M378K Introduction to Mathematical Statistics

Homework assignment #5

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 5.1. *(10 points) Let X be a continuous random variable with the cumulative distribution function denoted by F_X and the probability density function denoted by f_X.*

Express the cumulative distribution function and the density of the random variable $\tilde{X} = X^2$ in terms of F_X and f_X.

Problem 5.2. *(10 points) Let Y be lognormal with parameters $\mu = 1$ and $\sigma = 2$, i.e., let $Y \stackrel{(d)}{=} e^X$ with $X \sim N(\mu, \sigma)$.*

Define $\tilde{Y} = 3Y$.

Find the median of $\tilde{Y}$, i.e., find the value m such that $\mathbb{P}[\tilde{Y} \leq m] = 1/2$.

Problem 5.3. *(10 points) Let T denote the time for a call center employee to respond to any single telephone call. We model the random variable T by uniform distribution on the interval $(48, 72)$ with the time being measured in seconds. Let R denote the* **rate** *at which the call center employee responds to queries expressed in the number of customers* per minute.

Does the random variable R have a density? If so, find the density of R.

Problem 5.4. *(30 points) Let X, Y and Z be independent and uniformly distributed on $(0, 1)$. Find the density function of $W = X + Y + Z$.*