W: 02/20/2019.

Problem. Assume that the current stock price equals \$100.

The stock price @ any later date is modeled as lognormally dist'd.

According to your model:

$$\begin{cases} \cdot \mathbb{P}[S(1/4) < 95] = 0.2358 \\ \cdot \mathbb{P}[S(1/2) < 110] = 0.6026 \end{cases}$$

What is the expected value of the time-1 stock price?

→: For the lognormal stock price:

$$S(T) = S(0) e^{\underbrace{(\alpha - \delta - \frac{\sigma^2}{2})}_{\mu}\cdot T + \sigma\sqrt{T}\cdot Z} \qquad Z \sim N(0,1)$$

rate of appreciation

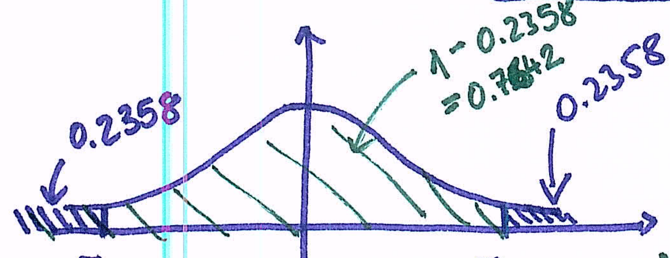
$$\mathbb{E}[S(T)] = S(0) e^{(\alpha-\delta)\cdot T}$$

$$\Rightarrow \mathbb{E}[\underset{T=1}{S(T)}] = S(0) e^{\mu + \frac{\sigma^2}{2}}$$

Focus on $\mathbb{P}[S(1/4) < 95] = 0.2358$

From the std normal tables:

$$N(z_{0.2358}) = \mathbb{P}[Z < z_{0.2358}] = \underline{0.2358}$$



$z_{0.2358}$ $\qquad -z_{0.2358} = N^{-1}(0.7642) = 0.72$

$\Rightarrow \quad z_{0.2358} = -0.72$

$$\mathbb{P}[\ Z < -0.72\ ] = 0.2358$$

$$\mathbb{P}[\sigma\sqrt{T}\cdot Z < \sigma\sqrt{T}\cdot(-0.72)] = 0.2358$$

$$\mathbb{P}[\ \mu\cdot T + \sigma\sqrt{T}\cdot Z < \mu\cdot T + \sigma\sqrt{T}\cdot(-0.72)] = 0.2358$$

$$\mathbb{P}[\underbrace{S(0)e^{\mu T + \sigma\sqrt{T}\cdot Z}}_{S(T)} < S(0)e^{\mu\cdot T + \sigma\sqrt{T}(-0.72)}] = 0.2358$$

Take $T = 1/4$.

$$\mathbb{P}[\ S(1/4) < \underbrace{S(0)e^{\mu(1/4) + \sigma\sqrt{1/4}(-0.72)}}_{=95}] = 0.2358$$

$$\Rightarrow \quad S(0)e^{\mu(1/4) + \sigma(1/2)(-0.72)} = 95 \quad (\mathrm{I})$$

Get $z_{0.6026} = N^{-1}(0.6026) = 0.26$

$$\Rightarrow \quad S(0)e^{\mu(1/2) + \sigma\sqrt{1/2}\cdot(0.26)} = 110 \quad (\mathrm{II})$$

$\boxed{S(0) = 100}$

$$\begin{cases} \mu\cdot\frac{1}{4} + \sigma\left(\frac{1}{2}\right)(-0.72) = \ln(0.95) \\ \mu\cdot\frac{1}{2} + \sigma\sqrt{\frac{1}{2}}\ (0.26) = \ln(1.1) \end{cases}$$

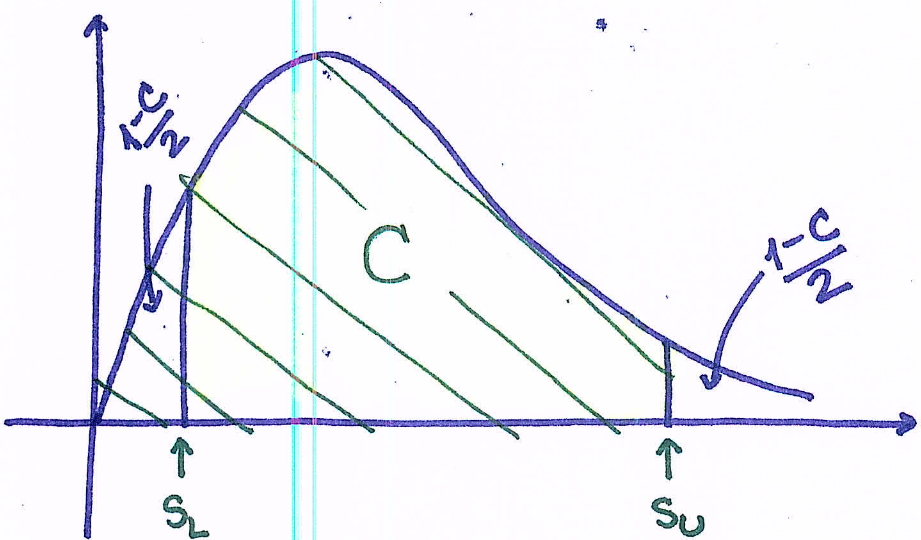
$$\Rightarrow \boxed{\sigma = 0.2189 \ ; \quad \mu = 0.1101}$$

$$\Rightarrow \mathbb{E}[S(1)] = 100\, e^{0.1101 + \frac{(0.2189)^2}{2}} = \dots = 114.35 \ \blacksquare$$

LogNormal "Confidence" Intervals

By design : • two-sided
• symmetric

Given a probability, i.e., a "confidence" level $C \in (0,1)$



The stock-price "confidence" interval is (S_L, S_U) such that:

With $z^* = N^{-1}\left(\frac{1+C}{2}\right)$, we set

$$S_U = S(0)e^{(\alpha-\delta-\frac{\sigma^2}{2})\cdot T+\sigma\sqrt{T}\,z^*}$$

$$S_L = S(0)e^{(\alpha-\delta-\frac{\sigma^2}{2})\cdot T+\sigma\sqrt{T}\,(-z^*)}$$

50. Assume the Black-Scholes framework.

You are given the following information for a stock that pays dividends continuously at a rate proportional to its price.

(i) The current stock price is 0.25.

(ii) The stock's volatility is 0.35. $= \sigma$

(iii) The continuously compounded expected rate of stock-price appreciation is 15%.

$\alpha - \delta$

Calculate the upper limit of the 90% lognormal confidence interval for the price of the stock in 6 months.

$z^* = N^{-1}(0.95) = 1.645$

(A) 0.393

(B) 0.425

(C) 0.451

(D) 0.486

(E) 0.529

$S_U = 0.25e^{(0.15 - \frac{0.35^2}{2})\cdot(\frac{1}{2}) + 0.35\sqrt{\frac{1}{2}}\cdot(1.645)}$

$S_U = 0.393 \Rightarrow$ (A) ■

51-53. DELETED

54. Assume the Black-Scholes framework. Consider two nondividend-paying stocks whose time-t prices are denoted by $S_1(t)$ and $S_2(t)$, respectively.

You are given:

(i) $S_1(0) = 10$ and $S_2(0) = 20$.

(ii) Stock 1's volatility is 0.18.

(iii) Stock 2's volatility is 0.25.

(iv) The correlation between the continuously compounded returns of the two stocks is –0.40.

(v) The continuously compounded risk-free interest rate is 5%.

(vi) A one-year European option with payoff $\max\{\min[2S_1(1), S_2(1)] - 17, 0\}$ has a current (time-0) price of 1.632.

Consider a European option that gives its holder the right to sell either two shares of Stock 1 or one share of Stock 2 at a price of 17 one year from now.

Calculate the current (time-0) price of this option.

35) You own a share of a nondividend-paying stock and will hold it for a period of time. You want to set aside an amount of capital as a percentage of the initial stock price to reduce the risk of loss at the end of the holding period.

w/out earning interest.

You are given:

i) The stock price follows a lognormal distribution.
ii) The annualized expected rate of return on the stock is 15%. $\alpha = 0.15$
iii) The annualized stock volatility is 40%. $\sigma = 0.40$
iv) The investment period is 4 years. $T = 4$
v) The Value-at Risk (VaR) at the 3rd percentile for the capital plus the ending stock value equals the initial stock price.

Calculate the capital amount as a percentage of initial stock price.

(A) 57%

(B) 63%

(C) 71%

(D) 82%

(E) 91%

C... the amount of capital set aside

Given that

$$VaR_{0.03}(S(T)+\underset{\uparrow \atop \varphi \cdot S(0)}{C}) = S(0)$$

By def'n:

$$\mathbb{P}[S(T)+\varphi \cdot S(0) \leq S(0)] = 0.03$$

$$\mathbb{P}[S(T) \leq S(0)(1-\varphi)] = 0.03$$

Find $z^* = N^{-1}(0.03) = -N^{-1}(0.97) = -1.88$

$$\Rightarrow 1-\varphi = e^{(0.15-\frac{0.16}{2})\cdot 4 + 0.4\sqrt{4}(-1.88)}$$

$$\Rightarrow \varphi = 1-e^{-1.224} \quad \Rightarrow \varphi = 0.7059 \approx 71\%$$

$\Rightarrow$ (C) ■