UNIVERSITY OF TEXAS AT AUSTIN

# HW Assignment 11

## More on binomial option pricing. More on exotic options.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

**Problem 11.1.** (15 points) Consider the one-period binomial option pricing model. Let $V_C(0) > 0$ denote the price of a European call on a stock which pays continuous dividends. What is the impact on the value of European call option prices if the company decides to increase the dividend yield paid to the shareholders?

(a) The call option price will drop.
(b) The call option price will increase.
(c) The call option price will always remain constant.
(d) The impact on the price of the call cannot be determined using the binomial option pricing model.
(e) There is not enough information provided.

**Solution:** **(a)**
Let $\delta < \tilde{\delta}$ be the two dividend yields. Then, the risk-neutral price of the European call on the stock with the dividend yield $\delta$ equals

$$V_C(0) = e^{-rT}[p^*(S_u - K)_+ + (1-p^*)(S_d - K)_+]$$

with $p^* = (e^{(r-\delta)h} - d)/(u-d)$. On the other hand, the risk-neutral price of the European call on the stock with the dividend yield $\tilde{\delta}$ equals

$$\tilde{V}_C(0) = e^{-rT}[\tilde{p}^*(S_u - K)_+ + (1-\tilde{p}^*)(S_d - K)_+]$$

with $\tilde{p}^* = (e^{(r-\tilde{\delta})h} - d)/(u-d)$. We have

$$\delta < \tilde{\delta} \quad \Rightarrow \quad e^{(r-\delta)h} > e^{(r-\tilde{\delta})h} \quad \Rightarrow p^* > \tilde{p}^* \quad \Rightarrow \quad V_C(0) > \tilde{V}_C(0).$$

**Problem 11.2.** (5 points) Your portfolio consists of a long up-and-in call with the barrier at 50 and a long up-and-out call with the barrier at 50. Then, in our usual notation, the initial price of your portfolio equals:

(a) $V_C(0)$
(b) $F^P_{0,T}(S)$
(c) $S(T)$
(d) $V_P(0)$
(e) None of the above.

**Solution:** **(a)**

**Problem 11.3.** (8 points) Consider a non-dividend-paying stock whose current price is \$100 per share. Its volatility is given to be 0.25. You model the evolution of the stock price over the following year using a two-period forward binomial tree.

The continuously-compounded, risk-free interest rateis 0.04.

Consider a \$110-strike, one-year **down-and-in** put option with a barrier of \$90 on the above stock. What is the price of this option consistent with the above stock-price model?

(a) About \$10.23
(b) About \$11.55
(c) About \$11.78
(d) About \$11.90
(e) None of the above.

**Solution: (a)**
The up and down factors in the above forward binomial tree are

$$u = e^{0.02+0.25/\sqrt{2}} = 1.2089, \quad d = 0.8610.$$

The option is knocked-in only if the stock price goes down in the first step. So, the payoff of the option will be

$$\begin{aligned} V_{du} &= 5.92, && \text{if the path } \textit{down-up} \text{ is taken,} \\ V_{dd} &= 35.87, && \text{if the path } \textit{down-down} \text{ is taken,} \\ V_{uu} = V_{ud} &= 0, && \text{otherwise.} \end{aligned}$$

The risk-neutral probability of a single step up in the tree equals

$$p^* = \frac{1}{1+e^{0.3/\sqrt{2}}} = 0.4577.$$

So, the option price is

$$V(0) = e^{-rT}[(1-p^*)^2 V_{dd} + p^*(1-p^*)V_{du}] = 10.23.$$

**Problem 11.4.** (2 points) A compound call on a put option costs at most as much as the underlying put option itself. *True or false?*

**Solution: TRUE**

**Problem 11.5.** (5 points) Let $V_{C,C}(0)$ denote the price of a comopound call on a call. Let $V_{P,C}(0)$ denote the price of an otherwise identical comopound put on the same call option. Let $V_C(0)$ denote the price of the underlying call option. Let $S(0)$ denote the initial price of the underlying asset for this vanilla call option.

Which one(s) of the following inequalities is (are) **always** true for the above prices?

(a) $V_{C,C}(0) > V_{P,C}(0)$
(b) $V_{C,C}(0) - V_{P,C}(0) \leq V_C(0)$
(c) $V_{C,C}(0) > S(0)$
(d) $V_C(0) > S(0)$
(e) None of the above.

**Solution: (b)**
Directly from put-call parity for compound options.

**Problem 11.6.** (15 pts) For a two-period binomial model, you are given that:

(1) each period is one year;
(2) the current price of a non-dividend-paying stock $S$ is $S(0) = \$20$;
(3) $u = 1.2$, with $u$ as in the standard notation for the binomial model;
(4) $d = 0.8$, with $d$ as in the standard notation for the binomial model;
(5) the continuously compounded risk-free interest rate is $r = 0.04$.

Consider a **chooser** option such that its owner can decide after one year whether the option becomes a put or a call option with exercise date at time$-2$ and strike equal to \$20.

Find the price of the chooser option.

**Solution:** The risk-neutral probability is

$$p^* = \frac{e^{0.04} - 0.8}{1.2 - 0.8} = 0.6020.$$

When one constructs the two-period binomial tree, one gets

$$S_u = 24,\ S_d = 16,$$
$$S_{uu} = 28.80,\ S_{ud} = S_{dd} = 19.2,\ S_{dd} = 12.8$$

The call will be worth more than the put in the *up* node while the put will be worth more than the call in the *down* node. This means that the chooser option's owner will choose the call in the *up* node and will choose the put in the *down* node.

The possible payoffs of the call at the end of the second period are

$$V_{uu} = 8.80 \quad \text{and} \quad V_{ud} = 0.$$

So, taking the discounted expected value at the *up* node of the payoff with respect to the risk-neutral probability, we get that the price of this call (and, hence, the price of the chooser option) at the *up* node equals

$$V_u^{CH} = e^{-0.04} \times 8.80 \times 0.602 = 5.0899.$$

The possible payoffs of the put at the end of the second period are

$$V_{ud} = 0.80 \quad \text{and} \quad V_{dd} = 7.20.$$

So, taking the discounted expected value at the *down* node of the payoff with respect to the risk-neutral probability, we get that the price of this put (and, hence, the price of the chooser option) at the *down* node equals

$$V_d^{CH} = e^{-0.04}\left[0.80 \times 0.602 + 7.20 \times 0.398\right] = 3.21595.$$

Finally, the time$-0$ price of the chooser option equals

$$V_{CH}(0) = e^{-0.04}\left[5.0899 \times 0.602 + 3.21595 \times 0.398\right] = 4.1737. \tag{11.1}$$