

M339W: September 8th, 2021.

Subjective probabilities.

Our agents form conclusions/models about what the relative likelihoods are of the price of a particular asset @ a later date. Formally, they create a probabilistic model for the distribution of the time T asset price $S(T)$.

At least, the mean $E[S(T)]$ is to be paid attention to.

Assume: Agents invest in a portfolio (among those admissible in a particular market model) so that they maximize their expected profit according to their model.

Note: Our agents always have the option to invest @ the risk-free interest rate.

\Rightarrow In the least, the investors require a strictly positive expected profit.

UNIVERSITY OF TEXAS AT AUSTIN

Subjective expectations.**Problem 2.1. IFM Sample (Introductory) Problem #6.**

The following relates to one share of XYZ stock:

- The current price is 100. $S(0) = 100$
- The forward price for delivery in one year is 105. $F_{0,1} = 105$
- An investor who decides to long the forward contract denotes by P the expected stock price in one year.

Determine which of the following statements about P is TRUE.

- (A) $P < 100$
- (B) $P = 100$
- (C) $100 < P < 105$
- (D) $P = 105$
- (E) $P > 105$

$$P = \mathbb{E}[S(1)]$$

$$\mathbb{E}[\underbrace{\text{Profit (Long Forward)}}_{S(1) - F_{0,1}}] > 0$$

$$\Rightarrow \mathbb{E}[S(1)] > 105$$

$$\Rightarrow P > 105$$

Problem 2.2. IFM Sample (Introductory) Problem #38.

The current price of a medical company's stock is 75. The expected value of the stock price in three years is 90 per share. The stock pays no dividends. You are also given:

- The risk-free interest rate is positive. $r > 0$
- There are no transaction costs.
- Investors require compensation for risk. $\Rightarrow \mathbb{E}[\text{Profit}] > 0$

$$\mathbb{E}[S(3)] = 90$$

The price of a three-year forward on a share of this stock is X , and at this price an investor is willing to enter into the forward. Determine what can be concluded about X .

- (A) $X < 75$
- (B) $X = 75$
- (C) $75 < X < 90$
- (D) $X = 90$
- (E) $X > 90$

- Using the same argument as in the previous problem, we get $X = F_{0,3}$ $X < 90$
- By the formula for the forward price:

$$X = F_{0,3} = S(0)e^{3r} > S(0) = 75$$

\uparrow
 $r > 0$

Problem 2.3. IFM Sample (Introductory) Problem #70.

Investors in a certain stock demand to be compensated for risk. The current stock price is 100. The stock pays dividends at a rate proportional to its price. The dividend yield is 2%. The continuously compounded risk-free interest rate is 5%. Assume there are no transaction costs.

Let X represent the expected value of the stock price 2 years from today. Assume it is known that X is a whole number. Determine which of the following statements is true about X .

- (A) The only possible value of X is 105.
- (B) The largest possible value of X is 106.
- (C) The smallest possible value of X is 107.
- (D) The largest possible value of X is 110.
- (E) The smallest possible value of X is 111.

Say, the investor invests in 1 share.

Initial Cost: $S(0) = 100$

Payoff: The investor's wealth @ time 2.

The investor owns $e^{S \cdot T} = e^{0.04}$ shares of stock.
 \Rightarrow Their wealth is $e^{0.04} \cdot S(2)$

$$\begin{aligned} \text{Profit} &= \text{Payoff} - \text{FV}(\text{Initial cost}) \\ &= S(2)e^{0.04} - 100e^{0.05(2)} \end{aligned}$$

$$\mathbb{E}[S(2)]e^{0.04} > 100e^{0.05(2)} = 100e^{0.1} \quad / \mathbb{E}$$

$$\mathbb{E}[S(2)] > 100e^{0.1 - 0.04} = 100e^{0.06} = 106.18$$

Reconsider the binomial model!