

UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 10  
The  $\chi^2$ -distribution.

Provide your **complete solution** to the following problems.

**Problem 10.1.** (5 points) Let the random sample  $X_1, \dots, X_6$  be drawn from a normal distribution with mean 4 and variance 1. Define

$$Y = \sum_{i=1}^6 (X_i - 4)^2.$$

Using R, find the constant  $q$  such that

$$\mathbb{P}[Y \geq q] = 0.07.$$

**Problem 10.2.** (10 points) Let  $X_1, X_2, \dots, X_{11}$  be a simple random sample from a normal distribution with an **unknown** mean  $\mu$  and a known variance of 2. Let  $S^2$  denote the sample variance.

Using the  $\chi^2$ -distribution tables, find the constants  $a$  and  $b$  such that

$$\mathbb{P}[S^2 \leq a] = 0.025 \quad \text{and} \quad \mathbb{P}[S^2 \leq b] = 0.975.$$