University of Texas at Austin

Problem Set #3

Forward contracts. European call options. European put options.

3.1. **Forwards.**

Problem 3.1. (5 points) A soy-bean farmer shorts forward contracts on soy in an amount matching his crop volume and with delivery at harvest time. Then, he is considered:

(a) an arbitrageur.
(b) a broker.
(c) a speculator.
(d) a hedger.
(e) None of the above.

Solution: (d)

3.2. **Calls.**

Problem 3.2. The initial price of a non-dividend-paying asset is \$100. A six-month, \$95-strike European call option is available at a \$8 premium.

The continuously compounded risk-free interest rate equals 0.04.

What is the break-even point for this call option?

(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

Solution: (d)
We need to solve for s in

$$(s-95)_+ = 8e^{0.02} \quad \Rightarrow \quad s = 95 + 8e^{0.02} = 103.16$$

Problem 3.3. (5 points) A stock's price today is \$1000 and the annual effective interest rate is given to be 5%. You write a one-year, \$1,050-strike call option for a premium of \$10 while you simulataneously buy the stock. What is your **profit** if the stock's spot price in one year equals \$1,200?

(a) \$150.00
(b) \$139.90
(c) \$10.50
(d) - \$39.00
(e) None of the above.

Solution: (c)

$$S(T) - 1000(1.05) - (S(T)-K)_+ + 10(1.05) = 1050 - 990(1.05) = 10.50.$$

Problem 3.4. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for \$1,000.

The jeweler models the market price of gold in one year as follows:

Gold price in one year	Probability
750 per ounce	0.2
850 per ounce	0.5
950 per ounce	0.3

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of \$900 per ounce. The option costs \$100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelery produced.

Solution:

With $S(T)$ denoting the market price of gold at time $T = 1$, the jeweler's **hedged** profit per piece of jewelry can be expressed as

$$1,000 - \min(S(T), 900) - 100e^{0.05} = 894.873 - \min(S(T), 900).$$

So, her expected **hedged** profit equals

$$894.873 - \mathbb{E}[\min(S(T), 900)] = 894.873 - (0.2 \cdot 750 + 0.5 \cdot 850 + 0.3 \cdot 900) = 49.873.$$

3.3. **Puts.**

Problem 3.5. The initial price of the market index is \$900. After 3 months the market index is priced at \$915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of \$930, is \$8.00. What is the profit at expiration for a **long** put?

(a) \$15.00 loss
(b) \$6.90 loss
(c) \$6.90 gain
(d) \$15.00 gain
(e) None of the above.

Solution: (c)

The profit from a position is defined as the position's payoff minus the future value of the initial cost.
If $S(T) = 915$ denotes the price of the market index at time $T = 0.25$ (i.e., in three months), then the payoff of the long put is $(K - S(T))_+$, where $K = 930$ denotes the strike of the put. So, since $K > S(T)$, the payoff is

$$(930 - 915)_+ = 15.$$

The future value of the initial put premium is

$$8(1 + 0.004)^3 = 8.0964.$$

So, the profit is

$$15 - 8.0964 = 6.90.$$

Problem 3.6. **Sample FM(DM) #12**

Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4% **convertible semiannually**, and that the put costs 74.20 now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

A. 922.83
B. 924.32
C. 1,000.00
D. 1,075.68
E. 1,077.17

Solution: **(b)**
Method I. A quick and insightful way of solving this problem is by realizing that the long-put and the short-put profits are negatives of each other. So, the only way they can be equal is at the "break-even" point. We solve for s in

$$(K-s)_+ - V_P(0)\left(1+\frac{i^{(2)}}{2}\right) = (1000-s)_+ - 74.20(1.02) = 0.$$

The solution is $s = 924.32$.

Method II. This is the more pedestrian method. The long-put profit is

$$(K-s)_+ - V_P(0)\left(1+\frac{i^{(2)}}{2}\right) = (1000-s)_+ - 74.20(1.02).$$

The short-put profit is the exact negative of the expression above, i.e.,

$$-(K-s)_+ + V_P(0)\left(1+\frac{i^{(2)}}{2}\right) = -(1000-s)_+ + 74.20(1.02).$$

So, algebraically, we need to solve for s in the equation

$$\begin{aligned}(1000-s)_+ - 74.20(1.02) = -(1000-s)_+ + 74.20(1.02) \quad &\Leftrightarrow \quad 2(1000-s)_+ = 2\cdot 74.20(1.02)\\ &\Leftrightarrow \quad (1000-s)_+ = 74.20(1.02).\end{aligned}$$

We get the same answer as above, of course.

Problem 3.7. Farmer Shaun is producing sweet potatoes. He intends to harvest $10,000-$cartons' worth in six months. His total costs are \$12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of \$13 per carton and another with the strike price of \$15 per carton. Their premiums are \$0.15 and \$0.18, respectvely.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals \$14. What would Farmer Shaun's profit be if he had decided to hedge using the \$13-strike put versus his profit if he had decided to use the \$15-strike put to hedge?

Solution: Farmer Shaun's unhedged position has the following profit:

$$10,000(S(T)-12)$$

where $S(T)$ stands for the spot price of sweet potatoes in six months.

If he decided to hedge using put options, he would **long** the put. So, the profit of the \$13-strike-put hedge would be:

$$10,000(13-S(T))_+ - 10,000\times 0.15\times 1.04.$$

The profit of the \$15-strike-put hedge would be:

$$10,000(15-S(T))_+ - 10,000\times 0.18\times 1.04.$$

The profit of the hedged position with the given $S(T) = 14$ in the first case equals

$$10,000(14-12-0.15\times 1.04) = 18,440.$$

For the second insurance strategy, the profit is

$$10,000(14-12+(15-14)-0.18\times 1.04) = 28,128.$$