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Physics 5345
Final Project

Analysis of the L4 Lagrange Point of the Planar Circular Restricted Three Body Problem

Introduction

Kepler's Laws can be easily derived by applying Newtonian gravity to two astronomical bodies; the well-known solution in the reference frame of one of the bodies limits the motion of the other body to conic sections and has been known since the 17th century. Applying Newtonian gravity to three or more bodies is a much more difficult problem, with only a few exact solutions being possible and those only occur in special cases such as all of the objects having identical masses. Poincaré's treatment of the problem in the late 19th century found that the three body problem admits what we now know as chaotic behavior. [1]

The most studied version of the problem uses a system of three masses. To simplify the system, one mass (the "particle") is assumed to be essentially zero, so that while it is affected by the gravitational fields of the other two masses (the "primaries"), it does not affect their motion (the restricted three body problem). Even more simplification can be done by giving all three masses initial velocities in the same plane, so that they stay in that plane throughout their motion, and by working in the reference frame that rotates with the center of mass so that the two large masses are stationary (the planar and circular restrictions). [1]

Even with all of these simplifications and restrictions, the system still exhibits highly complex behavior. For my project, I chose to examine the behavior of the system with the particle having its initial position at the L4 Lagrange point with a small initial velocity.

Lagrange Points

The restricted three body problem has five points at which the gravitational and centrifugal forces are in equilibrium, known as Lagrange points, named after Joseph-Louis Lagrange who discovered what are now known as the L4 and L5 Lagrange points. The L1, L2, and L3 points lie along the line connecting the two primaries; L1 is located between the primaries, L2 is located on the opposite side of the less massive primary, and L3 is located on the opposite side of the more massive primary. All three of these points are saddle points, being unstable along the line the connecting the primaries and stable in the orthogonal directions. [2]

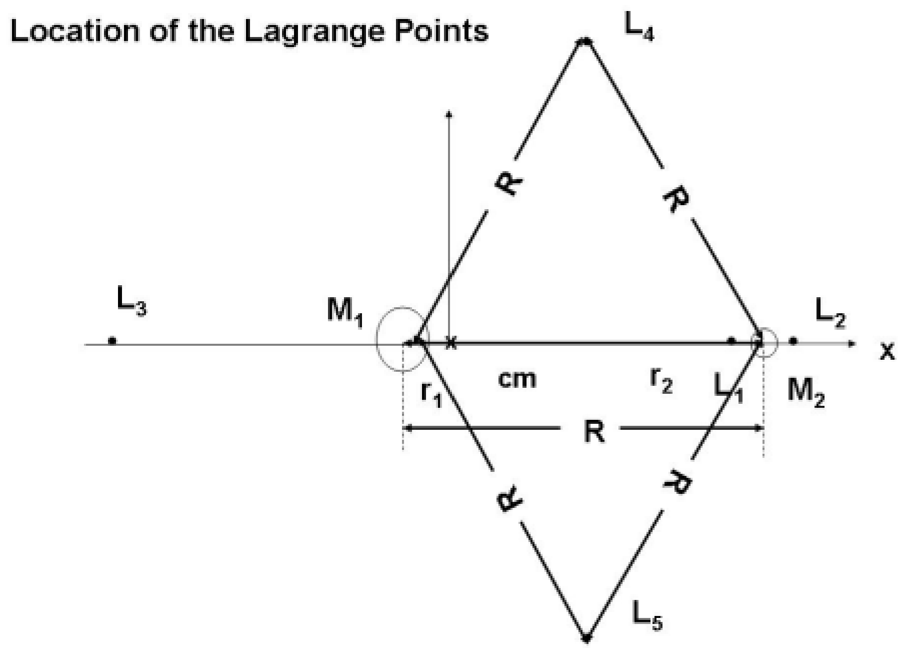


Figure 1: Location of the Lagrange Points [3]

The L4 and L5 points are at the vertex of equilateral triangles with the primaries located at the other two vertices. These points are semi-stable; that is, they are stable if the ratio of the masses of the primaries exceeds $25\left(\frac{1+\sqrt{1-4/625}}{2}\right) \approx 24.9599$. [2] Since the L4 and L5 points are symmetrical about the line connecting the primaries, the dynamics of the particle in their vicinities would be identical; therefore I only examined the L4 point since examining the L5 point would be redundant.

Normalization and Parameterization

Proper choice of units can greatly simplify the mathematics of the planar circular restricted three body problem. The unit of length is chosen so that the distance between the primaries is 1. Similarly, the unit of mass is chosen so that the total mass of the primaries is 1. Therefore, if the mass of one primary (M_1) is μ, the mass of the other primary (M_2) is 1-μ. Setting the center of mass at the origin requires that the position of M_1 be given by (-(1-μ),0), and the position of M_2 is given by (μ,0). [1]

A further simplification is made by choosing a unit of time so that the period of rotation of the primaries about their center of mass is 2π, which causes the angular velocity of the rotating reference frame to be 1. Furthermore, these choices of units causes the gravitational constant G to also be equal to 1. [1]

With these simplifications in place, the system can now be characterized by a single parameter, the ratio of the masses of the primaries, $c \stackrel{\text{def}}{=} \frac{M_1}{M_2}$. The values of the masses of the individual primaries as well as their positions can be derived from this, with $\mu = \frac{c}{c+1}$ and $1 - \mu = \frac{1}{c+1}$.

The general formula for the position of the L4 Lagrange point is given by $\left(\frac{R}{2}\left(\frac{M_1 - M_2}{M_1 + M_2}\right), \frac{\sqrt{3}}{2}R\right)$, where R is the distance between the two primaries [2]. In my normalized system, this reduces to $\left(\frac{1}{2}\left(\frac{c-1}{c+1}\right), \frac{\sqrt{3}}{2}\right)$.

Equations

By definition, the velocity of the particle in the x and y directions are given by $\dot{x} = v_x$ and $\dot{y} = v_y$, respectively. The acceleration of the system, however, is nonlinear and depends on both the position and velocity of the particle. There is a term for the force created by the gravitational field of each of the primaries; there is also a Coriolis term and a centrifugal term since the system is being formulated in a rotating reference frame. [4]

The equations for the system are:

$$\dot{x} = v_x$$

$$\dot{y} = v_y$$

$$\dot{v_x} = \frac{-M_1}{[(x + (1-\mu))^2 + y^2]^{3/2}}(x + (1-\mu)) + \frac{-M_2}{[(x-\mu)^2 + y^2]^{3/2}}(x-\mu) + 2v_y + x$$

$$\dot{v_y} = \frac{-M_1}{[(x + (1-\mu))^2 + y^2]^{3/2}}(y) + \frac{-M_2}{[(x-\mu)^2 + y^2]^{3/2}}(y) - 2v_x + y$$

Methods

The system was simulated in Matlab using the 4th order Runge-Kutta method. A program and associated functions found in Garcia (2000) [5] was modified for the purposes of mapping the trajectory of the particle, as well as mapping Poincaré sections, time series, and return maps. A program to look for possible bifurcations was constructed from scratch. All programs are given in the appendix.

Results

When I began working on this project, it was clear from the literature that the restricted three body problem does have chaotic solutions. However, the system is not chaotic for all mass ratios or initial conditions. It was not until late into the project that I came across a paper [6] that indicated that trajectories starting at or near the L4 point are not chaotic. I had hints that this might be the case from constructing a bifurcation diagram using the initial conditions

$(x, y, v_x, v_y) = \left(\frac{1}{2}\left(\frac{c-1}{c+1}\right), \frac{\sqrt{3}}{2}, 0.01, 0.01\right)$:

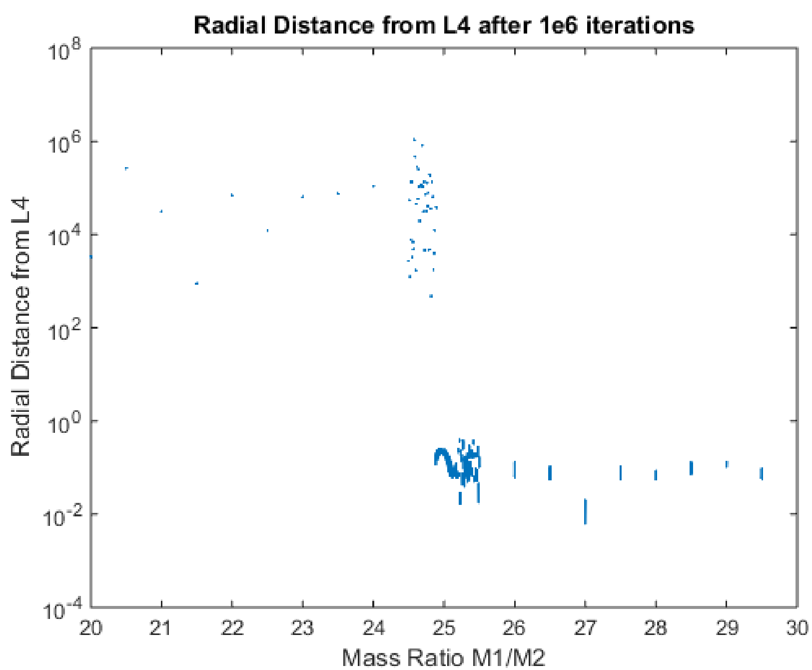


Figure 2: Bifurcation diagram run over $1X10^6$ time steps. The final 100 points are marked.

As you can see in figure 2, the bifurcation diagram does not show the type of bifurcation that would be expected of a system moving between a non-chaotic regime and a chaotic regime. For values of the mass ratio that are much smaller than the critical value of ≈24.9599, the particle is ejected from the system, although I will show further down that at least some of these trajectories contain chaotic transients. Since the particle is ejected from the system, it is not confined to a finite volume of phase space, which is a necessary condition for a system to be chaotic [7]. For values of the mass ratio that are less than 24.9599 but greater than about 24.6, the particle is in a nearly stable orbit around L4, but it is eventually ejected after a long enough time period (not shown in this figure). For values of the mass ratio above the critical value, the particle is in a stable orbit around L4; while these orbits can be quite complex, they are not chaotic, as I will demonstrate below. It should also be noted that there a finite number of initial conditions that will give a stable orbit, even when the mass ratio is above the critical value; for some initial conditions the particle is still ejected.

The remainder of this results section is an analysis of the behavior of the system under three different initial conditions for each of three different mass ratios. For all runs, the initial position of the particle was directly at the L4 Lagrange point, so only the initial velocities will be enumerated.

I. $\frac{M_1}{M_2} = 24$

a) $(v_x, v_y) = (0.01{,}0.01)$ ($2X10^4$ iterations)

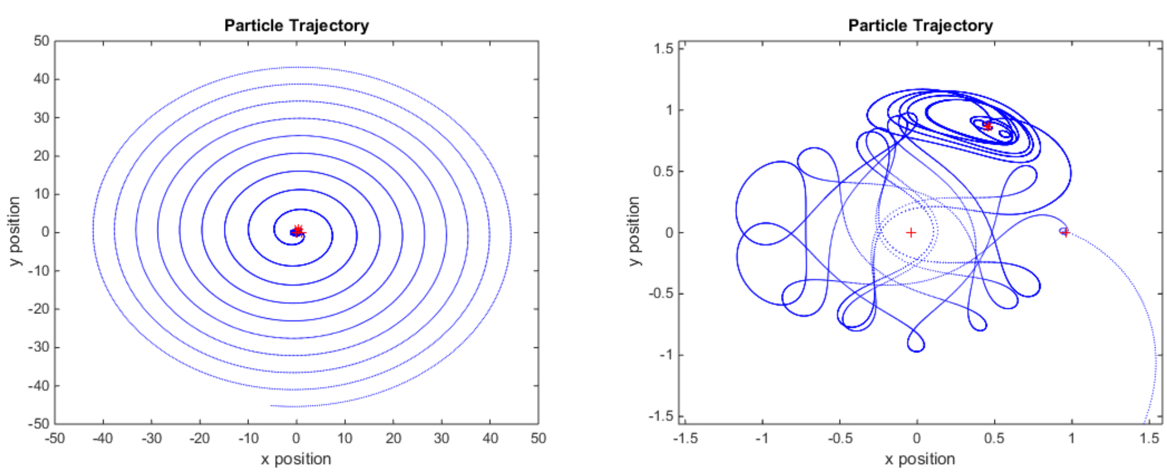


Figure 3: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of the primaries

While the particle does exhibit very complex motion at the beginning of its trajectory, this is only transient. It eventually settles into a clockwise spiral taking it out of the system entirely.

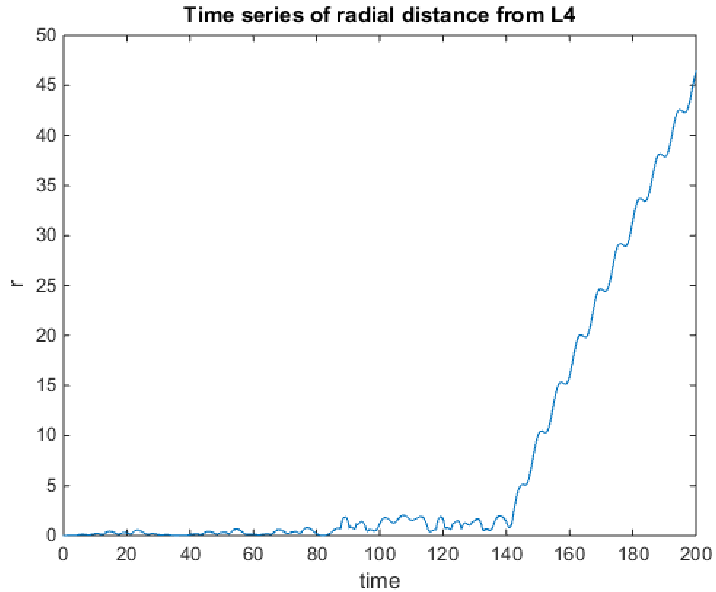
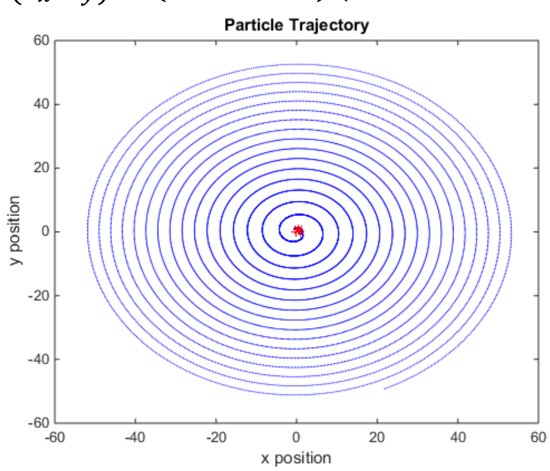


Figure 4: While the particle is initially near the L4 point, it begins to spiral outward starting at t≈140

b) $(v_x, v_y) = (-0.01, 0.01)$ (2X10[4] iterations)



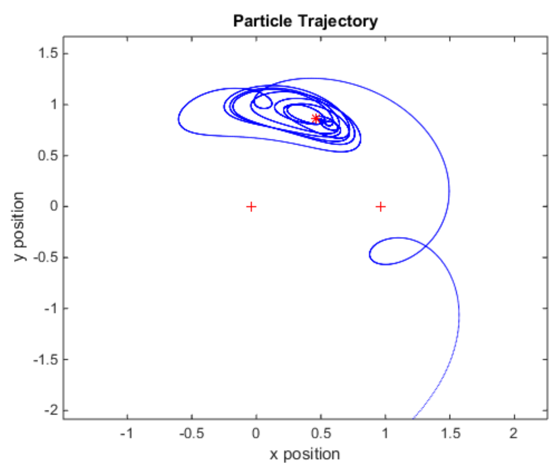


Figure 5: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of the primaries

Again, the particle is ejected from the system in a clockwise spiral. Note that the transient motion in the vicinity of the primaries is significantly different than in scenario I.a. Furthermore, as you can see in figure 6, the radial distance of the particle begins to increase in a manner consistent with ejection sooner than in scenario I.a. (t≈100 instead of t≈140).

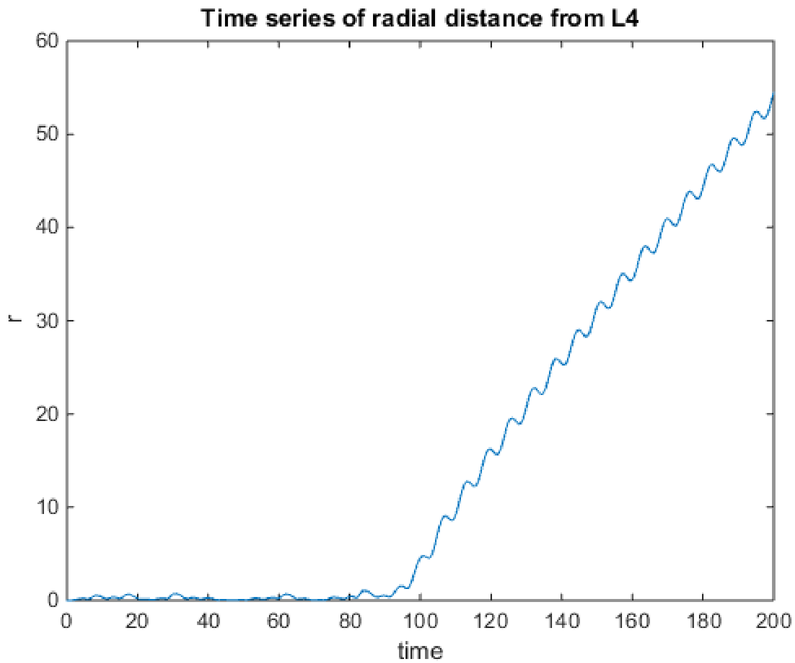


Figure 6: While the particle is initially near the L4 point, it begins to spiral outward starting at around t=140

c) $(v_x, v_y) = (0,0.01)$ ($4X10^4$ iterations)

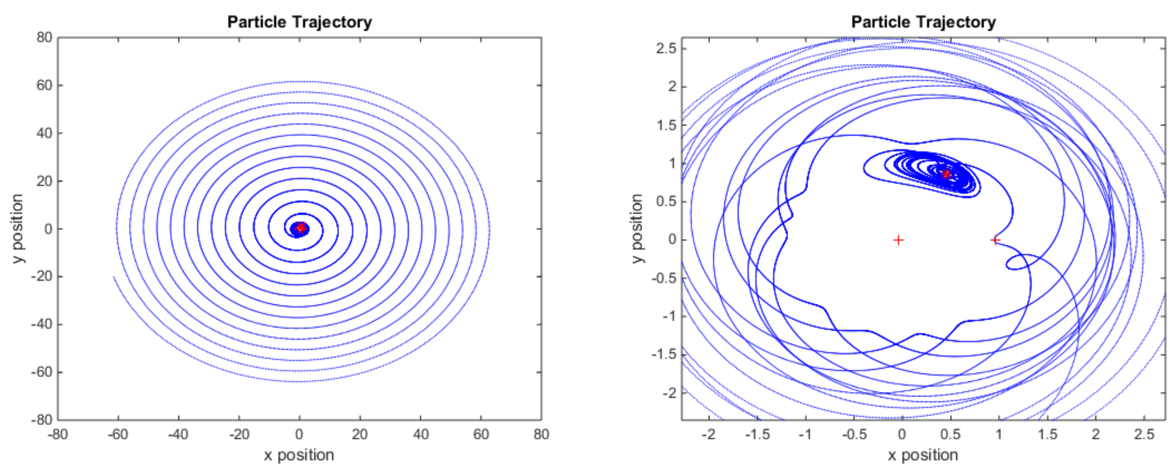


Figure 7: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of the primaries

Again, as the literature indicates, the particle that starts near the L4 point but with a mass ratio lower than the critical value is ejected. Under these initial conditions, however, it takes significantly longer, most likely due to the lower amount of initial kinetic energy.

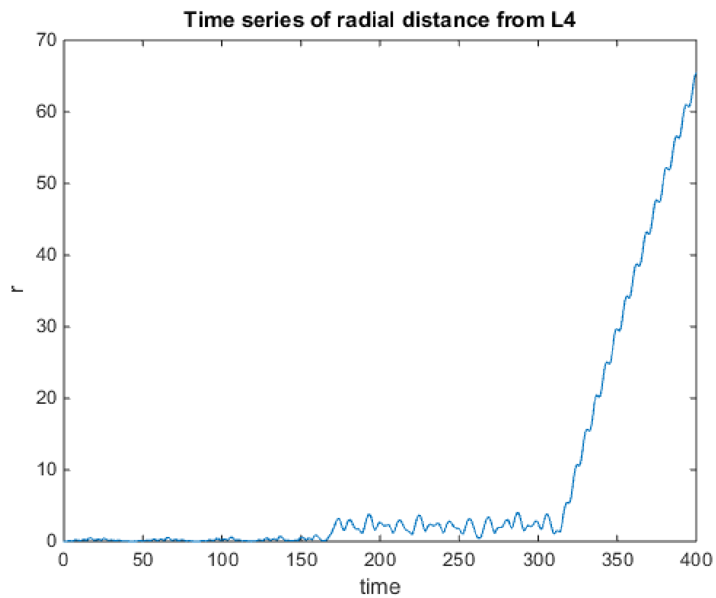


Figure 8: While the particle is initially near the L4 point, it begins to spiral outward starting at t≈310

II. $\frac{M_1}{M_2} = 24.9$

a) $(v_x, v_y) = (0.01, 0.01)$ (1X10^6 iterations)

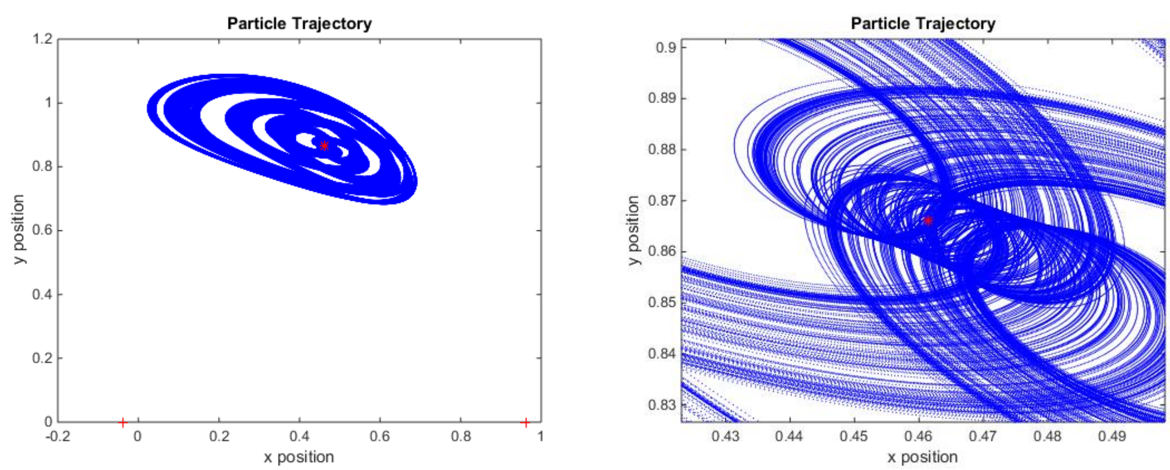


Figure 9: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of L4

The mass ratio of 24.9 is just under the critical value of ≈24.9599. If the system was going to exhibit chaotic behavior, it seems likely that this value of the parameter would be within that regime. And while the particle does exhibit a nearly periodic trajectory for more than 1X10^6 time steps, it is not truly chaotic as it is eventually ejected (not shown).

Even without running the program for the $>1.7X10^6$ time steps necessary for the particle to be ejected, analysis of the data shows that this trajectory is not chaotic.

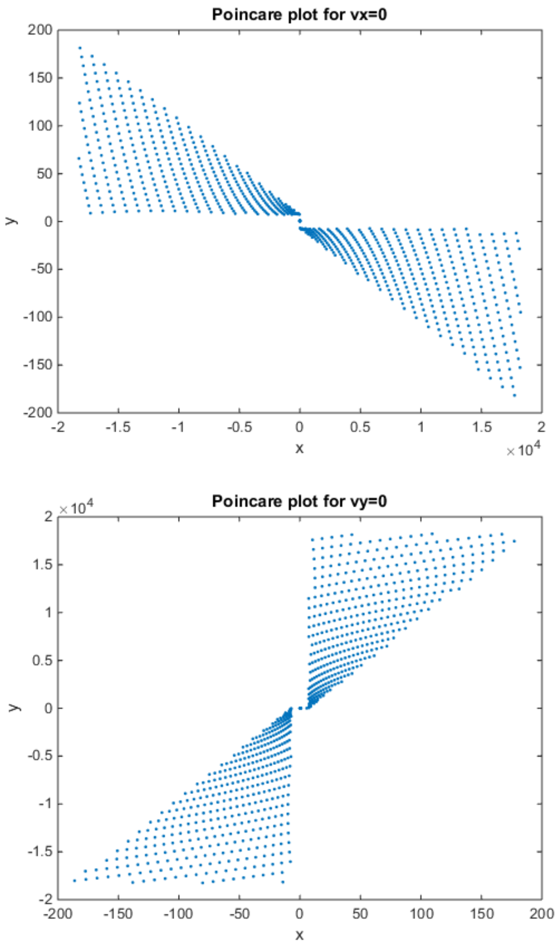


Figure 10: Poincaré plots: Upper Left: x and y plotted whenever $v_x=0$; Upper Right: y and v_y plotted whenever $v_x=0$; Bottom Left: x and y plotted whenever $v_y=0$; Bottom Right: x and v_x plotted whenever $v_y=0$

The Poincaré sections in figure 10 all show a regularity that would not occur if the behavior of the system was chaotic. Further evidence of the non-chaotic behavior is shown in figure 11, the return map for the radial distance from L4. Non-chaotic systems typically have return maps that are symmetrical around the 45° line, which is precisely what is seen.

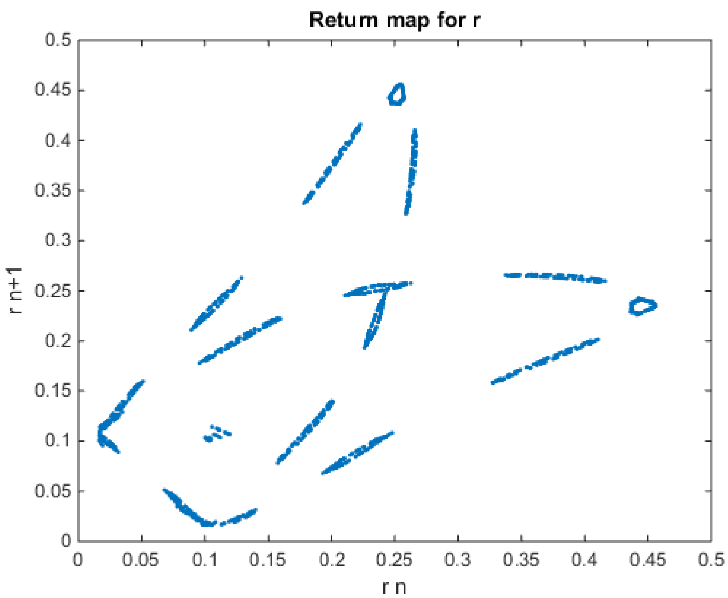


Figure 11: Return map of the radial distance from L4

b) $(v_x, v_y) = (-0.01, 0.01)$ ($1X10^6$ iterations)

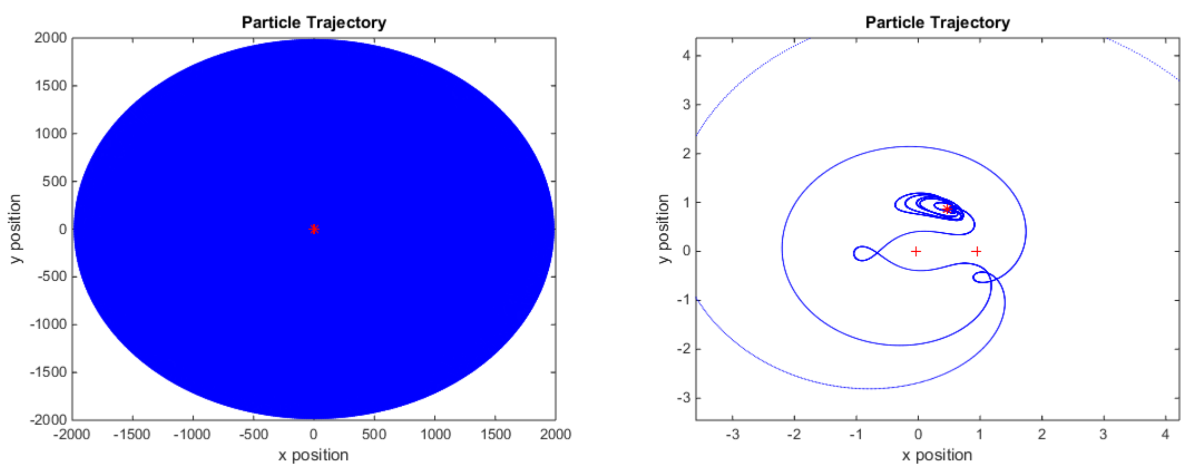


Figure 12: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of the primaries

Despite having the same amount of initial kinetic energy as the previous initial conditions, this trajectory is ejected much more quickly. Again, the Poincaré sections (figure 13) and return map (figure 14) show that the behavior is not chaotic.

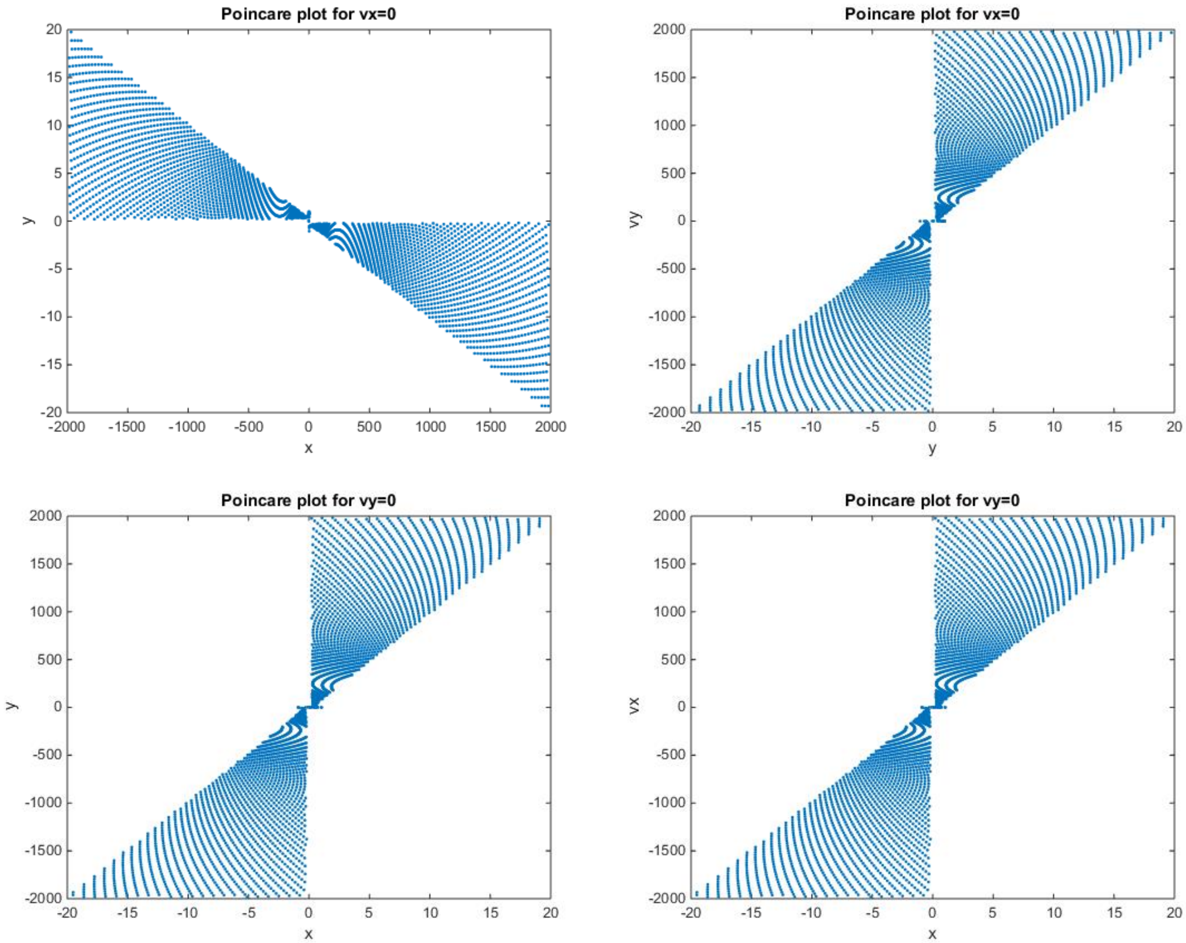


Figure 13: Poincaré plots: Upper Left: x and y plotted whenever v_x=0; Upper Right: y and v_y plotted whenever v_x=0; Bottom Left: x and y plotted whenever v_y=0; Bottom Right: x and v_x plotted whenever v_y=0

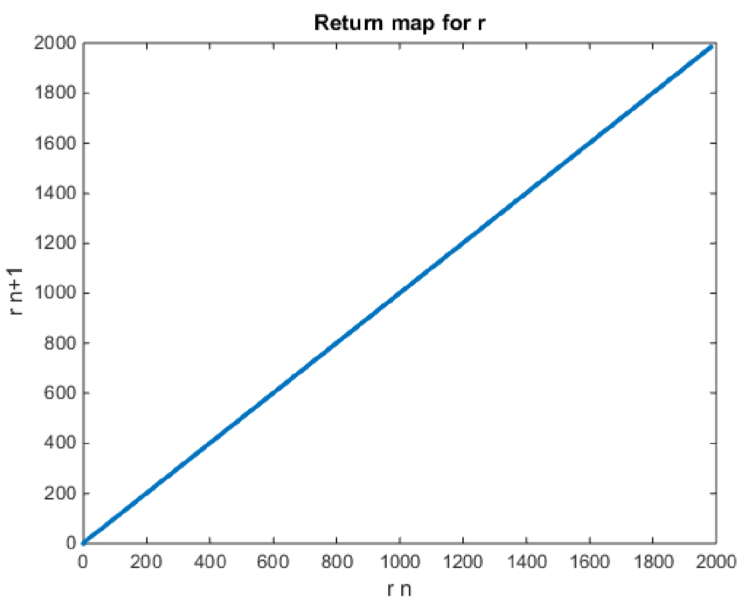


Figure 14: Return map of the radial distance from L4

c) $(v_x, v_y) = (0,0.01)$ ($1X10^6$ iterations)

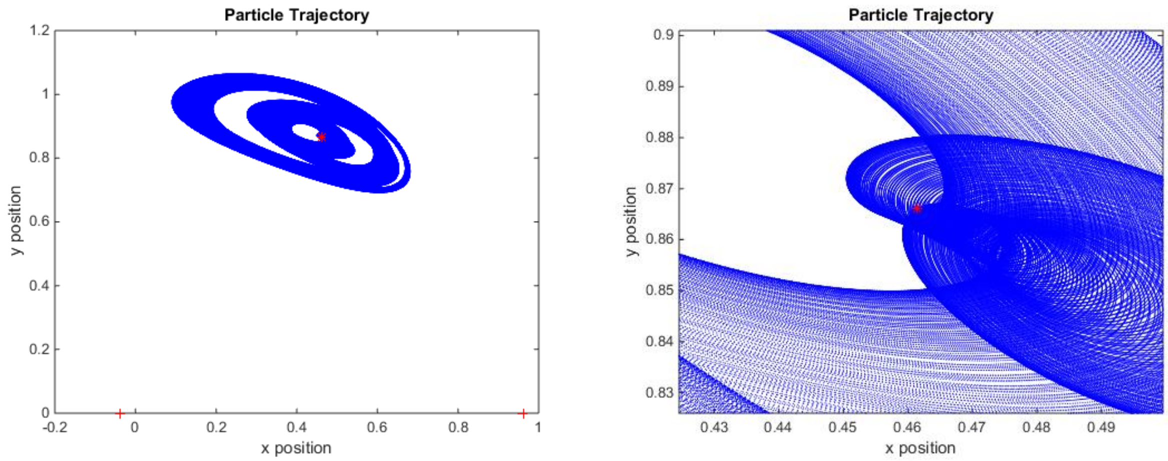


Figure 15: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of L4

Similar to II.a., $1X10^6$ time steps was not long enough to see the particle ejected from the system. However, just as in II.a. and II.b., the Poincaré sections and return map dispel the possibility that the system is in a chaotic state.

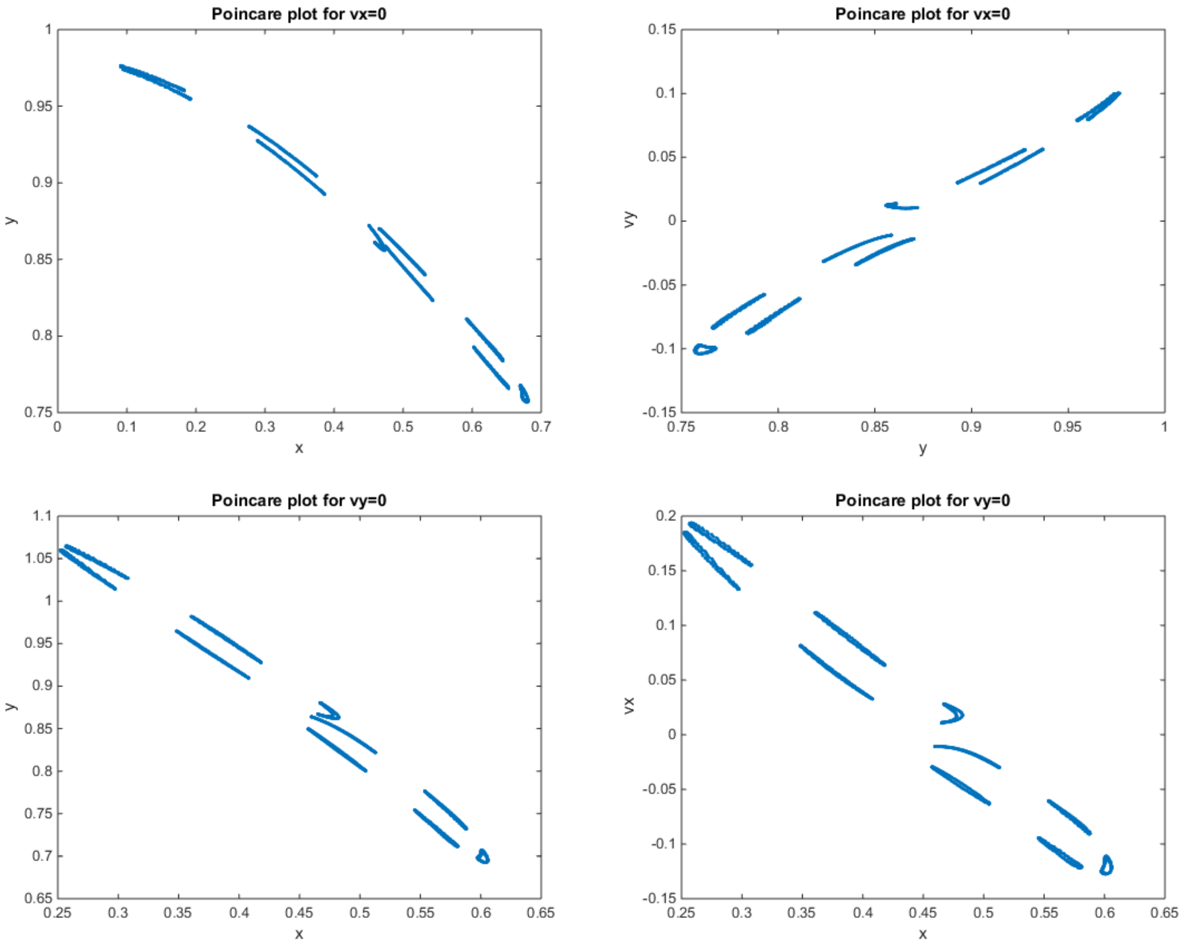


Figure 16: Poincaré plots: Upper Left: x and y plotted whenever v_x=0; Upper Right: y and v_y plotted whenever v_x=0; Bottom Left: x and y plotted whenever v_y=0; Bottom Right: x and v_x plotted whenever v_y=0

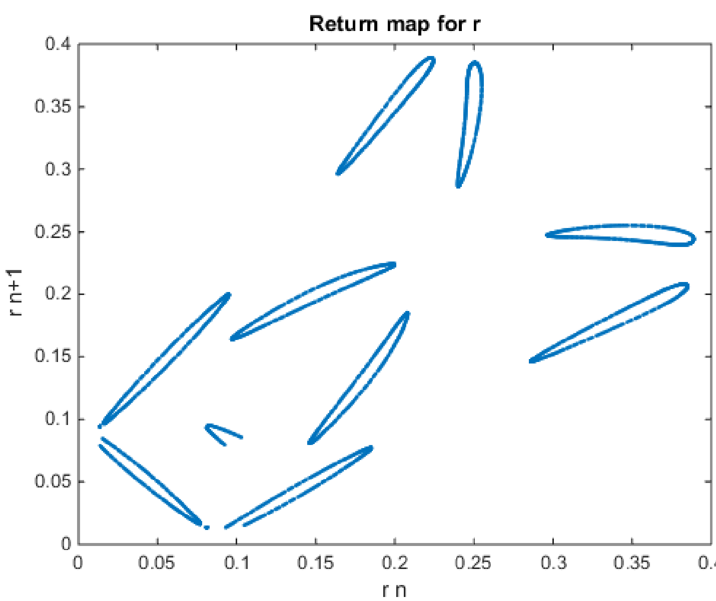


Figure 17: Return map of the radial distance from L4

III. $\frac{M_1}{M_2} = 30$

a) $\left(v_x, v_y\right) = (0.01,0.01)$ ($1X10^6$ iterations)

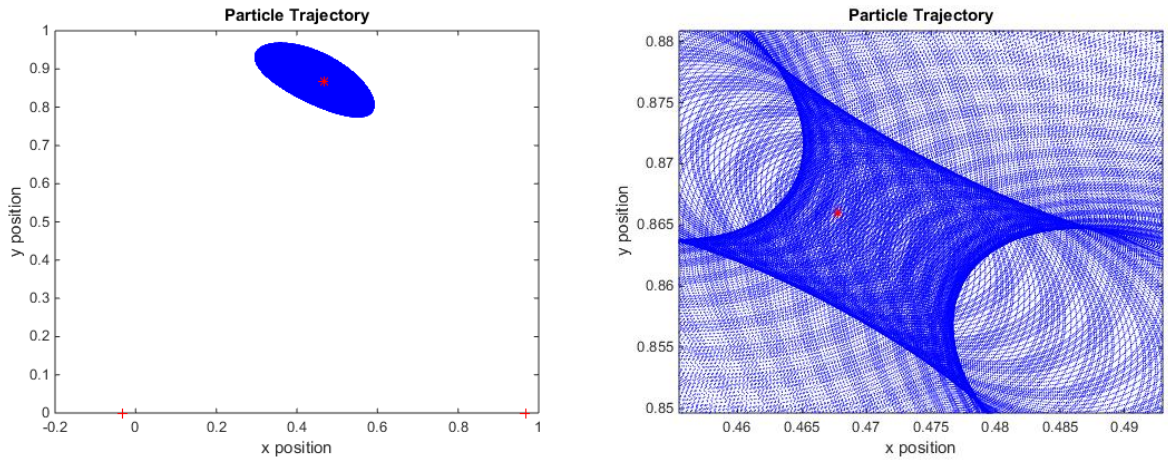


Figure 18: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of L4

Under these conditions, the particle is confined to a finite volume of phase space, which is one of the necessary conditions for chaos. However, again this possibility is dispelled by the Poincaré sections and return maps.

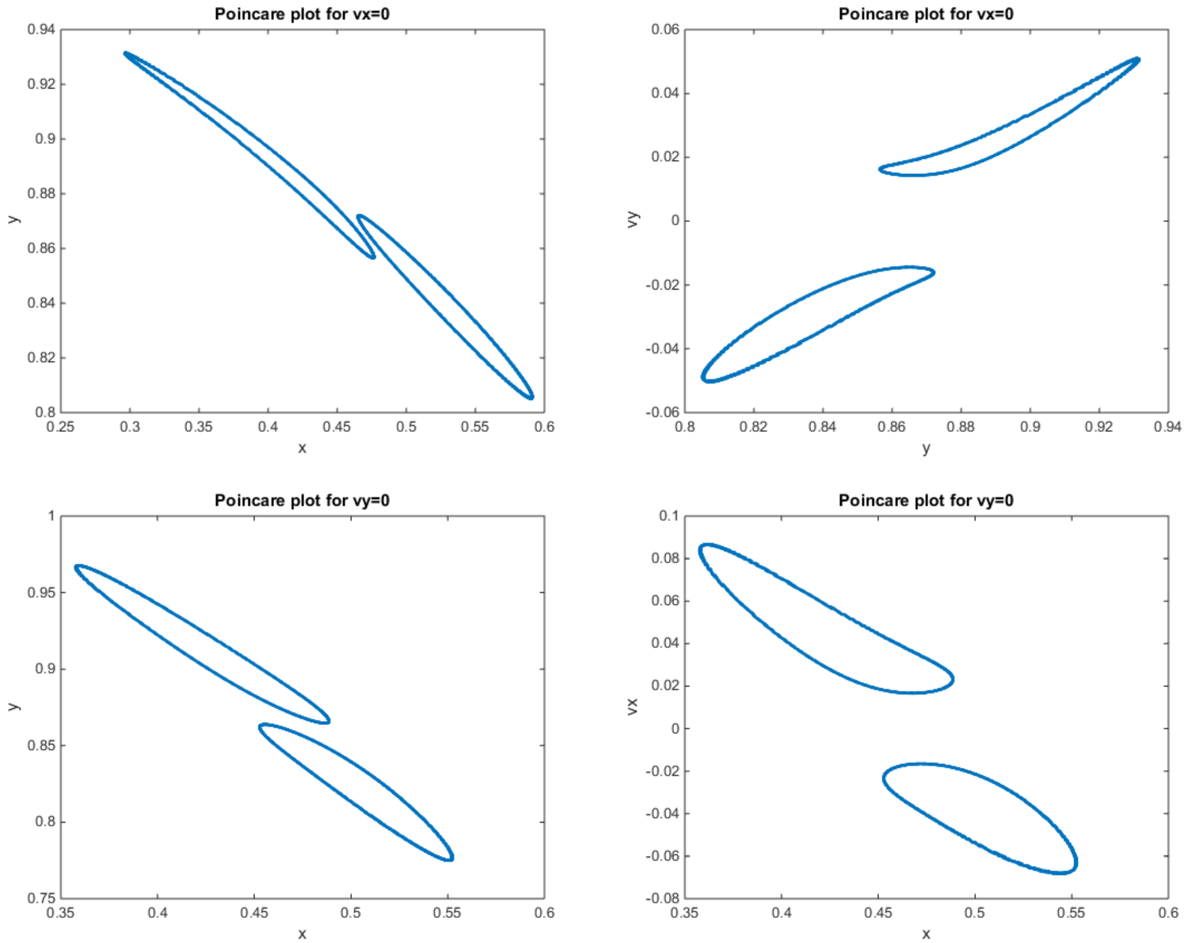


Figure 19: Poincaré plots: Upper Left: x and y plotted whenever v_x=0; Upper Right: y and v_y plotted whenever v_x=0; Bottom Left: x and y plotted whenever v_y=0; Bottom Right: x and v_x plotted whenever v_y=0

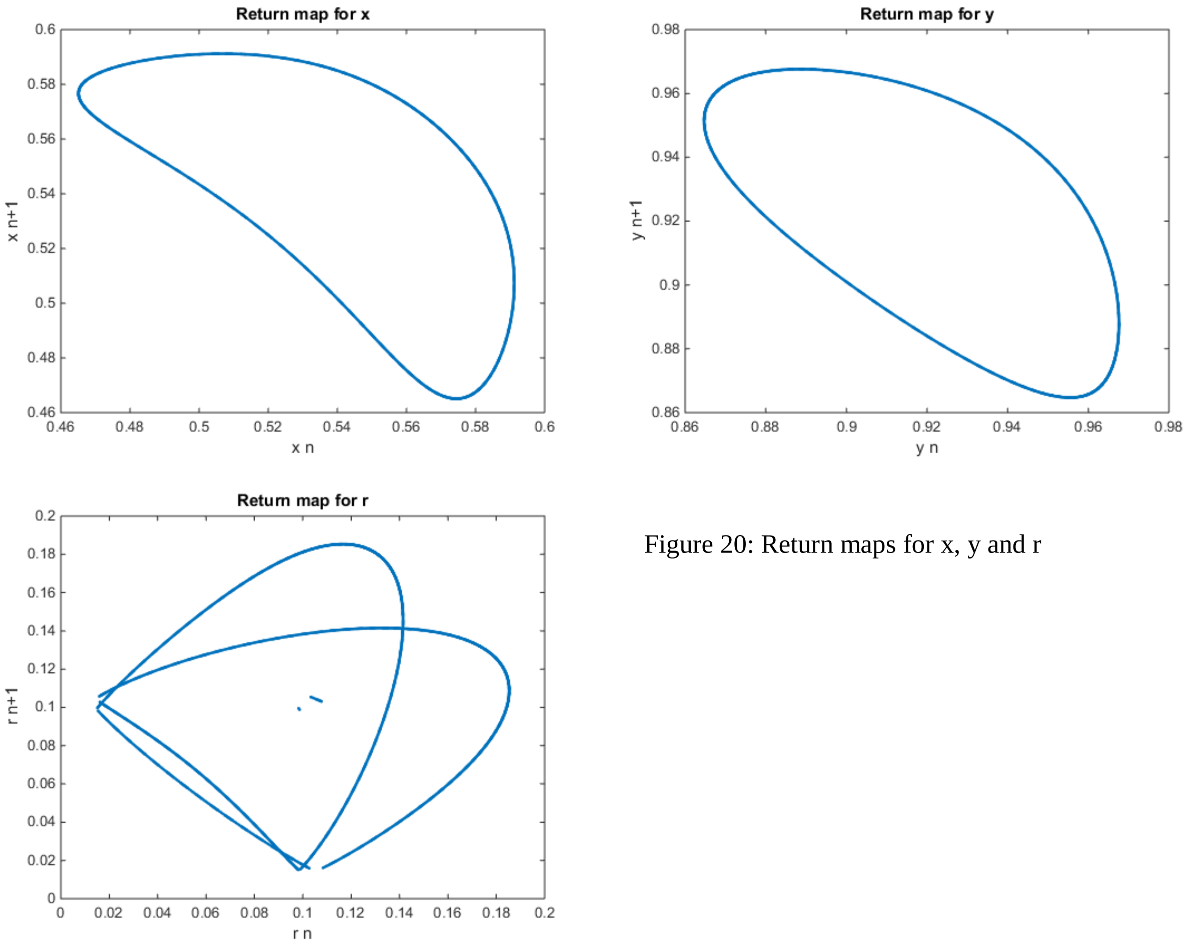


Figure 20: Return maps for x, y and r

What is particular interesting about the return maps for these conditions is that the x and y maps form closed loops, which would be expected for periodic motion. I have also included the time series for the x and y position as well as the radial distance from L4 (figure 21), and highlighted areas where the pattern appears to repeat.

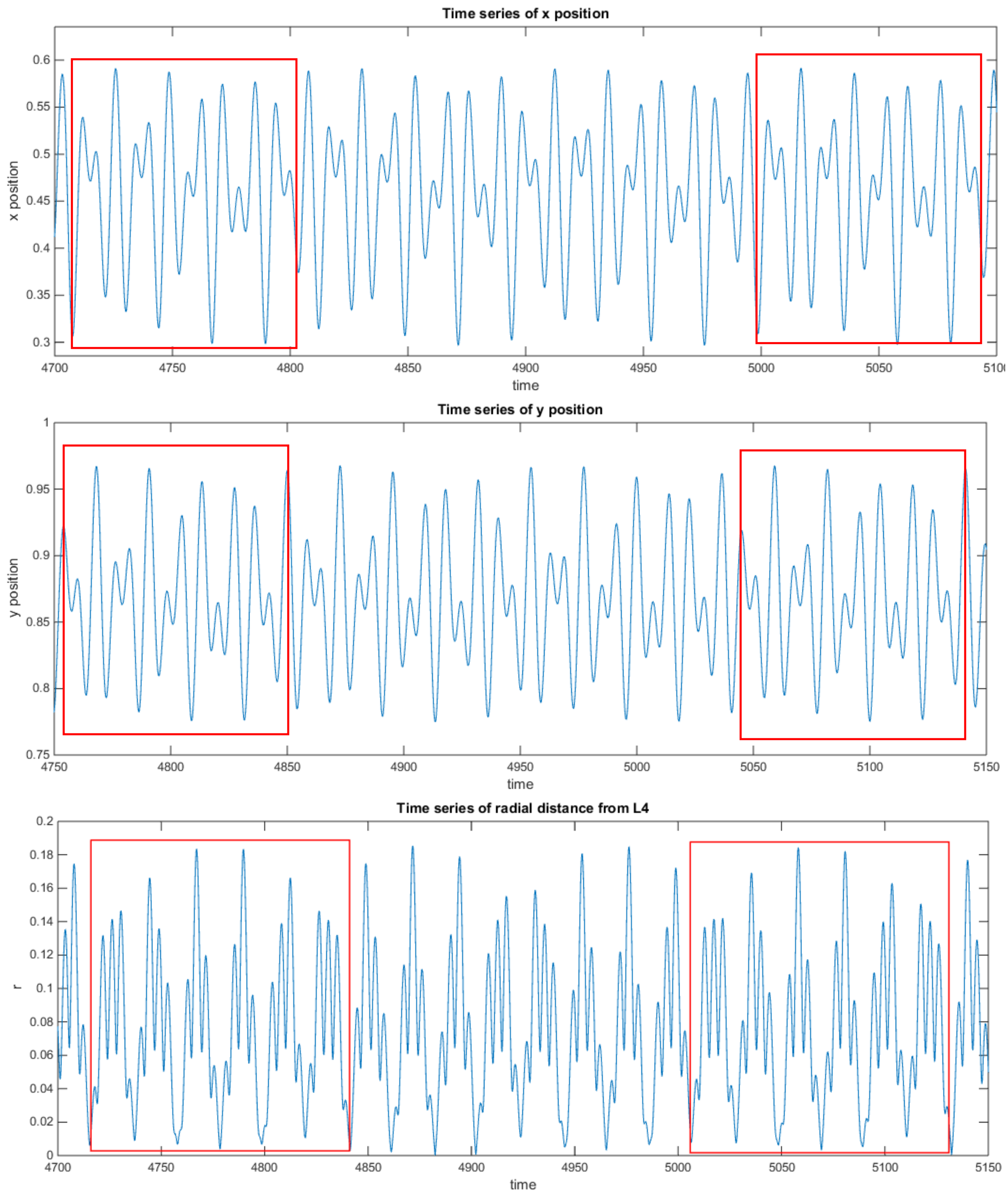


Figure 21: A portion of the time series for the position measurements. Top: Time series for x, highlighted area shows that pattern repeats; Middle: Time series for y, highlighted area shows that pattern repeats; Bottom: Time series for r, highlighted area shows that pattern repeats

Since these graphs show periodicity in the position measurements, it cannot be chaotic [7].

b) $(v_x, v_y) = (-0.01, 0.01)$ (1X10^6 iterations)

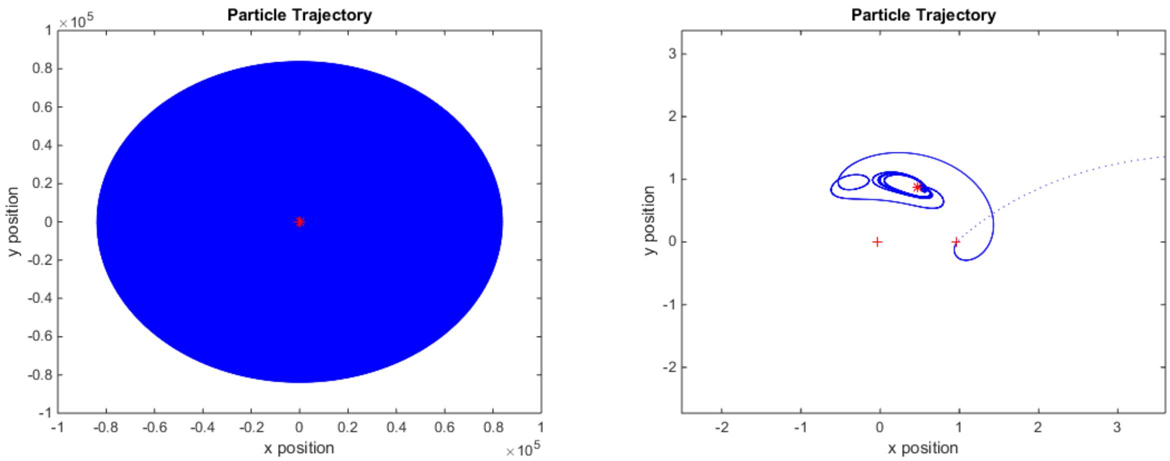


Figure 22: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of L4

Surprisingly, this choice of initial conditions leads to the particle being ejected, despite the L4 point being ostensibly stable for this mass ratio. Of course, it is not surprising that not every orbit in the vicinity of L4 is stable, as it should be obvious that objects with large initial velocities would not settle into stable orbits. However, these initial conditions have the same amount of initial kinetic energy as those in III.a. Even more striking is the fact that the time series of the radial distance from L4 (figure 23) shows that the particle is ejected relatively quickly, even more quickly than for the unstable mass ratio of 24.

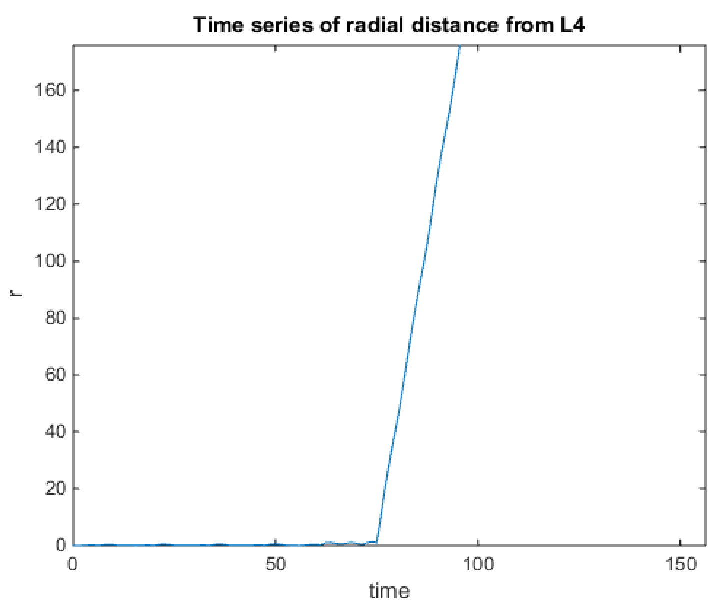


Figure 23: While the particle is initially near the L4 point, it begins to spiral outward starting at t≈75

Sensitivity to initial conditions is one of the criteria for chaos, but again, since the particle is ejected from the vicinity of the primary masses, this is not a chaotic regime.

c) $\left(v_x, v_y\right) = (0,0.01)$ ($1X10^6$ iterations)

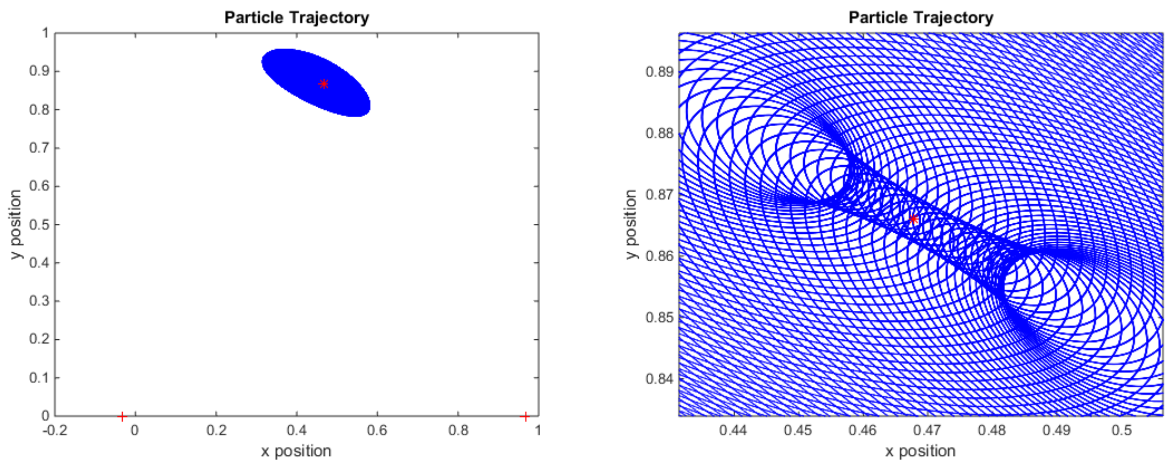


Figure 24: Left: total trajectory of the particle; Right: trajectory zoomed in around the vicinity of L4

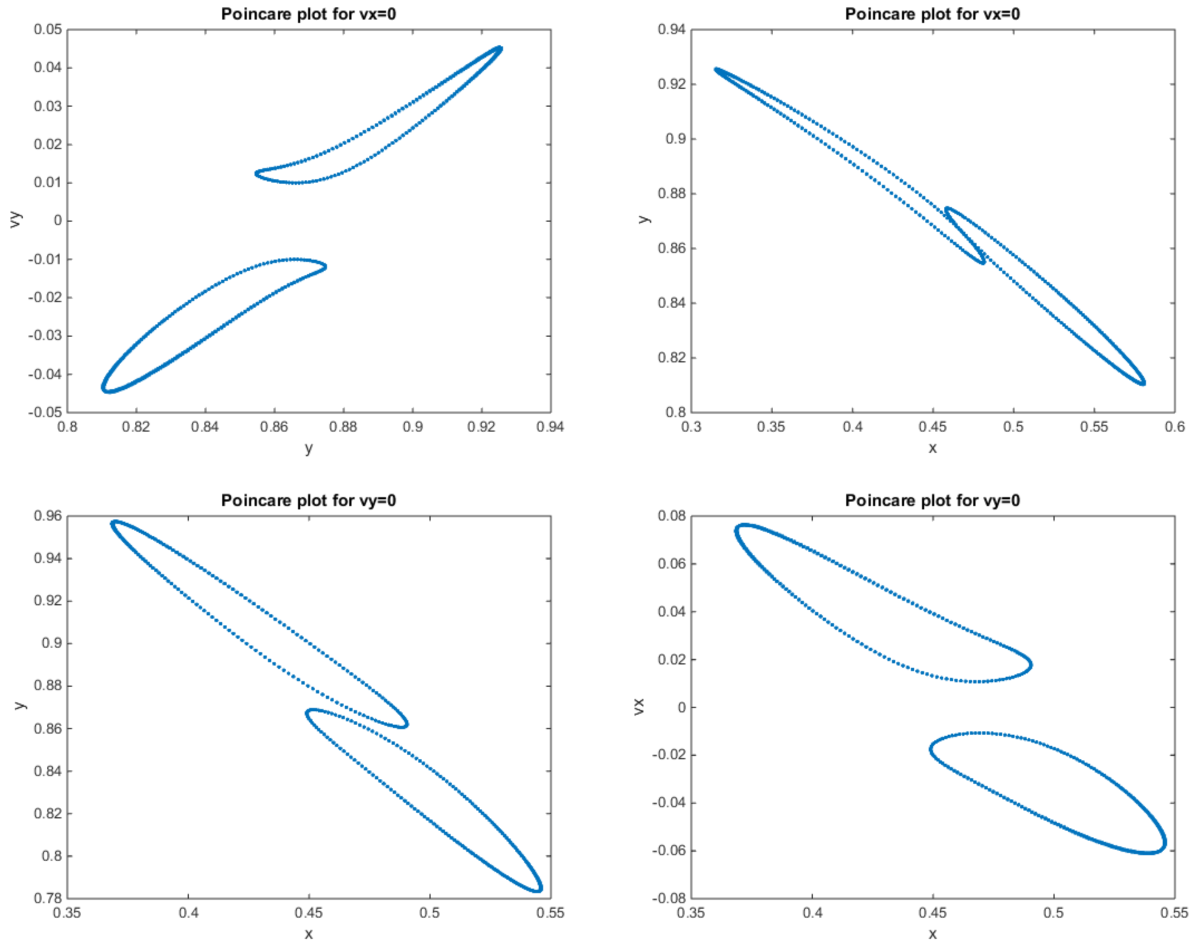
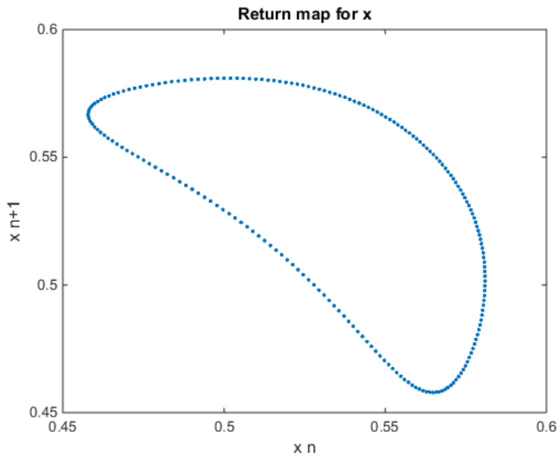
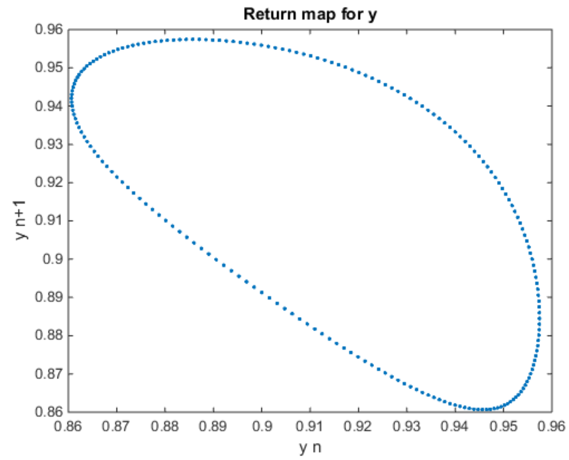


Figure 25: Poincaré plots: Upper Left: x and y plotted whenever v_x=0; Upper Right: y and v_y plotted whenever v_x=0; Bottom Left: x and y plotted whenever v_y=0; Bottom Right: x and v_x plotted whenever v_y=0





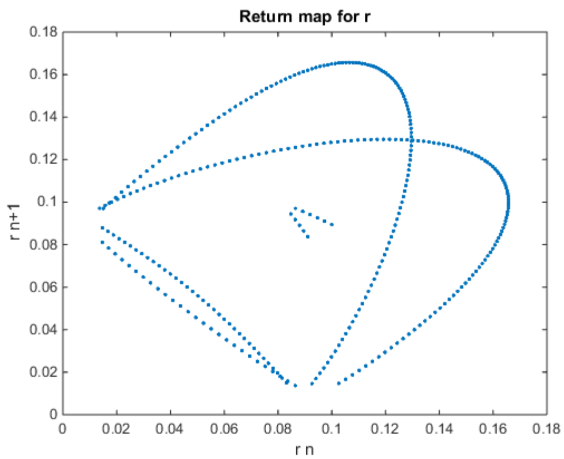


Figure 26: Return maps for x, y and r

Again, the closed loops of the return maps indicate that the particle's motion is periodic. Unfortunately, I was unable to determine over what time scale the motion was periodic. Given the fewer number of points on the return map, I suspect that the time scale is longer than it was for III.a., and it becomes difficult to see the pattern in the peaks when the length of the time axis being analyzed increases. Nevertheless, I am confident that the system is again periodic, eliminating the possibility of chaotic motion.

Conclusions

The planar circular restricted three body problem is complex system that will, under some conditions, exhibit chaotic behavior. Unfortunately, I chose to focus my analysis on regimes where the system is not chaotic. For all of the values of the parameters and initial conditions tested, the particle was either ejected from the system of settled into a periodic orbit; neither of those outcomes is chaotic. Given more time, I would have analyzed some other areas besides those near the L4 Lagrange point.

References

[1] Worthington, J. "*A Study of the Planar Circular Restricted Three Body Problem and the Vanishing Twist*," B.S. thesis, University of Sydney (2012).

[2] Cornish, N.J, "*The Lagrange Points*," WWW Document, (http://www.physics.montana.edu/faculty/cornish/lagrange.pdf) (2012).

[3] Widnall, S., "Exploring the Neighborhood: the Restricted Three-Body Problem," Lecture, Massachusetts Institute of Technology (2008).

[4] Marion, J.B., Thornton, S.T, *Classical Dynamics of Particles and Systems*, 4^{th} Ed. (Harcourt Brace & Company, USA, 1995), pp. 381-387.

[5] Garcia, A.L., Computer Programs (Matlab) **orbit.m**, **rk4.m**, and **gravrk.m**, *Numerical Methods for Physics*, 2^{nd} Ed., (Prentice-Hall, NJ, 2000), pp. 91-93.

[6] Wanex, L.F. *"Chaotic Amplification in the Relativistic Restricted Three-body Problem,"* Z. Naturforsch. **58a**, 13 – 22 (2003).

[7] Strogatz, S.H., *Nonlinear Dynamics and Chaos*, (Perseus Books Publishing, LLC, USA, 1994), pp. 323-324.

Appendix: Matlab code

The program **threebody.m** and the functions **gravrk.m** and **rk4.m** were modified from programs found in Garcia [5].

Function gravrk.m

```
function deriv = gravrk(s,t,M1,M2,rM1,rM2)
%  Returns right-hand side of Kepler ODE; used by Runge-Kutta routines
%  Inputs
%    s              State vector [r(1) r(2) v(1) v(2)]
%    t              Time (not used)
%    M1,M2,rM1,rM2  Parameters related to the primary masses
%  Output
%    deriv  Derivatives [dr(1)/dt dr(2)/dt dv(1)/dt dv(2)/dt]

%* Compute acceleration
r = [s(1) s(2)];  % Unravel the vector s into position and velocity
v = [s(3) s(4)];
r1=r-rM1;
r2=r-rM2;
accel = -M1*r1/norm(r1)^3-M2*r2/norm(r2)^3-2*[-v(2),v(1)]+r;

%* Return derivatives [dr(1)/dt dr(2)/dt dv(1)/dt dv(2)/dt]
deriv = [v(1) v(2) accel(1) accel(2)];
return;
```

Function rk4.m

```
function xout = rk4(x,t,tau,derivsRK,M1,M2,rM1,rM2)
%  Runge-Kutta integrator (4th order)
% Input arguments -
%   x = current value of dependent variable
%   t = independent variable (usually time)
%   tau = step size (usually timestep)
%   derivsRK = right hand side of the ODE; derivsRK is the
%              name of the function which returns dx/dt
%              Calling format derivsRK(x,t,M1,M2,rM1,rM2).
%   M1,M2,rM1,rM2 = extra parameters passed to derivsRK
% Output arguments -
%   xout = new value of x after a step of size tau
half_tau = 0.5*tau;
F1 = feval(derivsRK,x,t,M1,M2,rM1,rM2);
t_half = t + half_tau;
xtemp = x + half_tau*F1;
F2 = feval(derivsRK,xtemp,t_half,M1,M2,rM1,rM2);
xtemp = x + half_tau*F2;
F3 = feval(derivsRK,xtemp,t_half,M1,M2,rM1,rM2);
t_full = t + tau;
xtemp = x + tau*F3;
F4 = feval(derivsRK,xtemp,t_full,M1,M2,rM1,rM2);
xout = x + tau/6.*(F1 + F4 + 2.*(F2+F3));
return;
```

Program threebody.m

```
clear all;
%Masses of the two primary masses
c = input('Enter the ratio of the primary masses M1/M2: ');
mu = c/(1+c);
M1 = mu;
M2 = 1-mu;
%Positions of the two primary masses
rM1 = [-(1-mu),0];
rM2 = [mu,0];
%Set initial position and velocity of the object
r = [.5*(c-1)/(c+1),sqrt(3)/2];
v = [.01,.01];
state = [ r(1)  r(2)  v(1)  v(2) ];    % Used by RK4 routine
%Initialize time and counting indices
time = 0;
i=1;
j=1;
k=1;

%MAIN PROGRAM
%Loop over desired number of steps
tau = .01;
nStep = input('Enter the number of steps to calculate: ');
xplot=zeros(1,nStep);
yplot=zeros(1,nStep);
vxplot=zeros(1,nStep);
vyplot=zeros(1,nStep);
rplot=zeros(1,nStep);
tplot=zeros(1,nStep);
```

```
for iStep=1:nStep
%* Record values for plotting
xplot(iStep) = r(1);
yplot(iStep) = r(2);
vxplot(iStep) = v(1);
vyplot(iStep) = v(2);
tplot(iStep) = time;
rplot(iStep) = norm(r-[.5*(c-1)/(c+1),sqrt(3)/2]);

%Poincare section - plot position and y velocity every time vx=0
if (iStep>1)
if (vxplot(iStep)*vxplot(iStep-1)<0)
      xPxplot(j)=r(1);
      xPyplot(j)=r(2);
      xPvyplot(j)=v(2);
      j=j+1;
end
end
%Poincare section - plot position and x velocity every time vy=0
if (iStep>1)
if (vyplot(iStep)*vyplot(iStep-1)<0)
      yPxplot(k)=r(1);
      yPyplot(k)=r(2);
      yPvxplot(k)=v(1);
      k=k+1;
end
end

%* Calculate new position and velocity using RK4.
state = rk4(state,time,tau,'gravrk',M1,M2,rM1,rM2);
r = [state(1) state(2)];
v = [state(3) state(4)];
time = time + tau;

end

figure(1);
plot(xplot,yplot,'b.','MarkerSize',1);
hold on;
plot(rM1(1),rM1(2),'r+',rM2(1),rM2(2),'r+',xplot(1),yplot(1),'r*');
xlabel('x position'); ylabel('y position');
title('Particle Trajectory');

figure(2);
plot(xPxplot,xPyplot,'.');
xlabel('x'); ylabel('y');
title('Poincare plot for vx=0');

figure(3);
plot(xPyplot,xPvyplot,'.');
xlabel('y'); ylabel('vy');
title('Poincare plot for vx=0');

figure(4);
plot(yPxplot,yPyplot,'.');
```

```
xlabel('x'); ylabel('y');
title('Poincare plot for vy=0');

figure(5);
plot(yPxplot,yPvxplot,'.');
xlabel('x'); ylabel('vx');
title('Poincare plot for vy=0');

figure(6);
plot(tplot,xplot);
xlabel('time'); ylabel('x position');
title('Time series of x position');

figure(7);
plot(tplot,yplot);
xlabel('time'); ylabel('y position');
title('Time series of y position');

figure(8);
plot(tplot,rplot);
xlabel('time'); ylabel('r');
title('Time series of radial distance from L4');

figure(9);
n=1;
for i=2:(nStep-1)
if (xplot(i)>xplot(i-1)&&xplot(i)>xplot(i+1))
      retx(n)=xplot(i);
      n=n+1;
end
end
retx1=retx(1:end-1);
retx2=retx(2:end);
plot(retx1,retx2,'.');
xlabel('x n'); ylabel('x n+1');
title('Return map for x');

figure(10);
n=1;
for i=2:(nStep-1)
if (yplot(i)>yplot(i-1)&&yplot(i)>yplot(i+1))
      rety(n)=yplot(i);
      n=n+1;
end
end
rety1=rety(1:end-1);
rety2=rety(2:end);
plot(rety1,rety2,'.');
xlabel('y n'); ylabel('y n+1');
title('Return map for y');

figure(11);
n=1;
for i=2:(nStep-1)
if (rplot(i)>rplot(i-1)&&rplot(i)>rplot(i+1))
```

```
        retr(n)=rplot(i);
        n=n+1;
end
end
retr1=retr(1:end-1);
retr2=retr(2:end);
plot(retr1,retr2,'.');
xlabel('r n'); ylabel('r n+1');
title('Return map for r');
```

Program bifurcation.m

```
clear all;

%MAIN PROGRAM
c=20; %initial parameter value
ccount=1; %count the number of different values of c
plotcount=1;

while c<=30
mu = c/(1+c);
M1 = mu;
M2 = 1-mu;
%Positions of the two primary masses
rM1 = [-(1-mu),0];
rM2 = [mu,0];
%Set initial position and velocity of the object
r0 = [.5*(M1-M2),sqrt(3)/2]; %initial position is the L4 Lagrange point
r = r0;
v = [.01,.01];
state = [ r(1) r(2) v(1) v(2) ];    % Used by R-K routines

%Initialize time
time = 0;
%Loop over desired number of steps
tau = 0.01;
nStep = 1e6;
for istep=1:nStep
%* Calculate new position and velocity using RK4.
state = rk4(state,time,tau,'gravrk',M1,M2,rM1,rM2);
r = [state(1) state(2)];
v = [state(3) state(4)];
time = time + tau;

%Save the last point for plotting
if (istep>(nStep-100))
        xplot(plotcount)=r(1)-r0(1);
        yplot(plotcount)=r(2)-r0(2);
        rplot(plotcount)=norm(r-r0);
        cplot(plotcount)=c;
        plotcount=plotcount+1;
end
end

%change the parameter value
```

```
if c>=24.5 && c<25.5
c=c+.01;  %small increment near the bifurcation point
else
c=c+.5;   %larger increments further from the bifurcation point
end
ccount=ccount+1;
end

figure(1); clf;
plot(cplot,xplot,'.','MarkerSize',0.5);
xlabel('M1/M2'); ylabel('x position');

figure(2); clf;
plot(cplot,yplot,'.','MarkerSize',0.5);
xlabel('M1/M2'); ylabel('y position');

figure(3); clf;
plot(cplot,rplot,'.','MarkerSize',0.5);
xlabel('M1/M2'); ylabel('radial distance from L4');

figure(4); clf;
semilogy(cplot,xplot,'.','MarkerSize',0.5);
xlabel('M1/M2'); ylabel('x position');

figure(5); clf;
semilogy(cplot,yplot,'.','MarkerSize',0.5);
xlabel('M1/M2'); ylabel('y position');

figure(6); clf;
semilogy(cplot,rplot,'.','MarkerSize',0.5);
xlabel('M1/M2'); ylabel('radial distance from L4');
```