

A New Criterion for the Distribution of Normal Room Modes*

OSCAR JUAN BONELLO

Solidyne S.R.L., Buenos Aires, Argentina

A new criterion is proposed for the best distribution of normal room modes, with the objective of improving the acoustics of recording and broadcasting studios. A new system is analyzed for controlling isolated room modes to obtain rooms free of sound coloring. Applications are described, and the criterion is compared with others. A simple computer program performs the calculations.

0 INTRODUCTION

The acoustical behavior of rectangular rooms has been the subject of several studies. These provide sufficient information to enable the acoustical design of small- and medium-sized rooms. Nevertheless, several parameters are not perfectly defined in terms of evaluation criteria. One criterion, of utmost theoretical and practical importance, concerns the distribution of normal resonance modes in a given room. This matter is basic, particularly in the design of small broadcasting and television studios. The majority of sound studios are smaller than 120 m³, and the most common (and best studied) shape is the rectangular, with obvious building advantages. Other shapes have been tested, but their construction costs are higher and they have no advantage acoustically [1].

A rectangular room, subjected to an acoustical stimulus, behaves like a large number of mutually coupled resonators. Tones whose frequencies do not correspond exactly to any of the natural modes (eigentones) of the room drive several resonators simultaneously. The initial modes are widely spaced in frequency, and their number increases with the frequency cubed. For a correct reproduction of sound, we should have a large number of modes for each relative frequency interval (an octave, say). This requirement is fulfilled at medi-

um and high frequencies or in large concert halls and theaters. But at low frequencies and in small rooms, the eigentone spacing is very large, usually greater than half an octave. This spacing leads to peaks and valleys in the response curve of the reproduced sound, since the sound is the sum of relatively distant modes which constructively or destructively interfere. This effect is characteristic of very small studios, particularly when they have long reverberation times.

To eliminate or at least minimize the coloration, the eigentones should be spaced in order to avoid their concentration in some parts of the spectrum and their absence in others. The study of resonance modes and criteria for their spacing has been a classic subject in architectural acoustics [2]–[6]. Bolt [2]–[4], using the theory of numbers, recommended certain ratios between the dimensions, based on the concept of a frequency-spacing index. More recently a computer-aided analysis of the distribution of resonance modes led to differences with Bolt's recommendations while accepting part of his theory. These differences, marked by others [7], were recognized in practice by acoustical designers who used their own ratios derived from experience with good results [1], [7]–[9]. And yet, after 40 years of studies, and backed by theory, there were no valid empirical recommendations which could be used as guidelines for studio design. Our method is intended to fill this void by providing an easily applied criterion. The criterion is not statistical. It takes advantage of low-cost numerical computation provided by modern

* Presented at the 64th Convention of the Audio Engineering Society, New York, 1979 November 2–5.

calculators. It is based on calculus rather than prediction, and it uses analytical criteria that differ from their ancestors.

1 NORMAL RESONANCE MODES

Imagine a rectangular room extending in three dimensions, as illustrated in Fig. 1. In Cartesian coordinates the wave equation is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} + K^2 p = 0. \quad (1)$$

It can be shown [10], [11] that the pressure at any point in space, for the case of rigid walls, is

$$P_{n_x, n_y, n_z} = C \cos\left(\frac{n_x \pi x}{l_x}\right) \cos\left(\frac{n_y \pi y}{l_y}\right) \cos\left(\frac{n_z \pi z}{l_z}\right) e^{j2\pi f_n t} \quad (2)$$

where

f_n = resonance frequency of the $N(n_x, n_y, n_z)$ mode

C = arbitrary constant

l_x, l_y, l_z = three dimensions of room

n_x, n_y, n_z = integer numbers 0, 1, 2, 3, . . .

From Eq. (2) we can see that n_x, n_y, n_z indicate the number of zero-pressure planes which are present along the three axes x, y, z . The values of the normal frequencies of the room are given by the expression

$$f_n = \frac{c}{2} \sqrt{\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2} \quad (3)$$

where c is the velocity of sound (generally specified at 21°C as 344 m/s). Giving values to n_x, n_y, n_z starting from 0:0:0, the successive modes will be obtained. If l_x is the largest dimension, the mode of lowest frequency will be 1:0:0.

The total number of modes between 0 and frequency f will be, from [10],

$$N_f = \frac{4\pi}{3} V \left(\frac{f}{c}\right)^3 + \frac{\pi}{4} S \left(\frac{f}{c}\right)^2 + \frac{L}{8} \frac{f}{c} \quad (4)$$

where

V = volume of room, $= l_x l_y l_z$

S = internal surface, $= 2(l_x l_y + l_x l_z + l_y l_z)$

L = $4(l_x + l_y + l_z)$

The average density of eigenfrequencies, that is, the number of modes per hertz of bandwidth, is

$$\frac{\partial N_f}{\partial f} = \frac{4\pi V f^2}{c^3} \quad (5)$$

which shows that the density of modes increases with the frequency squared. Eq. (5) is valid only for frequencies high enough that statistical analysis can be applied.

For low frequencies the density of modes can be calculated by means of the differences between values given by Eq. (4), or better, through a direct process that will be explained further on. Eq. (5) suggests the cause of coloring problems in small rooms at low frequencies. The density of modes increases with the room volume. Accordingly, in a hall of 3000 m³ we have 50 times as many modes as those present in a broadcasting studio of 60 m³ for the same bandwidth and upper frequency. The influence of frequency is even more important than that of volume, since it appears as the square. It is for this reason that above 300 Hz sound coloring phenomena disappear.

If a room is driven in a normal mode and the excitation source is then disconnected, the pressure decreases exponentially according to the following equation [5], [11], [12]:

$$P_n = \frac{K}{k_n} e^{-k_n t} \cos \omega_n t \quad (6)$$

where

K = constant representing power, source location, and room volume

k_n = damping constant, representing room absorption

In addition, the steady-state sound pressure is given by

$$P_{n\max} = \frac{2K\omega}{\sqrt{4\omega_n^2 k_n^2 - (\omega^2 + \omega_n^2)^2}} \quad (7)$$

where ω_n is the normal angular frequency of that mode and ω is the driving angular frequency. The pressure versus frequency curve can be seen in Fig. 2. If in Eq. (7) we find the points at which the pressure drops 3 dB (half-power), we have

$$\Delta f = f_2 - f_1 = \frac{k_n}{\pi} \quad (8)$$

It is interesting to relate the coefficient k_n to the reverberation time RT. From Eq. (6) the time required for the pressure to drop 60 dB is $T_{60} = 6.91/k_n$ which, when substituted in to Eq. (8), yields

$$\Delta f = \frac{6.91}{\pi T_{60}} \quad (9)$$

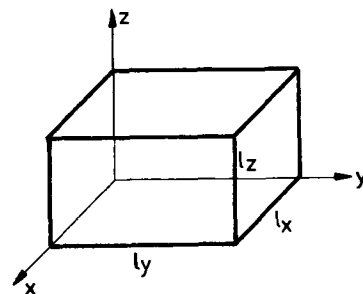


Fig. 1. Rectangular room extended in three dimensions.

showing that the bandwidth of the resonance modes is constant and independent of frequency *if* the reverberation time is also constant. The RT measured in a room is the average of the individual RT for each of the modes, but for practical purposes the RT used in Eq. (9) is the same when measured with pink noise. For rooms normally used as studios, the modal RTs are approximately constant with frequency. This constancy implies that the bandwidth will also be constant. For small rooms, the bandwidth given by Eq. (9) is between 3 and 10 Hz. In terms of relative bandwidth, measured in fractions of an octave and commonly used in electroacoustics, we have

$$\frac{\Delta f}{f} = 2^{1/2n} - 2^{-1/2n} \quad (10)$$

where $1/n$ is the bandwidth in octaves.

Taking into account Eqs. (9) and (10), we see that as RT increases, the bandwidth decreases. Rooms built with low RT in the bass, in accord with modern tendencies [13], will have wider bandwidths, advantageous for a larger number of responsive modes. On the other hand, for T_{60} constant, the selectivity of the modes increases with frequency; the bandwidth in octaves decreases. This is not important at high frequencies since the number of modes increases rapidly, as we have seen; but conversely the bass end is enhanced, since the octave bandwidth becomes large. For example, a typical value of $\Delta f = 5$ Hz implies a bandwidth of one-sixth octave at a frequency of 40 Hz, with the result that two or three modes within one-third octave will give a response free of coloring.

2 CRITERION SELECTION

An objective of the proposed criterion is to inform the designer whether or not the three dimensions for a given room are correct. If they are not, one or more of them should be changed, and the criterion applied once again. It should also inform the designer about the frequency band in which there will be coloring of sound in case the dimensions cannot be corrected.

The first step consists of calculating, by means of a

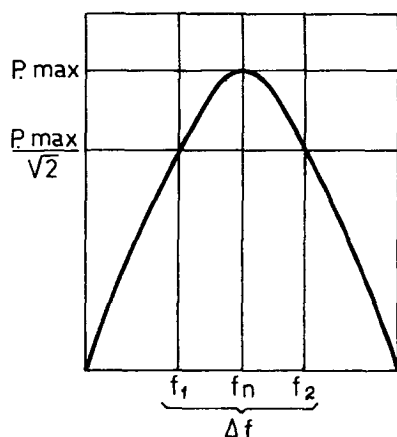


Fig. 2. Pressure versus frequency curve.

minicomputer or programmable calculator program, each of the lower resonance modes of the room. Eq. (3) is used for this calculation. It is not necessary to exceed a certain frequency or a certain order, due to considerations already noted and based on Eqs. (4) and (5). In the programs, which will be discussed later, the calculation is limited to the first 48 modes. Once the eigenfrequencies of the room are known, we analyze how many modes fall within each interval into which we divide the frequency spectrum. If we use the simplifying hypothesis that the ear is unable to discriminate modes within the interval, but only the sum of their contributions to the sound energy received within that band, it is necessary to know only the number of modes to obtain the total sound pressure. This implies that each mode, with its response to sound pressure given by Eq. (7) (Fig. 2), contributes in the same proportion to the band's energy, since it has the same maximum pressure and the same bandwidth given by Eq. (9). It is this concept of energy instead of frequency spacing that makes the criterion plausible.

What bandwidth shall we choose? Many arguments can be made in favor of narrow or wide bands. The value finally adopted is one-third octave. We use a relative bandwidth, not an absolute one, taking into account the logarithmic characteristic of auditory perception and the ear's response to musical intervals. We are also influenced by electroacoustical experience which indicates the usefulness of one-third octave as a minimum unit of bandwidth.

According to these concepts, the number of eigen-tones falling within each one-third octave between 10 Hz and 200 Hz is calculated; this number gives the modal density function per one-third octave, $D = F(f)$. The program plots modal density as ordinates and frequencies as logarithmic abscissas (the center frequencies of the one-third-octave bands). To analyze this curve, the following criterion is applied.

3 CRITERION

For optimum room dimensions the following conditions should be met:

1) The curve $D = F(f)$ should increase monotonically. Each one-third octave should have more modes than the preceding one (or, at least, an equal number if $D = 1$).

2) There should be no double modes. Or, at most, double modes will be tolerated only in one-third-octave bands with densities equal to or greater than 5.

Subsequent experience led us to accept, although reluctantly, the condition that two successive bands can have the same number of modes with D greater than 1. This condition simplifies the requirements, even though conditions 1) and 2) of the criterion are to be preferred in the design stage.

In Figs. 3 and 9 two computer plots of curve $D = F(f)$ are shown. Fig. 3 shows a room that complies satisfactorily with the criterion, while the room of Fig. 9 does not, and its dimensions are not recommended

for this room volume. Because of the limited number of calculated modes, the density curve increases up to a certain frequency and then begins to decrease. Accordingly, condition 1) should be applied only up to that frequency.

The calculation program for this criterion is simple and can be run on desk calculators. Our program is run on an HP 9815 with printer 9871. Initially Eq. (3) is calculated, increasing coefficients n_x, n_y, n_z one unit at a time. For example, after the value 3:0:0 we pass to 0:1:0 and then to 1:1:0. Thus n_x is increased in each subroutine, and once 3 is reached, the following digit advances. Coefficient n_y does the same with reference to n_z . The modes thus calculated are then printed. Finally all modes are classified in one-third-octave bands using the standard frequencies of ANSI S1.6. Even if the calculator lacks plotting facilities, the criterion can be applied using the number of modes in each band. It is also possible, in order to simplify calculations, to use Eq. (4) to obtain the number of modes directly as a difference between the frequency limits of each band, but this procedure is not as accurate as the use of Eq. (3).

4 COMPARING DIFFERENT CRITERIA

Over a period of four years, 35 broadcasting studios and recording rooms were successfully constructed using this criterion, and we believe that comparing it with other recommended and accepted parameters is important.

First we analyze the ratio recommended by Knudsen [8] for small studios, 1:1.25:1.6. This same ratio was recommended by Olson [9] and others. In Fig. 3 the computer plot for a room of 60 m³ shows a perfect fulfillment of the criterion. In Fig. 4 various ratios are analyzed which are inside and outside of Bolt's chart. All ratios have been standardized for 60 m³. Some comply with both criteria (such as ⑤ and ①). Others are acceptable even though outside of Bolt's values ⑥ or are not acceptable even though complying with Bolt's conditions ②. The lack of correspondence for some values has been noted by other authors [7], [14] and is due, besides other factors, to the room volume, an important parameter which our criterion takes into account. Thus the same relationship can be acceptable for one size of room and unacceptable for another.

For example, consider the ratio 1:1.5:2.4 in Fig. 5, recommended by Knudsen [8] for large studios. For a volume of 60 m³ this ratio is not acceptable. For a studio of 2000 m³, for which it was recommended, it is satisfactory (Fig. 6), although not ideal. Conversely, the ratio in Fig. 3, acceptable for small studios, is also appropriate for 200 m³ (Fig. 7) and for 400 m³ (Fig. 8). Another interesting example, which extends the series of satisfactory coincidences, is the ratio recommended by Knudsen for a large Hollywood studio, 1:1.45:3.27. Figs. 9 and 10 show that it is not acceptable for 60 m³ nor for 2000 m³. Nevertheless, and rather unexpectedly,

if dimensions are increased to obtain the 4850 m³ which corresponds exactly to the studio described by Knudsen, those dimensions become the best (Fig. 11).

From these comparisons we observe a coincidence between the values recommended by experience and those resulting from the proposed criterion. On the other hand, dimensional ratios once recommended, such as those of Sabine (2:3:5 and 1.6:3:4) [7], which are no longer used, are really inadequate, as Fig. 12 shows. It is even more interesting to note that several optimum relationships exist, without fame and recommendation, obtainable when they are needed and for each particular room, through application of the proposed criterion.

5 CONTROL OF ISOLATED MODES

There are cases in which the dimensions of a given room cannot be optimized for the simple reason that it is already built. It may not be possible even to modify its dimensions substantially. In these cases, undesirable modes can remain, and we are forced to minimize their RT. In other cases, as in new multitrack recording studios, it is convenient to have low RT in the bass to optimize separation between microphones [13]. To achieve good control of low frequencies, resonator panels of special design are used [15] with an internal structure such as the one shown in Fig. 13.

To achieve better control of certain modes, we locate these panels on the walls so that they coincide with areas of maximum pressure of the modal standing waves. A computer program was written to plot Eq. (2). The resulting chart represents the intersections of surfaces of constant pressure with a plane parallel to the floor at a given height. At zero pressure the surfaces become planes and the intersections become straight lines. Fig. 14 shows the mode 2:1:0 for an experimental

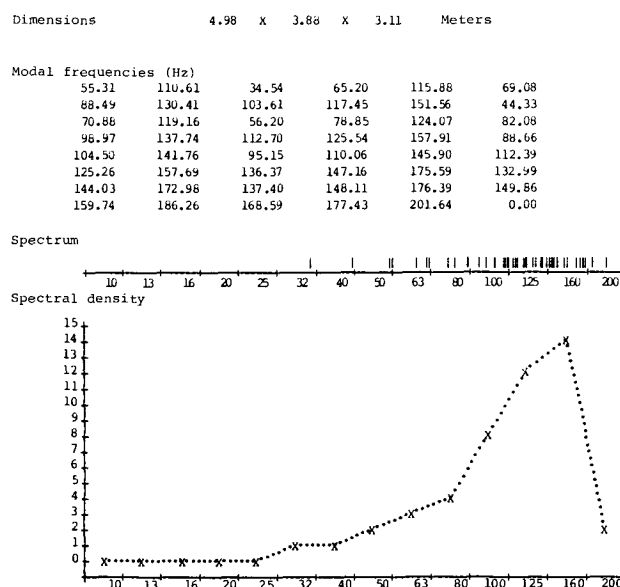


Fig. 3. Calculation of resonance modes. Application of the proposed criterion for a studio of 60 m³ with the relationship 1:1.25:1.6.

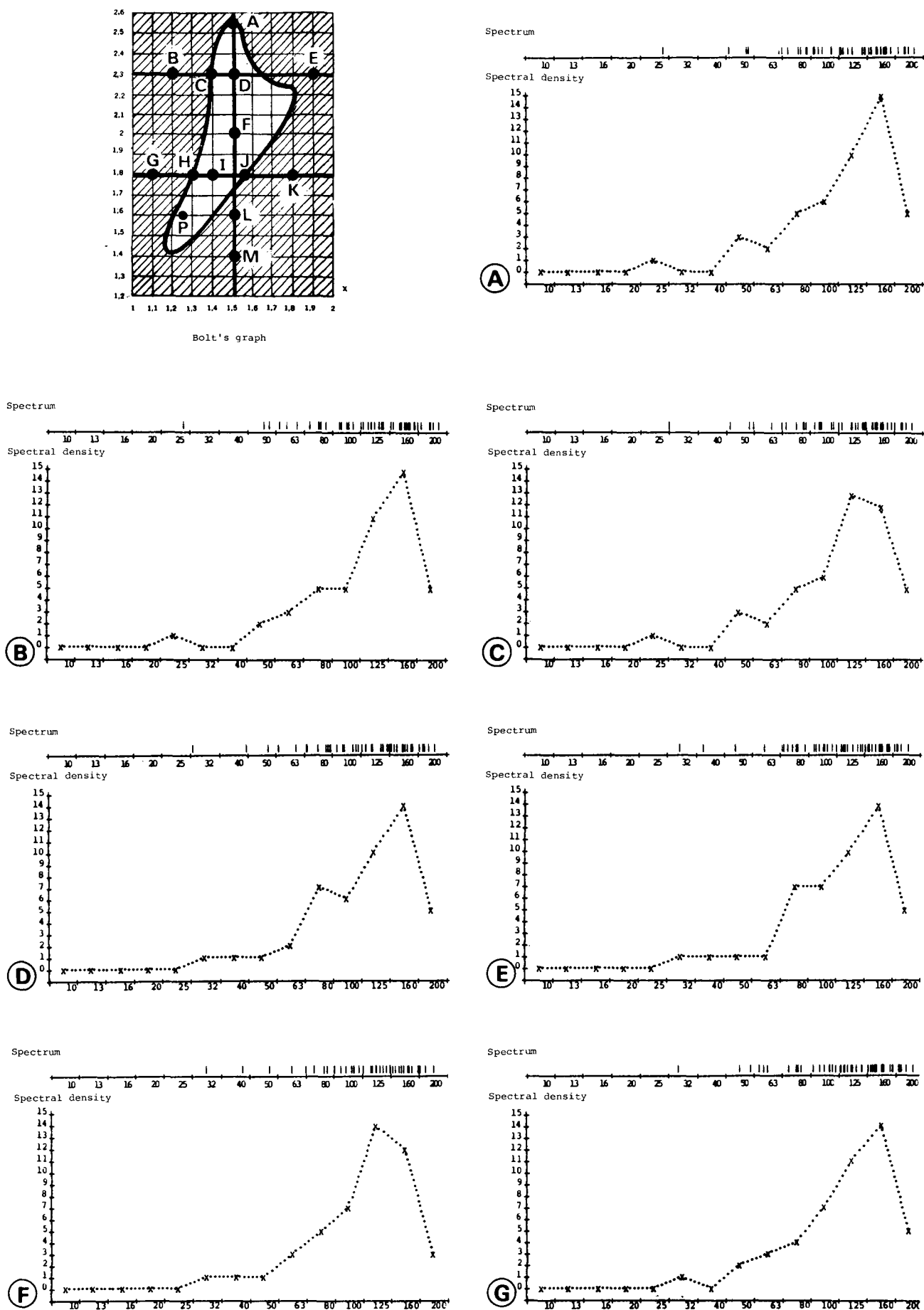
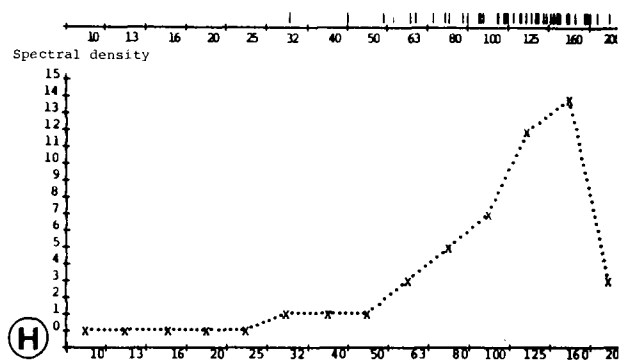


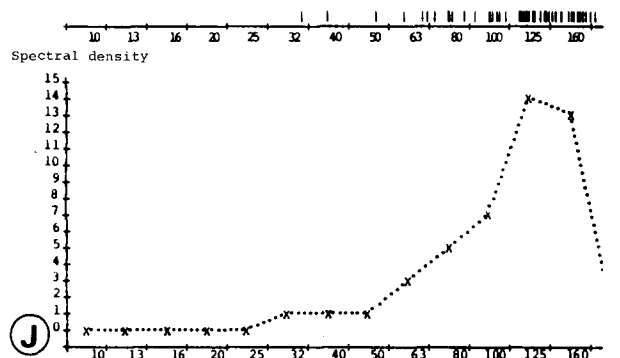
Fig. 4. Comparative analysis of Bolt's criterion with that proposed by author. The ratios have been standardized for 60 m³.

Fig. 4. (Continued).

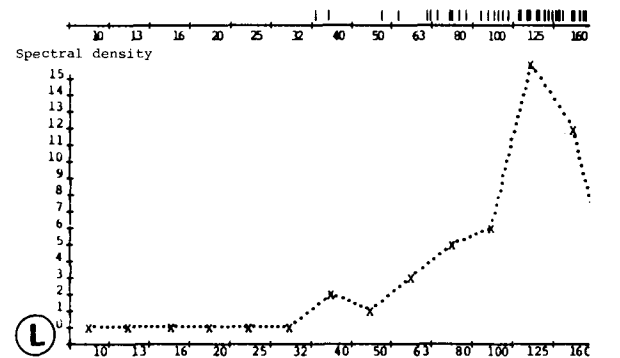
Spectrum



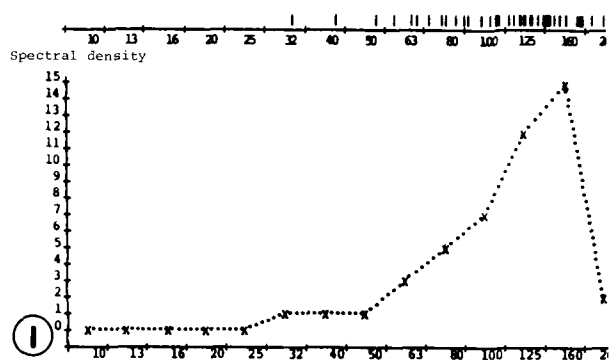
Spectrum



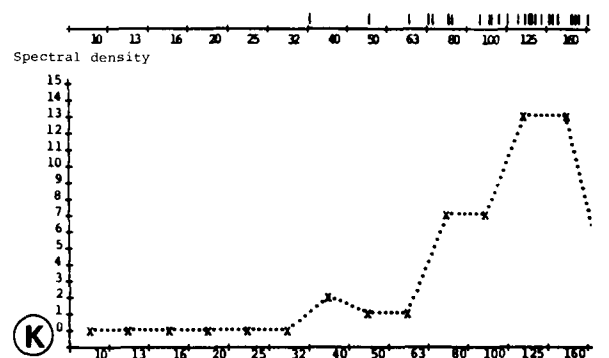
Spectrum



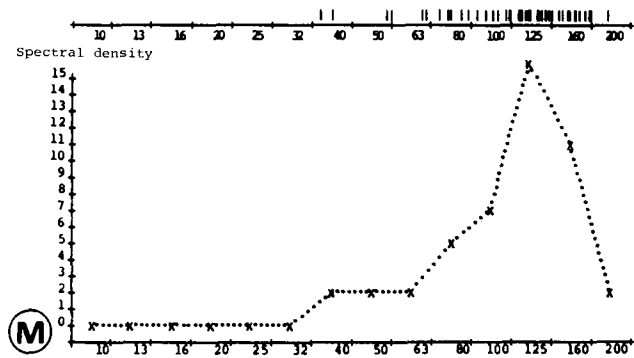
Spectrum



Spectrum



Spectrum

Fig. 4. Comparative analysis of Bolt's criterion with that proposed by author. The ratios have been standardized for 60 m³.

Dimensions 6.14 X 3.83 X 2.55 Meters

Modal frequencies (Hz)

67.45	134.90	28.01	73.04	137.78	56.03
87.65	146.07	84.04	107.76	158.94	44.91
81.03	142.18	52.93	85.74	144.91	71.80
98.52	152.82	95.29	116.74	165.16	89.82
112.32	162.07	94.08	115.76	164.47	105.86
125.52	171.48	123.00	140.28	182.56	134.73
150.67	190.66	137.61	153.25	192.70	145.91
160.75	198.72	158.79	172.52	208.36	0.00

Spectrum

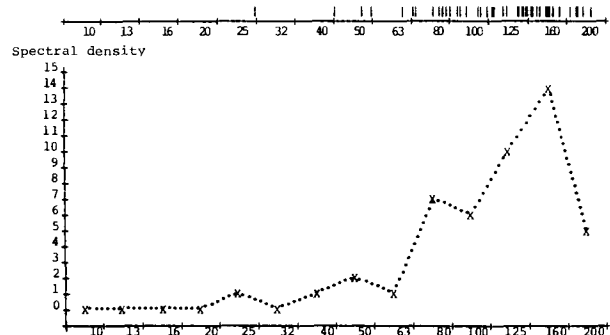


Fig. 5. Calculation of resonance modes.

Dimensions 19.74 X 12.33 X 8.22 Meters

Modal frequencies (Hz)

20.92	41.85	8.71	22.67	42.75	17.43
27.23	45.33	26.14	33.48	49.34	13.95
25.15	44.11	16.45	26.61	44.97	22.32
30.60	47.43	29.63	36.27	51.28	27.90
34.87	50.30	29.23	35.95	51.05	32.89
38.99	53.23	38.23	41.58	56.68	41.85
46.79	59.18	42.75	47.59	59.82	45.33
49.93	61.70	49.34	53.60	64.70	0.00

Spectrum

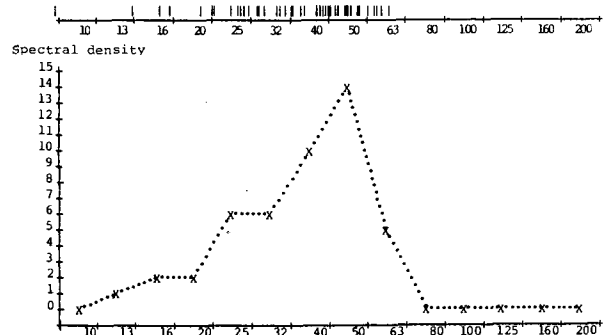


Fig. 6. Calculation of resonance modes.

studio. In this mode n_z equals zero, and the pattern is the same with the plane at any height. Not so with mode 3:2:2, which is shown at two different heights in Fig. 15. To show the effects of changing the locations of absorption panels within the experimental room ($6.1 \times 2.7 \times 2.6$ m), the RTs were measured for four panels in different positions, coinciding with maximum and minimum pressures (Fig. 14) for mode 2:1:0 at 85.08 Hz. The results are shown in Fig. 16 for a sinusoidal stimulus. In Fig. 17 the results are shown using pink noise filtered by octaves. Above 500 Hz the panel locations do not produce significant changes, since at that frequency the field is sufficiently diffuse. This technique of isolated-mode control, together with electronic or electroacoustical techniques [16], enables the designer to obtain rooms of the highest acoustical quality, even with reduced dimensions.

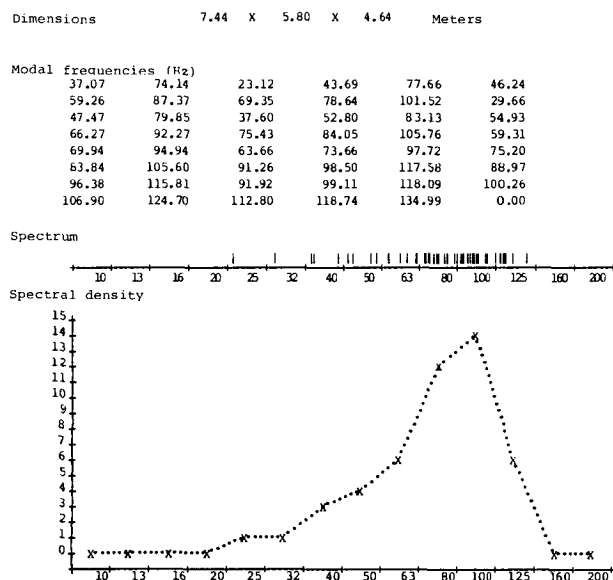


Fig. 7. Calculation of resonance modes.

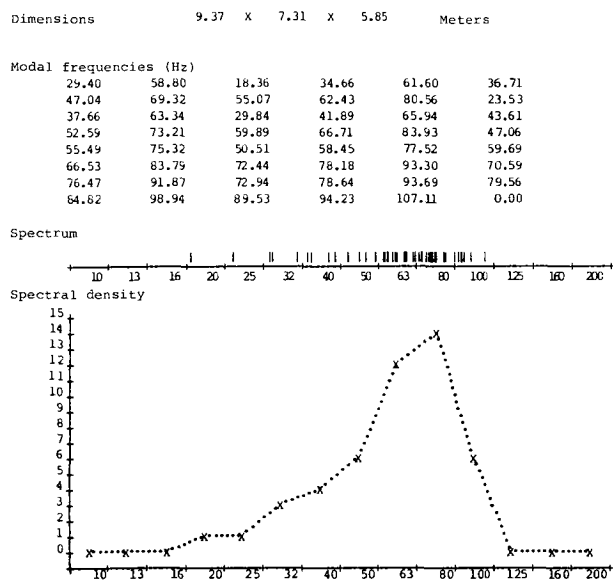


Fig. 8. Calculation of resonance modes.

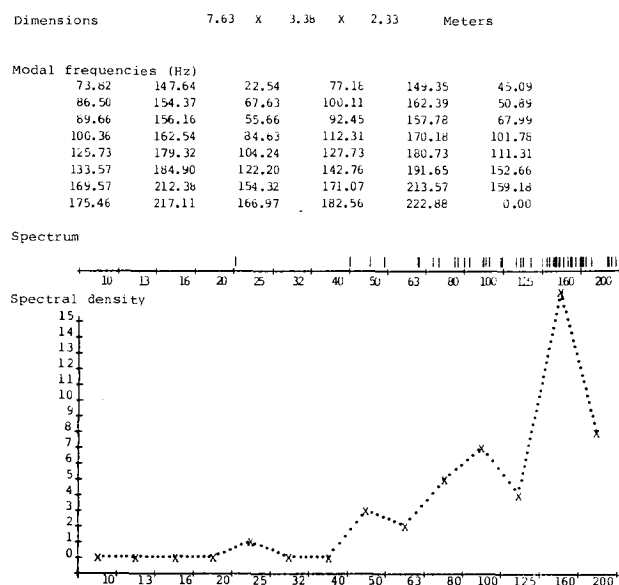


Fig. 9. Calculation of resonance modes.

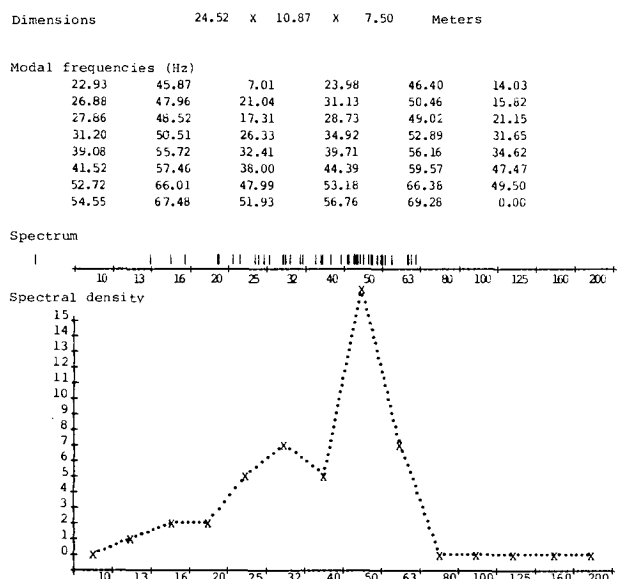


Fig. 10. Calculation of resonance modes.

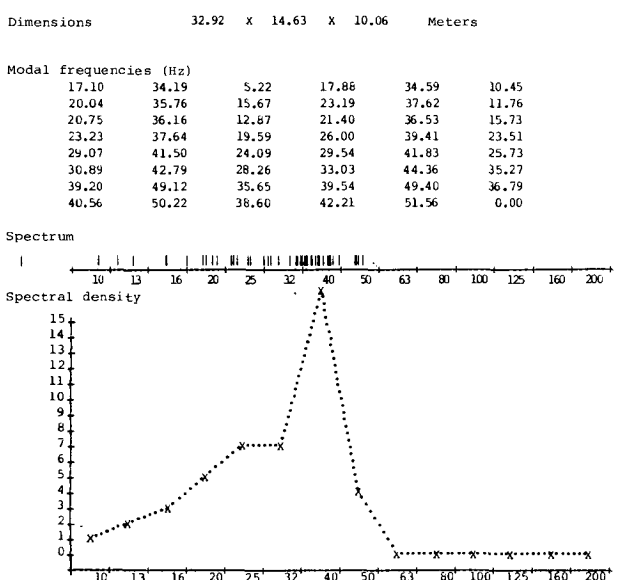


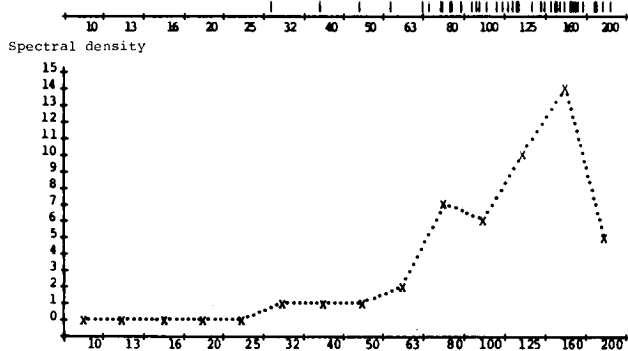
Fig. 11. Calculation of resonance modes.

Dimensions 5.85 x 4.39 x 2.34 Meters

Modal frequencies (Hz)

73.50	147.01	29.40	79.17	149.92	58.80
94.13	158.33	88.21	114.82	171.44	39.18
83.29	152.14	48.98	88.33	154.95	70.66
101.96	163.11	96.52	121.32	175.86	78.36
107.44	166.59	83.69	111.39	169.16	97.97
122.48	176.66	117.98	139.01	188.50	117.54
138.63	188.22	121.16	141.71	190.50	131.43
150.59	197.19	146.95	164.31	207.86	0.00

Spectrum



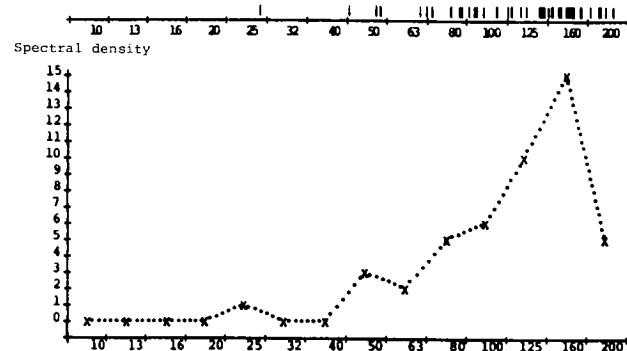
(a)

Dimensions 6.30 x 3.79 x 2.52 Meters

Modal frequencies (Hz)

68.25	136.51	27.30	73.51	139.21	54.60
87.41	147.02	81.90	106.62	159.19	45.38
81.96	143.85	52.96	86.39	146.42	71.00
98.49	153.87	93.64	115.87	165.54	90.77
113.56	163.93	94.78	116.80	166.19	105.92
126.01	172.78	122.26	140.02	183.25	136.15
152.30	192.80	138.86	154.73	194.72	146.69
161.79	200.38	158.89	172.93	209.47	0.00

Spectrum



(b)

Fig. 12. Calculation of resonance modes: (a) 1.6:3:4; (b) 2:3:5.

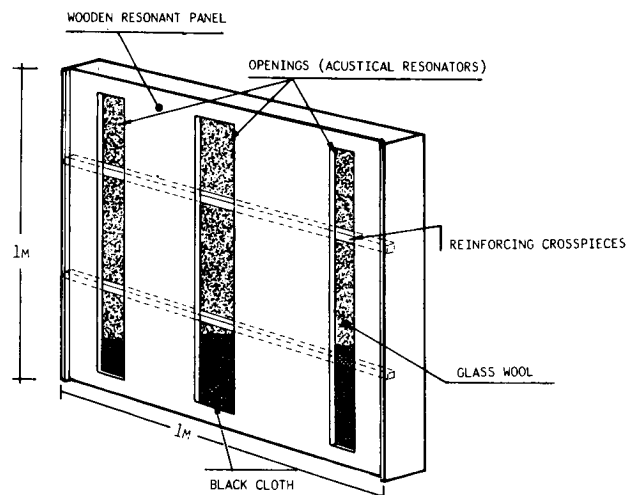
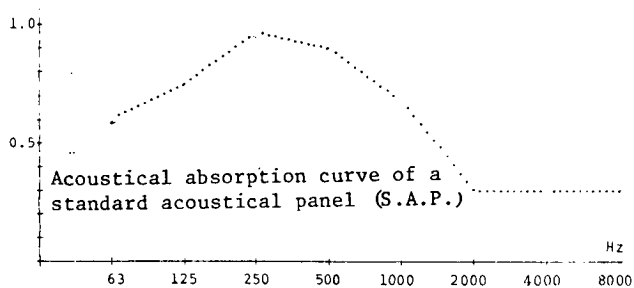


Fig. 13. Internal construction of a standard acoustical panel.

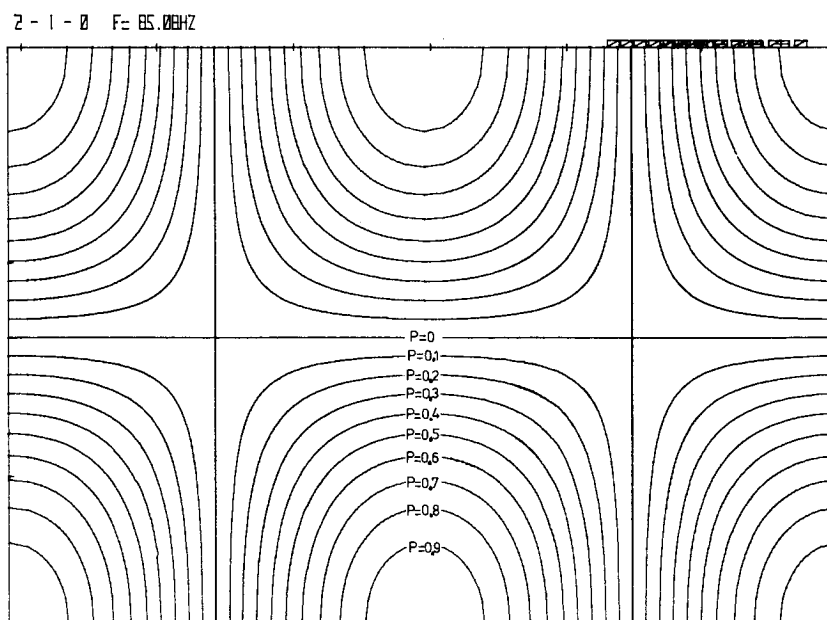


Fig. 14. Distribution of several values of pressure (computer output). Room size 6.1 x 2.7 x 2.6 m; mode 2:1:0.

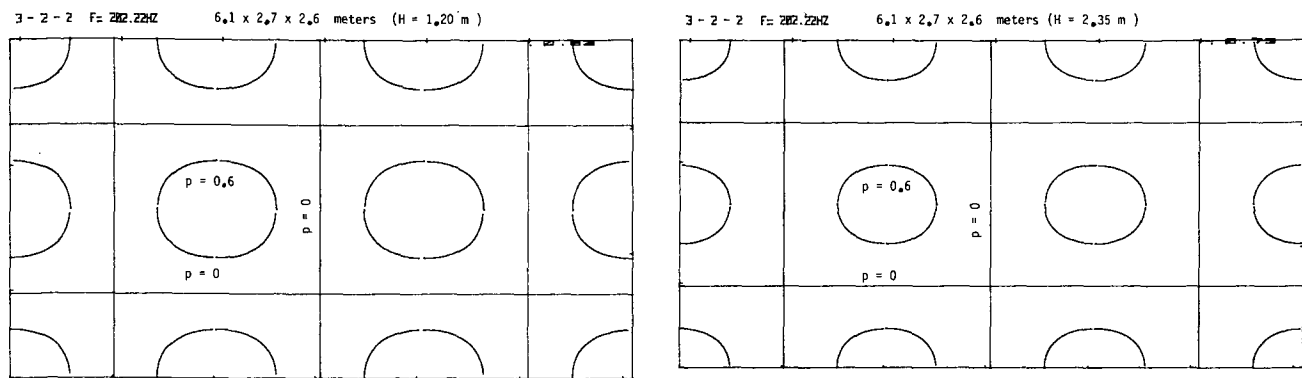


Fig. 15. Two plots of the same room at two different levels. $p = 0.6$.

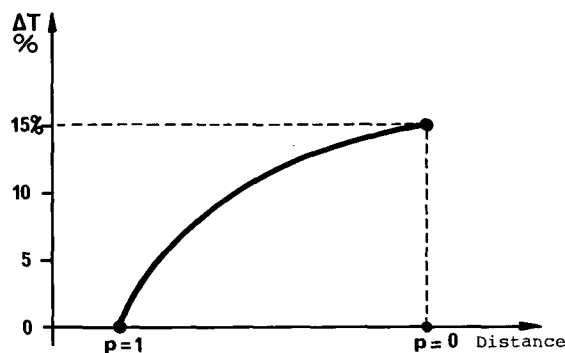


Fig. 16. Variation of RT of mode 2:1:0 (85 Hz), measured with a sinusoidal source, due to the movement of 4 m² of panels ($\alpha = 0.3$), which represents 20% of the total inside area of the room.

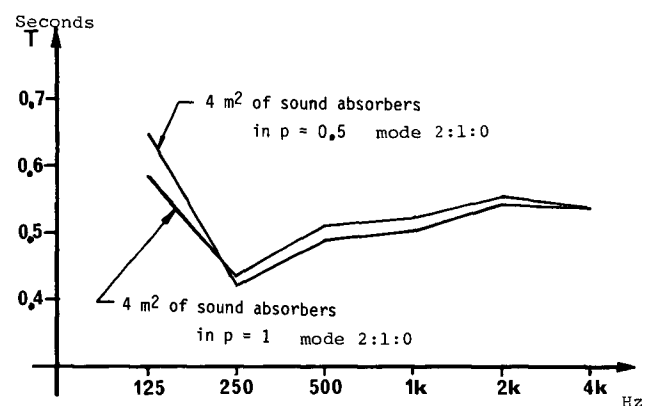


Fig. 17. Variation of the RT measured with pink noise, by octaves, for two different positions of the absorption panels.

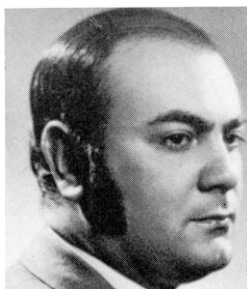
6 CONCLUSION

We are confident that this criterion, easy to apply, will dispel part of the mystery in constructing recording rooms of medium dimensions. The criterion gives designers a new calculation tool which has proved its efficacy in four years of applications.

7 REFERENCES

- [1] C. L. S. Gilford, "Acoustic Design of Talks Studios and Listening Rooms," *J. Audio Eng. Soc.*, vol. 27, pp. 17-31 (1979 Jan./Feb.).
- [2] R. H. Bolt, "Normal Modes of Vibration in Room Acoustics: Angular Distribution Theory," *J. Acoust. Soc. Am.*, vol. 11, pp. 74-79 (1939).
- [3] R. H. Bolt, "Note on Normal Frequency Statistics for Rectangular Rooms," *J. Acoust. Soc. Am.*, vol. 18, pp. 130-133 (1946).
- [4] R. H. Bolt, "Normal Frequency Spacing Statistics," *J. Acoust. Soc. Am.*, vol. 19, pp. 71-90 (1947).
- [5] F. V. Hunt, L. L. Beranek, and D. Y. Maa, "Analysis of Sound Decay in Rectangular Rooms," *J. Acoust. Soc. Am.*, vol. 11, pp. 80-94 (1939).
- [6] D. Y. Maa, "The Distribution of Eigentones in a Rectangular Chamber at Lower Frequency Ranges," *J. Acoust. Soc. Am.*, vol. 10, pp. 235-238 (1939).
- [7] M. Rettinger, *Acoustic Design and Noise Control*, vol. I (Chemical Publishing Co., New York, 1977), p. 88.
- [8] V. Knudsen and C. Harris, *Acoustical Designing in Architecture* (Wiley, New York, 1950), pp. 402-403.
- [9] H. F. Olson, *Music, Physics and Engineering* (Dover, New York, 1967), pp. 302-303.
- [10] H. Kuttruff, *Room Acoustics* (Applied Science Publishers, Barking, Essex, England, 1976).
- [11] L. L. Beranek, *Acoustics* (McGraw-Hill, New York, 1954), ch. 10.
- [12] L. L. Beranek, *Acoustics Measurements* (Wiley, New York, 1949), pp. 329-336.
- [13] M. Rettinger, "On the Acoustics of Multi-track Recording Studios," presented at the 49th Convention of the Audio Engineering Society, New York, 1974 September 9-12, preprint no. 795.
- [14] L. W. Sepmeyer, "Computed Frequency and Angular Distribution of the Normal Modes of Vibration in Rectangular Rooms," *J. Acoust. Soc. Am.*, vol. 37, pp. 413-423 (1965).
- [15] O. J. Bonello, "A New Computer Aided Method for the Complete Acoustical Design of Broadcasting and Recording Studios," *Proc. Int. Conf. on Acoustics, Speech, and Signal Processing, ICASSP 79*, Washington, 1979, pp. 326-329.
- [16] O. J. Bonello, "Modular Parametric Equalizer-Filter; A New Way to Synthesize Frequency Response," presented at the 55th Convention of the Audio Engineering Society, New York, 1976 October 29-November 1, preprint no. 1170.

THE AUTHOR



Oscar Juan Bonello was born in Buenos Aires in 1940. He received an M.S. degree in electrical engineering from the National University of Buenos Aires in 1969. His early work at the University involved accidental printing in magnetic tapes, the Doppler effect and solid-state physics.

In 1964 he founded Sistemas Solidyne, a company devoted to research in the electronics field and to the manufacturing of professional audio equipment. Mr. Bonello's work has included the design of audio consoles and signal processors for recording, broadcasting and television studios, development of a method of studio acoustic treatment and collaboration in developing the Argentine telecommunications standards.

Mr. Bonello designed the broadcasting network—

including all the audio systems—used during the satellite transmission of the 1978 World Football Games; 560 audio channels from five cities through three levels of control systems. The author of some 60 papers, he has several patents in his name. In 1973 he was awarded the Mención de Honor in the National Prize of Industrial Design. He is a member of the Centro de Diseñadores Industriales and Professional Member of the CIDI (Center of Research in Industrial Design).

A senior member of the IEEE, Mr. Bonello also is a member of the Audio Engineering Society, the American Acoustical Society, and the SMPTE. He is vice president and founding member of the Asociación Argentina de Acústica, and served as convention chairman of the last three Argentine Acoustical Conventions.