# M339 D: February 17th, 2023.

## European Put Options.

Usually, a right but not an obligation to sell!


option written          exercise date

**At time 0**: The writer and the buyer of the put agree on:

- the underlying asset: $S(t)$, $t \geq 0$,
- the strike/exercise price $K$,
- the exercise date $T$

The put premium $V_P(0)$ is paid by the put's buyer to the put's writer.

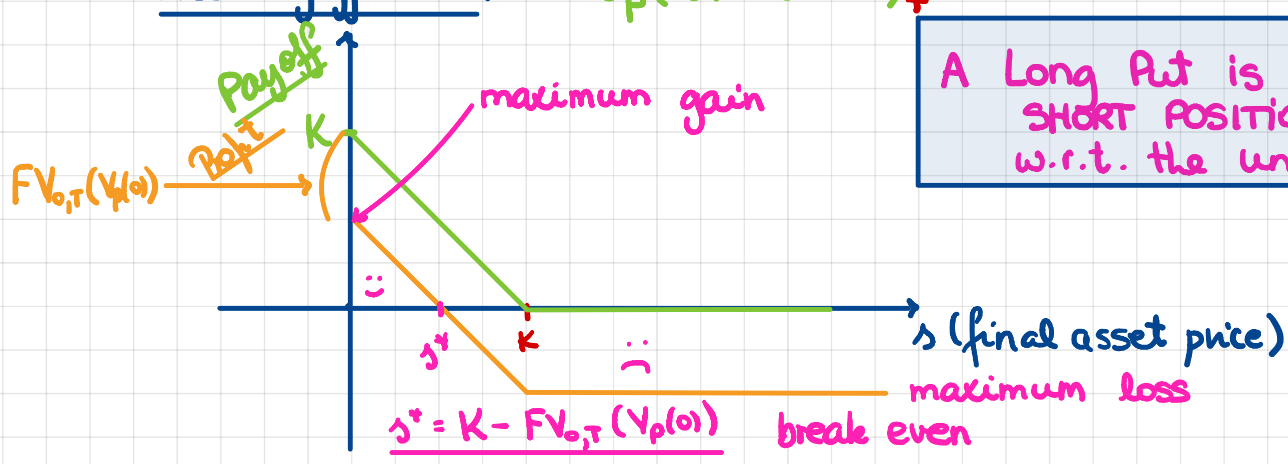
**At time T**:
- The put's owner has a right but not an obligation to sell one unit of the underlying asset for the strike price $K$.
- The put's writer is obligated to do what the put's owner decides.

The put owner's optimal behavior is:

| | | | Payoff |
|---|---|---|---|
| IF | $K > S(T)$, | then exercise | $K - S(T)$ |
| IF | $K \leq S(T)$, | then do NOT exercise | $0$ |

**The Payoff**: $V_P(T) = \max(K - S(T), 0) = (K - S(T))_+$

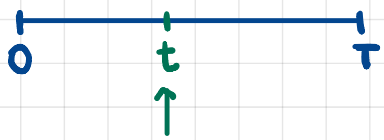
**The Payoff Fction**: $v_P(s) = (K - s)_+$




A Long Put is a SHORT POSITION w.r.t. the underlying.

## Moneyness.

Consider an option written @ time $0$ w/ exercise date @ time $T$




Imagine the cashflow that would happen to the option's owner should they exercise the option @ that time $t$.

call: $S(t) - K$

put: $K - S(t)$

If cashflow is

- $> 0$ we say the option is in the money
- $= 0$ we say the option is at the money
- $< 0$ we say the option is out of the money

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## Problem Set #6

European put options.

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**Problem 6.1.** The initial price of the market index is \$900. After 3 months the market index is priced at \$915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of \$930, is $\boxed{\$8.00.}$ What is the profit at expiration for a **long** put?

(a) \$15.00 loss
(b) \$6.90 loss
(c) \$6.90 gain
(d) \$15.00 gain
(e) None of the above.

$\longrightarrow$ :

$$(930-915)_+ = 15$$

$$8\cdot(1+0.004)^3 = \underline{8.10}$$

$$\overline{\phantom{xxxx}+6.90}$$

**Problem 6.2.** Sample FM(DM) #12
Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4% **convertible semiannually**, and that the put costs 74.20 now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

interest error → A. 922.83
B. 924.32
→ C. 1,000.00
call → D. 1,075.68
call + interest error → E. 1,077.17

: We're looking for the break-even price.

$$S^* = K - FV_{0,T}(V_P(0))$$

$$S^* = 1000 - 74.20(1.02) = \underline{924.32}$$

**Problem 6.3.** Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000−cartons' worth in six months. His total costs are $12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of $13 per carton and another with the strike price of $15 per carton. Their premiums are $0.15 and $0.18, respectvely.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals $14. What would Farmer Shaun's profit be if he had decided to hedge using the $13-strike put versus his profit if he had decided to use the $15-strike put to hedge?

Focus on Payoff (w/ production costs)

unhedged: $S(T)$

hedge: $(K-S(T))_+$

$+$

total hedged: $S(T)+(K-S(T))_+ =$

$$= \begin{cases} K, & \text{if } K > S(T) \\ S(T), & \text{if } K \leq S(T) \end{cases}$$

$= \max(K, S(T))$ Floor.

\$13-strike put: $\max(13,14) - 12 - 0.15(1.04) = 1.844$

\$15-strike put: $\max(15,14) - 12 - 0.18(1.04) = 2.812$