Hyperplanes.

Consider the set of all the points $(x,y)\in\mathbb{R}^2$ which satisfy the equation

$$\boxed{a\cdot x + b\cdot y + d = 0}$$

w/ a, b, and d all scalars and @ least one of a and be is $\neq 0$.

$$\boxed{a^2+b^2>0}$$

Say that $b\neq 0$.
Then, we can rewrite the above as:

$$\boxed{y = -\frac{a}{b}x - \frac{d}{b}}$$

The eq'n we remember from childhood.

The vector form is obtained through $x \leftrightarrow t$

$$(x,y) = \left(t, -\frac{a}{b}t - \frac{d}{b}\right) =$$

$$= t\underbrace{\left(1, -\frac{a}{b}\right)}_{\vec{v}} + \underbrace{\left(0, -\frac{d}{b}\right)}_{\vec{p}}$$

Return to: $\boxed{ax + by + d = 0}$

Define: $\vec{n} = (a,b)$

We can now write: w/ $\vec{x} = (x,y)$

$$\boxed{\vec{n}\cdot\vec{x} + d = 0}$$

Say that $\vec{p} = (p_1, p_2)$ is a point on the line

$$\Rightarrow \vec{n}\cdot\vec{p} + d = 0 \Rightarrow \boxed{d = -\vec{n}\cdot\vec{p}}$$

$$\Rightarrow \vec{n}\cdot\vec{x} - \vec{n}\cdot\vec{p} = 0$$

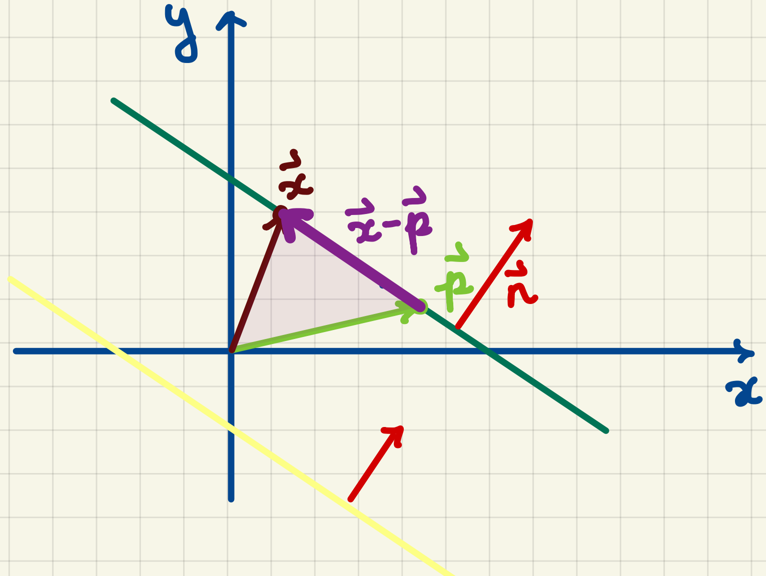
$$\Rightarrow \boxed{\vec{n}(\vec{x}-\vec{p}) = 0}$$

$\Rightarrow$ An equivalent condition for $\vec{x}$ being on the line is

$$\vec{n} \perp (\vec{x}-\vec{p})$$

THE NORMAL EQ'N

$\vec{n}$... normal vector



The hyperplane is the set of all the points $\vec{x} \in \mathbb{R}^2$ which satisfy the NORMAL EQUATION.

Now, we generalize to $\mathbb{R}^n$.

Def'n. Say that $\vec{n}$ and $\vec{p}$ are vectors in $\mathbb{R}^n$ w/ $\vec{n} \neq \vec{0}$

The set of all vectors $\vec{x}$ in $\mathbb{R}^n$ which satisfy the NORMAL EQUATION $\boxed{\vec{n}(\vec{x}-\vec{p})=0}$

is called a hyperplane through the point $\vec{p}$ normal to the vector $\vec{n}$.

More on Hyperplanes.

Example.

Suppose that L is line in $\mathbb{R}^2$ w/ the equation

$$\boxed{2x+3y=1.}$$

Then, a normal vector for L is $\vec{n}=(2,3)$.

We can easily find points on L: Say that $x=2 \Rightarrow y=-1$, i.e., the point $\boxed{\vec{p}=(2,-1)}$ is on L.

As a normal equation, all the points (x,y) on L must satisfy

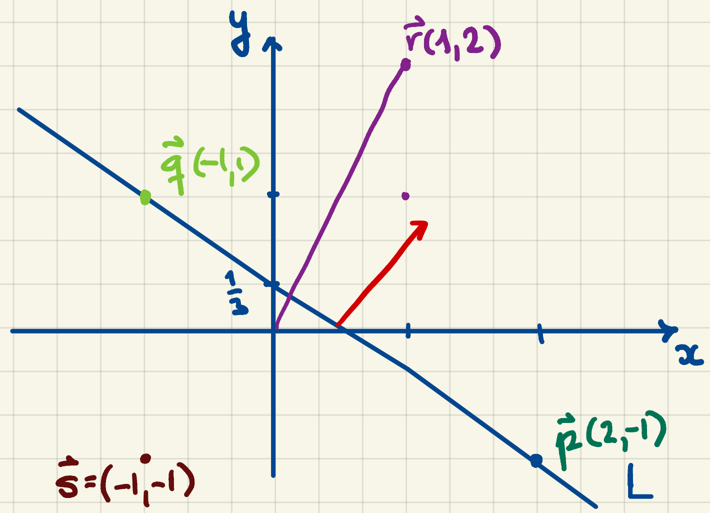
$$\boxed{\vec{n}\cdot(\vec{x}-\vec{p})=0} \qquad \left.\begin{aligned}(2,3)\cdot((x,y)-(2,-1))=0\\ \Longleftrightarrow \\ (2,3)\cdot(x-2,y+1)=0\end{aligned}\right\}$$

Let's find another point on L. Say, we denote it by $\boxed{\vec{q}=(q_1,q_2)}$

Pick $\boxed{q_1=-1 \Rightarrow q_2=1}$

We can check the normal equation: $(2,3)\cdot(-1-2,1+1) \stackrel{?}{=} 0$

$$(2,3)\cdot(-3,2) \stackrel{?}{=} 0$$
$$2(-3)+3\cdot 2 = 0$$



$$2x+3y=1$$
$$3y=-2x+1$$
$$y=-\frac{2}{3}x+\frac{1}{3}$$

What do we get for $\vec{r}=(1,2)$?

$$2(1)+3(2)-1=2+6-1=7>0$$

What do we get for $\vec{s}=(-1,-1)$?

$$2(-1)+3(-1)-1=-2-3-1=-6<0$$

Example. Find a point $\vec{p}$ on the plane $x+y-2z=6$ which lies closest to the origin.

$\longrightarrow$:

Q: Why is this a constrained optimization problem?

$\longrightarrow$: Function we're trying to minimize

$$\tilde{D}(x,y,z) = x^2+y^2+z^2$$

subject to: $x+y-2z=6$.

In general