M358K : October 18th, 2021.

# Hypothesis Testing : The Normal Case [cont'd].

## The Population Model

$$X \sim \text{Normal}(\boxed{\text{mean} = \mu}, \text{sd} = \sigma)$$

unknown (mean $= \mu$); given/known ($\text{sd} = \sigma$)

### Hypotheses.

**The Null Hypothesis.** $H_0: \mu = \mu_0$

**Alternative Hypothesis.** $H_a: \begin{cases} \mu < \mu_0 \\ \mu \neq \mu_0 \\ \mu > \mu_0 \end{cases}$

**Test Statistic:** $\bar{X} \sim \text{Normal}\left(\text{mean} = \mu, \text{sd} = \frac{\sigma}{\sqrt{n}}\right)$

Under the null hypothesis, i.e., if $\mu = \mu_0$,

$$\frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

With the observed value of the sample average $\bar{x}$, we can calculate its z-score:

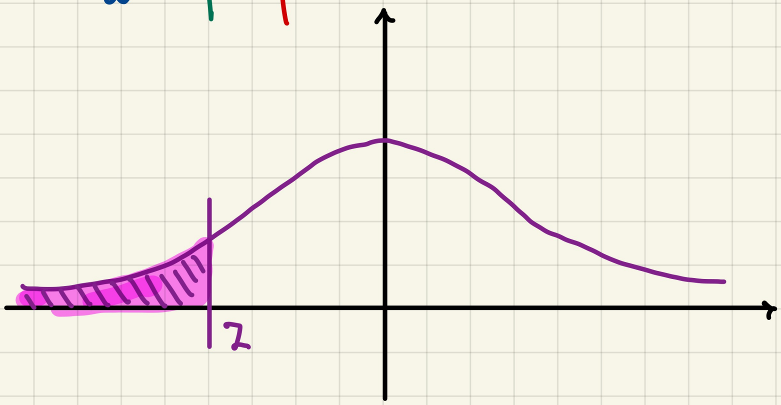
$$\boxed{z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}}$$

Q: What is the probability, under the null, of observing what was observed or something $\boxed{\text{more extreme}}$?

This probability is called the $\boxed{\text{p-value}}$

## Left-sided alternative

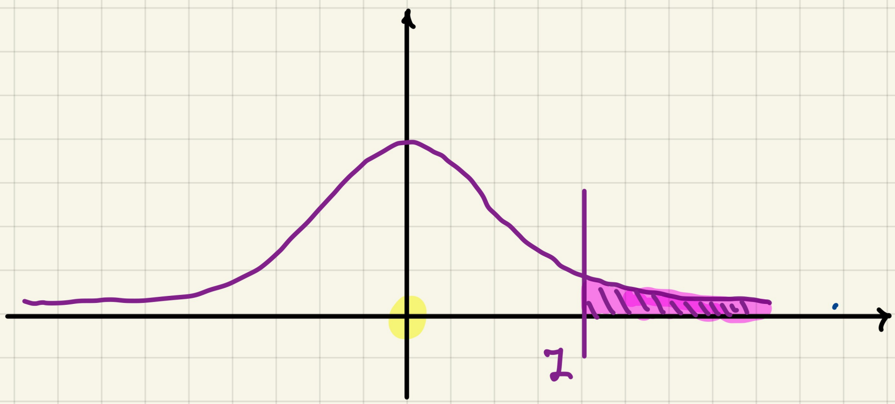
$H_a: \mu < \mu_0$




$\mathbb{P}[Z \leq z] = p\text{-value}$

## Right-sided alternative

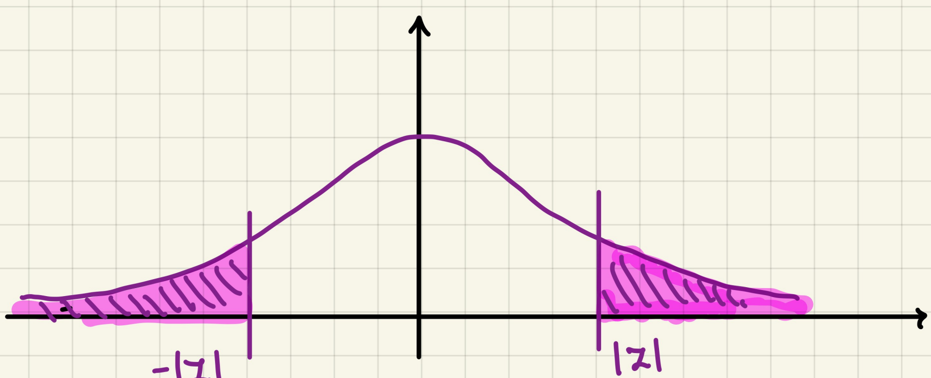
$H_a: \mu > \mu_0$




$\mathbb{P}[Z \geq z] = p\text{-value}$

## Two-sided alternative

$H_a: \mu \neq \mu_0$




$\mathbb{P}[Z \leq -|z|] + \mathbb{P}[Z \geq |z|] = 2 \cdot \mathbb{P}[Z \leq -|z|] = p\text{-value}$

University of Texas at Austin

# Problem Set # 10

## Hypothesis testing. $p$–value.

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**Problem 10.1.** The null hypothesis is a statement about the population parameter. *True or false?*

**Problem 10.2.** The null and alternative hypotheses are stated in terms of the statistics obtained from the random sample. *True or false?*

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Complete the following statements:

**Problem 10.3.** When we state the alternative hypothesis to look for a difference in a parameter in any direction, we are doing a ___two___-sided test.

**Problem 10.4.** When choosing between a one-sided alternative hypothesis and a two-sided alternative hypothesis, you should base the decision on ___the research question you're trying to answer.___

**Problem 10.5.** The ___smaller___ the $p$–value, the **stronger** the evidence against the null hypothesis provided by the data.

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Provide your **complete solution** for the following problems.

**Problem 10.6.** The square footage of several thousand apartments in a new development is advertised to be 1250 square feet, on average. A tenant group thinks that the apartments are smaller than advertised. They hire an engineer to measure a sample of apartments to test their suspicions. Let $\mu$ represent the "true" mean area (in square feet) of these apartments. What are the appropriate null and alternative hypotheses?

$H_0: \mu = 1250$ vs. $H_a: \mu < 1250$

**Problem 10.7.** Is the mean height for all adult American males between the ages of 18 and 21 now over 6 feet? Let $\mu$ denote the population mean height of all adult American males between the ages of 18 and 21. What are the appropriate null and alternative hypotheses?

$H_0: \mu = 6$ vs. $H_a: \mu > 6$

**Problem 10.8.** The hypotheses are $H_0 : \mu = 10$ versus $\boxed{H_a : \mu > 10.}$ The value of the test statistic for the population mean is $z = -2.12$. What is the corresponding $p$–value?

Right-sided : $\mathbb{P}[Z > -2.12] = 1 - \Phi(-2.12)$ 1-pnorm(-2.12)=0.982997

**Problem 10.9.** The value of the test statistic for a **two-sided** test for a population mean is $z = -2.12$. What is the corresponding $p$–value?

Two-sided : 2*pnorm(-2.12) = 0.034

## Tests of Significance.

Set $\alpha$... significance level

Typically: $\alpha = \boxed{0.05}$, $0.01$, $0.10$

Decision process:

If p-value $\leq \alpha$, we REJECT the null hypothesis.

If not, we FAIL TO REJECT the null hypothesis.

Note: The p-value corresponding to an observed value of the test statistic is the LOWEST significance level at which the null hypothesis would still be REJECTED.