TITLE OF THE THESIS

by

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I would like to thnk my supervisor.....

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1 Introduction

General introduction.

2 Option pricing

2.1 The fundamental theorem of asset pricing

2.2 The Black-Scholes model

Consider a given probability space $(\Omega, (\mathcal{F})_t, \mathbb{P})$ supporting a Brownian motion $(W_t)_{t\geq 0}$. In the Black-Scholes model, the stock price process $(S_t)_{t\geq 0}$ is the unique strong solution to the following stochastic differential equation:

$$\frac{\mathrm{d}S_t}{S_t} = r\mathrm{d}t + \sigma \mathrm{d}W_t, \qquad S_0 > 0, \tag{2.1}$$

where $r \geq 0$ denotes the instantaneous risk-free interest rate and $\sigma > 0$ the instantaneous volatility.

2.2.1 No interest rates

2.2.2 Including interest rates

A European call price $C_t(S_0, K, \sigma)$ with maturity $t > 0$ and strike $K > 0$ pays at maturity $(S_t - K)_+ = \max(S_t - K, 0)$. When the stock price follows the Black-Scholes SDE (2.1), Black and Scholes [1] proved that its price at inception is worth

$$C_t(S_0, K, \sigma) = S_0 \mathcal{N}(d_+) - K\mathrm{e}^{-rt}\mathcal{N}(d_-),$$

where

$$d_\pm := \frac{\log\left(S_0 \mathrm{e}^{rt}/K\right)}{\sigma\sqrt{t}} \pm \frac{\sigma\sqrt{t}}{2},$$

and where $\mathcal{N}$ denotes the cumulative distribution function of the Gaussian random variable.

Here is an example of how to insert a picture:

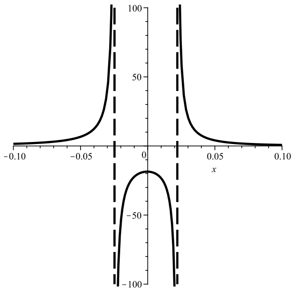

Figure 1: This is the caption for the figure.

or two side-by-side pictures:

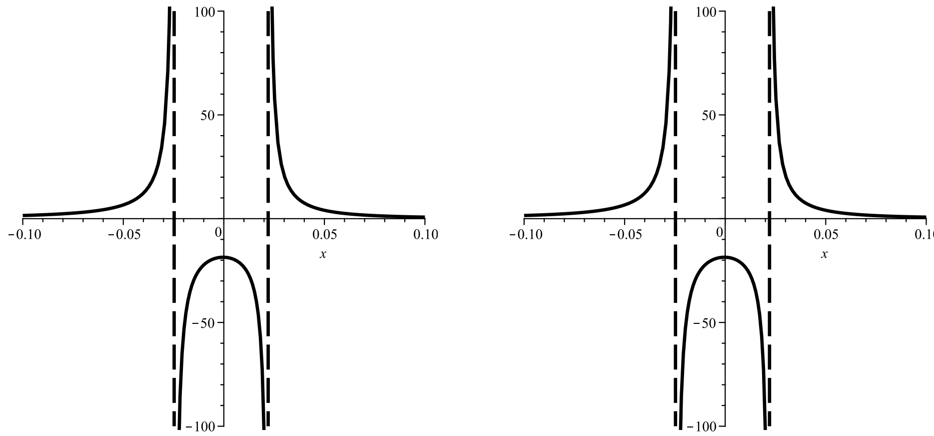

Figure 2: Blablabla

3 Model calibration

3.1 What is calibration?

Here is an example of a matrix in $A \in \mathcal{M}_n(\mathbb{R})$:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ a_{n1} & \dots & \dots & a_{1n}. \end{pmatrix}$$

3.2 Numerical methods for calibration

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A Review of stochastic calculus

A.1 Riemann integration

A.2 The Itô integral

B Some technical proofs

Conclusion

Conclusion if needed...

References

[1] F. Black and M. Scholes. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81 (3): 637-659, 1973.

[2] I. Karatzas and S.E. Shreve. Brownian Motion and Stochastic Calculus. Springer-Verlag, 1997.

[3] S. Karlin and H. Taylor. A Second Course in Stochastic Processes. Academic Press, 1981.

[4] P. Tankov. Pricing and hedging in exponential Lévy models: review of recent results. *Paris-Princeton Lecture Notes in Mathematical Finance*, Springer, 2010.

[5] D. Williams. Probability With Martingales. CUP, 1991.