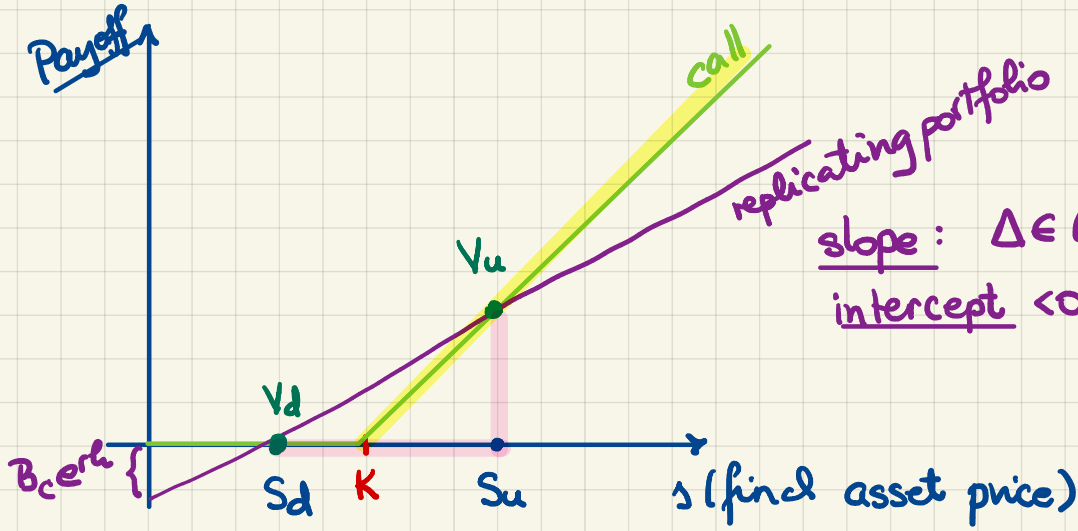


<u>slope</u> : $\Delta \in (0,1)$ => buying a fraction of a share

<u>intercept</u> <0 => borrowing

Problem 9.3. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$80 per share. In the model, it is assumed that the stock price can either go up by \$5 or down by \$4.

You use the binomial tree to construct a replicating portfolio for a 78-strike call option on the above stock. What is the stock investment in the replicating portfolio?

$K=78$

$$\Delta = \frac{V_u - V_d}{S_u - S_d}$$

Payoff

$S(0)$

$S_u = 85$ $\qquad V_u = (85-78)_+ = 7$

$S_d = 76$ $\qquad V_d = (76-78)_+ = 0$

$$\Delta = \frac{7-0}{85-76} = \frac{7}{9}$$

$\square$

Problem 9.4. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is $50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a 45−strike call on the above stock. What is the risk-free investment in the replicating portfolio?

$\longrightarrow$:

Payoff:

$S(0) = 50$

$S_u = 50(1.05) = 52.5$ $\quad V_u = (52.5-45)_+ = 7.5$

$S_d = 50(0.9) = 45$ $\quad V_d = (45-45)_+ = 0$

$$B = e^{-rh} \cdot \frac{u \cdot V_d - d V_u}{u-d} = e^{-0.04} \cdot \frac{1.05(0) - 0.9(7.5)}{1.05 - 0.9}$$

$$= \dots = \underline{-43.2355}$$

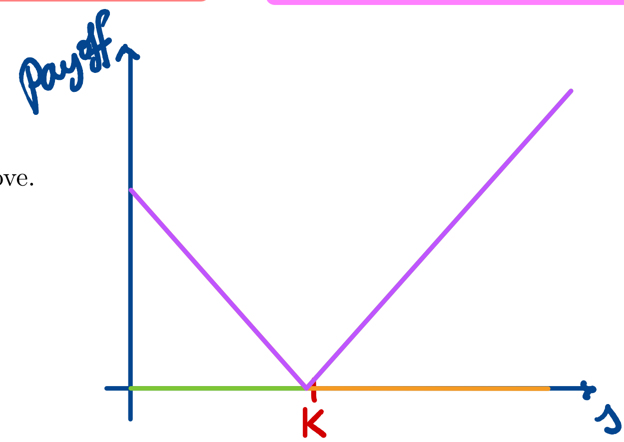
$\square$

Problem 9.5. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120 or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

A **straddle** consists of a long call and a long otherwise identical put. Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

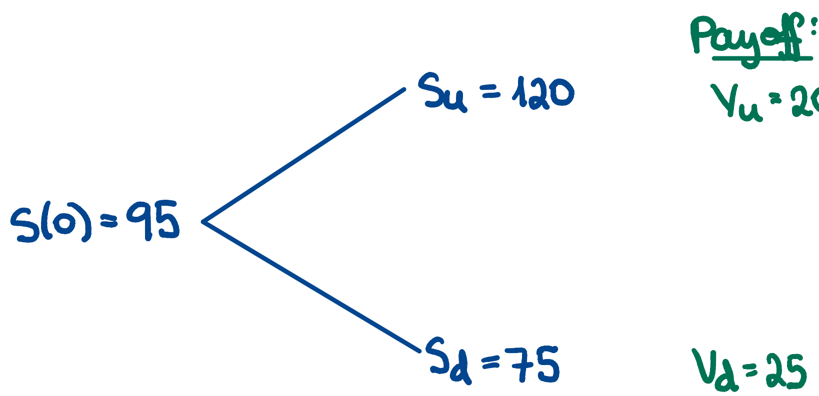
(a) About \$10
(b) About \$10.83
(c) About \$15.45
(d) About \$20.84
(e) None of the above.



Payoff f'tion of a straddle:

$$\boxed{v(s) = |s-K|}$$

In this problem: $\boxed{K=100}$



Replicating Portfolio:

$$\begin{cases} \Delta = \dfrac{V_u - V_d}{S_u - S_d} = \dfrac{20-25}{120-75} = -\dfrac{5}{45} = -\dfrac{1}{9} \\ B = e^{-rh} \cdot \dfrac{u \cdot V_d - d \cdot V_u}{u-d} = \end{cases}$$

$$= e^{-0.06} \cdot \frac{\frac{120}{\cancel{95}} \cdot 25 - \frac{75}{\cancel{95}} \cdot 20}{\frac{120}{\cancel{95}} - \frac{75}{\cancel{95}}} = e^{-0.06} \cdot \frac{120(25) - 75(20)}{45}$$

$$= \underline{31.392}$$

$$V(0) = \Delta \cdot S(0) + B = -\frac{1}{9}(95) + 31.392 = \underline{20.83} \quad \square$$

Start w/

$$V(0) = \Delta \cdot S(0) + B$$

$$V(0) = \frac{V_u - V_d}{\underbrace{S_u - S_d}_{\cancel{S(0)}(u-d)}} \cdot \cancel{S(0)} + e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u-d}$$

$$V(0) = \frac{1}{u-d}\left[V_u - V_d + e^{-rh}(u \cdot V_d - d \cdot V_u)\right]$$

$$V(0) = e^{-rh} \cdot \frac{1}{u-d}\left[e^{rh} \cdot V_u - e^{rh} \cdot V_d + u \cdot V_d - d \cdot V_u\right]$$

$$V(0) = e^{-rh} \cdot \frac{1}{u-d}\left[V_u(e^{rh} - d) + V_d(u - e^{rh})\right]$$