Problem 6.3. Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000−cartons' worth in six months. His total costs are $\boxed{\$12.00}$ per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of \$13 per carton and another with the strike price of \$15 per carton. Their premiums are \$0.15 and \$0.18, respectvely.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals \$14. What would Farmer Shaun's profit be if he had decided to hedge using the \$13-strike put versus his profit if he had decided to use the \$15-strike put to hedge?

Focus on Payoff: (w/out production costs)

unhedged: $S(T)$

hedge: $(K-S(T))_+$

$+$

total hedged: $S(T)+(K-S(T))_+ = \begin{cases} K & \text{if } K>S(T) \\ S(T) & \text{if } K\leq S(T) \end{cases}$

$= \boxed{\max(K, S(T))}$

$\boxed{\text{FLOOR.}}$

\$13 Payoff: $\max(13,14)=14$

$14-12-0.13\cdot(1.04)^{1/2} = 1.867$

\$15 $15-12-0.18(1.04)^{1/2} = 2.847$

$\times 10\,000$

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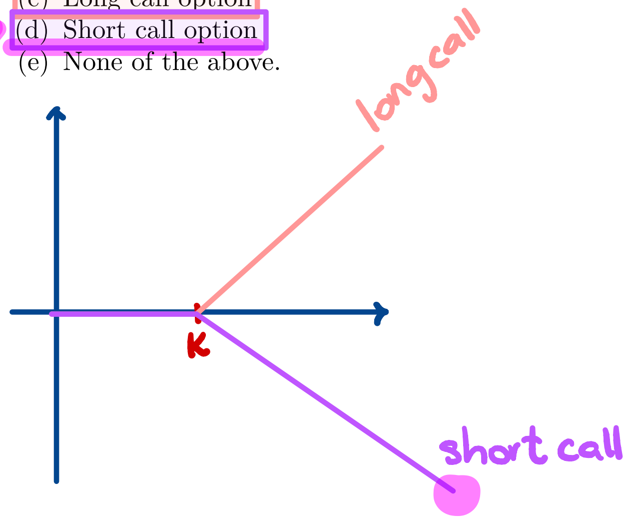
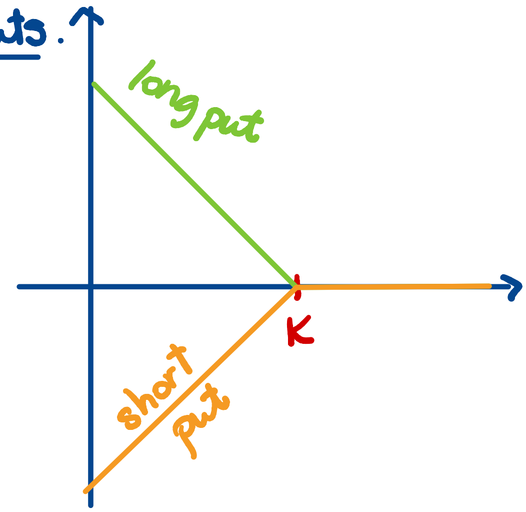
Problem 6.4. (2 points) In which of the following option positions is the investor exposed to an unlimited loss?

(a) Long put option
(b) Short put option
(c) Long call option
(d) Short call option
(e) None of the above.

Calls.



Puts.



Problem 6.5. (3 points) The initial price of the market index is \$1000. After 3 months the market index is priced at \$950. The nominal rate of interest convertible quarterly is 4.0%.
The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

(a) \$12.00 loss
(b) \$14.90 loss
(c) \$12.00 gain
(d) \$14.90 gain
(e) None of the above.

→ :

Payoff: $(K - S(T))_+ = (975 - 950)_+ = 25$

Profit: $25 - 10(1.01) = 14.90$

Problem 6.6. (3 points) *Source: Sample FM(DM) Problem #62.*

The stock price today equals \$100 and its price in one year is modeled by the following distribution:

$$S(1) \sim \begin{cases} 125 & \text{with probability } 1/2 \\ 60 & \text{with probability } 1/2 \end{cases}$$

The annual effective interest rate equals 3%.

Consider an at-the-money, one-year European put option on the above stock whose initial premium is equal to \$7.

What is the expected profit of this put option?

→ : $\mathbb{E}[V_P(T)] = ?$

$$V_P(T) \sim \begin{cases} (100-125)_+ = 0 & \text{w/ probab. } 1/2 \\ (100-60)_+ = 40 & \text{w/ probab. } 1/2 \end{cases}$$

$$\mathbb{E}[V_P(T)] = 40 \cdot \frac{1}{2} = 20$$

answer: $20 - 7 \cdot (1.03) = 12.79$ $\square$

Problem 6.7. Aunt Dahlia simultaneously purchased

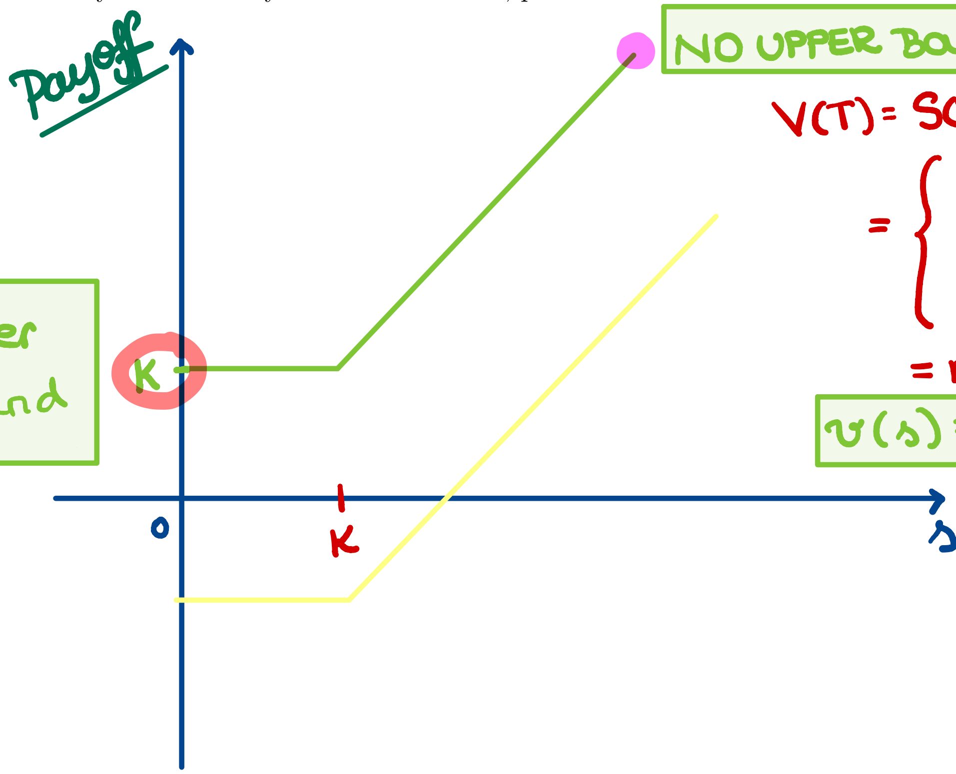
- one share of a market index at the current spot price of $1,000;
- one one-year, $1,050-strike put option on the above market index for the premium of $20.

Floor.

(i) (5 points) Is the above portfolio's payoff bounded from above? If you believe it is not, substantiate your claim. If you believe that it is, provide the upper bound.

(ii) (5 points) Is the above portfolio's payoff bounded from below? If you believe it is not, substantiate your claim. If you believe that it is, provide the lower bound.

NO UPPER BOUND.



Lower Bound

$$V(T) = S(T) + (K - S(T))_+$$

$$= \begin{cases} S(T) & \text{if } S(T) \geq K \\ K & \text{if } S(T) < K \end{cases}$$

$$= \max(S(T), K)$$

$$\boxed{v(s) = \max(s, K)}$$

Finite Probability Spaces.

... serve as environments for the possible paths that the asset price can take.

e.g.,

$$S(T) = \begin{cases} 120 & \text{w/ probab. } 1/6 \\ 80 & \text{w/ probab. } 1/2 \\ 50 & \text{w/ probab. } 1/3 \end{cases}$$

Q: What is the expected payoff of a 105-strike put on S?

$\longrightarrow$:

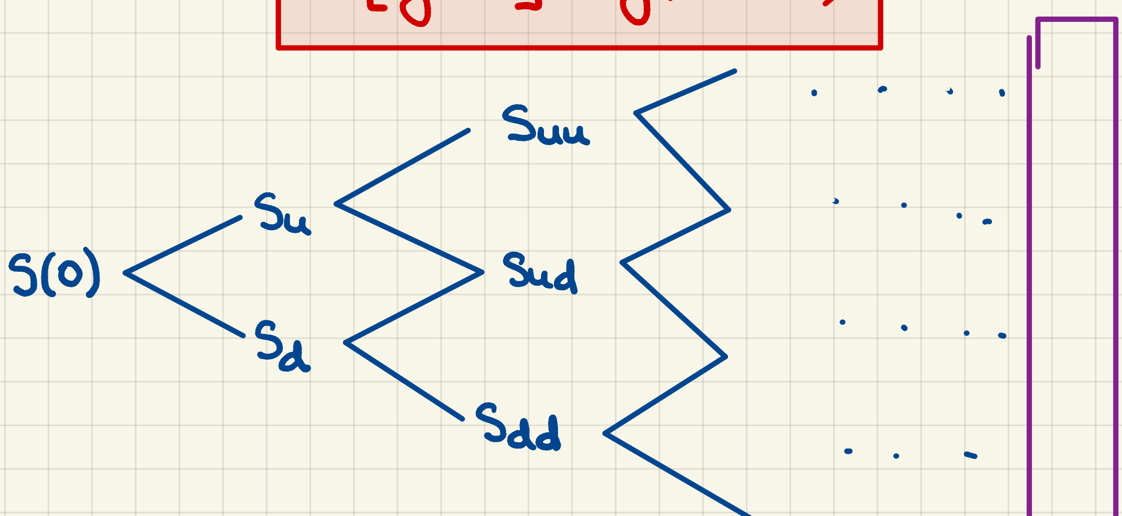
$$V_p(T) = \begin{cases} 0 & \text{w/ probab. } 1/6 \\ 25 & \text{w/ probab. } 1/2 \\ 55 & \text{w/ probab. } 1/3 \end{cases}$$

$$\mathbb{E}[V_p(T)] = 25 \cdot \left(\frac{1}{2}\right) + 55 \cdot \left(\frac{1}{3}\right) = \frac{185}{6}$$

In general:

$$\boxed{\mathbb{E}[g(X)] \neq g(\mathbb{E}[X])}$$

e.g.,



All the finitely many scenarios are called states of the world.

We assume that:

- each can happen, i.e., it has $\boxed{\text{probab} > 0}$

and

- they exhaust all possibilities, i.e., $\boxed{\sum \text{probab.} = 1}$

Arbitrage Portfolios.

Def'n. An arbitrage portfolio is a portfolio whose $\boxed{\text{profit}}$ is:

- nonnegative in all states of the world

and

- strictly positive in $\boxed{\text{at least one}}$ state of the world.

> Unless it is specified otherwise in a particular problem/example, we assume NO ARBITRAGE.