

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 4Probability. Forward contracts.

Please, provide your complete solutions to the following problems. Final answers only, without the justification, will earn zero points.

Problem 4.1. (10 points) Complete the following definition:

The two random variables X and Y are said to be **equal** if:

Solution: They are on the same probability space and

$$\mathbb{P}[X = Y] = 1,$$

i.e., X equals Y almost surely (with probability 1).

Problem 4.2. (10 points) Complete the following definition:

The two random variables X and Y are said to be **identically distributed** if:

Solution: They have the same cumulative distribution functions, i.e.,

$$F_X(x) = F_Y(x), \text{ for all } x$$

with F_X being the cumulative distribution function of the random variable X and F_Y being the cumulative distribution function of the random variable Y .

Problem 4.3. (10 points) Provide an example of a pair of random variables which are identically distributed, but **not** equal.

Solution: Answers will vary. One example is:

- X is defined as:

$$X = \begin{cases} 1 & \text{if the result of a fair cointoss is heads} \\ 0 & \text{if the result of a fair cointoss is tails} \end{cases}$$

- Y is defined as:

$$Y = \begin{cases} 1 & \text{if the result of a roll of a fair die is a prime number} \\ 0 & \text{if the result of a roll of a fair die is not a prime number} \end{cases}$$

Problem 4.4. (2 points) Derivative securities can only be used for hedging, i.e., they can only be bought and written by agents who already have a position in the underlying asset. *True or false?*

Solution: FALSE

There is no such requirement due to the possibility of cash settlement.

Problem 4.5. (2 points) The profit diagram and the payoff diagram for long positions in a forward contract are identical. *True or false? Why?*

Solution: TRUE

The initial cost is equal to zero.

Problem 4.6. (2 points) In our usual notation, the difference between the **profit** of a long forward contract and a long investment in one unit of the non-dividend-paying underlying asset equals the forward price. *True or false?*

Solution: FALSE

$$S(T) - F - S(T) + S(0)e^{rT} = S(0)e^{rT} - F.$$

Of course, if the available forward price is the no-arbitrage forward price, the profits are equal.

Problem 4.7. (4 points) We all enter derivative-security-like contracts on a daily basis. For instance, ordering a pizza can be understood as a forward contract on the pizza. Ignore time-limits on when the pizza should be delivered. Imagine that the pizza is to be delivered in 30 minutes **exactly**. Explain why the pizza ordering is similar to a forward contract.

Solution: Solutions will vary.

Problem 4.8. (5 points) Maryam bakes batches of cupcakes for a cupcake convention. She buys forward 21 pounds of raspberries from a local farmer at the forward price of \$5.60 per pound.

She projects to bake 336 cupcakes and sell each for \$3. The total and aggregate non-raspberry costs of baking the cupcakes are \$200.

If the market price of raspberries on the day of the cupcake convention is \$5.40, what is Maryam's profit?

Solution:

$$336 \times 3 - 21 \times 5.60 - 200 = 690.40.$$

Problem 4.9. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded, risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

Solution:

$$200 \times 10 - 200 \times 5 - 500 - 200 \times 0.50e^{0.02} = 397.98$$