PCA and K-Means Clustering

Milica Cudina

We consider the seeds data set. This data set contains measurements of seeds. First, we import the data set.

```
seeds<-read.csv("seeds_dataset.csv")</pre>
seeds
##
         V1
               ٧2
                      VЗ
                            ۷4
                                  ۷5
                                        ۷6
     15.26 14.84 0.8710 5.763 3.312 2.221 5.220
## 1
## 2 14.88 14.57 0.8811 5.554 3.333 1.018 4.956
## 3 14.29 14.09 0.9050 5.291 3.337 2.699 4.825
     13.84 13.94 0.8955 5.324 3.379 2.259 4.805
     16.14 14.99 0.9034 5.658 3.562 1.355 5.175
    14.38 14.21 0.8951 5.386 3.312 2.462 4.956
## 7 14.69 14.49 0.8799 5.563 3.259 3.586 5.219
     14.11 14.10 0.8911 5.420 3.302 2.700
## 9 16.63 15.46 0.8747 6.053 3.465 2.040 5.877
## 10 16.44 15.25 0.8880 5.884 3.505 1.969 5.533
## 11 15.26 14.85 0.8696 5.714 3.242 4.543 5.314
## 12 14.03 14.16 0.8796 5.438 3.201 1.717 5.001
## 13 13.89 14.02 0.8880 5.439 3.199 3.986 4.738
## 14 13.78 14.06 0.8759 5.479 3.156 3.136 4.872
## [ reached 'max' / getOption("max.print") -- omitted 196 rows ]
dim(seeds)
## [1] 210
#View(seeds)
```

We immediately see that there are missing data points. I will choose to omit those rows in the future analysis.

```
seeds=na.omit(seeds)
dim(seeds)
```

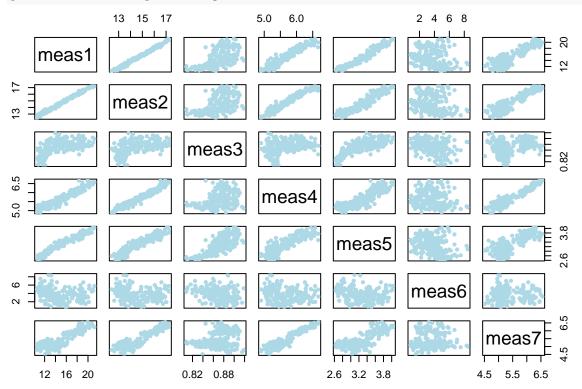
```
## [1] 203 7
```

I don't like how the columns are labeled, so I will change names for aesthetic reasons.

```
colnames(seeds)=c("meas1", "meas2", "meas3", "meas4", "meas5", "meas6", "meas7")
attach(seeds)
```

Here is the usual EDA.

plot(seeds, col="lightblue", pch=20)

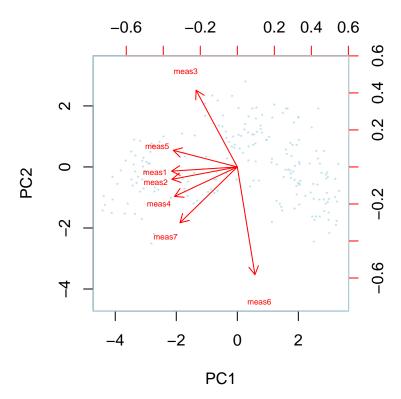


We do recognize the strong relationship between some measurements. We could perform K-means clustering on the entire data set.

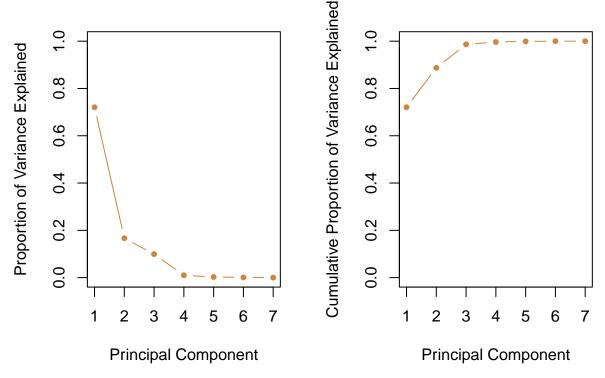
```
km.out <- kmeans(seeds, 2, nstart = 20)</pre>
km.out
## K-means clustering with 2 clusters of sizes 75, 128
##
## Cluster means:
         meas1
                   meas2
                               meas3
                                         meas4
                                                   meas5
                                                             meas6
                                                                        meas7
## 1 18.23107 16.08280 0.8844160 6.126080 3.671987 3.420161 5.965347
   2 12.95852 13.71328 0.8630281 5.355648 3.025430 3.880501 5.100070
##
##
   Clustering vector:
##
##
     1
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##
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                                                                         99 100 101 102 103
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    [ reached 'max' / getOption("max.print") -- omitted 103 entries ]
##
##
## Within cluster sum of squares by cluster:
## [1] 318.6342 651.6545
    (between_SS / total_SS = 63.3 %)
```

```
##
## Available components:
##
## [1] "cluster"
                                      "totss"
                                                                     "tot.withinss"
                       "centers"
                                                      "withinss"
## [6] "betweenss"
                       "size"
                                      "iter"
                                                      "ifault"
pre.clustering=km.out$cluster
However, visualization is "challenging" in a 7-dimensional space.
So, we want to start with PCA for this data set to reduce the dimension. Let's import the requisite library.
library(stats)
Let's look at the principal components analysis.
pr.out=prcomp(seeds,scale=TRUE)
pr.out$center
       meas1
                 meas2
                            meas3
                                      meas4
                                                meas5
                                                           meas6
                                                                     meas7
## 14.906502 14.588719
                        0.870930
                                   5.640291
                                             3.264305
                                                        3.710425
                                                                  5.419754
pr.out$scale
##
        meas1
                   meas2
                               meas3
                                          meas4
                                                     meas5
                                                                 meas6
                                                                            meas7
## 2.91985421 1.30985992 0.02333552 0.44356887 0.37841943 1.50445337 0.49274090
pr.out$rotation
                PC1
                             PC2
                                         PC3
                                                     PC4
                                                                  PC5
                                                                              PC6
## meas1 -0.4438797 -0.02827866 0.02487893 0.19823234 -0.19754734 -0.42733099
## meas2 -0.4409974 -0.08341264 -0.06128012 0.30190710 -0.16267355 -0.47768044
## meas3 -0.2786483 0.51826746 0.63993258 -0.33862828
                                                          0.32426621 -0.13864936
## meas4 -0.4238644 -0.19909129 -0.21365717 0.24983603 0.76819351 0.28331567
## meas5 -0.4324862 0.11204425 0.21527972 0.20512206 -0.47158921 0.69855470
## meas6 0.1192982 -0.72757429 0.66757050 0.09462566 0.03866974 -0.01705868
## meas7 -0.3871819 -0.37694281 -0.22015213 -0.80089937 -0.12389829 0.03805960
                  PC7
##
## meas1 -0.735268248
## meas2 0.670088948
## meas3 0.072103502
## meas4 -0.047383128
## meas5 0.040078258
## meas6 0.003587394
## meas7 0.036030449
```

What does the biplot tell us?



Let's look at the variance explained.



The values corresponding to our rows in terms of principal components are available through pr.out\$x.

pr.out\$x

```
PC1
                            PC2
                                         PC3
                                                        PC4
                                                                       PC5
##
       -0.27208727
##
                     0.81428921 -0.61045254
                                              0.4068678200
                                                             0.1108691927
  1
##
   2
        0.04374462
                     1.94324843
                                -0.62732194
                                              0.4193787646 -0.0422075100
##
   3
        0.49263170
                     1.91682186
                                 0.97891526
                                              0.0945618791
                                                             0.0051184569
##
  4
        0.62627162
                     1.94545680
                                 0.54327189
                                              0.2134810496 -0.0792880095
## 5
       -1.06194841
                     2.09018176
                                 0.10717166
                                              0.1260659263 -0.0214127618
## 6
        0.37282678
                     1.65279631
                                 0.47890409
                                              0.0841153996
                                                             0.0032071271
## 7
        0.18684029
                                 0.31745265
                                              0.1044517240
                     0.45446902
                                                             0.0715895980
##
  9
       -1.71585861
                     0.34378543 -0.95282241
                                             -0.2439069377
                                                             0.1343082487
##
  10
       -1.39463443
                     1.03957434 -0.35353894
                                             -0.0170437368
                                                             0.1002000159
       -0.02164777 -0.41103381
                                 0.32280885
                                              0.3572058770
                                                             0.1285946808
##
   11
  12
                     1.58478719 -0.38567551
##
        0.61068607
                                              0.1228737973
                                                             0.0156321674
                                 0.97274446
## 13
                     0.88444950
        0.96671193
                                              0.5288777755
                                                             0.2878919266
## 14
        0.95267769
                     0.89211163
                                 0.15734662
                                              0.4341716260
                                                             0.1895459811
##
  15
        1.04592550
                     0.98065151
                                 0.02147881
                                              0.4934043039
                                                             0.2367611943
##
                  PC6
                                 PC7
        0.0240907211
                       0.01353314270
##
  1
   2
        0.0166815256
                       0.00468348379
##
##
   3
       -0.0537045502
                       0.00449527985
##
  4
        0.2253545574
                       0.01013462950
## 5
        0.0488641611
                       0.00112007051
       -0.0244820342
                       0.00884895351
##
  6
##
  7
       -0.0588647805
                       0.02445094078
##
  9
        0.0959633857
                       0.02998744531
                       0.00846338822
##
        0.0614608024
  10
##
   11
       -0.1508217703
                       0.02455405863
## 12
       -0.0226973071
                       0.00771664945
```

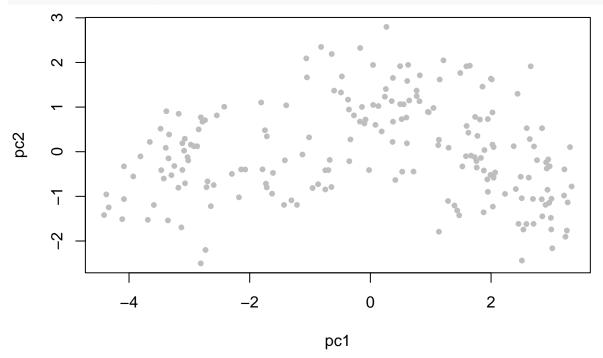
```
## 13 -0.0501568583 -0.01683378752
## 14 -0.0105925230 -0.00711376945
## 15 -0.0691114386 -0.01548348958
## [ reached 'max' / getOption("max.print") -- omitted 189 rows ]
```

We can isolate the first two principal components as follows:

```
pc1=pr.out$x[,1]
pc2=pr.out$x[,2]
```

Let's look at the scatterplot of these two vectors.

```
plot(pc1, pc2, col="grey",pch=20)
```



Now, let's perform K—means clustering to these data based on the first and second principal components.

```
pcs=cbind(pc1, pc2)
km.out=kmeans(pcs, 3, nstart=25)
```

Let's compare the clusters based on the entire data set and the first two principal components.

```
sum(km.out$cluster==pre.clustering)/length(pre.clustering)
```

```
## [1] 0.6305419
```

km.out\$cluster

```
7
                                                                                                       21
##
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```

```
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pre.clustering
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    [ reached 'max' / getOption("max.print") -- omitted 103 entries ]
What about the scatterplot?
```

```
plot(pcs, col = (km.out$cluster + 1),
    main = "K-Means Clustering Results with K = 2",
    xlab = "", ylab = "", pch = 20, cex = 1)
```

K-Means Clustering Results with K = 2

