4) You are given the following information about a portfolio consisting of stocks X, Y, and Z:

Stock	Investment	Expected Return
X	10,000	$\boxed{8\%}$
Y	15,000	$\boxed{12\%}$
Z	25,000	$\boxed{16\%}$

$\Sigma = 50{,}000$

Calculate the expected return of the portfolio.

$\longrightarrow$: R_P... the return of the total portfolio

$\mathbb{E}[R_P] = ?$

$$\mathbb{E}[R_P] = w_X \cdot \mathbb{E}[R_X] + w_Y \cdot \mathbb{E}[R_Y] + w_Z \, \mathbb{E}[R_Z]$$

(A) 10.8%

(B) 11.4%

(C) 12.0%

(D) 12.6%

(E) 13.2%

$$w_X = \frac{10K}{50K} = 0.2$$

$$w_Y = \frac{15K}{50K} = 0.3$$

$$w_Z = 0.5$$

$$\mathbb{E}[R_P] = 0.2(0.08) + 0.3(0.12) + 0.5(0.16) = 0.132$$

$\square$

The Volatility of a Two-Stock Portfolio.

We index the two securities in the portfolio by $i=1,2$.

$$\boxed{R_p = w_1 \cdot R_1 + w_2 \cdot R_2}$$

$$\Rightarrow \boxed{\mathbb{E}[R_p] = w_1 \cdot \mathbb{E}[R_1] + w_2\, \mathbb{E}[R_2]}$$

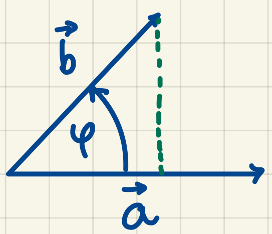
$$Var[R_p] = Var[w_1 \cdot R_1 + w_2 \cdot R_2]$$

$$= w_1^2 \cdot Var[R_1] + w_2^2\, Var[R_2] + 2 w_1 \cdot w_2 \cdot Cov[R_1, R_2]$$

By def'n:

$$Cov[R_1, R_2] = SD[R_1] \cdot SD[R_2] \cdot \boxed{corr(R_1, R_2)}$$

$$= \sigma_1 \cdot \sigma_2 \cdot \rho_{1,2}$$



scalar product:

$$\langle \vec{a}, \vec{b} \rangle = \vec{a} \cdot \vec{b}$$

$$= \|\vec{a}\| \cdot \|\vec{b}\| \cdot \boxed{\cos(\varphi)}$$

The volatility of the portfolio: $\boxed{\sigma_p = SD[R_p] = \sqrt{Var[R_p]}}$

3) You are given the following information about the annual returns of two stocks, X and Y:

i) The expected returns of X and Y are $E[R_X] = \boxed{10\%}$ and $E[R_Y] = \boxed{15\%}$.

ii) The volatilities of the returns are $\cancel{V_X}$ σ_X = 18% and $\cancel{V_Y}$ σ_Y = 20%.

iii) The correlation coefficient of the returns for these two stocks is $\boxed{0.25}$. ✓

iv) The expected return for a certain portfolio, consisting only of stocks X and Y, is 12%. !

Calculate the volatility of the portfolio return.

$\longrightarrow$: $\sigma_P = \sqrt{Var[R_P]}$

(A) 10.88%

(B) 12.56%

(C) 13.55%

(D) 14.96%

(E) 16.91%

$$\boxed{Var[R_P] = w_X^2 \cdot \sigma_X^2 + w_Y^2 \cdot \sigma_Y^2 + 2 \cdot w_X \cdot w_Y \cdot \sigma_X \cdot \sigma_Y \, \rho_{X,Y}}$$

$$0.12 = E[R_P] = w_X \cdot E[R_X] + w_Y E[R_Y]$$

$$0.12 = w_X \cdot (0.10) + \underbrace{w_Y}_{=1-w_X} \cdot (0.15)$$

$$0.12 = w_X(0.10 - 0.15) + 0.15$$

$$w_X = \frac{0.03}{0.05} = 0.6 \Rightarrow w_Y = 0.4$$

$$Var[R_P] = (0.6)^2 \cdot (0.18)^2 + (0.4)^2 \cdot (0.2)^2 + 2 \cdot (0.6) \cdot (0.4)(0.18)(0.2) \cdot (0.25) = \underline{0.022384}$$

$$\sigma_P = \sqrt{0.022384} = 0.1496 \quad \square$$

Volatility of an n-component Portfolio.

$$R_P = \boxed{w_1 \cdot R_1 + w_2 \cdot R_2 + \cdots + w_n \cdot R_n}$$

$$\Rightarrow \quad Var[R_P] = Cov[\boxed{R_P}, R_P]$$

$$= Cov[w_1 \cdot R_1 + w_2 \cdot R_2 + \cdots + w_n \cdot R_n , R_P]$$

$$= w_1 \cdot Cov[R_1, R_P] + w_2 \cdot Cov[R_2, R_P] + \cdots + w_n \cdot Cov[R_n, R_P]$$

$$= \sum_{i=1}^{n} w_i \cdot Cov[R_i, R_P]$$

2) You are given the following information about a portfolio with four assets.

Asset	Market Value of Asset	Covariance of asset's return with the portfolio return
I	40,000	0.15
II	20,000	-0.10
III	10,000	0.20
IV	30,000	-0.05

$\Sigma = 100K$

Calculate the standard deviation of the portfolio return.

$\rightarrow \cdot\ Var[R_p] = \cancel{X} = 0.4(0.15) + 0.2(-0.10) + 0.1(0.20) + 0.3(-0.05) = 0.045$

$w_I = \frac{40K}{100K} = 0.4$

$w_{II} = 0.2$

$w_{III} = 0.1$

$w_{IV} = 0.3$

$\sigma_p = \sqrt{0.045} = 0.212$

(A) 4.50%

(B) 13.2%

(C) 20.0%

(D) 21.2%

(E) 44.7%