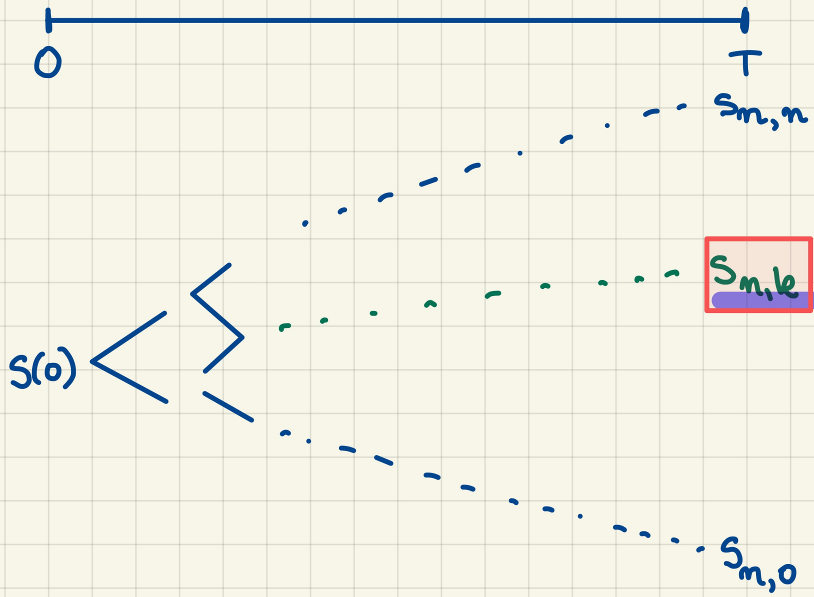
Multiple Binomial Periods.

T... exercise date of a European option
n... # of periods

} the length of each period $\boxed{h = \frac{T}{n}}$



$\boxed{s_{n,k}}$ ... k = # of steps up

$\boxed{\text{These } (n+1) \text{ values are the support of the r.v. } S(T).}$

$\Rightarrow$ for every $k = 0, 1, \dots, n$,

# of upsteps

$$s_{n,k} = S(0)\, u^k \cdot d^{n-k} = S(0) \cdot \left(\frac{u}{d}\right)^{k} \cdot d^n$$

Consider a European option w/ payoff f'tion $v(\cdot)$.

Then, the possible payoff values will be

$$\boxed{v_{n,k} = v(s_{n,k})}$$

p^*... the risk-neutral probability of an upstep

$\Rightarrow$ The risk-neutral probability of attaining the payoff $v_{n,k}$ is:

$$\binom{n}{k} (p^*)^k (1-p^*)^{n-k}$$

$\Rightarrow$ The risk-neutral option price:

$$\boxed{V(0) = e^{-rT} \sum_{k=0}^{n} \left(\binom{n}{k} (p^*)^k (1-p^*)^{n-k} \cdot v_{n,k} \right)}$$

Problem. Let the ccrfir be $\boxed{r = 0.10.}$ ✓
Let the initial price of a non-dividend-paying stock be \$100 per share.

$h = \frac{1}{5}$ You use a five-period binomial tree to model the stock price over the next year. Let $u = 1.04$ and $d = 0.96$.

What is the price of a one-year, $\boxed{\text{at-the-money}}$ European $\boxed{\text{call}}$ on the above stock?

$S(0) = K$ $\Rightarrow$ $\boxed{K = 100}$

$\longrightarrow$: Risk-neutral probability:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{0.10(1/5)} - 0.96}{1.04 - 0.96} = 0.7525$$

The relevant final stock prices in the tree are

$$S_{5,5} = S(0)\,u^5 = 100\,(1.04)^5 = 121.67 \Rightarrow v_{5,5} = 21.67$$

$$S_{5,4} = S(0)\,u^4 \cdot d = 100\,(1.04)^4 (0.96) = 112.31 \Rightarrow v_{5,4} = 12.31$$

$$S_{5,3} = S(0)\,u^3 \cdot d^2 = 100\,(1.04)^3 (0.96)^2 = 103.67 \Rightarrow v_{5,3} = 3.67$$

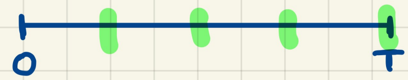
the remaining terminal nodes are all out-o-money.

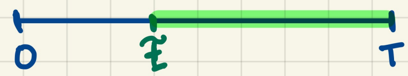
$$\Rightarrow V_C(0) = e^{-0.10}\Big(21.67\,(p^*)^5 + 12.31 \cdot 5 \cdot (p^*)^4 (1-p^*) + 3.67 \cdot \underbrace{\binom{5}{2}}_{=10} (p^*)^3 (1-p^*)^2 \Big) = \underline{10.0176} \qquad \square$$

Early exercise.

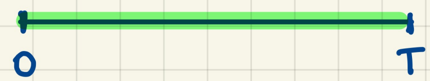
- European:
  no early exercise

- Bermudan:
  admissible early
  exercise on a subset of $[0,T]$





- American: allow early exercise
  for all $t \in [0,T]$



33.

Several years ago, John bought three separate 6-month options on the same stock.

- Option I was an American-style put with strike price 20.
- Option II was a Bermuda-style call with strike price 25, where exercise was allowed at any time following an initial 3-month period of call protection.
- Option III was a European-style put with strike price 30.

When the options were bought, the stock price was 20.

When the options expired, the stock price was 26.

The table below gives the maximum and minimum stock price during the 6 month period:

Time Period:	1st 3 months of Option Term	2nd 3 months of Option Term
Maximum Stock Price	24	28
Minimum Stock Price	18	22

John exercised each option at the optimal time.

Rank the three options, from highest to lowest payoff.

(A) I > II > III

(B) I > III > II

(C) II > I > III

(D) III > I > II

(E) III > II > I

I: Optimally exercised at the minimum stock price, i.e., at 18:

Payoff: $(20-18)_+ = 2$ ✓

II. Optimally exercised at the maximum stock price after call protection, i.e., @ 28:

Payoff: $(28-25)_+ = 3$ ✓

III. Payoff: $(30-26)_+ = 4$ ✓

34.

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4. For a stock, you are given:.

(i) The current stock price is $50.00.

(ii) $\delta = 0.08$

(iii) The continuously compounded risk-free interest rate is $r = 0.04$.

(iv) The prices for one-year European calls (C) under various strike prices (K) are shown below:

K	C
$40	$ 9.12
$50	$ 4.91
$60	$ 0.71
$70	$ 0.00

You own four special put options each with one of the strike prices listed in (iv). Each of these put options can only be exercised immediately or one year from now.

Determine the lowest strike price for which it is optimal to exercise these special put option(s) immediately.

(A) $40

(B) $50

(C) $60

(D) $70

(E) It is not optimal to exercise any of these put options.

The special put owner can

exercise the option NOW

⇒ value of immediate exercise: $K-50$

hold onto the option; it becomes a regular European put

?