Properties.

- $\boxed{p_X(x) = \underline{F_X(x) - F_X(x-)}}$
- $\sum_{x \in \mathbb{R}} p_X(x) = \underline{1}$

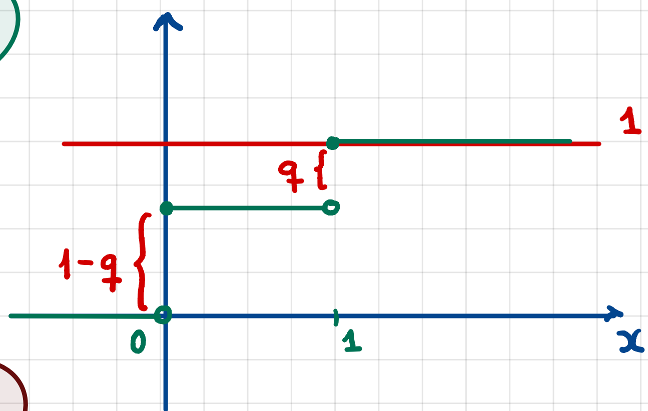
Example.

- **Bernoulli.**

Support $= \{0, 1\}$

$$X \sim \begin{cases} 0 & \text{w/ probability } 1-q \\ 1 & \text{w/ probability } q \end{cases}$$

cdf



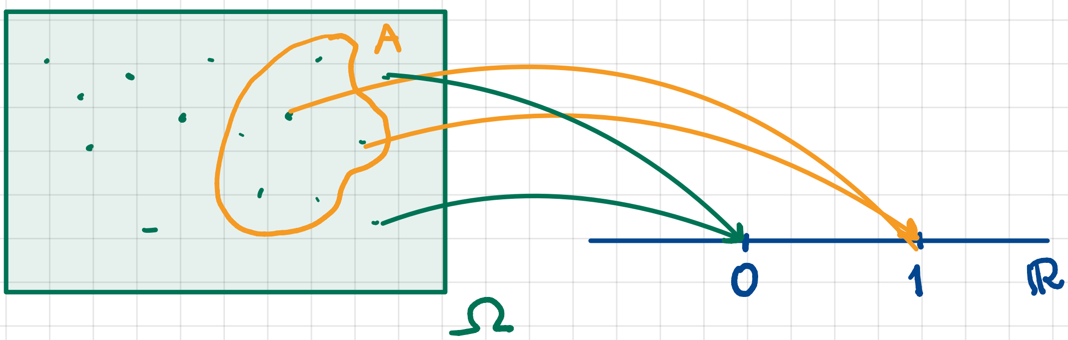
Step Function

pmf

$p_X(0) = 1-q$

$p_X(1) = q$

- **Indicator Random Variable.**



$$\mathbb{I}_A = \begin{cases} 1 & \text{if A happened} \\ 0 & \text{if A did not happen} \end{cases}$$

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A \sim \begin{cases} 1 & \text{w/ probability } \mathbb{P}[A] \\ 0 & \text{w/ } \quad -\|- \quad 1-\mathbb{P}[A] \end{cases}$$

Special Case.

In the special case where the support is (contained in) $\boxed{\{0,1,2,\dots\}}$, we say that the random variable is $\boxed{\mathbb{N}_0\text{-valued}}$

Then, it's convenient to write the pmf as a sequence.

Problem. We model the number of accidents N in a particular year so that we assume:

$$\boxed{p_N(n+1) = \frac{1}{5}\, p_N(n)} \quad \text{for all } n \geq 0$$

What is the probability that there is @ least one accident in that year?

→: $\mathbb{P}[\text{@ least one accident}] = 1 - \mathbb{P}[\text{no accidents}]$

$$= 1 - \mathbb{P}[N=0]$$

$$= 1 - \boxed{p_N(0)}$$

From our recursive property:

$$p_N(n+1) = \frac{1}{5}\, p_N(n) = \frac{1}{5}\left(\frac{1}{5}\, p_N(n-1)\right) = \left(\frac{1}{5}\right)^2 \cdot p_N(n-1)$$

$$= \dots = \left(\frac{1}{5}\right)^{n+1} p_N(0)$$

The pmf sums up to 1. So,

$$p_N(0) + p_N(1) + \dots = 1$$

$$\sum_{n=0}^{+\infty} p_N(n) = 1$$

$$\sum_{n=0}^{+\infty} \left(\left(\frac{1}{5}\right)^n \cdot p_N(0) \right) = 1$$

$$p_N(0) \cdot \boxed{\sum_{n=0}^{+\infty} \left(\frac{1}{5}\right)^n} = 1$$

$$\sum_{n=0}^{+\infty} \left(\frac{1}{5}\right)^n = \frac{1}{1-\left(\frac{1}{5}\right)} = \frac{5}{4}$$

$$\Rightarrow \boxed{p_N(0) = \frac{4}{5}}$$

answer : $1 - \frac{4}{5} = \frac{1}{5}$

$\square$