UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #6

European put options.

Problem 6.1. The initial price of the market index is \$900. After 3 months the market index is priced at \$915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of \$930, is \$8.00. What is the profit at expiration for a **long** put?

(a) \$15.00 loss
(b) \$6.90 loss
(c) \$6.90 gain
(d) \$15.00 gain
(e) None of the above.

Solution: (c)
The profit from a position is defined as the position's payoff minus the future value of the initial cost.
If $S(T) = 915$ denotes the price of the market index at time $T = 0.25$ (i.e., in three months), then the payoff of the long put is $(K - S(T))_+$, where $K = 930$ denotes the strike of the put. So, since $K > S(T)$, the payoff is

$$(930 - 915)_+ = 15.$$

The future value of the initial put premium is

$$8(1 + 0.004)^3 = 8.0964.$$

So, the profit is

$$15 - 8.0964 = 6.90.$$

Problem 6.2. Sample FM(DM) #12
Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4% **convertible semiannually**, and that the put costs 74.20 now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

A. 922.83
B. 924.32
C. 1,000.00
D. 1,075.68
E. 1,077.17

Solution: (b)
Method I. A quick and insightful way of solving this problem is by realizing that the long-put and the short-put profits are negatives of each other. So, the only way they can be equal is at the "break-even" point. We solve for s in

$$(K - s)_+ - V_P(0)\left(1 + \frac{i^{(2)}}{2}\right) = (1000 - s)_+ - 74.20(1.02) = 0.$$

The solution is $s = 924.32$.

Method II. This is the more pedestrian method. The long-put profit is

$$(K - s)_+ - V_P(0)\left(1 + \frac{i^{(2)}}{2}\right) = (1000 - s)_+ - 74.20(1.02).$$

The short-put profit is the exact negative of the expression above, i.e.,

$$-(K - s)_+ + V_P(0)\left(1 + \frac{i^{(2)}}{2}\right) = -(1000 - s)_+ + 74.20(1.02).$$

So, algebraically, we need to solve for s in the equation

$$\begin{aligned}(1000-s)_+ - 74.20(1.02) = -(1000-s)_+ + 74.20(1.02) \quad &\Leftrightarrow \quad 2(1000-s)_+ = 2\cdot 74.20(1.02)\\ &\Leftrightarrow \quad (1000-s)_+ = 74.20(1.02).\end{aligned}$$

We get the same answer as above, of course.

Problem 6.3. Farmer Shaun is producing sweet potatoes. He intends to harvest $10,000-$cartons' worth in six months. His total costs are \$12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of \$13 per carton and another with the strike price of \$15 per carton. Their premiums are \$0.15 and \$0.18, respectvely.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals \$14. What would Farmer Shaun's profit be if he had decided to hedge using the \$13-strike put versus his profit if he had decided to use the \$15-strike put to hedge?

Solution: Farmer Shaun's unhedged position has the following profit:

$$10,000(S(T)-12)$$

where $S(T)$ stands for the spot price of sweet potatoes in six months.

If he decided to hedge using put options, he would **long** the put. So, the profit of the \$13-strike-put hedge would be:

$$10,000(13-S(T))_+ - 10,000\times 0.15\times 1.04.$$

The profit of the \$15-strike-put hedge would be:

$$10,000(15-S(T))_+ - 10,000\times 0.18\times 1.04.$$

The profit of the hedged position with the given $S(T)=14$ in the first case equals

$$10,000(14-12-0.15\times 1.04) = 18,440.$$

For the second insurance strategy, the profit is

$$10,000(14-12+(15-14)-0.18\times 1.04) = 28,128.$$

Problem 6.4. (2 points) In which of the following option positions is the investor exposed to an unlimited loss?

(a) Long put option
(b) Short put option
(c) Long call option
(d) Short call option
(e) None of the above.

Solution: (d)
Just draw the payoff diagrams to convince yourselves.

Problem 6.5. (3 points) The initial price of the market index is \$1000. After 3 months the market index is priced at \$950. The nominal rate of interest convertible quarterly is 4.0%.
The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

(a) \$12.00 loss
(b) \$14.90 loss
(c) \$12.00 gain
(d) \$14.90 gain
(e) None of the above.

Solution: (d)
The profit is

$$(K - S(T))_+ - FV_{0,T}[V_P(0)] = (975 - 950)_+ - 10\left(1 + \frac{0.04}{4}\right) = 25 - 10.10 = 14.90.$$

Problem 6.6. (3 points) *Source: Sample FM(DM) Problem #62.*

The stock price today equals \$100 and its price in one year is modeled by the following distribution:

$$S(1) \sim \begin{cases} 125 & \text{with probability } 1/2 \\ 60 & \text{with probability } 1/2 \end{cases}$$

The annual effective interest rate equals 3%.

Consider an at-the-money, one-year European put option on the above stock whose initial premium is equal to \$7.

What is the expected profit of this put option?

Solution:

$$\frac{1}{2}(100 - 60) - 7(1.03) = 20 - 7.21 = 12.79.$$

Problem 6.7. Aunt Dahlia simultaneously purchased

- one share of a market index at the current spot price of \$1,000;
- one one-year, \$1,050-strike put option on the above market index for the premium of \$20.

(i) (5 points) Is the above portfolio's payoff bounded from above? If you believe it is not, substantiate your claim. If you believe that it is, provide the upper bound.

(ii) (5 points) Is the above portfolio's payoff bounded from below? If you believe it is not, substantiate your claim. If you believe that it is, provide the lower bound.

Solution: The payoff of the portfolio expressed in terms of the final asset price $S(T)$ is

$$V(T) = S(T) + (K - S(T))_+ = \min[K, S(T)] = K \wedge S(T).$$

(i) The payoff is **not** bounded from above since the stock price $S(T)$ may be arbitrarily large.

(ii) The payoff is bounded from below by the put option's exercise price K. This means that there is a guarantee of the minimum price the owner of the portfolio can fetch for the underlying asset. That's why this type of a portfolio is referred to as the *floor*.