LC PDE Derivation

Overview

This document summarizes the derivation of the nematic liquid-crystal (LC) director equation used in the LC Soliton Simulator. The formulation accounts for elastic, dielectric, and optical torques.

1. Free-Energy Density

The elastic energy is represented by the one-constant Frank approximation: $F_{el} = (K/2) |\nabla\theta|^2$. The dielectric and optical energy contributions add torque terms proportional to $\sin(2\theta)$.

2. Governing PDE

From torque balance, the director tilt $\theta(x, y, t)$ satisfies: $(\gamma\blacksquare / K)\ \partial\theta/\partial t = \nabla^2\theta + (\varepsilon\blacksquare\ \Delta\varepsilon_{RF} E^2 / 2K) \sin(2\theta) + (\varepsilon\blacksquare\ n_a^2 |E_{op}|^2 / 4K) \sin(2\theta)$.

3. Dimensionless Form

By defining $b = \varepsilon\blacksquare\ \Delta\varepsilon_{RF} V^2 / (8K)$ and $b_i = \varepsilon\blacksquare\ n_a^2 d^2 \blacksquare|E_{op}|^2\blacksquare / (16K)$, and scaling x, y by d/2, the PDE becomes: $\partial\theta/\partial t' = \nabla^2\theta + b \sin(2\theta) + b_i I(x, y) \sin(2\theta)$.

4. Mobility

The mobility parameter sets the time scale: mobility = $(K/\gamma\blacksquare) * (4 / d^2)$. The physical time step dt corresponds to seconds.