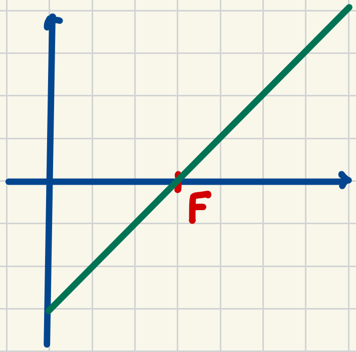
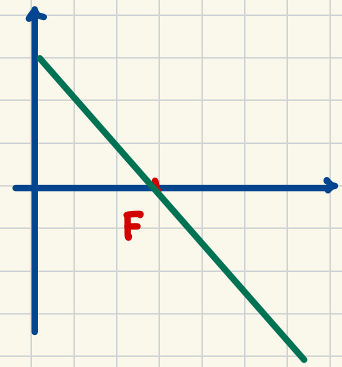
M339D: September 29th, 2025.

Review: An array of payoff curves.

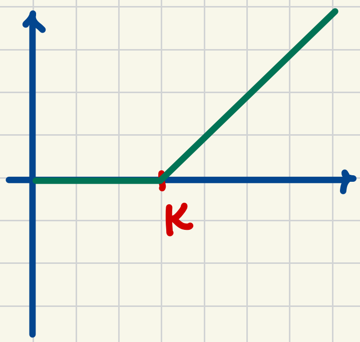
Long Forward $S(T)-F$



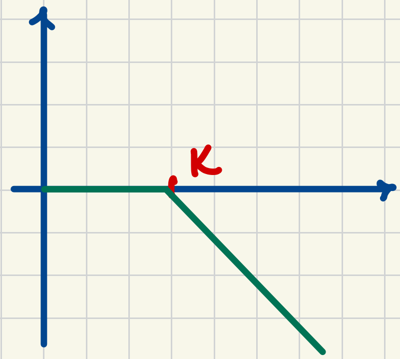
Short Forward $F-S(T)$



Long Call. $(S(T)-K)_+$

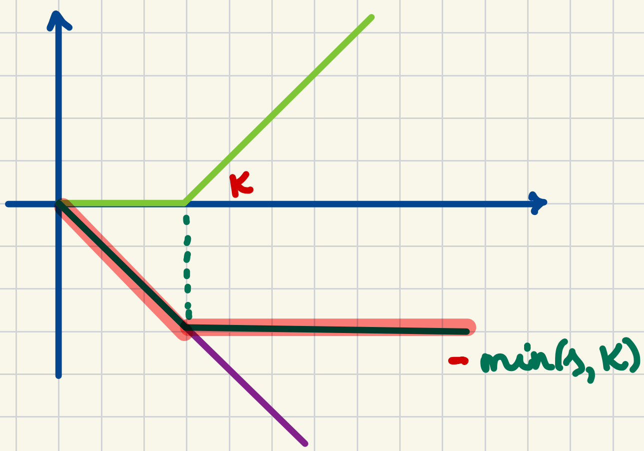


Short/Written Call. $-(S(T)-K)_+$



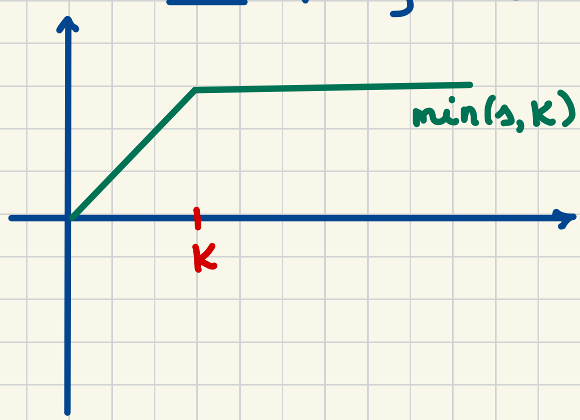
Cap.

$\begin{cases} \text{Long Call} \\ \text{Short Stock} \end{cases}$

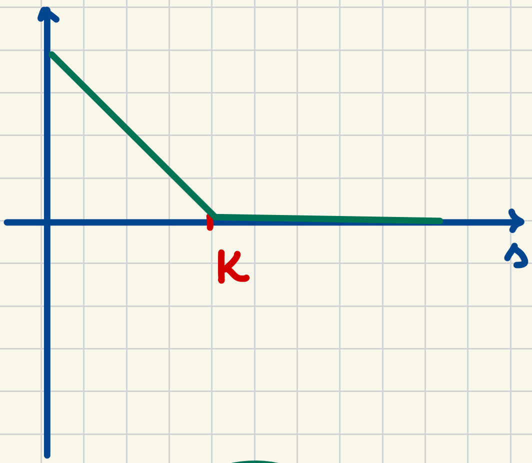


Covered Call.

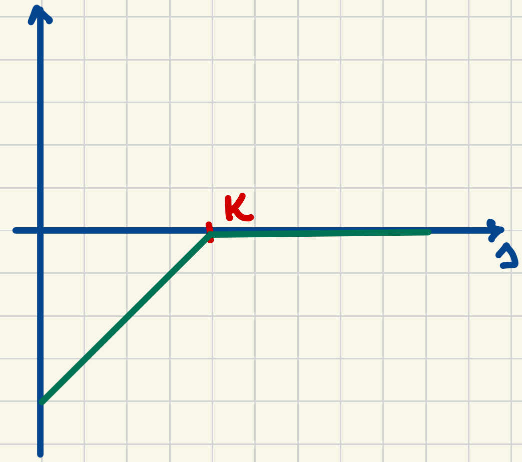
$\begin{cases} \text{Short Call} \\ \text{Long Stock} \end{cases}$



Long Put. $(K-S(T))_+$

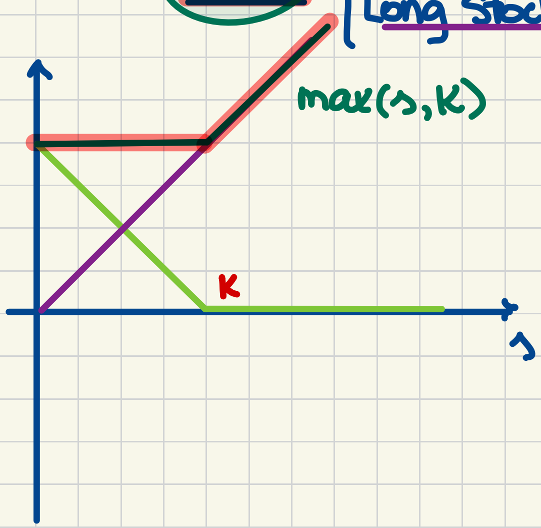


Short Put. $-(K-S(T))_+$



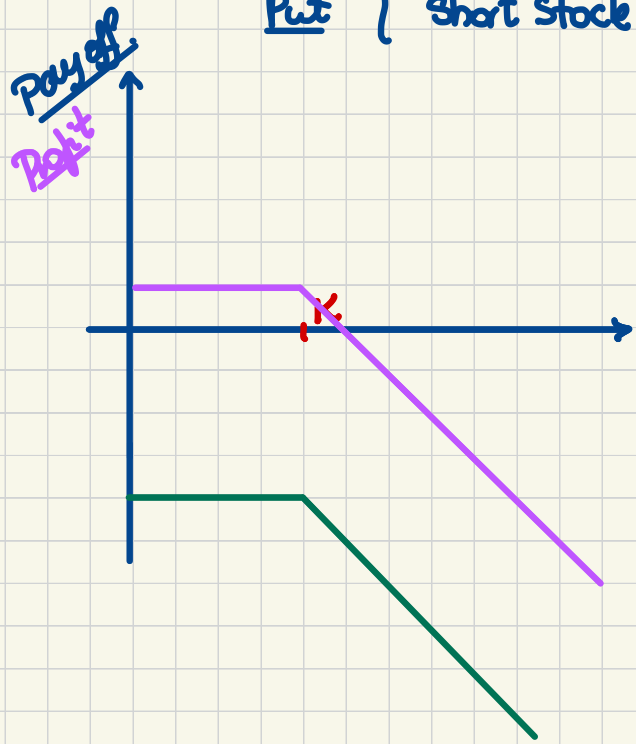
Floor.

$\left\{ \begin{array}{l} \text{Long Put} \\ \text{Long Stock} \end{array} \right.$



Covered Put

$\left\{ \begin{array}{l} \text{Short Put} \\ \text{Short Stock} \end{array} \right.$



Finite Probability Spaces.

... serve as environments for the possible paths that asset prices can take.

e.g.,

$$S(T) \sim \begin{cases} 120 & \text{w/ probab. } 1/6 \\ 80 & \text{w/ probab. } 1/2 \\ 50 & \text{w/ probab. } 1/3 \end{cases}$$

Q: What is the expected payoff of a put w/ strike 105?

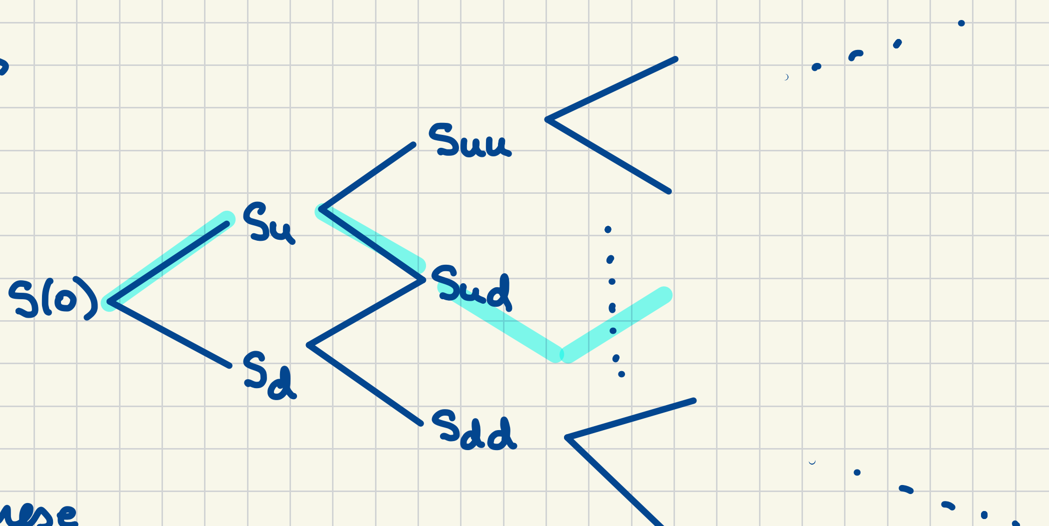
$\longrightarrow$: $\boxed{V_P(T) = (K - S(T))_+}$

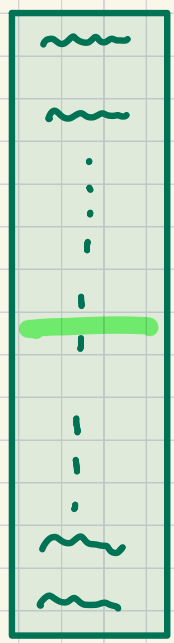
$$V_P(T) \sim \begin{cases} 0 & \text{w/ probab. } 1/6 \\ 25 & \text{w/ probab. } 1/2 \\ 55 & \text{w/ probab. } 1/3 \end{cases}$$

$$\mathbb{E}[V_P(T)] = 0 \cdot \frac{1}{6} + 25 \cdot \frac{1}{2} + 55 \cdot \frac{1}{3} = \ldots \quad \square$$

$\boxed{\text{Caveat:}}$ $\boxed{\mathbb{E}[g(X)] \neq g(\mathbb{E}[X])}$

e.g.,





All these finitely many scenarios are called states of the world.

We assume that:

- each can happen, i.e., probab > 0

and

- they exhaust all possibilities, i.e., $\sum$ probab = 1.