Cand.merc. Applied Economics and Finance
Department of Economics

Master Thesis

Myopic Loss Aversion and the Equity Premium Puzzle

An Alternative Explanation Applied to the Danish Stock Market

Student: Line Isager-Nielsen

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Instructor: Morten Bennedsen

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Executive Summary

This thesis presents an attempt to resolve the well-known equity premium puzzle using insight from behavioural finance – namely prospect theory and the concept known as myopic loss aversion. The notion is that the reason why economist have had such a hard time reconciling the predictions of standard expected utility theory to real world observations is that decision makers do not behave as suggested by the standard normative model. Rather a new descriptive theory is warranted since decision makers in their behaviour are observed to violate vital assumptions underlying utility maximisation.

The thesis firstly reviews the original equity premium puzzle from 1985, presenting the finding that observed stock returns in the US by far exceed the predictions of expected utility theory. Only an assumption of implausibly high risk aversion of agents is able to reconcile empirics and theory.

The thesis then proceeds to argue the failure of expected utility theory as a useful descriptive model of decision making behaviour. As the alternative, prospect theory is presented. A purely descriptive model that encompasses loss aversion, i.e. the notion that a loss to an agent is more hurtful than an equivalent win is pleasurable. Prospect theory is combined with myopia to build the alternative description of decision making behaviour. Myopia is related to mental accounting and captures the assertion that individual are constantly monitoring the success or failure of their financial dispositions. Thus the alternative description of behaviour is myopic loss aversion – agents are aggravated extraordinarily by losses and are frequently evaluating results. Stocks are volatile and will drop in value from time to time but over the long run the average return is high. Myopic loss averse investors will evaluate often and be hurt by volatility – by observing losses – and so they will view stocks as a less attractive investment than if they were rational. In turn they will demand a higher premium to invest in stocks. This is the rationale for why describing investors as myopic loss averse should result in a higher observed risk premium to stock investment.

In the empirical part of the thesis, the theoretical argument is put to work on Danish stock market data, where firstly it is confirmed that there exists an equity premium puzzle similar to the one documented in the US twenty years ago. Secondly a model based on prospect theory and myopic loss aversion is fitted to the Danish data to arrive at the implied factors of investor behaviour. The results show that the approach can reconcile the puzzle in Denmark if it is assumed that myopic investors evaluate results every year and are hurt approximately twice as hard by a loss than pleased by a similar gain. These assumption lead to the observed equity premium but also optimal asset allocations very similar to observed behaviour in the Danish market.

Content

1.0 Introduction

The *equity premium puzzle* is a paradox originally presented by Mehra and Prescott in 1985. They found that the risk adjusted return to US stock investment over a risk free investment had been much too high over the course to the twentieth century relative to what traditional theory would suggest. This puzzle has been thoroughly scrutinized over the years and numerous researchers have tried to reconcile the empirical evidence to economic theory. Many attempts have proven futile and the puzzle has held up remarkably well to advances in modern finance theory. The original analysis of the equity premium puzzle is based on expected utility theory, which has traditionally dominated the analysis of decision making under risk and has generally been accepted as a normative model of rational choice, and in turn, been applied as a descriptive model of economic behaviour. The equity premium puzzle is thus an illustration of a discrepancy between what the theory predicts and what is actually observed empirically.

Behavioural finance as an area of research has gained much support in the late twentieth century and has been positioned as an interesting alternative to many traditional ways of looking at finance. In this area, psychologists Daniel Kahneman and Amos Tversky have been quite influential and in 1979 they developed a concept of behavioural finance called *prospect theory* as an alternative formulation of investor behaviour and risk attitudes. In their research Kahneman and Tversky found that the predictions of expected utility theory sometimes fail because agents diverge from rationality and utility maximisation in their decision-making. Prospect theory is a descriptive model that captures how agents are actually observed to behave rather than predict how agents are supposed to behave.

In 1995, Benartzi and Thaler utilized prospect theory to present an approach called *myopic loss aversion* which consists of two behavioural concepts, namely loss aversion and mental accounting. They hypothesized that a combination of a strong aversion to losses (rather than risk per se) and frequent evaluation of portfolio returns is a more intuitive description of investor behaviour and in doing so presented a plausible explanation of the equity premium puzzle using empirical US data.

This thesis will explore the usefulness of this approach as a descriptive model of the Danish financial markets. Benartzi and Thaler have reached plausible results for the US financial markets but thus far none have thoroughly investigated whether the Danish financial market serves to be a similarly accommodating testing ground where myopic loss aversion can gain further support.

2.0 Problem Statement

The introduction above sets the stage for what I set out to do in this thesis. This chapter defines the actual objective of the thesis and how I intend to reach that objective. The next subsection more concretely defines the overall mission of the thesis and the issues that I seek to resolve in order to complete this mission. Based on this formulation of my objectives, I will present a motivated description of the structure of the thesis, i.e. review the different parts, why they are in the thesis, how the composition is motivated and the limits of my subject area.

2.1 Problem definition

As mentioned, the overall objective of this thesis is to investigate the usefulness of myopic loss aversion as a plausible explanation for the equity premium puzzle using Denmark as the empirical testing ground. Following this line of argumentation, I can pose the main question that I seek to answer in the thesis as the following;

> *Does myopic loss aversion as a description of investor behaviour constitute an improvement in explaining the equity premium in Denmark?*

Evidently, several subordinate tasks need to be addressed in order to shed light on this overall objective. Firstly, we must review the mechanics of the equity premium puzzle in order to answer the question of what causes the puzzle and why the behavioural finance field might theoretically resolve the paradox. Moreover, as a supportive question it is illustrative to address whether other alternative approaches have already successfully resolved the paradox or if the puzzle still holds. Part of answering this question is obviously to stipulate whether or not the puzzle is a product of restrictive stylised theoretical assumptions. It should be evident by now that I will argue that the assumptions regarding investor behaviour underlying the model is of vital importance. Secondly, it is of vital importance to argue the usefulness of behavioural finance and concretely the concepts of prospect theory and myopic loss aversion as descriptive models of the behaviour of individual agents. To this end, I will show how standard expected utility theory fails to describe the observed behaviour that is captured by prospect theory. In order to reach these conclusions I must investigate evidence on the actual behaviour of individuals and how this behaviour contradicts expected utility theory and I must dedicate efforts to reviewing prospect theory and argue how it constitutes an accommodating alternative. Further I will dedicate some effort to explaining the concepts that combined constitute myopic loss aversion, namely loss aversion and mental accounting. Finally, investigating the magnitude of the equity premium in Denmark is a central part of the overall objective because the result of such an investigation will serve as a benchmark for evaluating the effectiveness of my approach. Thus I will submit the Danish financial market to a test similar to the original test for the equity premium puzzle in the US.

So in order to answer the main question, the following sub questions must be analysed:

a. What is the equity premium puzzle?
b. Have other alternative approaches successfully resolved the paradox or does the puzzle still hold?
c. Is expected utility theory a useful descriptive model?
d. What is prospect theory and how does it differ from traditional expected utility theory?
e. What is loss aversion and mental accounting and how can these concepts theoretically resolve the puzzle?
f. What is the equity premium in Denmark and does it constitute a puzzle?

Based on this ground work I will be able to address the overall mission statement. The next section puts into concrete language the overall structure and motivation for the individual parts of the analyses to be made.

2.2 Methodology and Delimitations

This section provides an overview of and argumentation for the theoretical methodology applied throughout this thesis as well as the structure of the thesis and the delimitations that will be made.

The main objective of the thesis is to investigate whether or not myopic loss aversion constitutes a reasonable explanation to the equity premium puzzle. So at the very basis, I must account for, what the equity premium puzzle actually is. For this purpose I turn to Mehra and Prescott who were the first to document the puzzle, so naturally their model deserves some attention. In order to provide a more nuanced picture of the puzzle, I also chose to include a more statistical approach conducted by Kocherlakota (1996) because this provides me with a more direct tool to test if an equity premium puzzle exists in Denmark.

Throughout the last 21 years, since Mehra and Prescott first documented the puzzle, several attempts have been made trying to account for their findings. Some of these concerned potential adjustments to the empirical side of the puzzle, some concerned an exploration of different theoretical frameworks and yet others have focused on relaxing the key assumptions underlying the model. Myopic loss aversion belongs to the latter of these groups of attempts. So I delimit the discussion of the different attempts to those that have as their objective to relax the underlying assumptions.

At the hart of myopic loss aversion lies prospect theory which is a *descriptive* alternative to traditional rational decision theory, specifically expected utility theory. In order to see why we must abandon this traditional approach, I dedicate some effort to accounting for some empirical and experimental work showing how agents seem to violate the key axioms of rational decision theory. Obviously, this is done in order to give justification to prospect theory as a relevant alternative.
Prospect theory as such has been applied in many different contexts; however the main objective here is to account for the parts of prospect theory that are relevant for myopic loss aversion. Therefore I will not focus on the many different applications of prospect theory. As will become evident later on in the thesis, prospect theory is able to come up with some closed form functions, which I apply when investigating the myopic loss aversion model. These functions contain parameters estimated through experimental research conducted in US by the originators of prospect theory. Since no similar experiments have been conducted in Denmark, I assume that these estimates apply to the representative investor and therefore also to a Danish investor. It should be noted that the examination of prospect theory will be thorough. This is done due to the belief that since prospect theory is the "challenger" to traditional economic theory it is important to dedicate a significant focus to the deduction of the theory.

Myopic loss aversion (and prospect theory) belongs to the behavioural finance field. As such, this is a huge theoretical field and the purpose of this thesis is *not* to grasp the entire field of behavioural finance. Rather I limit myself to account for and discuss the parts relevant to myopic loss aversion, namely loss aversion and mental accounting. These two concepts can be applied in very different contexts, but the examination in the thesis will have as its purpose to clarify what the concepts are and how they are applied in the context of myopic loss aversion.

The empirical part of the thesis consists of an analysis of whether an equity premium puzzle exists in Denmark in which I perform a statistical test as described by Kocherlakota and whether myopic loss aversion can account for the magnitude of this premium in which I apply the model by Benartzi and Thaler. So, having provided the theoretical background I turn to the empirical analyses of this. The data material used in the empirical analyses will be described in the next section.

2.3 Data

In this section, I will present and discuss the data materiel that forms the basis for the empirical analyses in this thesis. The analyses are carried out on the Danish market and will be on monthly and annual stock, money market and bond returns[1]. In the analysis concerning the equity premium puzzle I apply the annual returns from the period 1971-2005 whereas in the analysis concerning myopic loss aversion I apply monthly data from January 1971- July 2006. This small inconsistency results from the desire to have as many monthly observations as possible in the latter analysis and the inconsistency of periods is of insignificant concern with regards to the results. The analysis period spreads over 35 years from January 1971 to July 2006, which results in 427 monthly observations and 35 annual observations[2].

2.3.1 Description of Data

For a description of the Danish stock market I use the MSCI Denmark Total Return Index[3] (in Danish Kroner). The total return index measures the market performance, including price performance and income from dividend payments. The dividends are reinvested the day the security is quoted ex-dividend (ex-date). The index covers 85% of free float[4] on the Danish stock market and is market cap.-weighted. Note that this index is not an all-share index. This means that there can be a risk of size- and selection bias. However, the index has the longest history relative to other Danish stock indices and so I find that the benefits of this longer index series as well as the fact that reinvested dividends are included outweigh the potential biases. Finally, in appendix A.1 I use scatter plots and Durbin Watson tests to show that the series exhibit no sign of autocorrelation or heteroskedasticity.

As the risk free asset, I use a data series constructed from two different sources. From January 1971 till December 1991, I use the discount rate from the National Bank (Nationalbankens diskonto) and from 1992 onwards, I have used a 3-month CIBOR rate.

A common discussion concerns how to define the risk free asset and no formal consensus has been developed. Some choose a short term money market interest rate[5], whereas others have applied government bonds with various durations[6]. The rationale for choosing a 3-month interest rate as a risk free asset in this thesis is twofold. Firstly, I find it hard to justify that for instance a 10-year government bond can be considered risk free. Though default risk seems rather unlikely (at least in my empirical field), reinvestment risk and interest rate risk, for instance, are still present. Secondly, with regards to my empirical investigation on whether or not an equity premium puzzle can be observed in Danish data, I wish to be completely faithful to the theoretical and empirical approach on

[1] Using closing prices
[2] End period being December 2005.
[3] A thorough description of the index can be found on msci.com
[4] MSCI defines the free float of a security as the proportion of shares outstanding that are deemed to be available for purchase in the public equity markets by international investors.
[5] E.g. The Federation of Danish Investment Associations (IFR), Kocherlakota (1996), Mehra and Prescott (1985/2003)
[6] The Danish National Bank uses the return of a 10 year Government Bond, Quarterly Report 2003

which I build my analysis and here 3-month T-bills are applied as a risk free asset. Therefore, I find that the most appropriate proxy of a risk free asset is a short money market interest rate.

An extensive investigation revealed that available data for returns on Danish Government Bonds back to 1971 does not seem to exist. So I construct an approximated data series from 1971 until December 1985 from whereon available data exists. Hereafter I use the JP Morgan Danish Government Bond Index +1. I find this to be the most appropriate due to the fact that it contains the most issues of available alternatives and has a duration of 4.5 years. From January 1971 to December 1985 I have used the monthly bond yields for 5-year Danish Government Bonds of which available data exists and calculated returns using the following expression.

$$R_t^n = \left[\frac{r_{t-1}}{12}\right] - \left[\frac{v}{1+r_{t-1}} \cdot (r_t - r_{t-1})\right],$$

where $R^n{}_t$ is the nominal bond return and v is an estimate of the duration of the bond – this variable has been set to 4.5. This expression approximates the monthly returns to 5-year bonds by one twelfth of the annualised bond yield for the previous month less an estimated price change. The price change is estimated as the modified duration times the yield change over the previous month, e.g. a one percent yield increase results in a price decrease corresponding to the duration.

The approximation approach and the size of the duration factor have been thoroughly discussed with the head of Fixed Income at Gudme Raaschou Asset Management, Henrik Qvistgaard. Since the approximation concerns government bonds and short term month-to-months yield changes, no factor for the convexity has been implemented. If I had been estimating returns using a self-constructed Danish callable mortgage bond, convexity had been an issue due to the high level of negative convexity in especially the high coupon Danish mortgage bond market. This high degree of negative convexity appears when the price is close to par and the option starts to represent value.

It is reasonable to ask how well this approximation works. Obviously, I cannot examine this prior to 1985, but I applied the methodology above to the period 1986-2005 in which data from the JP Morgan index is available. This investigation revealed that the average (arithmetic) annual return is only 5 bp higher for the JP Morgan index than for the constructed series (8.86% vs. 8.81%). However, it must be noted that the annual standard deviation is somewhat higher for the constructed series (6.84% compared to 4.26%). But with these considerations in mind, I find that the approximation works rather well. So from 1971-1985, I use yields from 5-year government bonds and calculate an approximated index. From 1986 onwards, I use J.P Morgan Danish Government Bond Index +1.

Per capita consumption has been calculated as the annual private consumption (all included) divided by the size of the annual Danish population.

In order to arrive at real returns and consumption growth, all series will be deflated with the consumer price index. It should be pointed out that the annual inflation rate is calculated by annualizing the monthly rates thereby giving me the exact annual inflation rate. Danish Statistics report the *average* annual inflation rate but I find this to be an inappropriate measure due to the fact that I wish to examine the actual *monthly* returns based on the actual *monthly* inflation.

Refer to appendix A.2 for an overview of the data series and their sources.

3.0 The Equity Premium Puzzle

This section presents the equity premium puzzle as originally posed by Mehra and Prescott in 1985 and later revisited in 2003. They showed how standard theory fails to produce the large equity premium that has been observed empirically in the US. Intuitively, since stocks are riskier than bonds they should command a higher return because investors will demand a premium for bearing this risk. This is supported by Mehra and Prescott qualitatively but at the quantitative level stocks are shown not to be sufficiently riskier than bonds to justify the observed equity premium.
I describe the updated model utilized by Mehra and Prescott (2003), the intuition behind it, and present their results for the US stock market for the period 1889-1978. The formal deduction of the central equations of the model is carried out in appendix A.3. In this section, I will also present the conclusions of Kocherlakota (1996) who supports the findings of Mehra and Prescott by testing the statistical significance of the equity premium puzzle on the US stock market (same period and data). He finds that the observed premium is significantly higher than what the model predicts for all reasonable levels of risk aversion. This approach will be the basis for the empirical investigation of Danish data in chapter 8.
Following this, I will give a brief overview of some of the most predominant alternative theoretical attempts that have been made to explain the magnitude of the equity premium. The common denominator for these attempts is that they deviate from the main assumptions underlying Mehra and Prescott's model in different ways.

3.1 The Representative Agent Model – Consumption CAPM

As opposed to the Capital Asset Pricing Model, in which it is assumed that the typical investor's consumption stream is perfectly correlated with the return to the stock market[7], Lucas (1978) described a so-called "representative" agent model of asset returns in which per capita consumption is perfectly correlated with the consumption stream of the typical investor. In this type of model, the risk of an asset can be measured using the covariance of its return with per capita consumption. The key idea in this type of model is that consumption today and consumption in some future period are treated as different goods. The relative prices of these different goods are equal to people's willingness to substitute between them and businesses' ability to transform these goods into each other.

In their paper from 1985, Mehra and Prescott described an empirical problem for the representative agent paradigm. They find, that in the period 1889-1978, the average annual real return to stocks has been about 7% whereas the average annual real return to T-bills has been only about 1%, They show that the difference in the covariance of these returns with consumption growth is only large enough to explain the difference in the average returns if investors are implausibly risk averse. And this is what they dubbed the equity premium puzzle; in a quantitative sense, stocks are not sufficiently riskier than T-bills to justify the spread in their returns.

3.1.1 Deduction of the Model

In the framework of Mehra and Prescott the following basic assumptions apply. 1) Investors are rational and have preferences associated with the "standard" utility function, and therefore are able to maximise expected future discounted utility. 2) Markets are complete, i.e. it is possible to insure against any possible situation. 3) There are no transaction costs associated with investing. The model is of the representative agent type, so the results for the representative agent are assumed to hold at the aggregate level as well.[8]

Agents maximise the following utility function describing the present expectation of all future consumption streams,

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t)\right], \text{ where } 0<\beta<1 \tag{3.1}$$

Utility is derived from consumption. Since the model holds at the aggregate level c_t is consumption per capita. The investors' time preference is captured by the discount factor, β, that people apply to

[7] Which in turn implies that a financial investor can measure an asset's risk by its covariance with the return to the stock market

[8] According to Constandinides (1982) this restrictive assumption that all agents have homogenous preferences is not vital since even models with hetorogenous agents produce similar results at the aggregate level.

the utility derived from future consumption. β is small if people are highly impatient and thus prefer consumption today rather than tomorrow.

Mehra and Prescott restrict the utility function to be of the form,

(3.2) $$U(c,\alpha) = \frac{c^{1-\alpha}-1}{1-\alpha}, \text{ where } \alpha>0.$$

This function is of the constant relative risk aversion (CRRA) type since $\frac{-U''(c_t)c_t}{U'(c_t)} = \alpha$.

This preference function links risk preferences with time preferences. Agents with CRRA preferences like to smooth consumption over various states of nature and they also prefer to smooth consumption over time, that is they dislike growth. This is because the coefficient of relative risk aversion is the reciprocal of the elasticity of intertemporal substitution[9]. The maximization behaviour of investors is what prices assets in equilibrium. The investor purchases financial assets if she can obtain a higher marginal utility from investing than from consuming today. So in equilibrium the marginal utility of the amount paid for the stock $p_tU'(c_t)$ must be equal to the present value of the expected utility $\beta E_t((p_{t+1} + y_{t+1})U'(c_{t+1}))$ in the next period, where p is the price of the security and y is the dividend it pays. I show in appendix A.3 that equating these two yields the following equilibrium expression for the expected return on stocks

(3.3) $$E(R_{e,t+1}) = R_{f,t+1} + COV_t\left(\frac{-U'(c_t), R_{e,t+1}}{E_t(U'(c_{t+1}))}\right),$$

where the return on stocks is $R_{e,t+1} = \frac{p_{t+1}+y_{t+1}}{p_t}$.

This result shows that the expected return on stocks is equal to the risk free rate plus a risk premium that depends on the covariance of marginal utility with stock returns. If stock returns are positively correlated with consumption this premium is high and vice versa.

The intuition behind why the equity premium is derived from the covariance of consumption and equity returns is that investors obtain different levels of utility from the same amount of consumption at different times. This follows from decreasing marginal utility, i.e. an asset that pays off when times are good and consumption is high will be considered less desirable than an asset that pays off a similar amount when times are bad and where additional consumption therefore is more highly valued. Similar, agents are assumed to seek smooth consumption paths over time and thus like as-

[9] Mehra and Prescott (2003) page 20

sets that pay off when consumption is low to fill the consumption gap. Contrarily, assets that pay off when consumption is already high, ruins the stability of the consumption path and are thus less valuable to investors, who in turn will demand a higher return to hold them. The question then is whether the covariance between equity returns and consumption growth is large enough to justify the empirically observed equity premium.

Now I proceed to derive the version of this relationship for the equity premium tested by Mehra and Prescott. I show in appendix A.3 that the optimum condition $p_t U'(c_t)=\beta E_t((p_{t+1} + y_{t+1})U'(c_{t+1}))$ yields the following expression for the expected return on stocks and bonds respectively

(3.4) $$E(R_{e,t+1}) = \frac{E_t(x_{t+1})}{\beta E_t(x_{t+1}^{1-\alpha})} \text{ and}$$

(3.5) $$R_{f,t+1} = \frac{1}{\beta E_t(x_{t+1}^{-\alpha})},$$

where x is the growth rate of consumption, i.e. $x_{t+1} = \frac{c_{t+1}}{c_t}$.

The process of consumption growth is assumed by Mehra and Prescott to be log normally distributed. This means that we have explicit expressions for the expected returns for stocks and risk free investments:[10]

(3.6) $$E_t(R_{e,t+1}) = \frac{e^{\mu_x + ½\sigma_x^2}}{\beta e^{(1-\alpha)\mu_x + ½(1-\alpha)^2\sigma_x^2}} \quad \Leftrightarrow \quad \ln E_t(R_{e,t+1}) = -\ln\beta + \alpha\mu_x - ½\alpha^2\sigma_x^2 + \alpha\sigma_x^2$$

(3.7) $$R_{f,t+1} = \frac{1}{\beta e^{-\alpha\mu_x + ½\alpha^2\sigma_x^2}} \quad \Leftrightarrow \quad \ln R_{f,t+1} = -\ln\beta + \alpha\mu_x - ½\alpha^2\sigma_x^2$$

In these expressions $\mu_x = E(\ln x)$, σ_x^2 = VAR(ln$x$), and ln$x$ is the continuously compounded growth rate of consumption. From this we get the models prediction of the equity premium:

(3.8) $$\ln E_t(R_{e,t+1}) - \ln R_{f,t+1} = \alpha\sigma_x^2 .$$

Thus the risk premium commanded by stock investment is the product of the coefficient of the investors' risk aversion and the variance of consumption growth.

Mehra and Prescott assume that in equilibrium the consumption growth path is perfectly correlated with equity returns, which means the equity premium is also equal to the coefficient of risk aversion

[10] I derive these expressions in appendix A.2.

times the *covariance* of stock returns and consumption growth, σ_{x,R_e}. That is, the result obtained from the model assuming log normally distributed consumption growth is equivalent to the general representation in equation (3.3).

3.1.2 Mehra and Prescott's Empirical Results – 1889-1978

In their original analysis, Mehra and Prescott used the following data series; the real return to the S&P 500, the real return to short term nominally risk free bonds[11], and the growth rate of per capita consumption. The sample statistics, Mehra and Prescott arrived at is shown below in table 3.1:

Table 3.1

Sample statistics – 1889-1978	
Mean Risk free rate, R_f	1.008
Mean return on equity, $E(R_e)$	1.0698
Mean growth rate of consumption $E(x)$	1.018
Variance of the growth rate of consumption, σ_x^2	0.00125
Mean equity premium $E(R_e)$-R_f	0.0618

From Mehra and Prescott (2003)

As can be seen from the table, the variance of the growth rate of consumption is 0.00125. And as we observe from equation (3.8), this will have to imply a very large coefficient of risk aversion, α, otherwise a high equity premium simply is not possible. What is further illustrated in the table is that the equity premium is calculated to be 6.18 percent p.a. Several studies[12] have argued that the coefficient of risk aversion is a small number in the range of 1-2. Mehra and Prescott use this insight to argue that it should at least be less than 10. So, if we for instance set α equal to 10 and β equal to 0.99, applying equation (3.7) yields:

$\ln R_{f,t+1} = -\ln 0.99 + 10 \cdot \ln(1.018) - ½ \cdot 10^2 \cdot 0.00125 = 0.12$ that is a risk free rate of 12.7%.

Now applying equation (3.8) we have $\ln E(R_e) = 10 \cdot 0.00125 + 0.12 = 0.1325$, which yields $E(R_e)$ = 1.141, that is a return on equity of 14.1%. This indicates an equity premium of 1.4% and even with a very high coefficient of risk aversion, this is far lower than the observed premium of 6.18%.

This circumstance is what Mehra and Prescott dubbed the equity premium *puzzle*. It is puzzling that even for parameter values, α and β, pushed to their very limits, there is a huge difference between what the model predicts and what is actually observed empirically. As stated before, a risk aversion of 10 is considered too large by several studies. Furthermore, the value of beta is set as liberally as

[11] 90-day T-bills from 1931-1978, T-certificates from 1920-31, and 60-90 day Commercial Paper prior to 1920.
[12] Following Mehra and Prescott (1985) page 154

possible because applying a beta value larger than one would indicate that people's subjective time preference (θ) is negative (since $\beta = \frac{1}{1+\theta}$). Obviously, this is counterintuitive since it would imply that investors are willing to pay to transfer consumption from today to tomorrow.

3.1.3 Further Validation of the Empirical Results

Kocherlakota (1996) utilises equation (3.4) and (3.5) to perform a statistical significance test of the findings of Mehra and Prescott using the same data. In appendix A.4, I illustrate that combining equation (3.4) and (3.5) yields the following expression:

$$(3.9) \qquad E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\left(R_{e,t+1} - R_{f,t+1}\right)\right] = 0$$

That is, when taking consumption risk into account, the equity premium should not be significantly different from zero. Kocherlakota proceeds by estimating the expectations on the left hand side of the equation by using the sample means of:

$$(3.10) \qquad e_{t+1} = \left[\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\left(R_{e,t+1} - R_{f,t+1}\right)\right]$$

And this relationship is then tested as the null hypothesis. That is, the sample means should not be significantly different from zero. Kocherlakota calculates the sample mean for different values of α, ranging from 0.0 to 10.0. As can be seen from the table below, for all values of alpha ≤ 8.5, the sample mean of e_t is significantly positive and therefore the null hypothesis is rejected for all values of alpha smaller than 8.5.

Table 3.2

The Equity Premium Puzzle

α	e	t-stat
0.0	0.0594	3.345
1.0	0.0560	3.173
5.0	0.0433	2.370
8.0	0.0341	1.715
8.5	**0.0326**	**1.607**
9.5	0.0295	1.395
10.0	0.0279	1.1291

Extract from Kocherlakota 1996

That is, in order for investors to be indifferent between investing in stocks and bonds, investors must be highly risk averse. So, Kocherlakota supports Mehra and Prescott's findings showing that only with an unrealistically high level of risk aversion the observed equity premium can be justified. That is, the premium for bearing aggregate risk accounts for little of the historic equity premium.[13] So, even though standard theory is consistent with the notion of risk, that stocks, on average, should earn a higher return than bonds, the quantitative predictions of the theory are an order of magnitude different from what have been documented in the empirical data above.

3.2 Historical Attempts to Explain the Equity Premium Puzzle

Over the last 20 years, several attempts to resolve the puzzle have been made. Generally the consensus is that the theoretical framework of Mehra and Prescott is robust and represents an integral part of modern macroeconomics and international finance. Thus any attempts to reconcile the apparent empirical defects of the representative agent model of asset returns must be based on the abandonment of at least one of the three key assumptions on which it is based (Kocherlakota 1996 page 43). These were 1) asset trading is costless, 2) asset markets are complete, and 3) agents have preferences associated with the 'standard' utility function. In this section, I discuss some of the attempts made to explain the equity premium puzzle by relaxing these assumptions. The chapter closes with a more thorough introduction to the alternative explanation that is the main topic of this thesis, namely myopic loss aversion.

3.2.1 Complete Markets Assumption

A key presumption underlying Mehra and Prescott's model is that the behaviour of per capita consumption growth is an appropriate proxy for the behaviour of individual consumption growth. This is true if it is assumed (as Mehra and Prescott do) that markets are complete.

The assumption that markets are complete implies that agents can insure against any contingency, e.g. fluctuations in labour income – income shocks. In the framework of the consumption based representative agent model of Mehra and Prescott (1985 and 2003), this means that agents can insure against fluctuations in their consumption stream. This assumption is vital in using the per capita consumption as a measure of consumption for the representative agent. Agents will use the financial markets to diversify away any idiosyncratic differences between their own consumption growth and aggregate consumption growth making the two series identical.

The rationale for why the abolishment of this assumption could explain the equity premium puzzle is as follows: If the reality is that markets are not complete and investors then are not able to com-

[13] Mehra and Prescott (2003), page 33

pletely hedge all possible fluctuation in their consumption stream, then they face a more volatile consumption stream than what is indicated by per capita consumption. (And as have previously been noted, investors want to smooth consumption over time and states.) Since the Mehra-Prescott model shows that the equity premium equals risk aversion times the variance of the consumption stream, the premium demanded by investors with higher consumption volatility would be higher. Indeed the main empirical finding of Mehra and Prescott was that the variance of consumption was too low to explain the premium. Weil (1992) studies a two-period model in which markets are not complete. This means, that variability in income must be fully reflected in the consumption pattern. He shows that the extra variability in individual consumption growth induced by the absence of markets helps explain the equity premium puzzle.

Kocherlakota (1996), however, argues that two-period models are incomplete in the sense that they do not capture the use of dynamic self-insurance; an intuitive process by which individuals (if assumed that they live for more than two periods) offset fluctuations in income and thus consumption by increasing (when income is high) or decreasing (when income is low) savings. That is, individuals need not absorb the income risk totally into current consumption. In this framework, investors are able to smooth consumption quite successfully if only income shocks are not highly persistent. If the income shock is permanent, dynamic self-insurance cannot play a role; income shocks must be fully absorbed into consumption. Heaton and D. Lucas (1995a) find that income shocks are in fact not persistent; rather the autocorrelation of idiosyncratic income shocks is around 0.5, which means that the income shock dies out after some time (an autocorrelation of 1 implying a permanent income shock).

Numerous empirical applications of dynamic incomplete markets models[14] confirm that individuals can closely approximate the allocations in the complete markets environment by dynamically self-insuring, i.e. equilibrium asset prices are very similar. So in conclusion, even though the complete markets assumption may seem unrealistic, the evidence shows that the equity premium need not be largely affected by market incompleteness.

3.2.2 No Transaction Costs Assumption

The model developed by Mehra and Prescott assumes that asset trading is costless, which means that there are no constraints on or costs associated with trading financial securities. This is not the case in the real world where the typical investor will face constraints on both borrowing and short sales. Thus, the relaxation of this assumption has been put forward as a possible resolution of the

[14] Following Kocherlakota, these are Telmer (1993), Lucas (1994), Heaton and Lucas (1995a), Macet and Singleton (1991).

equity premium puzzle. Kocherlakota (1996) argues that if investors are constrained on borrowing this leads to a lower demand for loans and so a lower interest rate. All else equal this implies a higher equity premium by simply increasing the theoretically predicted difference between the mean return to equities and the (now lower) interest rate. Heaton and D. Lucas (1995b), however, find that constraints on the trading activity of investors have little effect on the size of the equity premium. This is because the typical investors will face constraints both in the bond market and in the stock market. If not, investors could shift resources from one market to the other and hereby loosen the constraint. With parallel constraints on investment in bonds and stocks the expected return in both markets will be similarly lower thus preserving the equity premium.

The absence of trading costs in the Mehra and Prescott model is also possible to question since the real world features several levels of expenses associated with asset trading. If investors have long horizons the magnitude of trading costs will diminish over the life of the investment – consequently reducing the importance of these costs. If investors, however, are forced to sell investments prematurely, e.g. following a drop in labour income, the investment horizon is too short to fully amortize the costs. Thus, the equity premium should be higher in order to offset these costs. Research by Aiyagari and Gertler (1991) and Heaton and D. Lucas (1995a), however, shows that only a very large difference in the cost of equity trading relative to bond trading can explain the equity premium. Kocherlakota (1996) finds that this substantial difference in costs is not supported by empirical evidence and as such the relaxation of the assumption of costless trading cannot help resolve the equity premium puzzle.

3.2.3 Alternative Preference Structure

Thus far, we have seen that relaxing the assumptions regarding complete and frictionless markets have not helped refute the results of Mehra and Prescott. So, now we turn to the third key assumption underlying the model, which concerns the preferences of the representative agent.

I briefly review three different modifications to (3.1) and hereafter turn the attention to the main focus of this thesis, namely myopic loss aversion, which also constitutes an alternative in the "preference modification class".

3.2.3.1 Habit Formation

The standard preferences in (3.1) assume that the level of consumption in period *t-1* does not affect the marginal utility of consumption in period *t*. It could be argued that it is more natural to think that an individual who consumes a lot in period *t-1* will get used to this high level of consumption and therefore more strongly desire consumption in period *t*. A habit-formation utility function as presented by Constantinides (1990) captures this intuition: once an individual gets used to a certain standard of living, her consumption level forms a “habit”. This level will then become the benchmark to which she evaluates future consumption. Thus, it is the deviations from this benchmark that matters for the individual rather than the absolute level of consumption and the *utility* of current consumption will be a decreasing function of consumption yesterday. The implication of this approach is that demand for savings will be higher than in Mehra and Prescott’s model. This is because individuals for any given level of current consumption knows that the desire for future consumption is ever increasing. So a fair amount of savings is necessary. The consequence for the implied equity premium is not encouraging, however. The high demand for savings drives down interest rates and thus predicts a low empirical risk free rate, but unfortunately it is still necessary for individuals to be highly averse to consumption risk to explain the magnitude of the equity premium. Mehra and Prescott (2003) argue that the puzzle is not explained by emphasizing that with a moderate level of risk aversion (Constantinides presents α=2.81 as a solution) the sensitivity to consumption risk as measured by the coefficient of relative risk aversion is five times α. The reason for this is that although Constantinides finds that the model can generate a high equity premium at a relatively low level of risk aversion, it is necessary to assume that agents are extremely persistent in requiring current consumption to exceed previous consumption – Kocherlakota points out that the agents in Constantinides’ framework requires a large amount of consumption just to survive and thus will pay a lot to avoid small consumption gambles. Thus although aversion to wealth risk can be low, consumption risk aversion must still be implausibly high to explain the puzzle.

3.2.3.2 Keeping Up With the Joneses

Duesenberry (1949) assumes that agents’ utility not only depends on their own consumption as in (3.1) but also on the aggregate level of consumption in the economy. This type of preferences has been dubbed ‘keeping up with the Joneses”. Abel (1990) applies this type of preference in an attempt to explain the equity premium puzzle. In this setting the investment decision of an individual will depend on both the attitude towards own consumption risk and the variability of the general consumption growth in the society. Specifically, the utility function of agents includes individual consumption *relative to* per capita consumption at time t as well as time $t-1$.

It is then possible to estimate the risk aversion parameters associated with individual as well as per capita consumption. The model offers an explanation of the high equity premium, namely that investors need *not* be excessively averse to individual consumption risk as long as the sensitivity of marginal utility towards the variability in per capita consumption is sufficiently high. So, investors do not find stocks unattractive because they are highly averse to individual consumption risk but rather because they are very averse to per capita consumption risk. Kocherlakota notes that the insight gained from relaxing the preference structure of Mehra and Prescott in this direction, is limited. In the original set-up the only looming explanation of the puzzle was that investors were extremely risk averse. The relative consumption approach implies that this need not be the case. But instead investors are required to be extremely averse to any marginal variation in per capita consumption in order to explain the equity premium.

3.2.3.3 Generalised Expected Utility

A central assumption utilized in the standard preferences in (3.1) is that the coefficient of relative risk aversion is restricted to be equal to the reciprocal of the elasticity of intertemporal substitution. Consequently individuals who are sensitive to variation in consumption across different states are also averse to variability in consumption over time, i.e. will desire a smooth consumption path. Several studies suggest that this specification of preferences is too rigid and is the restriction that causes the equity premium puzzle.[15] Epstein and Zin (1989) develop the concept of generalised expected utility (GEU) preferences, which is a preference structure that allows the disentanglement of risk aversion from the elasticity of intertemporal substitution. In this model agents' utility depend partly on total wealth and the return on the agents total portfolio of assets (including real estate, human capital, etc.). This return is principally unobservable but Epstein and Zin use the market return as a proxy (specifically, the value-weighted return to the NYSE). In equilibrium, the equity premium depends on the covariance of asset returns with both consumption growth and the return on total assets or the market portfolio. (The model then has as its two limit cases the consumption CAPM and the standard CAPM). The key to the specification is that agents can be risk averse without wanting to smooth consumption over time and in their 1991 paper, Epstein and Zin claim to resolve the equity premium puzzle empirically. Mehra and Prescott (2003) counter this evidence by noting that they overstate the correlation between the return on total assets and the return on the market portfolio. Kocherlakota (1996) supports this notion by pointing out that the market portfolio underestimates the level of diversification of agents' total asset portfolios and so overestimates the correlation between the marginal rate of substitution and stock returns. This high covariance is the reason why Epstein and Zin can explain the puzzle with moderate risk aversion. Moreover, Kocherlakota further develops the framework of Epstein and Zin to a model where the assumption regarding total asset return is not required. He shows that, equivalent to standard utility, the preference structure of Epstein and Zin requires an implausibly high level of risk aversion to explain the puzzle.

[15] Epstein and Zin (1990) reference Hall (1985), Zin (1987) and Attanasio and Weber (1989). Mehra and Prescott (2003) note that there is no a priori reason why the parameters should be linked.

3.2.3.4 Myopic Loss Aversion

As I have shown above, there have been several theories trying to explain this large equity premium, but none, as it seems, has been able to fully account for the magnitude of the premium.

In 1995, B&T (B&T henceforth) set out to try to give an alternative explanation for the size of the equity premium – they called their attempt myopic loss aversion. Their explanation finds its foundation in the behavioural finance literature. The overall focus for behavioural finance is the integration of human psychology and economic theory. The human per se is in focus and this means that behavioural finance deviates from the more standard economic theory.

The concept of myopic loss aversion rests on two principles from behavioural finance. These are loss aversion and mental accounting. Loss aversion means that investors tend to be more sensitive to decreases in their wealth than increases. The concept of loss aversion originates from prospect theory, which is an alternative to expected utility theory. Prospect theory differs from expected utility theory in several aspects. First of all, it is a *purely descriptive* theory that makes *no normative* claims regarding how people ought to act. Rather it merely investigates how people actually do act. Moreover, as I will return to in chapter 5, in prospect theory outcomes are not evaluated in terms of final wealth; rather outcomes are evaluated as either a gain or a loss relative to a reference point.

The other behavioural concept is mental accounting. Mental accounting is a term that captures the cognitive and unconscious operations people use to organize, evaluate and keep track of financial activities. The notion is that people tend to make and evaluate decisions one at a time and place them in separate mental accounts rather than evaluate them in a broader context. In a financial perspective, this refers to how transactions are grouped both cross-sectionally (are securities evaluated one at a time or as portfolios) and intertemporally (how often are portfolios evaluated). When this narrow evaluation of the decisions and outcomes take place, financial investors will tend to make short-term decisions rather than adopt long-term policies regarding their investments and evaluate their gains and losses frequently (Thaler, Tversky, Kahneman and Schwartz 1997).

The combination of loss aversion and mental accounting constitutes the concept of myopic loss aversion. For financial investors this implies that they are averse to losses and evaluate their portfolios at very short horizons. And according to B&T, this combination can account for the magnitude of the equity premium.

They illustrate the concept of myopic loss aversion with a problem that Samuelson (1963) posed; He asked a colleague of his whether he would be willing to take a bet that would either pay $200 or -$100 with 50% chance. The colleague turned the bet down but said that he would be willing to take 100 of such bets. Samuelson showed that if a single bet is rejected, so must a whole sequence of such bets. Otherwise it would be an inconsistency of expected utility maximization.
Several things can be noted about this example, and I will return to it later for further discussion.
First, Samuelson quotes the following reason for why his colleague will not take the single bet: "I will not bet because I would feel the $100 loss more than the $200 gain." The behavioural finance translation of this would be "I am loss averse". Moreover, why is it that he likes a series of bets? That is, what mental accounting operations does he apply since a series of bets seem attractive when one single play is not?
Assume that Samuelson's colleague is characterized by loss aversion and have a utility function in which $U(x) = x$, if $x \geq 0$, and $2.5x$ if x is < 0 (and x being the *change* in wealth due to the bet). The 2.5 indicates a loss aversion factor of 2.5, i.e. losses are weighted 2.5 times as hard as gains.
Then the expected utility of one single bet is negative: $½(200) + ½(2.5)(-100) = -25$, and he will obviously turn down this bet. Hence, Samuelson will reject one bet, and even two or more, *if they are evaluated separately.* So, if each play of the bet is treated as a separate event, then two plays of the bet is twice as bad as one. But if two bets are combined into a portfolio, the expected utility of the bets are positive: $¼(400) + ½(100) + ¼(-500) = 25$. And as the number of repetitions increases the portfolio of bets become even more attractive! So Samuelson's colleague should accept any number of plays of this bet (>1) as long as he does not have to watch them being carried out, i.e. evaluate after each bet.

This means, that loss averse people (investors) are more willing to take on risk if they combine many bets (investments) together than if they consider them one at a time.
Returning to the equity premium puzzle, we see that this intuition can also be applied here by considering the problem facing an investor with the same utility function as described above. Imagine that an investor must choose between a risky asset offering a 7 percent expected return (like stocks) and a risk free asset offering 1 percent. By the same intuition as applied in the example above, the attractiveness of the risky asset will depend on the horizon of the investor. The longer the investor intends to hold on to the risky asset, the more attractive it will seem, as long as she do not evaluate the investment frequently.

So, two factors contribute to investors not being willing to bear the risk associated with investing in equities, loss aversion and a short evaluation period, i.e. the risk attitudes of loss averse investors depend on the frequency with which they reset their reference point, i.e. how often they count their money.

And this is what made B&T hypothesize that the concept of myopic loss aversion serves as an explanation of the equity premium puzzle.

To investigate this hypothesis, B&T asked the question: how often would an investor have to evaluate her portfolio (i.e. the gains and losses) in order to explain the magnitude of equity premium. They find that an evaluation period of approximately one year will account for the size of the premium and argue that this is a natural evaluation period for most investors to use. The way people evaluate gains and losses is plausibly influenced by the way information is presented to them. Since investors receive the most comprehensive reports from their brokers, mutual funds etc. once a year and individual investors file their taxes once a year, they argue that it is not unreasonable that gains and losses might be expressed as annual changes in value.

To give further support for this, B&T ask what combination of stocks and bonds will be optimal given this one-year evaluation period. They find that an optimal allocation to stocks is between 30% and 55%. Again, they find support for this in the observed behaviour of investors in the US financial markets, and this gives further evidence to the validity of myopic loss aversion as a plausible explanation to the equity premium puzzle. As a final plausibility test, they investigate whether the equity premium falls as the evaluation period increases and find support for this.

So, B&T find that the combination of loss aversion and mental accounting can explain the size of the equity premium, and hence it is no longer a puzzle. I will return to these analyses in chapter 7.

3.3 Chapter Summary

In this chapter, I have set the stage for my further analysis by reviewing the original findings of Mehra and Prescott. In a representative agent model based on standard expected utility theory, they showed that the premium demanded by agents when investing in stocks should be equal to risk aversion times the variance of consumption growth. The empirical problem with the specification was that only implausibly high levels of risk aversion could account for the size of the actual equity premium because the observed variance of consumption growth was simply too small. Kocherlakota supported these findings by performing a statistical analysis that showed that the equity premium was significantly higher than zero if investors were not extremely risk averse. Furthermore, I

reviewed several alternative attempts to reconcile the equity premium all with the mutual denominator that at least one of the vital assumptions of the representative agent model and the underlying expected utility theory had to be relaxed. Abolishing the assumption of complete markets proved only effective in explaining the puzzle if income shocks were implausibly long lasting or permanent since agents have the possibility of performing dynamic self-insurance. Transaction costs could potentially explain the spread in returns if the difference in the cost of equity trading relative to bond trading is very large. However, the empirical spread in trading costs did not support this.
Several alternative preference structures were discussed. The link between the coefficients of risk aversion and intertemporal substitution has been argued to cause the equity premium puzzle. Generalized Expected Utility (GEU) allowed a disentanglement of these two and led to a higher covariance between equity returns and consumption growth and thereby allowing a reconciliation of the equity premium and lower risk aversion. However, this high covariance seemed to be overestimated due to the specification of the model. The concepts of habit formation and 'keeping up with the Joneses' augmented the concept of consumption in the agents' utility functions to be either depended on past consumption or the consumption of others. Habit formation was able to explain the puzzle with low aversion to wealth risk but agents were still implausibly avers to consumption risk. Similarly, 'keeping up with the Joneses' could only reconcile the equity premium if investors were very sensitive to changes in the consumption of others.

Following this, I introduced the concept of myopic loss aversion as a possible explanation of the puzzle. This concept rests on two principals from behavioural finance; loss aversion and mental accounting. Loss aversion implied that investors were hurt more by losses than corresponding gains yielded pleasure. Mental accounting referred to the tendency of investors to evaluate their portfolios too frequently, dubbed myopia. The combination of these behavioural concepts into myopic loss aversion proved to constitute a promising possible explanation to the equity premium puzzle. The remainder of the theoretical part of this thesis will elaborate on the theoretical foundations upon which myopic loss aversion is built in order to perform the analysis on Danish data.

4.0 Expected Utility Theory and Beyond

The purpose of the following chapter is to lay the foundation for the development of prospect theory. Prospect theory is an alternative to expected utility theory as a theory of decision-making under risk. It differs from expected utility theory in being exclusively descriptive and in making no normative claims. It is designed to *explain* preferences, whether or not these can be rationalized. In order to see why prospect theory constitutes a relevant alternative to expected utility theory, we need to take a look at why and how expected utility theory seems to fail when confronted with empirical evidence.

Expected utility theory has been a dominant normative theory when trying to explain the equity premium puzzle. And as I argued above, the theory falls short when confronted with empirical data. So even though the theory has widely been accepted as a normative model of rational choice and widely applied as a descriptive model of economic behaviour, we still seem to face a puzzle.
This chapter starts out with a short review of rational decision-making and the axioms upon which the expected utility theory rests. Next I illustrate, through several empirical experiments, following Kahneman and Tversky (1979, 1984, 1992), that these axioms are often violated when people actually make decisions. These violations lead Kahneman and Tversky to develop prospect theory and later the extension called cumulative prospect theory (1992), which is the topic of the next chapter.

4.1 The Axioms of Expected Utility Theory

Expected utility theory rests upon the assumption that people are able to make rational choices under risk and uncertainty – rational decision making. This means that when faced with uncertainty or risk, people behave as if they were maximizing the expectation of some utility function of potential outcomes. And this maximization is based on rational preferences.[16]

Throughout this section, I will use the following terminology regarding risky choice. Choice under risk can be viewed as a choice between prospects/gambles, i.e. a prospect ($x_1, p_1; \ldots; x_n, p_n$) is a contract that gives you outcome x_i with probability p_i, where $p_1+p_2+\ldots+p_n=1$. Whenever an outcome has a probability of zero, that outcome will be omitted, so \$50 with probability 30 pct. will be written (50, 0.3), omitting the 70 pct. chance of winning nothing.

Turning now to expected utility theory, the application of rational decision making to choices between prospects is based on a series of characteristics, assumptions and axioms. I now give an overview of the key elements that have been seen to fail in a descriptive context.

At the very basis lies the assumption that agents maximise *expected* utility, which means that the utility associated with a given outcome is weighted by its *objective probability* in the overall utility function.

[16] Frank (2000)

Furthermore utility is based on *absolute* levels of wealth. So, the domain of the traditional utility function is *final states* rather than the gains or losses experienced along the way.

Expected utility theory incorporates the following normative descriptions of preferences.

i) *Risk aversion.* Agents are risk averse, i.e. they will always prefer a riskless prospect to any risky prospect yielding the same expected value. This is illustrated in a traditional utility function, which has the characteristic concave shape.

ii) *Transitivity.* Agents preferences are consistent so that if prospect A is preferred to B, which in turn is preferred to C, then transitivity governs that A will be preferred to C.

iii) *Substitution.* If prospect A is preferred to B, then an even chance to receive A or C is preferred to an even chance to receive B or C.[17]

iv) *Dominance.* If prospect A yields outcomes at least as high as B in all states, and higher in at least one state, then A dominates B and is preferred.

v) *Invariance.* The preference order of a series of prospects is invariant of the manner in which the prospects are presented – or framed, which is the term I will use in the following. As a consequence, two versions of a choice problem that are recognised to be equivalent when shown together should elicit the same preference even when shown separately.

As mentioned, these are the key tenets of rational decision-making, and thus expected utility theory, that I propose to revise in order to obtain an alternative descriptive theory of decision making. In the next section, the failure of some of these axioms in experimental evidence is reviewed.

4.2 Violations of the Axioms of Expected Utility Theory

In this section, I follow the pioneering work by Daniel Kahneman and Amos Tversky. Through several responses to hypothetical choice problems to both students and university faculty, they show that respondents often violate the assumptions underlying expected utility theory.

As mentioned before, in expected utility theory the utilities of outcomes are weighted by their probabilities. Kahneman and Tversky (1979) show that this principle is often violated by people's preferences because they overweight outcomes that are considered certain relative to outcomes that are merely probable – they labelled this the *certainty effect*.

[17] The substitution axiom of preferences is by some authors labelled cancellation.

An example of this is given below. People were given two pairs of choice problems in which they were asked to indicate what choices they prefer. N denotes the number of respondents who answered each problem, and the percentage of respondents accepting each prospect is in brackets. When significant at the 0.01 level, this is denoted by an asterisk[18].

Problem 1:

Prospect	Outcome	Probability	
A	2,500	0.33	
	2,400	0.66	[18]
	0	0.01	
B	2,400	1.00	
	0	0.00	[82]*
N = 72			

Problem 2:

Prospect	Outcome	Probability	
C	2,500	0.33	
	0	0.67	[83]*
D	2,400	0.34	
	0	0.66	[17]
N = 72			

Source: Kahneman and Tversky (1979), p. 265-266.

In the first problem the expected payoff of prospect A is 2,409 whereas the expected payoff of B is 2,400. In problem 2 the expected payoffs of prospect C and D are 825 and 816 respectively.

In problem 1, 82% of the respondents chose option B and in problem 2, 83% chose option C.
So the respondents showed a significant preference for B and C (in fact, 61% chose this *combination*). This represents a violation of expected utility theory since in problem 1 we have revealed the following preference order (where > denotes higher utility for): $u(2{,}400) > 0.33\ u(2{,}500) + 0.66\ u(2{,}400)$[19] $\Leftrightarrow 0.33\ u(2{,}500) < 0.34\ u(2{,}400)$. In problem 2, however, we have the opposite: $0.33\ u(2{,}500) > 0.34\ u(2{,}400)$. Thus, the choices are inconsistent and thereby a violation of expected utility theory. The proposition here is that the problem is the *certainty effect,* i.e. the fact that the respondents have a strong preference for the certain outcome, B, in problem 1. This means that the notion of weighting by the absolute probabilities as proscribed by expected utility theory does not hold. Note that problem 2 is actually just obtained from problem 1 by subtracting the prospect of winning 2,400 with probability 0.66. As mentioned though, from B also certainty is subtracted and substituted with risk.

The substitution axiom described above implies that your choice of a given prospect must not change when reducing the probabilities by a common factor. Kahneman and Tversky (1979) find violations of this substitution axiom as well. With reference to problem 1 and 2, this can be due to the certainty effect. If you reduce the probability of a certain and uncertain outcome by the same

[18] This notation is used throughout this thesis.
[19] $u(2{,}400) > 0.33\ u(2{,}500) + 0.66\ u(2{,}400)$ because a significant majority of respondents preferred prospect A.

factor, it will have a more significant effect on the certain outcome because reducing the probability of 1.00 hurts relatively more than reducing any other probability The results of the next choice problems, in which there are no certain outcomes, further illustrate the violation of the substitution axiom.

Problem 3:

Prospect	Outcome	Probability	
A	6,000	0.45	
	0	0.55	[14]
B	3,000	0.90	
	0	0.10	[86]*
N = 66			

Problem 4:

Prospect	Outcome	Probability	
C	6,000	0.001	
	0	0.999	[73]*
D	3,000	0.002	
	0	0.998	[27]
N = 66			

Source: Kahneman and Tversky (1979), p. 267.

For problem 3, the probability of obtaining the monetary outcome of prospect B is twice the size of A. This applies identically to problem 4. The actual monetary outcomes are the same for both problems. Then problem 4 is a probability mixture of 3, which means that subject to the substitution axiom preferences between A and B should be identical to preferences between C and D.

The results, however, are that in problem 3 respondents chose the outcome with the higher probability whereas in problem 4 people choose the prospect with the higher potential outcome; this inconsistency violates substitution.

According to Kahneman and Tversky, the reason for this is that when confronted with a substantial probability of winning, people choose the prospect where winning is more probable (problem 3, prospect B). But in a situation where the probability of winning is very, very small, most people choose the prospect with the largest potential gain (problem 4, prospect C.)

So even though the proportions between probabilities as well as outcomes are identical in the two problems, which means that the preference order of the two prospects should be the same, following directly from the substitution axiom, the focus switches to the monetary payoff when the probabilities become very small. A very central consequence of this emphasis on the monetary outcome, even though highly unlikely, is that the respondents *overweight the small probability*.

The next set of choice problems of this section combine to yield results that violate three tenets of expected utility theory, dominance, invariance and the emphasis on final states rather than gains and losses[20].

Problem 5:

Prospect	Outcome	Probability	
A	240	0.25	
	-760	0.75	[0]
B	250	0.25	
	-750	0.75	[100]
N = 86			

Source: Kahneman and Tversky (1984), p. 5.

Problem 6: "Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer."

Decision i)

Prospect	Outcome	Probability	
C	240	1.00	
	0	0.00	[84]
D	1,000	0.25	
	0	0.75	[16]
N = 150			

Decision ii)

Prospect	Outcome	Probability	
E	-750	1.00	
	0	0.00	[13]
F	-1,000	0.75	
	0	0.25	[87]
N = 150			

Source: Kahneman and Tversky (1984), p. 6.

In problem 5, all respondents made the obvious and rational decision. Prospect B clearly dominates A. In problem 6, the majority of respondents revealed their preference for the combination C and F. In fact, 73 percent of the respondents chose this combination. This combination, however, yields exactly prospect A in problem 5. To see this, realise that C combined with F gives 240 for certain plus a 75% chance of losing 1,000 and a 25% of a zero outcome, i.e. in total a 75% chance of losing 760 and a 25% chance of gaining 240 – this is equivalent to A. Similarly the rejected combination of D and E is equivalent to prospect B in problem 5. The D and E conjunction combines a 25% chance of gaining 1,000 with a certain loss of 750, i.e. a 25% chance of gaining 250 and a 75% chance of loosing 750 – again, this is exactly B.

As mentioned, the results of these problems imply failure of several of the assumptions underlying expected utility theory. First of all, the above analysis shows that the two problems are in fact identical. The different presentations or *framing* of the problems, however, cause respondents to alter their decision significantly. Thus the invariance axiom fails dramatically. The two representations

[20] Kahneman and Tversky (1984, page 5)

of this choice problem, which are recognised to be equivalent when shown together as in problem 5, should have elicited the same preferences when shown separately as in problem 6.
Furthermore, the choice of the combination C and F in problem 6 is a clear violation of dominance since, as shown, the combination is dominated by the D and E conjunction. Obviously, the framing of the choice problem causes this result, as respondents fail to realise that problem 6 boils down to problem 5 where dominance was evident. The result of problem 6, however, is that respondents have chosen what is equivalent to A over what is equivalent to B, i.e. they have chosen the dominated strategy over the dominant one.

The results also show that respondents fail to view the prospects beforehand as measured by *final wealth*. This would have induced subjects to aggregate and unveil the equivalence of the outcomes and thus the two choice problems. Instead, each prospect is viewed in isolation and the outcomes are evaluated as individual gains and losses relative to their initial reference point, rather than final states, leading to the inconsistent results above.

Finally, problem 6 above shows evidence of preferences that do not obey *risk aversion* as put forward by expected utility theory. Certainly in decision i) respondents chose the sure gain over a risky prospect with a potential gain. In decision ii), however, respondents chose the risky alternative over the certain loss implying risk-seeking behaviour. This behaviour is supported by the next set of results to choice problem experiments conducted by Kahneman and Tversky (1979). The implication is equivalent to problem 6, i.e. risk aversion holds in the domain of gains but fails in the domain of losses.

Table 4.1.

Positive prospects				Negative prospects			
Problem 7	(4,000, 0.80)	<	(3,000)	Problem 7’	(-4,000, 0.80)	>	(-3,000)
N = 95	[20]		[80]*		[92]*		[8]
Problem 8	(4,000, 0.20)	>	(3,000, 0.25)	Problem 8’	(-4,000, 0.20)	<	(-3,000, 0.25)
N = 95	[65]*		[35]		[42]		[58]

Source: Kahneman and Tversky (1979), p. 268.

Table 4.1 shows two choice problems ad the resulting preferences revealed by subjects in their decisions (preferences indicated by < and > signs) combined with the results of corresponding choice problems where outcomes are reversed to losses instead of gains. The key result is seen from problem 7 and 7’. When the sign of the outcomes are reversed, the preference of subjects suddenly shifts towards the risky prospect instead of the certain outcome even when the expected value is lower (-

3,200 versus –3,000). Thus, subjects exhibit risk-seeking behaviour in the domain of losses by revealing a marked distaste for the certain loss. This shows that the certainty effect found above not only indicates a strong preference for certain gains but also a strong aversion against certain losses. Further, when the certainty element is removed, as in problem 8', the preference is reversed. So, as in the domain of gains, the subjects in the domain of losses are markedly influenced by certainty relative to probability, only here they struggle to avoid the certainty. From these findings it is evident that risk aversion, as proposed by expected utility theory, must be augmented to a broader concept involving risk seeking behaviour in the domain of losses and risk aversion in the domain of gains.

4.3 Chapter Summary

In summary, I have reviewed experimental evidence on the failure of the main constituents of expected utility theory as a normative theory and more importantly key alternative implications of the behaviour of agents have been uncovered. We have seen that decision makers do not base their decisions on objective probabilities as asserted by expected utility theory, rather they overweight the importance of certain outcomes and small probabilities. This behaviour resulted in the violation of the substitution axiom. Moreover, we saw that final states are not the key determinants of choice; rather decisions are based on gains and losses relative to a reference point.

Furthermore, subjects were shown to be dependent on the framing of the choice problems with which they were faced, i.e. different representations of identical prospects yield different choices thus violating the invariance suggested by expected utility theory. Finally, I found that instead of being risk averse in general, subjects exhibit risk-seeking behaviour in the domain of losses. The combined effect of these attitudes towards risk, focusing on gains and losses, and being subject to framing effects was shown to result in the potential violation of the dominance axiom.

In the next section, prospect theory is presented as an alternative theory of choice that incorporates these key findings.

5.0 Prospect Theory

Prospect theory was originally formulated by Kahneman and Tversky (1979) and later extended to Cumulative Prospect Theory (Kahneman and Tversky 1992). In combination, these two developments form the basis for the representation of prospect theory below.

Prospect theory distinguishes two phases in a choice process: a framing phase, where the decision maker tries to organize and simplify the different alternatives and an evaluation phase, where she decides upon which alternative to choose. I start out describing the framing phase and subsequently present a formal model for the evaluation phase.

5.1 The Framing Phase

The framing phase is basically the phase in which the decision maker interprets the information given to her. This is done in a series of ways by applying some rules that are typically intended to simplify the decision problems presented. Thus at the end of the framing face, the decision maker is left with an adapted version of the prospect, which then constitutes the basis for evaluating and choosing. Kahneman and Tversky describe a series of operations that decision makers are found to go through. To capture the essence of the framing phase, I present a vital such operation and outline other examples.

Decision makers are found to code the outcomes of a given prospect. This was a key finding in the previous section as subjects were found to adapt the concept of gains and losses instead of final outcomes. This coding of outcomes means that the introduction of the reference point is an operation in the framing phase, and this operation determines the value of each prospect. As was seen this coding had decisive impact on the failure of expected utility theory and in the remainder of this thesis the focus on gains and losses will continuously serve as a key assumption and it will be included as part of a wider theoretical framework.

Further operations in this phase that decision makers may conduct include *combining* prospects or elements of prospects into joint problems. Also the opposite, decision makers may discard common factors shared by different prospects, termed *cancellation*. Related to cancellation is *segregation* where decision makers may opt to pull out riskless parts of an otherwise risky prospect. Finally, decision makers may subjectively frame probabilities to more simple representations – in essence this means rounding and an example is ignoring very small probabilities.

These are examples of operations that decision makers may perform in order to simplify the decision process in this phase.

We should note that it is in this framing phase that decision makers are potentially affected by the formulation of the choice problems they face – unfortunately also termed the 'framing' of prospects, i.e. the externally given representation of a given problem[21].
As Kahneman and Tversky note, the preference order of prospects need not at all be invariant of the way they are presented because different representation of identical problems may result in different framing by subject. This is what was seen above in problem 5 and 6.

Later on in this thesis, I will revisit the theoretical basis describing the mental shortcuts and behavioural patterns of the framing of risky prospects and their outcomes. This further development will have the framing phase of prospect theory as a subset of a more general theory concerning concepts such as mental accounting and choice bracketing. For now, I confine the interpretation to the one presented by Kahneman and Tversky – that the framing phase is the initial phase where a given problem or prospect is transformed by decision makers into an accommodating representation. The framed prospect serves to facilitate decision making in the next phase; the evaluation phase. I turn to the evaluation phase of prospect theory next.

5.2 The Evaluation Phase

In evaluating risky prospects, the decision maker seeks to maximize prospective value by utilizing a value function quite similar to the standard utility function but with two key differentiating features; Instead of basing decisions on objective probabilities, the decision maker uses subjective decision weights. Similarly, instead of basing decisions on utility of final states of wealth, she emphasises the value of relative outcomes (i.e. gains, losses or neutral outcomes – the maintenance of a status quo level).

This means that the modelling of a value function and a function of decision weights lies at the very heart of prospect theory. I turn to this next.

<u>Overall value of a prospect</u>

The decision-makers problem is to find the prospect with the highest prospective value, defined by the overall prospective value function:

[21] The meaning of the framing conducted by decision makers in the framing phase should not be confused with the framing concept described in the previous chapter regarding the way in which choice problems are *formulated*. Unfortunately, the term takes on two meanings, and in this particular context, framing refers to the way decision makers themselves frame the prospects and decisions upon which they will later evaluate. Kahneman and Tversky (1979) label the framing phase 'editing phase' and Thaler (1999) suggest 'parsing' or 'reframing' to clarify the distinction.

(5.1) $$V(x,p;y,q) = \pi(p)v(x) + \pi(q)v(y),$$

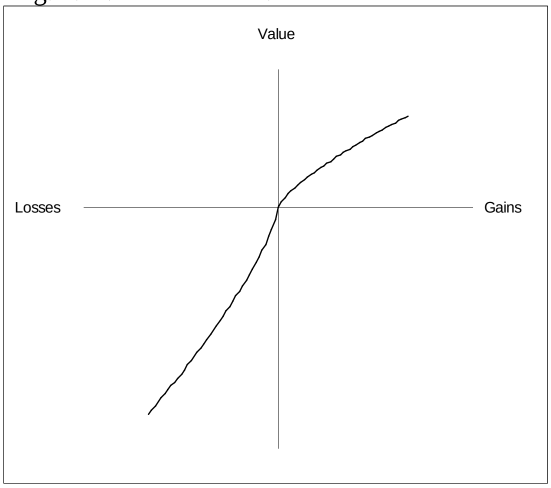
where x and y denotes the possible relative outcomes, p and q their respective probabilities. The value function defined on the outcomes is denoted by v and π is the subjective decision weight associated with the probabilities. The combination of v and π determine the *overall value* of a prospect.

The overall prospective value function above is a basic bilinear form, which emphasises the key elements and implications of prospect theory to be discussed in the following. So, it is used as a starting point for the discussion. But as will become evident throughout the next sections, an alternative overall value function will emerge.

5.3 The Value Function, *v*:

The experimental analyses reviewed above in section 4.2 lead Kahneman and Tversky (1979) to suggest three important characteristics of decision makers that have implications for the shape of the value function. A possible representation of the value function is depicted below.

Fig. 5.1 Value Function.



Reference dependence is imposed following from the evidence that the carriers of value, i.e. the factors that determines the perception of value, are *changes* in wealth rather than final states of wealth relative to a reference point (current wealth). So, value is a function of the asset position that serves as reference point and also the magnitude of the change (positive or negative) from that reference point.

Diminishing sensitivity is imposed following from the evidence that decision makers are found to be risk averse in the domain of gains but risk seeking in the domain of losses. Diminishing sensitivity to the size of gains imply that the value function, similarly to a standard utility function, is concave in the domain of gains. This means that the value attached to gains is increasing at a decreasing rate in the amount of the gains. In the domain of losses, diminishing sensitivity to the amount of the loss relative to the reference point implies that the value function is convex, i.e. the value of losses is decreasing at a diminishing rate.

It is important to note that these implications of risk aversion for gains and risk seeking for losses is a consequence of the interaction between the value function and the decision weights since the value attached to a given outcome is only part of the contribution to overall prospective value. The size of the corresponding decision weight is of vital importance. The characteristics of the function of decision weights (the weighting function) will be elaborated later.

Kahneman and Tversky (1979) present the following experimental result to underpin the shape of the value function.

Problem 9:

Prospect	Outcome	Probability	
A	6,000	0.25	[18]
	0	0.75	
B	4,000	0.25	[82]*
	2,000	0.25	
	0	0.50	
N = 66			

Problem 10:

Prospect	Outcome	Probability	
A	-6,000	0.25	[70]*
	0	0.75	
B	-4,000	0.25	[30]
	-2,000	0.25	
	0	0.50	
N = 66			

Source: Kahneman and Tversky (1979), p. 278

The results of these choice problems imply (using equation 5.1):

$$V(6{,}000, 0.25) < V(4{,}000, 0.25; 2{,}000, 0.25) \quad \Leftrightarrow \quad \pi(0.25)v(6{,}000) < \pi(0.25)\left[v(4{,}000) + v(2{,}000)\right]$$

and for problem 10:

$$V(-6{,}000, 0.25) > V(-4{,}000, 0.25; -2{,}000, 0.25) \quad \Leftrightarrow \quad \pi(0.25)v(-6{,}000) > \pi(0.25)\left[v(-4{,}000) + v(-2{,}000)\right]$$

hence

$v(6{,}000) < v(4{,}000) + v(2{,}000)$ implying concavity in the domain of gains.

$v(-6{,}000) > v(-4{,}000) + v(-2{,}000)$ implying convexity in the domain of losses and

Loss aversion is the final important assertion of decision-making behaviour with implication for the value function. It asserts that similar amount lost yields a higher negative value than the corresponding positive value associated with an identical gain. This means that the value function in the domain of losses is steeper than in the domain of gains.

To argue the intuition of this characteristic, Kahneman and Tversky find that the typical decision maker often find symmetric gambles like (x, 0.5; -x, 0.5) rather unappealing. [22] Furthermore, the desirability of this prospect is found to be decreasing in the amount of outcome x. That is, if $x > y \geq 0$, then the preference for (y, 0.5; -y, 0.5) is larger than for (x, 0.5; -x, 0.5). Again, applying equation 5.1 yields:

$$V(x,0.5;-x,0.5) < V(y,0.5;-y,0.5) \quad \Leftrightarrow \quad \pi(0.5)v(x) + \pi(0.5)v(-x) < \pi(0.5)v(y) + \pi(0.5)v(-y)$$

Dividing by $\pi(0.5)$ gives the following:

$$v(x) + v(-x) < v(y) + v(-y) \quad \Leftrightarrow \quad v(x) - v(y) < v(-y) - v(-x)$$

If we set y = 0, we get $v(x) < -v(-x)$ which shows that the value function is steeper in the domain of losses than in the domain of gains. We see this explicitly by letting y approach x to get the relationship between the slopes: $v'(x) < v'(-x)$.

Combining these three attributes shows that the value function has an asymmetric S shape that is steeper below the reference point (zero) than above it, as shown in fig. 5.1. Note that this shape implies that the steepest area for both gains and loses is at the reference point; this is where the decision maker is most sensitive to changes in relative outcomes. In section 5.4, I represent an explicit functional form of the value function following Kahneman and Tversky (1992).

5.4 The Weighting Function, π:

We now turn to the second cornerstone of prospect theory, the weighting function. In the illustration of the certainty effect described above, we saw that subjects did not base their decisions on objective probabilities. The weighting function is the functional form that models the objective probabilities into subjective decision weights $\pi(p)$, each of which are assigned to the value of a given outcome, $v(x)$. This means, that decision weights should not be interpreted as probability measures but rather as an emphasis parameter derived from the probability. The decision weight then measures the impact $v(x)$ has on the decision problem.

[22] Kahneman and Tversky (1979), page 279

There exists numerous formulations of the weighting function but in the following the specific functional will be of the rank-dependent or cumulative form (Kahneman & Tversky, 1992). This functional is defined upon the probabilities associated with the prospects *(x,p)*. To define the cumulative functional, the prospects are ranked in increasing order by their outcomes. The ranked prospects from $i = -m$ *to* n, are denoted (x_i, p_i) where the subscript denotes the rank but also the nature of the prospect, i.e. strictly negative prospects are in the domain $-m \leq i < 0$ and strictly positive prospects are in the domain $0 \leq i < n$. A strictly positive prospect is defined as one where all outcomes are non-negative whereas a strictly negative prospect has exclusively negative outcomes.

To derive the mechanics of the weighting function we introduce the strictly increasing function w defined on the probability distribution of a prospect (x_i, p_i) where w^+ is defined on the domain of strictly positive prospects and w^- is defined similarly on strictly negative prospects. At the margin $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$. Thus the weighting function is defined separately at the margins and in the positive and negative domain as

(5.2) $$\pi_n^+ = w^+(p_n), \qquad \pi_{-m}^- = w^-(p_{-m})$$

(5.3) $$\pi_i^+ = w^+(p_i + ... + p_n) - w^+(p_{i+1} + ... + p_n) \qquad 0 \leq i \leq n-1$$

(5.4) $$\pi_i^- = w^-(p_{-m} + ... + p_i) - w^-(p_{-m} + ... + p_{i-1}) \qquad 1-m \leq i \leq 0$$

This formulation states that the decision weight π_i^+ in the domain of gains is defined as the difference between the emphases put on all outcomes that are at least as good as or better than x_i and those that are strictly better than x_i. Similarly, the decision weight π_i^- associated with a negative outcome is the difference between the emphasis put on all outcomes at least as bad as x_i and those that are strictly worse. This means that the decision weights can be interpreted as a marginal contribution of the i^{th} outcome. At the end points the decision weight and w coincide since there exists no outcomes better than x_n or worse than x_{-m} respectively.

Then π is an increasing function of the probabilities. So, when the probability of an event increases, so does the value of π. At the margins, $\pi(0) = 0$ and $\pi(1) = 1$, which means that an event that simply cannot occur (p=0) is also weighted as 0 and all decision weights are normalized relative to the certain event[23].

[23] Obviously, this is a matter of choice. Setting the maximum decision weight equal to 1 naturally imposes consequent restrictions downwards on all other decision weights.

5.4.1 Behaviour of the Weighting Function

Having defined the mechanics of the weighting function, I now turn to the implications for the behaviour of the function. The patterns of behaviour and attitudes towards risk uncovered previously also govern the weighting function; this is important to note – the evidence presented by Kahneman and Tversky as in section 4.2 above is described by the weighting function as well as the value function. As mentioned, it is the interaction between the value attached to outcomes and the weight put on their probability of materializing that describes behaviour. The key findings from section 4.2 that apply to the weighting function as well, are the risk averse preferences for gains and risk seeking preferences for losses of moderate or high probability. Moreover, subjects were found to overweight small probabilities. Elaborating on this point, Kahneman and Tversky discovered, through an extensive empirical study, that the shape of the weighting function favours risk seeking for small probabilities of gains and risk aversion for small probabilities of loss. This is illustrated through an extract of the experimental evidence in the table below:

Table 5.1.

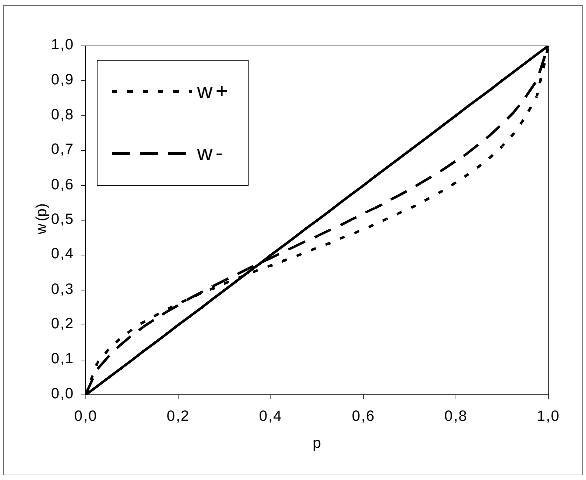
N=25	Gain		Loss	
	$P \leq 0.1$	$p \geq 0.5$	$P \leq 0.1$	$p \geq 0.5$
Risk seeking	**78**	10	20	**87**
Risk neutral	12	2	0	7
Risk averse	10	**88**	**80**	6

Source: Kahneman and Tversky (2000), p. 55.

The table shows, which portion of subjects that made risk seeking, risk averse, and neutral decisions. For example, for gains 78% of subjects made risk seeking choices for small probabilities. From this table we see that a fourfold pattern emerges; *for small probabilities*, the majority of the subjects were risk seeking in the domain of gains whereas the opposite occurred in the domain of losses. For high probabilities, this pattern is reversed. Here we notice that subjects acted risk aversive in the domain of gains and risk seeking in the domain of losses, confirming the previous findings.

The figure below depicts a weighting function in which these attributes are illustrated:

Fig. 5.2 Weighting Function



Kahneman and Tversky label these effects "the fourfold pattern of risk attitudes" and the weighting function is seen to capture this pattern. The figure shows that the patterns of w^+ and w^- respectively have similar cumulative representation as functions of the probabilities. They are concave at small values of *p* and they take on values that are larger than the corresponding *p*'s. Similarly they are convex for moderate and high values of *p* and take on values that are smaller than the corresponding values of *p*[24]. Note that moderate and high probabilities are underweighted due to the convexity of the weighting function. This corresponds to the certainty effect uncovered in section 4.2. All moderate and high probabilities are underweighted relative to certainty due to subjects' risk aversion in the domain of gains and risk seeking in the domain of losses.

In the following section, I represent explicit functional forms of the weighting- and value function in which these attributes are incorporated, following Kahneman and Tversky (1992).

5.5 Technical Formulations of Prospect Theory

The establishment of the fourfold pattern described above gave rise to a more quantitative description of (cumulative) prospect theory. Based on the empirical evidence, Kahneman and Tversky (1992) derived a value function, which is a two-part power function of the following form:

(5.5)
$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x < 0 \end{cases}$$

where, x denotes the relative outcome vis-à-vis the reference point and where α and $\beta < 1$.

[24] Note that for w+, the domain of gains, concavity and overweighting probabilities is equivalent to risk seeking behaviour; vice versa in the domain of losses.

Evidently, this representation of the value function includes the important patterns of risk attitudes derived above. First of all, the value of gains and losses respectively are determined separately. Secondly loss aversion is explicitly modelled into the function. The aversion towards losses is captured by the parameter λ. By the use of a non-linear regression procedure, Kahneman and Tversky estimated the parameter λ to be 2.25, which indicates a severe loss aversion. This means that losses hurt about twice as much as gains yield pleasure. Several authors in very different contexts have documented a loss aversion estimate of around 2[25]. Furthermore, the expressions imply that the attitudes towards risk are as determined earlier, i.e. sensitivity to both gains and losses is decreasing and so there is risk aversion in the domain of gains and risk seeking in the domain of losses. This is seen by the fact that gains are taken to the power of α, which is less than one and equivalently, losses are taken to the power of β. Kahneman and Tversky (1992) estimate both α and β to 0.88. The consequence is that the pattern of the value function is concave in the domain of gains but convex in the domain of losses. For a graphical illustration of this, I refer to fig. 5.1

The cumulative weighting functions were also fitted by Kahneman and Tversky (1992) using the following functional forms:

$$(5.6) \qquad w^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}} \qquad \text{and} \qquad w^{-}(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{1/\delta}}$$

Again, the probabilities associated with gains and losses respectively, are evaluated separately. The results from the estimates of γ and δ were 0.69 and 0.61. This is in accordance with the observation that the weighting function also exhibits diminishing sensitivity – w^{+} is steepest near the endpoints and shallower in the middle of the range – as described above. Moreover, these explicit functional forms obey the implications of prospect theory, i.e. they result in decision weights that are higher than p at small probabilities following a concave pattern and vice versa for moderate and high values of p. These functionals generate the graphs in fig. 5.2.

Having established the cumulative weighting function we can introduce a more general version of the overall prospective value in (5.1) using the value and weighting functions described above:

[25] For instance, Kahneman , Knetsch and Thaler (1990) investigate this in a purely deterministic context. Half a group of students were given a coffee mug while the other half did not receive a mug. Then a market for coffee mugs is conducted in which owners (non-owners) can sell (buy) a mug. The reservation prices for the two groups differed considerably, the median reservation price for the sellers was 2.5 times that of the buyers, indicating a loss aversion coefficient of 2.5.

(5.7) $$V = \sum_{i=-m}^{n} \pi_i v(x_i)$$

Here we see that the overall prospective value is defined as the sum of the value of the different prospects times their respective decision weights.

5.6 Chapter Summary

We have seen that several vital axioms of expected utility theory fails when confronted with empirical experiments. The goal of this chapter was to introduce an alternative theory that incorporated these findings, namely prospect theory.

Prospect theory distinguishes two phases in a decision process. In the framing phase, the decision maker applies different rules in order to organize and simplify the alternative prospects, thereby leaving her with adapted prospects which then are the basis for evaluating and choosing a given prospect.

The evaluation of prospects consisted of two distinct features. The decision maker bases her decisions on subjective decision weights rather than objective probabilities and emphasizes the value of relative outcomes as opposed to final wealth.

Three distinctive characteristics were shown to have influence on the shape of the value function. Reference dependence was imposed due to people focusing on relative outcomes rather than final states. Diminishing sensitivity was imposed due to the finding that people seem to be risk averse in the domain of gains but risk seeking in the domain of losses. This meant that the value function is concave in the domain of gains and the value of gains increases at a decreasing rate. In the domain of losses the value function is convex, i.e. the value of a loss decreases at a diminishing rate. The last property of the value function was loss aversion. The implication for the value function was that it is steeper in the domain of losses than in the domain of gains. These properties led to the development of an S-shaped value function.

The second cornerstone of the evaluation phase concerned the weighting function. The weighting function is the functional form that transforms the objective probabilities into subjective decision weights the decision maker can attach to the value of a given outcome. Kahneman and Tversky applied the cumulative weighting function, meaning that prospects are ranked in increasing order by their outcomes. We saw that decision weights were assigned separately to gains and losses and this led to the formulation that the decision weights in the domain of gains (losses) are defined as the difference between the emphasis put on all outcomes at least as good (bad) as or better than the i^{th} outcome and those that were strictly better (worse).

The interaction between the value attached to outcomes and the weight attached to the probability of it materializing is what describes decision maker-behaviour.

When analysing the behaviour of the weighting function, the fourfold pattern was described. Here it was shown that for gains subjects tended to make risk seeking choices for small probabilities and risk averse choices for large probabilities. For losses these observations were reversed in that for small probabilities the majority of subjects made risk averse decisions whereas they made risk seeking decisions for large probabilities.

The chapter closed with an overview of the technical formulations of the value- and weighting functions. Based on empirical evidence, Kahneman and Tversky were able to fit the functions thereby making explicit functions that captured the vital findings of prospect theory.

The concept of loss aversion expands the traditional perception of risk that is found in expected utility theory. Aversion to risk per se seems not to be the focus for decision makers; rather it is the aversion to losses. This concept will be elaborated on in the next chapter together with the second concept that constitutes myopic loss aversion, namely mental accounting.

6.0 Behavioural Finance

6.1 Loss Aversion

In chapter 5, I laid down the foundation for prospect theory. I established and illustrated that the overall value function has three main characteristics; *Reference dependence,* d*iminishing sensitivity* and *loss aversion*. Loss aversion implies that reductions in wealth, relative to this current reference point, are weighted much more heavily than increases in wealth (i.e. gains) and people are therefore more sensitive to decreases in their wealth than to increases. But what does this mean? And how is loss aversion actually observed? In this section, I will elaborate on the concept of loss aversion. Through references to several examples and empirical experiments that have been done in this field, I will give a more comprehensive explanation of what loss aversion is.

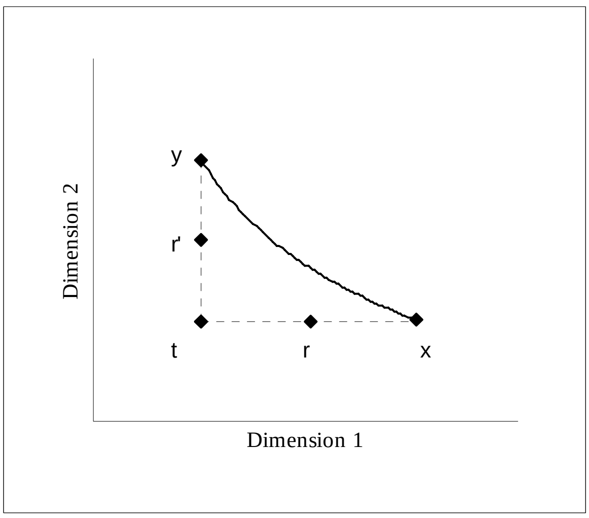
A direct illustration of loss aversion can be seen from the following example (Kahneman, Knetsch and Thaler 1991): A wine-lover purchased some Bordeaux wines several years ago at a price of $10 a bottle. This wine has now appreciated in value, so that a bottle in an auction would now sell at $200. The wine-lover actually drinks some of her fine wine, but she would neither be willing to sell the wine at the auction price nor buy an additional bottle at that price.

Thaler (1980) found that value apparently changes when a good is incorporated into an individual's endowment. He labelled this pattern the *endowment effect*. That is, people often demand much more to give up an object incorporated into their initial endowment than they would be willing to pay to acquire it. So, when the wine-lover does not want to sell the wine at an auction (even though her profit is 1,900%), this is due to her being loss averse; the pain of giving up the wine is so much larger than the pleasure of receiving the $200. The wine example also illustrates another implication of loss aversion, which has been dubbed the *status quo bias*. This means, that people have a strong preference for maintaining the status quo, and this bias makes the wine-lover averse to selling her wine, because the disadvantages of leaving it loom larger than advantages.
The two "anomalies" illustrated in the wine example will now be studied in more detail.

The examination follows Tversky and Kahneman (1991) complemented with examples following Kahneman, Knetsch and Thaler (1991). Through their "Reference-Dependence Model", Tversky and Kahneman (1991) showed that the choice between two options is affected by the reference point from which they are evaluated.

The Endowment Effect and Status Quo Bias are analysed by reference to figure 6.1 below. In the figure, there are two options, *x* and *y*, that differ on two different valued dimensions, dimension 1 and 2. As will become clear, the choice between these options will depend on the reference point from which they are evaluated. The reason for this is that the relative weight of the difference between x and y on dimension 1 and 2 varies with the location of the reference value on these attributes. Loss aversion implies that the impact of a difference on a dimension is greater when that specific difference is evaluated as a loss than when the same difference is evaluated as a gain.

Fig. 6.1



6.1.1 The Endowment Effect

As I discussed above, a direct consequence of loss aversion is that the pain associated with giving up a valued good is greater than the pleasure of receiving it. So, when it is more painful to give up an object than it is pleasurable to obtain it, selling prices will be significantly higher than buying prices.

Kahneman, Knetsch and Thaler (1991) conducted a test of this endowment effect in a classroom setting. A decorated mug was given to one third of the students (sellers) and they were told that they now were the owner of the mug (i.e. they now has the mug as part of their endowment) and asked to indicate whether they wished to (*x*) sell it (at different prices ranging from $0.5 to $9.5) or (*y*) if they wanted to take the mug home. The students who did not receive a mug (choosers) were given the option of receiving either a mug or a sum of money to be determined later. They indicated their preferences between the mug and sums of money ranging from $0.5 to $9.5.

The choosers and sellers actually face the same decision problem, *but their reference states differ*. Going back to figure 6.1, the choosers' reference state is *t*, and they face a positive choice between two options that dominates *t*, they can either receive a mug (which brings them to *y*) or they can receive the cash (which brings them to *x*).

The sellers, on the other hand, evaluate these options from the reference point *y* (they already have the mug). They choose between retaining the mug (staying at *y*), and giving up the mug (i.e. sell it to a chooser) in order to get the cash (which brings them down to *t* and out to *x*).

So, the mug is evaluated as a gain for the choosers but as a loss for the sellers.

Loss aversion implies that the price sellers are willing to accept in order to sell the mug differs from the price choosers are willing to pay in order to acquire the mug. And this is exactly what was found in the experiment. The median value of the mug for the sellers was $7.12 and $3.12 for the choosers[26]. That is, the sellers' reservation price was about twice the size of the choosers'[27].

The difference between these values reflects an immediate endowment effect, which is produced by giving an individual the property right over the mug. The owners' loss of the mug loom larger than the choosers' gain of the mug.

Another implication of the endowment effect is that people often treat opportunity costs differently than "out-of-pocket" costs, i.e. foregone gains are considered less painful than perceived losses.

[26] Note that this is inconsistent with standard economic theory which asserts that a person's willingness to pay for a good should equal their willingness to accept compensation to be deprived of the good

[27] Which gives further evidence for a loss aversion parameter of 2.25.

And according to rational decision theory, these should be treated similarly. An example of this is given below in table 6.1 in which it is illustrated that due to the endowment effect, it is easier to cut real wages during inflation periods.[28]

Table 6.1

	Question A: *A company is making a small profit. It is located in a community experiencing a recession with substantial unemployment but no inflation. The company decides to decrease wages and salaries 7% this year.*	Question B: *A company is making a small profit. It is located in a community experiencing a recession with substantial unemployment and inflation of 12%. The company decides to increase salaries only 5% this year.*
Acceptable:	37%	78%
Unfair:	63%	22%
N	125	129

Source: Kahneman, Knetsch and Thaler (1991)

In this case a 7% cut in real wages is judged fair when it is framed as a nominal wage increase (foregone gain of 5%), but very unfair when it is described as a nominal wage cut (a loss of 7%).

6.1.2 The Status Quo Bias

The retention of the status quo is an option in many decision problems. As was illustrated in the example with the mugs above, loss aversion causes a bias that favours the maintenance of the status quo over other options. Going back to figure 6.1, we see this in that a person who is indifferent between x and y from t, will prefer x over y from x (because she has got to give something up in dimension 1 in order to get to y) and y over x from y (because she has got to give something up in dimension 2).

In an experiment conducted by Knetsch (1989), two groups of undergraduate classes were asked to answer a questionnaire. The two groups were immediately giving a decorated mug respectively a chocolate bar as compensation. At the end of the experiment, the students in both classes were shown the alternative gift and were allowed to trade the gift they had received for the alternative gift. Since the transaction costs were at most minuscule, the fact that almost 90% of the students retained their gift was explained by the status quo bias. They would rather hold on to their initial gift, because the disadvantage of leaving it loomed larger than the pleasure associated with receiving the alternative gift.

[28] This is a direct representation of an experiment conducted by Kahneman, Knetsch and Thaler (1991)

6.1.3 Section Summary

In this section I have elaborated on the concept of loss aversion. I have shown two different effects, that both constitute examples of loss aversion. The endowment effect illustrated the fact that if a person has an object in her possession or endowment, the loss of giving up this object caused much more pain than it would have yielded pleasure to acquire it. It was shown, that the individuals' who already had a mug in their endowment, valued it at approximately twice the price of the subjects who did not posses a mug. That is, the owners' loss of the mug loomed larger than the choosers' gain of the mug. The status quo bias showed that, due to loss aversion, people tend to favour their status quo relative to other options if they have to give up something in order to achieve something else. This was graphically illustrated in figure 6.1 and exemplified through an experiment in which it was shown, that even though subjects had the opportunity to exchange the gift received, only 10% chose to do so. The above-mentioned experiments and examples illustrate that loss aversion, the endowment effect, and the status quo bias are important factors that deserve a fair amount of attention in descriptive analyses of decision, and- choice problems.

6.2 Mental Accounting

In the preceding section, the concept of loss aversion was clarified through the discussion of experiments and examples. I now turn to the elaboration of the second behavioural concept underlying myopic loss aversion, namely *mental accounting*.

Mental accounting, as defined by Thaler (1999), is a theoretical concept meant to capture the way individuals organise their thoughts, i.e. it is the set of cognitive operations that individuals use to structure, evaluate and keep track of for example financial activities. Defined as such, mental accounting has a lot of similarities with the objective accounting systems used by companies and serves the same purpose as these. Of course, in professional corporations, these accounting systems are governed by rules and legislation for accounting, which obviously is not the case for individuals. Still it is similar regarding the basic mechanics. This is because the very essence of mental accounting is that according to Thaler (1999) a given activity is assigned to a specific mental account and thus evaluated in the framework of that specific account. Thus the mental account is vital in determining how a given outcome then is perceived and experienced by an individual. Importantly following the definition of mental accounting these mental accounts can be defined in a variety of ways – from very specific to quite broad – and they can be balanced often or rarely. Below I will show how both of these concepts are important for decision-making and attitudes towards risk. All in all mental accounting is applied by individuals to organise and categorise decisions but is has consequences for decision making because it leads to a mental segregating of factors that might otherwise have been integrated (Rockenbach, 2003).

6.2.1 Mental Accounting and the Framing Phase of Prospect Theory

We have already seen evidence of mental accounting earlier on in this thesis. The framing phase of prospect theory, described above, is vitally linked to the concept of mental accounting. We might say that a key part of the framing conducted by individuals is setting up mental accounts or associating prospects with existing accounts. Earlier we saw the importance of the fact that individuals base their decisions upon relative outcomes instead of final states. This caused failure of the predictions of expected utility theory. The evaluation of gains and losses as separate outcomes is a reflection of mental accounting in that they are posted on different mental accounts. As noted by Thaler (1999), focussing on changes rather than wealth levels as in expected utility theory reflects the piecemeal nature of mental accounting. Transactions are often evaluated one at a time, rather than in conjunction with everything else. So, the determination of the reference point from which gains and losses are derived, which was a key part of the framing phase in prospect theory, results from mental accounting. Individuals will determine, on which mental account the prospect should be booked and as such define the reference point.

6.2.2 Choice Bracketing

As mentioned above a mental account can be very specific or widely defined. To elaborate this important point, I introduce another key concept regarding mental accounting, which is labelled *choice bracketing* (Read, Loewenstein and Rabin 1999). The concept of choice bracketing designates the grouping of individual choices together into sets, i.e. whether a series of choices are made one at a time or grouped together. A set of choices is bracketed together when the choices are made by taking into account the effect of each single choice on all the other choices in a set. *Narrow bracketing* is defined as the situation where sets are small and thereby only consists of one or few choices. When the sets are large, this is defined as *broad bracketing*. Moreover, choice bracketing refers the frequency with which mental accounts are evaluated, i.e. how often the reference point is reset. When choices are bracketed narrowly (broadly), the accounts are evaluated frequently (less frequently). A bracketing effect arises when outcomes chosen under narrow bracketing differ from those chosen under broad bracketing. So, when making many choices, the decision maker can either bracket them broadly by assessing the consequences of all taken together, or narrowly by making each choice in isolation. Because broad bracketing allow decision makers to take into account all the consequences of their actions, it generally leads to choices of higher utility than if the choices are bracketed narrowly (Thaler 1999). Because of this Thaler uses the term ‘myopia’ for narrow bracketing since “the frequent evaluations prevent the investors from adopting a strategy that would be preferred over an appropriately long time horizon” [29].

The concept of choice bracketing can be illustrated through the following example: Consider the decision whether to smoke or abstain (Read, Loewenstein and Rabin 1999). If choices are made one cigarette at the time, the expected pleasure from *each* cigarette can easily seem to outweigh the health consequences that appear trivial when evaluated one cigarette at the time. So lighting up the cigarette can be argued to be the best choice. But if 7.300 single-cigarette decisions a year are combined (this corresponds to a packet a day), the health consequences might appear less trivial and might outweigh the pleasure. The person who makes 7.300 individually inconsequential decisions to smoke, therefore, makes an aggregate choice that might have been rejected, if all the decisions had been bracketed together[30].

Obviously, bracketing too is a part of the framing process, as described above. Or rather, the concepts of narrow/broadly framing (Kahneman and Lovallo 1993) and bracketing are used intertwined.

The cigarette example clarified the concept of choice bracketing and now it is possible to show how narrow bracketing played a key role in the above rejections of the axioms of expected utility theory. Consider problem 6, decision *i* and *ii* in section 4.2. When the choices were presented in this way, the majority of subjects chose the less attractive option, C and F. This, we saw, was due to people being loss averse and giving disproportionate weight to outcomes that are certain. So, subjects were risk averse when making choice *i*, and risk seeking when making choice *ii*. But as we noted above, the other combination, D and E, dominated outcomes C and F. So the subjects apparently bracketed the two choices separately and treated each choice as if it had no connection to the other. Had they bracketed the choices broadly, they would have chosen differently. This was seen in problem 5, in which the choices were presented in aggregation and thus were broadly bracketed. Here *nobody* chose the dominated pair of C and F (i.e. prospect A in problem 5).

6.2.3 Section Summary

In this section the concept of mental accounting have been presented and discussed.

Mental accounting is defined as the cognitive operations decision makers use in order to structure and organize financial activities. A given activity is assigned to a specific mental account and then

[29] Page 200
[30] Had I only read this article when I was 13!

evaluated in the context of this account. Thus, the mental account is the catalyst for determining how an outcome is perceived and henceforth evaluated. A mental account can be defined in both broad and narrow terms, and it can be balanced frequently or rarely. Mental accounting played a significant role in the framing phase of prospect theory, for instance in decision makers basing their decisions on relative outcomes as opposed to final wealth. Gains and losses are posted on separate mental accounts resulting in decision makers evaluating them separately.

Closely related concepts are narrow and broad choice bracketing. Choice bracketing refers to whether a series of choices are made one at a time or grouped together and it refers to the frequency with which mental accounts are evaluated, i.e. how often the reference point is reset. When choices are bracketed narrowly, individuals evaluate decisions/outcomes one at a time and frequently. When choices are bracketed broadly individuals combine several decisions/outcomes into sets and evaluate them less frequently. Broad bracketing generally leads to choices of higher utility because it allows decision makers to implement a wider spectrum of consequences in their decision process. And this accounts for the term ‘myopia’ since frequent evaluations, i.e. narrow bracketing prohibits decision makers from making long-term strategies.

In the next section, the concepts of loss aversion and choice bracketing/myopia are discussed in combination in order to see how this combination can constitute a plausible explanation to the equity premium puzzle.

6.3 The Combination of Loss Aversion and Choice Bracketing

The consequences of narrow bracketing have now been illustrated, but let us consider what happens when it is combined with the concept of loss aversion described above. To illustrate, I recall the findings in the example with Samuelson’s colleague who declined the single bet with 50% chance to win $200 against 50% chance of loosing $100. The argument being that he would be hurt more by the loss that the corresponding gain would give him pleasure. He was, however, willing to take 100 of these bets. In section 4.2, I supplied the arguments that the reason for this irrationality was the difference between viewing each bet in isolation against aggregating them into a portfolio of gambles. Evidently, this is exactly the choice bracketing concept just presented combined with loss aversion. When bracketing is narrow, the bets are individually unattractive due to loss aversion, but if Samuelson’s colleague brackets broadly, the expected utility of the gamble is positive. So by accepting an aggregated prospect of 100 bets, he is revealed *not* to be myopic.

The example leads to the realisation of the potential consequences of myopic loss aversion as the combination of narrow bracketing and loss aversion. Since narrow bracketing leads individuals to scrutinise every single choice and outcome in isolation there will be a tendency to suffer more losses than if the bracketing was broad – especially if the prospect is stock investment. Continuing the discussion in this financial context, the narrow bracketing, all other things equal, yield more return data to be evaluated than when investments and returns are evaluated in aggregation. And due to this, losses may be experienced more often and thereby the effects of loss aversion will be much more severe. Recalling the value function of prospect theory, the loss part of the value function and thus the aggravation parameter λ is applied much more often. Conversely, it is obvious that the consequences of loss aversion will not be as severe if individuals succeed in bracketing broadly. Instead of taking the broad view and evaluating the long run mean return of stock investments the myopic investor brackets narrowly and evaluates for example the volatile monthly returns. Conversely, as Read, Loewenstein and Rabin (1999, page 180) emphasize, if investors evaluate their investments less frequently, i.e. bracket their investment choices more broadly, "the likelihood that they would see such losses would diminish, and the clear benefits of stocks would emerge". Following this intuition, clearly the myopic loss averse investor will have a less favourable attitude towards risky investments than if she was not myopic. This leads to the natural conclusion that such investors are less disposed to taking on the risk inherent in stock investments, i.e. myopic loss aversion inhibits risk taking (Thaler 1999). So for investors to participate in stock investments the equity premium must be higher than if they were not myopic and loss averse. And this is exactly what Bernatzi and Thaler (1995) hypothesize could explain the magnitude of the equity premium.

7.0 Empirical Analysis of Myopic Loss Aversion in the US

7.1 Introduction

Now that the foundations, upon which the concept of myopic loss aversion rest, have been put forward, it is time to focus on the application of myopic loss aversion. To recap from the previous chapters, myopic loss aversion consists of the combination of two behavioural concepts. The application of prospect theory gives rise to the concept of loss aversion, which serves as an alternative risk concept to the traditional economic concept of risk aversion.

Loss aversion refers to people's tendency to be more sensitive to losses than gains, i.e. losses hurt significantly more than gains yield pleasure and mental accounting refers to the narrow framing or bracketing activities people apply when making decisions.

The combination of the two behavioural concepts constitutes myopic loss aversion, which states that investors are averse to losses and evaluate their portfolios at very short horizons.

The goal of this chapter is to describe the analyses that B&T conducted in order to explain the equity premium puzzle. The objective of chapter 9 is then to apply this analysis on Danish data in order to see whether the concept of myopic loss aversion can account for the observed equity premium in Danish data and thereby see whether or not myopic loss aversion constitutes as a robust alternative explanation to a puzzle that has been debated for so long.

7.2 Empirical Hypothesis

As mentioned, B&T set out to explain the puzzle by hypothesizing that investors have prospect theory preferences, that they are loss averse and evaluate their portfolios frequently, i.e. the argument they developed is that the price of financial assets reflects the preferences of investors who are both myopic and loss averse (Thaler et al 1997).

The general question they ask is: If investors have prospect theory preferences, how often should they evaluate their portfolios to explain the equity premium puzzle?

Their answer to this question consists of the answering of the following sub questions: "How often would an investor with prospect theory preferences have to evaluate her portfolio in order to be indifferent between the historical distribution of returns on stocks and bonds?" And taking this evaluation period as given they ask: "For an investor with this evaluation period, what combination of stocks and bonds would maximize prospective utility?"

Since prospect theory, which is the basis for this analysis, cannot and should not be applied as a normative model, but rather a descriptive model in the sense that it tries to *describe* rather than predict the observed empirical behaviour, their analysis should by no mean be seen as such. B&T underline this and state that what they try to do is to investigate whether or not their hypothesis is *plausible*. That is, they perform a descriptive plausibility check consisting of the following elements. They take the observed empirical equity premium as a given, furthermore they accept the described model of prospective value maximisation with loss aversion as a given – including the parameters estimated by Kahneman and Tversky (1992), c.f. section 5.4 above. The test of the ability of the model to provide a plausible alternative explanation of the observed premium is then to derive the equilibrium evaluation horizon that makes investors indifferent between holding stocks and bonds and then scrutinise the plausibility of this result. This is the first part of the analysis. The second part is then to accept the derived evaluation horizon as a given input into the investment decision process of the investor and calculate the optimal asset allocation between stocks and bonds on this basis. The plausibility of the allocation obtained in relation to the typical allocation observed for real world investors will determine the final extent of support for the myopic loss aversion as a descriptive model of the equity premium.

Throughout this analysis, B&T apply Kahneman and Tversky's value and weighting functions as well as the attached parameters, which have been described in section 5.4.

7.3 Results of Benartzi and Thaler

7.3.1 Evaluation Period

The data they use for the analysis consists of historical monthly returns (1926-1990) on stocks, bonds and T-bills. They perform their analyses in four different ways. They compare stock returns to both T-bills and five-year government bond return and these comparisons are done in both nominal and real terms. They choose to focus on the five-year bond return rather than the T-bill return because they feel this is the closest substitute for a long-term investor. They also focus on nominal rather than real figures because returns are usually reported in nominal dollars. Moreover if investors were thinking in real terms they would never hold t-bills over any evaluation period because they would always yield negative prospective utility.

In order to find the equilibrium horizon, they compute the prospective utility of holding stocks, bonds and T-bills for evaluation periods starting at 1 month and then increasing one month at the time. The equilibrium evaluation period for nominal returns was found to be 13 months while for real returns it is around 10 – 11 months. So the evaluation period that makes investors indifferent between holding stocks or bonds is about 1 year[31].

B&T argue, that even though there exists no single universal evaluation period that applies to all investors, if they had to pick a single most plausible length for the evaluation period, one year might be it. Since investors file taxes annually, receive the most detailed and comprehensive reports from their broker, portfolio manager, mutual funds and retirement accounts once a year etc., one year is at least, as they argue, highly plausible. However

7.3.2 Optimal Asset Allocation

In order to test the rather intuitive justification for the evaluation period found, they calculate the allocation between stocks and bonds that yields the highest prospective utility. If this allocation is

[31] The equilibrium evaluation period between stocks and T-bills is about one month less in both nominal and real terms.

consistent with observed empirical allocation, the one-year period is accounted for. They test this by computing the prospective utility of each portfolio mix between 100% stocks and 100% bonds in 10% intervals. Again, they choose to focus on the nominal returns, but the results for real returns are similar.

This analysis revealed that the highest prospective utility was accomplished by allocating approximately 30-55 percent to stocks. To clarify whether this is consistent with observed behaviour, B&T compare their results to the allocation of the major pension funds and endowments in US. They find that such institutions invest, on average, 47 percent in bonds and the remainder in stocks. A similar allocation applies to individuals, where the allocation is about 50-50.

We have seen that for investors to be indifferent between investing in all stocks or bonds, the evaluation period is in the neighbourhood of one year. This evaluation period seems plausible and the observed asset allocation among financial institutions also seems consistent with the optimal allocation following from a one-year evaluation assumption.

This means, that if we accept the model, and here within the applied parameters for loss aversion etc. as well as the application of the data series, and thereby the observed equity premium, B&T account for and explain the size of this premium within the framework of myopic loss aversion. The implication of this is that there exists no puzzle since the magnitude of the premium is explained by the combination of investors' loss aversion and their frequent evaluation of their portfolios.

7.3.3 Implied Equity Premium

According to the analysis above, the equity premium is produced by a combination of loss aversion and frequent evaluations of the portfolio. Since loss aversion as such is a rather diffuse concept and is often not a very conscious consideration for the investor, it is difficult to alter this determinant. But the frequency with which a portfolio is evaluated is, as B&T put is, a policy choice that is more easily revised, i.e. a manipulation of the magnitude of myopia. If the likelihood of observing a loss is reduced (by increasing the evaluation period), then as the evaluation period increases the more attractive stocks become. So taking the loss aversion as given but altering the evaluation period would imply that the equity premium must fall as the evaluation period increases. A fall in the equity premium due to longer evaluation horizons would give further support for the robustness of myopic loss aversion explanation in the sense that if investors would evaluate their investments less frequent, the equity premium would be much lower than what is observed.

B&T showed that this was indeed the case; they showed that as the evaluation period increases the equity premium continuously falls[32]. So B&T are able to provide even more support for the one-year evaluation period.

7.4 Further Evidence of Myopic Loss Aversion

The analysis of myopic loss aversion in the previous section provides us with an explanation to the size of the equity premium. But as B&T point out; the analysis is more of a plausibility test than direct experimental or empirical evidence for the presence of myopic loss aversion.

So, one could argue that the evidence presented in their analysis is circumstantial (Gneezy and Potters 1997). This would mean that it is faced with the joint hypothesis problem in the sense that we cannot be sure whether the results are a function of the choice of theory and that that choice is simply not appropriate.

Several authors have addressed this question by designing direct and controlled experimental tests that can indicate whether the theory of myopic loss aversion holds. I will review some of these experiments in order to provide further evidence for the existence of myopic loss aversion.

Gneezy and Potters (1997) provided such a direct experimental test of the prediction of myopic loss aversion. In contrast to B&T, they do not estimate the period over which subjects evaluate financial outcomes, rather they manipulate this evaluation period in order to see if people's choices differ as a consequence of different evaluation periods. Of course, if the prediction of myopic loss aversion holds, then a frequent evaluation will result in less risky choices.

In Gneezy and Potters experiment, participants are subjected to the same sequence of choices – 12 identical but independent lotteries in which subjects were told that there was a probability of 2/3 of loosing an amount bet and 1/3 of winning 2.5 times an amount bet. Each participant was given 200 cents to bet in each lottery (they were permitted to bet accumulated money won in previous rounds). The subjects were divided into two different groups. A high-frequency group (H-group) where the subjects after *each round* were supplied with information about whether or not they had won the lottery and also the possibility of changing the amount to bet for the following lottery. The low-frequency group (L-group) was only given that information and the possibility of altering their amount to bet after *three rounds.* This was done in order to make subjects evaluate risky financial

[32] They use real returns on stocks and real returns on 5-year government bonds as the comparison asset

investments in a more aggregated way. So the H-group played the rounds one by one, whereas the L-group played the rounds in blocks of three.

Their prediction of this experiment is that if myopic loss aversion holds, the L-group would make more risky choices (because the longer evaluation period would make the trade-off between losses and gains more favourable to the risky choice – they are less likely to be deterred by the occurrence of losses), and thereby will their final amount of money earned be larger than the H-groups.

The experiment showed that in each round average bets were significantly larger for the L-group than for the H-group. And already in the first round, this difference was significant. The H-group bet 50.1% of their endowment and the L-group bet 66.7%. So it appears that right from the start, the design of the experiment was able to change subjects' attitude toward risk.

So Gneezy and Potters experiment provides yet another test of the validity and plausibility of myopic loss aversion as an alternative descriptive theory; a longer evaluation period makes risky bets look more attractive.

Another experimental study of myopic loss aversion was performed by Thaler, Tversky, Kahneman and Schwartz (1997). Two implications of myopic loss aversion were tested experimentally:

1. Investors who display myopic loss aversion will be more willing to take risks if they evaluate their investments less frequently and
2. If all payoffs are increased so much that they eliminate the change of getting a loss, investors will accept more risk (due to loss aversion).

In their experiment, in which 80 undergraduate students participated[33], the subjects were asked to think of themselves as portfolio managers, and were told that they would be required to allocate a portfolio of 100 shares between 2 investments – fund A and fund B. Fund A had risk and return characteristics similar to a value-weighted stock index and fund B had characteristics similar to 5-year bonds. This information, however, was not revealed to the participants. They had to experience this themselves along the way.

In order to test whether the participants experienced loss aversion, they were divided into different groups, each group with different evaluation horizons. Three groups would evaluate performance on a monthly, annual and five-year basis respectively and the last group would also evaluate on a monthly basis, but in this group returns were translated upward by 10% so that subjects always ex-

[33] From the University of California at Berkeley

perienced positive returns from both funds. Subjects in this last group were told that there was a high rate of inflation which was responsible for returns always being positive.

After each decision, the groups saw a bar graph that displayed the aggregated returns of each fund and their portfolio for the period(s) to which the decision applied.

As we now know, myopic loss aversion implies two predictions: First, the allocation to bonds should fall as the length of the evaluation period increases[34] (in essence myopia), that is, the subjects in the "monthly group" should allocate less to stocks than subjects in for instance the "five-year group". Second, the allocation to bonds should fall, when returns are transformed to eliminate losses (loss aversion), because then the implication of loss aversion that losses loom larger than gains would simply not exist in this context (this would be the case for the "inflated group").

Both predictions were supported by Thaler et al's experiment. The first prediction was documented in that the group that invested the most in bonds was the "one-month group", and thereby they earned the lowest return. And the groups that invested the least in bonds were the "one-year" and "five-year group" providing them with higher returns. The effect of myopia was studied by forcing some of the subjects to adopt a nonmyopic framing of the decisions and outcomes: Because they had to commit themselves to multiple periods and therefore received only infrequent feedback, the experience of losses was eliminated and also increased the preference for stocks.
The prediction of loss aversion was confirmed through the use of an "inflated group". That group actually invested the least in bonds of all the four groups, which clearly indicates that loss aversion plays a significant part in people's investment decisions.

The last experiment I will report was conducted by B&T (1996) where two groups of university employees were shown distributions of returns for two hypothetical retirement funds with the purpose of selecting one of these funds to invest in. The distributions were based on 10,000 random drawings of actual US stock and bond returns from the period 1926-1997. The first group was shown a distribution of annual returns, and they allocated 40% of their money in stocks. The second group was shown a simulated distribution of 30-year returns derived from the annual return data by drawing years at random. This group was given essentially the same information, however the allocation to stocks was 90% B&T argued that the subjects who saw the 1-year distribution made the

[34] The probability of observing a loss is lower when the frequency of evaluation is low.

wrong choice because they were fooled by myopic loss aversion into thinking that the probability of losses over the long run is higher than it is.

As seen, several experiments have been conducted that all contribute to the manifestation of the existence of myopic loss aversion. So, even though, as mentioned above, B&T do not formally test their hypothesis that myopic loss aversion explains the equity premium puzzle, the empirical and experimental evidence documented here gives further evidence to the existence of myopic loss aversion.

7.5 Chapter Summary

In this chapter, the concept of myopic loss aversion has been analysed in depth by introducing the analyses performed by B&T in 1995. As a purely descriptive method, they set out to investigate whether or not myopic loss aversion could contribute to a long-debated issue – the equity premium puzzle. By applying the concepts of loss aversion – by the use of prospect theory – and mental accounting, B&T were able to account for the magnitude of the equity premium. They set out by asking if investors have prospect theory preferences, how often would they have to evaluate their portfolios in order to explain the equity premium. By applying the explicit functions from prospect theory, they came up with an evaluation period of approximately 12 months. This evaluation period may appear reasonable on causal grounds, but they wanted to investigate whether or not it seemed consistent with investor behaviour in the US market. By taking these 12 months as given, they examined what combination of stocks and bonds would yield the highest prospective utility and found this to be 30-55 percent in stocks. This allocation was empirically observed, and thus they were able to conclude that an evaluation period of 12 months was accounted for. Now, if the equity premium is a product of loss aversion and frequent evaluations, it can be hypothesized that as the evaluation period is prolonged, the equity premium must fall. This was the final plausibility test of their model. If they were able to conclude that the equity premium falls as a consequence of a longer evaluation period, the model will constitute as a reasonable and plausible explanation the puzzle. And as hypothesized, they found that the equity premium fell as the evaluation period increased. All of these analyses was conducted on both real and nominal returns and by using both T-bills and 5-year government bonds, and all analyses came out with the same results, making their model seem rather robust.

So the conclusion that B&T were able to draw was that the combination of a very high sensitivity to losses (a loss aversion parameter of 2.25) combined with the tendency to evaluate portfolios fre-

quently (every 12 months) provides an explanation to the size of the premium, and hence the puzzle vanished.
In order to further investigate the relevance of the concept of myopic loss aversion, several experimental analyses in different contexts have been examined. As seen, all of these experiments have contributed to the manifestation of the existence of myopic loss aversion.
So, even though, as mentioned above, B&T do not formally test their hypothesis that myopic loss aversion explains the equity premium puzzle, the empirical and experimental evidence documented here gives further evidence to the existence of myopic loss aversion.

In chapter 9 I will perform B&T's analysis on Danish data. If I am able to arrive at similar results as they did, this provide us with further evidence of the robustness and validity of myopic loss aversion as an explanation of the equity premium puzzle. But first we return to the basis, namely the equity premium puzzle.

8.0 Empirical Analysis of the Equity Premium Puzzle in Denmark

I now turn to the more empirical part of this thesis. I want to investigate if an equity premium puzzle can be observed using Danish data. To perform this task I apply Kocherlakota's (1996) method for testing this, as described in section 3.1.3.

8.1 Data

The equity premium puzzle concerns the co-movement of the following three variables: The real return to stocks, the real return to a short-term risk free asset (for simplicity, this is referred to as "bonds") and the growth rate of real per capita consumption.

For this purpose I use the annual returns for MSCI Denmark and the annualized money market rate and data for consumption per capita.[35] In order to get the real returns, all series are deflated with the annual inflation rate. The data series span from January 1971 to December 2005, which gives me 35 annual observations. Realizing that the time period is not very long, I argue that with more than 30 observations[36], significant results can still be achieved.

[35] For a detailed discussion of the data series I refer to section 2.3.1
[36] Pindyck and Rubinfeld (1998), page 36

8.2 Empirical Results

Before moving into the statistical test of the presence of an equity premium puzzle, I present table 8.1 that contains some descriptive statistics for the three variables in which we see the sample means and the variance-covariance matrix for the Danish annual data.

Table 8.1

Summary Statistics

Danish Annual Data, 1971-2005

Sample Means			Sample Variance-Covariance		
			R_{stocks}	$R_{money\ market}$	C_t/C_{t-1}
R_{stocks}	11.3%	R_{stocks}	0.09498	0.0008553	0.0001710
$R_{money\ market}$	2.03%	$R_{money\ market}$	0.0008553	0.001145	0.00013066
C_t/C_{t-1}	1.53%	C_t/C_{t-1}	0.0001710	0.00013066	0.0007709

First of all, we notice that the observed equity premium here is 9.3%. But what is of great interest is that the covariance between consumption growth and stock returns and the covariance between consumption growth and bond returns are not very different. As shown in chapter 3, the results of Mehra and Prescott state that in order to account for a very large equity premium, stocks must covary greatly with consumption growth. As we see, this is not the case here. The covariance between consumption growth and stock returns is only slightly bigger than the covariance between consumption growth and bond returns. Qualitatively, this does imply a high(er) equity premium, since stocks are considered a poorer hedge against consumption risk. Thus, investors will demand a premium for taking on this risk. The question is whether the positive covariance is sufficiently large to explain the size of the equity premium.

In order to test these qualitative observations, I apply equation (3.10) and calculate for every year e_{t+1} for different values of α (going from 0 to 30 at 0.5 intervals). I then calculate the sample means and standard deviations as well as the matching t-statistic.

An extract from these calculations is shown in the table below, where we see the sample mean (e in %), standard deviation (σ in %) and t-stat. for different values of α. I have chosen to depict the results where there is a change in the t-stat (more on this later).

Table 8.2

The Equity Premium Puzzle - Danish Data 1971-2005

α	1.0	2.0	3.5	5.0	6.5	7.5	8.5	9.5	10.5	11.5	12.0	**12.5**	13.0	20.0	30.0
σ	30.3	30.0	29.5	29.2	28.8	28.7	28.5	28.4	28.4	28.3	28.3	**28.3**	28.3	29.2	33.1
e	9.16	9.03	8.85	8.69	8.54	8.45	8.36	8.28	8.20	8.13	8.10	**8.06**	8.03	7.69	7.44
t-stat	1.79	1.78	1.77	1.76	1.75	1.74	1.73	1.72	1.71	1.70	1.692	**1.685**	1.678	1.56	1.33

Several interesting things can be noted about the results depicted in the table. Most important, of course, is the fact that, when α is smaller than 12.5 the sample mean of e is significantly positive. This means that we (at a 5% significance level with 34 degrees of freedom) can reject the hypothesis that the average returns of stocks and bonds respectively are the same, after adjusting for risk. This tells us that only with very high risk aversion is the equity premium accounted for in Danish data, thus it can be concluded that in these data, we are facing a puzzle similar to the one documented by Kocherlakota.

Though I find significant results, some considerations must be addressed. As can be observed from the table, for all values of alpha, the estimated standard deviations are very high. In Denmark, the standard deviation has historically been somewhat higher (around 30%) than for instance in the US (around 20%). This leads to lower test sizes than Kocherlakotas, and thus less significant results. Moreover, the data series only span over 35 years and this combined with very volatile returns makes the t-stat smaller. However, we *do* still find significant results at 5%, so with the above mentioned considerations in mind, I draw the cautious conclusion that we have an equity premium puzzle in the data.

One more thing to notice about the results is that the size of the equity premium and *t*-stat is not affected much by changes in alpha. As noted earlier, the correlation between consumption growth and stock returns and the variance of consumption growth is very low, and this is why α has so little impact on the size of the premium. Mechanically, this is the case since with very low volatility of consumption, α is the exponent of something very close to one making the whole term less sensitive to the size of α. Intuitively, low correlation between stocks and consumption make stocks a better hedge against consumption risk, and as a hedge instrument risk aversion has less of an impact on the attractiveness of stocks.

8.3 Chapter Summary

In this chapter, I investigated if it could be concluded that an equity premium puzzle exists in Denmark. By applying a statistical analysis to this problem, I was able to conclude that we could not reject that a puzzle is present in Denmark. This was confirmed by the result that only with a risk aversion coefficient larger than 12 was the equity premium accounted for. And as previously stated, consensus is that this is implausibly high.

Some considerations regarding this conclusion were discussed. The *t*-statistics were much lower than the *t*-statistics using US data and were not found to be very sensitive to changes in α. These results were due to a very high standard deviation in Denmark in the analysed period and due to a very low correlation between consumption growth and stock returns. Moreover, the analysis only consists of 35 annual observations and this combined with very volatile returns produced smaller *t*-statistics. However, the conclusion was never the less drawn on a 5% significance level, so keeping these considerations in mind, I will conclude that an equity premium puzzle is present in Denmark in the analysed period.

9.0 Empirical Analysis of Myopic Loss Aversion in Denmark

9.1 Introduction

As seen in the previous chapter, the equity premium in Denmark for the period 1971-2005 was calculated to be 9.3%. And we saw that for this size to be explained, the investor would have to have a risk aversion of approximately 12.5. As noted earlier, this is by many considered to be too high.

In this chapter I turn to the empirical application of B&Ts model of myopic loss aversion in order to see if this model can reconcile the equity premium puzzle in Denmark. As B&T, I do not question the magnitude of the equity premium, rather I investigate whether or not the size can be accounted for by the concept of myopic loss aversion.

The overall disposition of the methodology follows B&T as described in chapter 7, i.e. firstly I derive the evaluation period that fits the Danish equity premium to myopic loss averse investors. Then I derive the optimal asset allocation of these investors based on this evaluation horizon. These results are evaluated against empirical evidence. Finally, I calculate implied equity premia for different evaluation horizons to uncover the decreasing pattern predicted by the theory. My goal with this empirical analysis is twofold. Obviously, I want to see if myopic loss aversion can account for the observed premium in Denmark. If I am able to arrive at plausible results which can be verified empirically by the behaviour of Danish investors, I have accomplished this. However, what is also important is the robustness of the model. For instance, B&T argue that their results are not affected by whether they use a T-bill or a 5 year government bond. Nor are the results altered by the use of either real or nominal returns. So I put the results for my primary data specifications into a wider context by comparing with the results for alternative data specifications but also test the sensitivity to the coefficient of loss aversion, i.e. I apply different "sensitivity analyses" concerning both the data series and the parameter estimates in order to see how sensitive my results are to changes in these inputs. I conclude this section by discussing the results obtained and more general limitations, perspectives and implications.

9.2 Methodology

9.2.1 Evaluation Period

The key to my analysis is the calculation of the prospective utility for a given evaluation period because this must be done numerous times, firstly to estimate the evaluation period where investors are indifferent between bonds and stocks – and later on for different allocations to bonds and stocks. The calculation of the prospective utility of an investor for a given allocation between bonds and stocks and evaluation period is a calculation heavy process and since I want to calculate more than 600 of these data points per set of data and parameter values, I utilize VBA programming to perform the calculations. The VBA functions, I have set up to calculate prospective utility are shown in appendix A.5.

When calculating prospective utility the inputs are stock returns, bond returns, asset allocation weights, the parameters of the model, i.e. α, β, λ, γ, δ, and finally the evaluation period.

Now I describe the calculations performed to calculate one value of prospective utility, i.e. for one evaluation period and asset allocation. Firstly, I have to calculate the compound returns of both stocks and bonds over each evaluation period denoted T, e.g. for stocks where r_i is the monthly return, I calculate $\prod_{i=1}^{i=T}(1+r_i)$ -1.

For each observation (compounded T-month return), the prospective value, $v_i(x)$, is calculated, i.e. the return is input in the below value function from section 5.4:

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta} & \text{if } x < 0 \end{cases}$$

The values $v_i(x)$ must be ranked in ascending in order to calibrate the weighting functions and determine the decision weight for each value. The ranking goes from to 0 to n, where the observation n has the highest value.

Based on the ranked series of values, I can determine the probabilities that need to be input in the weighting functions. Denote the probability of a value higher or as high as i by P_i , which then corresponds to $p_i + ... + p_n$ for gains in expression 5.3 in chapter 5 and similar for losses (expression 5.4). Denote the probability of a value *strictly* higher than i by P_i^*, which then corresponds to $p_{i+1} + ... + p_n$ for gains also in expression 5.3 and similar for losses. P_i is calculated from the ranked series for gains (losses) by dividing the number of observation that are as high (low) or higher (lower) than the i'th by the number of observations in all, so $P_i = \frac{n-i}{n}$ for gains and $P_i = \frac{i}{n}$ for losses. P_i^* is calculated for gains (losses) by dividing the number of observation that are *strictly* higher (lower) that the *i*'th by the number of observations in all, so $P_i^* = \frac{n-i-1}{n}$ for gains and $P_i^* = \frac{i-1}{n}$ for losses. Following the calculation of these probabilities, I can now utilize the weighting function of Kahneman and Tversky as specified in equation (5.6) to calculate the decision weights. In the domain of gains (if the calculated *T*-period compounded return is positive) I use P_i^* and P_i to calculate $w_i^+(P_i^*)$ and $w_i^+(P)$. Correspondingly, in the domain of losses I calculate $w_i^-(P_i^*)$ and $w_i^-(P)$. Now the decision weight to put on the prospective value of outcome *i* is $\pi_i^+ = w_i^+(P) - w_i^+(P_i^*)$ if that outcome is positive and $\pi_i^- = w_i^-(P) - w_i^-(P_i^*)$ if that outcome is negative. Now I have both the individual prospective value and the decision weight for each observation, so I calculate the impact of outcome *i* on overall prospective utility as $\pi_i^+ v(x_i)$ or $\pi_i^- v(x_i)$. Finally I sum all these impacts to arrive at overall prospective utility $V = \sum_{i=0}^{n} \pi_i^+ v(x_i)$.

I repeat this calculation for each evaluation period investigated. In my analysis I use evaluation periods from 1 to 30 months for both a pure stock portfolio and a pure bond portfolio. To uncover the evaluation period where stocks and bonds are equally attractive, the two functions of prospective utility as a function of the evaluation periods are plotted in a diagram. The point of intersection corresponds to the evaluation period, where investors are indifferent between investing in stocks or bonds.

9.2.2 Optimal Asset Allocation

The optimal allocation between stocks and bonds or the risk free asset is determined to garner support for the plausibility of the evaluation period. The methodology is extensively the same as described above, but now the derived evaluation period is fixed at the one estimated above. The prospective utility based on this horizon is then calculated for different allocations between stocks and bonds. I cover the range from 0-100% in increments of 5%-points (B&T use increments of 10%-points). I then plot stock exposure against prospective utility to find the optimum. This is the maximisation problem of the myopically loss averse investors when her myopia is assumed given. The asset allocation obtained is then subsequently evaluated against empirical survey data in order to speculate on the plausibility of the result and consequently on the evaluation period derived. Should the asset allocation fit the empirical surveys I will interpret this as support for the notion of myopic loss aversion as an explanation of the equity premium puzzle.

9.2.3 Implied Equity Premium

The final test is also one of plausibility. The implied equity premium should fall as the evaluation period increases. This follows from the mechanics of myopia and prospect theory preferences as scrutinized earlier. Hence, I calculate the implied equity premium for increasing evaluation horizons. The methodology is that for each evaluation period, I need to calculate prospective utility given this horizon. The calculation is conducted for stocks but also for the risk free asset where a premium has been added to the return for each month. This small premium must then be incrementally increased until the prospective utilities of stocks and the risk free asset plus premium are exactly equal. By doing this I find the equity premium needed to make myopic investors with prospect theory preferences indifferent between stocks and risk free investment, i.e. the premium implied by the theory. Concretely, since I use VBA programming to calculate prospective utility, I can use the Solver tool in Excel to determine the premium to add to monthly returns (the VBA functions specified in appendix A.5 recalculate prospective utility with each increment whereby the Solver tool can minimize the difference between the utility of stocks and the risk free asset plus premium). Once the utility of the two assets are equated by adding the resulting equity premium, I compound the monthly returns for each to annual returns and then calculate the mean annual return for stocks (unchanged) and risk free asset plus equity premium. The difference between these two is then the premium implied by the evaluation period used. This calculation is repeated for several different evaluation horizons and then the implied equity premia are plotted against the evaluation periods to evaluate if there is the suggested decreasing pattern.

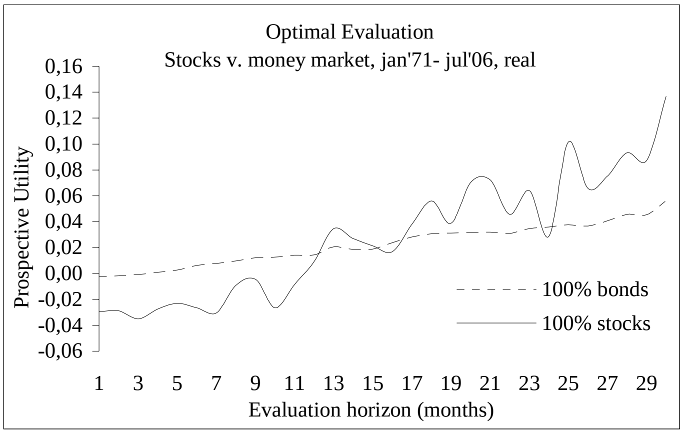
9.3 Empirical Results

9.3.1 Evaluation Period

Now I turn to the empirical results I have obtained by applying the methodology above. The analyses will be based on the same data series I applied in chapter 8, namely the MSCI Denmark real stock returns and the real return to risk free investments.

Figure 9.1 below shows how the prospective utility of investing 100% in stocks respectively the risk free asset evolves at different evaluation periods.

Fig. 9.1



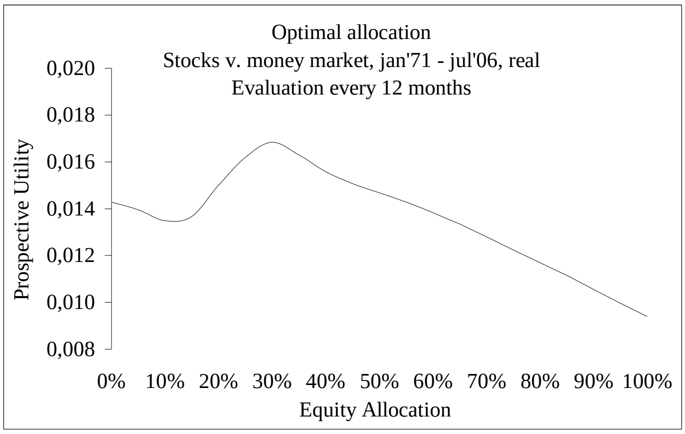
As expected, the longer the evaluation period the higher is the prospective utility from both asset classes since both yield positive long term average returns. Recall that as investors evaluate performance less frequently, they will experience negative returns less frequently as well, this in turn means that the loss aversion kicks in less frequently giving them higher prospective utility. Since stocks pay higher average returns than bonds in the long run but are more volatile, i.e. experience negative returns more often and of greater magnitude, the utility of stock investment is lower at very short evaluation horizons but increases to become higher for long horizons. The intersection point between the two illustrates the evaluation period at which the two assets are equally attractive for the myopic loss averse investor. In fig. 9.1, I find that this happens at an evaluation period of approximately 12 months, all though the course for the stock utility is quite volatile resulting in further intersections at around 16 and 24 months. However, these appear to be of insignificant character since they do not lie on the average upward trend of the line. This tells us that in order for myopic investors with prospect theory preferences to demand the observed equity premium, they must evaluate their portfolios on an annual basis. With more frequent evaluations stocks yield lower utility due to loss aversion kicking in too often (implying a higher required premium) and with less frequent evaluations stocks are more attractive since loss aversion is more rarely applied (implying a lower required premium). At 12-month evaluation, these two effects are balanced to make investors indifferent based on the empirically observed equity premium. Hence the first conclusion from my analysis of the Danish data is almost identical to the findings of B&T on US data; investors with prospect theory preference must evaluate their portfolios annually to generate the observed equity premium. The question then is, whether or not this is a plausible evaluation period. Using the intuition of B&T, annual evaluation seems plausible using casual evidence. Tax returns are filed annually, bank statements and financial reporting, etc. also follow annual reporting cycles. So the face value of my results seems justifiable, but in order to support the finding of a 12-month evaluation horizon more directly, however, I now take it as given and inspect the resulting asset allocation.

9.3.2 Optimal Asset Allocation

In order to check the reliability of this 12-month evaluation period found, we must determine what combination of the two assets maximizes prospective utility. The intuition is that if I find that the optimal asset allocation/portfolio composition, given the 12-month evaluation period, reconciles with the observed behaviour of Danish investors, then a 12-month evaluation period seems accounted for. And thereby it can be concluded that the concept of myopic loss aversion offers a reasonable explanation to the size of the equity premium.

Fig. 9.2 below graphs prospective utility as a function of the allocation to stocks given a 12-month evaluation period.

Fig. 9.2



The graph is not perfectly well behaved since the theory would indicate that prospective utility rises smoothly to its maximum and then falls hereafter. I find that at low levels of stock allocation further exposure actually decreases prospective utility. This means that the higher long run average return of stocks is not sufficient to outweigh the extra volatility induced by increasing the allocation to stocks at low levels. From 15% onwards, the relationship is as suggested by the theory. As can be seen, I find that prospective utility is maximized with the allocation of 30% to stocks and the remainder in the risk free asset. The question is then whether this is consistent with the observed behaviour of investors in Denmark. Table 9.1 below presents the key asset allocation figures as collected by the OECD regarding Danish pension funds for the period 1990-99 and by Kirstein Finansrådgivning for all institutional investors in 2006 and by The Federation of Danish Investment Associations (IFR) for institutional as well as retail investors in 2005.

Table 9.1 Asset allocation (pct. of total assets)

	1990-99, OECD[1)]	Kirstein, 2006[2)]	Institutional, IFR[3)]	Retail, IFR[3)]
Shares	31	26	32	38
Bonds	55	60	67	45
Other	14	14	1	16

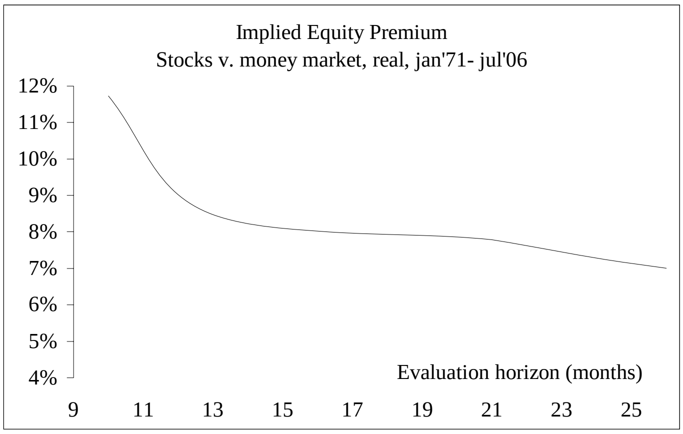
[1)] OECD, Financial Market Trends, No. 80, September 2001
[2)] Kirstein Finansrådgivning A/S. Investor Survey, 2006
[2)] The Federation of Danish Investment Associations (IFR), Market Statistic, December 2005

On average, the finding of an institutional allocation of approximately 30% of assets to stocks is supported by the data. The surveys report stock allocations for institutions of between 26% following Kirstein (2006) and 32% following IFR (2005). For retail investors the allocation to equity is slightly higher at 38%. It is important to note, however, that the data based on investment associations (mutual funds) must be used cautiously. To some degree the supply of and subsequently the demand for mutual funds will be driven by the sales and marketing efforts of financial institutions. Thus the minor overweight to equity funds might just be because these are more accommodating to market especially to retail investors. Overall, however, the plausibility test of the predictions of our model seems successful. The evaluation horizon that explains the equity premium produces an asset allocation that is supported by data on Danish investors. One important note at this stage is that the model predicts 30% to stocks and so 70% to risk free assets, i.e. money market securities. Table 3 shows that the alternatives to stocks for Danish investors are bonds and other investments (e.g. real estate, hedge funds or private equity) in the proportions 45-60% to bonds and 10-15% to other investments. Assuming that the impact of other investments is negligible, one must conjecture, however, that the 45-60% bonds are not all money market securities but rather higher yielding government and mortgage bonds. One could argue, though, that theoretically the results should be independent hereof since bonds, even though they have higher expected returns than the risk free asset, they also have higher expected risk. So on a risk adjusted basis bonds and bills constitute the same alternative to equity. B&T use bills and bonds interchangeably as alternatives to equity in their analyses. In section 9.4.2, I show the results for five year bonds in order to tests this invariability of the model. Now, however, I turn to estimating the equity premium implied by the model.

9.3.3 Implied Equity Premium

I have shown how the empirically observed equity premium can be seen as a result of loss aversion and frequent evaluations. Following the methodology of B&T, I now turn to estimating how much the equity premium implied by prospect theory preferences falls as the evaluation horizon increases. As was seen in fig. 9.1, the attractiveness of stocks increase, the less frequent their returns are evaluated, thus we would expect investors to demand a lower premium to equities at longer horizons. Fig. 9.3 below shows the estimated equity premium implied by the real Danish stock and risk free returns.

Fig. 9.3



As described in section 9.2, the premia are estimated numerically by fixing the evaluation period and then calibrating the equity premium that makes the prospective utility of holding 100% stocks equate the utility of 100% in the risk free asset. From the figure it is clear to see that the intuition holds. As the horizon increases, investors will demand a lower equity premium. Note that as was shown above, the observed equity premium of 9.3% is consistent with an evaluation horizon of 12 months. For longer evaluation periods the premium demanded falls to around 7-8% percent. At even longer evaluation horizons, not shown on the graph, the premium diminishes further, e.g. at 60 months the premium is 2.53%. So from these findings, the consequences of myopia are rather evident. If only investors could refrain from evaluating their portfolios so often, equities did not need to carry such a high premium. This analysis also supports the statement that if only investors were not myopic, then loss aversion by itself would not be able to produce the equity premium observed.

9.3.4 Section Summary

In this section, I found support for the theoretical framework as a descriptive model on Danish data. I found that if investors have prospect theory preferences then the model can explain the equity premium if investors are myopic evaluating their portfolios on an annual basis. Furthermore, this finding was supported by estimating the corresponding optimal asset allocation, which turned out to fit empirical survey data well. Finally, I showed that as suggested by the concepts of myopia and loss aversion, the equity premium demanded by investors actually falls as their evaluation period becomes longer. The results in this section, was produced using real returns of Danish stock and money market securities. In the following section, I will submit the model to different inputs in order to comment on the robustness of the results.

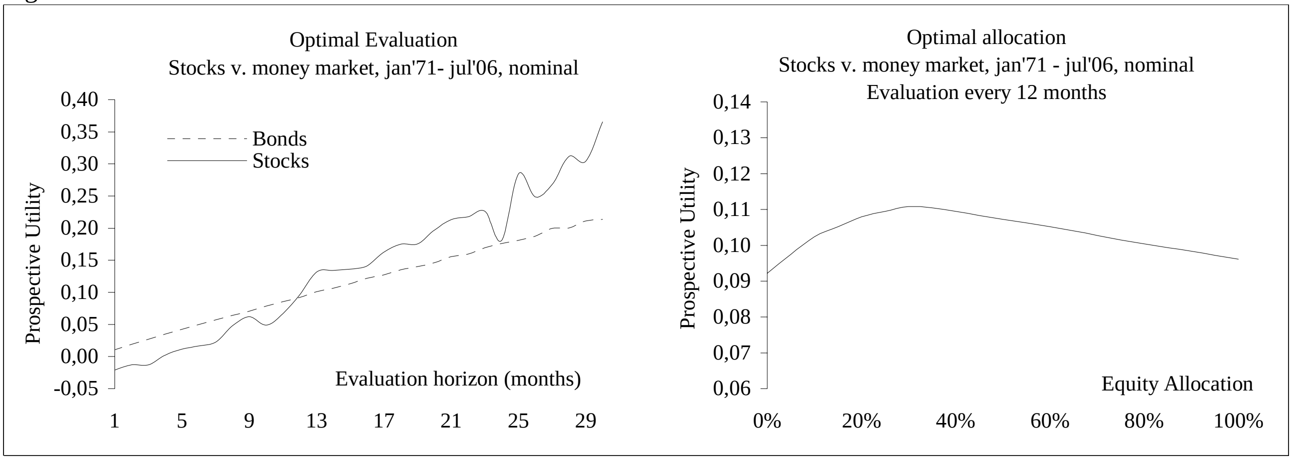
9.4 Sensitivity Analyses

In this section, I put the analysis on Danish data into further perspective by recalibrating the key results using different data inputs and at the end of the section by altering the parameterization of prospect theory preferences utilized.

9.4.1 Real Returns vs. Nominal Returns

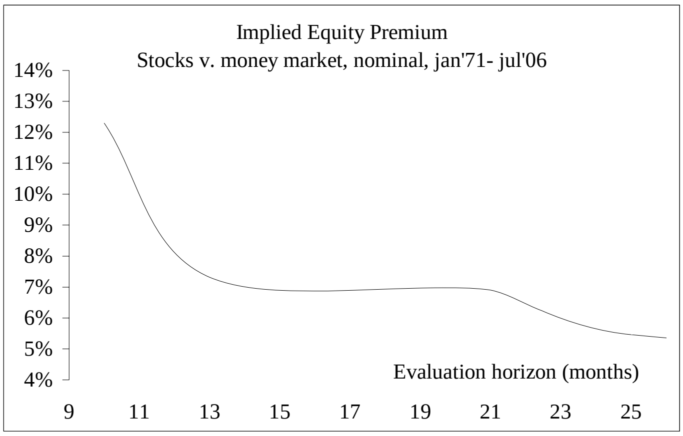
B&T find that their analyses and results are robust to estimations using both real and nominal return data. To gain further insight on the robustness of the model using Danish data, I present the results using nominal data in fig. 9.4 and 9.5 below.

Fig. 9.4



It is evident from the figure that the results using nominal returns support the findings above. The optimal evaluation period is found again to be approximately 12 months. Fixing this period and performing the optimization on the nominal data produces an optimal asset allocation of precisely 30% equity and 70% bonds. Fig. 9.5 shows the behaviour of the nominal implied equity premium as the evaluation period increases. Again the findings are similar as the premium diminishes if investors evaluate less frequently.

Fig. 9.5

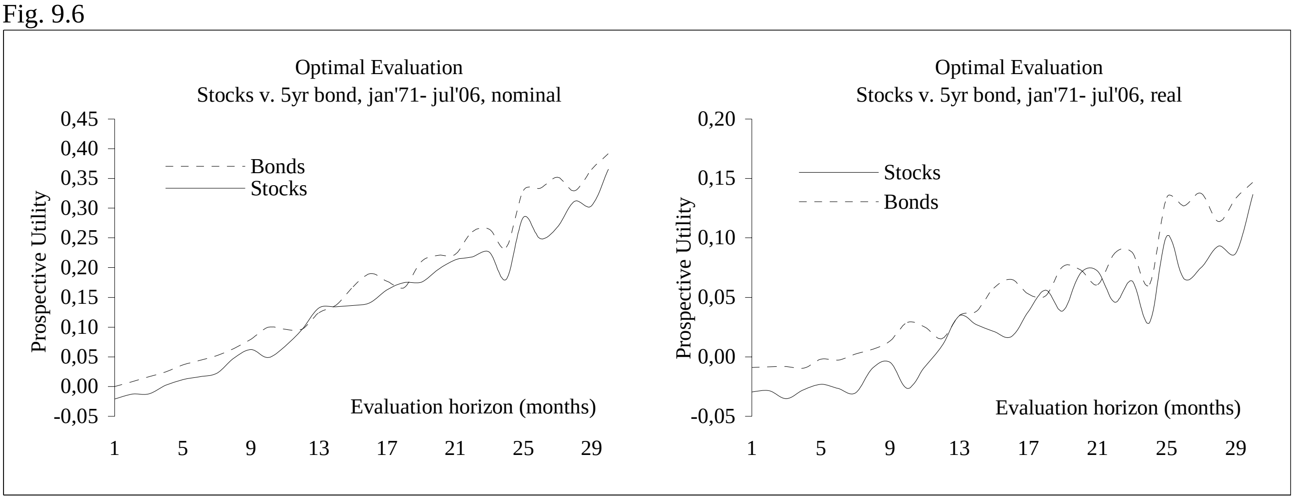


The risk premium in nominal terms over the period is 9.7%, which again is consistent with an evaluation period of approximately 12 months. If investors evaluate more rarely than annually, the premium required approaches 7% and with biannual evaluation the premium is down to 5%. With evaluation only every 5 years the premium is down to 2.52%. Hence, the results hold and are very similar when using nominal instead of real return data. So my results in this regard support the findings of B&T.

9.4.2 Money Market Returns vs. 5-Year Government Bond Returns

B&T make extensive use of 5-year government bond returns as the bond alternative to equity investments stating that T-bills are a less intuitive alternative for the representative investor. Fig. 9.6 shows the results of my analysis using a time series of 5-year bond returns instead of the money market return, real and nominal data respectively.

Fig. 9.6



The results based on 5-year bonds are much less accommodating than what I found above. For nominal data in the left panel of fig. 9.6, there is an apparent intersection between stocks and bonds around 13 months but the graphs are volatile and less well behaved. The prospective utility of equity investment does not decisively exceed bonds at any evaluation horizon. This means that in this framework investors will never hold equities over bonds. Using real data in the right panel further blurs the picture, as again the bonds seem to yield higher prospective utility irrespective of the evaluation horizon. It seems coincidental that they intersect at around 13 months, which in principal support the findings for stocks and risk free investment above. The calculation of optimal portfolios taking the 13 month evaluation period as given in both cases produces an optimal stock exposure of about 55%, i.e. higher than the 30% found when using the money market rate. The reason for this rather counterintuitive result is that bonds have been much riskier than the money market securities and have generated a higher mean return (7% in real terms). The volatile nature of the bond returns obviously makes them less attractive to myopically loss averse investors than the money market rate and consequently, I find the optimal allocation to stocks to be higher. On a pure 5-year bond portfolio, the loss aversion kicks in quite often too, and thus the higher mean returns on stocks as compared to bonds (11% compared to 7%) is more important, i.e. since investors will feel the aggravation of negative returns often, they might as well pick up the higher mean return of stocks. This is why stocks make up a larger portion of the optimal portfolio when using 5-year bonds instead of the money market rate. As mentioned, B&T find the same results for T-bills as for 5-year bonds. It can be argued that this should be the case since short term bills and longer term bonds should yield similar *risk adjusted* returns, i.e. the bond alternative against which stocks are evaluated should not matter. This is not an argument carried by B&T, however. They use 5-year bonds throughout their analysis and simply state that results for bills are similar. My position is that this argument is based on an assumption that the risk premium to bonds relative to bills is exactly equivalent to what expected utility theory predicts; an assumption that should not be trivial when investors are assumed to be myopic and have prospect theory preferences. Moreover, to explain the equity premium, I feel, as argued previously, that it is most proper to use the risk free rate. When comparing to bonds, the premium is not *pure* since bonds are risky resulting from reinvestment risk and price risk. Thus the results I obtain above can actually be viewed as quite intuitive because an investor who is loss averse will require a premium for the risk of observing negative returns on bond investments too – a premium that depends on the evaluation horizon.

9.4.3 Loss Aversion Parameter Value

In this section, I present my results of the analysis when the parameter of loss aversion as originally estimated by Kahneman and Tversky (1992) is altered. To this end I revert to my original basis, i.e.

the real returns of stocks and risk free investment. Table 8 below present the key findings for different values of the loss aversion coefficient λ originally set to -2.25.

Table 9.2

λ	-1.50	-2.00	-2.25	-2.50	-3.00
Evaluation period	4	11	12	17	30
Stock allocation	100%	30-45%	30%	5%	0%

As mentioned earlier, several authors suggested a loss aversion factor of around -2, and the estimation of Kahneman and Tversky from chapter 5 produced the -2.25 used so far. When recalibrating my analysis for each of the values of λ in table 9.2, I find that these values seem to generate the most accommodating results on Danish data as well. Remember that the findings state that to reconcile to the observed empirical equity premium investors with prospect theory preference must evaluate their portfolios with the evaluation periods found. This in turns implies the stated optimal allocation to stocks. The results with a loss aversion parameter of -2 are similar to my original conclusions based on -2.25 and as such it is possible to conclude that the theory does not rely heavily on which precise estimate is more correct. The results for other values of the loss aversion parameter show two things. Firstly, the model works as we should intuitively expect, i.e. for lower values for λ the evaluation period is low and optimal stock exposure is high. In contrast where loss aversion is high, the evaluation period is long and stock allocation low. Secondly, the analysis implies strong support for the notion of Kahneman and Tversky that λ is in the 2-2.25 range. The other values simply do not produce stock allocations that fit the empirical survey data of table 9.1 above. With low loss aversion, e.g. -1.5, the optimal evaluation period is four months, which makes sense since the investor is less sensitive to observing losses. The resulting allocation of 100% stocks is not plausible, though, and so -1.5 is not a viable measure of loss aversion in my data. The same can be said for very loss averse investors, as for a loss aversion parameter of -3, the optimal evaluation period is 30 months; people are so heavily aggravated by losses that they should only evaluate returns very rarely. Again the optimal allocation of zero percent stocks is not plausible and a λ-value of 3 can be rejected. This factor push analysis show that the results hold up to minor changes to the loss aversion parameter, which are in the range of what Kahneman and Tversky found in their original work, but parameter values that lie far from these values can be ruled out since they produce asset allocations that do not fit empirical evidence. So, these findings constitute rather strong support for the model and the value of λ.

9.4.4 Related Research

Two earlier master theses[37] from Copenhagen Business School have examined the presence of myopic loss aversion in Denmark. Both examined the period 1925-1999 in which they applied annual nominal stock returns and 1-year bond yields estimated by Nielsen & Risager (2001). This means that these analyses use annual returns and thus only focus on the estimation of the optimal asset allocation of Danish investors and the estimation of the implied equity premium. Since B&T's approach to estimating the evaluation period of investors is based on monthly returns, these theses are prohibited from performing the evaluation period estimation. As such my derivation of the evaluation period for Danish investors has no precedence to which it can be evaluated. The previous master thesis analyses on the area takes as a given the one year evaluation period found by B&T and assume that it also applies to Danish investors. Since, however, I find an evaluation period of approximately one year in my analysis, this assumption is somewhat vindicated at least in retrospect, and further comparisons of the work can be tentatively conducted.

The optimal asset allocation is in these theses also found to be approximately 30% to stocks, so this corresponds to my findings. So does the implied equity premium which, as it should according to the theory, falls as the evaluation period increases. Hence, all though these previous analyses are based on assumptions more so than my approach, it is possible to interpret the aggregate evidence as support for the model and approach. Of course, the support would be much stronger if the evaluation period was actually derived using the above methodology since even though my analysis shows that the evaluation period that solves the equity premium puzzle is 12 months, the assumption that this hold for the Nielsen and Risager data as well is not trivial. To sum up, the amount of related work in Denmark is limited and since it does not cover the complete analysis that I have performed, unambiguous conclusions upon and comparisons between their findings and mine should be conducted with caution.

9.4.5 Section Summary

The purpose of this section was to explore the consequences of different sensitivity analyses to the model. B&T were in their analyses able to find consistent results using both nominal and real returns and with both a short term risk free asset as well as 5-year government bonds, and thus were able to conclude on the robustness of the model. My main results were found to hold for nominal as well as real returns but when using 5-year bonds instead of the money market rate the results were rather different. So in this concern, my conclusions differed somewhat from the conclusion of B&T.

[37] Van Daalen and Thoroddsen (2005), Lalovic (2001)

I argued, however, that the results were not entirely counterintuitive due to the risk return characteristics of the bonds. Testing the value of the loss aversion parameter λ by repeating the complete analysis for different values of λ produced strong support of the original specification. First of all, it showed that assuming a loss aversion parameter of approximately 2 seemed reasonable. Only in this range, were results consistent with survey/empirical data. Moreover the parameter analysis showed that the model behaves as it is supposed to according to prospect theory. When the loss aversion parameter decreases, the corresponding allocation to stocks increases and vice versa.
Finally, I discussed some related empirical work on Danish data. And as mentioned, even though the conclusions to a large extent are similar to mine, there are some divergences that should be taken into consideration if one is to compare the results.

9.5 Discussion of Potential Limitations

In this section, I put forth some points of criticism one might raise against my empirical analyses.
First of all, the period under scrutiny is limited due to the lack of data available. B&T investigate a period of 64 years whereas my data series is just above half of that at 35 years since 1971. This critique is, of course, always a valid one since the results are only as robust or representative as the input data. My focus throughout has been on the theory and analysis and not on reconstructing data series for historical periods but an obvious route for further justification to the approach was to construct monthly returns for a longer period. Another potential caveat regarding the data set is the fact that the data series for 5-year bonds was constructed for part of the history. Obviously, this is not optimal as a basis for decisive conclusion. This means that the results for bonds should be concluded upon with caution. The data series for 5-year bonds is, however, not part of my primary subject area and as such is not a weakness of my *primary* conclusions. Moreover, as noted earlier, the comparison of the constructed and actual series for the period where both are available shows notable equivalence.

A further potential question regarding my approach concerns the plausibility test of the findings, where a central real world check was the evaluation of the optimal asset allocation against empirical survey data. The derivation of optimal asset allocation was based on a model of individual decision-making. But when investigating the empirical validity of the optimal asset allocation, most of the empirical observations were based on data for Danish pension funds and other institutional investors. An obvious question is whether these “professional” investors intuitively should be expected to display the traits of loss aversion and frequent evaluations. I will address this in the following. It is a fact that in the long run, stocks have outperformed bonds and T-bills (and the like). And it is not unreasonable to state that professional institutional investors such as pension funds are likely to exist infinitely. That is, their investment horizon is essentially infinite. So why do they not invest more in stocks than the approximately 30% found previously? I argue that even for professional institutions with very long horizons it is reasonable to assume that myopic loss aversion still applies because of the agency problems inherent in institutions. Even though a pension *fund* may exist for a very long time, the pension fund *manager* (the agent) is not likely to be in that job for the same length of time, so her individual horizon diverges from the funds horizon. First of all, she will have to make regular reports on the funding level and fund returns to senior management and the pension clients (the principals). So on a regular basis her performance is being evaluated either by senior staff or by the clients who both have the opportunity (if performance is poor) to sanction her either by her getting fired or the clients withdrawing their money. So, this short horizon can create a conflict between the manager and the principals. Secondly, most managers have personal concerns regarding their track record and potential bonuses. So this too can create a conflict between her short term evaluation horizon and "the optimal" more or less infinite evaluation horizon. So, even though an objection could be raised concerning the asset allocation of institutional investors, I argue that they too can display the traits of myopic loss aversion.

Finally, the theoretical model assumes that the parameter of loss aversion is fixed. That is loss aversion is assumed to be constant over time regardless of the dynamic character of returns. This is a matter of model choice and B&T choose to apply the functions and parameters of prospect theory. I will, however, put forth some thoughts concerning a more dynamic application of the concept of loss aversion. Some empirical evidence[38] seems to suggest that the degree of loss aversion depends on prior gains and losses; a loss that comes after prior gains is less painful than usual because it is cushioned by those earlier gains. And a loss that comes after other losses is more painful than usual; after being burned by the first loss agents become more sensitive to additional losses. Thaler and Johnson (1990) find through experimental evidence that a prior gain can increase an individual's willingness to accept gambles – the so-called "house money effect". And they also find that prior losses can decrease the willingness to take risks (the "snake-bite effect"). So with these observations in mind, it could seem reasonable to adjust the myopic loss aversion-model to take into account these different perceptions of risk thereby making it more dynamic. A problem facing this approach, however, is the question of which gains and losses this should be applied to. Should this be done on individual stocks (or other asset classes) or on the overall portfolio level? And here we

[38] Thaler & Johnson (1990), Barberis & Huang (2001), Nofsinger (2005)

return to the concept of mental accounting. This will depend on how and to what extent people bracket the outcomes. So depending on whether they bracket narrowly or broadly, this should be done on either the aggregated portfolio level or on the individual returns on the individual asset (Barberis and Huang 2001). To my knowledge, this dynamic approach to loss aversion has not been tested in the field of myopic loss aversion, but is in my opinion an important topic for further research thereby subjecting the concept of myopic loss aversion to further scrutiny.

9.6 Chapter Summary

In this chapter I have thoroughly examined the hypothesis originally posed by B&T. They hypothesized that if investors are loss averse and evaluate their portfolio returns too frequently, stocks are considered a very unattractive asset class due to the large possibility of experiencing a loss. So, for investors to be willing to hold stocks, they will demand a large equity premium. I began the empirical study by utilising the real returns to the MSCI Denmark and the risk free asset that resulted in an equity premium of 9.3%. For this equity premium to be explained I found that investors would have to evaluate their portfolios on an annual basis. As I argued, this did not seem highly unlikely since investors often receive their most comprehensive reporting annually. However, in order to test the plausibility of this, it was necessary to calculate the optimal asset allocation given the one-year evaluation period and then compare this to actual observed allocation among Danish investors. Taking the one-year evaluation period as given, I calculated the optimal asset allocation to be 30% to stocks. Several independent studies report that the allocation to stocks among both institutional and private investors is approximately 30% and the remainder to bonds and other assets. So when comparing to real world data, the asset allocation found and thus the one-year evaluation period is accounted for. If stocks are unattractive due to frequent evaluation of the portfolio returns, the opposite most hold as well. That is, stocks become more attractive as the evaluation period increases due to the lower probability of experiencing an aggregated loss, thus this would result in a lower equity premium. So as a further reliability test of the results of the model, I calculated the implied equity premium by manipulating the evaluation period. As hypothesized, I found that as the evaluation period was increased, the implied equity premium fell. So also in this respect, the model seemed to hold quite well.

So the main conclusion from the first part of the empirical analysis was that the equity premium is explained if investors have prospect theory preferences (with a loss aversion parameter of 2.25) and evaluate their portfolios on an annual basis. As such this supports the model and findings of B&T. However, in order to test the robustness of the model I subjected it to several sensitivity tests. When confronted with nominal short term money market returns rather than real returns, the model produced similar results both with regards to the evaluation period and the implied equity premium, thereby supporting the original results and conclusions. However when confronted with 5-year government bond returns (both nominal and real), the results were rather different. At no evaluation period was the prospective utility for stocks higher than for bonds. I argued how this to some extent could be rationalized with the approach since bonds too have had volatile returns with losses in some years. Thus myopic loss averse investors could be expected to demand a premium for bearing the risk of experiencing the aggravation of observing a loss on bonds. Following this argument, it seemed fitting that using bonds yield different results than using bills in my analysis. Consequently I will not use these results as a detractor from the support to the model and the overall case for the plausibility of my approach seems strong based on the empirical analyses. The final sensitivity test concerned the parameter for loss aversion. It was shown that a parameter value of around 2 produced reliable results and thus the assumption of a value of 2.25 was accounted for. Moreover the model was shown to behave according to theory since a lower value produced a lower evaluation period and higher stock allocation and vice versa.

Potential limitations of the approach were reviewed, pointing to the availability of sufficient data, which is always a caveat, but also to the use of survey data for both individuals and professional investors. I argued that due to the institutional factors and incentive structures characterizing large corporations and funds the occurrence of myopic loss aversion might be as frequent among professionals as among private individuals. A key area for improvement was the stationary or rigid nature of loss aversion. Arguably, integrating dynamic effects to loss aversion could constitute a potential route for further research.

10.0 Conclusion

The overall objective of this thesis has been to investigate if myopic loss aversion as a description of investor behaviour constitutes an improvement in explaining the equity premium in Denmark.

To reach this goal, I have covered extensive theoretical ground. Firstly, I investigated to nature of the original puzzle. The standard model derived from expected utility theory was incapable of reconciling the empirical equity premium. Agents had to be implausibly risk averse to fit the data. Reviewing alternative explanations showed that most attempts could be refuted as containing similar flaws regarding the assumptions of the investors' risk attitudes. The puzzle had held up well to the attempts to reconcile it. My approach to shed light on the equity premium puzzle was to focus on an alternative descriptive theory based on myopic loss aversion. To argue why, I showed how expected utility theory could be seen to fail as a useful descriptive model when confronted with the real world. I presented prospect theory as the answer to this inconsistency. Prospect theory was different by being exclusively a descriptive model and the theoretical basis of it, was derived from empirical findings. The defining features of prospect theory were that investors had subjective views of probabilities, focused on gains and losses compared to a neutral reference point rather than final wealth, and finally were assumed to be loss averse. Based on prospect theory, I presented myopic loss aversion as the description of investors' attitudes towards risk. I elaborated on the concept of loss aversion, showing that people seem to be more hurt by losses than corresponding gains would yield pleasure. This was captured and illustrated through the endowment effect and status quo bias. The second cornerstone of myopic loss aversion was built on mental accounting, a term that captures peoples' way of organizing and keeping track of financial activities. I introduced the concept of choice bracketing, which embraced the way people pool decisions and outcomes in either broad or narrow terms. Narrow bracketing referred to the situation where people evaluate outcomes one at a time and on frequent basis, and hence narrow bracketing explained the term myopia. Thus myopic loss aversion meant that agents had prospect theory preferences but also evaluated the outcomes of their decisions too frequently. This combination was expected to present an improvement capable of reconciling the puzzle since loss averse investors who evaluate returns often, i.e. are myopic, will demand a higher premium than a rational utility maximizing agent. I further elaborated on an analysis conducted by B&T showing that the US equity premium was explained if investors had prospect theory preferences with a loss aversion factor of 2.25 and evaluated their portfolio returns on an annual basis. Since the annual evaluation period was found to be consistent with empirically observed asset allocation for both institutional and private investors, B&T was able to conclude that the evaluation period derived was accounted for. As a further test of the model, the implied equity premium was derived. This analysis revealed that as the evaluation period is prolonged, the implied equity premium falls. This brought further evidence to the relevance of the model, and B&T concluded that myopic loss aversion constituted a relevant alternative in explaining the equity premium puzzle. To give further support to myopic loss aversion, I documented several experiments all showing how myopic loss aversion seems to exist in experimental settings.

In order to put this theory to work on Danish data in my empirical analysis, I first confirmed that there is an equity premium puzzle in Denmark. It is higher in absolute terms than what has been observed in the US but slightly less significant due to higher volatility in Denmark. Secondly, I put the model of myopic loss aversion to the test on the Danish stock market data. My conclusions were promising as the premium was found to be reconciled to the theory when investors used annual evaluation horizons. This finding was supported by the implied asset allocation of around 30% to stocks, which was found to fit surveys of Danish investor behaviour well. Moreover, the analysis of the implied equity premium revealed the same patterns as B&T found; the equity premium falls as the evaluation period is expanded. Sensitivity tests were applied in order to test the robustness of the results of the model. I documented similar results for both real and nominal returns, but they did not hold up to using bond data instead of risk free rates. I argued that the support for my approach was still strong, however, since myopic loss averse agents could be argued to demand a premium for carrying bond risk too. Moreover I investigated the effect of changing the loss aversion factor. Here it was shown that a factor of around 2 is consistent with empirical data, hence justifying the 2.25 originally found by Kahneman and Tversky. This analysis also showed that the model seems to behave quite well to changes in the loss aversion factor. As the loss aversion factor is reduced, so is the evaluation period and the allocation to stocks rises and vice versa. So, if investors have prospect theory preferences, evaluate their portfolio returns annually and allocate 30% of their investment to stocks, the magnitude of the equity premium is explained, and hence the puzzle resolved.

So my overall conclusion is that the use of myopic loss aversion represents an accommodating improvement in trying to resolve the equity premium puzzle. The outstanding issue is, however, whether or not it is the correct improvement. Even though the approach has success in reconciling the equity premium puzzle, it is unclear if the same will apply for other outstanding empirical issues in finance, macroeconomics, business cycle theory, etc. This is an obvious route for further research and only the success or failure of the approach in other areas can determine the thrust of its challenge to expected utility theory as the dominant descriptive model underlying most financial models.

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Appendix A.1. Autocorrelation and Heteroskedasticity

In this appendix, I investigate whether the stock returns of the MSCI Denmark can be characterized by either autocorrelation (serial correlation) or heteroskedasticity.

Scatter plots of the annual returns below are used to comment on heteroskedasticity and a Durbin Watson (DW) test of autocorrelation is conducted. In the DW test, I use the following test statistic:

$$DW = \frac{\sum_{t=2}^{T} (r_t - r_{t-1})^2}{\sum_{t=1}^{T} r_t^{\ 2}}$$

Real Returns

For real returns DW is calculated as 2.12, which means that the null hypothesis of no autocorrelation cannot be rejected at the 5% level of significance. Below is an abstract from a table of critical values for the DW test (at K=1 and n=35 as is the case here).

Critical values the Durbin-Watson statistic at the 5 pct. level

	K=1	
n	d_l	d_u
35	1.40	1.52

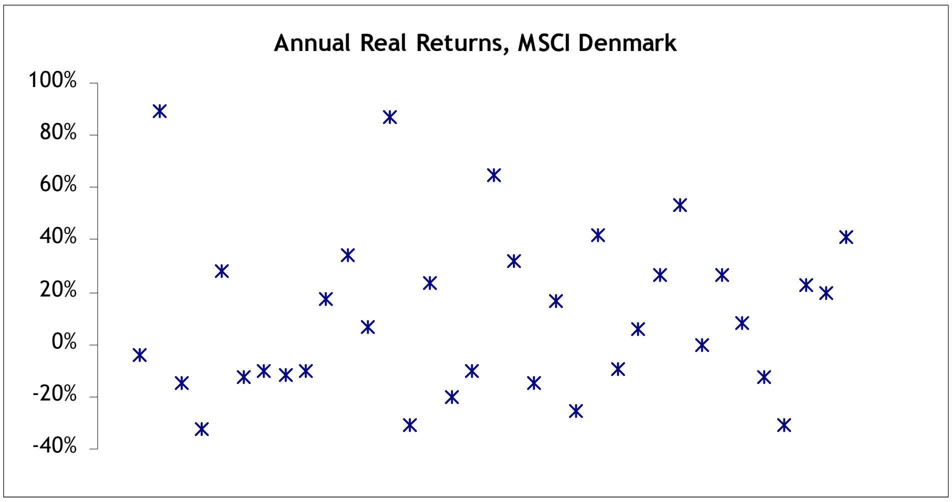
I overview the decision rules of the DW test using the correct critical values at n = 35 below.

Decision rules

Ranges	Values		Note:
0 to d_l:	0.00	1.40	H_0 rejected. Pos. serial corr.
d_l to d_u	1.40	1.52	Inconclusive
d_u to 4-d_u	1.52	2.48	H_0 not rejected
4-d_u to 4- d_l	2.48	2.60	Inconclusive
d-d_l to 4	2.60	4.00	H_0 rejected. Neg. serial corr.

The 2.12 falls in the region d_u to 4-d_u where the null cannot be rejected.

With regards to heteroskedasticity, we see from the graph below that there is no pattern indicating a non stationary variance (such as a trumpet pattern)



Nominal Returns

For nominal returns DW is calculated as 1.86, which means that the null hypothesis of no serial correlation cannot be rejected at the 5% level of significance. Below is an abstract from a table of critical values for the DW test (at K=1 and n=35 as is the case here).

Critical values the Durbin-Watson statistic at the 5 pct. level

	K=1	
n	d_l	d_u
35	1.40	1.52

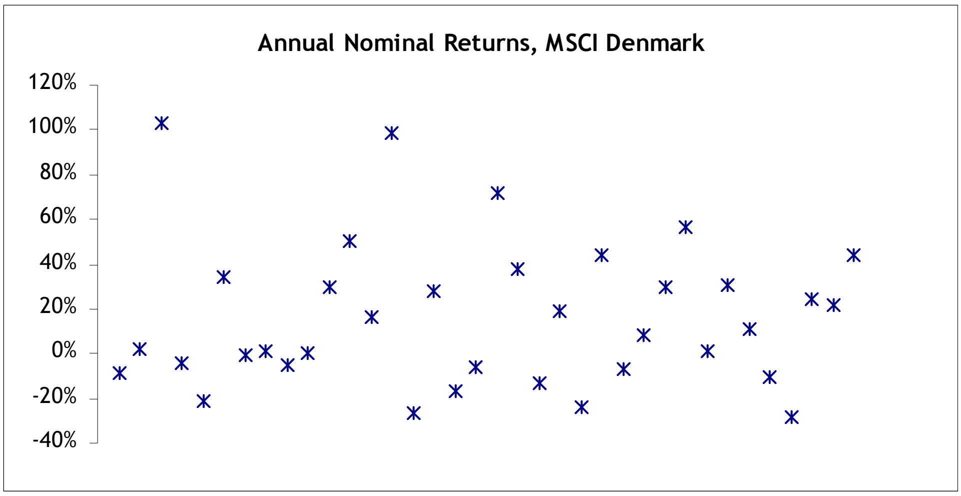
I overview the decision rules of the DW test using the correct critical values at n = 35 below.

Decision rules

Ranges	Values		Note:
0 to d_l:	0.00	1.40	H_0 rejected. Pos. serial corr.
d_l to d_u	1.40	1.52	Inconclusive
d_u to 4-d_u	1.52	2.48	H_0 not rejected
4-d_u to 4- d_l	2.48	2.60	Inconclusive
d-d_l to 4	2.60	4.00	H_0 rejected. Neg. serial corr.

The 1.86 falls in the region d_u to 4-d_u where the null cannot be rejected.

With regards to heteroskedasticity, we see from the graph below that there is no pattern indicating a non stationary variance (such as a trumpet pattern)


Annual Nominal Returns, MSCI Denmark
120%
100%
80%
60%
40%
20%
0%
-20%
-40%

Appendix A.2 Data, Periods and Sources

Table A.2.1

Asset Class:	Time Period:	Description:	Source:
Danish Stocks	Jan. 1971 – July 2006	Morgan Stanley Capital International (Denmark) incl. Net Dividends Reinvested	Bloomberg
Danish Government Bonds	Jan. 1971 – Dec. 1985	5-year Danish Yield	EcoWin (dnk14020)
	Jan. 1986 – July 2006	J.P Morgan Danish Government Bond Index +1	Bloomberg
Danish Money Market Rate	Jan. 1971 – Dec. 1991	Nationalbankens diskonto	The National Bank
	Jan. 1992 – July 2006	3-month CIBOR	Bloomberg
Inflation	Jan. 1971 – July 2006	Consumer Prices By Commodity, All Items, Total Index	EcoWin (dnk11801)
Population	Jan. 1971 – July 2006	Population Figures from the Censuses By Main Region and Time	Statistics Denmark (FT)
Private Consumption	Jan. 1971 – July 2006	Private Consumption by Group of Consumption and Price Unit	Statistics Denmark (ENS95)

Appendix A.3. Solving the Mehra Prescott Model.

This appendix contains the derivations for the model of chapter 3.

Agents have utility according to

(A.3.1) $$E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t)\right], \text{ where } 0<\beta<1$$

The specific utility function is given by

(A.3.2) $$U(c,\alpha)=\frac{c^{1-\alpha}-1}{1-\alpha}, \text{ where } \alpha>0$$

The first and second order derivatives at c_t are

(A.3.3) $$U'(c_t)=\frac{(1-\alpha)c_t^{-\alpha}(1-\alpha)-(c^{1-\alpha}-1)\cdot 0}{(1-\alpha)^2}=c_t^{-\alpha}$$

(A.3.4) $$U''(c_t)=-\alpha c_t^{-\alpha-1}$$

Thus the coefficient of relative risk aversion CRRA is

(A.3.5) $$\text{CRRA}=\frac{-U''(c_t)c_t}{U'(c_t)}=\frac{-(-\alpha c_t^{-\alpha-1})c_t}{c_t^{-\alpha}}=\frac{\alpha c_t^{-\alpha}}{c_t^{-\alpha}}=\alpha$$

The first order condition for utility maximisation is

(A.3.6) $$p_t U'(c_t) = \beta \mathrm{E}_t\left[(p_{t+1}+y_{t+1})U'(c_{t+1})\right]$$

$$\Updownarrow$$

(A.3.7) $$1 = \frac{1}{p_t U'(c_t)}\beta \mathrm{E}_t\left[(p_{t+1}+y_{t+1})U'(c_{t+1})\right]$$

$$\Updownarrow$$

(A.3.8) $$1 = \beta \mathrm{E}_t\left[\frac{(p_{t+1}+y_{t+1})U'(c_{t+1})}{p_t U'(c_t)}\right]$$

$$\Updownarrow$$

(A.3.9) $$1 = \beta \mathrm{E}_t\left[\frac{(p_{t+1}+y_{t+1})}{p_t}\frac{U'(c_{t+1})}{U'(c_t)}\right]$$

Introducing $R_{e,t+1}=\dfrac{(p_{t+1}+y_{t+1})}{p_t}$ yields

(A.3.10) $$1 = \beta \mathrm{E}_t\left[R_{e,t+1}\frac{U'(c_{t+1})}{U'(c_t)}\right]$$

The equivalent for risk free investment is

(A.3.11) $$1 = \beta \mathrm{E}_{\mathrm{t}}\left[\frac{U'(c_{t+1})}{U'(c_t)}\right] \mathrm{R}_{f,t+1}$$

since $R_{e,t+1}$ is known with certainty.

Taking individual expectations in (A.3.10) yields

(A.3.12) $$1 = \beta\left[\mathrm{E}_{\mathrm{t}}\left(\frac{U'(c_{t+1})}{U'(c_t)}\right)\mathrm{E}_{\mathrm{t}}\left(R_{e,t+1}\right) + COV\left(\frac{U'(c_{t+1})}{U'(c_t)}, R_{e,t+1}\right)\right]$$

$$\Updownarrow$$

(A.3.13) $$1 = \beta \mathrm{E}_{\mathrm{t}}\left(\frac{U'(c_{t+1})}{U'(c_t)}\right)\mathrm{E}_{\mathrm{t}}\left(R_{e,t+1}\right) + \beta COV\left(\frac{U'(c_{t+1})}{U'(c_t)}, R_{e,t+1}\right)$$

From (A.3.11) is follows that $\frac{1}{R_{f,t+1}} = \beta \mathrm{E}_{\mathrm{t}}\left(\frac{U'(c_{t+1})}{U'(c_t)}\right)$. Substituting this in (A.3.13) yields

(A.3.14) $$1 = \frac{1}{R_{f,t+1}}\mathrm{E}_{\mathrm{t}}\left(R_{e,t+1}\right) + \beta COV\left(\frac{U'(c_{t+1})}{U'(c_t)}, R_{e,t+1}\right)$$

$$\Updownarrow$$

(A.3.15) $$R_{f,t+1} = \mathrm{E}_{\mathrm{t}}\left(R_{e,t+1}\right) + \beta R_{f,t+1} COV\left(\frac{U'(c_{t+1})}{U'(c_t)}, R_{e,t+1}\right)$$

Again from (A.3.11) it follows that $R_{f,t+1} = \frac{1}{\beta}\mathrm{E}_{\mathrm{t}}\left(\frac{U'(c_t)}{U'(c_{t+1})}\right)$. Inserting on the right hand side yields

(A.3.16) $$R_{f,t+1} = \mathrm{E}_{\mathrm{t}}\left(R_{e,t+1}\right) + \beta\frac{1}{\beta}\mathrm{E}_{\mathrm{t}}\left(\frac{U'(c_t)}{U'(c_{t+1})}\right) COV\left(\frac{U'(c_{t+1})}{U'(c_t)}, R_{e,t+1}\right)$$

$$\Updownarrow$$

(A.3.17) $$R_{f,t+1} = \mathrm{E}_{\mathrm{t}}\left(R_{e,t+1}\right) + \mathrm{E}_{\mathrm{t}}\left(\frac{U'(c_t)}{U'(c_{t+1})}\right) COV\left(\frac{U'(c_{t+1})}{U'(c_t)}, R_{e,t+1}\right)$$

Since $U'(c_t)$ is known and does not covary with the future equity return, we have

(A.3.18) $$R_{f,t+1} = \mathrm{E}_{\mathrm{t}}\left(R_{e,t+1}\right) + \frac{U'(c_t)}{\mathrm{E}_{\mathrm{t}}\left(U'(c_{t+1})\right)} U'(c_t) COV\left(U'(c_{t+1}), R_{e,t+1}\right)$$

$$\Updownarrow$$

(A.3.19) $$R_{f,t+1} = \mathrm{E}_{\mathrm{t}}\left(R_{e,t+1}\right) + COV\left(\frac{U'(c_{t+1}), R_{e,t+1}}{\mathrm{E}_{\mathrm{t}}\left(U'(c_{t+1})\right)}\right)$$

$$\Updownarrow$$

(A.3.20) $$E_t\left(R_{e,t+1}\right) = R_{f,t+1} - COV\left(\frac{U'(c_{t+1}), R_{e,t+1}}{\mathrm{E}_t\left(U'(c_{t+1})\right)}\right)$$

$$\Updownarrow$$

(A.3.21) $$E_t\left(R_{e,t+1}\right) = R_{f,t+1} + COV\left(\frac{-U'(c_{t+1}), R_{e,t+1}}{\mathrm{E}_t\left(U'(c_{t+1})\right)}\right)$$

which is equation (3.3) in the text. The last rearrangement is intuitive since when c_t is high then $U'(c_t)$ is low. So since $COV(c_t, R_{e,t})$ is positive then $COV(U'(c_t), R_{e,t})$ is negative.

To solve the model explicitly revert to (A.3.6), the first order condition for utility maximisation,

(A.3.6) $$p_t U'(c_t) = \beta \mathrm{E}_t\left[(p_{t+1} + y_{t+1})U'(c_{t+1})\right]$$

$$\Updownarrow$$

(A.3.22) $$p_t = \frac{1}{U'(c_t)} \beta \mathrm{E}_t\left[(p_{t+1} + y_{t+1})U'(c_{t+1})\right]$$

$$\Updownarrow$$

(A.3.23) $$p_t = \beta \mathrm{E}_t\left[(p_{t+1} + y_{t+1})\frac{U'(c_{t+1})}{U'(c_t)}\right]$$

Introduce $x_{t+1} = \frac{c_{t+1}}{c_t}$ and use (A.3.3) to see that

(A.3.24) $$\frac{U'(c_{t+1})}{U'(c_t)} = \frac{c_{t+1}^{-\alpha}}{c_t^{-\alpha}} = \left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} = x_{t+1}^{-\alpha}$$

Insert into (A.3.23) to get

(A.3.25) $$p_t = \beta \mathrm{E}_t\left[(p_{t+1} + y_{t+1})x_{t+1}^{-\alpha}\right]$$

Mehra and Prescott assume that p_{t+1} is a function of y_t and is homogenous of degree one, so

(A.3.26) $$p_{t+1} = \mathrm{w}y_t$$

Again since the return on equity is given by $R_{e,t+1} = \frac{(p_{t+1} + y_{t+1})}{p_t}$ using (A.3.26) yields

(A.3.27) $$R_{e,t+1} = \frac{(wy_{t+1} + y_{t+1})}{wy_t} = \frac{(w+1)y_{t+1}}{wy_t} = \frac{w+1}{w}\frac{y_{t+1}}{y_t}$$

In the exchange economy of this model the equilibrium consumption is y_t so

(A.3.28) $$x_{t+1} = \frac{c_{t+1}}{c_t} = \frac{y_{t+1}}{y_t}$$

And consequently, combining (A.3.27) and (A.3.28) gives

(A.3.29) $$R_{e,t+1} = \frac{w+1}{w} x_{t+1}$$

Thus the expected equity return is

(A.3.30) $$E_t\left[R_{e,t+1}\right] = \frac{w+1}{w} E_t\left[x_{t+1}\right]$$

To proceed I determine $\frac{w+1}{w}$:

Substituting (A.3.26) into (A.3.25) yields

(A.3.31) $$wy_t = \beta \mathrm{E}_t\left[(wy_{t+1} + y_{t+1})x_{t+1}^{-\alpha}\right]$$

$$\Updownarrow$$

(A.3.32) $$wy_t = \beta \mathrm{E}_t\left[(w+1)y_{t+1}x_{t+1}^{-\alpha}\right]$$

$$\Updownarrow$$

(A.3.33) $$w = \beta \mathrm{E}_t\left[(w+1)\frac{y_{t+1}}{y_t}x_{t+1}^{-\alpha}\right]$$

Using (A.3.28) to substitute x_{t+1} for $\frac{y_{t+1}}{y_t}$ yields

(A.3.34) $$w = \beta \mathrm{E}_t\left[(w+1)x_{t+1}x_{t+1}^{-\alpha}\right]$$

$$\Updownarrow$$

(A.3.35) $$w = \beta \mathrm{E}_t\left[(w+1)x_{t+1}^{1-\alpha}\right]$$

$$\Updownarrow$$

(A.3.36) $$w = (w+1)\beta \mathrm{E}_t\left[x_{t+1}^{1-\alpha}\right]$$

$$\Updownarrow$$

(A.3.37) $$\frac{w}{(w+1)} = \beta \mathrm{E}_t\left[x_{t+1}^{1-\alpha}\right]$$

$$\Updownarrow$$

(A.3.38) $$\frac{w+1}{w} = \frac{1}{\beta \mathrm{E}_t\left[x_{t+1}^{1-\alpha}\right]}$$

Insert this into (A.3.30)

(A.3.39) $$E_t\left[R_{e,t+1}\right]=\frac{E_t\left[x_{t+1}\right]}{\beta \mathrm{E}_{\mathrm{t}}\left[x_{t+1}^{1-\alpha}\right]}$$

which is equation (3.4) in the text.

The equivalent for the risk free asset is determined as follows. Recall the general pricing equation

(A.3.10) $$p_t = \beta \mathrm{E}_{\mathrm{t}}\left[(p_{t+1}+y_{t+1})x_{t+1}^{-\alpha}\right]$$

Assume that the price of the risk free bond is q_t then

(A.3.40) $$q_t = \beta \mathrm{E}_{\mathrm{t}}\left[(p_{t+1}+y_{t+1})x_{t+1}^{-\alpha}\right]$$

The risk free bond has a certain value of one in the next period so $p_{t+1}+y_{t+1}=1$ and thus

(A.3.41) $$q_t = \beta \mathrm{E}_{\mathrm{t}}\left[x_{t+1}^{-\alpha}\right]$$

The one period return for the risk free bond is given by

(A.3.42) $$R_{f,t+1}=\frac{1}{q_t}$$

Inserting (A.3.41) yields

(A.3.43) $$R_{f,t+1} = \frac{1}{\beta \mathrm{E}_{\mathrm{t}}\left[x_{t+1}^{-\alpha}\right]}$$

which is equation (3.5) in the text.

Utilizing the explicit assumption that x_t is log normally distributed means that

(A.3.44) $$E_t\left(x_{t+1}\right)=e^{\mu_x+½\sigma_x^2}$$ and

(A.3.45) $$E_t\left(x_{t+1}^{1-\alpha}\right)=e^{(1-\alpha)\mu_x+½(1-\alpha)^2\sigma_x^2}$$

Hence inserting these in (A.3.39) we have that

(A.3.46) $$E_t(R_{e,t+1})=\frac{e^{\mu_x+½\sigma_x^2}}{\beta e^{(1-\alpha)\mu_x+½(1-\alpha)^2\sigma_x^2}}$$

$$\Updownarrow$$

(A.3.47) $$\ln E_t(R_{e,t+1})=\ln\left(e^{\mu_x+½\sigma_x^2}\right)-\ln\left(\beta e^{(1-\alpha)\mu_x+½(1-\alpha)^2\sigma_x^2}\right)$$

$$\Updownarrow$$

(A.3.48) $\ln E_t(R_{e,t+1}) = \mu_x + ½\sigma_x^2 - \ln\beta - \ln\left(e^{(1-\alpha)\mu_x + ½(1-\alpha)^2\sigma_x^2}\right)$

$\Updownarrow$

(A.3.49) $\ln E_t(R_{e,t+1}) = \mu_x + ½\sigma_x^2 - \ln\beta - (1-\alpha)\mu_x - ½(1-\alpha)^2\sigma_x^2$

$\Updownarrow$

(A.3.50) $\ln E_t(R_{e,t+1}) = -\ln\beta + \alpha\mu_x - ½\alpha^2\sigma_x^2 + \alpha\sigma_x^2$

which is expression (3.6) in the text

For the risk free bond we combine (A.3.43) and (A.3.45) to get

(A.3.51) $R_{f,t+1} = \dfrac{1}{\beta e^{-\alpha\mu_x + ½\alpha^2\sigma_x^2}}$

$\Updownarrow$

(A.3.52) $\ln R_{f,t+1} = \ln 1 - \ln\left(\beta e^{-\alpha\mu_x + ½\alpha^2\sigma_x^2}\right)$

$\Updownarrow$

(A.3.53) $\ln R_{f,t+1} = -\ln\beta - \ln\left(e^{-\alpha\mu_x + ½\alpha^2\sigma_x^2}\right)$

$\Updownarrow$

(A.3.54) $\ln R_{f,t+1} = -\ln\beta + \alpha\mu_x - ½\alpha^2\sigma_x^2$

which is expression (3.7) in the text

Calculate the equity premium by subtracting (A.3.54) from (A.3.50)

(A.3.55) $\ln E_t(R_{e,t+1}) - \ln R_{t+1} = -\ln\beta + \alpha\mu_x - ½\alpha^2\sigma_x^2 + \alpha\sigma_x^2 - \left(-\ln\beta + \alpha\mu_x - ½\alpha^2\sigma_x^2\right)$

$\Updownarrow$

(A.3.56) $\ln E_t(R_{e,t+1}) - \ln R_{t+1} = -\ln\beta + \alpha\mu_x - ½\alpha^2\sigma_x^2 + \alpha\sigma_x^2 + \ln\beta - \alpha\mu_x + ½\alpha^2\sigma_x^2$

$\Updownarrow$

(A.3.57) $\ln E_t(R_{e,t+1}) - \ln R_{t+1} = -½\alpha^2\sigma_x^2 + \alpha\sigma_x^2 + ½\alpha^2\sigma_x^2$

$\Updownarrow$

(A.3.58) $\ln E_t(R_{e,t+1}) - \ln R_{t+1} = \alpha\sigma_x^2$

which is expression (3.8) in the text

Appendix A.4. Deriving the Testable Expressions of Kocherlakota

This appendix contains the derivations of the expression in section 3.1.3.

From Mehra and Prescotts model we have the equations below (derived in appendix A.3)

(A.4.1) $$E_t\left[R_{e,t+1}\right]=\frac{E_t\left(x_{t+1}\right)}{\beta \mathrm{E}_t\left(x_{t+1}^{1-\alpha}\right)}$$

(A.4.2) $$R_{f,t+1}=\frac{1}{\beta \mathrm{E}_t\left(x_{t+1}^{-\alpha}\right)}$$

For equity rearrange (A.4.1)

(A.4.3) $$E_t\left[R_{e,t+1}\right]=E_t\left(x_{t+1}\right)\frac{1}{\beta \mathrm{E}_t\left(x_{t+1}^{1-\alpha}\right)}$$

$\Updownarrow$

(A.4.4) $$E_t\left[R_{e,t+1}\right]\beta \mathrm{E}_t\left(x_{t+1}^{1-\alpha}\right)=E_t\left(x_{t+1}\right)$$

$\Updownarrow$

(A.4.5) $$E_t\left[R_{e,t+1}\right]\beta \mathrm{E}_t\left(x_{t+1}^{1-\alpha}\right)E_t\left(x_{t+1}^{-1}\right)=1$$

$\Updownarrow$

(A.4.6) $$E_t\left[R_{e,t+1}\right]\beta \mathrm{E}_t\left(x_{t+1}^{-\alpha}\right)=1$$

$\Updownarrow$

(A.4.7) $$\beta \mathrm{E}_t\left(x_{t+1}^{-\alpha}R_{e,t+1}\right)=1$$

Use the fact that $x_{t+1}=\frac{c_{t+1}}{c_t}$ to get

(A.4.8) $$\beta \mathrm{E}_t\left(\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}R_{e,t+1}\right)=1$$

Equivalently for the risk free bond

(A.4.9) $$R_{f,t+1}=\frac{1}{\beta \mathrm{E}_t\left(x_{t+1}^{-\alpha}\right)}$$

$\Updownarrow$

(A.4.10) $$\beta \mathrm{E}_t\left(x_{t+1}^{-\alpha}\right)R_{f,t+1}=1$$

Again, use the fact that $x_{t+1} = \frac{c_{t+1}}{c_t}$ to get

(A.4.11) $$\beta \mathrm{E}_t\left(\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\right)R_{f,t+1} = 1$$

Now to derive the testable version of the equity premium subtract (A.4.11) from (A.4.8)

(A.4.12) $$\beta \mathrm{E}_t\left(\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} R_{e,t+1}\right) - \beta \mathrm{E}_t\left(\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\right)R_{f,t+1} = 0$$

$\Updownarrow$

(A.4.13) $$\mathrm{E}_t\left(\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} R_{e,t+1}\right) - \mathrm{E}_t\left(\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\right)R_{f,t+1} = 0$$

$\Updownarrow$

(A.4.14) $$\mathrm{E}_t\left(\left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\left(R_{e,t+1} - R_{f,t+1}\right)\right) = 0$$

which is equation (3.9) in the text.

Appendix A.5. VBA Functions.

Functions used to calculate utility as specified in Prospect Theory. Uses as inputs the asset allocation, i.e. the stock weight and the bond weight. When finding the evaluation period, these are set to 100%/0% and 0%/100% respectively for each evaluation period. When finding the optimal asset allocation, the weights are changed incrementally (by 5%) at each evaluation horizon.

```vba
Function PU(Dates As Range, StockReturns As Range, Stockweight As Double, BondReturns As Range, Bondweight As Double, alfa As Double, beta As Double, gamma As Double, my As Double, delta As Double, InvestmentHorizon As Integer) As Variant

Dim i, N, m As Integer
N = Dates.Rows.Count                                'Counts the number of observations - called N

If Int(N/InvestmentHorizon)<=N/InvestmentHorizon Then  'Ensures that we use only the maximum number of evalutation
  m = Int(N / InvestmentHorizon)                    'the remaining observations are omitted
Else: m = Int(N / InvestmentHorizon) - 1
End If

Dim CompDates()                                     'Creates a vector of evaluation dates, e.g. if the evaluation
ReDim CompDates(m - 1)                              'period is twelve months, the dates will be the first date and
                                                    'the date 12 months later, etc.
For i = 1 To m
  CompDates(i - 1) = Dates(InvestmentHorizon * i)
Next i


Dim StockReturnsComphorizon()                       'Creates a vector of compound stock returns. The calculation is
ReDim StockReturnsComphorizon(m - 1)                'performed by the function CompoundReturns below.

StockReturnsComphorizon = CompoundReturns(StockReturns, InvestmentHorizon, N)

Dim BondReturnsComphorizon()                        'Creates a vector of compound bond returns. The calculation is
ReDim BondReturnsComphorizon(m - 1)                 'performed by the function CompoundReturns below.

BondReturnsComphorizon = CompoundReturns(BondReturns, InvestmentHorizon, N)

Dim PortfolioReturns()                              'Creates a vector of compound portfolio returns using the weights specified
ReDim PortfolioReturns(m - 1, 1)                    'and the stock and bond returns calculated.

PortfolioReturns = PortfolioReturn(CompDates, StockReturnsComphorizon, BondReturnsComphorizon, Stockweight, Bondweight, m)

Dim v()                                             'The value function. Creates a vector of prospect values calculated on the
ReDim v(m - 1, 1)                                   'basis of the portfolio returns calculated above using the function
                                                    ' v(x)=x^α if x≥ 0 and v(x)=-λ(-x)^β if x<0.
For i = 0 To m - 1
  v(i, 0) = PortfolioReturns(i, 0)
  If PortfolioReturns(i, 1) >= 0 Then
    v(i, 1) = PortfolioReturns(i, 1) ^ alfa
  Else:
    v(i, 1) = gamma * (-PortfolioReturns(i, 1)) ^ beta
  End If
Next i

Dim Sortetv()                                       'Ranked value function. The rank dependend model obliges us to sort the
ReDim Sortetv(m - 1, 1)                             'vector of prospective values. This is done using the function DualSorter
                                                    'below. The ranked values are in the vector Sortetv where the worst is point 0,
Sortetv = DualSorter(v, 1)                          'which has the lowest prospective value and the best is m, which has the
                                                    'highest value
Dim P, Px, Wp, Wpx, WPdiff, Vg As Double            'Defines the weighting function and its variables

i = 0
PU = 0                                              'The overall value function.
                                                    'The ranked values are used to calculate the probabilities used to determine
For i = 0 To m - 1                                  'the decision weights in the weighting function.
  If Sortetv(i, 1) >= 0 Then                        'The domain of gains: P captures the outcomes as good as or better than i
    P = (m - i) / m                                 'P* captures the outcomes strictly better than i
    Px = (m - i - 1) / m
  Else
    P = (i + 1) / m                                 'The domain of losses: P captures the outcomes as bad as or worse than i
    Px = i / m                                      'P* captures the outcomes strictly worse than i
```

```
  End If

  If Sortetv(i, 1) >= 0 Then                          'The domain of gains: Calculates W+(P) and W+(P*) using expression XX
    Wp=(P^my)/((P^my+(1-P)^my))^(1/my)                'and XX in section XX.
    Wpx=(Px^my)/((Px^my+(1-Px)^my))^(1/my)
  Else
    Wp = (P^delta)/((P^delta+(1-P)^delta))^(1/delta)  'The domain of losses: Calculates W+(P) and W+(P*)
    Wpx=(Px^delta)/((Px^delta+(1-Px)^delta))^(1/delta) 'using expression XX in section XX
  End If

  WPdiff = Wp - Wpx                                   'Calculates π+i or π-i from the W's, i.e. the specific decision weight
  Vg = WPdiff * Sortetv(i, 1)                         'Calculates π+iv(xi), the contribution to overall prospective utility from the
                                                      'i'th outcome.
  PU = PU + Vg                                        'Aggregates all the individual contributions into the total overall value
                                                      'function and thus the total level of utility.
Next i

End Function
```

The prospective utility function above uses some subfunctions to calculate compounded returns for the relevant evaluation period and the portfolio return given the asset allocation supplied. For completeness, these are shown below along with a purely instrumental function that sorts a two dimensional array needed in the ranking of prospective values.

```
Function CompoundReturns(Data, H As Integer, ObsCount)

Dim i, j As Integer
Dim N, m As Integer

N = ObsCount
    If Int(N / H) <= N / H Then
      m = Int(N / H)
    Else: m = Int(N / H) - 1
End If

Dim rtn()
ReDim rtn(m - 1)

If N = m Then
  For j = 0 To m - 1
      rtn(j) = Data(j + 1)
  Next j
Else
  Dim factors()
  ReDim factors(H - 1)

  For j = 0 To (m - 1) * H Step H
      For i = 0 To H - 1
        factors(i) = 1 + Data(i + j + 1)
      Next i
      rtn(j / H) = Application.Product(factors) - 1
  Next j
End If

CompoundReturns = rtn

End Function
```

```
Function PortfolioReturn(Dates, Asset1_Returns, Asset2_Returns, Asset1_Weight As Double, Asset2_Weight As Double, N As Integer)

  Dim i As Integer

  Dim Pfrtn()
  ReDim Pfrtn(N - 1, 1)

    For i = 0 To N - 1
        Pfrtn(i, 0) = Dates(i)
        Pfrtn(i, 1) = Asset1_Weight * Asset1_Returns(i) + Asset2_Weight * Asset2_Returns(i)
    Next i

  PortfolioReturn = Pfrtn

End Function
```

```
Function DualSorter(ByRef arrArray, DimensionToSort)

   Dim row, j, StartingKeyValue, StartingOtherValue, _
      NewStartingKey, NewStartingOther, _
      swap_pos, OtherDimension
   Const column = 1

   If DimensionToSort = 1 Then
     OtherDimension = 0
  Else: DimensionToSort = 0
     OtherDimension = 1
  End If


    For row = 0 To UBound(arrArray, column) - 1

      StartingKeyValue = arrArray(row, DimensionToSort)
      StartingOtherValue = arrArray(row, OtherDimension)

      NewStartingKey = arrArray(row, DimensionToSort)
      NewStartingOther = arrArray(row, OtherDimension)

      swap_pos = row

      For j = row + 1 To UBound(arrArray, column)
         If arrArray(j, DimensionToSort) < NewStartingKey Then
            swap_pos = j
            NewStartingKey = arrArray(j, DimensionToSort)
            NewStartingOther = arrArray(j, OtherDimension)
         End If
      Next

      If swap_pos <> row Then
         arrArray(swap_pos, DimensionToSort) = StartingKeyValue
         arrArray(swap_pos, OtherDimension) = StartingOtherValue

         arrArray(row, DimensionToSort) = NewStartingKey
         arrArray(row, OtherDimension) = NewStartingOther

      End If
   Next

   DualSorter = arrArray
End Function
```
