Physics 4350 Computational Physics
Problem Set #5

For the first problem, provide a detailed written analysis with several graphs illustrating your results. Take this opportunity to demonstrate that you can use your code to research and analyze the chaotic behavior of a damped and driven nonlinear pendulum.

1. Study the shape of the "strange attractor" for the damped, driven, nonlinear pendulum for different initial conditions. Maintain the drive force at a fixed amplitude of $A_d = 1.2$ and calculate the attractors found for several different initial values of theta. Show that you obtain the same attractor even for different initial conditions, provided that these conditions are not changed by too much. Repeat your calculations for different values of the time step to be sure that it is sufficiently small that it does not cause any structure in the attractor. One set of parameters that will yield a chaotic plot are: $g = L = 1$, amplitude $A_d = 1.2$, frequency $f_d = 2/3$, and damping factor $q_d = 0.5$, although there are many others.

Programming notes:

Using the Euler-Cromer algorithm, the essential code for the oscillator is as follows.

```
accel  = -g_over_L*sin(theta)  -qd * omega + Ad * sin(fd * time);
omega  = omega + tau*accel;
theta  = theta + tau*omega;
```

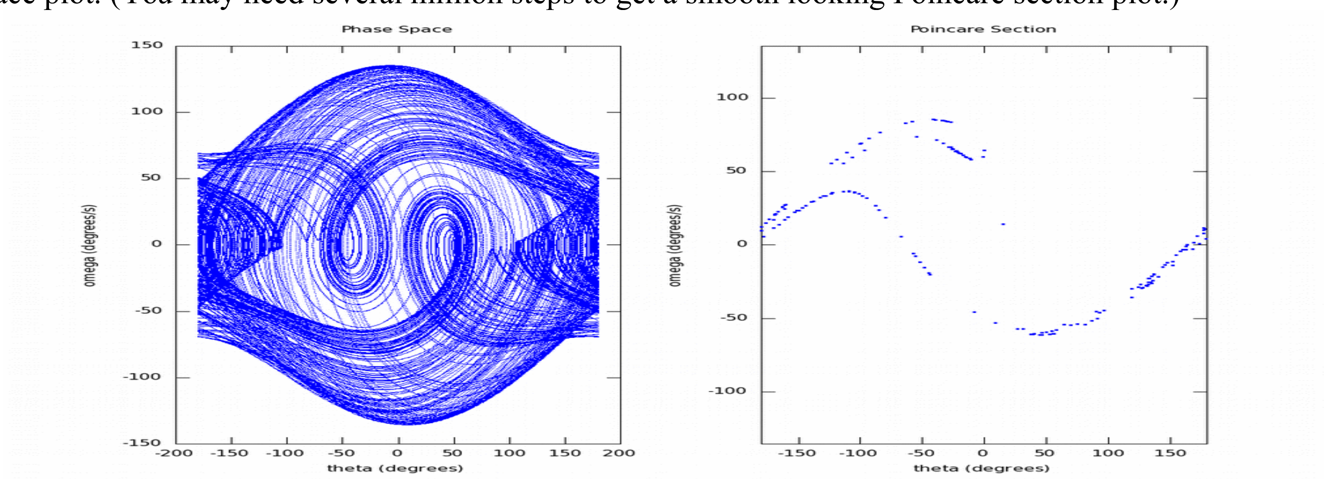
You may select points for the Poincare map using the following pseudo code. Note that pp*tau is the time, so fd*pp*tau is the argument to the driving term. Notice also the phase shift by 1/8 of a cycle.

```
if ( mod( fd*pp*tau – pi/4 , 2*pi ) < tau/2 )
       collect the relevant points for theta and omega plotting
endif
```

You may shift the points for theta to be in the interval [-180, 180] with the following code. The code shifts theta forward 180 degrees and then uses the "mod" function to map it to the interval [0,360], and then shifts it back to the interval [-180,180].

```
theta_bound = theta*180/pi+180;
theta_bound = mod(theta_bound,360) – 180;
```

The example graph below was generated using a step size of tau=0.01, theta0=11.46, A_0=1.2, f_d=2/3, and q_d=0.5. You may need to make your algorithm more efficient by not storing every data point for the phase space plot. (You may need several million steps to get a smooth looking Poincare section plot.)



2. Following the algorithm presented in class, write a matrix inverter using pivoting for an arbitrary NxN matrix. Demonstrate the use of the inverter for the following matrices:

(a) $\begin{pmatrix} 7 & 2 & 3 \\ 4 & 7 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ (c) $\begin{pmatrix} \text{1e-10} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 1 & 2 \end{pmatrix}$