Due on September 20^{th}, 2020
M339W/389W Financial Mathematics for Actuarial Applications
Fall 2021, University of Texas at Austin
Graduate Homework
Instructor: Milica Čudina

The following problems are meant to be the extra special homework assignment for graduate students. If you are taking this course as an undergraduate course, your score on will count as extra credit. Details are below.
A part of the homework involves simulations while the rest involves calculations (and a bit of simulations). You are required to use **R** to complete this part of the assignment. Please, do not include a list of your simulated values in your final submission.
If you are a graduate student, this HW will be taken into account when your HW average is calculated (see the First-Day Handout). It will have the weight of **two** regular homework assignments. If you are taking this course as an undergraduate course and you decide to attempt this assignment, then your score will serve as extra credit toward your final score in the course. More precisely, a full score in this homework will be worth 2 points in the final score in this course.

Problem 1.1. (5 points) The continuously compounded, risk-free interest rate is given to be 0.04. The current price of one ounce of gold is \$1830. According to Violet, the price $S(1)$ of one ounce of gold in one year can be modelled as follows:

$$S(1) \sim \begin{cases} \$1850 & \text{with probability } 1/5 \\ \$1900 & \text{with probability } 1/2 \\ x & \text{with probability } 3/10 \end{cases}$$

Assume that Violet is a rational, risk-averse investor. She is currently willing to enter a forward contract on gold. What can you say about the value of x in Violet's model?

Assume that there are no storage or insurance costs for gold.

Problem 1.2. (10 points) Consider a continuous-dividend-paying stock whose dividend yield is 0.03. The evolution of the price of this stock over the next year is modelled using a three-period binomial tree. Within each period, the stock price can go up by 10% or down by 10%.

The continuously compounded, risk-free interest rate is 0.03.

Consider an at-the-money American call option on the above stock which expires in one year. Find the ratio of the price of the option at time-0 to the stock price at time-0.

Problem 1.3. (10 points) Consider a continuous-dividend-paying stock whose current price is \$100. Its volatility is 0.20. The price of this stock at the end of one year is modelled using a one-period forward binomial tree.

The continuously compounded, risk-free interest rate equals 0.02.

Consider a one-year, \$90-strike American call option on this stock. For which values of the stock's dividend yield is it optimal to exercise this option immediately?

Before attempting the following problems, the student should read and understand Chapter #3. from the SOA study note available at `https://www.soa.org/globalassets/assets/Files/Edu/2018/ifm-21-18-study-note.pdf` on *Monte Carlo simulation*.

Problem 1.4. (25 points) A weighted spinner has four different regions: blue, red, green, and yellow. The blue is twice as likely as red, the red is twice as likely as green, and the green is twice as likely as yellow. You are invited to play the game with the following outcomes:

- if the spinner lands on blue, you lose \$5;
- if the spinner lands on red, you win \$2;
- if the spinner lands on green, you win \$10;
- if the spinner lands on yellow, you win \$20.

Use *Monte Carlo* simulation to figure out the fair price for entering this game.

Problem 1.5. (25 points) Let the current price of a non-dividend-paying stock be \$100. Let its volatility be 0.22. You model the evolution of this stock over the next year using a $25-$period forward binomial tree.

The continuously compounded, risk-free interest rate is 0.04.

Using *Monte Carlo* simulation, find the price of a one-year, at-the-money European call option on the above stock.

Problem 1.6. (25 points) Use *Monte Carlo* simulation to estimate the number π.