UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 6

Binomial option pricing: One period.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 6.1. (10 points) Let $S(0) = \$100$, $K = \$105$, $r = 8\%, T = 0.5$. Suppose that $u = 1.3$ and $d = 0.8$. Using the one-period binomial model, calculate the following:

a. (5 pts) The fair premium for a European put with the above characteristics.
b. (3 pts) The Δ in the corresponding replicating portfolio.
c. (2 pts) The amount B invested in the riskless asset in the replicating portoflio.

Solution: The risk-neutral probability is

$$p^* = \frac{e^{rT} - d}{u - d} = 0.48.$$

a. We start by drawing the one-period tree per the described model. According to the risk-neutral pricing formula in the binomial model, we have that the time-0 price of the put is

$$\begin{aligned} V_P(0) &= e^{-rT}\left[p^* V_u + (1-p^*)V_d\right] \\ &= e^{-0.08\cdot 0.5}\left[0.48\cdot 0 + 0.52\cdot 25\right] \\ &= 12.5. \end{aligned} \tag{6.1}$$

b.

$$\Delta = \frac{0 - 25}{100(1.3 - 0.8)} = -\frac{1}{2}.$$

c.

$$B = V_P(0) - \Delta S(0) = 12.5 + \frac{100}{2} = 62.5.$$

Problem 6.2. (5 points) Consider a non-dividend-paying stock with the current price of \$50.

The continuously-compounded, risk-free interest rateis 0.03.

You are using a one-period binomial tree to model the stock price at the end of the next quarter. You assume that the stock price can either increase by 0.04 or decrease by 0.02. What is the risk-neutral probability associated with this tree?

Solution: By definition, in our usual notation, we have

$$p^* = \frac{e^{rh} - d}{u - d} = \frac{e^{0.03(0.25)} - 0.98}{1.04 - 0.98} = 0.4588.$$

Problem 6.3. (5 pts) Consider a non-dividend-paying stock currently priced at \$100 per share.

The price of this stock in one year is modeled using a one-period binomial tree under the assumption that the stock price can either go up to 110 or down to 90.

Let the continuously-compounded, risk-free interest rateequal 0.04. What is the risk-neutral probability of the stock price going up?

Solution: (d)

$$p^* = \frac{100e^{0.04} - 90}{110 - 90} = 0.7041.$$

Problem 6.4. (10 points) Consider a non-dividend paying stock whose current price is \$52 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$72 or \$42 in one year.

The continuously-compounded, risk-free interest rateis 0.05.

Consider a \$50-strike, one-year European call option on the above stock. What is the call price consistent with the above stock-price model?

Solution: The risk-neutral probability of an up movement is

$$p^* = \frac{52e^{0.05} - 42}{72 - 42} = 0.4222.$$

So, the price of our call is

$$V_C(0) = e^{-0.05}[0.4222 \times (72 - 50)_+ + (1 - 0.4222) * (42 - 50)_+] = 8.8355.$$

Problem 6.5. (5 points) The current stock price is 40 per share. The price at the end of a three-month period is modeled with a one-period binomial tree so that the stock price can either increase by \$10, or decrease by \$4. The stock pays no dividends.

The continuously-compounded, risk-free interest rateis 0.05.

What is the stock investment in a replicating portfolio for three-month, \$40-strike European **straddle** on the above stock?

Solution: In our usual notation,

$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \left(\frac{10 - 4}{14}\right) \approx 0.4285714.$$

Problem 6.6. (5 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the Δ in the replicating portfolio of a single call option on that stock never exceeds 1. *True or false? Why?*

Solution: TRUE
The call's Δ will always be between 0 and 1.

Problem 6.7. (5 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the Δ in the replicating portfolio of a single put option on that stock is between -1 and 0. *True or false? Why?*

Solution: TRUE
The puts's Δ will always be between -1 and 0.

Problem 6.8. (5 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the risk-free component in the replicating portfolio of a single put option on that stock should be interpreted as lending. *True or false? Why?*

Solution: TRUE
The puts's B will always be positive and should be intrpreted as lending.