Name:

M339J: Probability models
University of Texas at Austin
Practice Problems for In-Term One
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

1.1. **TRUE/FALSE QUESTIONS.** *Please, note your final answer on the front page of this exam.*

Problem 1.1. Let X denote the outcome of a roll of a fair, regular icosahedron (a polyhedron with 20 faces) with numbers $1, 2, \cdots, 20$ written on its sides. Then $\mathbb{E}[X] = 15/2$. *True or false? Why?*

Problem 1.2. (2 pts) Let X be an exponential random variable. Then, its mean and its standard deviation are equal. *True or false?*

Problem 1.3. (2 points) For a random variable X and for a positive constant d, in our usual notation, we have

$$\mathbb{E}[X] = e_X(d)S_X(d) + \mathbb{E}[X \wedge d]\,. \tag{1.1}$$

True or false?

1.2. **Free-response problems.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.4. (10 points) A population of insureds consists of three types of people: α, β and γ. There is an equal number of Type α and Type β people in the population. The number of Type γ people is equal to the total number of the remaining two types of people. The probability that a Type α person makes at least one claim in a year is $1/5$. The probability that a Type β person makes at least one claim in a year is $2/5$. The probability that a Type γ person makes at least one claim in a year is $3/5$.

At the end of the year, a person is chosen at random from the population. It is observed that that person had at least one claim. What is the probability that the person was of Type β?

Problem 1.5. (15 points) Losses X follow a Pareto distribution with parameters $\alpha > 1$ and θ unspecified. For a positive constant c, determine the ratio of the mean excess loss function evaluated at $c\theta$ to the mean excess loss function evaluated at θ.

Problem 1.6. Let $\{X_n, n \geq 1\}$ be a sequence of independent random variables. Assume that all the variables in the sequence have the two-parameter Pareto distribution with $\theta = 10$ and $\alpha = 3$. For each n, define the random variable

$$Y_n = \frac{X_1^2 + X_2^2 + \cdots + X_n^2}{n}.$$

Does the limit of the sequence $\{Y_n, n \geq 1\}$ as $n \to \infty$ exist? If so, how much is it? If not, why not?

1.3. **MULTIPLE CHOICE QUESTIONS.** *Please, note your final answers on the front page of this exam.*

Problem 1.7. (5 pts) Let X be exponential with variance 225. Let $a = \mathbb{E}[|20 - X|]$. Then,

(a) $0 \leq a < 50$
(b) $50 \leq a < 150$
(c) $150 \leq a < 325$
(d) $325 \leq a < 550$
(e) None of the above.

Problem 1.8. Let E and F be two events on the same probability space. You know that $\mathbb{P}[E \cup F] = 0.75$ and $\mathbb{P}[E \cup F^c] = 0.85$. What is the probability of the event E?

(a) 0.5
(b) 0.6
(c) 0.65
(d) 0.7
(e) None of the above.

Problem 1.9. The time until the next bus arrives is a continuous random variable T with the density

$$f_T(t) = \begin{cases} \kappa(10 - t) & 0 < t < 10 \\ 0 & \text{otherwise} \end{cases}$$

for some constant κ. **Given** that you have already waited for 4 minutes, what is the probability that you will wait for at least another 4 minutes?

(a) 1/25
(b) 1/9
(c) 1/8
(d) 1/3
(e) None of the above.

Problem 1.10. Let X_1, X_2, and X_3 be independent, identically distributed random variables with the probability mass function

$$p_X(x) = \begin{cases} \frac{1}{4} & x = 0 \\ \frac{1}{2} & x = 1 \\ \frac{1}{4} & x = 2 \end{cases}$$

Find $\mathbb{P}[X_1 X_2 X_3 = 0]$.

(a) 27/64
(b) 1/8
(c) 31/64
(d) 37/64
(e) None of the above.

Problem 1.11. A recent study indicates that the annual cost of fertilizing a Japanese plum tree in Austin has a mean 100 with a variance of 20. A tax of 10% is introduced on fertilizer, i.e., fertilizer is made 10% more expensive. What is the variance of the new annual cost of fertilizing a Japanese plum tree in Austin after the tax is introduced?

(a) 20
(b) 22
(c) 23.1
(d) 24.2
(e) None of the above.

Problem 1.12. Let the random variable X have the cumulative distribution function given by

$$F_X(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{2}(x^2 - 2x + 2) & \text{for } 1 \leq x < 2 \\ 1 & \text{for } 2 \leq x \end{cases}$$

What is the expectation of X?

(a) 2/3

(b) 5/6
(c) 7/6
(d) 4/3
(e) None of the above.

Problem 1.13. Let the independent random variables X and Y have the same mean. You are given that coefficient of variation of X equals 2 and the coefficient of variation of Y equals 4. What is the coefficient of variation of the average of X and Y?

(a) 3/2
(b) $\frac{\sqrt{13}}{2}$
(c) 5/2
(d) There is not enough information to answer this problem.
(e) None of the above.