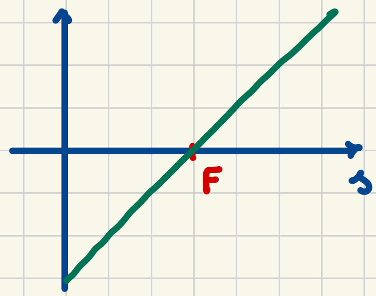
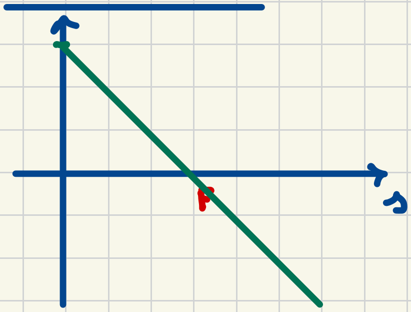
M339D: February 18th, 2025.

Review. An array of payoff curves.

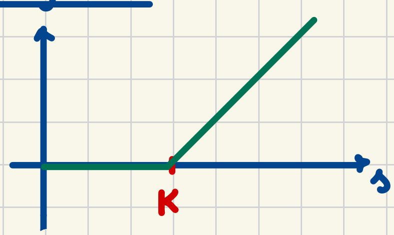
Long Forward. $S(T)-F$



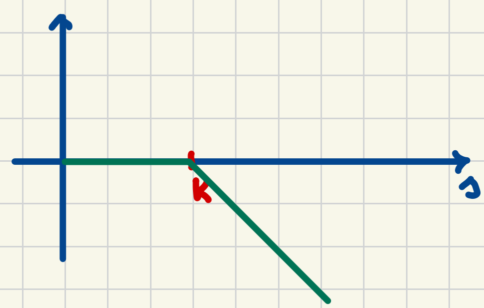
Short Forward. $F-S(T)$



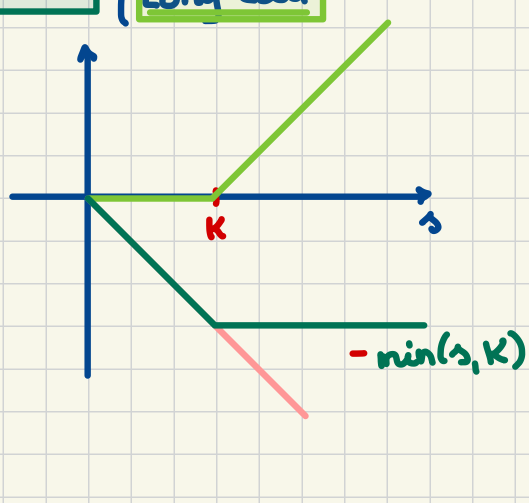
Long Call. $(S(T)-K)_+$



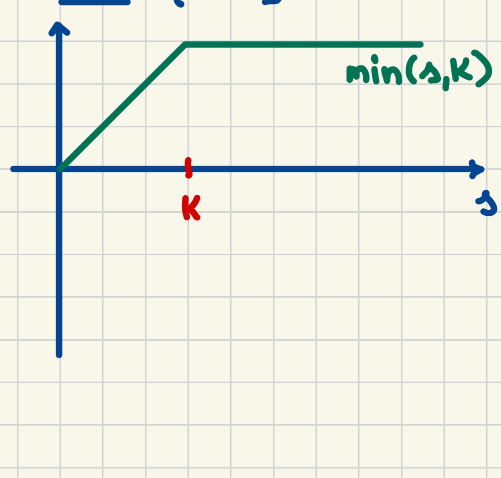
Short/Written Call. $-(S(T)-K)_+$



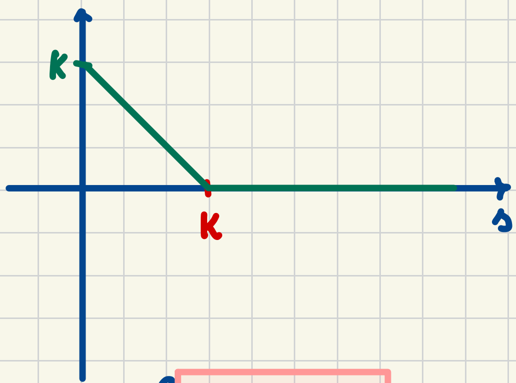
$\boxed{\text{Cap.}}$ $\begin{cases} \text{Short stock} \\ \text{Long Call} \end{cases}$



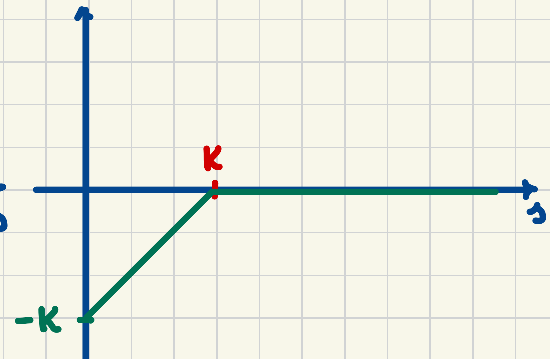
Covered Call. $\begin{cases} \text{Short Call} \\ \text{Long Stock.} \end{cases}$



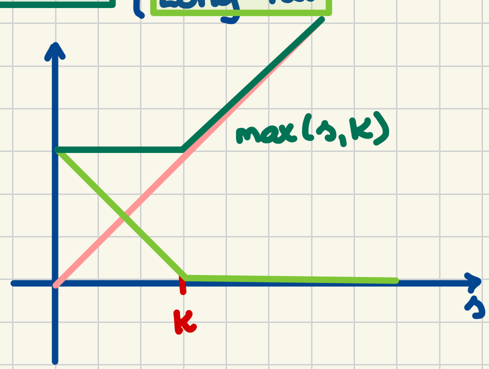
Long Put. $(K-S(T))_+$



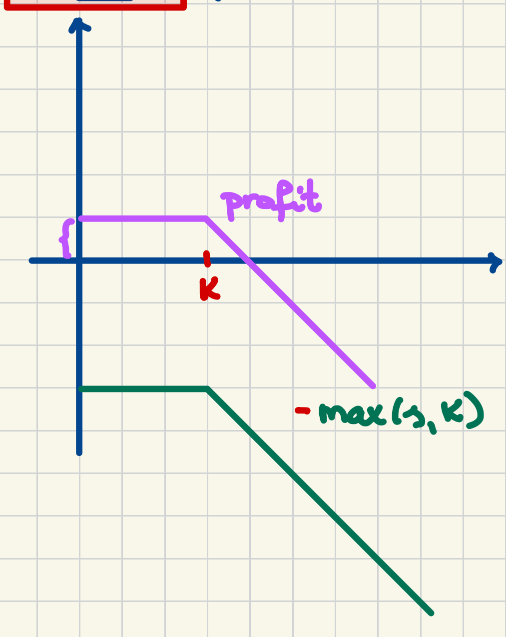
Short Put. $-(K-S(T))_+$



Floor. $\begin{cases} \text{Long Stock} \\ \text{Long Put} \end{cases}$



Covered Put $\begin{cases} \text{Short Put} \\ \text{Short Stock} \end{cases}$

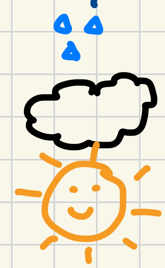


Finite Probability Space.

... serve as environments for the <u>possible paths</u> that the asset price can take.

e.g.,

$$S(T) \sim \begin{cases} 120 & \text{w/ prob. } \frac{1}{6} \\ 80 & \text{w/ prob. } \frac{1}{2} \\ 50 & \text{w/ prob. } \frac{1}{3} \end{cases}$$



Q: What is the expected payoff of a 105-strike put?

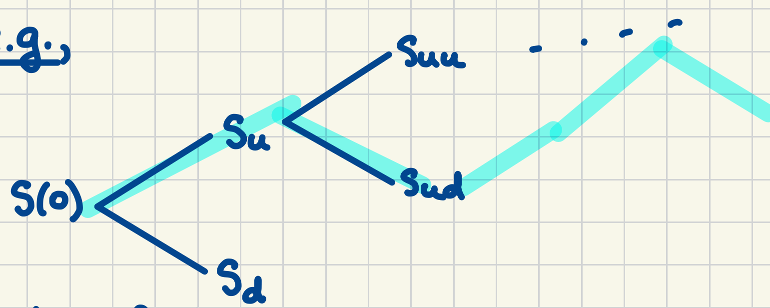
$\longrightarrow$: $\boxed{V_p(T) = (K - S(T))_+}$

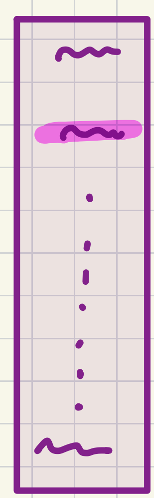
$$V_p(T) \sim \begin{cases} 0 & \text{w/ prob. } \frac{1}{6} \\ 25 & \text{w/ prob. } \frac{1}{2} \\ 55 & \text{w/ prob. } \frac{1}{3} \end{cases}$$

$$\mathbb{E}[V_p(T)] = 0 \cdot \frac{1}{6} + 25 \cdot \frac{1}{2} + 55 \cdot \frac{1}{3} = \dots \quad \square$$

Caveat: $\boxed{\mathbb{E}[g(X)] \neq g(\mathbb{E}[X])}$

e.g.,





All these finitely many scenarios are called **states of the world**.

We assume that

- each can happen, i.e., probab > 0

and

- they exhaust all possibilities, i.e., $\sum$ probab = 1

Arbitrage Portfolios.

Def'n. An **arbitrage portfolio** is a portfolio whose profit is

- nonnegative in ALL STATES OF THE WORLD, i.e., w/ PROBABILITY 1.

and

- strictly positive in AT LEAST ONE state of the world, i.e., w/ PROBABILITY > 0.

> Unless it's specified otherwise in a specific problem or example, we assume NO ARBITRAGE.