

UNIVERSITY OF TEXAS AT AUSTIN

Lecture 3

An introduction to forwards and options

3.1. **Forward contracts.** A *forward contract* is a **binding** agreement to buy/sell an underlying asset on an agreed-upon date in the future – the *delivery date* – at a fixed price set today. The forward contract specifies the following:

- the asset to be delivered,
- the **quantity** of the asset to be delivered,
- the delivery **logistics**: the time, date, place, e.g.,
- the **price** the buyer will pay at the time of delivery: the *forward price*,
- **type of settlement**: *cash settlement* or *physical delivery* (the former is preferable due to simplicity and lower transaction costs).

The person whose position is the **long forward** will be required to “buy” the underlying asset for the forward price. So, his/her payoff will be

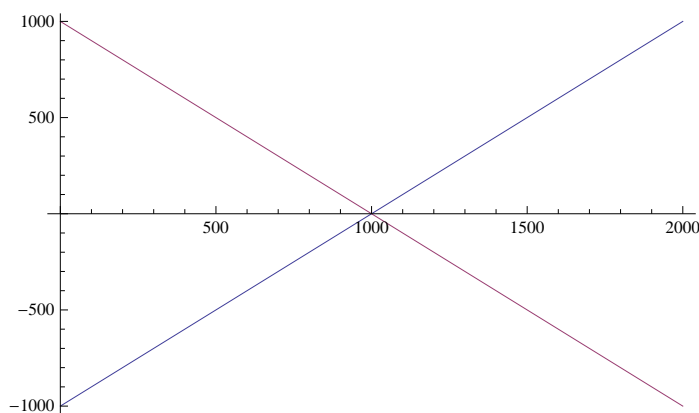
$$S(T) - F$$

where F denotes the forward price.

Analogously, if one has the **short forward**, one’s payoff is

$$F - S(T).$$

The payoff graphs for both positions are displayed below. The blue line is the **long** forward payoff, and the red line is the **short** forward payoff. The forward price is set to be $F = 1000$.



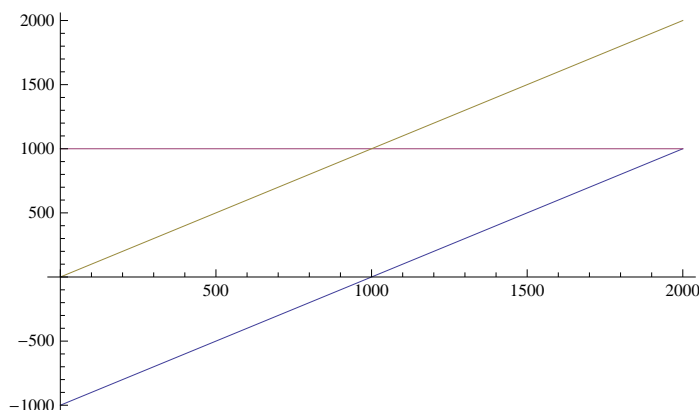
3.1.1. *Forward contract versus outright purchase.* The natural question to ask at this point is:

“What is the difference between the outright purchase of the underlying asset and the long forward contract on the same asset?”

First, let us go back to the way that the forward contract is agreed upon by the two parties. At no point in the discussion above did we mention the **initial cost** of the forward contract. The reason is simple – **there is no initial cost**. For a forward contract, the only action that happens at time-0 is the agreement reached which includes the stipulations in the forward

contract. However, there are no cash-flows taking place at time -0 . In particular, this means that the profit and the payoff curves **coincide** in the case of the forward contract.

Second, let us focus on the forward price F briefly. Imagine that the party with the long forward contract wants to be certain that he/she has sufficient funds at the delivery date T to pay the forward price. One way of ensuring this would be an investment at time -0 in a zero-coupon bond with maturity date T and redemption amount F . We can just think about this step as if he/she simply made a deposit into a savings account today and did nothing until the delivery date – then he/she withdraws the balance and uses it to purchase the underlying asset. With the purchase of the bond coupled with the long forward contract, one creates a simple *portfolio*. Let's try to see what the payoff curve for this portfolio is. Again, let $F = 1000$ for the sake of being able to have a concrete graph.



The blue line is the payoff of the long forward contract, and the red line is the payoff of the bond. The payoff of the entire portfolio is the **sum** of the payoffs of its components – the green line. If the green line reminds you of something, this is not accidental. In fact, we encountered the same payoff curve in the previous lecture. It corresponded to the **long stock** position – the simple **outright purchase** of a share of stock.

We have to emphasize that looking at the profit curve is not entirely straightforward. We have to take into account that the owner of the stock is entitled to any dividend payments. On the other hand, the holder of the forward contract does not receive dividend payments. We will return to these issues soon when we discuss the ways to come up with a **fair** forward price.

3.1.2. Credit risk. The term *credit risk* signifies the risk associated with the possibility that one of the counterparties in the contract (any contract – not just a forward contract) might fail to fulfill its financial obligations. To alleviate this risk and provide a sense of security needed to conduct the trades in the first place several steps might be taken. One example is “**standardization**” – certain types of contracts are popular/useful enough to become *exchange-traded*. There are two major consequences of this step: the contracts get very detailed and standard (collateral requirements are introduced, e.g.), and the **exchange** guarantees the transactions in conducts. However, not all contracts are traded frequently enough to warrant the above approach. The blanket term for the contract that are not exchange traded is *over-the-counter (OTC)* contracts. In their context, remedies for credit risk may include:

- credit checks,
- collateral,

- bank letter of credit, ...

3.1.3. *Suggested problems.* Sample FM problems: #10; McDonald: #2.7, #2.8, #2.9.

3.2. **European call options.** The mechanics of the *European call option* are the following:

- at time=0: (1) the contract is agreed upon between the buyer and the writer of the option,
 (2) the logistics of the contract are worked out (these include the underlying asset, the *expiration date* T and the *strike price* K),
 (3) the buyer of the option pays a *premium* to the writer of the call option;
 at time= T : the buyer of the option can **choose** whether he/she will purchase the underlying asset for the strike price K .

3.2.1. *The buyer's perspective.* The buyer of the European call option is the party who is entitled from the “**optionality**” in the term itself. For the buyer, the call is a **non-binding** agreement in the sense that he/she has the **right, but not the obligation** to buy the underlying asset on the expiration date. This feature is preserved in the overwhelming majority of derivative securities with the word “option” in their names.

We will consider all of the characters involved in our trades to be **rational** in the sense that they will always take the course of action which benefits them the most financially speaking. So, let us figure out what the rational buyer would do at time= T – whether he/she would purchase the asset or not. The buyer's behavior will be dictated by the price of the underlying asset at time= T .

If the final asset price exceeds the strike price, i.e., if $S(T) > K$, this means that the buyer is able to purchase the asset “cheaply”. In our perfectly liquid markets, he/she would be able to purchase the asset for K and immediately sell it for the market price of $S(T)$. The effect of these actions on his/her pocketbook is the payoff of $S(T) - K$.

To the contrary, if $S(T) \leq K$, the buyer would be better off purchasing the asset at the market price. So, he/she will simply walk away from the contract incurring the payoff of 0.

Combining the above two scenarios, we get the following expression for the buyer's payoff:

$$V(T) = \begin{cases} S(T) - K, & \text{if } S(T) \geq K \\ 0, & \text{if } S(T) < K. \end{cases}$$

After a (very short) while, it gets tedious to repeat the above expression, so we write

$$V(T) = \max(S(T) - K, 0) = (S(T) - K) \wedge 0.$$

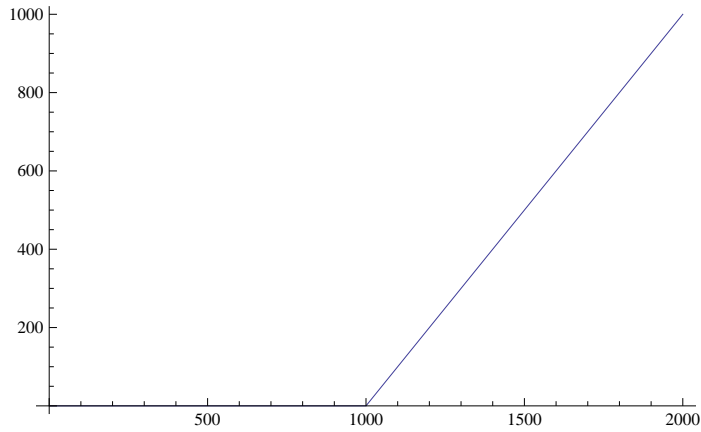
To make the notation even more compact we introduce the following shorthand

$$(x)_+ = x \wedge 0.$$

With this notation, we can finally write the buyer's payoff as

$$V(T) = (S(T) - K)_+.$$

For $K = 1000$, we get the following payoff curve:



Looking at the graph above, we see that the call option's payoff is increasing in the asset price having an infinite upward potential. On the other hand, there is no downside to the buyer for final asset prices lower than the strike. We should not forget that the buyer is supposed to pay the premium at $t = 0$. This will affect the profit curve.