UNIVERSITY OF TEXAS AT AUSTIN

Quiz #15

Exchange options.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 15.1. (5 points) **The minimum option**

Let $\mathbf{S} = \{S(t), t \geq 0\}$ and $\mathbf{Q} = \{Q(t), t \geq 0\}$ denote the prices of two risky assets. The payoff of the *minimum option* is given by

$$V_{min}(T) = \min(S(T), Q((T)).$$

Propose a replicating portfolio consisting of prepaid forward contracts on $\mathbf{S}$ and/or $\mathbf{Q}$, and exchange options on $\mathbf{S}$ and $\mathbf{Q}$.

Solution:

$$\begin{aligned} V_{min}(T) = \min(S(T), Q((T)) &= S(T) + \min(0, Q(T) - S(T)) \\ &= S(T) - \max(S(T) - Q(T), 0). \end{aligned}$$

So, an example of a replicating portfolio is

$$\begin{cases} \text{a prepaid forward contract on } \mathbf{S}\text{, and} \\ \text{a } \textbf{short} \text{ exchange call with } \mathbf{S} \text{ as underlying and } \mathbf{Q} \text{ as the strike asset} \end{cases}$$

Other examples are

$$\begin{cases} \text{a prepaid forward contract on } \mathbf{S}\text{, and} \\ \text{a } \textbf{short} \text{ exchange put with } \mathbf{Q} \text{ as underlying and } \mathbf{S} \text{ as the strike asset} \end{cases}$$

$$\begin{cases} \text{a prepaid forward contract on } \mathbf{Q}\text{, and} \\ \text{a } \textbf{short} \text{ exchange call with } \mathbf{Q} \text{ as underlying and } \mathbf{S} \text{ as the strike asset} \end{cases}$$

$$\begin{cases} \text{a prepaid forward contract on } \mathbf{Q}\text{, and} \\ \text{a } \textbf{short} \text{ exchange put with } \mathbf{S} \text{ as underlying and } \mathbf{Q} \text{ as the strike asset} \end{cases}$$

Problem 15.2. (3 points) Let our market model include two continuous-dividend-paying stocks whose time$-t$ prices are denoted by $S(t)$ and $Q(t)$ for $t \geq 0$. The current stock prices are $S(0) = 160$ and $Q(0) = 80$. The dividend yield for the stock S is $\delta_S = 0.06$ and the dividend yield for the stock Q is $\delta_Q = 0.03$.

The price of an exchange option giving its bearer the right to forfeit one share of Q for one share of S in one year is given to be \$11.

Find the price of a maximum option on the above two assets with exercise date in a year. Remember that the payoff of the maximum option is $\max(S(1), Q(1))$.

Solution: As we showed in class

$$V_{\max}(0) = F_{0,1}^P(Q) + V_{EC}(0, S, Q) = Q(0)e^{-\delta_Q} + V_{EC}(0, S, Q) = 80e^{-0.03} + 11 = 88.64.$$

Problem 15.3. (5 points) Assume that the continuously compounded interest rate equals 0.10.

Stock S has the current price of $S(0) = 70$ and does not pay dividends. Stock Q has the current price of $Q(0) = 65$ and it pays continuous dividends at the rate of 0.04.

An exchange option gives its holder the right to give up one share of stock Q for a share of stock S in exactly one year. The price of this option is \$11.50.

Another exchange option gives its holder the right to give up one share of stock S for a share of stock Q in exactly one year. Find the price of this option.

(a) About \$3.95
(b) About \$11.10
(c) About \$12.00
(d) About \$14.25
(e) None of the above.

Solution: (a)
By the generalized put-call parity, we get the price we are looking for should be

$$V_{EC}(Q(0), S(0), 0) = V_{EC}(S(0), Q(0), 0) + F^P_{0,T}(Q) - F^P_{0,T}(S) = 11.50 + 65e^{-0.04} - 70 = 3.95.$$

Problem 15.4. (2 pts) Consider two European exchange options both with exercise date T, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return.
On the other hand, consider the maximum option with the payoff

$$V_{max}(T) = \max(S(T), Q(T)),$$

and the minimum option with the payoff

$$V_{min}(T) = \min(S(T), Q(T)).$$

Then, in our usual notation,

$$V_{EC}(0, \mathbf{S}, \mathbf{Q}) + V_{EC}(0, \mathbf{Q}, \mathbf{S}) = V_{max}(0) + V_{min}(0).$$

Solution:
FALSE
If $S(T) \leq Q(T)$, the payoff of a long exchange option allowing you to give up a unit of Q and receive a unit of S is

$$V_{EC}(T, S(T), Q(T)) = (S(T) - Q(T))_+ = 0,$$

i.e., the option goes unexercised. On the other hand, the payoff of a long exchange option allowing you to give up a unit of S and receive a unit of Q is

$$V_{EC}(T, Q(T), S(T)) = (Q(T) - S(T))_+ = Q(T) - S(T).$$

So, the payoff of the portfolio whose price is on the left-hand side of (15.1) is simply $Q(T) - S(T)$.
The payoff of the portfolio whose initial cost is on the right-hand side of (15.1) is always $S(T) + Q(T)$.
So, it is impossible for the proposed equality in prices to always be true.