

Name:

M339D=M389D Introduction to Actuarial Financial Mathematics

University of Texas at Austin

**In-Term Exam III**

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**Notes:** This is a closed book and closed notes exam. The maximal score on this exam is 50 points.

**Time:** 50 minutes

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**Problem 3.1.** (5 points) Which one of the following statements is **TRUE**?

- (a) The payoff curve of a call bear spread is never positive.
- (b) A straddle has a nonnegative profit function.
- (c) A strangle can be replicated with a long put and a short call.
- (d) The payoff of the call bull spread is equal to the payoff of the put bull spread.
- (e) None of the other statements is TRUE.

**Problem 3.2.** (5 points) An investor buys a two-year (\$800, \$900)-strangle on gold. The price of gold two years from now is modeled using the following distribution:

\$750,   with probability 0.45,  
\$850,   with probability 0.4,  
\$925,   with probability 0.15.

What is the investor's expected payoff?

- (a) About \$23.25
- (b) About \$25.00
- (c) About \$26.25
- (d) About \$37.50
- (e) None of the above.

**Problem 3.3.** Let the continuously-compounded, risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$100 per share. The stock pays dividends continuously with a dividend yield of 0.02. In the model, it is assumed that the stock price can either go up by 3% or down by 4%.

You use the binomial tree to construct a replicating portfolio for a at-the-money, one-year European call on the above stock. What is the stock investment in the replicating portfolio?

- (a) Long 0.4201 shares.
- (b) Long 0.4286 shares.
- (c) Short 0.4201 shares.
- (d) Short 0.4286 shares.
- (e) None of the above.

**Problem 3.4.** (5 points) Let the continuously-compounded, risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is \$50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a \$48-strike, one-year European put on the above stock. What is the risk-free investment in the replicating portfolio?

- (a) Borrow \$21
- (b) Borrow \$20.18
- (c) Lend \$20.18
- (d) Lend \$21
- (e) None of the above.

**Problem 3.5.** You are given that the price of:

- a \$50-strike, one-year European call equals \$8,
- a \$65-strike, one-year European call equals \$2.

Both options have the same underlying asset. What is the maximal price of a \$56-strike, one-year European call such that there is no arbitrage in our market model?

- (a) \$4.40
- (b) \$5
- (c) \$5.60
- (d) \$6.02
- (e) None of the above.

**Problem 3.6.** We are given the following European-call prices for options on the same underlying asset:

\$50-strike	\$11
\$55-strike	\$6
\$60-strike	\$4

Assume that the continuously-compounded, risk-free interest rate is strictly positive. Which of the following portfolios would exploit an arbitrage opportunity stemming from the above stock prices?

- (a) The call bull spread only.
- (b) The call bear spread only.
- (c) Both the call bull and the call bear spread.
- (d) Neither the call bull or call bear spread, but there is an arbitrage opportunity.
- (e) There is no apparent arbitrage opportunity.

**Problem 3.7.** A portfolio consists of the following:

- one **short** one-year, 50–strike call option with price equal to \$8.50,
- one **long** one-year, 60–strike put option with price equal to \$6.75

All of the options are European and with the same underlying asset.

Assume that the continuously compounded, risk-free interest rate equals 0.04.

What is the portfolio's profit if the final price of the underlying asset equals \$55?

- (a) 1.75

- (b) 1.82
- (c) 6.82
- (d) 11.82
- (e) None of the above.

**Problem 3.8.** (5 points) *Source: Sample FM(DM) Problem #8.*

You believe that the volatility of a stock is higher than indicated by market prices for options on that stock. You want to speculate on that belief by buying or selling at-the- money options. What should you do?

- (a) Buy a straddle.
- (b) Buy a strangle.
- (c) Sell a straddle.
- (d) Buy a butterfly spread.
- (e) Sell a butterfly spread.

**Problem 3.9.** We are given the following European-call prices for options on the same underlying asset:

\$50-strike	\$10
\$55-strike	\$6
\$60-strike	\$4

Assume that the continuously-compounded, risk-free interest rate is strictly positive. Which of the following portfolios would exploit an arbitrage opportunity stemming from the above stock prices?

- (a) The call bear spread only.
- (b) The call bull spread only.
- (c) Both the call bull and the call bear spread.
- (d) Neither the call bull or call bear spread, but there is an arbitrage opportunity.
- (e) There is not enough information provided.

**Problem 3.10.** (5 points) Bertie constructs an asymmetric butterfly spread using call options with strikes 75, 78 and 90. It is constructed using  $m$  of the (75, 78) bull spreads and  $n$  (78,90) bear spreads. How much is  $m/n$ ?

- (a) 4
- (b) 2
- (c) 1/2
- (d) 1/4
- (e) None of the above.

**Problem 3.11.** (5 pts) Consider a non-dividend-paying stock currently priced at \$100 per share.

The price of this stock in one year is modeled using a one-period binomial tree under the assumption that the stock price can either go up to 110 or down to 90.

Let the continuously-compounded, risk-free interest rate equal 0.04. What is the risk-neutral probability of the stock price going up?

- (a) About 0.2969
- (b) About 0.3039
- (c) About 0.5000
- (d) About 0.7041
- (e) None of the above.

**Problem 3.12.** (5 points) The current stock price is 40 per share. The price at the end of a three-month period is modeled with a one-period binomial tree so that the stock price can either increase by \$5, or decrease by \$4. The stock pays dividends continuously with the dividend yield 0.04.

The continuously-compounded, risk-free interest rate is 0.10.

What is the price of a \$40-strike European **straddle** on the above stock?

- (a) 4.40
- (b) 3.30
- (c) 2.20
- (d) 1.10
- (e) None of the above.