M339J *Probability models*
University of Texas at Austin
HW #7
Instructor: Milica Čudina

Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 7.1. (5 points) The number of cars one sees passing by the local playground in an afternoon is modeled using a Poisson distribution with mean 25. The proportion of black cars in the stream is 1/5. The color of the cars is independent of the number of cars that drive by. What is the probability that exactly 5 black cars and exactly 10 non-black cars drive by in a particular afternoon?

Problem 7.2. (5 points) Let the number of customers N who walk into Hooper's store on a given day be Poisson with variance 30. The probability that a particular customer is a monster is 2/3. The number of customers is independent from whether they are monsters or not.

What is the probability that the total number of customers in a particular day is 25, **given** that the number of monster-customers equals 12?

Problem 7.3. (5 points) In a large population, the purple party and the mauve party are facing off in a two-party election. You are surveying people exiting from a polling booth and asking them if they voted purple. The probability that a randomly chosen person voted purple is 20%. What is the probability that exactly 15 people must be asked before you can find exactly 5 people who voted purple?

Problem 7.4. (20 points) A (6-sided) die is thrown and the number shown is written down (call it X). After that, a biased coin with the probability of *heads* equal to $1/(X+1)$ is tossed until the first *heads* appears.

i. (10 points) Compute the probability mass function for, as well as the expected of, the number of tosses.
ii. (10 points) Suppose that the number of tosses it took to get *heads* was observed, and it turned out to be equal to 5. The number on the die, on the other hand, is not known. What is the most likely number on the die?

Problem 7.5. (10 points) You repeatedly and independently spin a uniform spinner with three colored regions: yellow, blue, and red. After 1800 spins, you tally the number of times your spinner landed on red.

Independently from the spinner, you randomly extract balls from an urn containing 7 teal and 21 beige balls with replacement. After 1000 extractions, you tally the number of times a teal ball was extracted.

What is the variance of the difference between the number of times your spinner landed on red and the number of times a teal ball was extracted?

Problem 7.6. (5 points) *Source: Prof. Jim Daniel (personal communication).*
Let the random variable N be in the $(a, b, 0)$ class with $a = b = 3/4$. Find $Var[N]$.