UNIVERSITY OF TEXAS AT AUSTIN

# HW Assignment 12

## The last homework.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

**Problem 12.1.** (10 points) An investor invests $100,000$ in a contract with a *guaranteed minimum death benefit* and a *guaranteed minimum accumulation benefit*. The death-benefit guarantee will expire in three years and the accumulation-benefit guarantee stipulates that the minimum account value in three years is at least the original purchase price.

We assume that the death benefits are paid at the end of a year and that the surrenders happen during the year. Here are the expected counts of deaths and surrenders per 1000 annuitants which are expected to happen in the next three years:

| Year # | Deaths | Surrenders |
|---|---|---|
| 1 | 10 | 25 |
| 2 | 12 | 20 |
| 3 | 15 | 15 |

Let $V_P(0,T)$ denote the current price of a European put with strike $100,000$ and exercise date $T$. What is the expected combined value of the two guarantees in terms of $V_P(0,T)$ for $T = 1,2,3$?

**Solution:** The expected proportions of deaths in years $1, 2$, and $3$ are

$$\frac{10}{1000} = 0.01, \quad \frac{12}{1000} = 0.012, \quad \text{and} \quad \frac{15}{1000} = 0.015, \quad \text{respectively.}$$

So, the expected value of the GMDB is

$$0.01V_P(0,1) + 0.012V_P(0,2) + 0.015V_P(0,3).$$

All annuitants who neither die nor surender, are entitled to the GMAB. So, the expected proportion of recipients is

$$1 - \frac{10+25+12+20+15+15}{1000} = 0.903.$$

So, the expected value of the GMAB is $0.903V_C(0,3)$.

The combined value od the two guarantees is

$$0.01V_P(0,1) + 0.012V_P(0,2) + 0.918V_P(0,3).$$

**Problem 12.2.** (10 points) Consider a non-dividend-paying stock whose price $\mathbf{S} = \{S(t), t \geq 0\}$ is modeled using the Black-Scholes framework. Suppose that the current stock price equals \$100 and that its volatility is given to be 0.25.

The continuously compounded, risk-free interest rate is assumed to equal 0.04.

Consider a derivative security which entitles its owner to obtain a European call option on the above stock six months from today, i.e., at time $t_* = 1/2$. The call option is to be 3−month to expiration at time of delivery and have the strike equal to 105% of the time$-t^*$ price of the underlying asset. This contract is called a **forward start option**.

What is the price of the forward start option?

**Solution:** At time $t*$, the required Black-Scholes price of the call option equals

$$\begin{aligned} V_C(t^*) &= S(t^*)N(d_1) - 1.05S(t^*)e^{-r(T-t^*)}N(d_2) \\ &= S(t^*)(N(d_1) - 1.05e^{-0.01}N(d_2)) \end{aligned}$$

with

$$\begin{aligned} d_1 &= \frac{1}{0.125}\left[-\ln(1.05) + \left(0.04 - \frac{0.25^2}{2}\right) \times \frac{1}{4}\right] = -0.2478, \\ d_2 &= d_1 - \sigma\sqrt{T-t^*} = -0.2478 - 0.125 = -0.3728. \end{aligned}$$

So, $N(d_1) = 1 - N(0.25) = 1 - 0.5987 = 0.4013$ and $N(d_2) = 1 - N(0.37) = 1 - 0.6443 = 0.3557$ and Hence,

$$V_C(t^*) = S(t^*)(0.4013 - 1.05e^{-0.01} \times 0.3557) = S(t^*)0.31531.$$

So, one would need to buy exactly 0.031531 shares of stock to be able to buy the call option in question at time$-t^*$. This amount of shares costs \$3.1531.

**Problem 12.3.** (10 points) Assume CAPM. Assume that the risk-free interest rate equals 0.03 and that the market return equals 0.08. Consider a company which currently has 2 million shares outsanding with the stock price of \$20 per share. The equity rate is 0.09. The company has ten million dollars in debt with the debt beta equal to 0.4. The weighted average cost of capital is 0.0796. Calculate the capital tax rate.

**Solution:** The total amount of equity is $40 \times 10^6$. Since the total debt equals $10 \times 10^6$, we can conclude that

$$\frac{E}{E+D} = \frac{40 \times 10^6}{50 \times 10^6} = 0.8 \quad \text{and} \quad \frac{D}{E+D} = 0.2.$$

By CAPM, we know that

$$r_D = r_f + \beta_D(\mathbb{E}[R_{Mkt}] - r_f) = 0.03 + 0.4(0.08 - 0.03) = 0.05.$$

As we know, in general,

$$r_{wacc} = \left(\frac{E}{E+D}\right) r_E + \left(\frac{D}{E+D}\right) r_D(1 - \tau_C).$$

So,

$$0.0796 = 0.8(0.09) + 0.2(0.05)(1 - \tau_C) \quad \Rightarrow \quad 1 - \tau_C = \frac{0.0796 - 0.072}{0.01} = 0.76 \quad \Rightarrow \quad \tau_C = 0.24.$$

**Problem 12.4.** (10 points) Assume CAPM. This is the current capital structure of a particular company:

- The company has one million shares outstanding with the price of \$100 per share. The equity beta equals 1.2.
- The company has 50 million dollars in debt on which it pays 4% interest. The debt beta equals 0.2.

The company's earnings after interest has been paid equal ten million dollars.

There is a project the company's executives are thining about undertaking. The total investment for the project would be ten million dollars while the increase in the earnings would be two million dollars. The project would be financed with equity. Assuming that there is no effect on the debt beta, what is the new beta of equity?

**Solution:** We are told that $r_D = 0.04$. As for $r_E$, we are given that the total earnings are ten million dollars. The total value of the company's shares is 100 million dollars. So, $r_E = 0.10$. From the given information on the betas of equity and debt, we get the following system of equations:

$$\begin{aligned} 0.04 &= r_f + 0.2(\mathbb{E}[R_{Mkt}] - r_f) \\ 0.10 &= r_f + 1.2(\mathbb{E}[R_{Mkt}] - r_f) \end{aligned}$$

Subtracting the first equation from the second one, we get

$$0.06 = \mathbb{E}[R_{Mkt}] - r_f.$$

So, $r_f = 0.04 - 0.2(0.06) = 0.028$. Since the new project is financed with equity, the new total amount of equity equals 110 million dollars. The new earnings are 12 million dollars. Hence, the new return on equity equals $\tilde{r}_E = \frac{12}{110} = 0.1090909$. Again, we use CAPM to obtain the new beta of equity

$$\tilde{\beta}_E = \frac{\tilde{r}_E - r_f}{\mathbb{E}[R_{Mkt}] - r_f} = \frac{0.1090909 - 0.028}{0.06} = 1.351515.$$

**Problem 12.5.** (5 points) For stock $S_1$, you are given that its expected return equals 0.0494 and its $\beta$ is 0.32. For stock $S_2$, you are given that its expected return equals 0.0782 and its $\beta$ is 0.96. Both of these stocks lie on the *Security Market Line*. For stock $S_3$, you are given that its expected return equals 0.07 and its $\beta$ is 0.4. What is the $\alpha$ of stock $S_3$?

**Solution:** Since both $S_1$ and $S_2$ are on the **SML**, we know that

$$0.0494 = r_f + 0.32(r_m - r_f),$$
$$0.0782 = r_f + 0.96(r_m - r_f),$$

where $r_f$ stands for the risk-free interest rate and $r_m$ stands for the expected return of the market. Subtracting the first equation from the second one, we get

$$0.0288 = 0.64(r_m - r_f) \quad \Rightarrow \quad r_m - r_f = \frac{0.0288}{0.64} = 0.045.$$

The risk-free interest rate $r_f$ can, then, be calculated as

$$r_f = 0.0494 - 0.32(0.045) = 0.035.$$

Hence, the $\alpha$ of stock $S_3$ is

$$0.07 - 0.035 - 0.4(0.045) = 0.017.$$

**Problem 12.6.** (5 points) Which of the following statements is correct?

(a) Overconfidence bias generally does not result in a systematic trading bias.

(b) Disposition effect is a trait of uninformed individuals overestimating the precision of their knowledge.

(c) In the semi-strong form of the efficient market theory, prices reflect all private information.

(d) The momentum strategy consists of investing the stocks with lower returns and shorting the stocks with higher returns.

(e) Herd behavior does not result in a systematic trading bias.

**Solution: (a)**