M358K: November 18th, 2020.

Quiz #11.

Problem #1.

→: $H_0: p_1 = \cdots = p_6 = 1/6$

vs.

H_a: At least one of the population probabilities is different from the null.

Observed counts: in the table

Expected counts: $E_i = 10$ for all i

The observed value of the test statistic:

$$q^2 = \frac{2^2}{10} + \frac{1^2}{10} + \frac{10^2}{10} + \frac{2^2}{10} + \frac{10^2}{10} + \frac{1^2}{10}$$

$$q^2 = \frac{1}{10}(210) = 21$$

$df = k-1 = 6-1 = 5$

⇒ Reject the null!

Inference for numerical data

So far: • Normal population dist'n w/ an unknown mean μ & a known standard deviation σ

Simple random sample $X_1, X_2, \dots, X_n \sim N(\mu, \sigma^2)$ independent

Set $\bar{X} = \frac{1}{n}(X_1 + X_2 + \dots + X_n)$ ... the sample mean

$$\boxed{\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}} \sim \text{Normal}(\text{mean} = 0, \text{sd} = 1)$$

Q: What if σ is not known?

→: S... sample standard deviation

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

You want to use this statistic:

$$\frac{\bar{X} - \mu}{S/\sqrt{n}}$$

NOT NORMAL

a random variable

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim \boxed{t(df = n-1)}$$

t-distribution

Confidence Intervals for μ (small normal sample w/ σ unknown)

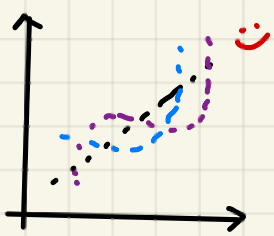
C... confidence level

$$\boxed{\text{point estimate}} \pm \text{margin}\cdot\text{of}\cdot\text{error}$$

$$\bar{x} \pm t^*(df = n-1) \cdot \frac{s}{\sqrt{n}}$$

To check for normality:

- plot the histogram
- box plot
- q·q·plot : plot the quantiles of the standard normal against the quantiles of the data in standard units



Example. [Ramachandran·Tsokos : available as an ebook in the library]

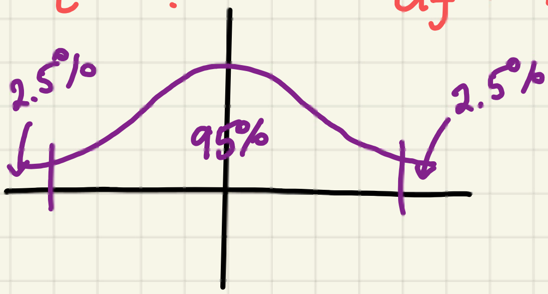
The scores of a random sample of 16 people had a sample mean of 540 and a sample standard deviation of 50. Construct a 95% confidence interval for the population mean μ of the score on the exam assuming that scores are normal.

corresponds to 2.5% upper tail

→: $\bar{x} = 540$ ✓

$s = 50$ ✓

$t^* = ?$ $df = 16 - 1 = 15$



$t^* = 2.131$

answer: $(513.36, 566.64)$

Check your answer against this :)