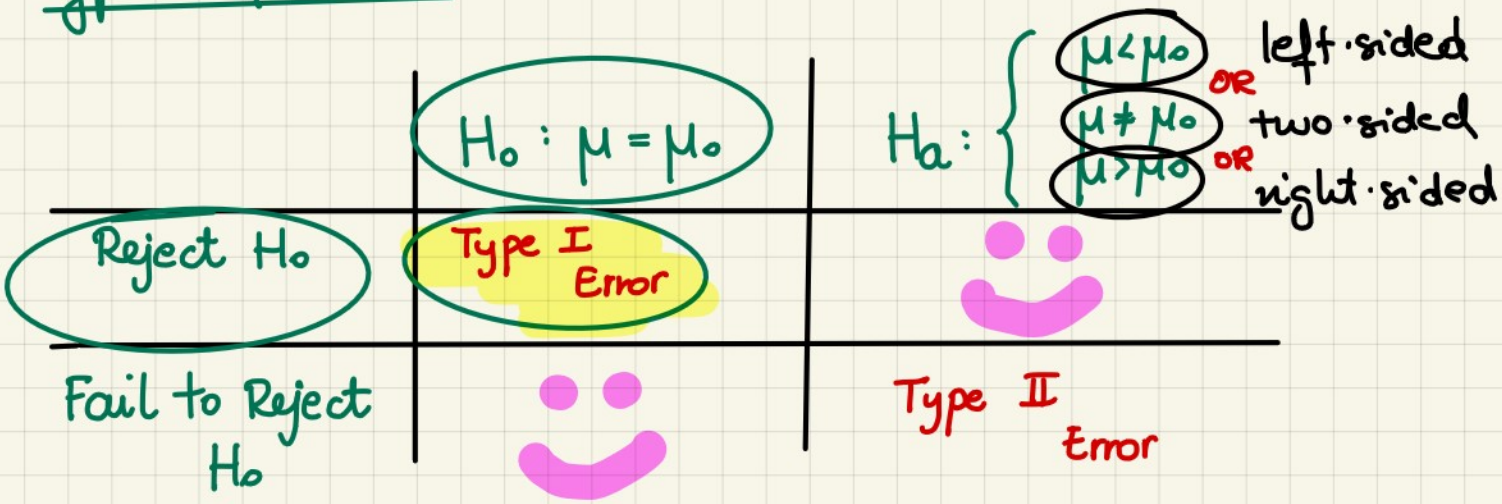
M358K: October 21st, 2020.

Types of Errors.



$$\mathbb{P}[\text{Type I Error}] = \mathbb{P}_0[\text{Reject } H_0] = \alpha$$

$\mathbb{P}_0$: under the null, i.e., if $\mu = \mu_0$

α: significance level

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Problem Set # 9

Types of errors.

Provide your **final answer only** for the following problems.

Problem 9.1. You perform 2000 significance tests using a significance level 0.10. Under the assumption that all of the null hypotheses for the 2000 significance tests are true, how many of the 2000 significance tests would you expect to **not** result in a Type I error?

a. 200
b. 1800
c. 2000
d. 0
e. None of the above.

The expected number of tests resulting in a Type I Error: $2000(0.10) = 200$.

Problem 9.2. A medical researcher is working on a new treatment for a certain type of cancer. The average survival time after diagnosis on the standard treatment is 2 years. In an early trial, she tries the new treatment on three subjects who have an average survival time after diagnosis of 4 years. Although the survival time has doubled, the results are not statistically significant, even at the 0.10 significance level.

Suppose, in fact, that the new treatment **does** increase the mean survival time in the population of all patients with this particular type of cancer. Which of the following statements is TRUE?

a. A Type I error occurred.
b. A Type II error occurred.
c. No error occurred.

Problem 9.3. An engineer has designed an improved light bulb. The previous design had a mean lifetime of 1200 hours. Using a sample of 2000 of the new bulbs, the sample average lifetime of this improved light bulb is found to be 1201 hours. Although the difference is quite small, the effect was statistically significant at the 0.05 level. Suppose that, in fact, there is no difference between the mean lifetimes of the previous design and the new design. Which of the following statements is TRUE?

a. A Type I error occurred.
b. A Type II error occurred.
c. No error occurred.

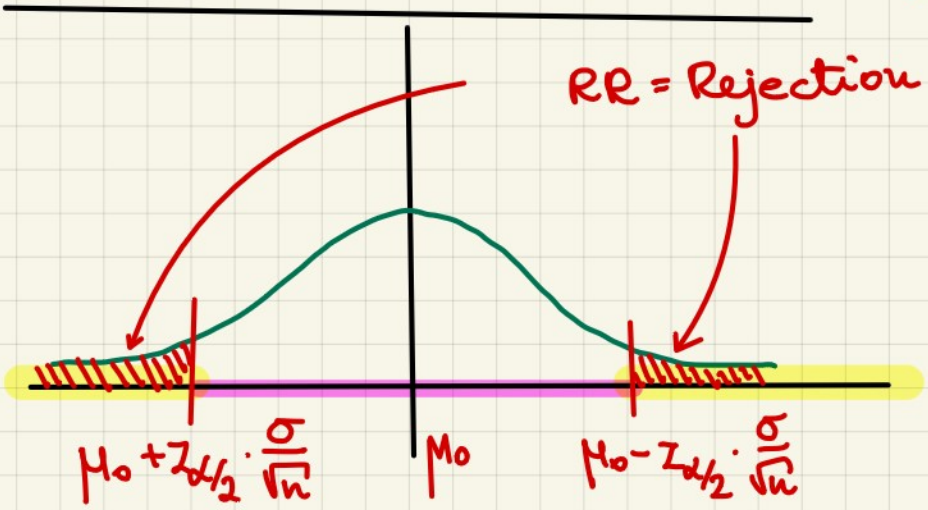
Power of Test.

Hypotheses:

$H_0: \mu = \mu_0$ vs. $H_a:$ $\mu < \mu_0$ left-sided; $\mu \neq \mu_0$ two-sided; $\mu > \mu_0$ right-sided.

Focus on the two-sided test:

α ... significance level



w/ $z_{\alpha/2} = \Phi^{-1}(\alpha/2)$

For instance, for $\alpha = 0.10$:

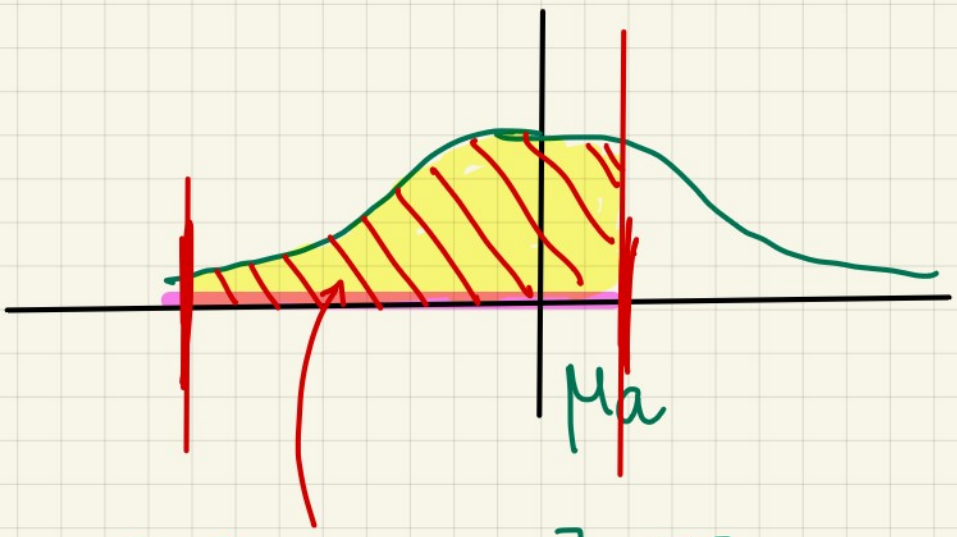
$$z_{\alpha/2} = z_{0.05} = -1.645 = \text{qnorm}(0.05)$$

$$RR = (-\infty, \mu_0 - 1.645\left(\frac{\sigma}{\sqrt{n}}\right)] \cup [\mu_0 + 1.645\left(\frac{\sigma}{\sqrt{n}}\right), +\infty)$$

Now, focus on the "fail-to-reject" region, i.e.,

$$\left(\mu_0 - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right), \mu_0 + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right)$$

Look @ one particular value from the alternative. Call it μ_a. What is the probability of failing to reject if that particular μ_a is the "true" value of the parameter?



$$\mathbb{P}_{\mu_a}[\text{Fail to Reject}] = \mathbb{P}[\text{Type II Error}] =: \beta$$

Note: The smaller the value of β, the better your test is!

Def'n: The power of the test w/ $\mu = \mu_a$ is defined as $1-\beta$.

Problem: A simple random sample of size 36 is gathered from a normal population w/ an unknown mean μ and a known standard deviation of 3. The sample average is 17.

We are testing:

$H_0: \mu = 15$ vs. $H_a: \mu > 15$

The significance level is 0.05.

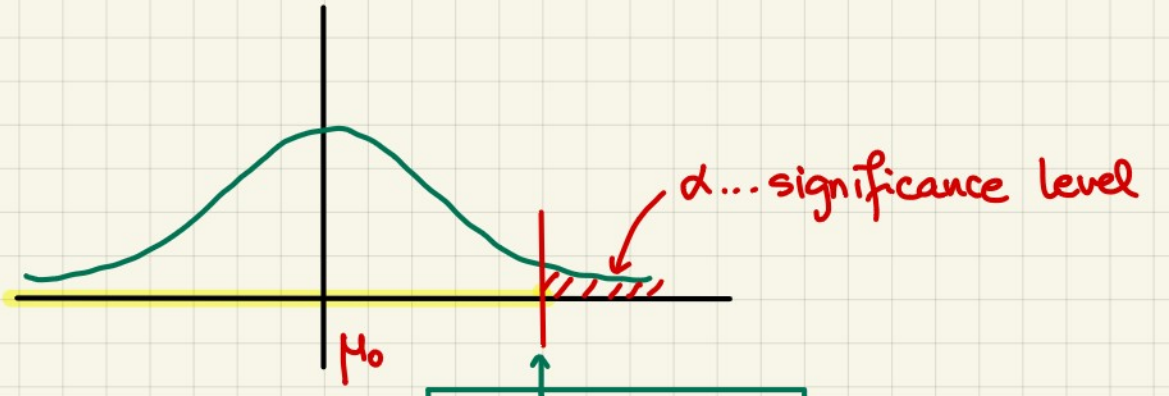
Find the power of this test @ $\mu_a = 16$.

→ : First: find the rejection region.

Second: find the probability that the sample mean hits the rejection region for $\mu = \mu_a = 16$.

From the structure of the alternative, we know that in raw units, the RR looks like:

$$RR = [\ ? \ , +\infty)$$ ✓



$$\mu_0 + z_{1-\alpha} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$$ ✓

w/ $z_{1-\alpha} = \Phi^{-1}(1-\alpha)$

In this problem:

$$RR = [\ 15 + 1.645\left(\frac{3}{\sqrt{36}}\right) , +\infty) = [15.8225 , +\infty).$$