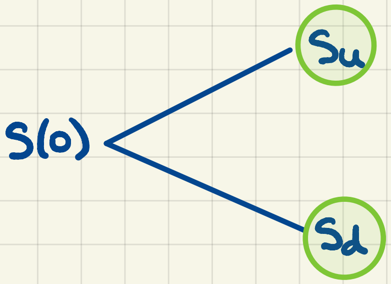
Graphical Interpretation.

Consider a European call w/ exercise date @ the end of the period and the strike price K such that $\boxed{S_d < K < S_u}$



Payoff

$V_u = S_u - K$

$\boxed{V_d = 0}$

Recall: The payoff f'tion of the call option: $v_c(s) = (s-K)_+$

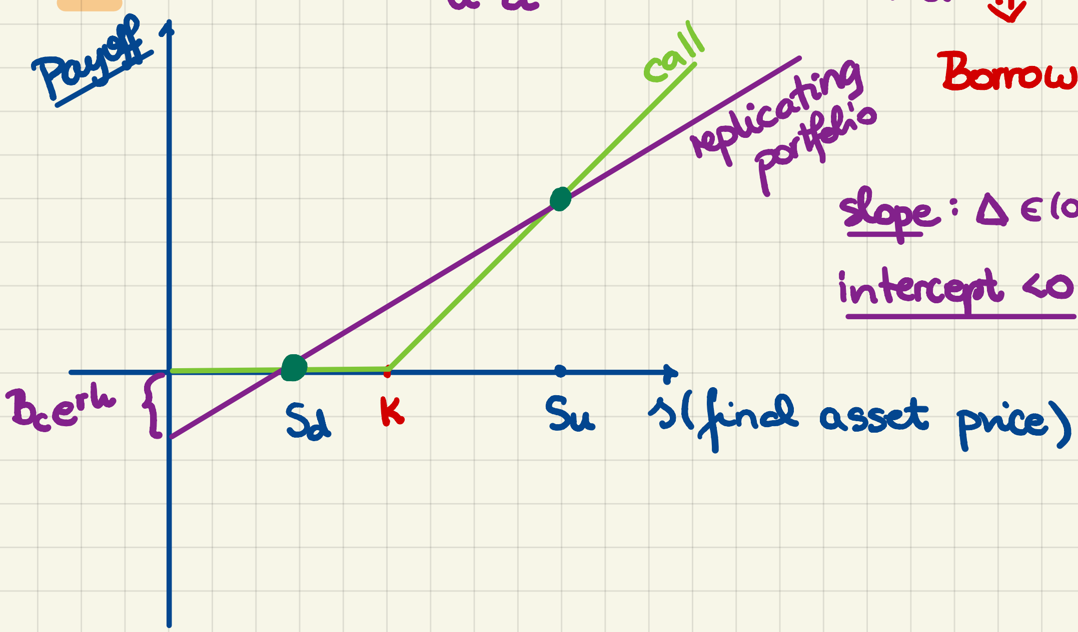
In the replicating portfolio:

- $\Delta_c = \dfrac{V_u - V_d}{S_u - S_d} = \dfrac{S_u - K}{S_u - S_d} \in (0,1)$

- $B_c = e^{-rh} \cdot \dfrac{u\cancel{V_d} - dV_u}{u-d} = -e^{-rh} \cdot \dfrac{dV_u}{u-d} < 0$ ($V_d = 0$)

$\Downarrow$

Borrowing!



slope: $\Delta \in (0,1)$ => buying a fraction of a share

intercept < 0 => borrowing

Problem 8.3. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$80 per share. In the model, it is assumed that the stock price can either go up by \$5 or down by \$4.

You use the binomial tree to construct a replicating portfolio for a 78−strike call option on the above stock. What is the stock investment in the replicating portfolio?

→ :

Payoff

$S(0) = 80$

$S_u = 85$ $\qquad V_u = (85-78)_+ = 7$

$S_d = 76$ $\qquad V_d = (76-78)_+ = 0$

$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{7-0}{85-76} = \frac{7}{9}$$

□

Problem 8.4. Let the continuously compounded risk-free interest rate be equal to $\boxed{0.04}$. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is \$50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a 45−strike call on the above stock. What is the risk-free investment in the replicating portfolio?

$\rightarrow$:

$S(0) = 50$

$S_u = 50(1.05) = 52.5$

$S_d = 50(0.90) = 45$

Payoff

$V_u = (52.5 - 45)_+ = 7.5$

$V_d = (45 - 45)_+ = 0$

$$B = e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u - d} = e^{-0.04} \cdot \frac{1.05(0) - 0.9(7.5)}{1.05 - 0.90} = -43.2355$$

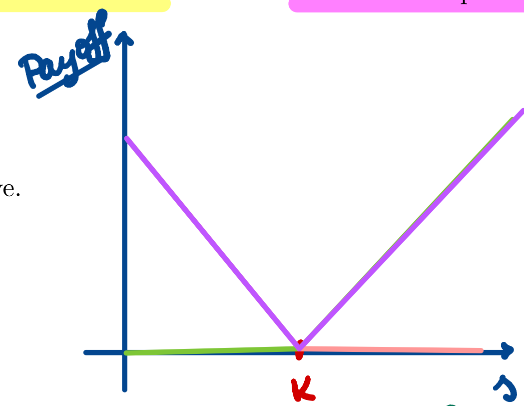
$\square$

Problem 8.5. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

A **straddle** consists of a long call and a long otherwise identical put. Consider a \$100-strike one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

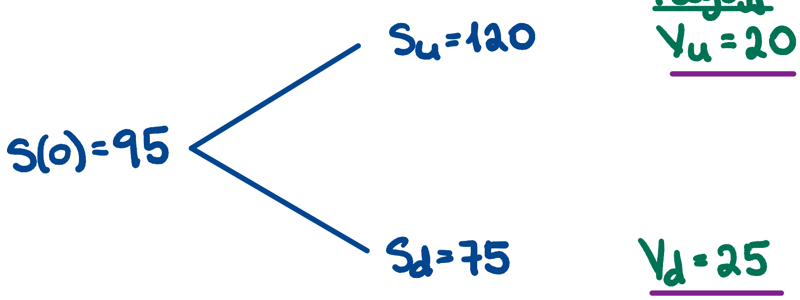
(a) About \$10
(b) About \$10.83
(c) About \$15.45
(d) About \$20.84
(e) None of the above.



Payoff f'tion of a straddle

$\boxed{v(s) = |s-K|}$

$K=100$



Replicating Portfolio

$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{20-25}{120-75} = -\frac{5}{45} = -\frac{1}{9}$$

$$B = e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u-d} = e^{-0.06(1)} \cdot \frac{\frac{120}{\cancel{95}} \cdot 25 - \frac{75}{\cancel{95}} \cdot 20}{\frac{120}{\cancel{95}} - \frac{75}{\cancel{95}}}$$

$$= e^{-0.06} \cdot \frac{120(25) - 75(20)}{45} = \underline{31.392}$$

$$V(0) = \Delta \cdot S(0) + B = -\frac{1}{9}(95) + 31.392 = \underline{20.83} \quad \square$$

Start w/

$$V(0) = \Delta \cdot S(0) + B$$

$$V(0) = \frac{V_u - V_d}{\underbrace{S_u - S_d}_{\cancel{S(0)}(u-d)}} \cdot \cancel{S(0)} + e^{-rh} \cdot \frac{u \cdot V_d - d \cdot V_u}{u-d}$$

$$V(0) = \frac{1}{u-d}\left[V_u - V_d + e^{-rh}(u \cdot V_d - d \cdot V_u)\right]$$

$$V(0) = e^{-rh} \cdot \frac{1}{u-d}\left[e^{rh} \cdot V_u - e^{rh} \cdot V_d + u \cdot V_d - d \cdot V_u\right]$$

$\vdots$ algebra