UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 1

Prerequisite material.

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 1.1. (5 points) Provide the definition of the *bias* of an estimator. What does it mean for the estimator to be *unbiased*? What about *biased*?

Problem 1.2. (5 points) Provide the definition of the *mean squared error (MSE)* of an estimator.

Problem 1.3. (10 points) Show that, for a point estimator $\hat{\theta}$,

$$MSE[\hat{\theta}] = Var_\theta[\hat{\theta}] + (bias(\hat{\theta}))^2.$$

Problem 1.4. (10 points) The Pareto distribution with parameters α and θ has the distribution function

$$F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha.$$

For integer k, its k^{th} moment is

$$\mathbb{E}[X^k] = \frac{\theta^k k!}{(\alpha-1)\dots(\alpha-k)}$$

A random variable X has a two-parameter Pareto distribution with parameters $\alpha = 4$ and θ (unknown, and to be estimated). Let $\hat{\theta} = 3X$ be our proposed estimator for the θ parameter, based on a random sample consisting of a single measurement. Find the mean squared error of this estimator.

Problem 1.5. (10 points) The gamma distribution with parameters α and β has mean $\alpha\beta$ and variance $\alpha\beta^2$.

Let the random variable X have the Gamma distribution with parameters $\alpha = 3$ and θ unknown (and to be estimated). A proposed estimator for the parameter θ based on a single observation X_1 of the above distribution is $\hat{\theta} = \frac{1}{3}X_1$. What is the **mean-squared error** of this estimator?

Problem 1.6. (10 points) Let Y_1, Y_2 be a random sample from the exponential distribution with the unknown parameter θ. The estimator $\hat{\theta}_2 = cY_{(1)}$ for θ is proposed. Find the constant c such that $\hat{\theta}_2$ is an unbiased estimator of θ.