M339G: November 13th, 2024.

Lines. Planes. Hyperplanes.

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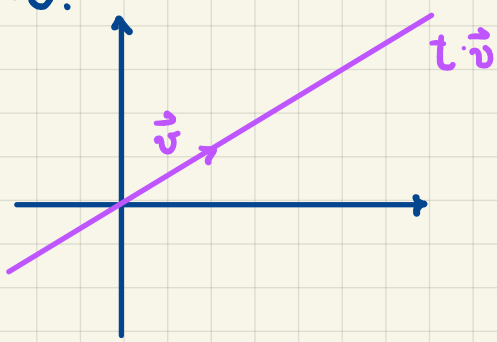
Lines in $\mathbb{R}^n$.

Start w/ $\vec{v}$, a non-zero vector in $\mathbb{R}^n$, i.e.,

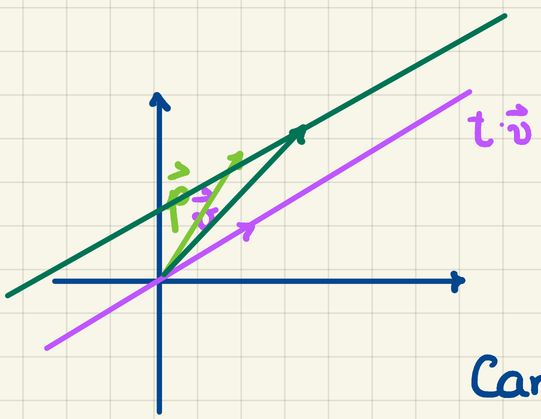
$$\vec{v} = (v_1, v_2, \dots, v_n)$$

For any scalar $t \in \mathbb{R}$, the vector $t \cdot \vec{v}$ will have:

- the same direction as $\vec{v}$ if $t > 0$,
- the opposite direction from $\vec{v}$ if $t < 0$,
- $\vec{0}$ if $t = 0$.



If I add a vector $\vec{p} \neq 0$, then I get a line shifted away from the origin.



$$\boxed{\left\{ t \cdot \vec{v} + \vec{p},\ -\infty < t < \infty \right\}}$$

VECTOR NOTATION

Can be expressed as parametric equations:

$$\boxed{\begin{aligned} y_1 &= t \cdot v_1 + p_1 \\ y_2 &= t \cdot v_2 + p_2 \\ &\vdots \\ y_n &= t \cdot v_n + p_n \end{aligned}}$$

Hyperplanes.

Consider a set of points (x,y) in $\mathbb{R}^2$ which satisfy the eq'n:

$$\boxed{a\cdot x + b\cdot y + d = 0} \quad \boxed{\star\star}$$

w/ a and b and d all scalars and @ least one of a and b different from zero.

Say that $b \neq 0$, then, we can rewrite the above as:

$$\boxed{y = -\frac{a}{b}\cdot x - \frac{d}{b}}$$

The eq'n we remember from childhood ☺

The vector form is obtained by setting $x \longleftrightarrow t$:

$$(x,y) = \left(t, -\frac{a}{b}t - \frac{d}{b}\right) = t\cdot\underbrace{\left(1, -\frac{a}{b}\right)}_{\vec{v}} + \underbrace{\left(0, -\frac{d}{b}\right)}_{\vec{p}}$$

Return to: $\boxed{ax + by + d = 0}$ $\star\star$

Define: $\boxed{\vec{n} = (a, b)}$

We can now write $\boxed{\vec{x} = (x,y)}$

$$\boxed{\vec{n}\cdot\vec{x} + d = 0}$$

Say that $\vec{p} = (p_1, p_2)$ is a point on this line.

$$\Rightarrow \vec{n}\cdot\vec{p} + d = 0 \quad \Rightarrow \boxed{d = -\vec{n}\vec{p}}$$

$$\Rightarrow \vec{n}\vec{x} - \vec{n}\vec{p} = 0$$

$$\Rightarrow \boxed{\vec{n}(\vec{x} - \vec{p}) = 0}$$

NORMAL EQUATIONS.