UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 2

Realized returns. Futures options. Currency options. Subjective expectations.

Problem 2.1. (5 pts) For a stock price that was initially \$55.00, what is the price after 4 years if the continuously compounded returns for these 4 years are $4.5\%, 6.2\%, 8.9\%$, and -3.2%?

Solution:

$$55e^{0.045+0.062+0.089-0.032} \approx 64.80.$$

Problem 2.2. (5 pts) A non-dividend-paying stock is valued at \$55.00. The annual expected return is 12.0% and the standard deviation of annualized returns is 22.0%. If the stock is lognormally distributed, what is the expected stock price after 3 years?

Solution: Let us denote the stock price today by $S(0)$ and that in three years by $S(3)$. According to the work we did in class, we need to calculate

$$\mathbb{E}[S(3)] = S(0)e^{3\alpha}$$

with α equal to the expected continuously compounded rate of return on the stock S. We are given in the problem that $\alpha = 0.12$. So, the answer is $55e^{0.36} \approx 78.83$.

Problem 2.3. (5 pts) For a stock price that was initially \$55.00, what is the price after 4 years if the observed continuously compounded returns for these 4 years are $4.5\%, 6.2\%, 8.9\%$, and 3.2%?

Solution:

$$55e^{0.045+0.062+0.089+0.032} \approx 69.08.$$

Problem 2.4. (10 points) Your goal is to price a call option on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is three months.
(ii) $u_F/d_F = 5/4$, where u_F is one plus the rate of gain on the futures price if it goes up, and d_F is one plus the rate of loss if it goes down.
(iii) The risk-neutral probability of an up move is $1/2$.
(iv) The initial futures price is 80.
(v) The continuously compounded risk-free interest rate is 5%.

Find the price of a half-year, 85-strike European call option on the futures contract.

Solution: We are given that

$$\frac{1}{2} = \frac{1-d_F}{u_F-d_F} = \frac{d_F^{-1}-1}{\frac{u_F}{d_F}-1} = \frac{d_F^{-1}-1}{\frac{5}{4}-1} \quad \Rightarrow \quad d_F^{-1} = \frac{9}{8} \quad \Rightarrow \quad d_F = \frac{8}{9} \quad \Rightarrow \quad u_F = \frac{10}{9}\,.$$

The prices in the futures-price tree are, thus,

$$\begin{array}{ccc} & & F_{uu} = 98.77 \\ & F_u = 88.89 & \\ F_0 = 80 & & F_{ud} = 79.01 \\ & F_d < 85 & \\ & & F_{dd} < 85 \end{array}$$

The option's price is

$$V_C(0) = e^{-0.025} \times \frac{1}{4} \times (98.77 - 85) = 3.3575.$$

Problem 2.5. (5 points) You are required to price a one-year, yen-denominated currency option on the USD. The exchange rate over the next year is modeled using a forward binomial tree with the number of periods equal to 4. Assume that the volatility of the exchange rate equals 0.1.

The continuously compounded risk-free interest rate for the yen equals 0.05, while the continuously compounded risk-free interest rate for the USD equals 0.02. What is the value of the so-called up factor u in the resulting forward binomial tree?

Solution:

$$u = e^{(r_{yen} - r_{\$})h + \sigma\sqrt{h}} = e^{(0.05-0.02)\frac{1}{4} + 0.1\sqrt{\frac{1}{4}}} = 1.0592.$$

Problem 2.6. (5 points) The evolution of a market index over the following year is modeled using a four-period binomial tree. We are given that the current value of the market index equals \$144, that its volatility equals 0.25, and that it pays dividends continuously.

You are tasked with constructing a four-period forward tree for the evolution over the following year of the forward price of the above market index with delivery at time−2.

What is the **down** factor d_F in the **forward price tree** for the **futures prices** on the stock?

Solution: In our usual notation,

$$d_F = de^{-(r-\delta)h} = e^{-\sigma\sqrt{h}} = e^{-0.25\sqrt{1/4}} = 0.8825.$$

Problem 2.7. (10 points) The evolution over the following year of futures prices with delivery at time 2 on a certain commodity are modeled using a one-period forward binomial tree. The volatility is given by 0.2. The continuously compounded risk-free interest rate is given to be 0.05.

Let the current futures price equal \$50. What is the price of a one-year, \$45-strike European put on the futures contract described above?

Solution: The up and down factors in the futures tree equal

$$u_F = e^{\sigma\sqrt{h}} = e^{0.2} = 1.2214, \quad d_F = e^{-\sigma\sqrt{h}} = 0.8187.$$

The futures price at the up node is

$$50 \times 1.2214 = 61.07.$$

The futures price at the down node is

$$50/1.2214 = 40.94.$$

So, the put gets exercised in the down node with the risk neutral probability of

$$1 - p^* = \frac{u_F - 1}{u_F - d_F} = 0.55.$$

So, the futures put option price is

$$V(0) = e^{-0.05} \times 0.55 \times (45 - 40.94) = 2.12345.$$

Problem 2.8. (5 points) The current price of a continuous-dividend-paying stock is \$80 per share. The stock's dividend yield is 0.02. According to your model, the expected value of the stock price in two years is \$90 per share. You are also given:

The risk-free interest rate exceeds the dividend yield.

The two-year forward price on a share of this stock is denoted by F. At this price you are willing to enter into the forward. What is the smallest range of values F can take according to the above information?

Solution: Using the fact that the investor is willing to enter a forward contract, we conclude that the forward contract's profit is positive. So,

$$\mathbb{E}[S(T)] > F \quad \Rightarrow \quad 90 > F$$

On the other hand, we know that

$$F = S(0)e^{(r-\delta)T} = 80e^{2(r-0.02)} > 80.$$

So, the most we can say about F is that $80 < F < 90$.