University of Texas at Austin

Problem Set #9

Binomial option pricing.

Problem 9.1. (2 points) In the setting of the one-period binomial model, denote by i the **effective** interest rate **per period**. Let u denote the "up factor" and let d denote the "down factor" in the stock-price model. If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage.

Solution: FALSE

Problem 9.2. In our usual notation, which of the parameter choices below creates a binomial model with an arbitrage opportunity?

(a) $u = 1.18, \quad d = 0.87, \quad r = 0.05, \quad \delta = 0, \quad h = 1/4$
(b) $u = 1.23, \quad d = 0.80, \quad r = 0.05, \quad \delta = 0.06, \quad h = 1/2$
(c) $u = 1.08, \quad d = 1, \quad r = 0.05, \quad \delta = 0.04, \quad h = 1$
(d) $u = 1.28, \quad d = 0.78, \quad r = \delta, \quad h = 2$
(e) None of the above.

Solution: (e)

Problem 9.3. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is $80 per share. The stock pays dividends continuously with a dividend yield of 0.02. In the model, it is assumed that the stock price can either go up by $5 or down by $4.

You use the binomial tree to construct a replicating portfolio for a $(78, 82)-$strangle on the above stock. What is the stock investment in the replicating portfolio?

(a) Long 0.1089 shares.
(b) Long 0.33 shares.
(c) Short 0.1089 shares.
(d) Short 0.33 shares.
(e) None of the above.

Solution: (a)
The two possible stock prices are $S_u = 85$ and $S_d = 76$. So, the possible payoffs of the strangle are $V_u = 3$ and $V_d = 2$. The Δ of the strangle, thus, equals

$$\Delta = e^{-\delta h} \frac{V_u - V_d}{S_u - S_d} = e^{-0.02} \frac{3-2}{85-76} = 0.108911. \tag{9.1}$$

Problem 9.4. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is $50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a $(45, 55)-$call bull spread on the above stock. What is the risk-free investment in the replicating portfolio?

(a) Borrow $45
(b) Borrow $43.24
(c) Lend $45
(d) Lend $43.24
(e) None of the above.

Solution: (b)
The two possible stock prices are $S_u = 52.5$ and $S_d = 45$. So, the possible payoffs of the call bull spread are $V_u = 7.5$ and $V_d = 0$. The risk-free investment B in the replicating portoflio of the call bull spread, thus, equals

$$B = e^{-rh}\frac{uV_d - dV_u}{u-d} = e^{-0.04}\frac{1.05(0) - 0.9(7.5)}{1.05 - 0.9} = -43.2355. \tag{9.2}$$

Problem 9.5. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

(a) About \$10
(b) About \$10.83
(c) About \$15.45
(d) About \$20.84
(e) None of the above.

Solution: (d)
The risk-neutral probability of an up movement is

$$p^* = \frac{95e^{0.06} - 75}{120 - 75} = 0.575.$$

So, the price of our straddle is

$$V(0) = e^{-0.06}[0.575 \times (120 - 100) + (1 - 0.575) * (100 - 75)] = 20.8366.$$

Problem 9.6. (5 points) Consider the one-period binomial option pricing model. Let $V_C(0) > 0$ denote the price of a European call on a stock which pays continuous dividends. What is the impact on the value of European call option prices if the company decides to increase the dividend yield paid to the shareholders?

(a) The call option price will drop.

(b) The call option price will increase.

(c) The call option price will always remain constant.

(d) The impact on the price of the call cannot be determined using the binomial option pricing model.

(e) There is not enough information provided.

Solution: (a)

Let $\delta < \tilde{\delta}$ be the two dividend yields. Then, the risk-neutral price of the European call on the stock with the dividend yield δ equals

$$V_C(0) = e^{-rT}[p^*(S_u - K)_+ + (1-p^*)(S_d - K)_+]$$

with $p^* = (e^{(r-\delta)h} - d)/(u-d)$. On the other hand, the risk-neutral price of the European call on the stock with the dividend yield $\tilde{\delta}$ equals

$$\tilde{V}_C(0) = e^{-rT}[\tilde{p}^*(S_u - K)_+ + (1-\tilde{p}^*)(S_d - K)_+]$$

with $\tilde{p}^* = (e^{(r-\tilde{\delta})h} - d)/(u-d)$. We have

$$\delta < \tilde{\delta} \quad \Rightarrow \quad e^{(r-\delta)h} > e^{(r-\tilde{\delta})h} \quad \Rightarrow p^* > \tilde{p}^* \quad \Rightarrow \quad V_C(0) > \tilde{V}_C(0).$$