M378K: January 23rd, 2026.

Named Discrete Dist'ns [cont'd].

Example. • Quality Control. Say, if a computer is defective or not.
• Insurance. An indicator of whether a deductible was met or not.

Bernoulli Dist'n.

$Y \sim B(p)$ w/ $p \in (0,1)$

y	0	1
$p_Y(y)$	$\underbrace{1-p}_{=q}$	p

Q: Say that we repeat independently the Bernoulli trials w/ the same success probability p a fixed number of times n.
We denote by Y the number of successes in those n trials.

$$S_Y = \cancel{?}\ \{0, 1, \dots, n\}$$

$$\boxed{p_Y = ?}$$

for $k = 0..n$,

$$p_Y(k) = \mathbb{P}[Y=k] = \boxed{\underbrace{\binom{n}{k}}_{=\frac{n!}{k!(n-k)!}}}\ \underline{p^k(1-p)^{n-k}}$$

BINOMIAL COEFFICIENT

The dist'n of Y is called the binomial dist'n.
We write $\boxed{Y \sim b(n,p)}$

M378K Introduction to Mathematical Statistics

Problem Set #3

Named discrete random variables.

Problem 3.1. Source: Sample P exam, Problem #125.
An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail. Calculate the probability that the system will overheat.

$\longrightarrow$: Y... # of components that overheat

$\mathbb{P}[Y \geq 2] = ?$

$Y \sim b(n=3, p=0.05)$

$$\mathbb{P}[Y \geq 2] = 1 - p_Y(0) - p_Y(1)$$

$$= 1 - \underbrace{\binom{3}{0}}_{=1}(0.05)^0(0.95)^3 - \underbrace{\binom{3}{1}}_{=3}(0.05)^1(0.95)^2$$

$$= \underbrace{\binom{3}{2}}_{=3}(0.05)^2(0.95)^1 + \underbrace{\binom{3}{3}}_{=1}(0.05)^3(0.95)^0$$

$$= 3(0.05)^2(0.95) + (0.05)^3 = \dots = 0.00725$$

$\square$

Geometric Dist'n.

Say we repeat independently Bernoulli trials w/ the same success probability p until the first success.

The random variable Y which denotes the number of failures until the first success is said to have the geometric dist'n w/ parameter p.

$$\boxed{Y \sim g(p)}$$

<u>Set</u>: $q = 1-p$

y	0	1	2	$\cdots$	k	$\cdots$
$p_Y(y)$	p	$q \cdot p$	$q^2 \cdot p$	$\cdots$	$q^k \cdot p$	$\cdots$

Q: $\mathbb{P}[Y>2] = 1 - \mathbb{P}[Y \leq 2]$

$$= 1 - p_Y(0) - p_Y(1) - p_Y(2)$$

$$= \underbrace{1 - p}_{} - q \cdot p - q^2 \cdot p$$

$$= q - qp - q^2 \cdot p$$

$$= q(\underbrace{1 - p}_{} - q \cdot p)$$

$$= q(q - q \cdot p)$$

$$= q \cdot q \cdot (1 - p)$$

$$= q^3 \quad \square$$

$\boxed{\text{In general:}}$ $\boxed{\mathbb{P}[Y>m] = q^{m+1}}$ ☆

Problem 3.2. Source: Sample P exam, Problem #462.
Each person in a large population independently has probability p of testing positive for diabetes where $0 < p < 1$. People are tested for diabetes, one person at a time, until a test is positive. Individual tests are independent. Determine the probability that m or fewer people are tested, given that n or fewer people are tested, where $1 \leq m \leq n$.

→: Y'... total # of people tested

SHIFTED geometric w/ parameter p

i.e., $\boxed{Y = Y' - 1 \sim g(p)}$

$$\mathbb{P}[Y' \leq m \mid Y' \leq n] = \mathbb{P}[Y+1 \leq m \mid Y+1 \leq n]$$

$$= \frac{\mathbb{P}[Y+1 \leq m, Y+1 \leq n]}{\mathbb{P}[Y+1 \leq n]}$$

$$= \frac{\mathbb{P}[\boxed{Y \leq m-1}, Y \leq n-1]}{\mathbb{P}[Y \leq n-1]}$$

$$= \frac{\mathbb{P}[Y \leq m-1]}{\mathbb{P}[Y \leq n-1]} =$$

$$= \frac{1 - \mathbb{P}[Y > m-1]}{1 - \mathbb{P}[Y > n-1]} = \frac{1 - q^m}{1 - q^n} \quad \square$$

Fact: $k, l \geq 0$

$$\mathbb{P}[Y > k+l \mid Y > k] = \frac{\mathbb{P}[Y > k+l, Y > k]}{\mathbb{P}[Y > k]} = \frac{\mathbb{P}[Y > k+l]}{\mathbb{P}[Y > k]} = \frac{q^{k+l+1}}{q^{k+1}}$$

$$= q^l = \mathbb{P}[Y > l-1]$$