04/17/2023.

Black-Scholes Practice.

Problem. Assume the Black-Scholes model.

For a European call option, the strike is $\boxed{S(0)e^{rT}}$ w/ T being the exercise date.

The price of a call option w/ $\boxed{\text{one year}}$ to exercise is $\boxed{0.6 \cdot S(0)}$.

Find the price of call option w/ $\boxed{\text{three months}}$ to exercise in terms of $S(0)$.

→ : For any T:

$$d_1 = \frac{\sigma\sqrt{T}}{2} = -d_2$$

$$V_c(0,T) = S(0)N\left(\frac{\sigma\sqrt{T}}{2}\right) - S(0)\cancel{e^{rT}}\cancel{e^{-rT}}N\left(-\frac{\sigma\sqrt{T}}{2}\right)$$

$$\boxed{V_c(0,T) = S(0)\left(2 \cdot N\left(\frac{\sigma\sqrt{T}}{2}\right) - 1\right)}$$ ✓

For $T=1$:

$$V_c(0,T=1) = 0.6 \cdot \cancel{S(0)} = \cancel{S(0)}\left(2 \cdot N\left(\frac{\sigma}{2}\right) - 1\right)$$

$$2N\left(\frac{\sigma}{2}\right) = 1.6 \Rightarrow N\left(\frac{\sigma}{2}\right) = 0.8$$

$$\Rightarrow \frac{\sigma}{2} = 0.84$$

For $T=\frac{1}{4}$:

$$V_c(0,T=\tfrac{1}{4}) = S(0)\left(2 \cdot N\left(\frac{\sigma\sqrt{\frac{1}{4}}}{2}\right) - 1\right)$$

$$= S(0)\left(2 \cdot N\left(\underbrace{0.84\left(\tfrac{1}{2}\right)}_{0.42}\right) - 1\right)$$

$$= S(0)\left(2 \cdot 0.6628 - 1\right)$$

$$= S(0) \cdot \underline{0.3256}$$

□

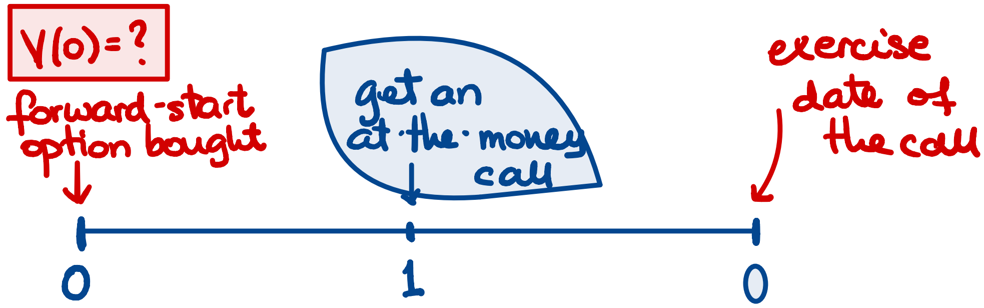
(A) 586

(B) 594

(C) 684

(D) 692

(E) 797



19. Consider a *forward start option* which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.

You are given:

(i) The European call option is on a stock that pays no dividends.

(ii) The stock's volatility is 30%. $\boxed{\sigma = 0.3}$

(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100. $\boxed{F_{0,1}(S) = 100}$

(iv) The continuously compounded risk-free interest rate is 8%. $r = 0.08$

Under the Black-Scholes framework, determine the price today of the forward start option.

(A) 11.90

(B) 13.10

(C) 14.50

(D) 15.70

(E) 16.80

At $t<T$:

$$V_c(t) = S(t)N(d_1(t)) - Ke^{-r(T-t)} \cdot N(d_2(t))$$

w/

$$d_1(t) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S(t)}{K}\right) + \left(r+\frac{\sigma^2}{2}\right)(T-t)\right] \star$$

and

$$d_2(t) = d_1(t) - \sigma\sqrt{T-t}$$

In this problem: $\boxed{t=1}$

$$V_c(1) = S(1)\cdot N(d_1(1)) - S(1)e^{-r(2-1)} \cdot N(d_2(1))$$

$$\boxed{V_c(1) = S(1)\left[N(d_1(1)) - e^{-r} \cdot N(d_2(1))\right]} \leftarrow$$

$$\text{w/ } d_1(1) = \frac{1}{0.3\sqrt{2-1}}\left[\cancel{\ln\left(\frac{S(1)}{S(1)}\right)} + \left(0.08 + \frac{0.09}{2}\right)\cdot(2-1)\right]$$

at-the-money

$$d_1(1) = \frac{0.08+0.045}{0.3} = \underline{0.4167}$$

$$d_2(1) = 0.4167 - 0.3\sqrt{2-1} = 0.1167$$

$$N(d_1(1)) = \text{pnorm}(0.4167) = \underline{0.66155}$$

$$N(d_2(1)) = \text{pnorm}(0.1167) = \underline{0.54645}$$

$$V_c(1) = S(1)\left(0.66155 - e^{-0.08} \cdot 0.54645\right) = S(1) \cdot \underline{0.15711}$$

At time 0, our forward start option is worth:

$$\boxed{0.15711 \cdot S(0)}$$

Our answer : $0.15711 \cdot \boxed{e^{-0.08} \cdot 100} = 14.50337$ ☐

We know $F_{0,1}(S) = S(0)e^r$

$$\Rightarrow \boxed{S(0) = F_{0,1}(S) \cdot e^{-r} = 100 \cdot e^{-0.08}}$$

33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

You are given:

(i) The continuously compounded risk-free interest rate is 8%.

(ii) The stock's volatility is 30%.

(iii) The current stock price is 45.

(iv) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

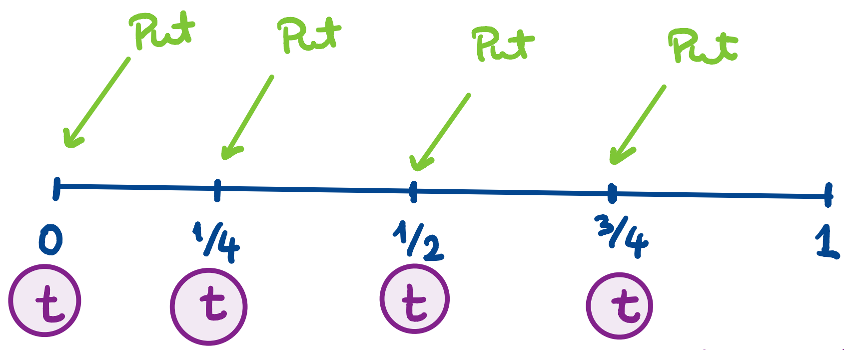
(A) 1.59

(B) 2.24

(C) 2.86

(D) .48

(E) 3.61



t… represents the valuation dates

34-39. DELETED

For each of the four puts in the rolling insurance strategy:

- one quarter-year to exercise
- $K_t = 0.9 \cdot S(t)$

For every t @ which a put option is received:

$$d_1(t) = \frac{1}{\sigma\sqrt{1/4}}\left[\ln\left(\frac{\cancel{S(t)}}{0.9\cdot \cancel{S(t)}}\right) + \left(r+\frac{\sigma^2}{2}\right)\left(\frac{1}{4}\right)\right]$$

$$d_1(t) = \frac{1}{0.3(0.5)}\left[-\ln(0.9) + \left(0.08 + \frac{0.09}{2}\right)\cdot(0.25)\right]$$

$d_1(\cancel{t}) = 0.9107$ $\qquad d_2 = 0.7607$

$N(-d_1) = \text{pnorm}(-0.9107) = \boxed{0.181217}$

$N(-d_2) = \text{pnorm}(-0.7607) = \boxed{0.2234072}$

$$V_p(t) = 0.9\,S(t)\,e^{-0.08(0.25)} \cdot 0.2234072 - S(t) \cdot 0.181217$$

$$V_p(t) = S(t)\Big(\underbrace{0.9 \cdot e^{-0.02} \cdot 0.2234072 - 0.181217}_{0.01587}\Big)$$

=> Note that for every "put·delivery" date $t = 0, 1/4, 1/2, 3/4$ the value of the put @ that time is equal to

$$\boxed{0.01587 \cdot S(t)}$$

Altogether:

$$\underbrace{4}_{\#\text{ of puts}} \cdot 0.01587 \cdot 45 \approx 2.86$$

is the price of the rolling insurance strategy. $\square$