

Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 4.1. (5 pts) Let the severity random variable X be continuous such that $f_X(x) > 0$ for all $x > 0$. Let Y^P denote the per payment random variable associated with X for some ordinary deductible $d > 0$. Then the random variable Y^P is

- (a) continuous.
- (b) discrete, but not degenerate (constant).
- (c) degenerate (constant).
- (d) mixed.
- (e) None of the above

Problem 4.2. (5 pts) Let the severity random variable X be continuous such that $f_X(x) > 0$ for all $x > 0$. Let Y^L denote the per loss random variable associated with X for some ordinary deductible $d > 0$. Then the random variable Y^L is

- (a) continuous.
- (b) discrete, but not degenerate (constant).
- (c) degenerate (constant).
- (d) mixed.
- (e) None of the above

Problem 4.3. (5 pts) The ground-up loss X is modeled by an exponential distribution with mean \$500. There is an ordinary deductible of $d = 200$. What is the expected value of the **per-loss** random variable?

Problem 4.4. (5 points) Let the severity random variable X be modelled using the Pareto distribution with parameters $\theta = 0.5$ and $\alpha = 6$. For a particular value of the ordinary deductible d , the expected value of the per-payment random variable Y^P is 10. What is the value of the deductible?

Problem 4.5. (5 points) For a random variable X and for a positive constant d , in our usual notation, we have

$$(4.1) \quad \mathbb{E}[X] = e_X(d)S_X(d) + \mathbb{E}[X \wedge d].$$

True or false? Why?

Problem 4.6. (5 points) Let $X \sim \text{Pareto}(\alpha = 3, \theta = 3000)$. Assume that there is a deductible of $d = 5000$. Find $\mathbb{E}[X \wedge d]$.

Problem 4.7. (10 points) Let X be the ground-up loss random variable whose density is given by

$$f_X(x) = \begin{cases} 0.01, & 0 \leq x \leq 80, \\ 0.03 - 0.00025x, & 80 < x \leq 120. \end{cases}$$

Let there be an ordinary deductible of $d = 20$.

Calculate $\mathbb{E}[X \wedge d]$.

Problem 4.8. (10 points) *Source: Problem 4.3 from "Loss Models".*

Assume that the claims r.v. X has a Pareto distribution with $\alpha = 2$ and θ unknown.

Claims for the following year are denoted by Y and will experience uniform inflation of 6%.

- (i) (2 points) Find the expression for the probability $\mathbb{P}[X > d]$ in terms of d and θ .
- (ii) (3 points) Find the expression for the probability $\mathbb{P}[Y > d]$ in terms of d and θ .
- (iii) (5 points) Find the expression for the ratio $\rho(d) = \frac{\mathbb{P}[X > d]}{\mathbb{P}[Y > d]}$ in terms of d and θ .

Find the limit of $\rho(d)$ as $d \rightarrow \infty$.