M378K Introduction to Mathematical Statistics Homework assignment #6

Please, provide your **final answer only** to the following problems.

Problem 6.1. (3 \times 7 = 21 points) *Identify the distributions with the following mgfs:*

- $\frac{2}{2-t}$.
- e^{2e^t-2} ,
- $e^{t(t-2)}$.
- $(3-2e^t)^{-1}$
- $\frac{1}{9} + \frac{4}{9}e^t + \frac{4}{9}e^{2t}$.
- $\frac{1}{t}(1-e^{-t})$.
- $\frac{1}{4}(e^{4t} + 3e^{-t})$

If the distribution has a name, give the name and the parameters. If it does not, give the pdf or the pmf (table).

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 6.2. (30 points) Solve **Problem 7.6.11** from the Lecture notes.

Problem 6.3. (18 *points*) Source: "Mathematical Statistics with Applications" by Wackerly, Mendenhal, Scheaffer.

Suppose that a random variable Y has a probability density function given by

$$f_Y(y) = \kappa y^3 e^{-y/2} \mathbf{1}_{(0,\infty)}(y)$$

- (i) (5 points) Find the value of κ that makes $f_Y(y)$ a density function.
- (ii) (3 points) Does Y have a χ^2 -distribution? If so, how many degrees of freedom?
- (iii) (5 points) What are the mean and standard deviation of Y?
- (iv) (5 points) (Extra credit) Using \mathbf{R} , find the probability that Y lies within 2 standard deviations of its mean?