Logistic Regression w/ 2 Categories in the Response.

We can represent one category by 0,
and the other by 1.

$$\mathbb{P}[Y=1 \mid X=x] = \cancel{X}\ p(X)$$

$$\ln\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta X$$

$$p(X) = \frac{e^{\beta_0+\beta X}}{1+e^{\beta_0+\beta X}}$$

$$\mathbb{P}[Y=1 \mid X=x] = \frac{e^{\beta_0+\beta_1 x_1+\cdots+\beta_p x_p}}{1+e^{\beta_0+\beta_1 x_1+\cdots+\beta_p x_p}}$$

$$\mathbb{P}[Y=0 \mid X=x] = \frac{1}{1+e^{\beta_0+\beta_1 x_1+\cdots+\beta_p x_p}}$$

Logistic Regression w/ K categories in the Response.

In the book:

$$\mathbb{P}[Y=K \mid X=x] = \frac{1}{1+\sum_{k=1}^{K-1} e^{\beta_{0k}+\beta_{1k} x_1+\cdots+\beta_{pk} x_p}}$$

For other categories $l \in \{1, 2, \ldots, K-1\}$

$$\mathbb{P}[Y=l \mid X=x] = \frac{e^{\beta_{0l}+\beta_{1l} x_1+\cdots+\beta_{pl} x_p}}{1+\sum_{k=1}^{K-1} e^{\beta_{0k}+\beta_{1k} x_1+\cdots+\beta_{pk} x_p}}$$