

1 Graph Model of Set Theory

- Directed graphs: $G = \langle V, A \rangle$
- A graph is **well-founded** if it has no looping paths and no infinite descending paths
- A graph is **extensional** if for any v_0, v_1 such that v_0 has the same incoming arrows as v_1 , $v_0 = v_1$
- Two graphs are isomorphic if there is a function (*isomorphism*) σ between them such that:
 - σ is a bijection (surjection + injection)
 - $v A_0 u \leftrightarrow \sigma(v) A_1 \sigma(u)$
- An automorphism is an isomorphism between some graph and itself (the identity is a trivial one)
- G is a subgraph of G' if $V \subseteq V'$ and $v_0 A v_1 \leftrightarrow v_0 A' v_1$ for all $v_0, v_1 \in V$
- G is *maximal* in some property Φ if G possesses Φ and there exists no graph G' such that:
 - G' possesses Φ ; and
 - G is a proper subgraph of G'
- Let G be a *maximal* well-founded graph with no non-trivial automorphisms
- Equivalently, G is a maximal well-founded graph which is extensional
- G is then an *intended model* of Set Theory

2 First Order Logic & Model Theory