Name:

M339J: Probability Models with Actuarial Applications
The University of Texas at Austin
"Mock" In-Term Exam II
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

Problem 2.1. (5 points) *Source: Sample P exam, Problem #30.*
An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. The number of claims filed has a Poisson distribution. Calculate the variance of the number of claims filed.

(a) $1/\sqrt{3}$
(b) 1
(c) $\sqrt{2}$
(d) 2
(e) None of the above.

Problem 2.2. (5 points) Once a tunnel drill breaks down, it takes at least a month to get a replacement. The waiting time T to get a new drill after that time has the following cumulative distribution function:

$$F_T(t) = \begin{cases} 1 - t^{-2} & \text{for } t > 1 \\ 0 & \text{otherwise} \end{cases}$$

The resulting cost to the construction company is $X = T^2$. Which of the following is a probability density function of the random variable X for $x > 1$?

(a) x^{-2}
(b) x^{-1}
(c) $\frac{1}{2}x^{-2}$
(d) $x^{-1/2}$
(e) None of the above.

Problem 2.3. (5 points) Let X be a two-point mixture. More precisely, let X be

- exponentially distributed with mean equal to 10 with probability 3/4;
- gamma distributed with parameters $\alpha = 4$ and $\theta = 10$ with probability 1/4.

What is the variance of the random variable X?

(a) 343.75
(b) 107.5
(c) 100
(d) 507.5
(e) None of the above.

Problem 2.4. (5 points) Let $X \sim Pareto(\alpha = 3, \theta = 3000)$. Assume that there is a deductible of $d = 5000$. Find the loss elimination ratio.

(a) 27/64
(b) 33/56
(c) 45/64
(d) 55/64
(e) None of the above.

Problem 2.5. (5 points) Assume that conditional on the random variable Λ, N has the Poisson distribution with variance Λ.

Let $\Lambda \sim U(0, 2)$.

Find the (unconditional) variance of N.

(a) 1/3
(b) 2/3
(c) 4/3
(d) 5/3
(e) None of the above.

Problem 2.6. (5 points) Let the ground-up loss X be exponentially distributed with mean \$800. An insurance policy has an ordinary deductible of \$100 and the maximum amount payable per loss of \$2500.
Find the expected value of the amount paid (by the insurance company) **per positive payment.**

(a) 654.85
(b) 764.85
(c) 800
(d) 864.85
(e) None of the above.

Problem 2.7. (5 points) Assume that the severity random variable X is exponentially distributed with mean 1400. There is an insurance policy to cover this loss. This insurance policy has

(1) a deductible of 200 per loss,
(2) the coinsurance factor of $\alpha = 0.25$, and
(3) the maximum amount payable by the insurer per loss of 700.

Calculate the expected value of the per payment random variable Y^P under this policy.

(a) 262.50
(b) 302.63
(c) 350
(d) 550
(e) None of the above.

Problem 2.8. (5 points) A model for the arrival time T for a particular event is initially an exponential distribution with mean 2 years. Upon reconsideration, this distribution is replaced with a spliced model whose density function:

(i) is proportional to the initial model's density function over $[0, 1]$,
(ii) is uniform over $[1, 3]$,
(iii) is continuous at 1,
(iv) is zero on $(3, \infty)$.

Calculate the probability of failure in the first year under the revised distribution.

(a) $1 - e^{-2}$
(b) $1 - e^{-1}$
(c) $e^{-\frac{1}{2}}$

(d) $1 - e^{-\frac{1}{2}}$

(e) None of the above.

Problem 2.9. (5 points) The ground-up loss random variable is denoted by X. An insurance policy on this loss has a **franchise** deductible of d and no policy limit. Then, the expected **policyholder** payment per loss equals ...

(a) $\mathbb{E}[X\mathbb{I}_{[X \leq d]}]$

(b) $\mathbb{E}[X \wedge d]$

(c) $\mathbb{E}[(X - d)_+]$

(d) $\mathbb{E}[X \wedge d] - d$

(e) None of the above.

Problem 2.10. (5 pts) Let X be the ground-up loss random variable. Assume that X has the exponential distribution with mean $5,000$.
Let B denote the expected payment per loss on behalf of an insurer which wrote a policy with a deductible of $1,500$ and with no maximum policy payment. Then,

(a) $B \approx 1,700$
(b) $B \approx 2,700$
(c) $B \approx 3,700$
(d) $B \approx 4,700$
(e) None of the above

Problem 2.11. (5 points) Let the number of customers N who walk into Hooper's store on a given day be Poisson with variance 30. The probability that a particular customer is a monster is 2/3. The number of customers is independent from whether they are monsters or not.

What is the probability that the total number of customers in a particular day is 25, **given** that the number of monster-customers equals 12?

(a) $e^{-10} \cdot \frac{10^{12}}{12!}$
(b) $e^{-10} \cdot \frac{10^{13}}{13!}$
(c) $e^{-30} \cdot \frac{30^{13}}{13!}$
(d) $e^{-30} \cdot \frac{30^{25}}{25!}$
(e) None of the above.

Problem 2.12. (5 points) In a large population, the purple party and the mauve party are facing off in a two-party election. You are surveying people exiting from a polling booth and asking them if they voted purple. The probability that a randomly chosen person voted purple is 20%. What is the probability that exactly 15 people must be asked before you can find exactly 5 people who voted purple?

(a) 0.022
(b) 0.034
(c) 0.046
(d) 0.052
(e) None of the above.