M339D: March 25th, 2022.

Put Option Price Monotonicity.

Claim. Put prices are **increasing** as functions of the strike, i.e., for $K_1 < K_2$, we have $\boxed{V_p(K_1) \leq V_p(K_2)}$

→: Assume, to the contrary, that there exist $K_1 < K_2$ such that

$$V_p(K_1) > V_p(K_2).$$

I. Suspicion. ✓

II. Propose an arbitrage portfolio.

- **write** the K_1-strike put
- **Long** the K_2-strike put

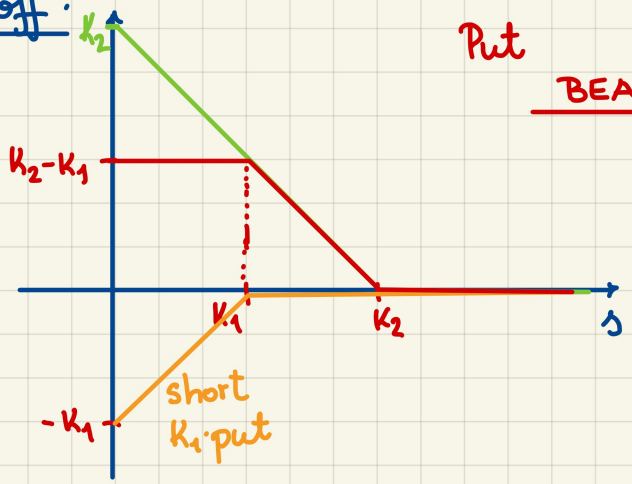
} Put BEAR Spread

III. Verification.

Initial inflow of cash.

Initial Cost: $-V_p(K_1) + V_p(K_2) < 0$

Payoff:



Put BEAR Spread

Payoff ≥ 0

$\Rightarrow$ Profit > 0, i.e., this is, indeed, an arbitrage portfolio!

"Cord · Slope" Bound.

Let $K_1 < K_2$.

$$0 \overset{\leq}{\uparrow} \begin{cases} V_C(K_1) - V_C(K_2) \\ V_P(K_2) - V_P(K_1) \end{cases} \leq PV_{0,T}(K_2 - K_1)$$

monotonicity ↑ — ↑ claim!

✓ **Calls.**

Assume, to the contrary, that there exist $K_1 < K_2$ such that

$$V_C(K_1) - V_C(K_2) > PV_{0,T}(K_2 - K_1).$$

$$\Leftrightarrow$$

$$V_C(K_1) > V_C(K_2) + PV_{0,T}(K_2 - K_1)$$

I. Suspicion. ✓

II. Propose an arbitrage portfolio.

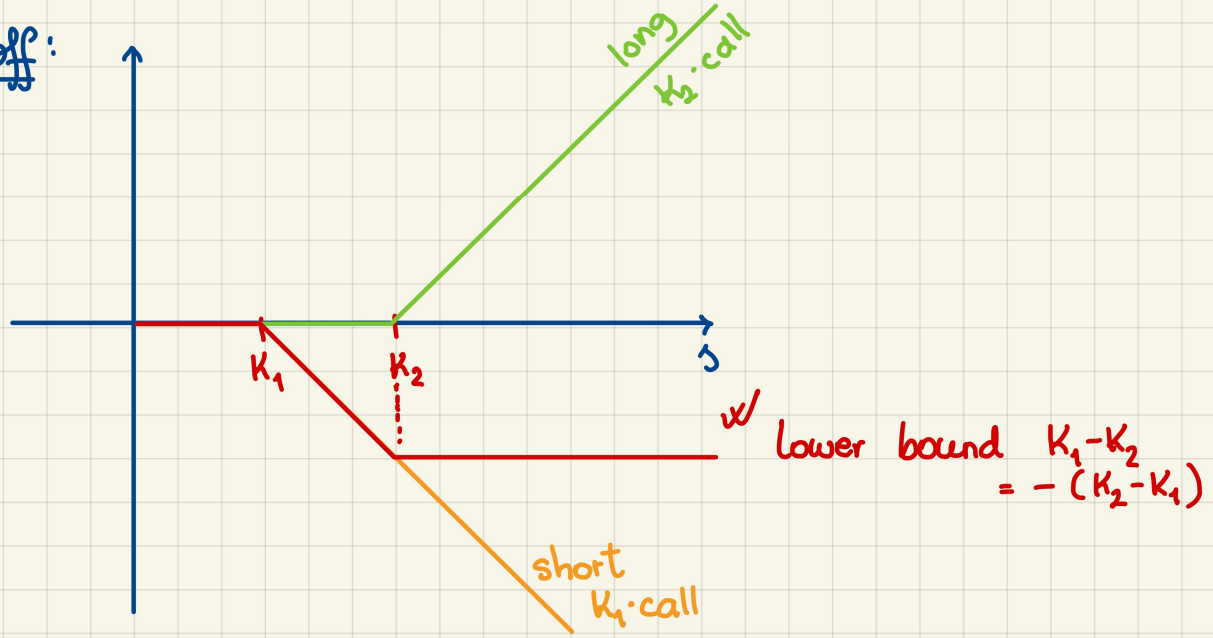
- write the K_1-strike call
- long the K_2-strike call

} call bear spread ✓✓

III. Verification.

Initial Cost: $-V_C(K_1) + V_C(K_2) < -PV_{0,T}(K_2 - K_1)$

Payoff:



✓✓ lower bound $K_1 - K_2 = -(K_2 - K_1)$

$$\text{Profit} \geq K_1 - K_2 - FV_{0,T}(\text{Init Cost}) > K_1 - K_2 + \cancel{FV_{0,T}}(+\cancel{PV_{0,T}}(K_2 - K_1)) = 0$$

$\Rightarrow$ Indeed, this is an arbitrage portfolio!

Puts.

Q: Which arbitrage portfolio would you propose if the above inequality is violated for puts?

$\longrightarrow$: What if there exist $K_1 < K_2$ such that

$$V_P(K_2) - V_P(K_1) > PV_{0,T}(K_2 - K_1) \quad ?$$

- long the K_1-strike put
- write the K_2-strike put

} Put BULL Spread.

Task: Go through the usual verification steps for this portfolio!

12. You are given:

(i) $C(K, T)$ denotes the current price of a K-strike T-year European call option on a nondividend-paying stock.

(ii) $P(K, T)$ denotes the current price of a K-strike T-year European put option on the same stock.

(iii) S denotes the current price of the stock.

(iv) The continuously compounded risk-free interest rate is r.

Which of the following is (are) correct?

monotonicity

PV(55-50) = PV(5) - $5e^{-rT}$

(I) $0 \le C(50, T) - C(55, T) \le 5e^{-rT}$

(II) $50e^{-rT} \le P(45, T) - C(50, T) + S \le 55e^{-rT}$

(III) $45e^{-rT} \le P(45, T) - C(50, T) + S \le 50e^{-rT}$

(A) (I) only

(B) (II) only

(C) (III) only

(D) (I) and (II) only

(E) (I) and (III) only

Focus on:

$P(45,T) - C(50,T) + S$

= Put-Call Parity

$C(45,T) - F^{P}_{0,T}(S) + PV_{0,T}(45) - C(50,T) + S$

$= C(45,T) - C(50,T) + 45e^{-rT}$

$\le 5e^{-rT}$

$\le 50e^{-rT}$

$0 \le 45e^{-rT}$