

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 2

Prerequisite material.

Problem 2.1. (10 points) Your goal is to price a call option on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

- (i) Each period is three months.
- (ii) $u_F/d_F = 5/4$, where u_F is one plus the rate of gain on the futures price if it goes up, and d_F is one plus the rate of loss if it goes down.
- (iii) The risk-neutral probability of an up move is $1/2$.
- (iv) The initial futures price is 80.
- (v) The continuously compounded risk-free interest rate is 5%.

Find the price of a half-year, 85-strike European call option on the futures contract.

Problem 2.2. (5 points) You are required to price a one-year, yen-denominated currency option on the USD. The exchange rate over the next year is modeled using a forward binomial tree with the number of periods equal to 4. Assume that the volatility of the exchange rate equals 0.1.

The continuously compounded risk-free interest rate for the yen equals 0.05, while the continuously compounded risk-free interest rate for the USD equals 0.02. What is the value of the so-called up factor u in the resulting forward binomial tree?

Problem 2.3. (5 points) The evolution of a market index over the following year is modeled using a four-period binomial tree. We are given that the current value of the market index equals \$144, that its volatility equals 0.25, and that it pays dividends continuously.

You are tasked with constructing a four-period forward tree for the evolution over the following year of the forward price of the above market index with delivery at time-2.

What is the **down** factor d_F in the forward price tree for the **futures prices** on the stock?

Problem 2.4. (10 points) The evolution over the following year of futures prices with delivery at time 2 on a certain commodity are modeled using a one-period forward binomial tree. The volatility is given by 0.2. The continuously compounded risk-free interest rate is given to be 0.05.

Let the current futures price equal \$50. What is the price of a one-year, \$45-strike European put on the futures contract described above?

Problem 2.5. (10 points) Consider a continuous-dividend-paying stock whose current price is \$100. The stock's dividend yield is 0.02 and its volatility is 0.25. The evolution of the price of this stock over the next half-year is modeled using a two-period forward binomial tree.

Let the continuously-compounded, risk-free interest rate be 0.04.

What is the price of a \$95-strike, half-year **American** put option on the above stock?

Problem 2.6. (10 points) Today's price of a continuous-dividend-paying stock is observed to be \$80. Its volatility is 0.2 and its dividend yield is 0.01. The evolution of this stock price over the following year is modelled using a three-period binomial tree such that the stock price can either go up by 2% or down by 1% at the end of every period. The continuously-compounded, risk-free interest rate is 0.04.

What is the price of an \$82-strike European put option on the above stock?