

NAME:

M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
Mock In-Term Exam I
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximum number of points on this exam is 50.

Time: 50 minutes

Problem 1.1. (5 points) The current price of a continuous-dividend-paying stock is \$80 per share. The stock's dividend yield is 0.02. According to your model, the expected value of the stock price in two years is \$90 per share. You are also given:

The risk-free interest rate exceeds the dividend yield.

The two-year forward price on a share of this stock is denoted by F . At this price you are willing to enter into the forward. What is the smallest range of values F can take according to the above information?

- (a) $F < 77$
- (b) $77 < F < 80$
- (c) $80 < F < 90$
- (d) $F > 90$
- (e) None of the above.

Problem 1.2. Consider a non-dividend-paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously-compounded, risk-free interest rate is 0.06.

Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

- (a) About \$10
- (b) About \$10.83
- (c) About \$15.45
- (d) About \$20.84
- (e) None of the above.

Problem 1.3. The current exchange rate is given to be \$1.25 per Euro and its volatility is given to be 0.15.

The continuously-compounded, risk-free interest rate for the US dollar is 0.03, while the continuously-compounded, risk-free interest rate for the Euro equals 0.06.

The evolution of the exchange rate over the following nine-month period is modeled using a three-period forward binomial tree.

What is the value of the so-called down factor in the above tree?

- (a) $d \approx 0.8586$
- (b) $d \approx 0.8982$
- (c) $d \approx 0.9208$
- (d) $d \approx 0.9347$
- (e) None of the above.

Problem 1.4. The evolution of a market index over the following year is modeled using a four-period binomial tree. We are given that the current value of the market index equals \$144, that its volatility equals 0.25, and that it pays dividends continuously.

You are tasked with constructing a four-period forward tree for the evolution over the following year of the forward price of the above market index with delivery at time-2.

What is the down factor d_F in the forward price tree for the futures prices on the stock?

- (a) 0.7788
- (b) 0.8825
- (c) 0.9914
- (d) There is not enough information given.
- (e) None of the above.

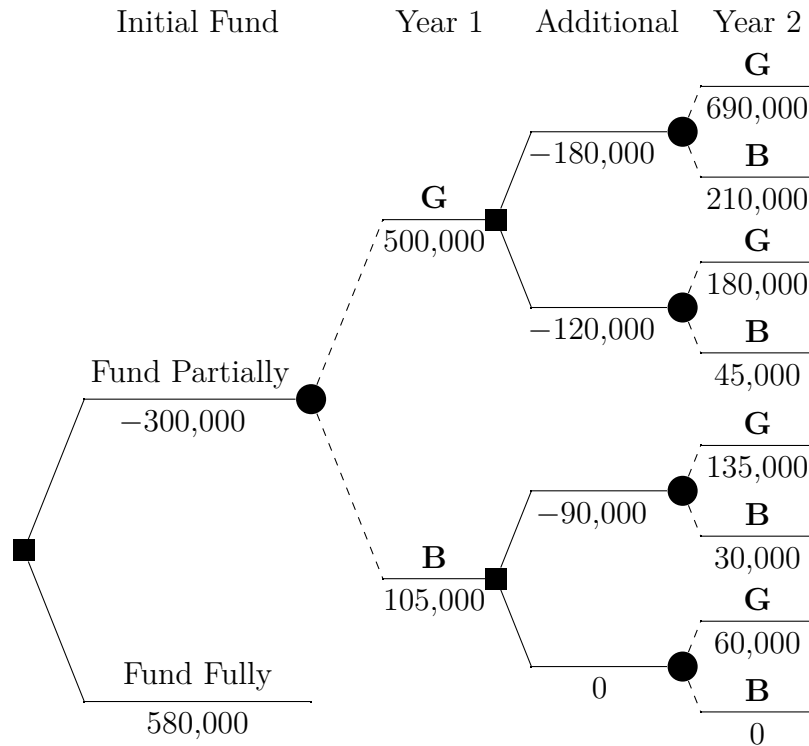
Problem 1.5. (5 points) Netflix is considering a cartoon series. When the production of two seasons is fully funded at time-0 the project has a net present value of 580,000.

The decision tree below shows the cash flows of the series when the promotion at the beginning of the Year 1 (i.e., at $t = 0$) is only partial with an option to provide different amounts of funding at the beginning of Year 2 (i.e., at $t = 1$) depending on how well the first season did.

This tree reflects two possible receptions of the two seasons at each information node (**G** = good, **B** = bad). The probability of the series being a success is given to be 1/2 and the probability of it being merely watchable is 1/2.

Assume the interest rate is 0%.

Find the **initial** (i.e., at $t = 0$) value of the option to fund partially.



- (a) 15000
- (b) 20000
- (c) 25000
- (d) 30000
- (e) None of the above.

Problem 1.6. *Source: Open Course Intro to Statistics.*

Cholesterol levels for women aged 20 to 34 follow an approximately normal distribution with mean 185 milligrams per deciliter (mg/dl). Women with cholesterol levels above 220 mg/dl are considered to have high cholesterol and about 18.5% of women fall into this category. What is the standard deviation of the distribution of cholesterol levels for women aged 20 to 34?

- (a) 38.9
- (b) 41.3
- (c) 43.7
- (d) 45.1
- (e) None of the above.

Problem 1.7. (5 points) A discrete-dividend-paying stock sells today for \$90 per share. The continuously compounded, risk-free interest rate is 0.05. The first dividend will be paid at in three months in the amount of \$2.50. The remaining dividends will be equal to \$2 and continue to be paid out quarterly for three more years. What is the **prepaid forward price** of this stock for delivery in eight months?

- (a) \$73.02
- (b) \$85.58
- (c) \$90
- (d) \$99.33
- (e) None of the above.

Problem 1.8. (5 points) *Source: Sample FM(DM) Problem #41.*

The current price of a non-dividend-paying stock is \$100. The **effective** risk-free interest rate equals 0.01. Which of the following portfolios has the highest initial cost?

- (a) Long a six-month, \$100-strike European put and short a six-month, \$100-strike European call.
- (b) Long a forward contract for the delivery of the above stock in six months.
- (c) Long a six-month, \$101-strike European put and short a six-month, \$101-strike European call.
- (d) Short a forward contract for the delivery of the above stock in six months.
- (e) Long a six-month, \$105-strike European put and short a six-month, \$105-strike European call.

Problem 1.9. The **writer** of a call option has ...

- (a) an obligation to sell the underlying asset at the strike price.
- (b) a right, but **not** an obligation, to sell the underlying asset at the strike price.
- (c) an obligation to buy the underlying asset at the strike price.
- (d) a right, but **not** an obligation, to buy the underlying asset at the strike price.
- (e) None of the above.

Problem 1.10. Consider a one-period forward binomial model for the stock-price movement over the following year. The current stock price is $S(0) = 100$, its dividend yield is 0.05 and its volatility is 0.3. The continuously compounded risk-free interest rate is given to be 0.05.

Consider American call options on this stock with the expiration date at the end of the period/year.

Which of the following is closest to the maximal (rounded to the nearest dollar) strike price K for which there is early exercise?

- (a) 76
- (b) 80

- (c) 100
- (d) 135
- (e) 180

Problem 1.11. In the setting of the one-period binomial model, denote by i the effective interest rate **per period**. Let u denote the “up factor” and let d denote the “down factor” in the stock-price model. Which of the following statements is the correct no-arbitrage condition for the binomial asset-pricing model?

- (a) $d < 1 + i < u$
- (b) $d < 1 < u$
- (c) $d < e^i < u$
- (d) $d = \frac{i}{1+i}$
- (e) None of the above.

Problem 1.12. (5 points) You roll a fair tetrahedron whose sides are labeled by 1, 2, 3, and 4 a total of 4000 times. What is the approximate probability that you see a 1 strictly more than 1025 times? There is no need to use the continuity correction.

- (a) 0.0446
- (b) 0.1287
- (c) 0.1456
- (d) 0.1814
- (e) None of the above.