M378K: 10/20/2025

The $\chi^2(n)$-distribution.

- If $Y \sim N(0,1)$, then $\boxed{W = Y^2 \sim \chi^2(df=1)}$

Its density is

$$f_W(w) = \frac{1}{\sqrt{2\pi w}} \cdot e^{-\frac{w}{2}} \cdot \mathbb{1}_{(0,\infty)}(w)$$

Its mgf is

$$\boxed{m_W(t) = (1-2t)^{-1/2} = \frac{1}{\sqrt{1-2t}}}$$

- $Y_1 \sim N(0,1)$, $Y_2 \sim N(0,1)$ independent

Set $\boxed{W = Y_1^2 + Y_2^2} \sim \chi^2(df=2)$

Let's find its mgf:

$$m_W(t) = m_{Y_1}(t) \cdot m_{Y_2}(t)$$

$$= \frac{1}{\sqrt{1-2t}} \cdot \frac{1}{\sqrt{1-2t}} = \frac{1}{1-2t}$$

From the HW, we know that $\boxed{W \sim E(\tau=2)}$

So, $\chi^2(df=2)$ is the same as $E(\tau=2)$.

Def'n. The χ^2-dist w/ n degrees of freedom is the dist'n of the sum

$$W = Y_1^2 + Y_2^2 + \cdots + Y_n^2$$

where

$Y_i \sim N(0,1)$ for $i = 1..n$ an independent.

We write

$$\boxed{W \sim \chi^2(df=n) = \chi^2(n)}$$

Note: $m_W(t) = \left(\frac{1}{\sqrt{1-2t}}\right)^n = (1-2t)^{-\frac{n}{2}} = \left(\frac{1}{1-2t}\right)^{\frac{n}{2}}$

The Gamma Distribution.

Def'n. A random variable Y is said to have the gamma distribution w/ parameters $k>0$ and $\tau>0$ if its mgf is of the form

$$m_Y(t) = \left(\frac{1}{1-\tau t}\right)^{k}$$

scale parameter

We write $\boxed{Y \sim \Gamma(k,\tau)}$.

k... shape parameter

Note: $\boxed{\begin{array}{l} \mathbb{E}[Y] = k\cdot\tau \\ Var[Y] = k\cdot\tau^2 \end{array}}$

Q: Say that $Y \sim \Gamma(1,\tau)$. What's another name for it?

$Y \sim E(\tau)$ ☺

Q: Say that $Y \sim \Gamma(\frac{n}{2}, 2)$. What's another name for it?

$Y \sim \chi^2(n)$ ☺

Q: $Y_1 \sim \Gamma(k_1,\tau)$, $Y_2 \sim \Gamma(k_2,\tau)$ independent

$$\boxed{Y_1+Y_2 \sim \Gamma(k_1+k_2,\tau)}$$