

## THE EQUITY PREMIUM A Puzzle\*

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Restrictions that a class of general equilibrium models place upon the average returns of equity and Treasury bills are found to be strongly violated by the U.S. data in the 1889–1978 period. This result is robust to model specification and measurement problems. We conclude that, most likely, an equilibrium model which is not an Arrow–Debreu economy will be the one that simultaneously rationalizes both historically observed large average equity return and the small average risk-free return.

### 1. Introduction

Historically the average return on equity has far exceeded the average return on short-term virtually default-free debt. Over the ninety-year period 1889–1978 the average real annual yield on the Standard and Poor 500 Index was seven percent, while the average yield on short-term debt was less than one percent. The question addressed in this paper is whether this large differential in average yields can be accounted for by models that abstract from transactions costs, liquidity constraints and other frictions absent in the Arrow–Debreu set-up. Our finding is that it cannot be, at least not for the class of economies considered. Our conclusion is that most likely some equilibrium model with a

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friction will be the one that successfully accounts for the large average equity premium.

We study a class of competitive pure exchange economies for which the equilibrium *growth* rate process on consumption and equilibrium asset returns are stationary. Attention is restricted to economies for which the elasticity of substitution for the composite consumption good between the year  $t$  and year  $t + 1$  is consistent with findings in micro, macro and international economics. In addition, the economies are constructed to display equilibrium consumption growth rates with the same mean, variance and serial correlation as those observed for the U.S. economy in the 1889–1978 period. We find that for such economies, the average real annual yield on equity is a maximum of four-tenths of a percent higher than that on short-term debt, in sharp contrast to the six percent premium observed. Our results are robust to non-stationarities in the means and variances of the economies' growth processes.

The simple class of economies studied, we think, is well suited for the question posed. It clearly is poorly suited for other issues, in particular issues such as the volatility of asset prices.<sup>1</sup> We emphasize that our analysis is not an estimation exercise, which is designed to obtain better estimates of key economic parameters. Rather it is a quantitative theoretical exercise designed to address a very particular question.<sup>2</sup>

Intuitively, the reason why the low average real return and high average return on equity cannot simultaneously be rationalized in a perfect market framework is as follows: With real per capita consumption growing at nearly two percent per year on average, the elasticities of substitution between the year  $t$  and year  $t + 1$  consumption good that are sufficiently small to yield the six percent average equity premium also yield real rates of return far in excess of those observed. In the case of a growing economy, agents with high risk aversion effectively discount the future to a greater extent than agents with low risk aversion (relative to a non-growing economy). Due to growth, future consumption will probably exceed present consumption and since the marginal utility of future consumption is less than that of present consumption, real interest rates will be higher on average.

This paper is organized as follows: Section 2 summarizes the U.S. historical experience for the ninety-year period 1889–1978. Section 3 specifies the set of economies studied. Their behavior with respect to average equity and short-term debt yields, as well as a summary of the sensitivity of our results to the specifications of the economy, are reported in section 4. Section 5 concludes the paper.

<sup>1</sup> There are other interesting features of time series and procedures for testing them. The variance bound tests of LeRoy and Porter (1981) and Shiller (1980) are particularly innovative and constructive. They did indicate that consumption risk was important [see Grossman and Shiller (1981) and LeRoy and LaCavita (1981)].

<sup>2</sup> See Lucas (1980) for an articulation of this methodology.

Table 1

Time periods	% growth rate of per capita real consumption		% real return on a relatively riskless security		% risk premium		% real return on S&P 500	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
1889–1978	1.83 (Std error = 0.38)	3.57	0.80 (Std error = 0.60)	5.67	6.18 (Std error = 1.76)	16.67	6.98 (Std error = 1.74)	16.54
1889–1898	2.30	4.90	5.80	3.23	1.78	11.57	7.58	10.02
1899–1908	2.55	5.31	2.62	2.59	5.08	16.86	7.71	17.21
1909–1918	0.44	3.07	–1.63	9.02	1.49	9.18	–0.14	12.81
1919–1928	3.00	3.97	4.30	6.61	14.64	15.94	18.94	16.18
1929–1938	–0.25	5.28	2.39	6.50	0.18	31.63	2.56	27.90
1939–1948	2.19	2.52	–5.82	4.05	8.89	14.23	3.07	14.67
1949–1958	1.48	1.00	–0.81	1.89	18.30	13.20	17.49	13.08
1959–1968	2.37	1.00	1.07	0.64	4.50	10.17	5.58	10.59
1969–1978	2.41	1.40	–0.72	2.06	0.75	11.64	0.03	13.11

## 2. Data

The data used in this study consists of five basic series for the period 1889–1978.<sup>3</sup> The first four are identical to those used by Grossman and Shiller (1981) in their study. The series are individually described below:

- (i) *Series P*: Annual average Standard and Poor's Composite Stock Price Index divided by the Consumption Deflator, a plot of which appears in Grossman and Shiller (1981, p. 225, fig. 1).
- (ii) *Series D*: Real annual dividends for the Standard and Poor's series.
- (iii) *Series C*: Kuznets–Kendrik–USNIA per capita real consumption on non-durables and services.
- (iv) *Series PC*: Consumption deflator series, obtained by dividing real consumption in 1972 dollars on non-durables and services by the nominal consumption on non-durables and services.
- (v) *Series RF*: Nominal yield on relatively riskless short-term securities over the 1889–1978 period; the securities used were ninety-day government Treasury Bills in the 1931–1978 period, Treasury Certificates for the

<sup>3</sup>We thank Sanford Grossman and Robert Shiller for providing us with the data they used in their study (1981).

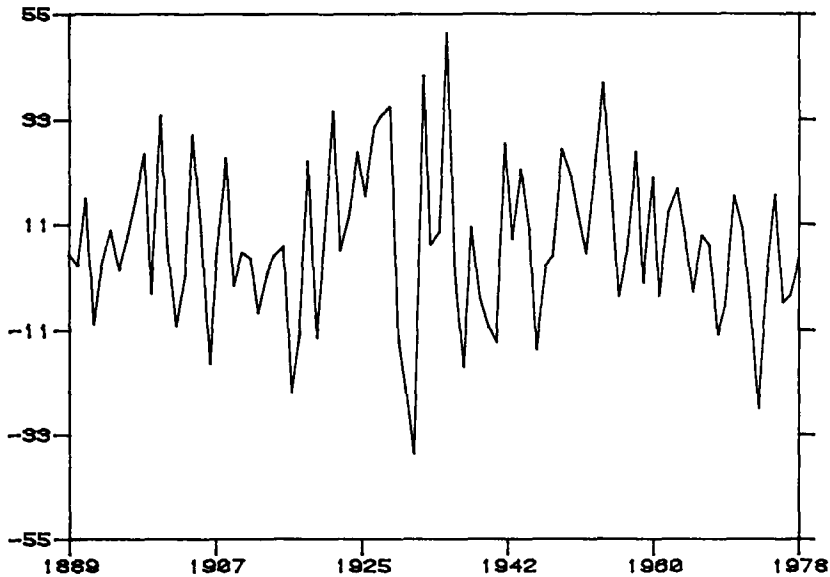


Fig. 1. Real annual return on S&P 500, 1889–1978 (percent).

1920–1930 period and sixty-day to ninety-day Prime Commercial Paper prior to 1920.<sup>4</sup>

These series were used to generate the series actually utilized in this paper. Summary statistics are provided in table 1.

Series  $P$  and  $D$  above were used to determine the average annual real return on the Standard and Poor's 500 Composite Index over the ninety-year period of study. The annual return for year  $t$  was computed as  $(P_{t+1} + D_t - P_t)/P_t$ . The returns are plotted in fig. 1. Series  $C$  was used to determine the process on the growth rate of consumption over the same period. Model parameters were restricted to be consistent with this process. A plot of the percentage growth of real consumption appears in fig. 2. To determine the real return on a relatively riskless security we used the series  $RF$  and  $PC$ . For year  $t$  this is calculated to be  $RF_t - (PC_{t+1} - PC_t)/PC_t$ .

This series is plotted in fig. 3. Finally, the Risk Premium ( $RP$ ) is calculated as the difference between the Real Return on Standard and Poor's 500 and the Real Return on a Riskless security as defined above.

<sup>4</sup>The data was obtained from Homer (1963) and Ibbotson and Singuefield (1979).

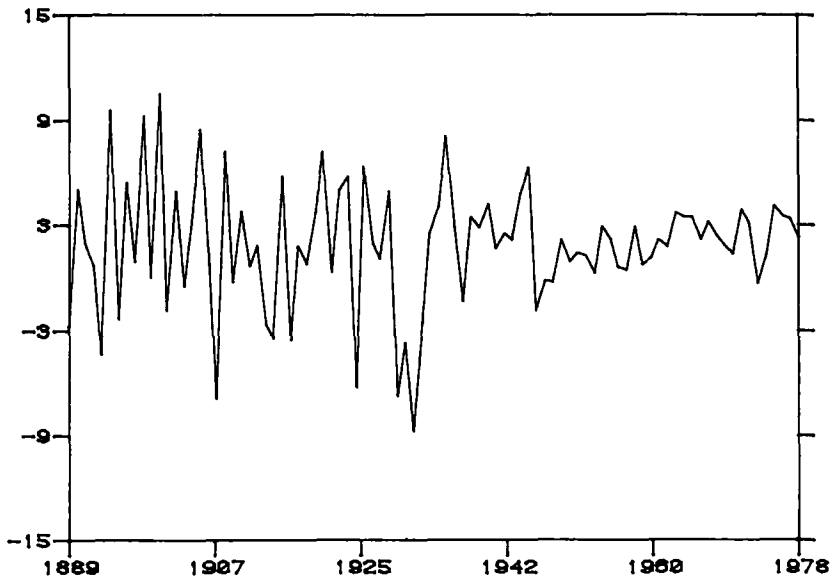


Fig. 2. Growth rate of real per capita consumption, 1889–1978 (percent).

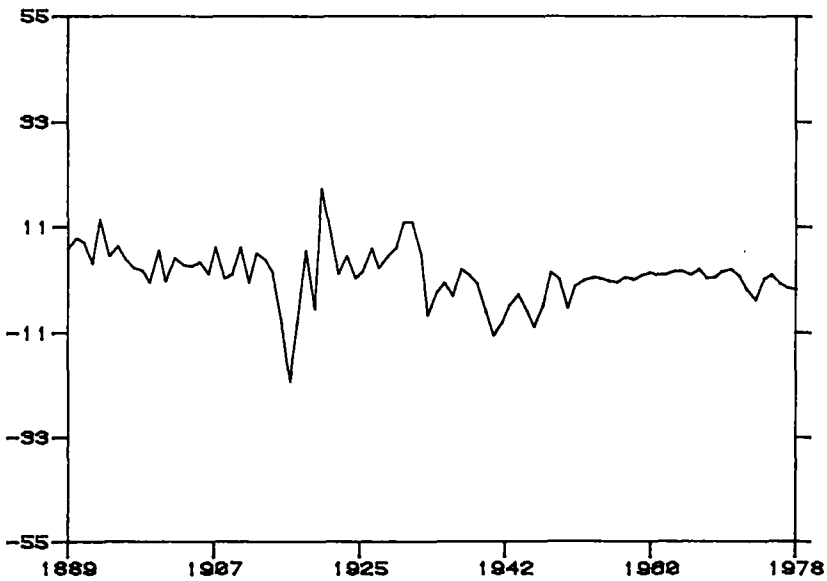


Fig. 3. Real annual return on a relatively riskless security, 1889–1978 (percent).

### 3. The economy, asset prices and returns

In this paper, we employ a variation of Lucas' (1978) pure exchange model. Since per capita consumption has grown over time, we assume that the *growth rate* of the endowment follows a Markov process. This is in contrast to the assumption in Lucas' model that the endowment *level* follows a Markov process. Our assumption, which requires an extension of competitive equilibrium theory, enables us to capture the non-stationarity in the consumption series associated with the large increase in per capita consumption that occurred in the 1889–1978 period.

The economy we consider was judiciously selected so that the joint process governing the growth rates in aggregate per capita consumption and asset prices would be stationary and easily determined. The economy has a single representative 'stand-in' household. This unit orders its preferences over random consumption paths by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\}, \quad 0 < \beta < 1, \quad (1)$$

where  $c_t$  is per capita consumption,  $\beta$  is the subjective time discount factor,  $E_0\{\cdot\}$  is the expectation operator conditional upon information available at time zero (which denotes the present time) and  $U: R_+ \rightarrow R$  is the increasing concave utility function. To insure that the equilibrium return process is stationary, the utility function is further restricted to be of the constant relative risk aversion class,

$$U(c, \alpha) = \frac{c^{1-\alpha} - 1}{1-\alpha}, \quad 0 < \alpha < \infty. \quad (2)$$

The parameter  $\alpha$  measures the curvature of the utility function. When  $\alpha$  is equal to one, the utility function is defined to be the logarithmic function, which is the limit of the above function as  $\alpha$  approaches one.

We assume that there is one productive unit producing the perishable consumption good and there is one equity share that is competitively traded. Since only one productive unit is considered, the return on this share of equity is also the return on the market. The firm's output is constrained to be less than or equal to  $y_t$ . It is the firm's dividend payment in the period  $t$  as well.

The growth rate in  $y_t$  is subject to a Markov chain; that is,

$$y_{t+1} = x_{t+1} y_t, \quad (3)$$

where  $x_{t+1} \in \{\lambda_1, \dots, \lambda_n\}$  is the growth rate, and

$$\Pr\{x_{t+1} = \lambda_j; x_t = \lambda_i\} = \phi_{ij}. \quad (4)$$

It is also assumed that the Markov chain is ergodic. The  $\lambda_i$  are all positive and  $y_0 > 0$ . The random variable  $y_t$  is observed at the beginning of the period, at which time dividend payments are made. All securities are traded ex-dividend. We also assume that the matrix  $A$  with elements  $a_{ij} \equiv \beta \phi_{ij} \lambda_j^{1-\alpha}$  for  $i, j = 1, \dots, n$  is stable; that is,  $\lim A^m$  as  $m \rightarrow \infty$  is zero. In Mehra and Prescott (1984) it is shown that this is necessary and sufficient for expected utility to exist if the stand-in household consumes  $y_t$  every period. They also define and establish the existence of a Debreu (1954) competitive equilibrium with a price system having a dot product representation under this condition.

Next we formulate expressions for the equilibrium time  $t$  price of the equity share and the risk-free bill. We follow the convention of pricing securities ex-dividend or ex-interest payments at time  $t$ , in terms of the time  $t$  consumption good. For any security with process  $\{d_s\}$  on payments, its price in period  $t$  is

$$P_t = E_t \left\{ \sum_{s=t+1}^{\infty} \beta^{s-t} U'(y_s) d_s / U'(y_t) \right\}, \quad (5)$$

as equilibrium consumption is the process  $\{y_s\}$  and the equilibrium price system has a dot product representation.

The dividend payment process for the equity share in this economy is  $\{y_s\}$ . Consequently, using the fact that  $U'(c) = c^{-\alpha}$ ,

$$\begin{aligned} P_t^e &= P^e(x_t, y_t) \\ &= E \left\{ \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{y_t^\alpha}{y_s^\alpha} y_s | x_t, y_t \right\}. \end{aligned} \quad (6)$$

Variables  $x_t$  and  $y_t$  are sufficient relative to the entire history of shocks up to, and including, time  $t$  for predicting the subsequent evolution of the economy. They thus constitute legitimate state variables for the model. Since  $y_s = y_t \cdot x_{t+1} \cdot \dots \cdot x_s$ , the price of the equity security is homogeneous of degree one in  $y_t$ , which is the current endowment of the consumption good. As the equilibrium values of the economies being studied are time invariant functions of the state  $(x_t, y_t)$ , the subscript  $t$  can be dropped. This is accomplished by redefining the state to be the pair  $(c, i)$ , if  $y_t = c$  and  $x_t = \lambda_i$ . With this

convention, the price of the equity share from (6) satisfies

$$p^e(c, i) = \beta \sum_{j=1}^n \phi_{ij} (\lambda_j c)^{-\alpha} [p^e(\lambda_j c, j) + c \lambda_j] c^\alpha. \quad (7)$$

Using the result that  $p^e(c, i)$  is homogeneous of degree one in  $c$ , we represent this function as

$$p^e(c, i) = w_i c, \quad (8)$$

where  $w_i$  is a constant. Making this substitution in (7) and dividing by  $c$  yields

$$w_i = \beta \sum_{j=1}^n \phi_{ij} \lambda_j^{(1-\alpha)} (w_j + 1) \quad \text{for } i = 1, \dots, n. \quad (9)$$

This is a system of  $n$  linear equations in  $n$  unknowns. The assumption that guaranteed existence of equilibrium guarantees the existence of a unique positive solution to this system.

The period return if the current state is  $(c, i)$  and next period state  $(\lambda_j c, j)$  is

$$\begin{aligned} r_{ij}^e &= \frac{p^e(\lambda_j c, j) + \lambda_j c - p^e(c, i)}{p^e(c, i)} \\ &= \frac{\lambda_j (w_j + 1)}{w_i} - 1, \end{aligned} \quad (10)$$

using (8).

The equity's expected period return if the current state is  $i$  is

$$R_i^e = \sum_{j=1}^n \phi_{ij} r_{ij}^e. \quad (11)$$

Capital letters are used to denote expected return. With the subscript  $i$ , it is the expected return conditional upon the current state being  $(c, i)$ . Without this subscript it is the expected return with respect to the stationary distribution. The superscript indicates the type of security.

The other security considered is the one-period real bill or riskless asset, which pays one unit of the consumption good next period with certainty.



From (6),

$$\begin{aligned}
 p_i^f &= p^f(c, i) \\
 &= \beta \sum_{j=1}^n \phi_{ij} U'(\lambda_j c) / U'(c) \\
 &= \beta \sum_{j=1}^n \phi_{ij} \lambda_j^{-\alpha}.
 \end{aligned} \tag{12}$$

The certain return on this riskless security is

$$R_i^f = 1/p_i^f - 1, \tag{13}$$

when the current state is  $(c, i)$ .

As mentioned earlier, the statistics that are probably most robust to the modelling specification are the means over time. Let  $\pi \in R^n$  be the vector of stationary probabilities on  $i$ . This exists because the chain on  $i$  has been assumed to be ergodic. The vector  $\pi$  is the solution to the system of equations

$$\pi = \phi^T \pi,$$

with

$$\sum_{i=1}^n \pi_i = 1 \quad \text{and} \quad \phi^T = \{\phi_{ji}\}.$$

The expected returns on the equity and the risk-free security are, respectively,

$$R^e = \sum_{i=1}^n \pi_i R_i^e \quad \text{and} \quad R^f = \sum_{i=1}^n \pi_i R_i^f. \tag{14}$$

Time sample averages will converge in probability to these values given the ergodicity of the Markov chain. The risk premium for equity is  $R^e - R^f$ , a parameter that is used in the test.

#### 4. The results

The parameters defining preferences are  $\alpha$  and  $\beta$  while the parameters defining technology are the elements of  $[\phi_{ij}]$  and  $[\lambda_i]$ . Our approach is to

assume two states for the Markov chain and to restrict the process as follows:

$$\begin{aligned}\lambda_1 &= 1 + \mu + \delta, & \lambda_2 &= 1 + \mu - \delta, \\ \phi_{11} &= \phi_{22} = \phi, & \phi_{12} &= \phi_{21} = (1 - \phi).\end{aligned}$$

The parameters  $\mu$ ,  $\phi$ , and  $\delta$  now define the technology. We require  $\delta > 0$  and  $0 < \phi < 1$ . This particular parameterization was selected because it permitted us to independently vary the average growth rate of output by changing  $\mu$ , the variability of consumption by altering  $\delta$ , and the serial correlation of growth rates by adjusting  $\phi$ .

The parameters were selected so that the average growth rate of per capita consumption, the standard deviation of the growth rate of per capita consumption and the first-order serial correlation of this growth rate, all with respect to the model's stationary distribution, matched the sample values for the U.S. economy between 1889–1978. The sample values for the U.S. economy were 0.018, 0.036 and  $-0.14$ , respectively. The resulting parameter's values were  $\mu = 0.018$ ,  $\delta = 0.036$  and  $\phi = 0.43$ . Given these values, the nature of the test is to search for parameters  $\alpha$  and  $\beta$  for which the model's averaged risk-free rate and equity risk premium match those observed for the U.S. economy over this ninety-year period.

The parameter  $\alpha$ , which measures peoples' willingness to substitute consumption between successive yearly time periods is an important one in many fields of economics. Arrow (1971) summarizes a number of studies and concludes that relative risk aversion with respect to wealth is almost constant. He further argues on theoretical grounds that  $\alpha$  should be approximately one. Friend and Blume (1975) present evidence based upon the portfolio holdings of individuals that  $\alpha$  is larger, with their estimates being in the range of two. Kydland and Prescott (1982), in their study of aggregate fluctuations, found that they needed a value between one and two to mimic the observed relative variabilities of consumption and investment. Altug (1983), using a closely related model and formal econometric techniques, estimates the parameter to be near zero. Kehoe (1984), studying the response of small countries balance of trade to terms of trade shocks, obtained estimates near one, the value posited by Arrow. Hildreth and Knowles (1982) in their study of the behavior of farmers also obtain estimates between one and two. Tobin and Dolde (1971), studying life cycle savings behavior with borrowing constraints, use a value of 1.5 to fit the observed life cycle savings patterns.

Any of the above cited studies can be challenged on a number of grounds but together they constitute an *a priori* justification for restricting the value of  $\alpha$  to be a maximum of ten, as we do in this study. This is an important restriction, for with large  $\alpha$  virtually any pair of average equity and risk-free returns can be obtained by making small changes in the process on consump-

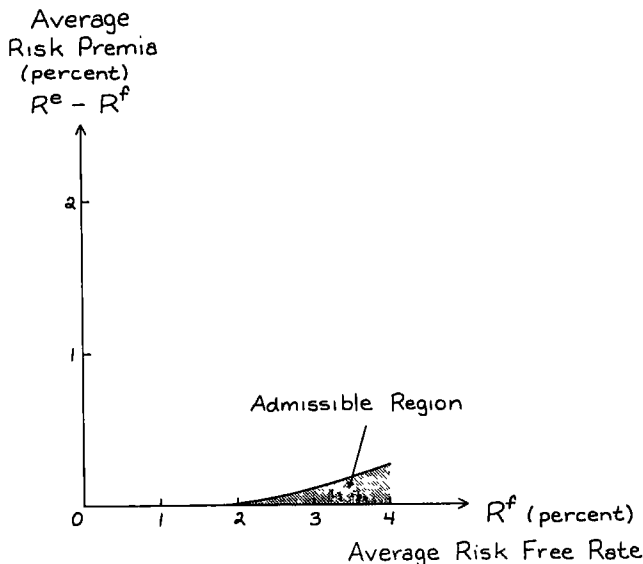


Fig. 4. Set of admissible average equity risk premia and real returns.

tion.<sup>5</sup> With  $\alpha$  less than ten, we found the results were essentially the same for very different consumption processes, provided that the mean and variances of growth rates equaled the historically observed values. An advantage of our approach is that we can easily test the sensitivity of our results to such distributional assumptions.

The average real return on relatively riskless, short-term securities over the 1889–1978 period was 0.80 percent. These securities do not correspond perfectly with the real bill, but insofar as unanticipated inflation is negligible and/or uncorrelated with the growth rate  $x_{t+1}$  conditional upon information at time  $t$ , the expected real return for the nominal bill will equal  $R_t^f$ . Litterman (1980), using vector autoregressive analysis, found that the innovation in the inflation rate in the post-war period (quarterly data) has standard deviation of only one-half of one percent and that his innovation is nearly orthogonal to the subsequent path of the real GNP growth rate. Consequently, the average realized real return on a nominally denoted short-term bill should be close to that which would have prevailed for a real bill if such a security were traded. The average real return on the Standard and Poor's 500 Composite Stock

<sup>5</sup>In a private communication, Fischer Black using the Merton (1973) continuous time model with investment opportunities constructed an example with a curvature parameter ( $\alpha$ ) of 55. We thank him for the example.

Index over the ninety years considered was 6.98 percent per annum. This leads to an average equity premium of 6.18 percent (standard error 1.76 percent).

Given the estimated process on consumption, fig. 4 depicts the set of values of the average risk-free rate and equity risk premium which are both consistent with the model and result in average real risk-free rates between zero and four percent. These are values that can be obtained by varying preference parameters  $\alpha$  between zero and ten and  $\beta$  between zero and one. The observed real return of 0.80 percent and equity premium of 6 percent is clearly inconsistent with the predictions of the model. The largest premium obtainable with the model is 0.35 percent, which is not close to the observed value.

#### 4.1. *Robustness of results*

One set of possible problems are associated with errors in measuring the inflation rate. Such errors do not affect the computed risk premium as they bias both the real risk-free rate and the equity rate by the same amount. A potentially more serious problem is that these errors bias our estimates of the growth rate of consumption and the risk-free real rate. Therefore, only if the tests are insensitive to biases in measuring the inflation rate should the tests be taken seriously. A second measurement problem arises because of tax considerations. The theory is implicitly considering effective after-tax returns which vary over income classes. In the earlier part of the period, tax rates were low. In the latter period, the low real rate and sizable equity risk premium hold for after-tax returns for all income classes [see Fisher and Lorie (1978)].

We also examined whether aggregation affects the results for the case that the growth rates were independent between periods, which they approximately were, given that the estimated  $\phi$  was near one-half. Varying the underlying time period from one one-hundredths of a year to two years had a negligible effect upon the admissible region. (See the appendix for an exact specification of these experiments.) Consequently, the test appears robust to the use of annual data in estimating the process on consumption.

In an attempt to reconcile the large discrepancy between theory and observation, we tested the sensitivity of our results to model misspecification. We found that the conclusions are not at all sensitive to changes in the parameter  $\mu$ , which is the average growth rate of consumption, with decreases to 1.4 percent or increases to 2.2 percent not reducing the discrepancy. The sensitivity to  $\delta$ , the standard deviation of the consumption growth rate, is larger. The average equity premium was roughly proportional to  $\delta$  squared. As the persistence parameter  $\phi$  increased ( $\phi = 0.5$  corresponds to independence over time), the premium decreased. Reducing  $\phi$  (introducing stronger negative serial correlation in the consumption growth rate) had only small effects. We also modified the process on consumption by introducing additional states that permitted us to increase higher moments of the stationary distribution of the

growth rate without varying the first or second moments. The maximal equity premium increased by 0.04 to 0.39 only. These exercises lead us to the conclusion that the result of the test is not sensitive to the specification of the process generating consumption.

That the results were not sensitive to increased persistence in the growth rate, that is to increases in  $\phi$ , implies low frequency movements or non-stationarities in the growth rate do *not* increase the equity premium. Indeed, by assuming stationarity, we biased the test *towards* acceptance.

#### 4.2. *Effects of firm leverage*

The security priced in our model does not correspond to the common stocks traded in the U.S. economy. In our model there is only one type of capital, while in an actual economy there is virtually a continuum of capital types with widely varying risk characteristics. The stock of a typical firm traded in the stock market entitles its owner to the residual claim on output after all other claims including wages have been paid. The share of output accruing to stockholders is much more variable than that accruing to holders of other claims against the firm. Labor contracts, for instance, may incorporate an insurance feature, as labor claims on output are in part fixed, having been negotiated prior to the realization of output. Hence, a disproportionate part of the uncertainty in output is probably borne by equity owners.

The firm in our model corresponds to one producing the entire output of the economy. Clearly, the riskiness of the stock of this firm is not the same as that of the Standard and Poor's 500 Composite Stock Price Index. In an attempt to match the two securities we price and calculate the risk premium of a security whose dividend next period is actual output less a fraction of expected output. Let  $\theta$  be the fraction of expected date  $t + 1$  output committed at date  $t$  by the firm. Eq. (7) then becomes

$$p^c(c, i) = \beta \sum_{j=1}^n \phi_{ij} (\lambda_j c)^{-\alpha} \left[ p^c(\lambda_j c, j) + c \lambda_j - \theta \sum_{k=1}^n \phi_{ik} c \lambda_k \right] c^\alpha. \quad (15)$$

As before, it is conjectured and verified that  $p^c(c, i)$  has the functional form  $w_i c$ . Substituting  $w_i c$  for  $p^c(c, i)$  in (15) yields the set of linear equations

$$w_i = \beta \sum_{j=1}^n \phi_{ij} \lambda_j^{-\alpha} \left[ \lambda_j w_j + \lambda_j - \theta \sum_{k=1}^n \phi_{ik} \lambda_k \right], \quad (16)$$

for  $i = 1, \dots, n$ . This system was solved for the equilibrium  $w_i$  and eqs. (10), (11), and (14) used to determine the average equity premium.

As the corporate profit share of output is about ten percent, we set  $\theta = 0.9$ . Thus, ninety percent of expected output is committed and all the risk is borne by equity owners who receive ten percent of output on average. This increased the equity risk premium by less than one-tenth percent. This is the case because financial arrangements have no effect upon resource allocation and, therefore, the underlying Arrow–Debreu prices. Large fixed payment commitments on the part of the firm do not reverse the test's outcome.

#### *4.3. Introducing production*

With our structure, the process on the endowment is exogenous and there is neither capital accumulation nor production. Modifying the technology to admit these opportunities cannot overturn our conclusion, because expanding the set of technologies in this way does not increase the set of joint equilibrium processes on consumption and asset prices [see Mehra (1984)]. As opposed to standard testing techniques, the failure of the model hinges not on the acceptance/rejection of a statistical hypothesis but on its inability to generate average returns even close to those observed. If we had been successful in finding an economy which passed our not very demanding test, as we expected, we planned to add capital accumulation and production to the model using a variant of Brock's (1979, 1982), Donaldson and Mehra's (1984) or Prescott and Mehra's (1980) general equilibrium stationary structures and to perform additional tests.

### **5. Conclusion**

The equity premium puzzle may not be why was the average equity return so high but rather why was the average risk-free rate so low. This conclusion follows if one accepts the Friend and Blume (1975) finding that the curvature parameter  $\alpha$  significantly exceeds one. For  $\alpha = 2$ , the model's average risk-free rate is at least 3.7 percent per year, which is considerably larger than the sample average 0.80 given the standard deviation of the sample average is only 0.60. On the other hand, if  $\alpha$  is near zero and individuals nearly risk-neutral, then one would wonder why the average return of equity was so high. This is not the only example of some asset receiving a lower return than that implied by Arrow–Debreu general equilibrium theory. Currency, for example, is dominated by Treasury bills with positive nominal yields yet sizable amounts of currency are held.

We doubt whether heterogeneity, per se, of the agents will alter the conclusion. Within the Debreu (1954) competitive framework, Constantinides (1982) has shown heterogeneous agent economies also impose the set of restrictions tested here (as well as others). We doubt whether non-time-additivity separable preferences will resolve the puzzle, for that would require consumptions near in

time to be poorer substitutes than consumptions at widely separated dates. Perhaps introducing some features that make certain types of intertemporal trades among agents infeasible will resolve the puzzle. In the absence of such markets, there can be variability in individual consumptions, yet little variability in aggregate consumption. The fact that certain types of contracts may be non-enforceable is one reason for the non-existence of markets that would otherwise arise to share risk. Similarly, entering into contracts with as yet unborn generations is not feasible.<sup>6</sup> Such non-Arrow-Debreu competitive equilibrium models may rationalize the large equity risk premium that has characterized the behavior of the U.S. economy over the last ninety years. To test such theories it would probably be necessary to have consumption data by income or age groups.

## Appendix

The procedure for determining the admissible region depicted in fig. 4 is as follows. For a given set of parameters  $\mu$ ,  $\delta$  and  $\phi$ , eqs. (10)–(14) define an algorithm for computing the values of  $R^e$ ,  $R^f$  and  $R^e - R^f$  for any  $(\alpha, \beta)$  pair belonging to the set

$$x = \{(\alpha, \beta): 0 < \alpha \leq 10, 0 < \beta < 1, \text{ and the} \\ \text{existence condition of section 3 is satisfied}\}.$$

Letting  $R^f = h_1(\alpha, \beta)$  and  $R^e - R^f = h_2(\alpha, \beta)$ ,  $h: X \rightarrow R^2$ , the range of  $h$  is the region depicted in fig. 4. The function  $h$  was evaluated for all points of a fine grid in  $X$  to determine the admissible region.

The experiments to determine the sensitivity of the results to the period length have model time periods  $n = 2, 1, 1/2, 1/4, 1/8, 1/16, 1/64$  and  $1/128$  years. The values of the other parameters are  $\mu = 0.018/n$ ,  $\delta = 0.036/\sqrt{n}$  and  $\phi = 0.5$ . With these numbers the mean and standard deviation of annual growth rates are 0.018 and 0.036 respectively as in the sample period. This follows because  $\phi = 0.5$  implies independence of growth rates over periods. The change in the admissible region were hundredths of percent as  $n$  varied.

The experiments to test the sensitivity of the results to  $\mu$  consider  $\mu = 0.014, 0.016, 0.018, 0.020$  and  $0.022$ ,  $\phi = 0.43$  and  $\delta = 0.036$ . As for the period length, the growth rate's effects upon the admissible region are hundredths of percent.

The experiments to determine the sensitivity of results to  $\delta$  set  $\phi = 0.43$ ,  $\mu = 0.018$  and  $\delta = 0.21, 0.26, 0.31, 0.36, 0.41, 0.46$  and  $0.51$ . The equity premium varied approximately with the square of  $\delta$  in this range.

<sup>6</sup>See Wallace (1980) for an exposition on the use of the overlapping generations model and the importance of legal constraints in explaining rate of return anomalies.

Similarly, to test the sensitivity of the results to variations in the parameter  $\phi$ , we held  $\delta$  fixed at 0.036 and  $\mu$  at 0.018 and varied  $\phi$  between 0.005 and 0.95 in steps of 0.05. As  $\phi$  increased the average equity premium declined.

The test for the sensitivity of results to higher movements uses an economy with a four-state Markov chain with transition probability matrix

$$\begin{bmatrix} \phi/2 & \phi/2 & 1-\phi/2 & 1-\phi/2 \\ \phi/2 & \phi/2 & 1-\phi/2 & 1-\phi/2 \\ 1-\phi/2 & 1-\phi/2 & \phi/2 & \phi/2 \\ 1-\phi/2 & 1-\phi/2 & \phi/2 & \phi/2 \end{bmatrix}.$$

The values of the  $\lambda$  are  $\lambda_1 = 1 + \mu$ ,  $\lambda_2 = 1 + \mu + \delta$ ,  $\lambda_3 = 1 + \mu$ , and  $\lambda_4 = 1 + \mu - \delta$ . Values of  $\mu$ ,  $\delta$  and  $\phi$  are 0.018, 0.051 and 0.36, respectively. This results in the mean, standard deviation and first-order serial correlations of consumption growth rates for the artificial economy equaling their historical values. With this Markov chain, the probability of above average changes is smaller and magnitude of changes larger. This has the effect of increasing moments higher than the second without altering the first or second moments. This increases the maximum average equity premium from 0.35 percent to 0.39 percent.

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