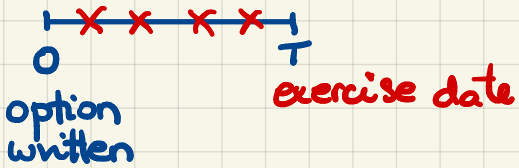
M339D: September 25th, 2024.

European Put Options.

Usually, a right but not an obligation to SELL the underlying!



At time 0: The writer and the buyer of the put agree on:

- the underlying asset: $S(t), t \geq 0$;
- the exercise date T;
- the strike/exercise price K

The put premium $V_P(0)$ is paid by the put's buyer to the put's writer.

At time T:

- The put's owner has the right, but not an obligation to SELL one unit of the underlying asset for the strike price K.
- The put's writer is obligated to do what the put's owner decides.

The put owner's optimal behavior is:

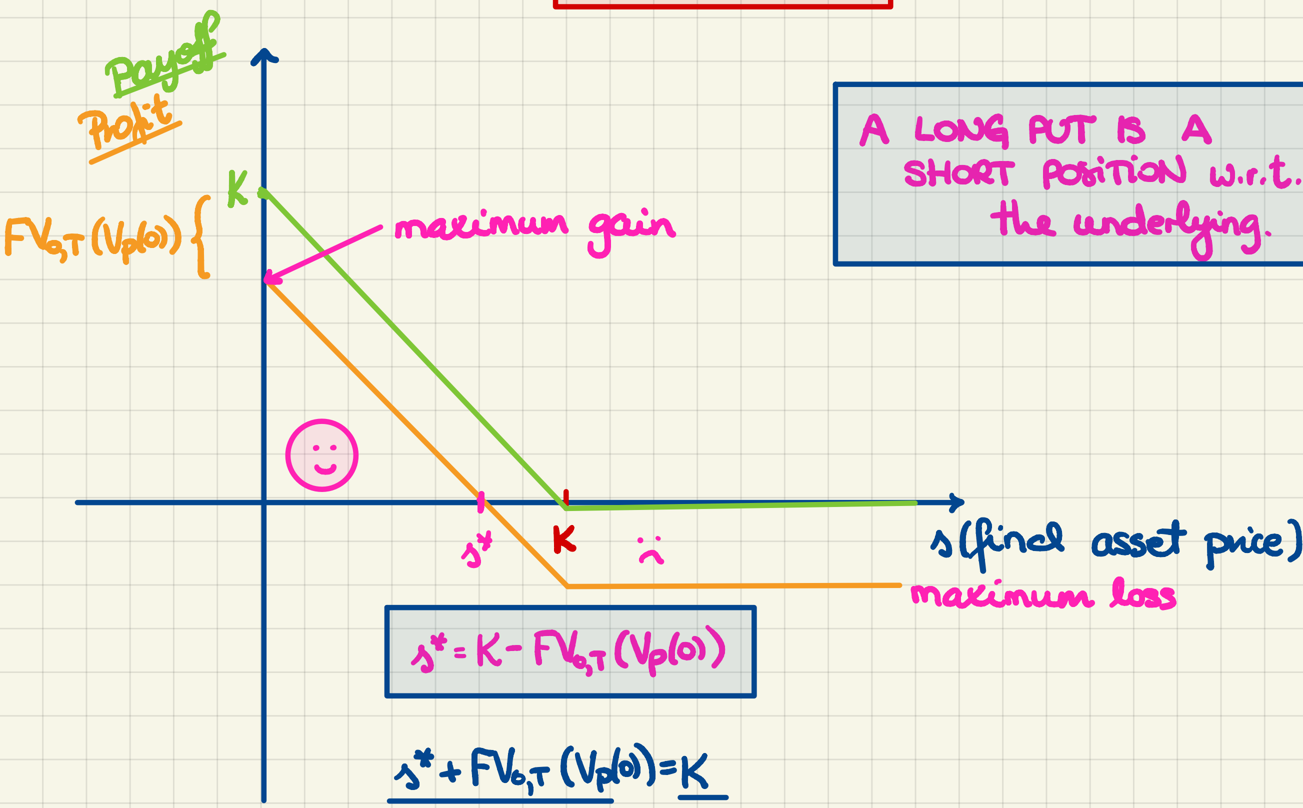
		Payoff
IF $K > S(T)$,	then exercise	$K - S(T)$
IF $K \leq S(T)$,	then do not exercise	0

The Payoff: $\boxed{V_P(T) = \max(K - S(T), 0) = (K - S(T))_+}$

The Payoff Function: $\boxed{v_P(s) = (K-s)_+}$

The Payoff Function:

$$\boxed{v_P(s) = (K-s)_+}$$



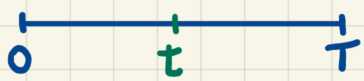
A LONG PUT IS A SHORT POSITION w.r.t. the underlying.

$$\boxed{s^* = K - FV_{0,T}(V_P(0))}$$

$$s^* + FV_{0,T}(V_P(0)) = K$$

Moneyness.

Consider an option written @ time·0 w/ exercise date @ time·T.



Imagine the cashflow that would happen to the option's owner were they to exercise it @ time·t

e.g.,
call: $S(t)-K$
put: $K-S(t)$

If cashflow is
- >0 we say the option is in·the·money
- $=0$ we say the option is **at the·money**
- <0 we say the option is out of·the·money

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Problem Set #6

European put options.

To distract :)

Problem 6.1. The initial price of the market index is \$900. After 3 months the market index is priced at \$915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of \$930, is \$8.00. What is the profit at expiration for a **long** put?

(a) \$15.00 loss
(b) \$6.90 loss
(c) \$6.90 gain
(d) \$15.00 gain
(e) None of the above.

$i^{(12)}$

$\Rightarrow$ effective monthly i.r. is $\frac{i^{(12)}}{12} = \frac{0.048}{12}$

$\longrightarrow$: Payoff: $(K-S(T))_+ = (930-915)_+ = 15$

Profit $= 15 - 8(1.004)^3 = 6.90$

$\square$

Problem 6.2. Sample FM(DM) #12
Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of $\boxed{1,000}$. Suppose that the nominal annual risk-free rate is 4% **convertible semiannually**, and that the put costs $\boxed{74.20}$ now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

A. 922.83
B. 924.32
C. 1,000.00
D. 1,075.68
E. 1,077.17

effective per half-year is $\frac{0.04}{2} = 0.02$

→: We're looking for the break-even point.

$$s^* = K - FV_{0,T}(V_P(0))$$

$$s^* = 1000 - 74.20(1.02) = \underline{924.32}$$

□

Problem 6.3. Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000−cartons' worth in six months. His total costs are $12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of $13 per carton and another with the strike price of $15 per carton. Their premiums are $0.15 and $0.18, respectvely.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals $14. What would Farmer Shaun's profit be if he had decided to hedge using the $13-strike put versus his profit if he had decided to use the $15-strike put to hedge?

Focus on the payoff: (w/out production costs)

unhedged: $S(T)$

hedge: $(K-S(T))_+$

total hedged: $S(T)+(K-S(T))_+ = \begin{cases} K & \text{if } K > S(T) \\ S(T) & \text{if } K \leq S(T) \end{cases} = \boxed{\max(K, S(T))}$

$\boxed{\text{FLOOR}}$

$13 Payoff: $\max(13,14)=14$

$14 - 12 - 0.15\cdot(1.04) =$ ______

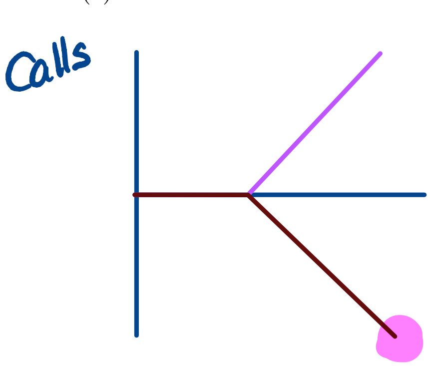
$15 Payoff: $\max(15,14)=15$

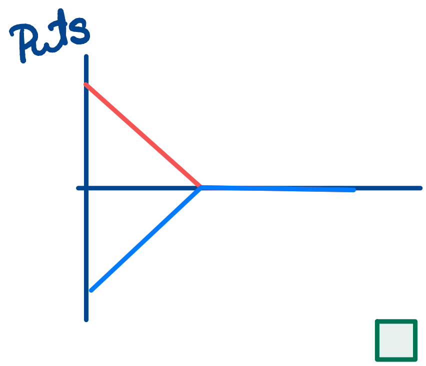
$15 - 12 - 0.18(1.04) =$ ______

$\times 10{,}000$

Problem 6.4. (2 points) In which of the following option positions is the investor exposed to an unlimited loss?

(a) Long put option
(b) Short put option
(c) Long call option
(d) Short call option
(e) None of the above.





Problem 6.5. (3 points) The initial price of the market index is \$1000. After 3 months the market index is priced at \$950. The nominal rate of interest convertible quarterly is 4.0%.
The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

(a) \$12.00 loss
(b) \$14.90 loss
(c) \$12.00 gain
(d) \$14.90 gain
(e) None of the above.

→:

Payoff: $(K - S(T))_+ = (975 - 950)_+ = 25$

Profit: $25 - 10(1.01) = 14.9$ $\square$

Problem 6.6. (3 points) *Source: Sample FM(DM) Problem #62.*

The stock price today equals \$100 and its price in one year is modeled by the following distribution:

$$S(1) \sim \begin{cases} 125 & \text{with probability } 1/2 \\ 60 & \text{with probability } 1/2 \end{cases}$$

The annual effective interest rate equals $\boxed{3\%}$.

Consider an at-the-money, one-year European put option on the above stock whose initial premium is equal to \$7.

What is the expected profit of this put option?

$\rightarrow$: $\boxed{\mathbb{E}[\text{Payoff}]} - FV_{0,1}(V_P(0))$

$\parallel$

$\mathbb{E}[V_P(T)]$

$$\boxed{V_P(T) = (K - S(T))_+}$$

$$V_P(T) \sim \begin{cases} (100-125)_+ = 0 & \text{w/ probab. } 1/2 \\ (100-60)_+ = 40 & \text{w/ probab. } 1/2 \end{cases}$$

$$\mathbb{E}[V_P(T)] = 40 \cdot (1/2) = 20$$

answer: $20 - 7 \cdot (1.03) = 12.79$ $\square$

Problem 6.7. Aunt Dahlia simultaneously purchased

- one share of a market index at the current spot price of \$1,000;
- one one-year, \$1,050-strike put option on the above market index for the premium of \$20. max(s,K)

(i) (5 points) Is the above portfolio's payoff bounded from above? If you believe it is not, substantiate your claim. If you believe that it is, provide the upper bound. NO

(ii) (5 points) Is the above portfolio's payoff bounded from below? If you believe it is not, substantiate your claim. If you believe that it is, provide the lower bound. YES