Lesson 01

Introduction

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Division of Mechanical System Engineering
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Content

1. Course Introduction
2. Definition of FEM
    - Conceptual and Structural Models
    - Workflow
    - History and Software Package
    - FEM Family
    - Element Types
    - Problem Classification
    - Advantages and Disadvantages

1. Course Introduction

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Email: mctrinh@jbnu.ac.kr & Mobile: 010-2177-6792

Time: Wednesday (수) 10:00 AM - 13:00 PM

Offline: Room 4329, Building No. 4

Prerequisite: Engineering Mathematics, Continuum Mechanics, Python

Course code: https://github.com/mctrinh/fem-class

Course evaluation:

Mid-term (40%) + Final-term (40%) + Attendance (15%) + Assignment (5%)

1. Course Introduction – References

J. N. Reddy, An Introduction to the Finite Element Method, McGraw Hill Series in Mechanical Engineering, 3rd edition, 2005

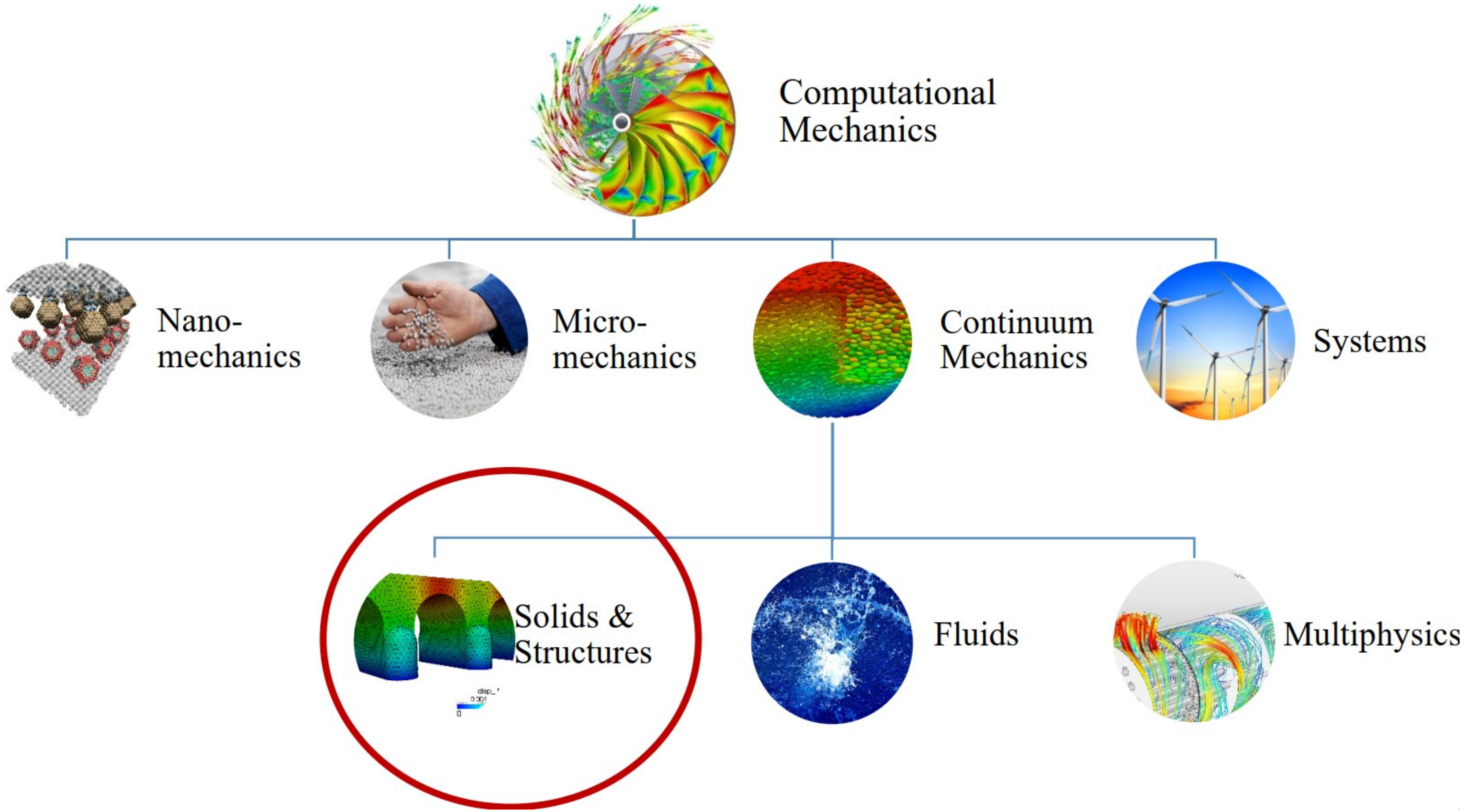
K. J. Bathe, Finite Element Procedures in Engineering Analysis, Prentice-Hall, 1982

E. Hinton and D. R. J. Owen, An Introduction To Finite Element Computations, Pineridge Press, 1979

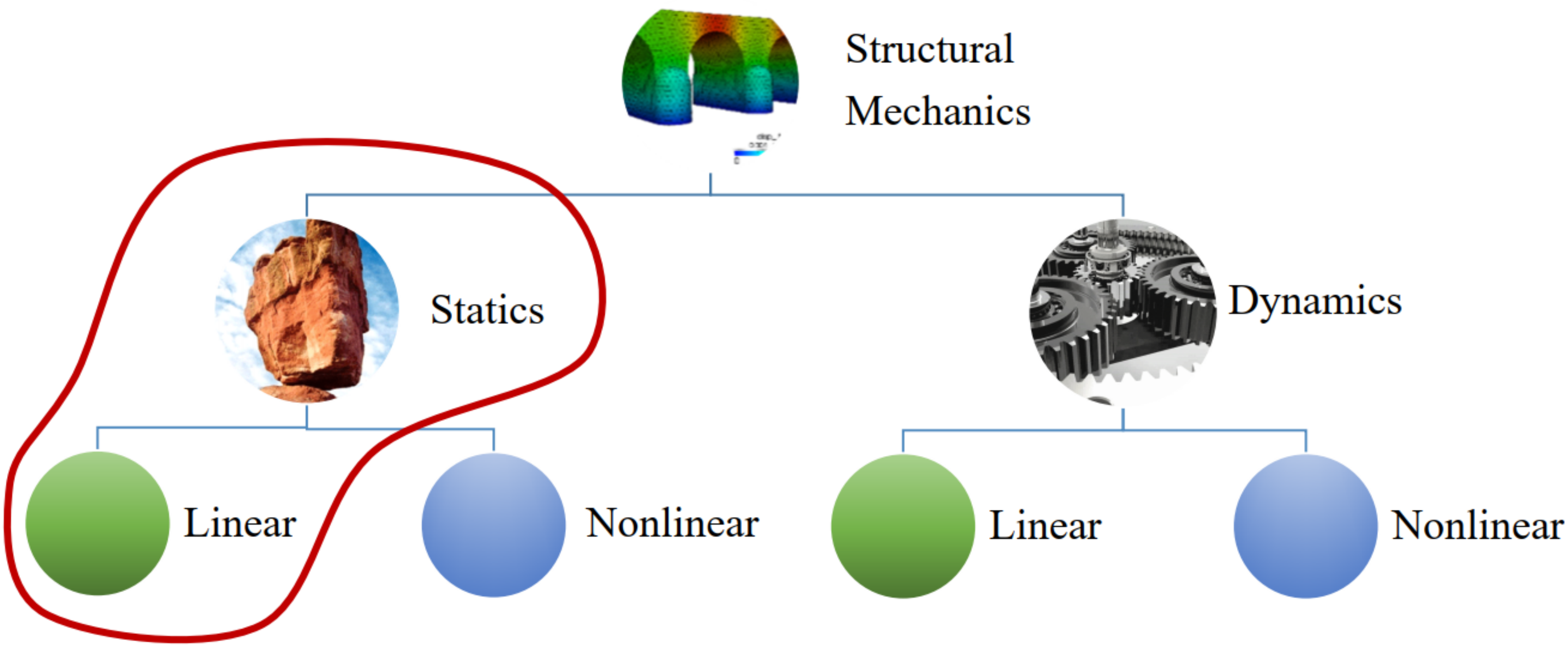
O. C. Zienkiewicz and *R. L. Taylor,* The Finite Element Method I, II, McGraw-Hill, 1989

M. A. Crisfield, Energy and Variational Methods in Applied Mechanics, John Wiley and Sons, 1984

1. Course Introduction – Computational Mechanics

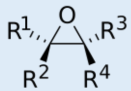
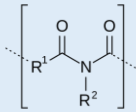


1. Course Introduction – Structural Mechanics

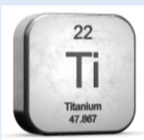


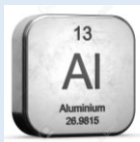
1. Course Introduction – Laminated composites

Matrix

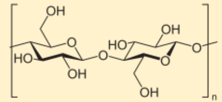

Epoxy *Polyimide* *Alumina*





Titanium *Aluminum*

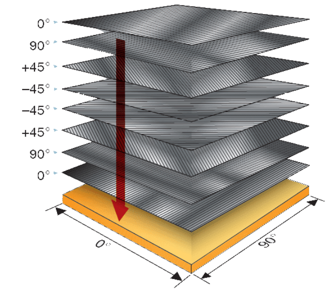
Fiber

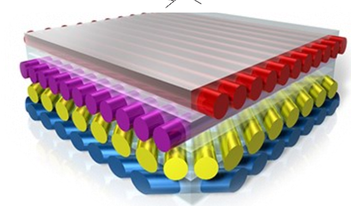


Cellulose *Graphite* *Glass*


Boron *Silicon carbide*

Laminate



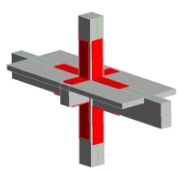
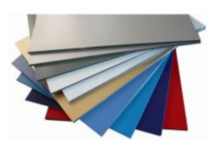


Application

Aircraft

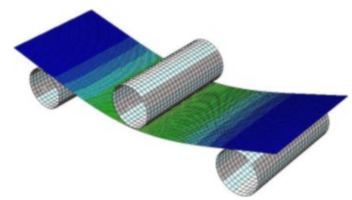
Automobile frame

Boat

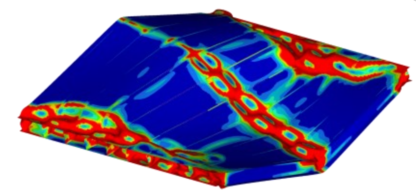



Civil components

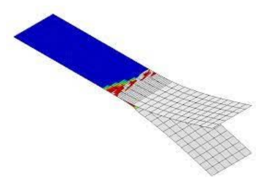
Analysis



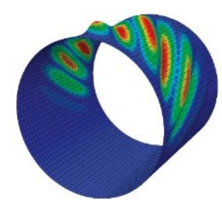
Bending



Large deformation



Delamination



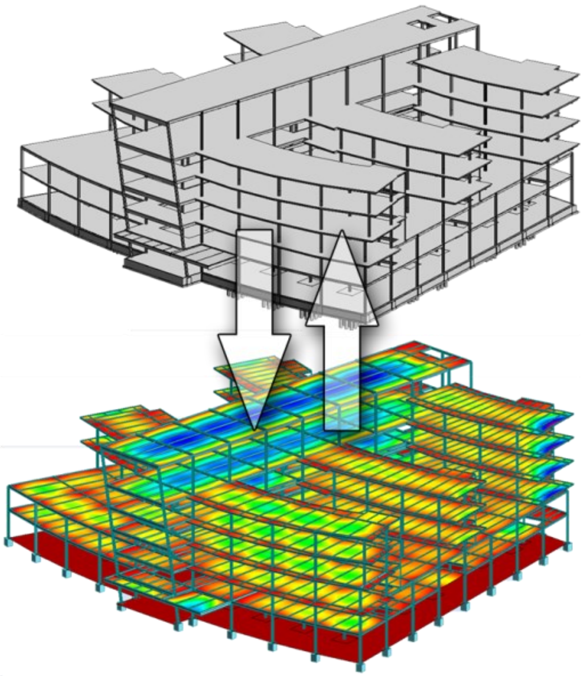
Buckling

Vibration

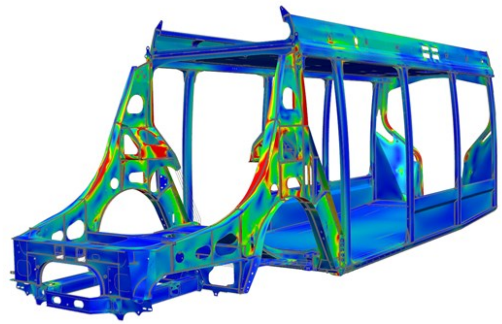
1. Course Introduction – Goals

- ❖ The Procedure of Finite Element Method.
- ❖ Variational Formulations.
- ❖ Formulating Finite Elements (Bar-Truss, Beam, Plate).
- ❖ Working with Python, Visual Studio Code, GitHub
- ❖ Solve Practical Problems using Python codes.
- ❖ Sample codes are available at: https://github.com/mctrinh/fem-code
- ❖ Discuss about research topics using Finite Element Analysis (i.e., Artificial Intelligence, Data-based Computational Mechanics, …)

2. Definition of FEM – Structural Analysis



@Engineersdaily



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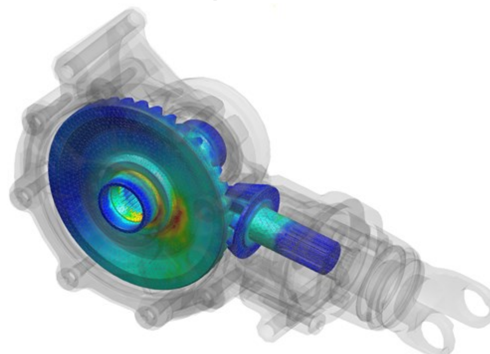

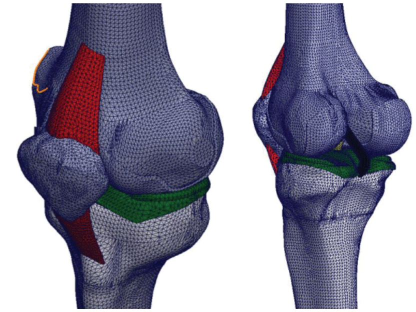

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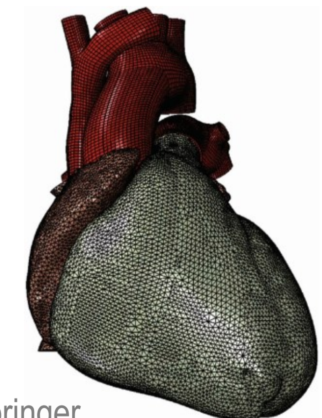
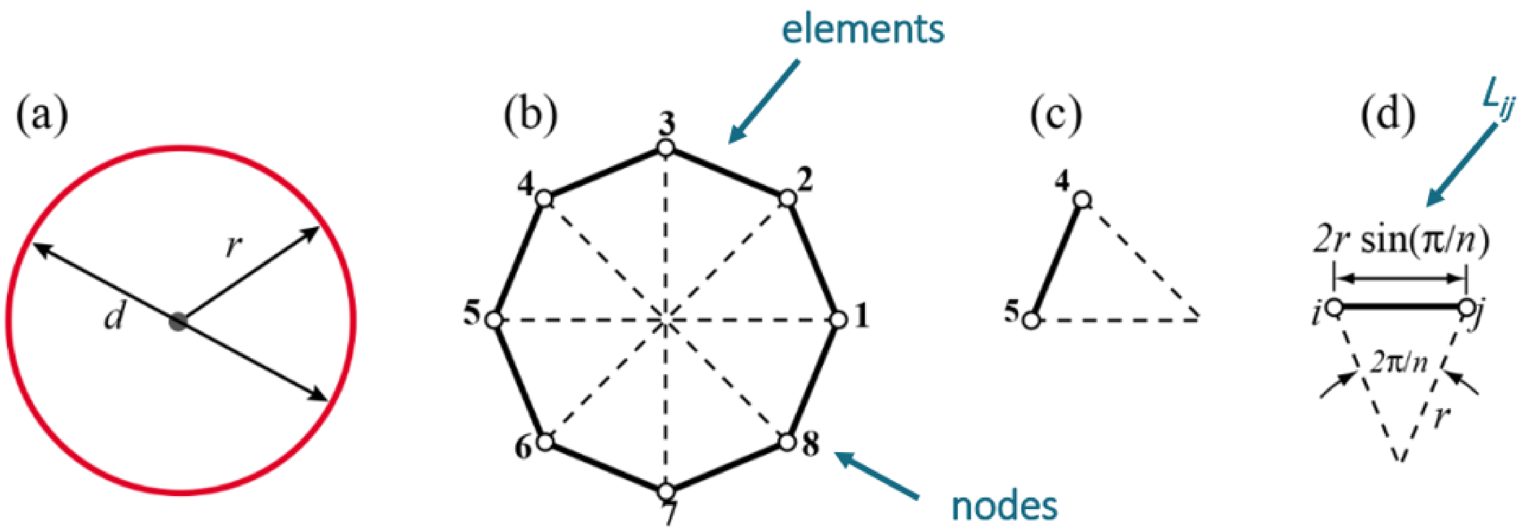


@Springer

The goal of Structural Analysis: Modeling a system by means of approximation.

How to do this? Need a simplified idealization, a conceptual model.

2. Definition of FEM – Finite Elements and Approximation



Find the perimeter *L* of a circle of diameter *d*

We know that $L = \pi d$.

Approximation via Inscribed Polygons

The element length is $L_{ij} = 2r\sin(\pi/n) = d\sin(\pi/n)$

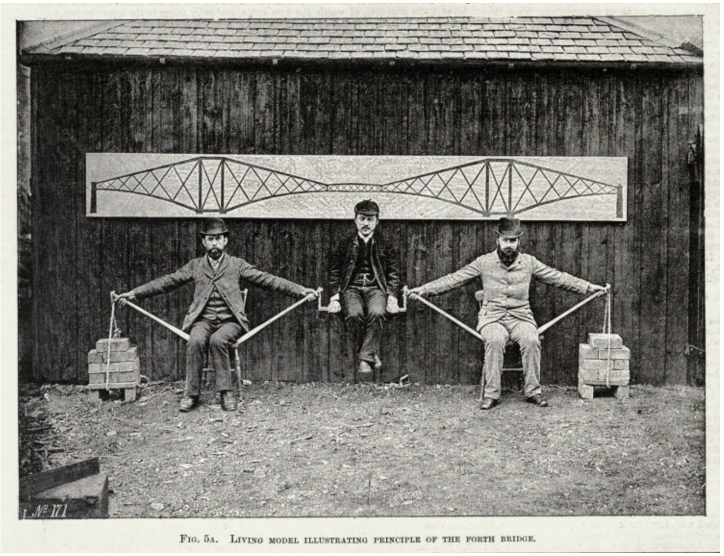
The polygon perimeter is $L_n = n L_{ij}$

Hence the approximation to π is $\pi_n = L_n/d = n\sin(\pi/n)$

n	$\pi_n = n\sin(\pi/n)$
1	0.00000000000000
2	2.00000000000000
4	2.82842712474619
8	3.06146745892071
16	3.12144515225805
32	3.13654849054593
64	3.14033115695475
128	3.14127725093277
256	3.14151380114430

2. Definition of FEM – Conceptual and Structural Models

Example of a conceptual model: How to model this bridge?



"A human-cantilever bridge"

Select Conceptual Model → Define Structural Model or Mathematical Model

Can we model this by hands?

Break down the system to 1D components (ex. truss, beams, supports)

"Different structural models could be set up using different complexity in our geometric and mathematical representation."

What if we want to model a critical component?

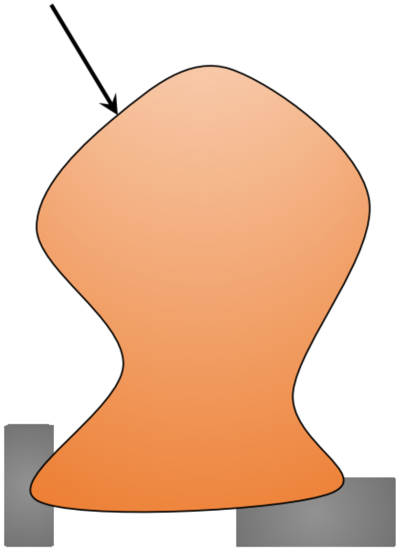
→

Can we still model this by hands?

"The task is now harder."

"It requires 2D or 3D elements used."

2. Definition of FEM – Problem Statement



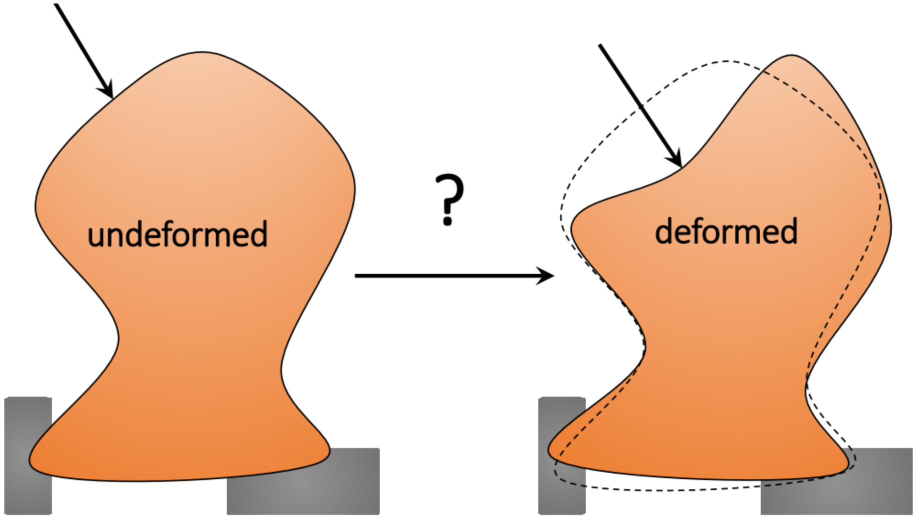
FEA was originally developed for solid mechanics !!!

Object: A Solid with known mechanical properties
(ex. a skyscraper, bio tissue …)

Main Features

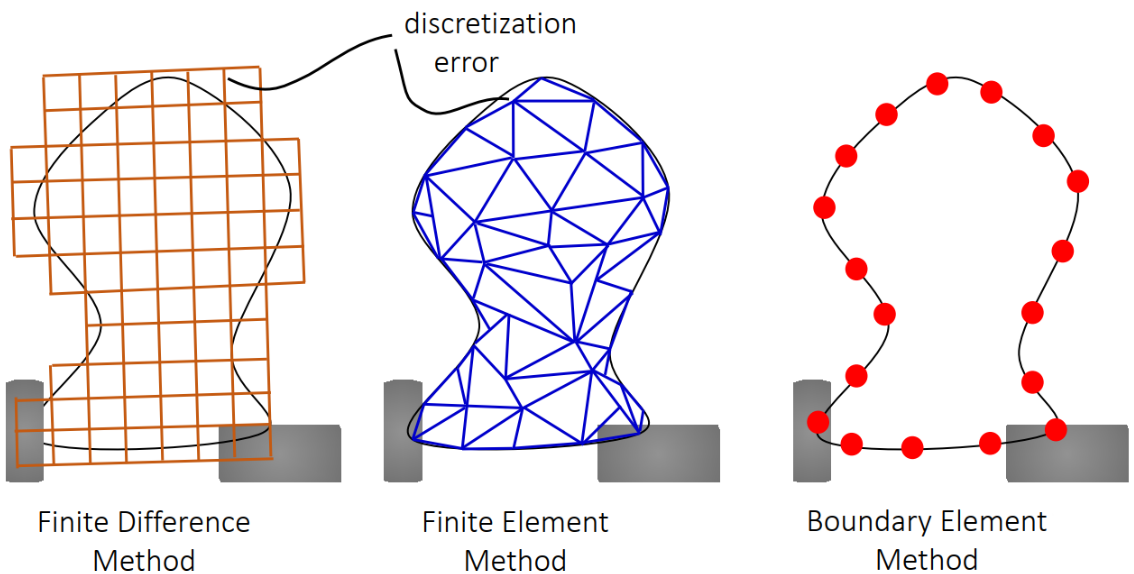
- *Acting Loads.*
- *Boundary:* The surface enclosing the geometry.
- *Solid:* Interior + Boundary.
- *Boundary conditions:* prescribed displacements/tractions on the boundary.

Problem Statement →



2. Definition of FEM – Approximate Complex Structures

- The geometry is **discretized** or split into a mesh of finite elements. Different ways to do.



- **Errors** = Formulation error + Discretization error + Numerical error

- Formulation error results from the use of elements that don't precisely describe the behavior of the physical problem. Can be reduced by improving the structural models to accurately describe the structure behaviors.

- Discretization error results from transforming the physical system (continuum) into a finite element model, and can be related to modeling the boundary shape, the boundary conditions, etc. Can be reduced by using finer mesh (more elements), or else by increasing the accuracy of the finite elements used.

- Numerical error occurs as a result of numerical calculation procedures and includes truncation errors and round off errors.

2. Definition of FEM – Workflow

- Start from a complex problem.

- Break it into pieces (elements).

- Use simple math to represent the force displacement relationship on each piece $\mathbf{f}_i = \mathbf{k}_i \mathbf{u}_i$.

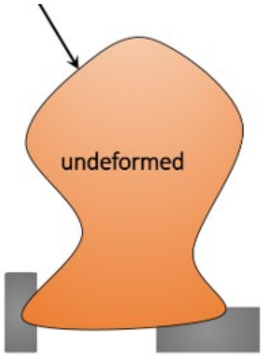
- Assemble the pieces into one global (matrix) equation.

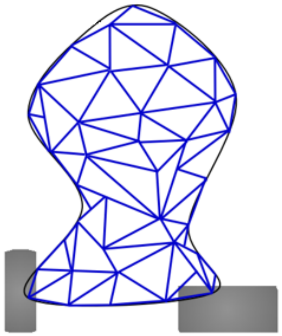
$$\mathbf{F} = \mathbf{K}\mathbf{U}$$

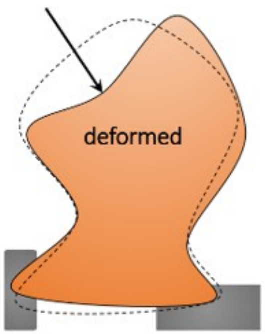
- Apply Boundary Conditions to constrain the problem.

$$\mathbf{F}_{BC} = \mathbf{K}_{BC}\mathbf{U}_{BC}$$

- Solve the problem to obtain quantities of interest such as displacements, stresses.





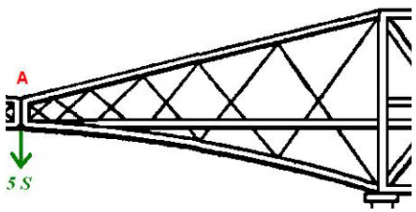


2. Definition of FEM – Workflow

FE workflow !

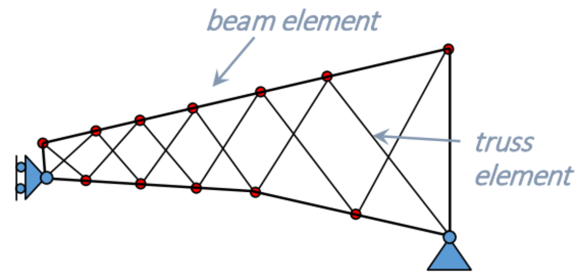
Physical Model

Describe the problem: Simplify a real engineering problem into a problem that can be solved by FEA.



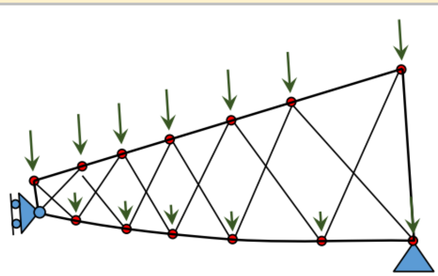
FE Model (Pre-processor)

Discretize/mesh the solid, define material properties, apply boundary conditions



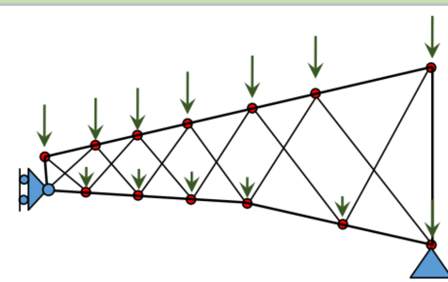
Results (Post-processor)

Obtain, visualize and explain the results



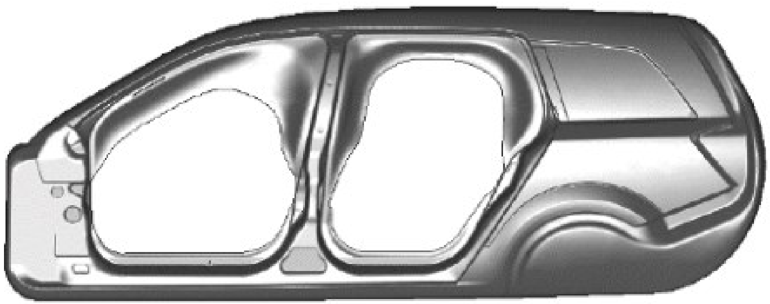
Theory (Solver)

Choose approximate functions, formulate linear equations, and solve equations. This solver is often viewed as black box in FE software.

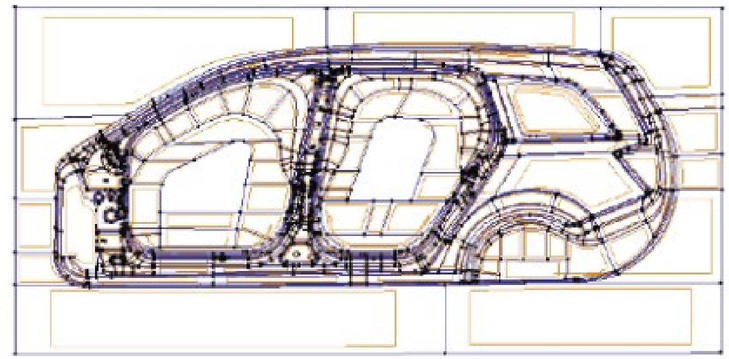


2. Definition of FEM – Example: Car body modeling

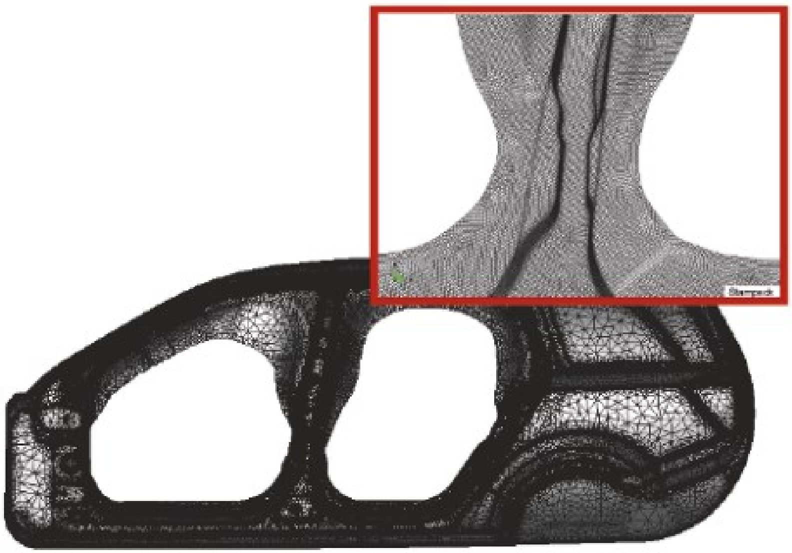
(a) Actual geometry of an automotive panel



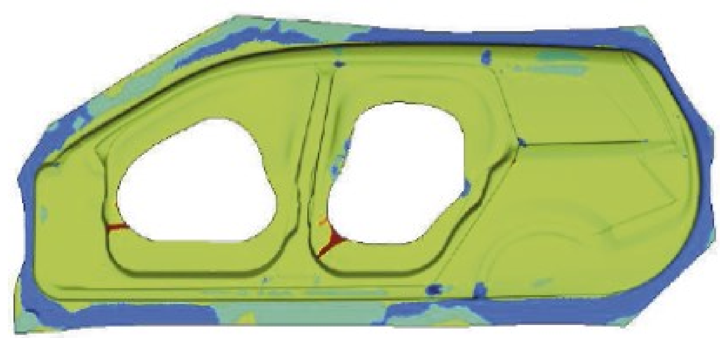
(b) CAD geometrical description by NURBS patches



@Fig 1.5 from Onate, Vol. 1



(c) Finite element mesh of 3-noded shell triangles discretizing the panel geometry



(d) FEM numerical results of the structural analysis showing the equivalent strain distribution

2. Definition of FEM

- ❖ Many problems in engineering and applied science are governed by differential or integral equations.

- ❖ The solutions to these equations would provide an exact, closed-form solution to the particular problem being studied.

- ❖ However, complexities in the geometry, properties and in the boundary conditions that are seen in most real-world problems usually means that an exact solution cannot be obtained or obtained in a reasonable amount of time.

- ❖ They are content to obtain approximate solutions that can be readily obtained in a reasonable time frame, and with reasonable effort. The FEM is one such approximate solution technique.

- ❖ The FEM is a numerical procedure for obtaining approximate solutions to many of the problems in engineering analysis.

- ❖ In the FEM, a complex region defining a continuum is discretized into simple geometric shapes called elements.

2. Definition of FEM

- The properties and the governing relationships are assumed over these elements and expressed mathematically in terms of unknown values at specific points in the elements called nodes.

- An assembly process is used to link the individual elements to the given system. When the effects of loads and boundary conditions are considered, a set of linear or nonlinear algebraic equations is usually obtained.

- Solution of these equations gives the approximate behavior of the continuum or system.

- The continuum has an infinite number of degree-of-freedom (DOF), while the discretized model has a finite number of DOF. This is the origin of the name, finite element method.

- The number of equations is usually rather large for most real-world applications of the FEM and requires the computational power of the digital computer. The FEM has little practical value if the digital computer were not available.

2. Definition of FEM – Features and Applications

Features

❖ The piecewise approximation of the physical field (continuum) on the finite elements provides good precision even with simple approximating functions. Simply increasing the number of elements can achieve increasing precision.

❖ The locality of the approximation leads to sparse equation systems for a discretized problem. This helps to ease the solution of problems having very large numbers of nodal unknowns. It is not uncommon today to solve systems containing a million primary unknowns.

Applications

- Mechanical/ Aerospace
- Structural Analysis
- Thermal/ Fluid Flow
- Electromagnetic
- Geomechanics
- Biomechanics
- …

2. Definition of FEM – Origin

- ❖ Courant in 1943 used minimum potential energy to study St. Venant torsion (Variational Method).
- ❖ The term finite element was first coined by Clough in 1960. In the early 1960s, engineers used the method for approximate solution of problems in stress analysis, fluid flow, heat transfer, and other areas.
- ❖ Argyris and Kelsey (1960) presented a paper.
- ❖ The first book on the FEM by Zienkiewicz and Chung was published in 1967.
- ❖ In the late 1960s and early 1970s, the FEM was applied to a wide variety of engineering problems. (on Mainframe Computers)
- ❖ The 1970s marked advances in mathematical treatments, including the development of new elements, and convergence studies.
- ❖ Microcomputers, pre- and postprocessors developed (1980s).
- ❖ Analysis of Large structural systems (1990s)

2. Definition of FEM – General purpose FE Package

- ❖ Most commercial FEM software packages originated in the 1970s (ABAQUS, ADINA, ANSYS, MARK) and 1980s (FENRIS, LARSTRAN 0, SESAM 0.).

- ❖ SAP (E.L. Wilson)
  - SAP I-V, SAP80, SAP90, SAP2000
  - Linear analysis
  - Natural frequency

- ❖ K. J. Bathe (Wilson's student)
  - Book with Wilson in 1976
  - NONSAP: material nonlinearity
  - ADINA (Automatic Dynamic Incremental Nonlinear Analysis

- ❖ P. V. Marcal (Brown University): MARC
- ❖ H. D. Habbit, Marcal's student
  - Worked at MARC corp.
  - Left to form own company HKS
  - ABAQUS

- ❖ NASTRAN (NASA Structural Analysis): MSC MASTRAN

- ❖ Search List of finite element software packages – Wikipedia for more information.

2. Definition of FEM – FEM Family

- Mixed FEM: extra independent variables are introduced as nodal variables during the discretization of a partial differential equation (PDE) problem.

- *hpk*-FEM: combines adaptively elements with variable size *h*, polygonal degree of the local approximation *p* and global differentiability of the local approximations *(k-1)* in order to achieve best convergence rates.

- X-FEM: The extended FEM, also known as generalized FEM (GFEM) or partition of unity method (PUM) is a numerical technique that extends the classical FEM approach by enriching the solution space for solutions to Des with discontinuous functions.

- Meshfree Method: Traditional simulation algorithms relied on a grid or a mesh, meshfree methods in contrast use the geometry of the simulated object directly for calculations.

- S-FEM: Smoothed FEMs are a particular class of numerical simulation algorithms for the simulation of physical phenomena.

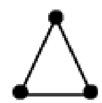
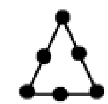
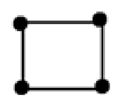
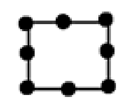
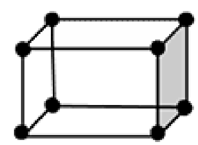
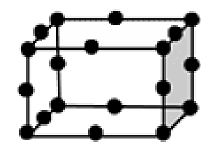
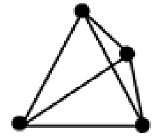
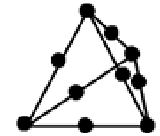
- Spectral Method: used in applied mathematics and scientific computing to numerically solve Dynamical Systems, often involving the use of the Fast Fourier Transform (FFT).

- Isogeometric Analysis: computational approach that offers the possibility of integrating FEA into conventional NURBS-based CAD design tools. (Hughes 205)

2. Definition of FEM – Element Types

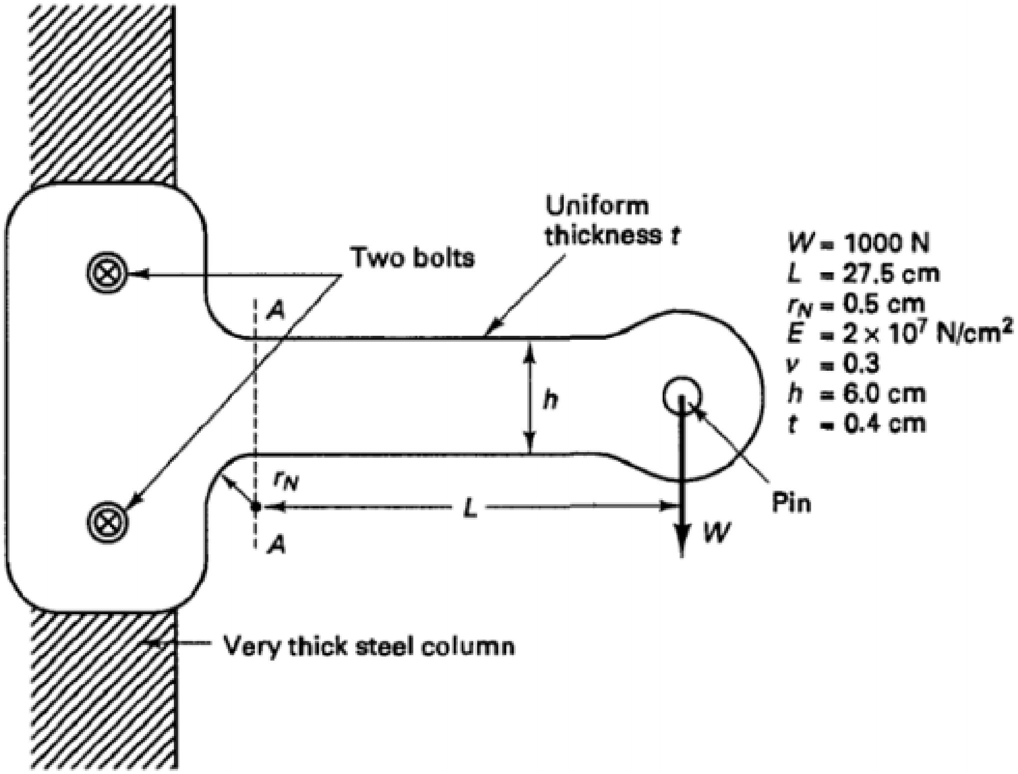
- The Finite Elements being used is decided when constructing the Structural Model from the Conceptual Model.

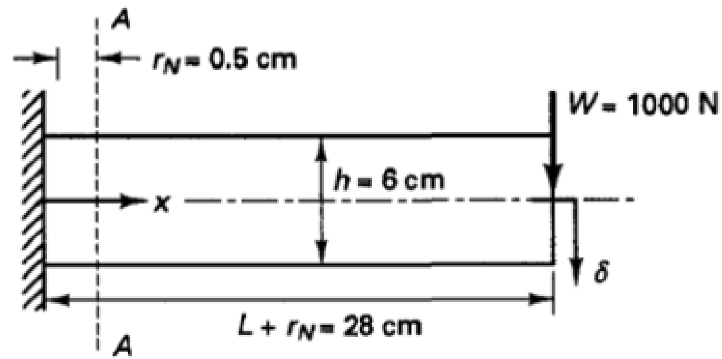
- The modelling accuracy depend significantly on the Finite Elements being used.

	Element Name	Element Shape First Order	 Second Order
1D Elements Line Element	Spring, Damper Beam, Truss		
2D Elements Surface Element	Shell, Plane2D		
			
3D Elements Volume element	Hexahedral		
	Tetrahedral		

2. Definition of FEM – A complex physical problem

❖ **Using a simple beam model**





$$M = WL$$

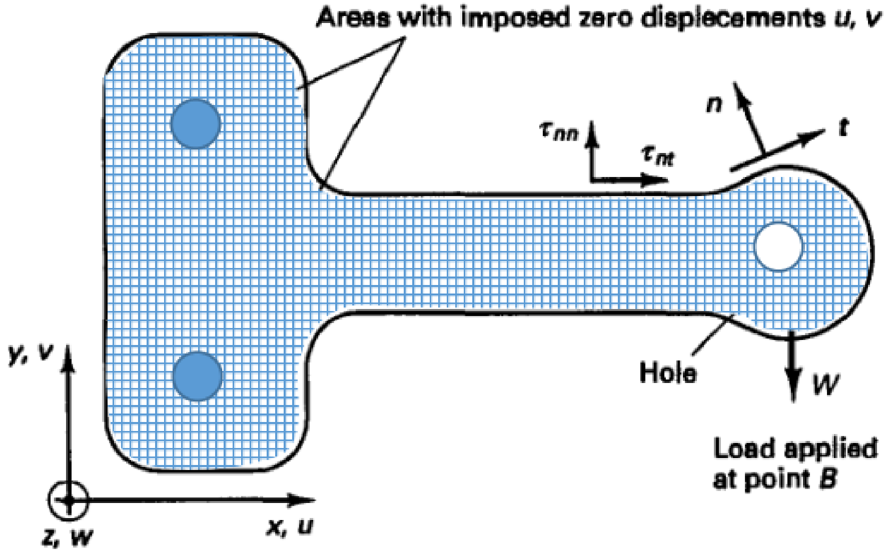
$$= 27{,}500 \text{ N cm}$$

$$\delta\big|_{\text{at load } W} = \frac{1}{3}\frac{W(L + r_N)^3}{EI} + \frac{W(L + r_N)}{\frac{5}{6}AG}$$

$$= 0.053 \text{ cm}$$

2. Definition of FEM – A complex physical problem

❖ **Using a 2D plane stress model – Reference model**



$$\delta|_{\text{at load } W} = 0.064 \text{ cm}$$

$$M|_{x=0} = 27{,}500 \text{ N cm}$$

Equilibrium equations (see Example 4.2)

$$\left.\begin{aligned} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= 0 \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} &= 0 \end{aligned}\right\} \text{ in domain of bracket}$$

$\tau_{nn} = 0,\ \tau_{nt} = 0$ on surfaces except at point B and at imposed zero displacements

Stress-strain relation (see Table 4.3):

$$\begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix}$$

E = Young's modulus, ν = Poisson's ratio

Strain-displacement relations (see Section 4.2):

$$\epsilon_{xx} = \frac{\partial u}{\partial x}; \qquad \epsilon_{yy} = \frac{\partial v}{\partial y}; \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

2. Definition of FEM – Problem Classification

Term

❖ Dependent Variables (Unknowns): The objective of analysis is to determine unknown function called dependent variables. When dependent variables are function of one independent variable (x), domain is line segment (1D)

❖ Continuity: Cm(w) is a domain if all its derivatives up to and including m-order exist and are continuous in D.

Problem Classification

❖ Boundary Value Problem
When dependent variables and their derivatives are required to take specified values on the boundary.

❖ Initial Value Problem (Dynamic Problem)
When dependent variables and their derivatives are generally time-dependent problems.

❖ Eigenvalue Problem
A differential equation governing the dependent unknown also contains an unknown parameter. Required to find both the dependent variable and the parameter such that the differential equations and the associated boundary conditions are satisfied.

2. Definition of FEM – Advantages

- Can readily handle complex geometry.
- Can handle complex analysis types: Vibration, transients, Nonlinear, Heat transfer, Fluids
- Can handle complex loading: Node-based loading (point loads), Element-based loading (pressure, thermal, inertial forces), Time or frequency dependent loading.
- Can handle complex restraints: Indeterminate structures can be analyzed.
- Can handle bodies comprised of nonhomogeneous materials: Every element in the model could be assigned a different set of material properties.
- Can handle bodies comprised of non-isotropic materials: Orthotropic, Anisotropic
- Special material effects are handled: Temperature dependent properties, Plasticity Creep
- Special geometric effects can be modeled: Large displacements, Large rotations, Contact (gap) condition.

2. Definition of FEM – Disadvantages

- Numerical problems
  - Computers only carry a finite number of significant digits.
  - Round off and error accumulation.
  - Can help the situation by not attaching stiff (small) elements to flexible (large) elements
- Susceptible to user-introduced modeling errors
  - Poor choice of element types.
  - Distorted elements.
  - Geometry not adequately modeled.
- Certain effects not automatically included
  - Buckling.
  - Large deflections and rotations.
  - Material nonlinearities.
  - Other nonlinearities.

Content

1. Course Introduction
2. Definition of FEM
    - Conceptual and Structural Models
    - Workflow
    - History and Software Package
    - FEM Family
    - Element Types
    - Problem Classification
    - Advantages and Disadvantages

thank you