RF Circuit Design

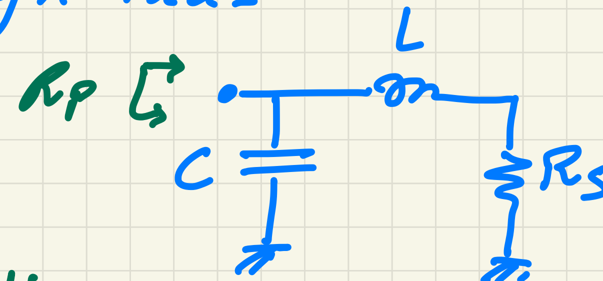
L10

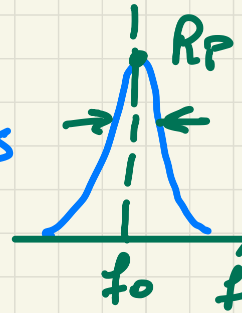
- L-match network design rules

Given ω_0, $\frac{R_P}{R_S}$

a. frequency

transformation ratio





1. $R_P = R_S(1+Q_L^2) \Rightarrow Q_L = \sqrt{\frac{R_P}{R_S} - 1} \Rightarrow$

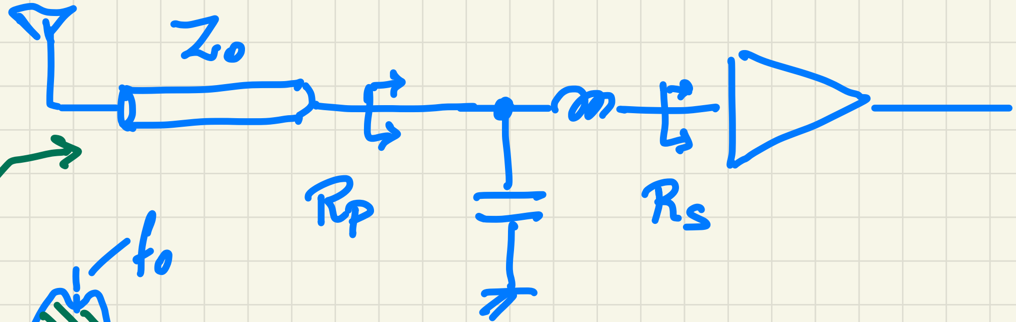
Large transformation $\Rightarrow$ Narrowband transform.

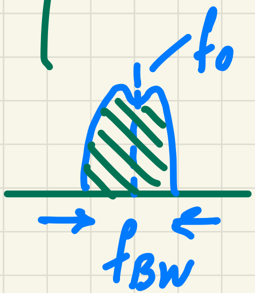
2. $Q_L = \frac{\omega_0 L}{R_S} \Rightarrow L = \frac{Q_L \cdot R_S}{\omega_0}$

R_P at $\omega_0 = \frac{1}{\sqrt{L_P C}}$

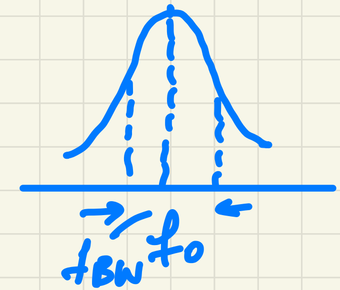
3. $\omega_0 = \frac{1}{\sqrt{L_P C}}$, $L_P = L \cdot \frac{1+Q_L^2}{Q_L^2} \Rightarrow C = \ldots$

e.g.
Rx

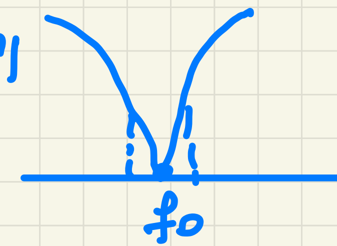




$R_P = |Z_{in}|$

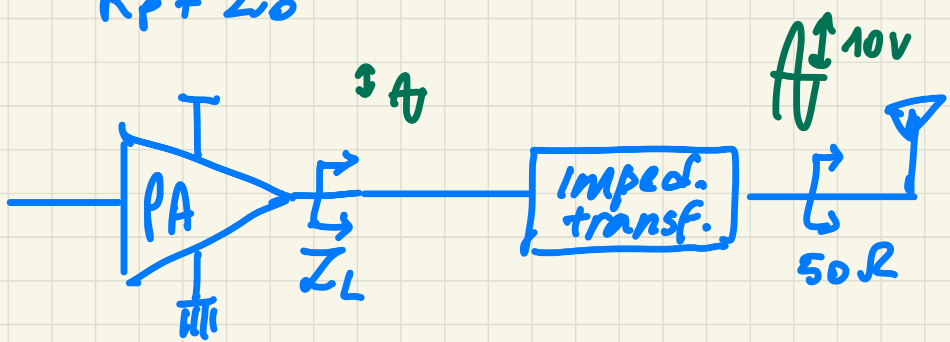


$R_P \neq Z_0 \Rightarrow$ Reflections



$$\Gamma = \frac{R_P - Z_0}{R_P + Z_0}$$

e.g.
TX

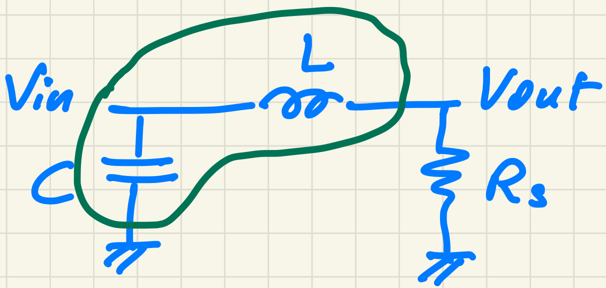


$$P = \frac{|V_0|^2}{2R} \qquad P = 1W \quad R = 50\Omega$$

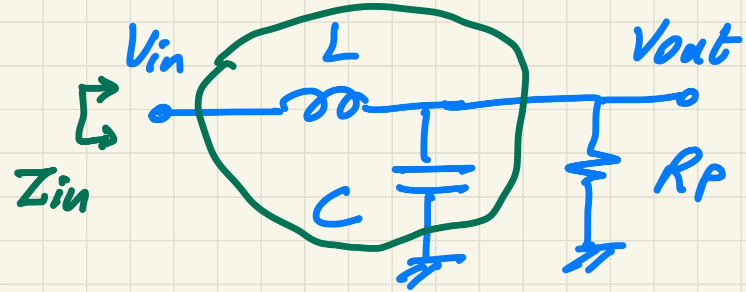
$$\Downarrow$$

$$V_0 = \sqrt{2PR} = 10V$$

Downward L-match network



UPWARD

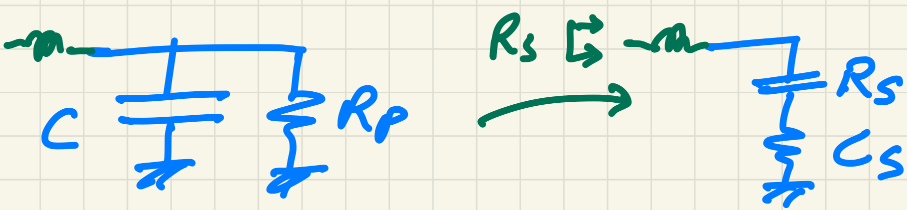


DOWNWARD

Lossless approximation : $|\bar{V}_{out}| \simeq Q \cdot |\bar{V}_{in}|$

$$Z_{in}(j\omega_0) \simeq \frac{R_p}{Q^2}$$

Accurate calculation : parallel-to-series transform

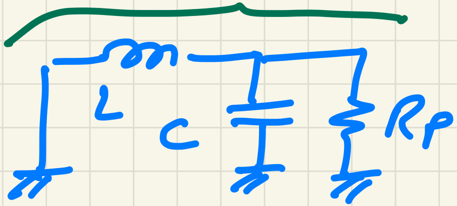


$R_s \searrow$

$C_s \nearrow$

$\frac{1}{j\omega C_s} \searrow$

$Q_c = \omega C R_p$



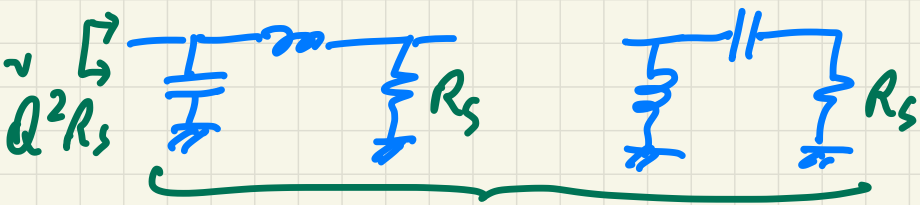
After calc. : at $\omega_0 = \frac{1}{\sqrt{L C_s}}$

$$\begin{cases} R_s = \dfrac{R_p}{1 + Q_c^2} \\ C_s = C \cdot \dfrac{1 + Q_c^2}{Q_c^2} \end{cases}$$

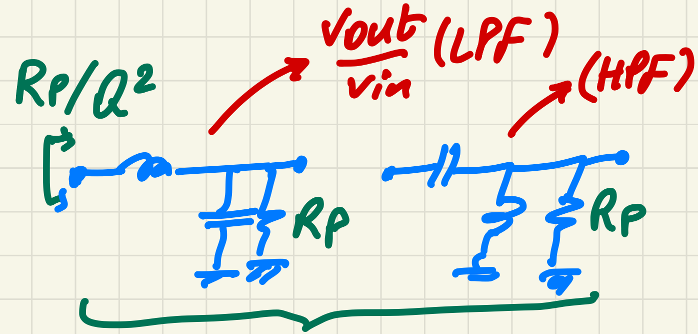
- Freqn. response
- DC blocking
- Absorption of stray cap

$Z_{in}(j\omega_0) = R_s$

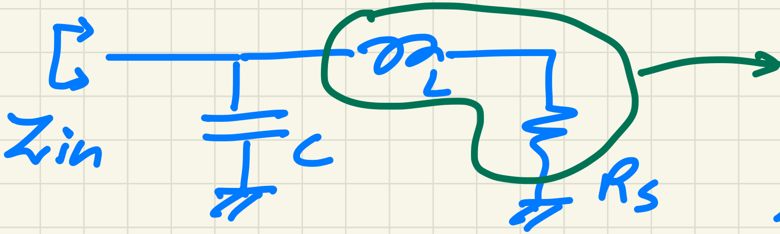
4 types of L-match



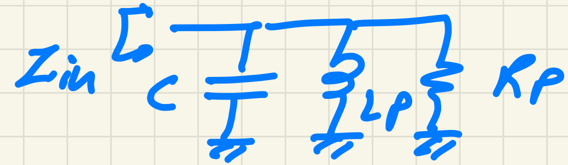
UPWARD



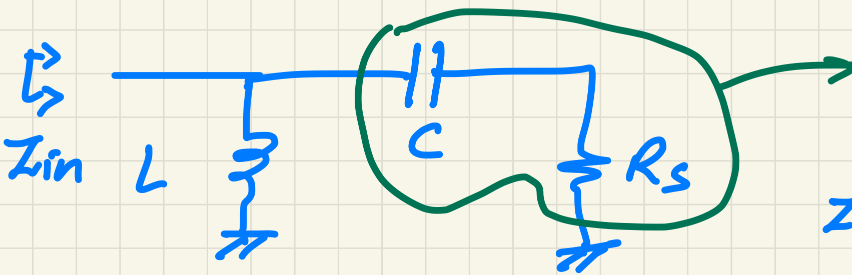
DOWNWARD



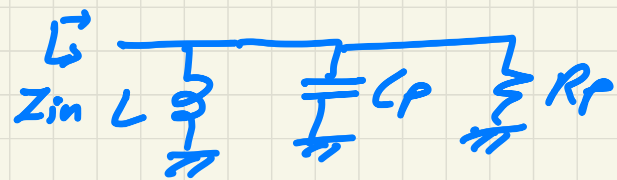
transformation :



$$Q_L = \frac{\omega_0 L}{R_s} \qquad \omega_0 = \frac{1}{\sqrt{C L_p}} \qquad \begin{cases} Z_{in}(j\omega_0) = R_p = R_s(1+Q_L^2) \\ L_p = L \cdot \dfrac{Q_L^2+1}{Q_L^2} \end{cases}$$

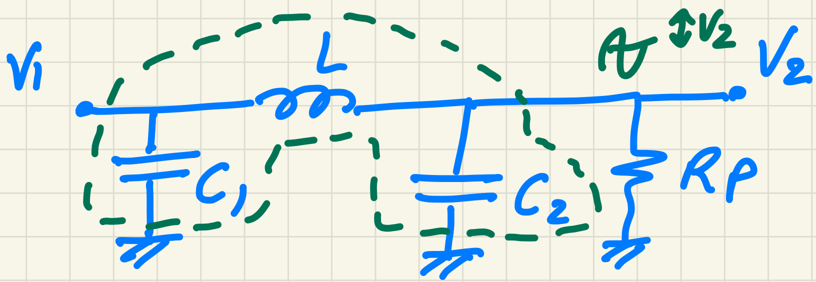


transformation :

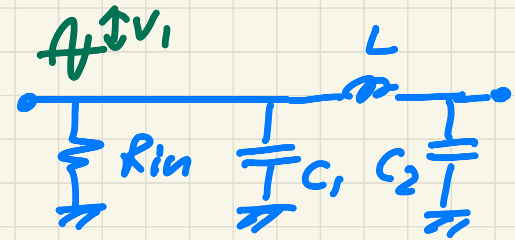


$$Q_C = \frac{1}{\omega R_s C} \qquad \omega_0 = \frac{1}{\sqrt{C_p L}} \qquad \begin{cases} R_p = R_s \cdot (1+Q_C^2) \\ C_p = C \cdot \dfrac{Q_C^2}{1+Q_C^2} \end{cases}$$

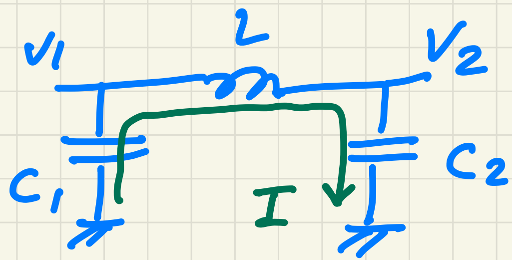
Π– match network or "Colpitts" network



equival.



- lossless approximation (physical interpretat.) $R_p \to \infty$

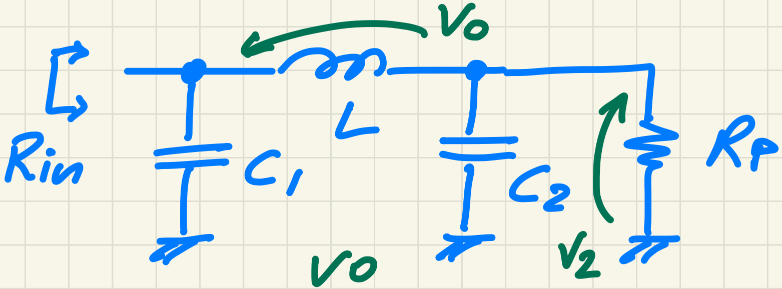


$$I = s C_2 \cdot V_2 \approx - s C_1 \cdot V_1$$

$$\Downarrow$$

$$\frac{V_1}{V_2} \approx -\frac{C_2}{C_1}$$

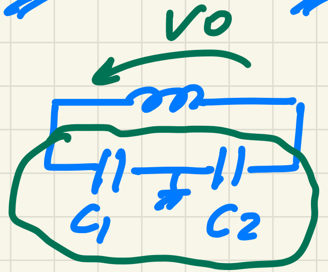
$$\underbrace{\frac{1}{2}\,\frac{V_1^2}{R_{in}}}_{\text{power dissipated in the equ. circ.}} = \underbrace{\frac{1}{2}\,\frac{V_2^2}{R_p}}_{\text{power dissip. in the orig. circ.}} \longrightarrow R_{in} = R_p \cdot \left(\frac{V_1}{V_2}\right)^2 \approx R_p \left(\frac{C_2}{C_1}\right)^2$$



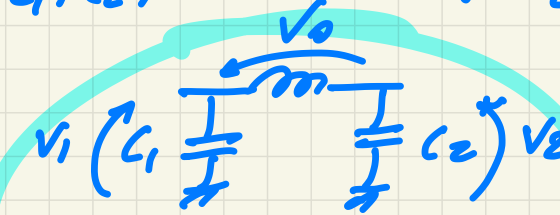
$$R_{in} = R_P \left(\frac{C_2}{C_1}\right)^2$$

$C_2 > C_1$ UPWARD

$C_2 < C_1$ DOWNWARD



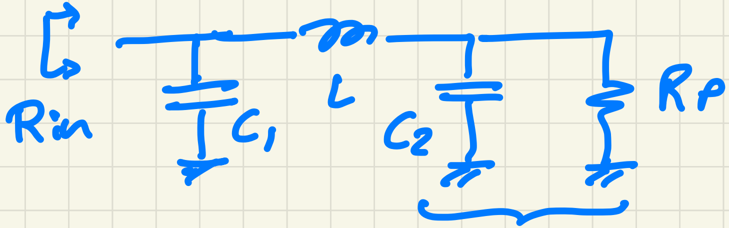
$$Q = \omega_0 \cdot \frac{E_{stored}}{P_{DISS}} \cong \omega_0 \cdot \frac{\dfrac{\cancel{C_1} C_2}{\cancel{C_1+C_2}}}{\left(\dfrac{C_1}{C_1+C_2}\right)^{\cancel{2}}} \cdot R_P = \omega_0 R_P C_2 \cdot \left(1+\frac{C_2}{C_1}\right)$$



$$E_{STORED} = \frac{1}{2} \cdot \frac{C_1 C_2}{C_1+C_2} \cdot |\bar{V}_0|^2$$

$$P_{DISS} = \frac{1}{2} \frac{|\bar{V}_2|^2}{R_P} \cong \frac{1}{2}\left(\frac{C_1}{C_1+C_2}\right)^2 \cdot |\bar{V}_0|^2 \Leftarrow$$

$$V_0 = V_1 - V_2 = -\frac{C_2}{C_1}V_2 - V_2$$
$$= -\left(1+\frac{C_2}{C_1}\right)\cdot V_2$$

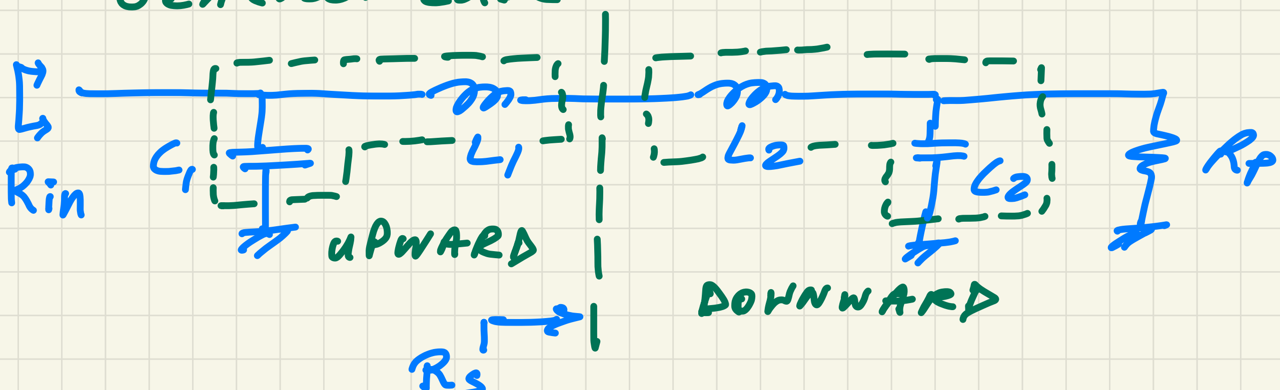


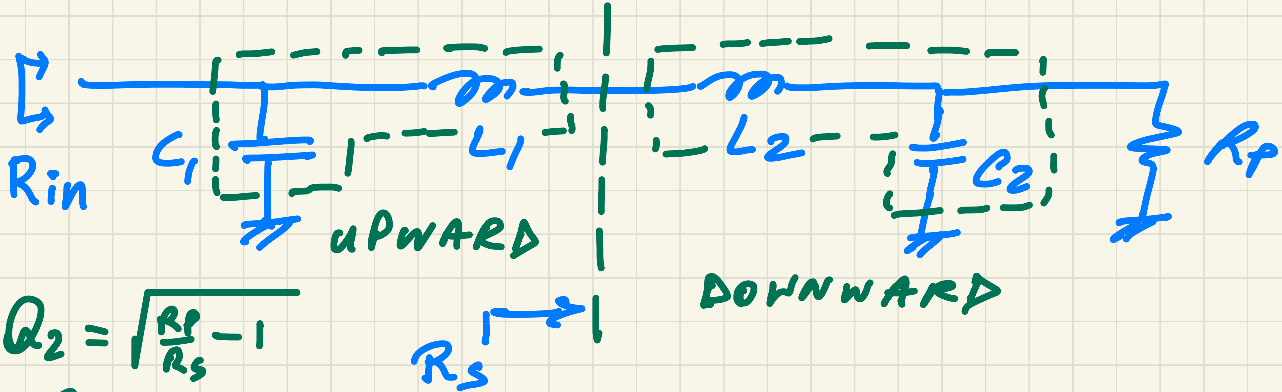
$$R_{in} \simeq R_P \cdot \left(\frac{C_2}{C_1}\right)^2$$

$$Q = \underbrace{\omega_0 C_2 R_P}_{\text{Q factor of equivalent L-match}} \cdot \underbrace{\left(1 + \frac{C_2}{C_1}\right)}_{\text{enhancement factor}}$$

Q - factor of π - match network > Q - factor of L - match

- General case





$$Q_2 = \sqrt{\frac{R_p}{R_s} - 1}$$

- $$R_s = \frac{R_p}{1 + Q_2^2} \quad \text{where} \quad Q_2 = \omega_0 R_p C_2 = \frac{\omega_0 L_2}{R_s}$$

- $$R_{in} = R_s \cdot (1 + Q_1^2) = R_p \cdot \frac{1 + Q_1^2}{1 + Q_2^2}$$

where $Q_1 = \dfrac{\omega_0 L_1}{R_s}$

$$Q_1 = \sqrt{\frac{R_{in}}{R_s} - 1}$$

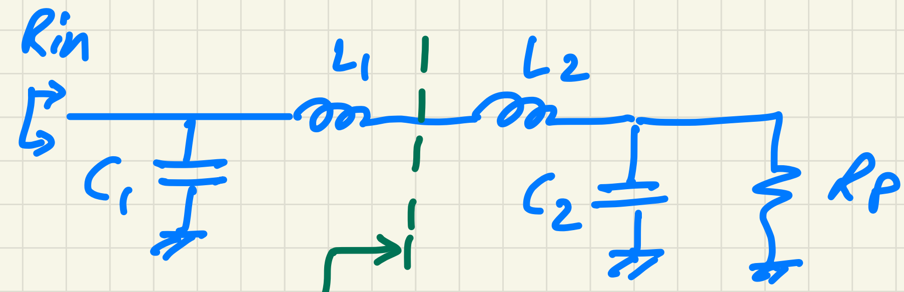
$\Rightarrow$

$$\frac{R_{in}}{R_p} = \frac{1 + Q_1^2}{1 + Q_2^2}$$

π - match network (Design Rules) :

Given parameters

ω_0 , $\frac{R_{in}}{R_P}$, Q



$$\omega_0 = \frac{1}{\sqrt{L_2 C_2 \cdot \frac{1+Q_2^2}{Q_2^2}}}$$

$$Q = \frac{\omega_0 (L_1 + L_2)}{R_s} = Q_1 + Q_2$$

1. $Q = Q_1 + Q_2 = \sqrt{\frac{R_{in}}{R_s} - 1} + \underbrace{\sqrt{\frac{R_P}{R_s} - 1}}_{Q_2} \Rightarrow R_s = \ldots$

Known   Known   Known

2. $L_1 + L_2 = \frac{Q \cdot R_s}{\omega_0}$

3. $Q_2 = \omega_0 R_P C_2 \Rightarrow C_2 = \frac{Q_2}{\omega_0 R_P}$

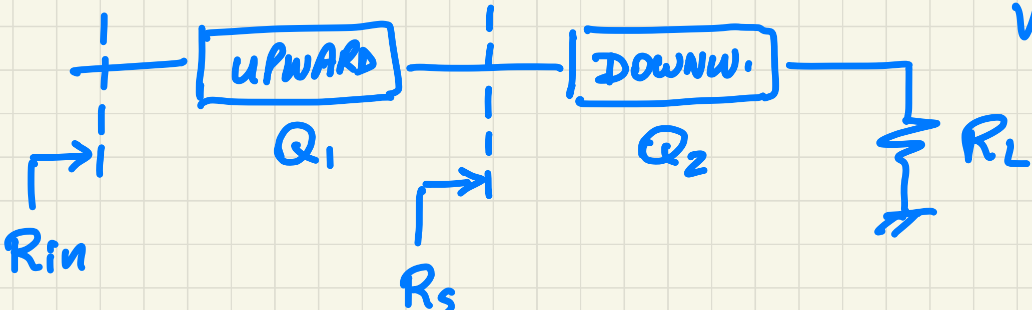
4. $Q_1 = \frac{\omega_0 L_1}{R_s} \Rightarrow L_1 = \frac{Q_1 R_s}{\omega_0}$

5. $$\omega_0 = \frac{1}{\sqrt{L_2 \underbrace{C_2 \frac{1+Q_2^2}{Q_2^2}}_{C_{2s}}}} = \frac{1}{\sqrt{\underbrace{L_1}_{L_{1P}} C_1 \cdot \underbrace{\frac{1+Q_1^2}{Q_1^2}}}}$$

$\swarrow$ $L_2 = \ldots$

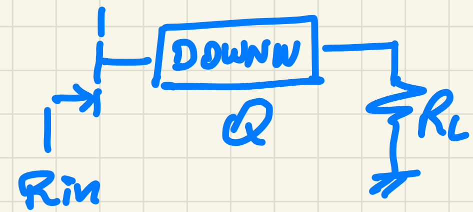
$\Rightarrow$ $C_1 = \ldots$

π-match



vs.

L-match



$R_{in} < R_L$

$R_{in} > R_s$

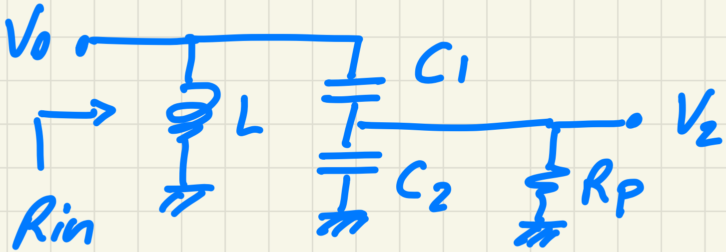
$R_s < R_L$

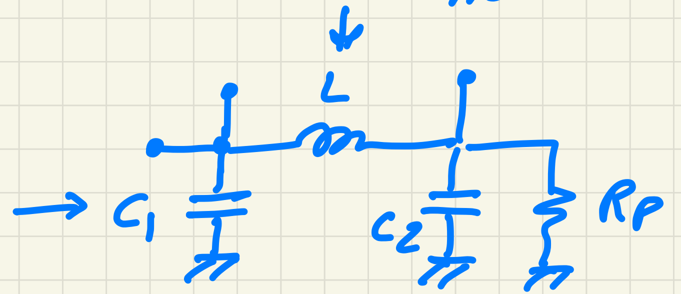
$$Q_2 = \sqrt{\frac{R_L}{R_s} - 1}$$

$$Q_1 = \sqrt{\frac{R_{in}}{R_s} - 1}$$

$$Q = \sqrt{\frac{R_L}{R_{in}} - 1}$$

Resonator with tapped capacitor "Colpitts" network



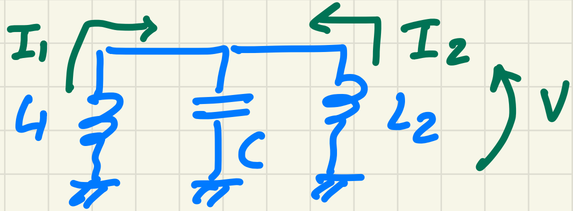


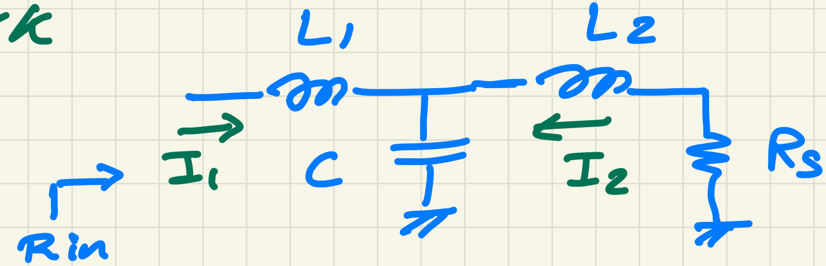
Low loss approximation : $R_P \to \infty$

$$\frac{V_2}{V_0} \approx \frac{C_1}{C_1 + C_2}$$

$$R_{in} = R_P \cdot \frac{V_0^2}{V_2^2} \cong R_P \cdot \left(1 + \frac{C_2}{C_1}\right)^2 \quad \text{UPWARD}$$

T- match network





lossless approx. $R_S \simeq 0$: $-V = sL_1 I_1 \simeq sL_2 I_2$

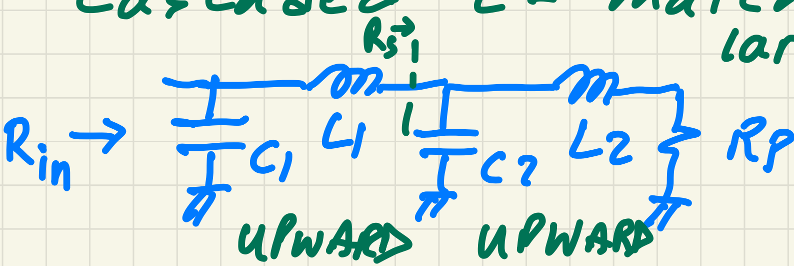
$$\Rightarrow \frac{I_1}{I_2} \simeq \frac{L_2}{L_1}$$

$$R_{in} = R_S \cdot \left(\frac{I_2}{I_1}\right)^2 \simeq R_S \cdot \left(\frac{L_1}{L_2}\right)^2$$

$$\begin{cases} Q_2 = \omega_0 L_2 / R_P \\ Q_1 = \dfrac{\omega_0 L_1}{R_S} \end{cases}$$

Cascaded L- match networks

large transformation → wider BW

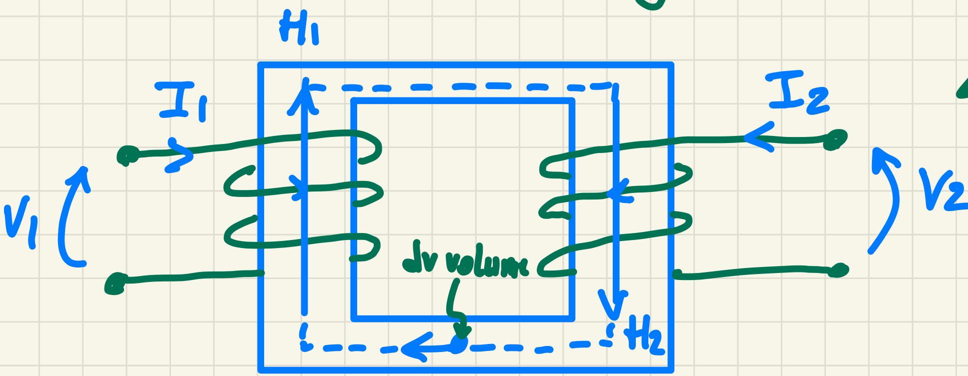


$$R_{in} = R_S(1+Q_1^2) =$$
$$= R_P(1+Q_2^2)(1+Q_1^2)$$

Impedance transformation (downward) → voltage / current amplification with passive netw.

1) Resonance

2) Inductor coupling (transformers)



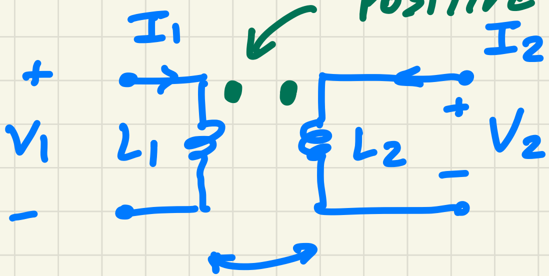
Case of POSITIVE MUTUAL ENERGY

$\bar{H}_1, \bar{H}_2$ same orientation

$$\overbrace{+\mu\, \bar{H}_1 \cdot \bar{H}_2 \cdot dv}^{\text{Mutual energy}}$$

$$\mathcal{E}_m = \frac{\mu}{2} \cdot |\underbrace{\bar{H}_1 + \bar{H}_2}_{\text{Total H fied}}|^2 \cdot dv = \underbrace{\frac{\mu}{2} |\bar{H}_1|^2 dv}_{\text{Energy coil 1}} + \underbrace{\frac{\mu}{2} |\bar{H}_2|^2 dv}_{\text{coil 2}}$$

POSITIVE MUTUAL ENERGY



$$\begin{cases} \Phi_1 = L_1 I_1 + M I_2 \\ \Phi_2 = M I_1 + L_2 I_2 \end{cases} \qquad \begin{cases} V_1 = \dot{\Phi}_1 \\ V_2 = \dot{\Phi}_2 \end{cases}$$

M is coupled inductance

$$\mathcal{E}_m = \int_0^t (\underbrace{V_1 I_1 + V_2 I_2}_{\text{power}}) dt' = \cdots =$$

POSITIVE MUT. ENERGY > 0

$$= \underbrace{\frac{1}{2} L_1 I_1^2(t)}_{\text{energy coil 1}} + \underbrace{\frac{1}{2} L_2 I_2^2(t)}_{\text{coil 2}} + \underbrace{M\, I_1(t) \cdot I_2(t)}_{\text{mutual energy}}$$

$M > 0$ if both I_1, I_2 enter or leave the dotted terminals