

# Optimization notes

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# Directional derivative

From a starting point  $\underline{x}_0$  and a given direction  $\underline{u}$ :

- $\underline{x}(\lambda) = \underline{x}_0 + \lambda \underline{u}$ 
  - $\lambda$  is a scalar.
- $d\underline{x} = \underline{u}d\lambda$ 
  - For a small change in  $\lambda$ .
- $F(\lambda) = f(\underline{x}_0 + \lambda \underline{u})$

$$\begin{aligned}dF &= df = (\nabla f(\underline{x}))^\top d\underline{x} \\&= (\nabla f(\underline{x}))^\top \underline{u}d\lambda = \nabla^\top f \underline{u} \lambda\end{aligned}$$

- $\frac{df}{d\lambda} = \nabla^\top f \underline{u}$ 
  - If  $f$  is minimized at  $\underline{x}^* = \underline{x}_0 + \lambda \underline{u}$ , then:
    - $\nabla f(\underline{x}^*)^\top \underline{u} = 0$
    - gradient  $f$  evaluated at the minimum point is orthogonal to  $\underline{u}$ .