M339D : November 15th, 2023.

Problem 12.3. (5 pts) Assume the Black-Scholes framework. Let the current price of a non-dividend-paying stock be equal to $S(0) = 95$ and let its volatility be equal to 0.35. Consider a European call on that stock with strike 100 and exercise date in 9 months. Let the risk-free continuously compounded interest rate be 6% per annum.

Denote the price of the call by $V_C(0)$. Then,

(a) $V_C(0) < \$5.20$

(b) $\$5.20 \leq V_C(0) < \$7.69$

(c) $\$7.69 \leq V_C(0) < \$9.04$

(d) $9.04 \leq V_C(0) < \$11.25$

(e) None of the above.

→ : 1st Calculate d_1 and d_2.

2nd Use the standard normal tables or R.

3rd Final pricing formula.

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T\right]$$

$$d_1 = \frac{1}{0.35\sqrt{3/4}}\left[\ln\left(\frac{95}{100}\right) + \left(0.06 + \frac{(0.35)^2}{2}\right)\cdot 0.75\right] = 0.13079 \approx 0.13$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.13079 - 0.35\sqrt{0.75} = -0.1723 \approx -0.17$$

$$N(d_1) = N(0.13) = 0.5517$$

$$N(d_2) = N(-0.17) = 0.4325$$

$$V_c(0) = S(0)N(d_1) - Ke^{-rT}N(d_2)$$

$$V_c(0) = 95 \cdot 0.5517 - 100e^{-0.06(0.75)} \cdot 0.4325$$

$$V_c(0) = 11.06$$

□

Problem 12.4. Assume the Black-Scholes setting. Let $S(0) = \$63.75$, $\sigma = 0.20$, $r = 0.055$. The stock pays no dividend and the option expires in 50 days (simplify the number of days in a year to 360).

What is the price of a \$60-strike European put?

(a) 0.66
(b) 0.55
(c) 0.44
(d) 0.37
(e) None of the above.

→ :

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T\right]$$

$$d_1 = \frac{1}{0.2\sqrt{\frac{5\cancel{0}}{36\cancel{0}}}}\left[\ln\left(\frac{63.75}{60}\right) + \left(0.055 + \frac{0.04}{2}\right)\cdot\frac{5\cancel{0}}{36\cancel{0}}\right]$$

$$d_1 = \underline{0.9531189} \approx 0.95$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.953189 - 0.2\sqrt{\frac{5}{36}} = \underline{0.8785833}$$

$$\Rightarrow d_2 \approx 0.88$$

$$N(-0.95) = \underline{0.1711}$$

$$N(-0.88) = \underline{0.1894}$$

$$V_P(0) = Ke^{-rT}\cdot N(-d_2) - S(0)N(-d_1)$$

$$V_P(0) = 60e^{-0.055\left(\frac{50}{360}\right)}\cdot 0.1894 - 63.75\cdot 0.1711$$

$$V_P(0) = \underline{0.3699}$$

$\square$

8. Let $S(t)$ denote the price at time t of a stock that pays no dividends. The Black-Scholes framework holds. Consider a European call option with exercise date T, $T>0$, and exercise price $S(0)e^{rT}$, where r is the continuously compounded risk-free interest rate.

$\boxed{K = S(0)e^{rT}}$

You are given:

(i) $S(0) = \$100$

(ii) $T = 10$

(iii) $\mathrm{Var}[\ln S(t)] = 0.4t, \quad t>0.$ $\longrightarrow$ $\boxed{\sigma = \sqrt{0.4}}$

Determine the price of the call option.

(A) \$7.96

(B) \$24.82

(C) \$68.26

(D) \$95.44

(E) There is not enough information to solve the problem.

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{\cancel{S(0)}}{\underbrace{K}_{\cancel{S(0)}e^{rT}}}\right) + \left(r+\frac{\sigma^2}{2}\right)\cdot T\right]$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[-\cancel{rT} + \cancel{rT} + \frac{\sigma^2 \cdot T}{2}\right] = \frac{\sigma\sqrt{T}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{\sigma\sqrt{T}}{2} - \sigma\sqrt{T} = -\frac{\sigma\sqrt{T}}{2}$$

$\sigma = \sqrt{0.4}, \quad T = 10$

$$d_1 = \frac{\sqrt{0.4}\sqrt{10}}{2} = 1 \quad \text{and} \quad d_2 = -1$$

$$V_c(0) = S(0)N(d_1) - Ke^{-rT}N(d_2)$$

$$V_c(0) = S(0)\,N(1) - S(0)\cancel{e^{rT}}\cdot\cancel{e^{-rT}}\cdot N(-1)$$

$$V_c(0) = S(0)\left(N(1) - \underbrace{N(-1)}_{1-N(1)}\right) = S(0)\left(2\cdot\underbrace{N(1)}_{0.8413} - 1\right)$$

$$V_c(0) = 100\,(2\cdot 0.8413 - 1) = 68.26 \qquad \square$$

Problem. Assume the Black-Scholes model.

For a European call option, the strike is $S(0)e^{rT}$ w/ T being the exercise date.

The price of a call option w/ one year to exercise is $\boxed{0.6\cdot S(0)}$.

Find the price of such a call option w/ three months to exercise in terms of $S(0)$.

→: For any T:

$$d_1 = \frac{\sigma\sqrt{T}}{2} = -d_2$$

↑ previous problem

$$V_c(0,T) = S(0)\cdot N\left(\frac{\sigma\sqrt{T}}{2}\right) - S(0)\cancel{e^{rT}}\cdot\cancel{e^{-rT}}\cdot N\left(-\frac{\sigma\sqrt{T}}{2}\right)$$

$$\boxed{V_c(0,T) = S(0)\left(2\cdot N\left(\frac{\sigma\sqrt{T}}{2}\right) - 1\right)}$$

For $T=1$:

$$V_c(0,T=1) = \cancel{S(0)}\left(2\cdot N\left(\frac{\sigma}{2}\right) - 1\right) = 0.6\cdot\cancel{S(0)}$$

$$2 \cdot N\left(\frac{\sigma}{2}\right) - 1 = 0.6$$

$$N\left(\frac{\sigma}{2}\right) = 0.8$$

$$\Rightarrow \quad \frac{\sigma}{2} = 0.84 \quad \Rightarrow \quad \sigma = 1.68$$

For $T = \frac{1}{4}$:

$$V_c\left(0, T = \frac{1}{4}\right) = S(0)\left(2N\left(\overbrace{\frac{\sigma\sqrt{\frac{1}{4}}}{2}}^{0.42}\right) - 1\right)$$

$$= S(0)\left(2 \cdot 0.6628\right) - 1\Big) = S(0) \cdot 0.3256$$

$\square$