M358K : October 11th, 2023.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 9

Confidence interval for the mean (the normal case).

Provide your **complete solutions** for the following problems.

Problem 9.1. A student uses computer software to generate 10 random numbers from a N(500, 100) distribution. From these ten numbers, she computes a 95% confidence interval. She then repeats this process (generating a new set of 10 random numbers from a N(500, 100) distribution each time) until she has produced 1000 such intervals. Which of the following is true?

a. Approximately 95% of the intervals will contain the value 500.
b. Approximately 95% of the intervals will contain the value 100.
c. Approximately 97.5% of the intervals will contain the true mean because the probability that a standard normal random variable is less than 1.96 is 0.975. She incorrectly used the formula for a 97.5% confidence interval.

Problem 9.2. A random sample is collected of size n from a population with standard deviation σ and with the data collected a 95% confidence interval is computed for the mean of the population. Which of the following would produce a new confidence interval with smaller width (smaller margin of error) based on these same data?

a. Increase σ.
b. Use a smaller confidence level.
c. Use a smaller sample size.

Problem 9.3. A random sample of 85 students in Chicago city high schools take a course designed to improve SAT scores. Based on these students, a 90%-confidence interval for the mean improvement in SAT scores for all Chicago city high school students is computed as $(72.3, 91.4)$ points. The correct interpretation of this interval is ...

a. 90% of the students in the sample improved their scores by between 72.3 and 91.4 points.
b. 90% of the students in the population should improve their scores by between 72.3 and 91.4 points.
c. Neither of the above.

Problem 9.4. You want to construct a 92% confidence interval. The correct z^* to use is ...

→ : qnorm(1.92/2) = 1.750686

Problem 9.5. *How well does a new medication reduce blood pressure relative to baseline?*

One hundred patients had their blood pressure measured before and after taking the drug. The average reduction in blood pressure for these patients is observed to be $\boxed{\bar{x} = 30}$ mm Hg. Assume that the reduction in blood pressure for the new medication follows a normal distribution with unknown mean μ and standard deviation $\sigma = 10$ mm Hg. A 90% confidence interval for μ is ...

a. 30 ± 1.645

✗ **b.** 30 ± 1.96 95% conf. int

✗ **c.** 30 ± 16.45 forget $\sqrt{n}$

pt. estimate ± margin of error

$\bar{x} = 30$

$z^* \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{100}} = 1$

qnorm(1.9/2) = 1.644854

Problem 9.6. To assess the accuracy of a kitchen scale, a standard weight known to weigh 1 gram is weighed a total of n times and the sample average $\bar{x}$ of the weighings is computed. Suppose the scale readings are normally distributed with unknown mean μ and standard deviation $\sigma = 0.01$ g. How large should the sample size n be so that a 90% confidence interval for μ has a margin of error of ± 0.0001?

a. 165

b. 27061

c. 38416

→ : $z^* = 1.645$

$$m.e. = 1.645\left(\frac{0.01}{\sqrt{n}}\right) \leq 0.0001$$

$$1.645 \leq 0.01\sqrt{n} \quad / \cdot 100$$

$$164.5 \leq \sqrt{n} \quad /^2$$

$$n \geq 27061$$

Problem 9.7. A hospital administrator wishes to study the mean birth weight of babies born at a particular hospital in 2014. She extracts the birth weights of all births in the hospital in 2014 and computes the 95% interval as (3200, 3700) grams. She wants to use this interval to describe the mean birth weight of babies born at this hospital in 2014.

What is your comment on this finding? Do you find it useful for the particular stated purpose? Why?

Hypothesis Testing.

Proof by Contradiction.

K... claim that we're trying to PROVE to be true

Q: What if K were not true?

Assume not K

fact Q ...... fact (not Q)

They cannot coexist!

We say we have reached a CONTRADICTION!

$\Rightarrow\Leftarrow$

↯

Our assumption of not K was wrong!

Hypothesis Testing.

Claim we're trying to SUBSTANTIATE.

μ.. the mean population parameter
(mean cholesterol level after new treatment)

μ_0... the null mean (a number)
(mean cholesterol lever before treatment)

$$\boxed{\mu < \mu_0}$$

Assume $\boxed{\mu = \mu_0}$

collect data
statistical analysis

Figure out the probability of seeing the data that we saw (or something more extreme) if $\boxed{\mu = \mu_0}$