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Focus on the Delta.

value f'n: $v(s, t, r, \sigma)$

Def'n. The Delta $\Delta(s, t) := \frac{\partial}{\partial s} v(s, t)$

Example. Overnight Purchase of a Non-Dividend-Paying Stock.

$v(s, t) = s$
stands for the time- t stock price
 $\Rightarrow \Delta(s, t) = 1$

Example. European Call

$v_c(s, t) = s \cdot N(d_1(s, t)) - K e^{-r(T-t)} N(d_2(s, t))$

w/ $d_1 = \frac{1}{\sigma \sqrt{T-t}} \left[\ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right]$

and $d_2 = d_1 - \sigma \sqrt{T-t}$

By def'n: $\Delta_c(s, t) = \frac{\partial}{\partial s} v_c(s, t)$

After the chain rule and product rule

$\Delta_c(s, t) = N(d_1(s, t)) > 0$

The positivity makes sense since the call is
long w.r.t. the underlying.