**Example 10.1.1.** Consider the following two data sets, both consisting of measurements of the same quantity (say the distance to Proxima Centauri) and in the same units, but made with two different methods.

method 1: 4.51, 4.52, 4.48, 4.49, 4.47, 4.53
method 2: 14.12, 1.30, 0.40, 2.50, 1.00, 3.18

If we use the sample mean $\bar{Y}$ as the (point) estimator for the "true" mean $\mu$, both of these data sets yield the same result, namely $\bar{Y} = 4.5$. It is clear, however, that the first method is more accurate and that, in general, one should trust the results produced by method 1 more than those obtained by method 2.

# Point vs. Interval Estimation.

An interval estimator is a pair
$$\hat{\Theta}_L \leq \hat{\Theta}_R$$
of point estimators.

Good traits:

- being narrow
- containing the true parameter $\theta$ w/ a high probability

First: Pick a confidence level $C = 0.95, 0.99$, another probab. close to 1

or pick a significance level $\alpha = 0.05, 0.01$, another probab. close to 0

Convention: $\boxed{C = 1-\alpha}$

The purpose:
$$\boxed{\mathbb{P}[\hat{\Theta}_L \leq \theta \leq \hat{\Theta}_R] = 1-\alpha = C}$$

Def'n. Consider a random sample $(Y_1, Y_2, \dots, Y_n)$ from a distribution $D$ which is parameterized by an unknown parameter $\theta$.
A pivotal quantity is a function of the data $(Y_1, Y_2, \dots, Y_n)$ and the parameter $\theta$ whose distribution does not depend on $\theta$.

Example. Say that $Y_i \sim N(\mu, 1)$, $i = 1..n$ in our random sample.

$\bar{Y}$ is a great estimator for $\mu$

$\bar{Y} \sim Normal(\mu, \frac{1}{\sqrt{n}})$

$\Rightarrow$ $\bar{Y}$ is NOT A PIVOTAL QUANTITY!

But $\boxed{\bar{Y} - \mu}$ is a PIVOTAL QUANTITY!

$\bar{Y} - \mu \sim Normal(0, \frac{1}{\sqrt{n}})$

**Example.** Consider a random sample $(Y_1, Y_2, \ldots, Y_n)$ from $N(\mu, \sigma)$ w/ both parameters unknown

$$\frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim \text{Normal}(0,1)$$

will be a pivotal quantity $\theta = (\mu, \sigma)$

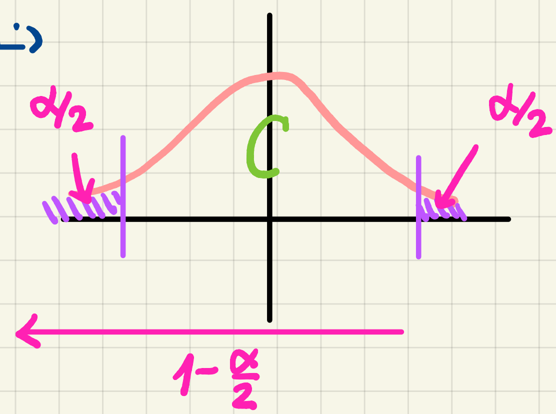
**Recipe.**

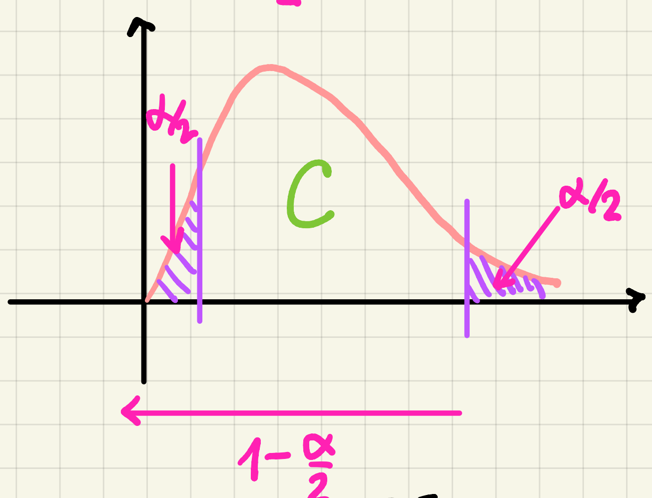
1. Consider a pivotal quantity $U$
2. Define

$$a = q_U(\alpha/2)$$

and $b = q_U(1-\alpha/2)$ with $q_U$ is the quantile f'tion of $U$

e.g.,




**Because:** $\mathbb{P}[a \leq U \leq b] = 1-\alpha$

3. **Algebra:**

$$a \leq \boxed{U} \leq b$$

Has $\Theta$ in its expression!

Undo the "rule" for U so that the final expression look like

$$\boxed{\hat{\Theta}_L} \leq \Theta \leq \boxed{\hat{\Theta}_R}$$

Works for 1D parameters.