M339W: October 13th, 2021.

41. Assume the Black-Scholes framework. Consider a 1-year European contingent claim on a stock.

$T=1$

You are given:

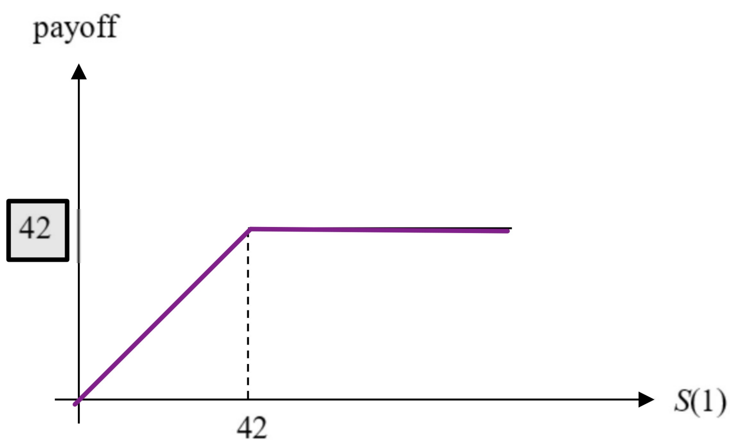
(i) The time-0 stock price is 45. $\boxed{S(0)=45}$

(ii) The stock's volatility is 25%. $\sigma=0.25$

(iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%. $\boxed{\delta=0.03}$

(iv) The continuously compounded risk-free interest rate is 7%. $\boxed{r=0.07}$

(v) The time-1 payoff of the contingent claim is as follows:



Calculate the time-0 contingent-claim elasticity.

(A) 0.24

(B) 0.29

(C) 0.34

(D) 0.39

(E) 0.44

$\Omega(S(0),0)=?$

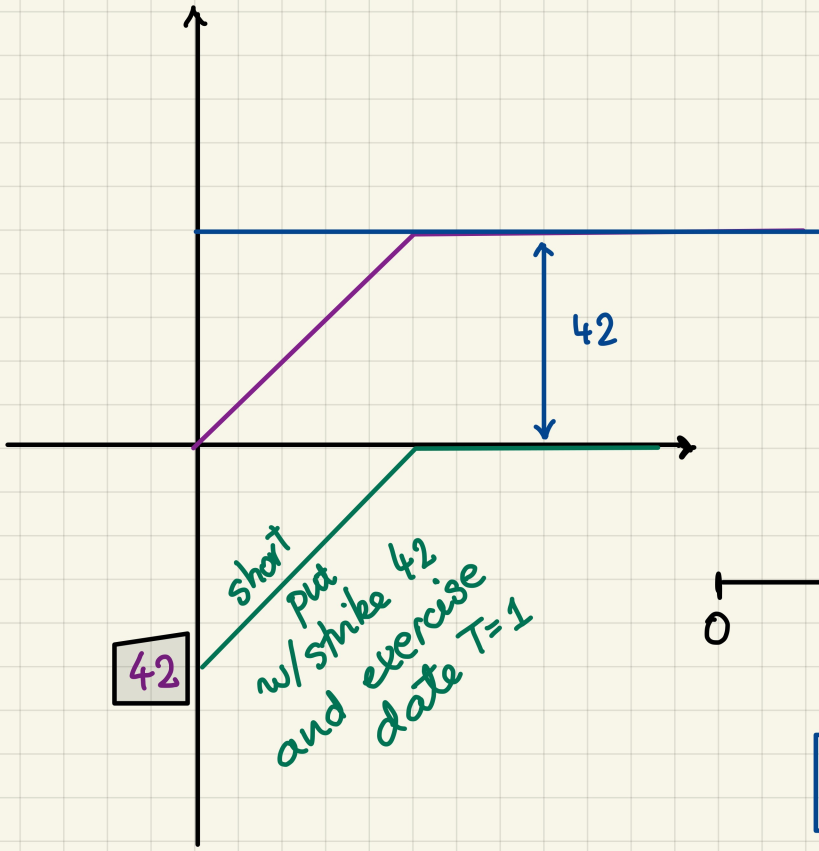
$$\Omega(S(0),0)=\frac{\Delta_{cc}(S(0),0)\,S(0)}{v_{cc}(S(0),0)}$$

by def'n

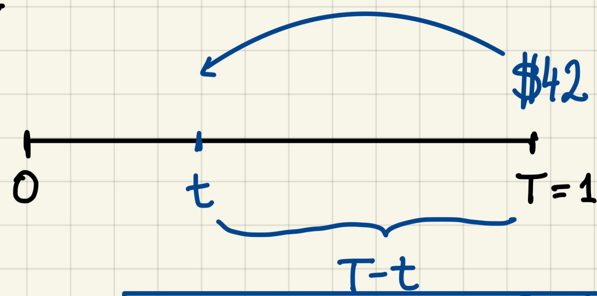
w/ cc... the shorthand for contingent claim.

First step:

Replicate this CC in terms of derivative securities whose B-S prices and Δ's we know the formulae for.



a zero-coupon bond redeemable @ time 1 for 42



$$\boxed{v_B(s,t) = 42e^{-r(T-t)}}$$

We designed our replicating portfolio for the CC:

$$v_{CC}(s,t) = v_B(s,t) - v_P(s,t)$$

$$\frac{\partial}{\partial s}\Big| \quad \Delta_{CC}(s,t) = -\Delta_P(s,t) = +\left(+e^{-\delta(T-t)}N(-d_1(s,t))\right)$$

$$\boxed{\Delta_{CC}(s,t) = e^{-\delta(T-t)}\cdot \underline{N(-d_1(s,t))}} \quad ✓$$

$$v_{CC}(s,t) = 42e^{-r(T-t)} - \left(42e^{-r(T-t)}\cdot N(-d_2(s,t)) - se^{-\delta(T-t)}\cdot N(-d_1(s,t))\right)$$

$$\boxed{v_{CC}(s,t) = 42e^{-r(T-t)}\cdot \underline{N(d_2(s,t))} + se^{-\delta(T-t)}\underline{N(-d_1(s,t))}}$$

<u>At time 0:</u>

$$d_1(S(0),0) = \frac{1}{0.25\sqrt{1}}\left[\ln\left(\frac{45}{42}\right) + \left(0.07-0.03+\frac{(0.25)^2}{2}\right)\cdot 1\right]$$

$$\boxed{d_1(S(0),0) = 0.56097}$$