M339D: February 2nd, 2024.

Payoff and Profit Curves.

Example. [Outright Purchase of Stock]

Initial cost: $S(0)$

Payoff: $S(T)$ — a random variable

Profit := Payoff − $FV_{0,T}$(Initial Cost)

= $S(T) - e^{rT} \cdot S(0)$

r... ccrfir

Inspiration ↓

Goal. To study the payoff and the profit as functions of the final asset price.

Introduce: s... an independent argument taking values in $[0, +\infty)$;
it stands for the FINAL ASSET PRICE,
i.e., it's a PLACEHOLDER for the random variable $S(T)$.

Now, we can define the PAYOFF FUNCTION which describes the dependence of payoff on the independent argument s.

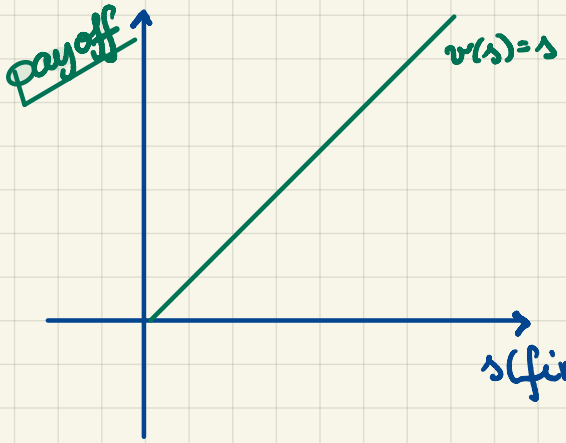
Notation: v... payoff function

$v : [0, +\infty) \longrightarrow \mathbb{R}$

$v(s)$... the agent's payoff is the final asset price is s

Example. [continued]

For the outright purchase: $v(s) = s$ identity f'tion



When we plot the payoff f'tion, we get the payoff curve or the payoff diagram.

In general, the profit function is: $v(s) - FV_{0,T}(\text{Initial Cost})$

For the outright purchase: $s - S(0)e^{rT}$

NO UPPER BOUND, i.e., unlimited growth potential

The payoff and profit curves are increasing

s^*... break-even point

Here: $s^* = S(0)e^{rT}$

Terminology. If the payoff/profit is increasing (not necessarily strictly) as a function of the final asset price s, we say that the portfolio is

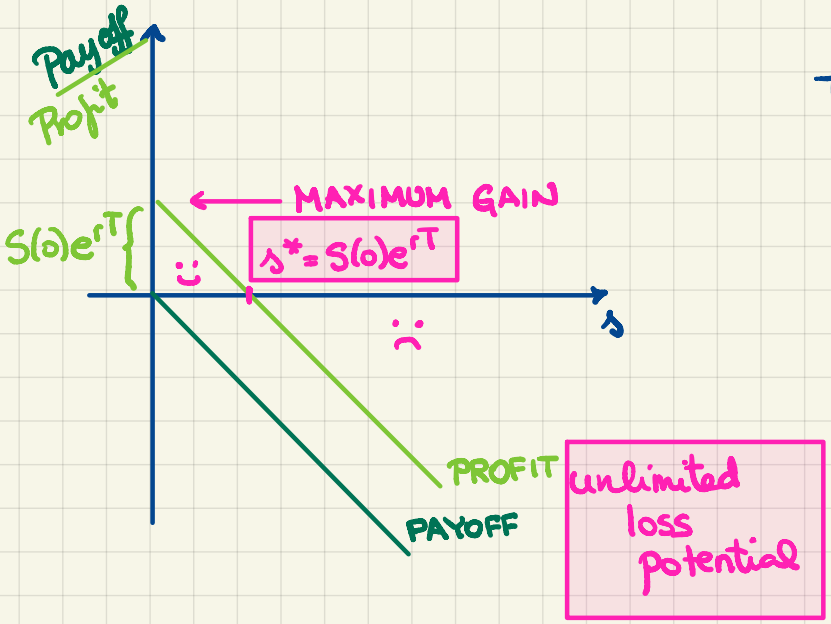
long with respect to the underlying asset.

Example. [A Short Sale]

Initial Cost: $-S(0)$

Payoff: $-S(T)$ $\Rightarrow$ payoff f'tion: $v(s) = -s$

$$\text{Profit} = -S(T) + FV_{0,T}(+S(0))$$
$$= -S(T) + S(0)e^{rT}$$



The profit/payoff is decreasing

⇓

The short sale is short w.r.t. the underlying.

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Problem Set 3

Payoff. Profit.

Problem 3.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time−1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time−0 and liquidate your investment at time−1?

→ :

Profit = Payoff − $FV_{0,1}$(Initial Cost)

$40 \cdot e^{0.04}$

$\mathbb{E}$

$\mathbb{E}[\text{Profit}] = \mathbb{E}[\text{Payoff}] - 40e^{0.04}$

$= \mathbb{E}[S(1)]$

$= 45 \cdot \left(\frac{1}{4}\right) + 42 \cdot \left(\frac{1}{2}\right) + 38 \cdot \left(\frac{1}{4}\right) = 41.75$

answer: $41.75 - 40e^{0.04} = 0.1176$

□