University of Texas at Austin

HW Assignment 8

Various positions. Binomial asset pricing.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 8.1. (5 points) Which one of the following statements is **TRUE**?

(a) The payoff curve of a call bear spread is never positive.

(b) A straddle has a nonnegative profit function.

(c) A strangle can be replicated with a long put and a short call.

(d) The payoff of the call bull spread is equal to the payoff of the put bull spread.

(e) None of the other statements is TRUE.

Solution: (a)

Problem 8.2. Let the continuously compounded, risk-free interest rate be 0.05. The current price of a particular stock is \$100 per share. You model the stock price at the end of one year as follows

$$S(1) \sim \begin{cases} 90 & \text{with probability } 2/5 \\ 105 & \text{with probability } 2/5 \\ 125 & \text{with probability } 1/5 \end{cases}$$

The price of the one-year, 100-strike call on the above stock is 13.60, while the price of the one-year, 120-strike call on the above stock equals 8.00. Bertie buys a one-year $(100, 120)$ call bull spread on the above stock. What is Bertie's expected profit?

Solution: The initial cost is $13.60 - 8.00 = 5.60$. The payoff of the (100,120) call bull spread has the following distribution:

$$V(1) \sim \begin{cases} 0 & \text{with probability } 2/5 \\ 5 & \text{with probability } 2/5 \\ 20 & \text{with probability } 1/5 \end{cases}$$

So, the expected payoff equals

$$\mathbb{E}[V(1)] = 5\left(\frac{2}{5}\right) + 20\left(\frac{1}{5}\right) = 6.$$

Finally, the expected profit is $6 - 5.60e^{0.05} = 0.1128819$

Problem 8.3. Consider three kinds of European call options on the same underlying asset and with the same exercise date. Their strikes are $100, 110$ and 120. The price of the \$100-strike call is 16.70. The price of the $120-$strike call is 4.50. A butterfly spread is constructed using the above call options. Its price is \$3.00 and the total number of options used to construct it is four. What is the price of the \$110-strike call option?

Solution: Let butterfly spread is evidently symmetric. It is contructed by buying one \$100-strike call and one \$120-strike call and writing two \$110-strike calls. So, if we denote the price of the \$110-strike call by x, we have that

$$16.70 - 2x + 4.50 = 3 \quad \Rightarrow \quad x = 9.10.$$

Problem 8.4. An investor bought a six-month, $(70, 80)$-put bear spread on an index. The \$70-strike, six-month put is currently valued at \$1, while the \$80-strike, six-month put is currently valued at \$8.

Assume that the continuously-compounded, risk-free interest rate equals 0.05.

What is the **break-even** final index price for the above put bear spread?

Solution: The break-even point We need to solve for s such that $70 < s < 80$, in

$$80 - s = (8 - 1)e^{0.025} \quad \Rightarrow \quad s = 72.82279$$

Problem 8.5. (5 points) Consider the ratio spread consisting of:

- five long \$40-strike, one-year calls on **S**,
- seven short \$60-strike, one-year calls on **S**.

You model the stock price at time-1 using the following model

$$S(1) \sim \begin{cases} \$35, & \text{with probability } 0.15 \\ \$45, & \text{with probability } 0.25 \\ \$55, & \text{with probability } 0.35 \\ \$65, & \text{with probability } 0.25 \end{cases}$$

What is the expected payoff of the ratio spread above?

Solution:

$$5(45 - 40)(0.25) + 5(55 - 40)(0.35) + 5(65 - 40)(0.25) - 7(65 - 60)(0.25) = 55$$

Problem 8.6. (5 points) An investor buys a two-year (\$800, \$900)-strangle on gold. The price of gold two years from now is modeled using the following distribution:

$$\begin{aligned} &\$750, \quad \text{with probability } 0.45, \\ &\$850, \quad \text{with probability } 0.4, \\ &\$925, \quad \text{with probability } 0.15. \end{aligned}$$

What is the investor's expected payoff?

Solution:

$$50 \times 0.45 + 25 \times 0.15 = 26.25.$$

Problem 8.7. A portfolio consists of the following:

- one **short** one-year, 50$-$strike call option with price equal to \$8.50,
- one **long** one-year, 60$-$strike put option with price equal to \$6.75

All of the options are European and with the same underlying asset.

Assume that the continuously compounded, risk-free interest rate equals 0.04.

What is the portfolio's profit is the final price of the underlying asset equals \$55?

Solution:

$$-(55 - 50)_+ + (60 - 55)_+ + (8.50 - 6.75)e^{0.04} = 1.82$$

Problem 8.8. (5 points) Bertie constructs an asymmetric butterfly spread using call options with strikes $75, 78$ and 90. It is constructed using m of the (75, 78) bull spreads and n (78,90) bear spreads. How much is m/n?

Solution: We have

$$\frac{m}{n} = \frac{\frac{90-78}{90-75}}{\frac{78-75}{90-75}} = 4.$$

Problem 8.9. (10 points) Assume that one of the no-arbitrage conditions in the binomial model for pricing options on a non-dividend paying stock S is violated. Namely, let

$$e^{r\cdot h} \leq d < u.$$

Illustrate that the above inequalities indeed violate the no-arbitrage requirement. In other words, construct an arbitrage portfolio and show that your proposed arbitrage portfolio is, indeed, and arbitrage portfolio.

Solution: There are multiple ways to illustrate arbitrage opportunities in the above set-up. We provide just one simple example.

Let today's stock-price be denoted by $S(0)$. We simply borrow $S(0)$ from the money market and buy one share of stock. After one period, according to the binomial model, the stock-price either rises to $S_u = uS(0)$ or drops to $S_d = dS(0)$.

Let us denote the value of our portfolio on the second day by X_u in the case the stock price went up and by X_d if the stock price went down. The values of our portfolio in those two cases are

$$X_u = -e^{rh} \cdot S(0) + uS(0) > 0$$
$$X_d = -e^{rh} \cdot S(0) + dS(0) \geq 0$$

We have non-negative payoffs in both cases and a strictly positive payoff in one of the cases. Hence, the above strategy constitutes arbitrage.