

Notes: This is a closed book and closed notes exam. The maximal score on the real exam will be 100 points.

There are many ways in which any single problem can be solved. The solutions herein are just one possible way to tackle the given problems.

Time: 50 minutes

All written work handed in by the student is considered to be
their own work, prepared without unauthorized assistance.

The University Code of Conduct

"The core values of The University of Texas at Austin are learning, discovery, freedom, leadership, individual opportunity, and responsibility. Each member of the university is expected to uphold these values through integrity, honesty, trust, fairness, and respect toward peers and community. As a student of The University of Texas at Austin, I shall abide by the core values of the University and uphold academic integrity."

"I agree that I have complied with the UT Honor Code during my completion of this exam."

Signature:

1.1. DEFINITIONS.

Problem 1.1. (10 points) Provide the definition of *mutually exclusive* events.

Problem 1.2. (10 points) Provide the definition of *independent* events.

1.2. TRUE/FALSE QUESTIONS.

Problem 1.3. (2 pts) A party of n people can arrange themselves around a circular table in $n!$ ways assuming rotations are interchangeable. *True or false?*

Problem 1.4. (2 pts) For every positive integer n , the number

$$\frac{(n+2)!}{n!}$$

is even. *True or false?*

Problem 1.5. (2 pts) If events E and F are disjoint, they are necessarily independent. *True or false?*

Problem 1.6. (2 pts) If events E and F are independent, then E^c and F are independent as well. *True or false?*

Problem 1.7. (2 pts) Let E and F be any two events. Then

$$\mathbb{P}[E \cup F] \geq \mathbb{P}[E] + \mathbb{P}[F].$$

True or false?

1.3. Free-response problems. Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.8. (4 pts) One tosses a coin three times and observes the sequence of heads (H) and tails (T) that appears. Write down the appropriate sample space S for this “experiment”.

(2 points) Let A be the event that two or more heads appear consecutively. Write down the event A as a set of elementary outcomes from S .

(2 points) Let B be the event that all tosses are the same. Write down the event B as a set of elementary outcomes from S .

(2 points) Let C denote the event that only heads appear. Express C in terms of A and B .

Problem 1.9. (8 points) A certain soccer team wins (W) with probability 0.6, loses (L) with probability 0.3 and ties (T) with probability 0.1. The team plays three games over one weekend. The outcomes of the games are assumed to be independent.

- (i) (3 pts) Determine the elements (elementary outcomes) of the event A that the team wins at least twice and does not lose.
- (ii) (5 pts) Find $\mathbb{P}[A]$.

Problem 1.10. (10 points) A box contains three coins; one coin is fair, one coin is two-headed and one coin is weighted so that the probability of heads is $1/3$. A coin is selected at random and tossed. Find the probability that the outcome of this coin-toss is heads.

Problem 1.11. (30 points) A piggy bank contains coins of three different types: T_1, T_2 and T_3 .

There are twice as many type T_1 coins as type T_2 coins, and twice as many type T_2 coins as type T_3 coins. The coins are indistinguishable to touch.

- (i) (10 points) A coin is extracted from the piggy bank at random. Let the probability that the coin is of type T_i be denoted by p_i for $i = 1, 2, 3$. Find p_1, p_2 and p_3 .
 - (ii) (10 points) Coins of type T_1 are fair, coins of type T_2 come up heads (H) when tossed with probability $3/10$, and coins of type T_3 come up heads (H) when tossed with probability $1/10$. A coin is drawn from the piggy bank at random and tossed. What is the probability that the result of the coin toss was heads?
 - (iii) (10 points) A coin is drawn from the piggy bank at random and tossed. It is observed that the result of the coin toss was tails (T). What is the probability that the coin was of type T_3 ?
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1.4. MULTIPLE CHOICE QUESTIONS.

Problem 1.12. (5 pts) A class has 12 boys and 4 girls. If three students are selected at random from this class, what is the probability that they are all boys?

- (a) $1/4$
- (b) $5/9$
- (c) $11/28$
- (d) $17/36$
- (e) None of the above

Problem 1.13. (5 pts) A pair of dice is thrown. Find the probability that the sum of the outcomes is 10 or greater if a 5 appears on the first die.

- (a) $1/6$
- (b) $1/4$
- (c) $1/3$
- (d) $1/2$
- (e) None of the above