M339D: February 9^{th}, 2024.

About Project #1.

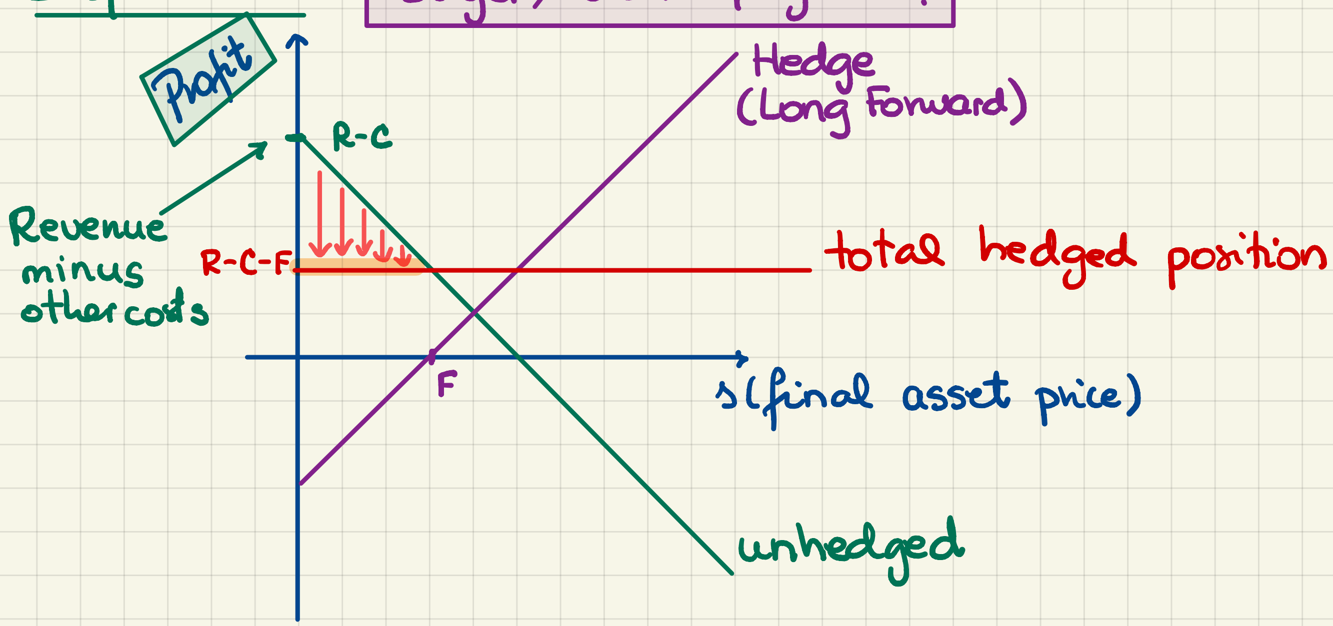
$\boxed{\text{Your rule:}}$ Keep 1/2 of wealth in stock.

inputs of function: wealth, s.end

output of function: 0.5*wealth /s.end

Inspiration.

Buyer/User of goods.

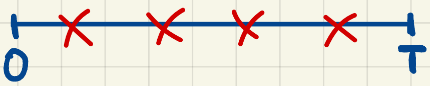


European Call Options.

↓ The option can be exercised, i.e., the cashflow can be collected only on the exercise date.

Usually, this means a right to buy the underlying asset.

Usually, the option's owner has the right but not an obligation to exercise the option.



Option written. EXERCISE DATE

At time 0:

- The writer of the option writes/shorts the call.
- The buyer of the option is said to long the call. They are referred to as the option's owner.
- The agreement:
  - the underlying asset: $S(t), t \geq 0$
  - the exercise date: T
  - K... the strike/exercise price
- $\boxed{\text{The buyer pays the premium to the writer.}}$

  $\boxed{V_c(0)}$

At time T:

- The call's owner has a right, but not an obligation to buy one unit of the underlying asset for the strike price K.
- The call's writer is obligated to do what the owner opts for.

Payoff = ?

We focus on the payoff of the long call, i.e., the payoff for the call's owner.

The call owner's rationale for whether to exercise is "maximising money in".

The criterion for exercise:

IF $S(T) \geq K$, <u>then</u> EXERCISE. $\Rightarrow$ Payoff $= S(T) - K$

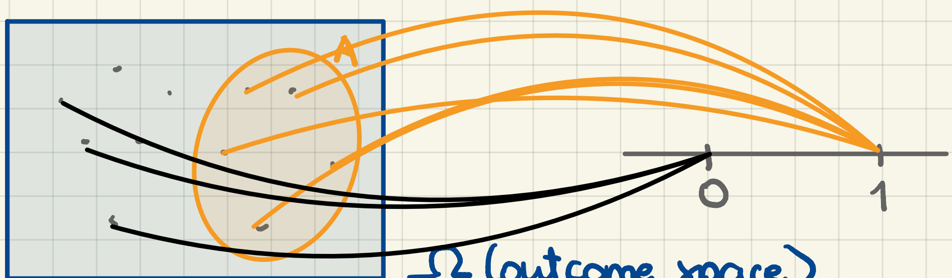
IF $S(T) < K$, <u>then</u> do NOT EXERCISE. $\Rightarrow$ Payoff $= 0$

<u>We introduce</u>:

$V_c(T)$ ... the random variable denoting the payoff of a long call

$$\Rightarrow \boxed{V_c(T) = \begin{cases} S(T) - K & \text{if } \boxed{S(T) \geq K} \\ 0 & \text{if } S(T) < K \end{cases}}$$

<u>Indicator Random Variables:</u>



Ω (outcome space)

ω ... elementary outcomes

A ... a "nice" subset of Ω, aka an <u>event</u>

<u>We define</u>:

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if A happened} \\ 0 & \text{if A did not happen} \end{cases}$$

$$\boxed{V_c(T) = (S(T) - K) \cdot \mathbb{I}_{[S(T) \geq K]}}$$

<u>Also</u>:

$$\boxed{V_c(T) = \max(S(T) - K, 0) = (S(T) - K) \vee 0}$$

maximum operator

Introduce:

The positive part function

$$x \longmapsto (x)_+ =: \max(x,0) = x \vee 0$$

$\Rightarrow$ $$\boxed{V_c(T) = (S(T) - K)_+}$$

$\Rightarrow$ the payoff f'tion: $\boxed{v_c(s) = (s-K)_+}$