M339D: April 7th, 2021.

HW#6 : Problem #1.

$i = 0.08$

$K_1 = 35$ $\quad V_c(0,35) = 9.12$

$K_2 = 40$ $\quad V_c(0,40) = 6.22$ $\quad T = 1$

$K_3 = 45$ $\quad V_c(0,45) = 4.08$

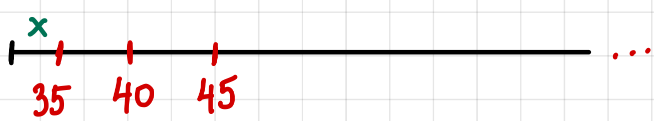
{ 45-strike has a higher profit than 40-strike
but lower profit than 35-strike }

Profit = Payoff − $FV_{0,T}$(Init Cost)

$\Rightarrow$ for a call:

$$(s-K)_+ - V_c(0,K)(1+i) \quad \text{for every strike } K.$$

$$(s-40)_+ - \overbrace{6.22\,(1.08)}^{6.72} < (s-45)_+ - \overbrace{4.08\,(1.08)}^{4.41} < (s-35)_+ - \overbrace{9.12\,(1.08)}^{9.85}$$



possible final stock prices

Case 0: $s < 35$ trivial

Case 1: $35 < s \leq 40$

Case 2: $40 < s \leq 45$

Case 3: $45 < s$ trivial

$$(s-35)_+ - 9.85 = s - 35 - 9.85$$

$$(s-45)_+ - 4.41 = -4.41$$

$$-4.41 < s - 44.85$$

$$40.44 < s$$

HW#6 : Problem #4:

$$S(T) \sim \begin{cases} S_u & \text{w/ probab. } 1/2 \\ S_d & \text{— '' —} \end{cases}$$

$S(0)$

Expected profit of a put:

$$\mathbb{E}[\text{Profit}] = \mathbb{E}[\text{Payoff} - FV_{0,T}(\text{Init. Cost})]$$
$$= \mathbb{E}[(K-S(T))_+] - FV_{0,T}(V_p(0))$$

put

$$(K-S(T))_+ \sim \begin{cases} (K-S_u)_+ & \text{w/ probab } 1/2 \\ (K-S_d)_+ & \text{w/ probab } 1/2 \end{cases}$$

$$\mathbb{E}[\text{Profit}] = (K-S_u)_+\left(\frac{1}{2}\right) + (K-S_d)_+\left(\frac{1}{2}\right) - FV_{0,T}(V_p(0))$$

Properties of Prices of European calls & Puts.

No Arbitrage

$\Downarrow$

Law of the Unique Price

So far (examples):

- Prepaid forward prices on stocks

$$F^P_{0,T}(S) = \begin{cases} S(0) & \text{no dividends} \\ S(0)e^{-\delta \cdot T} & \text{continuous div.} \\ S(0) - PV(Div) & \text{discrete div.} \end{cases}$$

- Put-Call Parity

$$V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K)$$

Equalities

Next task ☺

Inequalities involving call & put prices:

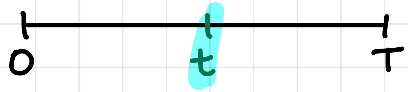
- bounds
- consider the call/put prices as functions of the strike K

monotonicity | "slope" bounds | convexity

Bounds on call/put prices.

T... exercise date

K... strike price



$S(t)$... time t stock price

$V_c(t)$... time t call price

$V_p(t)$... time t put price

Calls.

Lower bounds: $V_c(t) \geq 0$

By put-call parity:

$$V_c(t) - \underbrace{V_p(t)}_{\geq 0} = F^P_{t,T}(S) - PV_{t,T}(K)$$

$$V_c(t) \geq F^P_{t,T}(S) - PV_{t,T}(K)$$

$$\Rightarrow \boxed{V_c(t) \geq \max\left(F^P_{t,T}(S) - PV_{t,T}(K), 0\right)}$$

Q: Are both terms "meaningful"?

Q: What would you do if the above inequality is observed not to be true in a particular problem?