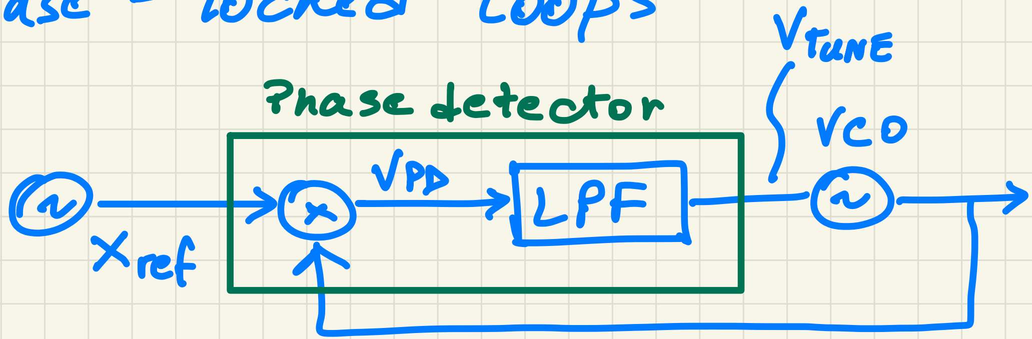
RF Circuit Design

LF

Phase-locked Loops



$$\begin{cases} x_{ref} = \sin \Phi_{ref} \\ x_{out} = \cos \Phi_{out} \end{cases}$$

$$V_{PD} = K_{PD} \sin \Phi_\varepsilon$$

- PD : multiplier + ideal filter
- VCO : linear tuning

$$\omega_{out} = \omega_{fr} + K_{VCO} \cdot V_{TUNE}$$

$$\underbrace{\dot{\Phi}_\varepsilon = \Delta\omega - K \sin \Phi_\varepsilon}_{\text{First-order}}$$

$$\Phi_\varepsilon \triangleq \Phi_{ref} - \Phi_{out}$$

$$\Delta\omega \triangleq \omega_{ref} - \omega_{fr}$$

$$K \triangleq K_{PD} \cdot K_{VCO} \quad [rad/s]$$

$\dot{\Phi}_\varepsilon = 0$ Equibrium (phase error is no longer variable $\Rightarrow$ it is constant)

$\omega_\varepsilon = \dot{\Phi}_\varepsilon = 0$ (frequency error: $\omega_{ref} - \omega_{out}$)

$\Rightarrow$ PLL is in "lock" state $\boxed{\omega_{out} = \omega_{ref}}$

$$\dot{\Phi}_\varepsilon = \Delta\omega - K \sin \Phi_\varepsilon = 0 \quad ; \quad \sin \Phi_\varepsilon = \frac{\Delta\omega}{K}$$

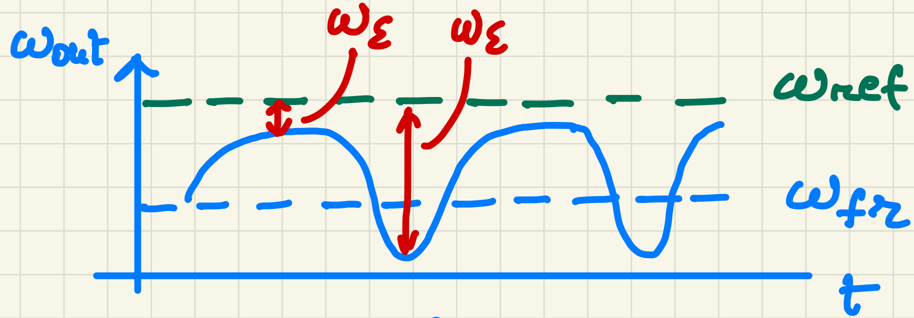
- IF $\left|\frac{\Delta\omega}{K}\right| < 1$: $\varphi_\varepsilon = \arcsin\left(\frac{\Delta\omega}{K}\right)$ stable equ. point

  $\Phi_\varepsilon(t) \rightarrow \varphi_\varepsilon$ "LOCK STATE"

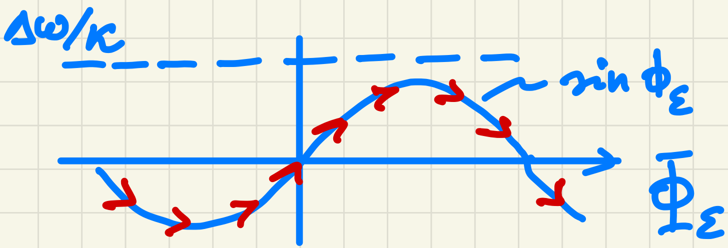
- IF $\left|\frac{\Delta\omega}{K}\right| > 1$: no equ. point "OUT-OF-LOCK"

$$\omega_{out}(t) = \omega_{fr} + K_{VCO} V_{TUNE} = \omega_{fr} + K \sin \Phi_\varepsilon(t)$$

If $\frac{\Delta\omega}{K} > 1 \Rightarrow \dot{\Phi}_\varepsilon > 0$



$$\omega_{out} = \omega_{fr} + K \sin \Phi_\varepsilon(t)$$



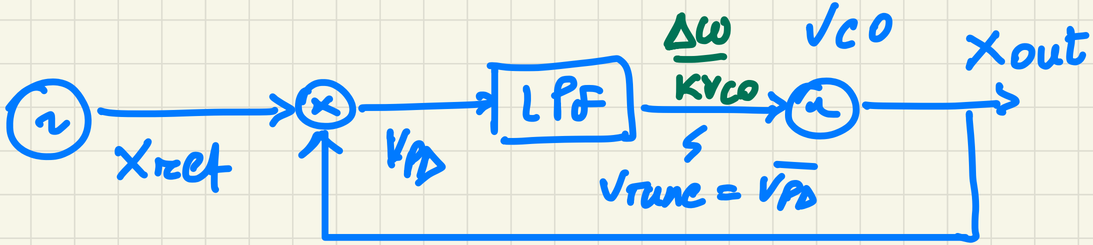
$$\omega_\varepsilon = \dot{\Phi}_\varepsilon$$

$$\omega_\varepsilon = \omega_{ref} - \omega_{out}$$

LOCK STATE $\Leftrightarrow$ $\left|\frac{\Delta\omega}{K}\right| < 1$ $\Leftrightarrow$ $-K < \Delta\omega < K$

LOCK RANGE : $\boxed{\Delta\omega_L = K}$

Intuitive interpretation



IMPOSING "LOCK"

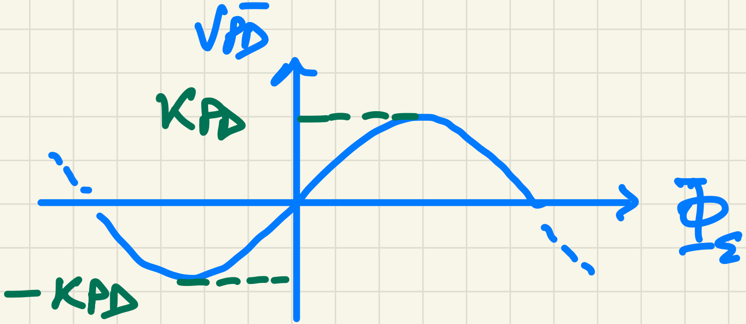
imposing equality at steady state

$$\omega_{out} = \omega_{fr} + K_{vco} V_{Tune} = \omega_{ref}$$

$$\Rightarrow \quad V_{tune} = \frac{\overbrace{\omega_{ref} - \omega_{fr}}}{K_{vco}} = \frac{\Delta\omega}{K_{vco}}$$

$$\overline{V_{PD}} = K_{PD} \cdot \sin \Phi_{\varepsilon} = \frac{\Delta\omega}{K_{VCO}} \; ; \quad \sin \Phi_{\varepsilon} = \frac{\Delta\omega}{\underbrace{K_{vco} K_{PD}}_{K}}$$

$$\Rightarrow \quad \sin \Phi_{\varepsilon} = \frac{\Delta\omega}{K}$$



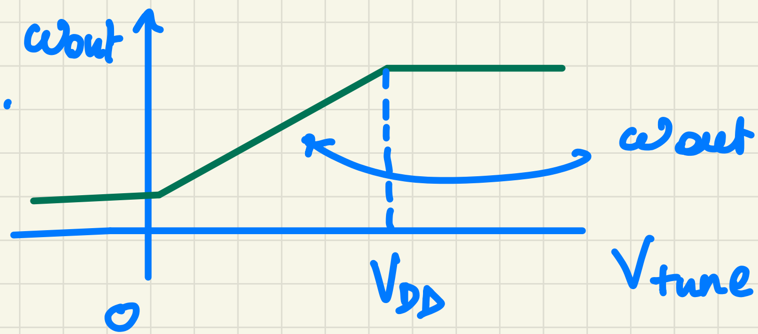
$$V_{PD} = \frac{\Delta\omega}{K_{VCO}}$$

to reach "LOCK"

limited dynamic range of PD $\Rightarrow$ limits lock range

In reality, the VCO also limits LOCK RANGE

e.g.



$\omega_{out} = \omega_{fr} + K_{VCO} V_{tune}$

Perturbation analysis : Linearization

$$\dot{\Phi}_\varepsilon = \Delta\omega - K \sin \Phi_\varepsilon$$

- $\left|\frac{\Delta\omega}{K}\right| < 1$ stable equil. exists

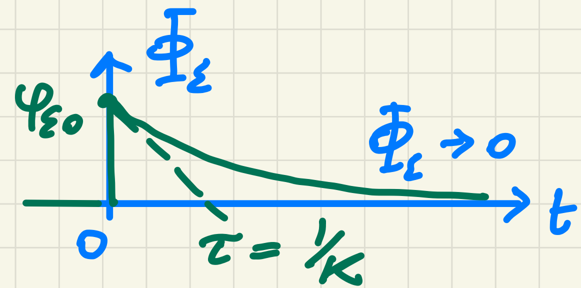
If $\Delta\omega = 0$:

- $\Delta\Phi_\varepsilon \ll 1$ rad small perturbation on Φ_ε

$$\dot{\Phi}_\varepsilon = - K \sin \Phi_\varepsilon$$

$$\xrightarrow[\text{linearization}]{} \quad \dot{\Phi}_\varepsilon = - K \Phi_\varepsilon \quad ; \quad \Phi_\varepsilon(t) = \varphi_{\varepsilon 0}\, e^{-K \cdot t}$$

$\tau = 1/K$ is the time constant of the system



Laplace transformation

- Input – to – Output transfer function of PLL

Φ_{out} vs. Φ_{ref}

$$\dot{\Phi}_\varepsilon = -K\,\Phi_\varepsilon$$

$$\Downarrow$$

$$s\,\cancel{\Phi_\varepsilon} = -K\,\cancel{\Phi_\varepsilon}$$

$\rightarrow$ pole at $s = -K$

$$\omega_{out} = \omega_{fr} + K_{vco}\, V_{tune}(t) =$$

$$= \underbrace{\omega_{fr} + K_{vco}\, V_{tune,0}} + K_{vco} \cdot v_{tune}(t)$$

$$= \omega_{out,0} + K_{vco} \cdot v_{tune}(t) =$$

$$= \omega_{out,0} + \underbrace{K_{vco} \cdot K_{PD} \cdot [\phi_{ref} - \phi_{out}]}$$

$$\underset{\text{ABSOLUTE PHASE}}{\Phi_{out}} = \int_{-\infty}^{t} \omega_{out}(t')\,dt' =$$

$$= \omega_{out,0} \cdot t + \underset{\text{EXCESS PHASE}}{\varphi_{out}(t)}$$

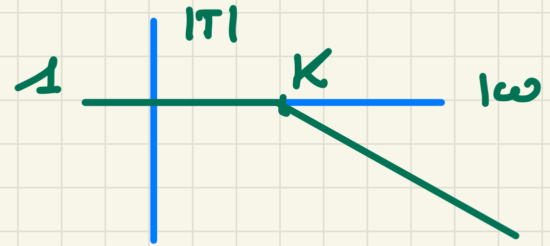
$$\Downarrow$$

$$\dot{\varphi}_{out}(t) = \underbrace{K_{VCO} \cdot K_{PD}}_{K} \left[\varphi_{ref}(t) - \varphi_{out}(t) \right]$$

$$\xrightarrow{\mathcal{L}} \quad s\,\Phi_{out}(s) = K \left[\Phi_{ref}(s) - \Phi_{out}(s) \right]$$

$$\boxed{\Phi_{out} / \Phi_{ref} = K / (s+K) = T(s)}$$

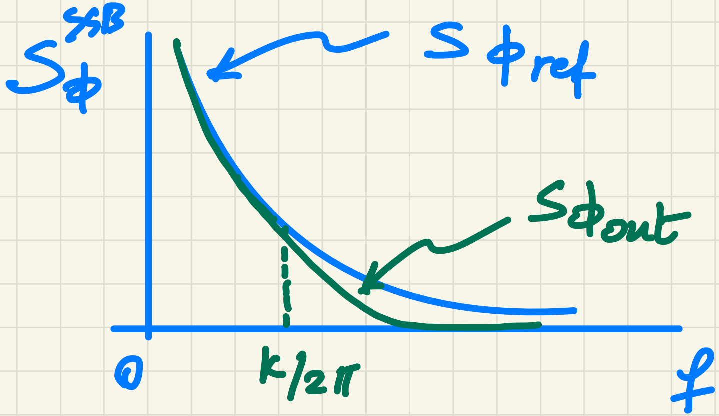
$$T(s) = \frac{K}{s + K}$$

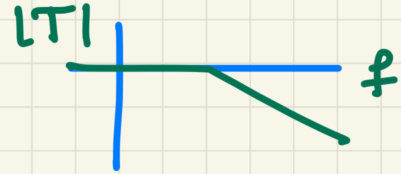


$$T(s) = \frac{\Phi_{out}}{\Phi_{ref}} = \frac{s \cdot \Phi_{out}}{s \cdot \Phi_{ref}} = \frac{\omega_{out}}{\omega_{ref}}$$

Interpretation: in this PLL, the VCO "follows" the phase and the frequency of the reference clock with $\omega_{BW} = K$

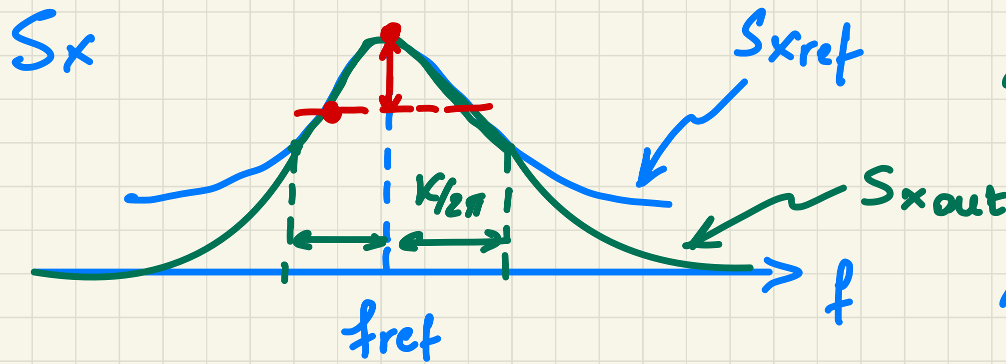
Only slow variations of ϕ_{ref} (or ω_{ref}) are followed by the VCO





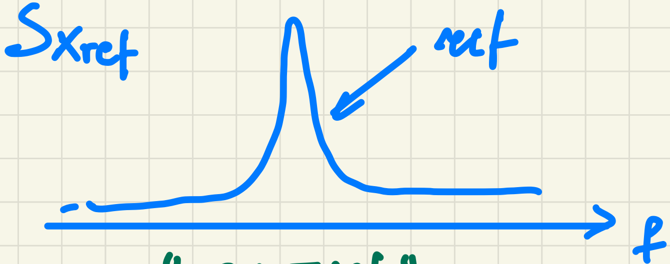
$$S_{\phi out} = |T(f)|^2 \cdot S_{\phi ref}$$

PLL: low-pass filtering of input phase noise

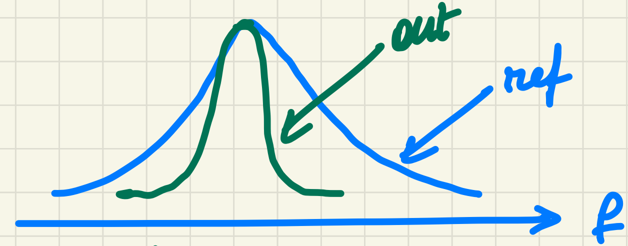


$$\mathcal{L}(f) \approx \frac{S_\phi^{SSB}}{2}$$

PLL: bandpass filtering of input signal



" CLEAN "



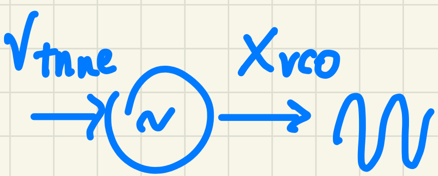
"DIRTY"

- <u>equivalent model</u> of linear PLL :

  - VCO :

$$\underbrace{\varphi_{out}(t)}_{\text{excessphase}} = \int_{-\infty}^{t} \overbrace{K_{vco} \cdot v_{tune}(t')}^{\text{variation of }\omega_{out}}\, dt' =$$

$$\overset{\mathcal{L}}{\Longrightarrow} \quad \Phi_{out}(s) = \frac{K_{vco}}{s} \cdot V_{tune}(s)$$

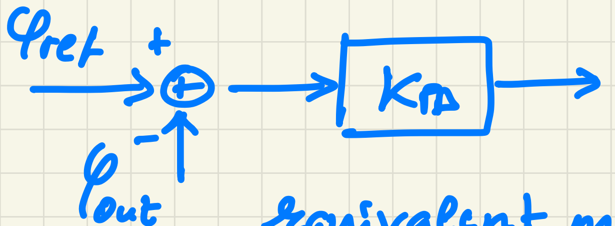


Real VCO

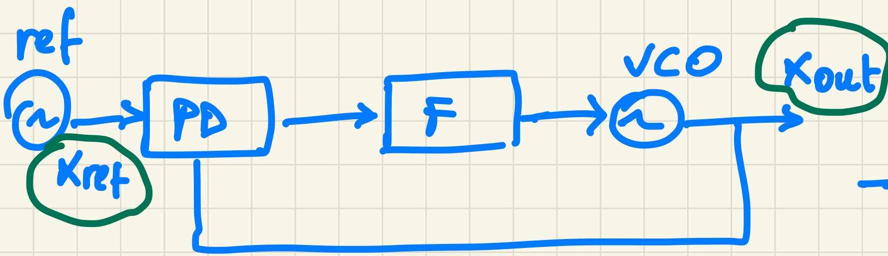
$\Longrightarrow$ $V_{tune} \longrightarrow \boxed{\frac{K_{vco}}{s}} \longrightarrow \Phi_{out}$ excess phase

Equivalent model of linear VCO

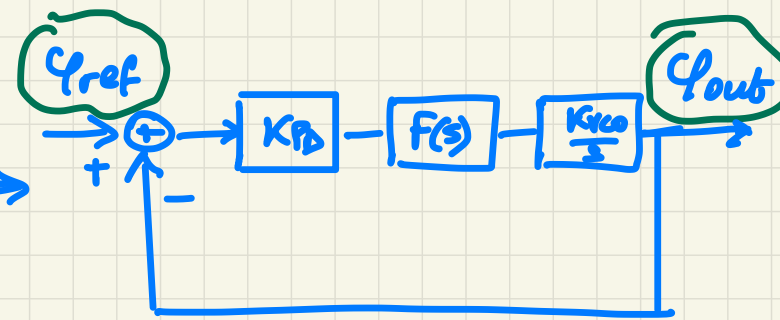
* PD : $V_{PD} = K_{PD} \cdot [\underbrace{\phi_{ref} - \phi_{out}}_{\phi_{\varepsilon}}]$ linear PD



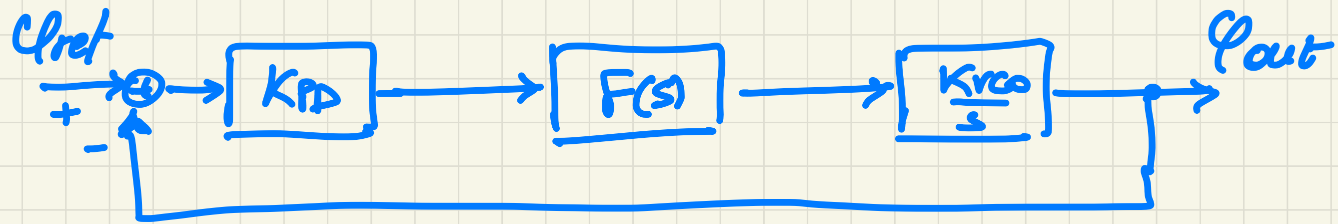
equivalent model of PD



PLL



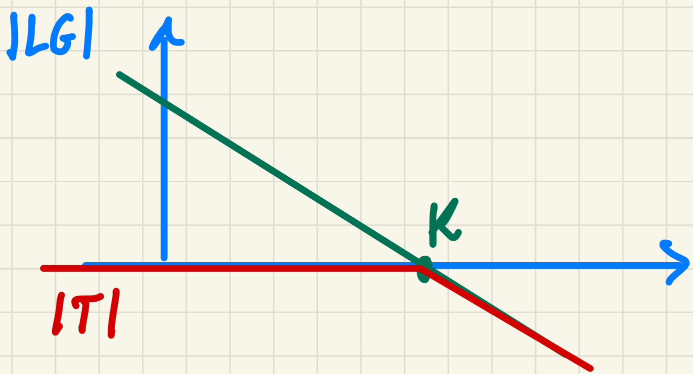
linear continuous-time model of a PLL



First-order PLL : $F(s) = 1$

"LOOP GAIN"

$$LG(s) = K_{PD} \cdot F(s) \cdot \frac{K_{VCO}}{s} = \frac{K}{s}$$



$$T_{IDEAL}(s) = 1$$

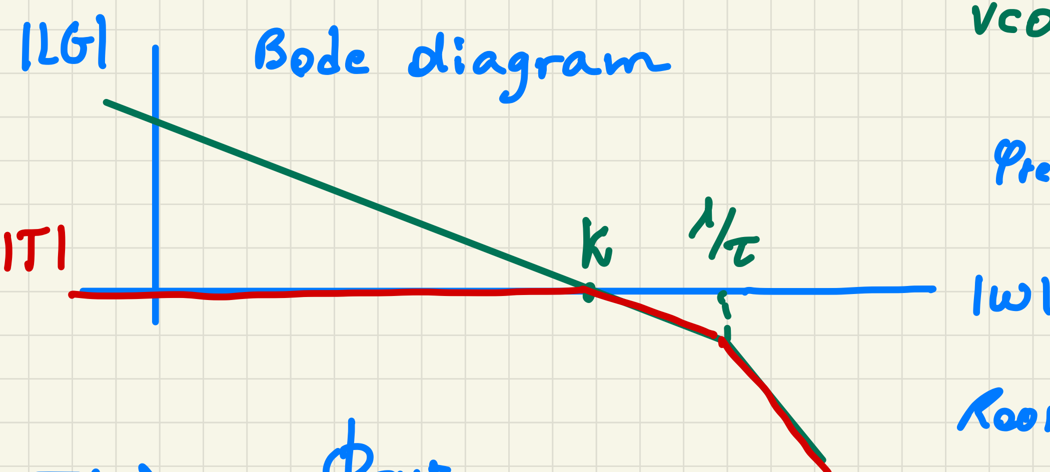
Hyp: • small phase error $\phi_\varepsilon \ll 1\,rad$

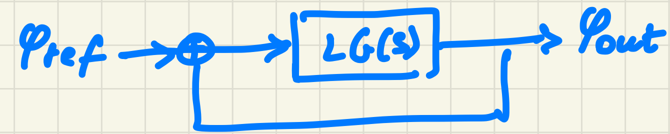
• averaged model of the PD $V_{PD} \simeq \langle V_{PD}(t) \rangle$

Second-order PLLs

$$F(s) = \frac{1}{1+s\tau} \Rightarrow LG(s) = \underbrace{\frac{K}{s}}_{\text{VCO}} \cdot \underbrace{\frac{1}{1+s\tau}}_{\text{FILTER}}$$

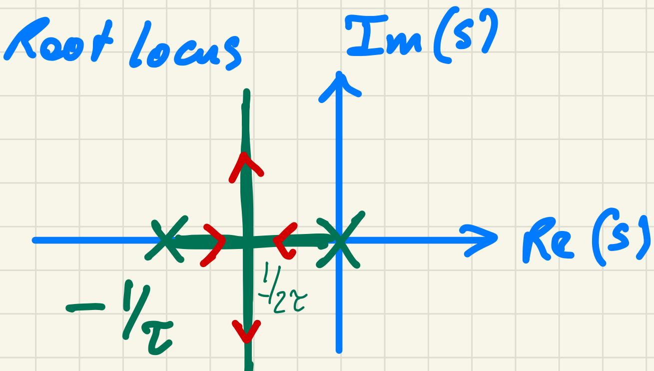
Bode diagram





$$T(s) = \frac{\Phi_{out}}{\Phi_{ref}} = \frac{LG(s)}{1+LG(s)}$$

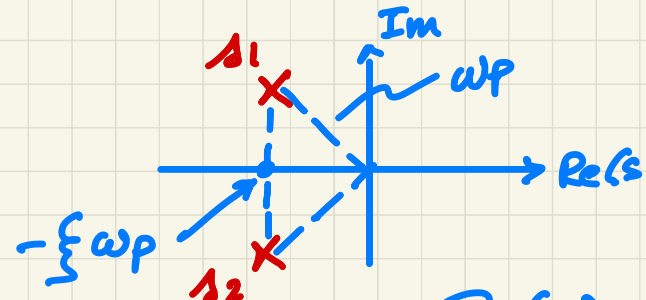
Root locus



$$T(s) = \frac{\frac{K}{s}\,\frac{1}{1+s\tau}}{1+\frac{K}{s}\,\frac{1}{1+s\tau}} = \frac{K}{s^2\tau + s + K} =$$

$$= \frac{1}{s^2\frac{\tau}{K} + s\frac{1}{K} + 1} = \frac{1}{s^2/\omega_p^2 + \frac{2\xi s}{\omega_p} + 1}$$

$$\Rightarrow \omega_p = \sqrt{\frac{K}{\tau}} \quad \text{AND} \quad \xi = \frac{1}{2\sqrt{K\tau}}$$



By choice of τ and K

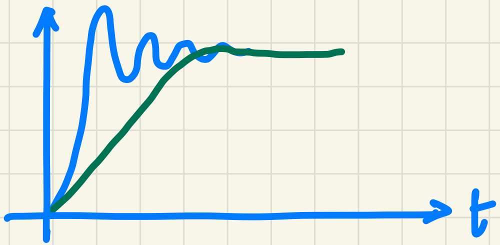
$\longrightarrow$ you can set ω_p and ξ

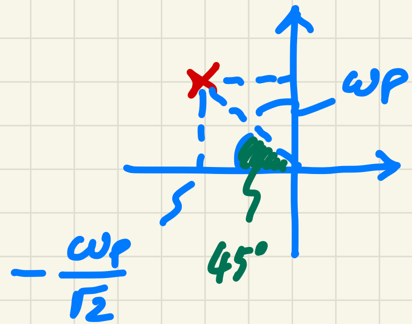
$$\xi = -\frac{Re(p)}{|p|}$$

Damping factor

$$s_{1,2} = -\xi\omega_p \pm j\sqrt{1-\xi^2}\cdot\omega_p$$

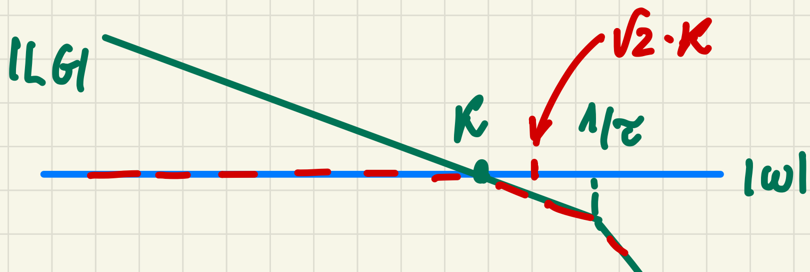
Closed loop poles $s_{1,2}$ at $45°$ in Gauss plane
(best trade off between overshoot and rise time)





$$\xi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.707..$$

$$\xi = \frac{1}{2\sqrt{K\tau}} = \frac{\sqrt{2}}{2} \Rightarrow K\tau = \frac{1}{2};$$



$$K = \frac{1}{2\tau} = \frac{1/\tau}{2}$$

Crossover of LG (K) one octave before the second pole (1/τ)

$$\underline{\underline{\omega_p}} = \sqrt{\frac{K}{\tau}} = \sqrt{\frac{1}{2\tau^2}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\tau} = \underline{\underline{\sqrt{2} \cdot K}}$$