

M339D: April 8th, 2022.

The Binomial Asset Pricing Model.

$S(0)$... the observable initial asset price

0

T

time horizon (i.e., exercise or expiration date of an option)

$S(0)$
ROOT
NODE

$$S_u = u \cdot S(0)$$

up node

$$S_d = d \cdot S(0)$$

down node

h ... length of a single period

one period:

$S(T) = S(h)$... a r.v. denoting the time-T stock price w/ two possible values: S_u and S_d

Market Model.

- riskless asset: @ the certifir r
- risky asset: continuous dividend paying asset w/ dividend yield δ

Imagine investing in one share of stock @ time 0:

wealth

$S(0)$

$$S_u = u \cdot S(0)$$

$$e^{\delta \cdot h} \cdot S_u = e^{\delta \cdot h} \cdot u \cdot S(0)$$

$$S_d = d \cdot S(0)$$

$$e^{\delta \cdot h} \cdot S_d = e^{\delta \cdot h} \cdot d \cdot S(0)$$

h

The no-arbitrage condition:

$$e^{\delta h} \cdot d \cdot S(0) < S(t) e^{rh} < e^{\delta h} \cdot u \cdot S(0) \quad | : e^{\delta h}$$

$$d < e^{(r-\delta)h} < u$$

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Problem Set #9

Binomial option pricing.

Problem 9.1. (2 points) In the setting of the one-period binomial model, denote by i the effective interest rate per period. Let u denote the “up factor” and let d denote the “down factor” in the stock-price model. If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage.

Actually: $d < e^{rh} \cdot e^{-\delta h} < u$

$$d < (1+i) e^{-\delta h} < u$$

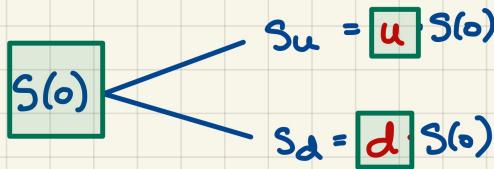
False!

Problem 9.2. In our usual notation, which of the parameter choices below creates a binomial model with an arbitrage opportunity?

- (a) $u = 1.18$, $d = 0.87$, $r = 0.05$, $\delta = 0$, $h = 1/4$
- (b) $u = 1.23$, $d = 0.80$, $r = 0.05$, $\delta = 0.06$, $h = 1/2$
- (c) $u = 1.08$, $d = 1$, $r = 0.05$, $\delta = 0.04$, $h = 1$
- (d) $u = 1.28$, $d = 0.78$, $r = \delta$, $h = 2$
- (e) None of the above.

Binomial Option Pricing.

Stock price tree.



populating
the tree

We want to price a European-style derivative security w/ the exercise date @ the end of the period.

It is completely determined by its payoff function: v(·)

e.g., for the call: $v_c(s) = (s - K)_+$

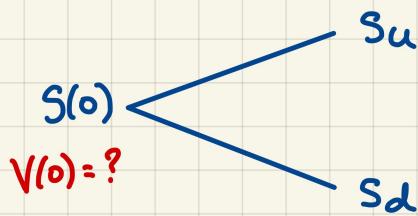
for the put: $v_p(s) = (K - s)_+$

⇒ The payoff the derivative security is a random variable :

$$V(T) := v(S(T))$$

payoff
 $V_u = v(S_u)$

e.g., for a call option
 $V_u = (S_u - K)_+$



$V_d = v(S_d)$

$V_d = (S_d - K)_+$

populating
pricing

In the binomial model, any derivative security can be replicated w/ a portfolio of this form:

- △ shares of stock {
 - △ > 0 buying
 - △ = 0 "nothing"
 - △ < 0 shorting }
 - B @ the ccfir r {
 - B > 0 lending (buying a bond)
 - B = 0 "nothing"
 - B < 0 borrowing (issuing a bond) }
- ↑
time · 0 holdings