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UNIVERSITY OF TEXAS AT AUSTINHW Assignment 5The Black-Scholes pricing formula.

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Please, provide your **complete solution** to the following problem(s):

**Problem 5.1.** (2 points) Let the stock price  $S(t)$  be modeled using the lognormal distribution. Define  $Y(t) = S(t)^3$ . Then, the random variable  $Y(t)$  is lognormal itself. *True or false? Why?*

**Problem 5.2.** (2 pts) Let the stochastic process  $S = \{S(t), t \geq 0\}$  represent the stock price as in the Black-Scholes model. Let its volatility term be denoted by  $\sigma$ . Then, the volatility parameter of the process  $Y(t) = 2S(t)$  is  $4\sigma$ .

**Problem 5.3.** (2 points) Under the risk-neutral probability measure, every option on a particular stock has the continuously compounded, risk-free interest rate as its mean rate of return. *True or false? Why?*

**Problem 5.4.** (2 points) The Black-Scholes option pricing formula can **always** be used for pricing American-type call options on non-dividend-paying assets. *True or false? Why?*

**Problem 5.5.** (2 points) The Black-Scholes option pricing formula can always be used for pricing American-type options. *True or false?*

**Problem 5.6.** (20 points) Let  $S(0) = \$100$ ,  $K = \$120$ ,  $\sigma = 0.3$ ,  $r = 0.08$  and  $\delta = 0$ .

- (8 pts) Let  $V_C(0, T)$  denote the Black-Scholes European call price for the maturity  $T$ . Does the limit of  $V_C(0, T)$  as  $T \rightarrow \infty$  exist? If it does, what is it?
- (8 pts) Now, set  $\delta = 0.001$  and let  $V_C(0, T, \delta)$  denote the Black-Scholes European call price for the maturity  $T$ . Again, how does  $V_C(0, T, \delta)$  behave as  $T \rightarrow \infty$ ?
- (4 pts) Interpret in a sentence or two the differences, if there are any, between your answers to questions in a. and b.

**Problem 5.7.** (20 points) Let  $S(0) = \$120$ ,  $K = \$100$ ,  $\sigma = 0.3$ ,  $r = 0$  and  $\delta = 0.08$ .

- (10 pts) Let  $V_C(0, T)$  denote the Black-Scholes European call price for the maturity  $T$ . Does the limit of  $V_C(0, T)$  as  $T \rightarrow \infty$  exist? If it does, what is it?
- (8 pts) Now, set  $r = 0.001$  and let  $V_C(0, T, r)$  denote the Black-Scholes European call price for the maturity  $T$ . Again, how does  $V_C(0, T, r)$  behave as  $T \rightarrow \infty$ ?
- (2 pts) Interpret in a sentence or two the differences, if any, between your answers to questions in a. and b.