UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 4

CAPM.

Provide your final answer only to the following problem(s):

Problem 4.1. (2 points) Under the **CAPM**, the expected return and the required return of the market portfolio are equal. *True or false?*

Solution: TRUE

Problem 4.2. (2 points) Assume CAPM. Then, the expected return of every available security equals its required return with respect to the market portfolio. *True or false?*

Solution: TRUE

Problem 4.3. (2 points) Assume the assumptions of CAPM. Then, the slope of the **capital market line (CML)** equals the Sharpe ratio of the market portfolio. *True or false?*

Solution: TRUE

Problem 4.4. (2 points) Under the **CAPM**, the expected return and the required return of the market portfolio are equal. *True or false?*

Solution: TRUE

Problem 4.5. (2 points) You are given the following information about stock X and a portfolio P:

- The annual effective risk-free rate is 5%.
- The portfolio's expected return is 0.10 and the its volatility is 0.2.
- The expected return of stock X is 6% and its volatility is 0.3.
- The correlation between the returns of stock X and the portfolio P is 0.2.

Then, the investor holding portfolio P should invest in stock X. *True or false?*

Solution: FALSE
The β for the stock X equals

$$\beta_X = \frac{0.3(0.2)}{0.2} = 0.3.$$

So, the stock X has a required return equal to

$$r_X = r_f + \beta_X(\mathbb{E}[R_m] - r_f) = 0.05 + (0.3)(0.10 - 0.05) = 0.05 + 0.015 = 0.065.$$

Since the expected return is smaler than the required return, one should not invest in stock X.

Problem 4.6. (2 points) Under the **CAPM**, the *beta* of the market portfolio is equal to one. *True or false?*

Solution: TRUE

Problem 4.7. (2 points) Assume the **CAPM** assumptions holds. Investors are only allowed to invest in stock A and stock B. One investor invests \$2,000 in stock A and \$3,000 in stock B. Another investor has \$5,000 invested in stock A. Then, necessarily, he has \$7,500 invested in stock B. *True or false?*

Solution: TRUE

Problem 4.8. (2 points) Assume the assumptions of **CAPM** hold. The risk premium of a zero-beta investment equals zero. *True or false?*

Solution: TRUE

Problem 4.9. (5 points) Assume the **Capital Asset Pricing Model** holds.

You are given the following information about stock X, stock Y, and the market:

- The required return and volatility for the market portfolio are 0.08 and 0.25, respectively.
- The required return and volatility for the stock X are 0.06 and 0.4, respectively.
- The correlation between the returns of stock X and the market is -0.25.
- The volatility of stock Y is 0.3.
- The correlation between the returns of stock Y and the market is 0.2.

Calculate the required return for stock Y.

(a) 0.0489
(b) 0.0542
(c) 0.0691
(d) 0.0734
(e) None of the above.

Solution: (c)
The βs of stocks X and Y are

$$\beta_X = \frac{0.4(-0.25)}{0.25} = -0.4,$$
$$\beta_Y = \frac{0.3(0.2)}{0.25} = 0.24.$$

So, the required return of stock X must satisfy

$$0.06 = r_X = r_f + (-0.4)(0.08 - r_f) \quad \Rightarrow \quad 0.06 = r_f - 0.032 + 0.4r_f$$
$$\Rightarrow \quad 1.4r_f = 0.092 \quad \Rightarrow \quad r_f = 0.0657.$$

Finally, the required return of stock Y equals

$$r_Y = 0.0657 + 0.24(0.08 - 0.0657) = 0.0691.$$

Provide your complete solution(s) to the following problem(s):

Problem 4.10. (10 points) Assume the **CAPM** holds.

Let the risk-free interest rate be 0.05 and let the expected return of a market portfolio be equal to 0.10.

Suppose that stock X has $\beta_X = 1.4$ and that stock Y has $\beta_Y = 0.8$. Using the risk-free asset, stock X, and stock Y, you create a portfolio such that the weight given to X equals the weight given to Y while the weight of the risk-free asset is 0.4. What is the expected return of this portfolio?

Solution: The β of the risk-free asset is zero. Hence, the β of the portfolio is

$$\beta_P = 0.3\beta_X + 0.3\beta_Y = 0.3(2.2) = 0.66.$$

So, realizing that the expected return of the portfolio equals its required return, we get

$$\mathbb{E}[R_P] = r_f + \beta_P(r_m - r_f) = 0.05 + 0.66(0.05) = 1.66(0.05) = 0.083.$$

Problem 4.11. (9 points) Suppose that your market consists exactly of the five different stocks whose information is given in the following table:

Stock	Price per share	Number of shares outstanding (in 10^6)
1	10	12
2	20	14
3	30	10
4	40	4
5	50	4

What are the portfolio weights in the market portfolio?

Solution:

The total market value of the shares of the five stocks are (in millions of \$):

$$MV_1 = 10 * 12 = 120,$$
$$MV_2 = 20 * 14 = 280,$$
$$MV_3 = 30 * 10 = 300,$$
$$MV_4 = 40 * 4 = 160,$$
$$MV_5 = 50 * 4 = 200.$$

The total market value of all the shares in the market is (in millions of \$):

$$MV = 120 + 280 + 300 + 160 + 200 = 1060.$$

The weights of the five stocks in the market portfolio are

$$w_1 = 120/1060 = 6/53, \quad w_2 = 280/1060 = 14/53, \quad w_3 = 300/1060 = 15/53,$$
$$w_4 = 160/1060 = 8/53, \quad w_5 = 200/1060 = 10/53.$$

Problem 4.12. (10 points) Consider a portfolio of four stocks as displayed in the following table:

Stock	Weight	Beta
1	0.1	1.2
2	0.2	1.4
3	0.5	1.0
4	0.2	β_4

The expected return of the portfolio is 0.12, the annual effective risk-free rate is 0.04, and the market risk premium is 0.06.

Assuming the **Capital Asset Pricing Model**, calculate β_4.

Solution: From the conditions on the first three stocks, we get

$$r_1 = 0.04 + 1.2(0.06) = 0.112,$$
$$r_2 = 0.04 + (1.4)(0.06) = 0.124,$$
$$r_3 = 0.04 + 1.0(0.06) = 0.1.$$

Therefore, from the condition on the portfolio's expected return, we get

$$0.1(0.112) + 0.2(0.124) + 0.5(0.1) + 0.2r_4 = 0.12 \quad \Rightarrow \quad r_4 = 0.17$$

Finally, we have

$$\beta_4 = \frac{0.17 - 0.04}{0.06} = \frac{13}{6} \approx 2.17.$$