UNIVERSITY OF TEXAS AT AUSTIN, DEPARTMENT OF MATHEMATICS
M358K - Applied Statistics

HOMEWORK #3 - ADDITIONAL PROBLEMS

Problem 3.1. (6 pts) Let Z be a standard normal random variable. Using the standard normal tables, calculate the following probabilities:

(i) (2 points) $\mathbb{P}[-1.23 < Z < 2.37]$
(ii) (2 points) $\mathbb{P}[|Z| < 0.5]$
(iii) (2 points) $\mathbb{P}[Z^2 > 2.56]$

Solution:

(i)

$$\begin{aligned}\mathbb{P}[-1.23 < Z < 2.37] &= \mathbb{P}[Z < 2.37] - \mathbb{P}[Z < -1.23] = \Phi(2.37) - (1 - \Phi(1.23)) \\ &= 0.9911 + 0.8907 - 1 = 0.8818.\end{aligned}$$

(ii)

$$\mathbb{P}[|Z| < 0.5] = \mathbb{P}[Z < 0.5] - \mathbb{P}[Z < -0.5] = 2\Phi(0.5) - 1 = 2(0.6915) - 1 = 0.383.$$

(iii)

$$\mathbb{P}[Z^2 > 2.56] = \mathbb{P}[|Z| > 1.6] = 2(\mathbb{P}[Z > 1.6]) = 2(1 - \Phi(1.6)) = 0.1096.$$

Problem 3.2. (9 points) *Source: Problem #139 from Moore-McCabe-Craig.*

The interquartile range (IQR) of a distribution is defined as the distance between the first and the third quartiles.

(i) (4 points) What is the IQR for the standard normal distribution? *Note: Do* **not** *interpolate in the standard normal tables!*
(ii) (5 points) What is the IQR for a normal distribution with mean μ and variance σ^2?

Solution:

(i) The value $z*$ of the third quartile can be obtained as

$$z^* = \Phi^{-1}(0.75) \approx 0.67$$

By the symmetry of the standard normal distribution, we have

$$-z^* = \Phi^{-1}(0.25) \approx -0.67$$

Therefore the IQR for the standard normal distribution is $0.67 - (-0.67) = 1.34$.

(ii) Any normal random variable $X \sim Normal(mean = \mu, variance = \sigma^2)$ can be represented as a linear transformation of the standard normal random variable Z. Namely, we have

$$X = \mu + \sigma Z.$$

So, the interquartile range is 1.34σ.