

M339 Y: April 15<sup>th</sup>, 2022.

## Compound Poisson.

### Homework Problem #6.1.

i.  $X \dots$  seventy

$$\text{Support}(x) = \{1, 2, 3\}$$

$$p_x(1), p_x(2), p_x(3) = ?$$

ii.

$$\boxed{\mathbb{E}[S] = 56}$$

w/

$$S = S_1 + S_2 + \dots + S_N \quad w/ \quad N \sim \text{Poisson}(\lambda=29)$$



$$\underbrace{\mathbb{E}[N]}_{29} \cdot \underbrace{\mathbb{E}[X]}_{?} = 56$$

$$29 \cdot (1 \cdot p_x(1) + 2 \cdot p_x(2) + 3 p_x(3)) = 56$$

iii.

$$\boxed{\text{Var}[S] = 126}$$

$$\text{Var}[S] = \underbrace{\mathbb{E}[N]}_{\lambda} \cdot \text{Var}[X] + \underbrace{\text{Var}[N]}_{\lambda} (\mathbb{E}[X])^2$$

$$\text{Var}[S] = \lambda \left( \text{Var}[X] + (\mathbb{E}[X])^2 \right)$$

$\underbrace{\mathbb{E}[X^2]}$

$$\boxed{\text{Var}[S] = \lambda \cdot \mathbb{E}[X^2]}$$

ALL COMPOUND POISSON!

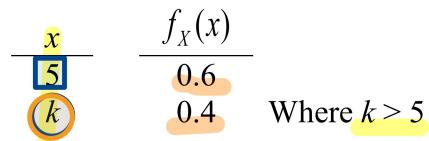
In this problem:

$$29 \cdot (1 \cdot p_x(1) + 4 \cdot p_x(2) + 9 \cdot p_x(3)) = 126$$

$$\boxed{p_x(1) + p_x(2) + p_x(3) = 1}$$

because pmf

280. A compound Poisson claim distribution has  $\lambda = 5$  and individual claim amounts distributed as follows:



The expected cost of an aggregate stop-loss insurance subject to a deductible of 5 is 28.03.

Calculate  $k$ .

$$\rightarrow: \mathbb{E}[(S-d)_+] = \boxed{\mathbb{E}[S]} - \boxed{\mathbb{E}[S^d]}$$

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

$$\begin{aligned} \bullet \quad \mathbb{E}[S] &= \lambda \cdot \mathbb{E}[X] = \\ &= 5 \cdot (5 \cdot 0.6 + k \cdot 0.4) \\ &= 2k + 15 \end{aligned}$$

281. DELETED

$$\bullet \quad \mathbb{E}[S^d] = ? \quad d=5$$

$$\text{Support}(S^d) = \{0, 5\}$$

$$S^d \sim \begin{cases} 0 & \text{w/ probab. } \frac{p_N(0)}{e^{-5}} \\ 5 & \text{w/ probab. } \frac{1-e^{-5}}{e^{-5}} \end{cases}$$

$$\mathbb{E}[S^d] = 0 \cdot e^{-5} + 5 \cdot (1 - e^{-5}) = 5(1 - e^{-5})$$

$$28.03 = \mathbb{E}[(S-5)_+] = 2k + 15 - 5(1 - e^{-5})$$

$$k = \frac{18.03 - 5e^{-5}}{2} = \underline{8.99816 = 9}$$

□

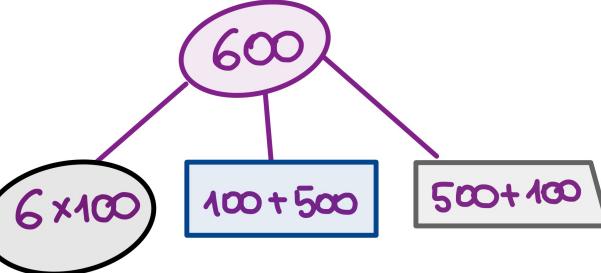
289. A compound Poisson distribution has  $\lambda = 5$  and claim amount distribution as follows:

| $x$  | $p(x)$ |
|------|--------|
| 100  | 0.80   |
| 500  | 0.16   |
| 1000 | 0.04   |

Calculate the probability that aggregate claims will be exactly 600.

- (A) 0.022
- (B) 0.038
- (C) 0.049
- (D) 0.060**
- (E) 0.070

$$\rightarrow : p_S(600) = ?$$



290. DELETED

291. DELETED

292. DELETED

293. DELETED

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299. DELETED

$$\begin{aligned}
 P[6 \text{ claims of } 100] &= P[N=6, X_1=X_2=X_3=X_4=X_5=X_6=100] \\
 &= p_N(6) (p_X(100))^6 = e^{-5} \cdot \frac{5^6}{6!} (0.80)^6 \\
 &= \underline{\underline{0.038331}}
 \end{aligned}$$

↑  
independence

$$P[\text{two claims w/ first claim of } 100 \text{ and second of } 500] =$$

$$\begin{aligned}
 &= p_N(2) \cdot p_X(100) \cdot p_X(500) = \\
 &= e^{-5} \cdot \frac{5^2}{2!} (0.80)(0.16) = \underline{\underline{0.01078}}
 \end{aligned}$$

$$P[\text{two claims w/ first claim of } 500 \text{ and second of } 100] =$$

$$\underline{\underline{0.01078}}$$

$$\text{answer : } 0.038331 + 2(0.01078) = \underline{\underline{0.05989}}$$



## A Few Compound Poissons.

We start w/  $n$  different streams of losses, each of them a compound Poisson and all independent.

$\{S_j, j=1..n\}$  are independent compound Poissons, i.e.,

for every  $j$ :  $N_j \sim \text{Poisson}(\lambda_j)$  }  
and  $X^j \sim \text{cdf } F_j$

More precisely, for stream  $j$ , the severity r.v.s are  
 $\{X_1^j, X_2^j, \dots, X_{N_j}^j, \dots\}$

$$\Rightarrow S_j = X_1^j + X_2^j + \dots + X_{N_j}^j$$

Set:  $S = S_1 + S_2 + \dots + S_n$

Thm.  $S$  is itself a compound Poisson w/

$$N \sim \text{Poisson}(\lambda = \lambda_1 + \lambda_2 + \dots + \lambda_n)$$

and w/ the severity r.v.s w/ the cdf

$$F_X(x) = \sum_{j=1}^n \frac{\lambda_j}{\lambda} F_j(x)$$

$n$ -point mixture of  $X_j$ 's.