Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Sample In-Term Exam III
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Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.

3.1. FREE-RESPONSE PROBLEMS.

Problem 3.1. Let the continuously compounded, risk-free interest rate be equal to 0.04. Consider a non-dividend-paying stock whose current price is 90. You model the evolution of this stock over the next six months using a two-period binomial tree assuming that the stock price can either increase or decrease by 10% in a single period. Consider a put-on-put option with the following characteristics:

- the exercise date of the compound option is in three months;
- the strike price of the compound option is \$2;
- the underlying put option is at-the-money at time−0 and its exercise date is in six months.

What is the price of the put-on-put option?

Problem 3.2. There are two European options on the same stock with the same time to expiration. The 90-strike call costs \$20 and the 100-strike call costs \$8.

Is there an arbitrage opportunity due to the above call prices?

Propose an arbitrage portfolio (if you concluded that it exists) and **verify** that your proposed portfolio is indeed an arbitrage portfolio.

Problem 3.3. Consider a continuous-dividend-paying stock whose current price is \$40 and whose dividend yield is 0.02. The price of stock in three months is modeled using a one-period binomial tree.

The continuously compounded, risk-free interest rate is 0.06.

According to the above stock-price model, the replicating portfolio of an at-the-money, three-month European call option consists of:

- 0.6 shares of stock, and
- borrowing \$20 at the risk-free interest rate.

What is the risk-free portion of the replicating portfolio for the otherwise identical put option?

3.2. MULTIPLE CHOICE QUESTIONS.

Problem 3.4. You construct an asymmetric butterfly spread using the following three types of European options on the same asset and with the same exercise date:

- a \$50-strike call,
- a \$60-strike call,
- a \$65-strike call.

You are told that there is exactly **one** short \$60-strike call in the asymmetric butterfly spread. What is the maximal payoff of the above butterfly spread?

(a) 0
(b) 10/3
(c) 5
(d) The payoff is not bounded from above.
(e) None of the above.

Problem 3.5. The following two one-year European put options on the same asset are available in the market:

- a \$50-strike put with the premium of \$5,
- a \$55-strike put with the premium of \$10.

The continuously compounded, risk-free interest rate is 0.04.

Which of the following positions certainly exploits the arbitrage opportunity caused by the above put premia?

(a) Put bull spread.
(b) Put bear spread.
(c) Both of the above positions.
(d) There is no arbitrage opportunity.
(e) None of the above.

Problem 3.6. (5 points) Which one of the following positions always has an infinite upward potential in the sense that the payoff diverges to positive infinity as the argument s (standing for the final stock price) tends to positive infinity?

(a) A long call option.
(b) A bear spread.
(c) A bull spread.
(d) A long butterfly spread.
(e) None of the above.

Problem 3.7. A long strangle position...

(a) is equivalent to a short butterfly spread.
(b) can be replicated with a short call and a long put with the same strike, underlying asset and exercise date.
(c) is always strictly more expensive than the straddle on the same underlying asset and with the same exercise date.
(d) is a speculation on the stock's volatility.
(e) None of the above.

Problem 3.8. You are given that the price of:

- a \$50-strike, one-year European call equals \$8,
- a \$65-strike, one-year European call equals \$2.

Both options have the same underlying asset. What is the maximal price of a \$56-strike, one-year European call such that there is no arbitrage in our market model?

(a) \$4.40
(b) \$5
(c) \$5.60
(d) \$6.02
(e) None of the above.

Problem 3.9. An investor bought a six-month, $(70, 80)$-put **bull** spread on an index. The \$70-strike, six-month put is currently valued at \$1, while the \$80-strike, six-month put is currently valued at \$8.

Assume that the continuously compounded, risk-free interest rate equals 0.02.

What is the **break-even** final index price for the above put bull spread?

(a) \$62.86
(b) \$71.84
(c) \$72.93
(d) \$73.23
(e) None of the above.

Problem 3.10. Consider a continuous-dividend-paying stock with the current price of \$50 and dividend yield 0.02.

The continuously compounded, risk-free interest rate is 0.05.

You are using a one-period binomial tree to model the stock price at the end of the next quarter. You assume that the stock price can either increase by 0.04 or decrease by 0.02. What is the risk-neutral probability associated with this tree?

(a) 0.3675
(b) 0.4588
(c) 0.5430
(d) 0.8409
(e) None of the above.

3.3. TRUE/FALSE QUESTIONS.

Problem 3.11. (2 points) The payoff curve of a **call bear** spread is never positive. *True or false?*

Problem 3.12. (2 pts) Suppose that the European options with the same maturity and the same underlying assets have the following prices:

(1) a 50−strike call costs \$9;
(2) a 55−strike call costs \$10;

Then, one should *acquire* a **call bear spread** to exploit the arbitrage since some of the monotonicity conditions for no-arbitrage are violated by the above premiums.

Problem 3.13. (2 points) A **long** strangle has a non-negative payoff function. *True or false?*

Problem 3.14. Suppose that prices of European calls on the same asset and with the same exercise date for varying strike prices are given by the following table

Strike	80	100	105
Call premium	22	9	5

Then, one can use a butterfly spread to exploit the violations of the no-arbitrage conditions exhibited by the prices in the above table. *True or false?*

Problem 3.15. (2 points) In the setting of the binomial asset-pricing model, let d and u denote the up and down factors, respectively. Moreover, let r denote the continuously compounded, risk-free interest rate. Let h denote the length of a single period in our model.

Then, if,

$$e^{\delta h}d < e^{rh} < e^{\delta h}u$$

then there is no possibility for arbitrage. *True or false?*

Problem 3.16. (2 pts) A European call option with strike K on a futures contract on a stock has the same value as the European call option with strike K on that same stock provided that the futures contract has the same expiration as the stock option. *True or false?*

Problem 3.17. Assume a positive risk-free interest rate. It's never optimal to exercise an American option on a non-dividend-paying stock early. *True or false?*

Problem 3.18. (2 points) The price of a **up-and-in** option is increasing as a function of its barrier (with every other input held fixed).

Problem 3.19. (2 points) The price of a geometric average price Asian call option is strictly greater the price of an otherwise identical arithmetic average price Asian call option. *True or false?*

3.4. MORE MULTIPLE CHOICE QUESTIONS.

Problem 3.20. The current exchange rate is given to be \$1.25 per Euro and its volatility is given to be 0.15.

The continuously compounded, risk-free interest rate for the US dollar is 0.03, while the continuously compounded, risk-free interest rate for the Euro equals 0.06.

The evolution of the exchange rate over the following nine-month period is modeled using a three-period forward binomial tree.

What is the value of the so-called down factor in the above tree?

(a) $d \approx 0.8586$
(b) $d \approx 0.8982$
(c) $d \approx 0.9208$
(d) $d \approx 0.9347$
(e) None of the above.

Problem 3.21. The current stock price is observed to be \$100 per share. The stock is projected to pay dividends continuously at the rate proportional to its price with the dividend yield of 0.03. The stock's volatility is given to be 0.23. You model the evolution of the stock price using a two-period forward binomial tree with each period of length one year.

The continuously compounded, risk-free interest rate is given to be 0.04.

What is the price of a two-year, \$101-strike **American** put option on the above stock consistent with the above stock-price tree?

(a) About \$6.62
(b) About \$8.34
(c) About \$8.83
(d) About \$11.11
(e) None of the above.

Problem 3.22. The current price of a continuous-dividend-paying stock is \$100 per share. Its volatility is given to be 0.2 and its dividend yield is 0.03.

The continuously compounded risk-free interest rate equals 0.04.

Consider a \$95-strike European put option on the above stock with nine months to expiration. Using a three-period forward binomial tree, find the price of this put option.

(a) \$2.97
(b) \$3.06
(c) \$3.59
(d) \$4.32
(e) None of the above.