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## Herramientas básicas

Fuente: J.E. Hopcroft, J.D. Ullman. Introduction to Automata Theory, Languages, and Computation. Addison-Wesley, (1979), pp. 10-11.

**1.1** In the tree of Fig. 1.4,

a) Which vertices are leaves and which are interior vertices? Leaves Vertice Root
b) Which vertices are the sons of 5? 7, 8
c) Which vertex is the father of 5? 3
d) What is the length of the path from 1 to 9? 4
e) Which vertex is the root? 1

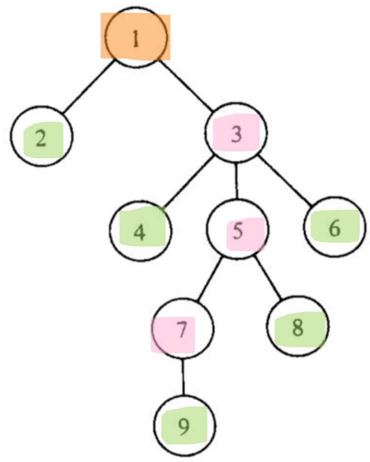


**Fig. 1.4** A tree.

**1.2** Prove by induction on $n$ that

a) $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$   b) $\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$

a) $\sum_{i=0}^{0} = 0 = \frac{0(0+1)}{2} = 0$   $\sum_{i=0}^{1} = 1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$$\sum_{i=0}^{n+1} \frac{n(n+1)}{2} + (n+1) = \frac{\overbrace{n(n+1)+2(n+1)}^{\text{Factor común}}}{2}$$

$$= \frac{n+1(n+2)}{2} = n+1\,\frac{((n+1)+1)}{2} = \boxed{\frac{n(n+1)}{2}}$$

b) $\sum_{i=0}^{k} i^3 = \left(\sum_{i=0}^{k} i\right)^2 = \frac{k^2(k+1)^2}{4}$

$\sum_{i=0}^{0} 0^3 = \frac{0^2(0+1)^2}{4} = (0)^2$   $\sum_{i=0}^{1} 1^3 = 0^3+1^3 = \frac{1^2(1+1)^2}{4} = \frac{1^2(4)}{4} = 1^2$

$$\sum_{i=0}^{k+1} i^3 = \sum_{i=0}^{k} i^3 + (k+1)^3 = \frac{k^2(k+1)}{4} + (k+1)^3 = \frac{k^2(k+1)}{4} + \frac{4(k^3+3k^2+3k+1)}{4}$$

$$= \frac{k^4+6k^3+13k^2+12k+4}{4} = \frac{(k+1)^2(k+2)^2}{4}$$

**1.7** Find the transitive closure, the reflexive and transitive closure, and the symmetr closure of the relation

$$\{(1, 2), (2, 3), (3, 4), (5, 4)\}.$$

$R^+ = [(1,2),(1,3),(1,4),(2,3),(2,4),(3,4),(5,4)]$

$R^* = [(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4),(5,4)]$

$R^\wedge = [(1,2),(2,1),(2,3),(3,2),(3,4),(4,3),(5,4),(4,5)]$

**S 1.8** Prove that any equivalence relation $R$ on a set $S$ partitions $S$ into disjoint equivalence classes.

Si $x \in S$, $x \in [x]$, entonces la unión de todas las clases de equivalencia es igual a $S$

Si $x, y, z \in S$ y $x \in [y] \cap [z]$, entonces $x \in [y]$, así que $x \sim y$ y por simetría $y \sim x$, también $x \in [z]$, entonces $x \sim z$, por ser transitiva entonces $y \sim z$. entonces $y \in [z]$. que significa que $[y] = [z]$, entonces si dos clases tienen un elemento en común, en realidad son la misma.