GEFCom2014 Probabilistic Solar Power Forecasting based on k-Nearest Neighbor and Kernel Density Estimator

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***Abstract*—Probabilistic forecasting provides quantitative information of energy uncertainty, which is very essential for making better decisions in power system operation with increasing penetration of wind power and solar power. On the basis of k-nearest neighbor and kernel density estimator method, this paper presents a general framework of probabilistic forecasts for renewable energy generation. Firstly, the *k*-nearest neighbor algorithm is modified to find the days with similar weather conditions in historical dataset. Then, kernel density estimator method is applied to derive the probability density from *k* nearest neighbors. This approach is demonstrated by an application in probabilistic solar power forecasting. The effectiveness of our proposed approach is validated with the real data provided by Global Energy Forecasting Competition 2014.**

***Index Terms*—solar power, deterministic forecasting, probabilistic forecasting, *k*-nearest neighbor, kernel density estimator.**

I. Introduction

With the consumption of electric energy all over the world, renewable energy, especially wind power and solar power, draw more and more attraction in many countries. However, the randomness and intermittent of wind power and solar power is a big challenge for the optimal use of renewable energy. Accurate predictions of electrical load, electrical price, wind power and solar power, are effective tools to satisfy the requirement of power system planning and operation. Recently much research has been published regarding the implementation of energy forecasting [1]. In spite of several years of intensive research in this field, energy forecasting cannot yet be accurate enough, which are mainly boiled down to the low predictability of weather behavior. For example, the accurate temperature prediction is becoming more and more important for electrical load forecasting [2]. Several researchers have shown that any forecasting method had its inherent and irreducible uncertainty [3][4].

Currently-used energy forecasting only gives the expectation of energy output, which belongs to deterministic prediction, spot prediction or point prediction. Recently, a new kind of forecasting method, probabilistic forecasting, becomes an active topic. Introduction on probabilistic forecasting in detail can be found in [5] for electrical load, [6] for electrical price, [7][8] for wind power, and [9] for solar power. Compared with point prediction, probabilistic forecasting can produce quantitative information on the associated uncertainty of energy output. Both scenario and interval information, which are crucial inputs of two-stage stochastic optimization [10] and robust optimization [11] respectively, can be provided by probabilistic forecasting. From August to December 2014, IEEE working group on energy forecasting organizes the Global Energy Forecasting Competition (GEFCom 2014), and the focus of this competition falls on probabilistic forecasting of electric load, electricity price, wind power and solar power [12].

In this work, a general framework of probabilistic forecasting based on *k*-Nearest Neighbors algorithm (*k*-NN) and Kernel Density Estimator (KDE) method is implemented to forecast the renewable power generation. *k*-NN algorithm helps us find the *k* closest training examples in the feature space. Then, point forecasting of renewable power output is the weighted average of values of *k* nearest neighbors. As for probabilistic prediction, we apply KDE method to forecast probability density of renewable power generation, and then the required 99 quantiles are derived from probability density function. An advantage of this approach is that it can provide both point and probabilistic forecasts. It also has a strong adaptability for both wind power and solar power forecasts. Solar power forecasting is chosen as an illustration in this paper to validate the effectiveness of our proposed forecasting approach.

This paper is organized as follows. The next section introduces theoretical methodology of general framework for probabilistic forecasting including *k*-NN and KDE method. Section III gives data description and some works in data preparation. Section IV introduces evaluation framework of probabilistic forecasting. Results of our proposed approach in solar forecasts are illustrated in Section V. This paper ends with conclusions in Section VI, with remarks on future developments.

This work was supported in part by the National Science Foundation of China (51277141).

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II. A General Framework of Probabilistic Forecasting for Renewable Energy Generation

A. KNN Algorithm

k-NN algorithm is a non-parametric method used for classification and regression [13]. The main idea of k-NN algorithm is that whenever a new point is to predict, its k nearest neighbors are chosen from the training data. Then, the prediction of new point can be the average of the values of its k nearest neighbors. The k-NN algorithm resembles the similar day approach for power load forecasting. The similar day approach is widely used by many utilities, which derive the future power load using the historical days with similar temperature and similar day type (e.g. weekday, weekend and holiday) [1]. For renewable energy forecasts based k-NN algorithm, the prediction of future renewable power output is derived from the historical days with similar hour and similar weather condition (e.g. wind speed or solar radiation). The foundation of k-NN algorithm could be boiled down to three main steps [14].

- Calculating the pre-defined distance between testing example and training example.
- Choosing k nearest neighbors who owes k smallest distances from training examples.
- Predicting the final renewable power output based on weighted averaging technique.

A distance measure is required to characterize the similarity between two instances. Commonly used distance metrics are Euclidean Distance and Manhattan Distance. In this work, the original Manhattan distance is enhanced by using weight. The weighted Manhattan distance is calculated by

$$D\left[X^{(i)}, X^{(j)}\right] = \sum_{n=1}^{r} w_n \left| x_n^{(i)} - x_n^{(j)} \right| \quad (1)$$

where $X^{(i)}$ and $X^{(j)}$ are two instances, each instance X has r attributions $X = [x_1, \dots, x_n, \dots, x_r]$, and w_n is the weight assigned to the n-th attribution. In original Manhattan distance, the weight w_n equals to 1, which means the equal contribution of each attribution to the distance D. However, in renewable power generation forecasting, the importance of each attribution is quite different. The weight w_n reflects the contribution of each variable to the distance, and would be determined by an optimization process (see the following subsection).

Once k nearest neighbors (i.e. the instances who own the k smallest distances) is determined, a prediction is made on the basis of the target values associated with them. Let $X^1, \dots, X^K$ denote k nearest instances that are nearest to testing instance X, and their power outputs are represented by $p^1, \dots, p^K$. The distance between X and k nearest neighbor follows the ascending order $d^1 \le \dots \le d^K$, where $d^k = D[X, X^k]$ $(k = 1, \dots, K)$. Then, point prediction of renewable power generation is calculated by an average weighted by exponential function.

$$\hat{p} = \sum_{k=1}^{K} \delta^k p^k = \frac{\sum_{k=1}^{K} e^{-d^k} \cdot p^k}{\sum_{k=1}^{K} e^{-d^k}} \quad (2)$$

where the weight equals to $\delta^k = e^{-d^k} / \sum_{k=1}^{K} e^{-d^k}$, p^k and d^k are the renewable power output and the distance associated with the instance X^k, respectively.

B. Coordinate Descent Algorithm

The value of weight in Eq. (1) is determined by an optimization process. For each instance X^i, let p^i denote the measurement of renewable power generation, and its prediction $\hat{p}^i$ is obtained by Eq. (2). Thus, some figures of merit can be applied for the assessment of prediction performance, i.e. Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). The Sum of Square Error (SSE) is used in this paper, which is more convenient for differential.

$$\mathrm{SSE} = \sum_{i=1}^{m} (p^i - \hat{p}^i)^2 \quad (3)$$

The prediction $\hat{p}^i$ is a function of weights $w_1, w_2, \dots, w_r$. Thus, how to determine the weights $w_1, w_2, \dots, w_r$ is converted into the minimization of Eq. (3).

$$\underset{w_1, w_2, \dots, w_r}{\operatorname{argmin}} \ \mathrm{SSE} = \underset{w_1, w_2, \dots, w_r}{\operatorname{argmin}} \sum_{i=1}^{m} [p^i - \hat{p}^i(w_1, w_2, \dots, w_r)]^2 \quad (4)$$

Minimizing SSE with respect to the weights $w_1, w_2, \dots, w_r$, i.e. Eq. (4), is achieved by the coordinate descent algorithm [15]. This algorithm is shown in Table I. In the innermost loop of coordinate descent algorithm, all the weights are fixed except for a specified weight w_n. Then, the re-optimization of SSE with respect to just the specified weight w_n is an unconstrained optimization problem, which can be solved efficiently by Newton-type algorithm.

TABLE I. Coordinate Descent Algorithm

	Algorithm 1: Coordinate Descent
1	Loop until convergence
2	For $n = 1, 2 \dots, r$
3	$\widehat{w}_n = \operatorname{argmin}_{w_n} \mathrm{SSE}(w_1, \dots, w_{n-1}, w_n, w_{n+1}, \dots w_r)$
4	End For
5	End Loop

The rolling optimization of SSE from the first weight w_1 to the last one w_r is viewed as one iteration. Coordinate descent algorithm begins with a randomly-chosen weight vector $\bar{\boldsymbol{w}} = (\bar{w}_1, \bar{w}_2, \dots, \bar{w}_r)$, and finishes when the relative change of weight vector $\boldsymbol{w}$ is less than a given threshold ε.

C. Probabilistic Forecasting based on KDE method

The point forecasting is given by Eq. (2). As far as probabilistic renewable power forecasts, the weighted Kernel Density Estimator (KDE) [16] is used to derive the probability density of renewable power generation on the basis of the k nearest neighbors obtained by k-NN algorithm. Given the historical renewable power output p^k $(k = 1, \dots, K)$ associated with k nearest neighbors, renewable power density is forecasted by

$$\hat{f}(p) = \frac{1}{Kh} \sum_{k=1}^{K} \delta^k G\left(\frac{p - p^k}{h}\right) = \frac{1}{Kh} \sum_{k=1}^{K} \frac{e^{-d^k} G\left(\frac{p - p^k}{h}\right)}{\sum_{k=1}^{K} e^{-d^k}} \quad (5)$$

where $G(.)$ is a kernel function and usually is chosen to be the $N(0,1)$ density, $G(.) = \phi(.)$. Bandwidth h can be estimated by plug-in bandwidth selector [16].

Then, the predictive density of renewable power generation is converted into 99 quantiles. They are 1%, 2%, …, 99% quantiles which are called by $q_1, q_2, \dots, q_{99}$. Given the distribution function of renewable power generation, the quantile q_a with a specified percentage a can be derived.

$$q_a = \hat{F}^{-1}\left(\frac{a}{100}\right) \tag{6}$$

where $\hat{F}^{-1}(.)$ is the inversion of the Cumulative Distribution Function (CDF). Quantiles can be employed for constructing the interval prediction. For example, the predictive interval of 95 % confidence level is $[q_5, q_{95}]$.

III. Exploratory Data Analysis For Probabilistic Solar Power Forecasting

A general framework of probabilistic forecasts for renewable energy generation is introduced in previous section. This framework can be used for both wind power and solar power forecasting. In this paper, probabilistic solar power forecasting is chosen as an example for illustrating the effectiveness of our proposed approach. We used the dataset of three adjacent solar farms provided by GEFCom 2014. The target variable is solar power generation, normalized by the nominal capacity of each solar farm. Competition organizers provide two parts of explanatory variables. The first part is the past solar power measurements whose temporal resolution is 1 hour. The second part is 12 independent variables from the European Centre for Medium-Range Weather Forecasts (ECMWF) Numerical Weather Prediction (NWP) output, whose temporal resolution is also 1 hour. Some description information of 12 weather variables is shown in Table II.

TABLE II. Description Information of 12 Weather Variables

Variable	Description	Abbreviate	Unit
VAR78	Total Column Liquid Water	TCLW	kg/m^2
VAR79	Total Column Ice Water	TCIW	kg/m^2
VAR134	Surface Pressure	SP	Pa
VAR157	Relative Humidity at 1000 mbar	R	%
VAR164	Total Cloud Cover	TCC	0-1
VAR165	10 Meter U Wind Component	10U	m/s
VAR166	10 Meter V Wind Component	10V	m/s
VAR167	2 Meter Temperature	2T	K
VAR169	Surface Solar Radiation Down	SSRD	J/m^2
VAR175	Surface Thermal Radiation Down	STRD	J/m^2
VAR178	Top Net Solar Radiation	TSR	J/m^2
VAR228	Total Precipitation	TP	m

Fig. 1 shows the histogram of solar power output at farm #1. It can be found that solar power output follows a distribution very far from Gaussian distribution, with a large amount of value clustered around the zero point, which results from none of solar generation at nighttime. Solar farms #2 and #3 are similar with the histogram in Fig. 1. A correlation analysis of solar power output is made among three farms. A graphical display of correlation matrix, which is called 'correlogram', is designed in this paper. Fig. 2 shows the correlogram of solar power output at farm #1-#3. All solar farms' names are plotted in the diagonal of correlogram. In Fig. 2, we can observe that there is strong correlation among three solar farms.

The scales of weather variable in Table II vary widely. For example, the value of variable 'VAR169' representing solar radiation ranges from 0 at night to 43000 in daytime. However, the temperature variable 'VAR167' varies between 270 and 310 in Kelvin, and the precipitation variable 'VAR228' varies from 0 to 0.04 in meter. Different scales of weather variables have great influences on k-NN algorithm. In order to establish equal importance for every weather variable, all weather variables are mean-centered and scaled to unit variance based on the mean and the standard variance of training dataset. This pre-treatment is called 'zero-mean and unit-variance'.

$$\hat{x}_t = \frac{x_t - \mu}{\sigma} \tag{7}$$

where μ and σ are the mean and standard deviation of variable x_t in training dataset.

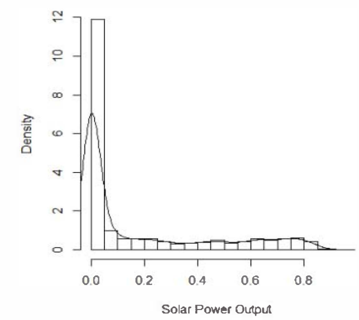

Figure 1. The histogram of solar power at solar farm #1. The black solid line is probability density of solar power output estimated by KDE method.

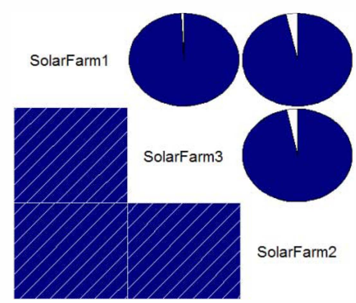

Figure 2. The correlogram of solar power output at farm #1-#3. Dark blue color and whitle solid line from upper right to lower left represents a positive correlation between two farm's solar output. The degree of this correlation is showed by the darkeness of blue color and the filling percentage in pie chart.

IV. Evaluation Framework

For point solar power forecasting, RMSE is a commonly-used metrics for the evaluation of prediction performance.

$$\mathrm{RMSE} = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(p^i - \hat{p}^i)^2} \tag{8}$$

where p^i and $\hat{p}^i$ are the observation and the prediction of solar power output, respectively.

All evaluation metrics used in point prediction evaluation are based on the discrepancy between the observation and the prediction. However, for probabilistic solar power forecasting, predictive information (e.g. probability density or a series of quantiles) can't be directly compared with the measurement. As a challenge, the evaluation of probabilistic forecasting has drawn much attention in recent years [17][18], including reliability and sharpness assessment. In this paper, we use evaluation metric of GEFCom 2014, and 99 quantile forecasts are to be evaluated by quantile score. For quantile forecast q_a with a specified percentage a, the quantile score Q is defined as

$$Q(q_a, p^i) = \begin{cases} \left(1 - \dfrac{a}{100}\right)(q_a - p^i) & \text{if } p^i < q_a \\ \dfrac{a}{100}(p^i - q_a) & \text{if } p^i \geq q_a \end{cases} \quad (9)$$

where p^i is the observation and $a = 1,2,\ldots,99$. To evaluate 99 quantile forecasts, the quantile score is then averaged over all quantiles from 1% to 99%. The lower the quantile score is, the better the probabilistic forecasts are.

V. CASE STUDY

A. Model Selection

Apart from the weight w_n in Eq. (1), the k value and the input variables also have great influence on prediction performance of the k-NN algorithm. Unfortunately, these two factors are quite difficult to be determined directly by an optimization. Cross validation is a commonly-used model selection method [19]. The k value, i.e. the number of nearest neighbors, is determined by a 7-fold cross validation. The possible k value is chosen from 100, 110 … 300. Quantile score Q is considered to evaluate the influence of k value on prediction performance. The experiment shows that the optimal k value is 200.

Another important case of model selection is to choose the proper input variables. The organizers supply 12 weather variables (see Table II). However, there are only a small part of weather variables which have strong relation with solar power generation. If too many irrelevant variables are included in k-NN algorithm, overfitting problem will be a potential problem. Given 12 features, there are 2^{12} possible feature subsets, and thus it is too expensive to evaluate and compare all 2^{12} possible feature subsets. In our work, a heuristic feature search procedure called forward search [19] is employed to find a proper feature subset. Table III shows the entire procedure of forward search algorithm. The first stage of forward search algorithm is to seek the best feature subset consisting of two variables, and the search result is HOUR and VAR169. In the second and third stage, VAR79 and VAR78 are added into the input subset, respectively. In the fourth stage, VAR157 is chosen as the candidate. Thus, the final feature subset is set as HOUR, VAR169, VAR79, VAR78 and VAR157.

TABLE III. FORWARD SEARCH PROCEDURE FOR THE BEST SUBSET OF INPUT VARIABLES AT SOLAR FARM #1

Stage	Feature Subset	Feature	Score Q
1	HOUR/VAR169	VAR169	0.013913
2	HOUR/VAR169/VAR79	VAR79	0.013708
3	HOUR/VAR169/VAR79/VAR78	VAR78	0.013410
4	HOUR/VAR169/VAR79/VAR78/VAR157	VAR157	0.013395

For two other solar farms #2 and #3, the configuration of k-NN algorithm is the same to that of solar farm #1. Finally, Table IV gives optimization results of all weights $w_1, \ldots, w_5$ for three solar power plants, which are provided by the coordinate descent algorithm.

TABLE IV. OPTIMAL WEIGHTS OF MANHATTAN DISTANCE

Farm	k	w_1	w_2	w_3	w_4	w_5
#1	200	1.65632	0.39948	1.35251	0.53952	0.21932
#2	200	1.87315	0.45264	1.69499	0.33925	0.22228
#3	200	1.85796	0.44050	0.82677	0.63108	0.32729

B. Deterministic Prediction Results

Fig. 3 shows parts of point prediction of solar power output at solar farm #1 in the 1st task of GEFCom2014. In Fig. 3, we can observe the bell shape of solar power generation from sunrise to sunset. Note that the timestamp in Fig. 3 and 4 are expressed by Universal Time Coordinated (UTC), not the local time. Thus, the maximum output of solar power generation isn't at midday. A good match between the observed and predicted values is obtained by the proposed k-NN algorithm.

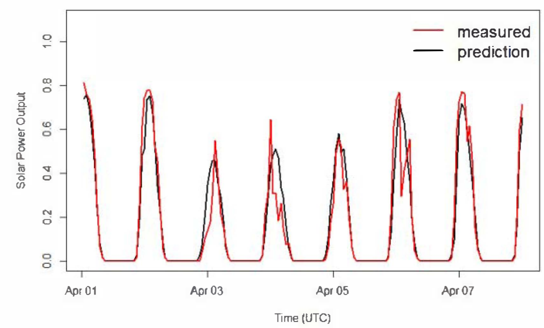

Figure 3. Point forecasting of solar power output at farm #1 from April 1st to April 7th 2013.

Evaluation results of point prediction in 5 tasks are listed in Table V. It shows that there is a little difference in RMSE between two tasks or two plants. These findings are understandable because the meteorological condition of solar power generation is quite different between two months. In addition, the maintenance of photovoltaic panels has great influences on the prediction accuracy. The average and standard variance of RMSE are given in the bottom of Table V. The small standard variance shows the robustness of prediction performance of k-NN algorithm, which is very valuable in industrial practice.

TABLE V. SUMMARY OF ACCURACY OF POINT PREDICTION IN FIVE TASKS OF GEFCOM 2014 SOLAR FORECASTING

Task	Period	RMSE		
		Farm #1	Farm #2	Farm #3
1	Apr 2013	0.088132	0.077563	0.075661
2	May 2013	0.071035	0.079000	0.068553
3	May 2013	0.076475	0.079952	0.083946
4	Jul 2013	0.101122	0.098185	0.092904
5	Aug 2013	0.089117	0.104147	0.126994
Average		0.085176	0.087769	0.089611
Standard Variance		0.011770	0.012438	0.022795

C. Probabilistic Prediction Results

In order to visualize results of probabilistic forecasting, 99 quantiles are converted into nine predictive intervals I_β ($\beta = 10,\ldots,90$). Fig. 4 gives probabilistic forecasts of solar power output at farm #1 in the first day of Task 1. Fig. 5, as a complement of Fig. 4, shows the shape of solar power predictive density at 24:00 on April 1st 2013. It can be found that prediction intervals are not symmetric around the median prediction. These results are understandable because the predictive density given by KDE approach is not symmetric. We can also observe that the width of predictive interval varies with the fluctuation of solar power output. Predictive interval has the tendency of becoming a little wider when solar power generation fluctuates sharply.

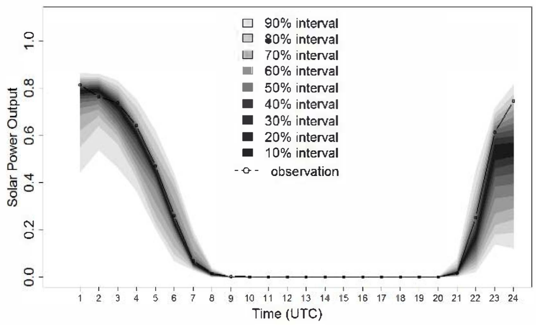

Figure 4. Probabilistic forecasting of solar power output at farm #1 on April 1st 2013, shown by a series of predictive intervals with different nominal converge rate. The measurements are displayed by black dot dash line.

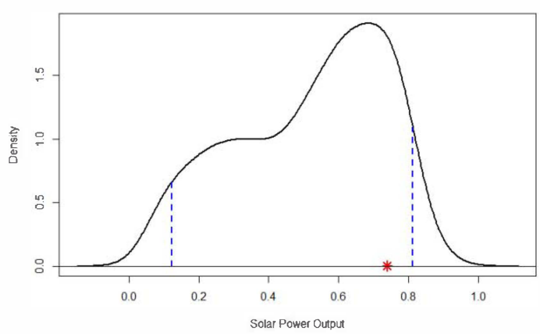

Figure 5. Density forecasting of solar power output at farm #1 at 24:00 (UTC) on April 1st 2013. The solid line denotes the predictive density. Two vertical dotted lines denote the 90% predictive interval I_{90}. The notation * denotes the measurment of solar power generation.

Evaluation results of probabilistic prediction in 5 tasks are listed in Table VI. The performance of proposed *k*-NN algorithm is quite well in Task 2 and 3, but a little poor in Task 4 and 5. The average and standard variance of quantile score are 0.0140 and 0.0026. Overall, the standard variance is acceptable for practical application. A small standard variance means that *k*-NN algorithm constantly give a good probabilistic forecasting of solar power output in different weather condition.

TABLE VI. SUMMARY OF QUANTILE SCORE OF PROBABILISTIC FORECASTING IN FIVE TASKS OF GEFCOM 2014

Task	Period	Score Q		
		Farm #1	Farm #2	Farm #3
1	Apr 2013	0.013646	0.013064	0.013299
2	May 2013	0.010751	0.012373	0.010904
3	May 2013	0.011650	0.013272	0.013477
4	Jul 2013	0.015350	0.016171	0.014242
5	Aug 2013	0.013778	0.016880	0.021629
Average		0.013035	0.014352	0.014710
Standard Variance		0.001831	0.002027	0.004064

VI. CONCLUSIONS

In this paper, we presented a general framework of probabilistic forecasting for renewable energy generation based on the *k*-Nearest Neighbor (*k*-NN) and Kernel Density Estimator (KDE) method. The *k*-NN algorithm helps us find the *k* closest training examples in the feature space. Our proposed approach can provide both point and probabilistic forecasts. Point prediction equals to the weighted average of values of *k* nearest neighbors. Probability density is derived from *k* nearest neighbors by means of KDE method. The effectiveness of proposed method has been validated with real dataset of Global Energy Forecasting Competition 2014. Results from evaluation shows that RMSE and quantile score are quite low, which verify the precision of the proposed forecasting method. This approach has the potential for practical application. In the future, more work will be made in error analysis which is necessary for further improving the existing approach.

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