深度學習
Deep Learning

Reinforcement Learning
強化學習

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2025/04/28

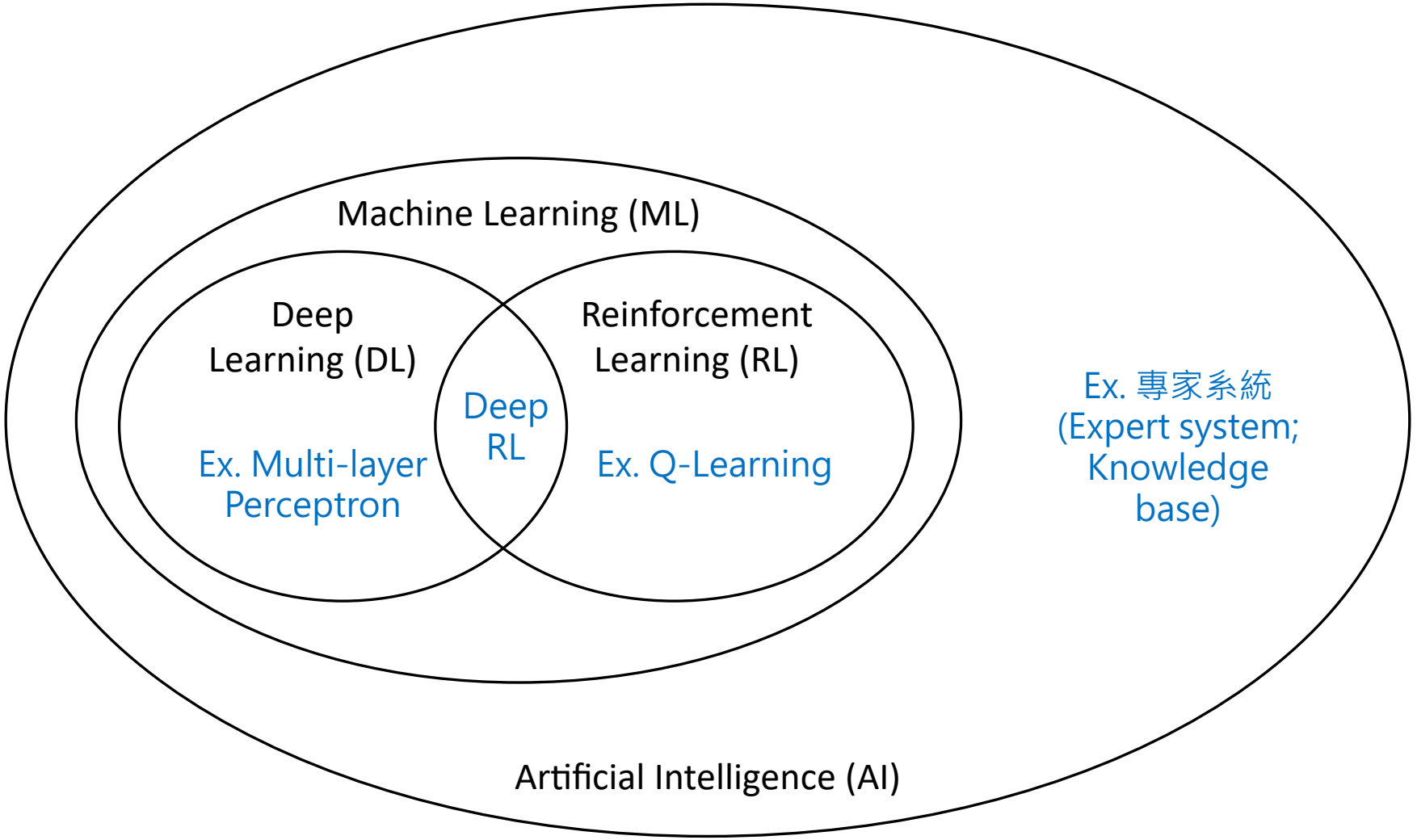
Course GitHub

Slido # DL_0428

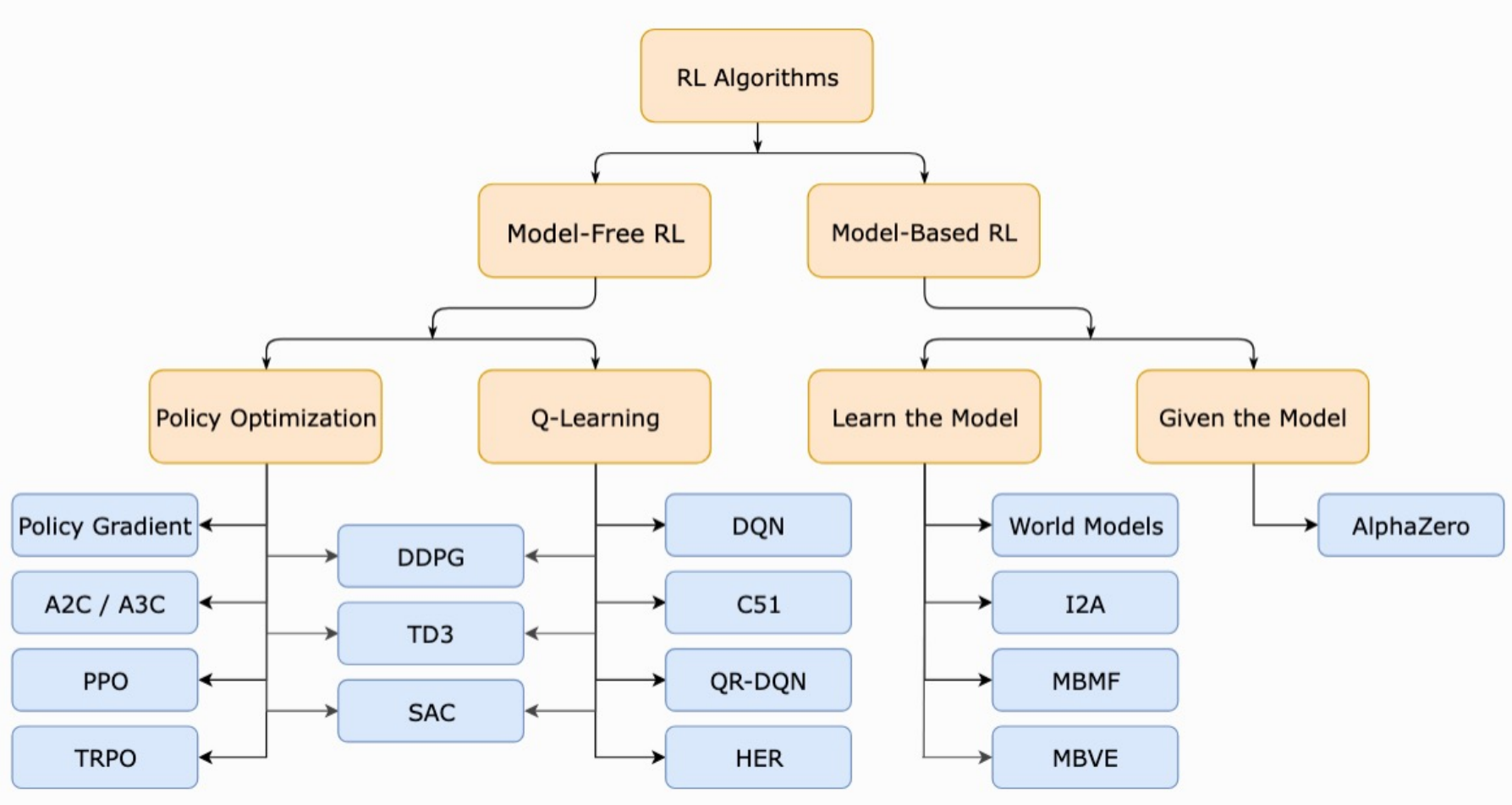
Outline

- Reinforcement Learning (RL) [100 min]
- Quiz [20 min]

Reinforcement Learning (RL) 歸類?



A Taxonomy of RL Algorithms



https://spinningup.openai.com/en/latest/spinningup/rl_intro2.html#a-taxonomy-of-rl-algorithms

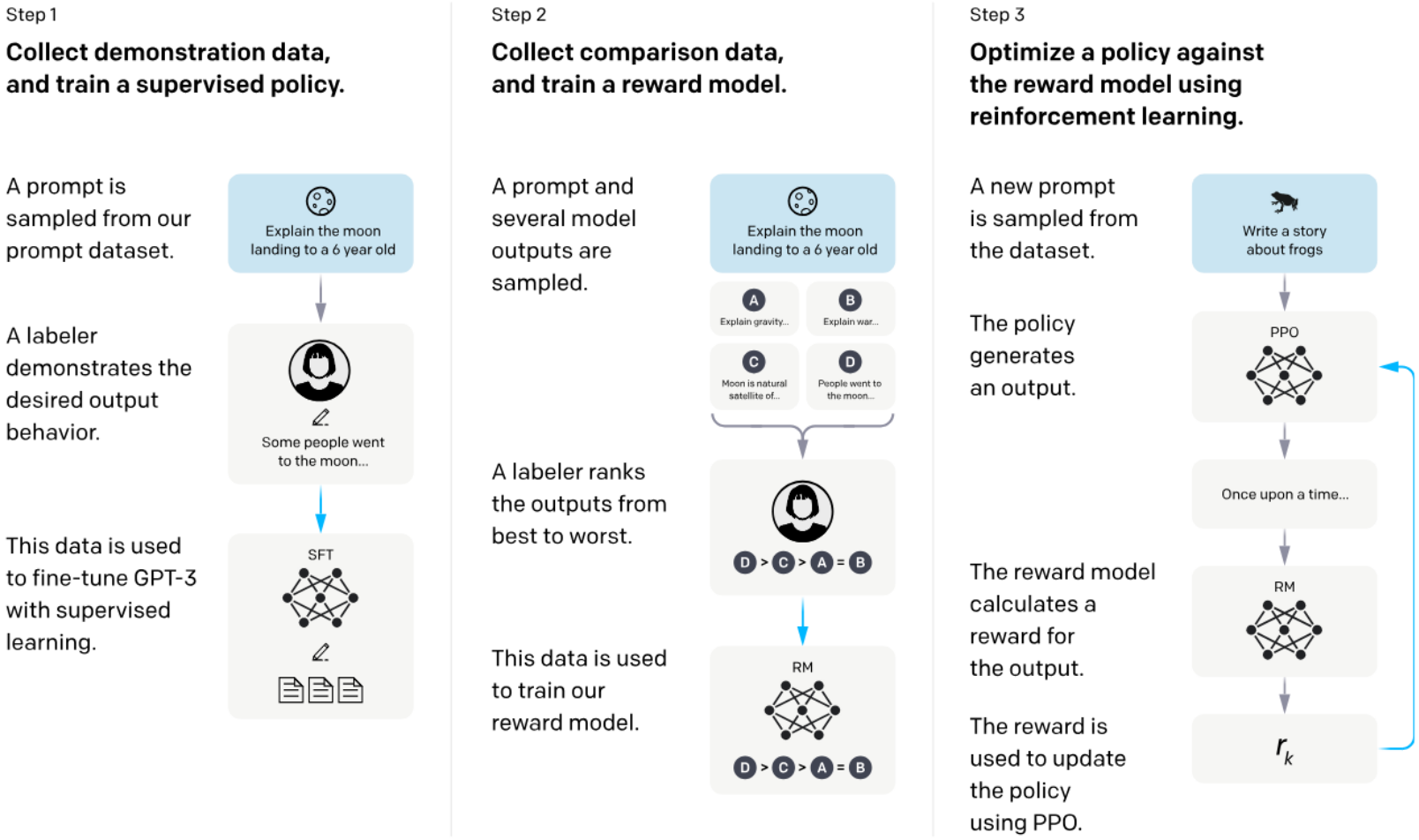
為什麼我們要學 RL (1): RL is everywhere

AlphaGo vs. Lee Sedol (2016)

- Paper: Silver, D., Huang, A., Maddison, C. J., Guez, A., Sifre, L., Van Den Driessche, G., ... & Hassabis, D. (2016). Mastering the game of Go with deep neural networks and tree search. nature, 529(7587), 484-489.
- Figure source: https://www.newyorker.com/tech/annals-of-technology/alphago-lee-sedol-and-the-reassuring-future-of-humans-and-machines

為什麼我們要學 RL (1): RL is everywhere

Ouyang, Long, et al. "Training language models to follow instructions with human feedback." NeurIPS 2022.



為什麼我們要學 RL (1): RL is everywhere

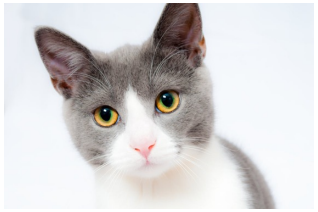
Using reinforcement learning.
Code: https://github.com/jiseongHAN/Super-Mario-RL

Atari

Figure from: Mnih, Volodymyr, et al. "Playing atari with deep reinforcement learning." NIPS (2013).

為什麼我們要學 RL (2): 訓練方式的不同

x



y

cat

dog

Machine Learning function

Supervised Learning

Machine Learning function

Reinforcement Learning
(不需要每個狀態都有標註)

Supervised Learning vs. Reinforcement Learning

- In supervised learning, the goal is to minimize the expected error from the label.
- In reinforcement learning, the goal is to maximize sum of reward.

Self-supervised learning 也是一種 supervised learning!

(Simplified) Path of Deep Reinforcement Learning

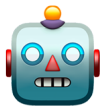
Elements of RL

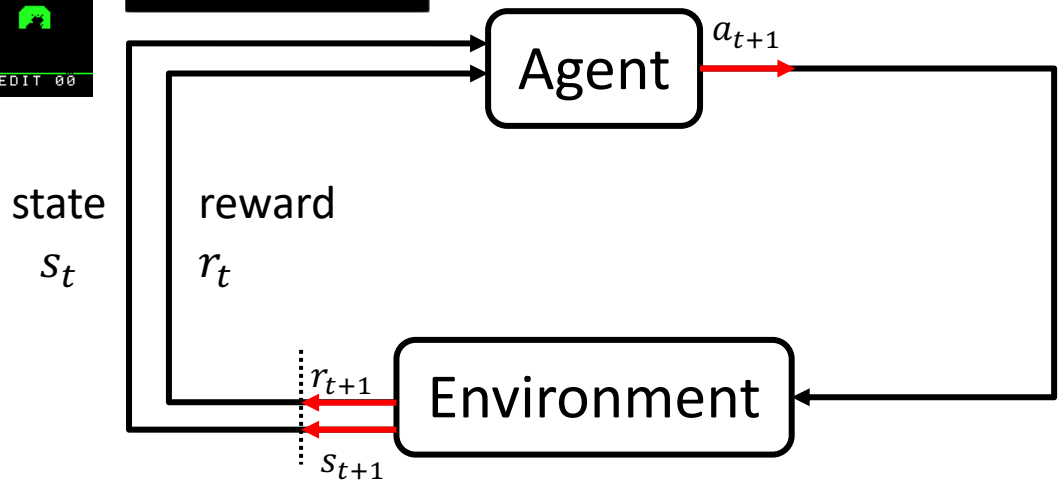
Policy Gradient Methods

Actor-Critic

On-policy and Off-policy

[Illustration] RL 的元素





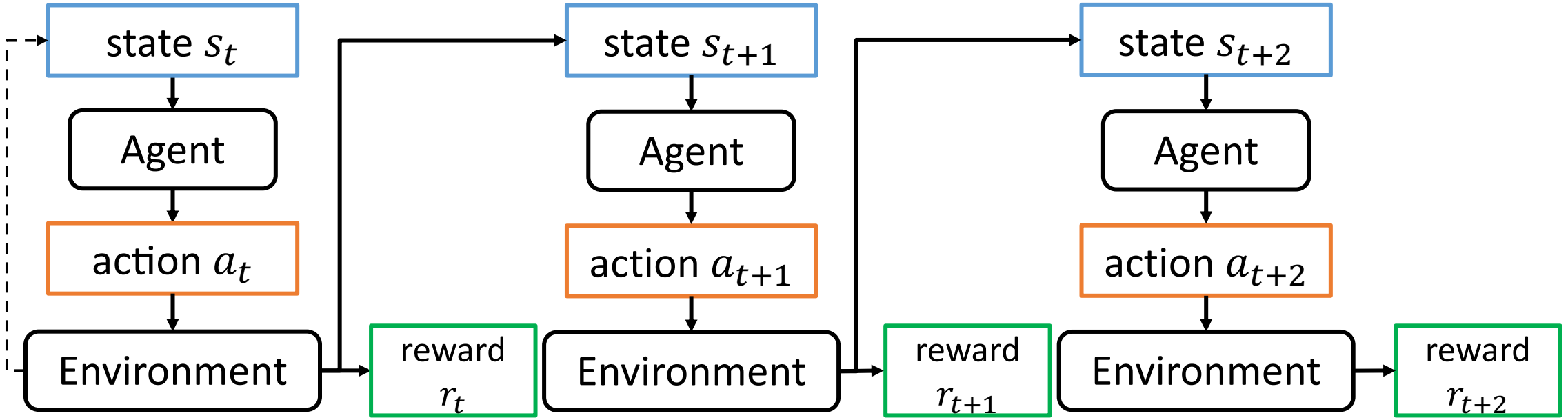
Rennie, Steven J., et al. "Self-critical sequence training for image captioning."
Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. 2017.

[Definitions] RL 的元素

符號	名稱	意義
t	時間點	狀態或 agent 執行動作的最小單位
s_t	狀態 (state)	在 t 時間點，agent 所觀察到的狀態，由環境 (environment) 所提供
a_t	動作 (action)	agent 在狀態 s_t 下選擇的動作
r_t	即時回饋 (reward)	agent 執行 a_t 後，由環境給的分數

Rennie, Steven J., et al. "Self-critical sequence training for image captioning."
Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. 2017.

[Illustration] RL 的元素



[Definition] Trajectory and Episode

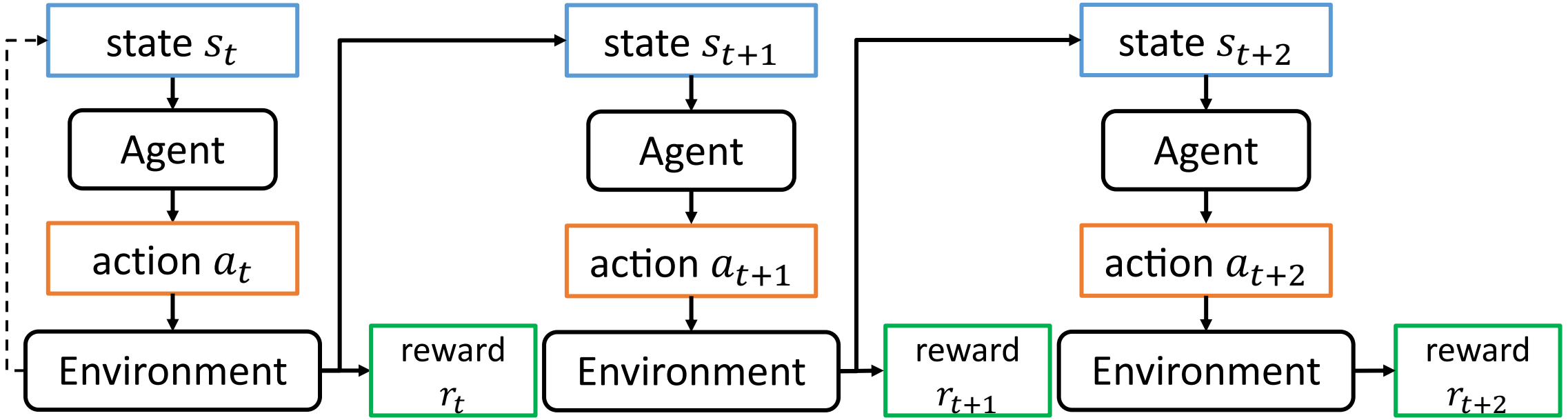
- Trajectory (軌跡, 通常 symbol 為 τ) 是指 agent 從環境中互動時，依序經歷的一連串狀態、動作和得到的回饋。

$$\tau = (s_1, a_1, r_1, s_2, a_2, r_2, \dots, s_T, a_T, r_T)$$

- Episode (回合, 通常無固定 symbol) 是指從「開始遊戲」到「遊戲結束」(或者達到某種終止條件，比如死亡、完成任務) 這段完整的互動過程
- 一條 trajectory，也就是一個 episode

https://ai.stackexchange.com/questions/7359/what-is-a-trajectory-in-reinforcement-learning

[Illustration] RL 的元素 (假設τ只有3個點)



$$\pi_\theta(\tau) = \cancel{p(s_1)} \times \pi_\theta(a_1|s_1) \times \pi_\theta(a_2|s_2) \times \pi_\theta(a_3|s_3) = \prod_{t=1}^{3} \pi_\theta(a_t|s_t)$$

[Definition] Cumulated reward

對 agent 來說：

輸入 s_t a_t

得到 r_t

reward r_1 | reward r_2 | reward r_3 | … | reward r_T

$$G_1 = r_1 + r_2 + r_3 + \cdots + r_T$$

$$G_2 = r_2 + r_3 + \cdots + r_T$$

$$G_3 = r_3 + \cdots + r_T$$

$$G_T = r_T$$

G_t: cumulated reward since the time step t

[Definition] Discounted cumulated reward

對 agent 來說：

輸入 s_t a_t

得到 r_t

γ: discount factor
(通常 $\gamma < 1$)

reward r_1 | reward r_2 | reward r_3 | ... | reward r_T

$$G_1 = r_1 + \gamma r_2 + \gamma^2 r_3 + \cdots + \gamma^{T-1} r_T$$

$$G_2 = r_2 + \gamma r_3 + \cdots + \gamma^{T-2} r_T$$

$$G_3 = r_3 + \gamma r_4 + \cdots + \gamma^{T-3} r_T$$

$$\vdots$$

$$G_T = r_T$$

G_t: cumulated reward since the time step t

(Simplified) Path of Deep Reinforcement Learning

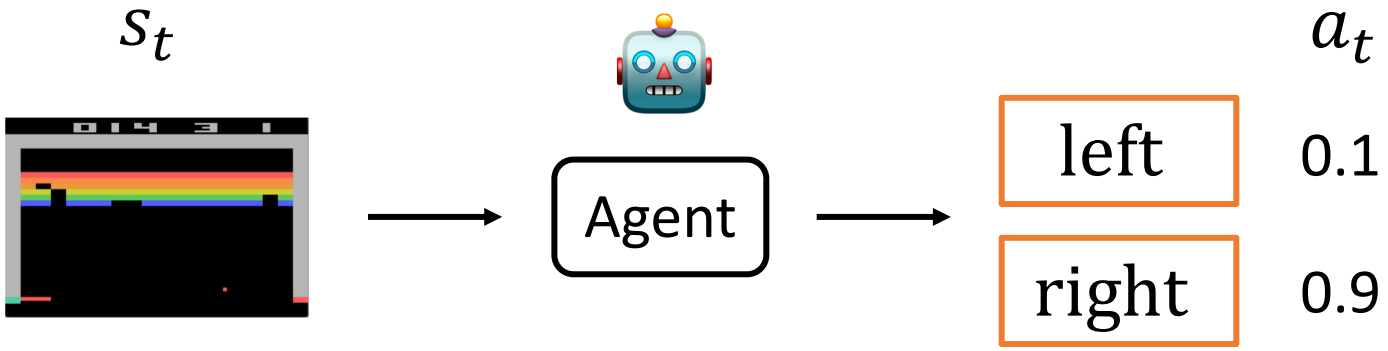
Elements of RL

Policy Gradient Methods

Actor-Critic

On-policy and Off-policy

[Definition] What is policy?



以打磚塊來說：$a_t \in \{\text{left}, \text{right}\}$

- Policy (策略) 是 agent 根據當前狀態來決定要採取哪個動作的行為準則
  - actor 是其中一種 agents
- Policy 通常以 π 為符號，在 DRL 中屬於可以被訓練的模型

如何訓練一個RL模型？

- Policy gradient

Step1: 收集資料

s_1	a_1	r_1
s_2	a_2	r_2
s_3	a_3	r_3
⋮		
s_T	a_T	r_T

Step2: 計算 reward 總和 $\sum_{t=1}^{T} r_t$

Step3: 更新模型

$$L = \sum_{t=1}^{T} r_t$$

累積的 reward 越大越好

如何訓練一個RL模型？(3-1)

Log-derivative trick:
http://www.math.ncu.edu.tw/~yu/smrcal100_1/boards/lec28_sc_100.pdf

- Policy gradient and gradient descent

Step3: 更新模型

$$L = \sum_{t=1}^{T} r_t = R(\tau)$$

$$L = \sum_{\tau} R(\tau) \cdot \pi_\theta(\tau)$$

期望值的概念

$$\nabla L = \sum_{\tau} R(\tau) \cdot \nabla \pi_\theta(\tau)$$

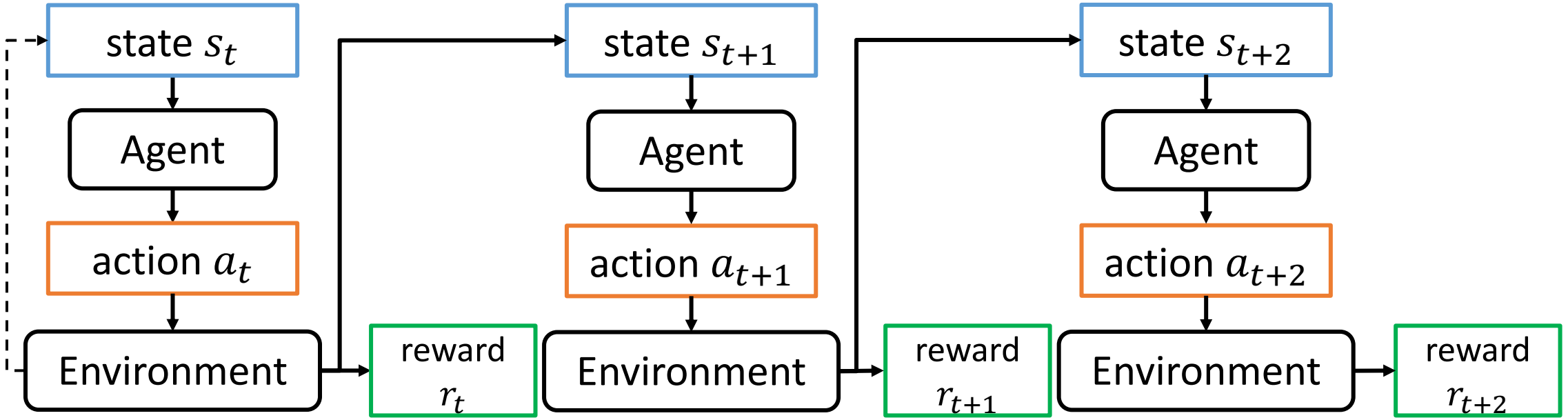
Log-derivative trick

$$= \sum_{\tau} R(\tau) \cdot \pi_\theta(\tau) \cdot \nabla \log \pi_\theta(\tau)$$

$$= \mathbb{E}_{\tau \sim \pi_\theta(\tau)}[R(\tau) \cdot \nabla \log \pi_\theta(\tau)]$$

從 $\pi_\theta(\tau)$ 機率分佈中採樣 (sampled) 的 τ
想成有可能你玩很多個回合，現在探討的是這組τ

[Illustration] RL 的元素 (假設τ只有3個點)



$$\pi_\theta(\tau) = \cancel{p(s_1)} \times \pi_\theta(a_1|s_1) \times \pi_\theta(a_2|s_2) \times \pi_\theta(a_3|s_3) = \prod_{t=1}^{3} \pi_\theta(a_t|s_t)$$

Log Derivative Trick

假設有一function $p_\theta(x)$

$$\log p'_\theta(x) = \frac{1}{p(x)} p'(x) \quad \Rightarrow \quad p'(x) = p(x) \cdot \log p'_\theta(x)$$

function取log的一次微分 (梯度)
對照上一頁： $\nabla \log \pi_\theta(\tau)$

function的一次微分 (梯度)
對照上一頁： $\nabla \pi_\theta(\tau)$

如何訓練一個RL模型？(3-2)

- Policy gradient and gradient descent

Step3: 更新模型

帶入動作和狀態

$$\pi_\theta(\tau) = \prod_{t=1}^{T} \pi_\theta(a_t|s_t)$$

$$\log \pi_\theta(\tau) = \log \prod_{t=1}^{T} \pi_\theta(a_t|s_t)$$

$$= \sum_{t=1}^{T} \log \pi_\theta(a_t|s_t)$$

$$\nabla L = \mathbb{E}_{\tau\sim\pi_\theta(\tau)}[R(\tau) \cdot \nabla \log \pi_\theta(\tau)]$$

$$= \mathbb{E}_{\tau\sim\pi_\theta(\tau)}\left[R(\tau) \cdot \nabla \sum_{t=1}^{T} \log \pi_\theta(a_t|s_t)\right]$$

$$= \mathbb{E}_{\tau\sim\pi_\theta(\tau)}\left[\sum_{t=1}^{T} R(\tau) \cdot \nabla \log \pi_\theta(a_t|s_t)\right]$$

Why Log Derivative Trick?

還沒有使用 log derivative trick 前:

$$\nabla L = \sum_{\tau} R(\tau) \cdot \nabla \pi_\theta(\tau)$$

$$\pi_\theta(\tau) = \rho(s_1) \times \pi_\theta(a_1|s_1) \times \pi_\theta(a_2|s_2) \times \cdots \times \pi_\theta(a_T|s_T) = \prod_{t=1}^{T} \pi_\theta(a_t|s_t)$$

$$\log \pi_\theta(\tau) = \log[\pi_\theta(a_1|s_1) \times \pi_\theta(a_2|s_2) \times \cdots \times \pi_\theta(a_T|s_T)] = \sum_{t=1}^{T} \log \pi_\theta(a_t|s_t)$$

相加 -> easier

如何訓練一個RL模型？(3-3)

- Policy gradient and gradient descent

Step3: 更新模型

$$\nabla L = \mathbb{E}_{\tau \sim \pi_\theta(\tau)} \left[\sum_{t=1}^{T} R(\tau) \cdot \nabla \log \pi_\theta(a_t|s_t) \right]$$

更新w：

$$w_N = w_{N-1} - \eta \underbrace{\frac{\partial L}{\partial w_{N-1}}}_{-\nabla L}$$

更新b：

$$b_N = b_{N-1} - \eta \underbrace{\frac{\partial L}{\partial b_{N-1}}}_{-\nabla L}$$

- η 代表 learning rate

把 G_t (Cumulated reward) 考慮進去

$$\nabla L = \mathbb{E}_{\tau \sim \pi_\theta(\tau)} \left[\sum_{t=1}^{T} R(\tau) \cdot \nabla \log \pi_\theta(a_t|s_t) \right]$$

$$G_t = \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'}$$

Replace total reward $R(\tau)$ with timestep-specific cumulative reward G_t

$$G_1 = r_1 + \gamma r_2 + \gamma^2 r_3 + \cdots + \gamma^{T-1} r_T$$

$$G_2 = r_2 + \gamma r_3 + \cdots + \gamma^{T-2} r_T$$

$$G_3 = r_3 + \gamma r_4 + \cdots + \gamma^{T-3} r_T$$

$$\vdots$$

$$G_T = r_T$$

$$\nabla L = \mathbb{E}_{\tau \sim \pi_\theta(\tau)} \left[\sum_{t=1}^{T} G_t \cdot \nabla \log \pi_\theta(a_t|s_t) \right]$$

[Important Notes] Training 細節

- 通常一個 episode 更新一次 agent
- 每次 agent 更新後，資料要重新收集

Step1: 收集資料

$$\begin{array}{|c|c|c|}\hline s_1 & a_1 & r_1 \\ \hline s_2 & a_2 & r_2 \\ \hline s_3 & a_3 & r_3 \\ \hline \end{array}$$

$$\vdots$$

$$\begin{array}{|c|c|c|}\hline s_T & a_T & r_T \\ \hline \end{array}$$

(Simplified) Path of Deep Reinforcement Learning

Elements of RL

Policy Gradient Methods

Actor-Critic

On-policy and Off-policy

General Problem (1)

通常一個回合你會玩很久 ... (?)


t=1


t=2


t=3


t=4


t=5

...


t=T

a_1

每個t時間點的動作都會影響未來的s跟a，導致每個回合的G_t都有可能出現很大差異

a_1 做什麼？	左	右	不動
G_t	10	100	0

General Problem (2)

$$\nabla L = \mathbb{E}_{\tau \sim \pi_\theta(\tau)}[R(\tau) \cdot \nabla \log \pi_\theta(\tau)]$$

代表玩完整場遊戲才能知道 reward 高不高 (過程中的 actions 到底好不好)

有沒有可能每個 t 時間點都能算出一個 reward 分數？

[Definition] Critic

- Critic: 用來評價 state (或 state, action) 的函數，可以是個模型
  - 用來評價 state 的函數：Value Function (通常 symbol 為 V)
  - 用來評價 state, action 的函數：State-action Value Function (通常 symbol 為 Q)
- Critic 就是 value function，功能為未卜先知，預測「未來總 reward」

訓練 V 的方法 (1): Monte Carlo

- 直接 train 下去

t=1

s_1 → V (Value function) → $\hat{G}_1$ ⟷ G_1

越接近越好

$$\text{Mean squared error} = \left(\hat{G}_1 - G_1\right)^2$$

Sutton, R. S., & Barto, A. G. (2018). *Reinforcement Learning: An Introduction* (2nd ed.), Chapter 5: Monte Carlo Methods.

訓練 V 的方法 (2): Temporal Difference (TD)

- [Recap] Discounted cumulated reward

reward r_1 | reward r_2 | reward r_3 | … | reward r_T

$$G_1 = r_1 + \gamma r_2 + \gamma^2 r_3 + \cdots + \gamma^{T-1} r_T$$

$$G_2 = r_2 + \gamma r_3 + \cdots + \gamma^{T-2} r_T$$

$$G_3 = r_3 + \gamma r_4 + \cdots + \gamma^{T-3} r_T$$

$$\vdots$$

$$G_T = r_T$$

G_t: cumulated reward since the time step t

$$G_1 = \gamma G_2 + r_1$$

$$G_2 = \gamma G_3 + r_2$$

$$G_t = \gamma G_{t+1} + r_t$$

$$G_t - \gamma G_{t+1} = r_t$$

任兩個時間點的 G 的關係

訓練 V 的方法 (2): Temporal Difference (TD)

任兩個時間點的 G 的關係：

$G_t - \gamma G_{t+1} = r_t$

➡ 以相鄰的兩個 steps 的差異 train 下去

time step: t

s_t → V (Value function) → $\hat{G}_t$ $\hat{G}_t = V(s_t)$

模型訓練目標： $V(s_t) - \gamma V(s_{t+1})$ 越接近 r_t 越好

time step: t+1

s_{t+1} → V (Value function) → $\hat{G}_{t+1}$ $\hat{G}_{t+1} = V(s_{t+1})$

Sutton, R. S., "Learning to Predict by the Methods of Temporal Differences," *Machine Learning*, 1988.

V 如何幫助 actor 訓練？

$$\nabla L = \mathbb{E}_{\tau\sim\pi_\theta(\tau)}\left[\sum_{t=1}^{T} G_t \cdot \nabla \log \pi_\theta(a_t|s_t)\right]$$

Advantage function (優勢函數)

$$A_t = G_t - V(s_t)$$

- Monte Carlo

$$\nabla L = \mathbb{E}_{\tau\sim\pi_\theta(\tau)}\left[\sum_{t=1}^{T} (G_t - V(s_t)) \cdot \nabla \log \pi_\theta(a_t|s_t)\right]$$

$V(s_t)$ 學到預估的累積 reward

只要 actor 試著將 $G_t - V(s_t)$ 變大，就能產生更好的 action

V 如何幫助 actor 訓練？

$$\nabla L = \mathbb{E}_{\tau\sim\pi_\theta(\tau)}\left[\sum_{t=1}^{T} G_t \cdot \nabla \log \pi_\theta(a_t|s_t)\right]$$

Advantage function (優勢函數)

$$A_t = G_t - V(s_t)$$

- Temporal Difference (TD)

  訓練V時： $V(s_t) - \gamma V(s_{t+1})$ 越接近 r_t 越好

  訓練 actor 時： $r_t - V(s_t) + \gamma V(s_{t+1})$ 越大越好，因為 $V(s_t) - \gamma V(s_{t+1})$ 只是預估值

$$\nabla L = \mathbb{E}_{\tau\sim\pi_\theta(\tau)}\left[\sum_{t=1}^{T} (r_t + \gamma V(s_{t+1}) - V(s_t)) \cdot \nabla \log \pi_\theta(a_t|s_t)\right]$$

TD 是看即時 reward

(Simplified) Path of Deep Reinforcement Learning

Elements of RL

Policy Gradient Methods

Actor-Critic

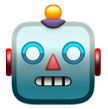
On-policy and Off-policy

[Recap, Important Notes] Training 細節

- 通常一個 episode 更新一次 actor
- 每次 agent 更新後，資料要重新收集

Question: 舊的資料是否能沿用呢？

Step1: 收集資料



s_1	a_1	r_1
s_2	a_2	r_2
s_3	a_3	r_3
⋮		
s_T	a_T	r_T

[Definition] On-policy and Off-policy

- On-policy：被訓練的 agent 跟資料是來自同一個 policy
  - 訓練資料來自目前的 policy (現在訓練 π_θ，資料來自 π_θ)
- Off-policy：被訓練的 agent 跟資料不是來自同一個 policy
  - 訓練資料來自不同的 policies (例如：現在訓練 $\pi_{\theta_{\text{new}}}$ 但資料來自 $\pi_{\theta_{\text{old}}}$)

On-policy and Off-policy

Hung-yi Lee: https://youtu.be/OAKAZhFmYoI?si=zmFUM46UJ4ONKchQ&t=90
On-policy: 阿光下棋
Off-policy: 佐為下棋、阿光在旁邊看

On-policy: 自己打game

Off-policy: 看實況主打game，自己偷學
https://www.youtube.com/watch?v=wmabM0dFGWU

Off-policy 的問題與 PPO

TRPO: Schulman, John, et al. "Trust region policy optimization." *International conference on machine learning*. 2015.
PPO: Schulman, John, et al. "Proximal policy optimization algorithms." arXiv preprint arXiv:1707.06347 (2017).

- 資料 (來自 $\pi_{\theta_{\text{old}}}$) 是舊的，但模型 ($\pi_{\theta_{\text{new}}}$) 是新的
  - $\pi_{\theta_{\text{new}}}$ 可能已經變強了，可能會被舊的資料誤導
- 有些 paper (如 TRPO) 會採用 importance sampling 的方法來限制 $\pi_{\theta_{\text{new}}}(a_t|s_t)$ 的變化

$$\frac{\pi_{\theta_{\text{new}}}(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}$$

- 但在 $\pi_{\theta_{\text{new}}}(a_t|s_t)$ 和 $\pi_{\theta_{\text{old}}}(a_t|s_t)$ 差異很大的情況下，訓練不穩定
  - -> 近代作法 PPO (Proximal Policy Optimization) 用範圍更加限制 importance sampling 的變化：

$$1-\epsilon \leq \frac{\pi_{\theta_{\text{new}}}(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)} \leq 1+\epsilon$$

- ϵ 為超參數，PPO 論文中設為 0.2

Summary

- Policy gradient: 讓 policy 透過 maximize reward 方向更新
- Value function: 預測一個 state 的「未來總 reward」
- On-policy vs. Off-policy: 模型「自己收資料」或「看舊資料學習」的差別
- PPO: 透過限制 policy 來小步更新幅度，慢慢變好

學習資源

- Sutton, R. S., & Barto, A. G. (2018). *Reinforcement Learning: An Introduction* (2nd ed.), Chapter 5: Monte Carlo Methods.
  - http://incompleteideas.net/book/the-book-2nd.html
- 李宏毅老師 Deep Reinforcement Learning, 2018 播放清單
  - https://youtube.com/playlist?list=PLJV_el3uVTsODxQFgzMzPLa16h6B8kWM_&si=qN41KpUMwMBVipiA
- OpenAI Spinning Up in Deep RL
  - https://spinningup.openai.com/en/latest/algorithms/ppo.html

Thank you!

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