M339J : March 21^{st}, 2022.

not typical

$X \sim \text{Poisson}(\text{mean} = \lambda = 3)$

284. A risk has a loss amount that has a Poisson distribution with mean 3.

An insurance covers the risk with an ordinary deductible of 2. An alternative insurance replaces the deductible with coinsurance α, which is the proportion of the loss paid by the insurance, so that the expected insurance cost remains the same.

Calculate α.

$\longrightarrow$: With the ordinary deductible $d=2$, the expected insurance cost is

$$\mathbb{E}[(X-2)_+] = \underbrace{\mathbb{E}[X]}_{=3} - \mathbb{E}[X \wedge 2]$$

(A) 0.22

(B) 0.27

(C) 0.32

(D) 0.37

(E) 0.42

285. You are the producer for the television show Actuarial Idol. Each year, 1000 actuarial clubs audition for the show. The probability of a club being accepted is 0.20.

The number of members of an accepted club has a distribution with mean 20 and variance 20. Club acceptances and the numbers of club members are mutually independent.

Your annual budget for persons appearing on the show equals 10 times the expected number of persons plus 10 times the standard deviation of the number of persons.

Calculate your annual budget for persons appearing on the show.

(A) 42,600

(B) 44,200

(C) 45,800

(D) 47,400

(E) 49,000

$$X \wedge 2 \sim \begin{cases} 0 & \text{w/ probab.} \\ 1 & \text{w/ probab.} \\ 2 & \text{w/ probab.} \end{cases}$$

$p_X(0) = p_0 = e^{-3}$

$p_X(1) = p_1 = e^{-3} \cdot \frac{3}{1!} = 3e^{-3}$

$\mathbb{P}[X \geq 2] = 1 - p_0 - p_1$

$= 1 - 4e^{-3}$

$$\mathbb{E}[X \wedge 2] = 0 \cdot e^{-3} + 1(3e^{-3}) + 2(1 - 4e^{-3})$$

$$= \underline{2 - 5e^{-3}}$$

$\mathbb{E}[(X-2)_+] = 3 - (2 - 5e^{-3}) = 1.25$

With the second insurance policy:

$\mathbb{E}[\alpha \cdot X] = \alpha\, \mathbb{E}[X] = 3\alpha$

$3\alpha = 1.25$

$\boxed{\alpha = 0.42}$

130. Bob is a carnival operator of a game in which a player receives a prize worth $W = 2^N$ if the player has N successes, $N = 0, 1, 2, 3, \ldots$ Bob models the probability of success for a player as follows:

(i) N has a Poisson distribution with mean Λ.

(ii) Λ has a uniform distribution on the interval (0, 4).

} A mixing dist'n.

Calculate $E[W]$.

(A) 5

(B) 7

(C) 9

(D) 11

(E) 13

$$\begin{cases} N \mid \Lambda = \lambda \sim \text{Poisson}(\lambda) \\ \Lambda \sim U(0,4) \end{cases}$$

$$\mathbb{E}[W] = \mathbb{E}[2^N] = \cancel{?} = \mathbb{E}\Big[\underbrace{\mathbb{E}[2^N \mid \Lambda]}_{\text{A function of the random variable } \Lambda}\Big]$$

131. DELETED

132. DELETED

Focus on:

$\mathbb{E}[2^N \mid \Lambda]$ is a pgf of $N \mid \Lambda$

Using the STAM tables:

$$\mathbb{E}[2^N \mid \Lambda] = e^{\Lambda(2-1)} = e^{\Lambda}$$

$$\mathbb{E}[2^N] = \mathbb{E}[e^{\Lambda}] = (\Lambda \sim U(0,4))$$

$$= \int_0^4 \frac{1}{4} \cdot e^{\lambda} d\lambda = \frac{1}{4}\int_0^4 e^{\lambda} d\lambda = \frac{1}{4}(e^4 - 1) = 13.4$$

$\square$

Theorem.

Let $N_1, N_2, \dots, N_\ell$ be independent, Poisson random variables w/ parameters $\lambda_1, \lambda_2, \dots, \lambda_\ell$, resp.

Set $\boxed{N := N_1 + N_2 + \cdots + N_\ell}$ ✓

Then:

$$N \sim \text{Poisson}(\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_\ell)$$

$\longrightarrow$: Focus on the pgf of N.

$$\begin{aligned}
P_N(z) = \mathbb{E}[z^N] &= \mathbb{E}\left[z^{N_1 + N_2 + \cdots + N_\ell}\right] \\
&= \mathbb{E}\left[z^{N_1} \cdot z^{N_2} \cdot \ldots \cdot z^{N_\ell}\right] \quad \text{independence!} \\
&= \mathbb{E}[z^{N_1}] \cdot \mathbb{E}[z^{N_2}] \cdots \mathbb{E}[z^{N_\ell}] \\
&= P_{N_1}(z) \cdot P_{N_2}(z) \cdots P_{N_\ell}(z) \quad \text{Poisson} \\
&= e^{\lambda_1(z-1)} \cdot e^{\lambda_2(z-1)} \cdots e^{\lambda_\ell(z-1)} \\
&= e^{(\lambda_1 + \lambda_2 + \cdots + \lambda_\ell)(z-1)}
\end{aligned}$$

$\square$

172. A policyholder has probability 0.7 of having no claims, 0.2 of having exactly one claim, and 0.1 of having exactly two claims. Claim amounts are uniformly distributed on the interval [0, 60] and are independent. The insurer covers 100% of each claim.

Calculate the probability that the total benefit paid to the policyholder is 48 or less.

(A) 0.320
(B) 0.400
(C) 0.800
(D) 0.892
(E) 0.924

173. In a given region, the number of tornadoes in a one-week period is modeled by a Poisson distribution with mean 2. The numbers of tornadoes in different weeks are mutually independent.

Calculate the probability that fewer than four tornadoes occur in a three-week period.

(A) 0.13
(B) 0.15
(C) 0.29
(D) 0.43
(E) 0.86

→: N... # of tornadoes in the three-week period

$\boxed{\mathbb{P}[N<4]=?}$

$N = N_1 + N_2 + N_3$ w/ $N_i \sim \text{Poisson}(\lambda_i = 2)$ independent

$N \sim \text{Poisson}(\lambda = 3 \cdot 2 = 6)$

174. An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail.

Calculate the probability that the system will overheat.

(A) 0.007
(B) 0.045
(C) 0.098
(D) 0.135
(E) 0.143

$$\mathbb{P}[N \le 3] = p_0 + p_1 + p_2 + p_3$$
$$= e^{-6} + e^{-6} \cdot \frac{6^1}{1!} + e^{-6} \cdot \frac{6^2}{2!} + e^{-6} \cdot \frac{6^3}{3!}$$
$$= e^{-6} + 6e^{-6} + 18e^{-6} + 36e^{-6}$$
$$= 61e^{-6} \approx 0.1512 \quad \square$$

Think about:
Say that you have a model for the total count of events coming from several different categories. Say that the total count is Poisson. Imagine you know the proportions of events from different categories. What is the model for the count in any particular individual category?