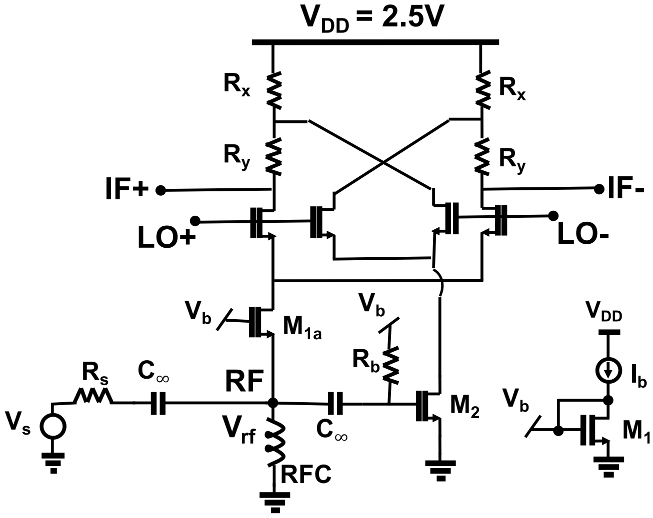
RF Circuit Design **Prof. Salvatore Levantino**

Available time: 2 hours Jul. 13, 2020

Problem #1

In the LNA/mixer cascade in figure, let the MOSFET aspect ratios $(W/L)_{M1a} = (W/L)_{M1b} = 500$, $(W/L)_{M2} = 2000$, and the resistor $R_b = 1\ \text{M}\Omega$. Let RFC inductance and C_∞ capacitances tend to infinity. Let the transistors threshold $V_T = 0.5\text{V}$, constants $½\mu_n C'_{ox} = 0.2\ \text{mA/V}^2$ and $\gamma/\alpha = 2/3$. The LO signal is such that it fully steers the current of the differential pairs at $f_{LO} = 1$ GHz, but maintains M1-M2 always in saturation region. V_S is a signal at $f_{RF} = 0.9$ GHz.



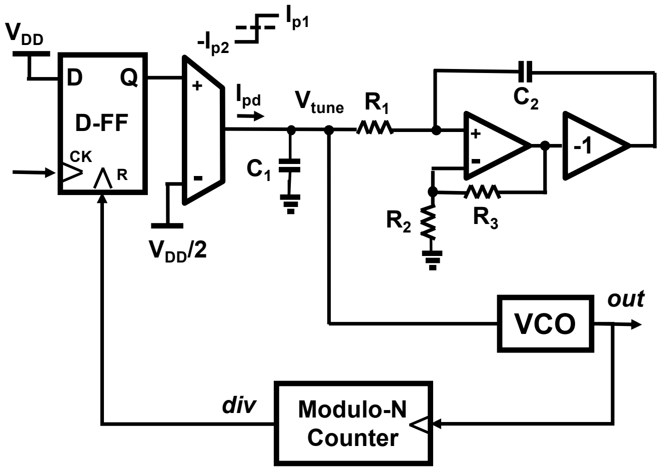
a) Calculate the **value of R_y/R_x ratio** which cancels the thermal noise of M_1 at IF port, after imposing conjugate matching at RF port.

b) Set the **value of R_x, R_y and I_b** to get conversion gain $G_C = 20\log(|V_{if}/V_{rf}|) = 28\text{dB}$ (where $V_{if} = V_{IF+} - V_{IF-}$), imposing the condition in a) and conjugate matching at RF port.

c) Calculate the **noise figure NF in dB** at f_{RF}, accounting only for the thermal noise of R_s, R_x, R_y resistors and M_1 and M_2 transistors. *(Assume R_s to model a resonator, whose noise is white passband around f_{RF}).*

Problem #2

The PLL in figure has VCO frequency ranging from 1.8GHz to 2.2GHz sweeping V_{tune} from 0 to $V_{dd} = 3.3\text{V}$. The phase detector is a D-type flip-flop (D-FF) sensitive to rising-edge transitions. Let the reference frequency $f_{ref} = 10\text{MHz}$ and the frequency division factor $N = 200$. Let $R_2 = 1\text{k}\Omega$, $C_1 = 80\text{pF}$, $C_2 = 100\text{pF}$. Let the charge-pump currents $I_{p1} = 50\mu\text{A}$ and $I_{p2} = 30\mu\text{A}$.

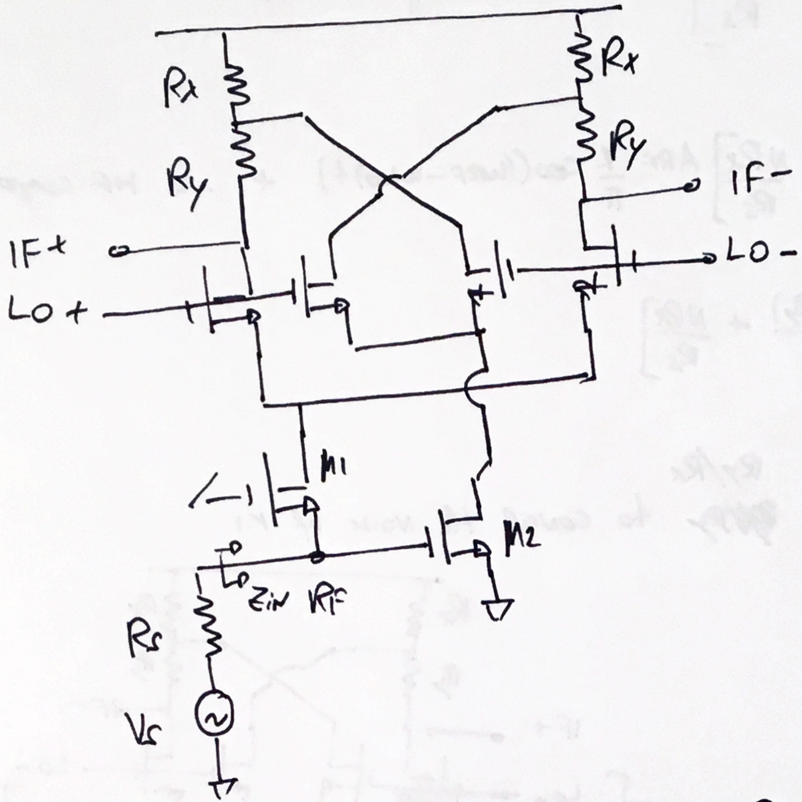


a) Calculate the **values of R_1 and R_3** to have (i) unity-gain frequency of PLL loop gain of 15 kHz, (ii) phase margin of 60 degree.

b) Calculate the **steady-state time shift (in seconds)** between *ref* and *div* signals. Evaluate **frequency (Hz) and level (dBc) of the dominant spur** in the output spectrum.

c) Determine **the value of the output phase noise PSD** accounting only for the thermal noise of R_1 at the offset frequency of 100Hz.

EX1 SOLUTION



a) • input matching: $Z_{IN} = 1/g_{m1} = 50\Omega = R_S$

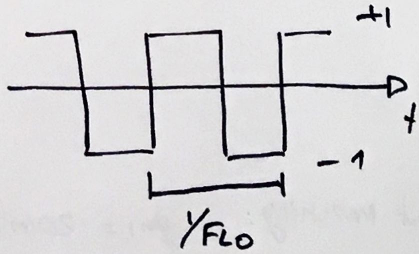
• conversion gain G^{RF-DIF}

$$V_{IF}(t) = \left[g_{m1}(R_X+R_Y) + g_{m2}R_X\right] V_{RF}(t)\, X_{LO}(t)$$

$$N = \frac{g_{m2}}{g_{m1}} = \frac{(W/L)_2}{(W/L)_1} = 4$$

$$V_{IF}(t) = \left[\frac{(R_X+R_Y)}{R_S} + \frac{N R_X}{R_S}\right] V_{RF}(t)\, X_{LO}(t)$$

$A_{RF}\cos(\omega_{RF}t)$



$$X_{LO}(t) \simeq 0 + \frac{4}{\pi}\cos(\omega_{LO}t) + 0 - \frac{4}{3\pi}\cos(3\omega_{LO}t) + \dots$$

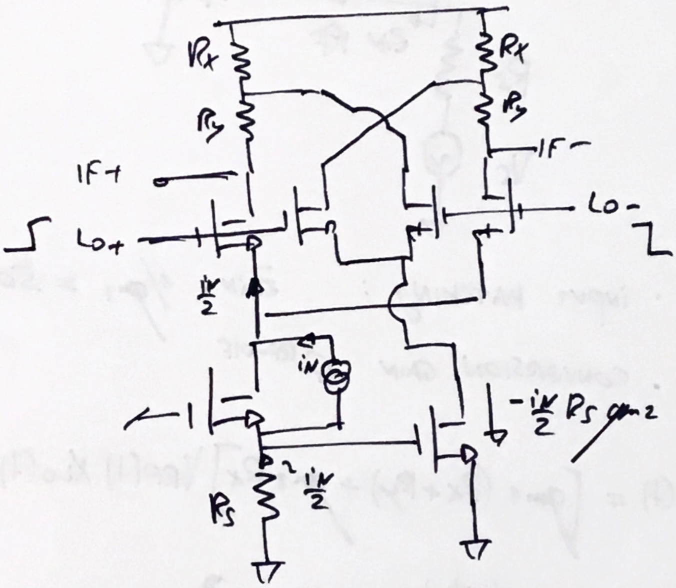
$$V_{IF}(t) \simeq \left[\frac{(R_x+R_y)}{R_S} + \frac{N R_x}{R_S}\right] A_{RF}\cos(\omega_{RF}t)\,\frac{4}{\pi}\cos(\omega_{LO}t)$$

$$\simeq \frac{1}{2}\left[\frac{(R_x+R_y)}{R_S} + \frac{N R_x}{R_S}\right] A_{RF}\,\frac{4}{\pi}\cos((\omega_{RF}-\omega_{LO})t) + \dots \text{ HF component}$$

$$G^{RF\text{-}DIF} = \frac{2}{\pi}\left[\frac{(R_x+R_y)}{R_S} + \frac{N R_x}{R_S}\right]$$

b) DETERMINE THE RATIO R_y/R_x ~~R_x/R_y~~ to cancel the noise of M_1

Consider $V_{LO} > 0$, the same result holds for $V_{LO} < 0$.



$$V_{IF} = \frac{i_N}{2}(R_x+R_y) - \frac{i_N}{2}R_S g_{m2} R_x = 0$$

$$\to (R_x+R_y) - N R_x = 0$$

$$R_y = R_x[N-1]$$

$$\to \frac{R_y}{R_x} = (N-1)$$

- input matching: $g_{m1} = 20\,mS = \frac{1}{50\Omega}$

$$I_b = \frac{(g_{m1}/2)^2}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1} = 1mA$$

• GAIN :
$$G^{RF\to IF} = \frac{2}{\pi}\frac{1}{R_S}\left[R_X + R_Y + N R_X\right] = \frac{2}{\pi R_S}\left[2 N R_X\right]$$

$R_Y = (N-1) R_X$

$$G = \frac{4 N R_X}{\pi R_S} = 10^{28/20} \rightarrow R_X \cong 247\,\Omega$$

$$R_Y = (N-1) R_X = 741\,\Omega$$

c) Noise Figure:

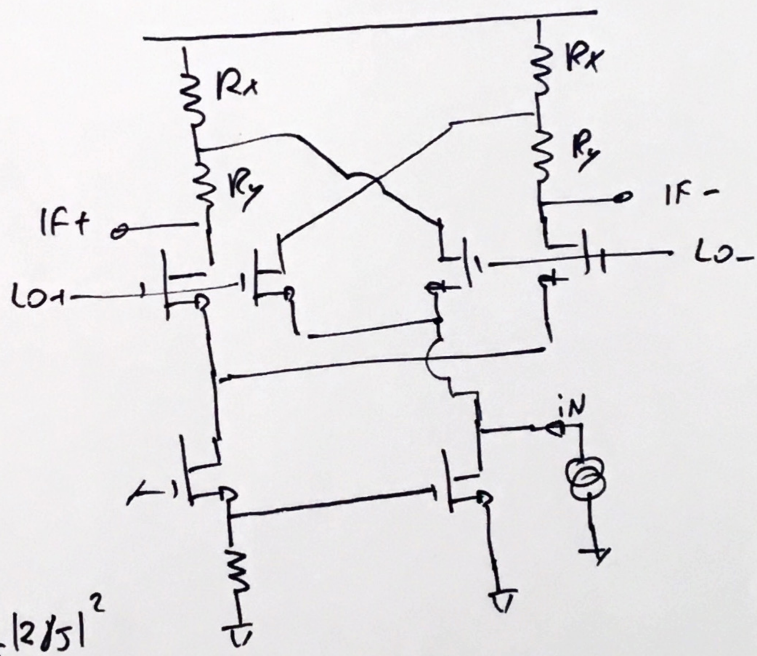
$M1 = \emptyset$

R_S: $S_{V_{IF}}^{R_S} = 4kT R_S \left[\frac{G^{RF\to IF}}{2}\right]^2$

R_X, R_Y: $S_{V_{IF}}^{R_X,R_Y} = 2 \times 4kT(R_X + R_Y)$

M2 |

$V_{IF}^{M2}(t) = i_N(t)\, x_{LO}(t) \cdot R_X$



$$S_{V_{IF}}^{M2}(f) = 4kT\frac{\gamma}{\alpha} N g_{m1} R_X^2 \sum_{J=1}^{+\infty}\frac{1}{2}|2\gamma_J|^2$$

coefficients Fourier Series.

$$\sum_{J=1}^{+\infty}\frac{|2\gamma_J|^2}{2} = \int_0^{+\infty} S_{V_{LO}}(f)\,df = \frac{1}{T_{LO}}\int_{-T_{LO}/2}^{T_{LO}/2} |x_{LO}(t)|^2 = 1$$

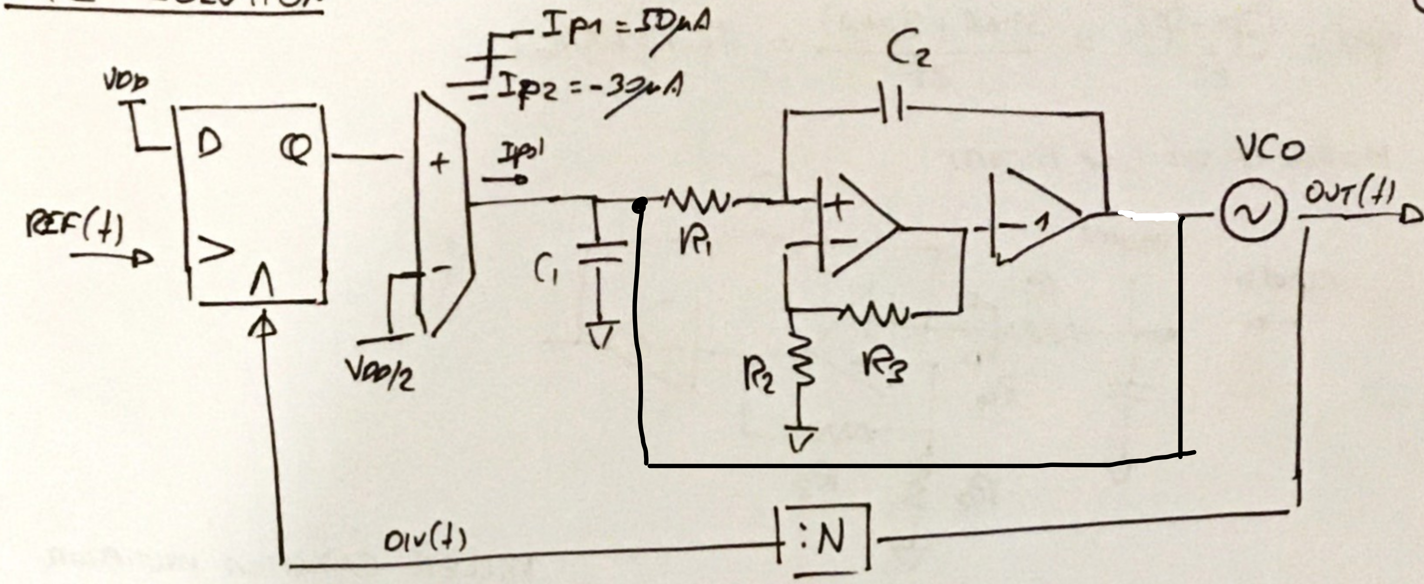
PARSEVAL'S THEOREM

$$S_{V_{IF}}^{M2}(f) = 4kT\frac{\gamma}{\alpha} g_{m1} N R_x^2$$

$$NF = 1 + \frac{2 \times \cancel{4kT}(R_x+R_y)}{\cancel{4kT} R_s \left(\frac{G}{2}\right)^2} + \frac{\cancel{4kT}\frac{\gamma}{\alpha} g_{m1} N R_x^2}{\cancel{4kT} R_s \left(\frac{G}{2}\right)^2}$$

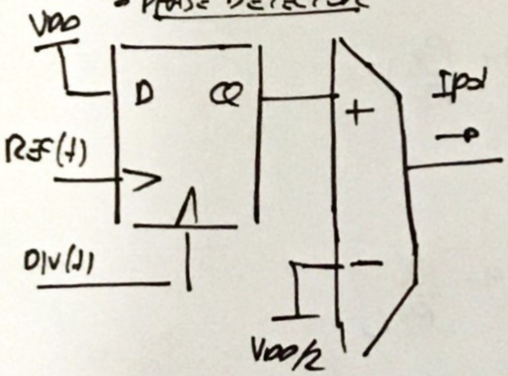
$$= 1 + \frac{2(R_x+R_y) + \frac{\gamma}{\alpha}\frac{NR_x^2}{R_s}}{R_s\left(\frac{G}{2}\right)^2} = 2.21\,dB$$

EX2 - SOLUTION

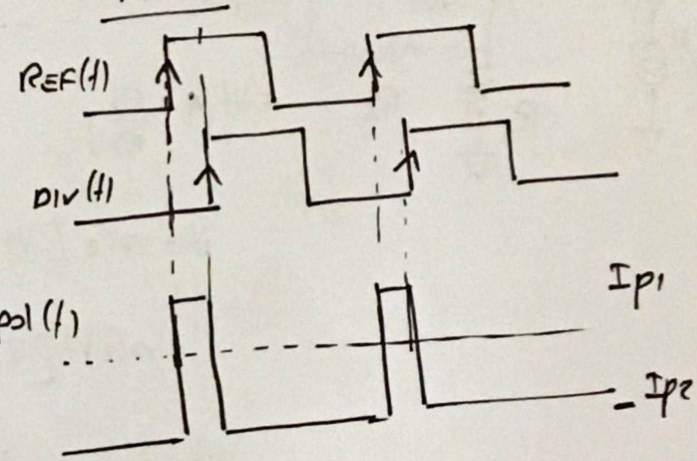


a) Eq Model in phase domain

- PHASE DETECTOR



$t_\varepsilon > 0$

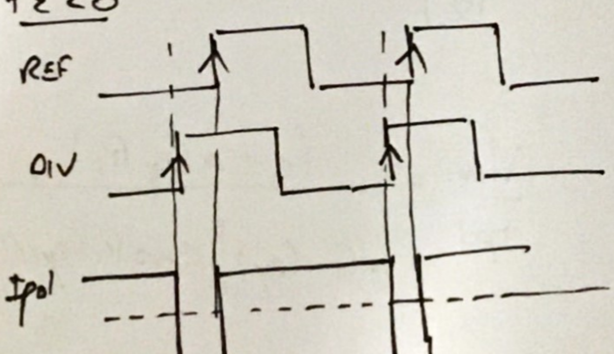


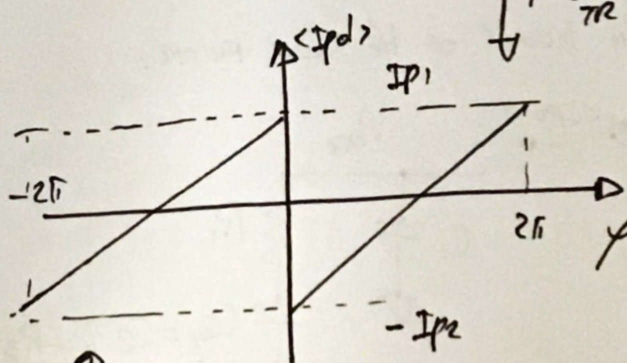
$$\langle I_{pd} \rangle = \frac{1}{T_R} \int_{-T_R/2}^{T_R/2} I_{pd}(t)\, dt \longrightarrow \langle I_{pd} \rangle = I_{p1} \frac{t_\varepsilon}{T_R} - I_{p2} \frac{[T_R - t_\varepsilon]}{T_R}$$

$$= (I_{p1} + I_{p2}) \frac{t_\varepsilon}{T_R} - I_{p2}.$$

$$\varphi = \frac{2\pi}{T_R} t_\varepsilon$$

$t_\varepsilon < 0$





$$\langle I_{pd} \rangle = -I_{p2} \frac{t_\varepsilon}{T_R} + I_{p1} \frac{(T_R - t_\varepsilon)}{T_R}$$

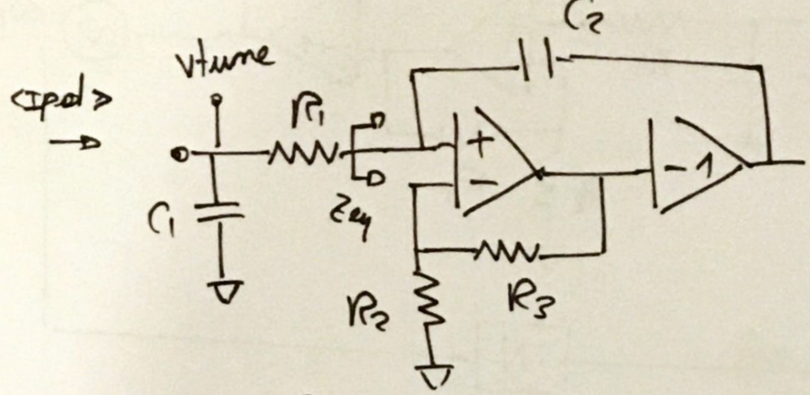
$$= -(I_{p1} + I_{p2}) \frac{t_\varepsilon}{T_R} + I_{p1}$$

$$\varphi = \frac{2\pi}{T_R} t_\varepsilon$$

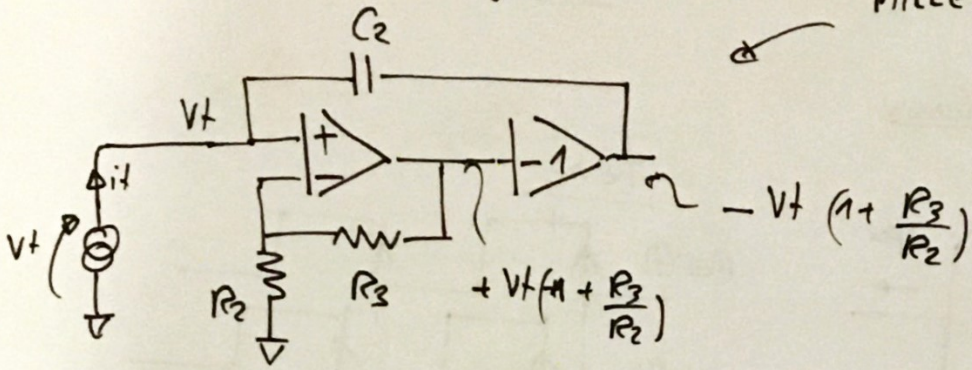
$$\boxed{K_{pd} = \frac{d \langle I_{pd} \rangle}{d\varphi} = \frac{(I_{p1} + I_{p2})}{2\pi}}$$

$$K_{pd} = \frac{(I_{p1} + I_{p2})}{2\pi} = \frac{(50\mu A + 30\mu A)}{2\pi} = 12.73\,\mu A/rad$$

- MODEL OF THE LOOP FILTER:



MILLER CAPACITOR MULTIPLIER



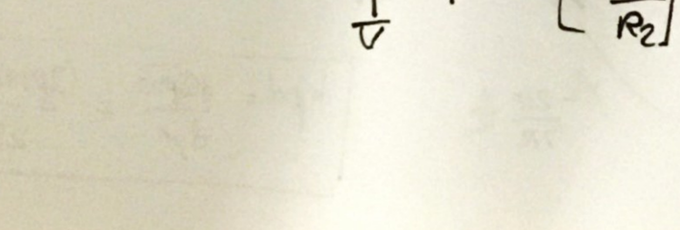
$$i_t = sC_2\left[V_t + V_t\left(1+\frac{R_3}{R_2}\right)\right]$$

$$= sC_2V_t\left[2+\frac{R_3}{R_2}\right]$$

$$\frac{V_t}{i_t} = \frac{1}{sC_2\left[2+\frac{R_3}{R_2}\right]} = \frac{1}{sC_{eq}}$$

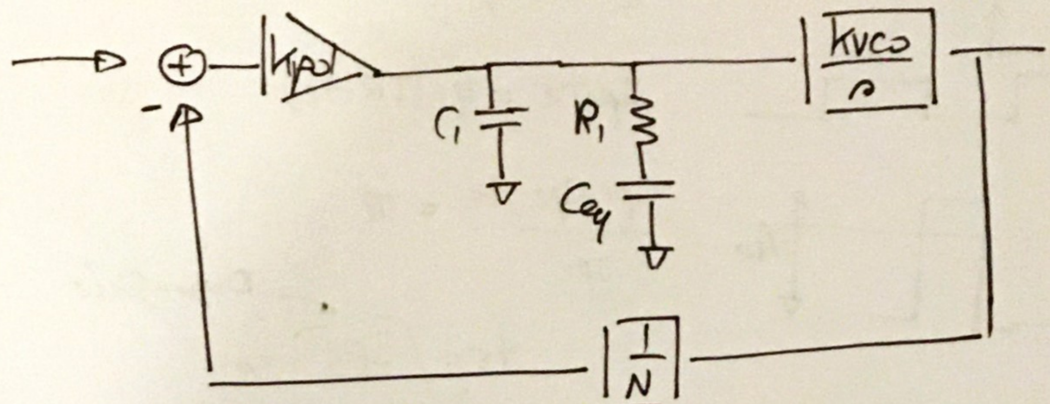
$$C_{eq} = C_2\left[2+\frac{R_3}{R_2}\right]$$

eq Model of the Loop FILTER:



$$\frac{V_{tune}}{I_{pd}} = \frac{(1 + sC_{eq}R_1)}{s(C_1 + C_{eq})\left[1 + sR_1(C_{eq}//C_1)\right]}$$

• COMPLETE PHASE MODEL:



$$K_{VCO} = \frac{(2.2 - 1.8)\,GHz \cdot 2\pi}{3.3V} = 761.6\ \frac{Mrad/s}{V}$$

$$G_{loop}(s) = K_{pd}\ \frac{(1 + sR_1C_{eq})}{s(C_1 + C_{eq})\left[1 + sR_1(C_{eq}//C_1)\right]}\ \frac{K_{VCO}}{s}\ \frac{1}{N}$$

- $f_{UGF} = 15KHz$ SIZE R_1 and C_0, R_3.
- $PM = 60°$

$$|G_{loop}(w_{UGF})| \simeq \frac{K_{pd} R_1 K_{VCO}}{w_{UGF} \cdot N} = 1 \rightarrow \boxed{R_1 = \frac{(2\pi\ 15KHz)\ \overset{200}{N}}{K_{pd} K_{VCO}} = 1.94\ K\Omega}$$

$$f_z = \frac{1}{2\pi R_1 C_{eq}}$$

$$f_p = \frac{1}{2\pi R_1 (C_{eq}//C_1)} \simeq \frac{1}{2\pi R_1 \underset{80pF}{C_1}} = 1.023 MHz$$

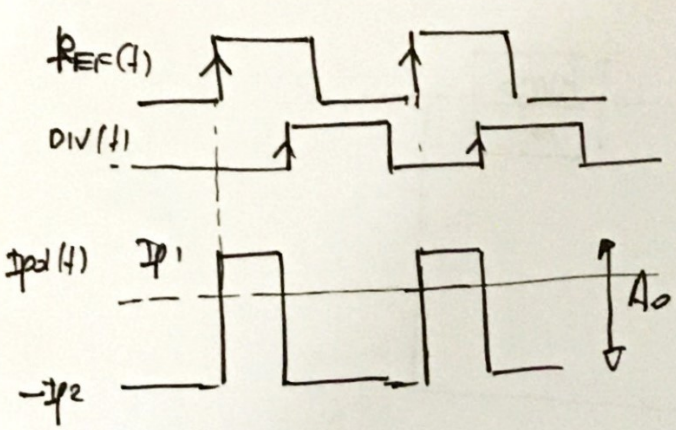
$$PM = 180 - 90 - 90 + atan\left(\frac{f_{UGF}}{f_z}\right) - \underbrace{atan\left(\frac{f_{UGF}}{f_p}\right)}_{0.84°} = 60$$

$$f_z = \frac{f_{UGF}}{tan(60 + 0.84°)} = 8.37 KHz$$

$C_2 = 100pF$
$R_2 = 1K\Omega$

$$C_{eq} = \frac{1}{2\pi f_z R_1} = 9.8 nF \longrightarrow C_{eq} = C_2\left[1 + \frac{R_3}{R_2}\right] \rightarrow \boxed{R_3 = 96 K\Omega}$$

b) TIME ERROR @ STEADY STATE:



@ STEADY STATE

$$I_{p1} t_\varepsilon = I_{p2}(T_R - t_\varepsilon)$$

$$\frac{(I_{p1}+I_{p2})\, t_\varepsilon}{I_{p2}} = T_R$$

Duty-Cycle

$$t_\varepsilon = \left[\frac{I_{p2}}{(I_{p1}+I_{p2})}\right] T_R$$

$$= \frac{30}{80}\, 100\,nsec = 37.5\,nsec.$$

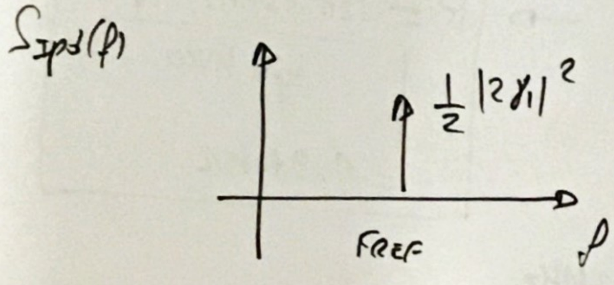
- FREQUENCY AND LEVEL OF THE DOMINANT SPUR.

$$I_{pd}(t) = \gamma_0 + 2|\gamma_1| \cos(w_{REF}t + \Delta\phi_1) + \cdots$$

AVERAGE $= \emptyset$

$$\gamma_1 = A_0 \cdot \cancel{\phi}\, \frac{\sin(\pi D)}{\pi \cancel{\phi}}$$

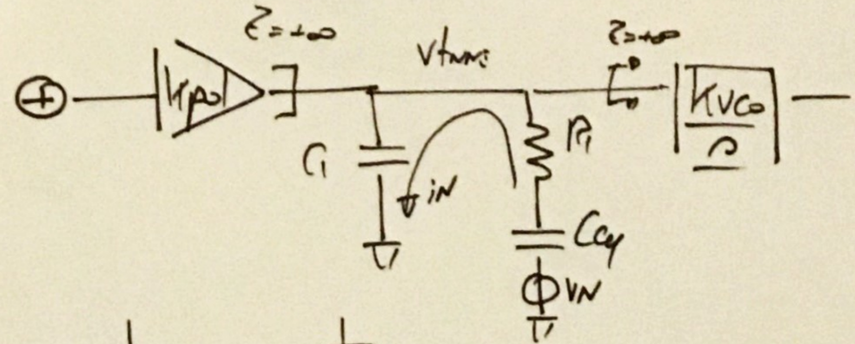
$$= (I_{p1} + I_{p2}) \cdot \frac{0{,}924}{\pi} = 23.53\,\mu A$$



- OUTPUT LEVEL:

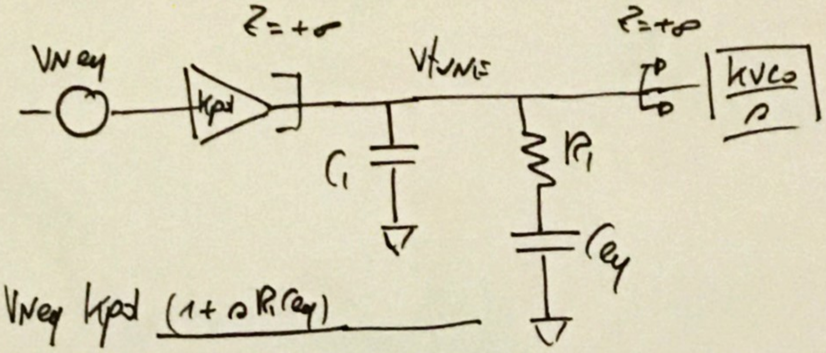
$$\mathcal{L}(\Delta f = F_{REF}) = \frac{S_\phi(F_{REF})}{2} = \frac{1}{2}\left(\frac{1}{2}|2\gamma_1|^2\right) \left|\frac{K_{VCO}}{(2\pi F_{REF})^2 C_1}\right|^2 = -24.92\,dBc.$$

c) <u>OUTPUT PHASE NOISE DUE TO R_1 THERMAL NOISE.</u> @ 100Hz OFFSET



$$V_{tune} = V_N \cdot \frac{1}{\left(R_1 + \frac{1}{sC_1} + \frac{1}{sC_{eq}}\right)} \frac{1}{sC_1} \simeq \frac{1}{(1+sR_1C_1)} V_N$$

- <u>input-referred.</u>

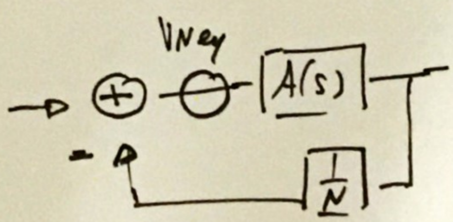


$$V_{tune} = V_{Neq} K_{pd} \frac{(1+sR_1C_{eq})}{s(C_1+C_{eq})\left[1+sR_1(C_1//C_{eq})\right]}$$

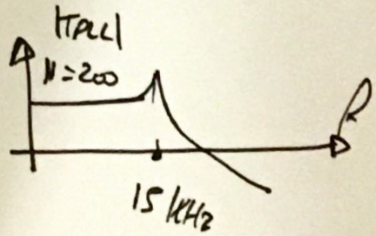
$$\simeq V_{Neq} K_{pd} \frac{(1+sR_1C_{eq})}{sC_{eq}\left[1+sR_1C_1\right]}$$

$$V_{Neq} = \frac{V_{tune}\, sC_{eq}\left[1+sR_1C_1\right]}{K_{pd}(1+sR_1C_{eq})} = \frac{V_N\, sC_{eq}}{K_{pd}(1+sR_1C_{eq})}$$

- <u>CLOSED-LOOP</u>:



$$S_{\phi out}(f) = S_{VNeq}(f)\,|T_{PLL}|^2$$



@ 100Hz

$$S_{\phi out}(f) = 4kTR_1 \frac{(2\pi 100Hz)^2 C_{eq}^2}{K_{pd}^2} N^2 = -125{,}22 \text{ dBc/Hz}$$