Static Grammar Inference for Ad Hoc Parsers

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Parsing, the process of structuring a linear representation according to a given grammar, is a fundamental activity in software engineering. While formal language theory has provided theoretical foundations for parsing, the most common kind of parsers used in practice are written *ad hoc*. They use common string operations for parsing, without explicitly defining an input grammar. These ad hoc parsers are often intertwined with application logic and can result in subtle semantic bugs. Grammars, which are complete formal descriptions of input languages, can enhance program comprehension, facilitate testing and debugging, and provide formal guarantees for parsing code. Writing grammars, however, can be tedious and error-prone. Inspired by the success of type inference in programming languages, we propose a general approach for static inference of input string grammars from unannotated ad hoc parser source code. We approach this problem as an intersection of refinement typing and abstract interpretation. We use refinement type inference to synthesize logical and string constraints that represent parsing operations, which we then interpret with an abstract semantics into formal grammars. Our contributions include a core calculus λ_Σ for representing ad hoc parsers, a formulation of grammar inference as refinement inference, an abstract interpretation framework for solving refinement variables, and a set of abstract domains for efficiently representing the kinds of numeric and string values encountered during ad hoc parsing. We implement our approach in the Panini system and evaluate its efficacy on a benchmark of 125 ad hoc parsers.

1 INTRODUCTION

Parsing is one of the fundamental activities in software engineering. It is an activity so common that pretty much every program performs some kind of parsing at one point or another. Yet in every-day software engineering, only a small minority of programs, mainly compilers and some protocol implementations, make use of formal grammars to document their input languages or use formalized parsing techniques such as combinator frameworks [Leijen and Meijer 2001] or parser generators [Johnson and Sethi 1990; Parr and Quong 1995; Warth and Piumarta 2007]. The vast majority of parsing code in software today is *ad hoc*.

Ad hoc parsers are pieces of code that use combinations of common string operations like `slice`, `index`, or `trim` to effectively perform parsing. A programmer manipulating strings in an ad hoc fashion would probably not even think about the fact that they are actually writing a parser. These string-manipulating programs can be found in functions handling command-line arguments, reading configuration files, or as part of any number of minor programming tasks involving strings, often deeply entangled with application logic—a phenomenon known as *shotgun parsing* [Momot et al. 2016]. They have also been shown to produce subtle and difficult to identify semantic bugs [Eghbali and Pradel 2020; Kapugama et al. 2022].

A *grammar* is a complete formal description of all values an input string may assume. It can elucidate the corresponding parsing code, revealing otherwise hidden features and potentially subtle bugs or security issues. By focusing on *data* rather than code, grammars provide a high-level perspective, allowing programmers to grasp an input language directly, without being distracted by the mechanics of the parsing process and the intricacies of imperative string manipulation. Augmenting regular documentation with formal grammars can increase program comprehension by providing alternative representations for a programming task [Fitter and Green 1979; Gilmore and Green 1984]. Because a grammar is also a *generating device*, it is possible to construct any sentence of its language in a finite number of steps—manually or in an automated fashion. Being able to reliably generate concrete examples of possible inputs is invaluable during testing and

2018. 2475-1421/2018/1-ART1 $15.00
https://doi.org/

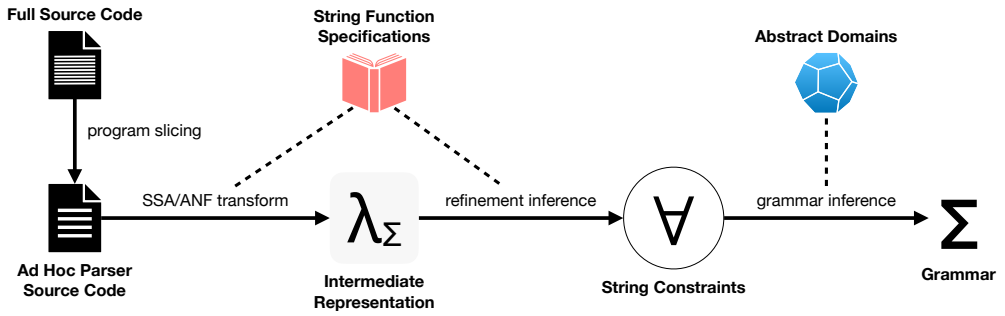

Fig. 1. The complete Panini system.

debugging. But despite providing all these benefits, hardly anyone ever bothers to write down a grammar, even for more complex ad hoc parsers. Grammars share the same fate as most other forms of specification: they are tedious to write, hard to get right, and seem hardly worth the trouble—especially for such small pieces of code like ad hoc parsers.

But there is a form of specification, one wildly more successful than grammars, that we can draw inspiration from: *types*. Formal grammars are similar to types, in that an ad hoc parser without a grammar is very much like a function without a type signature—it might still work, but you will not have any guarantees about it before actually running the program. Types have one significant advantage over grammars, however: most type systems offer a form of *type inference*, allowing programmers to omit type annotations because they can be automatically recovered from the surrounding context [MacQueen et al. 2020, § 4]. If we could infer grammars like we can infer types, we would reap all the rewards of having a complete specification of our program's input language.

Our Contribution. In this paper, we present a general approach for static inference of string grammars from unannotated ad hoc parser source code. We pose the problem of grammar inference as a sub-goal of refinement type inference. During type inference, we synthesize logical constraints that declaratively represent the parsing operations performed on the input string. We then interpret this complex first-order formula using an abstract semantics, in order to find a minimal sub-constraint to use as an input string refinement, i.e., a grammar.

In brief, the contributions of our work are:

- A core calculus λ_Σ for representing ad hoc parsers.
- A formulation of grammar inference as refinement inference.
- An abstract interpretation framework for solving string refinement variables.
- A set of abstract domains related to ad hoc parsing.
- An implementation of our approach in the Panini system.

2 OVERVIEW

Figure 1 presents a schematic overview of Panini,[1] our end-to-end grammar inference system. At the center of our approach is λ_Σ, a language-agnostic intermediate representation for ad hoc parsers. It is powerful enough to represent all relevant parsing operations and simple enough to enable straight-forward refinement type inference. The refinement type system of λ_Σ allows us to synthesize constraints over a parser's input string—i.e., it allows us to infer a parser's grammar.

The Panini system can be separated into a front- and a back-end. In the front-end, ad hoc parsers written in a general-purpose programming language, e.g., Python, are translated into λ_Σ programs.

[1]Named in honor of the Sanskrit grammarian Pāṇini [Bhate and Kak 1991], as well as the delicious Italian sandwiches.

Table 1. Python operations as axiomatic λ_Σ specifications.

Python	λ_Σ specification	
`assert b`	assert : $\{b : \mathbb{B} \mid b\} \to \mathbb{1}$	assertion
`a == b`	eq : $(a : \mathbb{Z}) \to (b : \mathbb{Z}) \to \{c : \mathbb{B} \mid c \Leftrightarrow a = b\}$	integer equality
`len(s)`	length : $(s : \mathbb{S}) \to \{n : \mathbb{N} \mid n = \lvert s\rvert\}$	string length
`s[i]`	charAt : $(s : \mathbb{S}) \to \{i : \mathbb{N} \mid i < \lvert s\rvert\} \to \{c : \mathbb{Ch} \mid c = s[i]\}$	character at index
`s == t`	eqChar : $(s : \mathbb{Ch}) \to (t : \mathbb{Ch}) \to \{b : \mathbb{B} \mid b \Leftrightarrow s = t\}$	character equality

In the back-end, those λ_Σ programs are statically analyzed and their input string constraints extracted. *This paper is about the back-end.* In short order, we will describe the λ_Σ calculus, its refinement type system, our grammar inference algorithm, and the underlying abstract domains. To situate this work, we first want to briefly sketch the front-end process.

2.1 The Front End: From Source to λ_Σ

After locating an ad hoc parser slice, it is first translated into static single assignment (SSA) form [Braun et al. 2013] and then, via an SSA-to-ANF transformation [Chakravarty et al. 2004], into a Panini program. This requires a library of string function specifications that map the source language's string operations to equivalent λ_Σ functions. Note that it is not necessary to have actual λ_Σ *implementations* of these operations. We only need axiomatic specifications—in the form of type signatures—to capture those properties of the original functions that are necessary to synthesize string grammars. Table 1 shows some examples of such axioms, based on the semantics of certain Python functions. We will use these axioms in the examples throughout this paper.

The front-end transformations (parser slicing, source-to-λ_Σ translation) and accompanying axiomatic string function specifications need to be defined and implemented (and proven correct) only once per source programming language. For the remainder of this paper, we will assume a Python-to-λ_Σ transformation and a library of Python string function specifications.

2.2 The Back End: From λ_Σ to Grammar

λ_Σ is a simple λ-calculus with a refinement type system in the style of *Liquid Types* [Rondon et al. 2008; Vazou et al. 2014]. Refinement types allow us to extend base types with logical constraints. This is useful to precisely describe subsets of values, as well as track complex relationships between values, all on the type level. For example, the type of natural numbers can be defined as a subset of the integers, $\mathbb{N} = \{\nu : \mathbb{Z} \mid \nu \geq 0\}$, and we can give a precise definition of the length function on strings using a dependent function type and the string length operator $\lvert\square\rvert$ of the refinement logic,

$$\text{length} \;:\; (s : \mathbb{S}) \to \{n : \mathbb{N} \mid n = \lvert s\rvert\}.$$

Checking a refinement type reduces to proving a so-called *verification condition* (VC), a first-order constraint in the refinement logic generated by the type system. The validity of the VC entails the correctness of the program's given and inferred types [Nelson 1980]. For example, in order to check whether $\{x : \mathbb{Z} \mid x > 42\}$ (the type of all numbers greater than forty-two) is a subtype of $\mathbb{N}$, the VC constraint $\forall x.\ x > 42 \Rightarrow x \geq 0$ has to be verified. For verification to remain practical, the refinement logic is typically chosen to allow *satisfiability modulo theories* (SMT) [Barrett et al. 2021], which means VCs can be discharged using an off-the-shelf constraint solver such as Z3 [De Moura and Bjørner 2008]. Our system uses quantifier-free linear arithmetic with uninterpreted functions (QF_UFLIA) [Barrett et al. 2016] for its refinement predicates, extended with a theory of operations over strings [Berzish et al. 2017].

```
assert s[0] == "a"
```

$$\lambda(s : \mathbb{S}).\ \textbf{let}\ x = \text{charAt}\ s\ 0\ \textbf{in}\ \textbf{let}\ p = \text{eqChar}\ x\ \text{`a'}\ \textbf{in}\ \text{assert}\ p$$

$$\forall s.\ \kappa(s) \Rightarrow 0 < |s| \wedge \forall x.\ x = s[0] \Rightarrow \forall p.\ (p \Leftrightarrow x = \text{`a'}) \Rightarrow p$$

Fig. 2. A simple Python expression (left) and the equivalent λ_Σ program (middle) with an incomplete verification condition (right) for its inferred type $\{s : \mathbb{S} \mid \kappa(s)\} \to \mathbb{1}$.

Refinement Inference. To infer a refinement type for any given term, one must find a predicate that describes (at most) all possible values the term could have during any (successful) run of the program. To facilitate this, refinement type systems typically first infer the basic shapes of all types in the program, using standard type inference à la Hindley-Damas-Milner [Damas and Milner 1982; Hindley 1969], with placeholder variables standing in for as-yet-unknown refinement predicates. These placeholder variables—variously called "κ variables" [Cosman and Jhala 2017], "Horn variables" [Jhala and Vazou 2020], or "liquid type variables" [Rondon et al. 2008]—are also present in the VC at this point and prevent it from being discharged (since they are unknown). The type system then tries to find the strongest satisfying assignments for all refinement variables in the VC constraints, in order to both validate the VC and complete the inferred type.

Example 2.1. The simple λ_Σ program **if** true **then** 1 **else** 2 can be inferred to have an incomplete type of shape $\{\nu : \mathbb{Z} \mid \kappa(\nu)\}$, where κ is an unknown refinement variable, together with the VC

$$(\text{true} \Rightarrow \forall \nu.\ \nu = 1 \Rightarrow \kappa(\nu)) \wedge (\text{false} \Rightarrow \forall \nu.\ \nu = 2 \Rightarrow \kappa(\nu)).$$

In order to complete the type, we now have to find an assignment for κ. With such a simple constraint, the refinement solver can easily infer the correct assignment $\kappa(\nu) \mapsto \nu = 1$, which validates the VC and produces the final type $\{\nu : \mathbb{Z} \mid \nu = 1\}$.

Grammar Inference. If a κ variable stands for an input string refinement, we call this a *grammar variable*, because its solution must be some finite description of all strings that are accepted by the program, i.e., a grammar. To be practical, we would like this grammar to be as complete as possible.

Example 2.2. To illustrate, consider the Python expression in Figure 2 (on the left). Assuming the function specifications from Table 1, we can transform this expression to an equivalent λ_Σ program (in the middle) with an inferred top-level refinement type of $\{s : \mathbb{S} \mid \kappa(s)\} \to \mathbb{1}$ and a VC (on the right) that closely matches the program. In order to complete both the VC and the top-level type, we have to find an appropriate assignment for the grammar variable κ. The assignment must take the form of a single-argument function constraining the string s. It is clear that choosing $\kappa(s) \mapsto \text{true}$, i.e., allowing *any* string for s, is not a valid solution because it does not satisfy the VC. On the other hand, choosing $\kappa(s) \mapsto \text{false}$ trivially validates the VC, but it implies that the function could never actually be called, as no string satisfies the predicate false. One possible assignment could be $\kappa(s) \mapsto s =$ "a", which only allows exactly the string "a" as a value for s. While this validates the VC and produces a correct type in the sense that it ensures the program will never go wrong, it is much too strict: we are disallowing an infinite number of other strings that would just as well fulfill these criteria (e.g., "aa", "ab", and so on). The correct assignment is $\kappa(s) \mapsto s[0] = \text{`a'}$, which ensures that the first character of the string is "a" but leaves the rest of the string unconstrained. This is equivalent to the regular language $\mathsf{a}\Sigma^*$, where Σ is any letter from the input alphabet. Note that the solution is a minimized version of the top-level consequent in the VC.

The key insight that allows us to find suitable assignments for grammar variables in a general manner is that the top-level VC for a parser will always be of the form $\forall s.\ \kappa(s) \Rightarrow \varphi$, where s is the input string and φ is a constraint that precisely captures all parsing operations the program performs on s. By simply taking $\kappa(s) \mapsto \varphi$, the VC becomes a trivially valid tautology and the string refinement captures exactly those inputs that the parser accepts. But clearly this solution is practically useless: we want a succinct predicate in a grammar-like form, but the top-level VC consequent φ is a complex term in first-order logic that is basically identical to the program code itself; it does not lead to any further insight about the parser's actual input language.

However, we can use φ as a starting point and reduce it into a simpler predicate by performing an *abstract interpretation* of φ. Our approach utilizes an abstract semantics of first-order formulas over string constraints in order to soundly eliminate quantifiers and approximate the parser's input language. The precision of this approximation depends on the language complexity of the parser, the ability of the refinement inference system to synthesize program invariants, and the expressiveness of the underlying abstract value domains.

Example 2.3. To give an idea of how grammar inference works, we present a brief example.

The program shown in Figure 3a is a simple parser written in Python that checks if the first character of the input string is an 'a' and, if so, asserts that the length of the string must be one and thus the string must be exactly "a"; otherwise, if the first character is not an 'a', then the program asserts that the second character must be a 'b', with no further restrictions on the input string. Note that the Python string indexing operations `s[0]` and `s[1]` are themselves types of assertions, since they will cause the program to crash if the index is out of bounds. This behavior is captured in the λ_Σ equivalent of the source program via its function axioms (Table 1).

Figure 3b shows the parser's top-level typing derivation (§ 3). The refinement inference judgement, applied to the parser function and its axioms, results in an incomplete refinement type containing an unsolved κ variable, and a verification condition of the form $\forall s.\ \kappa(s) \Rightarrow \varphi$, which tells us that this κ variable is indeed a grammar variable. We can see that the VC's consequent φ captures all of the parsers explicit and implicit constraints over its program variables.

Finally, Figure 3c shows a (possible) step-by-step reduction of φ into a simple quantifier-free predicate using abstract interpretation (§ 4). First, the innermost quantifiers $\forall v_1$, $\forall n$, $\forall v_2$, and $\forall y$ are eliminated and their quantified variables replaced by semantically equivalent expressions. Then we eliminate $\forall p_1$, followed by $\forall x$. At this point, there are no more quantified variables and we can fully abstract each occurrence of the only free variable s. Finally, we normalize the quantifier-free predicate into a single abstract string relation, equivalent to a regular expression.

3 REFINEMENT INFERENCE

We now give a formalization of λ_Σ, our core abstraction for representing ad hoc parsers. The language and its type system were heavily inspired by the Sprite tutorial language by Jhala and Vazou [2020], and incorporate ideas from various other refinement type systems [Cosman and Jhala 2017; Dunfield and Krishnaswami 2021; Montenegro et al. 2020]. Our main contribution in this area is an extended refinement variable solving procedure that synthesizes precise grammars for input string constraints (§§ 3.3 and 4).

3.1 Syntax

λ_Σ is a small λ-calculus in A-normal form (ANF) [Bowman 2022; Flanagan et al. 1993]. It exists solely for type synthesis and its programs are neither meant to be executed nor written by hand. Its syntax, collected in Figure 4, is thus minimal and has few affordances.

```
def parser(s):
  if s[0] == "a":
    assert len(s) == 1
  else:
    assert s[1] == "b"
```

```
>>> parser("")      IndexError
>>> parser("a")     ✓
>>> parser("b")     IndexError
>>> parser("ab")    AssertionError
>>> parser("bb")    ✓
```

$$\begin{array}{l}\textbf{parser} = \boldsymbol{\lambda}(s : \mathbb{S}).\\ \quad \textbf{let } x = \text{charAt } s\ 0 \textbf{ in}\\ \quad \textbf{let } p_1 = \text{eqChar } x\ \text{`a'} \textbf{ in}\\ \quad \textbf{if } p_1 \textbf{ then}\\ \quad\quad \textbf{let } n = \text{length } s \textbf{ in}\\ \quad\quad \textbf{let } p_2 = \text{eq } n\ 1 \textbf{ in}\\ \quad\quad \text{assert } p_2\\ \quad \textbf{else}\\ \quad\quad \textbf{let } y = \text{charAt } s\ 1 \textbf{ in}\\ \quad\quad \textbf{let } p_3 = \text{eqChar } y\ \text{`b'} \textbf{ in}\\ \quad\quad \text{assert } p_3\end{array}$$

(a) A Python program (left) and its λ_Σ equivalent (right).

$$\overbrace{\Gamma}^{\text{axioms}} \vdash \text{parser} \nearrow \overbrace{\{s : \mathbb{S} \mid \kappa(s)\} \to \mathbb{1}}^{\text{incomplete refinement}} \dashv \overbrace{\forall s.\ \kappa(s) \Rightarrow \varphi}^{\text{verification condition}}$$

$$\begin{array}{l}\varphi \doteq (\forall v_1.\ v_1 = 0 \Rightarrow v_1 \geq 0 \wedge v_1 < |s|) \wedge\\ \quad (\forall x.\ x = s[0] \Rightarrow (\forall p_1.\ p_1 = \text{true} \Leftrightarrow x = \text{`a'} \Rightarrow\\ \quad\quad (p_1 = \text{true} \Rightarrow (\forall n.\ n \geq 0 \wedge n = |s| \Rightarrow n = 1)) \wedge\\ \quad\quad (p_1 = \text{false} \Rightarrow (\forall v_2.\ v_2 = 1 \Rightarrow v_2 \geq 0 \wedge v_2 < |s|) \wedge\\ \quad\quad\quad (\forall y.\ y = s[1] \Rightarrow y = \text{`b'}))))\end{array}$$

(b) An incomplete refinement typing of the λ_Σ program above.

$$\begin{array}{lll}\varphi & \overset{1}{\rightsquigarrow} & |s| > 0 \wedge (\forall x.\ x = s[0] \Rightarrow (\forall p_1.\ p_1 = \text{true} \Leftrightarrow x = \text{`a'} \Rightarrow\\ & & \quad (p_1 = \text{true} \Rightarrow |s| = 1) \wedge (p_1 = \text{false} \Rightarrow |s| > 1 \wedge s[1] = \text{`b'})))\\ & \overset{2}{\rightsquigarrow} & |s| > 0 \wedge (\forall x.\ x = s[0] \Rightarrow (x = \text{`a'} \wedge |s| = 1) \vee (x \neq \text{`a'} \wedge |s| > 1 \wedge s[1] = \text{`b'}))\\ & \overset{3}{\rightsquigarrow} & |s| > 0 \wedge ((s[0] = \text{`a'} \wedge |s| = 1) \vee (s[0] \neq \text{`a'} \wedge |s| > 1 \wedge s[1] = \text{`b'}))\\ & \overset{4}{\rightsquigarrow} & s \in \Sigma^+ \wedge ((s \in \text{a}\Sigma^* \wedge s \in \Sigma) \vee (s \in (\Sigma \backslash \text{a})\Sigma^* \wedge s \in \Sigma^2\Sigma^* \wedge s \in \Sigma\text{b}\Sigma^*))\\ & \overset{5}{\rightsquigarrow} & s \in (\text{a} + (\Sigma \backslash \text{a})\text{b}\Sigma^*)\end{array}$$

(c) Possible steps in the abstract interpretation of φ: (1) eliminate $\forall v_1, \forall n, \forall v_2, \forall y$; (2) eliminate $\forall p_1$; (3) eliminate $\forall x$; (4) abstract s; (5) normalize. Highlights indicate changes relative to previous step.

Fig. 3. An example of grammar inference.

Values are either primitive constants or variables. **Terms** are comprised of values and the usual constructs: function applications, function abstractions, bindings, recursive bindings, and branches. Applications are in ANF as a natural result of the SSA translation performed on the original source code (see § 2.1), but we also generally enforce this in the syntax to simplify the typing rules (§ 3.2). λ_Σ terms have no type annotations, except on $\boldsymbol{\lambda}$-binders and recursive **rec**-binders, whose (base) types we assume are inferred in the pre-processing phase or provided by the programmer.

The primitive **Base Types** are $\mathbb{1}$ (unit), $\mathbb{B}$ (Boolean), $\mathbb{Z}$ (integer), $\mathbb{Ch}$ (character), and $\mathbb{S}$ (string). **Types** are formed by decorating base types with refinement predicates, or by constructing (dependent) function types, whose output types can refer to input types.

Predicates are terms in a Boolean logic, with the usual Boolean connectives, plus (in)equality relations and arithmetic comparisons between predicate expressions, membership queries for regular expression matching, applications of unknown κ variables, and existential quantifiers, which arise during the Fusion phase of κ solving (§ 3.3). Both κ applications and existentials are not part of the user-visible surface syntax. **Expressions** within predicates are built from lifted values and functions, in particular linear integer arithmetic and operations over strings, e.g., length $|\square|$, character-at-index $\square[\square]$, substring $\square[\square..\square]$, etc. To simplify the presentation, predicate expressions are not further syntactically stratified, but are assumed to always occur well-typed (which is assured by the implementation).

VC generation (§ 3.2) results in **Constraints** that are Horn clauses [Bjørner et al. 2015] in Negation Normal Form (NNF), basically tree-like conjunctions of refinement predicates, where each root-to-leaf path is a Constrained Horn Clause (CHC). This representation of VCs is due to Cosman and Jhala [2017], who cleverly employ the constraints' nested scoping structure to make κ solving tractable.

3.2 Type System

The main purpose of the type system of λ_Σ is to generate constraints, in particular input string constraints for parser functions. Thus, the type system is focused on inference/synthesis, rather than type checking. Terms need only be minimally annotated, at λ-abstractions and recursive bindings. The types of applied functions need to be known, however, and available in the typing context (see the discussion of axioms, § 2.1 and Table 1). Our system borrows heavily from Liquid Haskell [Vazou et al. 2014] and the expositions given by Cosman and Jhala [2017] and Jhala and Vazou [2020]. We combine typing rules and VC generation into one syntax-driven declarative system of inference rules, given in Figure 5 and discussed below.

$\boxed{t_1 \leqslant t_2 \dashv c}$ ***Subtyping.*** A type t_1 is a subtype of t_2 (meaning, the values denoted by t_1 are subsumed by t_2), if the entailment constraint c is satisfied. In the Sub/Base case, this means the refinement predicate of t_1 must imply the predicate of t_2, for all possible values the types can have. In the Sub/Fun case, where the contra-variant input constraint is joined with the co-variant output constraint, we add an additional implication to the output constraint, strengthening it with the supertype's input predicate. This is done using a *generalized implication* operation $(x :: t) \Rightarrow c$, which simply ensures that only base types are bound to quantifiers in the refinement logic.

$\boxed{\Gamma \vdash t \rhd \hat{t}}$ ***Template Generation.*** To enable complete type synthesis for all intermediate terms, it is sometimes necessary to turn a type t into a template $\hat{t}$, where the refinement predicate is denoted by a placeholder variable whose resolution is deferred (see §§ 2.2, 3.3, and 4). The rule Kap/Base introduces a fresh κ variable, representing an n-ary relation between the type itself and all variables in the current environment Γ. Usually this environment is empty, but if t is a function, Kap/Fun recursively generates input and output templates, extending the environment along the way.

Values	v	::=	$x, y, z, \ldots$	variables
		|	$\ldots$ varies $\ldots$	constants
Terms	e	::=	v	value
		|	$e\ v$	application
		|	$\boldsymbol{\lambda}(x : b).\ e$	abstraction
		|	**let** $x = e_1$ **in** e_2	binding
		|	**rec** $x : t = e_1\ \ e_2$	recursion
		|	**if** v **then** e_1 **else** e_2	branch
Base Types	b	::=	$\mathbb{1} \mid \mathbb{B} \mid \mathbb{Z} \mid \mathbb{Ch} \mid \mathbb{S}$	
Types	t	::=	$\{x : b \mid p\}$	refined base
		|	$(x : t_1) \rightarrow t_2$	dependent function
Predicates	p	::=	true | false	Boolean constants
		|	$p_1 \wedge p_2 \mid p_1 \vee p_2$	connectives
		|	$p_1 \Rightarrow p_2 \mid p_1 \Leftrightarrow p_2$	implications
		|	$\neg p$	negation
		|	$w_1 = w_2 \mid w_1 \neq w_2$	(in)equality
		|	$w_1 < w_2 \mid w_1 \leq w_2$	arithmetic comparison
		|	$w \in \mathrm{RE}$	regular language membership
		|	$\kappa(\overline{v})$	κ application
		|	$\exists(x : b).\ p$	existential quantification
Expressions	w	::=	v	value
		|	$f(\overline{w})$	function
Constraints	c	::=	p	predicate
		|	$c_1 \wedge c_2$	conjunction
		|	$\forall(x : b).\ p \Rightarrow c$	universal implication

Fig. 4. Syntax of λ_Σ terms, types, and refinements.

$\boxed{\Gamma \vdash e \nearrow t \dashv c}$ ***Type/Constraint Synthesis.*** Given a typing context Γ mapping values to types, and a term e, we can synthesize a type t whose correctness is implied by the constraint c. The rule Syn/Con synthesizes built-in primitive types, denoted by $\mathsf{prim}(c)$ in the obvious way, e.g., $\mathsf{prim}(0) \stackrel{\text{def}}{=} \{\nu : \mathbb{Z} \mid \nu = 0\}$ and so on. Syn/Var retrieves the type of a variable from the current context, using *selfification* to produce the most precise possible type [Ou et al. 2004]. The self function lifts the variable into the refinement, allowing each occurrence of the variable in different branches of the program to be precisely typed.

$$\mathsf{self}(x, t) \stackrel{\text{def}}{=} \begin{cases} \{\nu : b \mid p \wedge \nu = x\} & \text{if } t \equiv \{\nu : b \mid p\}, \\ t & \text{otherwise.} \end{cases}$$

$\boxed{t_1 \leqslant t_2 \dashv c}$ **Subtyping**

$$\frac{}{\{v_1 : b \mid p_1\} \leqslant \{v_2 : b \mid p_2\} \dashv \forall (v_1 : b).\ p_1 \Rightarrow p_2[v_2 := v_1]}\ \textsc{Sub/Base}$$

$$\frac{s_2 \leqslant s_1 \dashv c_i \qquad t_1[x_1 := x_2] \leqslant t_2 \dashv c_o}{(x_1 : s_1) \to t_1 \leqslant (x_2 : s_2) \to t_2 \dashv c_i \wedge ((x_2 :: s_2) \Rightarrow c_o)}\ \textsc{Sub/Fun}$$

$$(x :: t) \Rightarrow c \stackrel{\text{def}}{=} \begin{cases} \forall (x : b).\ p[v := x] \Rightarrow c & \text{if } t \equiv \{v : b \mid p\}, \\ c & \text{otherwise.} \end{cases}$$

$\boxed{\Gamma \vdash t \rhd \hat{t}}$ **Template Generation**

$$t \rhd \hat{t} \stackrel{\text{def}}{=} \varnothing \vdash t \rhd \hat{t}$$

$$\frac{\kappa \text{ is a fresh variable of sort } b \times \overline{t}}{\overline{x : t} \vdash \{v : b \mid p\} \rhd \{v : b \mid \kappa(v, \overline{x})\}}\ \textsc{Kap/Base} \qquad \frac{\Gamma \vdash t_1 \rhd \hat{t}_1 \qquad \Gamma[x \mapsto t_1] \vdash t_2 \rhd \hat{t}_2}{\Gamma \vdash (x : t_1) \to t_2 \rhd (x : \hat{t}_1) \to \hat{t}_2}\ \textsc{Kap/Fun}$$

$\boxed{\Gamma \vdash e \nearrow t \dashv c}$ **Type/Constraint Synthesis**

$$\frac{\Gamma(x) = t}{\Gamma \vdash x \nearrow \mathsf{self}(x, t) \dashv \mathsf{true}}\ \textsc{Syn/Var} \qquad \frac{\mathsf{prim}(c) = t}{\Gamma \vdash c \nearrow t \dashv \mathsf{true}}\ \textsc{Syn/Con}$$

$$\frac{\Gamma \vdash e \nearrow (y : t_1) \to t_2 \dashv c_e \qquad \Gamma \vdash x \nearrow t_x \qquad t_x \leqslant t_1 \dashv c_x}{\Gamma \vdash e\ x \nearrow t_2[y := x] \dashv c_e \wedge c_x}\ \textsc{Syn/App}$$

$$\frac{\tilde{t}_1 \rhd \hat{t}_1 \qquad \Gamma[x \mapsto \hat{t}_1] \vdash e \nearrow t_2 \dashv c_2}{\Gamma \vdash \boldsymbol{\lambda}(x : \tilde{t}_1).\ e \nearrow (x : \hat{t}_1) \to t_2 \dashv (x :: \hat{t}_1) \Rightarrow c_2}\ \textsc{Syn/Lam}$$

$$\frac{\begin{array}{c}\Gamma \vdash e_1 \nearrow t_1 \dashv c_1 \\ \Gamma[x \mapsto t_1] \vdash e_2 \nearrow t_2 \dashv c_2 \qquad t_2 \rhd \hat{t}_2 \qquad t_2 \leqslant \hat{t}_2 \dashv \hat{c}_2\end{array}}{\Gamma \vdash \textbf{let}\ x = e_1\ \textbf{in}\ e_2 \nearrow \hat{t}_2 \dashv c_1 \wedge ((x :: t_1) \Rightarrow c_2 \wedge \hat{c}_2)}\ \textsc{Syn/Let}$$

$$\frac{\tilde{t}_1 \rhd \hat{t}_1 \qquad \begin{array}{c}\Gamma[x \mapsto \hat{t}_1] \vdash e_1 \nearrow t_1 \dashv c_1 \\ \Gamma[x \mapsto t_1] \vdash e_2 \nearrow t_2 \dashv c_2 \qquad t_2 \rhd \hat{t}_2\end{array} \qquad \begin{array}{c} t_1 \leqslant \hat{t}_1 \dashv \hat{c}_1 \\ t_2 \leqslant \hat{t}_2 \dashv \hat{c}_2 \end{array}}{\Gamma \vdash \textbf{rec}\ x : \tilde{t}_1 = e_1\ \ e_2 \nearrow \hat{t}_2 \dashv ((x :: \hat{t}_1) \Rightarrow c_1 \wedge \hat{c}_1) \wedge ((x :: t_1) \Rightarrow c_2 \wedge \hat{c}_2)}\ \textsc{Syn/Rec}$$

$$\frac{\Gamma \vdash x \nearrow \mathbb{B} \qquad \begin{array}{c}\Gamma \vdash e_1 \nearrow t_1 \dashv c_1 \\ \Gamma \vdash e_2 \nearrow t_2 \dashv c_2\end{array} \qquad t_1 \rhd \hat{t} \qquad \begin{array}{c} t_1 \leqslant \hat{t} \dashv \hat{c}_1 \\ t_2 \leqslant \hat{t} \dashv \hat{c}_2 \end{array}}{\Gamma \vdash \textbf{if}\ x\ \textbf{then}\ e_1\ \textbf{else}\ e_2 \nearrow \hat{t} \dashv (x = \mathsf{true} \Rightarrow c_1 \wedge \hat{c}_1) \wedge (x = \mathsf{false} \Rightarrow c_2 \wedge \hat{c}_2)}\ \textsc{Syn/If}$$

Fig. 5. Typing rules for λ_Σ.

Syn/App synthesizes the result type of a function application, where the given input can be a subtype of the function's declared input type. In the result type, the declared input variable is replaced by the given input, using standard capture-avoiding substitution. Note that because our terms are in ANF, no arbitrary expressions are introduced into the type during this substitution. The synthesized VC is simply a conjunction of the function's VC and the constraint created by the subtyping judgement. Syn/Lam produces a function type whose input refinement is a fresh κ variable, based on the annotation on the $\boldsymbol{\lambda}$-binder. Syn/Let first synthesizes the type t_1 for the bound term, then the type t_2 for the expression's body under an environment where x is bound to t_1. To ensure that x does not escape its scope, the whole **let**-expression is given the templated type $\hat{t}_2$, a supertype of t_2 with a κ variable in place of its refinement. Syn/Rec is similar to Syn/Let, with the addition that we first assume the bound term's type as a placeholder $\hat{t}_1$ based on the annotation $\tilde{t}_1$ on the binder, before synthesizing it as t_1, allowing for recursion in the bound term. Syn/If synthesizes the type of the whole conditional as a supertype of both branches. In the VC, we imply the **then**-branch's constraints if the condition is true, and the **else**-branch's constraints if the condition is false. The κ variable in the templated supertype $\hat{t}$ allows this path-sensitive information to travel back upwards.

3.3 Variable Solving

Before the synthesized VCs can be sent off to an SMT solver to be proven valid, we have to replace all κ variables with concrete refinement predicates. Some of these variables represent input string refinements and this is the point where we need to find the grammars describing those strings.

Algorithm 1 gives a high-level overview of the VC solving procedure. We start with an incomplete VC constraint c (i.e., a constraint containing κ variables) and an initial κ assignment σ (usually empty). We use Fusion [Cosman and Jhala 2017] and predicate abstraction [Rondon et al. 2008] to deal with non-grammar κ variables. For grammar variables, we use our abstract interpretation approach detailed in § 4 to find an abstract solution for κ, i.e., a grammar. If the complete VC, with all κ variables replaced by their solutions, is satisfiable (modulo theories) then the inferred type is valid and σ contains concrete assignments for all of the type's refinement variables. Otherwise, no valid solution could be found, either because there does not exist one (e.g., the program has a type error) or due to insufficient invariants or limits of the abstract domain.

Algorithm 1 Solve an incomplete VC and find all κ assignments

```
1:  procedure Solve(c, σ)
2:      c ← σ(c)
3:      c ← Fusion(c)              ▷ eliminate local acyclic κ variables [Cosman and Jhala 2017]
4:      σ ← σ ∪ Liquid(c)          ▷ solve residual non-grammar κ variables [Rondon et al. 2008]
5:      c ← σ(c)
6:      for all constraints in c of the form ∀(s : 𝕊). κ(s) ⇒ φ do
7:          σ̂ ← [[φ]]♯             ▷ infer grammar using abstract interpretation [§ 4]
8:          σ ← σ [κ(s) ↦ s ∈ σ̂(s)]
9:          c ← σ(c)
10:     end for
11:     if Satisfiable(c) then return Valid else return Invalid
12: end procedure
```

4 GRAMMAR INFERENCE

Given a program P with a partially inferred refinement type $\{s : \mathbb{S} \mid \kappa(s)\} \to \mathbb{1}$ and an incomplete verification condition of the form $\forall(s : \mathbb{S}).\ \kappa(s) \Rightarrow \varphi$, our goal is to find an assignment for κ that both validates the VC and meaningfully refines the type of s. Trivially, the assignment $\kappa(s) \mapsto \text{false}$ always validates the VC, but is hardly a meaningful refinement. Assuming that P is a parser, we ideally want an assignment for κ that constrains s in a way that

(1) disallows any string rejected by P (*soundness*) and
(2) allows all strings accepted by P (*completeness*).

In other words, we want a refinement for s that is a *grammar* for P, i.e., a finite description of the (possibly infinite) set of strings contained in the language $L(P)$.

Luckily, the VC's consequent φ already appears to be what we are looking for: it is a first-order formula over a free string variable s that exactly captures the semantics of the parsing operations performed by P on s. Assuming that φ itself does not contain any other unsolved κ variables (these having been eliminated by prior inference and solving steps, e.g., Fusion and predicate abstraction, see § 3.3), as well as the correctness of all involved axiomatic string function specifications (Table 1), then, under some model $\mathfrak{M}$ of the refinement logic (e.g., linear integer arithmetic and basic string operations), by construction,

$$\mathfrak{M}, [s \mapsto t] \models \varphi \quad \iff \quad P \text{ successfully parses } t \quad \iff \quad t \in L(P).$$

The parser P represented by φ accepts some particular string t if and only if φ is true under an assignment of s to t. The set of all strings that satisfy φ is exactly the language $L(P)$ accepted by P. Thus, φ is technically a complete grammar for P.

Practically, however, φ makes for a rather poor grammar. It is a first-order formula close in size and structure to the parsing program P itself. While $\kappa(s) \mapsto \varphi$ is an ideal assignment in the sense that it is both sound and complete, it results in a rather impractical refinement that would puzzle a human programmer. The presence of quantifiers within φ complicates any further analysis.

In order to obtain a better grammar for $L(P)$, we will use φ as a starting point to derive a quantifier-free predicate that is much simpler than φ but semantically equivalent. They key components of our approach are:

(1) an abstract interpretation of certain first-order string formulas as grammars (§ 4.1),
(2) an extended constraint syntax that mixes symbolic and abstract representations (§ 4.2),
(3) an abstract semantics of relations between predicate expressions (§ 4.3),
(4) a procedure for eliminating quantified variables by abstract substitution (§ 4.4),
(5) abstract value representations of all base types (§ 5).

4.1 Abstract Interpretation

Background. Abstract interpretation [Cousot and Cousot 1977, 1979] is a well-established framework for formalizing static program analyses. It involves the sound approximation of all possible states of a program, usually trading precision for efficiency. The concrete semantics of a program $\mathcal{P}$ are defined by a semantic function $[\![\square]\!] : \mathcal{P} \to \wp(C)$, which produces a power set of concrete values C. The concrete domain $\wp(C)$ can be approximated by an abstract domain $\mathcal{A}$, with an abstraction function $\alpha : \wp(C) \to \mathcal{A}$ and a concretization function $\gamma : \mathcal{A} \to \wp(C)$ mapping elements between the domains. Usually, $\mathcal{A}$ is a complete lattice $\langle \mathcal{A}, \sqsubseteq, \sqcap, \sqcup, \bot, \top \rangle$ and the two domains form a Galois connection $\langle \wp(C), \subseteq \rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle \mathcal{A}, \sqsubseteq \rangle$, which intuitively means that relationships between elements of $\wp(C)$ also hold between the corresponding abstracted elements of $\mathcal{A}$. We can then define an abstract semantics $[\![\square]\!]^\sharp : \mathcal{P} \to \mathcal{A}$ to abstractly interpret programs and directly produce abstract

Table 2. Summary of abstract domains in Panini

base type				abstract domain	
unit	$\wp(\mathbb{1})$	$=$	$\hat{\mathbf{1}}$	the two-element lattice	§ 5.1
Booleans	$\wp(\mathbb{B})$	$=$	$\hat{\mathbf{B}}$	Boolean subset lattice	§ 5.2
integers	$\wp(\mathbb{Z})$	$\supseteq$	$\hat{\mathbf{Z}}$	open-ended interval lists	§ 5.3
characters	$\wp(\mathbb{Ch})$	$=$	$\hat{\mathbf{C}}$	Unicode character sets	§ 5.4
strings	$\wp(\mathbb{S})$	$\supseteq$	$\hat{\mathbf{S}}$	regular expressions over $\hat{\mathbf{C}}$	§ 5.5

values. The abstract interpretation is *sound* iff, for all programs $\mathcal{P}$, $\alpha([\![\mathcal{P}]\!]) \sqsubseteq [\![\mathcal{P}]\!]^\sharp$, and it is also *complete* iff $\alpha([\![\mathcal{P}]\!]) = [\![\mathcal{P}]\!]^\sharp$. Completeness in this context means that the abstract semantics incurs no loss of precision relative to the underlying abstract domain, i.e., the abstract semantics can take full advantage of the whole domain. Finally, an abstract interpretation is *exact* iff $[\![\mathcal{P}]\!] = \gamma([\![\mathcal{P}]\!]^\sharp)$, meaning that the abstraction loses no information and the abstract semantics exactly captures the concrete semantics of the program [Cousot 1997; Giacobazzi and Quintarelli 2001].

Parser Semantics. In our case, the kind of program we want to abstractly interpret is a parser P represented by a first-order formula φ. We take the concrete semantics $[\![\varphi]\!]$ to be an assignment σ of free variables to values, and in particular denote the parser's input string by the free variable s, such that

$$\mathfrak{M}, \sigma \models \varphi \quad \Longleftrightarrow \quad \sigma \in [\![\varphi]\!] \quad \Longleftrightarrow \quad \sigma(s) \in L(P).$$

Trying to compute $[\![\varphi]\!]$ directly will generally result in an infinite set of values. Hence our desire for an abstract semantics $[\![\varphi]\!]^\sharp$ that produces a finite approximation of this set such that

$$\mathfrak{M}, \hat{\sigma} \models \varphi \quad \Longleftarrow \quad \hat{\sigma} = [\![\varphi]\!]^\sharp \quad \Longrightarrow \quad \hat{\sigma}(s) \subseteq L(P),$$

where $\hat{\sigma}$ is an assignment of free variables to abstract values. In particular, $\hat{\sigma}(s)$ is now a grammar describing (a subset of) the strings in $L(P)$.

The completeness of the approximation $\hat{\sigma}$ depends on the underlying abstract domains. We give a brief summary of the domains currently used by Panini in Table 2 and provide complete definitions in § 5. Note that our abstract string domain $\hat{\mathbf{S}}$ represents sets of strings with regular expressions. This means that our approach must under-approximate any $L(P)$ above regular in the Chomsky hierarchy [Chomsky and Schützenberger 1963]. For super-regular languages, we can only infer a partial grammar representing a regular subset, if one exists. However, if $L(P)$ is (at most) regular, then we can infer a complete grammar for P.

Using our abstract interpretation, we can construct a finite but quantifier-free solution for the refinement variable,

$$\kappa(s) \mapsto s \in \hat{\sigma}(s), \quad \text{where } \hat{\sigma} = [\![\varphi]\!]^\sharp.$$

The definition of the abstract semantics function $[\![\square]\!]^\sharp$ is given in Figure 6. It depends on a novel variable-focused relational semantics $[\![\square]\!]\!\uparrow_{\square}$ and a quantifier elimination procedure $\mathrm{qelim}(\square)$. We describe these in the remainder of this section, after establishing some syntactic extensions to λ_Σ constraints.

4.2 Extended Constraint Syntax

We extend the formal syntax of predicates and constraints from Figure 4 to include abstract values and relations. The extended syntax is given in Figure 7.

Constraints φ, in addition to Boolean constants and the usual logical connectives, include both universal and existential quantification, and generalized relations between predicate expressions.

$$[\![\varphi]\!]^\sharp \doteq \{x \mapsto [\![\varphi]\!]\uparrow_x \mid x \in \mathrm{vars}(\varphi)\}$$

$$\begin{aligned}
[\![P_1 \wedge P_2]\!]\uparrow_x &\doteq [\![P_1]\!]\uparrow_x \sqcap [\![P_2]\!]\uparrow_x && \text{if } P_1, P_2 \text{ quantifier-free}\\
[\![P_1 \vee P_2]\!]\uparrow_x &\doteq [\![P_1]\!]\uparrow_x \sqcup [\![P_2]\!]\uparrow_x && \text{if } P_1, P_2 \text{ quantifier-free}\\
[\![\neg P]\!]\uparrow_x &\doteq \neg\, [\![P]\!]\uparrow_x && \text{if } P \text{ quantifier-free}\\
[\![\varphi]\!]\uparrow_x &\doteq [\![\mathsf{qelim}(\varphi)]\!]\uparrow_x\\
[\![\rho]\!]\uparrow_x &\doteq \begin{cases} \omega, & \text{as defined by domain;}\\ \langle x\colon \rho\rangle, & \text{otherwise.}\end{cases}
\end{aligned}$$

Fig. 6. Abstract semantics of λ_Σ constraints.

Constraints	φ	::=	true | false	Boolean constants
		|	$\varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$	connectives
		|	$\varphi_1 \Rightarrow \varphi_2 \mid \varphi_1 \Leftrightarrow \varphi_2$	implications
		|	$\neg\varphi$	negation
		|	$\exists x.\, \varphi$	existential quantification
		|	$\forall x.\, \varphi$	universal quantification
		|	ρ	relations
Relations	ρ	::=	$\omega_1 \bowtie \omega_2$ where $\bowtie \in \{=, \neq, <, \leq, \in, \notin, \not\parallel, \parallel\}$	
Expressions	ω	::=	$x, y, z, \ldots$	variables
		|	c	concrete value
		|	$\hat{c}$	abstract value
		|	$\langle x\colon \rho\rangle$	abstract relation
		|	$f(\overline{\omega})$	function

Fig. 7. Extended syntax of λ_Σ constraints.

There are no κ applications at this point. The base types of all bound variables are known, but we elide them from the presentation to reduce clutter.

Expressions ω, in addition to variables and concrete values, now include abstract values $\hat{c}$ and abstract relations $\langle x\colon \rho\rangle$. Abstract values $\hat{c}$ are taken from our abstract domains (§ 5) and are representations of potentially infinite sets of concrete values. For example, the abstract value $[1, \infty]$ represents an infinite set of integers $\{1, 2, 3, \ldots\}$ and the abstract string $\Sigma^*\mathsf{a}$ represents the set of all strings that end with the character 'a'. For functions, we assume the usual notational conveniences, e.g., we write $a + b$ for $+(a, b)$ and $|s|$ for $\mathrm{str.len}(s)$ and so on. If an abstract value appears in a function, it lifts the whole expression into the abstract realm, e.g., $x + [1, \infty]$ represents the set of expressions $\{x + 1, x + 2, x + 3, \ldots\}$. Abstract relations $\langle x\colon \rho\rangle$ similarly represent sets of values, specifically those values that are described by the given relation. For example, the abstract relation

$\langle x \colon x > 5 \rangle$ represents "the set of all values x for which $x > 5$ is true" and $\langle i \colon s[i] = c \rangle$ represents the set of all indexes i at which the string in variable s contains the character in variable c.

Relations ρ are comprised of the usual (in)equality and arithmetic comparisons, as well as set (non)inclusion ($\in, \notin$) and (non-)empty intersection ($\between, \|$). The latter are simply abbreviations for common set operations, with

$$A \between B \quad \equiv \quad A \cap B \neq \varnothing, \quad \text{meaning "}A \text{ and } B \text{ have at least one element in common,"}$$
$$A \,\|\, B \quad \equiv \quad A \cap B = \varnothing, \quad \text{meaning "}A \text{ and } B \text{ have no elements in common."}$$

Since abstract values are essentially sets, relating them works the same way. Note the distinction between $x \in [0, \infty]$ ("x is a member of the set of natural numbers"), $x = [0, \infty]$ ("x is the set of natural numbers"), and $x \between [0, \infty]$ ("x has at least one element in common with the set of natural numbers").

Normalization. Both expressions and relations can be normalized by partial evaluation. If abstract values are involved, the semantics of the particular abstract domains apply. If possible, relations are fully abstracted using their relational semantics (§ 4.3).

$$\begin{array}{rcll} 1 + x - 2 & \rightsquigarrow & x - 1 & \\ |\text{"abc"}| & \rightsquigarrow & 3 & \\ x - 1 \in [1, 5] & \rightsquigarrow & x \in [1, 5] + 1 & \rightsquigarrow \quad x \in [2, 6] \\ x < 5 & \rightsquigarrow & x \in [-\infty, 4] & \\ |s| + 1 > 2 & \rightsquigarrow & |s| > 1 & \rightsquigarrow \quad s \in \Sigma^+ \\ [1, 2] \between \langle i \colon s[i] = \text{`a'} \rangle & \rightsquigarrow & s\big[[1, 2]\big] \ni \text{`a'} & \rightsquigarrow \quad s \in \Sigma\Sigma?\text{a}\Sigma^* \end{array}$$

In the remainder, we assume that expressions and relations are always fully normalized.

4.3 Relational Semantics

The concrete semantics $[\![\rho]\!]$ of a single relation are all those assignments of the relation's free variables that make the relation true, e.g.,

$$[\![x > 3]\!] \doteq \{x \mapsto 4, x \mapsto 5, \dots\},$$
$$\Big[\!\Big[|s| > 3\Big]\!\Big] \doteq \{\dots, s \mapsto \text{"abaa"}, s \mapsto \text{"abab"}, \dots\}$$
$$\Big[\!\Big[|s| > x\Big]\!\Big] \doteq \left\{\dots, \begin{pmatrix} s \mapsto \text{"ab"} \\ x \mapsto 0 \end{pmatrix}, \begin{pmatrix} s \mapsto \text{"ab"} \\ x \mapsto 1 \end{pmatrix}, \dots\right\}.$$

Unfortunately, constructing an abstract semantics even for such simple relations is decidedly non-trivial. It requires complex relational domains to capture just a limited set of constraints between multiple variables [Cousot and Halbwachs 1978; Logozzo and Fähndrich 2010; Simon et al. 2003].

Fortunately, we do not actually need a full abstract semantics that condenses relations into singular abstract values. For our purposes, it is sufficient to consider relational semantics on a per-variable basis. To this end, we introduce a variable-focused abstract semantics function $[\![\rho]\!]\!\uparrow_x$ that for a given variable x occurring free in ρ produces an abstract expression whose concrete values are exactly those that could be substituted for x to make ρ true, i.e.,

$$\mathfrak{M}, [x \mapsto c] \models \rho \quad \Longleftrightarrow \quad c \in [\![\rho]\!]\!\uparrow_x .$$

Abstract relations $\langle x \colon \rho \rangle$ provide a convenient "default" implementation,

$$[\![\rho]\!]\!\uparrow_x \doteq \langle x \colon \rho \rangle ,$$

since by definition they exactly capture the abstract semantics of the relation ρ for the given variable x. But we can often do better than this. Depending on the underlying abstract domains

and domain-specific knowledge about the involved operations, more specific definitions of $[\![\square]\!]\uparrow_\square$ are possible (§ 5). For example, for the domains of regular expressions and integers, we can define precise abstractions for simple string length relations,

$$\left[\!\left[|s| = n\right]\!\right]\uparrow_s \doteq \Sigma^n, \qquad \left[\!\left[|s| \geq n\right]\!\right]\uparrow_s \doteq \Sigma^n\Sigma^*, \qquad \left[\!\left[|s| \leq n\right]\!\right]\uparrow_s \doteq \Sigma^0 + \Sigma^1 + \cdots + \Sigma^n,$$

where n is a meta-variable standing for some concrete integer value. With these and other domain-specific semantics, we can construct variable-focused abstractions for the earlier examples. Note how in all cases the abstraction effectively eliminates the chosen variable:

$$[\![x > 3]\!]\uparrow_x \doteq [4, \infty] \qquad \left[\!\left[|s| > 3\right]\!\right]\uparrow_s \doteq \Sigma^4\Sigma^* \qquad \left[\!\left[|s| > x\right]\!\right]\uparrow_s \doteq \langle s\colon\ |s| > x\rangle$$
$$\left[\!\left[|s| > x\right]\!\right]\uparrow_x \doteq |s| - [1, \infty]$$

4.4 Quantifier Elimination

Figure 8 presents our procedure for eliminating quantifiers by abstract substitution. The top-level function qelim traverses a given constraint to eliminate all of its quantifiers. During this traversal, we apply standard logical transformations to simplify the problem.

$$\forall x.\varphi \iff \neg\exists x.\neg\varphi \qquad \text{(DeMorgan's law)}$$
$$\exists x.\ \vee\wedge\rho \iff \vee\exists x.\ \wedge\rho \qquad \text{(distributivity of } \exists \text{ over } \vee\text{)}$$

The actual variable elimination happens in qelim1, where we only need to consider a single conjunctive set of relations R. To eliminate a particular variable x from the relations in this set, we first compute the x-relative relational semantics $[\![\rho]\!]\uparrow_x$ for all relations ρ in the subset of relations in R that contain x as a free variable. This results in a set E of (abstract) expressions, all representing some aspect of x in R. The expressions in E do not contain the variable x but they might contain other free variables. We now generate all unordered pairwise combinations of expressions in E and create a new equality relation between each pair of expressions (using a relational operator appropriate for the pair's types), resulting in a set of relations $\hat{R}$ that contain no reference to x yet preserve the semantics of the original conjunction of relations. Finally, we return $\hat{R}$ together with those relations in R that did not contain x in the first place.

5 ABSTRACT DOMAINS

We now define abstract domains for each of the base types in our system. Each domain efficiently captures (possibly infinite) sets of concrete values of the corresponding type, lifts certain operations of the type to the abstract domain, and defines some abstract relational semantics $[\![\square]\!]\uparrow_\square$ for expression involving those operations.

To simplify the definitions and avoid boilerplate repetition, we generally assume that expressions have already been arranged in a uniform manner and are fully normalized (§ 4.2). We also forego subscripting domain-specific operations and elements if there is no ambiguity, e.g., we write $\top$ instead of differentiating $\top_{\mathbb{1}}, \top_{\mathbb{B}}$, etc.

5.1 Unit

The abstract unit type $\hat{\mathbf{1}}$ simply adds a bottom element, forming the two-element lattice, with

$$\alpha = \gamma = \mathrm{id}, \qquad [\![x = \mathsf{unit}]\!]\uparrow_x \doteq \mathsf{unit}, \qquad [\![x \neq \mathsf{unit}]\!]\uparrow_x \doteq \bot.$$

$$
\begin{aligned}
\mathsf{qelim}(\exists x.\ \varphi) &\doteq \bigvee \left\{ \mathsf{qelim1}(x, R) \;\middle|\; R \in \mathsf{dnf}(\mathsf{qelim}(\varphi)) \right\} \\
\mathsf{qelim}(\forall x.\ \varphi) &\doteq \mathsf{qelim}(\neg \exists x.\ \neg \varphi) \\
\mathsf{qelim}(\varphi_1 \wedge \varphi_2) &\doteq \mathsf{qelim}(\varphi_1) \wedge \mathsf{qelim}(\varphi_2) \\
\mathsf{qelim}(\varphi_1 \vee \varphi_2) &\doteq \mathsf{qelim}(\varphi_1) \vee \mathsf{qelim}(\varphi_2) \\
\mathsf{qelim}(\neg \varphi) &\doteq \neg \mathsf{qelim}(\varphi) \\
\mathsf{qelim}(\rho) &\doteq \rho \\
\mathsf{qelim}(\mathsf{true}) &\doteq \mathsf{true} \\
\mathsf{qelim}(\mathsf{false}) &\doteq \mathsf{false}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{qelim1}(x, R) &\doteq \hat{R} \cup \{ \rho \in R \mid x \notin \mathsf{vars}(\rho) \} \\
&\text{where} \\
\hat{R} &= \left\{ \omega_1 \bowtie \omega_2 \;\middle|\; (\omega_1, \omega_2) \in \binom{E}{2} \right\} \quad \bowtie \in \{=, \in, \ni, \not\parallel\} \\
E &= \left\{ [\![\rho]\!]\!\uparrow_x \;\middle|\; \rho \in R, x \in \mathsf{vars}(\rho) \right\}
\end{aligned}
$$

Fig. 8. Quantifier elimination

5.2 Booleans

The Boolean subset lattice $\langle \wp(\mathbb{B}), \subseteq \rangle$ is a complete abstraction of the Boolean values, adding $\varnothing$ and $\{\mathsf{true}, \mathsf{false}\}$ as bottom and top elements, respectively, and forming a complete complemented lattice via subset inclusion and set complement. For consistency with the other definitions, we call this abstraction $\hat{\mathbf{B}}$ and will use the notation $\langle \hat{\mathbf{B}}, \sqsubseteq, \bot, \top, \sqcup, \sqcap, \neg \rangle$ for the lattice elements and operations. The abstract semantics are defined by

$$
\alpha = \gamma = \mathrm{id}, \qquad [\![x = b]\!]\!\uparrow_x \doteq \{b\}, \qquad [\![x \neq b]\!]\!\uparrow_x \doteq \{\neg b\}.
$$

5.3 Integers

Any concrete set of contiguous integers $\{x \in \mathbb{Z} \mid a \leq x \leq b\}$ can be represented efficiently as an interval $[a, b]$, even allowing for a or b to be $\pm\infty$. The domain of integer intervals

$$
\mathbf{IZ} \stackrel{\text{def}}{=} \left\{ [a, b] \;\middle|\; a, b \in \mathbb{Z} \cup \{-\infty, \infty\} \text{ and } a \leq b \right\}
$$

forms a pseudo-semi-lattice $\langle \mathbf{IZ}, \sqsubseteq, \top, \sqcup, \sqcap \rangle$ with a bounded meet but no $\bot$ element:

$$
\begin{aligned}
[a, b] \sqsubseteq [c, d] &\iff c \leq a \wedge b \leq d \\
\top &= [-\infty, \infty] \\
[a, b] \sqcup [c, d] &= [\min(a, c), \max(b, d)] \\
[a, b] \sqcap [c, d] &= [\max(a, c), \min(b, d)]
\end{aligned}
$$

We can also define some standard operations and convenient relations between intervals:[2]

$$[a,b] + [c,d] = [a+c, b+d] \qquad [a,b] \text{ precedes } [c,d] \iff b < (c-1)$$
$$[a,b] - [c,d] = [a-d, b-c] \qquad [a,b] \text{ is before } [c,d] \iff b < c$$
$$[a,b] \text{ contains } [c,d] \iff a \le c \wedge d \le b$$
$$[a,b] \text{ overlaps } [c,d] \iff a \le c \wedge c \le b \wedge b < d$$

While **IZ** can abstractly represent infinite contiguous sets, such as those defined by a single inequality relation like $\{x \in \mathbb{Z} \mid x > 5\}$, it can not represent even finite non-contiguous sets like $\{1, 5, 7\}$ or inequalities like $\{x \in \mathbb{Z} \mid x \neq 2\}$. Thus the connection between $\mathbb{Z}$ and **IZ** is only partial:

$$\alpha : \wp(\mathbb{Z}) \hookrightarrow \mathbf{IZ} \qquad \gamma : \mathbf{IZ} \to \wp(\mathbb{Z})$$
$$\alpha\big(\{x \in \mathbb{Z} \mid a \le x \le b\}\big) = [a,b] \qquad \gamma\big([a,b]\big) = \{x \in \mathbb{Z} \mid a \le x \le b\}$$

To completely represent *non*-contiguous sets of integers, we can use ordered lists of non-overlapping intervals; e.g., $\{1, 2, 3, 7, 8, \dots\}$ can be represented as $[1, 3 \mid 7, \infty]$. As long as the number of gaps between intervals is bounded, $\wp(\mathbb{Z})$ can be efficiently abstracted by the domain

$$\hat{\mathbf{Z}} \stackrel{\text{def}}{=} \left\{ x_1 x_2 \cdots x_n \;\middle|\; x_i \in \mathbf{IZ} \text{ and } x_i \text{ is before } x_{i+1} \text{ and } i \le n \right\},$$

which forms a complete complemented lattice $\langle \hat{\mathbf{Z}}, \sqsubseteq, \bot, \top, \sqcup, \sqcap, \neg \rangle$, with $\bot = \varnothing$ and $\top = [-\infty, \infty]$ and the operations defined below. We use Haskell list notation $(x : xs)$ to peel off (or add on) the first interval x in a list, with xs denoting the remaining intervals.

$$(x : xs) \sqsubseteq (y : ys) \iff (x \sqsubseteq_{\mathbf{IZ}} y \wedge xs \sqsubseteq (y : ys)) \vee (y \text{ is before } x \wedge (x : xs) \sqsubseteq ys)$$

$$(x : xs) \sqcup (y : ys) = \begin{cases} x : (xs \sqcup (y : ys)) & \text{if } x \text{ precedes } y \\ y : ((x : xs) \sqcup ys) & \text{if } y \text{ precedes } x \\ ((x \sqcup_{\mathbf{IZ}} y) \sqcup xs) \sqcup ys & \text{otherwise} \end{cases}$$

$$(x : xs) \sqcap (y : ys) = \begin{cases} xs \sqcap (y : ys) & \text{if } x \text{ is before } y \\ (x : xs) \sqcap ys & \text{if } y \text{ is before } x \\ (x \sqcap_{\mathbf{IZ}} y) : ((x : xs) \sqcap ys) & \text{if } x \text{ contains } y \text{ or } y \text{ overlaps } x \\ (x \sqcap_{\mathbf{IZ}} y) : (xs \sqcap (y : ys)) & \text{if } y \text{ contains } x \text{ or } x \text{ overlaps } y \\ (x \sqcap_{\mathbf{IZ}} y) : (xs \sqcap ys) & \text{otherwise} \end{cases}$$

$$\neg[-\infty, b_1 \mid a_2, b_2 \mid \dots \mid a_n, \infty] = [b_1 + 1, a_2 - 1 \mid b_2 + 1, a_3 - 1 \mid \dots \mid b_{n-1} + 1, a_n - 1]$$
$$\neg[-\infty, b_1 \mid a_2, b_2 \mid \dots \mid a_n, b_n] = [b_1 + 1, a_2 - 1 \mid \dots \mid b_{n-1} + 1, a_n - 1 \mid b_n + 1, \infty]$$
$$\neg[a_1, b_1 \mid a_2, b_2 \mid \dots \mid a_n, \infty] = [-\infty, a_1 - 1 \mid b_1 + 1, a_2 - 1 \mid \dots \mid b_{n-1} + 1, a_n - 1]$$
$$\neg[a_1, b_1 \mid a_2, b_2 \mid \dots \mid a_n, b_n] = [-\infty, a_1 - 1 \mid b_1 + 1, a_2 - 1 \mid \dots \mid b_{n-1} + 1, a_n - 1 \mid b_n + 1, \infty]$$

The standard arithmetic operations are lifted into $\hat{\mathbf{Z}}$ via pointwise mapping of the equivalent operations on **IZ**. We assume a standard semantics for abstract integer expressions, allowing us to reduce and rewrite arithmetic expressions into a canonical form, e.g., $[3, 5] + 1 \rightsquigarrow [4, 6]$.

We can construct the abstraction function $\alpha : \wp(\mathbb{Z}) \to \hat{\mathbf{Z}}$ for any finite subset of integers $X \in \wp(\mathbb{Z})$ by first sorting all elements of X in ascending order and then identifying all non-overlapping intervals of consecutive integers. This strategy does not work if X is infinite, and in

[2] These interval relations are reminiscent of the temporal interval algebra of Allen [1983]. Our definitions of "precedes" and "overlaps" are identical to Allen's, whereas "is before" corresponds to Allen's "precedes or meets," and our "contains" is equivalent to Allen's "contains or equals or is started by or is finished by."

any case is rather inefficient. Likewise, the concretization function $\gamma : \hat{\mathbf{Z}} \to \wp(\mathbb{Z})$, defined as

$$\gamma(x_1 x_2 \cdots x_n) = \bigcup_{i \le n} \gamma_{\mathbf{IZ}}(x_i),$$

is not very practical. Fortunately, for our use case we only need to abstract and concretize sets of integers defined via relational predicates, which is easily tractable, even with infinite bounds. The corresponding semantic functions are defined below.

$$[\![x = a]\!]\uparrow_x \doteq [a, a] \qquad [\![x \ge a]\!]\uparrow_x \doteq [a, \infty] \qquad [\![x \le a]\!]\uparrow_x \doteq [-\infty, a]$$
$$[\![x \ne a]\!]\uparrow_x \doteq [-\infty, a-1 \mid a+1, \infty] \qquad [\![x > a]\!]\uparrow_x \doteq [a+1, \infty] \qquad [\![x < a]\!]\uparrow_x \doteq [-\infty, a-1]$$

5.4 Characters

Abstract characters, i.e., sets of possible characters, are a common component of string grammars—whitespace, for example, is usually defined as a set of certain invisible characters. While the size of any string alphabet is always bounded and thus the maximum number of possibilities for a single character is finite, these bounds can be quite large—the Unicode standard currently defines 149 878 characters [Unicode Consortium 2023]. Additionally, it is often desirable to define elements of a string by what characters are *not* allowed to be there. Thus the need for an efficient abstract representation of large sets of (im)possible characters.

Formally, we can define the domain of abstract characters $\hat{\mathbf{C}}$ via the alphabet subset lattice $\langle \wp(\Sigma), \subseteq \rangle$, with the usual operations and elements $\langle \hat{\mathbf{C}}, \sqsubseteq, \bot, \top, \sqcup, \sqcap, \neg \rangle$ (cf. abstract Booleans $\hat{\mathbf{B}}$). We use the familiar notation Σ interchangeably with $\top$, indicating the set of all characters of the alphabet. We use set difference to indicate exclusion, e.g., $\Sigma \backslash \{\mathtt{a}, \mathtt{b}\}$ means the set of all characters excluding 'a' and 'b'. For singleton sets like $\{\mathtt{a}\}$ we will usually drop the braces and just write $\mathtt{a}$. Note that $\bot = \varnothing$ is *not* equivalent to the empty string; rather, it is the empty set of characters, representing a space that is impossible to fill or a character that is impossible to produce. The abstract semantics of $\hat{\mathbf{C}}$ are defined by

$$\alpha = \gamma = \mathrm{id}, \qquad [\![x = c]\!]\uparrow_x \doteq \{c\}, \qquad [\![x \ne c]\!]\uparrow_x \doteq \Sigma \backslash \{c\}.$$

5.5 Strings

To abstractly represent infinite sets of strings, we define a domain $\hat{\mathbf{S}}$ of regular expressions over abstract characters $\hat{\mathbf{C}}$, consisting of the empty language $\varnothing$, the empty string ε, abstract character literals $\hat{c} \in \hat{\mathbf{C}}$ such that $\hat{c} = \{c_1, c_2, \ldots, c_n\} \equiv c_1 + c_2 + \cdots + c_n$, concatenation $\square \cdot \square$ (usually elided), alternation $\square + \square$, the Kleene star $\square^*$, and optionals $\square^? \equiv \varepsilon + \square$. We additionally allow the common abbreviations $\square^+ = \square\square*$ and $\square^n = \square\square\cdots\square$ where $\square$ is repeated n times, with $n < 1$ resulting in $\varnothing$ and $\square^0 = \varepsilon$.

The domain $\hat{\mathbf{S}}$ forms a complete complemented lattice $\langle \hat{\mathbf{S}}, \sqsubseteq, \bot, \top, \sqcup, \sqcap, \neg \rangle$ with $\bot = \varnothing$, $\top = \Sigma^*$, and the standard operations on regular sets [Hopcroft and Ullman 1979]. The language $L(\hat{s})$ describes all strings generated/recognized by a regular expression $\hat{s} \in \hat{\mathbf{S}}$, thus $\gamma(\hat{s}) = L(\hat{s})$. We can of course only abstract sets of strings that form a regular language, i.e., $\alpha(S) = \hat{s} \iff S = L(\hat{s})$. While α is not generally realizable [Gold 1967], we can define an abstract semantics over string relations, shown in Figure 9, that allows us to abstract all strings expressed via relations between common string operations.

6 IMPLEMENTATION AND EVALUATION

We have implemented our approach in the Panini prototype system, available at https://anonymous.4open.science/r/panini. It includes a small default set of λ_Σ axioms mimicking the semantics of common Python string functions. Our implementation is written in Haskell and uses the Z3 theorem

$$\left[\!\left[|s| \in \hat{n}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{\hat{n}} \qquad \Box^{\hat{n}} = \bigsqcup_{[a,b]\in\hat{n}} \Box^{[a,b]}$$

$$\left[\!\left[|s| \notin \hat{n}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{\neg\hat{n}} \qquad \Box^{[a,b]} = \begin{cases} \Box^{a}\Box^{*} & \text{if } b = \infty \\ \Box^{a} + \Box^{a+1} + \cdots + \Box^{b} & \text{otherwise} \end{cases}$$

$$\left[\!\left[s[i] \in \hat{c}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{i}\hat{c}\Sigma^{*} \qquad \left[\!\left[s[i] \notin \hat{c}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{[0,i]} + \Sigma^{i}(\neg\hat{c})\Sigma^{*}$$

$$\left[\!\left[s[i] \in \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{[0,i]} \qquad \left[\!\left[s[i] \notin \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{i+1}\Sigma*$$

$$\left[\!\left[s[\hat{\imath}] \between \hat{c}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{\hat{\imath}}\hat{c}\Sigma^{*} \qquad \left[\!\left[s[\hat{\imath}] \parallel \hat{c}\right]\!\right]^{\uparrow}_{s} \doteq \prod_{[a,b]\in\hat{\imath}} \begin{cases} \Sigma^{[0,a]} + \Sigma^{a}(\neg\hat{c})^{+} & \text{if } b = \infty \\ \Sigma^{[0,a]} + \Sigma^{a}(\neg\hat{c})^{b-a+1}\Sigma^{*} & \text{otherwise} \end{cases}$$

$$\left[\!\left[s[\hat{\imath}] \between \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{\min(\hat{\imath})} \qquad \left[\!\left[s[\hat{\imath}] \parallel \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{\max(\hat{\imath})}\Sigma^{*} \quad \text{if } \max(\hat{\imath}) \neq \infty$$

$$\left[\!\left[s\left[|s| - i\right] \in \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \varnothing \qquad \left[\!\left[s\left[|s| - i\right] \notin \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{*}$$

$$\left[\!\left[s\left[|s| - i\right] \in \hat{c}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{*}\hat{c}\Sigma^{i-1} \qquad \left[\!\left[s\left[|s| - i\right] \notin \hat{c}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{*}(\neg\hat{c})\Sigma^{i-1} + \Sigma^{[0,i-1]}$$

$$\left[\!\left[s\left[|s| - \hat{\imath}\right] \between \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \varnothing \qquad \left[\!\left[s\left[|s| - \hat{\imath}\right] \parallel \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{*}$$

$$\left[\!\left[s\left[|s| - \hat{\imath}\right] \between \hat{c}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{*}\hat{c}\Sigma^{\hat{\imath}-1}$$

$$\left[\!\left[s\left[|s| - \hat{\imath}\right] \parallel \hat{c}\right]\!\right]^{\uparrow}_{s} \doteq \prod_{[a,b]\in\hat{\imath}} \begin{cases} (\neg\hat{c})^{*}\Sigma^{a-1} + \Sigma^{[0,a-1]} & \text{if } b = \infty \\ \left[\!\left[s\left[|s| - [a, b-1]\right] \parallel \hat{c}\right]\!\right]^{\uparrow}_{s} \Sigma + \varepsilon & \text{otherwise} \end{cases}$$

$$\left[\!\left[s[i..j] \in \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{[0,i]} \qquad \left[\!\left[s[i..j] \notin \varnothing\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{j+1}\Sigma^{*}$$

$$\left[\!\left[s[i..j] \in \hat{t}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{i}(\Sigma^{j-i+1} \sqcap \hat{t})\Sigma^{*} \qquad \left[\!\left[s[i..j] \notin \hat{t}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{[0,i]} + \Sigma^{i}(\Sigma^{j-i+1} \setminus \hat{t})\Sigma^{*}$$

$$\left[\!\left[s[\hat{\imath}..\hat{\jmath}] \between \hat{t}\right]\!\right]^{\uparrow}_{s} \doteq \Sigma^{\hat{\imath}}(\Sigma^{\hat{\jmath}-\hat{\imath}+1} \sqcap \hat{t})\Sigma^{*} \qquad \left[\!\left[s[i..j] \parallel \hat{t}\right]\!\right]^{\uparrow}_{s} \doteq \neg\left[\!\left[s[i..j] \between \hat{t}\right]\!\right]^{\uparrow}_{s}$$

Fig. 9. Abstract relational semantics for strings. Abstract values are denoted $\hat{\Box}$. We assume single characters have been lifted to singleton character sets, e.g., $s[i] = c \rightsquigarrow s[i] \in \hat{c}$ where $\hat{c} = \{c\}$.

solver [De Moura and Bjørner 2008] and is modular with respect to the abstract domains used during grammar inference. Panini can be used as a library, a standalone batch-mode command-line application, or interactively via a read-eval-print loop.

6.1 Efficient Regular Expressions

An important aspect for the practical viability of our approach is an efficient implementation of the underlying abstract domains, in particular abstract strings $\hat{\mathbf{S}}$ and abstract characters $\hat{\mathbf{C}}$. The machine representation of $\hat{\mathbf{C}}$ is based on complemented PATRICIA tries [Kmett 2012; Morrison 1968; Okasaki and Gill 1998], which enable compact representation of negated sets while allowing all

Table 3. Evaluation Results

Type	Parser Example		Total	Successful Exact	Successful Under	Failed Empty	Failed Stuck	Time (ms)
Straight	getAddrSpec	`[^<>]*<[^>]*>.*`	61	60	0	0	1	155 ±249
+ Cond.	Figure 3	`a|[^a]b.*`	28	28	0	0	0	178 ±246
+ Loops	lsb_check	`0*1`	36	7	3	13	13	785 ±556
			125	95	3	13	14	342 ±461

common set operations. For $\hat{\mathbb{S}}$, we implemented an efficient regular expression type whose literals are abstract characters from $\hat{\mathbb{C}}$ and whose operations, like regular intersection and complement, are performed purely algebraically, without going through finite automata. Our approach uses Brzozowski derivatives [Antimirov 1996; Brzozowski 1964] and is based on the equation-solving technique by Acay [2018], using Arden's lemma [Arden 1961] and Gaussian elimination to solve a system of regular equations. It is similar to the approach by Liang et al. [2015], but makes use of the local mintermization trick employed by Keil and Thiemann [2014] to effectively compute precise derivative over large alphabets. To keep regular expressions as concise and human-readable as possible, we aggressively simplify intermediate expressions using the transformation rules and heuristics described by Kahrs and Runciman [2022].

6.2 Experiments

To provide insights into the efficacy and applicability of the Panini prototype for inferring grammars from ad hoc parser implementations, we curated a benchmark dataset comprising 125 ad hoc parsers. The dataset encompasses a diverse set of parser implementations over regular grammars, ensuring a comprehensive evaluation of Panini across different structural paradigms.

```
def getAddrSpec(email):
  b1 = email.index('<',0)+1
  b2 = email.index('>',b1)-1
  return email[b1:b2]
```

```
getAddrSpec : {email : 𝕊 | ⋆} → 𝕊
getAddrSpec = λ(email : 𝕊).
  let b1 = index email '<' in
  let p1 = ge b1 0 in
  let _ = assert p1 in
  let b2 = add b1 1 in
  let v1 = index email '>' in
  let v2 = ge v1 b2 in
  let _ = assert v2 in
  let v3 = sub v1 1 in
  slice email b2 v3
```

Fig. 10. A straight-line ad hoc parser.

Methodology. For each ad hoc parser in the dataset, we used Panini to automatically infer a grammar from the parser's λ_Σ source code representation. We assessed the efficacy and performance overheads of Panini in terms of both the accuracy of inferred grammars and the computational resources required for the inference process. The inferred grammars were compared against manually curated ground truth grammars to assess the accuracy of Panini's inference capabilities. We differentiate between exact matches with the ground truth grammar; successful under-approximations, which correctly identify a subset of allowed strings; trivial under-approximations, i.e., undesired inferences of the empty language $\emptyset$; and stuck inferences, where Panini is unable to proceed with inference due to missing semantics for certain combinations of expressions or other limitations of the abstract domains. We discuss the underlying reasons for under-approximations and failed inference cases. We also measured the computational overhead incurred during the grammar inference process, particularly

wall-clock execution time. All benchmarks were run on an iMac with a 3.5 GHz Intel Core i5 processor and 24 GB RAM, using Z3 4.8.10 for SMT solving.

Our complete benchmark dataset, along with scripts for reproducibility, is made publicly available as an artifact to ensure the transparency and replicability of our experimental findings. The reproducibility package includes the source code of ad hoc parsers, ground truth grammars, and instructions for using Panini for grammar inference.

```
def lsb_check(s):
  i = 0
  while i < len(s)-1:
    assert s[i] == "0"
    i += 1
  assert s[i] == "1"

lsb_check : {s : 𝕊 | ⋆} → 𝟙
lsb_check = λ(s : 𝕊).
  let n1 = length s in
  let n2 = sub n1 1 in
  rec w1 : ℤ → 𝟙 = λ(i1 : ℤ).
    let p1 = lt i1 n2 in
    if p1 then
      let c1 = charAt s i1 in
      let p2 = eqChar c1 '0' in
      let _ = assert p2 in
      let i2 = add i1 1 in
      w1 i2
    else
      unit
  in
    let _ = w1 0 in
    let c2 = charAt s n2 in
    let p3 = eqChar c2 '1' in
    assert p3
```

Fig. 11. An ad hoc parser with loops.

Results. To facilitate a structured analysis, we classified the parsers within our benchmark dataset into three overarching categories based on their structural features: *Straight-line Programs* are ad hoc parsers characterized by linear execution flow without branching constructs such as conditionals or loops. Figure 10 shows an example of such a program, used to parse the angle-bracketed *addr-spec* part of an email address with a display name. *Programs with Conditionals* are ad hoc parsers that incorporate conditional statements to make decisions based on input characteristics. We saw such a program in the example of Figure 3. *Programs with Loops and Conditionals* are ad hoc parsers containing iterative constructs alongside conditional statements for string manipulation tasks. Figure 11 presents a parser looping over a bit-string to ensure that only the least-significant bit is set.

Table 3 presents a comprehensive overview of our experimental results, showcasing an illustrative example representative of each ad hoc parser category, together with summary statistics on the accuracy and performance of Panini's grammar inference. Out of the 125 parser programs in our dataset, we achieve exact inference for all but one straight-line program, all purely conditional programs, and 7 out of 36 programs containing loops. We infer safe under-approximations of the grammars for 3 loop programs, fail to find a meaningful refinement beyond the empty language for 13 loop programs, and get stuck without results on the remaining 13. Our main obstacle for complete inference of all loop programs are insufficient invariants. Loop invariants are discovered using classical refinement inference while solving non-grammar variables (§ 3.3) and our prototype implementation is limited in this regard. We plan to incorporate better classical invariant generation as part of our future work (§ 6.3).

The performance of grammar inference is in the sub-second range on average. Most of the inference time is spent during classical refinement inference, where SMT solving is the biggest bottleneck. However, our implementation is not optimized in this area and we conjecture there are a lot of low-hanging fruits that could significantly improve performance by large constant factors.

6.3 Current Limitations and Future Work

Panini is a prototype implementation of our approach to grammar inference for ad hoc parsers using refinement types and abstract interpretation, as discussed in this paper. It is a proof-of-concept

that shows the viability of our approach, but it is not yet a practical end-user system. We see three main areas of improvement as part of future work:

(1) The biggest limit of our system at the moment lies in the solving of non-grammar κ variables by the classical refinement inference machinery, specifically the generation of loop invariants. Our prototype implements purely conjunctive predicate abstraction [Rondon et al. 2008], which is limited in the shape of invariants that can be produced and is highly dependent on a good set of candidate qualifiers. Our qualifier extraction heuristics are very basic and we are working on extending them. More generally, we plan on integrating external state-of-the-art invariant generators to improve the classical refinement inference phase and in turn increase the amount of grammars we can more accurately infer.
(2) The abstract interpretation we perform is bounded by the limits of the underlying abstract domains. For example, our integer domain $\hat{\mathbf{Z}}$ cannot efficiently represent congruence classes, e.g., infinite sets of even or odd numbers. We can recover some of this power by using a functional representation with additional semantics, e.g., a built-in mod function that generates additional constraints over its integer arguments, which can then be resolved by regular grammar inference in certain cases. However, this approach has its limits and cannot be extended to all manner of relational invariants. By design, our system is modular in the choice of underlying abstract domains, and we are working on extending the capabilities of Panini by adding more extensive abstract domains.
(3) The λ_Σ language is by design minimal, in order to provide a common intermediate representation for a wide range of ad hoc parser implementations and to simplify many aspects of refinement and grammar inference. However, its lack of language features makes it difficult to easily capture many real-world parser programs. We are currently working on extending the λ_Σ language to add support for polymorphism and user-defined data types and are investigating how such features interact with abstract interpretation and our grammar solving approach.

7 RELATED WORK

Dynamic Grammar Inference for Fuzzing. Obtaining input grammars of programs has been heavily pursued by the *fuzzing* community [Manes et al. 2019; Zeller et al. 2021] for use in *grammar-based fuzzing* [Aschermann et al. 2019; Holler et al. 2012]. The idea behind fuzzing is simple: test programs by bombarding them with (systematically generated) random inputs and see if anything breaks. But generating good fuzz inputs is hard, because in order to penetrate into deep program states, one generally needs valid or near-valid inputs, meaning inputs that pass at least the various syntactic checks and transformations—i.e., ad hoc parsers—scattered throughout a typical program. In grammar-based fuzzing, valid inputs are specified with the help of language grammars, shifting the problem to that of obtaining accurate grammars or language models. Black-box approaches try to infer a language model by poking the program with seed inputs and monitoring its runtime behavior [Bastani et al. 2017; Godefroid et al. 2017]. This has some theoretical limits [Angluin 1987; Angluin and Kharitonov 1995] and the amount of necessary poking (i.e., membership queries) grows exponentially with the size of the grammar. White-box approaches make use of the program code and can thus use more sophisticated techniques like taint tracking to monitor data flow between variables [Höschele and Zeller 2016] or observing character accesses of input strings by tracking dynamic control flow [Gopinath et al. 2020]. These approaches can produce fairly accurate and human-readable grammars, at least in test settings, but they rely on dynamic execution and thus require complete runnable programs. Furthermore, the accuracy of such dynamically inferred grammars cannot be guaranteed.

String Constraint Solving. Precise formal reasoning over strings can be accomplished using *string constraint solving* (SCS), a declarative paradigm of modeling relations between string variables and solving attendant combinatorial problems [Amadini 2021]. It is usually assumed that collecting string constraints requires some kind of (dynamic) symbolic execution [Kausler and Sherman 2014], and practical SCS applications are generally concerned with the inverse of our problem: modeling the possible strings a function can return or express [Bultan et al. 2018], instead of the strings a function can accept. In our approach, we use purely static means (viz. refinement type inference) to essentially collect input string constraints (see § 3), which we then simplify/solve in ways not dissimilar but nonetheless different from traditional SCS techniques (see § 4). There have been many recent advances in SCS for SMT [Abdulla et al. 2015; Kan et al. 2022; Kiezun et al. 2009; Trinh et al. 2014, 2020; Zheng et al. 2013]. We particularly make use of string theories embedded in the Z3 constraint solver [Berzish et al. 2017] in our implementation (§ 6).

Related work in (dynamic) symbolic execution make use of constraint solvers over string domains at their core to reason about how strings are manipulated in programs, with applications ranging from generating test inputs [Bjørner et al. 2009; Cadar et al. 2008; Li et al. 2011] and detecting vulnerabilities (e.g., cross-site scripting, SQL injection) [Holík et al. 2017; Loring et al. 2017; Saxena et al. 2010]. These approaches differ from our work on two specific aspects. First, they rely on traces from dynamic executions to infer more precise constraints, while we are able to reason about string constraints statically. Second, in the case of detecting vulnerabilities, they reason about the resulting output induced through string operations, and do not infer a grammar over the input language, which is our main goal.

Abstract Domains. Abstract string domains approximate strings to track information precisely enough to analyze particular behaviors of interest while only preserving relevant information. Most of the existing work in string domains differs in what kind of behavior is of interest and how the approximation is achieved efficiently.

Costantini et al. [2015] introduces a suite of different abstract semantics for concatenation, character inclusion, and substring extraction (particularly pre- and suffixes). In their work, they explicitly discuss the trade-off between precision and efficiency. Amadini et al. [2020] review the dashed string abstraction, an approach that considers strings as blocks of characters and the constraints on these blocks, which has shown good performance on benchmarks involving constraints on string length, equality, concatenation, and regular expression membership. M-String [Cortesi and Olliaro 2018] considers a parametric abstract domain for strings in the C programming language by leveraging abstract domains for the content of a string and for expressions to infer when a string index position corresponds to an expression of interest. While most of the existing work has focused on approximating a single variable, very recent work by Arceri et al. [2022] focuses on relational string domains that try to capture the relation between string variables and expressions for which we cannot compute static values, such as user input.

There have been a multitude of abstract domains that aim at specific target languages, such as JavaScript [Amadini et al. 2017; Jensen et al. 2009; Kashyap et al. 2014; Park et al. 2016].

Precondition Inference. Despite a wide variety of approaches for computing preconditions [Barnett and Leino 2005; Cousot et al. 2013; Dillig et al. 2013; Padhi et al. 2016; Seghir and Kroening 2013], we are not aware of any that focus specifically on string operations, or that would allow us to reconstruct an input grammar in a way suitable for our envisioned applications.

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