M339W: December 1st, 2021.

Example. Let the risk-free interest rate be equal to 0.02 and let the market risk premium be 0.04.
The current stock price is 50 per share w/ one million shares outstanding.
The company has 25 million in debt w/ the debt cost of capital of 0.05. ✓
The equity beta is 1.2.
The weighted average cost of capital is 0.058.
What is the corporate tax rate?

$\longrightarrow$:

- Find the return on equity. ✓

CAPM $\Rightarrow$ $r_E = r_f + \beta_E (\mathbb{E}[R_{Mkt}] - r_f)$

$$r_E = 0.02 + 1.2(0.04) = 0.068$$

- $$r_{wacc} = \frac{E}{E+D} r_E + \frac{D}{E+D} \cdot r_D (1 - T_c)$$

$\frac{E}{E+D} \overset{?}{=}$ and $\frac{D}{E+D} \overset{?}{=}$

Given: $\left.\begin{array}{l} E = 50 \cdot 10^6 \\ D = 25 \cdot 10^6 \end{array}\right\} \Rightarrow \frac{E}{E+D} = \frac{2}{3}$ and $\frac{D}{E+D} = \frac{1}{3}$

$$0.058 = \frac{2}{3} \cdot 0.068 + \frac{1}{3} \cdot (0.05)(1 - T_c)$$

$$3(0.058) = 2(0.068) + 0.05 - 0.05 \cdot T_c$$

$$0.05\, T_c = 2(0.068) + 0.05 - 3(0.058)$$

$$T_c = 0.24$$

Options Embedded in Insurance Products.

Read the study Note!

... variable annuities.

I. Guaranteed Minimum Death Benefit (GMDB)

K... guaranteed minimum amount paid @ death;
If K is equal to the original premium, this is called the return of premium guarantee

S_T ... the account value @ time T

If the policyholder dies @ time T_x, then the amount paid equals

$$\boxed{\max(S_{T_x}, K)}$$

Note that

$$\max(S_{T_x}, K) = S_{T_x} + \boxed{\max(0, K - S_{T_x})}$$

like the payoff of a put

Let f_{T_x} be the density of the time of death T_x of x-year-olds.

$$\boxed{\int_0^{+\infty} P(t) \cdot f_{T_x}(t)\,dt}$$

w/ $P(t)$... the price of the put w/ strike K and exercise date t.

38) An insurance company has a variable annuity linked to the S&P 500 index. A guaranteed minimum death benefit (GMDB) specifies the beneficiary will receive the greater of the account value and the original amount invested, if the policyholder dies within the first three years of the annuity contract. If the policyholder dies after three years, the beneficiary will receive the account value.

Out of every 1000 policies sold, the company expects 10 deaths in each of years one, two, and three. Thus they also expect that 970 will survive the first three years. Assume the deaths occur at the end of the year.

You are given the following at-the-money European call and put option prices, expressed as a percentage of the current value of the S&P 500 index.

Duration (years)	Call Price	Put Price
1	18.7%	15.8%
2	26.2%	20.6%
3	31.6%	23.4%

Calculate the expected value of the guarantee when the annuity is sold, expressed as a percentage of the original amount invested.

Deaths are assumed discrete.
So, the integral becomes a sum:

$$0.01(0.158) + 0.01(0.206) + 0.01(0.234)$$

$$\approxeq 0.01(0.6) = 0.6\%$$

(A) 0.23%

(B) 0.32%

(C) 0.52%

(D) 0.60%

(E) 0.76%

In addition, there can be an earnings-enhanced death benefit. This benefit is a percentage of the excess above the guarantee of the account value (if any). The payoff will be:

$$\boxed{p\% \boxed{\max(S_{T_x} - K, 0)}}$$

like the payoff of a call option

The value of this benefit can be expressed as:

$$p\% \int_0^{+\infty} C(t) \, f_{T_x}(t) \, dt$$

w/ $C(t)$... the price of a call w/ strike K and exercise date t.