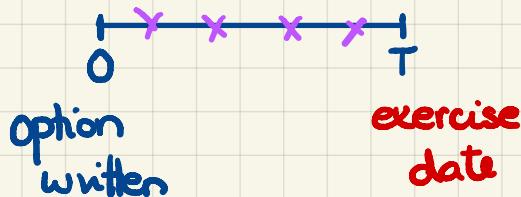


M3398: February 14th, 2024

European Put Options.

Usually, a right but not an obligation to sell!



At time 0: The writer and buyer of the put agree on:

- the underlying asset: $S(t)$, $t \geq 0$;
- the exercise date T ;
- the strike/exercise price K

The put premium $V_p(0)$ is paid by the put's buyer to the put's writer.

At time T :

- The put's owner has the right, but not an obligation, to SELL one unit of the underlying asset for the strike price K .
- The put's writer is obligated to do what the put's owner decides.

The put's owner's optimal behavior is:

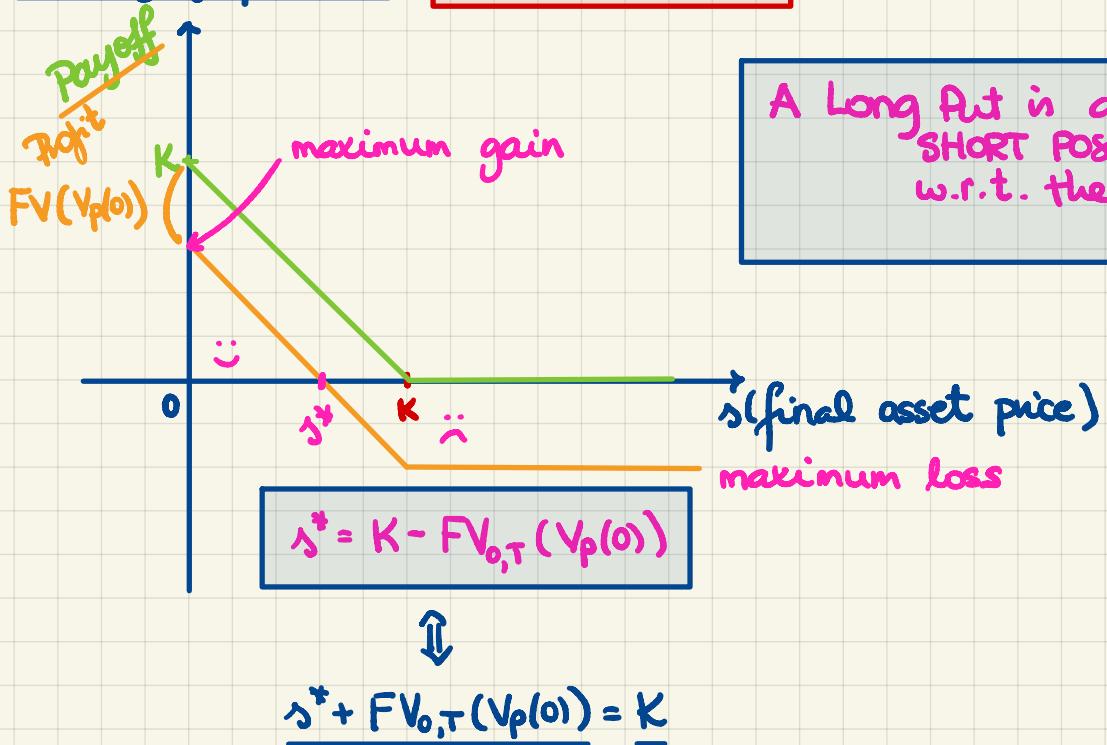
IF $K > S(T)$, then exercise $\frac{\text{Payoff}}{K - S(T)}$

IF $K \leq S(T)$, then do not exercise 0

The Payoff: $V_p(T) = \text{Max}(K - S(T), 0) = (K - S(T))_+$

The Payoff function: $v_p(s) = (K - s)_+$

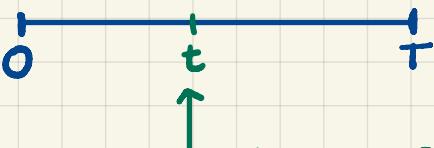
The Payoff function: $v_p(s) = (K-s)_+$



A Long Put is a SHORT POSITION w.r.t. the underlying.

Moneyness.

Consider an option written @ time 0 w/ exercise date @ time T.



Imagine the cashflow that would happen to the option's owner were they to exercise @ that time T.

e.g.

call: $S(t) - K$

put: $K - S(t)$

If cashflow is $\begin{cases} > 0 & \text{we say the option is in-the-money} \\ = 0 & \text{we say the option is at-the-money} \\ < 0 & \text{we say the option is out-of-the-money} \end{cases}$

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #6
European put options.

Problem 6.1. The initial price of the market index is \$900. After 3 months, the market index is priced at \$915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of \$930, is \$8.00. What is the profit at expiration for a long put?

- (a) \$15.00 loss
- (b) \$6.90 loss
- (c) \$6.90 gain
- (d) \$15.00 gain
- (e) None of the above.

$$\rightarrow : \text{Payoff: } (K - S(T))_+ = (930 - 915)_+ = 15$$

$$\text{Profit: } 15 - 8(1.004)^3 = 6.90$$

Problem 6.2. Sample FM(DM) #12

Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4% convertible semiannually, and that the put costs 74.20 now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

- A. 922.83
- B. 924.32
- C. 1,000.00
- D. 1,075.68
- E. 1,077.17

We're looking for the break-even point.

$$\Delta^* = K - FV_{0,T} (V_p(0))$$

$$\Delta^* = 1000 - 74.20(1.02) = \underline{924.32}.$$



Problem 6.3. Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000—cartons' worth in six months. His total costs are \$12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of \$13 per carton and another with the strike price of \$15 per carton. Their premiums are \$0.15 and \$0.18 respectively.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals \$14. What would Farmer Shaun's profit be if he had decided to hedge using the \$13-strike put versus his profit if he had decided to use the \$15-strike put to hedge?

Focus on Payoff: (w/out production costs)

$$\begin{aligned} \text{unhedged: } & S(T) \\ \text{hedge: } & (K - S(T))_+ \end{aligned} \quad \left. \right\} +$$

$$\text{total hedged: } S(T) + (K - S(T))_+ =$$

$$= \begin{cases} K & \text{if } K > S(T) \\ S(T) & \text{if } K \leq S(T) \end{cases}$$

$$= \boxed{\max(K, S(T))}$$

FLOOR.

\$13

$$\text{Payoff: } \max(13, 14) = 14$$

$$14 - 12 - 0.15 \cdot (1.04)^{1/2} = \underline{1.867} \quad) \times 10,000$$

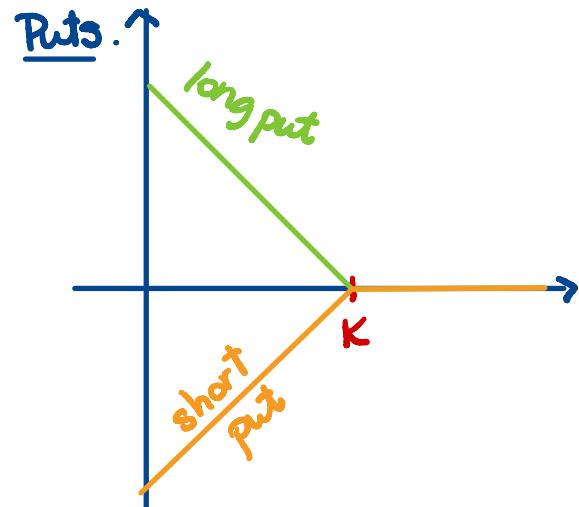
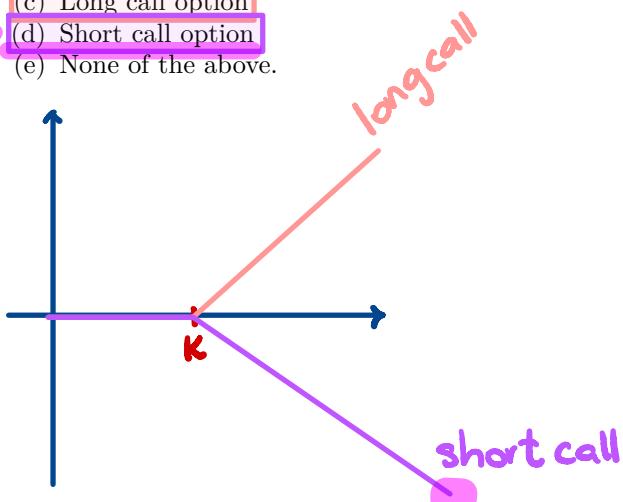
\$15

$$15 - 12 - 0.18 \cdot (1.04)^{1/2} = 2.847 \quad)$$

Problem 6.4. (2 points) In which of the following option positions is the investor exposed to an unlimited loss?

- (a) Long put option
- (b) Short put option
- (c) Long call option
- (d) Short call option
- (e) None of the above.

Calls .



Problem 6.5. (3 points) The initial price of the market index is \$1000. After 3 months the market index is priced at \$950. The nominal rate of interest convertible quarterly is 4.0%. The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

- (a) \$12.00 loss
- (b) \$14.90 loss
- (c) \$12.00 gain
- (d) \$14.90 gain
- (e) None of the above.

→ : Payoff: $(K-S(T))_+ = (975-950)_+ = 25$
Profit: $25 - 10(1.01) = 14.90$

Problem 6.6. (3 points) *Source: Sample FM(DM) Problem #62.*

The stock price today equals \$100 and its price in one year is modeled by the following distribution:

$$S(1) \sim \begin{cases} 125 & \text{with probability } 1/2 \\ 60 & \text{with probability } 1/2 \end{cases}$$

The annual effective interest rate equals 3%

Consider an at-the-money, one-year European put option on the above stock whose initial premium is equal to \$7.

What is the expected profit of this put option?

→: $\mathbb{E}[V_p(T)] = ?$

$$V_p(T) \sim \begin{cases} (100 - 125)_+ = 0 & \text{w/ probab. } 1/2 \\ (100 - 60)_+ = 40 & \text{w/ probab. } 1/2 \end{cases}$$

$$\mathbb{E}[V_p(T)] = 40 \cdot \frac{1}{2} = 20$$

answer: $20 - 7 \cdot (1.03) = 12.79$ □

Problem 6.7. Aunt Dahlia simultaneously purchased

- one share of a market index at the current spot price of \$1,000;
- one one-year, \$1,050-strike put option on the above market index for the premium of \$20.

Floor.

- (i) (5 points) Is the above portfolio's payoff bounded from above? If you believe it is not, substantiate your claim. If you believe that it is, provide the upper bound.
- (ii) (5 points) Is the above portfolio's payoff bounded from below? If you believe it is not, substantiate your claim. If you believe that it is, provide the lower bound.

