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Review on probabilistic forecasting of wind power generation

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ABSTRACT

The randomness and intermittence of wind resources is the biggest challenge in the integration of wind power into the power system. Accurate forecasting of wind power generation is an efficient tool to deal with such problem. Conventional wind power forecasting produces a value, or the conditional expectation of wind power output at a time point in the future. However, any prediction involves inherent uncertainty. In recent years, several probabilistic forecasting approaches have been reported in wind power forecasting studies. Compared to currently wide-used point forecasts, probabilistic forecasts could provide additional quantitative information on the uncertainty associated with wind power generation. For decision-makings in the uncertainty environment, probabilistic forecasts are optimal inputs. A review of state-of-the-art methods and new developments in wind power probabilistic forecasting is presented in this paper. Firstly, three different representations of wind power uncertainty are briefly introduced. Then, different forecasting methods are discussed. These methods are classified into three categories in terms of uncertainty representation, i.e. probabilistic forecasts (parametric and non-parametric), risk index forecasts and space-time scenario forecasts. Finally, requirements and the overall framework of the uncertainty forecasting evaluation are summarized. In addition, this article also describes current challenges and future developments associated with wind power probabilistic prediction.

Contents

Abbreviations: WPF, Wind Power Forecasting; PDF, Probability Density Function; CDF, Cumulative Distribution Function; QR, Quantile Regression; KDE, Kernel Density Estimation; ISO, Independent System Operator; RS-AR, Regime-Switching Autoregression; MS-AR, Markov-Chain Regime-Switching Autoregression; AR-GARCH, Autoregression-Generalized Autoregressive with Conditional Heteroscedasticity; CPAR, Conditional Parametric Autoregression; LQR, Local Quantile Regression; SQR, Spline Quantile Regression; QRF, Quantile Regression Forest; BMA, Bayesian Model Averaging; MRI, Meteo-Risk Index; NPRI, Normalized Prediction Risk Index; AI, Artificial Intelligence; NWP, Numerical Weather Prediction; CRPS, Continuous Ranked Probability Score

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1. Introduction

To protect the environment and reduce consumption of conventional energy resources, the installed capacity of renewable energy, wind power in particular, is ever-growing all over the world. Usually, wind power generation is considered as 'non-dispatchable' due to the randomness and intermittence involved, which brings about a great impact on power system operation in various aspects, e.g. power system stability, ancillary service, and power quality. In this aspect, forecasting wind power output is an efficient tool to tackle these problems and bring more and more wind power into power system. Accurate forecasting of wind power can also provide technical support for wind power trading in electricity market, thus producing significant economic benefit.

The currently-used wind power forecasting (WPF) produces only a conditional expectation of wind power output, and is a deterministic prediction (or spot prediction) in nature. Until now, much research has been carried out for improving the accuracy of these prediction methods, which has been published in other reviews [1,2]. In fact, it is extremely difficult, if not impossible, to get the whole knowledge of future events, especially the capricious atmospheric behavior. Therefore, any prediction approach has its inherent and irreducible uncertainty. Compared with the deterministic forecasting, the approach that provides probabilistic information on future events may offer advantages if it is difficult to achieve the accurate spot forecasting. Recently, uncertainty forecasting or probabilistic forecasting has drawn more and more attention in the development of prediction theory [3]. It has found wide applications in weather prediction [4], risk management in economics and finance [5], epidemiological studies [6]. For example, every quarter, the Bank of England issues probabilistic forecasts of Gross Domestic Product (GDP) and Consumer Price Index (CPI) of future three years on its website [7].

Studies have indicated that WPF was not precise enough and the accuracy of WPF varied with time, resulting in remarkable uncertainty for wind power forecasting. In the last decade, the uncertainty forecasting method has been introduced into the wind power generation, and the theory of wind power uncertainty forecasting has also been established. Unlike the conventional wind power forecasting that usually only produces a single value of future wind power output, the wind power uncertainty forecasting can give much information on uncertainty, and is very useful for power system operation with volatile wind power. Gneiting [8] proposed that for a large family of decision-making problems, the optimal decision was directly related to the quantile of conditional predictive distribution. This generalized conclusion provided a theoretical foundation for the wind power uncertainty forecasting to be applied to power system operation. With the help of stochastic optimization, uncertainty information of wind power output has been used in the decision-making problems related to reserve requirement [9–11], trading strategy of wind power [12–15], unit commitment considering wind power uncertainty [16–18], energy storage sizing [19], and optimal dispatch of wind-hydro power plants [20]. These studies show that the penetration rate of wind power generation has increased substantially after the uncertainty forecasting was applied to the power system operation.

Evaluation of wind power forecasts is also of great importance to both deterministic and probabilistic forecasting methods. In general, evaluating wind power forecasts is made up of four functions: performance assessment, model diagnosis, model selection, and model ranking. There are several criteria for evaluating wind power spot forecasts, e.g. Mean Absolute Error (MAE), Mean Square Error (MSE), and Root Mean Square Error (RMSE) [21]. The evaluation of wind power spot forecasts is on the basis of the discrepancy between predictive and measured values. However, it is more difficult to evaluate wind power uncertainty forecasts because the information on uncertainty (e.g. predictive density) cannot be compared with the measured value directly. As a prominent challenge, the evaluation of wind power uncertainty forecasts has drawn much attention among the researchers in recent years.

This paper gives a detailed review of state-of-the-art methods and new developments in wind power uncertainty forecasting. It is organized as follows. The uncertainty of wind power generation is analyzed briefly in Section 2. A detailed definition of wind power forecasting is also given in this section. In Section 3, we qualitatively investigate the influence of wind power forecasting on electricity price. In order to prove the necessity of uncertainty forecast in wind power generation, a representative example of decision-making problems, i.e. offering strategy of wind power in electricity market, is also described in Section 3. Section 4 deals with three different uncertainty representations of wind power output. They are probabilistic forecasting, risk index, and scenario. In Section 5, based on the mathematical methodologies employed, various techniques of wind power probabilistic forecasting are classified into two categories, i.e. parametric and non-parametric. Each technique with its pros and cons is discussed in detail. Risk index forecasting and scenario forecasting are discussed in Sections 6 and 7, respectively. Section 8 presents the verification of wind power uncertainty forecasting, focusing on the evaluation criterion and framework. Section 9 contains a discussion that covers the existing challenge of wind power uncertainty forecasting, the development of wind power forecasting in smart grid context, and the accommodation of more wind power generation through electricity market under the framework of stochastic optimization. This paper ends with a conclusion in Section 10.

2. Wind power forecasts: uncertainty and definition

2.1. Uncertainty analysis

The generation of electric power from wind resource depends on the atmospheric process in nature. In short, wind is caused by differences in atmospheric pressure. When the difference in atmospheric pressure exists, air moves from the higher to the lower pressure system, which results in winds. There are many factors that can affect the generation of wind resource, i.e. terrain, temperature and friction. Therefore, the dynamic behavior of wind is quite complex, resulting in high variability of wind power generation. A detailed introduction to the physical conception involved in wind power generation and forecasts could be found in Lange et al. [22].

Much research has been carried out on the modeling and forecasting of wind power generation. Initially, estimating the marginal distribution of wind speed (e.g., Weibull and Gaussian) was the subject of much research [23]. Then, some dynamic models (e.g., linear time-series) were proposed for the prediction of wind speed [24]. Nowadays, it is widely believed that the modeling of wind power generation and forecasts should be stochastic and dynamic in short-term, middle-term and long-term [25]. One popular method is that wind power generation can be modeled by a stochastic process $P_{t,s} = \{P(t,s)\}$ which involves the spatial and temporal scale. This stochastic process is made up of many discrete points observed in time t and space s. In practice, it often reduce to a temporal process $P_t = \{P(t)\}$. The stochastic process of future wind power output could be applied to describe wind power forecasts. It follows that

$$\hat{\boldsymbol{P}}_{t+k|t,s} = \{\hat{P}(t+k|t,s),\ k \in K,\ s \in S\} \tag{1}$$

where, the set S is a collection of m locations $S = \{s_1, s_2, \cdots, s_m\}$, and the set K is a collection of n look-ahead times $K = \{k_1, k_2, \cdots, k_n\}$.

However, the high variability of wind power generation brings much difficulty for power system operation. Wind speed is the meteorological variable of most relevance to wind power generation. Mur-Amada et al. [26] employed the frequency domain analysis to investigate the character of wind speed variability. They found that the wind speed time-series demonstrated high fluctuations in a wide range of frequencies, as shown in Table 1. It could be found that wind power generation is a non-linear and non-stable process. Meanwhile, complex features of wind speed variability results in the low predictability of wind power generation.

Table 1
Fluctuations of wind speed time-series in frequency domain.

Frequency domain	Time domain	Caused by
Extra-low	Months	Climatic changes and human activities
Low	Days	General changes of weather patterns
Middle	Hours	Thermal exchange between ground and atmosphere
High	Minutes	Local meteorological effect (e.g. convection)
Extra-high	Seconds	Turbulence effects of wind speed

Table 2
Classification of wind power uncertainty forecasting in terms of forecasting horizon.

Forecasting horizon	Time scale	Application
Very short-term	From seconds to minutes	Wind turbine control Power system frequency control
Short-term	From hours to days	Economic dispatch Reserve requirement Day-ahead electricity market
Medium-term	From days to weeks	Unit commitment Maintenance scheduling
Long-term	From weeks to months or years	Wind power planning Power system planning

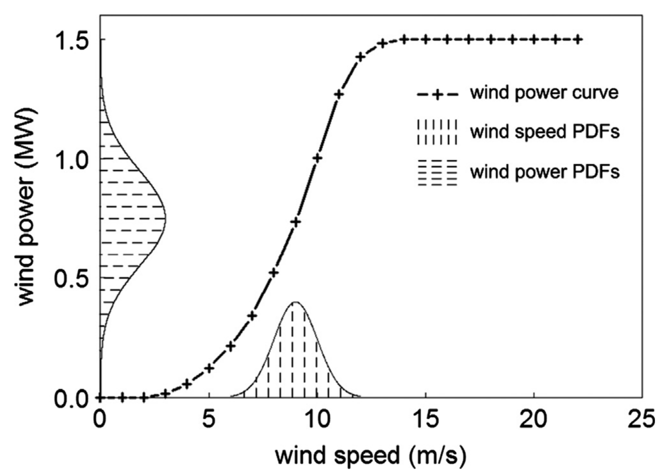

Fig. 1. Theoretical wind power curve of the VENSYS 77 (1.5 MW) wind turbine installed at Beidaqiao II wind farm in Gansu province, China.

For a single wind turbine, the wind power curve is used to describe the conversion from wind to electric power. Wind power curves for different types of turbines almost have the same shape, i.e. S-shaped curve charactered by cut-in, rated and cut-out wind speeds, as shown in Fig. 1. Due to the lack of the whole knowledge about atmospheric behavior, weather prediction has more or less uncertainty. This uncertainty of the wind speed forecast is usually amplified by the non-linear wind-to-power conversion process. In Fig. 1, wind speed density is assumed to follow standard normal distribution. It could be seen that wind power distribution is more dispersed after the wind-to-power conversion. Therefore, for wind power forecasting, a large part of forecasts error directly comes from the prediction of meteorological variable (especially wind speed).

The theoretical wind power curve (as shown in Fig. 1) is obtained in the condition of free wind without considering changes in weather situations. However, in practice, the actual wind power curve looks quite different from the theoretical one, as shown in Fig. 2. A non-parametric approach (i.e., local polynomial regression) to the modeling of the actual wind power curve could be found in Nielson et al. [27]and Pinson et al. [28]. There are many explanations for the dispersion of the actual power curve, as shown below.

- Impacts of other meteorological variables, e.g. wind direction and air density.

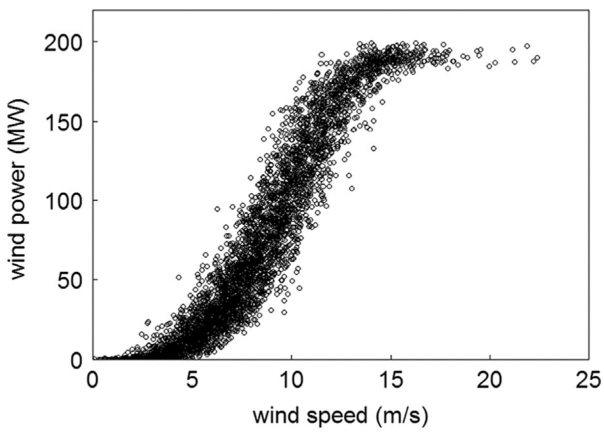

Fig. 2. Actual wind power curve for Beidaqiao II wind farm in Gansu province, China from January to June in 2013.

- Wake effects from neighboring wind turbines, resulting in changes of wind.
- Changes in the environment of wind farm, aging of wind turbine component.

Wind speed and wind-to-power curve are the two factors of most relevance to wind power generation. Both of them exhibit high degree of variability, and become two main sources of uncertainty in wind power forecasts. This uncertainty results in the limited predictability of wind power generation and the low accuracy of conventional WPF. In order to consider the uncertainty of wind power generation better, the relevant forecasts should be developed in the probabilistic framework.

2.2. Definitions

To give forecasting result at time t, the forecaster should collect much useful information. In this paper, the set Ω_t is used to denote this information. The set Ω_t usually contains three types of information.

- Historical measurements of wind power output $\Omega_t^{o_1} = \{p_t, p_{t-1}, \cdots, p_{t-l}\}$.
- Historical measurements of explanatory variables $\Omega_t^{o_2} = \{x_t, x_{t-1}, \cdots, x_{t-l}\}$, e.g. relevant meteorological variables.
- Forecasts of explanatory variables $\Omega_t^{f} = \{\hat{x}_{t+k|t}\}$, e.g. Numerical Weather Prediction (NWP).

For very short-term forecast horizon, historical measurements of wind power output are sufficient for the operation of WPF. While for short-term and further forecast horizon, weather prediction of wind components is necessary to improve the quality of WPF. Given a statistical model g and its parameters $\boldsymbol{\theta}$, the problem of point forecasting at time t for look-ahead time $t+k$ could be formulated as

$$\hat{p}_{t+k|t} = g(p_t, p_{t-1}, \cdots, p_{t-l};\ x_t, x_{t-1}, \cdots, x_{t-l};\ \hat{x}_{t+k|t};\ \boldsymbol{\theta}) + e_t \qquad (2)$$

where e_t is a zero mean random variable. Common choices of statistical model g in WPF are time-series [24] and Artificial Intelligence (AI) approaches [29].

Unlike point predictions, which produce only the single-valued expectation of wind power output, uncertainty forecasts could provide additional quantitative information on the uncertainty associated with wind power generation. Such uncertainty could be represented by Probability Density Functions (PDFs) or Cumulative Distribution Functions (CDFs), as shown in Eqs. (3) and (4).

$$\hat{f}_{t+k|t} = f(p_{t+k};\ p_t, p_{t-1}, \cdots, p_{t-l};\ x_t, x_{t-1}, \cdots, x_{t-l};\ \hat{x}_{t+k|t};\ \boldsymbol{\theta}) \qquad (3)$$

$$\hat{F}_{t+k|t} = F(p_{t+k};\ p_t, p_{t-1}, \cdots, p_{t-l};\ x_t, x_{t-1}, \cdots, x_{t-l};\ \hat{x}_{t+k|t};\ \boldsymbol{\theta}) \qquad (4)$$

In practical terms, predictive PDFs $\hat{f}_{t+k|t}$ in Eq. (3) is translated into a set of density forecasts $\hat{d}_{t+k|t}^{(p_i)}$, and predictive CDFs $\hat{F}_{t+k|t}$ in Eq. (4) is translated into a set of quantile forecasts $\hat{q}_{t+k|t}^{(\alpha_i)}$.

$$\hat{f}_{t+k|t} = \{\hat{d}_{t+k|t}^{(p_i)};\ 0 \le p_1 < p_2 < \cdots < p_i < \cdots < p_m \le 1\} \qquad (5)$$

$$\hat{F}_{t+k|t} = \{\hat{q}_{t+k|t}^{(\alpha_i)};\ 0 \le \alpha_1 < \alpha_2 < \cdots < \alpha_i < \cdots < \alpha_m \le 1\} \qquad (6)$$

Density locations p_i in Eq. (5) and nominal proportions α_i in Eq. (6) are spread over the unit interval. Several methods have been proposed for wind power uncertainty forecasts, e.g. Kernel Density Estimation (KDE) [30] and Quantile Regression (QR) [31], which would be introduced in Sections 5–7.

3. Wind power forecasts and decision-making applications

3.1. Impact of wind power forecasts on electricity prices

Forecasting information is widely used as inputs to several decision-making problems related to power system operation. In recent year, many researchers have called for the participation of wind power producers in electricity market in the same way as conventional power producers [32–34]. However, the liberalized electricity market involving wind power generation still face many technical challenges, especilly for some decision-making problems (e.g., market-clearing mechanisms and offering strategies). As mentioned in Section 2, wind power generation exhibits high variability and low predictability. Zeineldin [35] have shown that the variability of wind power generation and the forecasting inaccuracy could have much influence on electricity prices. The inaccurate wind power forecasts can lead to either underestimation or overestimation of market prices, resulting in additional savings for load customers or additional revenues for power producers. Experiences in Texas USA [36] and South Australia [37] also supported the opinion that the level of wind power output had great influence on spot price, and the inaccurate point forecasts enlarged the variance of spot price.

In this paper, for simplicity, the impact of wind power forecasts on market prices is demonstrated qualitatively in the supply and demand curve. Compared with conventional power generation, wind power generation has the lowest short-run marginal cost. But on the other hand, wind power point forecasts are not accurate enough, leading to the sharp fluctuation of wind power output. Fig. 3 shows the supply and demand of electricity market under the assumption that the short-run marginal cost of wind power producers is zero. It could be found that great changes take place in Market Clearing Prices (MCP) when wind power output moves from low to high level. These results suggest that wind power producers and other renewable energy producers are likely to become potential drives in electricity market. With high penetration of wind power generation and stochastic load consumption (for instance electric vehicle) in power grid, more uncertainty should be dealt with in electricity market. The accuracy of wind power point forecasts is directly related to revenues of all power produces in market. However, wind power point forecasts are not precise enough and its accuracy varied with time, resulting in remarkable uncertainty. In the uncertain market environment, it's far from sufficient for only providing the inaccurate wind power point forecasts. Quantitative information on all

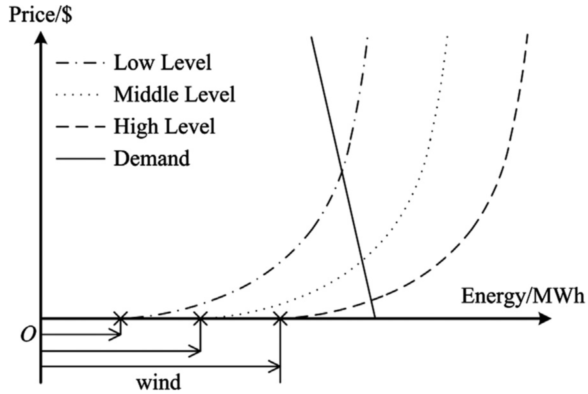

Fig. 3. Supply and demand of electricity market when great changes take place in wind power output.

sources of uncertainty is quite important for decision-making problems. Compared to currently wide-used point forecasts, which produce only single-valued expectation of wind power output, uncertainty forecasts could provide additional quantitative information on the uncertainty associated with the expectation. Therefore, uncertainty forecasts techniques would play important roles in the future electricity market.

3.2. Optimal bidding strategy of wind power producers

In order to prove the necessity of developing wind power uncertainty forecasts, an example of decision-making problems is analyzed in this section. How to design the optimal bidding strategy in electricity market is an important decision-making problem [12–15], which is directly related to benefits of wind power producers. In this paper, we focus on studying the participation of wind power producers in the competitive electricity market. This pool-based electricity market is composed by the day-ahead market and the real-time market which are all run by Independent System Operator (ISO). All power producers would participate in the electricity market under same rules including wind power producers. In the day-ahead market, each power producer has to submit its bid for energy of the next operating day to the ISO, and takes financial responsibility for the deviation from the submitted bid. In PJM day-ahead market, bids of the next operating day from 0:00 a.m. to 12:00 p.m. should be submitted before 12:00 a.m. every day [38]. Therefore, the forecasting horizon of wind power forecasts is from 12 to 36 h ahead. The PJM day-ahead market clears at 4 p.m. on the day. Then, all bids and offers are combined to form the aggregate supply and demand curve. The market clearing price is determined by the intersection of supply and demand curve.

Let e_b and e_p be the bidding and the actual amount of wind power output. Wind power producer can get the income from the selling of actual power generation e_p, but has to pay for the deviation from its bid e_b. Therefore, the total revenue I_k for a given time k could be formulated as

$$I_k(e_p, e_b) = \begin{cases} e_p \hat{p}_s - (e_b - e_p) \cdot \hat{c}_- & \text{if} \quad e_p \le e_b \\ e_p \hat{p}_s - (e_p - e_b) \cdot \hat{c}_+ & \text{if} \quad e_p > e_b \end{cases} \tag{7}$$

where $\hat{p}_s$ is the predictive electricity price, $\hat{c}_+$ and $\hat{c}_-$ are the predictive unit costs for downward and upward deviation from the submitted bid respectively. The actual wind power production e_p is regarded as a random variable. Similar formulations of wind farm's revenue could also be found in Bremnes [31] and Pinson et al. [39].

If only point forecasts are available in WPF, these point forecasts are the best bids in the day-ahead market. Wind power producers could not obtain maximal benefits unless point forecasts are perfect and error-free, i.e. $e_b = \hat{e}_p = e_p$. However, this request is unrealistic even for the development of state-of-the-art wind power point forecasts. However, if WPF produce probabilistic information, e.g. predictive distributions of wind power output e_p, we can develop more advanced bidding strategies under the stochastic optimization framework. Our objective of designing the optimal bidding strategy is to maximize benefits of wind farms. Due to the randomness and intermittence involved, there is often considerable uncertainty about the revenue of wind power generation. For a rational participator in electricity market, one reasonable choice is to maximize his expected revenue. Then, the optimal bidding problem at the day-ahead market is given by

$$e_b^* = \arg\max_{e_b} E[I_k(e_p, e_b)] \tag{8}$$

Solving this stochastic optimization problem, we obtain a closed-form solution, i.e. the optimal bidding amount of wind power producers at time k.

$$e_b^* = \hat{F}_{t+k|t}^{-1}\left(\frac{\hat{c}_+}{\hat{c}_+ + \hat{c}_-}\right) \tag{9}$$

where $\hat{F}_{t+k|t}^{-1}$ is the predictive CDFs given at time t for look-ahead time $t+k$. These results suggest that the optimal bidding amount is equal to a special quantile produced by wind power uncertainty forecasts in Eq. (6). The nominal proportion of this special quantile is a function of the predictive unit costs, i.e. $\alpha = \hat{c}_+/(\hat{c}_+ + \hat{c}_-)$.

In the practical electricity market, optimal bidding problem would be further complicated by adding constraints into Eq. (8), e.g. considering spatial–temporal correlation of wind power generation, respecting power system operation constrains. It seems that different types of bidding strategies depend on the forecasting information about future wind power generation. However, advanced bidding strategies based on uncertainty forecasts, instead of point forecasts, have higher value for trading wind power through the electricity market. Simulation results in Pinson et al. [12,39] demonstrated that considering bidding strategies based on probabilistic forecasts allowed a multi-MW wind farm in Dutch electricity market to increase its revenue. In addition to trading strategy of wind power, uncertainty forecasts could also be used as inputs to other decision-making problems, e.g. reserve requirement [9–11], unit commitment and economic dispatch [16–18], energy storage sizing [19], and optimal dispatch of wind-hydro plants [20]. With more and more stochastic power penetration in power system, there are many sources of uncertainty in decision-making problems. Quantitative information about forecasts uncertainty is a better choice for inputs of decision-makings. This type of information can only be generated by uncertainty forecasts. Therefore, energy-related forecasts method should be developed in the probabilistic framework.

4. Uncertainty representation of wind power forecasting

In wind power uncertainty forecasting, three main representations of the uncertainty in WPF are usually considered, i.e. probabilistic forecasting [31], risk index [40], and scenario forecasting [41]. Probabilistic forecasting is the most commonly used uncertainty representation. Risk index, a user-oriented representation, can be easily understood by end-users of WPF. The spatial–temporal correlation of uncertainty information is considered in scenario forecasting, which is often applied in dynamic decision-making problems. In general, researchers are not interested in which uncertainty representation is the best. The focus of much research is usually on the most suitable representation. Therefore, the chosen of wind power uncertainty representation depends on the actual situation.

4.1. Probabilistic forecasting

In this representation, wind power output is viewed as random variable. The uncertainty of wind power output can be expressed by probability measure as follows:

- PDFs and CDFs.
- Quantiles and intervals.
- Discrete probabilities.
- Moments of probability distribution, e.g. mean, variance and skewness.

Under the framework of stochastic optimization, PDFs or CDFs of wind power output is widely used for decision-making problems of power system operation. Wind power output can be seen as a random variable following predictive PDFs or CDFs. Meanwhile, PDFs or CDFs is also the most general representation in wind power probabilistic forecasting. Other representations (e.g. quantiles and intervals) can be derived from PDFs or CDFs. Interval is the most visualized representation, and can provide the range which wind power output lies in with a specified probability. Therefore, interval forecasting is the representation that attracts the most attention of end-users of WPF. Different approaches of wind power probabilistic forecasting are introduced in Section 5.

4.2. Risk index

Wind power spot forecasting can be directly used for power system operation. However, uncertainty forecasting gives the probabilistic information of wind power output, which is a more complicated form in comparison with spot forecasting. Such information could not be directly used for power system operation. Risk indices of WPF can be defined as the expected level of forecasting error. Higher risk value means less reliable forecasting result. The indices can be expressed in the form of color codes (e.g. red–green–yellow) or integer values (e.g. from 1 to 5), which is similar to widely used warning signals for extreme or disastrous weather. When managing wind power generation, one can have the knowledge of wind power forecasting uncertainty through risk indices. When risk indices are high, some preventive tools (e.g. increasing reserve capacity) would be taken for reducing the potential risk caused by forecasting error. Details about different definitions of risk indices are introduced in Section 6.

4.3. Scenario

Wind power uncertainty forecast is generated every look-ahead time independently, without considering the correlation among different look-ahead time points. This implies that the uncertainty forecasting is a time-independent approach and is not able to provide information about the variation of wind power uncertainty over the forecasting horizon. For power system operation, however, there are a lot of time-dependent and multi-stage decision-making problems such as dynamic economic dispatch, the coordination between wind farms and energy storage systems, and trading strategy in multi-market with different closing times. These dynamic decision-making problems inevitably involve time-dependent information.

The temporal correlation of wind power output at different look-ahead time points can be modeled by joint PDFs. However, it is difficult to use joint PDFs directly for decision-making. Fortunately, as an alternative, scenario is suitable for such decision-making problems. It contains a series of spot forecasting results in a period of future time. Therefore, a series of scenarios are also referred to as time trajectories of wind power output. In addition to the temporal correlation, the spatial correlation among wind farms in different geographical positions can also be considered. For example, at the time t, the jth scenario of wind power output P in space s for future k time points can be written as

$$\hat{\mathbf{P}}^{(j)} = \{\hat{P}^{(j)}(t+\mathbf{k}|t,\ \mathbf{s}),\ \mathbf{k}\in K,\ \mathbf{s}\in S\} \quad j=1,\ 2,\ \cdots,\ J \tag{10}$$

where J is the number of all scenarios. Information about the spatial–temporal correlation of forecasting uncertainty is very useful for congestion management and probabilistic load flow. Section 7 introduces how to produce several scenarios of wind power output.

5. Wind power probabilistic forecasting

Similar to the deterministic forecasting, different approaches of wind power uncertainty forecasting can be classified into four categories in terms of forecasting horizon, i.e. very short-term, short-term, medium-term, and long-term forecasting Table 2 gives some information about these four categories of uncertainty forecasting.

Probabilistic forecasts are the most used representation of the uncertainty in WPF, which is introduced in this section. The two other forms, i.e. risk index and spatial–temporal scenario, would be reviewed in Sections 6 and 7. For wind power output, probabilistic forecasts usually take the form of probability density function. Compared to currently wide-used point forecasts, which produce only single-valued expectation of wind power output, probabilistic forecasts could provide additional quantitative information on the uncertainty associated with the expectation. Parametric and non-parametric approaches are two main techniques to construct predictive distributions.

Parametric approaches are based on the assumed shape of predictive density, e.g. Gaussian or Beta. In parametric approaches, predictive distribution is fully charactered by the parameter set $\boldsymbol{\theta}$, for instance that Gaussian distribution is described by location parameter μ and scale parameter σ^2. Wind power point forecasts are usually used as the estimation of location parameter μ. While for the scale parameter σ^2, several estimators have been proposed, e.g. non-linear time-series and artificial intelligence. Without any assumption of distribution shape, in non-parametric approaches, predictive PDFs or CDFs are estimated at a finite number of points, as shown in Eqs. (5) and (6). Then through the interpolation among these points, we can obtain the full description of predictive PDFs or CDFs. Some non-parametric approaches have been applied in probabilistic forecasts, e.g. quantile regression and kernel density estimation.

Advantages of parametric approach are that the distribution shape only depends on a few parameters, resulting in the simplified estimation and low computational costs. However, sometimes the assumption of wind power distribution shape may be not reasonable, and the shape of predictive density changes with time. This phenomenon would influence the effectiveness of parametric approach. For example, it is difficult to deal with multi-modal distribution for the conventional parametric approach. Without any assumption of distribution shape, the non-parametric approach is referred to as distribution-free. Compared with parametric approach, there are many more densities or quantiles to be estimated for non-parametric approach. Sometimes, a specific model is required to be identified and trained for each quantile of the predictive distribution. Therefore, taking into account the computational cost, certainly it is more expensive to run non-parametric model than parametric one. A general summary of different probabilistic forecasting approaches is given in Table 3. Each of probabilistic forecasting approaches would be introduced in this section.

Table 3
Classification of wind power probabilistic forecasting in terms of mathematical methodology.

Classification	Method	Remarks
Parametric	Homoscedastic time-series Heteroscedastic time-series Artificial intelligence	– Assumption of distribution shape – More competitive for very short-term forecasting – Low computational costs due to the simplified estimation
Non-parametric	Quantile regression Kernel density estimation Ensemble forecasting Artificial intelligence	– Without any assumption of distribution shape – A data-driven approach requiring a lot of samples – More competitive for short and medium term forecasting – high computational costs due to the complicated estimation

5.1. Parametric approach

Parametric approach assumes that predictive distribution follows the pre-defined shape which can be described by analytical expression. For instance, Gaussian distribution is fully described by location parameter μ and scale parameter σ^2, which can be represented as

$$\hat{f}_{t+k|t} = f(p_{t+k}, \mu, \sigma^2) \tag{11}$$

The simplest parametric approach of wind power uncertainty forecasting was presented in [42], which assumed that forecasting error followed Gaussian distribution with zero mean and given standard deviation. In the field of parametric forecasts approaches, four aspects are subjects of much research, including

- shape assumption of predictive distribution;
- estimator of location parameter;
- estimator of scale parameter;
- parameter evaluation theory.

5.1.1. Shapes of wind power predictive density

The assumption of predictive distribution shape is also the focus of much research. Gaussian and Beta distributions are two widely used choices. Lange et al. [43] found the conditional distribution of wind speed forecasting error could be modeled by Gaussian distribution. Bludszuweit [44,45] found the kurtosis of wind power forecasting error distribution was large, and varied with look-ahead time. This phenomenon was also called fat-tailed. Luig et al. [46] and Bofinger et al. [47] argued that wind power output should be viewed as the double-bounded variable, instead of unlimited variable following Gaussian distribution. For wind power uncertainty forecasting, in comparison with Gaussian distribution, Beta distribution was a more reasonable choice due to its large kurtosis and its range from 0 to 1. Apart from Gaussian and Beta distributions, Pinson [48] proposed another distribution shapes, i.e. the modified Generalized Logit-Normal Distribution (GL-normal) distribution. This novel shape of wind power forecasting distribution is a discrete-continues mixture consisting of GL-normal distribution in the interval [0,1] and two probability masses at the boundaries of the interval [0,1].

5.1.2. Estimators of local parameter

In parametric approaches, wind power point forecasts are usually used as the estimators of location parameter. Many researchers have reported on a number of point forecasting approaches in the past 3 decades. In this paper, we only choose and review some latest approaches in this research field. Details about wind power point forecasts could be found in other reviews [2]. Wind power point forecasts can be divided into two categories by the used information set Ω_t in forecasting.

- Usage of historical measurements Ω_t^o, including wind power output $\Omega_t^{o_1}$ and meteorological variable $\Omega_t^{o_2}$
- Usage of historical measurements Ω_t^o and weather forecasts Ω_t^f.

Due to the inertia of the local atmosphere process, the former model only using historical measurments Ω_t^o is more competitive for very short-term forecasting (i.e., for look-ahead time less than a few hours). High temporal resolution (e.g. 15 min) in NWP models is required for very short-term forecasting. However, it is very expensive to operate NWP models frequently. Therefore, forecasting models based on historical measurements would be preferred in very short-term WPF. When weather forecasting information Ω_t^f is added into forecasting models as explanatory variables, forecasting horizon of wind power point forecasts can be extended to the short-term range (i.e., 24–48 h).

(a) *Estimators based on historical measurements*. Non-linear time-series is the main approach introduced in this part. Researchers have showed that wind speed time-series was non-stationary, which meant that its statistical characteristic changed with time [49]. Therefore, two important nonlinear time-series models, i.e. Regime-Switching Autoregression (RS-AR) and Conditional Parametric Autoregression (CPAR), become the focus of much research. The basic viewpoint of RS-AR models is the assumption that the stochastic process switches between a finite number of discrete models. These discrete models are also referred to as regimes, which could be interpreted as different states. Therefore, the crucial issue of RS-AR models is how to distinguish and determine the current regime. The Self-Exciting Threshold Autoregressive (SETAR) model was proposed [50], and its regime was determined by a piecewise function of the latest wind speed and direction. In the Markov-Chain Regime-Switching Autoregression (MS-AR) model proposed by Pinson [51,52], the switch from one regime to another was controlled by a first order hidden Markov Chain. Spatial correlation of wind speed is also considered into wind power point forecasts. The quality of point forecasts could be improved by using meteorological observations in the vicinity of wind farm. This type of point forecasts is also referred to as off-site predictors. Under the framework of RS-AR model, Gneiting et al. [53] proposed to define regimes as the measurements of wind direction at the meteorological station close to wind farm.
Researchers have found that the uncertainty of WPF would be affected by forecasting horizon, predictive value and weather situation [54–56]. Therefore, developing a situation-dependent, conditional uncertainty forecasting is more suitable. In CPAR models, autoregression coefficients are modeled by

functions of some meteorological variables, instead of constant parameters. These functions are also referred to as coefficient functions, which are used to describe the non-stability of wind power time-series. Therefore, key issues of CPAR models are how to select explanatory variable of coefficient function and how to define the coefficient function. Gallego et al. [50] showed that the coefficient function could be assumed as sinus-shaped or linear-shaped when the explanatory variable was selected as the latest wind direction or wind speed, respectively. Pinson [48] proposed CPAR models for estimating the location parameter of GL-normal distribution, and the explanatory variable of CPAR model was selected as wind direction measurement.

(b) *Estimators based on historical measurements and weather forecasts*. For further look-ahead times, forecasting information from NWP can be added in all non-linear models mentioned above, e.g. RS-AR and CPAR. Nielsen [57] proposed an extended model, i.e. Conditional Parametric Autoregression eXtraneous (CPARX). For the short look-ahead time, outputs of CPARX model result from the combination of weather forecasts and autoregressive model based on historical measurements. For the long look-ahead time, CPARX model is mainly controlled by weather forecasts. Wind power point forecasts based on artificial intelligence approaches (e.g., neural network and machine learning) have been extensively examined in recent years. Both historical measurements and weather forecasts can be used as inputs of neural network or machine learning. From the perspective of machine learning, WPF belongs to the nonlinear regression problem. In general, the Gaussian process is the basic assumption of uncertainty forecasting based on machine learning. To consider the heteroscedasticity of wind speed time-series, Kou et al. [58] proposed a sparse heteroscedastic Gaussian process. Kou et al. [59] also proposed a warped Gaussian process to characterize the non-Gaussian wind power output. To avoid over-learning phenomenon in maximum likelihood estimation, Yang et al. [60] applied the sparse Bayesian learning algorithm. In the case of neural network, Siderates et al. [29] employed Radial Basis Function Neural Network (RBFNN) and Self-Organized Map (SOM) to predict the wind power uncertainty caused by NMP models, atmosphere stability and spot forecasting models. Under the framework of Recurrent Neural Network (RNN), Felder et al. [61] obtained probability density of the future wind power output.

5.1.3. Estimators of scale parameter

In conventional AR models, the scale parameter of predictive PDFs is equal to the variance of white noise term, and does not change with time. However, wind speed exhibits high degree of variability for all temporal scale, which means that wind power time-series is a non-linear and non-stationary process. The assumption of constant variance of white noise in AR models is not effective. The variance should be allowed to vary in time-series models. A better choice is Autoregression-Generalized Autoregressive with Conditional Heteroscedasticity (AR-GARCH) model. In AR-GARCH models, an autoregressive moving average model is assumed for the variance of white noise. Therefore, AR-GARCH models could provide a description of the conditional heteroscedasticity in wind speed time-series. The Autoregressive Fractionally Integrated-GARCH model was proposed to characterize the spatial correlation of wind speed time-series [62]. The Autoregressive Integrated Moving Average-GARCH model was used to obtain wind power probabilistic forecasting [63]. Under the framework of Cartesian coordinate, Vector Autoregressive Moving Average-GARCH was employed to describe bivariate wind vector time-series [64]. The MS-AR-GARCH model was introduced to characterize the spatial correlation and conditional heteroscedasticity in wind power time-series [65]. Artificial intelligence approaches are another suitable choice that could take into account the heteroscedasticity of wind power time-series. Both location and scale parameters of predictive PDFs could be defined as outputs of neural network or machine learning. For instance, Khosravi et al. [66,67] constructed a feedforward neural network between wind power measurements and prediction residuals.

5.1.4. Parameter evaluation theory

For all parametric models in wind power probabilistic forecasts, parameters are usually estimated by Maximum Likelihood (ML) or Least Square (LS) method. Gneiting et al. [68] found that Continuous Ranked Probability Score (CRPS) was a proper scoring rule which could verify the whole performance of probabilistic forecasts. They proposed the minimum CRPS approach which was more suitable for parameter estimation of probabilistic forecasting model. In addition, an adaptive estimation method based on the exponential forgetting process was also proposed by Pinson [52]. In adaptive estimation method, it is allowed for long-term smooth changes of model parameters which are affected by several factors (e.g. seasonality, climate, aging, failures and maintenance). Another benefit is that adaptive estimation method could reduce computational costs.

5.2. Non-parametric approach

Without any assumption of distribution shape, the non-parametric approach is distribution-free. In non-parametric approaches, predictive PDFs or CDFs of wind power output is translated into a number of density forecasts or a set of quantile forecasts respectively, as shown in Eqs.(5) and (6). Several non-parametric methods have been proposed for wind power probabilistic forecasts. The most well-known and wide-used approach is the adaptive resampling of Pinson et al. [69]. They constructed the empirical distribution of wind power point forecasts error in similar conditions. Other non-parametric approaches of probabilistic forecasts include KDE, QR and AI methods. Apart from historical measurements, the usage of forecasts information (e.g. wind power point forecasts or NWP) is also required in non-parametric approaches. Then, some typical non-parametric forecasting approaches would be introduced as follows.

5.2.1. Quantile regression method

A series of predictive quantiles, instead of conditional expectation, could be obtained in QR models. The relationship between predictive quantile and explanatory variable is the focus of much research. In general, weather prediction information from NWP could be chosen as explanatory variable. In the wind power uncertainty forecasting, three main QR models are Local Quantile Regression (LQR), Spline Quantile Regression (SQR), and Quantile Regression Forest (QRF).

In LQR approaches, the dependence of predictive quantile on explanatory variables is modeled by a linear regression in the neighborhood of explanatory variable [31,70]. Through giving higher weighting factor, samples close to explanatory variable have more influence than those far from explanatory variable. In SQR models, predictive quantile could be modeled by a series of nonlinear smooth functions, e.g. cubic B-Spline functions [71]. Meanwhile, a time-adaptive estimation algorithm for SQR models was proposed for reducing computational complexity and speeding up the solution [72]. Juban et al. [73] proposed QRF models which was an extension of Regression Forest (RF) based on classifications and regression trees. In QRF-based wind power uncertainty forecasting, the estimation of the whole conditional distribution was obtained from weighted observations of wind power output. In order to consider the impact of varying weather regimes, Anastasiades et al. [74] extracted various variability indices from wind power time-series, and employed these variability indices as explanatory variables of QR models.

QR method is a non-parametric approach without any assumption of distribution shape. However, for each predictive quantile, a special QR model should be configured and trained independently, which leads to a larger amount of computation. Meanwhile, the independent training of each QR model may lead to crossing quantiles. Therefore, for avoiding the cross quantile phenomenon, some constraint conditions should be added in the parameter estimation of QR model.

5.2.2. Kernel density estimation method

KDE method is popular among non-parametric approaches of PDFs estimation. The basic viewpoint of KDE method is generating a smooth histogram to estimate PDF of random variable. A curve representing the contribution of data point to probability density is placed at each data point. PDF estimation can be obtained by adding up all curves of data points. Therefore, KDE is a data-driven and non-parametric approach. The KDE-based PDF estimation could be formulated as

$$\hat{f}_X(x) = \frac{1}{N \cdot h} \sum_{i=1}^{N} K\left(\frac{x - X_i}{h}\right) \tag{12}$$

where K represents the kernel function, h represents the bandwidth parameter, X_i is the data point, and N is the sum of data points. For wind power uncertainty forecasting, conditional PDFs estimation is often required, i.e. predicting wind power distribution when conditional variables are known to particular values. Explanatory variables could be chosen from prediction information of NWP. Crucial issues of KDE methods include

- How to select explanatory variables which have high correlation with wind power output.
- How to select the proper kernel function in terms of the type of explanatory variable.
- How to determine the optimal bandwidth, which plays an important role of quality control in PDFs estimation.

The chosen explanatory variable from NWP models should have high correlation with wind power output. In general, the linear correlation between explanatory variable and wind power output could be measured by correlation coefficient and cross validation. The mutual information in information theory was employed to describe the nonlinear correlation between meteorological variable and wind power output [75]. Wind speed at 50 m, gust wind direction at 10 m, temperature and humidity were selected as explanatory variables.

The shape of kernel function depends on the type of random variable. There are four types of explanatory variables in wind power uncertainty forecasting. Therefore, Bessa et al. [30,76–78] proposed four types of kernel functions as shown in Table 4.

Table 4
Different types of kernel functions in KDE.

Type of random variable	Example	Kernel function
Variable bounded between 0 and 1	Wind power	Beta
Variable bounded between 0 and $+\infty$	Wind speed	Gamma
Variable without any limitation	Temperature	Gaussian or bi-weight
Circular or periodic variable	Wind direction	Von Mises distribution

Table 5
Widely-used skill scores for uncertainty forecasting.

Type of random variable	Uncertainty representation	Skill score
Univariate	PDFs	Logarithmic score
	CDFs	CRPS
	Quantiles	Tick or check loss score
	Intervals	Interval score
Multivariate	Multivariate PDFs	Energy score
	Ensembles or scenarios	

Bandwidth has a great influence on the accuracy of probability density estimation. A high bandwidth leads to over-fitting, i.e. over-smoothed density estimation with less concentration. A small bandwidth leads to under-fitting, i.e. unsmoothed density estimation with too many peaks. Zhang et al. [79] employed Asymptotic Mean Integrated Square Error (AMISE) as the optimality criterion. The multivariate plug-in selector was applied to solve the optimization problem of minimizing AMISE. Qin et al. [80] proposed to minimize Integrated Square Error (ISE) of two kernel estimation. Then, optimal bandwidth estimation was converted to an unconstrained optimization problem. This approach did not require any assumption of the shape of true probability density.

Apart from three main issues mentioned above, other research on KDE method has attracted much interest in recent years. Wind power curve is not a strictly increasing function. Therefore, after the wind-to-power conversion, the predictive PDF of wind power output is a mixture including a probability density in the interval and two probability masses at boundaries. The modified KDE method was proposed to predict a discrete-continuous PDF of wind power output [81]. A novel KDE approach based on copula method was proposed to increase numerical stability [76,77]. When measurements of wind power output arrived one after another, for simultaneously updating the predictive PDF of wind power output, an exponential forgetting process was introduced into KDE method [64,77,78].

In summary, KDE method could provide the whole information of wind power predictive PDF, instead of the discrete CDF consisting of a set of predictive quantiles. Meanwhile, KDE method could easily deal with the multi-modal distribution which is frequently observed in wind power generation. However, KDE method is the large sample technique, and requires a great many samples. For newly installed wind farms, the lack of wind power output measurements might limit the application of KDE method.

5.2.3. Probabilistic forecasting based on ensemble

With the development of ensemble forecasting, NWP models have changed a lot in recent years. Ensemble forecasting could provide several equiprobable scenarios of meteorological variable, instead of only a scenario supplied by conventional NWP. Four approaches are used to generate ensemble forecasting, i.e. (1) several running results of NWP with small differences in initial condition, (2) several running results of NWP with small differences in numerical representation of the atmosphere, (3) several NMP models developed by different institutes, (4) several prediction results given at different time origins in the past [82]. Ensemble forecasts provide a series of spatial–temporal trajectory of meteorological variables. Through the wind-to-power conversion shown in Figs. 1 and 2, we can obtain several scenarios of wind power generation. Scenarios of wind power generation could be treated as samples following predictive distribution [83].

Therefore, for wind power probabilistic forecasting based on ensemble, there are two crucial issues as follows.

- How to estimate the wind power curve, which is very useful for the wind-to-power scenario conversion.
- How to estimate PDFs of wind power output on the dataset of wind power ensembles.

Ensemble forecasting systems usually give information of several meteorological variables, which could not be directly applied in wind power probabilistic forecasting. Therefore, research on estimating wind power curve is necessary for the transformation from original ensembles to wind power ensembles. Taylor et al. [62] applied the single and deterministic wind power curve. Nielsen et al. [84] proposed the logistical-shaped wind power curve based on non-linear regression. To model the non-linear and non-stationary wind power curve, Pinson et al. [85] proposed local polynomial regression method. This approach was non-parametric without any assumption about the shape of wind power curve.

In the field of wind power distribution forecasting based on wind power ensembles, KDE and Bayesian Model Averaging (BMA) methods have been the subject of much research in recent years. Under the framework of KDE, wind power predictive density could be modeled by a weighted sum of kernel functions. A mean-variance model based on logistic function was proposed to describe the mean and variance of Gaussian kernel function [85]. Under the framework of BMA, the basic idea is that ensemble members correspond to several conditional PDFs. Predictive PDF of wind power could be modeled by a weighted sum of these conditional PDFs. A higher weighting factor would be assigned to the ensemble member who has the capability to produce accurate forecasting. BMA method is similar to KDE method. However, weighting factors of BMA method are variable, instead of constant. BMA method has been successfully applied to the probabilistic forecasting of wind speed [86] and surface wind direction [87]. Apart from KDE and BMA method, Gneiting et al. [68] employed Gaussian distribution to construct the predictive PDF of wind power directly. The linear regression was used to model the mean and variance of predictive distribution.

For medium-term and long-term predictions, wind power probabilistic forecasting based on ensemble performs better than that based on conventional NWP. Therefore, wind power probabilistic forecasting based on ensemble is very suitable for medium-term and long-term planning problems, wind energy trading in the medium-term and long-term range, unit commitment, and maintenance scheduling.

5.2.4. Artificial intelligence approach

With the development of spot forecasting based on artificial intelligence techniques, some researchers also introduced artificial intelligence approaches (e.g., neural network and machine learning) into wind power probabilistic forecasting. In the non-parametric framework, outputs of artificial intelligence approaches are directly defined as a set of predictive quantiles in Eq. (6), instead of location and scale parameters in parametric approaches. Then, predictive interval could be constructed by two predictive quantiles, $\hat{I}_{t+k|t} = \left[\hat{q}^{(\alpha_L)}_{t+k|t}, \hat{q}^{(\alpha_U)}_{t+k|t}\right]$. AI models are trained through minimizing the defined cost function. For AI approaches in the non-parameter framework, the cost function is directly related to the evaluation of predictive interval. Khosravi et al. [66,88,89] proposed a new scoring rule for the evaluation of interval forecasts, the so-call Coverage Width-based Criterion (CWC). According to the definition, CWC highly penalizes the situation that interval forecasts are not probabilistic calibrate. Predictive intervals with narrow width would obtain high values of CWC. Wan et al. [90,91] proposed a linear combination of Average Coverage Error (ACE) and interval score to verify interval forecasts.

Approaches based on neural network and machine learning play great roles in the development of probabilistic forecasting methodologies. Wind power probabilistic forecasts are more difficult to be evaluated than common-used point forecasts. A number of scoring rules have been proposed, aiming to assess the overall quality of probabilistic forecasting with a single value. Such scoring rules are required to be proper so that the interval forecasts with higher skill could be rewarded effectively. The propriety of scoring rule ensures that the perfect forecasts obtain the best score value. Details of the evaluation of wind power uncertainty forecasting would be presented in Section 8.

6. Risk index forecasting of wind power generation

As mentioned in Section 4.2, risk index is a single value, and reflects the expected level of wind power forecasting error. The definition of risk index is the focus of much research in the field of risk index forecasting. In general, the predictability of WPF would be high if the atmosphere situation is stable. Therefore, several measurements of atmosphere stability were proposed in the study of risk index definition.

Meteo-Risk Index (MRI) [40] and Normalized Prediction Risk Index (NPRI) [92] were proposed to describe the spread of ensemble members. In definitions of MRI and NPRI, the distance between two ensemble members was measured with the Euclid norm and the weighted standard deviation, respectively. For NPRI, the weighting factor would be high if the ensemble member has the capability to generate accurate weather forecasting. NPRI could supply the spread information of ensemble members in the forecasting horizon, compared with MRI only providing such information at a given look-ahead time. MRI and NPRI could also be used to fine-tune predictive interval of wind power output. For example, predictive interval could be narrowed if MRI or NPRI is high. Apart from MRI and NPRI, MaxMin and MaxMinMax were proposed to describe the difference between the maximum and minimum of ensemble members [93].

7. Wind power space-time scenario forecasting

As mentioned in Section 4.3, for a specified wind farm, probabilistic forecasting is usually generated at every look-ahead time independently. It provides the marginal uncertainty information of wind power output. However, the temporal and spatial correlations of wind power probabilistic forecasting are required to describe the dynamic change of wind power uncertainty. When taking into account the spatial–temporal dependency, point forecasts, probabilistic forecasts and risk index forecasts are all suboptimal inputs to decision-making problems. Therefore, the full description about the distribution of the spatial–temporal process $\hat{\boldsymbol{P}}_{t+k|t,s}$ is required, for instance multivariable PDFs. However, it is difficult to directly use joint PDFs in decision-makings. Fortunately, as an alternative, scenario is a suitable input to a large class of stochastic optimization approaches in decision-making problems. Meanwhile, scenarios or trajectories are ideal tools to visualize the multivariate probabilistic distribution.

How to characterize the interdependence structure of wind power output is of great importance to scenario forecasting. Gaussian copula method is the main approach to model such interdependence structure. Under the framework of copula method, several random variables constituting probabilistic forecasting series could be converted to a single multivariate Gaussian variable $\boldsymbol{X}$. The interdependence structure of wind power output is modeled by the covariance matrix Σ of multivariate Gaussian distribution [41]. Besides, this method could be extended to study the spatial correlation of forecasting uncertainty [94]. When considering the spatial–temporal dependence of wind power output, a multi-dimensional variable $\boldsymbol{X}$ could be constructed as

$$X_{t+k|t,s} = \Phi^{-1}(\hat{F}_{t+k|t,\,s}(p_{t+k|t,\,s})) \quad k \in K,\ s \in S \tag{13}$$

$$\boldsymbol{X} = (\{X_{t+k|t,\,s}\}_{k \in K,\,s \in S})^{\mathrm{T}} \sim N(0,\ \Sigma) \tag{14}$$

where, the set S is a collection of m locations $S=\{s_1,\ s_2,\ \cdots,\ s_m\}$, the set K is a collection of n look-ahead times $K=\{k_1,\ k_2,\ \cdots,\ k_n\}$, $\hat{F}_{t+k|t,\,s}$ is the predictive marginal distribution of wind power output. The multi-dimensional variable $\boldsymbol{X}$ follows multivariate Gaussian distribution. The covariance matrix Σ describes the spatial and temporal interdependence structure of wind power generation. Elements in the covariance matrix Σ can be estimated by parametric or non-parametric approaches, e.g. exponential smoothing. Morales et al. [95] introduced the multivariate ARMA model and employed the variance–covariance matrix to characterize the spatial–temporal correlation of wind speed distribution. The diagonal and off-diagonal parts of variance–covariance matrix were used to describe the temporal and spatial correlation, respectively. Tastu et al. [96] proposed the non-linear regime-switching model to describe the interdependence of wind power forecasting error. In Gaussian copula methods, many scenarios of wind power output could be generated through the Monte Carlo simulation.

Scenario forecasting could produce the time-dependent information among different look-ahead times, and is very useful for dynamic stochastic optimization problems. To accurately represent the wind power uncertainty, an enormous number of scenarios have to be generated through the Monte Carlo simulation. Too many scenarios lead to the increase of computational complexity and computation time when solving the dynamic stochastic optimization model. Therefore, several researchers have studied the reduction technique of wind power scenarios, which is a tradeoff between amount and accuracy. Clustering technique [97] and Kantorovich distance technique [98] are two widely used approaches. In clustering techniques, several scenarios are grouped into a cluster in terms of the defined distance between two scenarios. In Kantorovich distance techniques, the scenario with minimum Kantorovich distance is eliminated until achieving the required number of scenarios.

Some research has reported that the wind speed correlation among different wind farms might decrease the wind power uncertainty [99]. Meanwhile, the wind speed correlation has been considered in some research on power system reliability evaluation [100,101]. Therefore, scenario forecasting of wind power generation is very suitable for decision-making problems with temporal and spatial correlation. However, when the dimensionality increases (considering too much look-ahead time and too many wind farm locations), the volume of the covariance matrix Σ increases so fast that the parameter estimation becomes quite difficult. This phenomenon is also referred to as the curse of dimensionality in high-dimensional spaces, which limits the universality of application of Gaussian copula method in space-time scenario forecasts.

8. Evaluation of wind power uncertainty forecasting

Evaluation of wind power uncertainty forecasting could point out deficiencies of forecasting models, suggest potential improvements to prediction methods and establish a clear ranking approach of different forecasting methodologies. As mentioned in Section 1, evaluation of uncertainty forecasting is more difficult than that of spot forecasting. Therefore, evaluation approaches of deterministic and uncertainty forecasting are quite different. Two crucial issues of evaluating wind power uncertainty forecasting involve

- How to define required properties of the uncertainty forecasting evaluation.
- How to establish the overall framework of the uncertainty forecasting evaluation.

8.1. Required property

Reliability, sharpness and skill score are considered as the main required properties of probabilistic forecasting [102]. Firstly, wind power probabilistic forecasts should be probabilistic calibrate. This is an important request for probabilistic forecasts to be used in decision-making problems. Results in Section 3 suggest that the optimal bidding amount is equal to the predictive quantile with special nominal proportion. The probabilistic bias of quantile forecasts would result in the sub-optimality of decision-making results. In addition, ensuring probabilistic calibration in forecasts is also the precondition for generating the spatial–temporal trajectory of wind power generation such as that of Section 7. Secondly, wind power probabilistic forecasts should be sharp, which means that the predictive interval width should be as small as possible. The narrow predictive interval is more informative and more competitive than the wide one. The narrower predictive interval enables prediction end-users to make decision more easily. Therefore, the sharp wind power uncertainty forecasting would be popular with decision-makers of power system operation. Thirdly, it is found that the quality of uncertainty forecasts is not determined unilaterally by a certain property, but by reliability and sharpness together. Sometimes two properties seem to be conflicting in the evaluation of probabilistic forecasts. In practical, forecasting end-users often want to use a unique evaluation criterion that could describe the quality of forecasts method. Similar to evaluation criteria of wind power spot forecasting (e.g. MAE, MSE and RMSE), the skill score could provide the whole information of uncertainty forecasting performance involving reliability and sharpness. Such skill score is required to be proper so that the forecasts with higher skill could obtain a higher score. The propriety of skill score ensures that the perfect forecasts obtain the best score. These three properties would be introduced in detail as follows.

8.1.1. Reliability

Reliability is referred to as the statistical consistency of predictive distribution and observations. Therefore, it is a joint property of forecasting distribution and measurements [103]. The focus of reliability evaluation has been on the nominal proportion of predictive quantile in recent years. In the dataset containing enough measurements of wind power output, the empirical proportion of predictive quantile could be obtained through comparing predictive quantile and measurements. For example, as shown in Fig. 4, five of the ten measurements are lower than the specified predictive quantile $q^{(50\%)}$. Then, the empirical proportion of predictive quantile is 50% and equal to the nominal proportion. However, it is only an ideal case that predictive quantile possesses the highest reliability. In practice, for the reliable predictive quantile, the empirical proportion should be as close as possible to the nominal one.

Diagnostic tools of reliability evaluation involve Quantile–Quantile (Q–Q) diagram [84], reliability diagram [102], Probability

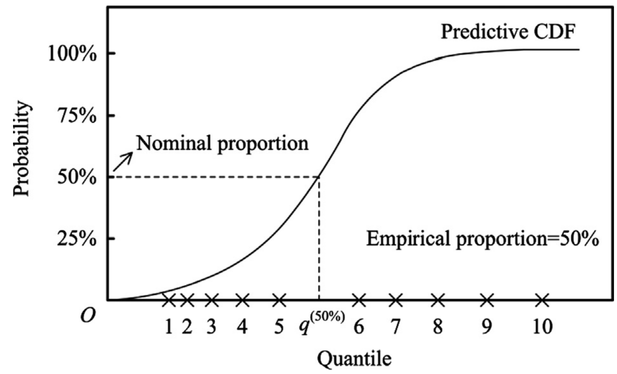

Fig. 4. Example of predictive distribution with the highest reliability. The solid line denotes predictive CDF. Ten measurements of wind power output are denoted by cross symbols and numbered from 1 to 10.

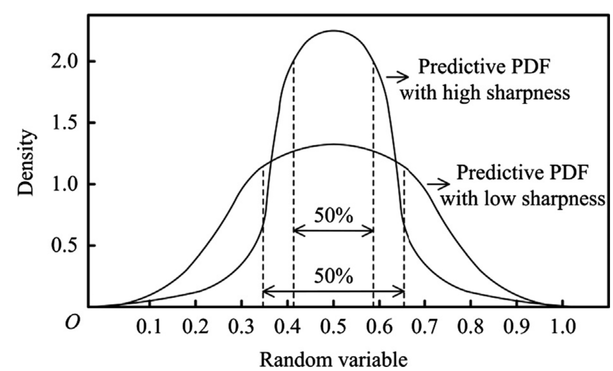

Fig. 5. Example of predictive distribution with high or low sharpness. The solid line denotes predictive PDF. Prediction interval with 50% confidence level is denoted by the dotted line.

Integral Transform (PIT) [103]. In Q–Q diagrams, horizontal and vertical coordinates are predictive and empirical quantiles, respectively. If wind power uncertainty forecasting possesses high reliability, the curve of Q–Q diagram should be as close as possible to the diagonal line. In reliability diagrams, horizontal and vertical coordinates are nominal and empirical proportions of predictive quantile, respectively [102]. Similar to Q–Q diagrams, the closer to the diagonal line, the more reliable. The reliability diagram with consistency bars was proposed to characterize sampling effects and serial correlation [104]. Uncertainty forecasting would be confirmed to be reliable if the empirical proportion of predictive quantile lies within consistency bars. The PIT relates to the conclusion that data values could be converted to random variables following the uniform distribution. If the uncertainty forecasting is reliable, PIT values should follow the uniform distribution.

8.1.2. Sharpness

Sharpness is referred to as the concentration of predictive distribution. It is just a single property of predictive distribution [103]. In general, the distribution of predictive interval width is the focus of sharpness evaluation. For example, as shown in Fig. 5, the predictive interval width should be as small as possible if the forecasting result is a sharp predictive distribution.

Diagnostic tools of sharpness evaluation involve the box plot [31] and the average width of predictive interval [102,103]. Several quantiles of predictive interval width distribution could be showed in the box plot [31]. If wind power uncertainty forecasting possesses high sharpness, quantiles of predictive interval width distribution should be as concentrated as possible. The averaged width of predictive interval could also be used to measure the sharpness [103]. In the δ diagram proposed in [102], horizontal and vertical coordinates were the nominal coverage rate and averaged width of predictive interval, respectively. The averaged width of predictive interval should be as small as possible if wind power uncertainty forecasting is sharp.

8.1.3. Skill score

Skill scores could be defined in terms of different scoring rules. Scoring rules are required to be proper so that the perfect forecasts obtain the best score. In the evaluation of wind power uncertainty forecasting, common wide-used skill scores are summarized in Table 5.

Different skill scores are constructed in terms of random variable types and uncertainty representations. The logarithm score is highly sensitive to outliers and extreme events, which may lead to the occurrence of an infinite skill score in the evaluation of wind power uncertainty forecasting [105]. Compared with the logarithm score, CRPS is more robust when dealing with outliers and extreme events [103]. The tick or check loss score and the interval score were proposed to evaluate the predictive quantile and predictive interval of wind power output, respectively [106]. Energy score is the multivariate generalization of CRPS and quite useful for the evaluation of multivariate random variable, e.g. (u, v)-wind [107]. With the help of energy score, the direct comparison of predictive performance could be made among different types of forecasting information, e.g. deterministic forecasting, probabilistic forecasting and discrete ensemble forecasting. To increase the participation of forecasting end-users in the evaluation of uncertainty forecasting, the novel evaluation based on pre-defined event was proposed [108]. Events which catch much interest of forecasting end-users would be considered at first. For example, the capacity to generate much electric power in a period of time is very important for wind farms connected to power grid. Therefore, the long-lasting event, a kind of pre-defined events, is defined as wind power output being larger than 50% of the installed capacity in a successive period of 6 h.

8.2. Evaluation framework

After defining required properties of the uncertainty forecasting evaluation, it is necessary to establish the evaluation framework and decide which property should be evaluated at first. Pinson et al. [102] pointed out that reliability was the primary requirement of probabilistic forecasting evaluation, and should be evaluated in the first step. The sharpness represented the inherent value of probabilistic forecasting models. On the basis of satisfying the reliability requirement, the performance of forecasting models could be ranked through making comparisons of sharpness. Gneiting et al. [103] also proposed an important evaluation principle, i.e. maximizing the sharpness of predictive distribution subjected to the reliability. The whole performance of uncertainty forecasting could be compared by skill score directly. However, the reliability requirement could not be always satisfied even if the forecasting model possesses the competitive skill score. Therefore, at the first stage in the evaluation of wind power uncertainty forecasting, the reliability evaluation plays the role of basic assessment and should be carried out firstly. If the forecasting model is confirmed to be reliable, comparative assessments involving sharpness and skill score could be executed at the second stage.

How to assess probabilistic forecasts is still seen as the major difficulty in the implementation of wind power uncertainty forecasts. Results in Section 3 suggest that the quality of probabilistic forecasts has much influence on the optimality of decisions. Apart from reliability, sharpness and skill score as reviewed in Section 8, further verifications of probabilistic forecasts are required in the future research. The uncertainty of wind power generation is situation-dependent and usually affected by many factors, e.g. forecasting horizon, the predictive value of wind speed. Therefore, conditional evaluation of wind power uncertainty forecasting is required in the future study. This required property in evaluation is also referred as to the resolution [102], which is defined as the capability of generating different uncertainty forecasting in terms of predictive conditions. Meanwhile, how to evaluate multivariate PDFs or spatial–temporal trajectories is also a crucial issue of concern. In the multivariate framework, the evaluation of multivariate PDFs would be affected by the curse of dimensionality in high-dimensional spaces. This open question is also very valuable to be studied in future studies.

9. Discussion

With the rapid development of the uncertainty forecasting method, the application of uncertainty forecasting in power system engineering might happen in the future. Today, forecasts end-users may still prefer to get wind power single-value forecasts because point forecasts are more easily to be understood and accepted. It is much expensive to generate uncertainty forecasts, and is hard to verify. Nowadays, probabilistic information coming from uncertainty forecasts is still difficult to be appreciated by users. However, after integrating more and more stochastic power generation in power system, traditional point forecasts cannot satisfy the requirement of uncertainty information in decision-making problems. Therefore, methodologies of wind power forecasts should be developed in the probabilistic framework. The development of uncertainty forecasting would be of great benefit to increase the penetration of wind power generation. However, the study on wind power uncertainty forecasting is still in the early stage. Much problem and challenge still exists in this field.

9.1. Development of new wind power uncertainty forecasts

How to improve the quality of wind power forecasting is still the biggest challenge currently. Forecasting of wind power generation is a cross-disciplinary research, which requires the combination of expertise in meteorology, statistics, and power systems engineering. Through studying the better description of atmospheric physics and the higher spatial–temporal resolution of NWP, meteorological institutions are expected to provide more accurate prediction of weather variables. Meanwhile, there are many approaches to improve the accuracy of existing wind power prediction models. Quite few studies have been performed on wind spatial–temporal dynamic behaviors and its influence on wind power forecasts. Wind farms are distributed geographically in a region. They construct a huge network of measurement equipments, which could capture the information on the spatial–temporal propagation of wind power forecasts errors [96,109]. Taking into account geographically distributed information, we can develop non-linear models to character spatial–temporal dynamics of wind power generation. Therefore, most of existing wind power forecasts systems should be optimized because they only use on-site information and run locally for a single wind farm. Some researchers have found that the quality of wind speed point forecasts could be improved by using meteorological observations in the vicinity of wind farm [53]. In the future, we should devote more attention to the modeling of wind spatial–temporal dynamics, and integrate the off-site information (coming from neighboring wind farms) into existing wind power forecasts systems. Forecasts systems of all wind farms in a region would be developed and operated together. There is no requirement to recognize and estimate forecasts models for each single wind farm of interesting.

KDE method is popular among non-parametric approaches. KDE method could provide the whole information of wind power predictive distribution. Meanwhile, KDE method could easily deal with the multi-modal situation which is frequently observed in wind power generation. However, there still exist some problems in direct applications of the conventional KDE for probabilistic forecasts. Wand et al. [110] found the conventional KDE works well for near-Gaussian distributions but not for ones that are significantly different from Gaussian. Studies have indicated that the asymmetric distribution of wind power output was found to be heavy-skewed and heavy-tailed [45]. Therefore, wind power distribution is quite different from Gaussian and could be categorized as ill-conditioned PDF. It is difficult to accurately estimate such an ill-conditioned distribution in the conventional KDE method. In the last two decades, some modifications of kernel density estimator were proposed, for instance transformation-based KDE [111]. Transforming a variable to facilitate model estimation is a common used approach in statistical analyses. After the transformation of original data, the new distribution followed by transformed data is closer to the Gaussian distribution. The benefit of this transformation is the bias reduction of KDE method in the transformed scale. Wind power density is a distribution with compact support. Therefore, KDE model suffers from boundary effect problem [112]. The main reason of boundary problem for the conventional KDE is that, at a boundary point, kernel mass falls outside the support of the estimated PDF, resulting in the loss of probability density. One can also think that boundary effect problems are caused by the discontinuity of wind power distribution across the boundary [113]. The compact support of wind power density has significantly adverse impact on the performance of KDE model. In order to eliminate boundary error of KDE model, we introduce boundary kernel method [114], which is a linear combination of two types of kernel function. As a result, the density would not leak outside the boundary of PDF.

Compared with conventional statistical approaches, the performance of wind power spot forecasting could be improved by artificial intelligence techniques. In recent year, direct quantile forecasts based on AI approach have drawn much attention [29,66,90]. AI models are trained through minimizing the defined cost function. For AI approaches in the non-parameter framework, the cost function is directly related to the evaluation of predictive intervals or quantiles. Regardless of structures of AI models (i.e., neural network or machine learning), the crucial issue is to ensure the propriety of scoring rule when evaluating predictive intervals or quantiles. It is suggested that future research of probabilistic forecasts based on artificial intelligence approaches should be built on the foundation of proper scoring rule. Gneiting et al. [68] found that CRPS was a proper scoring rule which could verify the whole performance of probabilistic forecasts. Compared with most of existing scoring rules in AI models, CRPS-based cost function may be a better choice. Full descriptions of predictive CDFs are required in decision-making problems such as that of Section 3. This means that a specific AI model needs to be set-up and trained for every quantiles in Eq. (6), resulting in the high computational cost. Since these AI models are trained independently, they may result in crossing quantiles. This is not desirable for both theoretical and practical applications. In the future, for avoiding crossing quantile, some constraints should be added in parameter estimation of AI models. Meanwhile, how to reduce computational cost of AI-based quantile forecasts is also an open question of concern.

9.2. Accommodating more wind power through electricity market

Similar to wind power spot forecasting, how to improve the quality of wind power uncertainty forecasting is still the most crucial problem. However, the quality only represents the statistic property of uncertainty forecasting model. Predictive end-users and power system operators usually have more interest in the value of wind power uncertainty forecasting. This value could be described by great benefits from the application of uncertainty forecasting in power system operation. Therefore, more attention should be paid to the value of wind power uncertainty forecasting in the future study. Results in Section 3 suggest that probabilistic forecasts, instead of point forecasts, are optimal inputs to decision-making problems. It is believed that the usage of wind power uncertainty forecasts could gain more benefits for forecasts end-user. With the application of wind power uncertainty forecasts in power system decision-markings, more forecasts end-user will accept and appreciate uncertainty forecasts in the future.

In recent year, many researchers have called for the participation of wind power producers in electricity market in the same way as conventional power producers. Researchers believed that more renewable energy generation could be consumed in the liberalized electricity market [32–34]. With high penetration of stochastic power generation in power grid, more uncertainty should be dealt with in the conventional electricity market operation. On the base of respecting power system operation constrains, in order to accommodate more stochastic power generation through electricity market, some innovations are required in designing market clearing and market operation. Uncertainty forecasting technique and stochastic optimization method would play important roles in the revolution of electricity market. In the uncertain market environment, quantitative information on all sources of uncertainty is quite important for making the best decision. Therefore, in future studies, uncertainty forecasts could be extended to other energy-related forecasts. Apart from wind power generation, we also require uncertainty forecasts of photovoltaic power generation, electric vehicle load and demand–response relationship. The probabilistic forecasting information is an optimal input to relevant decision-making problems in electricity market. Then, the modeling and the solving of decision-making problems should be built under stochastic optimization framework, instead of the deterministic one. In Section 3, the optimal bidding strategy is obtained through maximizing the expected revenue of wind farm. In addition, other stochastic modeling methods could be applied in decision-making problems. The well-known chance-constrained model [115] is a feasible scheme for adding power system operation constrains into decision-making problems. Apart from some specific forms of expectation and chance-constrained models, we cannot obtain a closed-form solution when solving most of stochastic optimization problems. Therefore, stochastic optimization problems are usually solved by intelligent optimization algorithms, e.g. Genetic Algorithms (GA) and Particle Swarm Optimization (PSO). Finally, great challenges could be foreseen in the future electivity market involving different types of uncertainty. Therefore, a better interaction among mathematicians, economists, meteorologists and power system engineers is required in the research of market design, market operation, uncertainty forecasts and stochastic optimization.

9.3. *Wind power forecasts in smart grid context*

With wide range deployment of smart grid technologies, wind power forecasts are facing both opportunities and challenges. Firstly, in smart grid context, high-speed communication networks enable utilities to gather more data related to wind power generation than ever before. The big data available may help us better understand the spatial–temporal dynamic of wind power generation and further develop new wind power forecasts. Utilities are facing analytic issues with making sense and taking advantage of the big data. In the future, data mining technology may be a hot topic in the research of wind power forecasts. Through the automatic or semi-automatic analysis of large quantities of data, data mining technology help us to extract previously unknown interesting patterns (such as dependencies). These patterns can then be seen as a summary of input data, and may be used in further analysis (such as prediction), which benefits the improvement of the quality in wind power forecast. Secondly, in order to reduce reserve requirements and increase renewable energy penetration, a new trend in smart grid is the higher temporal resolution when dispatching power generation. This translates to a new challenge that we should develop wind power forecasts models at higher temporal resolution, for instance a few minutes. However, for most of conventional NWP models, a severe limitation is its temporal resolution (only 1 h). In future works, new meteorological observations at a higher spatial–temporal resolution should be introduced into wind power forecasts. For instance, Trombe et al. [116] proposed to apply observations of weather radar in wind power forecasts models. Other useful meteorological observations include information from weather satellite.

10. Conclusion

Wind power generation is a non-linear and non-stable process. It could be modeled as a stochastic process. Both wind speed and wind-to-power curve exhibit high degree of variability, which become two main sources of uncertainty in wind power forecasts. Forecasting information is widely used as input to several decision-making problems related to power system operation. Unlike point prediction, which produces only single-valued expectation of wind power output, uncertainty forecasts could provide additional quantitative information on the uncertainty. With more and more stochastic power integrated in power grid, wind power uncertainty forecasts are better choices for decision-makings in the uncertain environment. Therefore, energy-related forecasts method should be developed in the probabilistic framework. Based on uncertainty representations employed, various techniques of wind power uncertainty forecasting are classified into three categories, i.e. probabilistic forecasting, risk index forecasting, and scenario forecasting. Evaluation of uncertainty forecasting is more difficult than that of spot forecasting, which requires defining some properties and establishing evaluation framework. Researches on wind power uncertainty forecasting have been very active in recent years. The rapid increase of installed capacity of wind power in the past 15 years requires the continuous progress of wind power prediction techniques. Wind power uncertainty forecasting is a novel approach and very suitable to describe the randomness and intermittence of wind resources. In power system operation, uncertainty forecasting approaches could reduce the economic and technical risk caused by wind power uncertainty. Therefore, it is necessary to carry out more research on wind power uncertainty forecasting in the future.

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