UNIVERSITY OF TEXAS AT AUSTIN

The binomial asset-pricing model. Extra-credit homework assignment 5

Binomial pricing of European options.

Please, provide your **complete solutions** to the following problems:

Problem 5.1. (2 points) In the setting of the one-period binomial model, denote by i the **effective** interest rate **per period**. Let u denote the "up factor" and let d denote the "down factor" in the stock-price model. If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage. *True or false? Why?*

Problem 5.2. (2 points) In the binomial asset pricing model, the number of shares Δ of the underlying asset in the replicating portfolio for a **put** option is always positive. *True or false? Why?*

Problem 5.3. (2 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the Δ in the replicating portfolio of a single call option on that stock never exceeds 1. *True or false? Why?*

Problem 5.4. (2 points) In the binomial asset pricing model, the replicating portfolio for a put option has a bond investment which is equivalent to borrowing at the risk-free interest rate. *True or false? Why?*

Problem 5.5. (2 points) In our usual notation, let $S(0) = 40, r = 0.08, \sigma = 0.3, \delta = 0$. You need to construct a 2−period forward binomial tree for the above stock with every period in the tree of length $h = 0.5$. Then, $u > 1.45$. *True or false? Why?*

Problem 5.6. (10 points) In our usual notation, which of the parameter choices below creates a binomial model with an arbitrage opportunity?

(a) $u = 1.18, \quad d = 0.87, \quad r = 0.05, \quad \delta = 0, \quad h = 1/4$
(b) $u = 1.23, \quad d = 0.80, \quad r = 0.05, \quad \delta = 0.06, \quad h = 1/2$
(c) $u = 1.08, \quad d = 1, \quad r = 0.05, \quad \delta = 0.04, \quad h = 1$
(d) $u = 1.28, \quad d = 0.78, \quad r = \delta, \quad h = 2$
(e) None of the above.

Problem 5.7. (4 pts) Consider a non-dividend-paying stock currently priced at \$100 per share.

The price of this stock in one year is modeled using a one-period binomial tree under the assumption that the stock price can either go up to 110 or down to 90.

Let the continuously compounded, risk-free interest rate equal 0.04. What is the risk-neutral probability of the stock price going up?

Problem 5.8. (5 points) The current stock price is 40 per share. The price at the end of a three-month period is modeled with a one-period binomial tree so that the stock price can either increase by \$10, or decrease by \$4. The stock pays dividends continuously with the dividend yield 0.04.

The continuously compounded, risk-free interest rate is 0.05.

What is the stock investment in a replicating portfolio for three-month, \$40-strike European **straddle** on the above stock?

Problem 5.9. (6 points) Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded, risk-free interest rate is 0.06.

Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

Problem 5.10. (5 points) Assume that the a stock price is modeled using a one-period forward binomial tree with the length of a single period equal to three months. According to this model, the stock price can take either the value of \$55, or the value of \$40 in exactly three months. Calculate the volatility of the stock price.

Problem 5.11. (10 points) The current price of a continuous-dividend-paying stock is \$100 per share. Its volatility is given to be 0.2 and its dividend yield is 0.03.

The continuously compounded risk-free interest rate equals 0.06.

Consider a \$95-strike European put option on the above stock with nine months to expiration. Using a three-period forward binomial tree, find the price of this put option.