SORBONNE UNIVERSITÈ

Computer Science

Image Analysis and Processing

Practical work on:

# Mathematical morphology

Student:

**Maria Cristina Centorame**

Academic year 2023/2024

# Chapter 1

# Mathematical morphology on gray-scale images

## 1.1 Dilation, Erosion, Opening and Closing: application and properties

The morphological operations of dilation, erosion, opening and closing have been applied by varying the size and the shape of the structuring elements. The following structuring elements where given: disk, square, diamond, line. It can be shown that by applying the dilation on an image the objects in the image are inflated and the background is deflated thus in a binary image as the "cellbin.bmp"(see Figure 1.1) the white regions tend to be expanded and by fixing the shape of the structuring element for example to a disk the operation will result in a more significant dilation effect(see Figure 1.2). In addition, the shape of the chosen structuring element will influence the direction of the dilation: Figure 1.3 shows how a square or diamond-shaped element expands equally in all directions, while a line-shaped element expands along its orientation.
On the other hand, the erosion effect consists in deflating the objects contained in the image and in inflating the background as depicted in Figure 1.4 where the erosion is applied to the "bat200.bmp" image(see original image 1.1) with a disk as a structuring element and becomes more evident increasing the size of the disk. Moreover, even the erosion if performed with a circular element, erodes uniformly from all sides, while with a line-shaped element erodes along its orientation(1.5) .
Shifting the focus to opening and closing operations, the former involves an initial erosion followed by a dilation, primarily employed for noise reduction and fine detail removal thus darkening narrow bright zones as it happens in Figure 1.6 In contrast, the latter combines dilation followed by erosion and serves the purpose of sealing small holes and gaps so, as a result, on the "retina2.gif" image it brights narrow dark zones.(1.7)

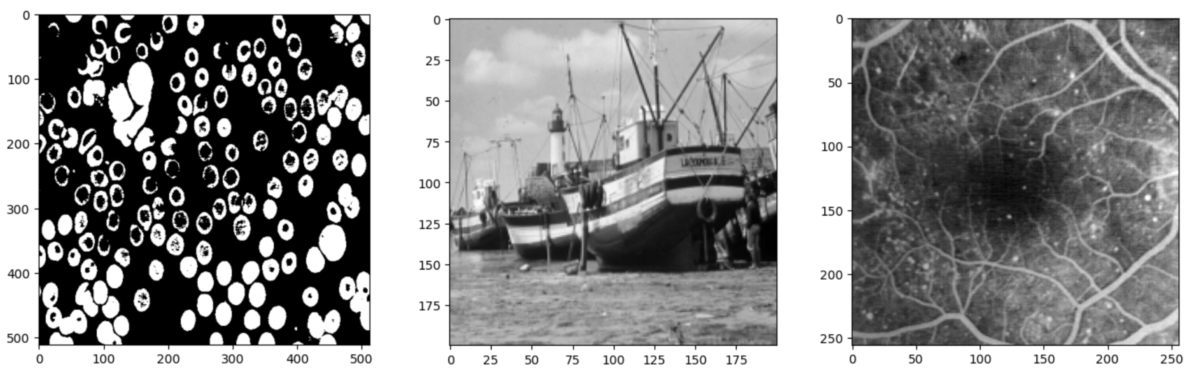

Figure 1.1: Original image "cellbin.bmp"(left), "bat200.bmp"(center),"retina2.gif"(right) "

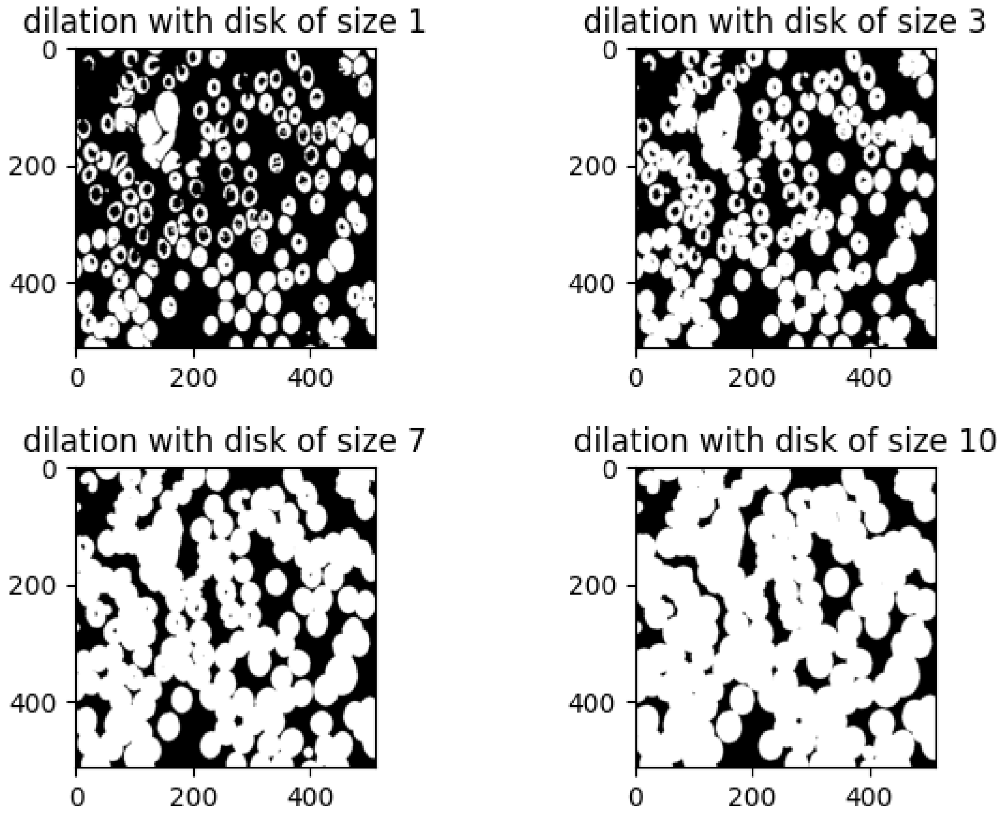


Figure 1.2: Dilation of image "cellbin.bmp" with a disk of different sizes

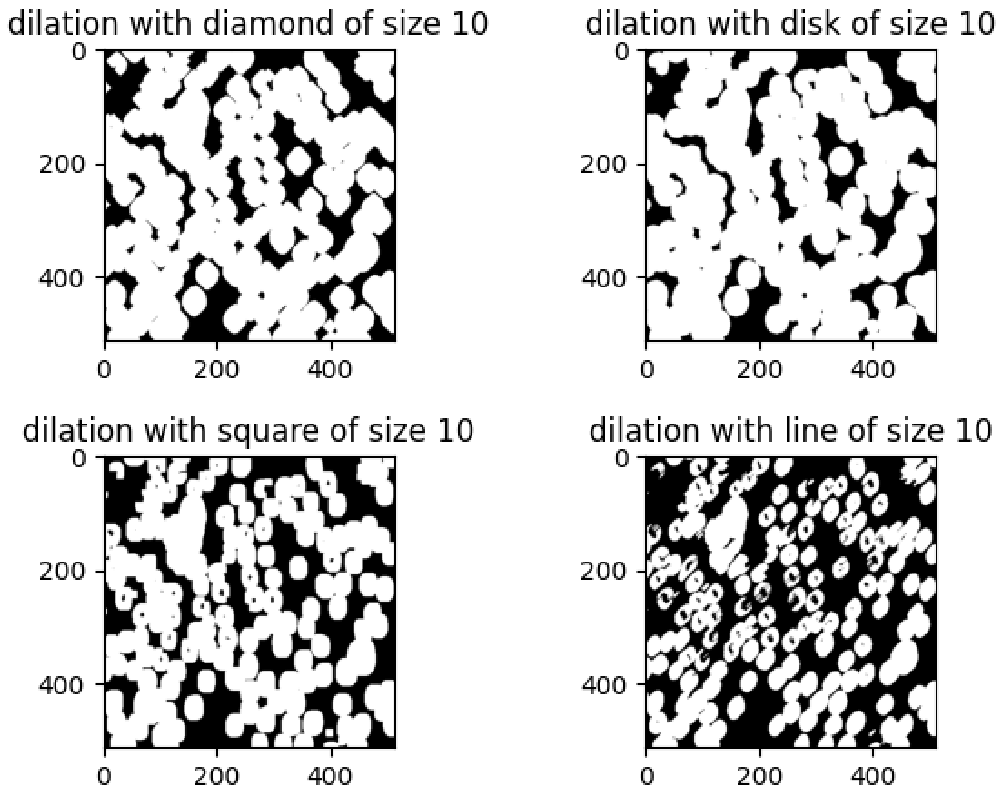


Figure 1.3: Dilation of image "cellbin.bmp" with different shapes of the structuring element (the line has an orientation of 45°)

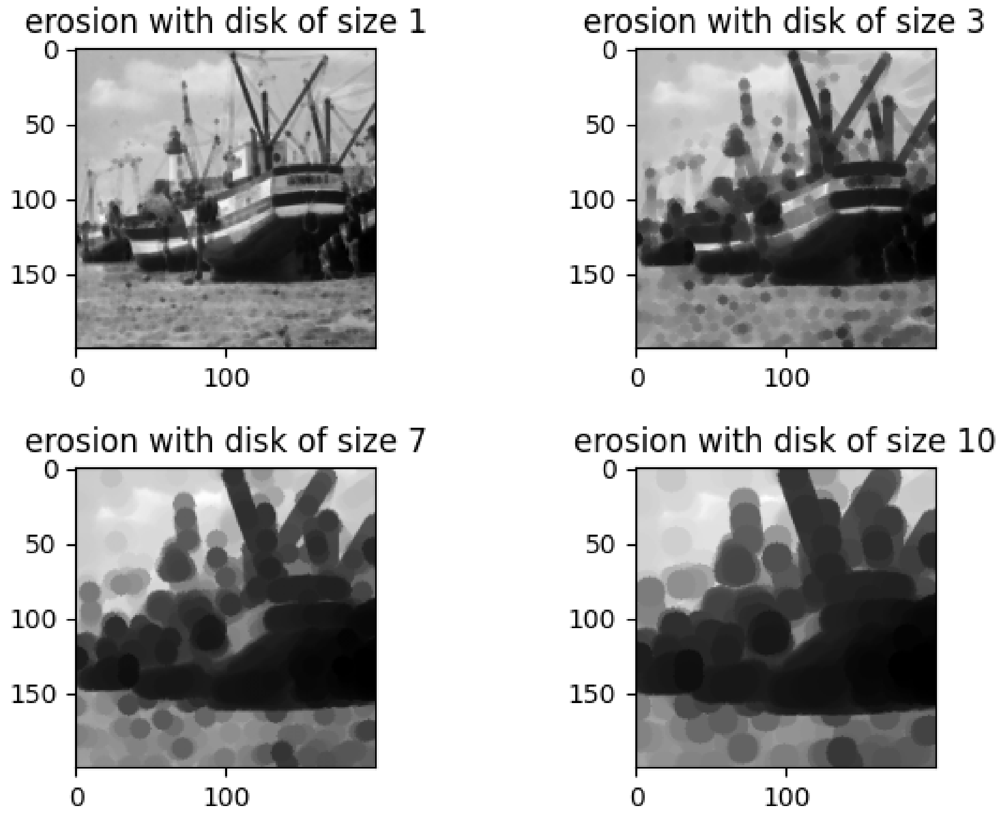


Figure 1.4: Erosion of image "bat200.bmp" with a disk of different sizes

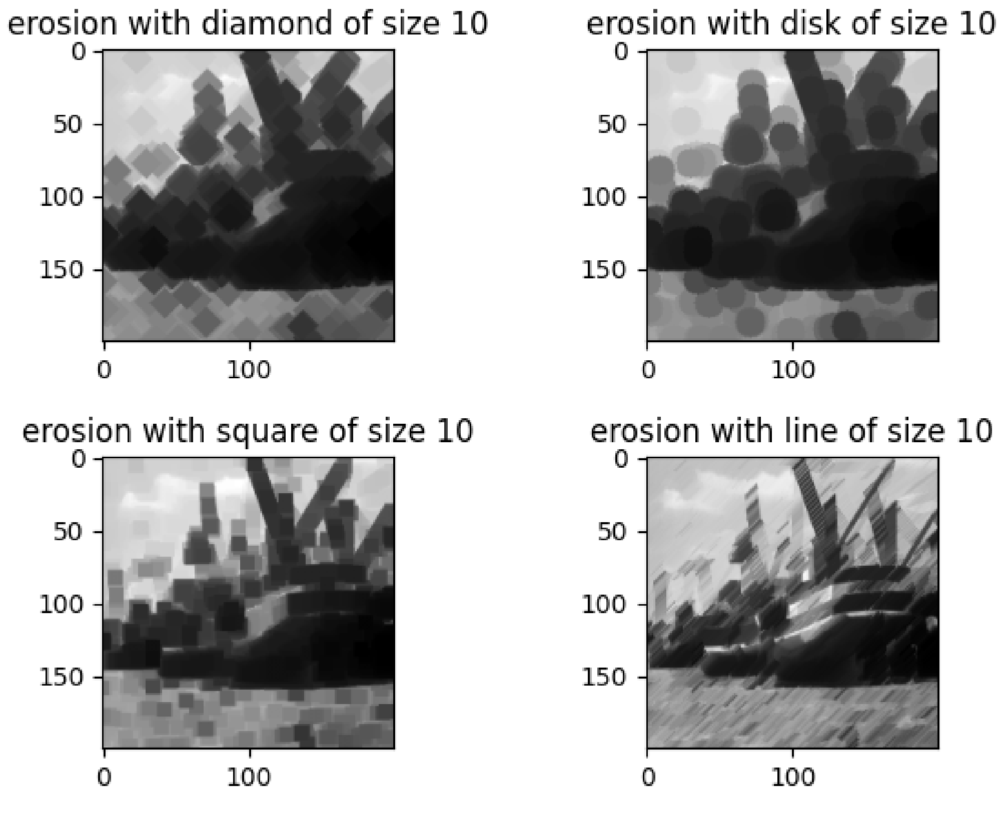


Figure 1.5: Erosion of image "bat200.bmp" with different shapes of the structuring element (the line has an orientation of 45°)

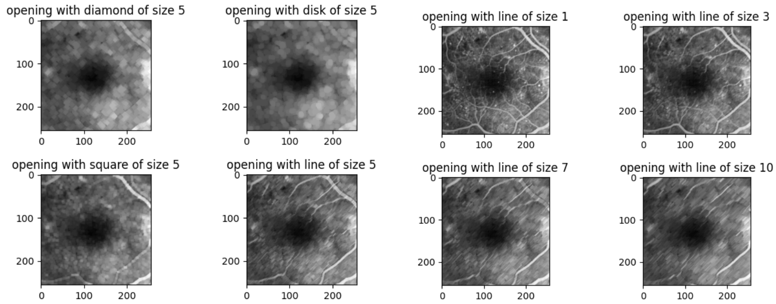


Figure 1.6: Opening applied to image "retina2.gif with different shapes and a fixed size(left), with a line as structural element in different sizes(right)"

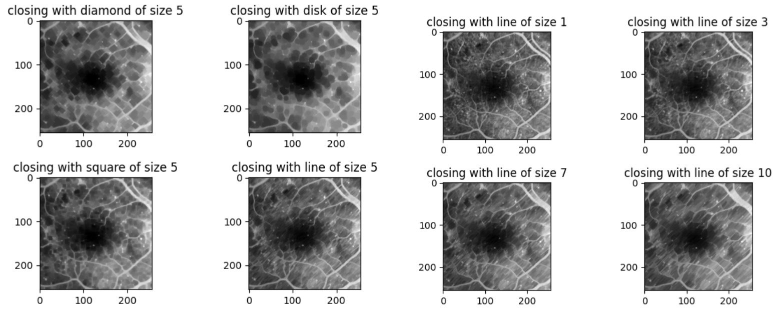


Figure 1.7: Closing applied to image "retina2.gif with different shapes and a fixed size(left), with a line as structural element in different sizes(right)"

## 1.2 The Idempotent property

Dilation and opening operations are idempotent, meaning that applying the operation multiple times with the same structuring element does not change the result after a certain point. The succession of a dilation by a square of size 3x3 followed by a dilation by a square of size 5x5 is equivalent to a single dilation operation with a larger square structuring element of size 5x5. Similarly, the succession of an opening by a square of size 3x3 followed by an opening by a square of size 5x5 is equivalent to a single opening operation with a larger square structuring element of size 5x5.

Continuously applying dilation using the same structuring element will extend the object until it reaches its maximum expansion, determined by the size of the structuring element. Subsequent dilations with the same structuring element will have no effect on the object's shape or size.

And the similar principle holds for opening: iterative opening operations with the same structuring element will persist in eliminating noise and fine details until no further changes are possible. Further openings with the same structuring element will yield no alterations in the outcome. See Figure 1.8and Figure 1.9 for the result from both the operations. As expected, they are equal.

## 1.3 The top-hat transform

The top-hat transform is obtained by taking the difference between the original image and its opening and it's used to highlight the details or small structures that might be obscured within the foreground objects. Increasing the size of the structural element the bright structures are more enhanced and using shapes which cover more directions as square and diamond rather than line the top_hat transform is able to catch better the structures. This comparison is shown in Figure 1.10 and 1.11.

The dual operation to the top-hat transform is the bottom-hat transform. It is obtained by taking the difference between the closing of the original image and the original image itself. As shown in Figure 1.12 and Figure 1.13, it enhances dark structures or regions within the image by reducing the intensity of the bright ones, making them less prominent.

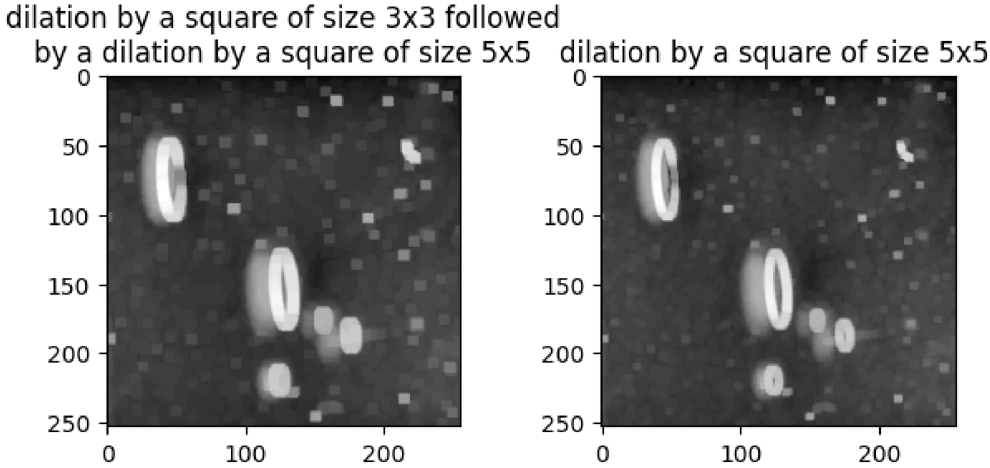


Figure 1.8: Dilation by a square of size 3x3 followed by a dilation by a square of size 5x5 on image "bulles.bmp"(left);
Dilation by a square of size 5x5(right) on image "bulles.bmp".

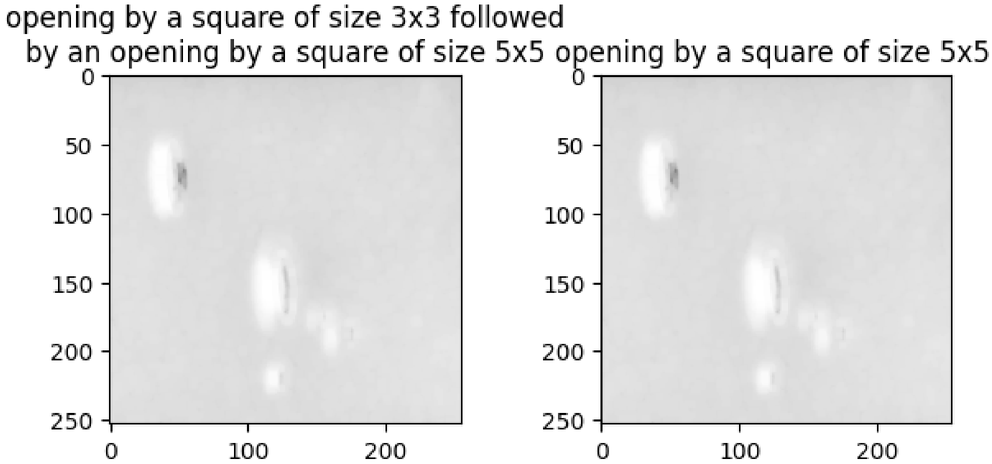


Figure 1.9: Opening by a square of size 3x3 followed by an opening by a square of size 5x5 on image "bulles.bmp"(left);
Opening by a square of size 5x5 on image "bulles.bmp"(right).

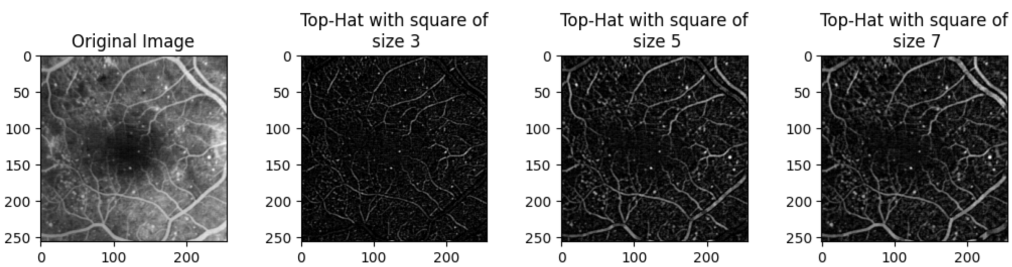


Figure 1.10: Top-hat transform on image "retina2.gif" with a square of different sizes

## 1.4 Opening with segments of different orientations

We define segments in several directions, applied an opening with each of them as a structuring element, and then computed the point-wise maximum of the results. We can conclude that the point-wise maximum of the results effectively highlights linear structures or patterns in the image, and it can provide valuable information about the orientation and location of such structures.(see Figure

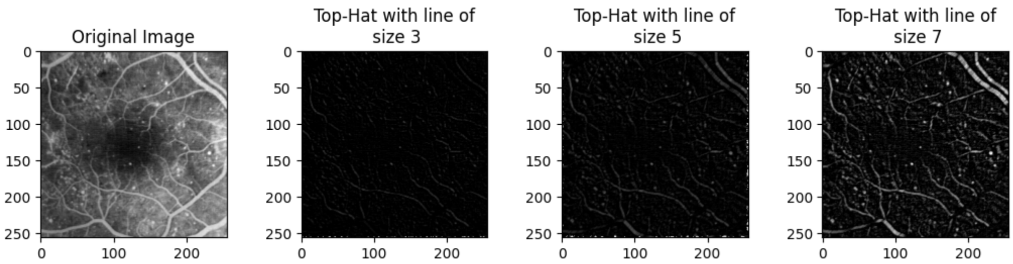


Figure 1.11: Top-hat transform on image ”retina2.gif” with a line of different sizes

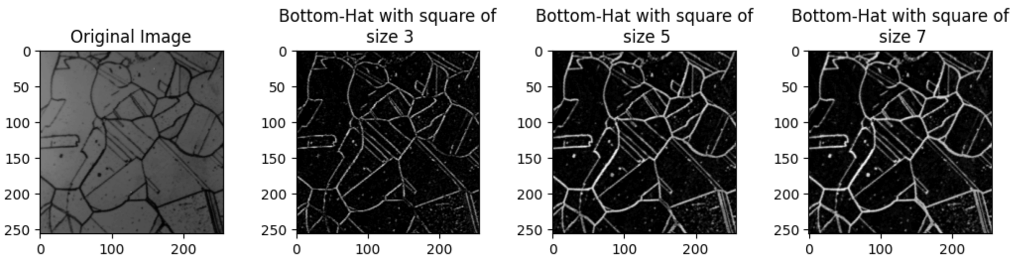


Figure 1.12: Bottom-hat transform on image ”laiton.bmp” with a with a square of different sizes

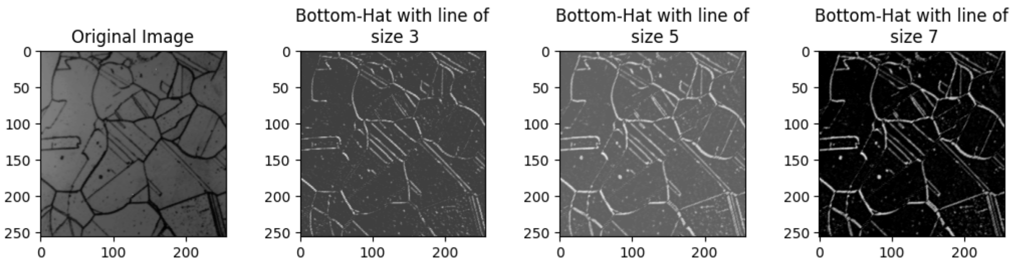


Figure 1.13: Bottom-hat transform on image ”laiton.bmp” with a with a line of different sizes

1.14)

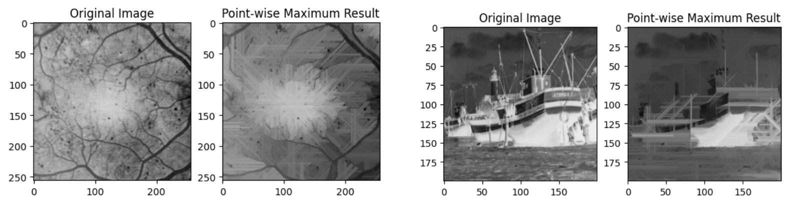


Figure 1.14: Point-wise maximum of openings applied with segments of different orientations on image ”retina2.gif”(left) and on image ”bat200.bmp”(right)

# Chapter 2

# Alternate sequential filters

The implemented alternate sequential filter is a sequence of morphological operations that combines opening and closing operations with varying structuring elements of different sizes and shapes. By repeatedly applying this sequence with progressively larger structural elements, one can observe how larger-scale structures become more prominent, and the selection of the structural element's shape dictates which features are accentuated.

For instance, a square structural element may accentuate horizontal and vertical structures, while a circular (disk-shaped) structural element may highlight circular features. (see Figure 2.1)

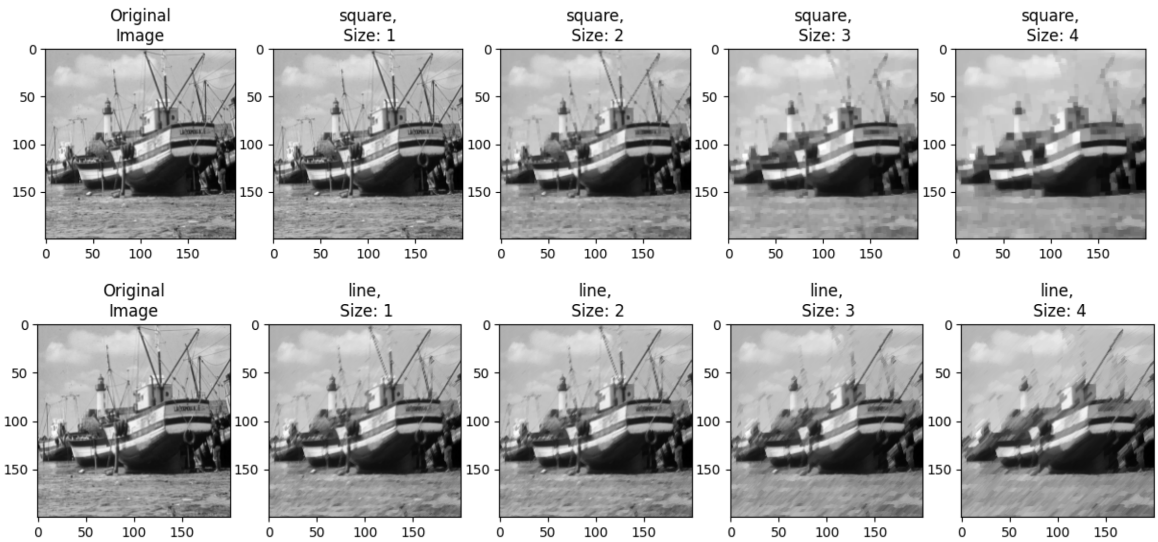


Figure 2.1: Application of the alternate sequential filter on image "bat200.bmp"

# Chapter 3

# Reconstruction

## 3.1 Reconstruction of an image

The reconstruction from an opening has been applied in order to recover small details of objects that are partially preserved by the opening. Indeed, the original image of Figure 3.1 contains various structures, including vessels and details thus the purpose of reconstruction is to retain the missing details due to the opening and enhancing them by comparing the opening result with the original image so it removes dark features smaller than the structuring element, without altering the shape and reconstructs connected components from the preserved features.

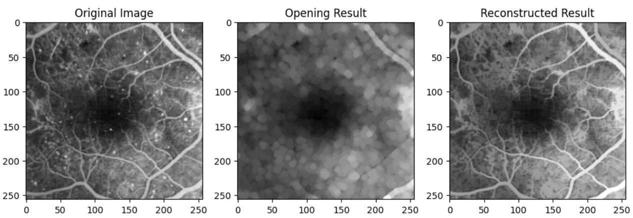


Figure 3.1: Reconstruction from opening of image "retina2.gif"

## 3.2 Sequence of operators for a reconstruction by erosion

Another type of reconstruction is the reconstruction by erosion and it can be obtained using the duality principle from reconstruction by dilation. The sequence of operations that makes a reconstruction by erosion consists in:

$f_0 = fVg$
$f_1 = E(f_0, B_1)Vf$
$f_2 = E(f_1, B_1)Vf$
...
$f_n = f_{n-1}$
where $f$ is original image
$g$ is the seed
$f_i$ is the reconstructed image at iteration i
$B_1$ is an elementary structuring element of size

Since reconstruction by erosion and reconstruction by dilation are dual operations in mathematical morphology and the duality principle states that we can obtain one from the other by taking the complement of the input image and the complement of the marker image, here follows how to obtain the reconstruction by erosion from the reconstruction by dilation: invert the image and the marker, apply reconstruction by dilation to the inverted input image and the inverted marker image, apply the reconstruction by dilation using the inverted images and in the end, apply the duality principle to convert the reconstruction by dilation into a reconstruction by erosion $E(-D(-f, B), g)$.

## 3.3 Alternate sequential filter with a reconstruction step

A reconstruction step after the opening and the closing has been added to the alternate sequential filter with the advantage of preserving important features, noise reduction, mantaining object connectivity.
(see Figure 3.2)

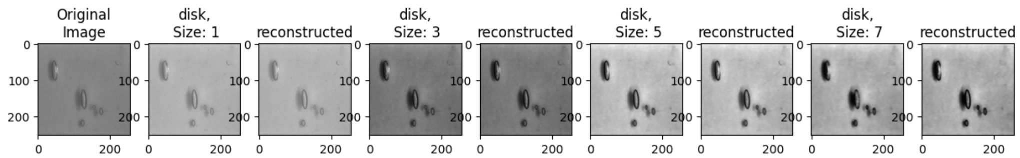


Figure 3.2: Alternate sequential filter with a reconstruction step on "bulles.bmp" image

# Chapter 4

# Segmentation

## 4.1 Computation of the morphological gradient

The morphological gradient is a useful operation in morphological image processing that makes the watershed consider the image local characteristics, in particular it highlights the edges or boundaries of objects within an image. It is computed as the difference between a dilation and an erosion of the image using an elementary structuring element. The result of this process is that dark regions emerge where the original image had sharp transitions or boundaries, emphasizing the object edges. (see Figure 4.1)

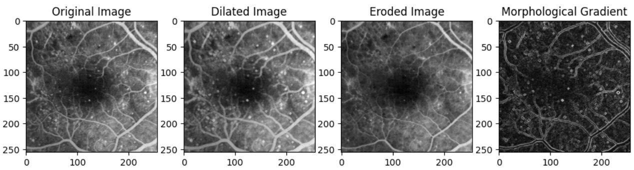


Figure 4.1: Morphological gradient of "retina2.gif" image

## 4.2 Watershed of the gradient

The watershed algorithm has been applied to the gradient image to segment the objects. The watershed lines are superimposed on the original image, with regions separated by these lines. Each region represents a different segmented object.
Note that the segmentation primarily relies on the gradient information derived from the morphological gradient image. Therefore, wherever the gradients are notably pronounced, the watershed lines are positioned, leading to the partitioning of objects. Moreover, as we can see in Figure 4.2, segmentation can sometimes lead to over-segmentation, which means it may detect a greater number of regions or boundaries than intended. You may notice the presence of numerous small regions in the result.

## 4.3 Watershed with filtering

To improve the result of the watershed segmentation, an appropriate morphological filtering has been applied to the original image and to the gradient image before computing the watersheds. The application of a closing operation to the image is an appropriate choice since, by smoothing the image, it eliminates small inconsistencies in the image and provides more consistent gradient information leading to an overall better and cleaner result than the one from a raw-gradient based approach. This result is visible in Figure 4.3.

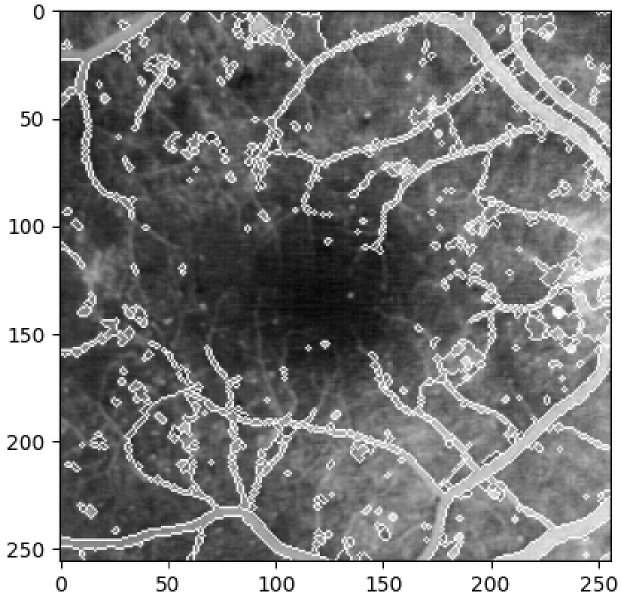

Figure 4.2: Watershed of the gradient of "retina2.gif" image

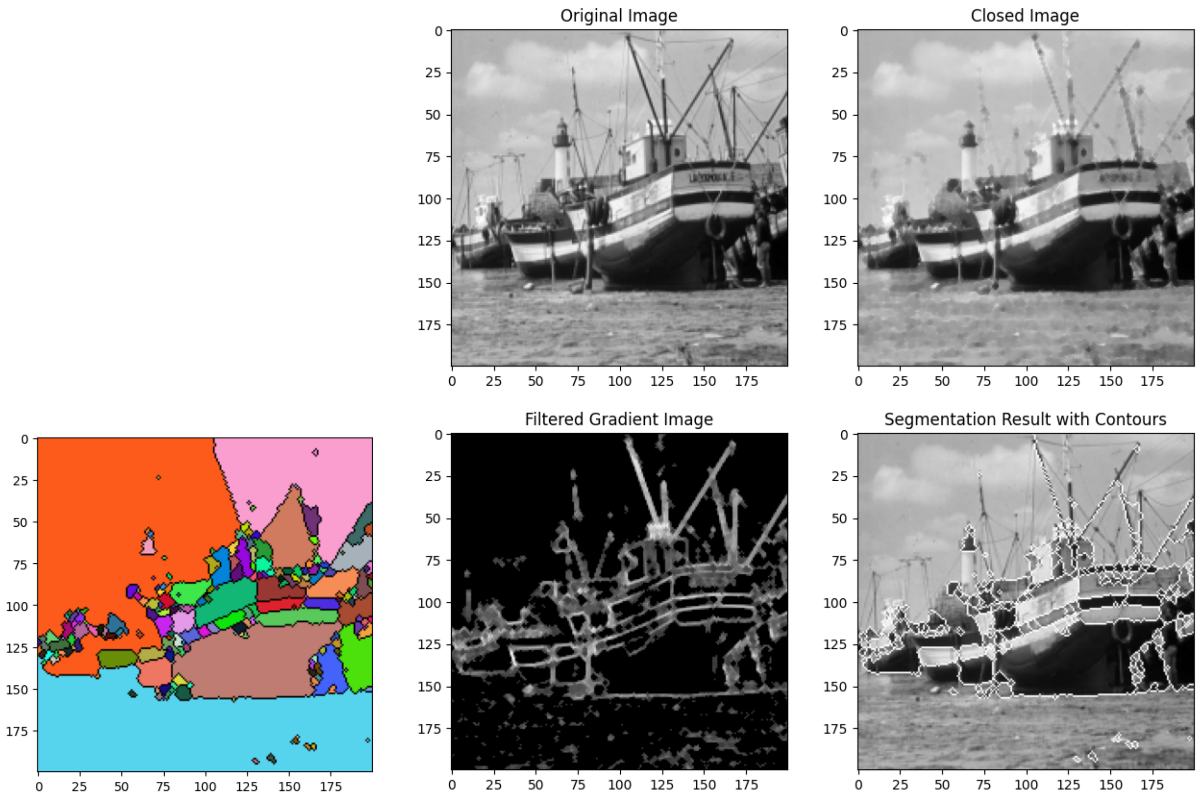


Figure 4.3: Watershed with filtering of "bat200.bmp" image

## 4.4 Eliminate regional minima

This process should help in suppressing small or insignificant regions that might be incorrectly segmented as separate objects. The sequence of operations consists in the computation of the morphological gradient, the retaining of pixels with values greater than a specified threshold to enhance gradient information in regions with strong gradients, a reconstruction step applied to the thresholded gradient with the added constant to eliminate regional minima that are below the constant level and in the end, the watershed transform. This approach helps focusing on the meaningful objects in the image and the choice of the threshold and dynamic constant influence the segmentation result. (see Figure 4.4)

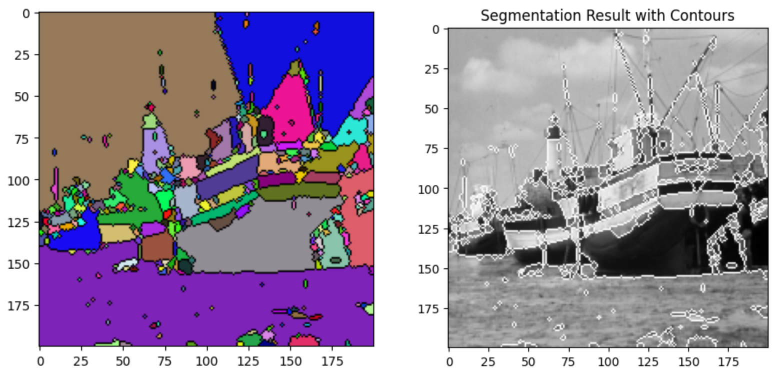


Figure 4.4: Watershed with deletion of regional minima applied to "bat200.bmp" image

## 4.5 Watershed with manually defined markers

The following approach has been applied on image "laiton.bmp": Markers have been manually defined using inside and outside markers which are represented by a binary image where the inside markers are set to 0 and the outside markers are set to 255; the image $I'$ has been obtained by taking the element-wise minimum between the thresholded gradient image and the marker image; a reconstruction operation has been performed on I' from the markers and finally the watershed trasform has been applied on the reconstructed image.
The manually defined markers guide the watershed segmentation by specifying the regions of interest through the inside markers and the boundaries through the outside ones.

## 4.6 Specific application: segmentation of the black lines in image "bulles.bmp"

For this aim, we made sure the watershed transform was preceded by a bottom_hat operation in order to darken the black zones in the image to enhance the gradient and lead to a more precise segmentation. (see Figure 4.5)

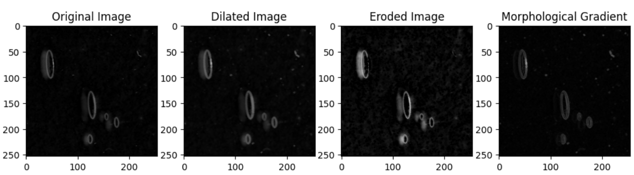


Figure 4.5: Morphological gradient of the bottom-hat transform of "bulles.bmp" image