UNIVERSITY OF TEXAS AT AUSTIN

Lecture 4

The Inverse Transformation (Simulation) Method

Proposition 4.1. *Let X be a continuous random variable with the cumulative distribution function F_X and probability density function f_X.*

Assume that $f(x) > 0$ for all positive x and zero elsewhere.

Define $Y = F_X(X)$.

Then, $Y \sim U(0,1)$.

Proposition 4.2. *Let $U \sim U(0,1)$ and let F be a cumulative distribution function.*

Define $X = F^{-1}(U)$.

Then, the random variable X has the cumulative distribution function F.

An Informal Implementation.

1. Set F to be the cdf of the distribution from which we want to simulate values. "Figure out" F^{-1}; this can be analytic or numerical.
2. Draw the simulated values from the unit uniform $U(0,1)$:
$$u_1, u_2, \dots, u_n$$
3. Apply F^{-1} to the simulated values to obtain
$$x_1 = F^{-1}(u_1), x_2 = F^{-1}(u_2), \dots, x_n = F^{-1}(u_n)$$
The $x_1, x_2, \dots, x_n$ are the simulated values from your target distribution.

Example 4.3. In the exponential case $X \sim Exponential(\theta)$, we have already obtained the analytic expression for the quantile function F_X^{-1}. It is
$$F_X^{-1}(y) = -\theta \ln(1-y)$$
So, with $\{u_i, i = 1, \dots, n\}$ generated from the unit uniform, the x_i defined as
$$-\theta \ln(1-u_1) \quad \text{for } i = 1, \dots, n$$
will be simulated values from the exponential distribution with parameter θ.