

UNIVERSITY OF TEXAS AT AUSTIN

Lecture 9

Introduction to Derivative Securities

9.1. What are derivative securities? Consider an asset, such as shares of stock, or ounces of gold, or barrels of oil, or bonds, or currencies...The important thing is that this asset is traded so that its price is readily available. We will call this asset the *underlying asset*. Typically, the worth, i.e., the *price* of this asset will be denoted by $S = \{S(t) : t \geq 0\}$. We can agree on the notation, but, what kind of an object is S ?

It is useful for S to be:

- time-dependent,
- not necessarily deterministic.

This type of a mathematical object is called a *stochastic process*. One can think about it as a random function or a random trajectory. We will not get into technical intricacies too much, but it is a good idea to be aware of this set-up from the start.

Now that we have established the structure of our underlying assets, it will be easier to talk about what the value (worth, price) of derivatives on those underlying assets will be. Since the price of the underlying asset changes as time goes by (possibly in a random fashion), it is evident that the value of the derivative on that asset will do the same. Since **at first** we will not be concerned with the evolution of the derivative price as time goes by, we will just focus on a certain time-horizon T at which the said derivative *expires*. The worth of the derivative at that time is called the *payoff* of the derivative and we will denote it by $V(T)$. The definition of the derivative above means that $V(T)$ is a function of the observed values of $S(t)$ between times 0 and T .

More precisely, we approach the payoff generally, and write $V(T) = v(S(t), 0 \leq t \leq T)$ where v is referred to as the *payoff function*. This level of generality will usually not be necessary as we will look at simpler derivative securities. Examples of derivatives we want to look at are options and forwards (futures).

We will define their payoff functions precisely as we introduce them and as we learn that we need them to create a more useful market-model. The function v will usually have the domain R_+ or R_+^n for some $n \in \mathbb{N}$. This brings us to our next point: the rationale for the introduction of derivatives.

9.2. Uses of derivatives.**9.2.1. Risk management.**

Example 9.1. Imagine that you are a farmer. You grow crops and have a good idea of the fixed costs for growing crops. However, when it comes to the price at which you will be able to sell your crops, you are not that confident. There are simply too many variables to take into consideration: the weather, the yields in faraway places, government subsidies, novel uses for the cultures you are growing, health fads, ...You would like to be able to protect yourself from the variability in crop prices stemming from *risks* outside of your control. In other words you would like to *hedge* against those risks; to purchase a sort of an insurance

policy. In this way, another party would “take over” a part of the financial risk to which you are exposed. To compensate them for that, you would, perhaps, have to pay a price.

Example 9.2. Consider an air carrier. It is plausible that the labor costs, maintenance costs and fees are more-or-less stable. It is also a nuisance to be changing the ticketing schemes and prices depending on a day-to-day basis. However, the oil prices are not very stable, and oil is needed to move the airplanes. This company would doubtlessly be interested in somehow reducing the variability in the future prices they would have to pay for oil. They would engage in a contract allowing them to pay, say, a predetermined price for a barrel of oil, as opposed to the market price.

There are countless examples such as the ones listed above. In the first example the “insurance contract” the farmer would get engaged in would have the final worth at the time of the harvest dependent on the actual observed price of the crops. In the second example, the worth of the contract depends on the observed price of a barrel of oil. The idea is clear, and this rationale for the existence of derivatives is sensible. We will spend a lot of time looking at derivatives precisely as risk management tools.

Example 9.3. Risk reduction can at times arrive in rather bizarre packages. Several years ago there were articles in the press about health-insurance companies trading in derivatives on the stocks of fast-food companies. You can fill in the blanks yourselves...

9.2.2. *Speculation.* Derivatives’ prices are contingent on the worth of another asset. More importantly, they can be traded regardless of whether the parties involved actually have an immediate exposure to risk in that asset. This means that they can be used as “bets” – not merely as risk management tools. In this sense, derivatives are a form of *speculation* on the market.

9.2.3. *Employee compensation.* A somewhat controversial practice of issuing call-option-like derivative securities as compensation (and stimulation) to employees has been present in practice for a while.

9.2.4. *Transaction cost reduction.* *Transaction costs* are defined as costs incurred by trading. They can be implemented in many different ways: fixed or per trade, proportional to the value or a fixed amount, etc. At times, derivatives (as a single contract) can incur smaller transaction costs than an equivalent set of multiple trades would. This is rare, but possible.

9.2.5. *Regulatory arbitrage.* Derivatives we will look into are simple, sensible and heavily traded. However, there may be novel contracts (even bespoke ones) that are complex and might be designed to benefit one of the parties more or to by-pass existing regulations. This will be of interest to you if you want to get into the field of forensic financial analysis. We will not focus on these financial instruments at all.