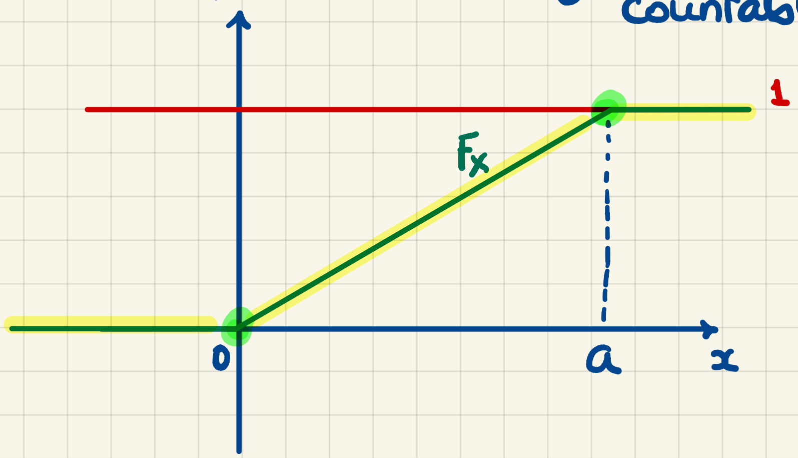
M358K : September 22nd, 2023.

Continuous Random Variables [Review].

Def'n. A random variable X is said to be continuous if its cumulative distribution f'tion F_X is:

(i) continuous everywhere;

(ii) differentiable everywhere except @ at most countably many points.

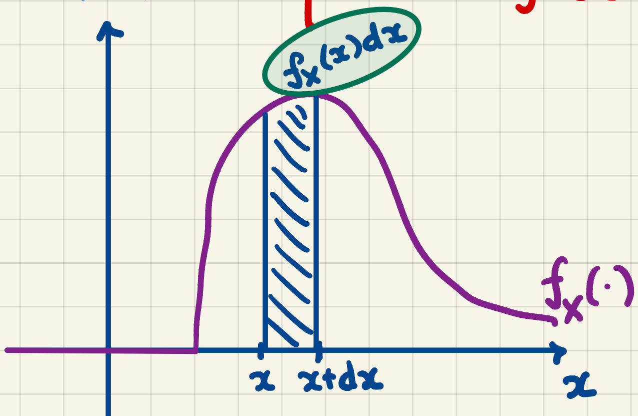


$X \sim Uniform(0, a)$

Def'n. Any function $f_X : \mathbb{R} \to [0, +\infty)$ such that

$$\boxed{f_X(x) := F_X'(x)}$$, for all x where the derivative exists,

is called the probability density function (pdf) of X.



Q: $$\mathbb{P}[a < X \leq b] = \int_a^b f_X(x)\,dx \overset{FTC}{=} F_X(b) - F_X(a)$$

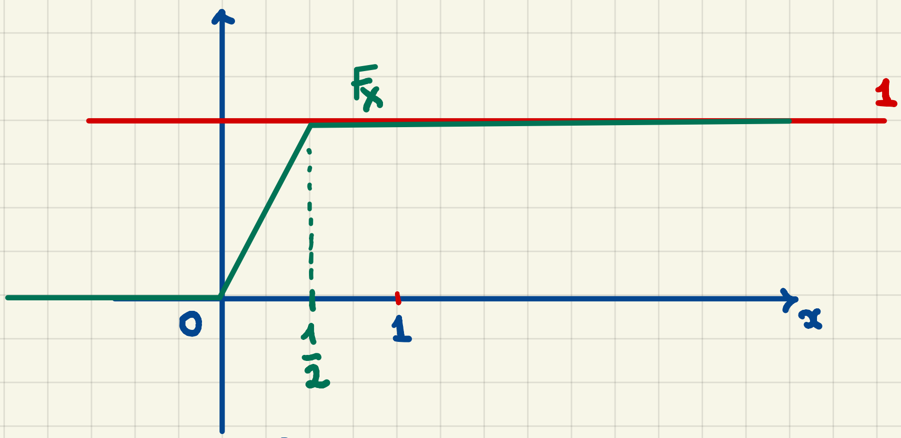
$$\mathbb{P}[a < X \leq b] = \mathbb{P}[X \leq b] - \mathbb{P}[X \leq a]$$

Q: X is continuous $\Rightarrow$ $\boxed{\mathbb{P}[X=x]=0}$

Q: $\boxed{\int_{-\infty}^{+\infty} f_X(x)\,dx = \underline{1}}$

Q: Is it possible for $\boxed{f_X(x) > 1}$ for some x? Yes.

Example. $\boxed{X \sim U(0,\frac{1}{2})}$ $\mathbb{E}[X] = \frac{1}{4}$



In general:

$Y \sim U(\alpha,\beta)$

$\mathbb{E}[Y] = \frac{\alpha+\beta}{2}$

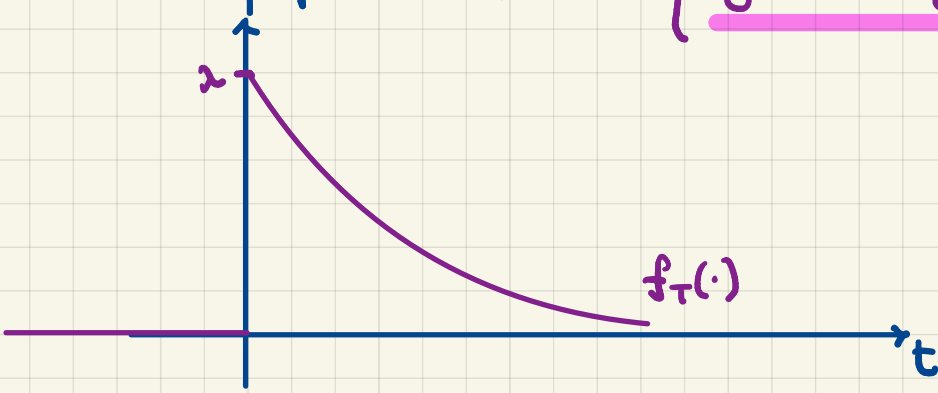
$$F_X(x) = \begin{cases} 0 & \text{for } x<0 \\ 2x & \text{for } x \in [0,\frac{1}{2}] \\ 1 & \text{for } x > \frac{1}{2} \end{cases}$$

$$f_X(x) = \begin{cases} 0 & \text{for } x<0 \\ \boxed{2} & \text{for } x \in (0,\frac{1}{2}) \\ 0 & \text{for } x > \frac{1}{2} \end{cases}$$

Example. Exponential Distribution. $T \sim Exp(\lambda)$ $\boxed{\mathbb{E}[T] = \frac{1}{\lambda}}$

w/ a positive parameter λ

Its pdf is: $f_T(t) = \begin{cases} \lambda e^{-\lambda t} & t>0 \\ 0 & \text{otherwise} \end{cases}$



Note: If $\lambda = 3.14$, then $f_T(t) \approx 3.14$ for t close to zero and $t>0$