M339D: December 6th, 2024.

Focus on the Delta [cont'd].

Review

Def'n. With $v(s,t)$ being the value f'tion of your portfolio, the **delta** of the portfolio is

$$\boxed{\Delta(s,t) = \frac{\partial}{\partial s} v(s,t)}$$

Example. A European call in the B·S Model:

The call price:

$$v_c(s,t) = s \cdot \boxed{N(d_1(s,t))} - Ke^{-r(T-t)} \cdot N(d_2(s,t))$$

w/
$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{s}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right]$$

and
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$$\boxed{\Delta_c(s,t) = N(d_1(s,t))} \checkmark$$

Example. A European put in the B·S Model

The put price:

$$v_p(s,t) = Ke^{-r(T-t)} \cdot N(-d_2(s,t)) - s\,N(-d_1(s,t))$$

$$\boxed{\Delta_p(s,t) = -N(-d_1(s,t))} < 0 \checkmark$$

Puts are short w.r.t. underlying.

Put-Call Parity.

$$\frac{\partial}{\partial s} \Big| \quad v_c(s,t) - v_p(s,t) = s - Ke^{-r(T-t)}$$

$$\Delta_c(s,t) - \Delta_p(s,t) = 1$$

$$\Delta_p(s,t) = \Delta_c(s,t) - 1 = N(d_1(s,t)) - 1 = -N(-d_1(s,t)) \checkmark$$

(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

$T=\frac{1}{4}$ $K=41.5$

8. You are considering the purchase of a 3-month 41.5-strike ~~American~~ European call option on a nondividend-paying stock.

You are given:

(i) ~~The Black-Scholes framework holds.~~

(ii) The stock is currently selling for 40.

(iii) The stock's volatility is $\boxed{30\%}$. ✓

(iv) The current call option delta is $\boxed{0.5}$.

Determine the current price of the option.

$\boxed{v_c(S(0),0)=?}$

$$v_c(S(0),0)=S(0)\boxed{\Delta_c(S(0),0)}-Ke^{-0T}\underbrace{N(d_2(S(0),0))}_{N(-0.15)}$$

(A) $20-20.453\int_{-\infty}^{0.15} e^{-x^2/2}\,dx$ ✗

(B) $20-16.138\int_{-\infty}^{0.15} e^{-x^2/2}\,dx$ ✗

(C) $20-40.453\int_{-\infty}^{0.15} e^{-x^2/2}\,dx$ ✗

(D) $\boxed{16.138}\int_{-\infty}^{0.15} e^{-x^2/2}\,dx-20.453$

(E) $40.453\int_{-\infty}^{0.15} e^{-x^2/2}\,dx-20.453$

$$\Delta_c(S(0),0)=0.5$$

$$\boxed{N(d_1(S(0),0))=0.5}$$

$$\Downarrow$$

$$\boxed{d_1(S(0),0)=0}$$

$$\Rightarrow d_2(S(0),0)=0-0.3\sqrt{\tfrac{1}{4}}=\boxed{-0.15}$$

$$\frac{1}{\sigma\sqrt{T}}\underbrace{\left[\ln\left(\frac{40}{41.5}\right)+\left(r+\frac{0.09}{2}\right)\left(\frac{1}{4}\right)\right]}_{=0} = 0$$

$$\frac{1}{4}(r+0.045) = \ln\left(\frac{41.5}{40}\right)$$

$$r = 4\ln\left(\frac{41.5}{40}\right) - 0.045 = \underline{0.1032}$$

$$v_c(S(0),0) = \underbrace{40\cdot 0.5}_{20} - \underbrace{41.5\,e^{-0.1032(0.25)}}_{40.453}\cdot \underbrace{N(-0.15)}_{1-N(0.15)}$$

$$v_c(S(0),0) = 20 - 40.453\left(1-N(0.15)\right)$$

$$= \boxed{40.453}\cdot \underbrace{N(0.15)}_{\int_{-\infty}^{0.15} f_Z(x)\,dx} - 20.453$$

$$\int_{-\infty}^{0.15} f_Z(x)\,dx$$

$$\parallel$$

$$\int_{-\infty}^{0.15} \boxed{\frac{1}{\sqrt{2\pi}}}\, e^{-\frac{x^2}{2}}\,dx$$

$\square$

Delta Hedging.

Market Makers.

- immediacy
- inventory

} $\Rightarrow$ exposure to risk $\Rightarrow$ hedge

Say, a market maker writes an option w/ the value f'tion

$$\boxed{v(s,t)}$$