Strong Law of Large Numbers (SLLN).

Let $\{X_k, k=1,2,\dots\}$ be a sequence of independent, identically distributed r.v.s.

Assume: $\boxed{\mu_X := \mathbb{E}[X_1] < \infty}$

Then,

$$\boxed{\frac{X_1+X_2+\cdots+X_n}{n} \xrightarrow[n\to+\infty]{} \mu_X}$$

If a f'tion g is such that $g(X_1)$ is well-defined,

and $\mathbb{E}[g(X_1)]<\infty$,

then,

$$\boxed{\frac{g(X_1)+g(X_2)+\cdots+g(X_n)}{n} \xrightarrow[n\to\infty]{} \mathbb{E}[g(X_1)]}$$

Monte Carlo.

Recipe:
- Draw simulated values of a random variable from a specific dist'n.
- Apply a f'tion to the simulated values.
- Calculate the arithmetic average of the obtained quantities.

We get the value which is "close to" the theoretical expectation.

Precision:

- $$\mathrm{Var}\left[\frac{X_1+X_2+\cdots+X_n}{n}\right] = \frac{1}{n^2}\mathrm{Var}[X_1+X_2+\cdots+X_n] \quad \text{(independent)}$$
$$= \frac{1}{n^2}\left(\mathrm{Var}[X_1]+\mathrm{Var}[X_2]+\cdots+\mathrm{Var}[X_n]\right) \quad \text{(identically dist'd)}$$
$$= \frac{1}{n^{\cancel{2}}}\cancel{n}\cdot\mathrm{Var}[X_1] = \frac{\mathrm{Var}[X_1]}{n}$$

- $$\boxed{SD\left[\frac{X_1+\cdots+X_n}{n}\right]=\frac{SD[X_1]}{\sqrt{n}}}$$

To increase the precision by a factor η, the number of <u>variates</u> must increase by a factor of η^2.