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Combatting Gerrymandering with Computation

A Review of Markov Chain Models for Redistricting

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Quantitative Methods of Redistricting

DeFord, Duchin, and Solomon (2021) present a novel computational approach to redistricting, optimizing the division of geographical regions into contiguous, population-balanced electoral districts – in effort to address **gerrymandering**.

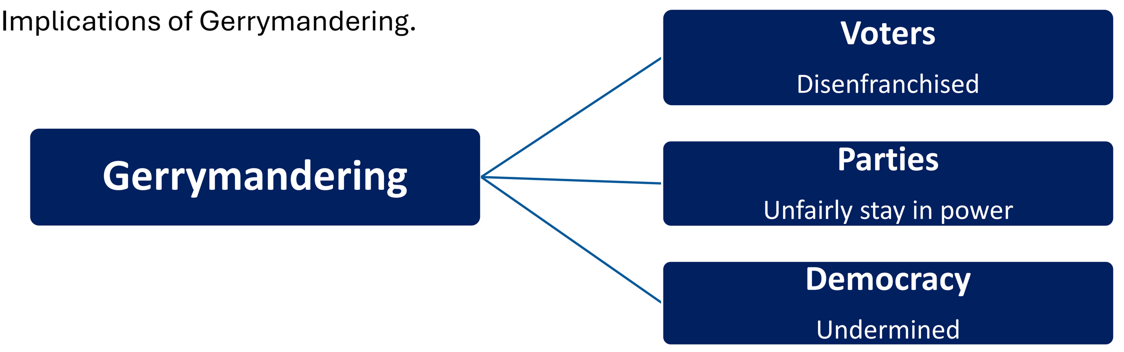
'Recombination' (**ReCom**) offers an efficient means to sample from near-infinite potential district configurations, while integrating required legal and policy criteria. Surpassing traditional Ising-Flip redistricting methods in scale, efficiency, and computational speed, DeFord, Duchin, and Solomon (2021) leverage Markov chains and spanning tree structures to produce viable districting plans for the Virginia House of Delegates with fewer iterations.

The ReCom method holds promise for redistricting efforts worldwide, providing an optimized tool to support the formation of fair electoral districts.

Gerrymandering vs. Computation

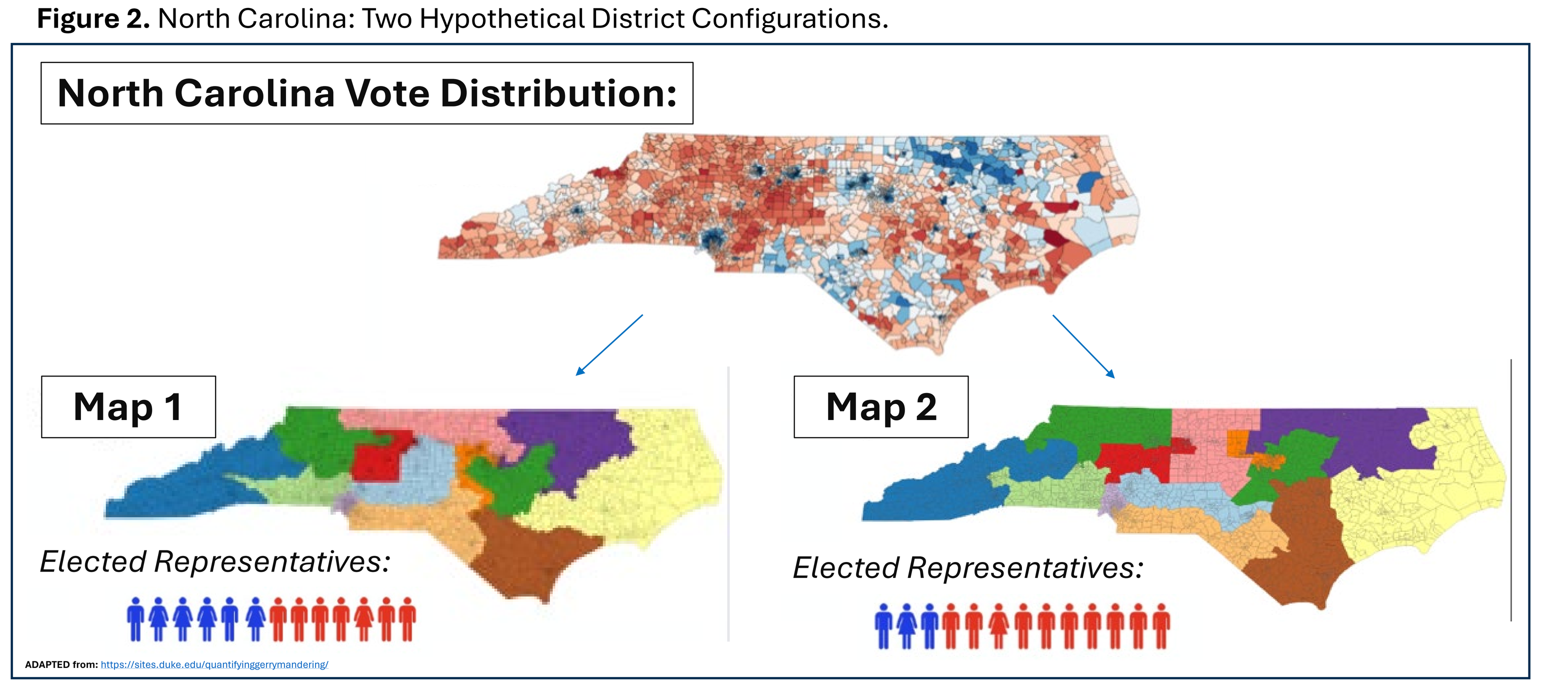
Control of electoral district lines can confer a surprising degree of power over election outcomes – even under real-world conditions. Abusing this power is called **gerrymandering.** (2)

Figure 1. The Implications of Gerrymandering.



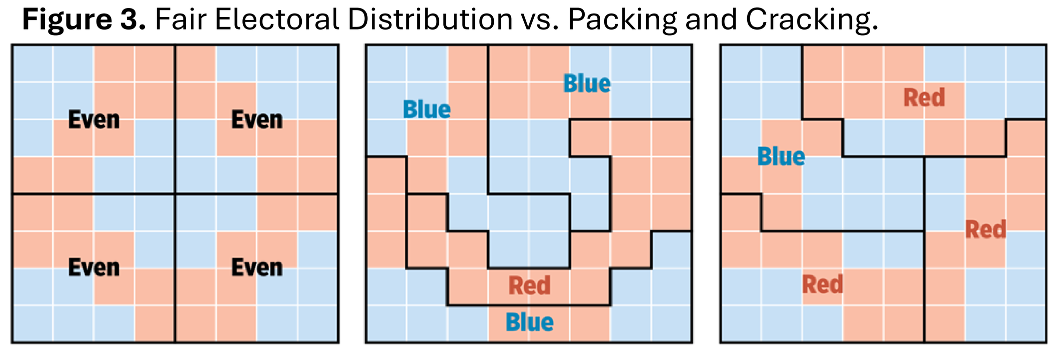
Traditionally, redistricting maps were drawn by hand. With algorithms, the creation and contestation of gerrymandering has become a **science**.

Figure 2. North Carolina: Two Hypothetical District Configurations.



Types of Gerrymandering

Figure 3. Fair Electoral Distribution vs. Packing and Cracking.



Even distribution
2 red, 2 blue
In the example above, there are an even number of red and blue voters to be split among four districts. Drawing four squares creates districts that split red and blue voters equally.

Packing
1 red, 3 blue
Packing concentrates members of a party in a single district, allowing the other party to win the others. Here, one district is packed entirely with red voters. Blue voters have a majority in the three others.

Cracking
3 red, 1 blue
Cracking splits a group among multiple districts to dilute their voting power. Above, blue voters are divided in such a way that red voters have a slim majority in three of the districts.

SOURCE: Azavea

Staff Graphic

Step 1: Establish the Probability Distribution

Operationalize rules for viable maps (*non-uniform distribution*):

- Identify a range of potential redistricting plans for the state – by encoding mathematised legal, policy, and spatial requirements into a scoring function.
- **Probability that a given map** – which satisfies the criteria – **will be added to sample:**
  Weighted likelihood that each map should be chosen to be part of the comparison sample.

$$\underbrace{P(\xi)}_{\text{Probability of Plan}} = \frac{1}{Z} e^{-\beta \overbrace{J(\xi)}^{\text{Plan Score}}}$$

- **Score Function:** $J(\xi) = \underbrace{a_0 J_{\text{compact}}(\xi) + a_1 J_{\text{VRA}}(\xi) + a_2 J_{\text{mun}}(\xi) + a_3 J_{\text{pop}}(\xi) + \dots}$
  ∑ of *independent factors.*

District Compactness Score

$$J_{\text{compact}}(\xi) = \underbrace{\left(\frac{\text{Perimeter}^2}{\text{Area}}\right) \geq 4\pi \approx 12.5}$$

Legislation

(e.g., Voting Rights Act 1965)

Communities of Interest

$$J_{\text{mun}}(\xi) = \#\ \text{Ousted People}$$

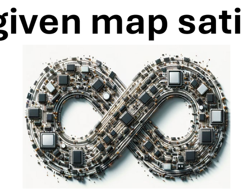
Equal Population Score

$$J_{\text{pop}}(\xi) = \sum_{i=1}^{n} (\text{pop}(\xi_i) - \text{pop}_{\text{ideal}})^2$$

Table 1. Standard Compactness Measures.

Measure	Compact	Non-compact
Reock		
Convex Hull		
Polsby-Popper		??

∑ All measures encoded to create **log-likelihood function** that indicates **how well a given map satisfies the criteria**.



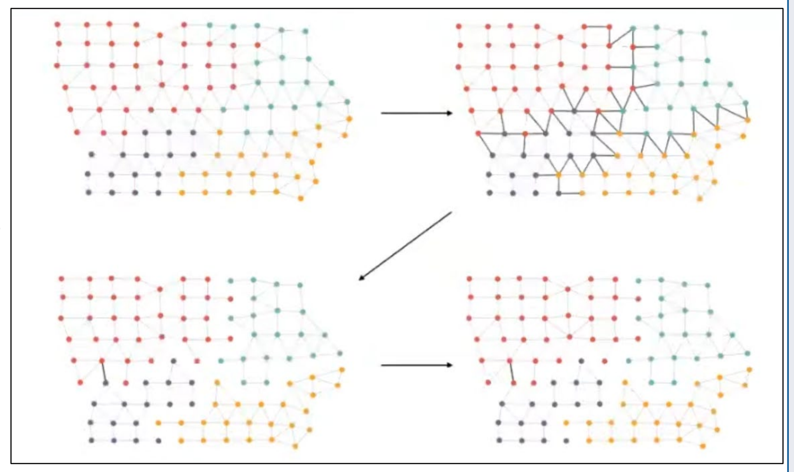
number of possibilities (*unsolvable*)

- There are more districting permutations than there are **atoms in the human body** (on average, 7 x 10^{27} atoms).

Step 2: Build Ensemble (sample collection of maps)

Sampling Algorithms: Advanced algorithms that chain functions to generate maps matching probability distribution.

Figure 4. Iowa Redistricting via Flip Markov Chain.



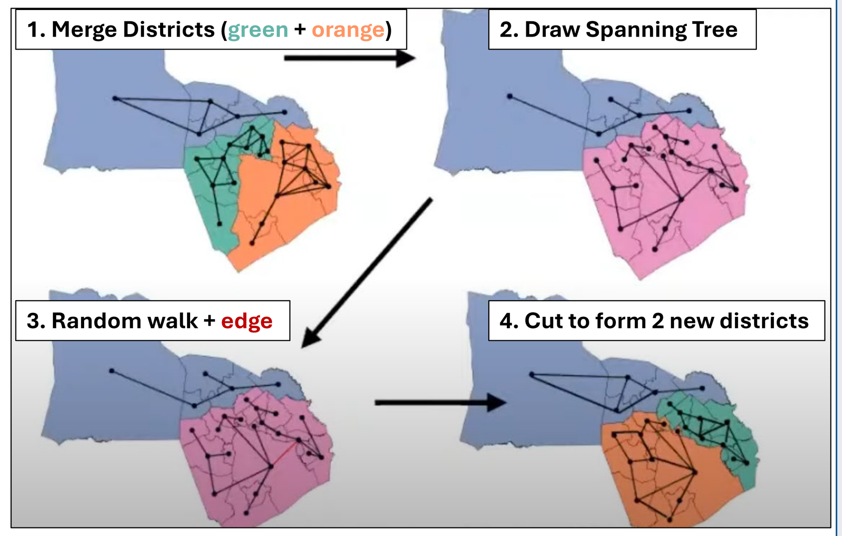
Single-node Ising (Flip MCMC):

- Flip the district of a boundary precinct, reassigning one node at a time. Repeat.
  - Most natural, simplistic (edge flipping), and explainable approach.
  - Allows for local adjustments to boundaries without altering full map.

Global Merge-Split Move (ReCom MCMC):

- Recombines two districts and then randomly splits them in two by cutting an edge of a spanning tree.

Figure 5. One Step of ReCom Markov Chain.



1. **Merge two districts** at random, or according to specified rule.
2. **Draw a Spanning Tree** – a tree that reaches all points within a graph, ensuring there's exactly one path between any two points.
3. **Randomly walk** across the tree, and find an edge (**red line**).
4. **Cut the edge**: two new districts.
   - Produces **compact districts** with efficiency, speed, and reduced computational requirements.
   - Adheres to traditional districting criteria without the need for user-defined thresholds – advantageous for legal applications. (23)

Acknowledgements

- *Dr. Jonathan Christopher Mattingly* [Duke University]: https://services.math.duke.edu/~jonm/
- *Quantifying Gerrymandering Group* [Lecture Series]: https://sites.duke.edu/quantifyinggerrymandering/

Step 3: Visualize & Analyze Results

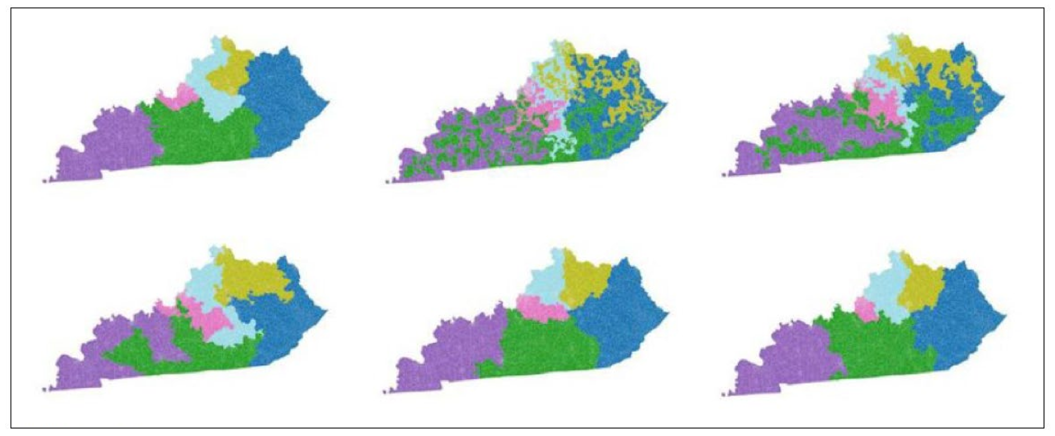
Results contingent on Distribution Type (Step 1):

- **Uniform Distribution** (*no criteria*): units have equal probability of selection.
  - e.g., Using Flip MCMC, equal chance of boundary unit being flipped.
- **Non-uniform Distribution:** non-equal probability configuration.
  - e.g., **ReCom** *explicitly* uses non-uniform distributions to consider complex constraints and goals (compactness, population balance etc.)

Flip MCMC (*non-uniform*): **produces non-compact, highly correlated ensembles**.

- Even when modified with simulated annealing (optimization), Flip MCMC produces negligible differences in output (boundary segments remain fixed).

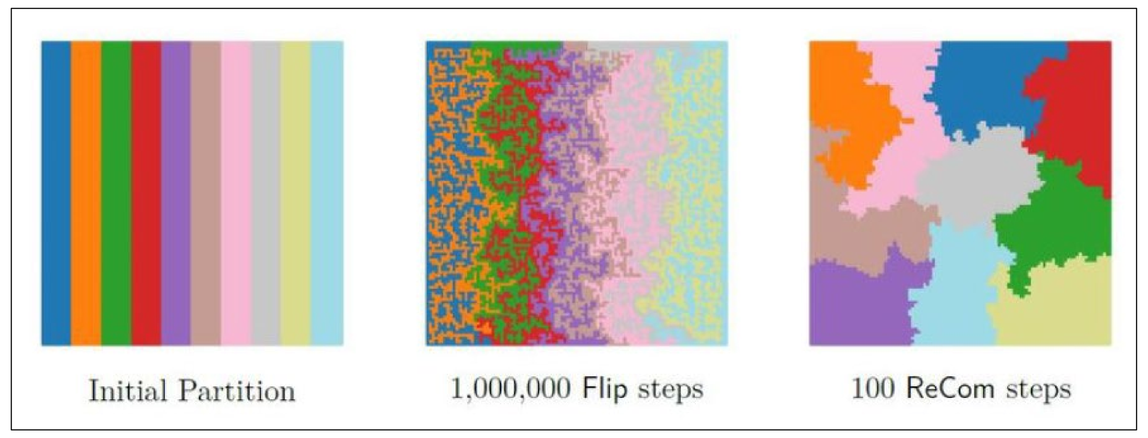
Figure 6. Flip MCMC: Tennessee and Kentucky samples after 100,000 steps.



SOURCE: DeFord, Duchin, and Solomon (2021)

ReCom (*non-uniform*): **produces compact, non-correlated ensembles.**

Figure 7. 1,000,000 Flip MCMC steps vs. 100 ReCom MCMC steps.



SOURCE: DeFord, Duchin, and Solomon (2021)

Broader Implications: Canadian Context

While deliberate cuts between electoral districts affect the outcome of elections, **systematic boundary *gerrymander-like* effects can still emerge** in the absence of controlled gerrymandering.

- The effects of gerrymandering on electoral representation, competitiveness, and outcomes can occur in single-member plurality systems **even when the parties have no control over the design** of the district boundaries (Cochrane, Ferraro, and Snider 2022).

Canadian district requirements that generate **gerrymander-like distortions**:

1. Boundaries should be drawn around *communities of interest*;
2. Redrawn as little as possible to accommodate population change;
3. Political parties' district targets differ (overlapping but different) – resulting in targeting efforts paying dividends for one party, while the other party fails.

Advances in computational redistricting methods hold promise for future research into Canadian boundary design effects.

References

1. Cochrane, Christopher, Stefan Ferraro, and Meghan Snider. "District Competitiveness, Representativeness, and the Short Game at the Margins in Recent Canadian Elections, 2015-2021," 1-24, (working paper 2022).
2. DeFord, Daryl, Moon Duchin, and Justin Solomon. 2021. "Recombination: A Family of Markov Chains for Redistricting." *Harvard Data Science Review* 3 (1). https://doi.org/10.1162 /99608f92.eb30390f.