



Problem 1

```
In [41]: import numpy as np
import matplotlib.pyplot as plt
import networkx as nx

from sklearn.cluster import KMeans, SpectralClustering
from sklearn.datasets import make_circles
from sklearn.metrics import adjusted_rand_score

seed = 42
np.random.seed(seed)
```

part a

```
In [42]: # Setting up SSBM graph
n = 30
k = 3
A = 0.7
B = 0.1

sizes = [n // k] * k

p_matrix = np.full((k, k), B)
np.fill_diagonal(p_matrix, A)

def create_ssrbm_graph(seed):
    G = nx.stochastic_block_model(sizes, p_matrix, seed=seed)
    return G, nx.to_numpy_array(G)
```

```
In [43]: # clustering

def train_kmeans(mat, n_clusters, seed):
    kmeans = KMeans(n_clusters=n_clusters, random_state=seed)
    return kmeans.fit_predict(mat)

def train_spectral(mat, n_clusters, seed):
    spectral_ssrbm = SpectralClustering(
        n_clusters=n_clusters,
        affinity="precomputed",
        assign_labels="kmeans",
        random_state=seed,
    )
    return spectral_ssrbm.fit_predict(mat)
```

```
In [44]: # plot results

def visualize_clustering(G, true_labels, labels_kmeans, labels_spectral):
    pos = nx.spring_layout(G, seed=seed)
```

```

fig, axes = plt.subplots(1, 3, figsize=(16, 4.5))

# plot ground truth
nx.draw_networkx(
    G,
    pos=pos,
    node_color=true_labels,
    node_size=220,
    with_labels=True,
    font_size=8,
    ax=axes[0],
)
axes[0].set_title("Ground truth communities")

# plot kmeans
nx.draw_networkx(
    G,
    pos=pos,
    node_color=labels_kmeans,
    node_size=220,
    with_labels=True,
    font_size=8,
    ax=axes[1],
)
axes[1].set_title(f"k-means on adjacency rows")

# plot spectral clustering
nx.draw_networkx(
    G,
    pos=pos,
    node_color=labels_spectral,
    node_size=220,
    with_labels=True,
    font_size=8,
    ax=axes[2],
)
axes[2].set_title(f"Spectral clustering")

plt.tight_layout()
plt.show()

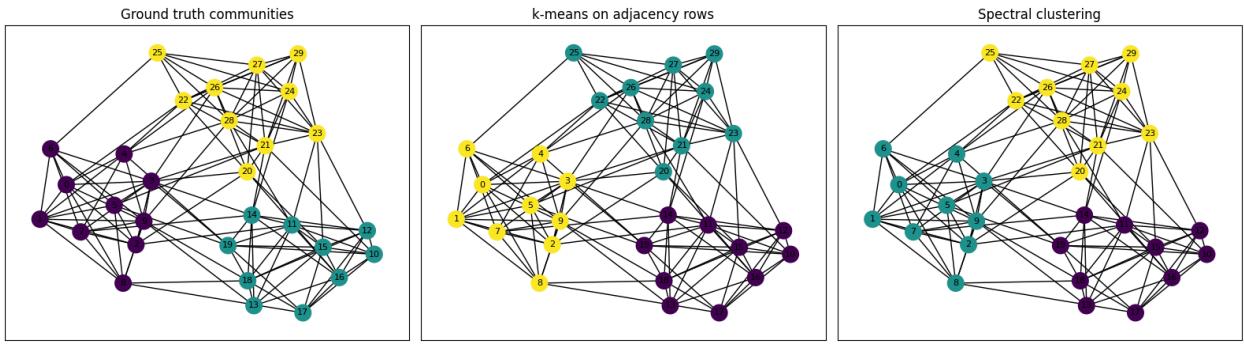
```

In [45]:

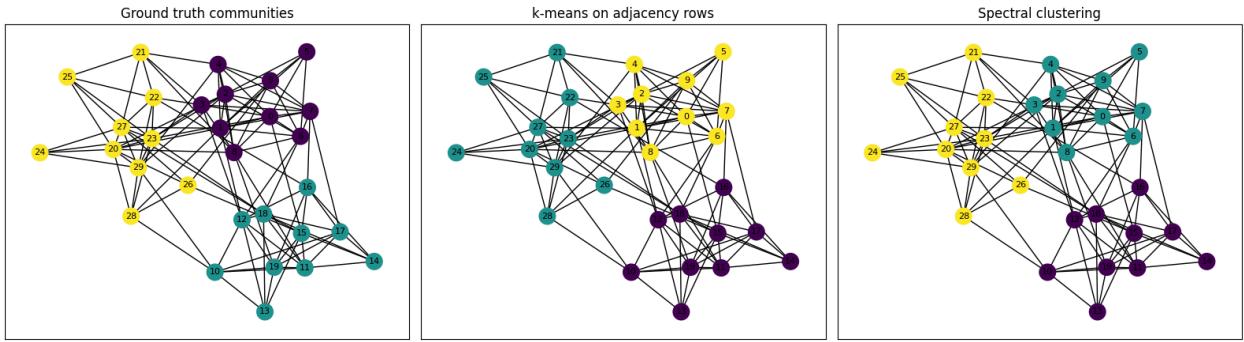
```

G_ssbbm, mat_ssbbm = create_ssbbm_graph(seed)
true_labels_ssbbm = []
for i in range(k):
    true_labels_ssbbm.extend([i] * (sizes[i]))
true_labels_ssbbm = np.array(true_labels_ssbbm)
labels_kmeans_ssbbm = train_kmeans(mat_ssbbm, k, seed)
labels_spectral_ssbbm = train_spectral(mat_ssbbm, k, seed)
visualize_clustering(G_ssbbm, true_labels_ssbbm, labels_kmeans_ssbbm, labels_spec

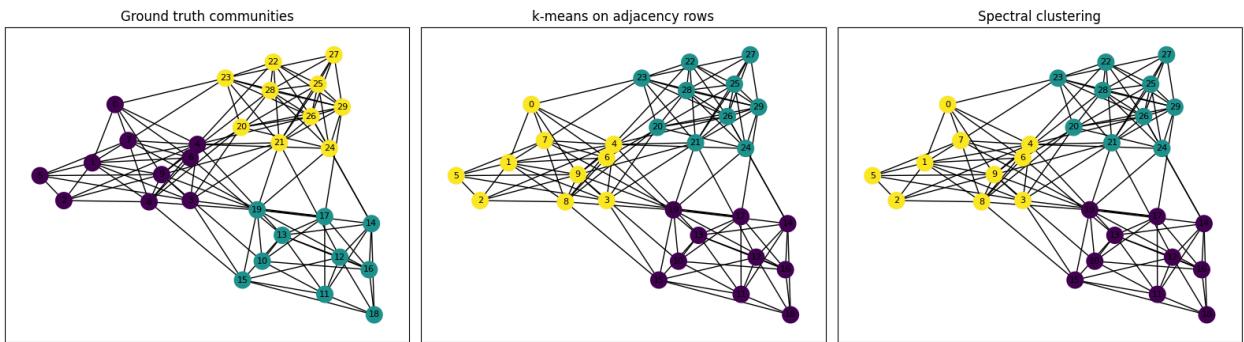
```



```
In [46]: G_ssbbm, mat_ssbbm = create_ssbbm_graph(420)
true_labels_ssbbm = []
for i in range(k):
    true_labels_ssbbm.extend([i] * (sizes[i]))
true_labels_ssbbm = np.array(true_labels_ssbbm)
labels_kmeans_ssbbm = train_kmeans(mat_ssbbm, k, seed)
labels_spectral_ssbbm = train_spectral(mat_ssbbm, k, seed)
visualize_clustering(G_ssbbm, true_labels_ssbbm, labels_kmeans_ssbbm, labels_spectral_ssbbm)
```

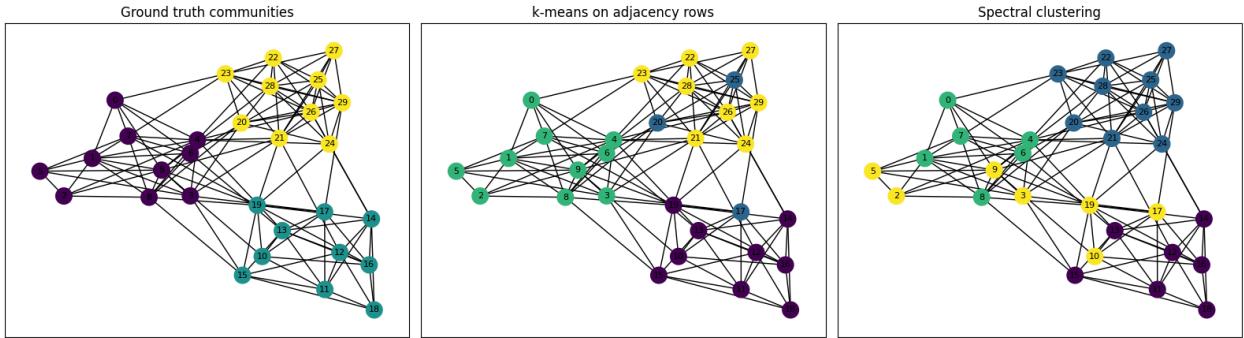


```
In [47]: G_ssbbm, mat_ssbbm = create_ssbbm_graph(67)
true_labels_ssbbm = []
for i in range(k):
    true_labels_ssbbm.extend([i] * (sizes[i]))
true_labels_ssbbm = np.array(true_labels_ssbbm)
labels_kmeans_ssbbm = train_kmeans(mat_ssbbm, k, seed)
labels_spectral_ssbbm = train_spectral(mat_ssbbm, k, seed)
visualize_clustering(G_ssbbm, true_labels_ssbbm, labels_kmeans_ssbbm, labels_spectral_ssbbm)
```

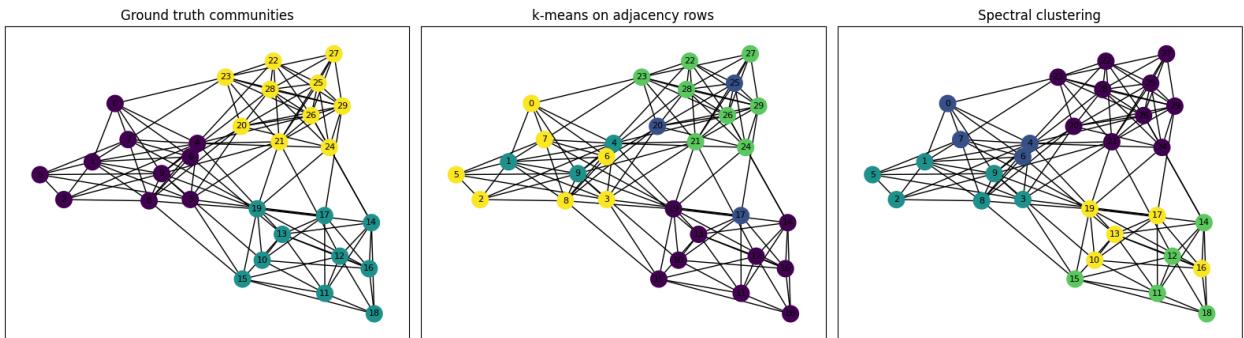


```
In [48]: labels_kmeans_ssbbm = train_kmeans(mat_ssbbm, 4, seed)
labels_spectral_ssbbm = train_spectral(mat_ssbbm, 4, seed)
```

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visualize_clustering(G_ssbbm, true_labels_ssbbm, labels_kmeans_ssbbm, labels_spec
```



```
In [51]: labels_kmeans_ssbbm = train_kmeans(mat_ssbbm, 5, seed)
labels_spectral_ssbbm = train_spectral(mat_ssbbm, 5, seed)
visualize_clustering(G_ssbbm, true_labels_ssbbm, labels_kmeans_ssbbm, labels_spec
```



```
In [49]: from sklearn.metrics import adjusted_rand_score
```

```
avg_ari_kmeans = 0
avg_ari_spectral = 0
low, high = 2, 10
for n_clusters in range(low, high + 1):
    labels_kmeans_ssbbm = train_kmeans(mat_ssbbm, n_clusters, seed)
    labels_spectral_ssbbm = train_spectral(mat_ssbbm, n_clusters, seed)
    ari_kmeans_ssbbm = adjusted_rand_score(true_labels_ssbbm, labels_kmeans_ssbbm)
    ari_spectral_ssbbm = adjusted_rand_score(true_labels_ssbbm, labels_spectral_ssbbm)
    avg_ari_kmeans += ari_kmeans_ssbbm
    avg_ari_spectral += ari_spectral_ssbbm
    print(f"k-means ARI: {ari_kmeans_ssbbm:.3f}, Spectral ARI: {ari_spectral_ssbbm:.3f}")
avg_ari_kmeans /= (high - low + 1)
avg_ari_spectral /= (high - low + 1)
print(f"Average k-means ARI: {avg_ari_kmeans:.3f}, Average spectral ARI: {avg_ari_spectral:.3f}")
```

```
k-means ARI: 0.554, Spectral ARI: 0.554 for 2 clusters
k-means ARI: 1.000, Spectral ARI: 1.000 for 3 clusters
k-means ARI: 0.848, Spectral ARI: 0.672 for 4 clusters
k-means ARI: 0.717, Spectral ARI: 0.708 for 5 clusters
k-means ARI: 0.643, Spectral ARI: 0.551 for 6 clusters
k-means ARI: 0.547, Spectral ARI: 0.556 for 7 clusters
k-means ARI: 0.464, Spectral ARI: 0.315 for 8 clusters
k-means ARI: 0.416, Spectral ARI: 0.290 for 9 clusters
k-means ARI: 0.351, Spectral ARI: 0.186 for 10 clusters
Average k-means ARI: 0.615, Average spectral ARI: 0.537
```

part b

```
In [50]: n_circles = 500
noise = 0.01
k_circles = 2

X, y_true = make_circles(
    n_samples=n_circles,
    factor=0.8,
    noise=noise,
    random_state=seed,
)

kmeans_circles = KMeans(n_clusters=k_circles, random_state=seed, n_init=20)
labels_kmeans_circles = kmeans_circles.fit_predict(X)

spectral_circles = SpectralClustering(
    n_clusters=k_circles,
    affinity="nearest_neighbors",
    n_neighbors=12,
    assign_labels="kmeans",
    random_state=seed,
)
labels_spectral_circles = spectral_circles.fit_predict(X)

ari_kmeans_circles = adjusted_rand_score(y_true, labels_kmeans_circles)
ari_spectral_circles = adjusted_rand_score(y_true, labels_spectral_circles)

print(f"Circle ARI (k-means): {ari_kmeans_circles:.3f}")
print(f"Circle ARI (spectral): {ari_spectral_circles:.3f}")

fig, axes = plt.subplots(1, 3, figsize=(16, 4.5))

axes[0].scatter(X[:, 0], X[:, 1], c=y_true, s=18, cmap="tab10")
axes[0].set_title("Ground truth circles")
axes[0].set_aspect("equal")

axes[1].scatter(X[:, 0], X[:, 1], c=labels_kmeans_circles, s=18, cmap="tab10")
axes[1].set_title(f"k-means\nARI={ari_kmeans_circles:.3f}")
axes[1].set_aspect("equal")
```

```

axes[2].scatter(X[:, 0], X[:, 1], c=labels_spectral_circles, s=18, cmap="tab10")
axes[2].set_title(f"Spectral clustering\nARI={ari_spectral_circles:.3f}")
axes[2].set_aspect("equal")

for ax in axes:
    ax.set_xlabel("x")
    ax.set_ylabel("y")

plt.tight_layout()
plt.show()

```

Circle ARI (k-means): -0.002

Circle ARI (spectral): 1.000

/Users/marco/school/cs144/.venv/lib/python3.11/site-packages/sklearn/manifol
d/_spectral_embedding.py:324: UserWarning: Graph is not fully connected, spectr
al embedding may not work as expected.
warnings.warn(

