

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #6

European put options.

Problem 6.1. The initial price of the market index is \$900. After 3 months the market index is priced at \$915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of \$930, is \$8.00. What is the profit at expiration for a **long** put?

- (a) \$15.00 loss
- (b) \$6.90 loss
- (c) \$6.90 gain
- (d) \$15.00 gain
- (e) None of the above.

Solution: (c)

The profit from a position is defined as the position's payoff minus the future value of the initial cost.

If $S(T) = 915$ denotes the price of the market index at time $T = 0.25$ (i.e., in three months), then the payoff of the long put is $(K - S(T))_+$, where $K = 930$ denotes the strike of the put. So, since $K > S(T)$, the payoff is

$$(930 - 915)_+ = 15.$$

The future value of the initial put premium is

$$8(1 + 0.004)^3 = 8.0964.$$

So, the profit is

$$15 - 8.0964 = 6.90.$$

Problem 6.2. Sample FM(DM) #12

Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4% **convertible semiannually**, and that the put costs 74.20 now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

- A. 922.83
- B. 924.32
- C. 1,000.00
- D. 1,075.68
- E. 1,077.17

Solution: (b)

Method I. A quick and insightful way of solving this problem is by realizing that the long-put and the short-put profits are negatives of each other. So, the only way they can be equal is at the “break-even” point. We solve for s in

$$(K - s)_+ - V_P(0) \left(1 + \frac{i^{(2)}}{2} \right) = (1000 - s)_+ - 74.20(1.02) = 0.$$

The solution is $s = 924.32$.

Method II. This is the more pedestrian method. The long-put profit is

$$(K - s)_+ - V_P(0) \left(1 + \frac{i^{(2)}}{2} \right) = (1000 - s)_+ - 74.20(1.02).$$

The short-put profit is the exact negative of the expression above, i.e.,

$$-(K - s)_+ + V_P(0) \left(1 + \frac{i^{(2)}}{2} \right) = -(1000 - s)_+ + 74.20(1.02).$$

So, algebraically, we need to solve for s in the equation

$$\begin{aligned}(1000 - s)_+ - 74.20(1.02) &= -(1000 - s)_+ + 74.20(1.02) &\Leftrightarrow & 2(1000 - s)_+ = 2 \cdot 74.20(1.02) \\ & &\Leftrightarrow & (1000 - s)_+ = 74.20(1.02).\end{aligned}$$

We get the same answer as above, of course.

Problem 6.3. Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000—cartons' worth in six months. His total costs are \$12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of \$13 per carton and another with the strike price of \$15 per carton. Their premiums are \$0.15 and \$0.18, respectively.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals \$14. What would Farmer Shaun's profit be if he had decided to hedge using the \$13-strike put versus his profit if he had decided to use the \$15-strike put to hedge?

Solution: Farmer Shaun's unhedged position has the following profit:

$$10,000(S(T) - 12)$$

where $S(T)$ stands for the spot price of sweet potatoes in six months.

If he decided to hedge using put options, he would **long** the put. So, the profit of the \$13-strike-put hedge would be:

$$10,000(13 - S(T))_+ - 10,000 \times 0.15 \times 1.04.$$

The profit of the \$15-strike-put hedge would be:

$$10,000(15 - S(T))_+ - 10,000 \times 0.18 \times 1.04.$$

The profit of the hedged position with the given $S(T) = 14$ in the first case equals

$$10,000(14 - 12 - 0.15 \times 1.04) = 18,440.$$

For the second insurance strategy, the profit is

$$10,000(14 - 12 + (15 - 14) - 0.18 \times 1.04) = 28,128.$$

Problem 6.4. (2 points) In which of the following option positions is the investor exposed to an unlimited loss?

- (a) Long put option
- (b) Short put option
- (c) Long call option
- (d) Short call option
- (e) None of the above.

Solution: (d)

Just draw the payoff diagrams to convince yourselves.

Problem 6.5. (3 points) The initial price of the market index is \$1000. After 3 months the market index is priced at \$950. The nominal rate of interest convertible quarterly is 4.0%.

The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

- (a) \$12.00 loss
- (b) \$14.90 loss
- (c) \$12.00 gain
- (d) \$14.90 gain
- (e) None of the above.

Solution: (d)

The profit is

$$(K - S(T))_+ - FV_{0,T}[V_P(0)] = (975 - 950)_+ - 10 \left(1 + \frac{0.04}{4}\right) = 25 - 10.10 = 14.90.$$

Problem 6.6. (3 points) *Source: Sample FM(DM) Problem #62.*

The stock price today equals \$100 and its price in one year is modeled by the following distribution:

$$S(1) \sim \begin{cases} 125 & \text{with probability } 1/2 \\ 60 & \text{with probability } 1/2 \end{cases}$$

The annual effective interest rate equals 3%.

Consider an at-the-money, one-year European put option on the above stock whose initial premium is equal to \$7.

What is the expected profit of this put option?

Solution:

$$\frac{1}{2}(100 - 60) - 7(1.03) = 20 - 7.21 = 12.79.$$

Problem 6.7. Aunt Dahlia simultaneously purchased

- one share of a market index at the current spot price of \$1,000;
 - one one-year, \$1,050-strike put option on the above market index for the premium of \$20.
- (i) (5 points) Is the above portfolio's payoff bounded from above? If you believe it is not, substantiate your claim. If you believe that it is, provide the upper bound.
- (ii) (5 points) Is the above portfolio's payoff bounded from below? If you believe it is not, substantiate your claim. If you believe that it is, provide the lower bound.

Solution: The payoff of the portfolio expressed in terms of the final asset price $S(T)$ is

$$V(T) = S(T) + (K - S(T))_+ = \max[K, S(T)] = K \vee S(T).$$

- (i) The payoff is **not** bounded from above since the stock price $S(T)$ may be arbitrarily large.
- (ii) The payoff is bounded from below by the put option's exercise price K . This means that there is a guarantee of the minimum price the owner of the portfolio can fetch for the underlying asset. That's why this type of a portfolio is referred to as the *floor*.