LC Soliton Simulator — Solver Reference

API Reference

advance_theta_timestep() — One IMEX (implicit–explicit) time step for the transient LC director PDE.
theta_newton_step() — One Newton update toward the steady-state director solution.

Both solvers support coherent and incoherent beam combinations via the coh flag.

Acronyms and Solver Terms

PCG — Preconditioned Conjugate Gradient: iterative linear solver for large sparse systems.
SPD — Symmetric Positive Definite: ensures PCG convergence.
LM shift — Levenberg–Marquardt shift: adds τI to stabilize the Jacobian.
IMEX — Implicit–Explicit scheme: implicit linear, explicit nonlinear update.
DST — Discrete Sine Transform: applies Dirichlet boundaries in spectral space.
UFFT/UIFFT — Unitary FFT / inverse FFT for energy-preserving transforms.
Jacobian — Derivative of the residual $F(\theta) = (-\Delta\theta) - K\cdot\sin(2\theta)$.
Line search — Step control ensuring residual reduction.
Preconditioner — Approximate inverse improving convergence speed.
LM damping — Adaptive variant of LM shift for step-size control.
Mobility — Relaxation rate $M = K/\gamma_1\cdot 4/d^2$ setting the physical time scale.

Conceptual Summary

The transient solver integrates the overdamped LC equation:
$\partial\theta/\partial t = \nabla^2\theta + (b + b_i\, I(x,y)) \sin(2\theta)$
using an IMEX scheme for stability.

The steady-state solver iterates toward:
$\nabla^2\theta + (b + b_i\, I(x,y)) \sin(2\theta) = 0$
via Newton–PCG steps, applying an LM shift to keep the operator SPD.

Both solvers are GPU-ready (CuPy), coherence-aware (via intens(amp, coh)), and consistent with the package spectral framework.