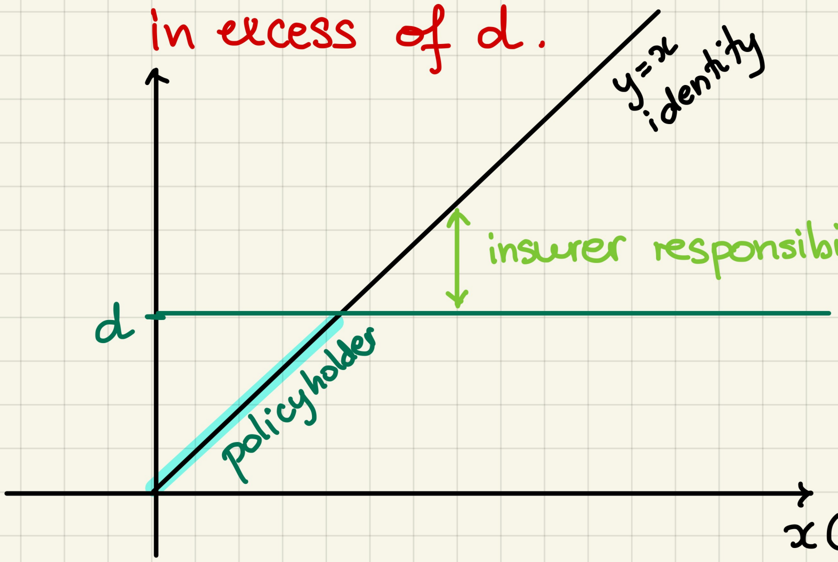
M339J: February 8th, 2021.

Review:

X ... (ground-up) loss; a random variable w/ a non-negative support

d ... deductible, i.e., the amount until which the policyholder is responsible for the losses; the insurance company is (for an ordinary deductible) responsible for everything in excess of d.



$$\text{insurer responsibility} = \begin{cases} x-d & \text{if } x>d \\ 0 & \text{if } x \le d \end{cases}$$

Def'n. Let X be a loss random variable such that
$$\mathbb{P}[X>d]>0.$$
The excess loss (random) variable, usually denoted by Y^P, is defined as

$$\boxed{Y^P = X-d \mid X>d}$$

Example. $X \sim$ Exponential (θ)

What's the distribution of Y^P?

→ : Support of Y^P is $(0, +\infty)$.

Let's find its survival function.

The interesting case are all the positive arguments.

Take $y > 0$.

$$S_{Y^P}(y) = \mathbb{P}[Y^P > y]$$
$$= \mathbb{P}[X - d > y \mid X > d] =$$
$$= \mathbb{P}[X > y + d \mid X > d]$$

Using the Memoryless property of the Exponential!

$$\boxed{S_{Y^P}(y) = \mathbb{P}[X > y]} = S_X(y)$$

$\Rightarrow$ $Y^P \sim$ Exponential (θ)

Def'n. The mean excess loss function is defined as

$$e_X(d) = \mathbb{E}[Y^P].$$

We calculate it as:
$$\boxed{e_X(d) = \frac{\mathbb{E}[(X-d)\cdot \mathbb{I}_{[X>d]}]}{\mathbb{P}[X>d]}}$$

Important final note: Y^P is most most frequently refered to as the PER PAYMENT random variable.

Def'n. The left censored and shifted random variable, usually denoted by Y^L, is defined by

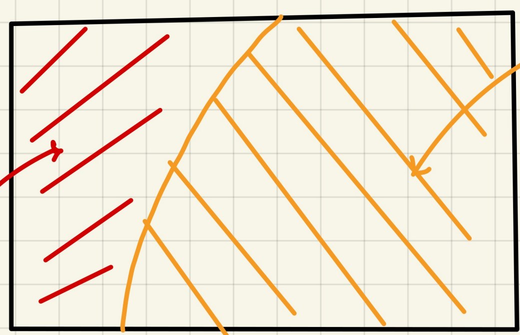
$$\boxed{Y^L = \begin{cases} 0 & X \le d \quad (\Leftrightarrow \; X-d \le 0) \\ X-d & X > d \quad (\Leftrightarrow \; X-d > 0) \end{cases}}$$

More notation: $Y^L = (X-d)\cdot \mathbb{I}_{[X>d]}$

More frequently, we use the positive part function, i.e.,

$$(\xi)_+ = \begin{cases} \xi & \text{if } \xi > 0 \\ 0 & \text{if } \xi \le 0 \end{cases}$$

$$\boxed{Y^L = (X-d)_+}$$



The deductible is exceeded.

Ω ... all possible scenarios

Y^P is not even defined on this portion of the probab. space

Y^L is defined and equal to zero here.

Important Final Note: Y^L is usually called the

PER LOSS RANDOM VARIABLE.

Example. $X \sim \text{Exponential}(\theta)$

What is the dist'n of Y^L?

$\longrightarrow$: What is the support of Y^L?

$[0, +\infty)$

Q: What is $\mathbb{P}[Y^L = 0] = ?$

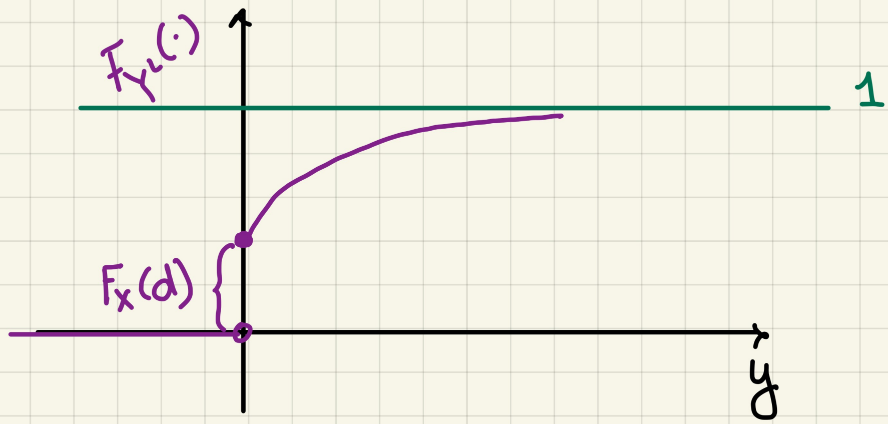
$$Y^L = 0 \iff X - d < 0$$

$$\mathbb{P}[Y^L = 0] = \mathbb{P}[X - d < 0] = F_X(d) = 1 - e^{-\frac{d}{\theta}}$$

Look @ any $y > 0$:

Law of Total Probability

$$\begin{aligned}
F_{Y^L}(y) &= \mathbb{P}[Y^L \le y] = \\
&= \mathbb{P}[Y^L \le y, X \le d] + \mathbb{P}[Y^L \le y, X > d] \\
&= \mathbb{P}[0 \le y, X \le d] + \mathbb{P}[X - d \le y, X > d] \\
&= \mathbb{P}[X \le d] + \mathbb{P}[d < X \le d + y] \\
&= \mathbb{P}[X \le d + y] = F_X(d + y) = 1 - e^{-\frac{d+y}{\theta}}.
\end{aligned}$$



Think about:
- The differences between Y^L and Y^P.
- Whether exponential is actually important.
- What kind of r.v. is Y^L?