M339G: October 28th, 2024.

Bivariate Normal in the Matrix Notation.

Consider a bivariate normal pair (U, V).

In 2D, we can place the means into a vector

$$\begin{pmatrix} \mu_U \\ \mu_V \end{pmatrix}$$

and the variances/covariances in a matrix

$$\Sigma = \begin{bmatrix} \sigma_U^2 & \sigma_U \sigma_V \rho \\ \sigma_U \sigma_V \rho & \sigma_V^2 \end{bmatrix}$$ (positive definite)

Then, the joint density of (U, V) can be written as:

$$f_{U,V}(u, v) = \frac{1}{2\pi} \boxed{\frac{1}{(\det(\Sigma))^{1/2}}} \exp\left(-\frac{1}{2} \boxed{\begin{pmatrix} u-\mu_U \\ v-\mu_V \end{pmatrix}}^T \Sigma^{-1} \begin{pmatrix} u-\mu_U \\ v-\mu_V \end{pmatrix}\right)$$

Multivariate Normal Density.

Let $\mathbb{X} = (X_1, X_2, \dots, X_p)^T$ be

$$\text{Normal}\left(\text{mean} = \mu = (\mu_1, \mu_2, \dots, \mu_p)^T, \ \Sigma = \begin{bmatrix} \sigma_1^2 & & \text{Cov} \\ & \ddots & \\ \text{Cov} & & \sigma_p^2 \end{bmatrix}\right)$$

w/ Σ positive definite

Then,

$$f_{\mathbb{X}}(\underbrace{x_1, x_2, \dots, x_p}_{x}) = \frac{1}{(2\pi)^{p/2}} \frac{1}{(\det(\Sigma))^{1/2}} \exp\left(-\frac{1}{2} (x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

for all $x \in \mathbb{R}^p$