M339D: October 1st, 2025.

Arbitrage Portfolio.

Def'n. An arbitrage portfolio is a portfolio whose profit is:

- nonnegative in ALL states of the world, i.e., w/ probability 1,

and

- strictly positive in AT LEAST one state of the world, i.e., w/ probability > 0.

Unless it's specified otherwise in a specific problem/example, we assume NO ARBITRAGE.

Law of Unique Price.

Assume that the payoffs of two static portfolios A and B are equal, i.e.,

$$\boxed{V_A(T) = V_B(T)}$$

T... time horizon (temporarily fixed)

random variable

Claim. $\boxed{V_A(0) = V_B(0)}$.

Proof. Assume, to the contrary, that

$$V_A(0) \neq V_B(0)$$

Without loss of generality,

$$\boxed{\underbrace{V_A(0)}_{\text{relatively cheap}} < \underbrace{V_B(0)}_{\text{relatively expensive}}}$$

Diagnosis.

Propose an arbitrage portfolio:

- Long Portfolio A
- Short Portfolio B

} Total Portfolio.

Verify that this is, indeed, an arbitrage portfolio.

- Initial Cost (Total Portfolio) = $\underline{V_A(0) - V_B(0)} < 0$ ⇝ Inflow of money @ time 0.

- Payoff (Total Portfolio) = $V_A(T) - V_B(T) = 0$

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= 0 - FV_{0,T}(\underbrace{V_A(0) - V_B(0)}_{<0}) > 0$$

↑ Indeed, this is an ARBITRAGE PORTFOLIO!

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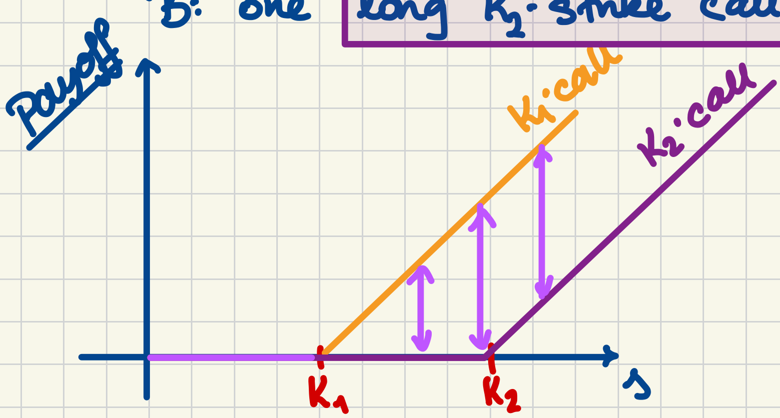
Corollary. If $V_A(T) \geq V_B(T)$, then $\boxed{V_A(0) \geq V_B(0)}$

Example. $K_1 < K_2$

A: one long K_1-strike call
B: one long K_2-strike call

} w/ the same underlying asset and exercise date, and European



The payoff of the K_1-strike call dominates the payoff of the K_2-strike call.

$\Rightarrow$ **The K_1-call costs @ least as much as the K_2-call.**

In Math:

$$K_1 < K_2 \Rightarrow V_c(0, K_1) \geq V_c(0, K_2)$$

As a function of the strike price, call prices are <u>decreasing</u>.