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M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Mock In-Term Exam II
Instructor: Milica Čudina

All written work handed in by the student is considered to be
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Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

2.1. **TRUE/FALSE QUESTIONS.**

Problem 2.1. (5 points) You are using a one-period binomial asset-pricing model to model the evolution of the price of a particular stock. Assume that, in our usual notation, $S_d < K < S_u$ for a European call option. Then, the risk-free component in the replicating portfolio of a single call option on that stock should be interpreted as lending. *True or false? Why?*

Solution: FALSE
We know that

$$B = e^{-rh}\frac{uV_d - dV_u}{u-d} = -e^{-rh}\frac{dV_u}{u-d}.$$

So, the call's B will always be negative and should be interpreted as borrowing.

Problem 2.2. (5 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the Δ in the replicating portfolio of a single call option on that stock never exceeds 1. *True or false? Why?*

Solution: TRUE
The call's Δ will always be between 0 and 1. More precisely, consider

$$\Delta_C = \frac{V_u - V_d}{S_u - S_d} = \frac{(S_u - K)_+ - (S_d - K)_+}{S_u - S_d}.$$

The numerator is between 0 and $S_u - S_d$ which completes the proof.

Problem 2.3. (5 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the Δ in the replicating portfolio of a single put option on that stock is between -1 and 0. *True or false? Why?*

Solution: TRUE
The put's Δ will always be between -1 and 0. By definition,

$$\Delta_P = \frac{V_u - V_d}{S_u - S_d} = \frac{(K - S_u)_+ - (K - S_d)_+}{S_u - S_d}.$$

The numerator is non-positive and at least $S_d - S_u$.

2.2. **FREE-RESPONSE PROBLEMS.**

Problem 2.4. (10 points) The current stock price is given to be $S(0) = 30$ and its volatility is 0.3 The continuously-compounded, risk-free interest rate is 0.12.

(i) (2 points) What is the expected stock price in three months under the risk-neutral probability measure?

(ii) (3 points) What is the median stock price in three months under the risk-neutral probability measure?

(iii) (5 points) Find the risk-neutral probability that the stock price in three months is less than \$32.

Solution:

(i)
$$\mathbb{E}^*[S(1/4)] = S(0)e^{r/4} = 30.91364$$

(ii)
$$S(0)e^{(r-\frac{\sigma^2}{2})/4} = 30.56781$$

(iii) First, we calculate d_2. We get
$$d_2 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T\right] = \frac{1}{0.3\sqrt{\frac{1}{4}}}\left[\ln\left(\frac{30}{32}\right) + \left(0.12 - \frac{0.09}{2}\right) \times \frac{1}{4}\right] = -0.30525.$$

Then, we use the standard normal tables to obtain
$$\mathbb{P}[S(1/4) < 32] = N(-d_2) \approx N(0.31) = 0.6217 \tag{2.1}$$

Problem 2.5. (15 points) The current price of a non-dividend paying stock is \$100. Its evolution over the following year is modeled using a three-period binomial tree under the assumption that the price can increase by 2% or decrease by 0.5% over each period. The continuously compounded, risk-free interest rate is 0.03.

What is the price of a one-year, \$101-strike call option on this stock?

Solution: The length of every period is $h = 1/3$. In our usual notation, we have that the definition of the risk-neutral probability reads as

$$p^* = \frac{e^{rh} - d}{u - d} = \frac{e^{0.03/3} - 0.995}{1.02 - 0.995} = 0.6020067.$$

The possible final stock prices are

$$S_{uuu} = S(0)u^3 = 106.1208,$$
$$S_{uud} = S(0)u^2 d = 103.5198.$$

The other two terminal nodes are out-of-the-money. So, the price of the call option is

$$V_C(0) = e^{-0.03}\left((106.1208 - 101)(p^*)^3 + (103.5198 - 101)(3)(p^*)^2(1 - p^*)\right) = 2.142334$$

Problem 2.6. (10 points) A non-dividend-paying stock is currently priced at \$100 per share. One-year, \$102-strike European call and put options on this stock have equal prices. What is the continuously-compounded annual risk-free rate of interest?

Solution: By put-call parity, we have

$$K = F_{0,T}(S) = S(0)e^{rT} \quad \Rightarrow \quad r = \frac{1}{T}\ln\left(\frac{K}{S(0)}\right).$$

So,

$$r = \frac{1}{T}\ln\left(\frac{K}{S(0)}\right) = \ln(102/100) = 0.01980263.$$

Problem 2.7. (10 points) A derivative security has the payoff function given by

$$v(s) = \begin{cases} 10 & \text{if } s < 90 \\ 0 & \text{if } 90 \leq s < 100 \\ 20 & \text{if } 100 \leq s \end{cases}$$

Its exercise date is in one year. You model the time−1 price of the underlying asset as

$$S(1) \sim \begin{cases} 85 & \text{with probability } 1/4 \\ 95 & \text{with probability } 1/2 \\ 105 & \text{with probability } 1/4 \end{cases}$$

What is the expected payoff of the above derivative security?

Solution:

$$10\left(\frac{1}{4}\right) + 20\left(\frac{1}{4}\right) = \frac{30}{4} = 7.5$$