Impact of Deductibles on Claim Frequency

The Set-Up.

INDEPENDENT

Let $\{X_j, j = 1, 2, \dots\}$ be the severity random variables corresponding to ground-up losses.

Let N^L be a $\mathbb{N}_0$-valued random variable representing the number of losses (in a particular time interval).

Introduce a coverage modification such that

$$\mathbb{P}[X_j \text{ results in a claim}] =: v \quad \text{for all } j$$

Say, we have a deductible d and we have:

$$\boxed{v = \mathbb{P}[X_j > d]} \quad \text{for all } j$$

For every j, we introduce

$$\boxed{I_j = \begin{cases} 1 & \text{if } X_j > d \\ 0 & \text{if } X_j \le d \end{cases}}$$

$$\Rightarrow \boxed{I_j \sim \text{Bernoulli}(q = v)} \quad \text{for all } j$$

and $\{I_j, j = 1, 2, \dots\}$ are independent.

Let N^P denote the number of pmts.

Then,

$$\boxed{N^P = I_1 + I_2 + \dots + I_{N^L}}$$

$$\Rightarrow P_{N^P}(z) \underset{\text{compounding}}{=} P_{N^L}(P_{I_1}(z)) = P_{N^L}((1 - v) + v \cdot z)$$

$$\boxed{P_{N^P}(z) = P_{N^L}(1 + v(z - 1))}$$

In the $(a,b,0)$ class, i.e., where N^L has the pgf of the form $P_{N^L}(z) = P_{N^L}(z; \theta) = \boxed{B(\theta(z-1))}$

w/ θ a parameter and B a deterministic f'tion.

$$\Rightarrow \quad P_{N^P}(z) = P_{N^L}\left(1+v(z-1)\right) = B\left(\theta\left(\cancel{1}+v(z-1)-\cancel{1}\right)\right)$$
$$= B\left(\theta \cdot v\,(z-1)\right)$$

$\Rightarrow$ We conclude that N^P has the same dist'n as N^L, w/ the value of a parameter $\boxed{\theta' = \theta \cdot v.}$

Note: The Poisson thinning is a special case.

39. You are given:

(i) The frequency distribution for the number of losses for a policy with no deductible is negative binomial with $r = 3$ and $\beta = 5$.

$\boxed{N^L \sim NegBin(r=3, \beta=5)}$

(ii) Loss amounts for this policy follow the Weibull distribution with $\theta = 1000$ and $\tau = 0.3$.

$X \sim Weibull(\theta=1000, \tau=0.3)$

Determine the expected number of payments when a deductible of 200 is applied.

N^P... # of pmts after the deductible is applied

$E[N^P] = ?$

(A) Less than 5

(B) At least 5, but less than 7

(C) At least 7, but less than 9

(D) At least 9, but less than 11

(E) At least 11

$\Downarrow$

$N^P \sim NegBin(r = 3, \beta' = \beta \cdot v)$

$v = S_X(200) = e^{-\left(\frac{200}{\theta}\right)^{\tau}} = e^{-\left(\frac{200}{1000}\right)^{0.3}} = 0.5395$

$E[N^P] = 3 \cdot 5 \cdot (0.5395) \cong 8.0925$

86. Aggregate losses for a portfolio of policies are modeled as follows:

(i) The number of losses before any coverage modifications follows a Poisson distribution with mean λ.

(ii) The severity of each loss before any coverage modifications is uniformly distributed between 0 and b.

The insurer would like to model the effect of imposing an ordinary deductible, d $(0 < d < b)$, on each loss and reimbursing only a percentage, c $(0 < c \le 1)$, of each loss in excess of the deductible.

It is assumed that the coverage modifications will not affect the loss distribution.

The insurer models its claims with modified frequency and severity distributions. The modified claim amount is uniformly distributed on the interval $[0, c(b-d)]$.

Determine the mean of the modified frequency distribution.

$N^P \sim$ Poisson (mean $= \lambda' = \lambda v$)

$v = \mathbb{P}[X > d] = \dfrac{b-d}{b}$

$\mathbb{E}[N^P] = \lambda \cdot \dfrac{b-d}{b}$

(A) λ

(B) λc

(C) $\lambda \dfrac{d}{b}$

(D) $\lambda \dfrac{b-d}{b}$

(E) $\lambda c \dfrac{b-d}{b}$

Aggregate Loss Models.

The Individual Risk Model.

Let $\{X_j, j=1,2,\dots,n\}$ be of independent (but not necessarily identically dist'd) r.v.s.

Set $S = X_1 + X_2 + \cdots + X_n$.

Then, S represents $\boxed{\text{aggregate losses.}}$