UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 12

Black-Scholes pricing.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 12.1. The current price of a continuous-dividend paying stock is observed to be $50 per share while its volatility is given to be 0.34. The dividend yield is projected to be 0.02.

The continuously compounded, risk-free interest rate is 0.05.

Consider a European call option with the strike price equal to $40 and the exercise date in three months.

Using the Black-Scholes pricing formula, find the value $V_C(0)$ of this option at time−0.

(a) $9.08

(b) $9.80

(c) $10.55

(d) $14.10

(e) None of the above.

Solution: (c)

In our usual notation,

$$
\begin{aligned}
d_1 &= \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r - \delta + \frac{1}{2}\sigma^2\right)T\right] \\
&= \frac{1}{0.34\sqrt{1/4}}\left[\ln\left(\frac{50}{40}\right) + \left(0.05 - 0.02 + \frac{1}{2}\times 0.34^2\right)\times\frac{1}{4}\right] = 1.44, \\
d_2 &= d_1 - \sigma\sqrt{T} = 1.27.
\end{aligned}
$$

The standard normal tables give us

$$N(d_1) = 0.9253, \quad N(d_2) = 0.8983.$$

Finally,

$$V_C(0) = S(0)e^{-\delta T}N(d_1) - Ke^{-rT}N(d_2) = 10.55.$$

Problem 12.2. (5 points) Consider a continuous-dividend-paying stock whose current price equals $100 per share. The stock's dividend yield equals 0.01 while its volatility equals 0.25.

The continuously compounded risk-free interest rate is given to be 0.05.

Using the Black-Scholes model, calculate the price of a $98-strike, three-month European put option on the above stock.

(a) $3.37

(b) $3.80

(c) $4.55

(d) $5.10

(e) None of the above.

Solution: (a)

In our usual notation, the Black-Scholes pricing formula reads as

$$V_P(0) = Ke^{-rT}N(-d_2) - S(0)e^{-\delta T}N(-d_1)$$

with

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T\right],$$
$$d_2 = d_1 - \sigma\sqrt{T}.$$

Using the provided data, we get

$$d_1 = \frac{1}{0.25\sqrt{\frac{1}{4}}}\left[\ln\left(\frac{100}{98}\right) + \left(0.05 - 0.01 + \frac{0.25^2}{2}\right) \times \frac{1}{4}\right] = 0.3041,$$
$$d_2 = 0.3041 - 0.125 = 0.1791.$$

Hence,

$$N(-d_1) = 1 - N(d_1) = 1 - 0.6179 = 0.3821, \quad \text{and} \quad N(-d_2) = 1 - N(d_2) = 1 - 0.5714 = 0.4286.$$

Finally,

$$V_P(0) = 98e^{-0.05/4} \times 0.4286 - 100e^{-0.01/4}0.3821 = 3.3664.$$

Problem 12.3. (5 points) The current price of a continuous-dividend-paying stock is given to be \$92. The stock's volatility is 0.35 and its dividend yield is 0.02.

The continuously compounded risk-free interest rate is 0.05.

Consider a \$90-strike European call option on the above stock with exercise date in a quarter-year.

What is the Black-Scholes price of this call option?

(a) 5.05
(b) 7.66
(c) 7.71
(d) 7.89
(e) None of the above.

Solution: (d)

$$d_1 = 0.26, d_2 = 0.08.$$

So,

$$V_C(0) = 92e^{-0.02/4} \times 0.6026 - 90e^{-0.05/4} \times 0.5319 \approx 7.89.$$