Name:

M339J/M389J: Probability Models with Actuarial Applications
The University of Texas at Austin
Sample Problems for In-Term Exam III
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Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.

3.1. **TRUE/FALSE QUESTIONS.**

Problem 3.1. (2 pts) Let N have a mixture distribution with the mixing variable Λ.
More precisely, let

$$N \mid \Lambda = \lambda \sim Poisson(\lambda)$$

with $\Lambda \sim Gamma(\alpha, \theta)$. Then,

$$N \sim NegBin(\alpha, \theta).$$

True or false?

Solution: **TRUE** with $r = \alpha, \beta = \theta$

Problem 3.2. (2 points) With our usual assumptions in place, assume that the claim count follows the Poisson distribution with parameter $\lambda = 10$.
The severities of individual claims are such that on average half of the claims are expected to have claim amounts equal to 50 and the other half are expected to have the claim amounts equal to 100.
Then, the total losses have the expected value 750. *True or false?*

Solution: TRUE

Problem 3.3. (2 points) In the compound aggregate loss model, with our usual notation and with the usual assumptions in place, we have that the claim count distribution is binomial with $m = 40$ and $q = 1/4$, and that the p.m.f. of the individual losses is

$$p_X(10) = p_X(20) = 0.3, p_X(50) = 0.4.$$

Then, $\mathbb{E}[S] \leq 290$. *True or false?*

Solution: TRUE
In fact,

$$\mathbb{E}[S] = \mathbb{E}[N]\mathbb{E}[X]$$

with $N \sim bin(m = 40, q = 1/4)$ and X as in the problem statement. We have

$$\mathbb{E}[N] = mq = 40(1/4) = 10$$
$$\mathbb{E}[X] = 10 \cdot 0.3 + 20 \cdot 0.3 + 50 \cdot 0.4 = 29.$$

So,

$$\mathbb{E}[S] = \mathbb{E}[N]\mathbb{E}[X] = 10 \cdot 29 = 290 \leq 290.$$

3.2. **Free-response problems.** *Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.*

Problem 3.4. A Poisson distribution is used to fit frequency data. You are given that:

- there was a total of 20 observations less than or equal to 1;
- there were 6 observations equal to 2;
- there were 4 observations equal to 3;
- there were no observations greater than or equal to 4.

What is the maximum likelihood estimate for the mean of this Poisson distribution?

Solution: The likelihood function is

$$L(\lambda) = (e^{-\lambda}(1+\lambda))^{20}(e^{-\lambda}\frac{\lambda^2}{2!})^6(e^{-\lambda}\frac{\lambda^3}{3!})^4 \propto e^{-30\lambda}(1+\lambda)^{20}\lambda^{24}.$$

The likelihood function is

$$l(\lambda) = const - 30\lambda + 20\ln(1+\lambda) + 24\ln(\lambda).$$

Differentiating, we obtain

$$l'(\lambda) = -30 + \frac{20}{1+\lambda} + \frac{24}{\lambda}.$$

We equate the above to zero and solve for λ in

$$-15\lambda(1+\lambda) + 10\lambda + 12(1+\lambda) = 0,$$

i.e.,

$$15\lambda^2 - 7\lambda - 12 = 0 \quad \Rightarrow \quad \lambda = \frac{7+\sqrt{49+720}}{30} = 1.157695.$$

Problem 3.5. Let the number of losses be negative binomial with mean 6 and variance 24. The losses have the Pareto distribution with $\theta = 100$ and $\alpha = 6$. The number of losses and loss amounts are assumed to be independent. There is an ordinary deductible of 25 on individual losses. What is the variance of the number of claims?

Solution: We are given that $N^L \sim NegBinomial(r, \beta)$. We are also given that $r\beta = 6$ and $r\beta(1+\beta) = 24$. Whence, we obtain that $r = 2$ and $\beta = 3$. As we derived in class, the number of claims will also be negative binomial with parameters $r^P = r$ and $\beta^P = \beta v$ where v denotes the probability that an individual loss meets the deductible. For the Pareto distribution with the given parameter values, we have

$$\mathbb{P}[X > 25] = \left(\frac{100}{100+25}\right)^6 = (0.8)^6 = 0.262144.$$

So, the variance of the number of claims is

$$2(3(0.262144))(1 + 3(0.262144)) = 2.809815.$$

Problem 3.6. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 1.

Let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the following p.m.f.

$$p_X(100) = 1/2, p_X(200) = 3/10, p_X(300) = 1/5.$$

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Find the expected value of the **policyholder's** payment for a stop-loss insurance policy with an ordinary deductible of 200, i.e., calculate $\mathbb{E}[S \wedge 200]$.

Solution: Note that S has the support of the form $\{0, 100, 200, 300, \dots\}$. So,

$$\mathbb{E}[S \wedge 200] = 100\mathbb{P}[S = 100] + 200\mathbb{P}[S \geq 200].$$

Next,

$$\begin{aligned} \mathbb{P}[S = 0] &= \mathbb{P}[N = 0] = e^{-1}, \\ \mathbb{P}[S = 100] &= \mathbb{P}[N = 1, X_1 = 100] = 0.5e^{-1}, \\ \mathbb{P}[S \geq 200] &= 1 - \mathbb{P}[S = 0] - \mathbb{P}[S = 100] = 1 - 1.5e^{-1}. \end{aligned}$$

So,

$$\mathbb{E}[S \wedge 200] = 100 \cdot 0.5e^{-1} + 200(1 - 1.5e^{-1}) = 200 - 250e^{-1} \approx 108.03.$$

Problem 3.7. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 20.

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be the Gamma distribution with parameters $\alpha = 2.5$ and $\theta = 3,000$.

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.

Using the normal approximation, find the amount of the premium π such that the total aggregate losses S exceed the premium with the probability of at most 1%.

Solution: Let $\mu_S = \mathbb{E}[S]$ and $\sigma_S = \sqrt{Var[S]}$. Then, using the normal approximation, we have

$$0.01 = \mathbb{P}[S > \pi] = \mathbb{P}\left[\frac{S - \mu_S}{\sigma_S} > \frac{\pi - \mu_S}{\sigma_S}\right] \approx 1 - \Phi\left(\frac{\pi - \mu_S}{\sigma_S}\right)$$

where Φ denotes the c.d.f. of the standard normal distribution. From the tables for Φ, we get

$$\pi = \mu_S + 2.33\sigma_S.$$

From the given information on the severity r.v.s, we obtain

$$\begin{aligned}
\mathbb{E}[X] &= \theta\alpha = 7,500 \\
\mathbb{E}[X^2] &= \theta^2\alpha(\alpha+1) = 3000^2 \cdot 2.5 \cdot 3.5 = 7.875 \cdot 10^7, \\
\mathbb{E}[N] &= Var[N] = 20.
\end{aligned}$$

So,

$$\begin{aligned}
\mu_S &= \mathbb{E}[S] = \mathbb{E}[X]\mathbb{E}[N] = 20 \cdot 7500 = 150,000, \\
\sigma_S^2 &= Var[S] = Var[X]\mathbb{E}[N] + Var[N]\mathbb{E}[X]^2 = \lambda \cdot \mathbb{E}[X^2] = 20 \cdot 7.875 \cdot 10^7 = 15.75 \cdot 10^8.
\end{aligned}$$

Hence,

$$\pi = 150000 + 2.33 \cdot 3.96863 \cdot 10^4 \approx 242469.$$

Problem 3.8. (10 points) The frequency random variable N is assumed to have a Poisson distribution with a mean of 2. Individual claim severity random variable X has the following probability mass function

$$p_X(100) = 0.6, \quad p_X(200) = 0.3, \quad p_X(300) = 0.1.$$

Let the above be the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$, and Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$. Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$. Calculate the probability that S is exactly equal to 3.

Solution:

$$\begin{aligned}
\mathbb{P}[S = 300] &= \mathbb{P}[N = 1, X_1 = 300] + \mathbb{P}[N = 2, X_1 = 100, X_2 = 200] \\
&\quad + \mathbb{P}[N = 2, X_1 = 200, X_2 = 100] + \mathbb{P}[N = 3, X_1 = 100, X_2 = 100, X_3 = 100] \\
&= 2e^{-2} \cdot 0.1 + 2 \cdot \frac{2^2}{2} e^{-2} \cdot 0.6 \cdot 0.3 + \frac{2^3}{3 \cdot 2} \cdot e^{-2} \cdot 0.6^3 \\
&= e^{-2}(0.2 + 0.72 + 0.288) = 1.208e^{-2} \approx 0.1635.
\end{aligned}$$

Problem 3.9. (6 points) In the compound model for aggregate claims, let the frequency random variable N have the probability (mass) function

$$p_N(0) = 0.4, p_N(1) = 0.3, p_N(2) = 0.2, p_N(4) = 0.1.$$

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the probability (mass) function $p_X(1) = 0.3$ and $p_X(2) = 0.7$.
Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Calculate $\mathbb{E}[(S-2)_+]$.

Solution: We intend to use the equality

$$\mathbb{E}[(S-2)_+] = \mathbb{E}[S] - \mathbb{E}[S \wedge 2].$$

Using

$$\begin{aligned}\mathbb{E}[N] &= 0.4 \cdot 0 + 0.3 \cdot 1 + 0.2 \cdot 2 + 0.1 \cdot 4 = 0.3 + 0.4 + 0.4 = 1.1,\\ \mathbb{E}[X] &= 0.3 \cdot 1 + 0.7 \cdot 2 = 0.3 + 1.4 = 1.7,\end{aligned}$$

we get

$$\mathbb{E}[S] = \mathbb{E}[N]\mathbb{E}[X] = 1.1 \cdot 1.7 = 1.87.$$

On the other hand,

$$\begin{aligned}\mathbb{E}[S \wedge 2] &= \mathbb{P}[S > 0 + \mathbb{P}[S > 1]\\ &= (1 - F_S(0)) + (1 - F_S(1)).\end{aligned}$$

From the problem statement, we conclude that

$$\begin{aligned}F_S(0) &= \mathbb{P}[S \leq 0] = \mathbb{P}[S = 0] = \mathbb{P}[N = 0] = 0.4,\\ F_S(1) &= \mathbb{P}[S \leq 1] = \mathbb{P}[S = 0] + \mathbb{P}[S = 1] = \mathbb{P}[N = 0] + \mathbb{P}[N = 1, X_1 = 1]\\ &= 0.4 + 0.3 \cdot 0.3 = 0.49.\end{aligned}$$

Finally,

$$\mathbb{E}[S \wedge 2] = 0.6 + 0.51 = 1.11.$$

and

$$\mathbb{E}[(S-2)_+] = 1.87 - 1.11 = 0.76.$$

3.3. MULTIPLE CHOICE QUESTIONS.

Problem 3.10. (5 pts) Let us denote the claim count r.v. by N. We are given that N is a mixture random variable such that

$$N \,|\, \Lambda = \lambda \sim Poisson(\lambda)$$

while Λ is Gamma distributed with mean 2 and variance equal to 4. Then,

(a) $F_N(1) \leq 0.1$
(b) $0.1 < F_N(1) \leq 0.2$
(c) $0.2 < F_N(1) \leq 0.3$
(d) $0.3 < F_N(1) \leq 0.5$

(e) None of the above

Solution: (e)
We have shown in class that the distribution of N is negative binomial. Let us find its parameters r and β. Using the fact that $N \mid \Lambda$ is Poisson, we get

$$r\beta = \mathbb{E}[N] = \mathbb{E}[\mathbb{E}[N \mid \Lambda]] = \mathbb{E}[\Lambda] = 2,$$
$$r\beta(1+\beta) = Var[N] = \mathbb{E}[Var[N \mid \Lambda]] + Var[\mathbb{E}[N \mid \Lambda]] = \mathbb{E}[\Lambda] + Var[\Lambda] = 6.$$

So, $\beta = 2$ and $r = 1$. Finally, using our tables, we get

$$F_N(1) = p_N(0) + p_N(1) = \frac{1}{3} + \frac{2}{9} = \frac{5}{9} > \frac{1}{2}.$$

Problem 3.11. (5 points) Let the number of typos in the new edition of the *"Lord of the Rings"* trilogy be denoted by N and modeled using the Poisson distribution with mean 80. Some typos involve elves' names and others do not involve the elves' names. The number of typos and the type of the typo are assumed to be independent.

Assume that the probability that an observed typo involves an elf's name equals $1/5$.

Find the probability that the number of typos involving elves' names is 2, given that the **total** number of typos in the trilogy equals 5.

(a) 0.0406
(b) 0.0842
(c) 0.1632
(d) 0.2048
(e) None of the above.

Solution: (d)
Let N denote the total number of typos and let N_e denote the r.v. which stands for the number of typos involving elves' names. According to our extension of the *"Thinning" theorem*,

$$N_e \mid N = 5 \sim Binomial(m = 5, q = 1/5).$$

We have that

$$\mathbb{P}[N_e = 2 \mid N = 5] = \binom{5}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 = 10\left(\frac{4^3}{5^5}\right) = 0.2048$$

Problem 3.12. (8 points) Let N be an $\mathbb{N}_0$-valued random variable from the $(a, b, 0)$ class. You are given that

$$p_0 = 0.049787, \quad p_1 = 0.149361, \quad \text{and} \quad p_2 = 0.224042.$$

Find p_6.

(a) 0.04
(b) 0.05

(c) 0.06
(d) 0.07
(e) None of the above.

Solution: (b)
From the given information, we conclude that

$$\frac{p_1}{p_0} = 3 = a + \frac{b}{1} = a + b, \quad \text{and} \quad \frac{p_2}{p_1} = 1.5 = a + \frac{b}{2}.$$

So, $b = 3$ and $a = 0$. We conclude that the distribution of N is Poisson with parameter $\lambda = 3$. Finally,

$$p_6 = e^{-3} \times \frac{3^6}{6!} = 0.050409.$$

Problem 3.13. (5 points) Let S be the aggregate claims random variable. You are given the following:
(i) $\mathbb{E}[(S-100)_+] = 15$,
(ii) $\mathbb{E}[(S-120)_+] = 10$,
(iii) $\mathbb{P}[80 < S \leq 120] = 0$.
Find $\mathbb{E}[(S-105)_+]$.

(a) $\mathbb{E}[(S-105)_+] \approx 10.75$
(b) $\mathbb{E}[(S-105)_+] \approx 12.75$
(c) $\mathbb{E}[(S-105)_+] \approx 13.75$
(d) $\mathbb{E}[(S-105)_+] \approx 15.75$
(e) None of the above

Solution: (c)
By the Interpolation Theorem,

$$\mathbb{E}[(S-105)_+] = \frac{3}{4}\mathbb{E}[(S-100)_+] + \frac{1}{4}\mathbb{E}[(S-120)_+] = 11.25 + 2.5 = 13.75.$$

Problem 3.14. (5 points) Consider the following individual observed values:

$$5, 8, 10$$

of a random variable X whose distribution function is given by $F_X(x) = 1 - (1/x)^p$ for $x > 1$ and an unknown parameter p. Let $\hat{p}$ denote the Maximum Likelihood Estimate of p based on the above observed values. Then,

(a) $\hat{p} \approx \frac{3}{2}$
(b) $\hat{p} \approx \frac{3\ln(20)}{2}$
(c) $\hat{p} \approx \frac{3\ln(20)}{4}$
(d) $\hat{p} \approx \frac{3}{2\ln(20)}$

(e) None of the above

Solution: (d)
The density function of X is

$$f_X(x) = F_X'(x) = -(-p)x^{-p-1} = px^{-p-1} \qquad x > 1.$$

So, the likelihood function has the form

$$\begin{aligned} L(p) &= p5^{-p-1} \cdot p8^{-p-1} \cdot p10^{-p-1} \\ &= p^3(5 \cdot 8 \cdot 10)^{-p-1}, \end{aligned}$$

and the loglikelihood function is

$$l(p) = 3\ln(p) - (p+1)\ln(400).$$

Differentiating with respect to p, we get

$$l'(p) = 3/p - \ln(400).$$

Setting the above equal to zero and solving for p, we get

$$\hat{p} = 3/\ln(400) = 3/(2\ln(20)).$$

Problem 3.15. (5 points) Consider the following individual observed values

$$5, 10$$

and the one right censored value 8 of a random variable X whose distribution function is given by $F_X(x) = 1 - (1/x)^p$ for $x > 1$ and an unknown parameter p. Let $\hat{p}$ denote the Maximum Likelihood Estimate of p based on the above observed values. Then,

(a) $\hat{p} \approx \frac{1}{\ln(20)}$
(b) $\hat{p} \approx \frac{3\ln(20)}{2}$
(c) $\hat{p} \approx \frac{3}{4\ln(20)}$
(d) $\hat{p} \approx \frac{3}{2\ln(20)}$
(e) None of the above

Solution: (a)
The density function of X is

$$f_X(x) = F_X'(x) = -(-p)x^{-p-1} = px^{-p-1} \qquad x > 1.$$

So, the likelihood function has the form

$$\begin{aligned} L(p) &= p5^{-p-1} \cdot 8^{-p} \cdot p10^{-p-1} \\ &= p^2(5 \cdot 10)^{-p-1}8^{-p}, \end{aligned}$$

and the loglikelihood function is

$$l(p) = 2\ln(p) - (p+1)\ln(50) - p\ln(8).$$

Differentiating with respect to p, we get

$$l'(p) = 2/p - \ln(400).$$

Setting the above equal to zero and solving for p, we get

$$\hat{p} = 2/\ln(400) = 2/(2\ln(20)) = 1/\ln(20).$$

Problem 3.16. You fit a distribution with the following density function:

$$f_X(x) = (p+1)x^p, \quad 0 < x < 1, p > -1.$$

As usual, your observations are denoted by

$$x_1, x_2, \ldots, x_n.$$

What is the expression for the maximum likelihood estimate of the parameter p?

(a) $-\frac{n}{\sum_{i=1}^n \ln(x_i)} - 1$
(b) $-\frac{1}{n}\sum_{i=1}^n \ln(x_i)$
(c) $\frac{1}{n}\sum_{i=1}^n x_i$
(d) $\frac{\sum_{i=1}^n \ln(x_i)}{n} - 1$
(e) None of the above.

Solution: (a)
The likelihood function is

$$L(p) = \prod_{i=1}^n f_X(x_i;p) = \prod_{i=1}^n ((p+1)x_i^p) = (p+1)^n \left(\prod_{i=1}^n x_i\right)^p.$$

Hence, the log-likelihood function is

$$\ell(p) = \ln(L(p)) = n\ln(p+1) + p\left(\sum_{i=1}^n \ln(x_i)\right).$$

The derivative of the log-likelihood function is

$$\ell'(p) = \frac{n}{p+1} + \left(\sum_{i=1}^n \ln(x_i)\right).$$

Equating the above to zero, and solving for p, we obtain

$$\frac{n}{p+1} = -\sum_{i=1}^n \ln(x_i) \quad \Rightarrow \hat{p}_{MLE} = -\frac{n}{\sum_{i=1}^n \ln(x_i)} - 1\,.$$

Problem 3.17. You have observed the following three loss amounts:

$$190, \quad 90, \quad 60$$

Four other loss amounts are known to be less than or equal to 60. Losses follow an inverse exponential distribution. Calculate the maximum likelihood estimate of the population mode based on the above data.

(a) 10.125
(b) 12.378
(c) 15.044
(d) 20.232
(e) None of the above.

Solution: (c)
The cumulative distribution function of the inverse exponential distribution is given in the STAM tables:

$$F_X(x;\theta) = e^{-\theta/x},$$

while its probability density function has the form:

$$f_X(x;\theta) = \frac{\theta e^{-\theta/x}}{x^2}, \quad x > 0,$$

So, the likelihood function in this problem has the form

$$\begin{aligned} L(\theta) &= f_X(190;\theta) f_X(90;\theta) f_X(60;\theta)\,(F_X(x;\theta))^4 \\ &\propto (\theta e^{-\theta/190})(\theta e^{-\theta/90})(\theta e^{-\theta/60})\left(e^{-\theta/60}\right)^4 \\ &= \theta^3 e^{-\theta\left(\frac{1}{190}+\frac{1}{90}+\frac{1}{60}+\frac{1}{15}\right)}. \end{aligned}$$

Thus, the log-likelihood function can be written as

$$\ell(\theta) = c + 3\ln(\theta) - \theta\left(\frac{1}{190} + \frac{1}{90} + \frac{1}{60} + \frac{1}{15}\right)$$

where c stands for an irrelevant constant. Differentiating the log-likelihoos function with respect to θ, we obtain

$$\ell'(\theta) = \frac{3}{\theta} - \left(\frac{1}{190} + \frac{1}{90} + \frac{1}{60} + \frac{1}{15}\right).$$

We equate the above expression to zero and solve for θ. We get

$$\frac{3}{\theta} = \frac{1}{190} + \frac{1}{90} + \frac{1}{60} + \frac{1}{15} \quad \Rightarrow \quad \hat{\theta}_{MLE} = \frac{3}{\frac{1}{190} + \frac{1}{90} + \frac{1}{60} + \frac{1}{15}} = 30.08797654.$$

According to the STAM tables, the mode of the inverse exponential distribution is $\theta/2$. So, our answer is 15.044.

Problem 3.18. *Source: Sample STAM Problem#196.*
You are given the following 10 bodily injury losses (before the deductible is applied):

Loss amount	Number of losses	Policy limit
200	3	500
400	4	800
> 800	3	800

Past experience indicates that these losses follow a two-parameter Pareto distribution with parameters α unknown and $\theta = 1,000$. Calculate the maximum likelihood estimate of α.

(a) 1.9145
(b) 2.307
(c) 2.853
(d) 3.089
(e) None of the above.

Solution: (a)
According to the STAM tables, the density function and the distribution function of the two-parameter Pareto are

$$f_X(x) = \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}} \quad \text{and} \quad F_X(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha.$$

So, the likelihood function with the above 7 unmodified data points and 3 right-censored data points is

$$\begin{aligned} L(\alpha) &= \left(\frac{\alpha 1000^\alpha}{(200+1000)^{\alpha+1}}\right)^3 \left(\frac{\alpha 1000^\alpha}{(400+1000)^{\alpha+1}}\right)^4 \left(\frac{1000}{800+1000}\right)^{3\alpha} \\ &= \left(\frac{\alpha 1000^\alpha}{(1200)^{\alpha+1}}\right)^3 \left(\frac{\alpha 1000^\alpha}{(1400)^{\alpha+1}}\right)^4 \left(\frac{1000}{1800}\right)^{3\alpha} \\ &\propto \alpha^7 (1000)^{10\alpha}(1200)^{-3\alpha}(1400)^{-4\alpha}(1800)^{-3\alpha}. \end{aligned}$$

So, the log-likelihood can be written as

$$\ell(\alpha) = 7\ln(\alpha) + 10\alpha\ln(1000) - 3\alpha(\ln(1200)+\ln(1800)) - 4\alpha\ln(1400) + c$$

where c denotes an irrelevant constant. Differentiating with respect to α, we obtain

$$\ell'(\alpha) = \frac{7}{\alpha} + 10\ln(1000) - 3(\ln(1200)+\ln(1800)) - 4\ln(1400).$$

Equating the above expression to zero, and solving for α, we get

$$\alpha = \frac{7}{-10\ln(1000) + 3(\ln(1200)+\ln(1800)) + 4\ln(1400)} = 1.914548969.$$

Problem 3.19. *Source: Sample STAM Problem #152.*
You are given the following information:

(i) A sample of losses is: $400, 600, 700, 900, 1000$
(ii) No information is available about losses of 300 or less.
(iii) Losses are assumed to follow an exponential distribution with mean θ.

Calculate the maximum likelihood estimate of θ based on the above data.

(a) 420

(b) 520

(c) 720

(d) 920

(e) None of the above.

Solution: (a)
This is a case of truncated data. We have worked out in class that in the exponential case with data truncated at d, the expression for the maximum likelihood estimate of θ reads as

$$\hat{\theta}_{MLE} = \bar{x} - d.$$

With the given data, we get

$$\hat{\theta}_{MLE} = \frac{400 + 600 + 700 + 900 + 1000}{5} - 300 = 420.$$