

Name:

M339J/M389J: Probability Models with Actuarial Applications

The University of Texas at Austin

Practice Problems for the Final Exam

Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 200 points.

Problem 4.1. (15 points) Consider a mortality study for subjects between the ages of 60 and 80. Three specific lives contributed these data to the study:

Life #1 entered the study at the age of 60 and was still alive at the end of the study.

Life #2 entered the study at the age of 60 and died between the ages of 70 and 71.

Life #3 entered the study at the age of 60 and died exactly at age 74.5.

The Gompertz law is specified via its force of mortality which has to be of the form $\mu_x = Bc^x$ where $B > 0$ and $c > 1$ are its parameters.

What are the values of the likelihood and the loglikelihood functions produced by the above data for parameter values $B = 4 \times 10^{-6}$ and $c = 1.1$?

Problem 4.2. (5 pts) Given that the hazard rate function of a random variable X equals $h_X(x) = 1/x$ for $x > 1$, we can conclude that

- (a) $0 \leq f_X(2) < 1/4$
- (b) $1/4 \leq f_X(2) < 1/3$
- (c) $1/3 \leq f_X(2) < 1/2$
- (d) $1/2 \leq f_X(2) < 1$
- (e) None of the above.

Problem 4.3. (10 points) Denote the random variable corresponding to the age-at-death (in the absence of censoring) by X . In our usual notation, you are given the following table of observations of death-times, surrender times and times of entry into the study:

j	d_j	x_j	u_j
1	0	20	-
2	22	25	-
3	5	30	-
4	0	-	10
5	0	-	12
6	20	35	-
7	5	22	-
8	0	30	-
9	10	22	-
10	20	-	30

- (a) (5 pts) Use the Kaplan-Meier estimator to estimate the probability $\mathbb{P}[X > 21]$.
- (b) (5 pts) Find the Nelson-Aalen estimate for $H_X(25)$.