

M339D: April 2nd, 2021.

Options on Currencies.

→: underlying asset ... foreign currency (FC)

→: the price of the underlying asset:

the exchange rate, i.e.,

$x(t)$, $t \geq 0$ (worth of 1 unit of FC in terms of DC)

• r_D ... ccrfir for DC

• r_F ... ccrfir for FC

Repaid forward on a FC.

Repaid forward: pay $F_{0,T}^P(x)$
i.e., the prepaid forward price

T ... delivery date

receive 1 unit of FC

←
no arbitrage

Outright purchase: buy $e^{-r_F \cdot T}$ of FC
cost: $e^{-r_F \cdot T} \cdot x(0)$

own 1 unit of FC

$$\Rightarrow F_{0,T}^P(x) = x(0) e^{-r_F \cdot T} \quad \dots \text{DC-denominated}$$

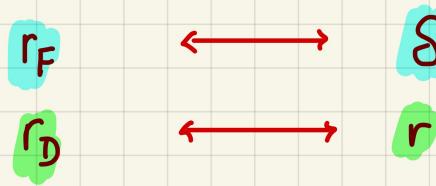
$$\Rightarrow F_{0,T}(x) = F_{0,T}(F_{0,T}^P(x)) = e^{r_D \cdot T} \cdot e^{-r_F \cdot T} \cdot x(0) \\ \Rightarrow F_{0,T}(x) = x(0) e^{(r_D - r_F) \cdot T}$$

Remember: For continuous dividend paying stocks:

$$F_{0,T}^P(S) = S(0) e^{-S \cdot T}$$

$$F_{0,T}(S) = S(0) e^{(r - S) \cdot T}$$

=> Analogy: exchange rate \longleftrightarrow price of cont. div. paying stock



European calls & puts

w/ strike price K & exercise date T

↑
DC · denominated ✓

Payoffs:

$$V_C(T) = (x(T) - K)_+$$

$$V_P(T) = (K - x(T))_+$$

Goal: Put-call parity for currency options!

Portfolio A:

- long call
- short put

} both on the foreign currency (FC) w/ strike K and exercise date T

Portfolio B:

- long prepaid forward on the FC w/ delivery @ time T
- borrow the $PV_{0,T}(K)$ @ the risk-free rate r_D to be repaid @ time T

$$\Rightarrow V_A(T) = x(T) - K \quad \boxed{=}$$

$$V_B(T) = x(T) - K \quad \boxed{=}$$

$$\Rightarrow V_C(0) - V_P(0) = x(0) e^{-r_F \cdot T} - K e^{-r_D \cdot T}$$

Put-Call Parity

Stock options

(vanilla calls/puts)

shares ↓ ↑ cash

$$\Rightarrow \underline{\text{Call}}: V_c(T) = (S(T) - K)_+$$

$$\underline{\text{Put}}: V_p(T) = (K - S(T))_+$$



Introduce a new option type:

stock#1 ↓ ↑ stock#2

FC ↓ ↑ DC

$$\Rightarrow \underline{\text{Call}}: V_c(T) = (x(T) - K)_+$$

$$\underline{\text{Put}}: V_p(T) = (K - x(T))_+$$



Exchange Options

... calls/puts where **BOTH** the underlying asset & the strike are **RISKY ASSETS.**

Notation: S and Q denote the two **risky assets**

For now: $\begin{cases} S \dots \text{the underlying asset} \\ Q \dots \text{the } \underline{\text{strike}} \text{ asset} \end{cases}$

Exchange call:

At time T , the owner of the call has a right but NOT an obligation

to RECEIVE 1 share of S

and GIVE UP 1 share of Q

$$\Rightarrow \underline{\text{The payoff}}: V_{EC}(T, S, Q) = (S(T) - Q(T))_+$$

↑ **strike asset**
underlying asset

Exchange put:

At time T , the owner of the put has a right but NOT an obligation

to GIVE UP 1 share of S

and RECEIVE 1 share of Q

$$\Rightarrow \text{The payoff: } V_{EP}(T, S, Q) = (Q(T) - S(T))_+$$

Thinking Task: Discover the special symmetry which exists for exchange options. :-)