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Markov regime-switching quantile regression models and financial contagion detection

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HIGHLIGHTS

- We propose a Markov regime switching quantile regression model for the first time.
- A simulation study of the model is conducted and shows that the MLE method is efficient.
- An empirical analysis, which focuses on the detection of financial contagion is presented.
- In a crisis situation, the interdependence between US and EU countries dramatically increases.

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ABSTRACT

In this paper, we propose a Markov regime-switching quantile regression model, which considers the case where there may exist equilibria jumps in quantile regression. The parameters are estimated by the maximum likelihood estimation (MLE) method. A simulation study of this new model is conducted covering many scenarios. The simulation results show that the MLE method is efficient in estimating the model parameters. An empirical analysis is also provided, which focuses on the detection of financial crisis contagion between United States and some European Union countries during the period of sub-prime crisis from the angle of financial risk. The degree of financial contagion between markets is subsequently measured by utilizing the quantile regression coefficients. The empirical results show that in a crisis situation, the interdependence between United States and European Union countries dramatically increases.

1. Introduction

In financial practice, it is of great importance to know the dependence structure among various financial market indices. Such study is critical for risk management, asset pricing and portfolio optimization (McNeil et al., 2015; Sharpe et al., 1999). Early works on analyzing the dependence structure have primarily focused on examining the simple correlation coefficients between market indices (Billio and Pelizzon, 2003). However, it has been widely accepted and also verified that there exists extensive non-linear and asymmetric dependence structure among financial markets (Patton, 2006; Okimoto, 2008). Thus the copula method was introduced for studying financial interdependence (Rodriguez, 2007; Durante and Jaworski, 2010). However, none of those approaches has considered detecting the contagion effect using a specific quantile of financial return. It is well known that the essence of financial contagion is the risk spillover, and the most popular risk measure is Value at Risk (VaR), which can be considered as a given quantile of return distribution of one asset or portfolios. Thus, in this paper, the financial contagion is detected from the perspective of conditional financial risk via a quantile regression model.

Quantile regression has recently attracted increasing research attention in finance. It allows one to examine the effect of explanatory variables on the dependent variable at quantiles of the dependent variable's conditional distribution, and was initially introduced as a robust regression technique when the typical assumption of normality of the error term might not be strictly satisfied (Koenker and Bassett, 1978). Because the risk measure VaR is a specific negative quantile, it can be estimated by quantile regression model directly. Engle and Manganelli (2004) developed conditional autoregressive Value at Risk (CAViaR) models, a class of models suitable for estimating conditional quantile in dynamic

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settings, which were used to estimate the market risk. Chuang et al. (2009) and Lee and Li (2012) applied quantile regression to model the dependence of financial variables, e.g. trading volume and return volatility. While adopting the quantile regression method, the financial literature can only prove the existence of interdependence by analyzing the significance of regression coefficients (Baur and Schulze, 2005). A notable exception is Ye and Miao (2012), who proposed a new approach to detect the financial contagion by examining the time-varying regression coefficients based on non-parametric quantile regression, which enables us to capture the dynamic process of interdependence by time between different countries.

It is noted the possibility of the existence of regime switching effects is not taken into consideration in the above papers that have focused on quantile regression. In order to address this problem, a Markov regime-switching quantile regression model is considered in this paper. Markov switching models are traced back to Quandt (1958). Later, Goldfeld and Quandt (1973) proposed a useful regime-switching regression model, in which the latent variable controlling the regime changes follows a Markov chain. Hamilton (1989) extended the Markov regime-switching model to dependent data, such as AR models. For its advantage of capturing the probability of jumps in different equilibria (Hardy, 2001), the Markov regime-switching model has become one of the most popular models in economic research. Recently, Markov regime-switching models have also been applied in various fields, which include regime-switching vector autoregressive models (Krolzig, 2013), regime-switching regression models with endogenous switching (Kim et al., 2008), regime-switching copula models (Stoeber and Czado, 2012), and so on.

The important applications of quantile regression model can be seen in risk management, portfolio optimization, and asset pricing. For example, Value at Risk, which is the most important measure of financial risk, can be estimated by quantile regression model. The proposed model can measure abrupt changes of financial risk under different regimes. Because of its unique characteristics in capturing parameter jumps, the Markov regime-switching model has been implemented by numerous literatures in the application of detecting financial contagion (Jeanne, 1997; Jeanne and Masson, 2000; Chollete et al., 2009; Guo et al., 2011). In this paper, we present supportive simulation and empirical evidences to show that the Markov regime-switching quantile regression model is well suited for this task.

We allow the regression coefficients to vary when the Markov state changes. In this way, it enables us to examine the variation tendency from the filter probability graph and to measure the degree of financial contagion through changes in regression coefficients under different states. The financial contagion will be detected from the angle of financial risk (VaR) changing over time, which is reflected in the changes of regression coefficients under different Markov states. If the risk in one state (crisis period) is significantly larger than that in the other state (pre-crisis period), we conclude that financial contagion exists.

In this paper, we make two contributions to the literature. The first contribution is that we develop a Markov regime-switching quantile regression model to consider the regime-switching effect methodologically. The parameters of linear quantile regression model are estimated by Asymmetric least absolute deviation method. In this paper we present an estimate method of the parameters by Maximum Likelihood Estimation (MLE). Our second contribution is empirical in nature. The financial risk is usually measured by Value at Risk, which can be estimated by Markov regime-switching quantile regression model directly. In this paper, the financial contagion is detected from the angle of financial risk (VaR) changing under different Markov states by the proposed model.

The rest of the article is organized as follows. The Markov switching quantile regression model is briefly introduced in Section 2; In Section 3, simulation studies of the proposed model are provided. In addition, an empirical case is presented which aims to examine the financial contagion effects during sub-prime crisis period. Finally, the article is concluded by a brief discussion.

2. Markov switching quantile regression

A Markov switching quantile regression model is primarily based on the linear quantile regression. In the first part of this section, we present a brief introduction of linear quantile regression. We then introduce a linear quantile regression model, an extension of traditional linear regression model, which assumes a linear relationship between the conditional quantile of dependent variable for each given quantile and independent variables.

2.1. Linear quantile regression model

Let Y be a dependent variable, and $y_1, \ldots, y_n$ be its n observations. The distribution of Y is denoted by $F(y) = P(Y \leqslant y)$. Thus for all the $\tau \in (0, 1)$, the τth quantile of Y is defined as:

$$Q_\tau(Y) = \inf\{y : F(y) \geqslant \tau\}.$$

Let explanatory variable $\boldsymbol{x}_t = (x_{t,1}, \ldots, x_{t,p})^T$ be a $p \times 1$ random vector for $t = 1, \ldots, n$. Then the linear quantile regression model is written as:

$$y_t = \boldsymbol{x}_t^T \boldsymbol{\beta}_\tau + u_{t,\tau}, \quad t = 1, 2, \ldots, n, \tag{1}$$

where the error term $u_{t,\tau}$ satisfies that $Q_\tau(u_{t,\tau}|\boldsymbol{x}_t) = 0$. Thus Y's τth conditional quantile is given by $Q_\tau(Y|\boldsymbol{x}_t) = \boldsymbol{x}_t^T \boldsymbol{\beta}_\tau$. In analogy to Eq. (1), the estimation of quantile regression coefficients is formulated as the solution to a minimization problem (Koenker and Bassett, 1978):

$$L_{\tau,n}(\boldsymbol{\beta}_\tau) = \sum_{t=1}^{n} \rho_\tau(y_t - \boldsymbol{x}_t^T \boldsymbol{\beta}_\tau) \tag{2}$$

with the check function

$$\rho_\tau(\mu) = \begin{cases} \tau\mu, & \mu \geq 0, \\ (\tau - 1)\mu, & \mu < 0. \end{cases}$$

Since $\rho_\tau(u)$ is not differentiable at the origin, there is no explicit solution to the minimization of Eq. (2). However, the problem has been solved in Koenker et al. (1994), who suggested an algorithm called the interior point algorithm that has been widely used in estimating quantile regression coefficients.

Koenker and Machado (1999) proposed to apply a skew distribution for quantile regression, which was named as asymmetric Laplace distribution (ALD). Thus minimizing $L_{\tau,n}(\boldsymbol{\beta}_\tau)$ in Eq. (2) with respect to parameters $\boldsymbol{\beta}_\tau$ is identical to maximizing the likelihood based on the following asymmetric Laplace probability density (ALPD):

$$f(y; \boldsymbol{x}, \boldsymbol{\beta}_\tau, \tau, \sigma) = \frac{\tau(1-\tau)}{\sigma} \exp\left(-\frac{\rho_\tau(y - \boldsymbol{x}^T \boldsymbol{\beta}_\tau)}{\sigma}\right) \tag{3}$$

for a given τ. It is noted that if $\tau = 0.5$, Eq. (3) becomes the symmetric Laplace (double exponential) density function of $Y - \boldsymbol{x}^T \boldsymbol{\beta}_\tau$.

After obtaining the estimates of regression coefficients $\hat{\boldsymbol{\beta}}_\tau$, the conditional quantile function under the assumption of linear dependence can be formed as

$$Q_\tau(Y|\boldsymbol{x}) = \boldsymbol{x}^T \hat{\boldsymbol{\beta}}_\tau.$$

2.2. Markov regime-switching quantile regression model

In this paper, the existence of regime switching effects is taken into consideration in quantile regression model.

$$Pr(S_{t-1}=j|\Omega_{t-1},I_{t-1};\boldsymbol{\theta})=\sum_{i=1}^{2}Pr(S_{t-1}=j,S_{t-2}=i|\Omega_{t-1},I_{t-1};\boldsymbol{\theta})$$
$$=\sum_{i=1}^{2}\frac{f(y_{t-1}|S_{t-1}=j,S_{t-2}=i,\Omega_{t-1},I_{t-2};\boldsymbol{\theta})Pr(S_{t-1}=j,S_{t-2}=i|\Omega_{t-1},I_{t-2};\boldsymbol{\theta})}{f(y_{t-1}|\Omega_{t-1},I_{t-2};\boldsymbol{\theta})}$$
$$=\sum_{i=1}^{2}\frac{f(y_{t-1}|S_{t-1}=j,\Omega_{t-1};\boldsymbol{\theta})Pr(S_{t-2}=i|\Omega_{t-2},I_{t-2};\boldsymbol{\theta})p_{ij}}{f(y_{t-1}|\Omega_{t-1},I_{t-2};\boldsymbol{\theta})}.$$

Box I.

Let $\{(y_t,\boldsymbol{x}_t),\ t=1,\dots,n\}$ be a sample of size n, where y_t is the tth observation of the dependent variable Y and $\boldsymbol{x}_t$ is the tth p-vector of observed explanatory variables. Consider the following model

$$y_t=\alpha+\boldsymbol{x}_t'\boldsymbol{\beta}_{S_t}+u_{t,\tau},\quad t=1,\dots,n, \tag{4}$$

where α is the intercept parameter, S_t is the state variable that takes value 1 or 2, $\boldsymbol{\beta}_{S_t}$ is a p-vector of unknown regression coefficients, which implies different effects to the quantile of explained variable of explanatory variable under different regimes. σ is an unknown scale parameter, and $u_{t,\tau}$, $t=1,\dots,n$, are independent and identically distributed (i.i.d.) random errors with probability density function $\tau(1-\tau)\exp\{-\rho_\tau(u)/\sigma\}/\sigma$ (the τth quantile of $u_{t,\tau}$ is zero). The model (4) is a quantile regression with regime-switching coefficients. If we further require that the probability of changing regime depends only on the current regime, not on the history of the process, the resulting model becomes a Markov regime-switching quantile regression model.

Assume that there are a total of k number of regimes. For simple presentation, we only study the case that $k=2$ here considering that the two-regime case is a popular specification in applied work, and can be extended to a case with more states directly.

Let the switching probability from regime i to regime j be p_{ij}, i.e.,

$$p_{ij}=Pr(S_t=j|S_{t-1}=i),\quad i=1,2;\ j=1,2.$$

It is obvious that the probability of Y_t staying in regime j at time t can only be inferred from currently available information $I_t=\{y_1,y_2,\dots,y_t\}$, which is $Pr(S_t|I_t)$, also known as filtered probability.

Next, the maximum likelihood estimation of the model will be proposed.

Let $\Omega_t=(\boldsymbol{x}_t,\boldsymbol{x}_{t-1},\dots,\boldsymbol{x}_1)^T$ be a matrix and $I_t=(y_1,y_2,\dots,y_t)^T$ be a vector containing observations observed through time t, and let $\boldsymbol{\theta}=(\alpha,\ \boldsymbol{\beta}_1^T,\boldsymbol{\beta}_2^T,\sigma,p_{11},p_{22})^T$ be the vector of model parameters.

The parameter set $\boldsymbol{\theta}$ can be estimated by the maximum likelihood, which is similar to the case of the Markov regime-switching log-normal model with two regimes (Hamilton, 1989; Kim and Nelson, 1999; Hardy, 2001). The details are given below.

Denote the probability density function of the tth observation given Ω_t and I_{t-1} by $f(y_t|\Omega_t,I_{t-1};\boldsymbol{\theta})$. Then the likelihood of $\{y_1,\dots,y_n\}$ is

$$L(\boldsymbol{\theta})=f(y_1|\Omega_1;\boldsymbol{\theta})f(y_2|\Omega_2,I_1;\boldsymbol{\theta})f(y_3|\Omega_3,I_2;\boldsymbol{\theta})\cdots f(y_n|\Omega_n,I_{n-1};\boldsymbol{\theta}).$$

We can compute $f(y_t|\Omega_t,I_{t-1},\boldsymbol{\theta})$ as follows:

$$\begin{aligned}f(y_t|\Omega_t,I_{t-1};\boldsymbol{\theta})&=\sum_i\sum_j f(y_t|S_t=i,S_{t-1}=j,\Omega_t,I_{t-1};\boldsymbol{\theta})\\&\quad\times Pr(S_t=i,S_{t-1}=j|\Omega_t,I_{t-1};\boldsymbol{\theta})\\&=\sum_i\sum_j f(y_t|S_t=i,\Omega_t,I_{t-1};\boldsymbol{\theta})\\&\quad\times Pr(S_t=i,S_{t-1}=j|\Omega_t,I_{t-1};\boldsymbol{\theta})\\&=\sum_i\sum_j f(y_t|S_t=i,\Omega_t;\boldsymbol{\theta})\\&\quad\times Pr(S_t=i,S_{t-1}=j|\Omega_t,I_{t-1};\boldsymbol{\theta}).\end{aligned} \tag{5}$$

Here

$$f(y_t|S_t=i,\Omega_t;\boldsymbol{\theta})=\frac{\tau(1-\tau)}{\sigma}\exp\left[-\frac{\rho_\tau(y_t-\alpha-\boldsymbol{x}_t^T\boldsymbol{\beta}_i)}{\sigma}\right]$$

as in Eq. (3). The weighting probability in (5) is computed recursively by applying Bayes Rule given initial unconditional probabilities $Pr(S_0=i)$ with $i=1,2$ as follows:

$$\begin{aligned}&Pr(S_t=i,S_{t-1}=j|\Omega_t,I_{t-1};\boldsymbol{\theta})\\&\quad=Pr(S_{t-1}=j|\Omega_{t-1},I_{t-1};\boldsymbol{\theta})Pr(S_t=i|S_{t-1}=j,\Omega_t,I_{t-1};\boldsymbol{\theta}),\end{aligned} \tag{6}$$

where $Pr(S_t=i|S_{t-1}=j,\Omega_t,I_{t-1};\boldsymbol{\theta})=p_{ji}$ is the transition probability between regimes, and $Pr(S_{t-1}=j|\Omega_{t-1},I_{t-1};\boldsymbol{\theta})$ can be found from the previous recursion, and is equal to equation given in Box I.

We can then apply an optimization method to find the estimates $\hat{\boldsymbol{\theta}}$ which maximize $L(\boldsymbol{\theta})$.

The proposed model generalizes the linear quantile regression model, and assumes a nonlinear relationship between the conditional quantile of dependent variable for each given quantile and independent variables. The model assumes the parameters change under different regimes, and can describe the complicated market behavior better. The new model can also be used as an analytic tool for wage and income studies in labor economics and so on.

3. Simulation

A simulation study of the proposed methodology is given in the section. We first describe how to generate a Markov chain with finite states and an asymmetric Laplace distributed random variable, respectively. We then proceed to the simulation study. The details are given below.

3.1. Generation of a Markov chain with finite states

We adopt the Markov chain simulation method provided by Perlin (2012). For simple presentation, the total of two states are considered here. Let p_{ij} denote the transition probability that the process lies in State j at time t, given that it was in State i at time $t-1$, where $i=1,2; j=1,2$. Assume that the initial state is i, i.e., $s_0=i$. Put $j=1$. A Markov chain $\{s_t\}$ of length N can be generated as follows:

Step 1. Generate a random number z from the standard uniform distribution $U(0,1)$.
Step 2. When $z\le p_{s_{j-1}s_{j-1}}$, the Markov chain remains at the current state, i.e., $s_j=s_{j-1}$. Otherwise, the Markov state jumps to a new state, i.e., $s_j=(s_{j-1}+1, \bmod 2)$.
Step 3. Repeat Steps 1–2 N times to obtain a Markov chain of length of N.

3.2. Generation of an asymmetric Laplace distributed random variable

Denote the asymmetric Laplace distribution with the following density

$$\tau(1-\tau)\exp\{-\rho_\tau(\xi-\mu)/\sigma\}/\sigma$$

as $ALD(\tau,\mu,\sigma)$. It can be seen that if $\Xi \sim ALD(\tau,\mu,\sigma)$, then there exists $Z \sim ALD(\tau,0,1)$ such that Ξ has the same distribution as $\mu+\sigma Z$. By Yu and Zhang (2005), if W_1 and W_2 are independent and identically standard exponential distributed, then $W_1/\tau - W_2/(1-\tau)$ has the distribution $ALD(\tau,0,1)$. Since it is easy to generate a standard exponential distributed random variable, $ALD(\tau,\mu,\sigma)$ distributed random variables can hence be generated easily for given μ and σ.

3.3. Simulation details and results

We can now conduct a simulation study to investigate the performance of maximum likelihood when it is used to estimate parameters $\boldsymbol{\theta}$ of a Markov regime-switching quantile regression model given below:

$$y_t = \alpha + \boldsymbol{x}_t'\boldsymbol{\beta}_{S_t} + u_{t,\tau}, \quad t = 1,\ldots,n,$$

where α is the intercept parameter, S_t is the state variable, $\boldsymbol{\beta}_{S_t}$ is a p-vector of unknown regression coefficients, σ is an unknown scale parameter, and $u_{t,\tau},\ t=1,\ldots,n$, are independent and identically $ALD(\tau,0,\sigma)$ distributed random errors. Even though there is no need to limit the number of regimes to 2 in the simulation study, for the purpose of simple illustration, the number of regimes is chosen as $k=2$. The following parameter setting is used in the simulation implementation:

- $\boldsymbol{x}_i = (x_{i1}, x_{i2})^T$ is independently generated from the normal distribution with mean 0.5 and standard deviation 0.2;
- $\alpha = 0.1, \beta_1 = -0.5, \beta_2 = 0.3$;
- $\sigma = 0.2, \tau = 0.05,\ 0.25,\ 0.65,\ 0.8$;
- $p_{11} = p_{22} = 0.9$.

We remark that the intercept term α and the asymmetric Laplace distribution scale parameter σ are set to be invariant as Markov regime switches, which thus provide a contrast between regime-independent and regime-dependent effects. Here, β_1 and β_2 are assigned different opposite value, which is consistent with actual situation. The opposite value implies different effects to the quantile of the explained variable of the explanatory variable under different regimes such as bear and bull markets.

For each $\tau = 0.05,\ 0.25,\ 0.65,\ 0.8$ and each $n = 100, 500$, and 1000, we generate a sample $\{(y_t, \boldsymbol{x}_t),\ t = 1,\ldots,n\}$ with 1000 repetitions. As discussed in Section 2.2, we find the maximum likelihood estimates of the parameters $\boldsymbol{\theta}$. Tables 1–2 display the average bias and standard deviation of the differences between the actual parameter values and their estimates. It can be seen the biases and standard deviations are relatively small, which tend to decrease as the number of simulations increases. We can conclude that it is appropriate to use the proposed methodology given in Section 2.2 to estimate parameters of a Markov regime-switching regression model.

Table 1
Simulation results.

τ	n	p_{11}		p_{22}	
		Bias	Std	Bias	Std
0.5	100	−0.0481	0.1395	−0.0551	0.1825
	500	−0.0440	0.1301	−0.0568	0.1871
	1000	−0.0412	0.1197	−0.0446	0.1801
0.25	100	−0.0025	0.0562	−0.0037	0.1049
	500	−0.0151	0.1348	−0.0091	0.1241
	1000	−0.0075	0.1231	0.0017	0.0847
0.65	100	−0.0806	0.2364	−0.0929	0.2671
	500	−0.0751	0.2302	−0.0729	0.2221
	1000	−0.0704	0.2167	−0.0797	0.2384
0.8	100	−0.0716	0.2469	−0.1236	0.3074
	500	−0.0612	0.2356	−0.0822	0.2567
	1000	−0.0488	0.2223	−0.0687	0.2412

4. Empirical study

As commented previously, the most popular risk measure is VaR, which is defined as a lower quantile of financial return distribution and can be estimated by quantile regression model directly. In this section, we demonstrate that the proposed Markov quantile regression model can be used not only to detect the existence but also analyze the trend of financial crisis contagion between highly related countries during the period of a crisis. To ensure the Markov property, the weekly log-returns of the US S&P 500 index, France CAC 40 index, and Germany DAX 30 index from January 14, 2005 to October 31, 2008 are utilized, which covers both a pre-crisis period and crisis period in a wide range. All the data are available in *China Stock Market and Accounting Research Database*.

To find out the relationship between the log-returns of the S&P 500 index time series and each of two main European country index time series during the sub-prime crisis period in terms of VaRs, we assume that there are two regimes, namely non-crisis regime (regime 1) and crisis regime (regime 2). Thus weekly log-returns of the CAC 40 index and the DAX 30 index are respectively fitted to a two-state Markov regime quantile regression model with the same explanatory variable, weekly log-return of the S&P 500 index. For convenience, the first fitting and the second fitting are respectively denoted by $\mathbb{C}-CAC$ and $\mathbb{C}-DAX$. Put $\tau = 0.2$. We remark that only the regression coefficient varies with regimes. We then proceed to obtain the maximum likelihood estimates $\hat{\boldsymbol{\theta}} = (\hat\alpha, \hat\beta_1, \hat\beta_2, \hat\sigma, \hat p_{11}, \hat p_{22})^T$ of the model parameters $\boldsymbol{\theta} = (\alpha, \beta_1, \beta_2, \sigma, p_{11}, p_{22})^T$.

In Table 3, we report the maximum likelihood estimates of model parameters (the columns under "estimate"), along with t-statistics (the columns under "t-stat"), and the maximum log likelihood values ("$\log L$"), where the computation of a t-statistic for the significance test is referred to Hamilton (1994) and Kim and Nelson (1999). As displayed in this table, the significant test results indicate that the model is suitable for modeling the weekly log-returns of the CAC 40 index and the DAX 30 index against the same explanatory variable, weekly log-return of the S&P 500 index. By the same table, it can also be observed that there exist nonignorable differences between the estimates $\hat\beta_1$ and $\hat\beta_2$ for both $\mathbb{C}-CAC$ and $\mathbb{C}-DAX$, which indicates that the regression coefficient does vary with regimes. In addition, one may further notice that for both $\mathbb{C}-CAC$ and $\mathbb{C}-DAX$, the estimate $\hat\beta_1$ is smaller than the estimate $\hat\beta_2$, which implies in regime 2, the behavior of the CAC 40 index or the DAX 30 index is more similar to the behavior of the S&P 500 index than in regime 1. Since regime 2 is the crisis regime, therefore the risk in regime 2 is larger than that in regime 1. Denote the weekly log-return of the S&P 500 index by r_t. By the relation between VaR and conditional quantile, it follows that

$$\mathrm{VaR}_t = -\hat\alpha - \hat\beta_i r_t.$$

Thus we can also conclude that the conditional risk in regime 2 is higher than that in regime 1 because $\hat\beta_2$ is larger than $\hat\beta_1$.

We now examine the financial contagion effect by combining smoothed probabilities (Refs.: Hamilton, 1994 and Kim and Nelson, 1999) with the actual time of the sub-prime crisis. The smoothed estimates of $Pr(S_t = 2)$ for both $\mathbb{C}-CAC$ and $\mathbb{C}-DAX$ are displayed

Table 2
Simulation results continued.

τ	n	β_1		β_2		α		σ	
		Bias	Std	Bias	Std	Bias	Std	Bias	Std
0.5	100	0.0778	0.2308	0.0467	0.2248	0.0179	0.0988	−0.0007	0.0068
	500	0.0187	0.2332	0.0401	0.2446	0.0125	0.1022	−0.0006	0.0068
	1000	0.0231	0.2367	0.0389	0.2364	0.0045	0.1015	−0.0002	0.0065
0.25	100	0.0114	0.1017	0.0151	0.0943	−0.0083	0.0467	−0.0005	0.0070
	500	0.0111	0.0994	0.0194	0.1041	−0.0077	0.0412	−0.0006	0.0069
	1000	0.0074	0.0934	0.0141	0.0944	−0.0065	0.0449	−0.0004	0.0067
0.65	100	0.0466	0.1531	−0.0052	0.1193	−0.1155	0.0536	0.0011	0.0089
	500	0.0517	0.1443	0.0049	0.1189	−0.0087	0.0494	0.0017	0.0083
	1000	0.0429	0.1436	0.0098	0.1141	−0.0114	0.0503	0.0015	0.0082
0.8	100	0.0799	0.1722	0.0335	0.1181	−0.0327	0.0610	0.0011	0.0076
	500	0.0738	0.1798	0.0374	0.1442	−0.0247	0.0537	0.0008	0.0080
	1000	0.0781	0.1436	0.0351	0.1432	−0.0302	0.0604	0.0008	0.0074

Table 3
Empirical results.

Parameter	CAC 40		DAX 30	
	Estimate	t-stat	Estimate	t-stat
p_{11}	0.9380	1.4660	0.9769	2.2343**
p_{22}	0.9203	1.8744*	0.9682	3.4570***
β_1	0.8123	14.4409***	0.6585	11.7226***
β_2	1.2357	17.8704***	1.1939	21.7261***
α	−0.0091	−10.7398***	−0.0078	−8.8768***
σ	0.0040	12.0907***	0.0041	14.9056***
log L	540.6600	–	527.0400	–

* Significance: 0.1.
** Significance: 0.05.
*** Significance: 0.01.

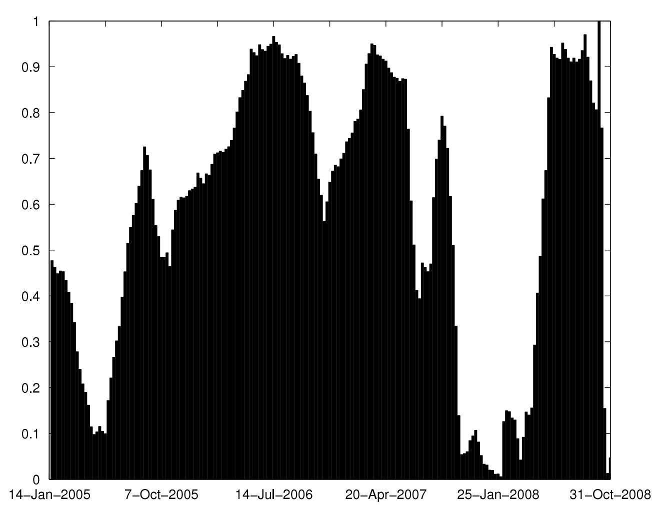

Fig. 1. Smoothed estimates of $Pr(S_t = 2)$ for $\mathbb{C} - CAC$ with $\tau = 0.2$.

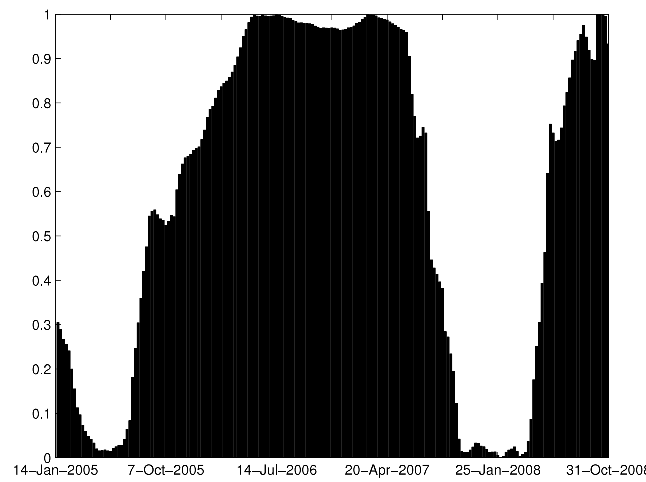

Fig. 2. Smoothed estimates of $Pr(S_t = 2)$ for $\mathbb{C} - DAX$ with $\tau = 0.2$.

respectively in Figs. 1 and 2. From these two figures, it can be seen that the probabilities have kept staying high levels from year 2006 to 2007, which appears to be consistent with the mainstream views of the sub-prime crisis outbreak period. The increases in the regression coefficients and filter probabilities indicate that in the crisis period, the probabilities that the US crisis is contagious to European countries' financial markets have surged. In other words, to some extent, there exists financial contagion between US and main European countries during the sub-prime crisis as demonstrated in this study.

In addition, there is a large decline near the end of year 2007 in both smoothed probability figures, and subsequently the probabilities recovered to the previous levels. This phenomenon can probably be explained as follows: At the end of the first climax of the sub-prime crisis, the lower tail risks tend to decline due to the various measures against financial crisis issued by these countries. However, it appears that all of these countries have underestimated the severity of the sub-prime crisis, and hence the second surge in the crisis took place in 2008, and reached the second climax around September 2008, which coincides to the bankruptcy of Lehman Brothers by time. Maybe for the sake of being part of European Union, both of the European countries have shown strong consistence and similarity in the time-varying smoothed probabilities. According to all the above, it suggests that our empirical results are quite consistent with the historical situation, in other words, the Markov regime-switching quantile regression model is well-suited for such empirical studies.

5. Discussion

In this paper, we develop a Markov regime-switching quantile regression model for assessing the quantile effects of stock returns. As a natural generalization, the Markov regime-switching quantile autoregressive model can be similarly constructed and the model parameters can also be estimated by the MLE method. Even though two regimes are considered in this paper, three or more regimes can be dealt with similarly. As another extension, the intercept may also vary with regimes. To see if it is necessary, the likelihood ratio test can be applied.

To conclude this paper, we would like to remark that quantile regression is not the only methodology to model random variables' quantile risk, in which practical exercises have focused on low quantiles. Instead, many other methods such as copula, extreme value theory can also be useful. In particular, the Markov regime-switching copula, as proposed in Stoeber and Czado (2012), seems to be very useful since it is applicable in multivariate setting, which has inspired us to consider to extend the proposed model to the quantile vector regression with regime-switching in the future study. Since a quantile vector autoregression could model dependence structure in an arbitrary quantile vector, by adding regime switching to this model, the resulting model will be well suited for modeling the cross-effects between random variables' quantiles that are regime-dependent.

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