M378K Introduction to Mathematical Statistics Problem Set #5 Continuous distributions.

Problem 5.1. Source: Sample P exam, Problem #33.

The lifetime of a machine part has a continuous distribution on the interval (0,40) with probability density function f_X , where

$$f_X(x) \propto \frac{1}{(10+x)^2}$$

on the interval.

Calculate the probability that the lifetime of the machine part is less than 6.

Problem 5.2. Source: Sample P exam, Problem #419.

A customer purchases a lawnmower with a two-year warranty. The number of years before the lawnmower needs a repair is uniformly distributed on [0,5]. Calculate the probability that the lawnmower needs no repairs within 4.5 years after the purchase, given that the lawnmower needs no repairs within the warranty period

$$f_Y(y) = 2y\mathbf{1}_{[0,1]}(y)$$

What is the expected value of this random variable?