

Name:

M339J/M389J: Probability Models with Actuarial Applications

The University of Texas at Austin

Practice Problems for In-Term Exam II

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Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.

2.1. Prior material. Please, solve problems **1.8, 1.9, 1.10, 1.11, 1.12, 1.21, and 1.22** from the practice problem set posted for the first in-term exam on the course website.

2.2. TRUE/FALSE QUESTIONS.

Problem 2.1. (2 points) Any negative binomial distribution has the memoryless property. *True or false?*

Problem 2.2. (2 points) With our usual assumptions in place, assume that the claim count follows the Poisson distribution with parameter $\lambda = 10$.

The severities of individual claims are such that on average half of the claims are expected to have claim amounts equal to 50 and the other half are expected to have the claim amounts equal to 100.

Then, the total losses have the expected value 750. *True or false?*

Problem 2.3. (2 points) In the compound aggregate loss model, with our usual notation and with the usual assumptions in place, we have that the claim count distribution is binomial with $m = 40$ and $q = 1/4$, and that the p.m.f. of the individual losses is

$$p_X(10) = p_X(20) = 0.3, p_X(50) = 0.4.$$

Then, $\mathbb{E}[S] \leq 290$. *True or false?*

2.3. MULTIPLE CHOICE QUESTIONS.

Problem 2.4. (5 points) *Source: Sample P exam, Problem #262.*

The number of traffic accidents occurring on any given day in Coralville is Poisson distributed with mean 5. The probability that any such accident involves an uninsured driver is 0.25, independent of all other such accidents. Calculate the probability that on a given day in Coralville there are no traffic accidents that involve an uninsured driver.

- (a) 0.007
- (b) 0.010
- (c) 0.124
- (d) 0.237
- (e) 0.287

Problem 2.5. (5 points) A group dental policy has a negative binomial claim count distribution with mean 20 and variance 100. What is the probability that there is at most 1 claim?

- (a) 0.0016
- (b) 0.0024
- (c) 0.0032
- (d) 0.0048
- (e) None of the above.

Problem 2.6. (5 points) Let the number of typos in the new edition of the "*Lord of the Rings*" trilogy be denoted by N and modeled using the Poisson distribution with mean 80. Some typos involve elves' names and others do not involve the elves' names. The number of typos and the type of the typo are assumed to be independent.

Assume that the probability that an observed typo involves an elf's name equals $1/5$.

Find the probability that the number of typos involving elves' names is 2, given that the **total** number of typos in the trilogy equals 5.

- (a) 0.0406
- (b) 0.0842
- (c) 0.1632
- (d) 0.2048
- (e) None of the above.

Problem 2.7. (8 points) Let N be an \mathbb{N}_0 -valued random variable from the $(a, b, 0)$ class. You are given that

$$p_0 = 0.049787, \quad p_1 = 0.149361, \quad \text{and} \quad p_2 = 0.224042.$$

Find p_6 .

- (a) 0.04
- (b) 0.05
- (c) 0.06
- (d) 0.07
- (e) None of the above.

2.4. Free-response problems. Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 2.8. Let the number of losses be negative binomial with mean 6 and variance 24. The losses have the Pareto distribution with $\theta = 100$ and $\alpha = 6$. The number of losses and loss amounts are assumed to be independent. There is an ordinary deductible of 25 on individual losses. What is the variance of the number of claims?

Problem 2.9. Let the independent random variables X_1, X_2 and X_3 all have the following probability mass function:

$$p_{X_1}(-1) = 1/4, \quad p_{X_1}(0) = 1/2, \quad p_{X_1}(1) = 1/4.$$

Let $X = 1 + X_1 X_2 X_3$. What is the probability generating function of X ?

Problem 2.10. Let the random variable X have the moment generating function given by

$$M_X(t) = (1 - 2t)^{-3}$$

Define the random variable \tilde{X} as $\tilde{X} = 5X + 1$. What is the moment generating function $M_{\tilde{X}}$ of \tilde{X} ?

Problem 2.11. *Source: "Probability" by Jim Pitman.*

Consider the average

$$\bar{X}_n = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$$

of n independent random variables, with each $X_i, i \in \mathbb{N}$ uniformly distributed over $[0, 1]$. Find the n such that $\mathbb{P}[\bar{X}_n < 0.51]$ is approximately 90%.

Problem 2.12. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 20.

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be the Gamma distribution with parameters $\alpha = 2.5$ and $\theta = 3,000$.

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$. Define the aggregate loss as $S = \sum_{j=1}^N X_j$.

Using the normal approximation, find the amount of the premium π such that the total aggregate losses S exceed the premium with the probability of at most 1%.