M339D: April 16th, 2025.

8. Let $S(t)$ denote the price at time t of a stock that pays no dividends. The Black-Scholes framework holds. Consider a European call option with exercise date T, $T > 0$, and exercise price $\boxed{\underset{K}{S(0)e^{rT}}}$, where r is the continuously compounded risk-free interest rate.

You are given:

(i) $S(0) = \$100$

(ii) $T = 10$

(iii) $\text{Var}[\ln S(t)] = 0.4t$, $t > 0$. $\Rightarrow$ $\boxed{\sigma = \cancel{x}\sqrt{0.4}}$

Determine the price of the call option.

(A) $7.96

(B) $24.82

(C) $68.26

(D) $95.44

(E) There is not enough information to solve the problem.

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{\boxed{K}}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T\right]$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\underbrace{\ln\left(\frac{\cancel{S(0)}}{\cancel{S(0)}e^{rT}}\right)}_{\cancel{-rT}} + \left(\cancel{r} + \frac{\sigma^2}{2}\right)\cdot T\right] = \frac{1}{\sigma\sqrt{T}}\cdot\frac{\sigma^2\cdot T}{2} = \frac{\sigma\sqrt{T}}{2} = \frac{\sqrt{0.4}\sqrt{10}}{2} = 1$$

$$d_2 = d_1 - \sigma\sqrt{T} = -\frac{\sigma\sqrt{T}}{2}$$

$$V_c(0) = S(0)N(d_1) - Ke^{-rT}N(d_2) = S(0)N(d_1) - S(0)\cancel{e^{rT}}\,\cancel{e^{-rT}}N(d_2)$$

$$V_c(0) = S(0)\left(N(d_1) - N(d_2)\right) = S(0)\left(N(d_1) - (1 - N(d_1))\right)$$

$$V_c(0) = S(0)\left(2N(d_1) - 1\right) = 100(2\cdot N(1) - 1)$$

$$V_c(0) = 100(2\cdot 0.8413 - 1) = \underline{68.26}$$

Problem. Assume the Black-Scholes framework.
For a European call, the strike is $S(0)e^{rT}$ where T is the exercise date.
(The price of such a call w/ one year to exercise is $0.6 \cdot S(0)$).

Find the price of such a call option w/ three months to exercise in terms of $S(0)$.

→: From the previous problem:

$$V_c(0,T) = S(0)\left(2N\left(\frac{\sigma\sqrt{T}}{2}\right) - 1\right)$$

For $T=1$:

$$V_c(0,T=1) = S(0)\left(2N\left(\frac{\sigma}{2}\right) - 1\right) = 0.6 \cdot S(0)$$

$$\Rightarrow 2N\left(\frac{\sigma}{2}\right) - 1 = 0.6$$

$$\Rightarrow N\left(\frac{\sigma}{2}\right) = 0.8$$

$$\Rightarrow \frac{\sigma}{2} = 0.84 \Rightarrow \boxed{\sigma = 1.68}$$

For $T=\frac{1}{4}$:

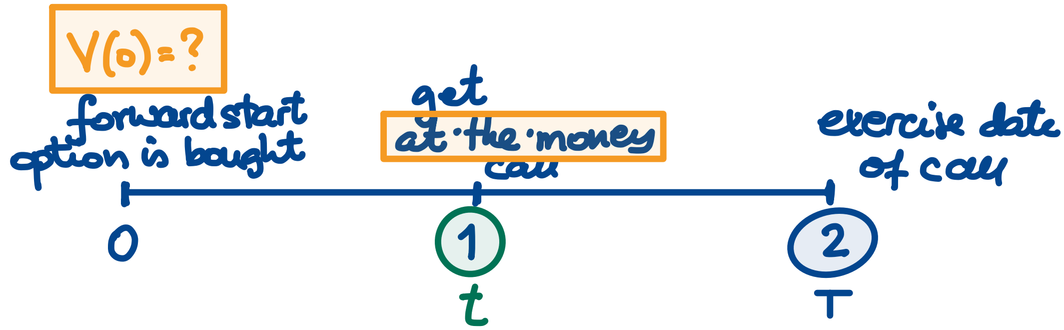
$$V_c(0,T=\tfrac{1}{4}) = S(0)\left(2N\left(\frac{\sigma\sqrt{\frac{1}{4}}}{2}\right) - 1\right)$$

0.42

$$= S(0)(2 \cdot 0.6628 - 1) = 0.3256 \cdot S(0)$$

□

(A) 586
(B) 594
(C) 684
(D) 692
(E) 797



19. Consider a *forward start option* which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.

You are given:

(i) The European call option is on a stock that pays no dividends.

(ii) The stock's volatility is 30%.

(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100.

$F_{0,1}(S) = S(0)e^{r\cdot 1} = 100 \Rightarrow S(0) = 100e^{-0.08}$

(iv) The continuously compounded risk-free interest rate is 8%.

Under the Black-Scholes framework, determine the price today of the forward start option.

(A) 11.90
(B) 13.10
(C) 14.50
(D) 15.70
(E) 16.80

At $t<T$:

$$V_c(t) = S(t)\cdot N(d_1(t)) - Ke^{-r(T-t)}\cdot N(d_2(t))$$

w/

$$d_1(t) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S(t)}{K}\right) + \left(r+\frac{\sigma^2}{2}\right)\cdot(T-t)\right]$$

and

$$d_2(t) = d_1(t) - \sigma\sqrt{T-t}$$

In this problem. $\boxed{t=1}$

$$V_c(1) = S(1)\cdot N(d_1(1)) - \underbrace{K}_{S(1)}e^{-r(2-1)}\cdot N(d_2(1))$$

$$V_c(1) = S(1)\cdot \boxed{\left(N(d_1(1)) - e^{-r}\cdot N(d_2(1))\right)}$$

w/

$$d_1(1) = \frac{1}{0.3\sqrt{2-1}}\left[\cancel{\ln\left(\frac{S(1)}{S(1)}\right)} + \left(0.08 + \frac{0.09}{2}\right)(2-1)\right]$$

@ the money

$$d_1(1) = \frac{0.08 + 0.045}{0.3} = \frac{0.125}{0.3} = \underline{0.42}$$

$$d_2(1) = d_1 - 0.3\sqrt{2-1} = \underline{0.12}$$

$N(d_1(1)) = 0.6628$

$N(d_2(1)) = 0.5478$

$$V_c(1) = S(1)\left(0.6628 - e^{-0.08} \cdot 0.5478\right) = S(1) \cdot 0.1571$$

At time 0, our forward start option is worth

$$S(0) \cdot 0.1571$$

$\Rightarrow$ answer: $100e^{-0.08} \cdot 0.1571 = \boxed{14.50}$ □