Physics 4350 Computational Physics
Problem Set #4

1. Garcia, chapter 3, problem 3. Additional notes for the solution:
   (a) To show that your program confirms Kepler's third law, make a plot with a^3 on the x-axis and T^2 on the y-axis. You will need to calculate the period for several orbits to do this, and you must do a minimum of six orbits (data points).
   (b) For part (c), calculate the requested quantities for all your orbits in part (b).

2. (Orbital decay problem) An object in low Earth orbit (LEO) is about 350 km above the surface of the Earth. If a spacecraft is in a circular orbit, with no drag from the atmosphere, the tangential velocity is $\sqrt{GM/r}$. One orbit at 350 km is made in *about* an hour. This orbit distance in units of "earth radii" is 350/6371=0.0549. The orbital decay of the International Space Station is approximately 2 km per month, at an orbit height of about 350 km. This means that without an orbital boost, the height of the orbit would drop to 348 km in 30 days. The mass of the ISS is 370,000 kg, and it has an effective cross sectional area of about 500 m^2 and a C coefficient of about 0.8. Use the adaptive Runge-Kutta algorithm with an error parameter of 10^{-6} and interpolation for accurate orbit times, and answer the following questions.

   (a) What is the constant *GM* in units of "earth radii" and days?
   (b) How many orbits does the space station make without drag in 30 days? Give your result to two decimal places.
   (c) Modify the orbit program to include drag as given by Eq. 2.2 in the text:
   $$\vec{F}_a = \frac{-1}{2} C_d \rho A |\vec{v}| \vec{v}$$
   Calculate the orbital decay for various values of the density, and determine the density needed to obtain a decay of 2 km per month. What is the density in units of kg/m^3? *Note: for this problem, you may assume that the density does not change appreciably over the orbital drop in one month, i.e. the density is constant over the 2 km drop. Hint: It will be easiest to calculate the required drop for one orbit. Finally, you will have to program the constant GM in units of Earth radii and days. If you input an object in LEO of 350 km, it will be a height of 1.0549 in the appropriate units.*