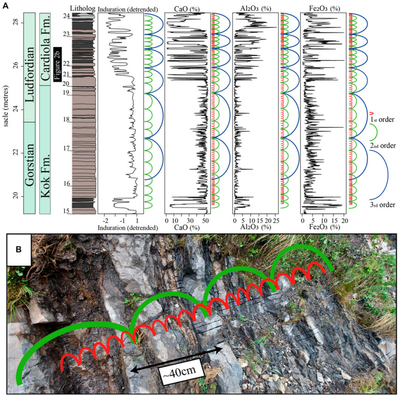
Michel Crucifix

the code and source of this presentation are available at
`$url$`
Commit: *commit*

Plan

Bundles in the rocks



M. Arts et al. In: *Frontiers in Earth Science* (2024)

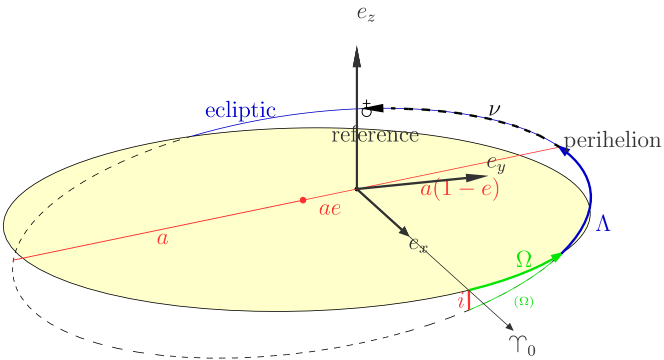
- The whole idea of cyclostratigraphy is to identify bundle structures, which are organised according to a hiearchy of periods.
- Where does this come from ?

Plan

Suppose the Sun and the Earth are alone in the Univere

- You want to descibe their motion
- The Sun is so big that it is practically the center of mass
- So we take it as the center of our framework. We define a *reference plane* and a *reference direction*.

The angles and variables describing the orbit (Keplerian elements)



The beauty of the Keplerian orbit

is that all Keplerian elements are constant, except the "mean longitude" ℓ, which varies in a somehow tricky way

- Celestial mechanics is an *art* of chosing the right variables to make the solution look as simple as possible. Astronomers look for *constant of motions* : things that are unchanged along the trajectory.
- The approach used since the XIXth century (angle-action) is a bit more abstract but the idea is there.

the "mean longitude" is linked to angles you can really measure:

$$\ell = E - e \sin E .$$

$$\tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

Plan

You terrified me; why do you say all of this?

- ... because Sun and Earth are not alone !
- In practice, these equations of a, i etc. will need to be modified to take into account *perturbations* by other planets. But how ?
- This ambitious program was developed by Lagrange (1736 – 1813) and Laplace (1749 – 1827), and many others.
- Astronomers have persued the goal of a *general theory* that describes planetary motion is one that is supposed to be valid over a *very long* time range : the orbital variables do not go to zero or infinity over time.

Lagrange and Laplace had already a go!

Indeed ! Already Lagrange and Laplaced introduced the idea of two abstract vectors (slight re-interpretation):

- the *eccentricity* vector $\mathbf{e}$. It has a size e and points to the perihelion.
- the *inclination* vector $\mathbf{i}$: It has a size $\sin i$ and points to the ascending node.

\end{document}

The averaging process

- To make a long (and complicated) story short: we are concerned here about mutual interactions of orbital planets over many thousands of years. So to first approximation, we care about the mean gravitational field generated by a planet over its whole orbit (Gauss' Keplerian rings).
- So in a way we are interested about how elements which describe the orbit (the "osculating elements") influence each other.
- This averaging process is still used today (though not always).
- Laplace and Lagrange found that to a *very first* approximation, the movements of $\mathbf{e}$ and $\mathbf{i}$ can be obtained by solving two sets of equations:

Linear equations

$$\begin{pmatrix} \dot{\mathbf{e}}_1 \\ \dot{\mathbf{e}}_2 \\ \dot{\mathbf{e}}_3 \\ \dot{\mathbf{e}}_4 \\ \dot{\mathbf{e}}_5 \\ \dot{\mathbf{e}}_6 \end{pmatrix} = G \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \\ \mathbf{e}_4 \\ \mathbf{e}_5 \\ \mathbf{e}_6 \end{pmatrix} \qquad \begin{pmatrix} c\dot{\mathbf{i}}_1 \\ \dot{\mathbf{i}}_2 \\ \dot{\mathbf{i}}_3 \\ \dot{\mathbf{i}}_4 \\ \dot{\mathbf{i}}_5 \\ \dot{\mathbf{i}}_6 \end{pmatrix} = K \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \\ \mathbf{e}_4 \\ \mathbf{e}_5 \\ \mathbf{e}_6 \end{pmatrix}$$

where the numbers correspond to the different planets. Nowadays, we call this an "eigenvalue" problem: the vector e and i will oscillate with angular velocities that are the eigenvalues of the matrices G and K.

In plain terms:

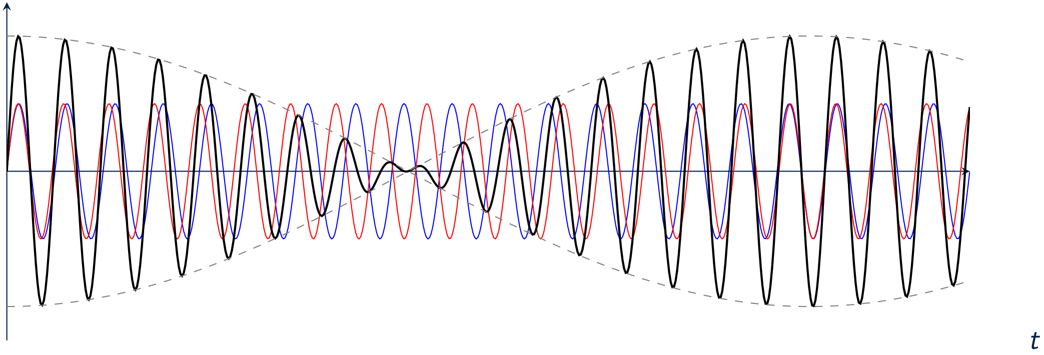
This means that given *inital conditions* (which set the phases), the e and i vectors follow quasi periodic movements:

$$e \sin \Pi = \sum a_i \sin g_i t + \phi_t$$
$$\sin i \sin \Omega = \sum b_i \sin s_i t + \phi_t$$

these are the famous g and s terms.

Beating

Amplitude



Exercise 1 : Beatings with more periods

Sine Waves and Beating

Introduction

In this document, we will explore how simple sine waves can interact to produce **beating patterns** and how these can be visualised using a **continuous wavelet transform**. This exercise is designed to show the phenomenon of **amplitude modulation** clearly.

Before starting, make sure you have installed the

The beating hiearchy of eccentricity

Exercise 2 : Understanding the hierarchy of eccentricity beatings

Calculating Eccentricity Periods Introduction

In this exercise, we will use a set of fundamental frequencies, often denoted as \(g_i\), to calculate a simplified version of the orbital eccentricity, \(e\). These frequencies are derived from celestial mechanics and describe the long-term behavior of planetary orbits.

```
# Load the vector of g_terms from Laskar's
```

Plan

What's wrong with the linear problem

- Linear theory: perturbations by others orbits, which are considered to be non-perturbed
- In reality, they danse together: It implies
  - that there are many more possible frequencies than the 8 'g' and 's' + background
  - that 'g' and 's' move
  - resonances are possible (dancing together). In general: combinations of 'g' and 's' = 0 are dangerous
  - entering in resonance and resonance escapes are partly inpredictable

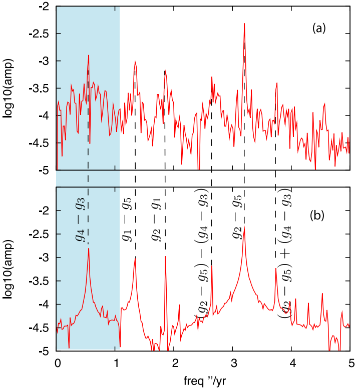
Mathematical approach (work in progress)

1. Analytical approaches :

   solution $= \sum_i a_i \sin(b_i t + phi_i)$

   - Historically : Le Verrier, Hill, Bretagnon, and Laskar's thesis Ch. 1
   - shows of g and s combine and contaminate both e and i
   - Used in the BER78 solution
   - Analytical averaging : Laskar 1988
   - Numerical averaging (Mogaverno and Laskar, using in Hoang et al).
   - First allowed to show resonance, drifts, and cahos
   - by design: respect energy conservation
   - Quinn, Laskar 93 and Laskar 04 (hybrid)
   - Zeebe 2023 (first to be fully open source, based on models available)
   - Best to reassess resonances involving the inner planets

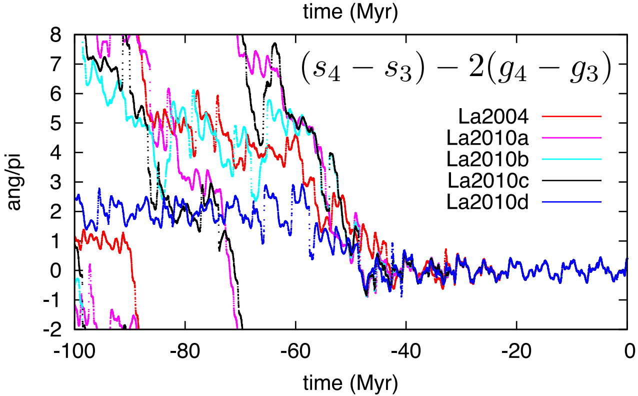
IF I HAVE TIME : SHOW ANALYTICAL VS LA10



{J. Laskar et al. In: *Astronomy and Astrophysics* (2011) see also exercise by ACDS}

The (g3-g4) - 2(s3-s4) resonance

J. Laskar et al. In: *Astronomy and Astrophysics* (2011)



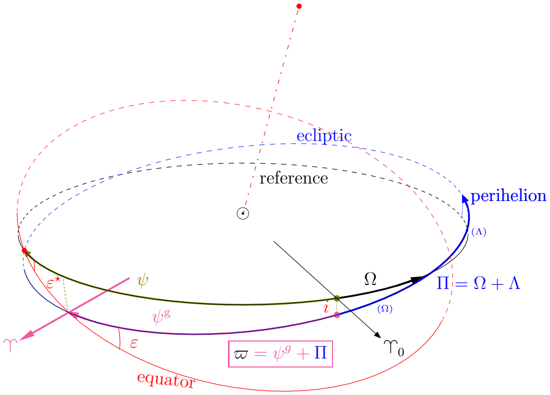
We introduce here a crucial resonance, which we will further inspect once we have explained what is obliquity and how it is computed.
{J. Laskar et al. In: *Astronomy and Astrophysics* (2011) see also qmd exercise 3}

The 404-ka bomb

STILL TO DO
{R. E. Zeebe and M. L. Lantink. In: *The Astronomical Journal* (2024)}

Plan

Climatic precession



Climatically relevant

- What counts, for climate, is insolation that you receive at the different *seasons*.
- Seasons depict the course of the *altitude* of the Sun.
- The reference is the point (node) at which the equator and the ecliptic intersect (Υ).
- The relevant angle for precession is thus the *longitude of the perihelion* ϖ
- The relevant angle for "inclinaison" is the obliquity ε.

For climate (and cyclostratrigraphy) we care about the following quantities

ϖ : heliocentric longitude of the Sun e : Eccentricity ε : Obliquity

How does that link to planetary precession ?

- And to very first order, *general precession* has a regular rate of $k = 1$ revolution every 23 000 years

What it implies for our frequencies ?

Again, *to very first order* (linear theory)

- Eccentricity: *differences of g terms*
- ϖ : $g + k$ terms
- Obliquity : $s + k$ terms

Obliquity beats

Obliquity : $s + k$ terms
Obliquity beats $=$ differences in oblquity terms $\rightarrow$ differences in s terms
Eccentricity beats $=$ differences in precession terms $\rightarrow$ differences in g terms

Exercise 3 : Visualise the g3-g4 - 2(s3-s4) resonance

Resonance between obliquity and eccentricity

```r
# Load the package
require(palinsol)
require(gtseries)
```

Objective of the exercise

In this exercise, we will explore the famous **(g3−g4) − 2(s3−s4) resonance**, which couples obliquity and

Exercise 3

non-linear effects on precession

Looking further general precession ψ is caused by - the torque by the Sun and Moon - thus depends (slightly) in e (for the Earth-Sun distance) and on the angle between equator and moon orbit - both vary (the Moon has a constant tilt with respect to the ecliptic. . .)

- What this says:
  - climatic precssion more *variable* when e is small (cf. Huybers 2008)

Mathematical approaches

1. Analytical approach (manipulate all terms; use trigonometric laws etc.)
   - find the origin of all frequencies
   - Sharaf - Budnikova ; Berger's thesis ; R code available from my team
2. Numerical integration driven by planetary solution
   - e.g. Laskar and Robutel ; Neron de Surgy (accounts for tidal dissipation)
   - Fortran code ; Julia code available from my team.
   - similar approach (based on Quinn) implemented in Zeebe's theme

Resonances and chaos due to precession (also!)

(still to do)

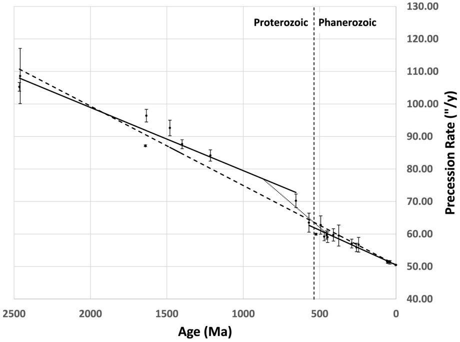
And multi-million-year trend !

By its action on tides, the Moon acts as if it was trying to 'brake' (slow-down) Earth's rotation

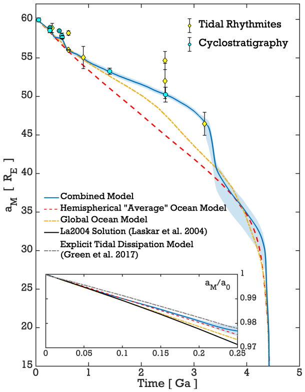
- Over the years: Earth's slows down and becomes rounder (less torque $\rightarrow$ less precession)
- Moon gains speed $\rightarrow$ accelerates $\rightarrow$ escapes the Earth

This 'brake' effect is not constant (it is particluarly high at this moment; this is not tenable backward in time), prompting evolution models of dissipation. Resonances are possible.

Two models are currently popular:



D. Waltham and M. Green. In: *Earth and Planetary Science Letters* (2024)



M. Farhat et al. In: *Astronomy & Astrophysics* (2022)

Understand the consequences on the main precession periods

- The periods of eccentricitiy ($g-$ differences) are unaffected by precession (this is planetary)
- The periods of precession ($g - k$ terms) are (k is the precession rate)

Thus the *ratio* gives you an indication on the precession rate. If you know the LOD, you get the Earth-moon distance.