Random Vectors.

Say, we are interested in two (or more) r.v.s as a PAIR (or VECTOR), i.e., we look @

$$(Y_1, Y_2)$$

Then, we must not only look @ their $\boxed{\text{"individual"}}$ dist'ns, but also @ their association.

Example. Y_i ... cointoss for $i=1,2$ of fair coins

independence

$\{Y_1 = H, Y_2 = H\}$ $\{Y_1 = T, Y_2 = H\}$

$\{Y_1 = H, Y_2 = T\}$ $\{Y_1 = T, Y_2 = T\}$

complete dependence

$\{Y_1 = H, Y_2 = H\}$

$\{Y_1 = T, Y_2 = T\}$

Discrete 2D Environment.

The Joint Distribution Table.

$X \backslash Y$	y_1	y_2	$\cdots$	y_j	$\cdots$	y_ℓ	
x_1							$p_X(x_1) = \sum_{j=1}^{\ell} p_{1j}$
x_2							$\vdots$
$\vdots$							
x_i				p_{ij}			$p_X(x_i) = \sum_{j=1}^{\ell} p_{ij}$
$\vdots$							$\vdots$
x_m							
	The Marginal Dist'n of Y						

The MARGINAL Dist'n of X

$p_{ij} = \mathbb{P}[X = x_i, Y = y_j]$, i.e., the joint pmf

for all i,j

X and Y are independent iff

$$\boxed{p_{ij} = p_X(x_i) \cdot p_Y(y_j) \text{ for all } i,j}$$

Example.

We independently throw two dice and record the results as Y_1 and Y_2, resp.

joint pmf: $p_{ij} = \frac{1}{36}$ for all $1 \le i, j \le 6$

Define $\boxed{Z = Y_1 + Y_2}$

Q: What is the joint dist'n table for (Y_1, Z)?

$Y_1 \backslash Z$	2	3	4	5	6	7	8	9	10	11	12
1	$\frac{1}{36}$	$\frac{1}{36}$					0	0	0	0	0
2	0		$\boxed{\frac{1}{36}}$					0	0	0	0
3	0	0							0	0	0
4	0	$\boxed{0}$	0							0	0
5	0	0	0	0							0
6	0	0	0	0	0						
	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Joint Distributions: The Continuous Case.

Recall: For a continuous r.v. Y w/ a pdf f_Y, we can calculate probabilities as

$$\mathbb{P}[Y \in [a,b]] = \mathbb{P}[a \leq Y \leq b] =$$
$$= \int_a^b f_Y(y)\,dy \quad \text{for all } a \leq b$$

In multiple dimensions.

Say that the random vector $(Y_1, Y_2, \ldots, Y_n)$ is jointly continuous w/ density

$$\boxed{f_{Y_1,\ldots,Y_n}}$$

Then,

$$\mathbb{P}[Y_1 \in [a_1,b_1], Y_2 \in [a_2,b_2], \ldots, Y_n \in [a_n,b_n]] =$$
$$= \int_{a_1}^{b_1}\int_{a_2}^{b_2}\cdots\int_{a_n}^{b_n} f_{Y_1,\ldots,Y_n}(y_1,y_2,\ldots,y_n)\,dy_n \cdots dy_2\,dy_1$$