

UNIVERSITY OF TEXAS AT AUSTIN

Problem set 1

Problem 1.1. Let E and F be any two events. Then, $\mathbb{P}[E \cup F] \leq \mathbb{P}[E] + \mathbb{P}[F]$. *True or false? Why?*

Problem 1.2. Let E and F be any two events. If $\mathbb{P}[E] = \mathbb{P}[F] = \frac{2}{3}$, then E and F cannot be mutually exclusive. *True or false? Why?*

Problem 1.3. Let E and F be any two events with positive probability. If $\mathbb{P}[E|F] < \mathbb{P}[E]$, then $\mathbb{P}[F|E] < \mathbb{P}[F]$. *True or false? Why?*

Problem 1.4. If events E and F are independent and events F and G are independent, then E and G are independent as well. *True or false? Why?*

Problem 1.5. The four standard blood types are distributed in a populations as follows:

$$\begin{array}{ll} A - 42\% & O - 33\% \\ B - 18\% & AB - 7\% \end{array}$$

Assuming that people choose their mates independently of their blood type, find the probability that the people in a randomly chosen couple from this population have different blood types.

Problem 1.6. Let X denote the outcome of a roll of a fair, regular icosahedron (a polyhedron with 20 faces) with numbers $1, 2, \dots, 20$ written on its sides. Then $\mathbb{E}[X] = 15/2$. *True or false? Why?*

Problem 1.7. The minimum of two independent exponential random variables is also exponential. *True or false? Why?*