M339D : February 28th, 2025.

Strong Law of Large Numbers (SLLN).

Let $\{X_k : k = 1, 2, \dots\}$ be a sequence of independent and identically distributed r.v.s

Assume: $\boxed{\mu_X := \mathbb{E}[X_1] < \infty}$

Then,

$$\boxed{\frac{X_1 + X_2 + \dots + X_n}{n} \xrightarrow[n \to \infty]{} \mu_X}$$

Let g be a function such that $g(X)$ is well-defined and $\mathbb{E}[g(X)] < \infty$.

Then,

$$\boxed{\frac{g(X_1) + \dots + g(X_n)}{n} \xrightarrow[n \to \infty]{} \mathbb{E}[g(X)]}$$

Monte Carlo.

Recipe.

- Draw simulated values of a r.v. w/ a specific dist'n.
- Apply a f'tion to the simulated values.
- Calculate the arithmetic average of the obtained quantities.

We get a value $\boxed{\text{close to}}$ the theoretical expectation.

Precision.

$$\mathrm{Var}\left[\frac{X_1 + \dots + X_n}{n}\right] = \frac{1}{n^2} \mathrm{Var}[X_1 + \dots + X_n] \quad \text{(independent!)}$$

$$= \frac{1}{n^2}\left(\mathrm{Var}[X_1] + \dots + \mathrm{Var}[X_n]\right) \quad \text{(identically dist'd!)}$$

$$= \frac{1}{n^{\cancel{2}}} \cancel{n} \cdot \mathrm{Var}[X_1] = \frac{\mathrm{Var}[X_1]}{n}$$

$$SD\left[\frac{X_1 + \dots + X_n}{n}\right] = \frac{SD[X_1]}{\sqrt{n}}$$

To increase our precision by a factor η, we must increase our number of variates by a factor η^2.