UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 10
Power of Test.

Provide your **complete solution** for the following problems.

Problem 10.1. As the sample size increases, the power of a test will increase. *True or false? Why?*

Problem 10.2. (2 points) Consider a two-sided hypothesis test for the population mean of a normal population. Then, the power of the test is symmetric with respect to the null mean. *True or false? Why?*

Problem 10.3. (2 points) Let μ denote the population mean μ of a normally distributed population model with a known σ. At a given significance level α, we are testing

$$H_0 : \mu = \mu_0 \quad vs. \quad H_a : \mu < \mu_0.$$

Let μ_a and μ_a' be two values in the alternative such that $\mu_a < \mu_a'$. Then, the power of the test at the alternative μ_a exceeds the power of the test at the alternative μ_a'. *True or false? Why?*

Problem 10.4. The time needed for college students to complete a certain mirror-symmetry puzzle is modeled using a normal distribution with a mean of 30 seconds and a standard deviation of 3 seconds. You wish to see if the population mean time μ is changed by vigorous exercise, so you have a group of nine college students exercise vigorously for 30 minutes and then complete the puzzle.
i. What are your null and alternative hypotheses?
ii. What is the rejection region at the significance level 0.01?
iii. What is the power of your test at $\mu = 28$ seconds?

Problem 10.5. (10 points) You believe that the mean pancake consumption at the pancake jamboree is more than 16 per person. So, you decide to test your hypothesis. You model the pancake consumption as normally distributed with an unknown mean and with variance equal to 4. The plan is to collect the information on the number of pancakes consumed from a sample of 64 people. Since you want to have everything ready for the big day, you work out the rejection region right away and you get $(16.4375, \infty)$.

(i) (5 points) What is the significance level used to obtain the above rejection region?

(ii) (5 points) What is the power of the above test at the alternative mean of 17?