(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

$\boxed{T = \frac{1}{4}}$ $\boxed{K = 41.5}$ European

8. You are considering the purchase of a 3-month 41.5-strike ~~American~~ call option on a nondividend-paying stock.

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40. $S(0) = 40$

(iii) The stock's volatility is 30%. $\sigma = 0.30$

(iv) The current call option delta is 0.5. $\Delta_c(S(0), 0) = 0.5$

$= N(d_1(S(0),0))$

Determine the current price of the option. $v_c(S(0),0) = ?$

(A) $20 - 20.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(B) $20 - 16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(C) $20 - 40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(D) $16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

(E) $40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

$$v_c(S(0),0) = S(0)\cdot \boxed{N(d_1(S(0),0))} - Ke^{-rT}\cdot N(\boxed{d_2(S(0),0)})$$

$$= d_1(S(0),0) - \sigma\sqrt{T}$$

$$= 0 - 0.3\sqrt{1/4}$$

$$= -0.15$$

$$N(d_1(S(0),0)) = 0.5$$

$$\Rightarrow \boxed{d_1(S(0),0) = 0} \checkmark \Rightarrow$$

$$\frac{1}{\sigma\sqrt{T}}\Big[\underbrace{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T}_{=0}\Big] = 0$$

$$\ln\left(\frac{40}{41.5}\right) + \left(r + \frac{0.09}{2}\right)\cdot\frac{1}{4} = 0 \qquad /\cdot 4$$

$$r + 0.045 = 4\cdot\ln\left(\frac{41.5}{40}\right)$$

$$r = 4\ln\left(\frac{41.5}{40}\right) - 0.045 = \underline{0.10226} \checkmark$$

$$v_c(S(0),0) = 40\cdot(0.5) - \underline{41.5e^{-0.10226(1/4)}}\cdot \underbrace{N(-0.15)}_{0.4404}$$

$$= 20 - \underline{40.453}\,(1 - N(0.15))$$

$$= 20 - 40.453 + 40.453\cdot N(0.15)$$

$$= 40.453\cdot \underbrace{N(0.15)}_{\int_{-\infty}^{0.15}\varphi(z)\,dz} - 20.453$$

$$\int_{-\infty}^{0.15}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}\,dz$$

$$= \underbrace{40.453\cdot\frac{1}{\sqrt{2\pi}}}_{16.138}\int_{-\infty}^{0.15} e^{-\frac{z^2}{2}}\,dz - 20.453 \implies (D)$$

$\square$

Delta-Hedging.

Market Makers.

- immediacy
- inventory

$\Rightarrow$ exposure to risk $\Rightarrow$ hedge

Say, a market maker writes an option whose value f'tion is

$$v(s,t)$$

At time-0, they wrote the option. So, they get $\boxed{v(S(0),0)}$.

At time-t, the value of the market maker's position is

$$\boxed{-v(s,t)}$$

To (partially) hedge this exposure to risk, they construct a portfolio which has zero sensitivity to small perturbations in the stock price. Formally speaking, their goal is to create a

delta-neutral portfolio,

i.e., a portfolio for which $\boxed{\Delta_{Port}(s,t)=0}$

Theoretically, with continuous rebalancing w/ no transaction costs it's possible.
Practically, continuous rebalancing is impossible and there are transaction costs.

In particular, @ time-0, we want to trade so that

$$\boxed{\Delta_{Port}(S(0),0)=0}.$$

The most straightforward strategy is to trade in the shares of the underlying asset.

At time-t let $N(s,t)$ denote the required number of shares in the portfolio necessary to maintain Δ-neutrality.

The total value of the portfolio is:

$$v_{Port}(s,t) = N(s,t)\cdot s - v(s,t)$$

$$\frac{\partial}{\partial s}\Big| \quad \Delta_{Port}(s,t) = N(s,t) - \Delta(s,t) = 0$$

Δ-neutrality

$$\boxed{N(s,t) = \Delta(s,t)}$$

Example. A market maker writes a call option @ time-0.

At time-t, the market maker's position is:

$$-v_C(s,t)$$

$\Rightarrow$ They have to maintain $N(s,t) = \Delta_C(s,t)$ in the Δ-hedge.

$\Rightarrow$ In particular, @ time-0:

$$N(S(0),0) = N(d_1(S(0),0)) > 0,$$

i.e., the market maker should long this much of a share.

Example.