Ordinary deductible

Let X be the loss random variable

- **"Definition":** An ordinary deductible modifies a random variable into either the

  excess loss, or

  the **left censored and shifted** random variable depending on whether the result of applying the deductible is per payment or per loss, respectively.
- So, in the already introduced notation, the **per-payment** random variable is

$$Y^P = \begin{cases} \text{undefined} & X \leq d \\ X - d & X > d \end{cases}$$

  while the **per-loss** random variable equals

$$Y^L = \begin{cases} 0 & X \leq d \\ X - d & X > d \end{cases}$$

Ordinary deductible: Densities of Per payment and per loss random variables

$$f_{Y^P}(y) = \frac{f_X(y+d)}{S_X(d)} \qquad y > 0$$

$$\begin{aligned} p_{Y^L}(0) &= F_X(d) \\ f_{Y^L}(y) &= f_X(y+d) \qquad y > 0 \end{aligned}$$

- From the densities, one can get all the other relevant functions associated with the two random variables (see Section 8.2 in the textbook)

Franchise deductible

- **"Definition":** A franchise deductible modifies the **ordinary deductible** by adding the deductible when there is a positive amount paid.
- So, reusing the already introduced notation, the **per-payment** random variable is

$$Y^P = \begin{cases} \text{undefined} & X \leq d \\ X & X > d \end{cases}$$

while the **per-loss** random variable equals

$$Y^L = \begin{cases} 0 & X \leq d \\ X & X > d \end{cases}$$

Franchise deductible: Densities of Per payment and per loss random variables

$$f_{Y^P}(y) = \frac{f_X(y)}{S_X(d)} \quad y > d$$

$$\begin{aligned} p_{Y^L}(0) &= F_X(d) \\ f_{Y^L}(y) &= f_X(y) \qquad y > d \end{aligned}$$

- From the densities, one can get all the other relevant functions associated with the two random variables (see Section 8.2 in the textbook)

Expected costs

This result is an extension of the formula we already used in class:

- **Theorem:**
  For an ordinary deductible d on the loss random variable X, the expected cost per loss is

$$\mathbb{E}[X] - \mathbb{E}[X \wedge d]$$

and the expected cost per payment is

$$\frac{\mathbb{E}[X] - \mathbb{E}[X \wedge d]}{1 - F_X(d)}.$$

For a franchise deductible the expected cost per loss is

$$\mathbb{E}[X] - \mathbb{E}[X \wedge d] + d(1 - F_X(d))$$

and the expected cost per payment is

$$\frac{\mathbb{E}[X] - \mathbb{E}[X \wedge d]}{1 - F_X(d)} + d$$