STAT 240
Personal Notes

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Chapter 1: What is Probability?

RANDOM EXPERIMENTS (1.1)

A "random experiment" is the process of obtaining a random observed result.

Random experiments can be split into two types:

① Controlled experiments; and
eg flipping a coin, rolling a die

② Observational studies.
eg # of students taking STAT 270 in F2021

FEATURES OF RANDOM EXPERIMENTS

Note that random experiments have the following common features:

① The outcomes/results cannot be predicted with certainty; and

② All the possible outcomes are known beforehand with certainty.

SAMPLE SPACE (1.2)

OUTCOME

An "outcome" is an observed result of interest from a random experiment.
eg the number rolled after rolling a die.

SAMPLE SPACE

The "sample space" of a random experiment is the set of all possible distinct outcomes of said experiment.
eg when rolling a 6-sided die:
$S = \{1, 2, 3, 4, 5, 6\}$

EVENTS

An "event" of a random experiment is a group or set of outcomes of said experiment; ie subsets of the sample space.

There are two types of events:

① Simple events – consist of one outcome
eg rolling a 1 on a 6-sided die:
$S = \{1, \dots, 6\}$ $E = \{1\}$

② Compound events – consist of multiple outcomes
eg rolling an odd number on a 6-sided die:
$S = \{1, \dots, 6\}$ $E = \{1, 3, 5\}$

Note that

① Two simple events will never occur simultaneously;
eg can never roll a 1 & 3 at the same time with one die

② A compound event occurs if and only if one of "its" simple events occurs; and
eg odd # rolled $\Leftrightarrow$ 1 rolled or 3 rolled or 5 rolled (on a 6-sided die)

③ Two compound events can occur simultaneously.
eg 3 rolled $\Rightarrow$ { odd number rolled ($E = \{1, 3, 5\}$); and
multiple of 3 rolled ($E = \{3, 6\}$).

DEFINITIONS OF PROBABILITY (1.3)

"Probability" is a quantitative measure of how likely an event is to occur.

CLASSICAL DEFINITION

The "classical definition" of probability states that each distinct outcome in the sample space is equally likely to occur.

In this case, the probability of an event E is equal to

$$P(E) = \frac{\#\text{ of ways event can occur}}{\#\text{ of outcomes in the sample space.}}$$

eg roll a 6-sided die once.
E = number is odd.
$\Rightarrow E = \{1, 3, 5\}$, $S = \{1, 2, 3, 4, 5, 6\}$.
So $P(E) = \frac{3}{6} = \frac{1}{2}$.

RELATIVE FREQUENCY DEFINITION

The "relative frequency" definition of probability states that the probability of an event occurring is the proportion it occurs in a very long series of repetitions of the experiment.
eg rolling a 6-sided die 300 times
$\Rightarrow$ 3 shows up 49 of those 300 times
$\Rightarrow$ so $P(\text{die} = 3) \approx \frac{49}{300} \approx \frac{1}{6}$.

SUBJECTIVE PROBABILITY DEFINITION

In the "subjective probability" definition of probability, the probability of an event is determined by an opinion (ie what a person thinks the probability is).
eg the probability of COVID-19 being eradicated by 2022.

Note that this plays a role in fields like "Bayesian Statistics".

DISCRETE PROBABILITY MODELS (1.4)

In discrete probability models:

① The sample space S satisfies $|S| \leq |\mathbb{N}|$;
ie there are either a finite or countably infinite number of basic events; and

② Each probability p_i satisfies $0 \leq p_i \leq 1$; and

③ The probabilities of each basic event sum to 1;
ie $\sum p_i = 1$.

CLASSIC DISCRETE MODELS (1.5)

In classic discrete models:

① The sample space S satisfies $|S| < |\mathbb{N}|$ (ie it is finite); and

② All basic events are equally likely to occur;
ie $P(a_1) = \dots = P(a_{|S|}) = \frac{1}{|S|}$.