UNIVERSITY OF TEXAS AT AUSTIN

Quiz #4

Prerequisite material.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 4.1. (5 points) *Source: Sample P exam, Problem #185.*
A student takes an examination consisting of 20 true/false questions. The student knows the correct answer to n of the questions and guesses the answers to the rest at random. The conditional probability that the student knows the answer to a question, given that the student answered it correctly, is 0.824. Calculate n.

(a) 8
(b) 10
(c) 14
(d) 16
(e) 18

Solution: (c)

Problem 4.2. (5 points) The probability mass function p_X of a discrete random variable X is given by

$$p_X(x) = \begin{cases} 1/2, & \text{for } x = 1 \\ 1/3, & \text{for } x = 2 \\ 1/6, & \text{for } x = 4 \end{cases}$$

Find $\mathbb{E}[|X - 3|]$.

(a) -7/6
(b) 1/6
(c) 11/6
(d) 3/2
(e) None of the above.

Solution: (d)
The random variable $|X - 3|$ has the following distribution

$$|X - 3| \sim \begin{cases} 1 & \text{with probability } 1/2 \\ 2 & \text{with probability } 1/2 \end{cases}$$

So, its expectation is 3/2.

Problem 4.3. (5 points) Maria lives in Austria and receives her salary in Euro. She decides to spend 1000 Euro and let the proceeds of the exchange accrue interest at the USD continuously compounded, risk-free interest rate. She will withdraw the balance in three months and exchange it back to Euros. Assume that there were no intermediate deposits or withdrawals. You know the following:

- The initial exchange rate is 1.19 USD per Euro.
- The USD continuously compounded, risk-free interest rate is equal to $r_\$ = 0.02$.
- The Euro continuously compounded, risk-free interest rate is equal to $r_e = 0.06$.

Given that the exchange rate at the end of the three months equals 1.23 USD per Euro, how much (in Euro) does Maria receive?

(a) 972.33

(b) 982.10

(c) 987.02

(d) 1195.97

(e) None of the above.

Solution: (a)
Maria spends 1000 Euro. So, she receives 1190 USD. The balance at the end of the three-month period is

$$1190e^{0.02/4} = 1195.965.$$

Taking into account the final exchange rate, her payoff is

$$1195.965/1.23 = 972.3293.$$