Normal confidence intervals

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First, I will set up the **confidence level**.

```
c=0.80
```

The critical value associated with this confidence level is

```
z.star=qnorm((1+c)/2)
z.star
```

```
## [1] 1.281552
```

Next, let's draw a sample of size $n = 100$ from the standard normal distribution.

```
n=100
srs=rnorm(n,0,1)
srs
```

```
##   [1]  0.25915167  0.11574200  0.59661785 -2.02795107 -0.24443083 -0.20591710
##   [7] -0.29727253 -1.15537109  0.56656248 -0.35627697 -0.04845604  0.54829681
##  [13] -1.08815161  2.25616429 -0.78452691  1.40776535  0.01766353  0.52491032
##  [19] -0.42369809 -0.73728414  0.81626525  1.16310236 -0.39934987  1.56520853
##  [25] -1.02647741  0.34854508  0.06181104 -1.74755104  0.23116746  0.50514287
##  [31] -0.58277265  0.35207234  1.44518542  0.01908338 -0.86368531 -0.48716906
##  [37]  0.97661264 -1.91886908 -0.90158009  0.15655673  0.45549280 -0.56760510
##  [43] -0.15442038 -0.76253076  0.67671333 -0.49388745  0.93609208 -1.49288371
##  [49]  1.80873080  0.76739031 -1.25419465 -0.64859834  1.04288131 -0.28334203
##  [55]  0.89544044 -1.59135820  0.98804300  0.50324165  0.54601610 -1.24311680
##  [61] -0.40033010 -1.32671492  1.60649798  0.20808914 -1.58969698 -0.10651805
##  [67]  1.18959102 -1.22709328  1.05522855 -0.55071794 -1.41968514 -0.35362659
##  [73] -0.60653550 -0.93872165  0.12638084 -1.42603960 -0.99896707  1.17981710
##  [79]  0.20328784  2.25245683  0.50930854  1.49466296 -0.74866986  0.19082804
##  [85] -0.22638792  0.88784647  0.23501857 -1.30506714  0.12570302  0.11572848
##  [91] -1.76027562 -0.69627705  0.65474335 -0.47835905  0.22924477 -0.11696630
##  [97]  1.66978777 -2.48790154  2.05050297  1.22168875
```

What's the value of the point estimate for the mean in this sample?

```
x.bar=mean(srs)
x.bar
```

```
## [1] -0.02793197
```

Assume that the σ_X parameter is known. What's the value of the standard error?

```
s.e=1/sqrt(n)
s.e
```

```
## [1] 0.1
```

What's the value of the margin of error?

```
m.e=z.star*s.e
m.e
```

```
## [1] 0.1281552
```

Then the confidence interval will be $\bar{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)$. Is the theoretical mean in the confidence interval?

```
abs(x.bar-0)<m.e
```

```
## [1] TRUE
```

What if we repeat this procedure a number of times? Say, 1000?

```
n=100
n.sim=1000
in.conf.int=numeric(0)
for (i in 1:n.sim){
  srs=rnorm(n,0,1)
  x.bar=mean(srs)
  in.conf.int=append(in.conf.int,as.numeric(abs(x.bar-0)<m.e))
}
mean(in.conf.int)
```

```
## [1] 0.786
```