M378K Introduction to Mathematical Statistics
Problem Set #15
Relative efficiency.

Definition 15.1. *Given two unbiased estimators $\hat{\theta}_1$ and $\hat{\theta}_2$, the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$ is defined as*

$$\mathit{eff}(\hat{\theta}_1, \hat{\theta}_2) = \frac{\mathrm{Var}[\hat{\theta}_2]}{\mathrm{Var}[\hat{\theta}_1]}\,.$$

Problem 15.1. *Let Y_1, Y_2 be a random sample from the exponential distribution with the unknown parameter θ.*

(i) The estimator $\hat{\theta}_1 = (Y_1 + Y_2)/2$ for θ is proposed. What is its variance?

(ii) The estimator $\hat{\theta}_2 = cY_{(1)}$ for θ is proposed. Find the constant c such that $\hat{\theta}_2$ is an unbiased estimator of θ. What is its variance?

(iii) Calculate the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$.