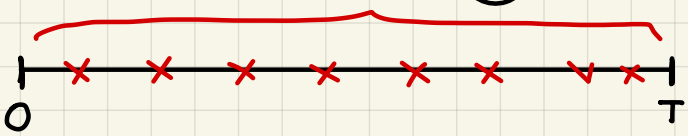
M3398: February 5th, 2021.

Static Portfolios.

Step #1. Decide who your protagonist is!

Step #2. Set up the time-line!

Static: (No) Intermediate Cashflows!



"today"
Cashflow from the perspective of your agent.
!!
THE INITIAL COST

"time horizon"
Cashflow/Wealth from the perspective of your agent.
!!
PAYOFF

PROFIT := PAYOFF − $FV_{0,T}$(INITIAL COST)

- IF Profit > 0, we call it a GAIN.
- IF Profit < 0, we call it a LOSS.
- IF Profit = 0, we say that we BROKE EVEN.

Example. [Investing in a zero-coupon bond.]

T... the maturity date

C... the redemption amount

r... ccrfir

Q: What is the initial cost?

→: $P = Ce^{-r \cdot T}$ ... the bond's price

Q: What is the payoff? →: C

Q: What is the profit?

→: Profit = Payoff − $FV_{0,T}$ (Initial Cost)

$= C - e^{rT}(Ce^{-rT}) = 0$ ■

Example. [TAKING A SIMPLE LOAN]

L ... the loan amt

T ... the loan's term, i.e., the time @ which the loan must be repaid in full

r ... ccrfir

Initial Cost = $-L$ (the negative sign is since your agent is TAKING a loan, i.e., RECEIVING L @ time-0)

Payoff = $-Le^{rT}$ (the negative sign is since your agent is GIVING UP the amt @ time-T)

Profit = Payoff − $FV_{0,T}$(Init. Cost)

$= -Le^{rT} - e^{rT} \cdot (-L) = 0$ ■

Example. [THE OUTRIGHT PURCHASE OF A NON-DIVIDEND-PAYING STOCK]

The Timeline:

⇒ Initial Cost = $S(0)$

Payoff = $S(T)$ a random variable

Introduce: s ... an independent argument taking values in $[0, +\infty)$; denotes the FINAL ASSET PRICE (a "placeholder" for the random variable $S(T)$).

Now, we can define the PAYOFF FUNCTION which describes the dependence of the investor's payoff on the independent argument s ($\leftrightarrow S(T)$).

Notation: v ... payoff function

Typically, $v : [0, +\infty) \longrightarrow \mathbb{R}$

$\Rightarrow$ $v(s)$ is the investor's payoff if the final asset price is s.

$\Rightarrow$ In the previous example, the payoff function is

$v(s) = s$ the identity function

When we graph the payoff function, we get the PAYOFF CURVE (or PAYOFF DIAGRAM):