Name:

M339J: Probability models
University of Texas at Austin
Solution: More Practice Problems for In-Term One
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is ?? points.
Time: 50 minutes

Problem 1.1. (5 pts) Let the severity random variable X be continuous such that $f_X(x) > 0$ for all $x > 0$. Let Y^P denote the per payment random variable associated with X for some ordinary deductible $d > 0$. Then the random variable Y^P is

(a) continuous.
(b) discrete, but not degenerate (constant).
(c) degenerate (constant).
(d) mixed.
(e) None of the above

Solution: (a)

Problem 1.2. (5 pts) Let the severity random variable X be continuous such that $f_X(x) > 0$ for all $x > 0$. Let Y^L denote the **per loss** random variable associated with X for some ordinary deductible d. Then the random variable Y^L is

(a) continuous.
(b) discrete, but not degenerate (constant).
(c) degenerate (constant).
(d) mixed.
(e) None of the above

Solution: (d)
The c.d.f. of Y^L has a single jump at 0.

Problem 1.3. (5 points) *Source: Sample STAM Problem #309.*
The random variable X represents the random loss, before any deductible is applied, covered by an insurance policy. The probability density function of X is given by

$$f_X(x) = 2x, \quad 0 < x < 1.$$

Payments are made subject to a deductible d where

$$0 < d < 1$$

. The probability that a claim payment is less than 0.5 is equal to 0.64. Calculate the value of the deductible d.

(a) 0.1
(b) 0.2
(c) 0.3
(d) 0.4
(e) None of the above

Solution: (c)
The cumulative distribution function corresponding to the given density is $F_X(x) = x^2$ for $0 < x < 1$. We are given that

$$\mathbb{P}[Y^L < 0.5] = 0.64 \quad \Rightarrow \quad \mathbb{P}[Y^L \geq 0.5] = \mathbb{P}[X - d \geq 0.5] = 0.36.$$

So,

$$F_X(d + 0.5) = 0.36 \quad \Rightarrow \quad (d + 0.5)^2 = 0.64 \quad \Rightarrow \quad d = 0.3.$$

Problem 1.4. (5 pts) *Source: Prof. Jim Daniel (personal communication).*
The ground-up loss X is modeled by an Exponential distribution with mean \$500. There is an ordinary deductible of $d = 100$. What can you say about the expected value of the per-loss random variable?

(a) It is less than 100.
(b) It is more than 100, but less than 250.
(c) It is more than 250, but less than 375.
(d) It is more than 375, but less than 500.
(e) None of the above

Solution: (d)
Let

$$Y^L = (X - d)_+$$

with $X \sim Exp(\theta = 500)$ and $d = 100$. Then,

$$\begin{aligned}
\mathbb{E}[Y^L] &= \mathbb{E}[(X - d)\mathbb{I}_{[X>d]}] \\
&= \int_d^\infty (x - d)\frac{1}{\theta}e^{-\frac{x}{\theta}}\, dx \\
&= \int_0^\infty y\frac{1}{\theta}e^{-\frac{y+d}{\theta}}\, dx \\
&= e^{-\frac{d}{\theta}} \int_0^\infty y\frac{1}{\theta}e^{-\frac{y}{\theta}}\, dx \\
&= \theta e^{-\frac{d}{\theta}} = 500e^{-1/5} \approx 409.37.
\end{aligned}$$

Problem 1.5. (5 points) Let a severity random variable X be uniform over $[0, 100]$. An insurance policy is written to cover X. This policy has an ordinary deductible d. With the deductible, the expected value of the per loss random variable under the policy is 36% of what it would be with no deductible. What is the value of the deductible?

(a) 30
(b) 40
(c) 50
(d) 60
(e) None of the above.

Solution: (b)
Without the deductible, the expected payment is 50. So, the expected payment with the deductible equals 18. We have

$$18 = \mathbb{E}[(X-d)_+] = \mathbb{E}[(X-d)\mathbb{I}_{[X>d]}] = \mathbb{E}[X-d|X>d]S_X(d).$$

However, $X - d|X > d \sim U(0, 100 - d)$. So,

$$18 = \frac{100-d}{2} \cdot \frac{100-d}{100} = \frac{(100-d)^2}{200} \quad \Rightarrow \quad (100-d)^2 = 200(18) = 3600 \quad \Rightarrow \quad 100-d = 60 \quad d = 40.$$

Problem 1.6. *Source: An old CAS exam; I think.*
Let X be the loss random variable such that $\mathbb{P}[X = 3] = \mathbb{P}[X = 12] = 0.5$. For a deductible d, you know that the expected value of the per loss random variable equals 3. How much is d?

Solution: Clearly $3 < d < 12$. So,

$$3 = \mathbb{E}[(X-d)_+] = 0.5(12-d) \quad \Rightarrow \quad d = 6.$$

Problem 1.7. *Source: An old exam 4.*
Losses follow a Pareto distribution with parameters θ and $\alpha > 1$. Determine the ratio of the mean excess loss function at $d = 2\theta$ to the mean excess loss function at $d = \theta$.

Solution: It was established in class that for $X \sim Pareto(\alpha, \theta)$, we have $e_X(d) = \frac{d+\theta}{\alpha-1}$. So, our answer is

$$\frac{e_X(2\theta)}{e_X(\theta)} = \frac{\frac{2\theta+\theta}{\alpha-1}}{\frac{\theta+\theta}{\alpha-1}} = \frac{3}{2}.$$

Problem 1.8. Claim sizes follow a Pareto distribution with parameters $\alpha = 0.5$ and $\theta = 10,000$. Determine the mean excess loss at 10,000.

Solution: Since $\alpha < 1$, the mean excess loss is infinite.

Problem 1.9. *Source: An old CAS exam 3.*
Losses follow an exponential distribution with parameter θ. For a deductible of 100, the expected payment per loss is 2,000. Which of the following in the expected payment per loss for a deductible of 500.

(a) θ

(b) $\theta(1 - e^{-500/\theta})$

(c) $2000e^{-400/\theta}$

(d) $2000e^{-5\theta}$

(e) $\frac{2000e^{-500/\theta}}{1-e^{-100/\theta}}$

Solution: (c)
We are given that

$$\theta e^{-\frac{100}{\theta}} = 2000$$

We need to calculate

$$\theta e^{-\frac{500}{\theta}} = \frac{2000}{e^{-\frac{100}{\theta}}} e^{-\frac{500}{\theta}} = 2000e^{-400/\theta}$$