M339J: February 16th, 2022.

More on Y^P and Y^L.

Recall our set-up:

- X... (ground-up) loss; severity
- d... the (ordinary) deductible
- the per payment r.v.

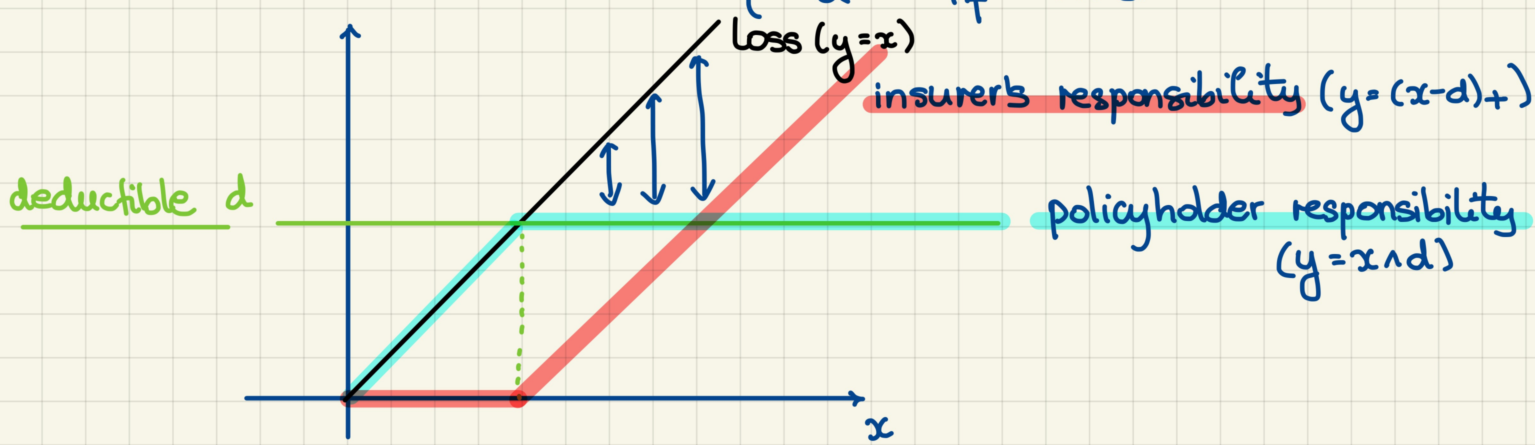
$$Y^P = X-d \mid X>d$$

- the per loss r.v.

$$Y^L = (X-d)_+$$

Def'n. The limited loss (random) variable is $\boxed{X \wedge d}$.

Note: $X \wedge d = \min(X,d) = \begin{cases} X & \text{if } X<d \\ d & \text{if } X \geq d \end{cases}$



Algebraically: $\boxed{x \wedge d + (x-d)_+ = x}$

$\Rightarrow$ For X being our severity r.v., we get

$$X \wedge d + (X-d)_+ = X$$

Apply the expectation to this equality:

$$\mathbb{E}[X \wedge d] + \mathbb{E}[(X-d)_+] = \mathbb{E}[X]$$

$$\Rightarrow \boxed{\mathbb{E}[(X-d)_+] = \mathbb{E}[X] - \mathbb{E}[X \wedge d]} = \mathbb{E}[Y^L]$$

$$\mathbb{E}[Y^P] = \mathbb{E}[X-d \mid X>d] = \frac{\mathbb{E}[(X-d)\cdot \mathbb{I}_{[X>d]}]}{\mathbb{P}[X>d]}$$

(assume $\mathbb{P}[X>d]>0$)

$$\mathbb{E}[Y^P] = \boxed{\frac{\mathbb{E}[X]-\mathbb{E}[X\wedge d]}{\mathbb{P}[X>d]}} = e_X(d)$$

100. The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution:

$$F(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \quad x \geq 0$$

The insurance policy pays amounts up to a limit of 1000 per claim.

Calculate the expected payment under this policy for one claim.

(A) 57

(B) 108

(C) 166

(D) 205

(E) 240

101. The random variable for a loss, X, has the following characteristics:

x	$F(x)$ cdf	$E(X \wedge x)$
0	0.0	0
100	0.2	91
200	0.6	153
1000	1.0	331

$\boxed{X \wedge 1000 = X}$

$\boxed{\text{Maximum value that } X \text{ can take is } 1000.}$

Calculate the mean excess loss for a deductible of 100.

(A) 250

(B) 300

(C) 350

(D) 400

(E) 450

$$e_X(100) = \mathbb{E}[Y^P] = \mathbb{E}[X - 100 \mid X > 100]$$

$$= \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge 100]}{1 - F_X(100)}$$

$$= \frac{331 - 91}{1 - 0.2} = 240 \cdot \frac{5}{4} = 300$$

160. You are given a random sample of observations:

0.1 0.2 0.5 0.7 1.3

You test the hypothesis that the probability density function is:

$$f(x) = \frac{4}{(1+x)^5}, \quad x > 0$$

Calculate the Kolmogorov-Smirnov test statistic.

(A) Less than 0.05

(B) At least 0.05, but less than 0.15

(C) At least 0.15, but less than 0.25

(D) At least 0.25, but less than 0.35

(E) At least 0.35

161. DELETED

162. A loss, X, follows a 2-parameter Pareto distribution with $\alpha = 2$ and unspecified parameter θ. You are given:

$$\boxed{E[X-100 \mid X>100] = \frac{5}{3} E[X-50 \mid X>50]}$$

Calculate $E[X-150 \mid X>150]$.

(A) 150

(B) 175

(C) 200

(D) 225

(E) 250

$\longrightarrow$: $\underline{X \sim \text{Pareto}(\alpha, \theta)}$

$$\mathbb{E}[X-d \mid X>d] = \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge d]}{S_X(d)}$$

$$= \frac{\frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left(1 - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1}\right)}{\left(\frac{\theta}{d+\theta}\right)^{\alpha}}$$

$$= \frac{\frac{\theta}{\alpha-1}\left(\cancel{1} - \cancel{1} + \cancel{\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}}\right)}{\left(\frac{\theta}{d+\theta}\right)^{\cancel{\alpha}}}$$

$$= \frac{\frac{\cancel{\theta}}{\alpha-1}}{\frac{\cancel{\theta}}{d+\theta}} = \frac{d+\theta}{\alpha-1}$$

<u>In this problem:</u> $\alpha = 2$

$$\frac{100+\theta}{\cancel{2-1}} = \frac{5}{3} \cdot \frac{50+\theta}{\cancel{2-1}}$$

$$300 + 3\theta = 250 + 5\theta$$

$$2\theta = 50$$

$$\boxed{\theta = 25}$$

<u>answer:</u> $150 + 25 = 175$ ■

13. The loss severity random variable X follows the exponential distribution with mean 10,000.

Determine the coefficient of variation of the ~~excess loss variable~~ $Y = \max(X - 30000, 0)$.

$$\frac{\sigma_Y}{\mu_Y}$$

(A) 1.0

(B) 3.0

(C) 6.3

(D) 9.0

(E) 39.2

Steps: 1. $\mu_Y = ?$

2. $E[Y^2] = ?$

$\xrightarrow{3.}$ σ_Y

$$Y^L \begin{cases} \xrightarrow{X>d} \; Y^P & \text{w/ probab. } S_X(d) \\ \xrightarrow{X\leq d} \; 0 & \text{w/ probab. } F_X(d) \end{cases}$$

In this problem: $Y^P \sim \text{Exponential}(\theta)$

↑ memoryless property