M339J *Probability models*
University of Texas at Austin
HW #10
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Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 10.1. (5 points) *Source: Sample STAM Problem #213.*
For an insurance portfolio, you are given that:

(i) The number of claims N has the probability mass function:

$$p_N(0) = 0.1, \quad p_N(1) = 0.4, \quad p_N(2) = 0.3, \quad \text{and} \quad p_N(3) = 0.2.$$

(ii) Each claim amount has a Poisson distribution with mean 3.
(iii) The number of claims and claim amounts are mutually independent.

Calculate the variance of aggregate claims.

Problem 10.2. (5 points) *Source: Sample STAM Problem #287.*
For an aggregate loss random variable S, you are given that

(i) The number of claims N has a negative binomial distribution with parameters $r = 16$ and $\beta = 6$.
(ii) The claim amounts $X_j, j \geq 1$, are uniformly distributed on the interval $(0, 8)$.
(iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium π such that the probability that aggregate losses will exceed the premium is 5%.

Problem 10.3. (10 points) *Source: Based on Problem #165 from sample C Exam.*
Consider the following collective risk model:
(i) The claim count random variable N is Poisson with mean 3.
(ii) The severity random variable has the following probability (mass) function:

$$p_X(1) = 0.6, \; p_X(2) = 0.4.$$

(iii) As usual, individual loss random variables are mutually independent and independent of N.

Assume that an insurance covers **aggregate losses** subject to a deductible $d = 3$.
Find the expected value of aggregate payments for this insurance.

Problem 10.4. (10 points) *Source: Sample STAM Problem #280.*
A compound Poisson claim distribution has the parameter λ equal to 5 and individual claim amounts X distributed as follows:

$$p_X(5) = 0.6 \quad \text{and} \quad p_X(9) = 0.4.$$

What is the expected cost of an aggregate stop-loss insurance subject to a deductible of 5?

Problem 10.5. (5 points) Aggregate losses for a portfolio of policies are modeled as follows:

(i) The number of losses before any coverage modifications follows a geometric distribution with mean β.
(ii) The severity of each loss before any coverage modifications is uniformly distributed between 0 and ω.

The insurer would like to model the effect of imposing an ordinary deductible d such that $0 < d < \omega$ on each loss and reimbursing only a percentage α, such that $0 < \alpha < 1$ of each loss in excess of the deductible.

It is assumed that the coverage modifications will not affect the loss distribution. The insurer models its claims with modified frequency and severity distributions.

What is the mean of the modified frequency distribution?

Problem 10.6. (5 points) A group insurance policy has a negative binomial claim count distribution with mean 200 and variance 600.

The severity random variable X has the following probability mass function:

$$p_X(60) = p_X(120) = p_X(160) = p_X(200) = 1/4.$$

There is a per-loss deductible of 100. Calculate the expected total claim payment.

Problem 10.7. (5 points) Let the loss count random variable have the Poisson distribution with parameter λ. The losses are assumed to be uniform on $(0, a)$. The losses are all mutually independent and independent from the loss count random variable.

There is a per-loss deductible of d such that $d < a$. What is the variance of aggregate claim payments? Express your answer in terms of λ, a and d.