Untitled

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Mixed Integer Linear Programming Models

- We have four ways of

One model for each α-quantile

One model for each α-quantile - Resume

In this part, we investigate the usage of MILP to select which variables are included in the model, by using a constraint which limits them to a number of K. This means that only K coefficients $\beta_{p\alpha}$ may have nonzero values, for each α-quantile. This assumption is modeled with binary variables $z_{p\alpha}$, which indicates whether $\beta_{p\alpha}$ is included or not.

One model for each α-quantile - Formulation

$$\min_{\beta_{0\alpha}, \beta_{\alpha}, z_{p\alpha} \varepsilon^+_{t\alpha}, \varepsilon^-_{t\alpha}} \quad \sum_{\alpha \in A} \sum_{t \in T} \left(\alpha \varepsilon^+_{t\alpha} + (1-\alpha) \varepsilon^-_{t\alpha} \right) \tag{1}$$

$$\text{s.t} \quad \varepsilon^+_{t\alpha} - \varepsilon^-_{t\alpha} = y_t - \beta_{0\alpha} - \sum_{p=1}^{P} \beta_{p\alpha} x_{t,p}, \qquad \forall t \in T, \forall \alpha \in A, \tag{2}$$

$$\varepsilon^+_{t\alpha}, \varepsilon^-_{t\alpha} \geq 0, \qquad \forall t \in T, \forall \alpha \in A, \tag{3}$$

$$-M z_{p\alpha} \leq \beta_{p\alpha} \leq M z_{p\alpha}, \qquad \forall \alpha \in A, \forall p \in P, \tag{4}$$

$$\sum_{p=1}^{P} z_{p\alpha} \leq K, \qquad \forall \alpha \in A, \tag{5}$$

$$z_{p\alpha} \in \{0, 1\}, \qquad \forall \alpha \in A, \forall p \in P, \tag{6}$$

$$\beta_{0\alpha} + \beta_{\alpha}^T x_t \leq \beta_{0\alpha'} + \beta_{\alpha'}^T x_t, \qquad \forall t \in T, \forall (\alpha, \alpha') \in A \times A, \alpha < \alpha', \tag{7}$$

Defining groups for α-quantiles - Resume

- Now, adding groups of quantiles.
- Each probability α belongs to a group g.
- The total number of groups is limited to G.
- Incorporation of new integer variables.
- Total number of valid solution falls.

Defining groups for α-quantiles - Formulation

$$\min_{\beta_{0\alpha},\beta_{\alpha},z_{p\alpha}\varepsilon^{+}_{t\alpha},\varepsilon^{-}_{t\alpha}} \quad \sum\nolimits_{\alpha\in A}\sum\nolimits_{t\in T}\left(\alpha\varepsilon^{+}_{t\alpha}+(1-\alpha)\varepsilon^{-}_{t\alpha}\right) \tag{8}$$

$$\text{s.t} \quad \varepsilon^{+}_{t\alpha}-\varepsilon^{-}_{t\alpha}=y_t-\beta_{0\alpha}-\sum\nolimits_{p=1}^{P}\beta_{p\alpha}x_{t,p}, \qquad \forall t\in T,\forall\alpha\in A, \tag{9}$$

$$\varepsilon^{+}_{t\alpha},\varepsilon^{-}_{t\alpha}\geq 0, \qquad \forall t\in T,\forall\alpha\in A, \tag{10}$$

$$-Mz_{p\alpha g}\leq\beta_{p\alpha}\leq Mz_{p\alpha g}, \qquad \forall\alpha\in A,\forall p\in P,\forall g\in G \tag{11}$$

$$z_{p\alpha g}:=2-(1-z_{pg})-I_{g\alpha} \tag{12}$$

$$\sum\nolimits_{p=1}^{P}z_{pg}\leq K, \qquad \forall g\in G, \tag{13}$$

$$\beta_{0\alpha}+\beta^{T}_{\alpha}x_t\leq\beta_{0\alpha'}+\beta^{T}_{\alpha'}x_t, \qquad \forall t\in T,\forall(\alpha,\alpha')\in A\times A,\alpha<\alpha', \tag{14}$$

$$\sum_{g\in G}I_{g\alpha}=1, \qquad \forall\alpha\in A, \tag{15}$$

$$I_{g\alpha},z_{pg}\in\{0,1\}, \qquad \forall p\in P,\quad\forall g\in G, \tag{16}$$

Defining groups by the introduction of switching variable - Formulation

$$\min_{\beta_{0\alpha}, \beta_\alpha, z_{p\alpha} \varepsilon^+_{t\alpha}, \varepsilon^-_{t\alpha}} \quad \sum\nolimits_{\alpha \in A} \sum\nolimits_{t \in T} \left(\alpha \varepsilon^+_{t\alpha} + (1-\alpha) \varepsilon^-_{t\alpha} \right) \tag{17}$$

$$\text{s.t} \quad \varepsilon^+_{t\alpha} - \varepsilon^-_{t\alpha} = y_t - \beta_{0\alpha} - \sum\nolimits_{p=1}^{P} \beta_{p\alpha} x_{t,p}, \qquad \forall t \in T, \forall \alpha \in A, \tag{18}$$

$$\varepsilon^+_{t\alpha}, \varepsilon^-_{t\alpha} \geq 0, \qquad \forall t \in T, \forall \alpha \in A, \tag{19}$$

$$-M z_{p\alpha} \leq \beta_{p\alpha} \leq M z_{p\alpha}, \qquad \forall \alpha \in A, \forall p \in \{1, \ldots, P\}, \tag{20}$$

$$\sum\nolimits_{p=1}^{P} z_{p\alpha} \leq K, \qquad \forall \alpha \in A, \tag{21}$$

$$z_{p\alpha} \in \{0, 1\}, \qquad \forall \alpha \in A, \forall p \in \{1, \ldots, P\}, \tag{22}$$

$$\beta_{0\alpha} + \beta_\alpha^T x_t \leq \beta_{0\alpha'} + \beta_{\alpha'}^T x_t, \qquad \forall t \in T, \forall (\alpha, \alpha') \in A \times A, \alpha < \alpha', \tag{23}$$

$$z_{p\alpha} - z_{p\alpha+1} \leq m_{p\alpha}, \qquad \forall \alpha \in A', \qquad \forall p \in P \tag{24}$$

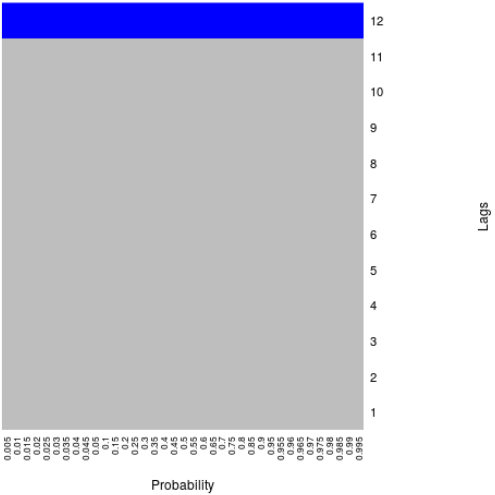
$$\sum\nolimits_{\alpha \in A'} r_\alpha \leq |G| - 1 \tag{25}$$

$$\tag{26}$$

where $A' = A \setminus \{|A|\}$

Results

- As there are much less possibilities when $K = 1$, every method gets pretty fast to the optimum result. Selecting the 12^{TH} lag was the best choice.



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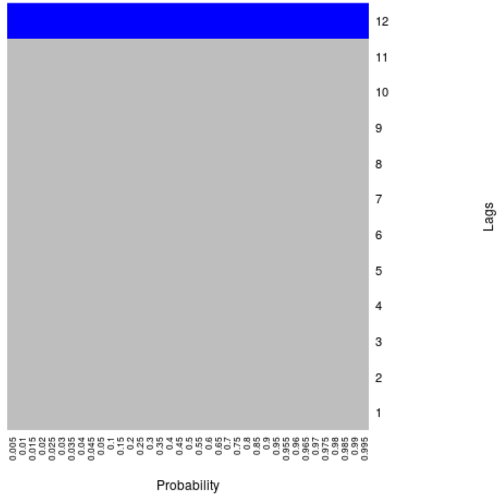

Results

- When

Results

- We start to notice, from $K = 2$, that by letting