UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #6

Binomial option pricing.

Problem 6.1. In the setting of the one-period binomial model, denote by i the **effective** interest rate **per period**. Let u denote the "up factor" and let d denote the "down factor" in the stock-price model.
If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage.

Solution: FALSE

Problem 6.2. In our usual notation, does this parameter choice create a binomial model with an arbitrage opportunity?
$u = 1.18, \quad d = 0.87, \quad r = 0.05, \quad \delta = 0, \quad h = 1/4$

Solution: No.

Problem 6.3. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$80 per share. In the model, it is assumed that the stock price can either go up by \$5 or down by \$4.

You use the binomial tree to construct a replicating portfolio for a 78−strike call option on the above stock. What is the stock investment in the replicating portfolio?

Solution: The two possible stock prices are $S_u = 85$ and $S_d = 76$. So, the possible payoffs of the call are $V_u = 8$ and $V_d = 0$. The Δ of the call, thus, equals

$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{8 - 0}{85 - 76} = \frac{8}{9}\,. \tag{6.1}$$

Problem 6.4. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is \$50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a 45−strike call on the above stock. What is the risk-free investment in the replicating portfolio?

Solution: The two possible stock prices are $S_u = 52.5$ and $S_d = 45$. So, the possible payoffs of the call are $V_u = 7.5$ and $V_d = 0$. The risk-free investment B in the replicating portoflio of the call bull spread, thus, equals

$$B = e^{-rh}\frac{uV_d - dV_u}{u - d} = e^{-0.04}\frac{1.05(0) - 0.9(7.5)}{1.05 - 0.9} = -43.2355. \tag{6.2}$$

Problem 6.5. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

A **straddle** consists of a long call and a long otherwise identical put. Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

(a) About \$10
(b) About \$10.83

(c) About \$15.45
(d) About \$20.84
(e) None of the above.

Solution: (d)
The risk-neutral probability of an up movement is

$$p^* = \frac{95e^{0.06} - 75}{120 - 75} = 0.575.$$

So, the price of our straddle is

$$V(0) = e^{-0.06}[0.575 \times (120 - 100) + (1 - 0.575) * (100 - 75)] = 20.8366.$$