M339D: March 21st, 2022.

Properties of Prices of European Calls and Puts.

No Arbitrage

$\Downarrow$

Law of the Unique Price

Equalities (so far):

- Prepaid forward prices on stocks.

$$F^P_{0,T}(S) = \begin{cases} S(0) & \text{no div.} \\ S(0)e^{-\delta T} & \text{cont div.} \\ S(0) - PV(Div) & \text{discrete div.} \end{cases}$$

- Put-Call Parity ✓

$$\boxed{V_c(0) - V_p(0) = F^P_{0,T}(S) - PV_{0,T}(K)}$$

Next: Inequalities

- bounds
- look @ call/put prices as f'tions of the strike K

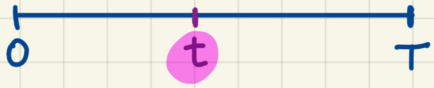
monotonicity

"cord-slope" bounds

convexity

Bounds on call/put prices.

$\begin{cases} T \ldots \text{ exercise date} \\ K \ldots \text{ strike price} \end{cases}$



$S(t) \ldots$ time t stock price
$V_c(t) \ldots$ time t call price
$V_p(t) \ldots$ time t put price

Calls.

✓ **Lower Bounds.**

- $V_c(t) \geq 0$
- By put-call parity: $V_c(t) - \boxed{V_p(t)} = F^P_{t,T}(S) - PV_{t,T}(K)$, where $V_p(t) \geq 0$

$$V_c(t) \geq F^P_{t,T}(S) - PV_{t,T}(K)$$

$$\Rightarrow \boxed{V_c(t) \geq \max\left(F^P_{t,T}(S) - PV_{t,T}(K), 0\right)}$$

Both terms are meaningful!

Q: What if the above inequality is violated?

$\longrightarrow$: Case #1. $V_c(t) < 0 \Rightarrow$ "buy" the call

Case #2. $V_c(t) < F^P_{t,T}(S) - PV_{t,T}(K)$

We propose to exploit the arbitrage opportunity:

- buy the call
- short the prepaid forward

Cost @ time t: $V_c(t) - F^P_{t,T}(S) < -PV_{t,T}(K)$

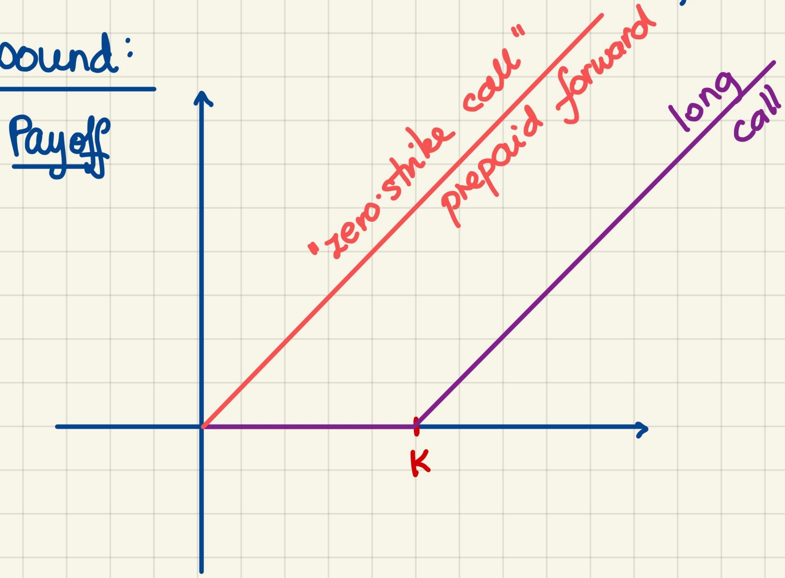
Payoff @ time T: $(S(T) - K)_+ + (-S(T)) =$

$= -\min(S(T), K) \geq -K$

Profit: $\underbrace{-\min(S(T), K)}_{\geq -K} - FV_{t,T}(\text{Cost @ time } t)$

$$> \quad -K \quad + \cancel{FV_{t,T}}(\cancel{PV_{t,T}}(K)) = 0$$

Upper bound:



The payoff of the prepaid forward DOMINATES the payoff of the call option:

$$\Downarrow$$

$$\boxed{F^P_{t,T}(S) \geq V_c(t)}$$

Q: Assume that the stock pays dividends continuously w/ $\boxed{\delta > 0}$. Consider European calls on this stock w/ varying exercise dates T and everything else the same.

$$V_c(t, \underset{\substack{\uparrow \\ \text{exercise date}}}{T}) \xrightarrow[T\to\infty]{} \underline{\;0\;}$$