Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
In-Term Exam III
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

Problem 3.1. (5 points) Which one of the following statements is **TRUE**?

(a) The payoff curve of a call bear spread is never positive.
(b) A straddle has a nonnegative profit function.
(c) A strangle can be replicated with a long put and a short call.
(d) The payoff of the call bull spread is equal to the payoff of the put bull spread.
(e) None of the other statements is TRUE.

Solution: (a)

Problem 3.2. (5 points) An investor buys a two-year (\$800, \$900)-strangle on gold. The price of gold two years from now is modeled using the following distribution:

$$\begin{aligned} &\$750, \quad \text{with probability } 0.45,\\ &\$850, \quad \text{with probability } 0.4,\\ &\$925, \quad \text{with probability } 0.15. \end{aligned}$$

What is the investor's expected payoff?

(a) About \$23.25
(b) About \$25.00
(c) About \$26.25
(d) About \$37.50
(e) None of the above.

Solution: (c)

$$50 \times 0.45 + 25 \times 0.15 = 26.25.$$

Problem 3.3. Let the continuously-compounded, risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$100 per share. The stock pays dividends continuously with a dividend yield of 0.02. In the model, it is assumed that the stock price can either go up by 3% or down by 4%.

You use the binomial tree to construct a replicating portfolio for a at-the-money, one-year European call on the above stock. What is the stock investment in the replicating portfolio?

(a) Long 0.4201 shares.
(b) Long 0.4286 shares.

(c) Short 0.4201 shares.
(d) Short 0.4286 shares.
(e) None of the above.

Solution: (a)
The two possible stock prices are $S_u = 103$ and $S_d = 96$. So, the possible payoffs of the call are $V_u = 3$ and $V_d = 0$. The Δ of the call, thus, equals

$$\Delta = e^{-\delta h}\frac{V_u - V_d}{S_u - S_d} = e^{-0.02}\frac{3-0}{103-96} = \tag{3.1}$$

Problem 3.4. (5 points) Let the continuously-compounded, risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is \$50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a \$48-strike, one-year European put on the above stock. What is the risk-free investment in the replicating portfolio?

(a) Borrow \$21
(b) Borrow \$20.18
(c) Lend \$20.18
(d) Lend \$21
(e) None of the above.

Solution: (c)
The two possible stock prices are $S_u = 52.5$ and $S_d = 45$. So, the possible payoffs of the put are $V_u = 0$ and $V_d = 3$. The risk-free investment B in the replicating portoflio of our put, thus, equals

$$B = e^{-rh}\frac{uV_d - dV_u}{u-d} = e^{-0.04}\frac{1.05(3) - 0.9(0)}{1.05 - 0.9} = \tag{3.2}$$

Problem 3.5. You are given that the price of:

- a \$50-strike, one-year European call equals \$8,
- a \$65-strike, one-year European call equals \$2.

Both options have the same underlying asset. What is the maximal price of a \$56-strike, one-year European call such that there is no arbitrage in our market model?

(a) \$4.40
(b) \$5
(c) \$5.60
(d) \$6.02
(e) None of the above.

Solution: (c)
Using the convexity of call price with respect to the strike, we get the following answer:

$$\frac{3}{5}\times 8 + \frac{2}{5}\times 2 = \frac{24+4}{5} = 5.60.$$

Problem 3.6. We are given the following European-call prices for options on the same underlying asset:

\$50-strike	\$11
\$55-strike	\$6
\$60-strike	\$4

Assume that the continuously-compounded, risk-free interest rate is strictly positive. Which of the following portfolios would exploit an arbitrage opportunity stemming from the above stock prices?

(a) The call bull spread only.
(b) The call bear spread only.
(c) Both the call bull and the call bear spread.
(d) Neither the call bull or call bear spread, but there is an arbitrage opportunity.
(e) There is no apparent arbitrage opportunity.

Solution: (b)

Problem 3.7. A portfolio consists of the following:

- one **short** one-year, $50-$strike call option with price equal to \$8.50,
- one **long** one-year, $60-$strike put option with price equal to \$6.75

All of the options are European and with the same underlying asset.
Assume that the continuously compounded, risk-free interest rate equals 0.04.
What is the portfolio's profit is the final price of the underlying asset equals \$55?

(a) 1.75
(b) 1.82
(c) 6.82
(d) 11.82
(e) None of the above.

Solution: (b)

$$-(55-50)_{+}+(60-55)_{+}+(8.50-6.75)e^{0.04}=1.82$$

Problem 3.8. (5 points) *Source: Sample FM(DM) Problem #8.*
You believe that the volatility of a stock is higher than indicated by market prices for options on that stock. You want to speculate on that belief by buying or selling at-the- money options. What should you do?

(a) Buy a straddle.
(b) Buy a strangle.
(c) Sell a straddle.
(d) Buy a butterfly spread.
(e) Sell a butterfly spread.

Solution: (a)
Since the options must be at-the-money, the strikes must be equal (and equal to the current stock price). So, the only contract which fits is a straddle. In order to have a positive profit for final stock prices that are far from the current stock price, it has to be long straddle.

Problem 3.9. We are given the following European-call prices for options on the same underlying asset:

\$50-strike	\$10
\$55-strike	\$6
\$60-strike	\$4

Assume that the continuously-compounded, risk-free interest rate is strictly positive. Which of the following portfolios would exploit an arbitrage opportunity stemming from the above stock prices?

(a) The call bear spread only.
(b) The call bull spread only.
(c) Both the call bull and the call bear spread.
(d) Neither the call bull or call bear spread, but there is an arbitrage opportunity.
(e) There is not enough information provided.

Solution: (e)

Problem 3.10. (5 points) Bertie constructs an asymmetric butterfly spread using call options with strikes $75, 78$ and 90. It is constructed using m of the (75, 78) bull spreads and n (78,90) bear spreads. How much is m/n?

(a) 4

(b) 2

(c) 1/2

(d) 1/4

(e) None of the above.

Solution: (a)
We have

$$\frac{m}{n} = \frac{\frac{90-78}{90-75}}{\frac{78-75}{90-75}} = 4.$$

Problem 3.11. (5 pts) Consider a non-dividend-paying stock currently priced at \$100 per share.

The price of this stock in one year is modeled using a one-period binomial tree under the assumption that the stock price can either go up to 110 or down to 90.

Let the continuously-compounded, risk-free interest rate equal 0.04. What is the risk-neutral probability of the stock price going up?

(a) About 0.2969
(b) About 0.3039
(c) About 0.5000
(d) About 0.7041

(e) None of the above.

Solution: (d)

$$p^* = \frac{100e^{0.04} - 90}{110 - 90} = 0.7041.$$

Problem 3.12. (5 points) The current stock price is 40 per share. The price at the end of a three-month period is modeled with a one-period binomial tree so that the stock price can either increase by \$5, or decrease by \$4. The stock pays dividends continuously with the dividend yield 0.04.

The continuously-compounded, risk-free interest rate is 0.10.

What is the price of a \$40-strike European **straddle** on the above stock?

(a) 4.40
(b) 3.30
(c) 2.20
(d) 1.10
(e) None of the above.

Solution: (a)
The risk-neutral probability is

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{S(0)e^{(0.10-0.04)(0.25)} - S_d}{S_u - S_d} = \frac{40e^{(0.06)(0.25)} - 36}{45 - 36} = 0.5116136.$$

The possible payoffs are $V_u = 5$ and $V_d = 4$. So,

$$V(0) = e^{-0.10/4}[5p^* + 4(1 - p^*)] = 4.400221$$