1. **Executive Summary – brief description of the issues you identified and your recommendation(s)**
    a. Our objective was to minimize the busing cost per student across 6 areas and 3 schools, while holding to the appropriate grade percentage thresholds for each school and not violating the school capacities. Solving this model gave us a total cost of $555,600. Our optimal solution involved sending all 450 students from area 1 to school 2, sending 432 students from area 2 to school 2 and 168 students from area 2 to school 3, 4 students from area 3 to school 1 and 218 from area 3 to school 2 and 328 from area 3 to school 3, sending all 350 students from area 4 to school 1, sending 364 kids from area 5 to school 1 and 136 to school 3, sending 82 kids from area 6 to school 1 and 368 to school 3.
    b. By conducting what-if analyses on road construction costs, our sensitivity analysis in part 2 revealed that increasing busing costs 10% for area 6 schools would raise the student busing costs to a range within the limit of the optimal solution we initially identified. For this reason, with the increased busing costs, we still recommend the same number of students from each area be sent to the different schools as we did in our initial analysis.
    c. In order to evaluate the cost effectiveness of installing a portable classroom to increase a school's capacity, we investigated the school constraint duals and their respective dual ranges in order to find the largest cost minimization margin over the classroom leasing cost. We recommend the school board install one portable classroom in school 2, as it would lead to a busing cost decrease of $3,333.33, and an overall cost decrease of $833.33 after adding in the cost of leasing the room ($2500).
2. **Model description - explain/write out your model formulation. Document what the model does and the inputs necessary**
    a. We used a linear programming model in R to find the optimal solution which was the lowest possible bussing cost per student we could achieve while satisfying all of the constraints
    b. Our objective function is to minimize the bussing cost per student, represented by the equation $300x_{11} + 0x_{12} + 700x_{13} + 0x_{21} + 400x_{22} + 500x_{23} + 600x_{31} + 300x_{32} + 200x_{33} + 200x_{41} + 500x_{42} + 0x_{43} + 0x_{51} + 0x_{52} + 400x_{53} + 500x_{61} + 300x_{62} + 0x_{63}$
    c. We have 3 capacity constraints that represent the capacity of each school

   i. $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} <= 900$ (school 1)
   ii. $x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} <= 1100$ (school 2)
   iii. $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} <= 1000$ (school 3)

d. We have 6 constraints that represent the total number of students in each area and the fact that they need to be divided between the three schools
   i. $x_{11} + x_{12} + x_{13} = 450$ (area 1)
   ii. $x_{21} + x_{22} + x_{23} = 600$ (area 2)
   iii. $x_{31} + x_{32} + x_{33} = 550$ (area 3)
   iv. $x_{41} + x_{42} + x_{43} = 350$ (area 4)
   v. $x_{51} + x_{52} + x_{53} = 500$ (area 5)
   vi. $x_{61} + x_{62} + x_{63} = 450$ (area 6)

e. We have 18 constraints to represent the restriction that each grade (6th-8th) must make up greater than 30% but less than 36% of each grade in each of the 3 schools
   i. Minimum for 6th grade in school 1 (30%)
      1. $.02x_{11} + .07x_{21} + 0x_{31} -.02x_{41} + .09x_{51} + .04x_{61} >= 0$
   ii. Maximum for 6th grade in school 1 (36%)
      1. $-.04x_{11} + .01x_{21} - .06x_{31} -.08x_{41} + .03x_{51} - .02x_{61} <= 0$
   iii. Minimum for 7th grade in school 1 (30%)
      1. $.08x_{11} - .02x_{21} + .02x_{31} +.1x_{41} + .04x_{51} - .02x_{61} >= 0$
   iv. Maximum for 7th grade in school 1 (36%)
      1. $.02x_{11} - .08x_{21} - .04x_{31} +.04x_{41} - .02x_{51} - .08x_{61} <= 0$
   v. Minimum for 8th grade in school 1 (30%)
      1. $0x_{11} + .05x_{21} + .08x_{31} +.02x_{41} - .03x_{51} + .08x_{61} >= 0$
   vi. Maximum for 8th grade in school 1 (36%)
      1. $-.06x_{11} - .01x_{21} + .02x_{31} -.04x_{41} - .09x_{51} + .02x_{61} <= 0$
   vii. Minimum for 6th grade in school 2 (30%)
      1. $.02x_{12} + .07x_{22} + 0x_{32} -.02x_{42} + .09x_{52} + .04x_{62} >= 0$
   viii. Maximum for 6th grade in school 2 (36%)
      1. $-.04x_{12} + .01x_{22} - .06x_{32} -.08x_{42} + .03x_{52} - .02x_{62} <= 0$
   ix. Minimum for 7th grade in school 2 (30%)
      1. $.08x_{12} - .02x_{22} + .02x_{32} +.1x_{42} + .04x_{52} - .02x_{62} >= 0$
   x. Maximum for 7th grade in school 2 (36%)
      1. $.02x_{12} - .08x_{22}- .04x_{32} +.04x_{42}- .02x_{52} - .08x_{62} <= 0$
   xi. Minimum for 8th grade in school 2 (30%)
      1. $0x_{12} + .05x_{22} + .08x_{32} +.02x_{42} - .03x_{52} + .08x_{62} >= 0$
   xii. Maximum for 8th grade in school 2 (36%)

1. $-.06x_{12} - .01x_{22} + .02x_{32} -.04x_{42} - .09x_{52} + .02x_{62} <= 0$

xiii. Minimum for 6th grade in school 3 (30%)

1. $.02x_{13} + .07x_{23} + 0x_{33} -.02x_{43} + .09x_{53}+ .04x_{63} >= 0$

xiv. Maximum for 6th grade in school 3 (36%)

1. $-.04x_{13} + .01x_{23} - .06x_{33} -.08x_{43} + .03x_{53} - .02x_{63} <= 0$

xv. Minimum for 7th grade in school 3 (30%)

1. $.08x_{13} - .02x_{23} + .02x_{33} +.1x_{43} + .04x_{53} - .02x_{63} >= 0$

xvi. Maximum for 7th grade in school 3 (36%)

1. $.02x_{13} - .08x_{23}- .04x_{33} +.04x_{43}- .02x_{53} - .08x_{63} <= 0$

xvii. Minimum for 8th grade in school 3 (30%)

1. $0x_{13} + .05x_{23} + .08x_{33} +.02x_{43} - .03x_{53} + .08x_{63} >= 0$

xviii. Maximum for 8th grade in school 3 (36%)

1. $-.06x_{13} - .01x_{23} + .02x_{33} -.04x_{43} - .09x_{53} + .02x_{63} <= 0$

xix. Constraints for infeasible solutions

1. $x_{21}, x_{43}, x_{52} = 0$

xx. Non-negativity constraint

1. $X_{ij} >= 0$ (i = areas 1-6, j = schools 1-3)

xxi. Integer constraint

1. We set the R function all.int = TRUE because all of the decision variables must have integer values in the final solution because partial students can't be assigned to a school

3. **Answers to the various questions asked and detailed explanation of the results**
   a. 1. Based on the constraints for each of the areas and schools, the best solution by area would be to send: 450 kids from area 1 to school 2; 432 kids from area 2 to school 2 and 168 to school 3; 4 kids from area 3 to school 1, 218 to school 2, and 328 to school 3; 350 kids from area 4 to school 1; 364 kids from area 5 to school 1 and 136 to school 3; and 82 kids from area 6 to school 1 and 368 to school 3. This would add up to 800 students in school 1 (100 below capacity), 1100 students in school 2 (full capacity), and 1000 students in school 3 (full capacity). This solution would result in the minimum busing costs possible under our constraints at $555,600.
   b. 2a. If the busing costs from area 6 to school 1 increase by 10% due to road construction, going from $500 per student to $550 per student, the optimal solution will remain the same. In carrying out a sensitivity

analysis, we found that the busing cost per student for this school and area could increase up to $680 before it would affect our optimal solution and require that the model be resolved.

c. 2b. If the busing costs from area 6 to school 2 also increased by 10%, going from $300 per student to $330 per student, the optimal solution would also still remain the same. The sensitivity analysis revealed that the cost from this area to this school could increase infinitely and still not affect the optimal solution. This is because our optimal solution already assigned 0 students from area 6 to school 2 due to the other constraints in place, so increasing costs would still ensure that no kids from this area were sent to school 2. Additionally, if the busing costs from area 6 to school 3 increased by 10%, there would be no impact because the cost for this school from area 6 is already $0 per student. This number could still increase up to $200 per student according to our sensitivity analysis and our optimal solution would remain. Overall, the road construction increasing busing costs would not disrupt our original plan for the school board's decision of how to organize the district.

d. 3a. In order to investigate the cost effectiveness of installing portable classrooms, we looked at LP_model$duals for the 3 school capacity constraints. The duals gave us insight into how much an increase in students would increase or decrease the busing cost. School 1 had a dual of 0, which meant it was irrelevant to consider further as adding capacity for additional students wouldn't decrease the cost of busing. School 2 and 3 both had negative duals (-166.66 and -133.33 respectively), which meant adding capacity for students could help minimize busing costs. Adding one classroom to school 2 (20 students x -166.66) decreases the busing cost by $3333.33, which would ultimately reduce costs by $833.33 after adding in the cost of leasing a classroom. Adding one classroom to school 3 (20 students x -133.33) decreases the busing cost by $2666.6, which would ultimately reduce costs by $166.6 after adding in the cost of leasing a classroom. Thus, adding a classroom to school 2 creates a wider margin of cost minimization than school 3. We then explored the ranges that these duals applied to minimize cost.

e. 3b. The dual range for school 2 was 1099 to 1129, which meant that at most 29 students could be added to the school for this cost decrease to hold true, but since the portable classrooms only accommodate 20 students, we recommend only adding one portable classroom for school

2. School 3 did not have a lower or upper bound. Despite this, we do not recommend that the school district invest in portable classrooms for school 3 because the cost decrease of only $166.60 per classroom is not very significant and would likely not be worth the hassle.

4. **Appendix of R code**