UNIVERSITY OF TEXAS AT AUSTIN

Quiz #15

The loss elimination ratio. Poisson.

Please, provide your **complete solutions** to the following questions:

Problem 15.1. (6 points) Losses have an exponential distribution with a mean of 1,000. There is a deductible of 500. Determine the amount by which the deductible should be raised in order to double the loss elimination ratio.

Solution: Using our tables, we get that the loss elimination ratio with the deductible of 500 equals

$$\frac{\mathbb{E}[X \wedge 500]}{\mathbb{E}[X]} = \frac{1000(1 - e^{-500/1000})}{1000} = 1 - e^{-1/2} \approx 0.39347.$$

We want to find the new deductible d^* for which

$$\frac{\mathbb{E}[X \wedge d^*]}{\mathbb{E}[X]} = \frac{1000(1 - e^{-d^*/1000})}{1000} = 1 - e^{-d^*/1000} = 2 \cdot 0.39347 = 0.78694.$$

We get $d^* \approx 1,546$.

Problem 15.2. (2 pts) The Poisson distribution has the memoryless property. *True or false? Why?*

Solution: FALSE

Problem 15.3. (2 pts) Let $N_1, N_2, \ldots, N_\ell$ be independent, Poisson random variables with respecive parameters $\lambda_1, \lambda_2, \ldots, \lambda_\ell$. Then, the random variable $N := N_1 + N_2 + \cdots + N_\ell$ is also Poisson with the parameter $\lambda = \max(\lambda_1, \lambda_2, \ldots, \lambda_\ell)$. *True or false? Why?*

Solution: FALSE
The parameter of the Poisson random variable N is actually $\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_\ell$.

Problem 15.4. (5 points) *Source: Prof. Jim Daniel, personal communication.*
Let the number of car accidents in a calendar year by a group of drivers be denoted by N and modeled using the Poisson distribution with mean 10.
Assume that the probability that the damage in any single accident is at most \$1,000 equals 2/5.
The number of accidents and the damages caused are assumed to be independent.
Find the probability that the number of accidents in one year with damage greater than \$1000 is 5, given that the number of accidents in that year with damage at most \$1000 equals 100.

Solution: Let N_1 denote the r.v. which stands for the number of accidents with damage of at most \$1,000, and let N_2 be the number of accidents with damage exceeding \$1,000. According to the *"Thinning" theorem*, N_1 and N_2 are independent and

$$N_1 \sim Poisson(\frac{2}{5} \cdot 10 = 4),$$
$$N_2 \sim Poisson(\frac{3}{5} \cdot 10 = 6).$$

We are ready to calculate the conditional probability

$$\mathbb{P}[N_2 = 5 \,|\, N_1 = 100].$$

Since N_1 and N_2 are independent, this probability equals

$$\mathbb{P}[N_2 = 5] = e^{-6} \cdot \frac{6^5}{5!} \approx 0.16.$$