# M339J: March $2^{nd}$, 2022.

## Splicing.

**Def'n.** We say that a random variable $X$ has a **k-component spliced distribution** if its probability density function $f_X$ has the following form:

$$f_X(x) := \begin{cases} a_1 f_1(x) & c_0 < x < c_1 \\ a_2 \cdot f_2(x) & c_1 < x < c_2 \\ \vdots & \vdots \\ a_k \cdot f_k(x) & c_{k-1} < x < c_k \end{cases}$$

- for $k$ a fixed positive integer,
- for every $1 \leq j \leq k$, the weight $a_j$ is positive,
- $a_1 + a_2 + \cdots + a_k = 1$
- for every $1 \leq j \leq k$, the function $f_j$ is a density and $f_j(x) = 0$ for all $x \in (c_{j-1}, c_j]$.

**211.** An actuary for a medical device manufacturer initially models the failure time $T$ for a particular device with an exponential distribution with mean 4 years. $\boxed{\theta=4}$

This distribution is replaced with a spliced model whose density function: $\tilde{T}$

(i) is uniform over [0, 3]

(ii) is proportional to the initial modeled density function after 3 years

(iii) is continuous

Calculate the probability of failure in the first 3 years under the revised distribution.

(A) 0.43

(B) 0.45

(C) 0.47

(D) 0.49

(E) 0.51

**212.** For an insurance:

(i) The number of losses per year has a Poisson distribution with $\lambda = 10$.

(ii) Loss amounts are uniformly distributed on (0, 10).

(iii) Loss amounts and the number of losses are mutually independent.

(iv) There is an ordinary deductible of 4 per loss.

Calculate the variance of aggregate payments in a year.

(A) 36

(B) 48

(C) 72

(D) 96

(E) 120

$\longrightarrow$: In the ~~old~~ old model, the failure time $T$ has the density:

$$f_T(t) = \frac{1}{4} e^{-\frac{t}{4}} \quad \text{for } t > 0$$

In the new model, the failure time $\tilde{T}$ has this density:

$$f_{\tilde{T}}(t) = \begin{cases} c & \text{if } \underline{0 < t < 3} \quad ✓ \\ \varkappa \cdot e^{-\frac{t}{4}} & \text{if } \underline{3 \leq t < +\infty} \quad ✓ \end{cases}$$

w/ constants $c$ and $\varkappa$ chosen so that:

- $f_{\tilde{T}}$ is, indeed, a density, i.e., it integrates to 1;
- $f_{\tilde{T}}$ is continuous, i.e., $\boxed{c = \varkappa \cdot e^{-\frac{3}{4}}}$ ✓

$$1 = \int_0^{+\infty} f_{\tilde{T}}(t)\,dt = \int_0^3 c\,dt + \int_3^{+\infty} \varkappa e^{-\frac{t}{4}}\,dt$$

$$1 = 3c + \varkappa(-4)e^{-\frac{t}{4}}\Big]_{t=3}^{+\infty}$$

$$1 = 3c + 4\varkappa\left(0 + e^{-\frac{3}{4}}\right)$$

$$1 = 3c + 4\boxed{\varkappa e^{-\frac{3}{4}}} \quad \Rightarrow \quad 1 = 3c + 4c = 7c \quad \Rightarrow \quad \boxed{c = \frac{1}{7}}$$

$$\mathbb{P}[\tilde{T} \leq 3] = 3c = \frac{3}{7} = 0.43$$ ■

**207.** For an insurance:

(i) Losses have density function

$$f(x) = \begin{cases} 0.02x, & 0 < x < 10 \\ 0, & \text{elsewhere} \end{cases}$$

(ii) The insurance has an ordinary deductible of 4 per loss. $\boxed{d=4}$

(iii) $Y^P$ is the claim payment per payment random variable.

Calculate $E[Y^P]$.

$$\boxed{Y^P = X-d \mid X>d}$$

(A) 2.9

(B) 3.0

(C) 3.2

(D) 3.3

(E) 3.4

Q: What is the density of $Y^P$ in terms of $f_X, F_X, S_X$?

→: For $y>0$:

$$F_{Y^P}(y) = \mathbb{P}[Y^P \le y]$$

$$= \mathbb{P}[X-d \le y \mid X>d]$$

$$= \frac{\mathbb{P}[X \le d+y,\ X>d]}{\mathbb{P}[X>d]}$$

$$= \frac{\mathbb{P}[d < X \le d+y]}{S_X(d)}$$

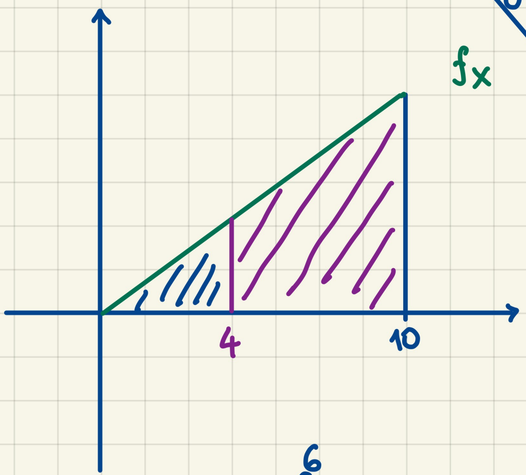
$$= \boxed{\frac{F_X(d+y) - F_X(d)}{S_X(d)}}$$

$$f_{Y^P}(y) = F'_{Y^P}(y) = \frac{F'_X(d+y)}{S_X(d)} = \frac{f_X(d+y)}{S_X(d)}$$

**208.** DELETED

In this problem:

$$\mathbb{E}[Y^P] = \int_0^6 y \cdot f_{Y^P}(y)\, dy$$




$$S_X(4) = 1 - F_X(4)$$

$$= 1 - \frac{1}{\cancel{2}} \cdot \overset{2}{\cancel{4}} \cdot \underbrace{(0.02)\cdot 4}_{f_X(4)} = 0.84$$

$$f_{Y^P}(y) = \frac{0.02(4+y)}{0.84} = \frac{1}{42}(y+4)$$

$$\mathbb{E}[Y^P] = \int_0^6 y \cdot \frac{1}{42}(y+4)\, dy = \frac{1}{42}\int_0^6 (y^2+4y)\, dy$$

$$= \frac{1}{42}\left(\frac{y^3}{3} + \frac{4y^2}{2}\right)\Bigg|_{y=0}^{6} = \frac{1}{42}\left(\frac{6^3}{3} + \frac{4\cdot 6^2}{2}\right)$$

$$= \frac{1}{42}(72+72) = \frac{24}{7} = 3.4 \;\blacksquare$$