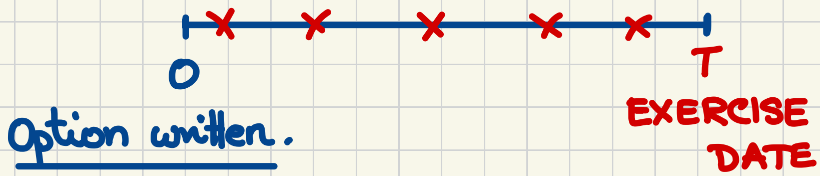
M339D : September 22nd, 2025.

European | **Call** | **Options.**

European ✓: The option can only be exercised, i.e., the transactions can take place only on the exercise date.

Call: Usually, this is a right to buy the underlying asset.

Options: Usually, the option's owner has the right but not an obligation to exercise the option.



At time 0:
- The writer of the option writes/shorts the call.
- The buyer of the call is said to long the call. They are referred to as the option's owner.
- The agreement:
  - the underlying asset: $S(t), t \geq 0$
  - the exercise date: T
  - K ... the strike/exercise price
- The buyer pays a premium to the writer. $\boxed{V_C(0)}$

At time T:
- The call's owner has a right but not an obligation to buy one unit of the underlying asset for the strike price K.
- The call's writer is obligated to do what the owner decides.

Payoff = ?

We focus on the payoff of the long call, i.e., the payoff for the call's owner.

The call owner's rationale for whether to exercise is to "maximize money in".

The criterion for exercise:

IF $S(T) \geq K$, then EXERCISE. $\Rightarrow$ Payoff $= S(T) - K$

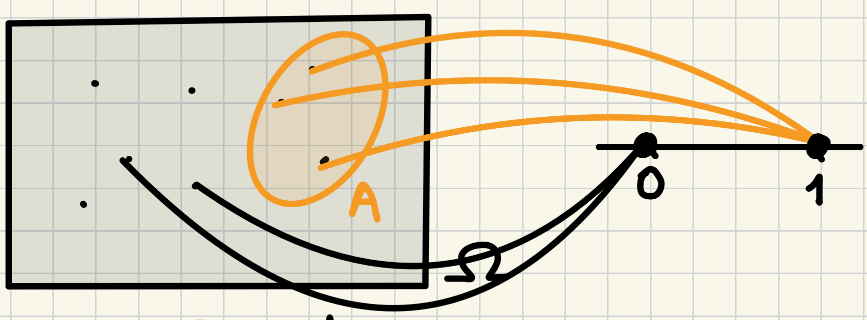
IF $S(T) < K$, then do not exercise. $\Rightarrow$ Payoff $= 0$

We introduce:

$V_c(T)$... the r.v. denoting the payoff of a long call

$$\Rightarrow \boxed{V_c(T) = \begin{cases} S(T)-K & \text{if } \boxed{S(T) \geq K} \\ 0 & \text{if } S(T) < K \end{cases}}$$

Indicator Random Variables.



Ω... outcome space

$\omega \in \Omega$... elementary outcomes

A... any "nice" subset of Ω aka an EVENT

We define:

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

$$\Rightarrow \boxed{V_c(T) = (S(T) - K) \cdot \mathbb{I}_{\boxed{[S(T) \geq K]}}}$$

Note: $S(T) \geq K \iff S(T) - K \geq 0$

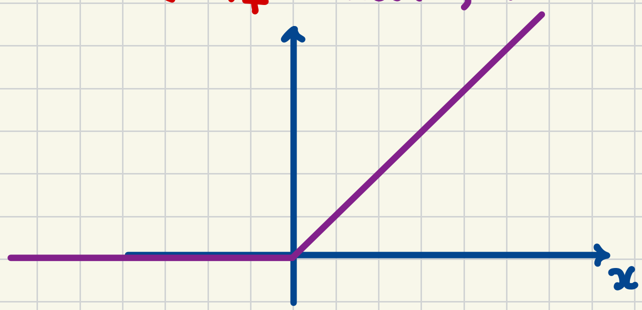
We could also write:

$$\boxed{V_c(T) = \max(S(T)-K, 0) = (S(T)-K) \vee 0}$$

$\vee$ ↑ MAXIMUM OPERATOR

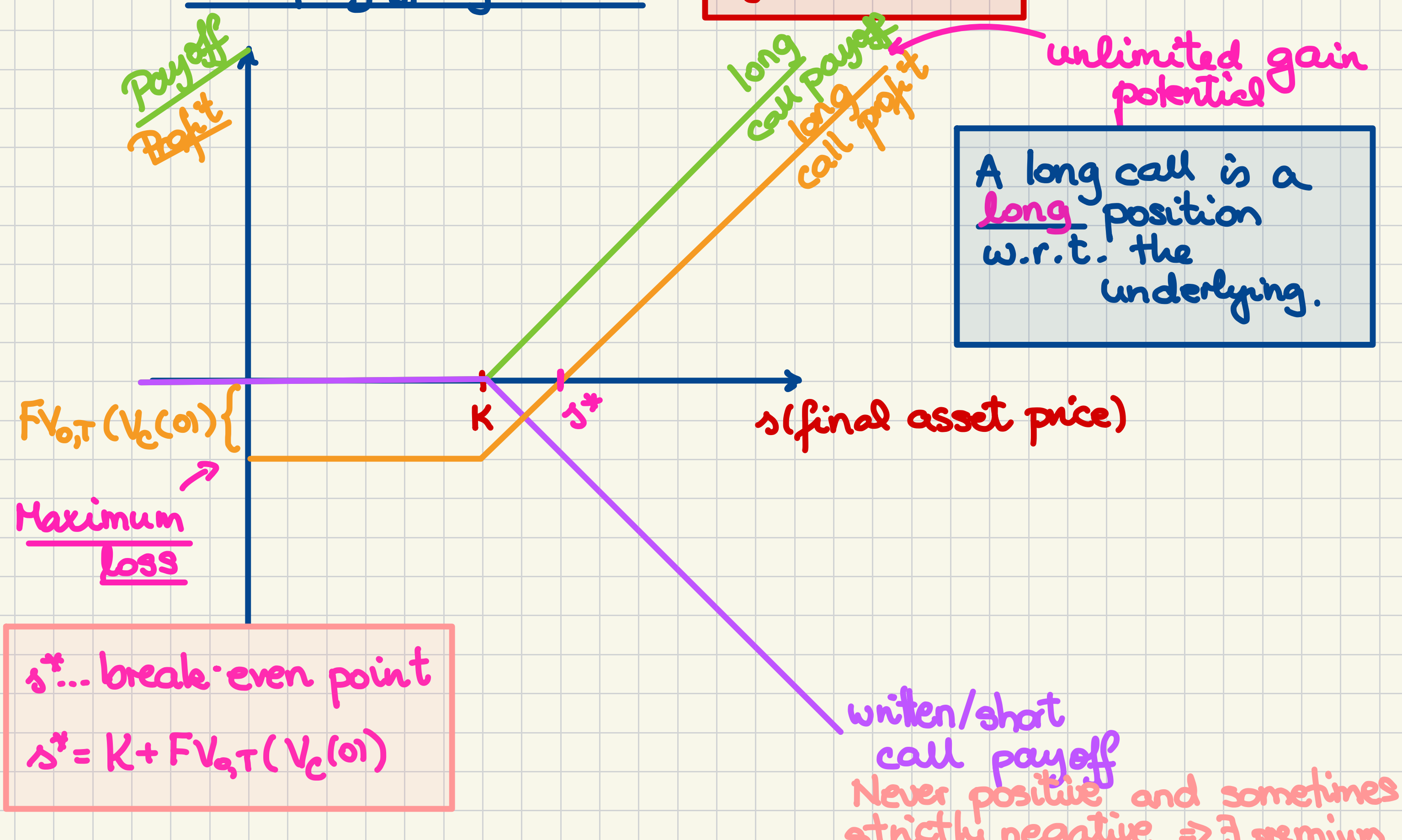
Introduce: The Positive Part Function.

$$x \longmapsto (x)_+ := \max(x,0) = x \vee 0$$



$$\Rightarrow \boxed{V_c(T) = (S(T)-K)_+}$$

$\Rightarrow$ the payoff function: $\boxed{v_c(s) = (s-K)_+}$



A long call is a **long** position w.r.t. the underlying.

s^*... break-even point

$s^* = K + FV_{0,T}(V_c(0))$