M339D: October 11th, 2024.

Strong Law of Large Numbers (SLLN).

Let $\{X_k, k=1,2,\dots\}$ be a sequence of independent and identically distributed r.v.s

Assume: $\boxed{\mu_X := \mathbb{E}[X_1] < \infty}$

Then,

$$\boxed{\frac{X_1+X_2+\cdots+X_n}{n} \xrightarrow[n\to\infty]{} \mu_X}$$

If a f'tion g is such that $g(X)$ is well-defined and $\mathbb{E}[g(X)]<\infty$,

then,

$$\boxed{\frac{g(X_1)+g(X_2)+\cdots+g(X_n)}{n} \xrightarrow[n\to\infty]{} \mathbb{E}[g(X)]}$$

Monte Carlo.

Recipe.

- Draw simulated values of a r.v. w/ a specific dist'n.
- Apply a f'tion to the simulated values.
- Calculate the arithmetic average of the obtained quantities.

We get a value which is $\boxed{\text{close to}}$ the theoretical expection.

Precision.

$$\mathrm{Var}\left[\frac{X_1+\cdots+X_n}{n}\right] = \frac{1}{n^2}\mathrm{Var}[X_1+\cdots+X_n] \quad \text{(independent!)}$$

$$= \frac{1}{n^2}\left(\mathrm{Var}[X_1]+\cdots+\mathrm{Var}[X_n]\right) \quad \text{(identically dist'd!)}$$

$$= \frac{1}{n^{\cancel{2}}}\cdot\cancel{n}\cdot\mathrm{Var}[X_1] = \frac{\mathrm{Var}[X_1]}{n}$$

$$\mathrm{SD}\left[\frac{X_1+\cdots+X_n}{n}\right] = \frac{\mathrm{SD}[X_1]}{\sqrt{n}}$$

To increase the precision by a fator η, we must increase the number of variates by η^2.