Taming the Noise in Reinforcement Learning via Soft Updates:

G-Learning

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Outline

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Stakes and purpose

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- Approach proposed : add a penalization to the cost/reward using Kullback-Leibler divergence (information theory). Thus, we can softly shifts from a randomized policy to a deterministic one.

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- Q-learning performs poorly in noisy environments. Why ? In early stages, the min/max operator brings a bias, which slow down the estimation.
- Approach proposed : add a penalization to the cost/reward using Kullback-Leibler divergence (information theory). Thus, we can softly shifts from a randomized policy to a deterministic one.
- This is G-learning, which [spoiler] performs better in noisy environments, regarding the results

Outline

Notation and hypothesis

- **S** and **A** are respectively the state space and the action space (finite).
- Stochastic Decision Process : $a_t \sim \pi(a_t|s_t)$(action), $c_t \sim \theta(s_t, a_t)$ (cost) and $s_{t+1} \sim p(s_t, a_t)$ (state) [non deterministic case]
- $V^\pi(s) = \sum_t \gamma^t E[c_t|s_0 = s]$
- $Q^\pi(s, a) = \sum_t \gamma^t E[c_t|s_0 = s, a_0 = a] = E_\theta[c|s, a] + \gamma E_p[V^\pi(s')|s, a]$
- Goal of Q-learning : find $Q^*(s, a) = \min_\pi Q^\pi(s, a)$
- In this paper, Q is model-free (table).

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Q-Learning algorithm

Q-Learning

The Bellman equation and the temporal differences lead to :

$$Q(s_t, a_t) \leftarrow (1 - \alpha_t) Q(s_t, a_t) + \alpha_t \Big(c_t + \gamma \sum_{a'} \pi(a'|s_{t+1}) Q(s_{t+1}, a') \Big) \quad (1)$$

with some learning rate $0 \leq \alpha_t \leq 1$ and the the following policy :

$$\pi(a|s) = \delta_{a, a^*(s)}; \; a^* = \underset{a}{\text{argmin}}\, Q(s, a)$$

If the exploration policy returns to each state-action pair infinitely many times and if the learning rates satisfies :

$$\sum_t a_t = \infty; \; \sum_t \alpha_t^2 < \infty$$

then Q converge to Q^* with probability 1.

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Optimistic bias

Q-learning induce a negative bias because of the *min* operator.

- Assume $\hat{Q}(s,a)$, is an unbiased estimate of $Q^*(s,a)$.
- Jensen inequality for f concave : $E[f(X)] \leq f(E[X])$
- $E[\min_a \hat{Q}(s,a)] \leq \min_a E(\hat{Q}(s,a)) = \min_a Q^*(s,a))$
- Equality only if argmin $\hat{Q}(s,a)$ is argmin $Q^*(s,a)$ with probability 1 $(Var(\hat{Q}(s,a)) = 0)$

There is an optimistic bias (winner's curse in auction theory) : the cost appear lower than it is : this is a problem.
Thus, at the end, the goal is to have a low $Var(\hat{Q}(s,a))$, by increasing the sample size which make the algorithm converge (but maybe very slowly).

Note : The expectation is with respect to any randomness in state transition, cost, exploration, or because of the use of a function approximation (not considered in the article). That's why this is a problem especially in noisy environments !

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The interplay of value bias and policy suboptimality

We consider the effect of the optimistic bias on V^{π} with $\pi = \text{argmin}_a(\hat{Q}(s,a))$ regarding the gap $\delta = Q^*(s,a') - V^*(s)$ where a' is sub-optimal.

- If $Var(\hat{Q}(s,a)) << \delta$, then, a' will be sub-optimal with high probability, as desired (Q-learning converge normally)
- If $Var(\hat{Q}(s,a)) >> \delta$, then, confusing such a' has a limited effect, since a' is near-optimal.
- If $Var(\hat{Q}(s,a)) \sim \delta$, then this is a problem ! "a" probably suboptimal, and propagation of bias between states via updating !

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A dynamic optimism-uncertainty loop

Usually, we use a ϵ-greedy exploration policy to accelerate the bias reduction : high variance is self corrected : it's called a "dynamic form of optimism under uncertainty" (did not catch this formulation). Optimism is generated by noise and self-corrected through exploration.

The purpose of this paper : explicitly represent the uncertainty and avoid the hard-min operator. This can be done by penalizing deterministic policies at the early learning stage.

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The Free-energy function F : Notations

We define :

- Stochastic policy prior : $\rho(a|s)$
- Informative cost of a learned policy : $\log \frac{\pi(a|s)}{\rho(a|s)}$ (penalizes deviations from prior and regularizes to avoid deterministic policy)
- Total discounted expected information cost : $I^\pi(s) = \sum_t \gamma^t E[g^\pi(s_t, a_t)|s_0 = s]$

Then, we define a new total cost called *free energy function*:

$$F^\pi(s) = V^\pi(s) + \frac{1}{\beta} I^\pi(s)$$

For the moment, β is fixed.

The Free-energy function G : Notations

Similarly to the function Q, we define the *state- action free-energy function* :

$$G^{\pi}(s,a) = E[c|s,a] + \gamma E[F^{\pi}(s',a)|s,a] \tag{2}$$
$$= \sum_t \gamma^t E[c_t + \frac{\gamma}{\beta} g^{\pi}(s_{t+1},a_{t+1}))|a_0 = a, s_0 = s] \tag{3}$$

The informative cost is not taken into account for the first step, since we have already chosen the first action a.

Relationship between G and F

We can compute F^π with G^π by computing the expected value under all action :

$$F^\pi(s) = \sum_a \pi(a|s)\left[\frac{1}{\beta}\log\frac{\pi(a|s)}{\rho(a|s)} + G^\pi(a,s)\right] \tag{4}$$

We fix s. Using this, F^π and G^π has a null gradient at :

$$\pi(a|s) = \frac{\rho(a|s)e^{-\beta G^{\pi(s,a)}}}{\sum_{a'}\rho(a'|s)e^{-\beta G^{\pi(s,a')}}} \tag{5}$$

which is the optimal policy.

Opimality G^*

Evaluated at the optimal policy, F^π becomes :

$$F^*(s) = -\frac{1}{\beta} \log \sum_a \rho(a|s) e^{-\beta G^*(s,a)} \tag{6}$$

We plug this expression into the definition of G^π and obtain the fixed-point equation for G^* :

$$G^*(s,a) = E[c|s,a] - \frac{\gamma}{\beta} E[\log \sum_a \rho(a|s) e^{-\beta G^*(s,a)}] = \mathbf{B}^*[G^*]$$

G-Learning algorithm

G-Learning : an off-policy TD algorithm

Analogous to the Q-learning algorithm, the G-learning algorithm is :

$$G(s_t, a_t) \leftarrow (1-\alpha_t)G(s_t, a_t)+\alpha_t\left(c_t-\frac{\gamma}{\beta}\log\left(\sum_{a'}\pi(a'|s_{t+1})Q(s_{t+1}, a')\right)\right) \tag{7}$$

with some learning rate $0 \leq \alpha_t \leq 1$.

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The role of the prior

The prior policy can encode any prior knowledge that we have about the domain. In the examples, the authors only use uniform prior.

This is a kind of regularization (cf. a Ridge-regularization is a $\mathbf{N(0,1)}$ prior) and we can enhance convergence by avoiding some exploration softly.

This soft-exploration formulation avoids using the min operator.

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Convergence

They use the following lemma :

Lemma

The operator $B^*[G]_{s,a}$ defined before verifies :

$$|\mathbf{B}^*[G_1]_{s,a} - \mathbf{B}^*[G_1]_{s,a}|_\infty < \gamma|G_1 - G_2|_\infty \tag{8}$$

They use also the fact that :

$$G_{t+1}(s_t, a_t) = (1 - \alpha_t)G_t(s_t, a_t) + \alpha_t(\mathbf{B}^*[G_t]_{(s,a)} + z_t(c_t, s_{t+1}))$$

where

$$z_t(c_t, s_{t+1}) = -\mathbf{B}^*[G_t]_{s_t,a_t} + c_t - \frac{\gamma}{\beta}\log\sum_a^{\prime} \rho(a'|s_{t+1})e^{-\beta G_t(s_{t+1},a')}$$

They note that $E[z_t] = 0$ which concludes the proof, given $|z_t < \infty|$.

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Scheduling β

For a fixed β the algorithm converges with probability 1. A few comments :

- When $\beta = \infty$, the equations for G^* and F^* degenerate into the equations for Q^* and V^*, and G-learning becomes Q-learning. (better in early stage cf. noisy Q function)
- When $\beta = 0$, the update policy π is equal to the prior ρ. This case, denoted Q^ρ-learning, converges to Q^ρ. (better in late stage because better policy than the prior)

Solution : smooth-transition from Q^ρ-learning to Q-learning

Oracle Scheduling

At each step, there is always a β for which the update rule such an unbiased estimate remains unbiased. They do not prove it, but give an intuition. Let G be an unbiased estimate of G^*

- G still unbisaed with the following update

$$c_t + \gamma G(s_{t+1}, a^*)$$

  where is the real true right action : $a^* = \text{argmin}_{a'} G^*(s_{_{t+1},a'})$
- If we update with the G-learning algorithm and $\beta = 0$, there is a positive bias :

$$G_{t+1} \leftarrow c_t + \gamma \sum_{a'} \rho(a'|s_{t+1}) G(s_{t+1}, a')$$

- If $\beta = \infty$, there is a negative bias :

$$c_t + \gamma min_{a'} G(s_{t+1}, a')$$

Practical scheduling

β is updated as following :

$$\beta_t = kt$$

Simple but efficient according to authors (as efficient than an updating using the Bellman-error)

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Related work

Other solutions to noisy environment :

- Double Q-learning : use of two estimators to update without bias.
- Advantage learning : learning $A(s, a) = Q(s, a) - V(s)$ seems to be faster than Q-learning in noisy environments
- Q-Learning with KL-divergence : very similar to G-learning.

Zoom on other KL-divergence techniques

There has been similar approaches using KL-divergence (penalty on information).

- Instead of using a prior ρ, they use the empirical distribution generated by the previous policy : *Jan Peters, Katharina Mulling, and Yasemin Altun. Relative entropy policy search. In AAAI, 2010.*
- Approach of [33] and [34] (references in the paper) : $\psi - learning$.

$$\psi(s_t, a_t) \leftarrow \psi(s_t, a_t) + \alpha_t(c_t + \gamma\bar{\psi}(s_{t+1}) - \psi(\bar{s_t}))i$$

with $\bar{\psi} = -\log \sum_a \rho(a|s)e^{-\psi(s,a)}$

6) Examples

Hyper-parameters :

- Learning rate :

$$\alpha_t = n_t(s_t, a_t)^{-\omega}$$

  where $n_t(s_t, a_t)$ is the number of times the pair (s_t, a_t) was visited. They choose $\omega = 0.8$.

- Discount factor :

$$\gamma = 0.95$$

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Description of the environment

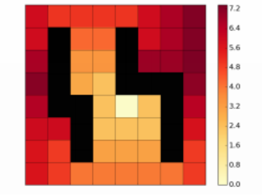

Figure: Gridworld domain. The agent can choose an adjacent square as the target to move to, and then may end up stochastically in a square adjacent to that target. The color scale indicates the optimal values V^* with a fixed cost of 1 per step.

For each case, the evolution over 250,000 algorithm iterations of the following three measures, averaged over N = 100 runs is computed.

Measures for the 1st example

Three measures :

- Empirical bias :

$$\frac{1}{nN}\sum_i\sum_s(V_{i,t}(s) - V^*(s)) \tag{9}$$

- Absolute error :

$$\frac{1}{nN}\sum_i\sum_s|V_{i,t}(s) - V^*(s)| \tag{10}$$

- Increase in cost-to-go, relative to the optimal policy :

$$\frac{1}{nN}\sum_i\sum_s(V^{\pi_{i,t}}(s) - V^*(s)) \tag{11}$$

Results

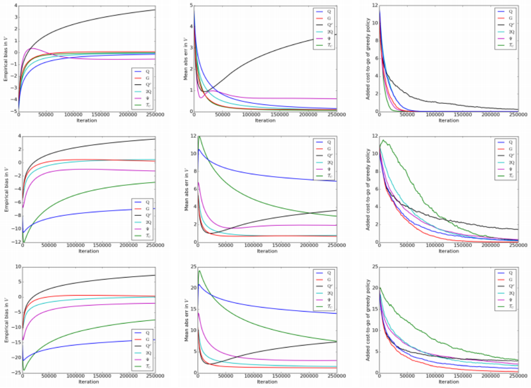

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Figure: Row 1 : cost is 1. Row 2 : cost is c $\sim N(1,2)$. Row 3: In each run, the domain is generated by drawing each E[c—s, a] uniformly over [1, 3]. The cost in each step is distributed as $N(E[c|s,a], 4^2)$.

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Game

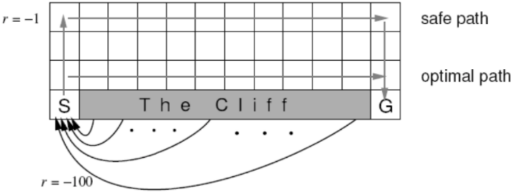

Figure: Cliff Waling environment

Cliff walking is a standard example in reinforcement learning , that demonstrates an advantage of on-policy algorithms such as SARSA and Expected-SARSA over off-policy learning approaches such as Q-learning.

Results

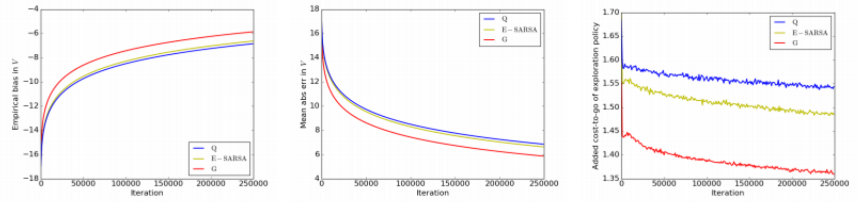

Figure: Cliff walking results

Conlusion

- Avoid the slow convergence in noisy environments caused by the bias generated (min operator)
- Explicit exploration
- Could be applied to other model-free setting, such as TD(λ)
- G-fit-learning ?

THANK YOU !