M339W: March 7th, 2022.

Option Elasticity.

Def'n. For any portfolio w/ value function $v(s,t)$, its **portfolio elasticity** is defined as:

$$\boxed{\Omega(s,t) := \frac{\Delta(s,t)\cdot s}{v(s,t)}}$$

In particular, if your portfolio consists of a single option, it's called **option elasticity**.

Example. A European Call.

Its Black-Scholes price:

$$v_C(s,t) = \underbrace{se^{-\delta(T-t)}\cdot N(d_1(s,t))}_{\Delta_C(s,t)} - Ke^{-r(T-t)}\cdot N(d_2(s,t))$$

$\Rightarrow$

$$\Omega_C(s,t) = \frac{\boxed{\Delta_C(s,t)}\cdot s}{s\cdot\boxed{\Delta_C(s,t)} - Ke^{-r(T-t)}N(d_2(s,t))} \geq 1$$

Example. A European Put.

Its BS price:

$$v_P(s,t) = Ke^{-r(T-t)}N(-d_2(s,t)) - se^{-\delta(T-t)}N(-d_1(s,t))$$

$$\Delta_P(s,t) = -e^{-\delta(T-t)}N(-d_1(s,t)) < 0$$

$\Rightarrow$

$$\Omega_P(s,t) = \frac{\Delta_P(s,t)\cdot s}{Ke^{-r(T-t)}N(-d_2(s,t)) + \Delta_P(s,t)\cdot s} < 0$$

Use for option elasticity:

σ_S ... stock volatility (in the B·S model: constant, deterministic)

We get the option volatility as

$$\boxed{\sigma_{opt}(s,t) = \sigma_S \cdot \left| \Omega_{opt}(s,t) \right|}$$

While σ_S is constant, the option volatility is NOT.

e.g., for a European call:

$$\sigma_c(s,t) = \sigma_S \underbrace{\left| \Omega_c(s,t) \right|}_{\geq 1} \geq \sigma_S$$

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

$\Omega_p(S(0),0) = ?$

$$\Omega_p(S(0),0) = \frac{\Delta_p(S(0),0)\,S(0)}{\boxed{V_p(S(0),0)}}$$

by def'n

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

21-24. DELETED

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of \$100.

The chooser option price is \$20 at time $t = 0$.

The stock price is \$95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of \$100.

You are given:

(i) The risk-free interest rate is 0.

(ii) $C(1) = \$4$.

Determine $C(3)$.

(A) \$ 9

(B) \$11

(C) \$13

(D) \$15

(E) \$17

$\longrightarrow$:

Investor A: $\quad v_A(s,t) = 2\cdot v_c(s,t) + v_p(s,t)$

$$\frac{\partial}{\partial s}\Big| \quad \Delta_A(s,t) = 2\cdot\Delta_c(s,t) + \Delta_p(s,t)$$

At time-0:

$$\cancel{5}\,9 = \frac{\Delta_A(S(0),0)\,S(0)}{v_A(S(0),0)} = \frac{(2\cdot\Delta_c(S(0),0) + \Delta_p(S(0),0))\cdot\cancel{45}\,9}{2\cdot(4.45)+1.9}$$

$$2\cdot\Delta_c(S(0),0) + \Delta_p(S(0),0) = \frac{10.8}{9} = 1.2 \quad \checkmark \quad (A)$$

Investor B: $\quad v_B(s,t) = 2\cdot v_c(s,t) - 3\cdot v_p(s,t)$

$$\frac{\partial}{\partial s}\Big| \quad \Delta_B(s,t) = 2\cdot\Delta_c(s,t) - 3\cdot\Delta_p(s,t)$$

At time-0:

$$2\cdot\Delta_c(S(0),0) - 3\cdot\Delta_p(S(0),0) = 3.4 \quad \checkmark \quad (B)$$

$$(A)-(B): \quad \Delta_p(S(0),0) + 3\Delta_p(S(0),0) = 1.2 - 3.4 = -2.2$$

$$4\,\Delta_p(S(0),0) = -2.2$$

$$\Delta_p(S(0),0) = -0.55$$

$$\Omega_p(S(0),0) = \frac{\Delta_p(S(0),0)\cdot S(0)}{v_p(S(0),0)} = \frac{(-0.55)\cdot 45}{1.9} = -13.0263$$