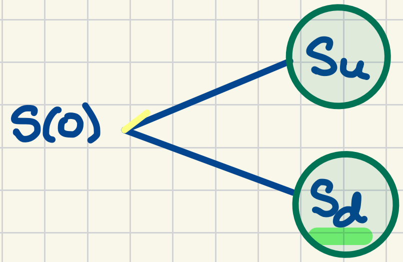
M339Q : October 20th, 2025.

Graphical Interpretation.

Consider a European call w/ exercise date @ end of the tree and the strike price K such that

$$\boxed{S_d < K < S_u}$$



PAYOFF:

$V_u = S_u - K$

$V_d = 0$

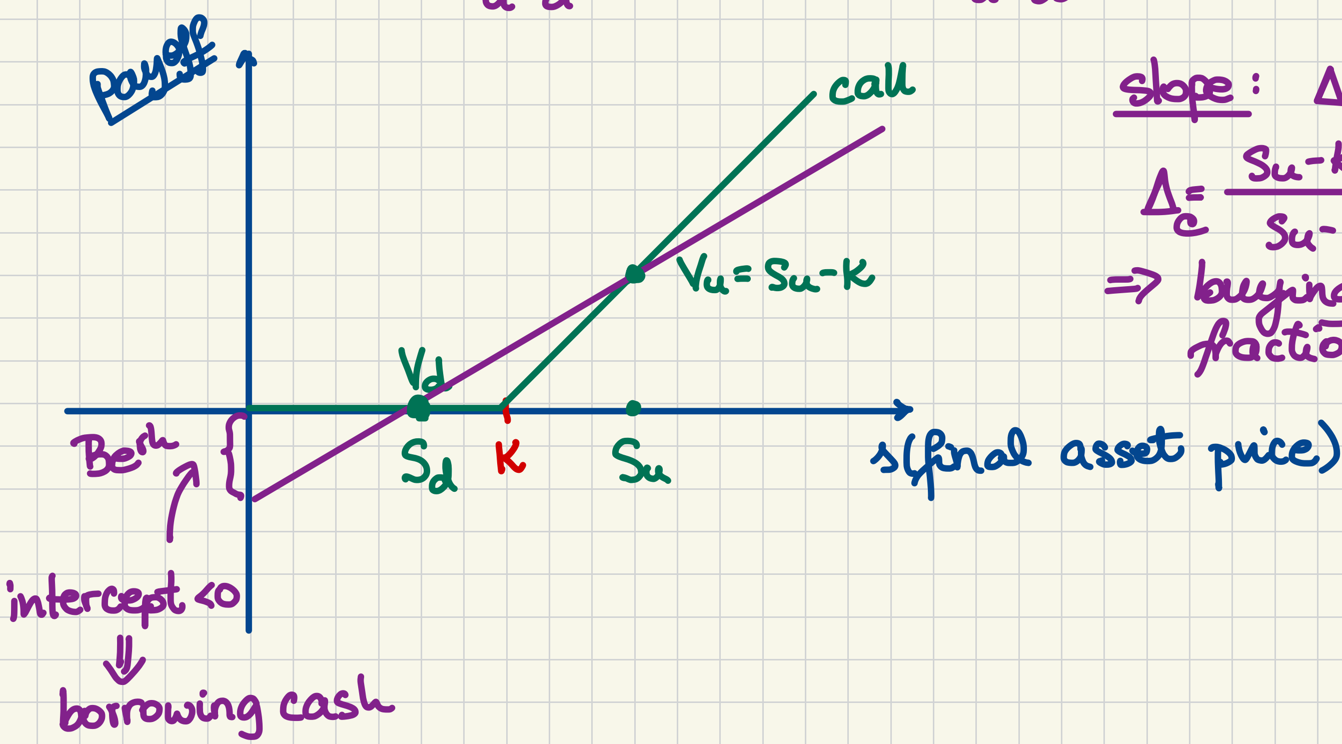
$$v_c(s) = (s-K)_+$$

In the replicating portfolio:

- $\Delta_c = \dfrac{V_u - V_d}{S_u - S_d} = \boxed{\dfrac{S_u - K}{S_u - S_d}} \in (0,1)$ Buy a fraction of a stock!

and

- $B_c = e^{-rh} \cdot \dfrac{u \cdot V_d - d \cdot V_u}{u-d} = - e^{-rh} \cdot \dfrac{d(S_u - K)}{u-d} < 0$ Borrowing!



slope: $\Delta \in (0,1)$

$$\Delta_c = \frac{S_u - K - 0}{S_u - S_d}$$

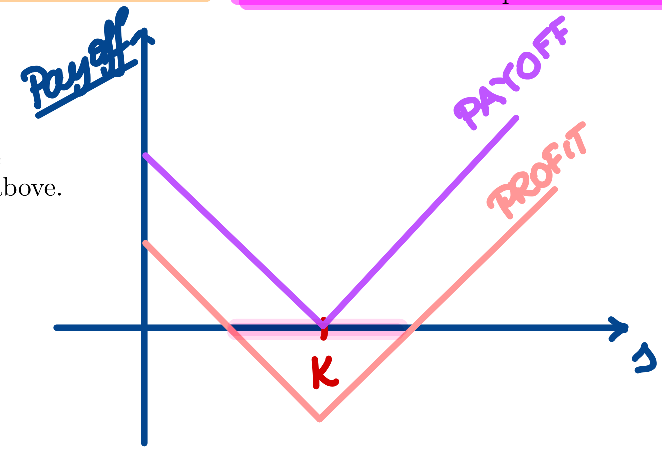
$\Rightarrow$ buying a fraction of a share

Problem 9.5. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

A **straddle** consists of a long call and a long otherwise identical put. Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

(a) About \$10
(b) About \$10.83
(c) About \$15.45
(d) About \$20.84
(e) None of the above.



Payoff f'tion of a straddle

$v(s) = |s-K|$

In this problem

$\boxed{K=100}$

$S(0)=95$

$S_u = 120$

$S_d = 75$

PAYOFF:

$V_u = 20$

$V_d = 25$

$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{20-25}{120-75} = -\frac{5}{45} = -\frac{1}{9}$$

$$B = e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u-d}$$

$$B = e^{-0.06} \cdot \frac{\frac{120}{\cancel{95}} \cdot 25 - \frac{75}{\cancel{95}} \cdot 20}{\frac{120}{\cancel{95}} - \frac{75}{\cancel{95}}} = e^{-0.06} \cdot \frac{120(25) - 75(20)}{45}$$

$$= 31.392$$

$$V(0) = \Delta \cdot S(0) + B = -\frac{1}{9} \cdot (95) + 31.392 = 20.84 \quad \square$$

Risk-Neutral Probability

Start w/ $\quad V(0) = \Delta \cdot S(0) + B$

$$V(0) = \frac{V_u - V_d}{\underbrace{S_u - S_d}_{\cancel{S(0)}(u-d)}} \cdot \cancel{S(0)} + e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u-d}$$

$$V(0) = \frac{1}{u-d}\left[(V_u - V_d) + e^{-rh}(u \cdot V_d - d \cdot V_u)\right]$$

$$V(0) = e^{-rh}\frac{1}{u-d}\left[e^{rh}V_u - e^{rh}V_d + u \cdot V_d - d \cdot V_u\right]$$

$$V(0) = e^{-rh}\frac{1}{u-d}\left[V_u(e^{rh} - d) + V_d(u - e^{rh})\right]$$

$$V(0) = e^{-rh} \cdot \left[\underbrace{\boxed{\frac{e^{rh}-d}{u-d}}}_{= p^*} \cdot V_u + \underbrace{\boxed{\frac{u-e^{rh}}{u-d}}}_{= 1-p^*} \cdot V_d\right]$$

Both positive (due to the no-arbitrage condition)!

Add up to 1!

We choose to interpret the two fractions as probabilities!
We define the risk-neutral probability of the stock price going up in a single period as

$$\boxed{p^* = \frac{e^{rh}-d}{u-d}}$$