M378K Introduction to Mathematical Statistics
Homework assignment #2

Please, provide your **final answer only** to the following problems.

Problem 2.1. *(5 points) Let X be a binomial random variable with parameters $n = 10$ and $p = 4/5$. Then*

(a) *If $Y = 2X$ then $\mathcal{S}_Y = \{0, 1, 2, 3, 4, \dots, 20\}$.*

(b) *If $Y = -X$ then Y is also binomial, but with parameters $n = 10$ and $p = 1 - \frac{4}{5} = \frac{1}{5}$.*

(c) *The support $\mathcal{S}_X$ of X is $10 \times \frac{4}{5} = 8$.*

(d) $\mathbb{P}[X] = 10 \times \frac{4}{5} = 8$.

(e) *None of the above.*

Problem 2.2. *(5 points) n people vote in a general election, with only two candidates running. The vote of person i is denoted by Y_i and it can take values 0 and 1, depending which candidate they voted for (we encode one of them as 0 and the other as 1). We assume that votes are independent of each other and that each person votes for candidate 1 with probability p. If the total number of votes for candidate 1 is denoted by Y, then*

(a) *Y is a geometric random variable*

(b) *Y^2 is a binomial random variable*

(c) *Y is uniform on $\{0, 1, \dots, n\}$*

(d) $\mathrm{Var}[Y] \leq \mathbb{E}[Y]$

(e) *None of the above.*

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 2.3. *(5 points) Let Y be a random variable such that*

$$\mathbb{P}[Y = 2] = 1/2, \mathbb{P}[Y = 3] = 1/3 \text{ and } \mathbb{P}[Y = 6] = 1/6.$$

How much is $\mathbb{E}[Y^2]$?

Problem 2.4. *(5 points) Let Y be a random variable such that*

$$\mathbb{P}[Y = 1] = 1/2, \mathbb{P}[Y = 3] = 1/3 \text{ and } \mathbb{P}[Y = 6] = 1/6.$$

With $|\cdot|$ denoting the absolute value, find $\mathbb{E}[|Y - 2|]$.

Problem 2.5. *(5 points) Let X be a Poisson random variable with parameter $\lambda > 0$. Express $\mathbb{P}[X \geq 3]$ in terms of λ.*

Problem 2.6. *(10 points) The probability that Janet makes a free throw is 0.6. What is the probability that she will make at least 16 out of 23 (independent) throws? Write down the answer as a sum - no need to evaluate it.*

Problem 2.7. *(15 points) A mail lady has $l \in \mathbb{N}$ letters in her bag when she starts her shift and is scheduled to visit $n \in \mathbb{N}$ different households during her round. If each letter is equally likely to be addressed to any one of the n households, what is the expected number of households that will receive no letters?*

Note: It is quite possible that some households will receive more than 1 letter.