Hopenhayn (1992) Replication

Model Setup

Timing

Household - Demand

Demand is given exogenously (partial equilibrium model):

$$D(p) = \bar{D}/p \tag{1}$$

Firms

Have access to the following production technology:

$$y = z_i n_i^{\eta} \tag{2}$$

Incumbents

Static firm profit maximisation problem, normalising $w = 1$.

$$\pi(z; p, w) = \max_{n} \{pzn^{\eta} - wn - wc_f\} \tag{3}$$

Labour demand

$$n(z; p, w) = \left(\frac{\eta pz}{w}\right)^{\frac{1}{1-\eta}} \tag{4}$$

Contemporaneous profit

$$\pi(z; p, w) = (1-\eta)(pz)^{\frac{1}{1-\eta}} \left(\frac{\eta}{w}\right)^{\frac{\eta}{1-\eta}} \tag{5}$$

Incumbent Value Function

$$V(z) = \pi(z; p) + \beta \max \left\{ \int V(z')dF(z'|z), 0 \right\} \tag{6}$$

Conjecture that $V(z)$ is monotonically increasing in z because $\frac{\partial \pi(z;p)}{\partial z} > 0$, and therefore $\exists$ $\bar{z}$ s.t. the expected future stream of profits is 0.

$$\mathbb{E}[V(z')|\bar{z}] = \int V(z')dF(z'|\bar{z}) = 0 \tag{7}$$

Entrants

$$V_e(z) = -c_e + \beta \int V(z)dG(z) \tag{8}$$

Free Entry Condition

Let $M \geq 0$ be mass of entrants. Firms enter if $V_e(z) \geq 0$. Entrance must drive profit to 0. Therefore in equilibrium we must have:

$$\beta \int V(z)dG(z) \leq c_e \tag{9}$$

With strict equality if $M > 0$.

Distribution of Firms

Let $\mu_t(z)$ be a measure of firms over productivity, z. Then:

$$\mu_{t+1}(z') = \int F(z'|z)(1 - \chi(z))d\mu_t + M_{t+1}G(z') \tag{10}$$

Where $\chi(z) = 1$ if an incumbent exits, 0 if else.

SRCE Algorithm

Hopenhayn (1992) with Aggregate Uncertainty

Individual States, $s = [z]$

Aggregate State, $S = [\mu, A]$

Firms

Have access to the following production technology, augmented by TFP shocks:

$$y = z_i A n_i^{\eta} \tag{11}$$

Where A follows a 2 state markov process s.t. $A \in \{G, B\}$

Incumbents

Static firm profit maximisation problem

$$\pi(z; S) = \max_n \{pzAn^{\eta} - w(S)n - c_f\} \tag{12}$$

Labour demand:

$$n(z; S) = \left(\frac{\eta p z A}{w}\right)^{\frac{1}{1-\eta}} \tag{13}$$

Labour supply is exogenous, N, and the wage rate clears the labour market:

$$N = \int n_i di = \int n(z;S) d\mu(z) \tag{14}$$

$$N = \int \left(\frac{\eta p z A}{w}\right)^{\frac{1}{1-\eta}} di = \left(\frac{\eta p A}{w}\right)^{\frac{1}{1-\eta}} \int z_i^{\frac{1}{1-\eta}} di \tag{15}$$

$$w(S) = \frac{\eta p A}{N^{1-\eta}} \left(\int z_i^{\frac{1}{1-\eta}} di \right)^{1-\eta} = \frac{\eta p A}{N^{1-\eta}} \left(\int z_i^{\frac{1}{1-\eta}} d\mu(z) \right)^{1-\eta} \tag{16}$$

So prices, w, depend on $S = (A, \mu)$. The agent must forecast the high dimension object, μ, to understand prices.

Incumbent Value Function

$$V(z;S) = \pi(z;S) + \beta \mathbb{E}_{S'|S} \left[\max \left\{ \int V(z';S') dF(z'|z), 0 \right\} \right] \tag{17}$$

$\exists$ a $z^*(S)$ s.t.

$$\mathbb{E}[V(z')|z^*(S)] = \mathbb{E}_{S'|S} \int V(z') dF(z'|z^*(S)) = 0 \tag{18}$$

Whereby if $z < z^*(S)$, exit, else stay.

Entrants

$$V_e(z;S) = -c_e + \beta \mathbb{E}_{S'|S} \int V(z';S') dG(z') \tag{19}$$