Frontiers in Networked Control	Lecture 4
Royal Institute of Technology (KTH)	26 April 2006

Decentralized Overlapping Control

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Problem

Find a formation keeping control for a system of N aerial vehicles. The problem is complicated to solve because each vehicle is affected by the dynamics/motion of its neighbors.

Solution strategy

Instead of treating the formation as one large system with information constraints and constraints on the internal dynamics, the problem is broken down and considered as an interconnected system with overlapping subsystems, each subsystem representing one vehicle. *Overlapping subsystems* here means that the subsystems share common components. Using the *inclusion principle*, the subsystems can be expanded into a higher dimensional space where the systems appear as disjoint. In this expanded system, decentralized control laws can be designed. The *inclusion principle* can then be used to contract the solution back to the original state space of the formation.

Example

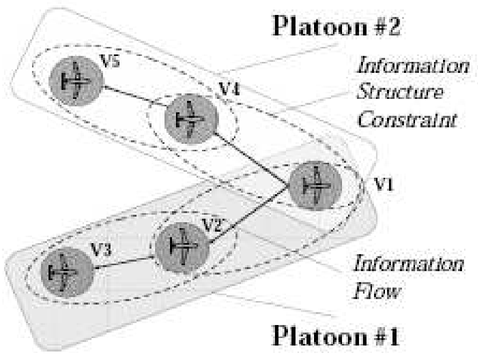

Figure 1: Formation of unmanned aerial vehicles.

Goal: *Keep the distance between planes constant.*

Mathematical model

Kinematic model of the planes:

$$\begin{aligned} \dot{x} &= v\cos\psi \\ \dot{y} &= v\sin\psi \\ \dot{\psi} &= \omega \end{aligned} \tag{1}$$

Define the vectors $z \in R^4$, $z^I \in R^2$ and $z^{II} \in R^2$ as

$$z = \begin{bmatrix} z^I \\ z^{II} \end{bmatrix} = \begin{bmatrix} x \\ y \\ v\cos\psi \\ v\sin\psi \end{bmatrix} \tag{2}$$

Expressed in the variable z_i, the **linearization of the nonlinear subsystem,** i, is

$$\dot{z}_i = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & 0_2 \end{bmatrix} z_i + \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} u_i \tag{3}$$

Change of variables

Assume that $v_{d_i} \in R^2$ is the desired speed for aircraft i and $d_{i-1} \in R^2$ is the constant desired distance between the $(i-1)$:st and the i:th aircraft. Introduce a **change of variables**:

- For $i = 1$ (the leader of the formation):

$$x_1^{II} = z_1^{II} - v_{d_1} \qquad \textit{(speed error)} \tag{4}$$

- For $i = 2, 3, ..., N$:

$$\begin{aligned} x_i^I &= z_{i-1}^I - z_i^I - d_{i-1} && \textit{(distance error)} \\ x_i^{II} &= z_i^{II} - v_{d_i} && \textit{(speed error)} \end{aligned} \tag{5}$$

Goal: *Drive the distance and speed errors, i.e.* $x_i^I \in R^2$ *and* $x_i^{II} \in R^2$, *to zero for all aircrafts.*

Assume that the desired speed is equal for all aircrafts in the formation, $v_{d_i} = v_d$. Then the linearized system corresponding to (3) becomes:

- For $i = 1$ (the leader of the formation):

$$\dot{x}_1^{II} = u_1 \tag{6}$$

- For $i = 2, 3, ..., N$:

$$\begin{aligned} \dot{x}_i^I &= x_{i-1}^{II} - x_i^{II} \\ \dot{x}_i^{II} &= u_i \end{aligned} \tag{7}$$

Expansion of the system

To demonstrate the *inclusion principle* we assume $N = 3$ in our case. The complete system can be written as $\dot{x} = Ax + Bu$ with

$$\begin{bmatrix} \dot{x}_1^{II} \\ \dot{x}_2^I \\ \dot{x}_2^{II} \\ \dot{x}_3^I \\ \dot{x}_3^{II} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1^{II} \\ x_2^I \\ x_2^{II} \\ x_3^I \\ x_3^{II} \end{bmatrix} + \underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{8}$$

Note: The variables of the subsystems are connected. Therefore, in order to construct a stabilizing control for the system, one must take the whole system into consideration simultaneously.

Expand the system by introducing variables $\tilde{x}_i$:

- For $i = 1$ (the leader of the formation):

$$\begin{aligned} \tilde{x}_1 &= x_1^{II} \\ \tilde{u}_1 &= u_1 \end{aligned} \tag{9}$$

- For $i = 2, 3, ..., N$:

$$\tilde{x}_i = \begin{bmatrix} x_{i-1}^{II} \\ x_i^I \\ x_i^{II} \end{bmatrix} \qquad \tilde{u}_i = \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix} \tag{10}$$

In the higher dimensional space, the system equations corresponding to (8) are:

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\tilde{A}_D} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \underbrace{\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}}_{\tilde{B}_D} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix} \tag{11}$$

Note: The three subsystems are now decoupled. It is possible to find stabilizing controls for each of the subsystems individually. These controls can be put together to form a control K that stabilizes the expanded system (11).

Contraction of the control

In order to be able to contract the control, K, back to the original state space, the expanded system must *include* the original system. Assume $\tilde{m} \geq m$, $\tilde{n} \geq n$ and that we have expansion/contraction matrices $V \in R^{\tilde{n}\times n}$, $U \in R^{n\times\tilde{n}}$, $R \in R^{\tilde{m}\times m}$, $Q \in R^{m\times\tilde{m}}$ such that

$$\begin{aligned} UV &= I \in R^{n\times n} \\ QR &= I \in R^{m\times m}. \end{aligned}$$

Definition (Inclusion principle): System $\tilde{S}$ *includes* system S if for any initial state x_0 and any input $u(t)$, we have $x(t; x_0, u) = U\tilde{x}(t, Vx_0, Ru)$.

For systems S and $\tilde{S}$ with static feedback control laws

$$\begin{aligned} u &= Kx, \quad K \in R^{m\times n} \\ \tilde{u} &= \tilde{K}\tilde{x}, \quad \tilde{K} \in R^{\tilde{m}\times\tilde{n}}, \end{aligned} \tag{12}$$

the condition for inclusion is satisfied if

$$\tilde{A}V = VA, \quad \tilde{B}R = VB, \quad and \quad \tilde{K}V = RK. \tag{13}$$

With our matrices $A, \tilde{A}, B$ and $\tilde{B}$ we can find V, U, R and Q (see paper) such that the conditions (13) are fulfilled if the control, K, satisfies

$$\tilde{K}V = RK. \tag{14}$$

In this case, a control, K, that stabilizes the three decoupled systems can be found. K can then be modified so that (14) can be solved while the stability of the system (11) is preserved:

$$\tilde{K}_{DM} = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_4 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & 0 & \tilde{K}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_5 & \tilde{K}_6 & \tilde{K}_4 \end{bmatrix} \tag{15}$$

Contraction of $\tilde{K}_{DM}$, using (14) gives a control for the system (8) in the original state space:

$$K_m = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 \\ \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_4 & 0 & 0 \\ 0 & 0 & \tilde{K}_5 & \tilde{K}_6 & \tilde{K}_4 \end{bmatrix} \tag{16}$$

References

[1] Stipanovic, D.M.; Inalhan, G.; Teo, R.; Tomlin, C.J, Decentralized overlapping control of a formation of unmanned aerial vehicles, in IEEE Conference on Decision and Control, 2002.