Efficient Portfolios.

A portfolio is said to be efficient if there is no other portfolio w/ a higher expected return and smaller or equal volatility.

A portfolio is said to be not efficient if there exists another portfolio w/ a higher expected return @ a smaller or equal volatility.

6) You are given the following information about the four distinct portfolios:

Portfolio	Expected Return	Volatility
P	3%	10%
Q	5%	10%
R	5%	15%
S	7%	20%

Determine which two of the four given portfolios are NOT efficient.

$\boxed{P < Q}$

since they have the same volatility and Q has a higher expected return

(A) P and Q

(B) P and R

(C) P and S

(D) Q and R

(E) Q and S

$\boxed{R < Q}$

since they have the same expected return but R has a higher volatility

The Effect of Correlation.

- If $\rho = 1$, then the feasible set is a straight line between the two assets.
- The higher the correlation, the smaller the curvature of the feasible set.
- If $\boxed{\rho = -1}$, then ...

<u>Claim.</u> There is a weight w of asset #1 such that the resulting portfolio is risk-free, i.e., its volatility is zero.

$\longrightarrow$: $\mathrm{Var}\left[w \cdot R_1 + (1-w) R_2\right] = 0$

$$w^2 \cdot \mathrm{Var}[R_1] + (1-w)^2 \cdot \mathrm{Var}[R_2] + 2w(1-w) \cdot \mathrm{Cov}[R_1, R_2] = 0$$

$$w^2 \cdot \sigma_1^2 + (1-w)^2 \cdot \sigma_2^2 + 2w(1-w) \underbrace{\boxed{\rho}}_{=-1} \sigma_1 \cdot \sigma_2 = 0$$

$$w^2 \cdot \sigma_1^2 - 2w(1-w)\sigma_1 \cdot \sigma_2 + (1-w)^2 \cdot \sigma_2^2 = 0$$

$$(w\sigma_1 - (1-w)\sigma_2)^2 = 0$$

$$w\sigma_1 - (1-w)\sigma_2 = 0$$

$$w\sigma_1 - \sigma_2 + w\sigma_2 = 0$$

$$\boxed{w = \frac{\sigma_2}{\sigma_1 + \sigma_2}}$$

$\square$

Short sales w/ two assets.