M358K: October 15th, 2021.

Hypothesis Testing.

Proof by Contradiction.

K... claim that I'm trying to prove to be true

Q: What if K were not true?

Assume not K

fact Q ; fact (not Q)

They cannot coexist!

We say we have reached a contradiction!

$\Rightarrow\Leftarrow$

$\lightning$

Our assumption of not K was wrong!

Hypothesis Testing.

Claim we're trying to substantiate.

μ... the population parameter representing the mean cholesterol level after treatment

μ_0... the mean cholesterol level before treatment (a number)

$\boxed{\mu < \mu_0}$ ALTERNATIVE HYPOTHESIS

Assume $\mu = \mu_0$ ← NULL HYPOTHESIS

collect data
statistical analysis

Figure out the probability of seeing the data that we saw (or something more extreme) if $\boxed{\mu = \mu_0}$

If this probability is "small", you have evidence to the contrary of $\boxed{\mu = \mu_0}$

The smaller this probability, the stronger the evidence.

The Normal Case.

Population model: $X \sim \text{Normal}(\boxed{\text{mean} = \mu}, \text{sd} = \sigma)$

mean: unknown and of interest — sd: known

Hypothesis testing procedure.

First: Set the hypotheses.

2nd Null Hypothesis: $H_0 : \mu = \mu_0$ ✓

1st Alternative Hypothesis. $H_a : \begin{cases} \mu < \mu_0 & \text{(lower or left sided)} \\ \mu \neq \mu_0 & \text{(two sided)} \\ \mu > \mu_0 & \text{(upper or right sided)} \end{cases}$

Second: Figure out the appropriate test statistic (TS).

Natural choice:

$$\bar{X} \sim \text{Normal}\left(\text{mean} = \mu, \text{sd} = \frac{\sigma}{\sqrt{n}}\right)$$

Under the null hypothesis, i.e., if $\mu = \mu_0$,

$$\frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

Third: Consider the observed value of the test statistic.

In this case, it's $\bar{x}$, i.e., the observed sample average.

Q: What's the probability of observing $\bar{x}$ or something $\boxed{\text{more extreme}}$ under the null?

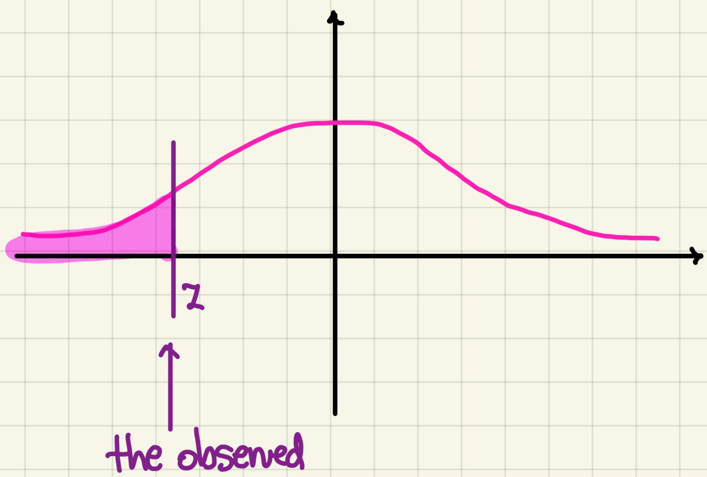
↖ The interpretation depends on the structure of the ALTERNATIVE HYPOTHESIS!

It's always useful to calculate the observed $\boxed{z\text{-score}}$, i.e.,

$$\boxed{z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}}$$

- Lower-sided alternative:

$H_a : \mu < \mu_0$



$\mathbb{P}[Z \leq z]$ ...

Def'n.

the p-value

the probability of seeing the observed value of the TS or something more extreme