EEP 547 Project Assignment

1. Project Guidelines

(1) In this project, you will work in your assigned groups. You will turn in one report and presentation PPT slides per group.

(2) Generate your report using MATLAB Publish. Submit your **report** in pdf and the corresponding **MATLAB code** and **Simulink models** on Canvas by the due date. Write the report including equations, explanation on what you did, and answers for all steps in Section 4. Using figures and tables to show the relevant details of any computation/derivation is also recommended. Use a readable font size with the normal (single) line space.

(3) Prepare a **presentation** that will be on the last day of class. Upload the PPT slides on Canvas. This presentation also includes a demonstration of the MinSeg robot. Your demonstration of the MinSeg robot can be done with no batteries (wired) or with batteries (stand-alone). The presentation and demonstration must be no more than **20 minutes**. The presentation covers the following topics: model, analysis of stability, controllability, observability, controller and estimator, the MinSeg robot with a sonar sensor, what worked and what did not, challenges, and any extra analysis you did.

(4) **Grading**: Report 35pts; Presentation 15pts (50% of your course grade).

2. Introduction

The goal of the project is implementing concepts and theories learned in the class with the MinSeg robot to control the movement and body attitude by state-space control methods.

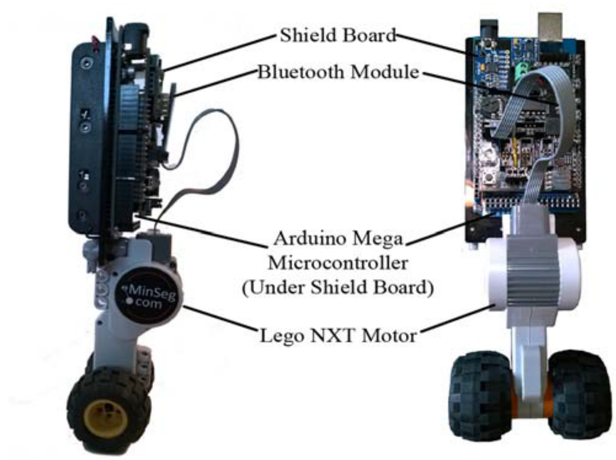

Figure 1 The test platform: MinSeg.

Throughout the project, you will implement the mathematical model using MATLAB and Simulink to balance the MinSeg robot. From the state-space representation of the system, the state variables can be defined, and the stability can be evaluated through numerical analysis using MATLAB. Since the MinSeg robot is equipped with accelerometers and gyroscope, an appropriate controller design via Simulink can improve the stability of MinSeg robot and achieve balancing.

3. A Dynamical Model of the MinSeg Robot

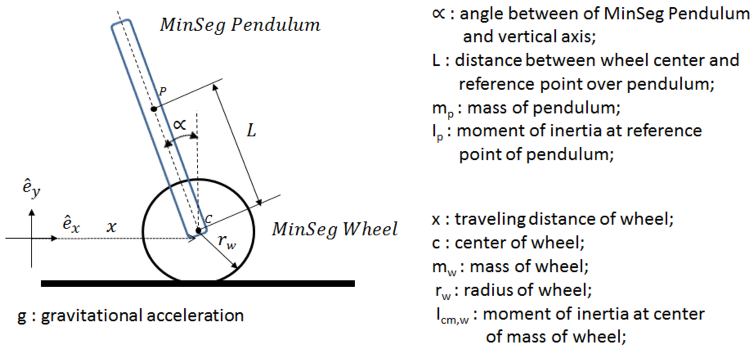

Figure 2 Mathematical model of MinSeg around the zero-equilibrium system.

Figure 2 depicts the motion of a MinSeg over the flat ground. In HW1, the equation of motion is given as (1).

$$\begin{bmatrix} -(I_p + m_p L^2) & m_p L\cos\alpha \\ m_p L\, r_w^2 \cos\alpha & -(I_{cm,w} + m_w r_w^2 + m_p r_w^2) \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} T_m - m_p L g \sin\alpha \\ T_m r_w + m_p L\, r_w^2 \dot{\alpha}^2 \cos\alpha \end{bmatrix} \tag{1}$$

The torque T_m was provided by the DC motor of the MinSeg.

The final relation between torque, input voltage and state variables are given as (2).

$$T_m = \frac{k_t}{R} V + \frac{k_t k_b}{R r_w} \dot{x} + \frac{k_t k_b}{R} \dot{\alpha}, \tag{2}$$

where V is applied voltage, R is the resistance of DC motor, k_t is torque constant and k_b is back-EMF constant. With (1) and (2), the state-space matrices A and B of this system are derived as (3) and (4).

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{gLm_p(i_{\mathrm{cmw}}+(m_p+m_w)r_w^2)}{i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2} & -\frac{k_bk_t(i_{\mathrm{cmw}}+r_w(m_wr_w+m_p(L+r_w)))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} & 0 & -\frac{k_bk_t(i_{\mathrm{cmw}}+r_w(m_wr_w+m_p(L+r_w)))}{Rr_w(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \\ 0 & 0 & 0 & 1 \\ \frac{gL^2m_p^2r_w^2}{i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2} & -\frac{k_bk_tr_w(i_p+Lm_p(L+r_w))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} & 0 & -\frac{k_bk_t(i_p+Lm_p(L+r_w))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \end{bmatrix} \tag{3}$$

$$B=\begin{bmatrix} 0 \\ -\frac{k_t(i_{\mathrm{cmw}}+r_w(m_wr_w+m_p(L+r_w)))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \\ 0 \\ -\frac{k_tr_w(i_p+Lm_p(L+r_w))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \end{bmatrix} \tag{4}$$

4. Procedures

Each step of analysis and implementation is listed below. The report must contain answers to each step in the order.

4.1 Linear Dynamical Model of the MinSeg Robot

Step 1 Since matrices A and B are determined as above, develop a linear continuous time state-space representation of the system.

$$\dot{x} = A\,x + B\,u$$
$$y = C\,x + D\,u$$

The state variables, x, is defined as x=$[\alpha\ \dot{\alpha}\ x\ \dot{x}]^T$, the input vector as u=V and the output vector as $y = x$=$[\alpha\ \dot{\alpha}\ x\ \dot{x}]^T$.

Step 2 Measure the physical parameters of your MinSeg as shown in Figure 2. Find creative ways to measure the weight in grams (home kitchen scale, post office, grocery store, etc.). Create a table to list the values of your measurement in SI unit. Set reference values of (k_t, K_b, R) as (k_t, K_b, R) = (0.3233 Nm/A, 0.4953 Vs/rad, 5.2628 ohms).

Step 3 Find the transfer function matrix of the linearized system.

Step 4 Find the characteristic polynomial and eigenvalues of matrix A.

Step 5 Is the system asymptotically stable? Is it marginally stable? Explain why.

Step 6 Find the poles of the transfer function. Is the system BIBO stable? Explain why.

4.2 Controllability and Observability of the System

Step 7 Find the controllability matrix of the linearized system. What is the rank of the controllability matrix? Is the linearized system controllable?

Step 8 Analyze the observability of the linearized system with the output vector as $y = x =[\alpha\ \dot{\alpha}\ x\ \dot{x}]^T$. Is the linearized system observable?

Step 9 Transform the linearized system into a *controllable canonical form* and *observable canonical form*.

4.3 State Estimator

Step 10 Develop a closed-loop state estimator (full-dimensional observer) for the open-loop system (no feedback yet) such that the poles of the observer are stable, and the dynamics of the observer is at least 6-8 times faster than the dynamics of the linearized model. Include in your report the value of the estimator gain, L.

Step 11 Develop a Simulink model of the linearized system (open-loop system with full-dimensional observer designed above). Add the state estimator derived in previous step to your Simulink model and set the initial conditions of the state estimator to $\hat{x}_0 = [0\ 0\ 0\ 0]$. Simulate the behavior of the system when a unit-step u(t) = 1, t ≥ 0 is applied at the input. Plot the estimated state-variables and output variables on the same graph.

4.4 Feedback Control

Step 12 Consider the case when the linearized system is stabilized by using feedback control. Using the *pole placement method*, via MATLAB, develop a proportional controller such that the poles of the closed-loop system are stable, and the dynamics of the closed-loop model is at least 4-6 times faster than the dynamics of the open-loop model. Include in your report the value of the proportional gain, K.

Step 13 Derive the state-space representation of the closed-loop system. Find the characteristic polynomial and the eigenvalues of the closed-loop system. Is this closed-loop system asymptotically stable?

Step 14 Develop a Simulink model of the linearized closed-loop system (no estimator) when the output of the system equals state variables $y = x$=$[\alpha\ \dot{\alpha}\ x\ \dot{x}]^T$. Add the proportional controller developed above to the Simulink model and simulate the response of the closed-loop system when a unit step u(t) = 1, t ≥ 0 is applied. Plot all the outputs on the same graph.

4.5 Feedback Control using State Estimator

Step 15 Combine the proportional feedback controller with the state estimator from 4.3 and 4.4 in Simulink. Simulate the system in to analyze its performance. Try using the estimator for states not measured (this will change your y(t)). Plot the error function and discuss your results.

4.6 PID Controller

Step 16 Demonstrate the feedback control system using a PID controller (You could try PI, PD, and PID.). Show that the PID controller balances your MinSeg robot. Show your separate Simulink model. This can be demonstrated in a video and live during the project presentation.

4.7 LQR Controller

Step 17 Demonstrate the feedback control system using an LQR controller. Show that the LQR controller balances your MinSeg robot. Show you separate Simulink model. This can be demonstrated in a video and live during the project presentation.

4.8 Adding a Sonar Sensor

Step 18 Use a sonar sensor to implement some type of feedback control. One option would be to implement an alert system on the balancing MinSeg robot, which generates sound as the distance between MinSeg robot and an object is less than certain threshold by using an ultrasonic sensor and a speaker. As the balancing robot approaches an object, the alert system should increase the volume, frequency, or tone of an alerting sound. This part is open for you to be creative to use the sonar sensor. Demonstrate this in a video or live during the project presentation.