M339D: September 23rd, 2024.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #5

European call options.

$V_C(0)$ $S(0)$ T K

Problem 5.1. The initial price of a non-dividend-paying asset is \$100. A six-month, \$95-strike European call option is available at a \$8 premium. The continuously compounded risk-free interest rate equals 0.04. What is the break-even point for this call option?

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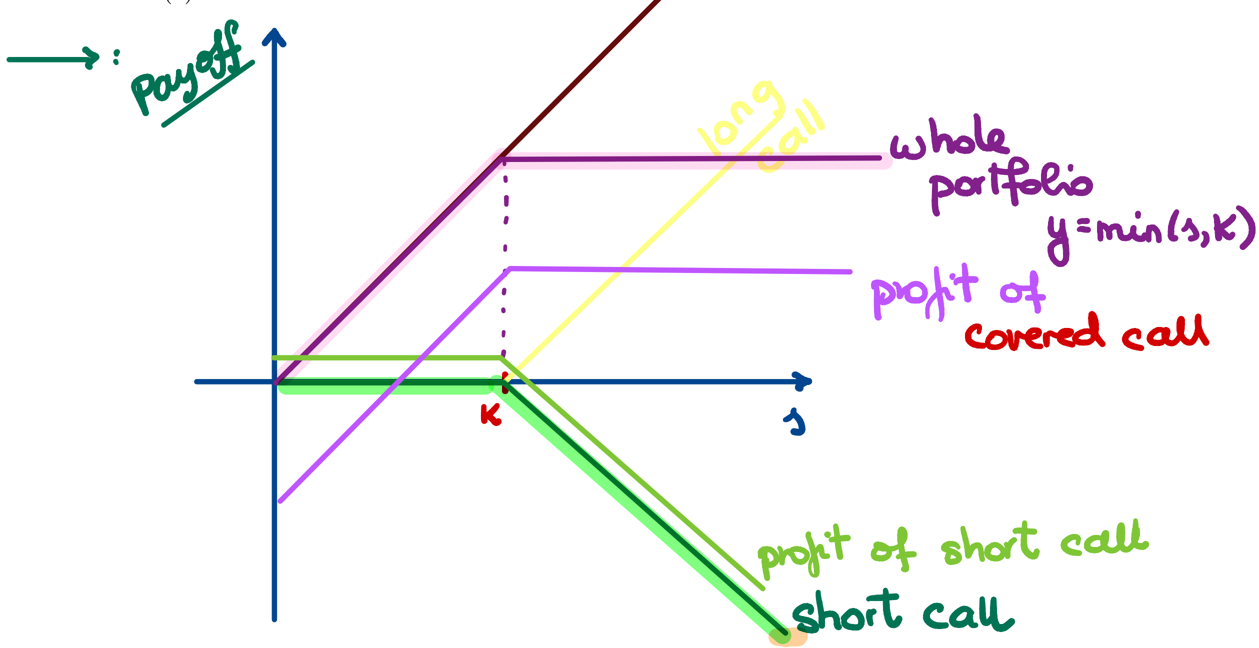
(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

$\longrightarrow$:

$$S^* = K + FV_{0,T}(V_C(0)) = 95 + 8e^{0.04\cdot(0.5)} = 103.16 \quad \square$$

Problem 5.2. (5 points) A stock's price today is $\boxed{\$1000}$ and the annual effective interest rate is given to be 5%. You write a one-year, \$1,050-strike call option for a premium of \$10 while you simultaneously buy the stock. What is your **profit** if the stock's spot price in one year equals \$1,200?

(a) \$150.00
(b) \$139.90
(c) \$10.50
(d) - \$39.00
(e) None of the above.



Algebra:

$$\text{Payoff} = -(S(T)-K)_+ + S(T) = \begin{cases} K & \text{if } S(T) \geq K \\ S(T) & \text{if } S(T) < K \end{cases}$$

$$= \min(S(T), K)$$

Covered call = Short Call + Long Underlying

In this problem:

$$\text{Payoff} = \min(1200, 1050) = 1050$$

$$\text{Initial Cost} = -10 + 1000 = 990$$

$$\text{Profit} = 1050 - 990(1.05) = 10.50$$

Problem 5.3. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for $\boxed{\$1,000}$.

The jeweler models the market price of gold in one year as follows:

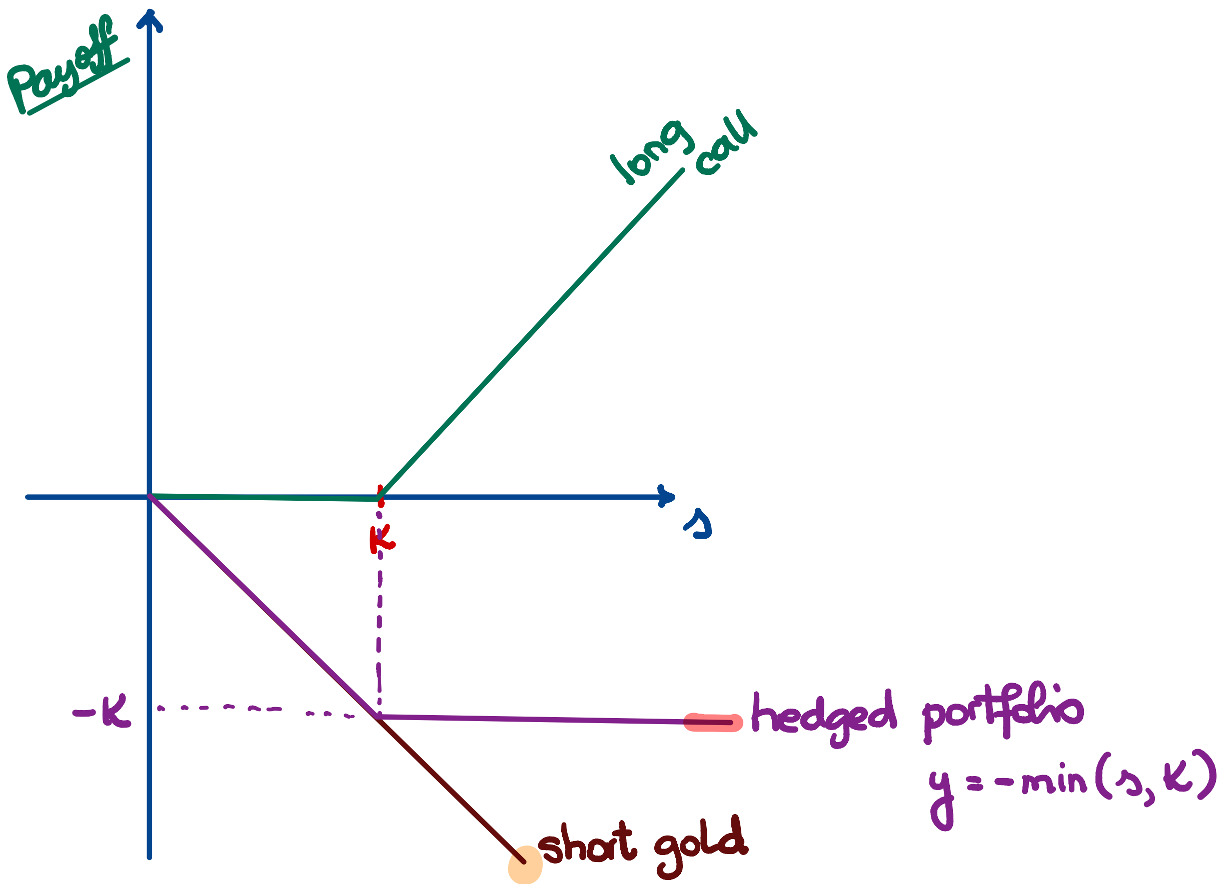
Gold price in one year	Probability	min(S(T), 900)
750 per ounce	0.2	750
850 per ounce	0.5	850
950 per ounce	0.3	900

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of \$900 per ounce. The option costs \$100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelry produced.

→ :



- short underlying
- long call

} CAP

Algebra

$$\text{Payoff(Total)} = \text{Payoff(Gold)} + \text{Payoff(Call)}$$
$$= -S(T) + (S(T) - K)_+$$

$$= \begin{cases} -K & \text{if } S(T) \geq K \\ -S(T) & \text{if } S(T) < K \end{cases} = -\min(S(T), K)$$

In this problem:

$$\mathbb{E}\ \Big|\ \text{Profit} = 1000 - \min(S(T), K) - 100e^{0.05}$$

$$\mathbb{E}[\text{Profit}] = 1000 - \boxed{\mathbb{E}[\min(S(T), K)]} - 100e^{0.05}$$

$$\mathbb{E}[\min(S(T), K)] = 750 \cdot 0.2 + 850 \cdot 0.5 + 900 \cdot 0.3 = 150 + 425 + 270 = 845$$

$$\mathbb{E}[\text{Profit}] = 1000 - 845 - 100e^{0.05} = 155 - 100e^{0.05} = \underline{49.873}$$

$\square$

Problem 5.8. *Source: Sample IFM (Derivatives - Intro), Problem#11*
The current stock price is \$40, and the effective annual interest rate is 8%.

You observe the following option prices:

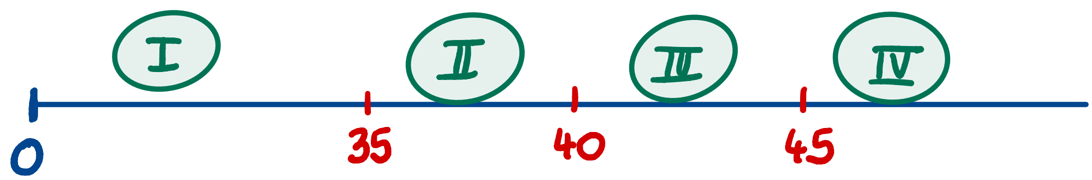
(1) The premium for a \$35-strike, 1-year European call option is \$9.12.
(2) The premium for a \$40-strike, 1-year European call option is \$6.22.
(3) The premium for a \$45-strike, 1-year European call option is \$4.08.

Assuming that all call positions being compared are **long**, at what 1-year stock price range does the \$45-strike call produce a higher profit than the \$40-strike call, but a lower profit than the \$35-strike call? Express your answer as an interval.

→ :

$$(s-40)_+ - 6.22(1.08) \le (s-45)_+ - 4.08(1.08) \le (s-35)_+ - 9.12(1.08)$$

$$(s-40)_+ - 6.72 \le (s-45)_+ - 4.41 \le \boxed{(s-35)_+} - 9.85$$



I $\quad -6.72 \le -4.41 \le -9.85$ No sol'ns here

II $\quad 0-6.72 \le 0-4.41 \le s-35-9.85$