NAME:

M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
Mock In-Term Exam I
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximum number of points on this exam is 50.
Time: 50 minutes

Problem 2.1. Assume the Black-Scholes setting.
Today's price of a non-dividend paying stock is \$65, and its volatility is 0.20.

The continuously-compounded, risk-free interest rate is 0.055.

What is the price of a three-month, \$60-strike European put option on the above stock?

(a) 0.66
(b) 0.59
(c) 0.44
(d) 0.37
(e) None of the above.

Problem 2.2. (5 points) A continuous-dividend-paying stock is valued at \$75.00 per share. Its dividend yield is 0.03. The time$-t$ realized return is modeled as

$$R(0,t) \sim N(\text{mean} = 0.035t, \text{variance} = 0.09t)$$

Find the probabilit y that the time-4 stock price exceeds today's stock price.

Problem 2.3. Assume the Black-Scholes model. According to your model, you expect the stock price in half a year to be \$84.10. The median stock price in half a year is \$83.26 according to that same model. What is the stock's volatility?

(a) 0.15

(b) 0.20

(c) 0.25

(d) Not enough information is given.

(e) None of the above.

Problem 2.4. (5 points) The current stock price is given to be $S(0) = 30$. The stock has the rate of appreciation 0.12 and volatility 0.3

Find the probability that the stock price in three months is less than \$32.

(a) 0.5218
(b) 0.5412

(c) 0.5846
(d) 0.6217
(e) None of the above.

Problem 2.5. (5 points) Assume the Black-Scholes model. Let the current price of a continuous-dividend-paying stock be \$80. Its dividend yield is 0.02 and its volatility is 0.25.

Let the continuously compounded, risk-free interest rate be 0.04.

Find the price of a $3-$month, \$75-strike European call option on the above stock.

(a) 6.84
(b) 7
(c) 7.22
(d) 7.51
(e) None of the above.

Problem 2.6. Suppose that the failure time (in seconds) of a certain component is modeled as lognormal random variable $Y = e^X$ such that the mean of X is -0.35 and its variance is 0.04.

What is the failure time t^* such that 95% of the components of the same type would still function after that time?

(a) 0.306
(b) 0.402
(c) 0.507
(d) 0.701
(e) None of the above.

Problem 2.7. (5 points) A stock is valued at \$55.00. The annual expected return is 12.0% and the standard deviation of annualized returns is 22.0%. If the stock is lognormally distributed, what is the expected stock price after 3 years?

(a) About \$78.83
(b) About \$88.83
(c) About \$98.83
(d) About \$108.83
(e) None of the above.

Problem 2.8. (5 pts)

A non-dividend-paying stock is currently valued at \$100 per share. Its annual mean rate of return is given to be 12% while its volatility is given to be 30%.

Assuming the lognormal stock-price model, find

$$\mathbb{E}[S(2) \,|\, S(2) > 95].$$

(a) \$86.55
(b) \$101.60
(c) \$152.35
(d) \$159.07
(e) None of the above.

Problem 2.9. Assume the Black-Scholes framework. For an at-the-money, T-year European call option on a non-dividend-paying stock you are given that its delta equals 0.5832. What is the delta of an otherwise identical option with exercise date at time $2T$?

(a) 0.62
(b) 0.66
(c) 0.70
(d) 0.74
(e) None of the above.

Problem 2.10. (5 points) Let the current price of a non-dividend-paying stock be denoted by $S(0)$. We model the time$-T$ stock price as lognormal. The mean rate of return on the stock is 0.12 and its volatility is 0.20. The continuously-compounded, risk-free interest rate is 0.04. You invest in one share of stock at time-0. and simultaneously deposit an amount $\varphi S(0)$ in a savings account. What should the proportion φ be so that the VaR at the level 0.05 of your total wealth at time-1 equals today's stock price $S(0)$?
(a) $\varphi = 0.1966$
(b) $\varphi = 0.5$
(c) $\varphi = 0.8034$
(d) $\varphi = 1$
(e) None of the above.

Problem 2.11. (5 points) Assume the Black-Scholes model. The current price of a continuous-dividend-paying stock is \$63 per share. Its dividend yield is 0.01 and its volatility is 0.25. Its mean rate of return is 0.10.

Consider a three-month,\$65-strike call option on the above stock. What is the probability that the option is in-the-money at expiration?

(a) 0.4483
(b) 0.4325
(c) 0.3936
(d) 0.4207
(e) None of the above.

Problem 2.12. Assume that the stock price follows the Black-Scholes model. You are given the following information:

- The current stock price is \$100.
- The mean rate of return on the stock is 0.15.
- The stock's dividend yield is 0.01.
- The stock's volatility is 0.35.
- The continuously-compounded, risk-free interest rate is 0.05.

Find

$$\mathbb{E}[S(1)\mathbb{I}_{[S(1)>80]}] .$$

(a) \$102.02
(b) \$108.19
(c) \$115.03
(d) \$126.71
(e) None of the above.