ETFs, Learning, and Information in Stock Prices

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Abstract

This paper studies how the introduction of ETFs, and the growth of ETF ownership, can change investors' learning behavior. I develop a rational-expectations model where agents decide (1) whether they want to become informed or not and (2) if informed, how to allocate their limited attention between learning about individual stocks and a systematic risk-factor. Introducing an ETF does not universally increase or decrease learning about systematic risk. If the volatility of the systematic risk-factor is large, risk aversion is high, or the cost of becoming informed is high, introducing the ETF leads investors to devote more attention to the systematic risk-factor. Otherwise, the ETF may lead investors investors to learn more about the individual stocks. I decompose the effect of introducing the ETF into 2 channels: (1) Changes in the share of agents who decide to become informed (2) Re-allocation of attention among informed investors. I then extend the model, allowing an intermediary to buy the underlying stocks and create more shares of the ETF. Finally, I link the model's predictions to empirical evidence on the growth of ETF ownership and less informative stock prices.

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1 Introduction

Sammon [2020a] shows that stock prices before earnings announcements have become less informative over the past 30 years. Between 1990 and 2018, pre-earnings trading volume dropped, the pre-earnings drift declined and the share of annual volatility on earnings days increased. Over the same time period, there was a boom in passive ownership. ETFs were first introduced in 1993, and have since grown rapidly, now owning almost 10% of the US stock market[1]. The proposed mechanism in Sammon [2020a] is that passive ownership decreases incentives to gather information on individual stocks. This paper develops a model which can rationalize these empirical findings.

I develop a model which features a systematic risk-factor that affects all assets, but initially there is no way to directly trade this risk. I then introduce an ETF, which is an asset only exposed to systematic risk. The effect of introducing the ETF on price informativeness can be decomposed into two parts: (1) Changes in the share of investors who decide to become informed (2) Changes in the share of attention investors devote to systematic risk. I find the sign of these effects is ambiguous. The ambiguity stems from the trade-off between earning high trading profits and avoiding portfolios that seems too risky. Which of these forces dominates depends on agents' effective risk aversion, and more broadly, the risk-bearing capacity of the economy.

Introducing the ETF can reduce the number of agents who decide to become informed by making uninformed investors better-off in equilibrium. This occurs when the risk-bearing capacity of the economy is relatively low i.e. if risk aversion is high, or if the systematic risk-factor is more volatile, relative to the individual stocks. Agents can only trade a limited number of assets (as in e.g. Merton [1987]), so the ETF is ex-ante less risky than all

[1] Another 5% of the market is owned by passive index mutual funds.

feasible portfolios of the underlying stocks, as such portfolios will always be exposed to some idiosyncratic risk. Because they have less precise posterior beliefs, uninformed investors are effectively more risk averse than informed investors, and thus prioritize holding lower risk and/or more diversified portfolios. As a result, uninformed agents make up the majority of long positions in the ETF, and are made better off by its introduction.

If the risk-bearing capacity of the economy is high i.e. both risk aversion and systematic risk are low, introducing the ETF can *increase* the number of agents who decide to learn. This is because without the ETF, investors cannot perfectly replicate the systematic risk-factor by buying or selling a portfolio of all the individual stocks. Informed investors see higher upside in betting on stocks, because idiosyncratic risk-factors are more volatile than the systematic risk-factor. Introducing the ETF allows agents to hedge out systematic risk when they bet on individual stocks, making learning about idiosyncratic risk-factors even more profitable. If risk aversion is sufficiently low, this force will dominate, and introducing the ETF can actually increase the number of agents who become informed in equilibrium.

Introducing the ETF can also lead to a re-allocation of attention because it makes learning about the systematic risk-factor relatively more profitable. This is for two reasons: (1) It is more profitable to learn about the systematic shock because informed investors can bet on it directly (2) Uninformed agents use the ETF's price as a signal for the systematic shock. All of the assets in the economy are exposed to this shock, so superior knowledge on this risk-factor can lead to large expected utility gains. To retain their information advantage over uninformed agents, informed investors may devote more attention to learning about systematic risk when the ETF is present.

Introducing the ETF also has implications for risk premia i.e. expected returns. Without the ETF, informed agents cannot perfectly hedge out systematic risk when betting on individual stocks, making all their investments riskier. In addition, without the ETF, uninformed agents are forced to hold more stock-specific risk. By making it easier to form low-risk portfolios, and allowing for more precise hedging, introducing the ETF can decrease expected returns in the economy.

Which of these forces dominates depends on model parameters. I focus on the effect of varying (1) risk aversion (2) the volatility of the systematic risk-factor, relative to the idiosyncratic risk-factor (3) the cost of becoming informed, which determines the share of investors who decide to become informed in equilibrium. Fixing the cost of becoming informed, introducing the ETF decreases the share of investors who decide to learn, as long as risk aversion or systematic risk are not too low. Fixing the share of agents who become informed, when risk aversion, the volatility of systematic risk, or the cost of becoming informed are high, introducing the ETF decreases learning about the individual stocks.

The next step is linking learning to price informativeness. I create model-analogues to the empirical informativeness measures in Sammon [2020a]: pre-earnings trading volume, the pre-earnings drift and the share of volatility on earnings days. Empirically, the growth of passive ownership has decreased pre-earnings price informativeness. This is shown causally through increases in passive ownership associated with the S&P 500 index additions and Russell 1000/2000 index rebalancing. This is consistent with the predictions of the model when risk aversion is high, and/or systematic risk is high.

The natural experiments in Sammon [2020a], however, are not perfect analogues to introducing an ETF in the model. To better link the model to the data, I propose a new natural experiment which relies on the staggered introduction of sector-specific ETFs. I find that when a sector ETF is introduced, firms in that sector experience a decrease in price informativeness, relative to firms outside of these sectors.

As an alternative way to link the model's predictions to the empirical results in Sammon [2020a], I develop an extension where the size of passive ownership can vary. I introduce an intermediary (she) who can buy stocks and put them into an ETF. By creating more shares of the ETF, however, she bears more basis risk. Thus, varying her risk aversion will endogenously change the amount of passive ownership. I present a calibration that shows a monotonic decrease in price informativeness as we (1) introduce the ETF and (2) increase the size of the ETF (by decreasing the intermediary's risk aversion).

This paper builds on two models. The version of the economy without the ETF is similar to Admati [1985], except I have added an additional risk and endogenous information choice. The version of the economy with the ETF is similar to Kacperczyk, Van Nieuwerburgh, and Veldkamp [2016], but I've (1) made the asset for trading systematic risk in zero average supply (2) added a fixed cost of becoming informed. The first change change is to better map their model to my setting: ETFs are just bundles of underlying shares, and their introduction does not actually increase the average amount of systematic risk in the economy. This remains true in the version of the model where the size of passive ownership can vary. The intermediary's technology ensures that the amount of systematic risk in the newly created shares of the ETF is exactly equal to the amount of systematic risk in the underlying stocks used to create these shares of the ETF.

The reasons behind the rapid growth of passive ownership are interesting, but somewhat outside the scope of this paper[2]. I take the introduction of an ETF as given, and study the effect on agents' learning behavior. This leaves the possibility that omitted factors led to both the introduction of ETFs and the empirical decrease in learning about individual stocks.

[2]In the version of the model where the ETF intermediary is present, the size of the ETF increases as her risk aversion decreases. One interpretation of this is that real-world intermediaries' (e.g. ETF arbitrageurs) ETF creation technology has improved (via improved high-frequency trading algorithms, increased liquidity, etc.), and this corresponds to an effective decrease in their risk aversion.

While a model cannot rule this out, quasi-experimental evidence in Sammon [2020a] and this paper suggest a causal relationship between increase in passive ownership and decreased price informativeness.

The paper is organized as follows: Section 2 sets up the model, and explains my numerical solution method. Section 3 studies how the model's predictions for the decision to become informed, and endogenous learning change as we vary (1) the volatility of systematic risk (2) risk aversion (3) cost of becoming informed. Section 4 lays out the model's predictions for the effect of introducing an ETF on price informativeness, and examines how predictions change as we vary the parameters of interest. It also relates the model's predictions to the introduction of Sector ETFs. Section 5 extends the model to allow the size of passive ownership to vary, and a presents calibration which matches the empirical results in Sammon [2020a]. Section 6 concludes.

2 Model

This section develops the baseline version of the model. The key model ingredients are (1) Assets are exposed to both idiosyncratic and systematic risk (2) Imperfectly informed agents (3) Endogenous information acquisition (4) Three periods. When the ETF is not present, the model is similar to an Admati [1985] economy with n idiosyncratic risks, one systematic risk and n assets. Because there are more risks than stocks, I need to solve for optimal attention allocation numerically. I then introduce an ETF, which is an asset that is only exposed to the systematic risk-factor. When the ETF is present, the economy is similar to the setting in Kacperczyk et al. [2016].

$t = 0$	• Agents make binary decision to pay c and become informed or stay uninformed. • If informed, decide how to allocate attention to the underlying risks
$t = 1$	Informed agents receive private signals. All agents submit demands
$t = 2$	Payoffs realized, agents consume

Table 1: Model Timeline

2.1 Setup

Timing

The model has three periods. At time 0, agents decide whether or not to become informed, and how to allocate their limited attention to the underlying risks. At time 1, agents receive signals about assets' terminal payoffs, and submit demand functions. At time 2, payoffs are realized and agents consume their terminal wealth. Table 1 presents a timeline of the events in the model. While the baseline setting only has one uncertainty event, Section A.7 of the Appendix presents an infinite horizon version of the model.

Agents

The model features three types of agents. There is a unit mass of rational traders which fall into two groups: informed and uninformed investors. They both have CARA preferences over time 2 wealth. At time 1, informed investors receive signals about the assets' time two payoffs. The precision of these signals depends on how informed agents allocate their limited attention. Uninformed traders can only learn about terminal payoffs through prices. The third set of agents are noise traders, which have random demand at time 1, and prevent prices from being fully informative.

Assets

Before introducing the ETF, there are n assets. Asset i has time 2 payoff:

$$z_i = a_i + f + \eta_i \tag{1}$$

where $\eta_i \overset{\text{iid}}{\sim} N(0, \sigma_i^2)$ and $f \sim N(0, \sigma^2)$. Each asset has $\overline{x}_i$, shares outstanding and noise trader demand shocks shocks $x_i \overset{\text{iid}}{\sim} N(0, \sigma_{i,x}^2)$. The η_i, f and x_i shocks are all jointly independent. In this economy there are $n + 1$ risks: one idiosyncratic risk for each asset, η_i, and one systematic risk, f[3].

For the baseline version of the model, I assume that $\sigma_i^2 = \sigma^2$, $\overline{x}_i = \overline{x}$ and, $\sigma_{i,x}^2 = \sigma_x^2$ i.e. all the assets are symmetric. This is not needed, but it simplifies the numerical technique for solving the model. For an extension where individual assets load differently on systematic risk, and have heterogeneous idiosyncratic risk and supply shocks, see Section A.1 of the Appendix.

Throughout the paper, I assume that the number of assets is sufficiently small so that idiosyncratic risk cannot be totally diversified away. As the number of assets grows to infinity, introducing an ETF has no effect. This model does not feature trading costs, so with an infinite number of stocks, agents could replicate the payoff of the ETF by buying an equal-weighted portfolio of all the individual assets. One can view this restriction to a small number of assets as a reduced-form way of modeling transaction costs: Trading the

[3]When the ETF is not present, there may be a version of this economy where asset returns have the same correlation structure, but there is no systematic risk-factor. The setup with a systematic risk-factor, however, is needed to make the learning technology comparable between economies when the ETF is and is not present. See Section A.5 of the Appendix for a more thorough discussion of this issue.

first n assets is free, but then trading costs go to infinity if the investor wanted to trade an additional asset (see e.g. Merton [1987]).

Signals

If agent j decides to become informed, they receive noisy signals at time 1 about the payoffs of the underlying *assets*:

$$s_{i,j} = (f + \epsilon_{f,j}) + (\eta_i + \epsilon_{i,j}) \tag{2}$$

where $\epsilon_{i,j} \overset{\text{iid}}{\sim} N(0, \sigma^2_{\epsilon_{i,j}})$, $\epsilon_{f,j} \sim N(0, \sigma^2_{\epsilon_{f,j}})$ and $\epsilon_{i,j}$ are independent for all permutations of i and j, as well as independent from $\epsilon_{f,j}$. As agent j allocates more attention to risk i, $\sigma^2_{\epsilon_{i,j}}$ decreases in a way that depends on the learning technology.

Learning

Agent j can allocate attention $K_{i,j}$ to risk-factor η_i or f to reduce signal noise:

$$\begin{gathered} s_{i,j} = (f + \epsilon_{f,j}) + (\eta_i + \epsilon_{i,j}) \\ \sigma^2_{\epsilon_{i,j}} = \frac{1}{\alpha + K_{i,j}}, \quad \sigma^2_{\epsilon_{f,j}} = \frac{1}{\alpha + K_{n+1,j}} \end{gathered} \tag{3}$$

where $\alpha > 0$. This differs from the setup in Kacperczyk et al. [2016], where the learning technology is $\sigma^2_{\epsilon_{i,j}} = \frac{1}{K_{i,j}}$. In my setting, I need $\sigma^2_{\epsilon_{i,j}}$ to be well defined even if agents devote no attention to asset i[4]. A way to think about α is that informed agents all have a "finger on the pulse" of the market, and know a little bit about each asset, even without explicitly devoting attention to it. I set $\alpha = 0.001$, and in unreported results, I find setting alpha anywhere between 0.0001 and 0.05 does not qualitatively change the results. Informed agents have a total attention constraint of $\sum_i K_i \leq 1$.

[4] This is because in my setting, the risk-factors are not fully separable. For example, if $\epsilon_{1,j}$ has infinite variance, but $\epsilon_{f,j}$ has finite variance, the variance of $s_{1,j}$ is still not well defined. In Kacperczyk et al. [2016], each of the rotated assets is only exposed to one risk, so devoting no attention to that risk leads to a precision of zero, but does not have spill-over effects on other assets.

Portfolio Choice

Define:

$$w_{2,j} = (w_{0,j} - \mathbb{1}_{informed,j}c) + \mathbf{q}_j'(\mathbf{z} - \mathbf{p}) \tag{4}$$

where $w_{0,j}$ is initial wealth, c is the cost of becoming informed (in dollars), $\mathbf{z}$ is the vector of terminal assets payoffs, $\mathbf{p}$ is the vector of time 1 prices and $\mathbb{1}_{informed,j}$ is an indicator equal to 1 if agent j decides to become informed. Here, and everywhere else in the paper, boldface is used to denote vectors.

Agent j submits demand $\mathbf{q}_j$ to maximize their time 1 objective function:

$$U_{1,j} = E_{1,j}[-exp(-\rho w_{2,j})] \tag{5}$$

where ρ is risk aversion. I use $E_{t,j}$ to denote the expectation with respect to agent j's time t information set. For informed agents, the time 1 information set is the vector of signals $\mathbf{s}_j$ and the vector of prices, $\mathbf{p}$. For uninformed agents, the time 1 information set is just prices.

Prices

Suppose we fix the information choice of informed investors at some set of $K_{i,j}$'s. Then, the model is equivalent to Admati [1985]. This is because, in the setup above, agents do not independently receive information about the $(n+1)^{th}$ risk-factor i.e. there is no $s_{f,j} = (f + \epsilon_{f,j})$. This means that agents think only in terms of asset payoffs, rather than risk-factor payoffs. For example, agent j's asset 1 signal is: $s_{1,j} = (f + \epsilon_{f,j}) + (\eta_1 + \epsilon_{1,j})$[5]. This is centered on $f + \eta_1$ so it is an unbiased signal about the payoff of asset 1. The variance of this signal is $var(\epsilon_{f,j}) + var(\epsilon_{1,j})$ because all signal noise is independent. All investors know the correlation structure of asset returns, so when agent j is calculating their posterior

[5] For clarity, in this example, I exclude all the mean payoff terms.

mean for asset 2, they still consider signal 1, as the assets are correlated via their common exposure to systematic risk. Further, when deciding what to learn about, agents understand that devoting attention to systematic risk will reduce the variance of all of their asset signals. For these reasons, I do not find this assumption too restrictive, but it is needed to solve the model, fixing information choices, using the closed form solutions in Admati [1985][6].

Define μ as the vector of a_i i.e. the vector of mean asset payoffs. Further define $\overline{\mathbf{x}}$ as the vector of $\overline{x}_i$ i.e. the vector of shares outstanding for each asset. Define $(n+1) \times (n+1)$ matrix Γ as:

$$\Gamma = \begin{bmatrix} 1 & 0 & \dots & 0 & 1 \\ 0 & 1 & \dots & 0 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \tag{6}$$

If we define η as the vector of η_i's and f (where f is the last entry), then the terminal payoffs can be written as $\mathbf{z} = \mu + \Gamma\eta$[7] Note that this includes a row for a hypothetical $(n+1)^{th}$ asset with payoff $z_n = a_n + f$ even though without the ETF, agents cannot trade that asset or observe its price. This row will be removed, but it is useful to include it here for comparing this setup to the economy where the ETF is present.

[6]Without this assumption, there is no closed-form solution for the price function, as discussed in Section 6 of Admati [1985]. To solve the model without this assumption, one would need to numerically solve for prices such that the market clears. The price function would be of the form $p = \tilde{A}_0 + \tilde{A}_1\eta + \tilde{A}_2 f + \tilde{A}_3\mathbf{x}$, where $\mathbf{x}$ is a vector of supply shocks. In unreported results, I find it difficult to solve for these A_i numerically, because it involves the product of one of the price coefficients A_1 with the inverse of another one of the price coefficients A_2^{-1}. This can lead to arbitrarily large offsetting entries in these matrices, and numerical instability of the solution.

[7]If the assets had different loadings on systematic risk, the 1's in the last column would be replaced by β_i's, i.e. the loadings of each stock on systematic risk, as discussed in the Appendix A.1.

Define the variance of asset payoffs, V as:

$$V = \Gamma \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 & 0 \\ 0 & \sigma_2^2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \sigma_n^2 & 0 \\ 0 & 0 & \dots & 0 & \sigma_f^2 \end{bmatrix} \Gamma' \tag{7}$$

Define the matrix of asset signal variances for agent j as:

$$S_j = \Gamma \begin{bmatrix} \frac{1}{\alpha+K_{1,j}} & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{\alpha+K_{2,j}} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{1}{\alpha+K_{n,j}} & 0 \\ 0 & 0 & \dots & 0 & \frac{1}{\alpha+K_{n+1,j}} \end{bmatrix} \Gamma' \tag{8}$$

I restrict to equilibria where all informed agents have the same attention allocation, so $S_j = S$ for all j i.e. all informed agents have the same attention allocation so $K_{i,j} = K_i$[8].

Define the variance-covariance matrix of asset noise shocks as $U = \sigma_x^2 \mathbf{I}_n$ where $\mathbf{I}_n$ is an $n \times n$ identity matrix. Define the vector of noise shocks as $\mathbf{x}$, which is normally distributed with mean zero and variance U. The available supply of each asset to informed and uninformed investors is $\overline{\mathbf{x}} + \mathbf{x}$ i.e. the number of shares outstanding plus/minus demand from noise traders.

In the economy without the ETF we need to remove the last row and last column of every matrix, as well as the last row of every vector. Any matrix with a superscript t has

[8]For discussions of non-symmetric equilibria, see e.g. Veldkamp [2011]

been *truncated* i.e. has had the last row and column removed. Any vector with a superscript t has had the last row removed.

Define Q^t as:

$$\frac{1}{\rho} \times \phi \times \left(S^t\right)^{-1} \tag{9}$$

where ϕ is the share of rational traders who decide to become informed at cost c.

The equation for equilibrium prices comes directly from Admati [1985]:

$$\begin{aligned}
\mathbf{p} &= A_0 + A_1\mathbf{z} - A_2(\overline{\mathbf{x}} + \mathbf{x}) \\
A_3 &= \frac{1}{\rho}\left(\left(V^t\right)^{-1} + Q^t * (U^t)^{-1} * Q^t + Q^t\right) \\
A_0 &= \frac{1}{\rho}A_3^{-1}\left(\left(V^t\right)^{-1}\mu^t + Q^t(U^t)^{-1}\overline{\mathbf{x}}^t\right) \\
A_1 &= A_3^{-1}\left(Q^t + \frac{1}{\rho}Q^t(U^t)^{-1}Q^t\right) \\
A_2 &= A_3^{-1}\left(\mathbf{I}_n + \frac{1}{\rho}Q^t(U^t)^{-1}\right)
\end{aligned} \tag{10}$$

Demands

Having solved for the price, we can solve for demands. In this section, we continue to use the *truncated* versions of all the model objects. To avoid excessive use of superscripts, I omit the t even though all the objects here have the last row/column removed.

Define the constant matrix $\gamma = \rho\left(A_2^{-1} - Q\right)$. There are separate demand functions for the informed and uninformed:

$$\begin{aligned}
\text{Uninformed: Demand} &= G_0 + G_{2,un}\mathbf{p} \\
\text{Informed, agent } j\text{: Demand} &= G_0 + G_1\mathbf{s_j} + G_{2,inf}\mathbf{p}
\end{aligned} \tag{11}$$

where $\mathbf{s}_j$ is the vector of signals received by agent j and:

$$\begin{aligned} G_0 &= A_2^{-1} A_0 \\ G_{2,un} &= \frac{1}{\rho}\gamma \\ G_{2,in} &= \frac{1}{\rho}\left(\gamma + S^{-1}\right) \\ G_1 &= \frac{1}{\rho} S^{-1} \end{aligned} \tag{12}$$

Many of objects in the demand function can be used to compute agents' posterior beliefs about mean asset payoffs. For informed agents, the posterior mean conditional on signals and prices is:

$$\begin{aligned} E_{1,j}[\mathbf{z}|\mathbf{s_j}, \mathbf{p}] &= B_{0,in} + B_{1,in}\mathbf{s_j} + B_{2,in}\mathbf{p} \\ V_{in}^a &= \left(V^{-1} + QU^{-1}Q + S^{-1}\right)^{-1} \\ B_{0,in} &= \rho V_{in}^a G_0 \\ B_{1,in} &= \rho V_{in}^a G_0 \\ B_{2,in} &= \mathbf{I}_n - \rho V_{in}^a G_{2,in}' \end{aligned} \tag{13}$$

For uninformed agents, the posterior mean conditional on prices is:

$$\begin{aligned} E_{1,j}[\mathbf{z}|\mathbf{p}] &= B_{0,in} + B_{2,un}\mathbf{p} \\ V_{un}^a &= \left(V^{-1} + QU^{-1}Q\right)^{-1} \\ B_{0,un} &= \rho V_{un}^a G_0 \\ B_{2,un} &= \mathbf{I}_n - \rho V_{un}^a G_{2,un}' \end{aligned} \tag{14}$$

Deciding to Become Informed

At time zero, agent j decides whether or not to pay c and become informed. They make this decision to maximize the time 0 objective function:

$$U_{0,j} = -E_0[ln(-U_{1,j})]/\rho \tag{15}$$

where the time 0 information set is the share of agents who decide to become informed. This simplifies to:

$$U_{0,j} = E_0 \left[E_{1,j}[w_{2,j}] - 0.5\rho Var_{1,j}[w_{2,j}] \right] \tag{16}$$

because time two wealth is normally distributed. See Section A.4 of the Appendix for a discussion of how these preferences differ from expected utility i.e. $U_{0,j} = E_{0,j}[U_{1,j}]$.

In this setting, I do not have closed-form solutions for $U_{0,informed}$ and $U_{0,uninformed}$, but I can obtain them through simulation. Solving for c directly would be computationally intensive, as the model would have to be re-solved at each proposed combination of c and share of informed investors to check that $U_{0,informed} = U_{0,uninformed}$. It is easier to solve for c by creating a grid for the share of informed agents between 0 and 1. Then, at each point on the grid, compute the difference in expected utility between informed and uninformed to back out c.

2.2 Equilibrium

As in Kacperczyk et al. [2016], I am going to assume a symmetric equilibrium. This means that all informed agents have the same $K_{i,j} = K_i$. There likely exist asymmetric equilibria, but I do not focus on them in this paper (see e.g. Veldkamp [2011]). In addition, I assume that assets 1 to n have the same: (1) Mean (2) Systematic risk (3) Idiosyncratic risk (4) Supply shock variance. This assumption reduces an otherwise n dimensional problem (the

$(n+1)^{th}$ dimension is accounted for by the total information constraint) to a two dimensional problem: Informed agents must only decide to allocate K_{n+1} attention to systematic risk, and $(1-K_{n+1})/n$ to each idiosyncratic risk-factor. This strong assumption is not needed, and it does not change any of the model's predictions, but it drastically speeds up the numerical solution method. For details, see Section A.1 of the Appendix, where I discuss how to solve a version of the model with this assumption relaxed.

Another possible issue is that the equilibria I find are not unique. Without closed form solutions, I cannot fully rule this out, but I have tried starting my numerical method at every point on the solution grid and I find it always converges to the same solution.

At time 1, given K_i's and the share of informed agents, the equilibrium is equivalent to that in Admati [1985]. At time zero, we know we are at an equilibrium if: (1) no informed or uninformed agent would improve their expected utility by switching to the other type and (2) no informed agent would improve their expected utility by re-allocating their attention. As discussed above, condition 1 is going to be met by construction, as I back out c for a given share of informed agents, to make the expected utility of both groups equal. I rely on condition 2 to develop my numerical method in the next subsection.

2.3 Numerical Method

Fixing the share of informed agents, I use the following algorithm to numerically solve for K_i's:

1. Start all agents at K^0

2. Consider an atomistic agent j who takes K^0 as given, and considers their expected utility by deviating to K_j^1 near K^0. These deviations are small increases/decreases in

the share of attention spent on the systematic risk-factor.

3. If j can be made better off, move all informed agents to K^1

4. Iterate on steps 2 and 3 until j can no longer improve their expected utility by deviating.

Discussion

At this point, it is not clear why a numerical method is needed to solve the model. Two possible alternative solution methods are (1) Adding the $(n+1)^{th}$ risk to Admati [1985]. This will not work, as discussed in the original paper, as there is no closed form solution for prices and demands with more risks than assets. (2) Deleting the $(n+1)^{th}$ asset from Kacperczyk et al. [2016]. This is not viable because the rotation used to isolate risk-factors and solve the model will not work if the number of risks is greater than the number of assets. Finally, we cannot use a benevolent central planner to solve the problem: I find that in the competitive equilibrium, attention is more concentrated on a small number of risks, relative to what would maximize total expected utility for informed *and* uninformed agents.

It also seems as though it should be possible to map the no-ETF economy to an economy with independent assets/risks via an eigendecomposition (see e.g. Veldkamp [2011]). Having done this, it would be straightforward to solve the model using the technique in Kacperczyk et al. [2016]. While this is possible, it still relies on numerical methods, to make sure that after reversing the rotation, the solution is feasible under the proposed learning technology. See Appendix section A.8 for more details.

2.4 Introducing the ETF

Introduce asset $n+1$, the ETF:

$$z_{n+1} = a_{n+1} + f \tag{17}$$

Asset $n+1$ has average supply $\overline{x}=0$, but is still subject to supply shocks $x_{n+1}=\tilde{x}_{n+1}+\sum_{z=1}^{n} x_z$ where $\tilde{x}_{n+1}$ has the same distribution as all the x_i for assets i equal 1 to n. This implies that the supply shock for the $(n+1)^{th}$ asset is the sum of the supply shocks to the n individual assets, as well as another independent supply shock $\tilde{x}_{n+1}$. I define the ETF noise shocks this way based Ben-David, Franzoni, and Moussawi [2018] and Chinco and Fos [2019], which document transmission in noise shocks between the ETFs and the underlying assets.

These assumptions on the supply of the ETF are important for two reasons (1) We need supply shocks in the ETF, otherwise its price would be a fully revealing signal for the systematic risk-factor (2) the ETF must be in zero average supply so its introduction does not increase average systematic risk[9]. If we assume that $\tilde{x}_{n+1}\sim N(0,\sigma_x^2)$, then the noise shock for the $(n+1)^{th}$ asset has total volatility $\sigma_{n,x}^2=(n+1)\times\sigma_x^2$. Define $\tilde{U}=\left(\Gamma'\right)^{-1}\sigma_x^2\mathbf{I}_{n+1}\left(\Gamma'\right)^{-1}$.

Informed agent j receives signals about the payoffs of all the underlying assets, including asset $n+1$:

$$\begin{aligned} s_{i,j} = (f+\epsilon_{f,j}) + (\eta_i+\epsilon_{i,j}) \text{ for } i=1,\ldots,n \\ s_{n+1,j} = (f+\epsilon_{f,j}) \end{aligned} \tag{18}$$

Note that $\epsilon_{f,j}$ in the first and second lines of this equation are identical i.e. there is only one systematic shock. The learning technology and total attention constraint are unchanged from the economy where the ETF is not present.

The price and demand functions are also unchanged from the setup without the ETF, but instead of using the truncated versions, we use the full versions i.e. use S instead of S^t,

[9] Although it does not increase the average supply of systematic risk, introducing the ETF does introduce additional noise-trader risk.

and use the new noise shock matrix $\tilde{U}$. We can also use the same numerical method to solve for the optimal allocation of attention, and cost of becoming informed.

Effect on posterior mean/variance

Introducing the ETF changes the way agents form beliefs about asset payoffs. Define $\mathbf{s_p} = \mathbf{z} + \epsilon_\mathbf{p}$ as the signal about asset payoffs contained in prices. From the price function, $\mathbf{s_p} = A_1^{-1}(\mathbf{p} - A_0)$, which implies that $\epsilon_\mathbf{p} = A_1^{-1}A_2(\overline{\mathbf{x}} + \mathbf{x})$ and $\Sigma_p = A_1^{-1}A_2U$ where U is the variance-covariance matrix of supply shocks. This implies that $\mathbf{s_p} \sim N(0, \Sigma_p)$. Without the ETF:

$$\underbrace{\widehat{\Sigma_j^{-1}}}_{\text{Posterior Precision}} = \underbrace{V^{-1}}_{\text{Prior Precision}} + \underbrace{\Sigma_p^{-1}}_{\text{Price Precision}} + \underbrace{S_j^{-1}}_{\text{Signal Precision}} \tag{19}$$

With the ETF, agents observe $s_{\mathbf{p},n+1}$ i.e. the signal about payoff of the $(n+1)^{th}$ asset contained in asset prices. This will change Σ_p^{-1} i.e. the price precision, but nothing else. This is because fixing attention allocation, introducing the ETF has no effect on S_j^{-1} for assets 1 to n. For any asset i, $s_{i,j} = (f + \epsilon_{f,j}) + (\eta_i + \epsilon_{i,j})$, so $var(s_{i,j}) = var(\epsilon_{f,j} + \epsilon_{i,j}) = var(\epsilon_{f,j}) + var(\epsilon_{i,j})$ by independence.

When the ETF is not present, the posterior mean of f will be:

$$\underbrace{E_{1,j}[\mathbf{z}]}_{\text{Posterior Mean}} = \underbrace{\hat{\Sigma}_j}_{\text{Posterior Variance}} \times \left(\underbrace{S_j^{-1}}_{\text{Precision on Asset Signals}} \mathbf{s_j} + \underbrace{\Sigma_p^{-1}}_{\text{Price Precision}} \mathbf{s_p} \right) \tag{20}$$

With the ETF, agents can separately weigh their signal for f by its own precision:

$$\underbrace{E_{1,j}[\mathbf{z}]}_{\text{Posterior Mean}} = \underbrace{\hat{\Sigma}_j}_{\text{Posterior Variance}} \times \left(\underbrace{S_j^{-1}}_{\text{Precision on Risk-Factor Signals}} \mathbf{s_j} + \underbrace{\Sigma_p^{-1}}_{\text{Price Precision}} \mathbf{s_p} \right) \tag{21}$$

where the terms that have changed are in color. To see how this works, we can use the eigendecomposition in Veldkamp [2011] to isolate the risk-factors. Pre-multiplying $\mathbf{z}$ by Γ, we create synthetic assets exposed to only one risk-factor:

$$\begin{aligned} \mathbf{z} = \mu + \Gamma\eta \leftrightarrow \tilde{\mathbf{z}} = \Gamma^{-1}\mu + \eta \\ \tilde{s}_i = \eta_i + \tilde{\epsilon}_i \text{ for } i = 1, \ldots, n \end{aligned} \tag{22}$$

With this rotation, the supply of the synthetic assets is $(\Gamma')^{-1}(\overline{\mathbf{x}} + \mathbf{x})$, but at this point, the signals may still be correlated. After another transformation to make the signals independent, we can solve for the equilibrium in this economy using the numerical technique in Kacperczyk et al. [2016][10], and then rotate back to the economy with payoffs $\mathbf{z}$ and signals $\mathbf{s}$. This rotation can be used as a check on the numerical method, as it allows me to compare my numerical solutions to the closed-form solutions in Kacperczyk et al. [2016]. In this rotated economy, it is clear that agents are going to independently use the $(n+1)^{th}$ signal, and the price of the $(n+1)^{th}$ asset to learn about $\mathbf{z}$, something they cannot do in the no-ETF world.

To quantify the effect of introducing the ETF on investors' posterior precisions, Table 2 contains selected entries of $\hat{\Sigma}$. Introducing the ETF always increases the precision of both the informed and uninformed for assets 1 to n.

Effect on learning trade-offs

When an agent is deciding whether to learn about systematic or idiosyncratic risk, they face the following trade-off: (1) Learning about systematic risk leads to a more precise posterior belief about every asset (2) But, volatility of systematic risk-factor (σ_f^2) is low, relative to idiosyncratic risk-factors (σ^2). This trade-off is affected by the presence of the ETF: If the ETF is not present, agents cannot take a bet purely on systematic risk, or idiosyncratic

[10] I would like to thank the authors for sharing their solution code with me

Panel A: Matching Cost of Becoming Informed

				Precision			
		Share Informed		Informed		Uninformed	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF	no ETF	ETF
0.1	0.2	0.05	0.2	1.82	2.24	1.66	2.06
0.1	0.5	0.35	0.2	2.04	2.06	1.93	1.94
0.25	0.2	0.5	0.2	1.85	1.87	1.74	1.82
0.25	0.5	0.5	0.2	1.78	1.87	1.69	1.82

Panel B: Share Informed at 10%

				Precision			
		Share Informed		Informed		Uninformed	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF	no ETF	ETF
0.1	0.2	0.1	0.1	1.85	2.05	1.70	1.88
0.1	0.5	0.1	0.1	1.75	1.90	1.64	1.83
0.25	0.2	0.1	0.1	1.76	1.87	1.65	1.82
0.25	0.5	0.1	0.1	1.71	1.87	1.62	1.82

Panel C: Share Informed at 30%

				Precision			
		Share Informed		Informed		Uninformed	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF	no ETF	ETF
0.1	0.2	0.3	0.3	2.20	2.54	2.05	2.37
0.1	0.5	0.3	0.3	1.96	2.30	1.85	2.16
0.25	0.2	0.3	0.3	1.79	1.92	1.68	1.84
0.25	0.5	0.3	0.3	1.73	1.88	1.64	1.83

Table 2: **Posterior Precision.** Diagonal entries of $\hat{\Sigma}$ for one of the stocks i.e. assets 1 to n. In panel A, the cost of being informed is chosen such that 20% of agents become informed when the ETF is present. In Panels B and C, the share of informed agents are fixed and 10% and 30% respectively. The "no ETF" column has the (1,1) entry of $\hat{\Sigma}$ when the ETF is not present, while the "ETF" column has the (1,1) entry of $\hat{\Sigma}$ after introducing the ETF.

risks[11].

To illustrate this trade-off, I present a few examples with only two stocks. Figure 1 shows this trade-off when there is no ETF and the assets are not exposed systematic risk i.e. $z_i = a_i + \eta_i$. The black line plots the excess profits of the informed agents in stock one, while the red line plots the excess profits of the informed agents in stock two. As we move to the right along the x-axis, informed agents are increasing their attention on stock 1. Initially, allocating more attention to stock one increases the informed agents' profit advantage in that stock, but eventually it hits a point of diminishing returns. The price becomes too informative about η_1, which is why the black line starts to slope down. Because the assets are symmetric, it is optimal for informed agents to allocate half their attention to each asset (vertical red line).

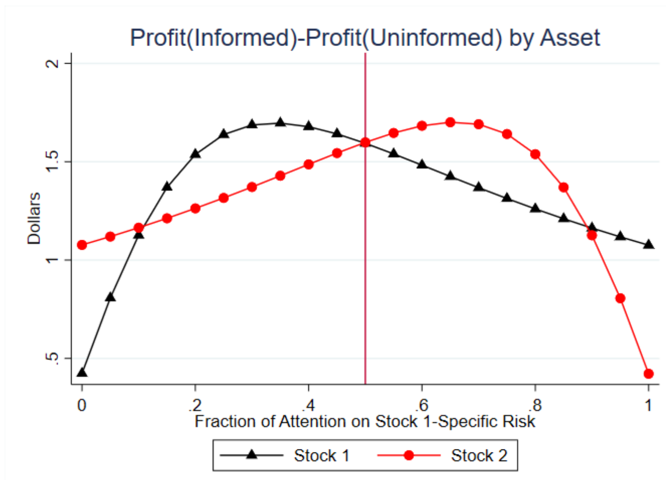

Figure 1: Vertical red line denotes optimal attention allocation. All other points are not equilibrium outcomes. 20% of investors are informed. Residual attention is on Stock 2-specific risk. $\rho = 0.1$, $\sigma^2 = 0.55$

Compare this to Figure 2, the case when there are two stocks, but they are both exposed

[11] Without the ETF, they cannot bet purely on an idiosyncratic risk, because they cannot perfectly hedge their exposure to systematic risk from holding that asset.

to a systematic risk-factor, which is less volatile than the stock-specific risks. Learning more about stock-specific risks (moving to the right along the x-axis) increases profits, but eventually there are diminishing returns for two reasons. One is that prices become too informative, which is what happened in the last example. The other is that the stocks are both exposed to systematic risk, and at some point, informed agents are not learning much about a risk that affects both stocks. The slopes are different to the right/left of the optimum attention allocation (red vertical line) because the volatility of the systematic risk is lower than that of the stock-specific risks.

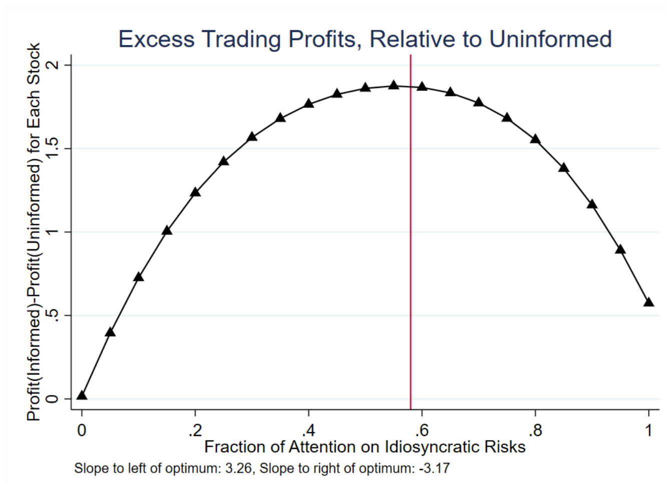

Figure 2: Two assets, systematic risk, no ETF. Vertical red line denotes optimal attention allocation. All other points are not equilibrium outcomes. 20% of investors are informed. Attention on stock-specific risks is equal. Residual attention is on systematic risk-factor. $\rho = 0.1$, $\sigma_f^2 = 0.2$, $\sigma^2 = 0.55$

Finally, Figure 3 shows what happens when there are two stocks, both exposed to systematic risk and idiosyncratic risk, and we introduce an ETF which is only exposed to systematic risk. Agents can now almost uniformly increase their profits on the stocks by learning more about them, because they will be able to isolate the stock-specific risk-factors. In equilibrium, informed agents learn more about stock-specific risks because there is more money to be made betting on η_i's – the stock specific risk-factors are more volatile than the systematic risk-factor f. And because the agents are not very risk averse, with a CARA risk-aversion, ρ, of 0.1, they don't mind loading up on these volatile stock-specific risks.

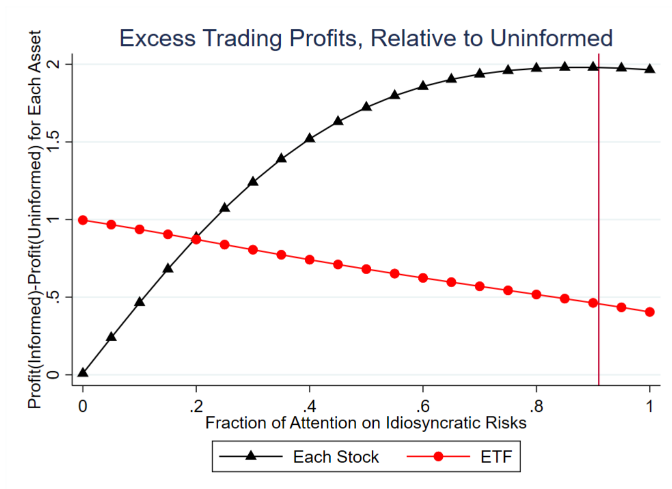

Figure 3: Vertical red line denotes optimal attention allocation. All other points are not equilibrium outcomes. 20% of investors are informed. Residual attention is on systematic risk-factor. ETF is in zero average supply. $\rho = 0.1$, $\sigma_f^2 = 0.2$, $\sigma^2 = 0.55$

2.5 Relating ETFs in the Model to ETFs in the Real-World

In this economy the ETF looks like a futures contract: it is a claim, in zero net supply, on the payoff of the systematic risk-factor. Futures contracts, however, have existed for much longer than ETFs. If ETFs were equivalent to futures contracts, then we would not expect to see any of the empirical effects of growing ETF ownership (see e.g. Sammon [2020a], Glosten, Nallareddy, and Zou [2016], Ben-David et al. [2018], Chinco and Fos [2019], among others). Defining an ETF the way I have in the model captures some features of the real-world, and misses others.

One thing it does capture is that ETFs make it easier for informed investors to bet on systematic risk. This is consistent with the fact that ETFs are more divisible than futures, which allows more investors to trade them. For example, E-mini S&P 500 futures trade at around $150,000 per contract, while SPY trades around $300 per share (as of June 1, 2020). The investors who benefit from this increased divisibility are not just retirees trading in their 401K's. According to Daniel Gamba, former head of Blackrock's ETF business (iShares) *"The majority of investors using ETFs are doing active management. Only about 30% of ETF investors look at these as passive funds..."* (2016). On a related point, ETFs have made it easier to hedge out/short systematic risk. According to Goldman Sachs Hedge Fund Monitor, "ETFs account for 27% of hedge funds short equity positions" (2016). This is important, as informed investors in the model will short the ETF when taking aggressive positions in individual stocks to hedge out their exposure to systematic risk, as we saw in Figure 3. This feature of the model is specific to the introduction of ETFs, relative to index mutual funds (which existed before ETFs), as (open-ended) mutual funds cannot be shorted.

Another way to link the ETF in the model to the real world comes from viewing f as a sector-specific risk, rather than an economy-wide risk. ETFs cover more indices and industries than futures. These sector ETFs are popular: as of June 1, 2020, there is over $170 Billion investment in State Street's 30 Sector ETFs. Another way to view the model is as introducing an ETF that offers cheap diversification for particular industry. I discuss this in more detail in Section 4.4, where I calibrate the model to match the empirical effects of introducing sector ETFs in the late 1990's.

The model clearly does not capture the creation/redemption mechanism, which is an important feature of ETFs that distinguishes it from index mutual funds and futures contracts. Other models like Cong and Xu [2016] have this feature. While I think this is an important channel, especially when talking about market-making in a Kyle [1985]-style model, I abstract away from this in my setting to focus on the decision to become informed and endogenous learning channels.

3 Model Predictions & Comparative Statics

In this section, I examine the effects of introducing the ETF on learning, and how sensitive the model's predictions are to input parameters.

3.1 Baseline Parameters

Table 3 contains the baseline parameters. I take most of them from Kacperczyk et al. [2016] with a few exceptions: (1) I have effectively set the gross risk-free rate r to 1 because I want to de-emphasize the effect of time-discounting (2) I have 8 idiosyncratic assets, instead of 2, so agents can better attempt to replicate the systematic risk-factor with a diversified portfolio of stocks before the ETF is introduced (3) I increase the supply of the stocks. In Kacperczyk et al. [2016], the supply of the $(n+1)^{th}$ risk-factor i.e. the supply of the ETF in the rotated economy is 15 units, and the supply of the two stock-specific risks is 1 unit each. This implies that there is systematic risk in the economy outside the systematic risk in the stocks: $\beta_1 \times$ (supply of asset 1) $+ \beta_2 \times$ (supply of asset 2) is less than 15.

I make the total supply of all idiosyncratic assets equal to 20, and split this equally among 8 stocks. I keep the number of stocks relatively small, because if there are too many stocks, introducing the ETF has no effect. In the limit, if there were an infinite number of stocks, agents could perfectly replicate the payoff of the ETF with the underlying securities, and there are no trading costs that prevent this behavior. We can view the small number of tradeable stocks as a reduced-form way of modeling transaction costs.

Mean asset payoff	a_i	15
Volatility of idiosyncratic shocks	σ_i^2	0.55
Volatility of noise shocks	σ_x^2	0.5
Risk-free rate	r	1
Initial wealth	w_0	220
Baseline Learning	α	0.001
# idiosyncratic assets	n	8
Coef. of risk aversion (low)	ρ	0.1
Coef. of risk aversion (high)	ρ	0.35
Vol. of systematic shocks (low)	σ_n^2	0.2
Vol. of systematic shocks (high)	σ_n^2	0.5
Total supply of idiosyncratic assets	$\overline{x}$	20

Table 3: **Baseline Parameters.**

I study four different scenarios based on lower/higher risk aversion, and lower/higher systematic risk. In this economy, increasing the share of agents who become informed (or decreasing the cost of becoming informed), and decreasing risk aversion have similar effects. This is because both of these changes are effectively increasing the *risk-bearing capacity* of the economy. See Appendix Section A.6 for more details on the exact relationship between the share of informed and risk aversion.

3.2 Effect of Introducing the ETF

Share informed

I want to understand how introducing the ETF affects the share of agents who decide to become informed. Figure 4 shows the relationship between the cost of becoming informed (in dollars) and the percent of rational investors who decide to become informed. When risk aversion ρ is low, systematic risk σ_n^2 is low, and the cost of becoming informed is not too low, more investors become informed after introducing the ETF. With these parameters, investors are willing to bet aggressively on their private signals, increasing the benefit of becoming informed. As we increase risk-aversion, however, for most costs of becoming informed, more investors learn when the ETF is not present. This is because for these parameter choices, introducing the ETF makes the uninformed investors relatively better off.

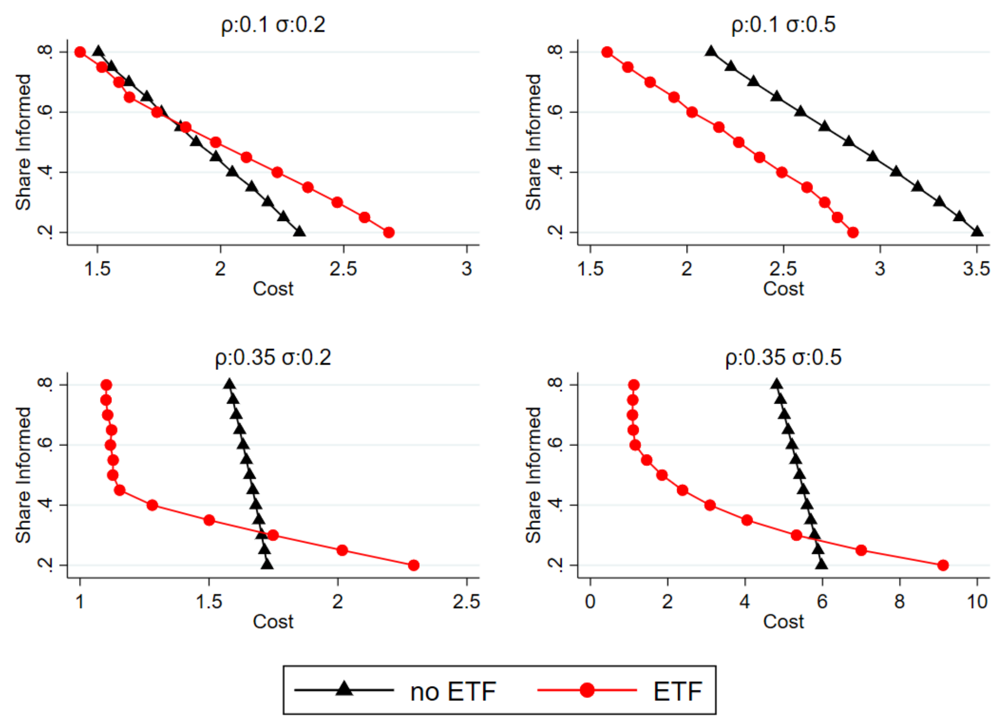

Figure 4: **Effect of introducing the ETF on learning.** In these plots, the x-Axis is the cost in dollars of becoming informed. The y-axis reports the share of agents who become informed in equilibrium at this cost. In the panels, ρ represents risk aversion, and σ represents the volatility of the systematic risk-factor i.e. σ_n^2.

Hedging Demand

One of the effects of introducing the ETF is that it allows informed investors to better isolate bets on signals about individual stocks. In the demand function, G_1 is a measure of how informed investors react to their own signals. Table 4 contains selected the entries of G_1. In the table, I fix the share informed at 50% to avoid mixing the effects of changing the share of informed agents and heding demand. When the share of informed agents changes, all agent's posterior precision matrices change as well, and this matters for how aggressive agents are in betting on any signals.

Because all the stocks have the same supply and have the same ex-ante risk, when the ETF is not present, G_1 is a symmetric matrix. The diagonal entries show how strongly investors react to signals about a particular asset. The off-diagonal entries show how investors may hedge such bets. The "No ETF" columns look at those entries of G_1. When an investor gets a good signal about a particular asset, they buy more of it. They hedge this position by shorting some of each of the other assets. For example in row 1, a 1 unit higher signal leads to 0.968 units more of that asset, and that is hedged by shorting -0.117 of each of the other 7 assets. Note that the investors does not fully hedge out systematic risk, as 0.968 is greater than 7 times -0.117 (recall that each stock has a unit loading on the systematic risk factor). One reason for this is because investors are getting a combined signal on the systematic and idiosyncratic components of the stock payoffs.

Compare this to the case with the ETF: The informed investor bets *more* aggressively on the stock for low values of risk aversion/systematic risk. But, in all cases, they hedge out all systematic risk with the ETF.

This result is not unique to how informed investors respond to their own signals. Both informed and uninformed investors change their behavior in response to the signal contained in prices. For details, see Appendix A.3.

Attention Allocation

Another key prediction of the model is that introducing the ETF affects attention allocation. As shown above, introducing the ETF can change the share of agents who decide to become informed, which makes it difficult to isolate the effect of attention re-allocation. In this sub-

ρ	σ_f^2	Shr. Inf.	No ETF $G_{i,i}$	No ETF $G_{i,j}$	No ETF $7 \times G_{i,j}$	ETF $G_{i,i}$	ETF $G_{i,j}$
0.1	0.2	0.5	0.968	-0.117	-0.817	1.260	-1.260
0.1	0.5	0.5	0.766	-0.069	-0.484	1.010	-1.010
0.25	0.2	0.5	0.290	-0.024	-0.171	0.274	-0.274
0.25	0.5	0.5	0.255	-0.019	-0.130	0.124	-0.124
0.35	0.2	0.5	0.189	-0.014	-0.100	0.046	-0.046
0.35	0.5	0.5	0.176	-0.012	-0.086	0.003	-0.003

Table 4: **Hedging Demand.** The share of informed agents are fixed and 50%. The "No ETF" columns are the entries of G_1 when the ETF is not present, while the "ETF" column is the entries of G_1 after introducing the ETF. The share of informed agents are fixed and 50%. There are 8 assets in the economy, so $7 \times G_{i,j}$ is the total hedging of systematic risk when betting on a stock-specific signal when the ETF is not present.

section, I fix the share of agents who decide to become informed, and look at intensive-margin learning effects.

Figure 5 shows the relationship between the share of agents who decide to become informed, and the share of attention allocated to systematic risk. When risk aversion is low, and systematic risk is low, introducing the ETF actually *decreases* learning about systematic risk. This is related to the hedging demand channel discussed above. Investors are willing to bet aggressively on their private signals, and can hedge out all systematic risk through an offsetting position of the same size in the ETF. As we increase systematic risk, the effect of introducing the ETF depends on the cost of becoming informed. Once risk aversion, systematic risk, or the share of investors learning is sufficiently high, introducing the ETF almost universally increases attention on systematic risk.

This illustrates a key trade-off for informed investors in the model: diversification vs.

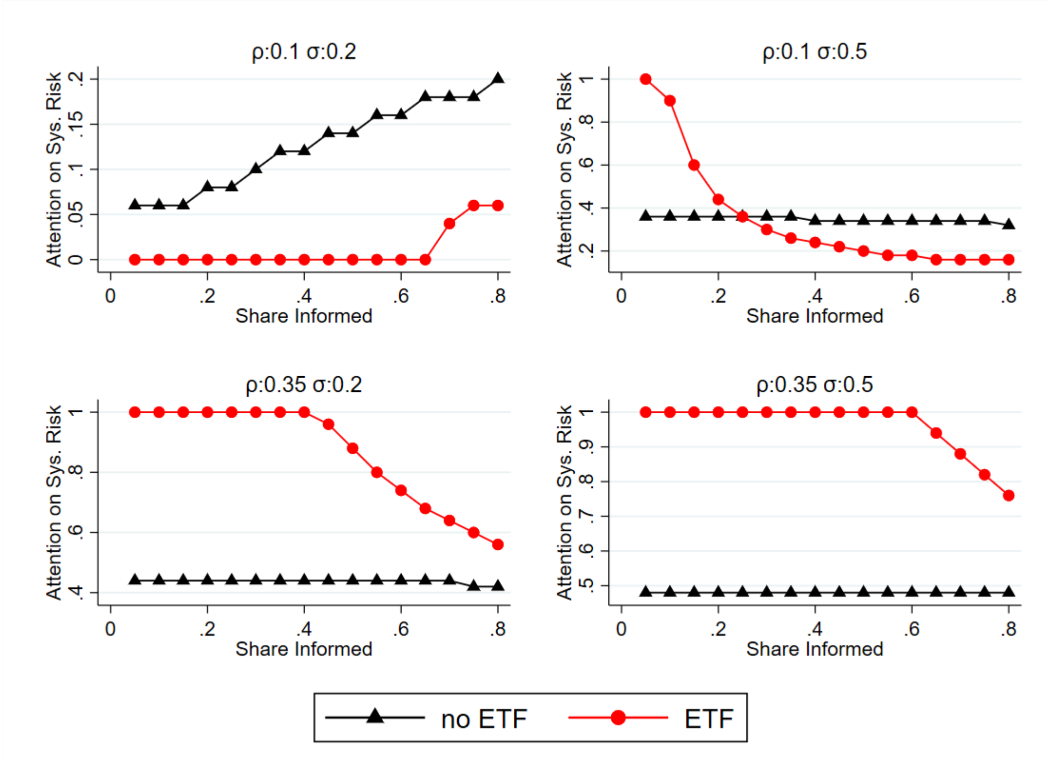

Figure 5: **Effect of Introducing the ETF on Attention Allocation (fixed share informed).** In these plots, the x-Axis is the the share of agents who become informed. The y-Axis is the share of agents attention devoted to systematic risk. All residual attention is allocated to stock-specific risks. In the panels, ρ represents risk aversion, and σ represents the volatility of the systematic risk-factor i.e. σ_n^2.

trading profits. If agents are risk averse, they generally care more about systematic risk because idiosyncratic risk can be diversified away. When we give them the ETF to trade on systematic risk directly, they want to learn even more about this economy-wide risk. If agents are closer to risk neutral they care more about profits than risk. When you give them the ETF, it lets them take more targeted bets on volatile individual securities, and they learn more about the stocks.

Risk Premia

If we fix the share of agents who become informed in equilibrium, introducing the ETF almost always decreases expected returns in the economy. This is not surprising, as the ETF increases the information in the economy i.e. it adds an $(n+1)^{th}$ public signal, the price of the ETF. Table 5 shows that introducing the ETF decreases average asset returns, as long as risk aversion and the volatility of systematic risk are not too high. Once we allow the share of informed agents to vary, however, risk premia can actually increase. This is because as the number of informed agents in the economy decreases, the effective risk-bearing capacity of the economy decreases, so risk premia must increase.

I view this risk premia as more of a modeling artifact than a testable prediction, and want to take out this effect when studying price informativeness. To do this, I work with market-adjusted returns: I calculate the returns of each asset as the actual return, minus the market returns, as discussed in Campbell, Lettau, Malkiel, and Xu [2001]. Market-adjusted returns are also used for all the empirical exercises in Sammon [2020a]. Whether or not the ETF is present, the market is defined as the average return of all the stocks, to ensure an apples-to-apples comparison[12].

3.3 Sensitivity to Parameter Choice

So far, I have focused on the four baseline parameter choices. In this sub-section, I want to examine how sensitive the model is to varying risk aversion and systematic risk.

In Figure 6 I fix the share of agents who decide to become informed at 20% (the baseline choice in Kacperczyk et al. [2016]), and look at the effect on learning about systematic risk. As risk aversion increases, learning about systematic risk increases. This is because as risk aversion increases, the agents' diversification motive starts to dominate their profit motive. The relationship is steeper in the economy with the ETF and when the volatility of the

[12]The results are unaffected if you define the market as the return of the ETF when it is present.

Panel A: Fix Share Informed

ρ	σ_f^2	Shr. Inf.	Risk Premium No ETF	ETF	Change(PP)
0.1	0.2	0.1	3.73%	3.71%	-0.02%
0.1	0.2	0.3	3.71%	3.59%	-0.12%
0.1	0.5	0.1	8.18%	8.19%	0.01%
0.1	0.5	0.3	8.09%	8.05%	-0.04%
0.35	0.2	0.1	14.33%	14.32%	-0.01%
0.35	0.2	0.3	14.28%	14.23%	-0.05%
0.35	0.5	0.1	35.98%	36.09%	0.11%
0.35	0.5	0.3	35.65%	35.94%	0.30%

Panel B: Fix Cost of Becoming Informed

ρ	σ_f^2	Risk Premium No ETF	ETF	Change(PP)
0.1	0.2	3.68%	3.38%	-0.30%
0.1	0.5	7.98%	8.19%	0.21%
0.35	0.2	14.23%	14.23%	0.00%
0.35	0.5	35.32%	35.94%	0.63%

Table 5: **Effect of introducing the ETF on Expected Returns.** In Panel A, the share informed is the same whether the ETF is present or not. In Panel B, the share informed when the ETF is not present is set to 50%. After introducing the ETF, the share informed are 0.55, 0.2, 0.3 and 0.3 in rows 1-4. The risk premium is defined as the average stock return between period 0 and period 2.

systematic risk factor is high.

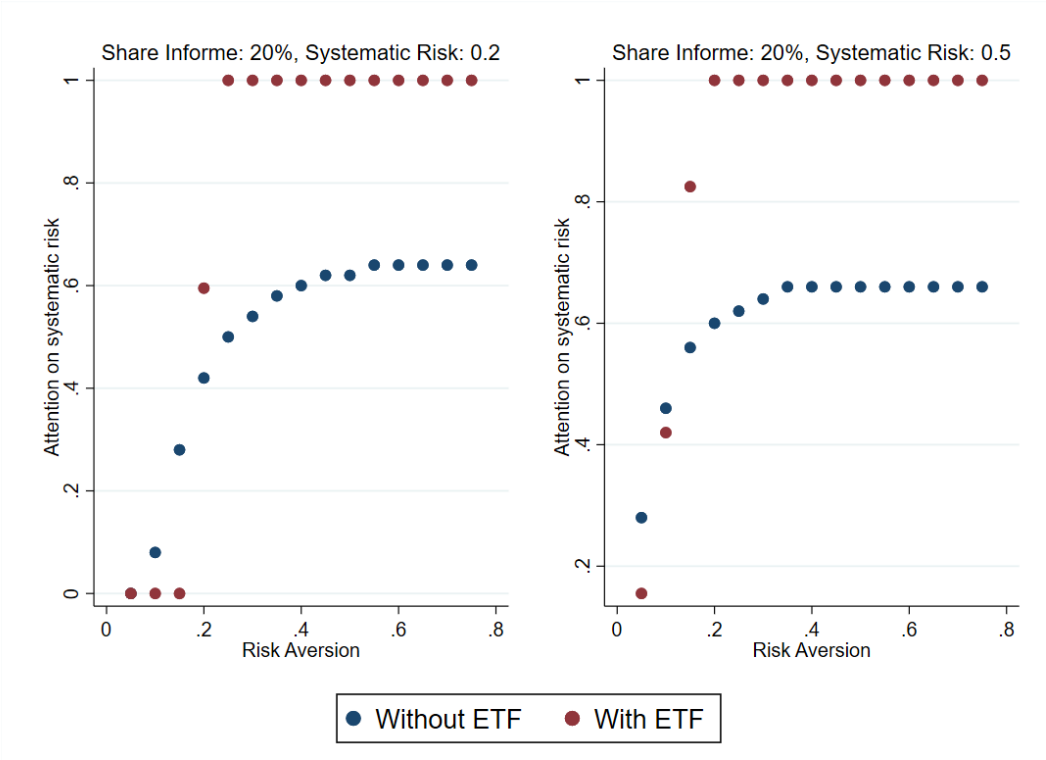

Figure 6: **Relationship between risk aversion and attention to systematic risk-factor.** In the left panel, σ_n^2 is set to 0.2, while in the right panel, σ_n^2 is set to 0.5

In Figure 7, I again fix the share of informed agents at 20% and vary σ_n^2. As expected, increasing systematic risk leads to increased learning about systematic risk. The effect is steeper when risk aversion is high and when the ETF is present.

3.4 Discussion

There are two main effects of introducing the ETF: (1) A re-allocation of attention, when we fix the share of agents who become informed in equilibrium, which I will call the *intensive margin* (2) A change in the share of agents who become informed, which I will call the

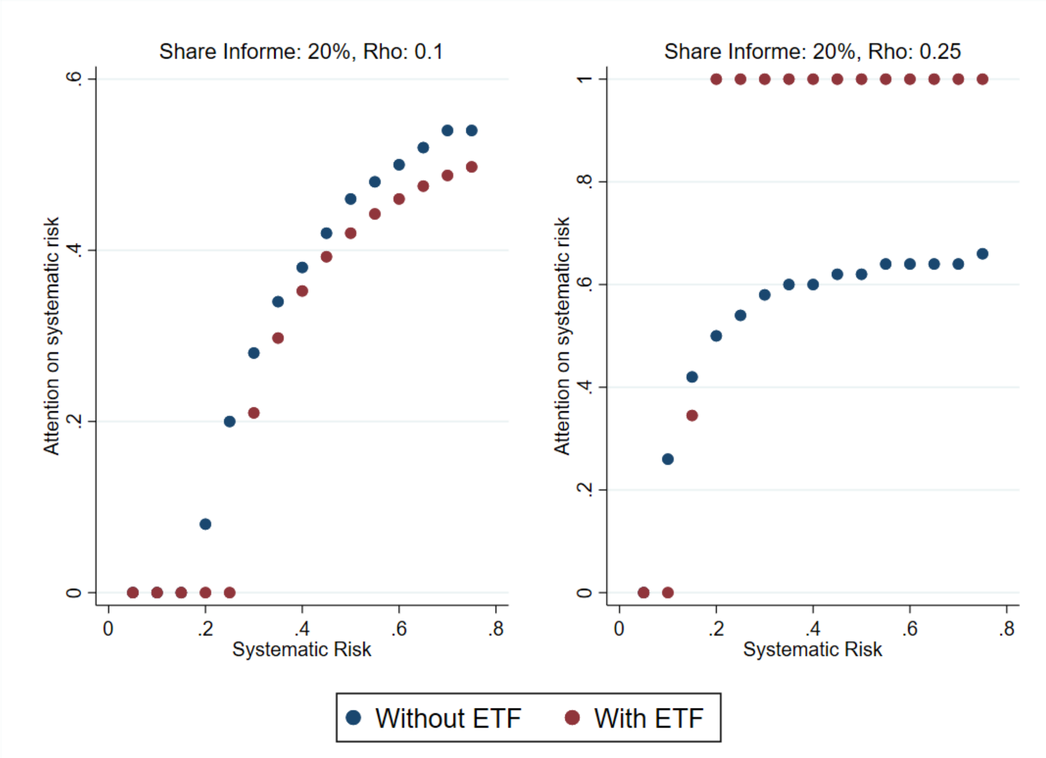

Figure 7: **Relationship between systematic risk and attention to systematic risk-factor.** In the left panel, risk aversion, ρ is set to 0.1, while in the right panel, risk aversion is set to 0.25.

extensive margin. Table 6 contains a summary of the intensive and extensive margin effects for my baseline parameter choices. These tables show that introducing the ETF has an ambiguous effect on both the intensive and extensive margins.

It is hard to take such ambiguous predictions to the data. Further, it is difficult to observe empirically which risks investors are learning about. In the next section, I will develop measures of price informativeness that are easier to take to the data.

Panel A: Intensive Margin

			Attention Allocation			
		Share	No ETF		ETF	
ρ	σ_f^2	Informed	Idio.	Sys.	Idio.	Sys.
0.1	0.2	0.5	86.0%	14.0%	100.0%	0.0%
0.1	0.5	0.5	66.0%	34.0%	80.0%	20.0%
0.35	0.2	0.5	56.0%	44.0%	12.0%	88.0%
0.35	0.5	0.5	52.0%	48.0%	0.0%	100.0%

Panel B: Extensive Margin

				Attention Allocation			
		Share Informed		No ETF		ETF	
ρ	σ_f^2	No ETF	ETF	Idio.	Sys.	Idio.	Sys.
0.1	0.2	0.5	0.55	78.0%	22.0%	100.0%	0.0%
0.1	0.5	0.5	0.2	58.0%	42.0%	56.0%	44.0%
0.35	0.2	0.5	0.3	44.0%	56.0%	0.0%	100.0%
0.35	0.5	0.5	0.3	36.0%	64.0%	0.0%	100.0%

Table 6: **Intensive and Extensive Margin Effects of Introducing the ETF.**

4 Price Informativeness

In this section, I explore the model's predictions for introducing an ETF on pre-earnings announcement price informativeness. To map the model to the empirical exercises in Sammon [2020a], I define $t = 1$ as the pre-earnings date, and $t = 2$ as the earnings date. The natural next step is to calculate a model-based measures of price-informativeness that could tell us something directly about the information content of prices. The issue is that these model-based measure of price informativeness are hard to measure in practice, and there is much debate about the "right" way to do this[13].

[13]For example, Grossman and Stiglitz [1980] defines price informativeness as a *conditional* covariance, which requires identifying the 'right' set of conditioning variables. Academic economists, still disagree on the right set of conditioning variables and whether this is the right definition of price informativeness. Bai, Philippon, and Savov [2016] measure price informativeness as the variance of fundamentals, conditional on prices. Dávila and Parlatore [2019] measure price informativeness as the variance of prices, conditional on

For this reason, I want to focus on the three model-free measures of price informativeness discussed in Sammon [2020b]. I create model analogues of these objects, and simulate the economy to determine the effect of introducing the ETF on these measures.

4.1 Defining Price Informativeness Measures

Pre-Earnings Volume

Although we are assuming a continuum of investors, when running the simulations, there are a finite number of traders, which I set to 10,000. Assume that at $t = 0$ all of the investors are endowed with $1/10,000^{th}$ of $\overline{x}$. Then we can think of trading volume as the difference between agents' initial holdings, and their holdings after submitting their demand at $t = 1$. This measure, however, would be contaminated by the effect of the noise shock, so I want to compute a measure similar to turnover. The measure of trading volume I use is the difference between initial holdings and final holdings, divided by the total supply of the asset, which *includes* the supply shock. I then take a weighted-average of this measure across informed an uninformed investors.

For now, I am going to focus on trading in the individual stocks. There are two main factors that affect trading volume in the model: (1) The share of investors who decide to become informed. As more investors become informed, there are more different signals in the economy, and thus more trading. Uninformed investors all submit the same demand, so in the limit with all uninformed investors, there would be no trading volume after accounting for the noise shock (2) Attention allocation. As more attention is devoted to the individual stocks, informed investors have more precise posterior beliefs, and are more willing to bet aggressively on their signals. We know that introducing the ETF both decreases incentives

fundamentals; effectively switching the left-hand-side and right-hand-side variables of a regression.

to become informed, and shifts attention toward the systematic risk-factor, so my prior is that it should decrease trading volume.

Pre-Earnings Drift

Define the pre-earnings drift:

$$DM = \begin{cases} \frac{1+r_{(0,1)}}{1+r_{(0,2)}} & \text{if } r_2 > 0 \\ \frac{1+r_{(0,2)}}{1+r_{(0,1)}} & \text{if } r_2 < 0 \end{cases} \tag{23}$$

where $r_{(0,t)}$ is the cumulative market-adjusted return from 0 to t. The pre-earnings drift will be near one when the return at $t = 2$ is small relative to the return at $t = 1$. $DM_{i,t}$ will be less than one when the $t = 2$ return is large, relative to the returns at $t = 1$. If r_2 is negative, this relationship would be reversed, which is why the measure is inverted when r_2 is less than zero[14]. To compute this measure, I save the prices at $t = 0$[15], $t = 1$ (calculated using the equilibrium in Admati [1985]), $t = 2$ (terminal payoffs), and compute returns as $r_{(t-n,t)} = \frac{p_t - p_{t-n}}{p_{t-n}}$ and $r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$. The pre-earnings drift measure would clearly be influenced by the risk premium channel, which is another reason why I work exclusively with market-adjusted returns.

Share of Volatility on Earnings Days

Define the share of volatility on earnings days as $r_2^2 / \left(r_1^2 + r_2^2\right)$. If prices are not informative before earnings announcements, we would expect earnings day volatility to be large, relative to total volatility. Note that this is not sensitive to using squared returns i.e. focusing on extreme observations – I find similar results working with absolute returns.

[14] This is similar to the price jump ratio in Weller [2017], but can be computed for all stocks. Weller has to filter out over 50% of earnings announcements because the denominator of his measure can be close to zero.

[15] The price at $t = 0$ is the price at the expected values of z and x i.e. prices if $x = 0$ and $z = 0$.

4.2 Introducing the ETF

We know that introducing the ETF changes the share of investors who become informed. To make the two settings comparable, I do the following: Calculate the cost in dollars of becoming informed such that in the world without the ETF, 50% of investors become informed in equilibrium. Then, find the closest cost of being informed on the grid for the world the ETF (This is the same exercise that I'm doing in Table 6). Table 7 contains the results for the price informativeness measures.

In the first scenario (green highlight), introducing the ETF *increases* per-earnings trading volume, increases the pre-earnings drift and decreases the share of volatility on earnings days. For these parameters, introducing the ETF seems to have made pre-earnings prices more informative. In the 2nd, 3rd and 4th scenarios (blue highlight), the ETF makes prices less informative across all measures. This decrease in pre-earnings price informativeness is likely due to a combination of two effects: (1) a decrease in the share of agents learning and (2) attention re-allocation away from stock-specific risks.

4.3 Relationship to Empirical Results

Sammon [2020a] finds that increasing passive ownership leads to (1) decreased pre-earnings trading volume (2) decreased pre-earnings drift (3) increased share of volatility on earnings days[16]. If we equate introducing an ETF in the model to the increases in passive ownership in the data, then we can compare the empirical results to the model's predictions. To this end, I do a calibration of the model, where I search over all possible values of (1) the share of informed agents when the ETF is not present (2) risk aversion (3) the volatility of the systematic risk factor. I select parameters to match the decrease in volume, decrease in drift,

[16] All results in Sammon [2020a] are robust to defining passive ownership only as ETFs.

	ρ	σ_f^2	No ETF	ETF	Change
Volume	0.1	0.2	1.4377	1.7168	0.2791
	0.1	0.5	1.4387	0.6964	-0.7423
	0.35	0.2	0.4192	0.3026	-0.1166
	0.35	0.5	0.4216	0.3026	-0.1191
Drift	0.1	0.2	96.82%	96.98%	0.16%
	0.1	0.5	96.70%	96.24%	-0.45%
	0.35	0.2	95.92%	95.88%	-0.04%
	0.35	0.5	95.22%	95.14%	-0.08%
Volatility	0.1	0.2	60.38%	56.89%	-3.48%
	0.1	0.5	60.46%	76.18%	15.72%
	0.35	0.2	74.70%	78.01%	3.32%
	0.35	0.5	75.24%	78.43%	3.19%

Table 7: **Effect of ETF on Price Informativeness, Matching Cost of Becoming Informed Across Economies.** The cost of becoming informed is set such that 50% of agents become informed before introducing the ETF.

and increase in volatility after a firm is added to the S&P 500 index. Specifically, I have the following objective function:

$$\min \frac{|drift_{model} - driftdata|}{|drift_{data}|} + \frac{|volume_{model} - volume_{data}|}{|volume_{data}|} + \frac{|volatility_{model} - volatility_{data}|}{|volatility_{data}|} \tag{24}$$

The empirical estimates I am trying to match are from the following regression:

$$\Delta Outcome_{i,t} = \alpha + \beta \times Treated_{i,t} + \gamma_t + \epsilon_{i,t} \tag{25}$$

where *Outcome* is pre-earnings volume, pre-earnings drift, or share of volatility on earnings days. The Δ denotes the change from before to after index addition. *Treated* equals 1 if a firm was added to the index, and is equal to zero if it is one of the control firms: firms in the same industry, and of similar size which are either (1) not in the index (2) already in the

	Treated vs. In/Out of Index		
	Volume	Drift	Volatility
Treated	-0.813**	-0.00534**	0.0179**
	(0.369)	(0.002)	(0.007)
Model	-0.075	-0.0001	0.019

Table 8: **S&P 500 Index Addition.** Model predictions use calibrated parameters: Cost of becoming informed is set so 90% learn in equilibrium when the ETF is not present, and 70% learn when the ETF is present. $\rho = 0.3$, σ_f^2=0.15. Regressions include month of index addition fixed effects.

index. The regression also includes month of index addition fixed effects γ_t (for full details of the S&P 500 index addition experiment, see Sammon [2020a]).

The results are in Table 8. The calibration is such that: (1) the cost of becoming informed is set so 90% learn in equilibrium when the ETF is not present. At this c 70% learn when the ETF is present (2) $\rho = 0.3$ and (3) σ_f^2=0.15.

The model is fairly close on matching the increase in volatility, is off by about a factor of 10 for matching the volume, and only gets the sign of the change in the drift correct. We can better match the data by increasing σ_n^2 to values near 0.4, but empirically, we did not observe the market to be almost as volatile as individual stocks between 1990 and 2018. Increasing σ_n^2 would also allow for a decrease in the share of agents who become informed when the ETF is not present to 50%, and the share who become informed after the introduction of the ETF to 40%.

While the model is consistent with the data, this is not the perfect comparison. Being added to an index, and having an increase in passive ownership (data), is clearly different from introducing an ETF, which effectively completes the market (model). In the next subsection, I propose a new empirical exercise which is closer to the model effect of introducing an ETF.

4.4 The Introduction of Sector ETFs

In the model, the ETF has two main effects on informed investors' trading: (1) it makes it easier to make bets on the systematic risk-factor and (2) it makes it easier to hedge systematic risk when betting on individual stocks. In this section, I propose a natural experiment designed to mimic the introduction of an ETF in the model.

4.4.1 History of Sector ETFs

Sector ETFs are ETFs that track specific industries, rather than the market as a whole. While there are many sector ETFs, the most well known are State Street's Sector SPDR Funds. Table 9 contains a list of the all the sector SPDRs. These funds were introduced in waves: The first set was introduced in 1999. The second wave, which were all sub-sects of the S&P 500 were introduced in 2005 and 2006. The third wave, also subsets of the S&P 500 was introduced in 2011, while the final few were introduced in 2015 and later. As of June 2020 there is over $170 Billion invested in these products.

4.4.2 Empirical Design

The introduction of sector ETFs seems likely to capture some of the features of introducing the ETF in the model. These are low fee products (expense ratios less than 50 basis points), so they make it easier to trade on systematic risk. These are also heavily used by hedge funds to short/hedge sector risks. According to Goldman Sachs Hedge Fund Monitor (2016), hedge funds are net short XRT, XLY, XBI, XOP, XLI, XLF, XLV, XLU and XLE – and this is only among sector ETFs they explicitly listed in their report. Further, these net short positions are not due to small long positions. For example, in XLF (the Financial Select Sector SPDR), hedge funds have net $712 million long and $2.4 *billion* short. The large

Name	Ticker	Founded	Expense Ratio	NAV	AUM
The Consumer Discretionary Select Sector Fund	XLY	3/31/1999	0.13%	$132.26	$14,012.89 M
The Consumer Staples Select Sector Fund	XLP	3/31/1999	0.13%	$60.54	$14,241.29 M
The Energy Select Sector Fund	XLE	3/31/1999	0.13%	$45.09	$12,164.02 M
The Financial Select Sector Fund	XLF	3/31/1999	0.13%	$26.16	$21,849.83 M
The Health Care Select Sector Fund	XLV	3/31/1999	0.13%	$102.82	$26,580.85 M
The Industrial Select Sector Fund	XLI	3/31/1999	0.13%	$74.39	$10,293.96 M
The Materials Select Sector Fund	XLB	3/31/1999	0.13%	$59.18	$5,209.07 M
The Technology Select Sector Fund	XLK	3/31/1999	0.13%	$102.42	$30,947.40 M
The Utilities Select Sector Fund	XLU	3/31/1999	0.13%	$61.55	$11,819.93 M
Bank ETF	KBE	12/30/2005	0.35%	$36.90	$1,666.27 M
Capital Markets ETF	KCE	12/30/2005	0.35%	$59.82	$25.42 M
Insurance ETF	KIE	12/30/2005	0.35%	$30.36	$629.94 M
Biotech ETF	XBI	3/31/2006	0.35%	$104.56	$4,577.23 M
Homebuilders ETF	XHB	3/31/2006	0.35%	$45.13	$857.47 M
Semiconductor ETF	XSD	3/31/2006	0.35%	$115.13	$518.10 M
Metals & Mining ETF	XME	6/30/2006	0.35%	$23.39	$457.38 M
Oil & Gas Equipment & Services ETF	XES	6/30/2006	0.35%	$43.91	$121.41 M
Oil & Gas Exploration & Production ETF	XOP	6/30/2006	0.35%	$66.61	$2,337.89 M
Pharmaceuticals ETF	XPH	6/30/2006	0.35%	$43.79	$249.63 M
Regional Banking ETF	KRE	6/30/2006	0.35%	$44.87	$1,503.21 M
Retail ETF	XRT	6/30/2006	0.35%	$44.21	$362.53 M
Health Care Equipment ETF	XHE	3/31/2011	0.35%	$88.00	$521.40 M
Telecom ETF	XTL	3/31/2011	0.35%	$73.55	$53.32 M
Transportation ETF	XTN	3/31/2011	0.35%	$57.37	$174.97 M
Aerospace & Defense ETF	XAR	9/30/2011	0.35%	$98.20	$1,571.20 M
Health Care Services ETF	XHS	9/30/2011	0.35%	$71.93	$90.64 M
Software & Services ETF	XSW	9/30/2011	0.35%	$110.64	$236.77 M
The Real Estate Select Sector Fund	XLRE	12/31/2015	0.13%	$37.43	$4,641.12 M
Internet ETF	XWEB	6/30/2016	0.35%	$98.83	$21.74 M
The Communication Services Select Sector Fund	XLC	6/29/2018	0.13%	$56.41	$9,803.64 M

Table 9: **List of Sector SPDR ETFs.** The expense ratio, Net Asset Value per share (NAV) and Assets Under Management (AUM) are as of 6/9/2020.

short interest in many ETFs by hedge funds may be due to the relatively low borrowing cost. According to Deutsche Bank, shorting SPY (the largest S&P 500 ETF) usually costs about 40 basis points, while shorting something riskier like the US consumer staples sector ETF (XLP) can run up to 72 basis points.

Given these features of sector ETFs, they seem like a reasonable empirical analogue to the introduction of ETFs in the model. With that in mind, I examine the effect of the introduction of the original set of sector ETFs in 1999. There are three groups of firms to compare (1) firms which were in the ETFs (2) firms in the same sector as the ETF, but were not part of the ETF basket (3) firms in sectors without ETFs.

I found that the firms which were added to the sector ETFs were mostly firms in the largest 20% of each industry. To construct a better control group I split firms up into quintiles of market capitalization by industry. The two groups of treated firms are (1) those in the ETF (2) those in the same 3-digit SIC industry as firms in the ETF and in the top 20% of market capitalization for these industries, but not in the ETF. The control group is going to be firms in 3-digit SIC industries that do not have sector ETFs, but are still in the top 20% of market capitalization for their own industry.

The empirical estimates I am trying to match are from the following regression:

$$Outcome_{i,t} = \alpha + \beta \times Treated_{i,t} \times Post_t + \gamma_t + \epsilon_{i,t} \tag{26}$$

where *Outcome* is pre-earnings volume, pre-earnings drift, or share of volatility on earnings days. $Post_t = 1$ for all year/quarters after the first quarter of 1999. I omit the second quarter of 1999 for the volume/drift regressions (which use quarterly data) in case of a temporary liquidity shock to these stocks as the result of the ETFs being introduced. For the volatility regression (which uses annual data), I omit all of 1999 for the same reason. $Treated$ equals 1 if a firm was in one of the ETFs, or in a sector with an ETF. It is equal to zero otherwise. The regression also includes time fixed effects, γ_t, and there is no uninteracted $Post_t$ term because of these time fixed effects. Observations are weighted by lagged market capitalization. Standard errors are clustered at the firm-level.

4.4.3 Empirical Results

Table 10 contains the regression results. After the introduction of sector ETFs, the treated firms had a decrease in pre-earnings volume, a decrease in pre-earnings drift, and an increase in the share of annual volatility on earnings days. The effect is slightly stronger among the treated firms that were members of the new sector ETFs[17]. This is consistent with the sector ETFs decreasing stock-level price informativeness. A calibration is in the row below the regression results. The calibrated parameters are: (1) informed share before ETF introduction at 90%, after ETF introduction 70% (2) Risk aversion (ρ) at 0.15, and (3) volatility of systematic risk (σ_n^2) at 0.3.

The calibration is able to quantitatively match the changes in volume and volatility, but can only qualitatively match the results for the pre-earnings drift. Part of this is due to the fact that in the data, I am using returns over 22 trading days to construct the drift, while in the model, there is only 1 day before the earnings announcement. The concept of information being slowly incorporated into prices might be better suited to a Kyle [1985]-style model, than the model in this paper.

[17]See Figure 16 in the Appendix for a visual version of these regressions.

	Volume	Drift	Volatility
$Treated \times Post$	-0.0776	-0.00715**	0.0128*
	(0.435)	(0.003)	(0.008)
Model	-0.0881	-0.00006	0.0112
In ETFs	343	343	343
In ETF Sectors	3316	3316	3316
Outside ETF Sectors	866	866	866
Time FE	YES	YES	YES

Table 10: **Effect of Introducing Sector ETFs.** Coefficients from: $Outcome_{i,t} = \alpha + \beta \times Treated_{i,t} \times Post_t + \gamma_t + \epsilon_{i,t}$ Observations are weighted by lagged market capitalization. Standard errors, clustered at the firm level, in parenthesis.

5 Varying the Size of ETF Ownership

Up to this point, the ETF has been in zero average supply, similar to a futures contract. This means that if an investor wants to go long the ETF, there needs to be another investor taking an exactly offsetting short position in the ETF. Unlike futures contracts, however, almost all ETFs are in positive supply: few have short interest equal to 100% or more of their AUM[18]. The mechanism for this is that investors can take a pre-specified basket of underlying securities and give them to an ETF custodian in exchange for shares of the ETF. These shares of the ETF then trade on the secondary market.

This is an important feature to capture when mapping the model's predictions to the empirical results in Sammon [2020a]. In the data, passive ownership is measured as the percent of each company's shares outstanding owned by passive funds. In this section, I will extend the model, allowing an intermediary to buy shares of the underlying stocks and convert them into shares of the ETF.

[18]See e.g. data here on the most shorted ETFs, as of 8/1/2020 only 3 ETFs have short interest greater than or equal to 100%.

5.1 Setup

As outlined above, I introduce a new player who can buy shares of the underlying stocks, and convert them into shares of the ETF. I am going to assume that, unlike the atomistic informed and uninformed investors, this intermediary is strategic: she understands that to create more shares of the ETF, she will have to buy more shares of the stocks, which will push up their *expected* prices. I emphasize *expected* because she still takes prices at $t = 1$ as given i.e. there is no price impact at $t = 1$. She does, however, account for changes in *expected* prices given her behavior.

I am also going to assume that she has to submit a market order at $t = 0$ i.e. she will have to decide how many shares of the ETF to create without knowing the $t = 1$ prices of any security. I discuss these two assumptions (strategic intermediary, submitting a market order) in the next subsection.

Her objective function is the same as the objective function for the informed and uninformed investors:

$$U_{0,j} = E_0 \left[E_{1,j}[w_{2,j}] - 0.5\rho^i Var_{1,j}[w_{2,j}]\right] \tag{27}$$

where ρ^i is the intermediary's risk aversion. I assume that because the stocks are symmetric, she must demand the same amount of each stock. If she buys v shares of every stock, this would take $v \times n$ units of systematic risk out of the economy. To ensure that the amount of systematic risk in the economy is constant, I assume this allows her to create $v \times n$ shares of the ETF. These assumptions imply that her only decision is how many shares of each stock to buy (v).

With this technology, the intermediary's payoff will be:

$$v\left(\sum_{i=1}^{n}(z_i - p_i) - n(z_{n+1} - p_{n+1})\right) \tag{28}$$

which is the average difference between the stocks' payoffs and their prices minus the difference between the ETF's payoff and its price, scaled by how many shares she creates. To create the ETF, she is essentially stripping out the idiosyncratic risk from an equal-weighted basket of the stocks, and bearing it herself. She sells the systematic risk from this basket to other investors.

This ETF creation technology does not exactly match the real world. ETF arbitrageurs don't hold on to the shares of the stocks they buy to create shares of the ETFs – they transfer them to an ETF custodian (e.g. State Street for the largest S&P 500 ETF SPY)[19]. This could be modeled by having the intermediary transfer the stocks she buys at $t = 1$ to another (new) agent, an ETF custodian, who gives her shares of the ETF, which she sells immediately at $t = 1$. With this setup, the intermediary would have no asset holdings at $t = 2$.

I find that with these alternative assumptions, all the qualitative results are unchanged. The quantitative difference is that creating shares of the ETF is less risky, so in equilibrium, the intermediary makes the ETF larger. This is because in this scenario, the intermediary is only exposed to risk on her market order i.e. that the average prices of the stocks is higher than the price of the ETF due to positive realizations of stock-specific risk-factors or negative realizations of the stock-specific noise trader shocks. The reason I do not use this

[19]For ETFs with many underlying securities, all idiosyncratic risk should be diversified away. For example, when creating shares of an S&P 500 ETF, there are 500 sources of idiosyncratic risk, substantially more than the 8 stocks in the baseline version of my model. The issue is that without transaction costs, if there are a large number of underlying assets, there would be no motivation to create the ETF in the first place.

alternative setup as the main specification is because with this ETF creation technology, the intermediary would be able to remove idiosyncratic risk from the economy by creating more shares of the ETF.

I solve for the optimal v numerically using the following procedure. First, I restrict v to be greater than or equal to zero. This almost never binds, but as mentioned in the introduction of this section, almost all ETFs are in positive supply. Then, I loop over all possible values of v between 0 and $\overline{x}$, and select the v which maximizes the intermediary's expected utility. The expectations in Equation 27 are computed by simulating 10,000 draws of the z and x shocks for each possible choice of v.

Having the intermediary submit a market order at $t = 0$ means that we can re-use the equilibrium price and demand functions from Section 2. Because this is a rational expectations equilibrium, all the agents know what v will be, given the model parameters. The only difference is that informed and uninformed agents will treat the supply of each stock as $\overline{x} - v$ and the supply of the ETF as $n \times v$. The new model timeline is in Table 11.

The size of the ETF depends on the intermediary's risk aversion, ρ^i. Figure 8 shows that as the intermediary's risk aversion increases, the number of shares of the ETF decreases.

The size of the ETF also depends on ρ, σ_n and the share of informed agents: if the risk-bearing capacity of the economy is low, investors will generally be willing to pay a higher price for the ETF, so the intermediary will create more shares. Figure 9 shows that as risk aversion of informed and uninformed investors increases, the equilibrium size of the ETF increases as well: The amount of the ETF created, as a function of ρ^i, shifts out to the right as we increase ρ.

$t = 0$	• Agents make binary decision to pay c and become informed or stay uninformed. • If informed, decide how to allocate attention to the underlying risks. • **Intermediary submits market order to buy v shares of the each stock.**
$t = 1$	• **Intermediary's market order clears, leaving $\overline{x} - v$ shares of each stock, and $v \times n$ shares of the ETF available for purchase** • Informed agents receive private signals. Informed and uninformed agents submit demands
$t = 2$	Payoffs realized, agents consume

Table 11: New Model Timeline. Differences from original timeline in bold.

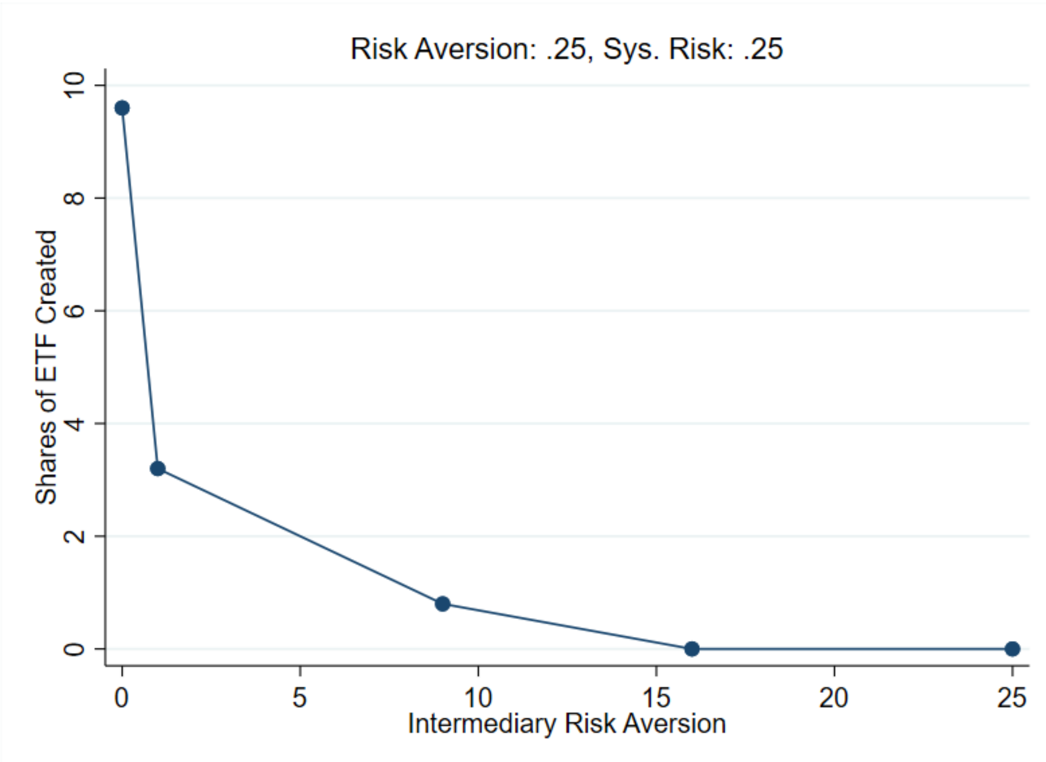

Figure 8: **Relationship between size of the ETF and the intermediary's risk aversion.** The share of informed agents is set to 50%.

5.2 Discussion

In this sub-section, I discuss (1) why I assumed the intermediary is 'strategic' and (2) why I assumed the intermediary submits a market order i.e. why her demand does not depend on prices.

The main reason for the first assumption is that I want the intermediary to be different from the informed/uninformed agents. Any of those agents could implement a trading strategy where they buy shares of the underlying stocks, and sell shares of the ETF. When risk aversion is low, informed investors will (collectively) implement a strategy like this. Given that the group of investors (informed or uninformed) 'creating' shares of the ETF

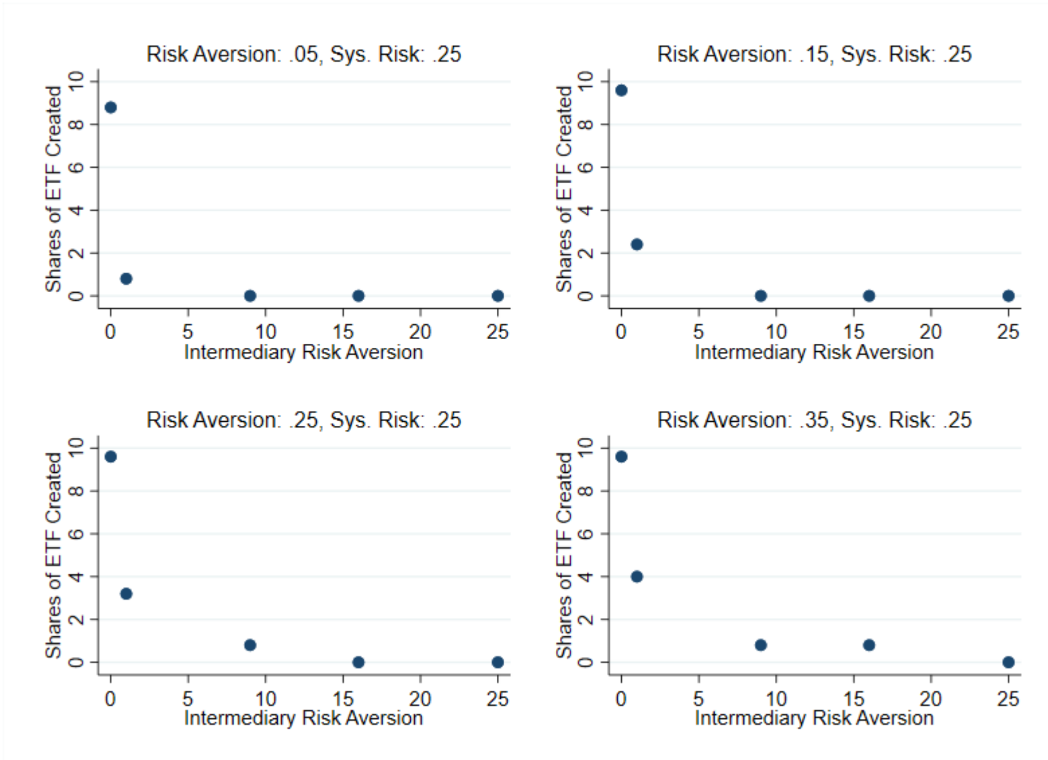

Figure 9: **Relationship between the size of the ETF and informed/uninformed investors' risk avesrion.** The share of informed agents is set to 50%, σ_n is set to 0.25.

(i.e. shorting the ETF when it is in zero average supply) is not always the same, it is not obvious how to define passive ownership. With my assumptions about the ETF creation process, passive ownership can be measured as the percent of shares of each stock purchased by the intermediary. This has the added benefit of a clearer link to the definition of passive ownership in Sammon [2020a].

A way to model non-strategic ETF creation would be to have a continuum of competitive agents who can create shares of the ETF for a fixed cost (this cost maps to the creation/redemption fee charged by ETF custodians). Because these agents are competitive, in equilibrium the ETF creators will make zero economic profit, and so will be indifferent to the number of shares they create. By making the ETF creator a monopolist I get a unique solution for the size of the ETF.

The second assumption is needed because of the first assumption. At $t = 1$, if the intermediary could have her demand depend on prices, say through a simple linear rule, there would be an interaction between a strategic agent (the intermediary) and atomistic agents (informed and uninformed investors). On top of that, informed and uninformed investors are learning from prices, while the intermediary, at least as she is defined now, does not. Without additional assumptions, it's not obvious that an equilibrium would exist.

5.3 Intensive and Extensive Margin Effects

Given the intensive/extensive margin effects of introducing the ETF were ambiguous, it seems that increasing the size of the ETF could also have an ambiguous effect on (1) the share of agents who become informed in equilibrium and (2) learning about the systematic risk-factor. In this subsection, I show that increasing the size of the ETF uniformly decreases the share of informed agents and uniformly increases learning about systematic risk, *relative* to the case where the ETF is in zero average supply.

Figure 10 relates the share of informed agents to the cost of becoming informed in three scenarios: (1) No ETF (blue diamonds) (2) ETF in zero net supply (red circles) and (3) ETF in positive supply (green crosses). For all four panels, I fix σ_n at 0.25, and vary ρ. The top right panel, when $\rho = 0.15$ and $\sigma_n = 0.25$, shows that there is an additional decrease in the share of informed agents when the ETF is in positive supply. Because there are 2.5 shares of each stock outstanding (20/8=2.5), and to create 8 shares of the ETF requires buying 1 share of each stock, passive ownership is 40% of the market (1/2.5=0.4). Note also that as we increase ρ, the intermediary endogenously creates more shares of the ETF. Figure 11 fixes ρ at 0.25, and varies σ_n. Again, the top left two panels show the additional decrease in the share of informed agents, relative to the zero ETF supply case, when we allow the ETF to be in positive supply.

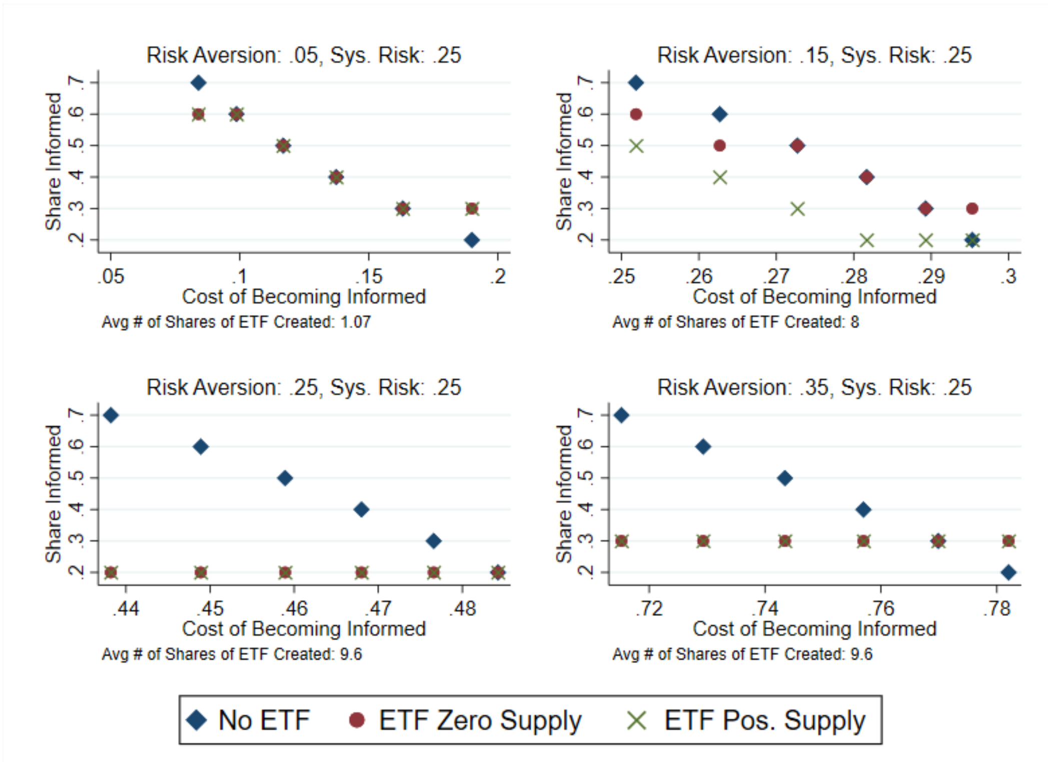

Figure 10: **Extensive Margin Effects (ρ).** Relationship between cost of becoming informed, and share of agents who become informed in equilibrium. The risk aversion of the intermediary, ρ^i is equal to 1. *Average # of Shares of ETF Created* is taken across all costs of being informed for each choice of ρ and σ_n

Figure 12 shows the intensive learning margin effect across these same three scenarios, fixing σ_n at 0.25. All the panels, except the top left, show the additional increase in attention on systematic risk when we allow the ETF to be in positive supply. Figure 13 shows the effect of varying σ_n, fixing ρ at 0.25, where the same pattern is present.

The average size of the ETF is smaller in these cases, relative to the averages taken in Figures 10 and 11. This is because I am taking the average over all shares of informed

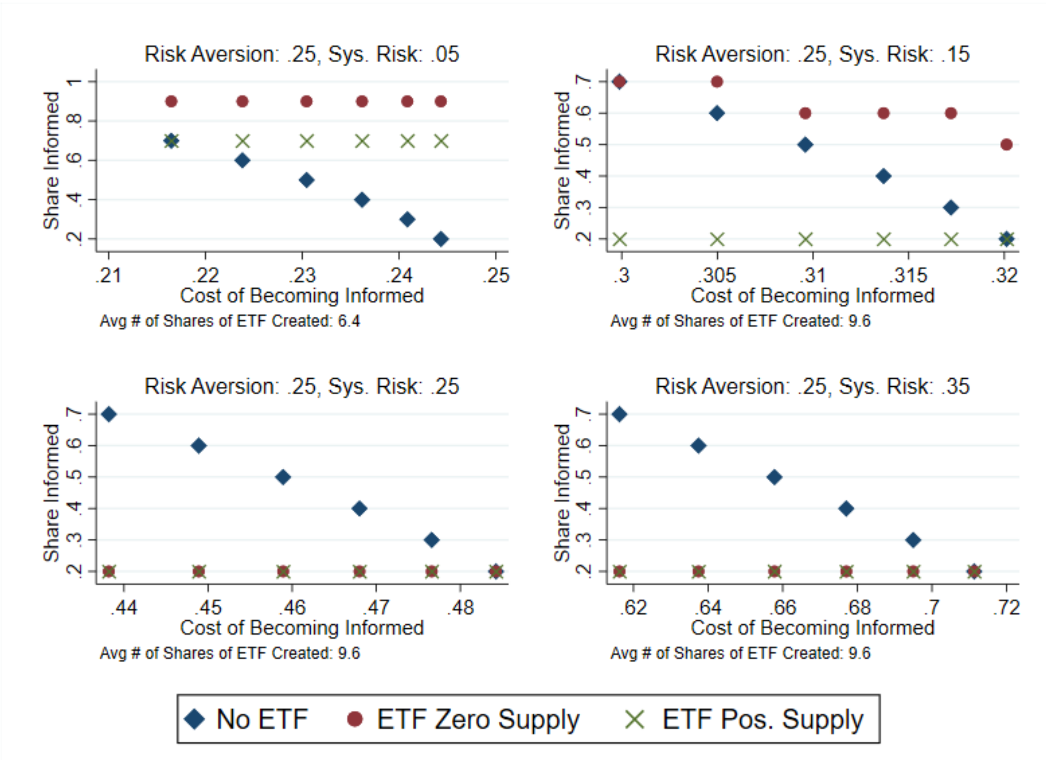

Figure 11: **Extensive Margin Effects (σ_n).** Relationship between cost of becoming informed, and share of agents who become informed in equilibrium. The risk aversion of the intermediary, ρ^i is equal to 1. *Average # of Shares of ETF Created* is taken across all costs of being informed for each choice of ρ and σ_n

agents between 0.2 and 0.7. For the most part, the share of informed agents is around 0.2 when I am matching economies on the cost of becoming informed. This implies that in these new averages, the average risk bearing capacity of the economy is larger, which implies less demand for the ETF as discussed above.

5.4 Effect on Price Informativeness

Figure 14 shows a calibration designed to match the empirical patterns in Sammon [2020a]. The vertical red line denotes the parameter set of interest. All three price informativeness

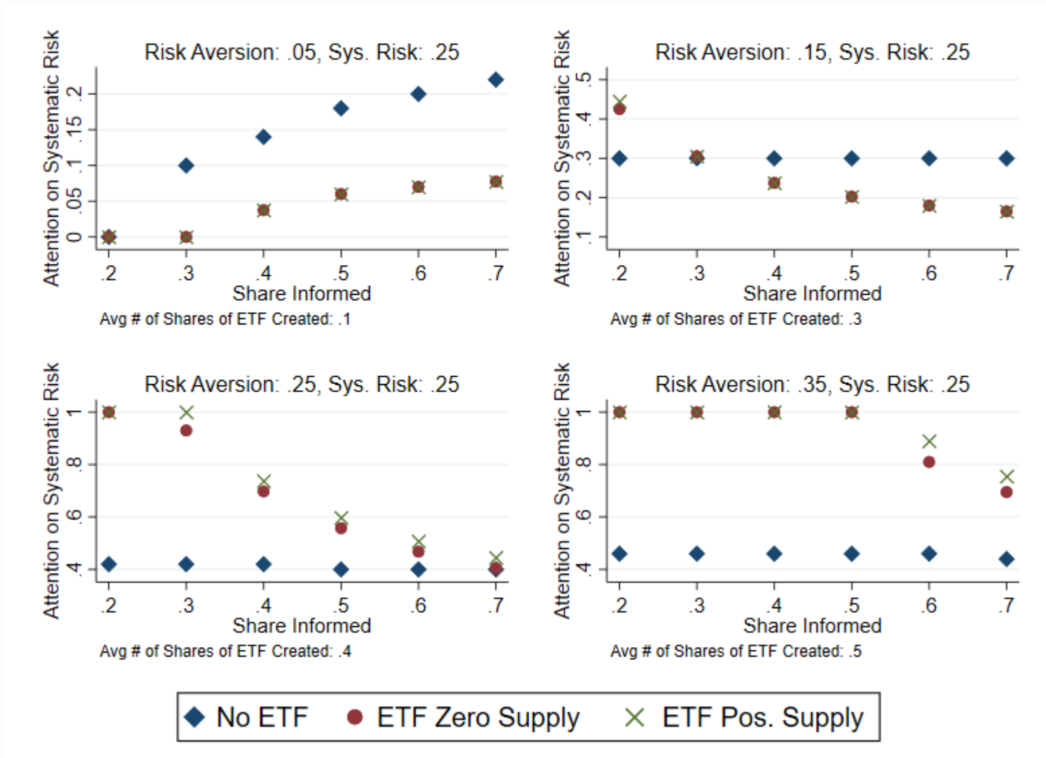

Figure 12: **Intensive Margin Effects (ρ).** Relationship between share of informed agents, and share of attention allocated to the systematic risk-factor. The risk aversion of the intermediary, ρ^i is equal to 1. *Average # of Shares of ETF Created* is taken across all shares of informed agents for each choice of ρ and σ_n.

measures decreases both as (1) we introduce the ETF in zero average supply and (2) as we increase the size of passive ownership to be 12% of the market. This is chosen to match the roughly 15% of the market owned by passive ownership, as documented in Sammon [2020a].

6 Conclusion

The introduction of ETFs has been one of the biggest changes in financial markets over the past 30 years. Given that a main function of financial markets is to aggregate information, it is important to know how their ability to perform this function changes as the landscape of

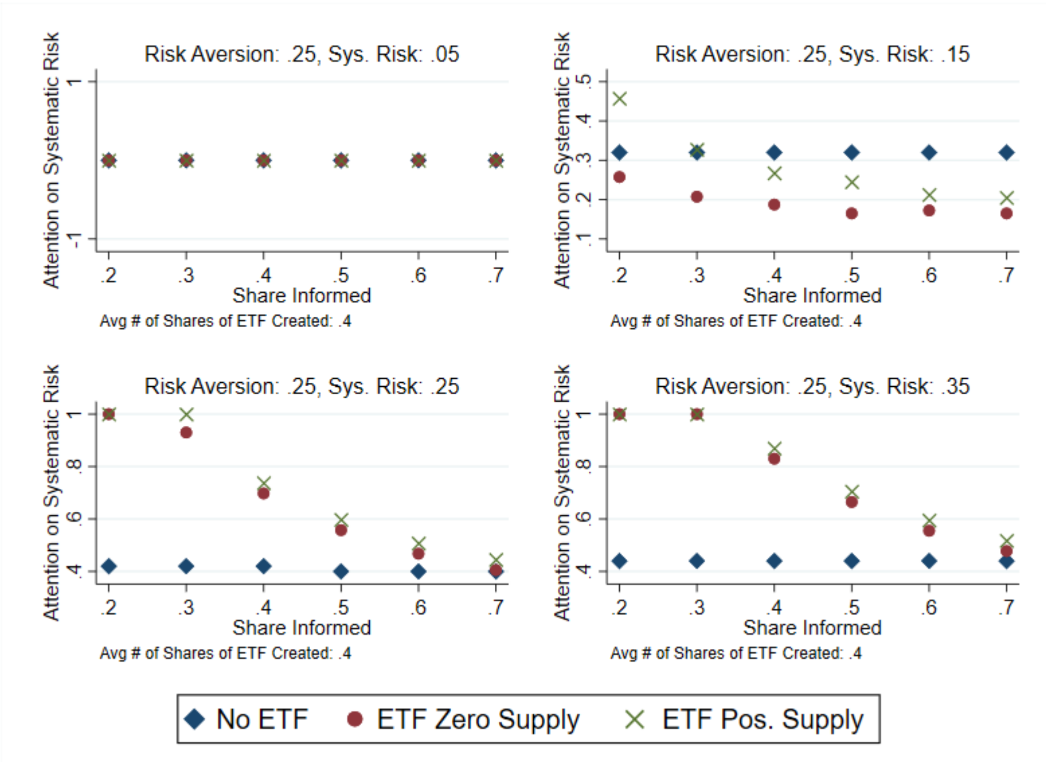

Figure 13: **Intensive Margin Effects (σ_n).** Relationship between share of informed agents, and share of attention allocated to the systematic risk-factor. The risk aversion of the intermediary, ρ^i is equal to 1. *Average # of Shares of ETF Created* is taken across all shares of informed agents for each choice of ρ and σ_n.

financial products changes. This paper focuses on the effect of introducing ETFs on learning, which in turn leads to changes in price informativeness for individual stocks.

I find three main effects of introducing ETFs: (1) Changes in the share of agents who decide to become informed (2) Changes in the attention of informed agents (3) Changes in risk premia.

I find that effect of introducing the ETF on pre-earnings price informativeness is ambiguous in the model, which is why empirical work is needed. Sammon [2020a] shows that increases in passive ownership cause decreases in price informativeness. The paper also

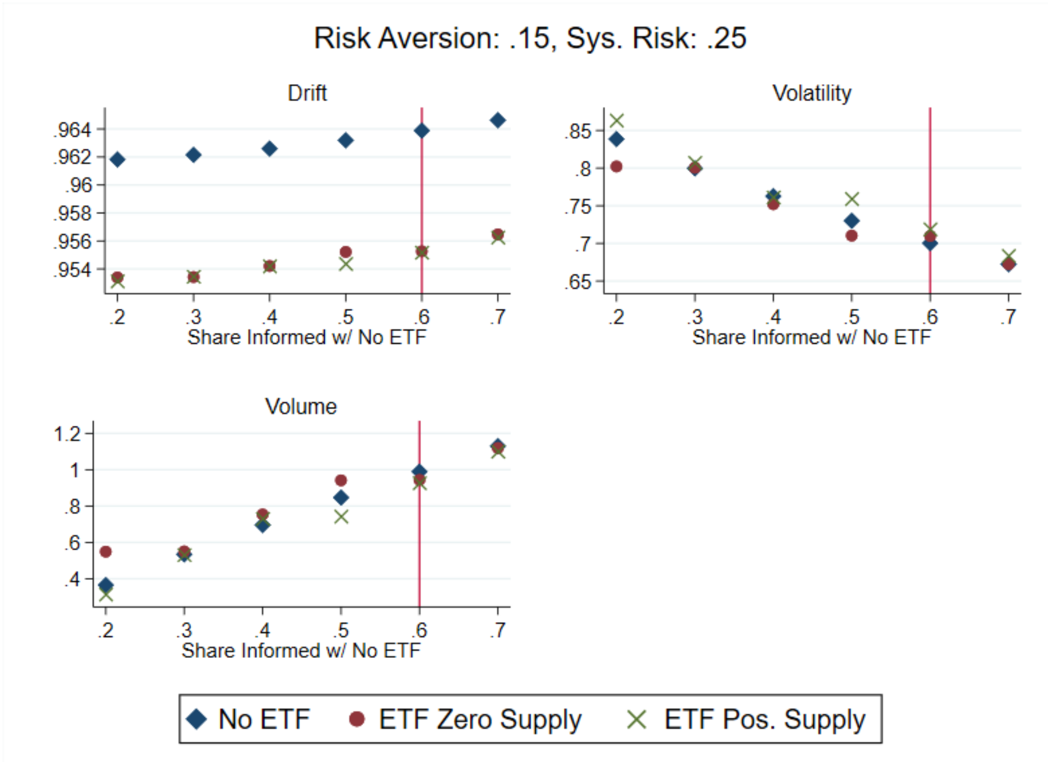

Figure 14: **Calibration.** Vertical red line is a calibration where passive ownership is 12% of each stock.

shows direct evidence on decreased information gathering, through analyst coverage and downloads of SEC filings. This is consistent with the model's predictions when risk aversion, or systematic risk are sufficiently high.

In this paper, I examine a new natural experiment to test the model's predictions: the introduction of sector ETFs. Empirically, stocks in sectors where the ETFs were introduced had decreases in price informativeness. The changes in volume and volatility can be quantitatively matched in the model, while the results for the pre-earnings drift can only be matched qualitatively. The calibration that matches the data features high risk aversion, similar to matching the results in Sammon [2020a].

The model could be enriched by creating two groups of stocks: those that are in the

ETF, and those whose systematic risk is spanned by the ETF, but are not part of the basket. For this to have an effect on the model, there would have to be some way for agents to convert shares of stock to the ETF and vice versa. This extension would yield more testable predictions for the effect of introducing the ETF on the different "treated" groups in the sector ETF experiment.

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A Appendix

A.1 Model where individual assets are not symmetric

Suppose each asset i now has the payoff:

$$z_i = a_i + \beta_i f + \eta_i \tag{29}$$

where β_i and $var(\eta_i)$ is different for each asset. In this setting, informed agents' choice is not just a trade-off between learning about systematic and idiosyncratic risk. To solve for information choice in this version of the model, I need to modify the numerical method:

1. Start all agents at K^0
2. Consider an atomistic agent j who takes K^0 as given, and considers their expected utility by deviating to K_j^1 on a $n \times n$ dimensional grid around K^0. Even though there are $(n+1)$ risks to learn about, we don't need the $(n+1)^{th}$ dimension because of the total information constraint.
3. Calculate the gradient numerically at K^0 using this grid of expected deviation utilities. Then, move j on the grid in the direction of the gradient.
4. If j's expected utility increased, move all informed agents to K_j^1
5. Iterate on steps 2-4 until j can no longer improve their expected utility by deviating.

This method works, and when the ETF is present, is able to match closed form solutions from Kacperczyk et al. [2016]. For $n > 3$, however, this method can take an extremely long time to find the solution. Given that heterogenous this does not drastically change pre-earnings volume, pre-earnings drift or earnings-day volatility, I focus on the symmetric asset case in the main body of the paper.

A.2 Expected utility of informed and uninformed

Table 12 contains information on the percentage difference in expected utility between informed and uninformed agents when the ETF is and is not present.

A.3 Sensitivity of Demand to Prices

This is also a type of hedging demand. Similar to the hedging from signals to informed agents, investors also use prices as a signal, and thus may do a similar hedging. Table 13 fixes the cost of becoming informed. Table 14 fixes the share of agents becoming informed.

A.4 Expected Utility

In line with Kacperczyk et al. [2016], I define agents' time 0 objective function as: $-E_0[ln(-U_{1,j})]/\rho$ which simplifies to: $U_0 = E_0\left[E_{1,j}[w_{2,j}] - 0.5\rho Var_{1,j}[w_{2,j}]\right]$. This simplification comes from the fact that (1) $w_{2,j}$ is normally distributed, and (2) $E[exp(ax)] = exp(a\mu_x + \frac{1}{2}a^2\sigma_x^2)$ where x is a normally distributed random variable with mean μ_x and standard deviation σ_x, and a is a constant. This objective function leads to a preference for an early resolution of uncertainty, relative to expected utility.

Too see how the log transformation, $-E_0[ln(-U_{1,j})]/\rho$, induces a preference for an early resolution of uncertainty relative to expected utility $E_0[U_{1,j}]$, we can follow Veldkamp [2011] and cast preferences as recursive utility (Epstein and Zin [1989]).

Panel A: Matching Cost of Becoming Informed

		Share Informed		Diff. in EU	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF
0.1	0.2	0.05	0.2	0.154%	0.163%
0.1	0.5	0.35	0.2	0.181%	0.177%
0.25	0.2	0.5	0.2	0.229%	0.229%
0.25	0.5	0.5	0.2	0.572%	0.571%

Panel B: Share Informed at 10%

		Share Informed		Diff. in EU	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF
0.1	0.2	0.1	0.1	0.154%	0.177%
0.1	0.5	0.1	0.1	0.226%	0.186%
0.25	0.2	0.1	0.1	0.251%	0.296%
0.25	0.5	0.1	0.1	0.727%	1.103%

Panel C: Share Informed at 30%

		Share Informed		Diff. in EU	
ρ	σ_n^2	no ETF	ETF	no ETF	ETF
0.1	0.2	0.3	0.3	0.132%	0.141%
0.1	0.5	0.3	0.3	0.190%	0.154%
0.25	0.2	0.3	0.3	0.237%	0.211%
0.25	0.5	0.3	0.3	0.650%	0.300%

Table 12: **Effect of Introducing the ETF on Expected Utility of Informed and Uninformed.** This table quantifies the effect of introducing the ETF on the expected utility of informed and uninformed agents. The columns of interest are under the header "Diff. in EU". The "no ETF" column is the % difference in expected utility between informed and uninformed agents when the ETF is not present. The ETF column repeats this exercise after introducing the ETF.

		Uninformed						
		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.05	0.2	6.333	-0.278	2.273	0.000	-2.273
0.1	0.5	0.35	0.2	1.764	-0.170	3.082	0.000	-3.082
0.25	0.2	0.5	0.2	2.380	-0.181	5.510	0.000	-5.510
0.25	0.5	0.5	0.2	2.550	-0.291	5.510	0.000	-5.510

		Informed						
		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.1	0.2	7.872	-0.489	4.023	0.000	-4.023
0.1	0.5	0.35	0.2	2.865	-0.270	4.307	0.000	-4.307
0.25	0.2	0.5	0.2	2.803	-0.218	5.710	0.000	-5.710
0.25	0.5	0.5	0.2	2.913	-0.317	5.710	0.000	-5.710

Table 13: **Sensitivity of Demand to Prices (fixed c).** Entries of $G_{2,inf}$ and $G_{2,un}$ for one of the stocks i.e. assets 1 to $n-1$. The cost of being informed is chosen such that 20% of agents become informed when the ETF is present. The "Own" columns are diagonal entries e.g. (1,1). The "Stock Hedge" column is one of the edge entries excluding the n^{th} e.g. (1,2) or (2,1). The "ETF Hedge" column is the n^{th} edge entry.

Panel A: Share Informed Fixed at 10%

Uninformed

		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.1	0.1	4.096	-0.036	4.040	0.000	-4.040
0.1	0.5	0.1	0.1	4.899	-0.528	7.656	0.000	-7.656
0.25	0.2	0.1	0.1	4.884	-0.464	6.270	0.000	-6.270
0.25	0.5	0.1	0.1	4.976	-0.601	6.270	0.000	-6.270

Informed

		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.1	0.1	5.597	-0.236	5.790	0.000	-5.790
0.1	0.5	0.1	0.1	5.979	-0.623	8.343	0.000	-8.343
0.25	0.2	0.1	0.1	5.299	-0.499	6.470	0.000	-6.470
0.25	0.5	0.1	0.1	5.331	-0.626	6.470	0.000	-6.470

Panel B: Share Informed Fixed at 30%

Uninformed

		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.3	0.3	1.774	0.059	1.581	0.000	-1.581
0.1	0.5	0.3	0.3	2.020	-0.197	1.950	0.000	-1.950
0.25	0.2	0.3	0.3	3.190	-0.266	4.018	0.000	-4.018
0.25	0.5	0.3	0.3	3.364	-0.393	4.914	0.000	-4.914

Informed

		Share Informed		No ETF Present		ETF Present		
ρ	σ_n^2	no ETF	ETF	Own	Stock Hedge	Own	Stock Hedge	ETF Hedge
0.1	0.2	0.3	0.3	3.198	-0.117	3.331	0.000	-3.331
0.1	0.5	0.3	0.3	3.121	-0.296	3.337	0.000	-3.337
0.25	0.2	0.3	0.3	3.614	-0.303	4.356	0.000	-4.356
0.25	0.5	0.3	0.3	3.728	-0.419	5.114	0.000	-5.114

Table 14: **Sensitivity of Demand to Prices (fixed share informed).** Entries of $G_{2,inf}$ and $G_{2,un}$ for one of the stocks i.e. assets 1 to $n-1$. In Panels A and B, the share of informed agents are fixed and 10% and 30% respectively. The "Own" columns are diagonal entries e.g. (1,1). The "Stock Hedge" column is one of the edge entries excluding the n^{th} e.g. (1,2) or (2,1). The "ETF Hedge" column is the n^{th} edge entry.

A.4.1 Formulation as Epstein-Zin Preferences I

Start by writing down the general formulation of Epstein-Zin preferences: $U_t = \left[(1-\beta)c_t^\alpha + \beta\mu_t\left(U_{t+1}\right)^\alpha\right]^{1/\alpha}$ where the elasticity of intertemporal substitution (EIS) is $1/(1-\alpha)$ and μ_t is the certainty equivalent (CE) operator. Note, the I've re-labeled what is usually ρ to α it to avoid confusion with the CARA risk aversion at time 1.

In my setting, all consumption happens at time 2, so let's simplify U_t from the perspective of $t = 0$. To further simplify things, set $\beta = 1$. Choose the von Neumann-Morgenstern utility index $u(w) = -exp(-\rho w)$ i.e. the CARA utility at time 1. We can then define the certainty equivalent operator $\mu_t(U_{t+1}) = E_t\left[-ln(-U_{t+1})/\rho\right]$. This μ_t is just the inverse function of the von Neumann-Morgenstern utility index. It makes sense to call this a certainty equivalent operator because it returns the amount of dollars for sure that would yield the same utility as the risky investment. Recall that $U_{1,j} = E_{1,j}[-exp(-\rho w_{2,j})]$ and wealth is normally distributed so $U_{1,j} = -exp(-\rho E_{1,j}[w_{2,j}] + 0.5\rho^2 Var_{1,j}[w_{2,j}])$

Starting with setting $\beta = 0$ and $c_1 = 0$: $U_0 = \left[\mu_0\left(U_1\right)^\alpha\right]^{1/\alpha}$

Substituting in the expression for the CE operator: $U_0 = \left[E_0\left[-ln(-U_1)/\rho\right]^\alpha\right]^{1/\alpha}$

Putting in our expression for U_1: $U_0 = \left[E_0\left[-ln(exp(-\rho E_{1,j}[w_{2,j}] + 0.5\rho^2 Var_{1,j}[w_{2,j}]))/\rho\right]^\alpha\right]^{1/\alpha}$

Simplifying: $U_0 = \left[E_0\left[(E_{1,j}[w_{2,j}] - 0.5\rho Var_{1,j}[w_{2,j}])\right]^\alpha\right]^{1/\alpha}$

Setting $\alpha = 1$ i.e. an infinite EIS: $U_0 = E_0\left[(E_{1,j}[w_{2,j}] - 0.5\rho Var_{1,j}[w_{2,j}])\right]$

which matches Equation 6 in Kacperczyk et al. [2016]. This shows that we can derive their utility function from Epstein-Zin preferences, but does make it totally clear what this transformation has to do with an early vs. late resolution of uncertainty.

A.4.2 Formulation as Epstein-Zin Preferences II

To make things clearer, let's work with a more well-known version of Epstein-Zin preferences: $V_t = \left((1-\beta)c_t^{1-\rho} + \beta[E_t(V_{t+1}^{1-\alpha})]^{(1-\rho)/(1-\alpha)}\right)^{1/(1-\rho)}$

Setting $t = 0$, $c_0 = 0$, $c_1 = 0$, $\beta = 1$: $V_0 = \left([E_0(V_1^{1-\alpha})]^{(1-\rho)/(1-\alpha)}\right)^{1/(1-\rho)}$

Notice that $c^{1-\alpha}$ is a version of Constant Relative Risk Aversion (CRRA) utility. CRRA utility simplifies to log utility if relative risk aversion is equal to 1. So, with this in mind, set $\alpha = 1$: $V_0 = \left(exp[E_0(ln[V_1])]^{(1-\rho)}\right)^{1/(1-\rho)}$

Set $\rho = 0$ (i.e. infinite EIS as we did above): $V_0 = exp[E_0(ln[V_1])]$

This is equivalent to maximizing: $V_0 = E_0(ln[V_1])$ because $exp(x)$ is a monotone function.

In my setting: $V_1 = E_1[exp(-\rho w)]$ i.e. time 1 utility times -1

So the final maximization problem is: $V_0 = -E_0(ln[-V_1])$

With Epstein-Zin, there is a preference for an early resolution of uncertainty if $\alpha > (1/EIS)$. As set up here, $\alpha = 1$ and $1/EIS = 0$, so agents have a preference for early resolution of uncertainty. For expected utility, we would set $\alpha = 0$, and then there would be no preference for early resolution of uncertainty.

A.4.3 Implications for Informed Investors

As I said above, $U_0 = E_0\left[(E_{1,j}[w_{2,j}] - 0.5\rho Var_{1,j}[w_{2,j}])\right]$ introduces a preference for the early resolution of uncertainty (see e.g. Veldkamp, 2011). There are two types of uncertainty in the model: (1) uncertainty about payoffs at $t = 2$, conditional on signals at $t = 1$ (2) uncertainty about portfolio you will hold at $t = 1$ from the perspective of $t = 0$. With these preferences, agents are not averse to uncertainty resolved before time two i.e. are not averse to the uncertainty about which portfolio they will hold.

An intuitive way to see this is that increases in expected variance, $E_0\left[Var_{1,j}[w_{2,j}]\right)$, linearly decrease utility. With expected utility, $-E_0[E_1[exp(-\rho w)]]$, simplifies to $-E_0[exp\left(-\rho E_{1,j}[w_{2,j}] + 0.5\rho\right.$
Because variance is always positive, utility is decreasing faster than linearly in expected variance.

A more nuanced argument requires a discussion of why learning about particular risks is useful. Expected excess portfolio return achieved through learning depends on the covariance between your portfolio q and asset payoffs $f-p$, $cov(q, f-p)$. Specializing in learning about one asset leads to a high covariance between payoffs and holdings of that asset. The actual portfolio you end up holding, however, can deviate substantially from the time 0 expected portfolio. Learning a little about every risk leads to smaller deviations between the realized and time 0 expected portfolio, but also lowers $cov(q, f-p)$.

With expected utility, investors are averse to time 1 portfolio uncertainty (i.e. risk that signals will lead them to take aggressive bets), so do not like portfolios that deviate substantially from $E_0\left[q\right]$ The utility cost of higher uncertainty from specialization offsets the utility benefit of higher portfolio returns, removing the "planning benefit" experienced by the mean-variance specification.

Recursive utility investors are not averse to risks resolved before time 2, so specialization is a low-risk strategy. Lowers time 2 portfolio risk by loading portfolio heavily on an asset whose payoff risk will be reduced by learning.

This also shows why it is desirable to introduce a preference for an early resolution of uncertainty in endogenous learning models. Think about an investor who wants to learn about AAPL. They do this so they can hold a lot of Apple (AAPL) when it does well, and hold little AAPL when it does poorly. An expected utility investor would be hesitant to learn too much about AAPL, because the fact that their portfolio will vary substantially depending on the signal they get seems risky to them.

A.5 Representation

We could re-write the payoffs as:

$$z_i = a_i + \eta_i \tag{30}$$

i.e. with no systematic component, but instead of having the η_i be *i.i.d.*, instead have them correlated in a way that replicates the strcture of the payoffs.

[TBA]

A.6 Risk-Bearing Capacity of the Economy

[TBA]

A.7 Dynamic Model

[TBA]

A.8 Solving a Rotated Version of the Model

1. Guess an initial total attention for informed investors

2. Solve orthogonal model with this total attention constraint

3. Loop over possible attention choices in un-rotated model

4. See if optimal attention from rotated model matches the guess after rotation i.e. $\Sigma_e = GL^*G'$ where $GLG' = \Sigma_e$ is the eigen-decomposition of the signal precision matrix and L^* is the optimal precision matrix in the rotated model

5. Loop over all possible max attention allocations for the orthogonal model until it matches desired total attention in the un-rotated model

Note, if assets are not independent need $\Sigma_e = \Sigma^{1/2} G L^* G \Sigma^{1/2}$, where Σ is the covariance matrix of asset payoffs.

This happens when the solution to the rotated model proposes values for K_i which do not satisfy the total information constraint. For example, suppose we have two assets and three risks. Using the notation in the appendix of Kacperczyk et al. [2016]:

$$\begin{aligned}
\text{Define: } \Sigma^{1/2} &= \text{Square root of } V\text{, the variance-covariance matrix of payoffs} \\
\text{Define: } \Sigma_s &= S\text{, the variance-covariance matrix of signals} \\
\text{Define: } \Sigma_s^1 &= \Sigma^{-1/2} \times \Sigma_s \times \Sigma^{-1/2} \\
\text{We can re-write: } \Sigma_s &= \Sigma^{1/2} \times G \times L \times G \times \Sigma^{1/2} \\
\text{where } G \text{ and } L \text{ come from the eigen-decomposition of } \Sigma_s^1 \\
\text{Define orthogonal signal matrix: } \tilde{\Sigma}_s &= G' \times \Sigma^{-1/2} \times \Sigma_s \times \left(\Sigma^{-1/2}\right)' \times G
\end{aligned} \tag{31}$$

This implies that:

$$\tilde{\Sigma}_s = \begin{bmatrix} 1/(\alpha + \tilde{K}_1) & 0 \\ 0 & 1/(\alpha + \tilde{K}_2) \end{bmatrix} \tag{32}$$

After solving the model, the optimal $\tilde{K}_i$ rotated back to the original economy may require K_i that do not satisfy $\sum_i \tilde{K}_i \leq 1$.

A.9 Increasing α

One of the effects of setting α to larger values is that in three of the four scenarios, we see a kink in the relationship between the cost of becoming informed and the share of agents who decide to learn when the ETF is present. To the right of the kink, the cost of becoming informed is high, so relatively few agents are becoming informed. Given that systematic risk affects all assets, informed agents initially devote all their attention to learning about this risk-factor. To the left of the kink, learning about the systematic risk-factor has become crowded, and informed agents start devoting some attention to the individual-asset risks.

Figure 15 focuses on the case where $\rho = 0.25$ and $\sigma_n^2 = 0.2$. The top panel shows two things: (1) The relationship between the cost of becoming informed, and the share of attention devoted to systematic risk [blue dots]. To the right of the kink, all attention is being devoted to the systematic risk-factor. (2) $U_{1,j}$ i.e. the time one objective function for informed [red squares] and uninformed agents [green triangles]. One of the counter-intuitive features of the kink is that the line is *steeper* once agents are devoting some attention to the idiosyncratic assets. For both informed and uninformed agents, the lines become steeper to the left of the kink.

The second panel shows why the slope changes: To the right of the kink informed and uninformed investors are making roughly the same profits on stocks, but informed investors are making significantly larger profits on the ETF. To the left of the kink, informed investors gain an advantage over uninformed investors on the individual stocks. This increases the relative benefit of becoming informed, which can explain the changes in slopes around the kink.

A.10 Trends Sector ETF

[TBA]

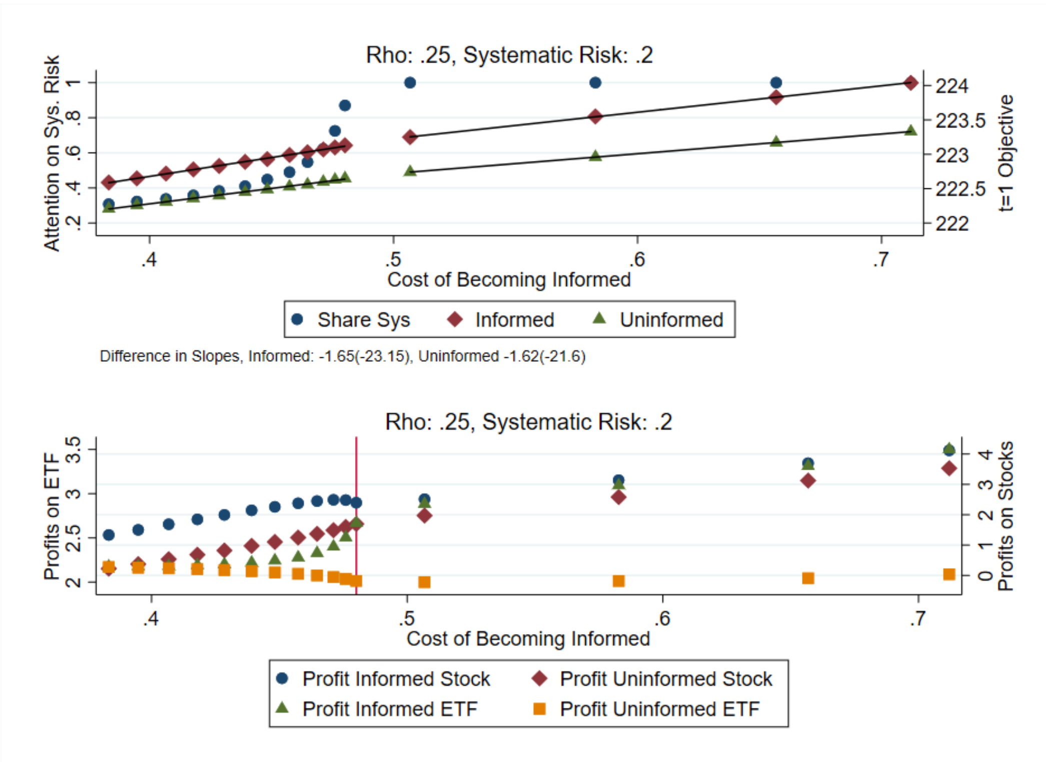

Figure 15: **Trading profits by asset type.**

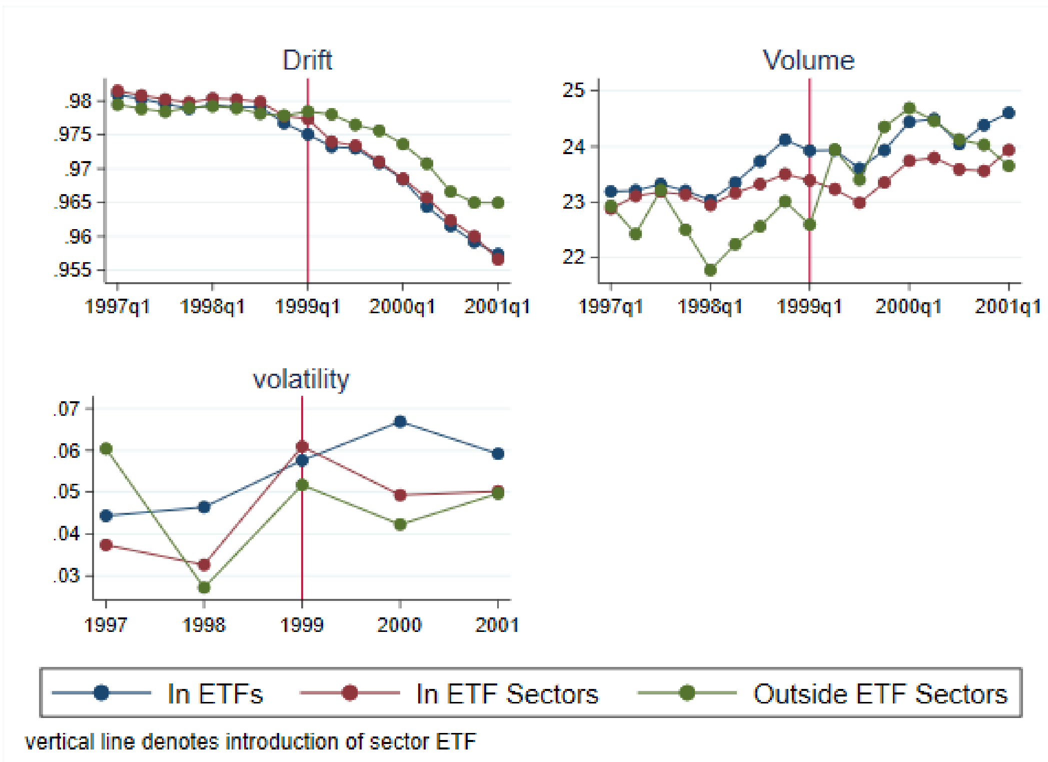

Figure 16: **Sector ETF Trends.**