

- 37) A policyholder owns a variable annuity contract with death benefit features defined as follows:
- Guaranteed minimum death benefit (GMDB) with return of premium: the greater of the account value and the initial investment will be paid when the policyholder dies.
 - Enhanced-income death benefit guarantee: 20% of the account value in excess of the initial investment amount will be added if the account value is greater than the initial investment when the policyholder dies.

Let T be the random variable denoting the future lifetime of the policyholder.

Let K be the initial investment amount of the variable annuity contract. **premium**

Let S_t be the value of the policyholder's account at time t .

You are given:

- T follows a distribution with probability density function $f(t), t > 0$.
- Given $T = t$, $p(t)$ is the payoff of a European put option based on account value S_t with strike price K .
- Given $T = t$, $c(t)$ is the payoff of a European call option based on account value S_t with strike price K .

Determine which one of the following statements is true.

- (A) The total death benefit payout for death at time t can be expressed as $\max(S_t, K) + 0.2 \times \max(K - S_t, 0)$. X
- (B) The total death benefit payout for death at time t can be expressed as $\max(S_t - K, 0) + 0.2 \times \int_0^\infty p(t)f(t)dt$. X
- (C) The expected value of the death benefit can be expressed as $K + \int_0^\infty c(t)f(t)dt + 0.2 \times \int_0^\infty p(t)f(t)dt$.
- (D) The expected value of the death benefit can be expressed as $K + \int_0^\infty p(t)f(t)dt + 0.2 \times \int_0^\infty c(t)f(t)dt$.
- (E) The expected value of the death benefit can be expressed as $K + 1.2 \times \int_0^\infty c(t)f(t)dt$.

Intended
answer!

$$\rightarrow: \frac{\max(S_t, K)}{\text{GMDB}} + \frac{0.20 \max(S_t - K, 0)}{\text{EEDB}}$$

$$= K + \frac{\max(S_t - K, 0) + 0.20 \max(S_t - K, 0)}{\text{EEDB}}$$

$$= K + 1.20 \underbrace{(S_t - K)_+}_{\text{like a call payoff}}$$

\Rightarrow the expectation is:

$$K + 1.2 \mathbb{E} \int_0^T C(t) f_T(t) dt$$



II. Guaranteed Minimum Accumulation Benefit (GMAB).

It guarantees a minimum value on the account at a pre-specified time m .

K ... the guaranteed amount

The GMAB will pay: $\max(K - S_m, 0)$ if the policyholder is still alive:

\Rightarrow Its total value:

$$P(m) \mathbb{P}[T_x \geq m]$$

w/ $P(m)$ as the price of the put w/ strike K and expiration @ time m .

III. Guaranteed Minimum Withdrawal Benefit (GMWB).

GMWB guarantees that policyholder can withdraw a particular pre-specified sum per year after they reach a certain age.

IV. Guaranteed Minimum Income Benefit (GMIB).

The price of a whole life annuity is guaranteed @ a particular age.

36) Determine which one of the following statements regarding guarantees on variable annuity products is FALSE:

- T (A) A guaranteed minimum death benefit (GMDB) with a return of premium guarantee is similar to a European put option with expiration contingent on the death of the policyholder or annuitant.
- T (B) A guaranteed minimum accumulation benefit (GMAB) with a return of premium guarantee is similar to a European put option with payment contingent on the policyholder surviving to the guarantee expiration date and the policy still being in force at that time.
- F (C) A guaranteed minimum withdrawal benefit (GMWB) provides a guarantee that the account value will not be less than the guaranteed withdrawal benefit base at any future time.
- T (D) A guaranteed minimum income benefit (GMIB) provides a guarantee on the future purchase rate for a traditional annuity.
- T (E) An earnings-enhanced death benefit is an optional benefit available with some variable annuity products that acts as a European call option with strike price equal to the original amount invested.