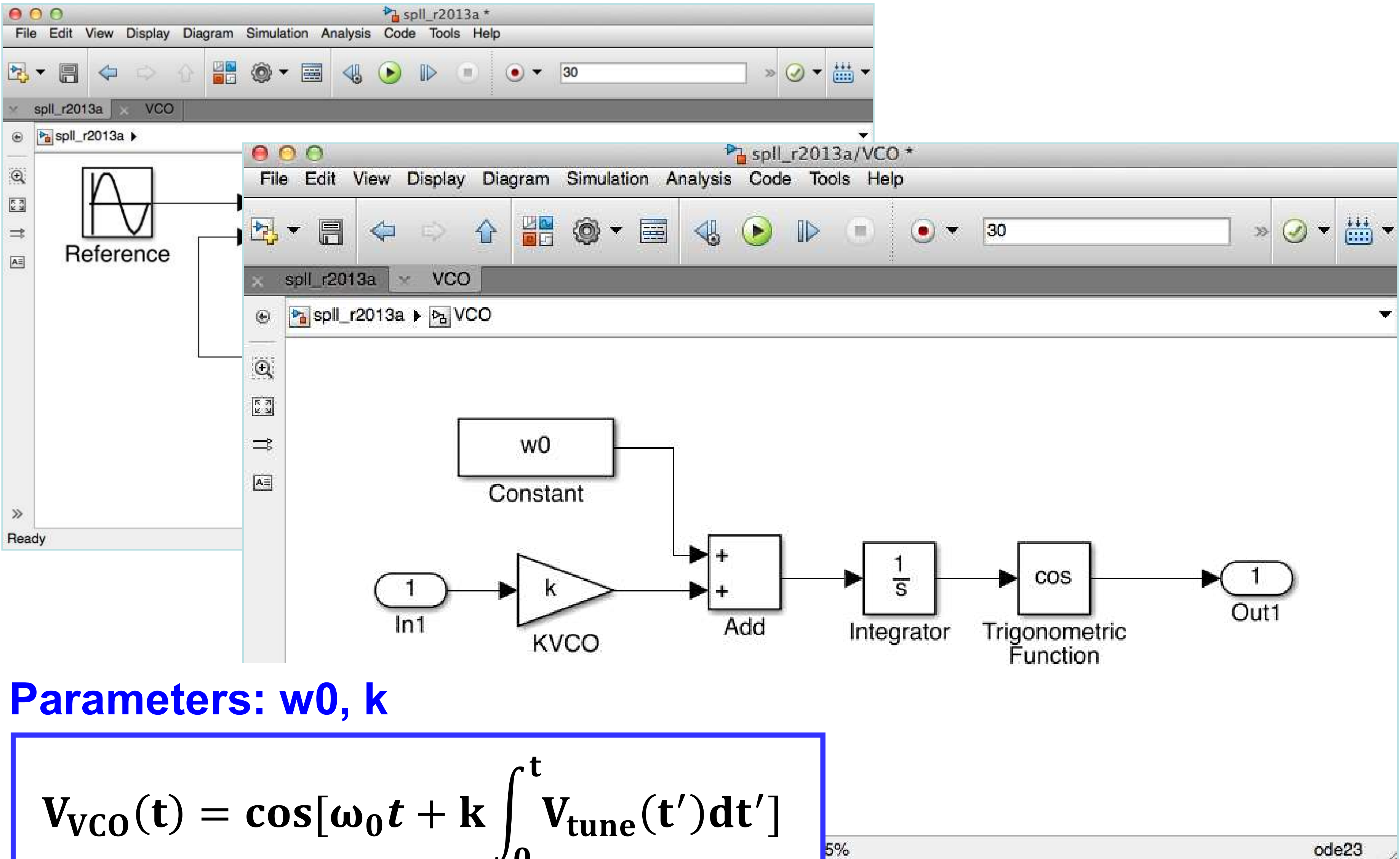


$$\begin{cases} V_{ref}(t) = \sin\left[(\omega_0 + \Delta\omega)t\right] \\ V_{vco}(t) = \cos\left[\omega_0 t + K_{vco}\int_0^t V_{tune}(t')dt'\right] \\ V_{pd}(t) = 2V_{ref}(t)\cdot V_{vco}(t) \\ V_{tune}(t) = V_{pd}(t) * g(t) \end{cases}$$

PLL with Analog Multiplier (Simulink)



VCO Model (Simulink)



Parameters: w0, k

$$V_{VCO}(t) = \cos[\omega_0 t + k\int_0^t V_{tune}(t')dt']$$

Parameter Set

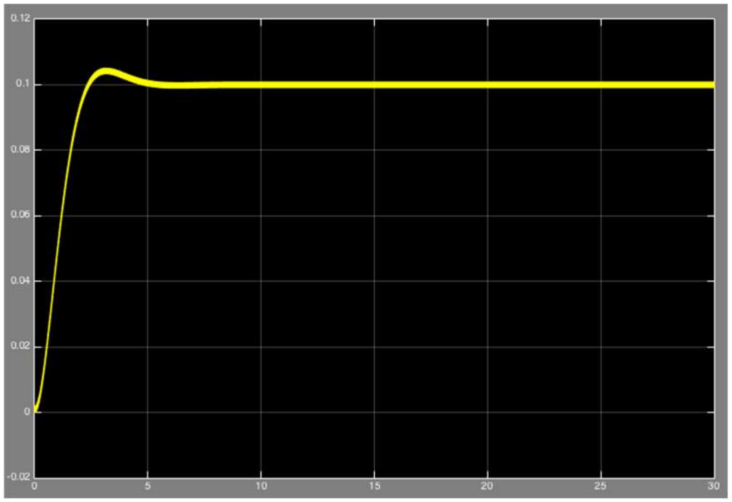
$$\begin{cases} \omega_n = \sqrt{k/\tau} \\ \zeta = \dfrac{1}{2\sqrt{k\tau}} \end{cases}$$

Critical damping

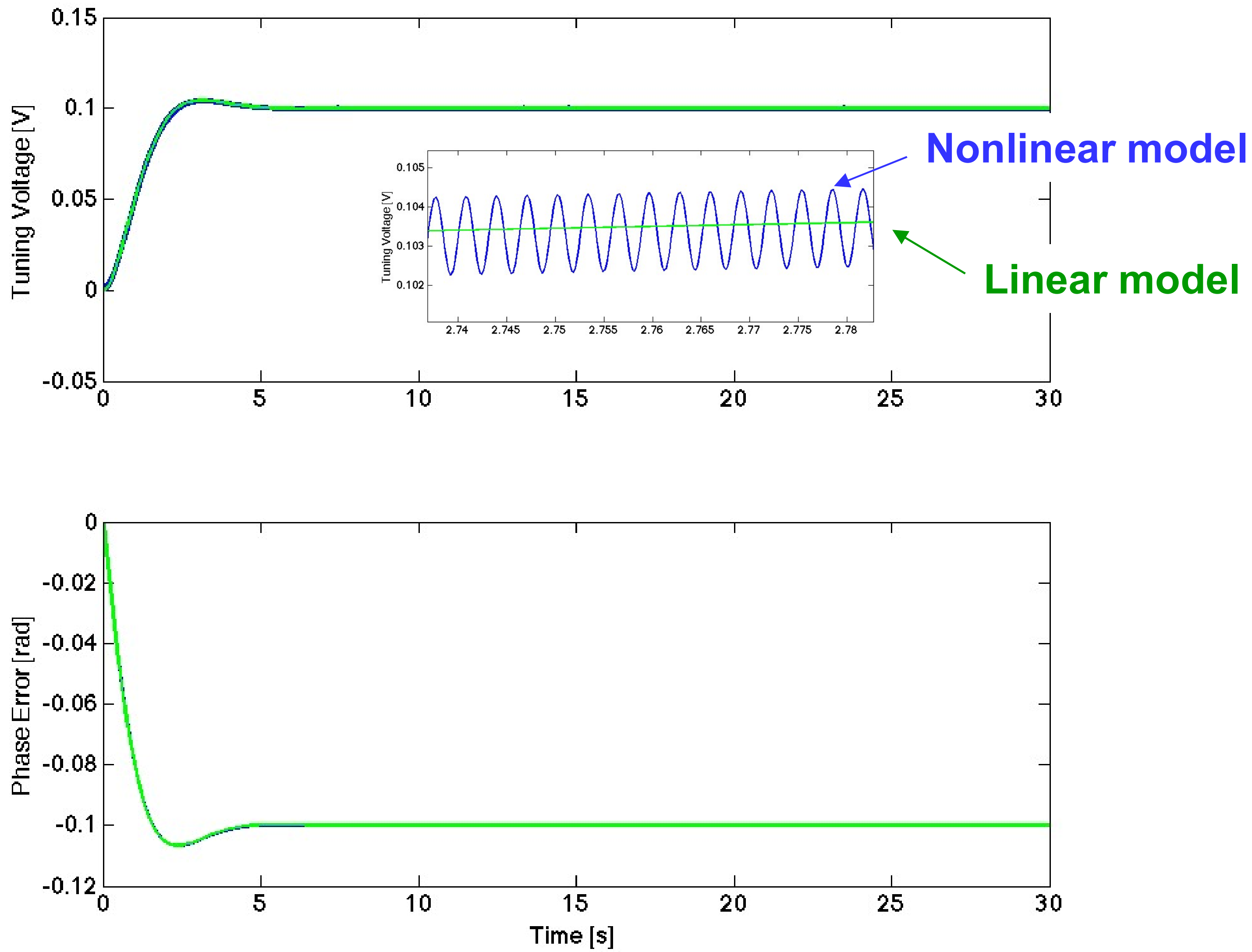
$$\zeta = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} \tau = \dfrac{1}{2k} \\ \omega_n = k\sqrt{2} \end{cases}$$

Parameter set:

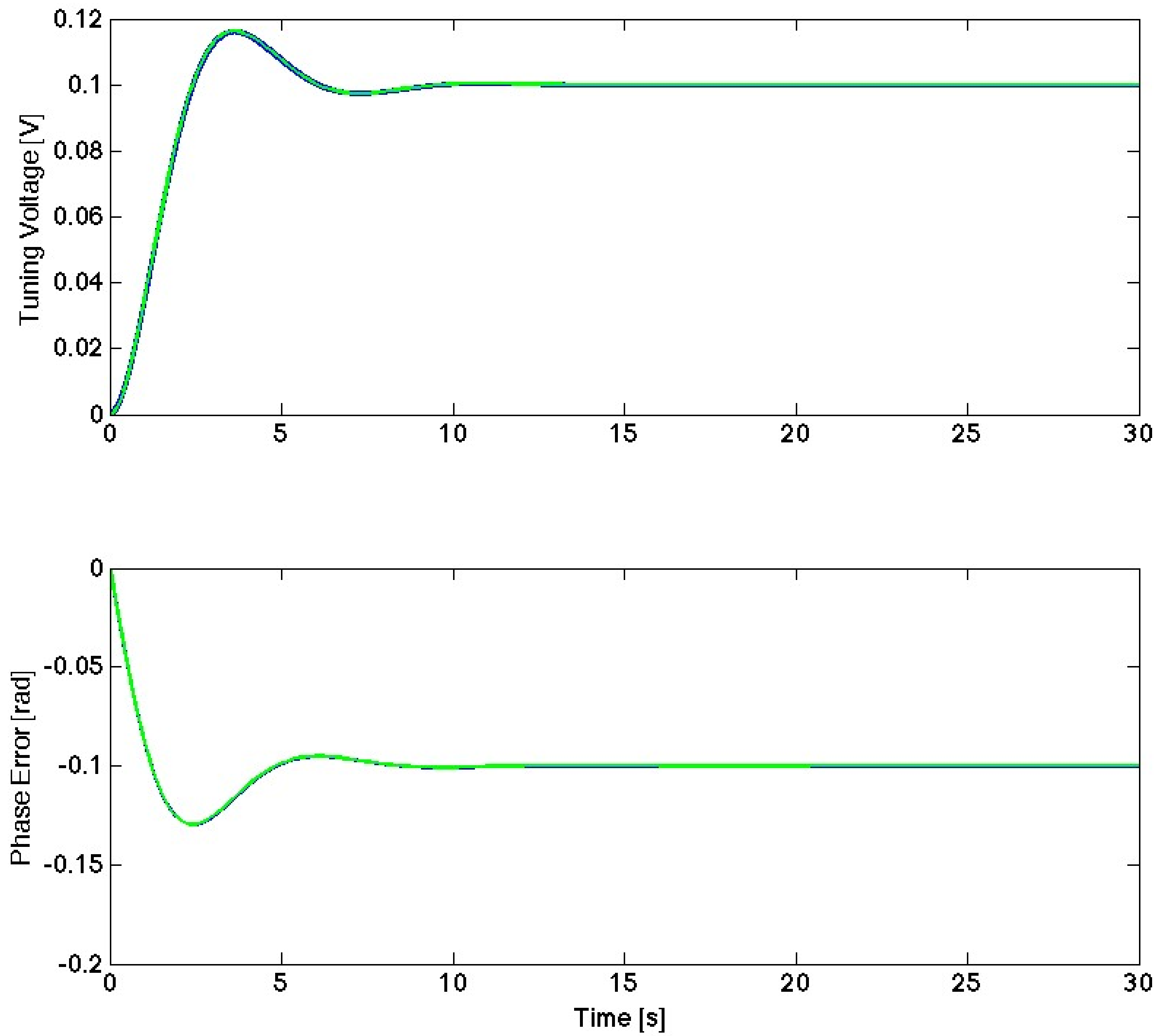
- **k = 1**
- **tau = 0.5**
- **tstop = 30**
- **w0 = 1e3**
- **dw = 0.1**



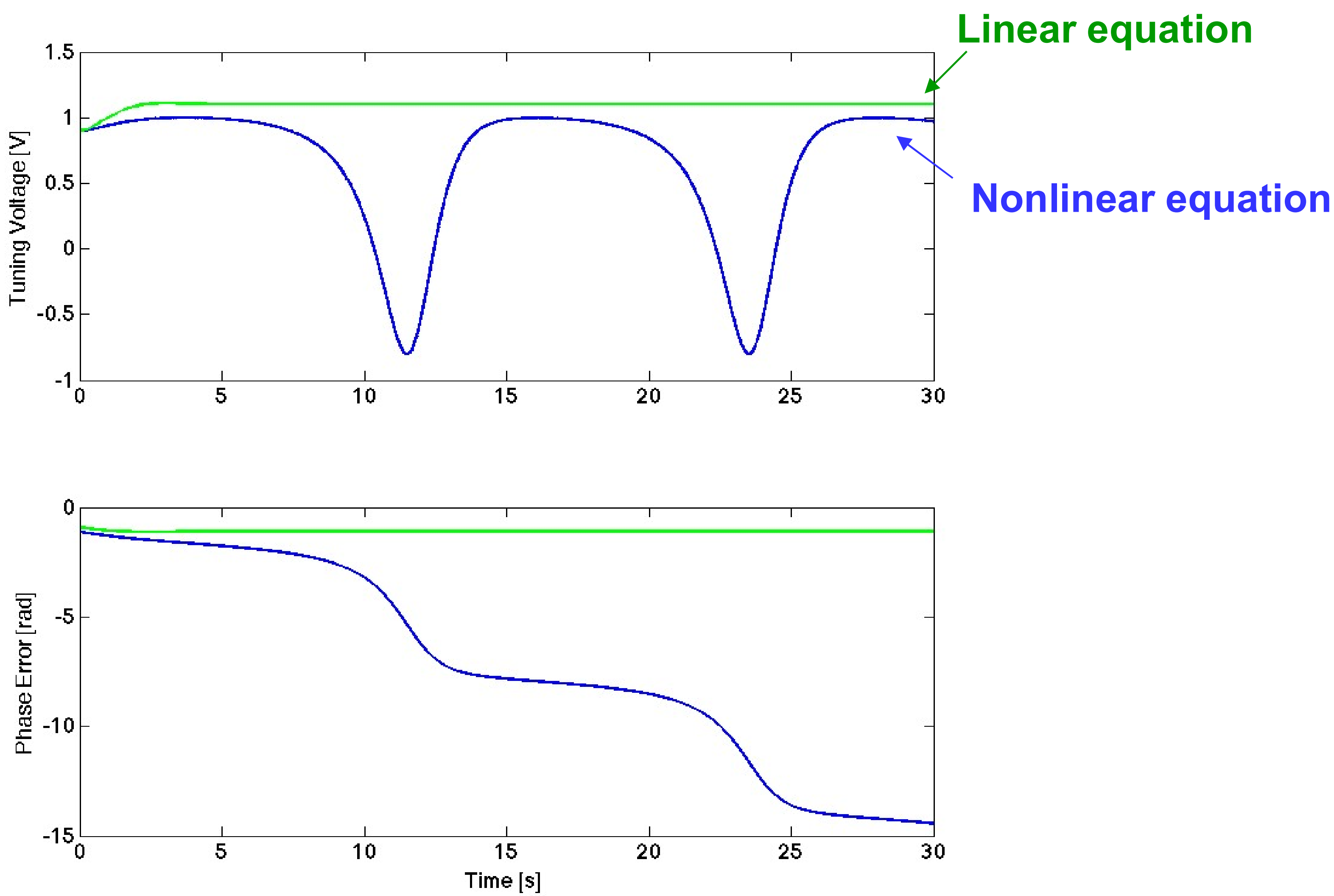
Nonlinear vs Linear (Maximum Flat Response)



Nonlinear vs Linear (Underdamped)



Nonlinear vs Linear (Out of Lock Range)



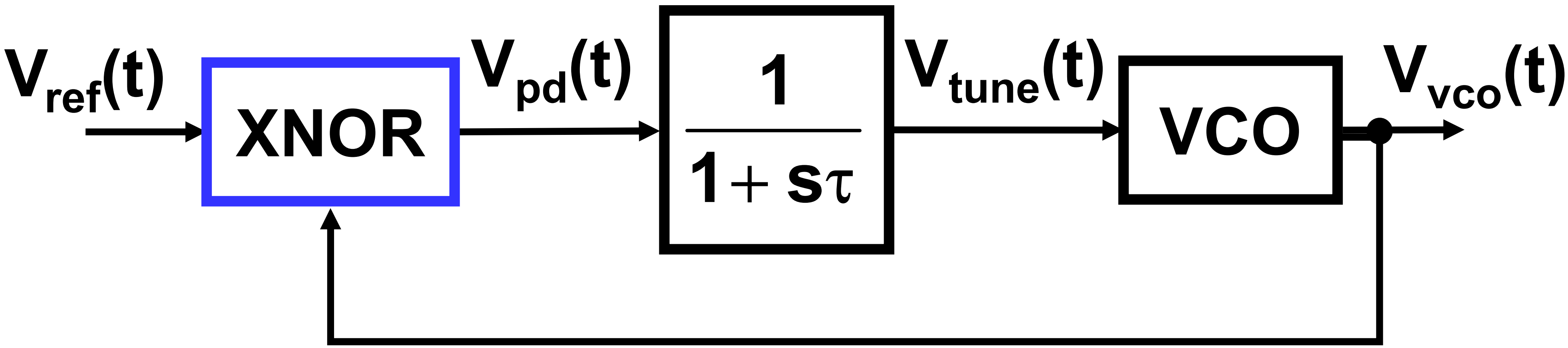
Exercise #2

- Estimate the ripple amplitude and frequency of the tuning voltage **(in critical damping and underdamped case) from theory and simulation.**

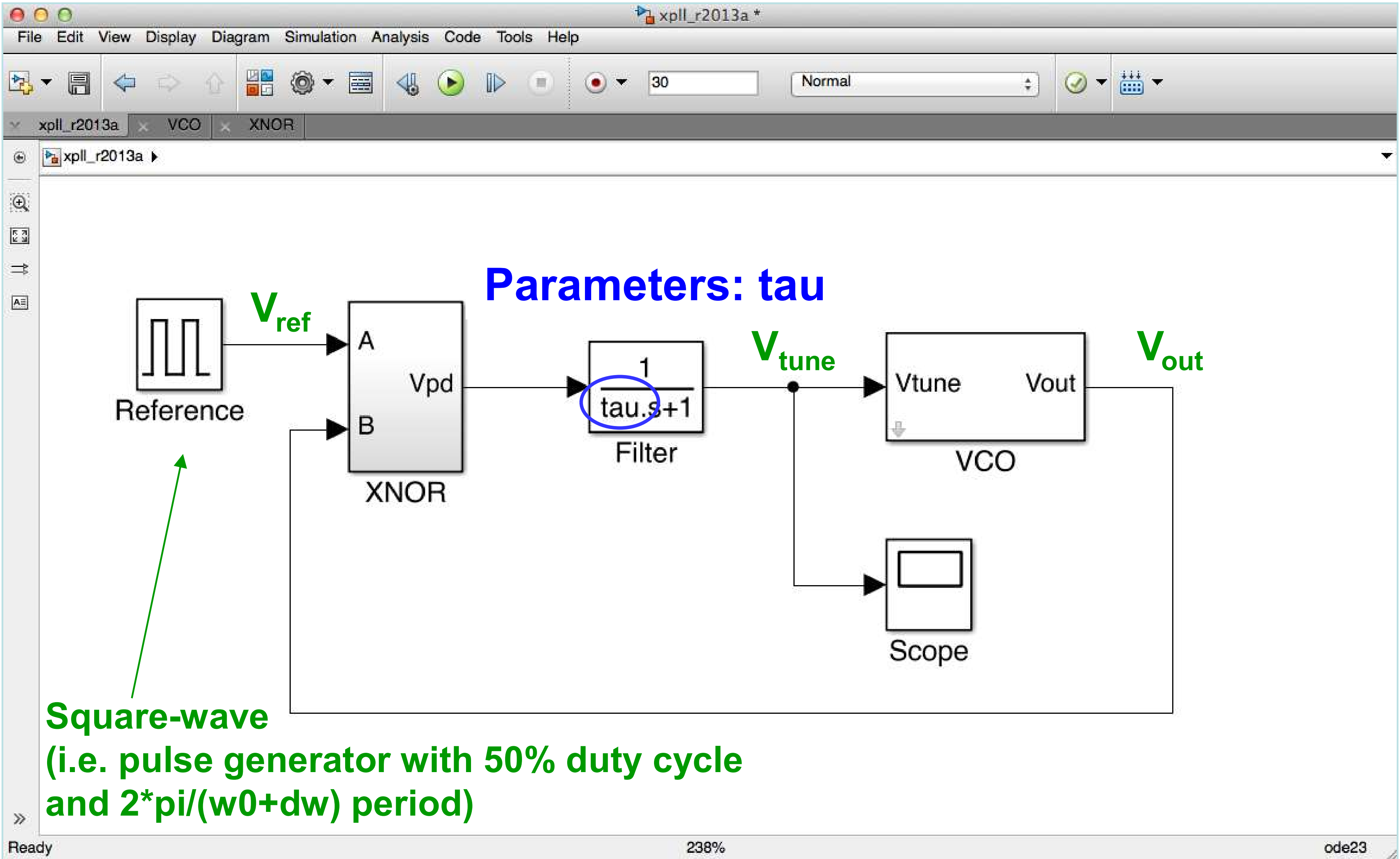
- Estimate the lock range and the capture range of the PLL **(in critical and underdamped case) from theory and simulation.**

Type-I PLL with XNOR

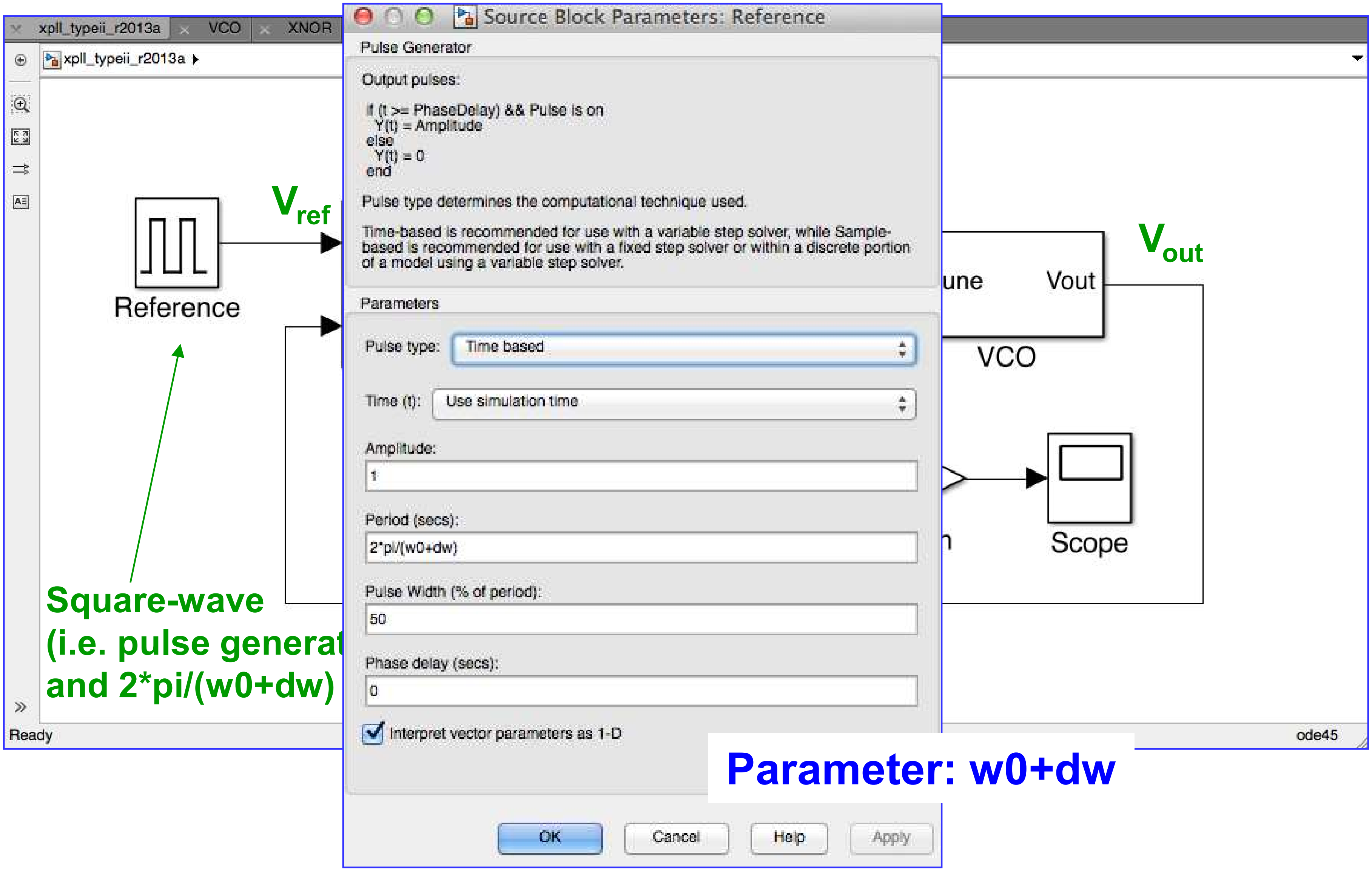
Type-I PLL with XNOR



Type-I PLL with XNOR (Simulink)



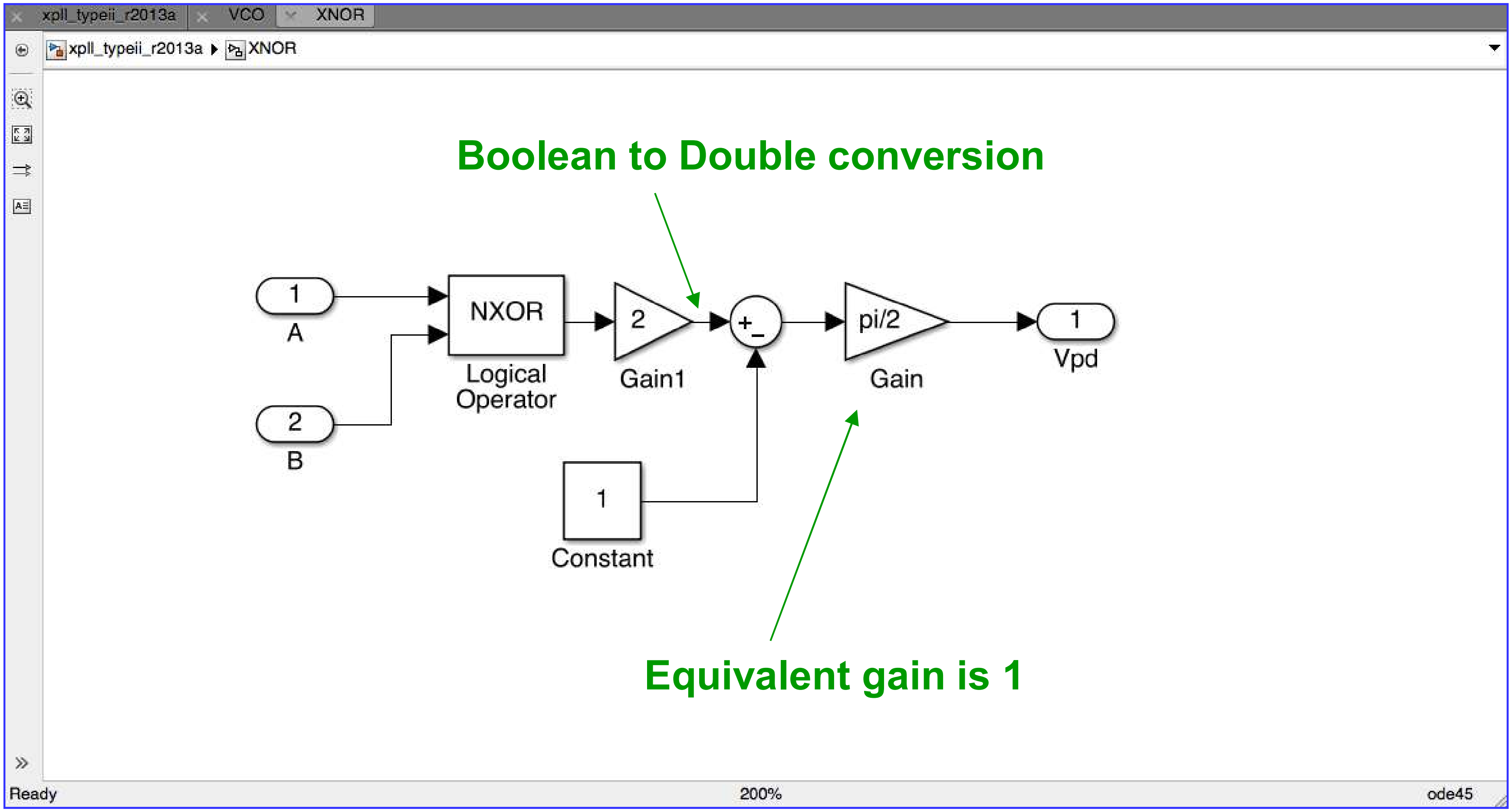
Reference Signal (Simulink)



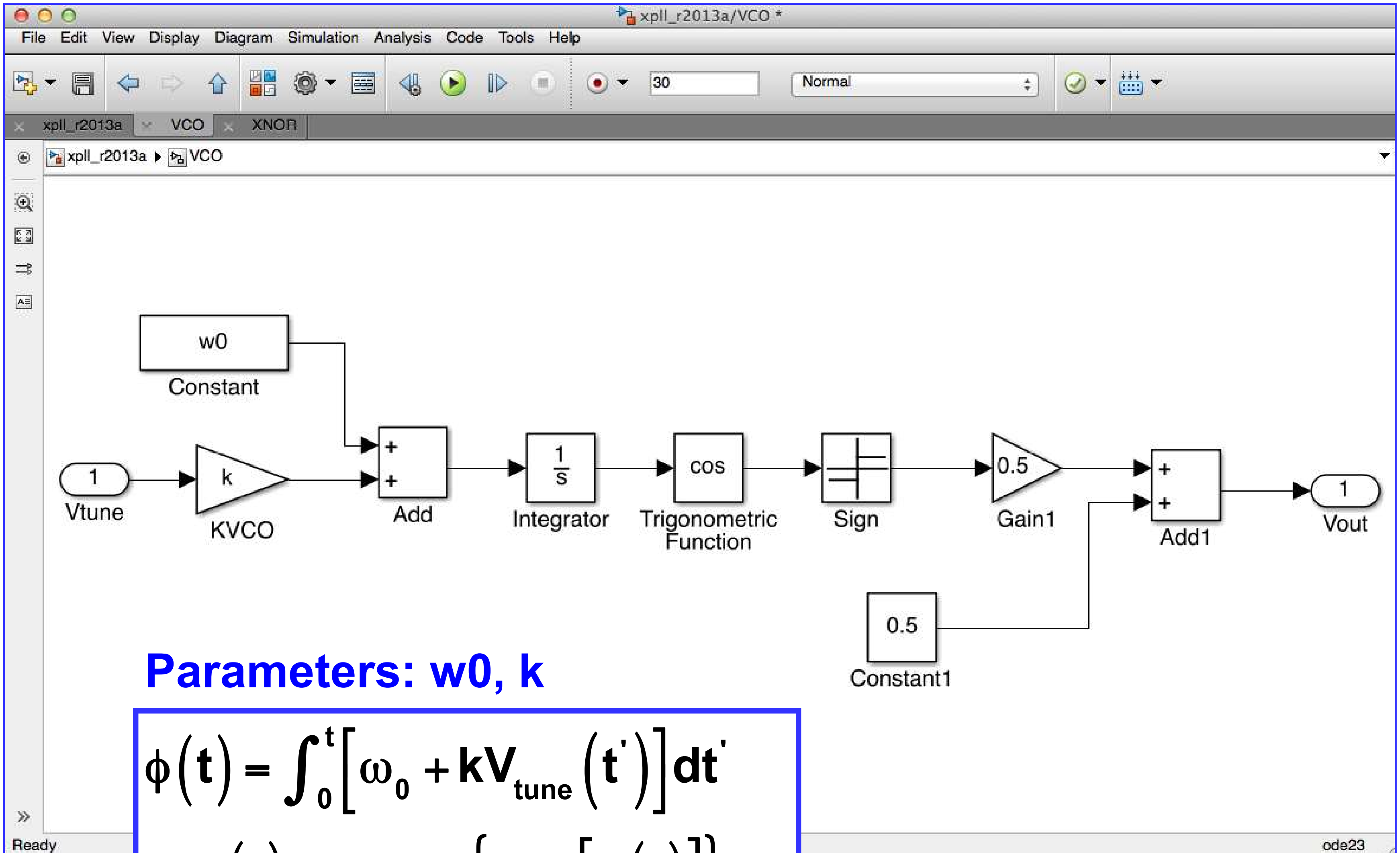
Square-wave
(i.e. pulse generat
and 2*pi/(w0+dw)

Parameter: w0+dw

XNOR Block (Simulink)



VCO (Simulink)

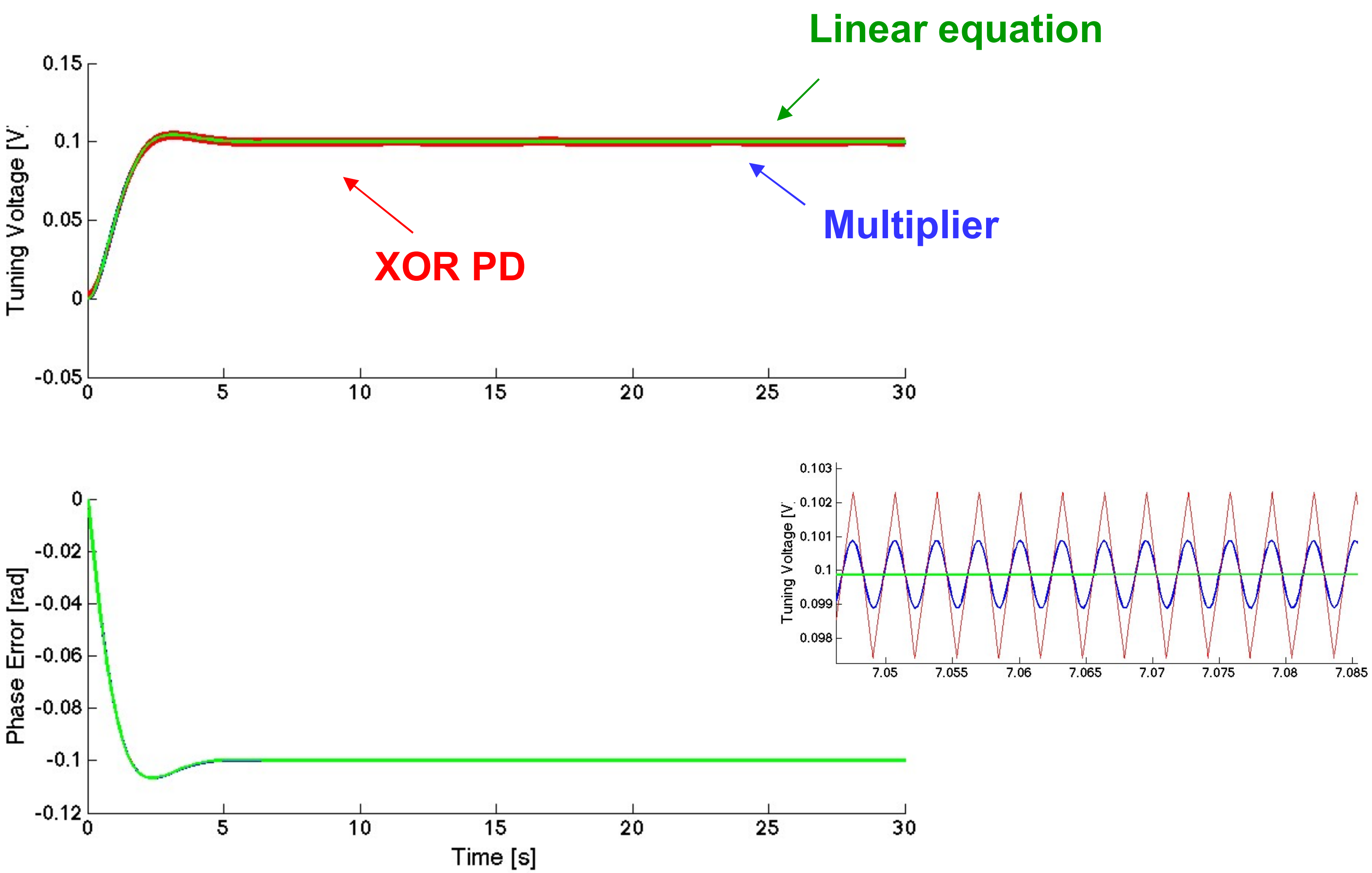


Parameters: w0, k

$$\phi(t) = \int_0^t \left[\omega_0 + kV_{tune}(t')\right]dt'$$

$$V_{out}(t) = \tfrac{1}{2}\,\mathrm{sign}\left\{\cos\left[\phi(t)\right]\right\} + \tfrac{1}{2}$$

Nonlinear vs Linear



Exercise #3

- Estimate the ripple amplitude and frequency of the tuning voltage (in critical damping and underdamped case) from simulation.

- Try to theoretically justify the previous empirical result