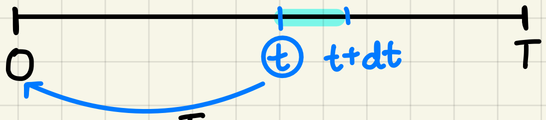
M339D: January 29th, 2021.

HW#1.

Problem 4.

Scenario A: continuous annuity w/ rate R



$$PV(cont.) = \int_0^T e^{-r\cdot t} R\,dt$$

w/ r the continuously compounded, risk-free interest rate

$$\Rightarrow \quad PV(cont.) = R\cdot\int_0^T e^{-r\cdot t}\,dt =$$

$$= R\cdot\left(-\frac{1}{r}\right)\cdot\left[e^{-r\cdot t}\right]_{t=0}^{T}$$

$$= R\cdot\left(-\frac{1}{r}\right)\left(e^{-r\cdot T}-1\right)$$

$$= R\cdot\frac{1-e^{-r\cdot T}}{r} = \bar{a}_{\overline{T}|r}$$

Scenario B: discrete annuity immediate w/ annual pmts equal to X

$$PV(disc.) = X\cdot a_{\overline{T}|i} = X\cdot\frac{1-v^T}{i}$$

w/ i ... the effective interest rate per period

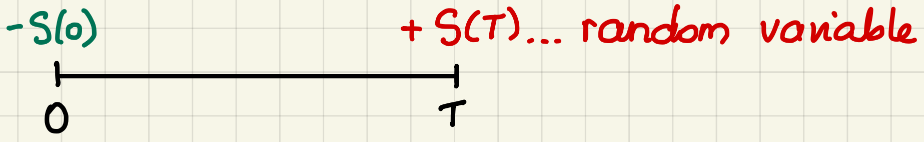
and $v = \frac{1}{1+i}$

In the problem: $R\cdot\frac{\cancel{1-e^{-r\cdot T}}}{r} = X\cdot\frac{\cancel{1-v^T}}{\boxed{i}}$

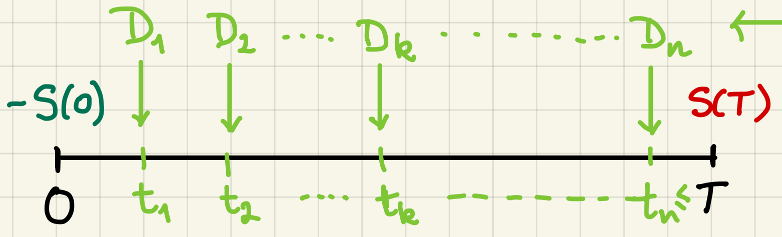
In terms of r, we have $\boxed{i = e^r - 1}$

Outright Purchase of one share of stock

Case #1.



Case #2. Discrete Dividends



dividend amounts

- expressed per share
- deterministic

Fixed deterministic times @ which the dividends are paid.

One should be interested in:

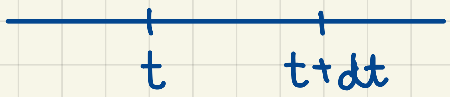
$$PV(\text{dividends}) = \sum_{k=1}^{n} D_k \cdot e^{-r \cdot t_k}$$

Case #3. Continuous dividends.

δ... dividend yield

The dividend amount paid to the shareholders during the time interval $(t, t+dt)$ is given as

$\delta \cdot S(t)\, dt$ per share owned.



observe : $S(t)$

Q: How would one calculate the total nominal amount of dividend paid over $[0,T]$?

→ : $\int_0^T \delta \boxed{S(t)}\, dt$ STOCHASTIC PROCESS.

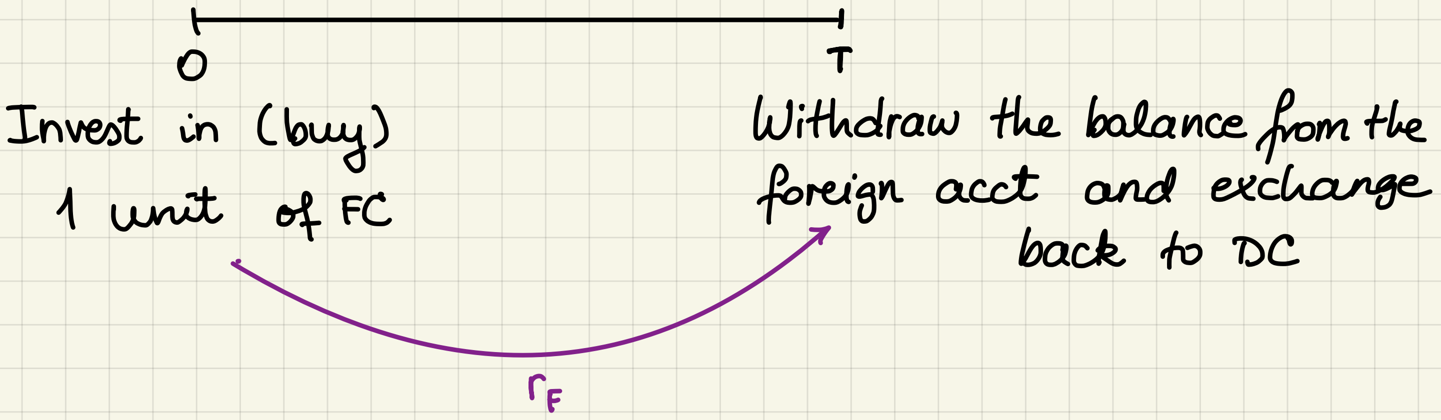
Foreign Currencies.

Domestic Currency (DC) ... its continuously compounded, risk-free interest rate is r_D

Foreign Currency (FC) ... its c.c. r.f. i.r. is r_F

$x(t), t \geq 0$ ... the EXCHANGE RATE from the FC to the DC;

i.e., $x(t)$ is the number of units of the DC we have to pay @ time t to receive one unit of the FC



Invest in (buy) 1 unit of FC

Withdraw the balance from the foreign acct and exchange back to DC

Until next time:
- Work on HW and Quiz.
- Read PS#2.

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Problem set 2

Foreign currencies.

Problem 2.1. Paige lives in Great Britain and receives her salary in GBP. She decides to spend 680 GBP and let the proceeds of the exchange accrue interest at the USD continuously compounded risk-free interest rate .

(i) Given that the initial exchange rate is 0.68 GBP per USD, how much (in USD) does Paige receive initially?

(ii) Given that the USD continuously compounded risk-free interest rate is equal to $r_{\$} = 0.02$, what is the balance in Paige's account six months after the initial transaction? Assume that there were no intermediate deposits or withdrawals.

(iii) Paige decides to withdraw the balance in her account at that time (still six months from the initial exchange) and exchange it back to GBP. Given that the exchange rate at that time equals 0.71 GBP per USD, how much (in GBP) does Paige receive?

(iv) Given that the GBP continuously compounded risk-free interest rate equals $r_{£} = 0.03$, what would have Paige's balance have been had she decided to simply deposit her initial investment in a GBP savings account?

Problem 2.2. You are given the following information:

- the current exchange rate is 8.71 Swedish Kronor (SEK) per USD;
- the SEK continuously compounded risk-free interest rate equals 0.04;
- the USD continuously compounded risk-free interest rate equals 0.02.

Niklas wants to have 10,000 USD exactly one year from now. He is going to buy USD today and deposit the proceeds in a USD saving account. He does not intend to make any withdrawals or deposits prior to the end of the one-year period. How many Swedish Kronor does Niklas need to spend in SEK today in order to purchase just enough USD so that the final balance in his USD savings account equals $10,000$?