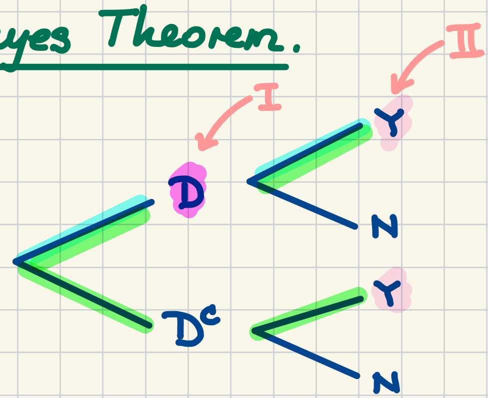
M339G: October 22nd, 2025.

Bayes Theorem.



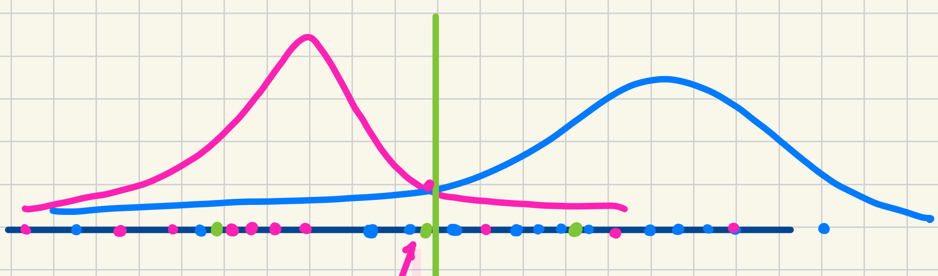
In m362k.

$$\mathbb{P}[D \mid Y] = ?$$

$$\mathbb{P}[D \mid Y] = \frac{\mathbb{P}[D \cap Y]}{\mathbb{P}[Y]} = \frac{\mathbb{P}[D] \cdot \mathbb{P}[Y \mid D]}{\mathbb{P}[Y]}$$

$$= \frac{\mathbb{P}[D] \cdot \mathbb{P}[Y \mid D]}{\mathbb{P}[D] \cdot \mathbb{P}[Y \mid D] + \mathbb{P}[D^c] \cdot \mathbb{P}[Y \mid D^c]}$$

Linear Discriminant Analysis.



Linear Discriminant Analysis w/ $p=1$.

Fisher 1936.

Goal: Classify observations into one of K classes ($K \geq 2$), i.e., figure out

$$p_k(x) = \mathbb{P}[Y=k \mid X=x]$$ posterior probability

Environment:

- π_k ... prior probability that a randomly chosen observation falls into category k, for $k=1..K$
- $f_k(x)$ ... density function of X for observations from class k, for $k=1..K$

choice, model

$f_k(x)dx$ ... the probability that X falls into $\boxed{(x, x+dx)}$ for points from class k, $k=1..K$

Then,

$$\mathbb{P}[Y=k \mid X=x] = \frac{\mathbb{P}[Y=k \text{ and } X=x]}{\mathbb{P}[X=x]} = \frac{\overbrace{\boxed{\mathbb{P}[Y=k]}}^{\pi_k} \cdot \boxed{\mathbb{P}[X=x \mid Y=k]}}{\boxed{\mathbb{P}[X=x]}}$$

$X \in (x, x+dx)$

Bayes Thm.

The Law of Total Probability

Now:

$$p_k(x) = \frac{\pi_k \cdot f_k(x)}{\sum_{l=1}^{K} \pi_l \cdot f_l(x)}$$

$\Rightarrow$ classify into

$$\boxed{k = \underset{l=1..K}{\operatorname{argmax}} \left(p_l(x)\right)}$$