UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 9

Bull/Bear spreads. Properties of European call/put prices.

9.1. **Bull and bear spreads.** Provide your final answer only to the following problem(s):

Problem 9.1. (5 points) The following three nine-month European put options are available in the market:

- a \$120-strike put with the premium of \$12,
- a \$127-strike put with the premium of \$10,
- a \$130-strike put with the premium of \$14.

Which of the following positions certainly exploits the arbitrage opportunity caused by the above put premia?

(a) Put bull spread.
(b) Put bear spread.
(c) Both of the above positions.
(d) There is no arbitrage opportunity.
(e) None of the above.

Solution: (c)

Please provide your **complete solutions** to the following problem.

Problem 9.2. (5 points) **Bear spreads in hedging**
An investor purchases a call option with an exercise price of \$55 for \$2.60. The same investor sells a call on the same asset with an exercise price of \$60 for \$1.40. At expiration, 3 months later, the asset price is \$56.75. All other things being equal and given a continuously compounded annual interest rate of 4.0%, what is the profit to the investor?

Solution: The total initial cost of establishing the investor's position is $2.60 - 1.40 = 1.20$. The future value of this amount at expiration is $1.20e^{0.25 \cdot 0.04} = 1.20e^{0.01} \approx 1.21$.
The payoff at expiration is

$$(S(T) - 55)_+ - (S(T) - 60)_+ = 1.75.$$

So, the profit is $1.75 - 1.21 = 0.54$.

Please, provide your **final answer only** for the following problem(s):

Problem 9.3. (5 points) An investor acquires a call bull spread consisting of the call with strike $K_1 = 100$ and $K_2 = 110$ and with expiration in one year. The initial price of the $100-$strike call option equals \$11.34, while the price of the $110-$strike option equals \$7.74. At expiration, it turns out that the stock price equals \$105. Given a continuously compouned annual interest rate of 5.0%, what is the profit to the investor?

(a) \$3.78 loss
(b) \$1.22 loss
(c) \$1.22 gain
(d) \$5 gain
(e) None of the above.

Solution: (c)
The total initial cost of establishing the investor's position is $11.34 - 7.74 = 3.60$. The future value of this amount at expiration is $3.60e^{0.05} = 3.78$. The payoff at expiration is

$$(S(T) - 100)_+ - (S(T) - 110)_+ = (105 - 100)_+ - (105 - 110)_+ = 5.$$

So, the profit is $5 - 3.78 = 1.22$.

Problem 9.4. (5 points) Consider a non-dividend-paying stock. Which of the following portfolios has the same profit as a $(40, 50)-$bull spread?

(a) A **long** $(40, 50)-$collar and a short stock.
(b) A **short** $(40, 50)-$collar and a long stock.
(c) A **long** $40-$strike call, a **written** $50-$strike put, and a long stock.
(d) A **long** $40-$strike call, a **written** $50-$strike put, and a short stock.
(e) None of the above.

Solution: (d)

9.2. **Butterfly spreads and convexity.** Please, provide your <u>**complete solution**</u> to the following problem(s):

Problem 9.5. (5 points) In a certain market, you are given that

the price of a $40-$strike European call option on an underlying asset S and with maturity T is \$11;
the price of a $50-$strike European call option on an underlying asset S and with maturity T is \$6;
the price of a $55-$strike European call option on an underlying asset S and with maturity T is \$3.

Let the risk-free interest rate be $r = 0.05$. A trader decides to construct the following portfolio:

(1) long one $40-$strike call option;
(2) short three $50-$strike European call options;
(3) long two $55-$strike calls.

Suppose that at time $T = 1$ the value of the asset S is $S(1) = 52$. What is the profit of the portfolio at time T?

Solution: The initial cost is

$$11 - 3 \cdot 6 + 2 \cdot 3 = -1.$$

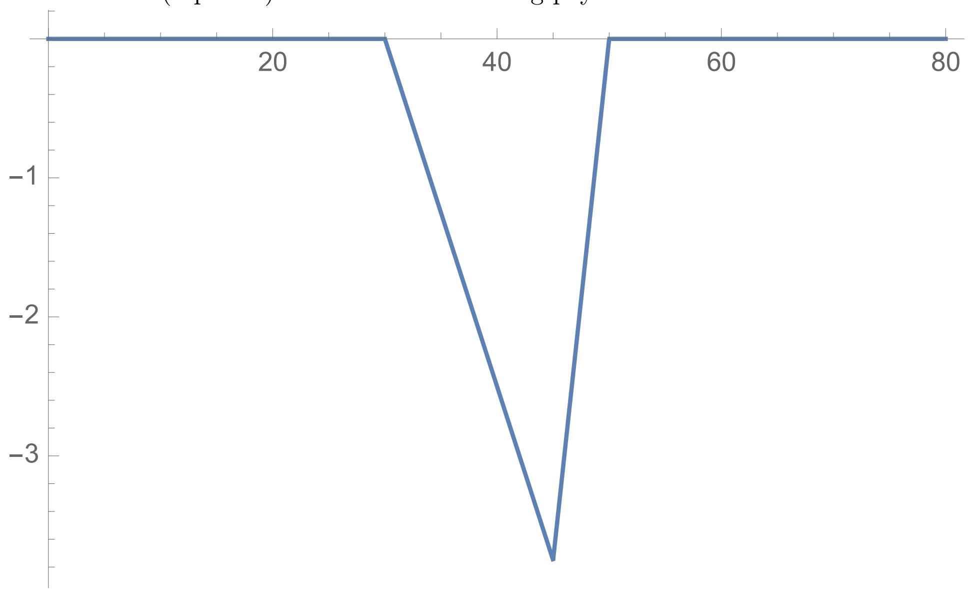
Note: The negative cost of -1 means that the trader can invest the one dollar *surplus* from the above trade in the money-market, i.e., *lend* the one dollar.

The payoff at maturity is

$$(S(T) - 40) - 3(S(T) - 50) + 2 \cdot 0 + 1 \cdot e^{rT} = e^{rT} + 2(55 - S(T)) \approx 7.05.$$

Provide your <u>**final answer**</u> only for the following problems.

Problem 9.6. (5 points) Consider the following payoff curve:



Which of the following positions has the above payoff?

(a) A long butterfly spread.
(b) A short butterfly spread.
(c) A long strangle.
(d) A short straddle.
(e) None of the above.

Solution: (b)

Problem 9.7. (5 points) Let $K_1 = 50, K_2 = 55$ and $K_3 = 65$ be the strikes of three European call options on the same underlying asset and with the same exipration date. Let $V_C(K_i)$ denote the price at time-0 of the option with strike K_i for $i = 1, 2, 3$.
We are given that $V_C(K_1) = 16$ and $V_C(K_3) = 1$. What is the maximum possible value of $V_C(K_2)$ which still does not violate the convexity property of option prices?

(a) $V_C(K_2) = 10$
(b) $V_C(K_2) = 11$
(c) $V_C(K_2) = 13$
(d) $V_C(K_2) = 15$
(e) None of the above.

Solution: (b)
Let $V_C(K_2) = C$. The convexity requirement for call-option prices is

$$\frac{16 - C}{55 - 50} \geq \frac{C - 1}{65 - 55}.$$

So,

$$10(16 - C) \geq 5(C - 1) \quad \Rightarrow \quad 33 \geq 3C \quad \Rightarrow \quad 11 \geq C.$$

Problem 9.8. (5 points) You are interested in purchasing a European call option on the underlying asset S which has expiration date $T = 1$ year and strike $K = 70$. Denote the price of this call by C.
You are given that the premiums for European call options with the same expiration date and the same underlying asset but with the strikes $K_1 = 60$ and $K_2 = 75$ are $C_1 = 10$ and $C_2 = 3$, respectively.
You know that there is no arbitrage in your market. Find the maximal price C that you might be charged so that none of the convexity inequalities are violated.

(a) $C = 16/3$
(b) $C = 6$
(c) $C = 25/3$
(d) $C = 28/3$
(e) None of the above.

Solution: (a)

$$C = 10 \times \frac{1}{3} + 3 \times \frac{2}{3} = \frac{16}{3}$$

Problem 9.9. (5 points) Consider three European put options on the same stock with the same exercise date. The put premium for the $32-$strike option is 2.50 and the put premium for the $37-$strike option is 6.50. What can you say about the $40-$strike put option?

(a) Its highest possible premium is \$8.90.
(b) Its lowest possible premium is \$8.90.
(c) Its highest possible premium is \$10.50
(d) Its lowest possible premium is \$10.50.
(e) None of the above.

Solution: (b)
To satisfy the convexity condition for put prices with respect to the strike, with x denoting the lowest possible 40−strike put price, we get

$$\frac{3}{8} \times 2.5 + \frac{5}{8}x = 6.50 \quad \Rightarrow \quad x = 8.9.$$

Problem 9.10. (5 points) Calculate the price of a long butterfly spread constructed using the following call options:

(1) a £3,925−strike call on the FTSE100 index which is being sold for £713.07;
(2) a £4,325−strike call on the FTSE100 index which is being sold for £496.46;
(3) a £4,725−strike call on the FTSE100 index which is being sold for £333.96.

Assume that the total number of the call options in your portfolio equals 4.

(a) £54.11
(b) £550.57
(c) £554.11
(d) £559.57
(e) None of the above

Solution: (a)
The long butterfly spread can be constructed by buying a certain number of calls with the lowest strike, buying a certain number of calls with the highest strike and writing a certain number of calls with the middle strike. Note that in the present problem, the middle strike is exactly the average of the lowest and highest strikes, so we are dealing with a symmetric butterfly spread. This position is constructed by buying one each of the call swith the "outer" strikes and writing two calls with the middle strike. The total cost equals

$$713.07 + 333.96 - 2 \cdot 496.46 = 54.11.$$