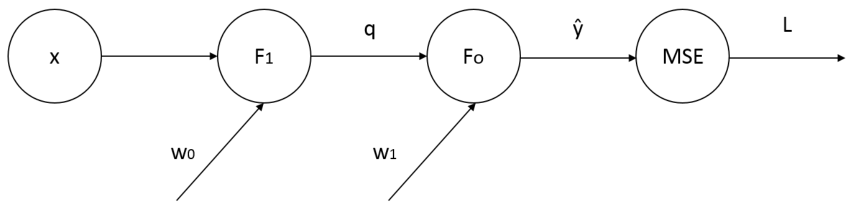

Figure 1: Graph

Exercises

Exercise-1 We can use two approaches to compute new values for the weights w_0 and w_1. The first one is to update the weights after each observation. The second is to take the average of the computed weights and thus, only update once.

Observations: $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (2, 3)$ Weights: $w_0 = 1$ and $w_1 = 2$

We use the MSE as loss function: $L = \frac{1}{2}(\hat{y} - y)^2$

1. Approach one:
   $\hat{y}_1 = w_1(w_0 x_1) = 2 * 1 * 1 = 2$
   $L = \frac{1}{2}(2 - 2)^2 = 0$
   As $L = 0$ the derivatives will all be zero and, therefore, the weights will not be updated. Thus, we continue to the next observation.
   $\hat{y}_2 = w_1(w_0 x_2) = 2 * 1 * 2 = 4$
   $L = \frac{1}{2}(4 - 3)^2 = \frac{1}{2}$

$$\begin{aligned}\frac{\partial L}{\partial \hat{y_2}} &= [\frac{1}{2}(\hat{y_2} - y_2)^2]' \\ &= 2 * \frac{1}{2}(\hat{y_2} - y_2) * 1 \\ &= \hat{y_2} - y_2 \\ &= 1\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial w_1} &= \frac{\partial \hat{y_2}}{\partial w_1}\frac{\partial L}{\partial \hat{y_2}} \\ &= w_0 x_2 * \frac{\partial L}{\partial \hat{y_2}} \\ &= 2 * \frac{1}{2} \\ &= 1\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial q} &= \frac{\partial \hat{y_2}}{\partial q}\frac{\partial L}{\partial \hat{y_2}} \\ &= w_1 * 1 \\ &= 2\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial w_0} &= \frac{\partial q}{\partial w_0}\frac{\partial L}{\partial q} \\ &= x_2 * \frac{\partial L}{\partial q} \\ &= 2 * 2 = 4\end{aligned}$$

Thus, $w_1^* = w_1 - 0.1 * 2 = 1.8$ and $w_0^* = w_0 - 0.1 * 4 = 0.6$.

2. Approach two: