Name:

M339J *Probability Models for Actuarial Applications*
Spring 2021
University of Texas at Austin
Mock In-Term Exam III
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

Problem 3.1. *Source: Problem #9.16 from "Loss Models".*
An insurer pays aggregate claims in excess of the deductible d. In return, they receive a stop-loss premium $\mathbb{E}[(S-d)_+]$. You model the aggregate losses S using a continuous distribution. Moreover, you are given the following information about the aggregate losses S:

(i) $\mathbb{E}[(S-100)_+] = 15$,
(ii) $\mathbb{E}[(S-120)_+] = 10$,
(iii) $\mathbb{P}[80 < S \leq 120] = 0$.

Find the probability that the aggregate claim amounts are less than or equal to 80.

(a) 1/4
(b) 1/2
(c) 3/4
(d) 5/8
(e) None of the above.

Problem 3.2. (5 points) *Source: Sample STAM Problem #213.*
For an insurance portfolio, you are given that:

(i) The number of claims N has the probability mass function:

$$p_N(0) = 0.1, \quad p_N(1) = 0.4, \quad p_N(2) = 0.3, \quad \text{and} \quad p_N(3) = 0.2.$$

(ii) Each claim amount has a Poisson distribution with mean 3.
(iii) The number of claims and claim amounts are mutually independent.

Calculate the variance of aggregate claims.

(a) 4.74
(b) 6.42
(c) 8.02
(d) 12.36
(e) None of the above.

Problem 3.3. (5 points) Suppose that the number N of customers visiting a fast food restaurant in a given morning is Poisson with mean 20. Assume that each customer purchases a drink with probability 3/4, independently from other customers, and independently from the value of N. Let N_1 be the number of customers who purchase drinks in that time interval and let N_2 be the number of customers that do not purchase drinks.

What is the probability that exactly 3 customers purchase a drink in a given morning, **given** that there is a total of 10 customers on that particular morning?

(a) 0.003
(b) 0.007
(c) 0.011
(d) 0.014
(e) None of the above.

Problem 3.4. (5 pts) We are using the aggregate loss model and our usual notation. The frequency random variable N is assumed to be geometric with mean equal to 3. The severity random variable is assumed to have the following probability mass function:

$$p_X(100) = 3/5, \quad p_X(200) = 3/10, \quad p_X(300) = 1/10.$$

The frequency random variable and the independent sequence of severity random variables are assumed to be independent. Find the probability that the total aggregate loss **exactly** equals 300.

(a) 0.09215
(b) 0.11415
(c) 0.10335
(d) 0.12745
(e) None of the above

Problem 3.5. (5 points) *Source: Sample STAM Problem #280.*
A compound Poisson claim distribution has the parameter λ equal to 5 and individual claim amounts X distributed as follows:

$$p_X(5) = 0.6 \quad \text{and} \quad p_X(9) = 0.4.$$

What is the expected cost of an aggregate stop-loss insurance subject to a deductible of 5?

(a) 20
(b) 24.27
(c) 28.034
(d) 33
(e) None of the above.

Problem 3.6. (5 points) *Source: Sample STAM Problem #289.*
A compound Poisson distribution has the parameter λ equal to 5 and the claim amount distribution as follows:

$$p_X(1) = 0.8, \quad p_X(5) = 0.1, \quad \text{and} \quad p_X(10) = 0.1$$

Calculate the probability that aggregate claims will be exactly equal to 6.

(a) 0.028712
(b) 0.03207
(c) 0.047813
(d) 0.051807
(e) None of te above.

Problem 3.7. (5 points) Let us denote the claim count r.v. by N. We are given that N is a mixture random variable such that

$$N \mid \Lambda = \lambda \sim Poisson(\lambda)$$

while Λ is Gamma distributed with mean 2 and variance equal to 4. Then,

(a) 5/9
(b) 8/27
(c) 1/2
(d) 4/9
(e) None of the above

Problem 3.8. (5 points) *Source: Prof. Jim Daniel (personal communication).*
Let the random variable N be in the $(a, b, 0)$ class with $a = b = 3/4$. Find $Var[N]$.

(a) 8
(b) 12
(c) 16
(d) 24
(e) None of the above

Problem 3.9. (5 points) You repeatedly and independently spin a uniform spinner with three colored regions: yellow, blue, and red. After 1800 spins, you tally the number of times your spinner landed on red.

Independently from the spinner, you randomly extract balls from an urn containing 7 teal and 21 beige balls with replacement. After 1000 extractions, you tally the number of times a teal ball was extracted.

What is the variance of the difference between the number of times your spinner landed on red and the number of times a teal ball was extracted?

(a) 587.5
(b) 212.5
(c) 650.0
(d) 150.0
(e) None of the above.

Problem 3.10. (5 points) *Source: Sample STAM Problem #287.*
For an aggregate loss random variable S, you are given that

(i) The number of claims N has a negative binomial distribution with parameters $r = 16$ and $\beta = 6$.
(ii) The claim amounts $X_j, j \geq 1$, are uniformly distributed on the interval $(0, 8)$.
(iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium π such that the probability that aggregate losses will exceed the premium is 5%.

(a) 526.2442
(b) 542.376
(c) 558.587
(d) 660.231
(e) None of the above.

Problem 3.11. (5 points) *Source: Based on Problem #165 from sample C Exam.*
Consider the following collective risk model:
(i) The claim count random variable N is Poisson with mean 3.
(ii) The severity random variable has the following probability (mass) function:

$$p_X(1) = 0.6,\ p_X(2) = 0.4.$$

(iii) As usual, individual loss random variables are mutually independent and independent of N.
Assume that an insurance covers **aggregate losses** subject to a deductible $d = 3$.
Find the expected value of aggregate payments for this insurance.

(a) 0.83
(b) 1.28
(c) 1.67
(d) 2.03
(e) None of the above.

Problem 3.12. (5 points) In the compound model for aggregate claims, let the frequency random variable N be negative binomial with parameters $r = 2$ and $\beta = 4$, and let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the probability (mass) function $p_X(1) = p_X(2) = 0.5$.
Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Calculate $\mathbb{P}[S \leq 2]$.

(a) 0.1232
(b) 0.0768
(c) 0.0944
(d) 0.1047
(e) None of the other offered answers are correct.