Quiz #13: Solutions

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Simulations

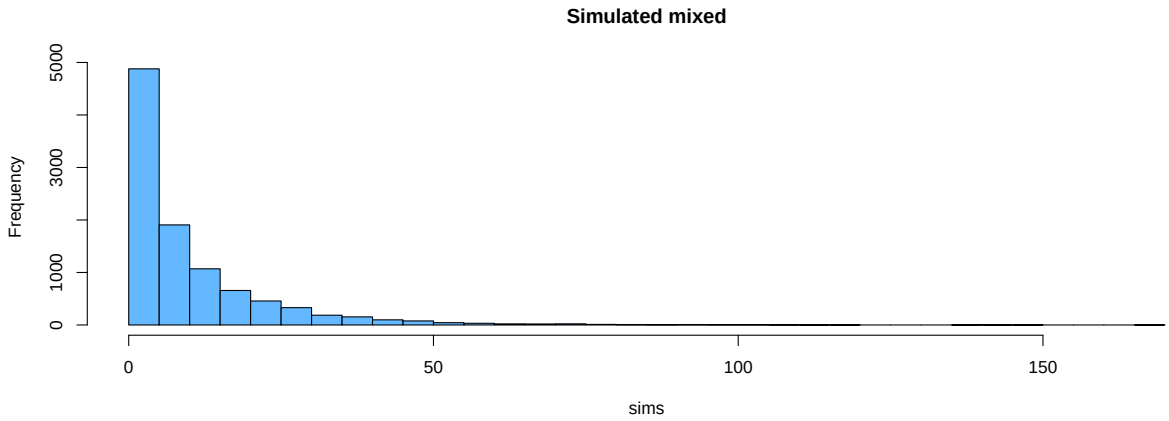
Problem 1. (15 points)

Consider the following two-step experiment. First you draw a the value of the mixing parameter Λ from a uniform distribution on $[0, 20]$. Then, you draw the simulated value from an exponential distribution with mean Λ.

(7 points) Repeat the above two-step experiment 10000 times and draw the histogram of the simulated values.

Solution:

```
a=0
b=20
nsims=10000
sims=numeric(0)
for (i in 1:10000){
  lambda=runif(1, min=a, max=b)
  new.sim=rexp(1, rate=1/lambda)
  sims=c(sims, new.sim)
}
hist(sims, breaks=25, col="steelblue1", main="Simulated mixed")
```



(2 points) Find the theoretical mean of the above distribution and compare it to the mean of the simulated values.

Solution: The theoretical mean is

$$\mathbb{E}[\mathbb{E}[X|\Lambda]] = \mathbb{E}[\Lambda] = 10$$

The average of the simulated values is

```
mean(sims)
## [1] 9.934101
```

(6 points) Find the theoretical variance of the above distribution and compare it to the variance of the simulated values.

Solution: The theoretical variance is

$$Var[X] = \mathbb{E}[Var[X|\Lambda]] + Var[\mathbb{E}[X|\Lambda]] = \mathbb{E}[\Lambda^2] + Var[\Lambda] = 2Var[\Lambda] + (\mathbb{E}[\Lambda])^2 = 2\left(\frac{20^2}{12}\right) + 10^2$$

Using `R` as a calculator, we get

```
2*20^2/12+10^2
## [1] 166.6667
```

The variance of the simulated values is

```
var(sims)
## [1] 169.9109
```