M339D: September 26th, 2025.

European Put Options.

Usually, a right but NOT an obligation to SELL an underlying @ the STRIKE PRICE!

At time 0: The writer and the buyer of the put agree on:

- the underlying asset: $S(t), t \geq 0$;
- the exercise date T;
- the strike/exercise price K.

The put premium $V_P(0)$ is paid by the put's buyer to the put's writer.

At time T:

- The put's owner has a right, but not an obligation to SELL one unit of the underlying asset for the strike price K.
- The put's writer is obligated to do what the put's owner decides.

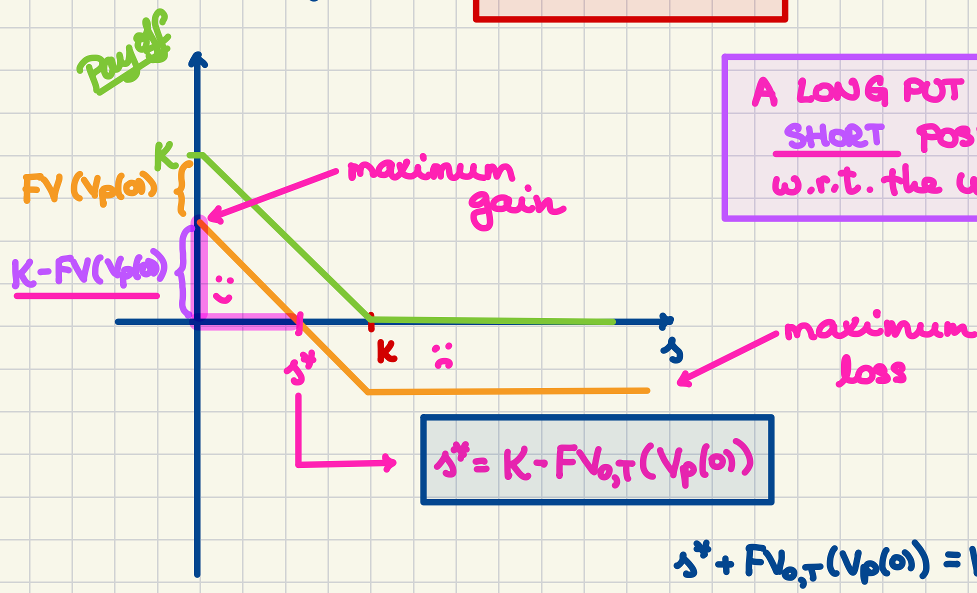
The put's owner's optimal behavior is:

	PAYOFF
IF $K > S(T)$, then exercise.	$K-S(T)$
IF $K \leq S(T)$, then do not exercise.	0

The payoff:

$$\boxed{V_P(T) = \max(K-S(T), 0) = (K-S(T))_+}$$

The payoff f'tion: $\boxed{v_p(s) = (K-s)_+}$



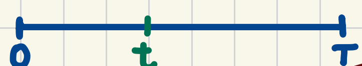
$\boxed{s^* = K - FV_{0,T}(V_p(0))}$

A LONG PUT IS A SHORT POSITION w.r.t. the underlying.

$$s^* + FV_{0,T}(V_p(0)) = K$$

Moneyness.

Consider an option written @ time·0
w/ an exercise date @ time·T.



Imagine the cashflow that would happen if the option were exercised @ time·t.

e.g.,
call: $S(t)-K$
put: $K-S(t)$

If the cashflow is
- >0, the option is in·the·money
- $=0$, —‖— $\boxed{\text{at·the·money}}$
- <0, —‖— out·of·the·money

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Problem Set #6

European put options.

Problem 6.1. The initial price of the market index is $900. After 3 months the market index is priced at $915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of $930, is $8.00. What is the profit at expiration for a **long** put?

DISTRACTION!

(a) $15.00 loss

(b) $6.90 loss

(c) $6.90 gain

(d) $15.00 gain

(e) None of the above.

$i^{(12)}$

$\Rightarrow$ effective monthly

$$j = \frac{i^{(12)}}{12} = 0.004$$

$$FV_{0,T}(V_P(0)) = 8(1.004)^3$$

$$\text{Payoff} = (K - S(T))_+ = (930 - 915)_+ = 15$$

$$\text{Profit} = 15 - 8(1.004)^3 = 6.90$$

$\square$

Problem 6.2. Sample FM(DM) #12

Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4% **convertible semiannually**, and that the put costs $\boxed{74.20}$ now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

ir A. 922.83

B. 924.32

PAYOFF C. 1,000.00

CALL D. 1,075.68

CALL+ir. E. 1,077.17

effective per half-year

$j = \frac{0.04}{2} = 0.02$

We're really looking for the break-even price.

$s^* = K - FV_{0,T}(V_P(0))$

$s^* = 1000 - 74.20(1.02) = 924.32$

Problem 6.3. Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000−cartons' worth in six months. His total costs are \$12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of \$13 per carton and another with the strike price of \$15 per carton. Their premiums are \$0.15 and \$0.18, respectvely.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals \$14. What would Farmer Shaun's profit be if he had decided to hedge using the \$13-strike put versus his profit if he had decided to use the \$15-strike put to hedge?

Focus on the payoff (w/out production costs

unhedged: $S(T)$

hedge: $(K-S(T))_+$

$+$

total hedged:

$$S(T)+(K-S(T))_+=\begin{cases} K & \text{if } K>S(T) \\ S(T) & \text{if } K\leq S(T)\end{cases} = \max(S(T),K)$$

$$\boxed{\text{FLOOR} = \text{long underlying} + \text{long put}}$$

\$13 Payoff $=\max(13,14)=14$

answer: $14-12-0.15\cdot(1.04)=$ ______

\$15 Payoff $=\max(15,14)=15$

$\times$ 10,000