M339W: October 1st, 2021.

3. You are asked to determine the price of a European put option on a stock. Assuming the Black-Scholes framework holds, you are given:

(i) The stock price is \$100. $S(0)=100$

(ii) The put option will expire in 6 months. $T=0.5$

(iii) The strike price is \$98. $K=98$

(iv) The continuously compounded risk-free interest rate is $r = 0.055$.

(v) $\delta = 0.01$

(vi) $\sigma = 0.50$

Calculate the price of this put option.

(A) \$3.50

(B) \$8.60

(C) \$11.90

(D) \$16.00

(E) \$20.40

1st

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r-\delta+\frac{\sigma^2}{2}\right)\cdot T\right]$$

$$d_1 = \frac{1}{0.5\sqrt{0.5}}\left[\ln\left(\frac{100}{98}\right) + \left(0.055-0.01+\frac{0.25}{2}\right)(0.5)\right]$$

$$d_1 = 0.29756$$

$$\Rightarrow \quad d_2 = d_1 - \sigma\sqrt{T} = 0.29756 - 0.5\sqrt{0.5}$$
$$= -0.05599$$

2nd

Put!

$$\boxed{N(-d_1) = 0.3830195}$$

$$N(-d_2) = 0.5223251$$

3rd

$$V_P(0) = Ke^{-rT}N(-d_2) - S(0)e^{-\delta T}N(-d_1)$$
$$= 98e^{-0.055(0.5)}(0.5223251)$$
$$- 100e^{-0.01(0.5)}(0.3830195) = 11.69$$

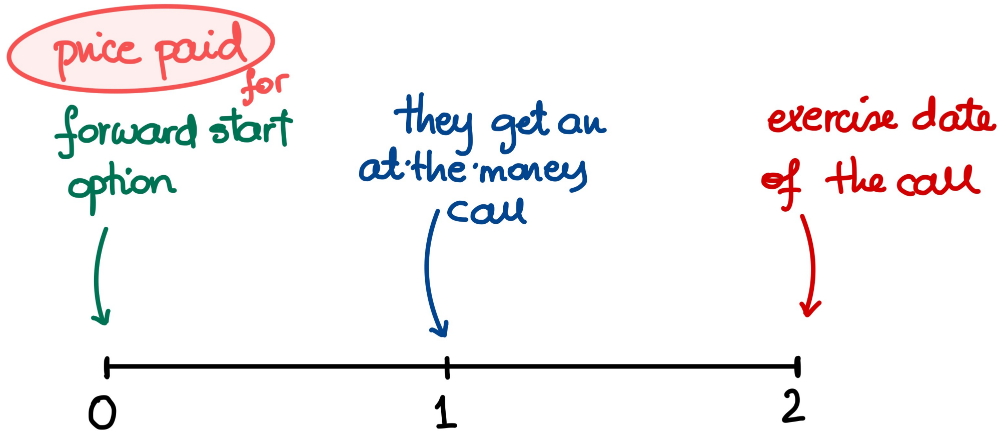
(A) 586

(B) 594

(C) 684

(D) 692

(E) 797



19. Consider a *forward start option* which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.

You are given:

(i) The European call option is on a stock that pays no dividends.

(ii) The stock's volatility is 30%. $\boxed{\sigma = 0.30}$

(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100. $\boxed{F_{0,1}(S) = 100}$

(iv) The continuously compounded risk-free interest rate is 8%. $r = 0.08$

Under the Black-Scholes framework, determine the price today of the forward start option.

(A) 11.90

(B) 13.10

(C) 14.50

(D) 15.70

(E) 16.80

At any time $t < T$:

$$V_c(t) = S(t)e^{-\delta(T-t)} \cdot N(d_1) - Ke^{-r(T-t)} \cdot N(d_2)$$

w/
$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S(t)}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)(T-t)\right]$$

and $d_2 = d_1 - \sigma\sqrt{T-t}$

In our problem:

$$V_c(1) = S(1) \cdot N(d_1) - S(1)e^{-0.08(2-1)} \cdot N(d_2)$$

w/
$$d_1 = \frac{1}{0.3\sqrt{2-1}}\left[\cancel{\ln\left(\frac{S(1)}{S(1)}\right)} + \left(0.08 + \frac{0.09}{2}\right)(2-1)\right]$$

$$d_1 = \frac{0.08 + 0.045}{0.30} \approx 0.41667$$

$$\Rightarrow d_2 = 0.41667 - 0.3\sqrt{2-1} = 0.11667$$

$$N(d_1) = 0.6615401$$

$$N(d_2) = 0.5464392$$

$$V_c(1) = S(1)\left((0.6615401) - e^{-0.08}(0.5464392)\right)$$

$$\boxed{V_c(1) = S(1) \cdot \underline{0.15711}}$$

At time 0, our forward start option must cost the same as 0.15711 prepaid forward contracts.

$$\Rightarrow V_{FS}(0) = 0.15711 \cdot F^P_{0,1}(S) = 0.15711 \cdot F_{0,1}(S) \cdot e^{-0.08(1)}$$

$$= 0.15711 \cdot 100\, e^{-0.08} = \underline{14.5034}$$

33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

You are given:

(i) The continuously compounded risk-free interest rate is 8%.

(ii) The stock's volatility is 30%.

(iii) The current stock price is 45.

(iv) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

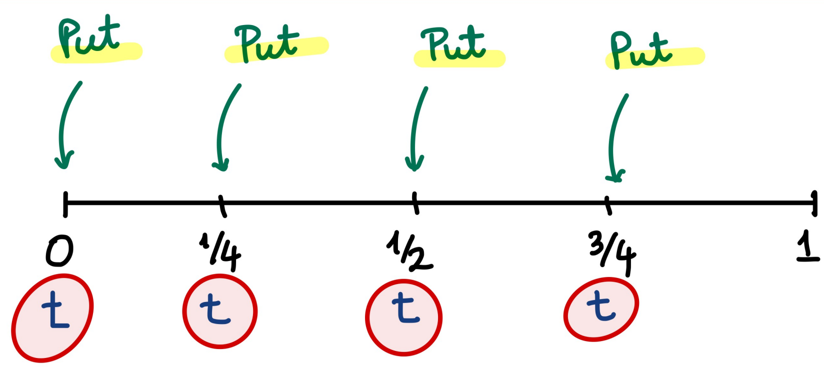
(A) 1.59

(B) 2.24

(C) 2.86

(D) .48

(E) 3.61



34-39. DELETED

For every put in the rolling insurance strategy, there is:

- one quarter to exercise
- $\boxed{K = 0.9 \cdot S(t)}$

For every t @ which a put option is received:

$$d_1(t) = \frac{1}{\sigma\sqrt{1/4}}\left[\ln\left(\frac{\cancel{S(t)}}{0.9\cancel{S(t)}}\right) + \left(r + \frac{\sigma^2}{2}\right)\left(\frac{1}{4}\right)\right]$$

$$d_1(t) = \frac{1}{0.3(0.5)}\left[-\ln(0.9) + \left(0.08 + \frac{0.09}{2}\right)\left(\frac{1}{4}\right)\right] = 0.9107,$$

for every t

$$d_2(t) = 0.9107 - 0.3(0.5) = 0.7607$$

$N(-d_1) = N(-0.9107) = 0.1812267$

$N(-d_2) = N(-0.7607) = 0.2234181$

$$\Rightarrow V_P(t) = 0.9 \cdot S(t) e^{-0.08(1/4)} \cdot 0.2234181 - S(t) \cdot 0.1812267$$

$$V_P(t) = S(t)\left(0.9 e^{-0.02} \cdot 0.2234181 - 0.1812267\right)$$

$$= S(t) \cdot 0.01586798$$

$\Rightarrow$ For every put·delivery date $t = 0, 1/4, 1/2, 3/4$, today's worth of the put option delivered on that date is

$$0.01586798 \cdot F^P_{0,t}(S)$$

prepaid forward price for delivery @ time t.

Since the stock pays no dividends, we know that $F^P_{0,t}(S) = S(0) = 45$

$\Rightarrow$ Altogether, the time·0 price of the rolling insurance strategy is

$$4 \cdot 45 \cdot 0.01586798 = 2.856236$$

↑

# of puts

■