M339D: October 6^{th}, 2023.

The Binomial Asset Pricing Model.

$S(0)$... the observable initial asset price

0 ———— T

time horizon (e.g., the exercise date of an option)

$$S(0) \nearrow \boxed{S_u = u \cdot S(0)} \text{ up node}$$
$$S(0) \searrow \boxed{S_d = d \cdot S(0)} \text{ down node}$$

ROOT NODE

By convention: $\boxed{u > d}$

u... up factor
d... down factor

h... length of a single period

one period $\Rightarrow$ $\boxed{S(T) = S(h)}$... a r.v. denoting the time-T stock price w/ two possible values: S_u and S_d

Returns:

As a random variable: simple return $= \boxed{\dfrac{S(T)-S(0)}{S(0)}}$

- $\dfrac{S_u - S(0)}{S(0)} = \dfrac{u \cdot \cancel{S(0)} - \cancel{S(0)}^{1}}{\cancel{S(0)}} = u - 1$
- $\dfrac{S_d - S(0)}{S(0)} = \dfrac{d \cdot \cancel{S(0)} - \cancel{S(0)}^{1}}{\cancel{S(0)}} = d - 1$