Regression II

Data Sciences Institute
Linear Regression, Classification, and Resampling

Learning objectives

- Explain the linear regression algorithm and contrast it with KNN regression.
- Fit simple and multivariable linear regression models on training data using Python.
- Evaluate the linear regression models on test data.
- Explain the impact of outliers and multicollinearity on linear regression.

Simple linear regression

$$y = mx + b$$

- **Predictor Variable (x)**:
  - The factor you're using to make the prediction (independent variable)
- **Response Variable (y):**
  - The outcome or result you're trying to predict (dependent variable)
- **Slope of the line (m)**:
  - How much y changes when x changes.
- **Y-intercept (b)**:
  - The starting value of y when x is 0.

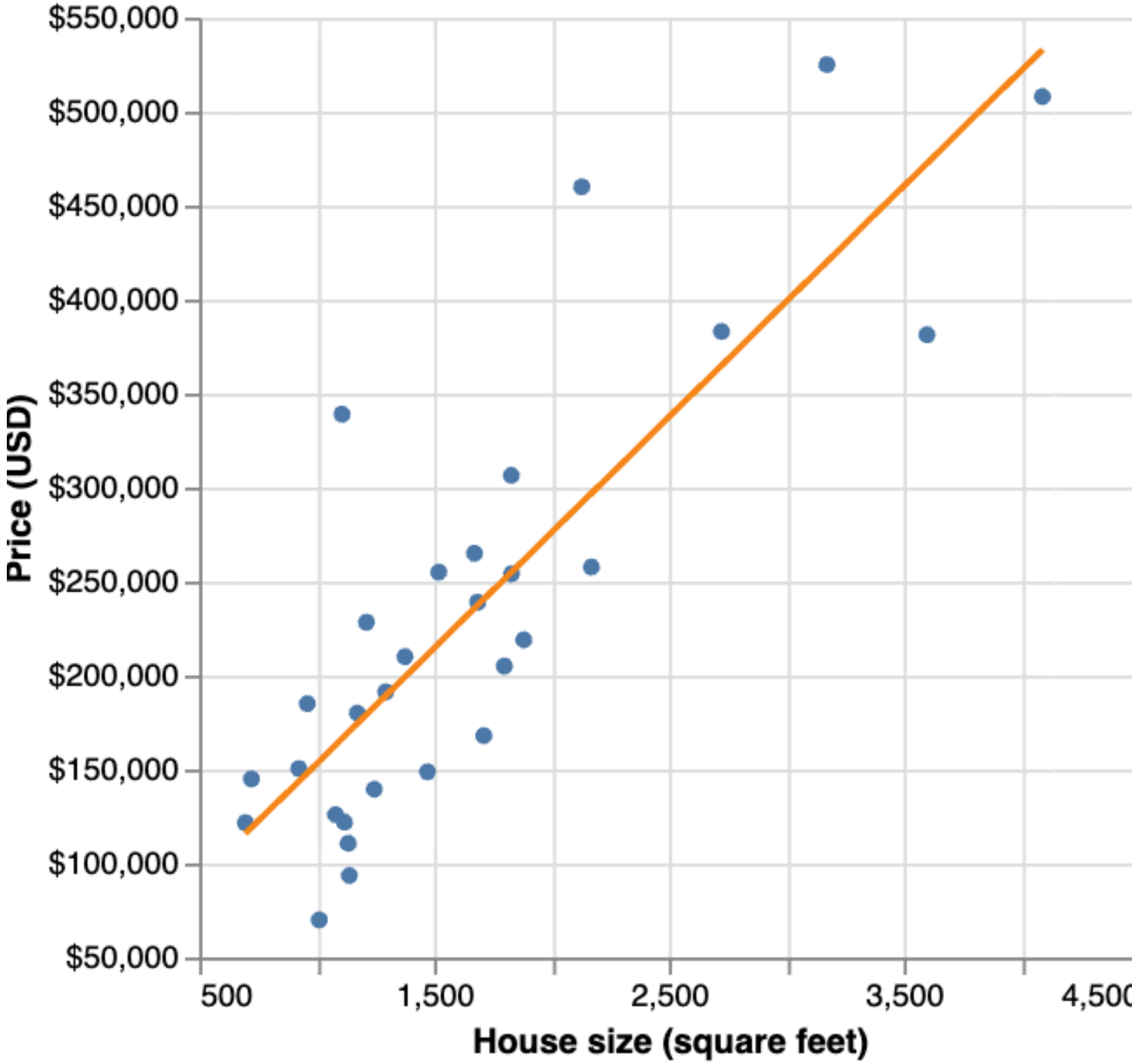
- KNN regression has limitations: poor prediction beyond the training data range and slower performance with larger datasets.
- Linear regression addresses these limitations: better prediction range and faster performance with larger datasets.
- Linear regression is widely used due to its interpretable mathematical equation linking predictors and response variables.
- Simple linear regression involves one predictor and one response variable and **predicts by creating a straight line of best fit through the training data.**

Example dataset

813 real estate transactions in Sacramento, California is the dataset we will be using, specifically for predicting whether the size of a house in Sacramento can be used to predict its sale price.

- *Key features:*
  - 813 observations (rows)
  - predictor of interest (sqft; house size, in livable square feet)
  - response variable of interest (house sale price, in USD)

- To decide whether the $350,000 asking price for the 2,000 square-foot house is fair, we can use our existing data to predict its likely sale price. However, since there are no exact observations for a 2,000 square-foot house in our dataset, we need a method to estimate the price.
- Using simple linear regression, we use the data we can draw a straight line of best fit through our existing data points.



- The equation for the straight line in simple linear regression is:

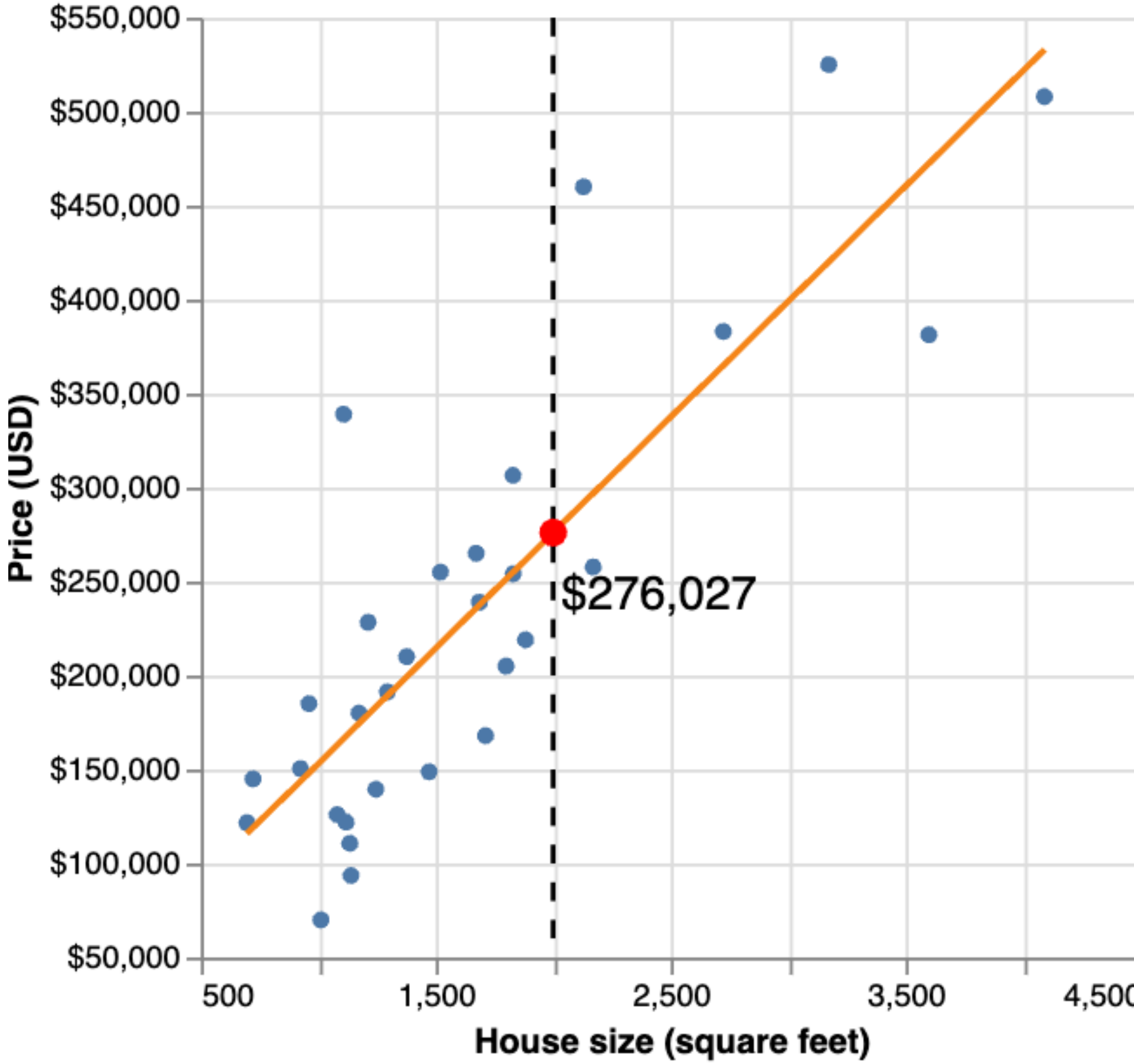
$$\text{house sale price} = \beta_0 + \beta_1 \text{house size}$$

where:

β_0 is the vertical intercept (price when house size is 0).
β_1 is the slope (rate of price increase as house size increases).

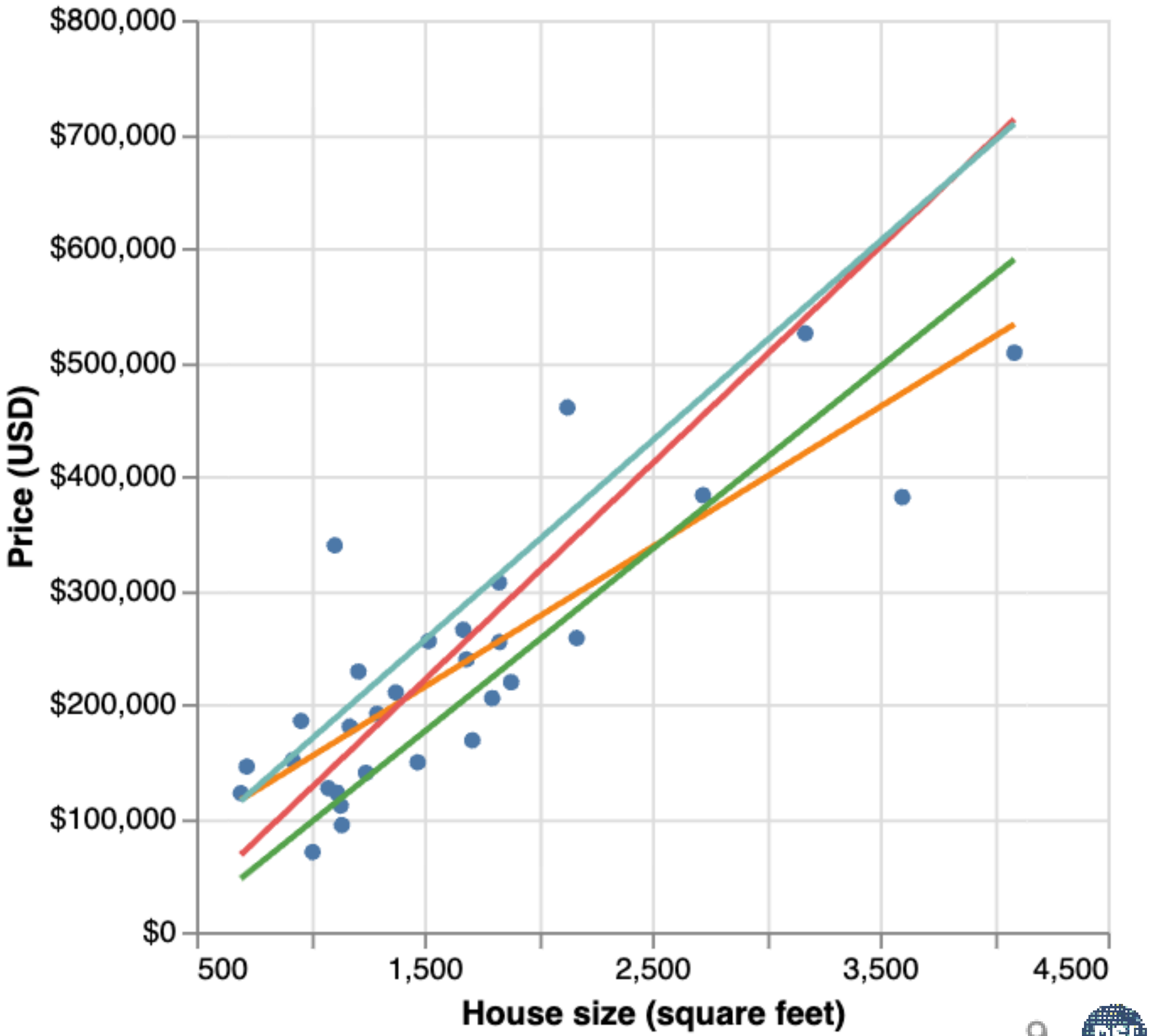
- Finding the line of best fit involves determining coefficients β_0 and β_1 that define the line.
    - β_0 represents the "base price,"
    - β_1 is the price increase per square foot.

- Once we have the coefficients, we can use the equation to evaluate the predicted sale price given the value we have for the predictor variable—here 2,000 square feet.
- Linear regression can predict extreme values (e.g., 6 million or -2,000 sq. ft.), but these predictions are unreliable.
- Make predictions within the original data range; extrapolate only when logically justified.

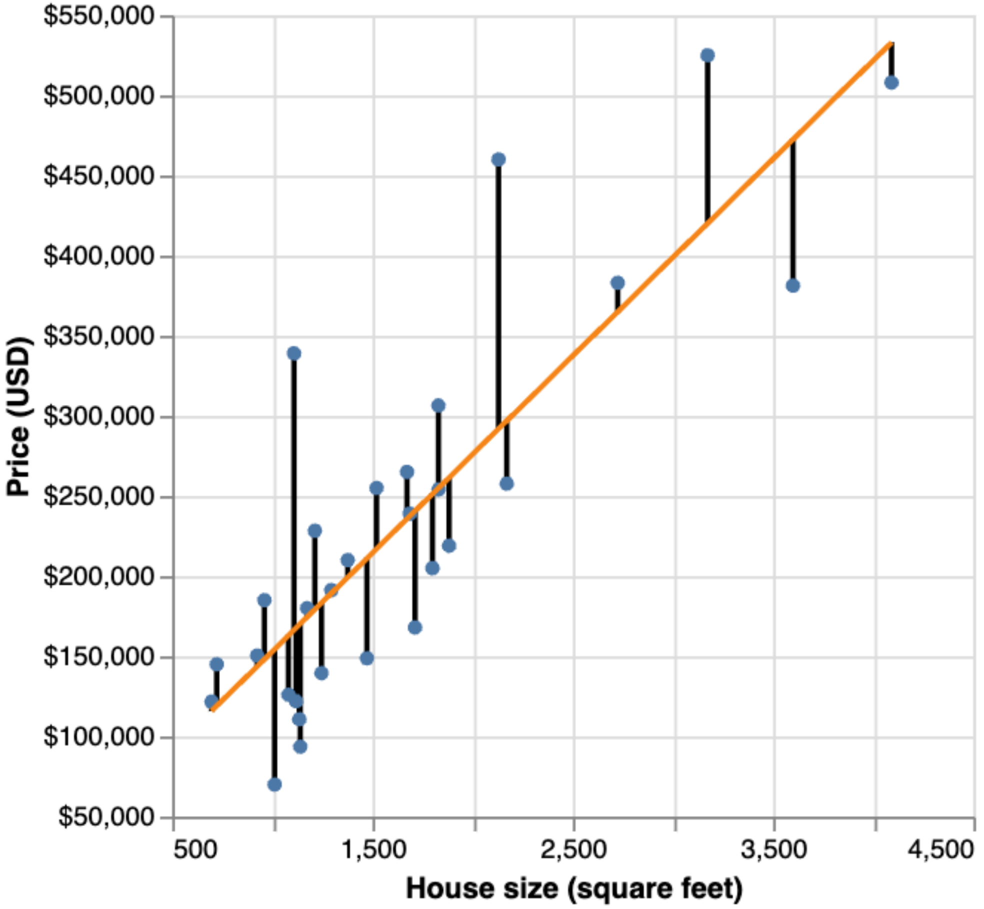


Choosing the line of best fit

Many different lines could be drawn through the data points, how do we choose the line of best fit?

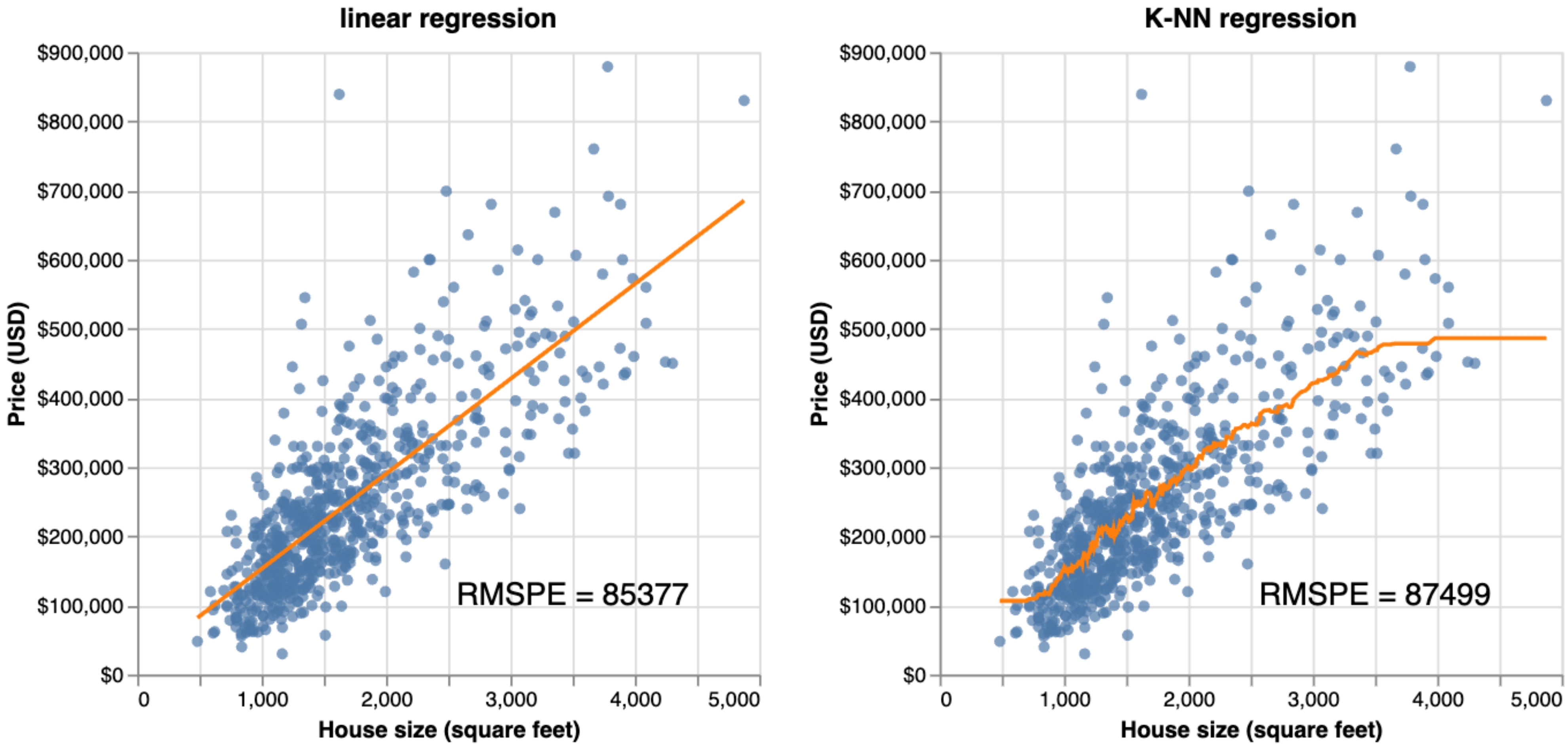


- Simple linear regression finds the line of best fit by minimizing the average squared vertical distance between the line and observed data points.
- This process is equivalent to **minimizing the Root Mean Squared Error (RMSE).**
- The predictive accuracy of the simple linear regression model is assessed using the Root Mean Squared Prediction Error (RMSPE), similar to KNN regression.



Comparing simple linear and KNN regression

- **Parameter Tuning:** No need for cross-validation to choose parameters in linear regression.
- **Data Preparation:** No need for standardization (centering and scaling) of the data in linear regression.



Visualization of the simple linear regression model predicting price from house size and the "best" K-NN regression model obtained from the same problem.

	Simple Linear Regression	KNN Regression
Line Shape	Straight line	Flexible, wiggly line
Interpretability	High (defined by intercept and slope)	Low (no clear interpretability due to flexibility)
Fit Quality	May underfit non-linear relationships	Better fit for non-linear relationships
RMSE/RMSPE	Lower on linear relationships	Lower on non-linear relationships
Extrapolation Behavior	Predicts with constant slope, can be inaccurate (e.g., negative prices)	Produces flat predictions at boundaries, may not match reality

Multivariable linear regression

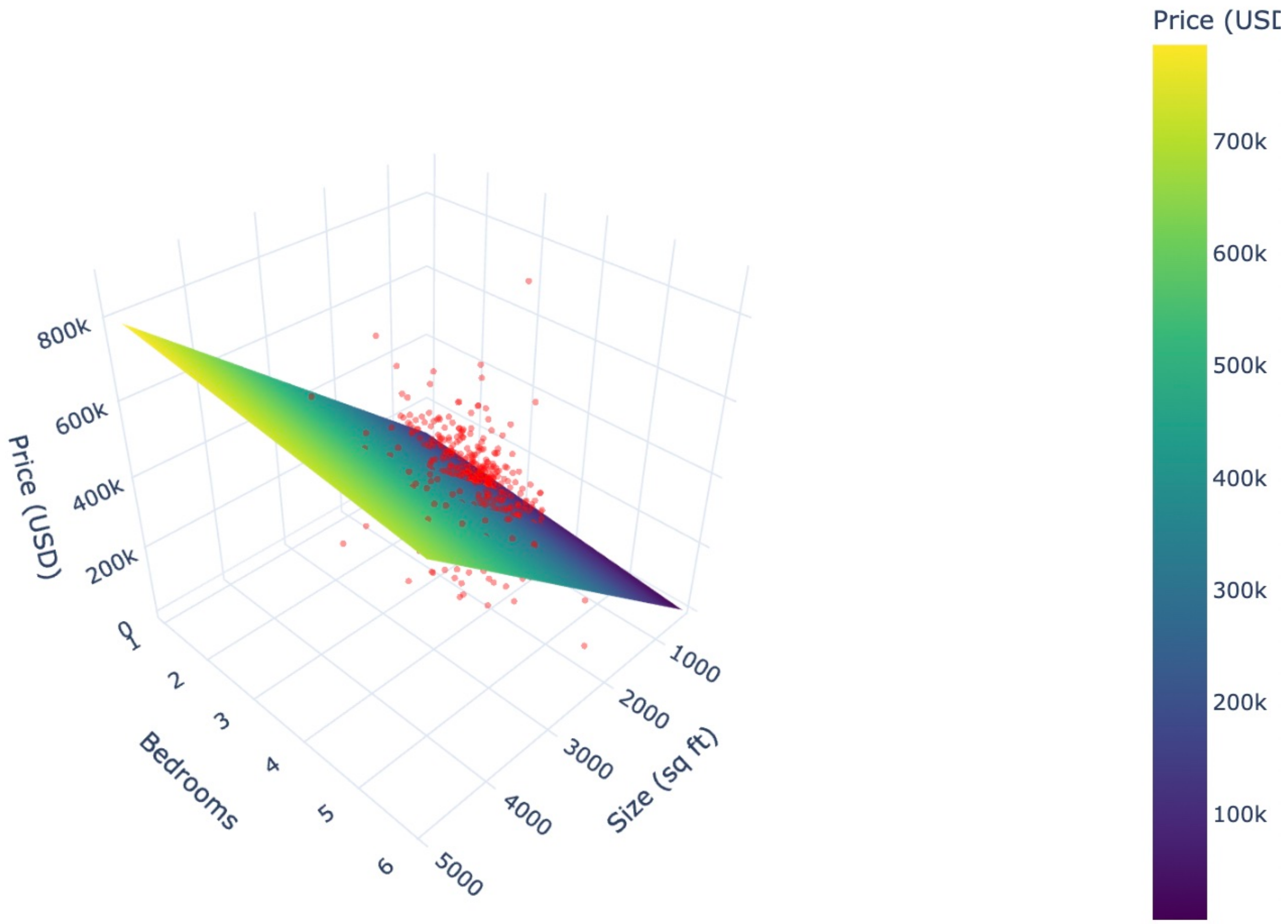
- Extends simple linear regression to multiple predictors.
- Similar to KNN regression, simply add more predictors to the training data.
- Each predictor variable *may* give us new information to help create our model.

  The only difference is the formula:

$$\text{house sale price} = \beta_0 + \beta_1 \text{house size} + \beta_2 \text{number of bedrooms}$$

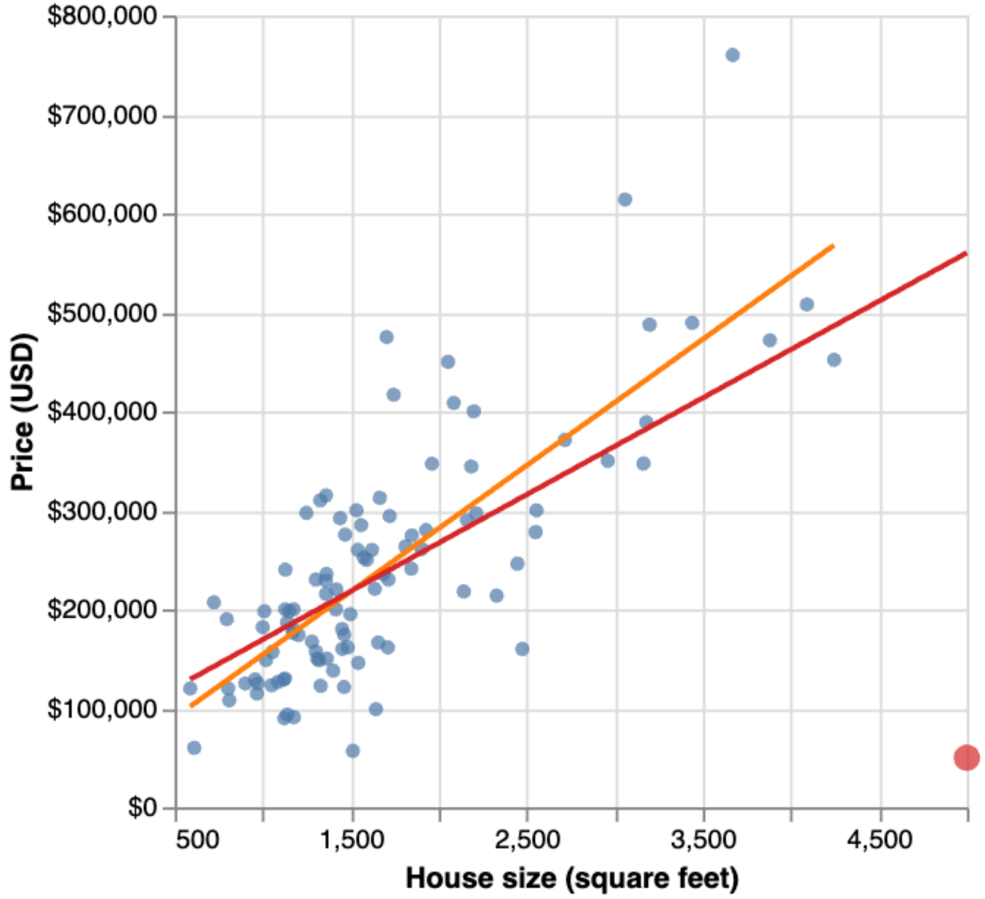
where:
β_0 is the vertical intercept (price when house size is 0 and number of bedrooms is 0).
β_1 is the slope (rate of price increase as house size increases when fixing everything else).
β_2 is the slope (rate of price increase as number of bedrooms increases when fixing everything else).

In the case of two predictors, we can plot the predictions made by our linear regression to create a plane of best fit.

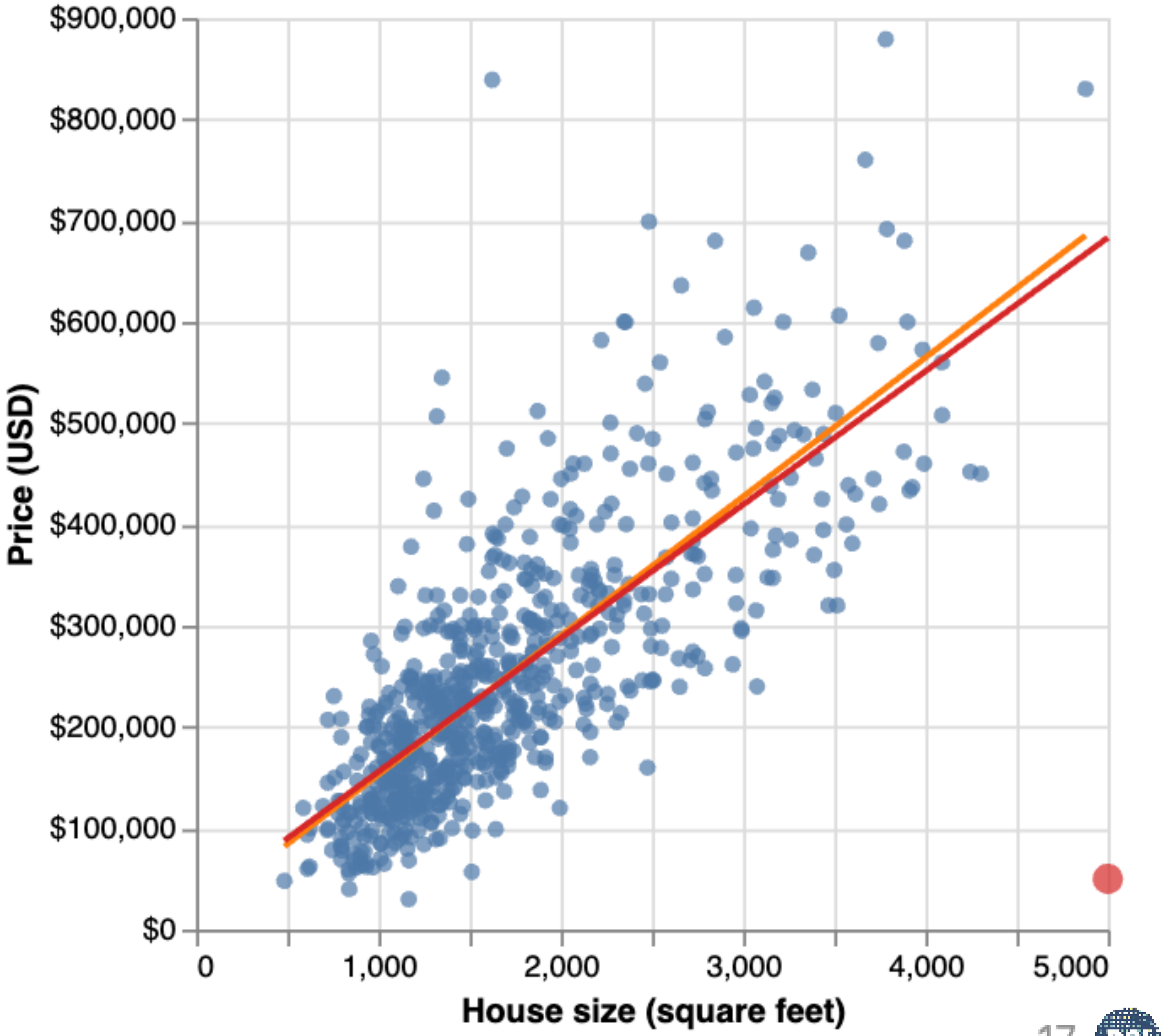


Outliers

- Data points with unusually high or low vertical distances from the line of best fit.
- Outliers can disproportionately influence the line of best fit.
- Identifying outliers accurately often requires advanced techniques.
- Eg., a single outlier (a 5,000 sq. ft. house sold for $50,000) dramatically alters the line of best fit, changing it from the original (**orange**) to a new line (**red**).

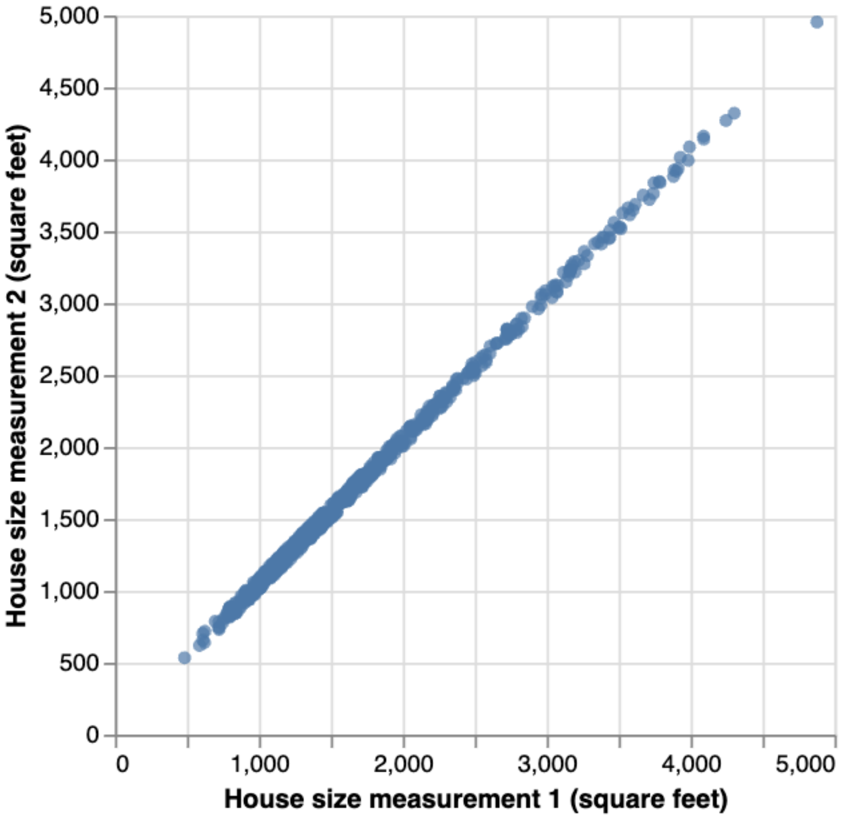


Fortunately, if you have enough data, the inclusion of one or two outliers—as long as their values are not too wild— will typically not have a large effect on the line of best fit.



Multicollinearity

- Occurs in multivariable linear regression when predictors are strongly linearly related.
- if predictors are highly correlated, the model's coefficients can become very sensitive to slight changes in the data.
- This sensitivity can lead to large variations in the estimated coefficients when using different data splits or subsets.
- Identifying multicollinearity often requires techniques like Variance Inflation Factor that are beyond the scope of this module.



`Putting it all together`

`linear regression with scikit-learn`