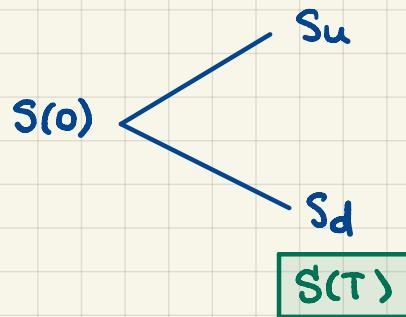
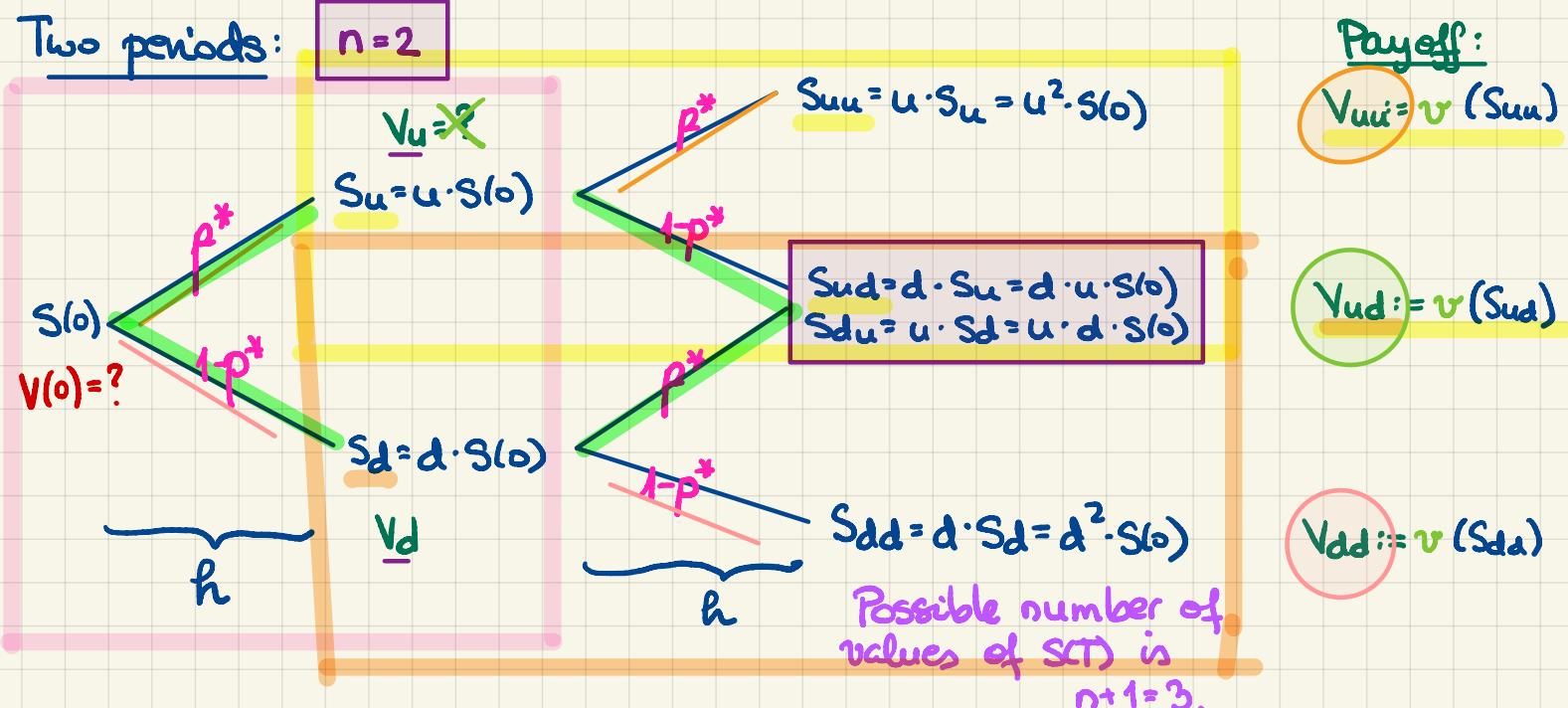
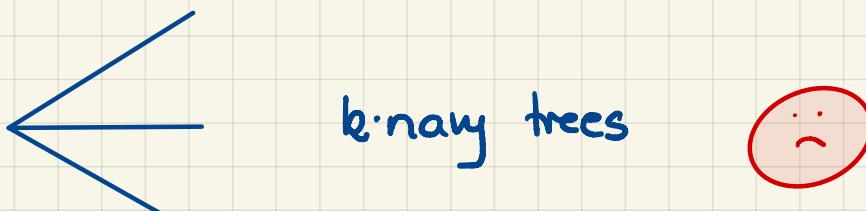


M339D: March 25th, 2024.

Inspiration.



Q: How can we make the model for $S(T)$ more complex?



$$0 \quad h = \frac{T}{2} \quad h = \frac{T}{2} \quad T$$

populating the tree →
← pricing the option

- up node:

replicating portfolio for the option:

$$\Delta_u = \frac{V_{uu} - V_{ud}}{S_{uu} - S_{ud}}$$

$$B_u = e^{-rh} \cdot \frac{u \cdot V_{ud} - d \cdot V_{uu}}{u-d}$$

=> the option's value @ the up node:

$$V_u = \Delta_u \cdot S_u + B_u = e^{-rh} \cdot [p^* \cdot V_{uu} + (1-p^*) \cdot V_{ud}]$$

w/

$$p^* = \frac{e^{rh} - d}{u - d}$$

- down node : Δ_d, B_d

$$\Rightarrow V_d = \Delta_d \cdot S_d + B_d = e^{-rh} \cdot [p^* \cdot V_{ud} + (1-p^*) \cdot V_{dd}] \quad w/$$

- Root node:

$$\Delta_0 = \frac{V_u - V_d}{S_u - S_d}$$

$$B_0 = e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u-d}$$

$$\Rightarrow V(0) = \Delta_0 \cdot S(0) + B_0 \quad \leftarrow$$

From the risk-neutral "perspective":

$$\begin{aligned} V(0) &= e^{-rh} \cdot [p^* \cdot V_u + (1-p^*) \cdot V_d] \\ &= e^{-rh} \cdot [p^* \cdot e^{-rh} (p^* \cdot V_{uu} + (1-p^*) \cdot V_{ud}) + \\ &\quad + (1-p^*) \cdot e^{-rh} ((p^* \cdot V_{ud}) + (1-p^*) \cdot V_{dd})] \\ &= \underbrace{e^{-rT}}_{\text{Discounting.}} \cdot [(p^*)^2 \cdot V_{uu} + 2p^*(1-p^*) \cdot V_{ud} + (1-p^*)^2 \cdot V_{dd}] \end{aligned}$$

Risk-Neutral Expectation of the Payoff.

Generally:

$$V(0) = e^{-rT} \mathbb{E}^* [V(T)]$$

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Problem Set #10

Binomial option pricing: Two or more periods.

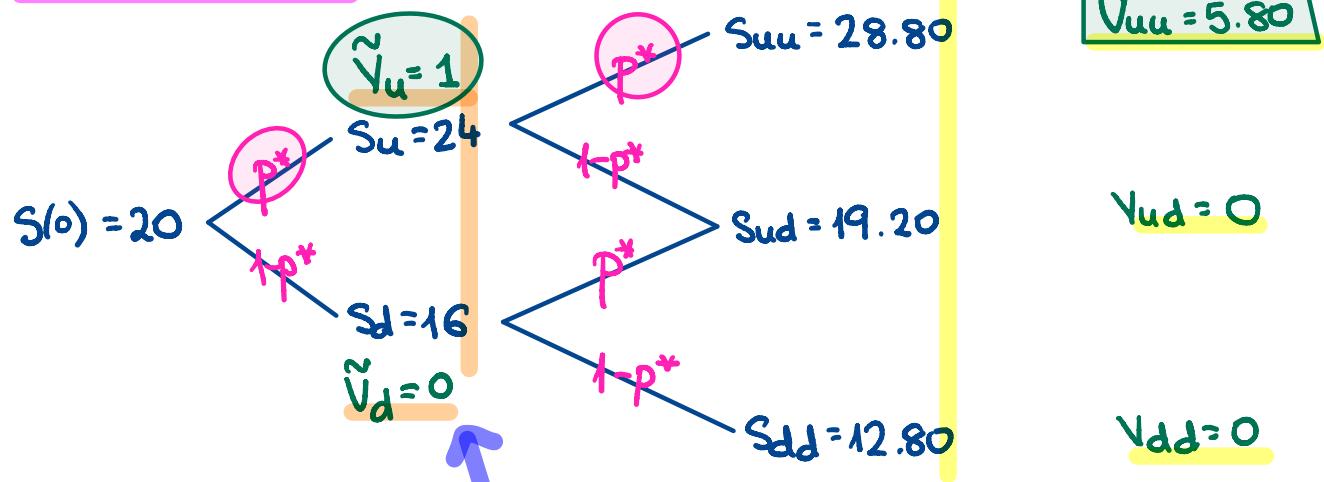
Problem 10.1. For a two-period binomial model, you are given that:

- (1) each period is one year;
- (2) the current price of a non-dividend-paying stock S is $S(0) = \$20$;
- (3) $u = 1.2$, with u as in the standard notation for the binomial model;
- (4) $d = 0.8$, with d as in the standard notation for the binomial model;
- (5) the continuously compounded risk-free interest rate is $r = 0.04$.

Consider a special call option which pays the excess above the strike price $K = 23$ (if any!) at the end of every binomial period.

Find the price of this option.

→ :

Risk-neutral Probability:

$$p^* = \frac{e^{r_h} - d}{u - d} = \frac{e^{0.04} - 0.8}{1.2 - 0.8} = 0.602$$

$$\tilde{V}(0) = e^{-0.04} \cdot p^* \cdot 1 = 0.5784$$

$$V(0) = e^{-0.04(2)} (p^*)^2 \cdot 5.80 = 1.9413$$

answer: the price of the special call is

2.5197

□

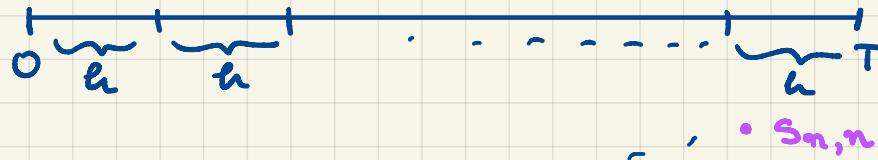
Multiple Binomial Periods.

T... exercise date of a European option

n... # of periods

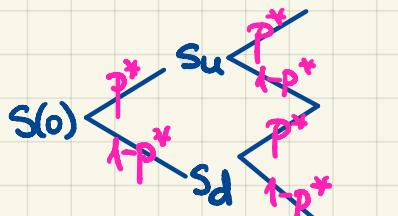
} the length of each period:

$$h = \frac{T}{n}$$



Payoffs

U_{n,n}



• $S_{n,k}$ w/ k ... # of steps up $v_{n,k}$

:

:

• $S_{n,1}$

• $S_{n,0}$

$v_{n,0}$

The $(n+1)$ values
are the support of $S(T)$

=> for every $k = 0, 1, \dots, n$:

$$S_{n,k} = S(0) u^k \cdot d^{n-k} = S(0) \left(\frac{u}{d}\right)^k \cdot d^n$$