

# A non-parametric quantile autoregressive model for wind power Firm Energy Certificate

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**Abstract**—In this article we use the framework of a non-parametric quantile regression model to generate forecasts of wind capacity factors of several quantiles. Such scenarios are then used as input to raise the distribution of the quantiles associated with each wind plant. In our proposed model we introduce a  $\ell_1$  penalty term in the objective function, in order to properly address an adaptive dependency structure for the  $\alpha$ -quantile time series. Hence, in contrast to the well-known quantile regression model, our proposed framework is not limited to linear functions. Computation experiments show a slight improvement when comparing our non-parametric framework to the benchmark.

**Index Terms**—Quantile regression, quantile autoregressive models, firm energy certificate, wind power,  $\ell_1$ -penalty term, time varying quantiles.

## I. INTRODUCTION

**W**ind Firm Energy Certificate (FEC) estimation impose several challenges. First and foremost, it is a quantile function of an aleatory quantity, named here on wind capacity factor (WP). Due to its non-dispatchable profile, accurate scenario generation model could reproduce a fairly dependence structure in order to the estimation of FEC. Secondly, as a quantile function, the more close to the extremes, more sensitive to sampling error.

In this work, we introduce a new non-parametric quantile autoregressive model with  $\ell_1$ -penalty term, in order to properly simulate FEC densities for several  $\alpha$ -quantiles.

### A. Review of the Brazilian Electricity Sector

In response to the growing demand of energy, Brazil began a period of major structural reforms in the 1990s [1]. Such reforms were positively accepted by private investors, which leads to numerous concession auctions for new projects [2]. Such environment had a mood turn over during 2001-2002, since the former security criteria were fully based on market mechanism and lead to a serious supply crisis. The outcomes of such crisis were the reduction of total load by 20% and an economic loss of tens of billions US dollars [3].

Therefore, as a response to aforementioned crisis, Brazilian government developed a new power sector model in 2004 [2]. The implementation of the reformed framework for Brazil's

electricity sector generally aims to provide the long-term system's expansion incentive, reduce uncertainty in generation companies future revenues, and mitigate unfavourable effects of the short-term market [4].

In addition, such system was applied to the Brazilian case based on mandatory reability contracts as an incentive to investors. These contracts are considered to be financial instruments, in the same spirit as forward contracts. Moreover, in order to provide a confidence sensibility over generation, all contracts ought to be covered by so-called "firm energy certificates" (FEC). These FECs are defined in GWh/year and represent the plant's physical energy production capacity. In a nutshell, the FEC of a certain plant is the maximum amount of energy that can be sold through contracts and establishes the reliability assured by the generator backing the contract. It is thus a critical parameter for the power plant's economic feasibility.

### B. Motivation and objectives

Accurate estimation of FEC values provide the investors safety regarding his cash-flows, since the main issue about introducing renewable energy into energy matrixes is its intermittent<sup>1</sup> profile. Regarding wind power, its production is fully conditioned to the wind speed, which is caused by difference in atmospheric pressure. Hence, prior knowledge over its future behaviour is essential when considering the generation of this renewable source.

Regarding wind plants, its FEC where fully detailed on a document produced by ANEEL in July of 2008 [XXX], which defines the firm energy certificate of a wind plant as the mean of the monthly production. As described in the mentioned document, the estimation of wind plants FEC is given by

$$FEC^{(\alpha)} = \sum_{m=1}^{12} \frac{E_m^{(\alpha)} \times h_m}{8760}, \quad (1)$$

where  $FEC^{(\alpha)}$  denotes the FEC of the wind plant certified with  $\alpha\%$  of confidence,  $h_m$  is the number of hours in the month  $m$  and  $E_m^{(\alpha)}$  denotes the mean of the monthly quantiles certified at  $\alpha\%$ . In practice, the generator provides 12 values of energy certified at  $\alpha\%$ . referring to the month  $m$ .

To give the reader a further understanding about Equation (1), we present in Figure 1 the calculated FEC's for a wind-plant located at the Brazilian northeast, named Icaraizinho. The estimatives  $E_m^{(\alpha)}$  for several  $\alpha \in \{50, 55, \dots, 95\}$  are plotted.

<sup>1</sup>When there is no human intervention over its production.

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## II. PARAMETRIC MODEL

Here we denoted as parametric model the well-known quantile regression model [5]. In contrast to the linear regression model through ordinary least squares (OLS), which provides only an estimation of the dependent variable conditional mean, quantile regression model yields a much more detailed information concerning the complex relationship about the dependent variable and its covariates, as defined by Equation (2),

$$\begin{aligned} \mathcal{Q}_y(\alpha|x_1, x_2, \dots, x_n) = & \beta_0(\alpha) + \beta_1(\alpha)x_1 + \beta_2(\alpha)x_2 + \dots \\ & + \beta_n(\alpha)x_n + F_\varepsilon^{-1}(\alpha), \end{aligned} \quad (2)$$

where  $F_\varepsilon$  denotes the error density function.

### A. Parameter estimation

The parameters of the quantile regression are estimated by a linear programming optimization problem that can be formulated as

$$\min_{\beta \in \mathbb{R}^p \times \mathbb{R}_+^{2n}} \{ \tau \mathbf{1}_n^T u + (1 - \tau) \mathbf{1}_n^T v | X\beta + u - v = y \}. \quad (3)$$

Here,  $X$  stands for the usual covariates matrix, the residual vector  $y - X\beta$  splits itself into negative and positive parts ( $u$  and  $v$ ).

## III. NON-PARAMETRIC MODEL

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## IV. COMPUTATIONAL EXPERIMENTS

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## V. XX

## VI. XX

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figures/GF\_SIM.pdf

Fig. 1. Percentiles  $E_m^{(\alpha)}$  for each month  $m$  and confidence criteria  $\alpha$ .