UNIVERSITY OF TEXAS AT AUSTIN

Lecture 9

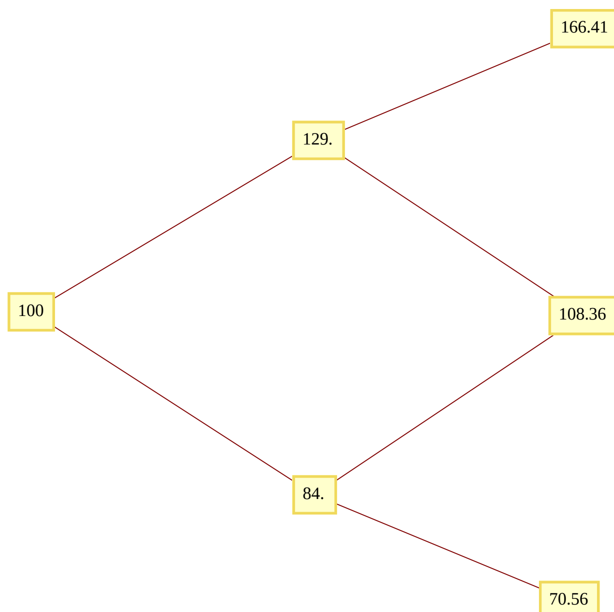
Example: Rebate option.

9.1. **Example.** Consider a non-dividend-paying stock whose initial price is $S(0) = 100$ and whose volatility is equal to $\sigma = 0.3$. We model the future evolution of this stock over the next year using a two-period forward binomial tree. Assume that the continuously compounded interest rate equals $r = 0.08$.

With the given parameters, the up and down factors equal

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{0.08/2+0.3/\sqrt{2}} = 1.29 \quad \text{and} \quad d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{0.08/2-0.3/\sqrt{2}} = 0.84.$$

So, we get the following tree:



A **rebate option** pays a rebate R at the expiration date T if the stock price ever reached the given level K during the time period $[0, T]$.

Let us use the above two-period binomial tree to calculate the price of a rebate option on the above stock with $T = 1, R = 20$ and $K = 110$. We intend to use the risk-neutral pricing formula, so we need the risk-neutral probability of the stock-price going up:

$$p^* = \frac{1}{1 + e^{\sigma\sqrt{h}}} = 0.447.$$

Even though the binomial tree is recombining, it is important to realize that the payoff of the rebate option depends on the path that the stock price takes through the tree. So, we have to look at the final payoff for the **trajectories** of the stock price. We get:

$$\begin{aligned} \text{up-up} &\Rightarrow R_{dd} = 20 \text{ with probability } (p^*)^2, \\ \text{up-down} &\Rightarrow R_{ud} = 20 \text{ with probability } p^*(1-p^*), \end{aligned}$$

down-up $\Rightarrow R_{du} = 0$ with probability $(1-p^*)p^*$,
down-down $\Rightarrow R_{dd} = 0$ with probability $(1-p^*)^2$.

The risk-neutral pricing principle gives us the initial value of the rebate option:

$$V_R(0) = e^{-0.08} \cdot 20 \cdot 0.0447 = 8.25.$$

Of course, we could have just moved throught the finished tree from left to right and simply find the probability of finding the first node above the level K. In our problem above, this happens after the first "coin-toss", i.e., if the stock price goes up it crosses 110 if it goes down, it never will. Convince yourselves that this is a correct procedure!