Optimization notes

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Directional derivative

From a starting point $\underline{x}_0$ and a given dirction $\underline{u}$:

- $\underline{x}(\lambda) = \underline{x}_0 + \lambda \underline{u}$
  - λ is a scalar.
- $d\underline{x} = \underline{u} d\lambda$
  - For a small change in λ.
- $F(\lambda) = f(\underline{x}_0 + \lambda \underline{u})$

$$\begin{aligned} dF = df &= (\nabla f(\underline{x}))^\top d\underline{x} \\ &= (\nabla f(\underline{x}))^\top \underline{u} d\lambda = \nabla^\top f \underline{u} \lambda \end{aligned}$$

- $\frac{df}{d\lambda} = \nabla^\top f \underline{u}$
  - If f is minimized at $\underline{x}^* = \underline{x}_0 + \lambda \underline{u}$, then:
    - $\nabla f(\underline{x}^*))^\top f \underline{u} = 0$
    - gradient f evaluated at the minimum point is orthogonalto $\underline{u}$.

Weierstrass Theorem

If $f(\underline{x})$ is continuous on a nonempty feasible set that is cloased and bounded, then $f(\underline{x})$ has a global minimum in this set.

- A set S is bounded if for any point $\underline{x}$ in S, we have $\underline{x}^\top \underline{x} < c$
    - c is a finite positive number.