Problem. [Sample STAM Problem #288.]

The r.v. N has the following mixed distn:

(i) With probability p, N has a binomial distn w/ $q_1 = \frac{1}{2}$ and $m_1 = 2$.

(ii) With probability $1-p$, N has a binomial distn w/ $q_2 = \frac{1}{2}$ and $m_2 = 4$.

Find the expression for $\mathbb{P}[N=2]$ in terms of p.

→:

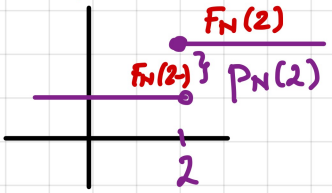
$$N \sim \begin{cases} N_1 \sim \text{bin}(q_1 = 0.5, m_1 = 2) & \text{w/ probab. } p \\ N_2 \sim \text{bin}(q_2 = 0.5, m_2 = 4) & \text{w/ probab. } 1-p \end{cases}$$

We seek: $\mathbb{P}[N=2] = p_N(2) =$

$$= F_N(2) - F_N(2-)$$

$$= F_N(2) - F_N(1)$$

the support are integers



Because N is a 2-pt mixture, we have

$$p_N(2) = p \cdot F_{N_1}(2) + (1-p) \cdot F_{N_2}(2) - \left(p \cdot F_{N_1}(1) + (1-p) \cdot F_{N_2}(1) \right)$$

$$p_N(2) = p \underbrace{\left(F_{N_1}(2) - F_{N_1}(1) \right)}_{p_{N_1}(2)} + (1-p) \underbrace{\left(F_{N_2}(2) - F_{N_2}(1) \right)}_{p_{N_2}(2)}$$

$$\boxed{p_N(2) = p \cdot p_{N_1}(2) + (1-p) \cdot p_{N_2}(2)}$$

Think about the generalization @ home?

Returning to our problem:

$$p_N(2) = p\left(\frac{1}{4}\right) + (1-p) \cdot \binom{4}{2} \cdot \left(\frac{1}{2}\right)^4$$

$$= p \cdot \frac{1}{4} + (1-p) \frac{4 \cdot 3}{2} \cdot \frac{1}{16_4} = \frac{p}{4} + \frac{3}{8} - \frac{3}{8}p = \frac{3}{8} - \frac{1}{8}p$$

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