M339W: October 29th, 2021.

Volatility of a "large" Portfolio.

$$R_P = \underbrace{w_1 \cdot R_1 + w_2 \cdot R_2 + \cdots + w_n \cdot R_n}$$

$$\Rightarrow \quad Var[R_P] = Cov[R_P, R_P]$$

$$= Cov[w_1 \cdot R_1 + \cdots + w_n \cdot R_n, R_P]$$

$$= w_1 \cdot Cov[R_1, R_P] + w_2 \cdot Cov[R_2, R_P] + \cdots + w_n \cdot Cov[R_n, R_P]$$

$$= \sum_{i=1}^{n} w_i \cdot Cov[R_i, R_P]$$

2) You are given the following information about a portfolio with four assets.

Asset	Market Value of Asset	Covariance of asset's return with the portfolio return
I	40,000	0.15
II	20,000	-0.10
III	10,000	0.20
IV	30,000	-0.05

$\Sigma = 100K$

Calculate the standard deviation of the portfolio return.

$w_{I} = 0.4,\ w_{II} = 0.2,\ w_{III} = 0.1,\ w_{IV} = 0.3.$

$Var[R_P] = 0.4 \cdot 0.15 + 0.2 \cdot (-0.10) + 0.1(0.2) + 0.3(-0.05) = 0.045$

$\sigma_p = \sqrt{0.045} = 0.212$

(A) 4.50%

(B) 13.2%

(C) 20.0%

(D) 21.2%

(E) 44.7%

11) You are given the following information about a portfolio that has two equally-weighted stocks, P and Q. $w_P = w_Q = \frac{1}{2}$

(i) The economy over the next year could be good or bad with equal probability.

(ii) The returns of the stocks can vary as shown in the table below:

Stock	Return when economy is good	Return when economy is bad	
P	10%	–2%	$\leftarrow R_P$
Q	18%	–5%	$\leftarrow R_Q$

Calculate the volatility of the portfolio return.

R_T ... return of the total portfolio

$R_T = \frac{1}{2}(R_P + R_Q)$

(A) 1.80%

(B) 6.90%

(C) 7.66%

(D) 8.75%

(E) 13.42%

$$R_T \sim \begin{cases} \underline{0.14} \text{ if good} & \text{w/ probab. } \underline{0.5} \\ \underline{-0.035} \text{ if bad} & \text{w/ probab. } \underline{0.5} \end{cases}$$

$Var[R_T] = \mathbb{E}[R_T^2] - (\mathbb{E}[R_T])^2$

- $\mathbb{E}[R_T] = 0.5(0.14) + 0.5(-0.035) = 0.0525$
- $\mathbb{E}[R_T^2] = 0.5(0.14)^2 + 0.5(-0.035)^2 = 0.0104125$

$\Rightarrow \quad Var[R_T] = 0.0104125 - (0.0525)^2 = 0.0076563$

$\Rightarrow \quad \sigma_T = \sqrt{0.0076563} = 0.0875.$

Diversification w/ an Equally-Weighted Portfolio.

$$w_i = \frac{1}{n} \quad \text{for all} \quad i = 1..n$$

$$\Rightarrow \quad R_p = \frac{1}{n}(R_1 + R_2 + \dots + R_n)$$

$$\Rightarrow \quad Var[R_p] = Var\left[\frac{1}{n}(R_1 + R_2 + \dots + R_n)\right]$$

$$= \frac{1}{n^2} \cdot Var[R_1 + R_2 + \dots + R_n]$$

$$= \frac{1}{n^2}\left(\sum_{i=1}^{n} Var[R_i] + \boxed{\sum_{i \neq j} Cov[R_i, R_j]}\right)$$

$$= \underbrace{\frac{1}{n} \cdot \boxed{\frac{1}{n} \cdot \sum_{i=1}^{n} Var[R_i]}}_{\xrightarrow{n \to \infty} 0} + \underbrace{\frac{1}{n^{\cancel{2}}} \cancel{n} \cdot (n-1) \boxed{\frac{1}{n(n-1)} \sum_{i \neq j} Cov[R_i, R_j]}}_{\xrightarrow{n \to \infty} \boxed{\text{Average covariance}}}$$

$$\frac{1}{n^2} n \cdot (n-1) = 1 - \frac{1}{n}$$

Average variance of the individual components (assume bdd)

Average covariance between the stocks in the portfolio

$n \to \infty$

0

$n \to \infty$

Average covariance

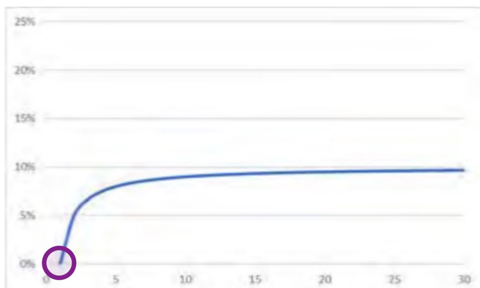
9) You are given the following information about an equally-weighted portfolio of n stocks:

(i) For each individual stock in the portfolio, the variance is 0.20.

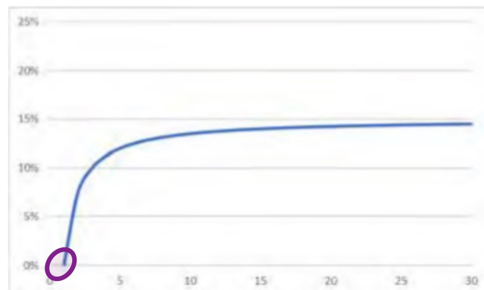
(ii) For each pair of distinct stocks in the portfolio, the covariance is 0.10.

Determine which graph displays the variance of the portfolio as a function of n.

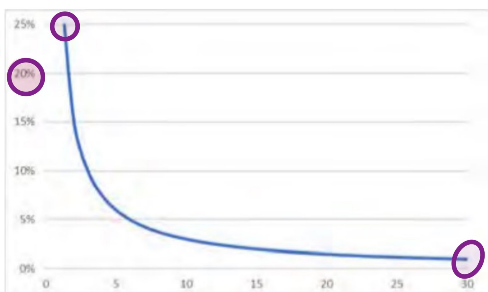
(A)



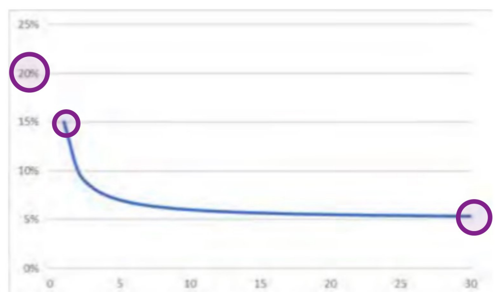
(B)



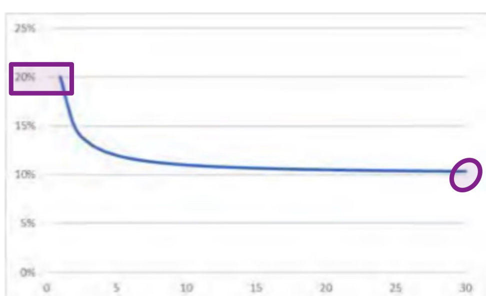
(C)



(D)



(E)



Diversification w/ a General Portfolio.

Assume: $w_i > 0$

Recall:

$$\sigma_P^{\cancel{2}} = Var[R_P] = \sum_{i=1}^{n} w_i \cdot Cov[R_i, R_P]$$

$$= \sum_{i=1}^{n} w_i \cdot \sigma_i \cdot \sigma_P \cdot \rho_{i,P}$$

$$= \cancel{\sigma_P} \cdot \sum_{i=1}^{n} w_i \cdot \sigma_i \cdot \rho_{i,P} \qquad / : \sigma_P$$

$$\sigma_P = \sum_{i=1}^{n} w_i \, \sigma_i \cdot \underbrace{\rho_{i,P}}_{\leq 1}$$

$$\boxed{\sigma_P \leq \sum_{i=1}^{n} w_i \cdot \sigma_i}$$

Equality only if all the investments are perfectly positively correlated.