ETFs, Learning, and Information in Stock Prices

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Abstract

This paper studies how the introduction of ETFs can change investors' allocation of limited attention. I develop a rational-expectations model where informed agents decide how much to learn about individual stocks, or a systematic risk-factor. Introducing an ETF does not universally increase or decrease learning about systematic risk. If the volatility of the systematic risk-factor is large, risk aversion is high, or the cost of becoming informed is high, introducing the ETF leads investors to devote more attention to the systematic risk factor. Otherwise, the ETF may lead investors investors to learn more about the individual stocks. I decompose the effect of introducing the ETF into 3 channels: (1) Changes in the share of agents who decide to become informed (2) Re-allocation of attention among informed investors (3) Decreases in risk premia. I link the model's predictions to empirical evidence on increased ETF ownership of stocks leading to less informative prices and decreased learning.

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1 Introduction

Sammon [2020a] shows that stock prices before earnings announcements have become less informative over the past 30 years. Between 1990 and 2018, there was a drop in pre-earnings trading volume, pre-earnings drift and an increase in the share in volatility on earnings days. Over the same time period, there was a boom in passive ownership. ETFs were first introduced in 1993, and have since grown rapidly, now owning almost 10% of the US stock market. The proposed mechanism in Sammon [2020a] is that passive ownership decreases incentives to gather information on individual stocks. This paper develops a model which can rationalize these empirical findings.

The model features systematic risk that affects all assets, but initially there is no way to directly trade this factor. I then introduce an ETF so agents can trade an asset only exposed to systematic risk. The effect of introducing the ETF on learning and price informativeness depends on model parameters because of three competing effects: (1) How many investors decide to become informed (2) What risks investors choose to learn about (3) Changes in risk premia.

Introducing the ETF can reduce the number of investors who decide to become informed because it makes uninformed investors better off. Without the ETF, uninformed investors can only learn about the systematic risk-factor through the prices of all the individual stocks. When the ETF is present, they can use the ETF's price to learn about the systematic shock. If we fix the cost of becoming informed, introducing the ETF can decrease the share of agents who decide to learn. This occurs if risk aversion is high, or if the systematic risk-factor is more volatile relative to the individual stocks. If both risk aversion and systematic risk are low, introducing the ETF can *increase* the number of agents who decide to learn. This is because without the ETF, investors cannot perfectly replicate the systematic risk-factor by buying a portfolio of all the individual stocks. Introducing the ETF allows agents to hedge out systematic risk when they bet on individual stocks, making learning about idiosyncratic risk-factors more profitable.

Introducing the ETF can lead to a re-allocation of attention because it makes learning about the systematic risk-factor relatively more profitable. This is for two reasons: (1) It is more profitable to learn about the systematic shock because informed investors can bet on it directly (2) Uninformed agents use the ETF's price as a signal for the systematic shock. All of the assets in the economy are exposed to this shock, so superior knowledge on this risk-factor can lead to large expected utility gains. To retain their information advantage over uninformed agents, informed investors may devote more attention to learning about systematic risk when the ETF is present.

Introducing the ETF universally decreases risk premia i.e. expected returns. Without the ETF, informed agents cannot perfectly hedge out systematic risk when betting on individual stocks, making all their investments riskier. In addition, uninformed agents receive no independent signal about the realization of the systematic shock, yet it has a large effect on their terminal wealth. By reducing uncertainty, and allowing for more precise hedging, introducing the ETF uniformly decreases expected returns in the economy.

Which of these forces dominates depends on model parameters. I focus on the effect of varying (1) risk aversion (2) the volatility of the systematic risk-factor, relative to the idiosyncratic risk factor (3) the cost of becoming informed, which determines the share of investors who decide to become informed. Fixing the cost of becoming informed, introducing the ETF decreases the share of investors who decide to learn, as long as risk aversion or systematic risk are not too low. Fixing the share of agents who become informed, when risk aversion, the volatility of systematic risk, or the cost of becoming informed are high, introducing the ETF decreases learning on the individual stocks. When the opposite is true, introducing the ETF decreases investor focus on the systematic risk-factor.

The next step is linking learning to price informativeness. I create model-analogues to the empirical informativeness measures in Sammon [2020a]: pre-earnings trading volume, the pre-earnings drift and the share of volatility on earnings days. Empirically, the growth of passive ownership has decreased pre-earnings price informativeness. This is consistent with the predictions of the model when risk aversion is high, systematic risk is high and the cost of becoming informed is high.

This paper builds on two models. The version of the economy without the ETF is similar to Admati [1985], except I have added an additional risk and endogenous information choice. The version of the economy with the ETF is similar to Kacperczyk, Van Nieuwerburgh, and Veldkamp [2016], but I've made the asset for trading systematic risk in zero average supply. This change is to better map their model to my setting: ETFs are just bundles of underlying shares, and their introduction does not actually increase the average amount of systematic risk.

The reasons behind the rapid growth of passive ownership are interesting, but outside the scope of this paper. I take the introduction of an ETF as given, and study the effect on agents' learning behavior. This leaves the possibility that omitted factors led to both the introduction of ETFs and a decrease in learning about individual stocks. While a model cannot rule this out, quasi-experimental evidence in Sammon [2020a] suggests a causal relationship between increase in passive ownership and decreased price informativeness.

The paper is organized as follows: Section 2 sets up the model, and explains my numerical solution method. Section 3 studies how the model changes as we vary (1) volatility of systematic risk (2) risk aversion (3) cost of becoming informed. Section 4 lays out the model's predictions for the effect of introducing an ETF, and examines how predictions change as we vary the parameters of interest. Section 5 concludes.

2 Model

This section begins by developing an economy without an ETF. When the ETF is not present, the model is essentially a Admati [1985] economy with $n-1$ idiosyncratic risks, one systematic risk and $n-1$ assets (stocks). Because there are more risks than stocks, I will need to solve for optimal attention allocation numerically. I then introduce an ETF so agents can directly trade on systematic risk. When the ETF is present, the economy is functionally equivalent to the setting in Kacperczyk et al. [2016].

2.1 Setup

Timing

The model is static, and has three periods. At time 0, agents decide whether or not to become informed, and how to allocate their limited attention to the underlying risks. At time 1, Agents receive signals about terminal payoffs, and submit demand functions. At time 2, payoffs are realized and agents consume their terminal wealth.

Agents

The model features three types of agents. There are unit mass of rational traders which fall into two groups: informed and uninformed investors. They both have CARA preferences over time 2 wealth. At time 1, informed investors receive signals about the assets' time two payoffs. The precision of these signals depends on how informed agents allocate their limited attention. Uninformed traders can only learn about terminal payoffs through prices. The third set of agents are noise traders, which have random demand at time 1, and prevent prices from being fully informative.

Assets

There are $n-1$ assets. Asset i has time 2 payoff:

$$f_i = a_i + z_n + z_i$$

where $z_i \sim N(0, \sigma_i^2)$. Each asset has average supply $\overline{x}_i$, and noise trader demand shocks shocks $x_i \sim N(0, \sigma_{i,x}^2)$. The z_i and x_i shocks are independent for all i. In this economy there are n risks: one idiosyncratic risk for each asset z_i, and one systematic risk z_n.

For the baseline version of the model, I assume that $\sigma_i^2 = \sigma^2$, $\overline{x}_i = \overline{x}$ and, $\sigma_{i,x}^2 = \sigma_x^2$ i.e. all the assets are symmetric. This is not needed, but it simplifies the numerical technique for solving the model. For an extension where individual assets load differently on systematic risk, and have heterogeneous idiosyncratic risk and supply shocks, see the Appendix A.1.

Throughout the paper, I assume that the number of assets is sufficiently small so that idiosyncratic risk cannot be totally diversified away. As the number of assets grows to infinity, introducing an ETF would have no effect. This model does not feature trading costs, so agents could replicate the payoff of the ETF by buying a portfolio of all the individual assets. One can view this restriction to trading a small, finite number of assets as a reduced-form way of modeling transaction costs: Trading the first $n-1$ assets is free, but then trading costs to go infinity if the investor wanted to trade an n^{th} asset.

Signals

If agent j decides to become informed, they receive noisy signals at time 1 about the payoffs of the underlying *assets*:

$$y_{i,j} = (z_n + \epsilon_{n,j}) + (z_i + \epsilon_{i,j})$$

where $\epsilon_{i,j} \sim N(0, var(\epsilon_{i,j}))$, and $\epsilon_{i,j}$ are independent for all permutations of i and j. As agent j allocates more attention to risk i, $var(\epsilon_{i,j})$ decreases.

Portfolio Choice

Define:

$$W_{2,j} = r\,(W_{0,j} - \mathbf{1}_{inf,j}c) + q_j(f - pr)$$

where $W_{0,j}$ is initial wealth, c is the cost of becoming informed, r is the gross risk-free rate, f is the vector of terminal assets payoffs, p is the vector of time 1 prices and $\mathbf{1}_{inf,j}$ is an indicator equal to 1 if agent j decides to become informed.

Agent j submits demand q_j to maximize their time 1 objective function:

$$E_{1,j}[-exp(-\rho W_{2,j})]$$

where ρ is risk aversion. I use $E_{t,j}$ to denote the expectation with respect to agent j's time t information set. For informed agents, the time 1 information set is the vector of signals y_j and the vector of prices, p. For uninformed agents, the time 1 information set is just prices.

Learning

Agent j can allocate attention $K_{i,j}$ to risk factor z_i to reduce signal noise:

$$y_{i,j} = (z_n + \epsilon_{n,j}) + (z_i + \epsilon_{i,j})$$
$$var(\epsilon_{i,j}) = \frac{1}{\alpha + K_{i,j}}, \quad var(\epsilon_{n,j}) = \frac{1}{\alpha + K_{n,j}}$$

where $\alpha > 0$. This differs from the setup in Kacperczyk et al. [2016], where the learning technology is $var(\epsilon_{n,j}) = \frac{1}{K_{n,j}}$. In my setting, I need $var(\epsilon_i)$ to be well defined even if agents devote no attention to asset i[1]. A way to think about α is that informed agents all have a "finger on the pulse" of the market, and know a little bit about each asset, even without explicitly devoting attention to it. Informed agents have a total attention constraint of $\sum_i K_i \leq 1$.

Prices and Market Clearing

Suppose we fix the information choice of informed investors. Then, the model is equivalent to Admati [1985]. This is because, in the setup above, agents do not independently receive information about the n^{th} asset i.e. there is no $y_{n,j} = (z_n + \epsilon_{n,j})$. In words, agents think only in terms of asset payoffs, rather than risk-factor payoffs. For example, agent j's asset 1 signal is: $y_{1,j} = (z_n + \epsilon_{n,j}) + (z_1 + \epsilon_{1,j})$[2]. This is centered on $z_n + z_1$ so it is an unbiased signal about the payoff of asset 1. The variance of this signal is $var(\epsilon_{n,j}) + var(\epsilon_{1,j})$ because all signal noise is independent. All investors know the correlation structure of asset returns, so when agent j is calculating the posterior mean of asset 2, they still consider signal 1, as the assets are correlated via systematic risk. Further, when deciding on what to learn, agents understand that devoting attention to systematic risk will reduce the variance of all of their asset signals. For these reasons, I do not find this assumption too restrictive, but it is needed to solve the model using the equilibrium in Admati [1985][3].

[1]This is because in my setting, the risk-factors are not fully separable. For example, if $\epsilon_{1,j}$ has infinite variance, but $\epsilon_{n,j}$ has finite variance, the variance of $y_{1,j}$ is still not well defined. In Kacperczyk et al. [2016], each of the rotated assets is only exposed to one risk, so devoting no attention to that risk leads to a precision of zero, but does not have spill-over effects on other assets.

[2]For clarity, in this example, I exclude all the mean payoff terms.

[3]Without this assumption, there is no closed-form solution for the price function, as discussed in Section 6 of Admati [1985]. To solve the model without this assumption, one would need to numerically solve for prices such that the market clears. The price function would be of the form $p = \tilde{A}_0 + \tilde{A}_1[z_1, \ldots, z_{n-1}] + \tilde{A}_2 z_n + \tilde{A}_3 \mathrm{x}$, where x is a vector of supply shocks. In unreported results, I find it difficult to solve for these A_i numerically, because it involves the product of one of the price coefficients A_1 with the inverse of another one of the price coefficients A_2^{-1}. This can lead to arbitrarily large offsetting entries in these matrices, and numerical instability of the solution.

Define μ as the vector of a_i i.e. the vector of mean asset payoffs. Further define $\overline{x}$ as the vector of $\overline{x}_i$ i.e. the vector of mean asset supplies. Define $n \times n$ matrix Γ as:

$$\Gamma = \begin{bmatrix} 1 & 0 & \dots & 0 & 1 \\ 0 & 1 & \dots & 0 & 1 \\ \dots & & & & \\ 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \tag{1}$$

If we define z as the vector of z_i's and z_n, then the terminal payoffs can be written as $f = \mu + \Gamma z$[4] Note that this includes a row for a hypothetical n^{th} asset with payoff $f_n = a_n + z_n$ even though without the ETF, agents cannot trade that asset or observe its price. This row will be removed, but it is useful to include it here for comparing this setup to the world with the ETF.

Define the variance of asset payoffs, V as:

$$V = \Gamma \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 & 0 \\ 0 & \sigma_2^2 & \dots & 0 & 0 \\ \dots 0 & 0 & \dots & 0 & \sigma_n^2 \end{bmatrix} \Gamma' \tag{2}$$

Define the matrix of asset signal variances for agent j as:

$$S_j = \Gamma \begin{bmatrix} \frac{1}{\alpha + K_{1,j}} & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{\alpha + K_{2,j}} & \dots & 0 & 0 \\ \dots 0 & 0 & \dots & 0 & \frac{1}{\alpha + K_{n,j}} \end{bmatrix} \Gamma' \tag{3}$$

[4]If the assets had different loadings on systematic risk, the 1's in the last column would be replaced by β_i's, i.e. the loadings of each stock on systematic risk, as discussed in the Appendix A.1.

I restrict to equilibria where all informed agents have the same attention allocation, so $S_j = S$ for all j i.e. all informed agents have the same attention allocation[5].

Define z as the vector of asset payoff shocks. Define the variance-covariance matrix of asset noise shocks as $U = \sigma_x^2 \mathbf{I}_{n-1}$ where $\mathbf{I}_{n-1}$ is an $n-1 \times n-1$ identity matrix. Define the vector of noise chocks as x, which is normally distributed with mean zero and variance U. The realized supply of each asset is $\overline{x} + x$.

This is the economy without the ETF, so we need to remove the last row and last column of every matrix, as well as the last row of every vector. Any matrix with a superscript t has been truncated i.e. has had the last row and column removed. Any vector with a superscript t has had the last row removed.

Define Q^t as:

$$\frac{1}{\rho} \times \phi \times \left(S^t\right)^{-1} \tag{4}$$

where ϕ is the share of rational traders who decide to become informed at cost c.

The equation for equilibrium prices comes directly from Admati [1985]:

$$\begin{aligned} p &= A_0 + A_1(\mu + \Gamma z) - A_2(\overline{x} + x) \\ A_3 &= \frac{1}{\rho}\left(\left(V^t\right)^{-1} + Q^t * (U^t)^{-1} * Q^t + Q^t\right) \\ A_0 &= \frac{1}{r\rho} A_3^{-1}\left(\left(V^t\right)^{-1}\mu^t + Q^t (U^t)^{-1}\overline{x}^t\right) \\ A_1 &= \frac{1}{r} A_3^{-1}\left(Q^t + \frac{1}{\rho} Q^t (U^t)^{-1} Q^t\right) \\ A_2 &= \frac{1}{r} A_3^{-1}\left(\mathbf{I}_{n-1} + \frac{1}{\rho} Q^t (U^t)^{-1}\right) \end{aligned} \tag{5}$$

Demands

[5] For discussions of non-symmetric equilibria, see e.g. Veldkamp [2011]

Having solved for the price, we can solve for demands. In this section, we continue to use the *truncated* versions of all the model objects. To avoid excessive use of superscripts, I omit the t even though all the objects here have the last row/column removed.

Define the constant $\gamma = \rho\left(A_2^{-1} - rQ\right)$. There are separate demand functions for the informed and uninformed:

$$\begin{aligned} \text{Uninformed: Demand} &= G_0 + G_{2,un}p \\ \text{Informed, agent } j\text{: Demand} &= G_0 + G_1 y_j + G_{2,inf}p \end{aligned} \tag{6}$$

where y_j is the vector of signals received by agent j and:

$$\begin{aligned} G_0 &= A_2^{-1}A_0 \\ G_{2,un} &= \frac{1}{\rho}\gamma \\ G_{2,in} &= \frac{1}{\rho}\left(\gamma + rS^{-1}\right) \\ G_1 &= \frac{1}{\rho}S^{-1} \end{aligned} \tag{7}$$

Many of objects in the demand function can be used to compute agents' posterior beliefs about mean asset payoffs. For informed agents, the posterior mean conditional on signals and prices is:

$$\begin{aligned} E_{1,j}[f|y_j, p] &= B_{0,in} + B_{1,in}y_j + B_{2,in}p \\ V_{in}^a &= \left(V^{-1} + QU^{-1}Q + S^{-1}\right)^{-1} \\ B_{0,in} &= \rho V_{in}^a G_0 \\ B_{1,in} &= \rho V_{in}^a G_0 \\ B_{2,in} &= r\mathbf{I}_{n-1} - \rho V_{in}^a G_{2,in}' \end{aligned} \tag{8}$$

For uninformed agents, the posterior mean conditional on prices is:

$$\begin{aligned} E_{1,j}[f|p] &= B_{0,in} + B_{2,un}p \\ V_{un}^a &= \left(V^{-1} + QU^{-1}Q\right)^{-1} \\ B_{0,un} &= \rho V_{un}^a G_0 \\ B_{2,un} &= r\mathbf{I}_{n-1} - \rho V_{un}^a G_{2,un}' \end{aligned} \tag{9}$$

Deciding to Become Informed

At time zero, agent j decides whether or not to pay c and become informed. They make this decision to maximize the time 0 objective function:

$$U_{0,j} = E_{0,j}\left[-exp(-\rho W_{2,j})\right]$$

Where the time 0 information set is the share of agents who decide to become informed. Maximizing CARA utility is equivalent to maximizing a mean-variance objective function. This means that expected utility at time 1, $U_{1,j}$, can be written as:

$$U_{1,j} = E_{0,j}\left[\rho E_{1,j}[W_{2,j}] - \frac{\rho^2}{2}V_{1,j}[W_{2,j}]\right]$$

where $V_{t,j}$ is the variance at time t conditional on agent j's information set. From Kacperczyk et al. [2016], we know that time expected 1 utility is equivalent to:

$$U_{1,j} = r\left(W_{0,j} - \mathbf{1}_{inf,j}c\right) + \frac{1}{2}E_{0,j}\left[\left[E_{1,j}[f] - pr\right]\hat{\Sigma}_j^{-1}\left[E_{1,j}[f] - pr\right]\right]$$

Because asset prices are functions of normally-distributed shocks, $E_{1,j}[f]-pr$ will be normally distributed. This implies that $\left[E_{1,j}j[f] - pr\right]\hat{\Sigma}_j^{-1}\left[E_{1,j}[f] - pr\right]$ will be distributed non-central

χ^2. If j is informed, the expected mean of this distribution is:

$$U_{1,j} = r\left(W_0 - c\right) + \frac{1}{2}trace\left(\hat{\Sigma}_j^{-1} V_{1,j}\left[E_{1,j}[f] - pr\right]\right) + \frac{1}{2}\left(\left[E_{1,j}[f] - pr\right]\hat{\Sigma}_j^{-1}\left[E_{1,j}[f] - pr\right]\right)$$

where $\hat{\Sigma}_j^{-1}$ is agent j's posterior precision.

If j is uninformed, expected utility at time 1 is:

$$E_0[U_{un}] = rW_0 + \frac{1}{2}trace\left(\hat{\Sigma}_j^{-1} V_{1,j}\left[E_{1,j}[f] - pr\right]\right) + \frac{1}{2}\left(\left[E_{1,j}[f] - pr\right]\hat{\Sigma}_j^{-1}\left[E_{1,j}[f] - pr\right]\right)$$

In equilibrium, $E_0[U_{in}] = E_0[U_{un}]$ so we can compute c by taking the difference and dividing by r. In this setting, I do not have closed-form solutions for the mean and variance of $E_{1,j}[f] - pr$, but I can obtain these through simulation. From the section on computing demand, $E_{1,j}[f] = B_{0,j} + B_{1,j}y + B_{2,j}p$ and the posterior precision is $\hat{\Sigma}_j^{-1} = inv(V_j^a)$.

Solving for c directly would be computationally intensive, as the model would have to be re-solved at each proposed combination of c and share of informed investors to check that $E_0[U_{in}] = E_0[U_{un}]$. It is easier to solve for c by creating a grid for the share of informed agents between 0 and 1. Then, at each point on the grid, compute the difference in expected utility between informed and uninformed to back out c.

2.2 Equilibrium

As in Kacperczyk et al. [2016], I am going to assume a symmetric equilibrium. This means that all informed agents have the same $K_{i,j} = K_i$. As discussed in Veldkamp [2011], there likely exist asymmetric equilibria, but I do not focus on them in this paper. In addition, I assume that assets 1 to $n-1$ have the same: (1) Mean (2) Systematic risk (3) Idiosyncratic risk (4) Supply shock variance. This assumption reduces an otherwise $n-1$ dimensional problem (the n^{th} dimension is accounted for by the total information constraint) to a two dimensional problem: Informed agents must only decide to allocate K_n attention to systematic risk, and $(1-K_n)/(n-1)$ to each idiosyncratic risk factor. This strong assumption is not needed, and it does not change any of the model's predictions, but it drastically speeds up the numerical solution method. For details, see Appendix A.1, where I discuss how to solve a version of the model with this assumption relaxed.

Another possible issue is that the equilibria I find are not unique. Without closed form solutions, I cannot fully rule this out, but I have tried starting my numerical method at every point on the solution grid and I find it always converges to the same place.

At time 1, given K_i's and the share of informed agents, the equilibrium is equivalent to that in Admati [1985]. K_i and the share informed will have an effect on S and Q, but everything else is equivalent. At time zero, we know we are at an equilibrium if: (1) no informed or uninformed agent would improve their expected utility by switching to the other type and (2) no informed agent would improve their expected utility by re-allocating their attention. As discussed above, condition 1 is going to be met by construction, as I back out c for a given share of informed agents, to make the utility of both groups equal. I rely on condition 2 when developing my numerical method in the next subsection.

2.3 Numerical Method

Fixing the share of informed agents, I use the following algorithm to numerically solve for K_i's:

1. Start all agents at K^0

2. Consider an atomistic agent j who takes K^0 as given, and considers their expected utility by deviating to K_j^1 near K^0. These deviations are small increases/decreases in the share of attention spent on the systematic risk-factor.

3. If j can be made better off, move all informed agents to K^1

4. Iterate on steps 2 and 3 until j can no longer improve their expected utility by deviating.

Discussion

At this point, it is not clear why a numerical method is needed to solve the model. Two possible alternative solution methods are (1) Adding the n^{th} risk to Admati [1985]. This will not work, as discussed in the original paper, as there is no closed form solution with more risks than assets. (2) Deleting the n^{th} asset from Kacperczyk et al. [2016]. This is not viable because the rotation used to isolate risk-factors and solve the model will not work if the number of risks is greater than the number of assets. Finally, we cannot use a benevolent central planner to solve the problem: I find that in the competitive equilibrium, attention is more concentrated on a small number of risks, relative to what would maximize total expected utility for informed *and* uninformed agents.

It also seems as though it should be possible to map the no-ETF economy to an economy with independent assets/risks via an eigendecomposition (see e.g. Veldkamp [2011]). Having done this, it would be straightforward to solve the model using the technique in Kacperczyk et al. [2016]. While this is possible, it does not always work, because after rotation, the solution may not be feasible under the proposed learning technology. This happens when the solution to the rotated model proposes values for K_i which do not satisfy the total information constraint. For example, suppose we have two assets and three risks. Using the notation in the appendix of Kacperczyk et al. [2016]:

$$\text{Define: } \Sigma^{1/2} = \text{Square root of } V\text{, the variance-covariance matrix of payoffs}$$

$$\text{Define: } \Sigma_s = S\text{, the variance-covariance matrix of signals}$$

$$\text{Define: } \Sigma_s^1 = \Sigma^{-1/2} \times \Sigma_s \times \Sigma^{-1/2}$$

$$\text{We can re-write: } \Sigma_s = \Sigma^{1/2} \times G \times L \times G \times \Sigma^{1/2}$$

$$\text{where } G \text{ and } L \text{ come from the eigen-decomposition of } \Sigma_s^1$$

$$\text{Define orthogonal signal matrix: } \tilde{\Sigma}_s = G' \times \Sigma^{-1/2} \times \Sigma_s \times \left(\Sigma^{-1/2}\right)' \times G$$

This implies that:

$$\tilde{\Sigma}_s = \begin{bmatrix} 1/(\alpha + \tilde{K}_1) & 0 \\ 0 & 1/(\alpha + \tilde{K}_2) \end{bmatrix}$$

After solving the model, the optimal $\tilde{K}_i$ rotated back to the original economy may require K_i that do not satisfy $\sum_i \tilde{K}_i \leq 1$.

2.4 Introducing the ETF

Introduce asset n, the ETF:

$$f_n = a_n + z_n$$

Asset n has average supply $\overline{x} = 0$, but is still subject to supply shocks $x_n = \tilde{x}_n + \sum_{z=1}^{n-1} x_z$ where $\tilde{x}_n$ has the same distribution as all the x_z for assets 1 to $n-1$. This implies that the supply shock for the n^{th} asset is the sum of the supply shocks to the $n-1$ individual assets, as well as another independent supply shock $\tilde{x}_n$. I define the ETF noise shocks this way based on evidence in Ben-David, Franzoni, and Moussawi [2018] and Chinco and Fos [2019].

These authors document transmission in noise shocks between the ETFs and the underlying assets, which is also a feature of my model.

Under this assumption, the noise shock for the n^{th} asset has total volatility $\sigma^2_{n,x} = n \times \sigma^2_x$. Define $\tilde{U} = (\Gamma')^{-1} \sigma^2_x \mathbf{I}_n (\Gamma')^{-1}$. We need supply shocks in the ETF, otherwise its price would be a fully revealing signal for the systematic risk-factor. Further, the ETF must be in zero average supply so its introduction does not increase average systematic risk[6].

Informed agent j receives signals about the payoffs of all the underlying assets, including asset n:

$$
\begin{aligned}
y_{i,j} &= (z_n + \epsilon_{n,j}) + (z_i + \epsilon_{i,j}) \\
y_{n,j} &= (z_n + \epsilon_{n,j})
\end{aligned}
$$

The learning technology and total attention constraint are unchanged.

The price and demand functions are unchanged from the setup without the ETF, but instead of using the truncated versions, we use the full versions i.e. use S instead of S^t, and use the new noise shock matrix $\tilde{U}$. We can also use the same numerical method to solve for the optimal allocation of attention, and cost of becoming informed. It is faster, however, to rotate the economy and solve the problem using the method in Kacperczyk et al. [2016]. I find that my numerical solution and their closed-form solution always yield identical results, regardless of where I start on the solution grid.

Effect on posterior mean/variance

Define $y_p = f + \epsilon_p$ as the signal about asset payoffs contained in prices. From the price function, $y_p = A_1^{-1}(p - A_0)$, which implies that $\epsilon_p = A_1^{-1} A_2(\overline{x} + x)$ and $\Sigma_p = A_1^{-1} A_2 U$ where U is the variance-covariance matrix of supply shocks. This implies that $y_p \sim N(0, \Sigma_p)$.

[6] Although it does not increase the average supply of systematic risk, introducing the ETF does introduce additional noise-trader risk.

Without the ETF:

$$\underbrace{\widehat{\Sigma_j^{-1}}}_{\text{Posterior Precision}} = \underbrace{V^{-1}}_{\text{Prior Precision}} + \underbrace{\Sigma_p^{-1}}_{\text{Price Precision}} + \underbrace{S_j^{-1}}_{\text{Signal Precision}}$$

With the ETF, agents observe $s_{p,n}$ i.e. the signal about payoff of the n^{th} asset contained in the price of the n^{th} asset. This will change the posterior precision to:

$$\underbrace{\widehat{\Sigma_j^{-1}}}_{\text{Posterior Precision}} = \underbrace{V^{-1}}_{\text{Prior Precision}} + \color{blue}{\underbrace{\Sigma_p^{-1}}_{\text{Price Precision}}} + \underbrace{S_j^{-1}}_{\text{Signal Precision}}$$

where the terms that have changed are in color. Conditional on learning being unchanged, introducing the ETF has no effect on S_j^{-1} for assets 1 to $n-1$. This is because $y_{i,j} = (z_n + \epsilon_{n,j}) + (z_i + \epsilon_{i,j})$, so $var(s_{i,j}) = var(\epsilon_{n,j} + \epsilon_{i,j}) = var(\epsilon_{n,j}) + var(\epsilon_{i,j})$ by independence.

When the ETF is not present, the posterior mean of f will be:

$$\underbrace{E_{1,j}[f]}_{\text{Posterior Mean}} = \underbrace{\hat{\Sigma}_j}_{\text{Posterior Variance}} \times \left(\underbrace{S_j^{-1}}_{\text{Precision on Asset Signals}} y_j + \underbrace{\Sigma_p^{-1}}_{\text{Price Precision}} s_p \right)$$

With the ETF, agents can separately weigh their signal for z_n by its own precision:

$$\underbrace{E_{1,j}[f]}_{\text{Posterior Mean}} = \underbrace{\hat{\Sigma}_j}_{\text{Posterior Variance}} \times \left(\color{red}{\underbrace{S_j^{-1}}_{\text{Precision on Risk-Factor Signals}}} \color{black}{y_j} + \color{blue}{\underbrace{\Sigma_p^{-1}}_{\text{Price Precision}}} \color{black}{s_p} \right)$$

To see how this works, we can use the eigendecomposition in Veldkamp [2011] to isolate the risk factors:

$$f = \mu + \Gamma z \leftrightarrow \tilde{f} = \Gamma^{-1}\mu + z$$

$$\tilde{y}_i = z_i + \tilde{\epsilon}_i \text{ for } i = 1, \ldots, n$$

Note that this rotation changes the supply of all the assets to $(\Gamma')^{-1}(\overline{x}+x)$. We can solve for the equilibrium in this economy using the numerical technique in Kacperczyk et al. [2016][7], and then rotate back to the economy with payoffs f and signals y. This rotation can be used as a check on the numerical method, as it allows me to compare my numerical solutions to the closed-form solutions in Kacperczyk et al. [2016]. In this rotated economy, it is clear that agents are going to independently use the n^{th} signal, and the price of the n^{th} asset to learn about z_n, something they do not do in the no-ETF world.

To make the effect of introducing the ETF on investors' posterior precisions, Table 1 contains selected entries of $\hat{\Sigma}$. Introducing the ETF always increases the precision of both the informed and uninformed for a generic asset i.e. assets 1 to $n-1$.

3 Comparative Statics

3.1 Baseline Parameters

Table 2 contains the baseline parameters. I take most of them from Kacperczyk et al. [2016] with a few exceptions: (1) I set the risk-free rate r to 1 because I want to de-emphasize the effect of time-discounting, as this is really a static model (2) I have 8 idiosyncratic assets, instead of 2, so agents can better attempt to replicate the systematic risk-factor with a diversified portfolio of stocks before the ETF is introduced (3) I change the asset supplies.

[7] I would like to thank the authors for sharing their solution code with me

Panel A: Matching Cost of Becoming Informed

				Precision			
		Share Informed		Informed		Uninformed	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF	no ETF	ETF
0.1	0.2	0.05	0.2	1.82	2.24	1.66	2.06
0.1	0.5	0.35	0.2	2.04	2.06	1.93	1.94
0.25	0.2	0.5	0.2	1.85	1.87	1.74	1.82
0.25	0.5	0.5	0.2	1.78	1.87	1.69	1.82

Panel B: Share Informed at 10%

				Precision			
		Share Informed		Informed		Uninformed	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF	no ETF	ETF
0.1	0.2	0.1	0.1	1.85	2.05	1.70	1.88
0.1	0.5	0.1	0.1	1.75	1.90	1.64	1.83
0.25	0.2	0.1	0.1	1.76	1.87	1.65	1.82
0.25	0.5	0.1	0.1	1.71	1.87	1.62	1.82

Panel C: Share Informed at 30%

				Precision			
		Share Informed		Informed		Uninformed	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF	no ETF	ETF
0.1	0.2	0.3	0.3	2.20	2.54	2.05	2.37
0.1	0.5	0.3	0.3	1.96	2.30	1.85	2.16
0.25	0.2	0.3	0.3	1.79	1.92	1.68	1.84
0.25	0.5	0.3	0.3	1.73	1.88	1.64	1.83

Table 1: **Posterior Precision.** Diagonal entries of $\hat{\Sigma}$ for one of the stocks i.e. assets 1 to $n-1$. In panel A, the cost of being informed is chosen such that 20% of agents become informed when the ETF is present. In Panels B and C, the share of informed agents are fixed and 10% and 30% respectively. The "no ETF" column has the (1,1) entry of $\hat{\Sigma}$ when the ETF is not present, while the "ETF" column has the (1,1) entry of $\hat{\Sigma}$ after introducing the ETF.

Mean asset payoff	a	15
Volatility of idiosyncratic shocks	σ_i^2	0.55
Volatility of noise shocks	σ_x^2	0.5
Risk-free rate	r	1
Initial wealth	W_0	220
Baseline Learning	α	0.05
# idiosyncratic assets	$n-1$	8
Coef. of risk aversion (low)	ρ	0.1
Coef. of risk aversion (high)	ρ	0.25
Vol. of systematic shocks (low)	σ_n^2	0.2
Vol. of systematic shocks (high)	σ_n^2	0.5
Total supply of idiosyncratic assets	$\overline{x}$	20

Table 2: **Baseline Parameters.**

In Kacperczyk et al. [2016], the supply of the n^{th} asset is 15, and the supply of the two individual stocks is 1 each. Instead, I make the total supply of all idiosyncratic assets equal to 20, and split this among 8 stocks. In unreported results, I have tried increasing and decreasing the number of idiosyncratic assets with the same total supply, and find it does not qualitatively change the results. I study four different scenarios based on lower/higher risk aversion, and lower/higher systematic risk.

3.2 Effect of Introducing the ETF

Share informed

I want to understand how introducing the ETF affects the share of agents who decide to become informed. Figure 1 shows the relationship between the cost of becoming informed (in dollars) and the percent of rational investors who decide to become informed. When risk aversion ρ is low, and systematic risk σ_n is low, more investors become informed after introducing the ETF. With low risk aversion, and low volatility of the n^{th} asset, investors are willing to bet aggressively on their private signals, increasing the benefit of becoming informed. As we increase risk-aversion, however, for most costs of becoming informed, more investors learn when the ETF is not present. This is because for these parameter choices, introducing the ETF makes the uninformed investors relatively better off[8].

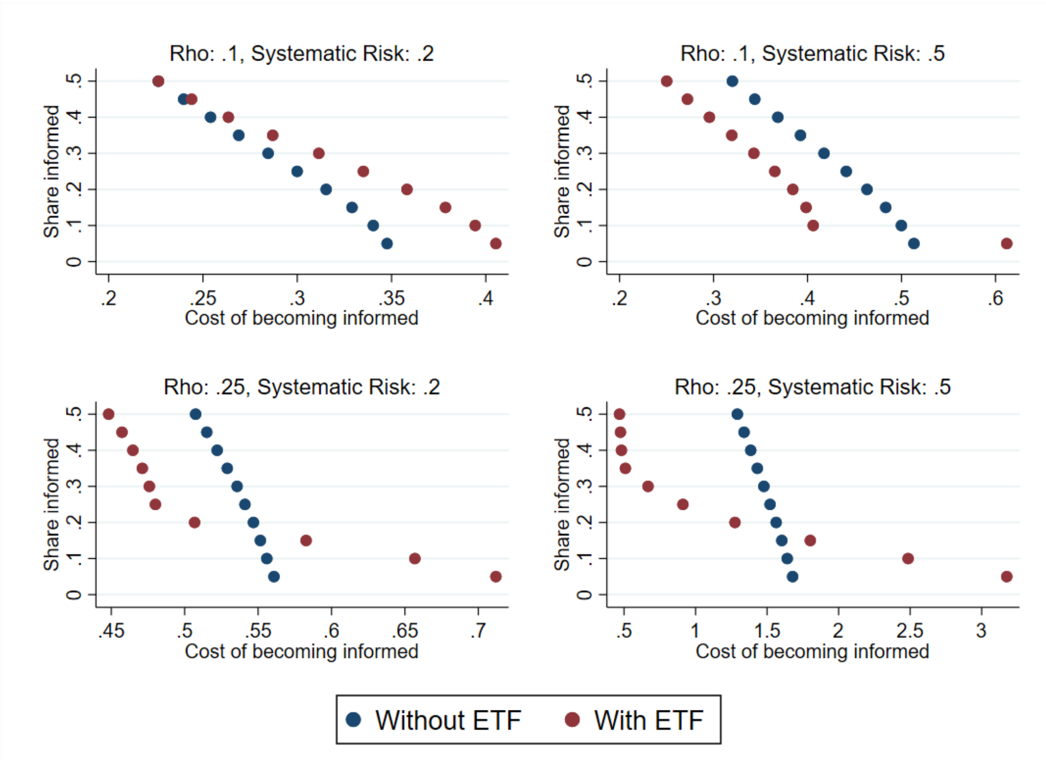

Figure 1: **Effect of introducing the ETF on learning.**

In three of the four scenarios, we see a kink in the relationship between the cost of becoming informed and the share of agents who decide to learn when the ETF is present. To the right of the kink, the cost of becoming informed is high, so relatively few agents are becoming informed. Given that systematic risk affects all assets, informed agents initially devote all their attention to learning about this risk-factor. To the left of the kink, learning

[8]For example, with ρ at 0.25, σ_n^2 at 0.5 and the share informed at 30%, the percent difference in expected utility between informed and uninformed agents decreases by more than half. A more thorough discussion of how uninformed fare, relative to the informed agents is in Appendix A.4.

about the systematic risk-factor has become crowded, and informed agents start devoting some attention to the individual-asset risks.

Figure 2 focuses on the case where $\rho = 0.25$ and $\sigma_n^2 = 0.2$. The top panel shows two things: (1) The relationship between the cost of becoming informed, and the share of attention devoted to systematic risk [blue dots]. To the right of the kink, all attention is being devoted to the systematic risk-factor. (2) $U_{1,j}$ i.e. the time one objective function for informed [red squares] and uninformed agents [green triangles]. One of the counter-intuitive features of the kink is that the line is *steeper* once agents are devoting some attention to the idiosyncratic assets. For both informed and uninformed agents, the lines become steeper to the left of the kink.

The second panel shows why the slope changes: To the right of the kink informed and uninformed investors are making roughly the same profits on stocks, but informed investors are making significantly larger profits on the ETF. To the left of the kink, informed investors gain an advantage over uninformed investors on the individual stocks. This increases the relative benefit of becoming informed, which can explain the changes in slopes around the kink.

Hedging Demand

One of the effects of introducing the ETF is that it allows informed investors to better isolate bets on signals about individual stocks. In the demand function, G_1 is a measure of how informed investors react to their own signals. Table 3 contains selected the entries of G_1. Because all the assets are assumed to have the same supply and risk, when the ETF is not present, G_1 is a symmetric matrix. The diagonal entries show how strongly investors react to signals about a particular asset. The off-diagonal entries show how investors may hedge such bets. The "No ETF Present" columns look at those entries of G_1. When an investor

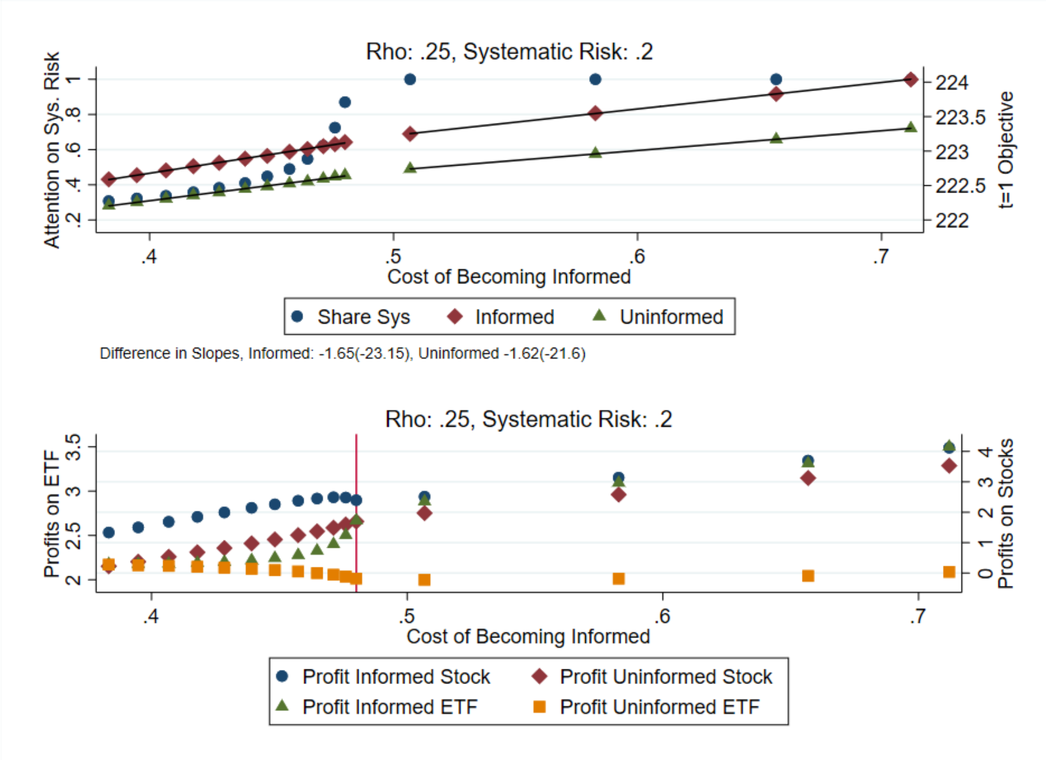

Figure 2: **Trading profits by asset type.**

gets a good signal about a particular asset, they buy more of it. They hedge this position by shorting some of each of the other assets. For example in row 1, a 1 unit higher signal leads to 1.54 units more of that asset, and that is hedged by shorting -0.21 of each of the other 7 assets. Note that the investors does not fully hedge out systematic risk, as 1.54 is greater than 7 times -0.21. One reason for this is because the investors are getting a combined signal on the systematic and idiosyncratic components of the stock payoffs.

Compare this to the case with the ETF: The informed investor bets *more* aggressively on the stock for low values of risk aversion/systematic risk. But, in all cases, they hedge out all systematic risk with the ETF. In addition, if we fix the cost of becoming informed, investors bet less aggressively on the idiosyncratic assets because introducing the ETF decreases the share of agents who become informed, so prices become less informative on average.

This result is not unique to how informed investors respond to their own signals. Both informed and uninformed investors change their behavior in response to the signal contained in prices. For details, see Appendix A.5.

Attention Allocation

Another key point of the paper is to understand how introducing the ETF affects attention allocation. As shown above, introducing the ETF can change the share of agents becoming informed, which makes it difficult to isolate the effect of attention re-allocation. In this subsection, I fix the share of agents who decide to become informed, and look at intensive-margin learning effects.

Figure 3 shows the relationship between the share of agents who decide to become informed, and the share of attention allocated to systematic risk. When risk aversion is low, and systematic risk is low, introducing the ETF actually *decreases* learning about systematic risk. This is related to the hedging demand channel discussed above. Investors are willing to bet aggressively on their private signals, and can hedge out all systematic risk through an offsetting position of the same size in the ETF. As we increase systematic risk, the effect of introducing the ETF depends on the cost of becoming informed. Once risk aversion, systematic risk, or the share of people learning is sufficiently high, introducing the ETF almost universally increases attention on systematic risk.

Risk Premia

Introducing the ETF decreases expected returns in the economy. This is not surprising, as the ETF increases the information in the economy i.e. it adds an n^{th} public signal. Table 4 shows that introducing the ETF universally decreases average asset returns. This can have a large effect on model outcomes. For example, the last row of Table 4 has expected

Panel A: Fixing Cost of Becoming Informed								
		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.05	0.2	1.54	-0.21	1.75	0.00	-1.75
0.1	0.5	0.35	0.2	1.10	-0.10	1.23	0.00	-1.23
0.25	0.2	0.5	0.2	0.42	-0.04	0.20	0.00	-0.20
0.25	0.5	0.5	0.2	0.36	-0.03	0.20	0.00	-0.20

Panel B: Share Informed at 10%								
		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.1	0.1	1.50	-0.20	1.75	0.00	-1.75
0.1	0.5	0.1	0.1	1.08	-0.10	0.69	0.00	-0.69
0.25	0.2	0.1	0.1	0.42	-0.03	0.20	0.00	-0.20
0.25	0.5	0.1	0.1	0.36	-0.02	0.20	0.00	-0.20

Panel C: Share Informed at 30%								
		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.3	0.3	1.42	-0.18	1.75	0.00	-1.75
0.1	0.5	0.3	0.3	1.10	-0.10	1.39	0.00	-1.39
0.25	0.2	0.3	0.3	0.42	-0.04	0.34	0.00	-0.34
0.25	0.5	0.3	0.3	0.36	-0.03	0.20	0.00	-0.20

Table 3: **Hedging Demand.** Entries of G_1 for one of the stocks i.e. assets 1 to $n-1$. In panel A, the cost of being informed is chosen such that 20% of agents become informed when the ETF is present. In Panels B and C, the share of informed agents are fixed and 10% and 30% respectively. The "Own" columns are diagonal entries e.g. (1,1). The "Stock Hedge" column is one of the edge entries excluding the n^{th} e.g. (1,2) or (2,1). The "ETF Hedge" column is the n^{th} edge entry.

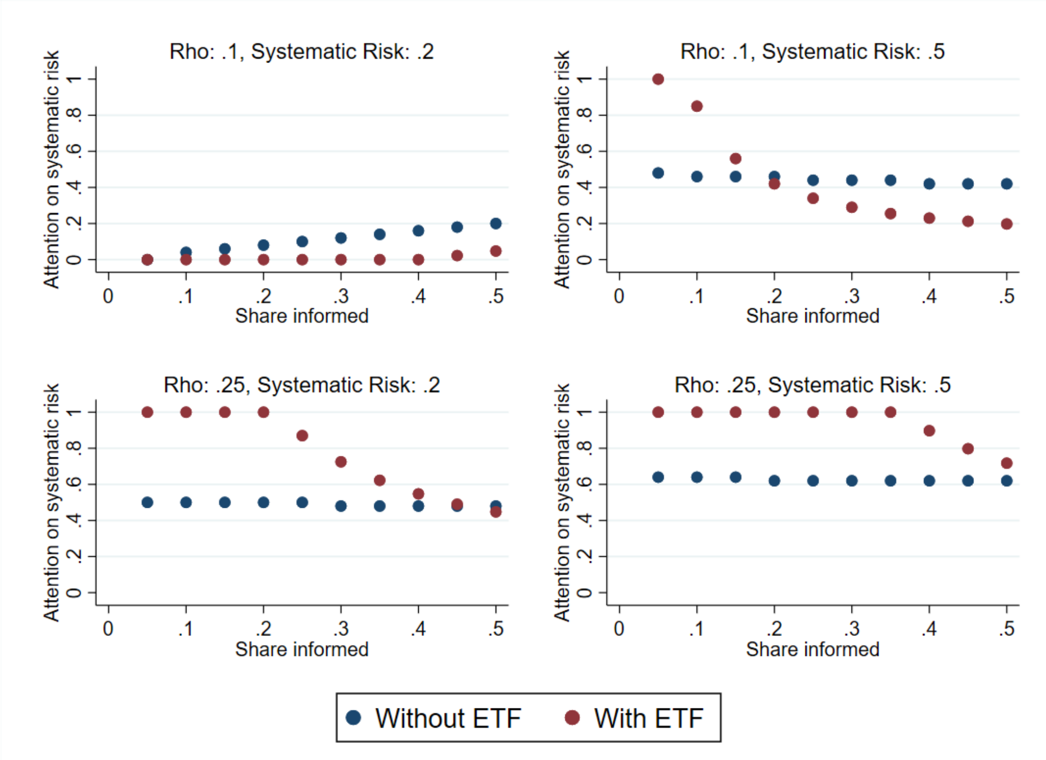

Figure 3: **Effect of Introducing the ETF on Attention Allocation (fixed share informed).**

returns decrease by over 50% after introducing the ETF. I view this risk premia as more of a modeling artifact than a testable prediction, and want to take out this effect when studying price informativeness. To do this, I work with market-adjusted returns: I calculate the returns of each asset net of the market returns, as discussed in Campbell, Lettau, Malkiel, and Xu [2001]. Market-adjusted returns are also used for all the empirical exercises in Sammon [2020a].

ρ	σ_n^2	Share Informed	Avg. Cumulative Return No ETF	With ETF
0.1	0.2	0.1	3.73%	3.69%
0.1	0.2	0.3	3.64%	3.45%
0.1	0.5	0.1	7.92%	4.72%
0.1	0.5	0.3	6.81%	4.11%
0.25	0.2	0.1	9.75%	9.18%
0.25	0.2	0.3	9.46%	7.57%
0.25	0.5	0.1	22.74%	18.84%
0.25	0.5	0.3	21.11%	9.10%

Table 4: **Effect of introducing the ETF on Expected Returns.**

3.3 Sensitivity to Parameter Choice

So far, I have focused on the four baseline parameter choices. In this sub-section, I want to examine how sensitive the model is to varying risk aversion and systematic risk.

In Figure 4 I fix the share of agents who decide to become informed at 20% (the baseline choice in Kacperczyk et al. [2016]), and look at the effect on learning about systematic risk. As risk aversion increases, learning about systematic risk increases. This is because systematic risk is the largest risk in the economy, as all assets are exposed to it. The relationship is steeper in the economy with the ETF and when risk aversion is high.

In Figure 5, I again fix the share of informed agents at 20% and vary σ_n^2. As expected, increasing systematic risk leads to increased learning about systematic risk. The effect is steeper when risk aversion is high and when the ETF is present.

4 Price Informativeness

In this sub-section, I explore the predictions of the model for introducing an ETF on pre-earning price informativeness. To map the model to the empirical exercises in Sammon

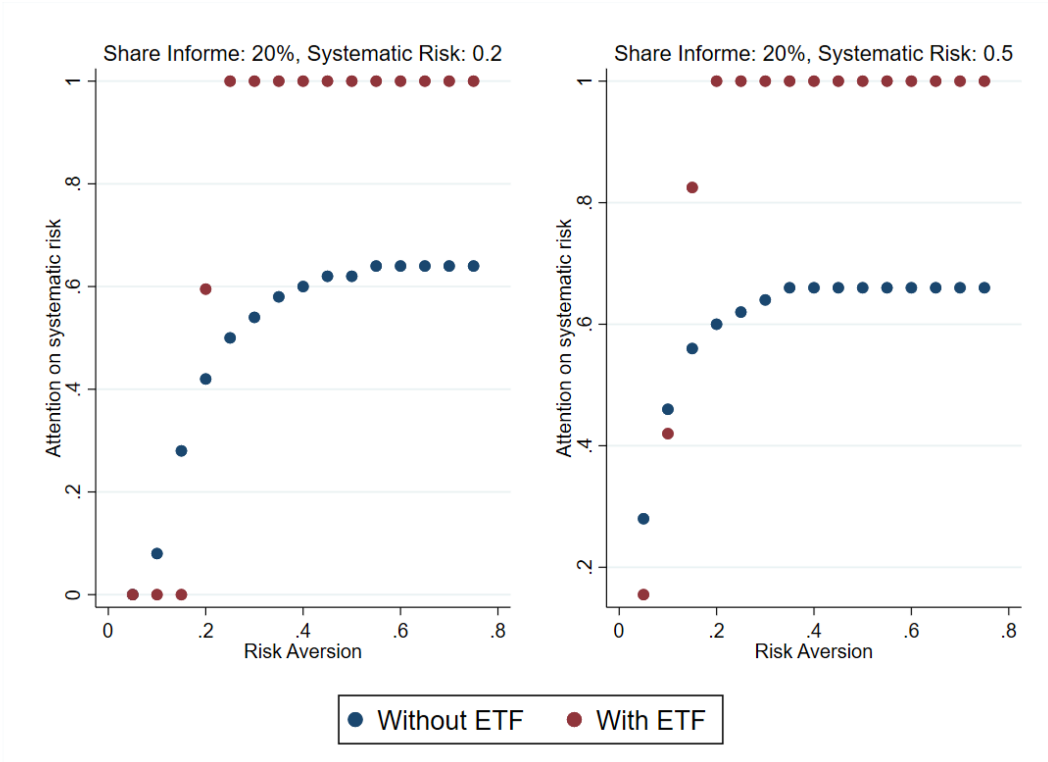

Figure 4: **Relationship between risk aversion and attention to systematic risk-factor.**

[2020a], I define $t = 1$ as the pre-earnings date, and $t = 2$ as the earnings date. The natural next step is to calculate a model-based measure of price-informativeness that could to tell us something directly about the information content of prices. The issue is that these model-based measure of price informativeness are hard to measure in practice, and there is much debate about the "right" way to do this.

For example, Grossman and Stiglitz [1980] defines price informativeness as a *conditional* covariance, which requires identifying the 'right' set of conditioning variables. Academic economists, still disagree on this. Bai, Philippon, and Savov [2016] measure price informativeness as the variance of fundamentals, conditional on prices. Dávila and Parlatore [2019] measure price informativeness as the variance of prices, conditional on fundamentals;

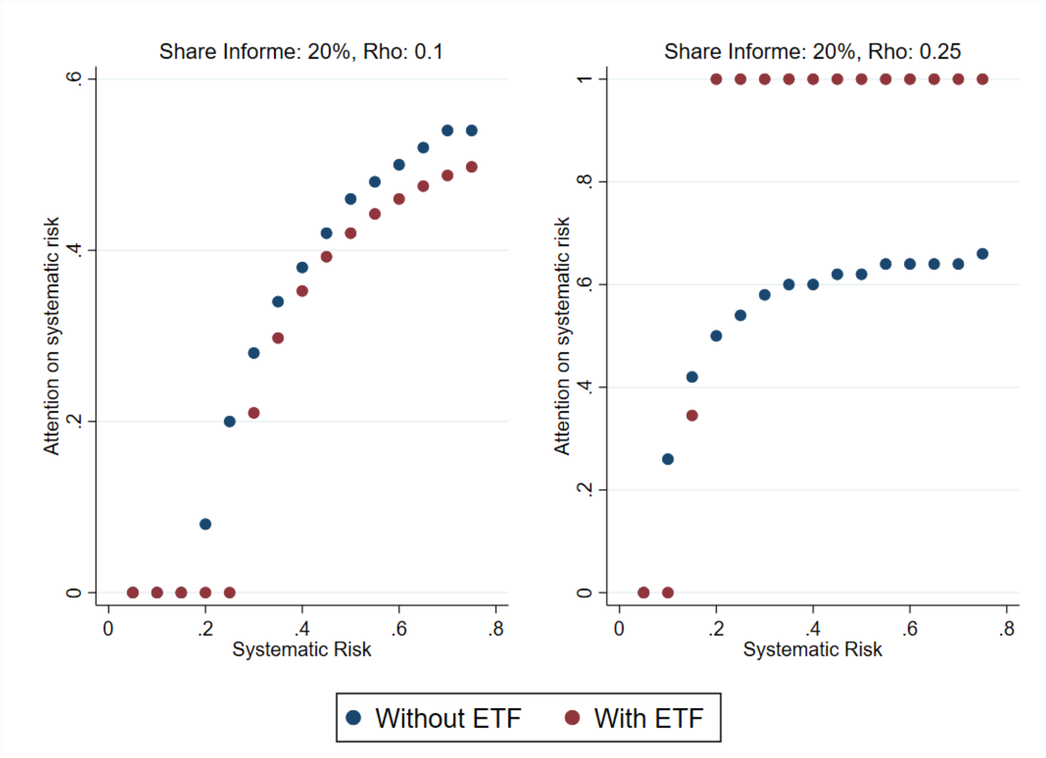

Figure 5: **Relationship between systematic risk and attention to systematic risk-factor.**

effectively switching the left-hand-side and right-hand-side variables of a regression.

For this reason, I want to focus on the three model-free measures of price informativeness discussed in Sammon [2020b]. I create model analogues of these objects, and simulate the economy to determine the effect of the ETF on these measures. I will examine the effect of introducing the ETF while varying the model on three dimensions: (1) risk aversion (2) volatility of systematic risk (3) share of agents who decide to become informed/cost of becoming informed.

4.1 Defining Price Informativeness Measures

In this sub-section, I work exclusively with market-adjusted returns to take out the risk-premium channel. See Appendix A.3 for the effect of introducing the ETF working with raw returns.

Pre-Earnings Volume

Although we are assuming a continuum of investors, when running the simulations, there are a finite number of traders, which I set to 10,000. Suppose we believe that at $t = 0$ all of the informed traders are endowed with $1/10,000^{th}$ of $\overline{x}$. Then we can think of trading volume as the difference between agents' initial holdings, and their holdings after submitting their demand at $t = 1$. This measure would be contaminated by the effect of the noise shock, so I want to compute a measure similar to turnover. To do this, I normalize the difference between initial holdings and final holdings by the total supply of the asset, *including* the supply shock. I then take a weighted-average of this measure across informed an uninformed investors.

For now, I am going to focus in trading in the individual stocks. There are two main factors that affect trading volume: (1) The share of investors who decide to become informed. As more investors become informed, there are more different signals, and thus more trading. Uninformed investors all submit the same demand, so in the limit with all uninformed investors, there would be no trading volume after accounting for the noise shock (2) Attention allocation. As more attention is devoted to the individual stocks, informed investors have more precise posterior beliefs, and are more willing to bet aggressively on their signals. We know that introducing the ETF both decreases incentives to become informed, and shifts attention tword the systematic risk-factor, so our prior is that it should decrease trading volume.

Pre-Earnings Drift

Define the pre-earnings drift:

$$DM = \begin{cases} \frac{1+r_{(0,1)}}{1+r_{(0,2)}} & \text{if } r_2 > 0 \\ \frac{1+r_{(0,2)}}{1+r_{(0,1)}} & \text{if } r_2 < 0 \end{cases} \tag{10}$$

where $r_{(0,t)}$ is the cumulative return from 0 to t. The pre-earnings drift will be near one when the return at $t = 2$ is small relative to the return at $t = 1$. $DM_{i,t}$ will be less than one when the $t = 2$ return is large, relative to the returns at $t = 1$. If r_2 is negative, this relationship would be reversed, which is why the measure is inverted when r_2 is less than zero. To compute this measure, I save the prices at $t = 0$[9], $t = 1$ (calculated using the equilibrium in Admati [1985]), $t = 2$ (terminal payoffs), and compute returns as $r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$. The pre-earnings drift measure would clearly be influenced by the risk premium channel, which is another reason to work exclusively with market-adjusted returns.

Share of Volatility on Earnings Days

Define the relative volatility of earnings days as $r_2^2 / \left(r_1^2 + r_2^2\right)$ If prices are not informative before earnings announcements, we would expect earnings day volatility to be large, relative to total volatility. Note that this is not sensitive to using squared returns i.e. focusing on extreme observations – it also works with absolute returns.

4.2 Exercise 1: Fix the Cost of Becoming Informed

We know that introducing the ETF changes investor's incentives to become informed. To make the two settings comparable, I do the following: Fix the share of informed agents in

[9]The price at $t = 0$ is the price at the expected values of z and x i.e. prices if $x = 0$ and $z = 0$.

		Share Informed		c	
ρ	σ_n^2	No ETF	ETF	No ETF	ETF
0.1	0.2	0.05	0.2	\$ 0.35	\$ 0.36
0.1	0.5	0.35	0.2	\$ 0.39	\$ 0.38
0.25	0.2	0.5	0.2	\$ 0.51	\$ 0.51
0.25	0.5	0.5	0.2	\$ 1.29	\$ 1.28

Table 5: **Matching Share Informed Across Economies.** This table shows for each set of baseline parameters how many agents would become informed without the ETF being present to match having 20% of agents become informed when the ETF is present.

the world with the ETF at 20%. Then, calculate the cost of becoming informed. After that, find the closest cost of being informed on the grid for the world without the ETF. Table 5 contains the results. If risk aversion and systematic risk are sufficiently low, more people become informed when the ETF is present. In the other cases, the share of agents who decide to become informed decrease when we introduce the ETF.

Table 6 shows the effects of introducing the ETF when fixing the cost of becoming informed. In the first scenario, introducing the ETF *increases* per-earnings trading volume, increases the pre-earnings drift and decreases the share of volatility on earnings days. For these parameters, introducing the ETF seems to have made pre-earnings prices more informative. In the 2nd and 3rd scenarios, the ETF makes prices less informative across all measures. In the last scenario, volume and volatility point to less informative prices, but the drift points to more informative prices. This is likely a combination of the share of agents learning and attention re-allocation effect. In this scenario, informed agents are only learning about systematic risk, so the risk premium on the individual assets is high.

		No ETF			With the ETF		
ρ	σ_n^2	Volume	Drift	Volatility	Volume	Drift	Volatility
0.1	0.2	0.210	0.963	0.876	0.846	0.965	0.710
0.1	0.5	1.104	0.965	0.676	0.686	0.963	0.768
0.25	0.2	0.603	0.961	0.740	0.165	0.961	0.857
0.25	0.5	0.650	0.958	0.750	0.165	0.959	0.857

		Change Introducing the ETF			t-Test		
ρ	σ_n^2	Volume	Drift	Volatility	Volume	Drift	Volatility
0.1	0.2	0.636	0.002	-0.167	558.41	20.19	-64.07
0.1	0.5	-0.418	-0.001	0.093	-472.67	-20.12	56.95
0.25	0.2	-0.438	-0.001	0.118	-874.14	-8.39	55.27
0.25	0.5	-0.484	0.001	0.108	-725.18	10.31	49.86

Table 6: **Effect of Introducing the ETF on Price Informativeness (fixed c).**

4.3 Exercise 2: Fix the Share of Informed Agents

We know that introducing the ETF changes the share of agents who decide to become informed. In 3/4 of the baseline parameter choices, the ETF decreases the share of agents who become informed for a fixed cost of becoming informed. Intuitively, as fewer agents become informed, price informativeness will go down. To isolate the effect of the attention re-allocation channel, I fix the share of informed agents, and compare the economies with/without the ETF.

Table 7 shows the results. Introducing the ETF always decreases the drift, and increases the volatility on earnings days. This is for a few reasons (1) learning channel - as show above, in many cases, for a fixed amount of informed agents, introducing the ETF decreases learning about idiosyncratic risk (2) risk aversion/hedging channel – when the ETF is present, agents are generally exposed to more risk, so they are less willing to be aggressively at $t = 1$. Volume is driven by disagreements, so this really isolates the learning channel, and shows when the

ρ	σ_n^2	Share Informed	No ETF Volume	No ETF Drift	No ETF Volatility	With the ETF Volume	With the ETF Drift	With the ETF Volatility
0.1	0.2	0.1	0.44	0.95	0.86	0.40	0.95	0.86
0.1	0.2	0.3	1.22	0.95	0.71	1.08	0.95	0.74
0.1	0.5	0.1	0.25	0.94	0.70	0.34	0.92	0.88
0.1	0.5	0.3	1.07	0.95	0.63	0.96	0.93	0.77
0.25	0.2	0.1	0.08	0.92	0.92	0.14	0.91	0.92
0.25	0.2	0.3	0.33	0.93	0.85	0.39	0.91	0.90
0.25	0.5	0.1	0.08	0.85	0.95	0.15	0.82	0.97
0.25	0.5	0.3	0.24	0.92	0.77	0.41	0.83	0.95

ρ	σ_n^2	Share Informed	Change After Introducing the ETF Volume	Change After Introducing the ETF Drift	Change After Introducing the ETF Volatility	t-Test Volume	t-Test Drift	t-Test Volatility
0.1	0.2	0.1	-0.04	0.00	0.00	-78.88	-14.97	5.31
0.1	0.2	0.3	-0.13	0.00	0.02	-145.69	-30.29	19.18
0.1	0.5	0.1	0.09	-0.02	0.18	203.70	-62.89	52.32
0.1	0.5	0.3	-0.11	-0.02	0.13	-119.39	-57.23	38.49
0.25	0.2	0.1	0.05	0.00	0.00	289.57	-35.22	0.25
0.25	0.2	0.3	0.06	-0.01	0.05	196.09	-73.11	22.05
0.25	0.5	0.1	0.07	-0.03	0.02	231.70	-96.82	13.35
0.25	0.5	0.3	0.16	-0.09	0.18	261.23	-202.86	65.62

Table 7: **Effect of Introducing the ETF on Price Informativeness (fixed share of informed agents).**

effect is ambiguous. The second panel in Table 7 examines the statistical significance of these changes.

4.4 Relationship to Empirical Results

Sammon [2020a] finds that increasing passive ownership leads to (1) decreased pre-earnings trading volume (2) decreased pre-earnings drift (3) increased share of volatility on earnings days. All results in that paper are robust to defining passive ownership only as ETFs. The model can explain these results if risk aversion is sufficiently high. It could also be that systematic risk is high, but we do not have evidence that is the case see e.g. the appendix of Sammon [2020a] which shows that if anything, systematic risk has decreased. It also requires that the cost of becoming informed is sufficiently high. While there is no clear empirical analogue to c, it seems unlikely that it has decreased over time for a few reasons (1) Firms have become increasingly complex, selling more products, operating in more geographic locations, and having more intertwined supply chains (2) There has been increased competition for information, leading to more advanced techniques such as drone fly-overs to learn about shipping volumes, car sales, etc.

5 Conclusion

The introduction of ETFs has been one of the biggest changes in financial markets over the past 30 years. Given that a main function of financial markets is to aggregate information, it is important to know how their ability to perform this function changes as financial technology changes. This paper focuses on the effect of introducing ETFs on learning, which in turn leads to changes in price informativeness for individual stocks.

I find three main effects of introducing ETFs: (1) Changes in the share of agents who decide to become informed (2) Changes in the attention of informed agents (3) Decreases in risk premia. These effects all depend on risk aversion, the relative riskiness of systematic risk, and the cost of becoming informed.

I find that effect of introducing the ETF on pre-earnings price informativeness is ambiguous, which is why empirical work is needed to test the predictions of the model. Sammon [2020a] shows that increases in passive ownership cause decreases in price informativeness. The paper also shows direct evidence on decreased information gathering, through analyst coverage and downloads of SEC filings. This is consistent with higher risk-aversion or systematic risk in the model.

References

Anat R Admati. A noisy rational expectations equilibrium for multi-asset securities markets. *Econometrica: Journal of the Econometric Society*, pages 629–657, 1985.

Jennie Bai, Thomas Philippon, and Alexi Savov. Have financial markets become more informative? *Journal of Financial Economics*, 122(3):625–654, 2016.

Itzhak Ben-David, Francesco Franzoni, and Rabih Moussawi. Do etfs increase volatility? *The Journal of Finance*, 73(6):2471–2535, 2018.

John Y Campbell, Martin Lettau, Burton G Malkiel, and Yexiao Xu. Have individual stocks become more volatile? an empirical exploration of idiosyncratic risk. *The Journal of Finance*, 56(1):1–43, 2001.

Alexander Chinco and Vyacheslav Fos. The sound of many funds rebalancing. 2019.

Eduardo Dávila and Cecilia Parlatore. Trading costs and informational efficiency. Technical report, National Bureau of Economic Research, 2019.

Sanford J Grossman and Joseph E Stiglitz. On the impossibility of informationally efficient markets. *The American economic review*, 70(3):393–408, 1980.

Marcin Kacperczyk, Stijn Van Nieuwerburgh, and Laura Veldkamp. A rational theory of mutual funds' attention allocation. *Econometrica*, 84(2):571–626, 2016.

Marco Sammon. Earnings announcements and the rise of passive ownership. Technical report, Kellogg School of Management, 2020a.

Marco Sammon. Etf introduction, learning and price informativeness. Technical report, Kellogg School of Management, 2020b.

Laura L Veldkamp. *Information choice in macroeconomics and finance.* Princeton University Press, 2011.

A Appendix

A.1 Model where individual assets are not symmetric

Suppose each asset i now has the payoff:

$$f_i = a_i + \beta_i z_n + z_i \tag{11}$$

where β_i and $var(z_i)$ is different for each assets. In this setting, informed agents' choice is not just a trade-off between learning about systematic and idiosyncratic risk. To solve for information choice in this version of the model, I need to modify the numerical method:

1. Start all agents at K^0

2. Consider an atomistic agent j who takes K^0 as given, and considers their expected utility by deviating to K_j^1 on a $n-1 \times n-1$ dimensional grid around K^0. Even though there are n risks to learn about, we don't need the n^{th} dimension because of the total information constraint.

3. Calculate the gradient numerically at K^0 using this grid of expected deviation utilities. Then, move j on the grid in the direction of the gradient.

4. If j's expected utility increased, move all informed agents to K_j^1

5. Iterate on steps 2-4 until j can no longer improve their expected utility by deviating.

This method works, and when the ETF is present, is able to match closed form solutions from Kacperczyk et al. [2016]. For $n > 3$, however, this method can take an extremely long time to find the solution. Given that heterogenous this does not drastically change pre-earnings volume, pre-earnings drift or earnings-day volatility, I focus on the symmetric asset case in the main body of the paper.

A.2 Return Predictability

One concern with a rational-expectations-equilibrium model like this is that there will be return predictability. In my setting, this would mean the return from $t = 0$ to $t = 1$ will have predictive power for the return from $t = 1$ to $t = 2$. I explicitly test this in the model, and find ... [add details on the test, and show there is no return predictability]

To test this, we can take the same simulation results from above, and run the following regression

$$r_{i,2} = a + br_{i,1} + \epsilon_i \tag{12}$$

To test whether returns at time 1 predict returns at time 2. Table 8 has the results. Seems like their is some predictability, but it is very small, note that even though the coefs are significant, the R-squareds are very tiny. I also constructed an alternative predictability metric called *Same Sign*. This is equal to 1 if r_1 and r_2 are the same sign. It is zero otherwise. I then do a test of whether this is equal to 0.5 i.e. whether or not returns are more/less likey to be the same sign. I find ABC

A.3 Effect of Introducing the ETF without market-adjusting returns

Table 9 shows the results when we do not market-adjust the returns. This is not that useful, because it is mixing a few of the key channels – especially the risk-aversion channel which is going to have a big effect on the drift and share of volatility on earnings days.

ρ	0.1	0.1	0.1	0.1	0.25	0.25	0.25	0.25
σ_n^2	0.2	0.2	0.5	0.5	0.2	0.2	0.5	0.5
Share Informed	0.1	0.3	0.1	0.3	0.1	0.3	0.1	0.3
				No ETF				
Return at $t = 1$	-0.370***	-0.164***	-0.540***	-0.270***	-0.905***	-0.675***	-1.068***	-0.854***
	(0.051)	(0.023)	(0.043)	(0.022)	(0.045)	(0.033)	(0.030)	(0.025)
Observations	10000	10000	10000	10000	10000	10000	10000	10000
R-Squared	0.005	0.005	0.016	0.015	0.04	0.04	0.113	0.111
%Same Sign	0.486	0.489	0.482	0.485	0.479	0.48	0.494	0.494
t-Stat	-2.761	-2.14	-3.562	-3.021	-4.143	-3.963	-1.16	-1.14
				with the ETF				
Return at $t = 1$	-0.276***	-0.112***	-0.0881***	-0.0838***	-0.537***	-0.312***	-0.435***	-0.168***
	(0.050)	(0.021)	(0.019)	(0.014)	(0.047)	(0.028)	(0.028)	(0.015)
Observations	10000	10000	10000	10000	10000	10000	10000	10000
R-Squared	0.003	0.003	0.002	0.004	0.013	0.013	0.024	0.013
%Same Sign	0.487	0.494	0.494	0.499	0.49	0.49	0.498	0.492
t-Stat	-2.661	-1.12	-1.1	-0.12	-2.06	-2.08	-0.32	-1.68

Table 8: **Return Predictability.** Define the return at t as $(p_t - p_{t-1})/p_{t-1}$. This table contains results from the regression of r_2 on r_1. The "% same sign" row is a calculate of what % of the time r_1 and r_2 have the same sign. The "t-Stat" row is a t-test of whether the % Same Sign is different from 50% i.e. what we would expect if prices were a martingale.

		No ETF			With the ETF		
ρ	σ_n^2	Volume	Drift	Volatility	Volume	Drift	Volatility
0.1	0.2	0.210	0.947	0.902	0.846	0.949	0.776
0.1	0.5	1.104	0.930	0.743	0.686	0.943	0.676
0.25	0.2	0.603	0.916	0.866	0.165	0.924	0.869
0.25	0.5	0.650	0.846	0.921	0.165	0.887	0.867

		Change Introducing the ETF			t-Test		
ρ	σ_n^2	Volume	Drift	Volatility	Volume	Drift	Volatility
0.1	0.2	0.636	0.002	-0.126	558.41	19.54	-55.53
0.1	0.5	-0.418	0.012	-0.066	-472.67	38.15	-18.53
0.25	0.2	-0.438	0.009	0.003	-874.14	42.71	1.25
0.25	0.5	-0.484	0.041	-0.054	-725.18	106.44	-25.56

Table 9: **Effect of introducing the ETF on price informativeness.** This table estimates the effect of introducing the ETF on pre-earnings volume, the pre-earnings drift and earnings-day volatility. In this table, the cost of being informed is fixed such that 20% of agents decide to become informed when the ETF is present.

A.4 Expected utility of informed and uninformed

A.5 Sensitivity of Demand to Prices

This is also a type of hedging demand. Similar to the hedging from signals to informed agents, investors also use prices as a signal, and thus may do a similar hedging. Table 11 fixes the cost of becoming informed. Table 12 fixes the share of agents becoming informed.

Panel A: Matching Cost of Becoming Informed

		Share Informed		Diff. in EU	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF
0.1	0.2	0.05	0.2	0.154%	0.163%
0.1	0.5	0.35	0.2	0.181%	0.177%
0.25	0.2	0.5	0.2	0.229%	0.229%
0.25	0.5	0.5	0.2	0.572%	0.571%

Panel B: Share Informed at 10%

		Share Informed		Diff. in EU	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF
0.1	0.2	0.1	0.1	0.154%	0.177%
0.1	0.5	0.1	0.1	0.226%	0.186%
0.25	0.2	0.1	0.1	0.251%	0.296%
0.25	0.5	0.1	0.1	0.727%	1.103%

Panel C: Share Informed at 30%

		Share Informed		Diff. in EU	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF
0.1	0.2	0.3	0.3	0.132%	0.141%
0.1	0.5	0.3	0.3	0.190%	0.154%
0.25	0.2	0.3	0.3	0.237%	0.211%
0.25	0.5	0.3	0.3	0.650%	0.300%

Table 10: **Effect of Introducing the ETF on Expected Utility of Informed and Uninformed.** This table quantifies the effect of introducing the ETF on the expected utility of informed and uninformed agents. The columns of interest are under the header "Diff. in EU". The "no ETF" column is the % difference in expected utility between informed and uninformed agents when the ETF is not present. The ETF column repeats this exercise after introducing the ETF.

		Uninformed						
		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.05	0.2	6.333	-0.278	2.273	0.000	-2.273
0.1	0.5	0.35	0.2	1.764	-0.170	3.082	0.000	-3.082
0.25	0.2	0.5	0.2	2.380	-0.181	5.510	0.000	-5.510
0.25	0.5	0.5	0.2	2.550	-0.291	5.510	0.000	-5.510

		Informed						
		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.1	0.2	7.872	-0.489	4.023	0.000	-4.023
0.1	0.5	0.35	0.2	2.865	-0.270	4.307	0.000	-4.307
0.25	0.2	0.5	0.2	2.803	-0.218	5.710	0.000	-5.710
0.25	0.5	0.5	0.2	2.913	-0.317	5.710	0.000	-5.710

Table 11: **Sensitivity of Demand to Prices (fixed c).** Entries of $G_{2,inf}$ and $G_{2,un}$ for one of the stocks i.e. assets 1 to $n-1$. The cost of being informed is chosen such that 20% of agents become informed when the ETF is present. The "Own" columns are diagonal entries e.g. (1,1). The "Stock Hedge" column is one of the edge entries excluding the n^{th} e.g. (1,2) or (2,1). The "ETF Hedge" column is the n^{th} edge entry.

Panel A: Share Informed Fixed at 10%

Uninformed

		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.1	0.1	4.096	-0.036	4.040	0.000	-4.040
0.1	0.5	0.1	0.1	4.899	-0.528	7.656	0.000	-7.656
0.25	0.2	0.1	0.1	4.884	-0.464	6.270	0.000	-6.270
0.25	0.5	0.1	0.1	4.976	-0.601	6.270	0.000	-6.270

Informed

		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.1	0.1	5.597	-0.236	5.790	0.000	-5.790
0.1	0.5	0.1	0.1	5.979	-0.623	8.343	0.000	-8.343
0.25	0.2	0.1	0.1	5.299	-0.499	6.470	0.000	-6.470
0.25	0.5	0.1	0.1	5.331	-0.626	6.470	0.000	-6.470

Panel B: Share Informed Fixed at 30%

Uninformed

		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.3	0.3	1.774	0.059	1.581	0.000	-1.581
0.1	0.5	0.3	0.3	2.020	-0.197	1.950	0.000	-1.950
0.25	0.2	0.3	0.3	3.190	-0.266	4.018	0.000	-4.018
0.25	0.5	0.3	0.3	3.364	-0.393	4.914	0.000	-4.914

Informed

		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.3	0.3	3.198	-0.117	3.331	0.000	-3.331
0.1	0.5	0.3	0.3	3.121	-0.296	3.337	0.000	-3.337
0.25	0.2	0.3	0.3	3.614	-0.303	4.356	0.000	-4.356
0.25	0.5	0.3	0.3	3.728	-0.419	5.114	0.000	-5.114

Table 12: **Sensitivity of Demand to Prices (fixed share informed).** Entries of $G_{2,inf}$ and $G_{2,un}$ for one of the stocks i.e. assets 1 to $n-1$. In Panels A and B, the share of informed agents are fixed and 10% and 30% respectively. The "Own" columns are diagonal entries e.g. (1,1). The "Stock Hedge" column is one of the edge entries excluding the n^{th} e.g. (1,2) or (2,1). The "ETF Hedge" column is the n^{th} edge entry.