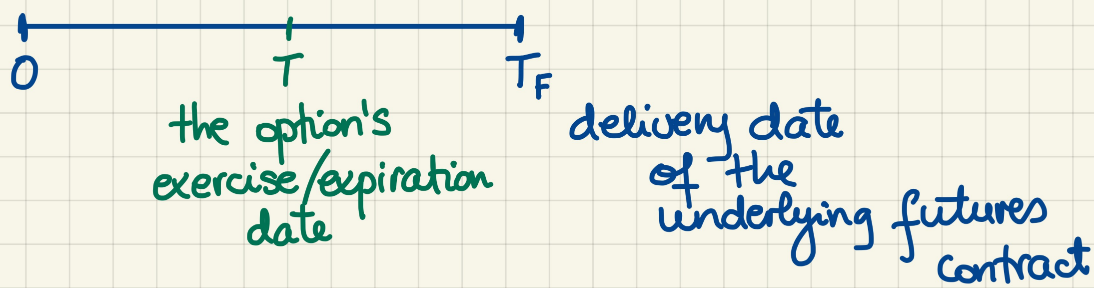
M339D: May 6th, 2022.

Binomial Pricing of Futures Options.

Review: Futures Contracts.

- tradeable "versions" of forward contracts
- liquid counterparts to forward contracts w/ observeable prices

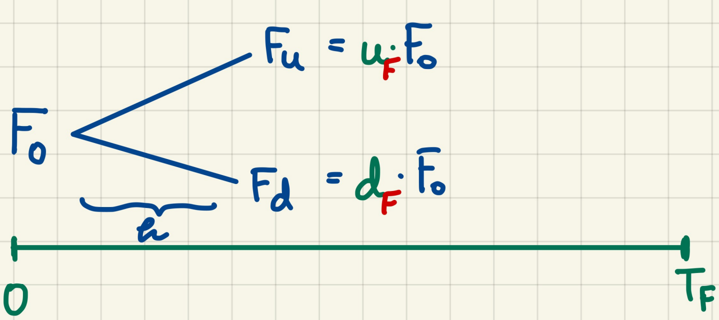
$\Rightarrow$ We write/buy options on futures contracts as the underlying.



Analogy: continuous dividend stocks $\longleftrightarrow$ futures contracts

δ dividend yield $\longleftrightarrow$ r ccrfir

Futures Tree:



Risk-neutral probability:

$$p^* = \frac{\overbrace{e^{(r-r)h}}^{1} - d_F}{u_F - d_F}$$

$$\boxed{p^* = \frac{1 - d_F}{u_F - d_F}} \longleftarrow$$

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months. $\boxed{h = \frac{1}{2}}$

(ii) $\boxed{u/d = 4/3,}$ where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3. $\boxed{p^* = \frac{1}{3}}$

(iv) The initial futures price is 80. $\boxed{F_0 = 80}$

(v) The continuously compounded risk-free interest rate is 5%. $r = 0.05$

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

$\boxed{\text{(E) 0.088}}$

$$(ii) \Rightarrow \frac{u_F}{d_F} = \frac{4}{3}$$

$$(iii) \Rightarrow p^* = \frac{1}{3} = \frac{1 - d_F}{u_F - d_F} = \frac{\frac{1}{d_F} - 1}{\frac{u_F}{d_F} - 1}$$

$$\frac{1}{d_F} - 1 = \frac{1}{9} \Rightarrow \frac{1}{d_F} = \frac{10}{9} \Rightarrow d_F = 0.9 \Rightarrow u_F = 1.2$$

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

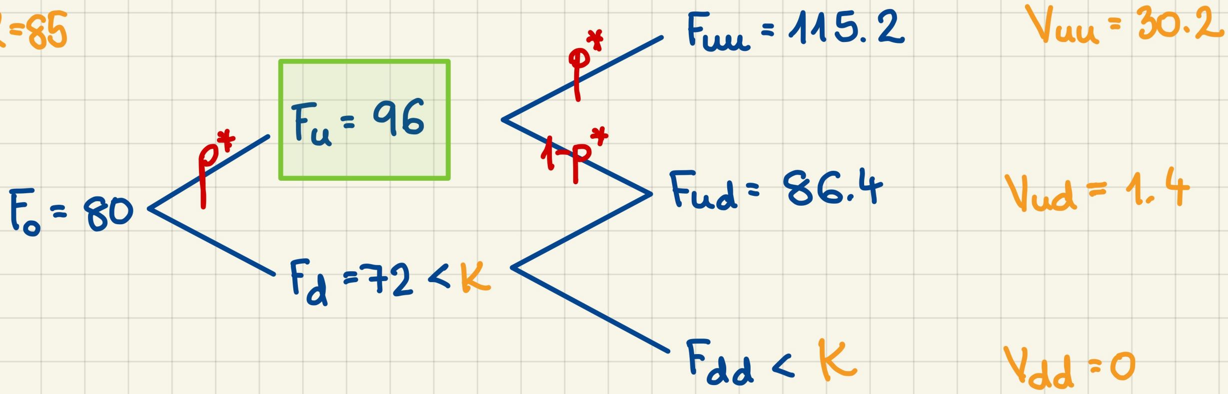
(i) The risk-free interest rate is constant.

(ii)

	Several months ago	Now
Stock price	\$40.00	\$50.00
Call option price	\$ 8.88	\$14.42
Put option price	\$ 1.63	\$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

$K = 85$



Focus on the up node:

- $IE_u = 96 - 85 = 11$
- $CV_u = e^{-0.05(0.5)}\left(\frac{1}{3}\cdot 30.2 + \frac{2}{3}\cdot 1.4\right) = 10.7284$

$$V_u^A - V_u^E = 11 - 10.7284 = 0.2716$$

The difference @ the root node:

$$C_{II} - C_I = e^{-0.05(0.5)}\cdot\frac{1}{3}\cdot 0.2716 = \underline{0.0883}$$

$\square$