M339W: March 31st, 2021.

Option Elasticity

Def'n. For any portfolio w/ the value $v(\cdots) =: v(s,t)$

$$\boxed{\Omega(s,t) := \frac{\Delta(s,t)\cdot s}{v(s,t)}}$$

is called the portfolio elasticity.

In particular, if your portfolio consists of a one single option, it's called option elasticity.

Example. [A EUROPEAN CALL]

Its Black-Scholes price is:

$$v_C(s,t) = \boxed{\underbrace{s e^{-\delta(T-t)}\cdot N(d_1(s,t))}_{\Delta_C(s,t)}} - Ke^{-r(T-t)}\cdot N(d_2(s,t))$$

$$\Omega_C(s,t) = \frac{s\cdot\Delta_C(s,t)}{s\cdot\Delta_C(s,t) - Ke^{-r(T-t)}N(d_2(s,t))} \geq 1$$

Use: σ_S ... stock volatility

We define the option volatility as

$$\boxed{\sigma_{opt}(s,t) = \sigma_S\cdot\left|\Omega_{opt}(s,t)\right|}$$

e.g., for a European call:

$$\sigma_C(s,t) = \sigma_S\cdot\underbrace{\left|\Omega_C(s,t)\right|}_{\geq 1} \geq \sigma_S$$

★ While σ_S is a constant, the option volatility is a true f'tion of $(\cdots)$ ★

Example. [A EUROPEAN PUT]

$$v_P(s,t) = Ke^{-r(T-t)}N(-d_2(s,t)) - s\,e^{-\delta(T-t)}\cdot N(-d_1(s,t))$$

$$\Delta_P(s,t) = -e^{-\delta(T-t)}N(-d_1(s,t))$$

$$\Rightarrow \quad \Omega_P(s,t) = \frac{\Delta_P(s,t)\cdot s}{Ke^{-r(T-t)}N(-d_2(s,t)) + \Delta_P(s,t)\cdot s} < 0$$

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

$S(0)=45$, $v_c(S(0),0)=4.45$, $v_p(S(0),0) = 1.9$

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

$\Omega_A(S(0),0) = 5$ $\Delta_B(S(0),0) = 3.4$

Calculate the current put-option elasticity.

$\Omega_P(S(0),0) = ?$

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

21-24. DELETED

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of $100.

The chooser option price is $20 at time $t = 0$.

The stock price is $95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of $100.

You are given:

(i) The risk-free interest rate is 0.

(ii) $C(1) = \$4$.

Determine $C(3)$.

(A) $ 9

(B) $11

(C) $13

(D) $15

(E) $17

$\longrightarrow$: Investor A: $\quad v_A(s,t) = 2\cdot v_C(s,t) + v_P(s,t)$

$$\frac{\partial}{\partial s}\Big| \qquad \Delta_A(s,t) = 2\cdot\Delta_C(s,t) + \Delta_P(s,t)$$

By its def'n:

$$\Omega_A(s,t) = \frac{\Delta_A(s,t)\cdot s}{v_A(s,t)}$$

$\Rightarrow$ At time 0:

$$5 = \Omega_A(S(0),0) = \frac{(2\Delta_C(S(0),0) + \Delta_P(S(0),0))\cdot 45}{2\cdot 4.45 + 1.9}$$

given in problem

$$\left(\boxed{2\cdot\Delta_C(S(0),0)} + \Delta_P(S(0),0)\right)\cdot\cancel{45}^{\cancel{9}\,1} = \cancel{5}^{1}\cdot(\cancel{10.80})^{1.2} \qquad (A)$$

Investor B: $\quad v_B(s,t) = 2\cdot v_C(s,t) - 3\cdot v_P(s,t)$

$$\frac{\partial}{\partial s}\Big| \qquad \Delta_B(s,t) = 2\cdot\Delta_C(s,t) - 3\cdot\Delta_P(s,t)$$

$\Rightarrow$ At time 0:

$$3.4 = \Delta_B(S(0),0) = \boxed{2\cdot\Delta_C(S(0),0)} - 3\,\Delta_P(S(0),0) \qquad (B)$$

given in problem

$(A)-(B) \quad \Rightarrow \quad 4\cdot\Delta_P(S(0),0) = 1.2 - 3.4 = -2.2$

$$\Rightarrow \quad \Delta_P(S(0),0) = \underline{-0.55}$$

$$\Omega_P(S(0),0) = \frac{-0.55\cdot 45}{1.9} = -13.03 \quad \checkmark$$

41. Assume the Black-Scholes framework. Consider a 1-year European contingent claim on a stock.

You are given:

(i) The time-0 stock price is 45.

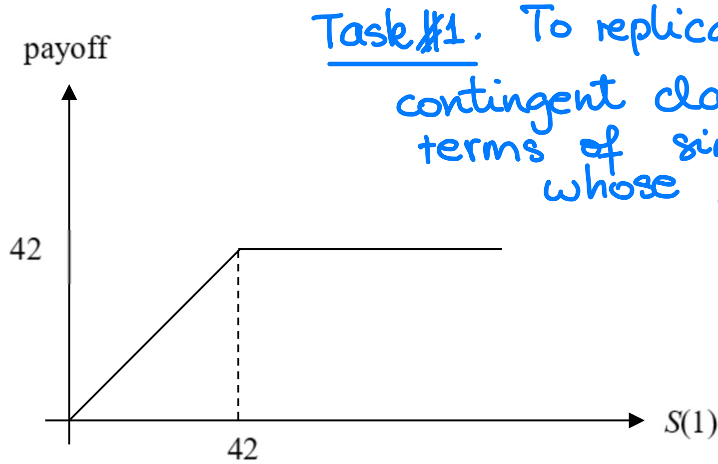
(ii) The stock's volatility is 25%.

(iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%.

(iv) The continuously compounded risk-free interest rate is 7%.

(v) The time-1 payoff of the contingent claim is as follows:

Task#1. To replicate this contingent claim (CC) in terms of simpler contracts whose Δ's we know :)



Think about this!

Calculate the time-0 contingent-claim elasticity.

(A) 0.24

(B) 0.29

(C) 0.34

(D) 0.39

(E) 0.44