

Named Discrete Distribution.

Def'n. Bernoulli trials have two possible outcomes.

They are also known as indicators (or indicator r.v.s).

Usually, the outcomes are encoded by

$$\begin{cases} 1 & \text{for "success"} \\ 0 & \text{for "failure"} \end{cases}$$

Example. Y_i ... result of a throw of a die for $i = 1, 2$.

$$S_{Y_1} = S_{Y_2} = \{1, 2, 3, 4, 5, 6\}$$

Define $W = Y_1 + Y_2$

$$I = \begin{cases} 1 & \text{if } W \geq 9 \\ 0 & \text{if } W < 9 \end{cases}$$

Example. • Quality control, say, whether a lightbulb is deficient or not
• Indicator of whether a deductible is met

Example. Bernoulli distribution.

y	0	1
$P_Y(y)$	$1-p$	p

$$Y \sim b(p) \text{ w/ } p \in (0, 1)$$

Example Say that we repeat independently the Bernoulli trial w/ the same p a fixed number of times n . Then, we count the total number of successes Y . This distribution is the Binomial distribution.

$$Y \sim b(n, p)$$

$$\text{for } k \in \{0, \dots, n\}: P_k = P[Y=k] = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

M378K Introduction to Mathematical Statistics

Problem Set #1

Named discrete random variables.

Problem 1.1. Source: Sample P exam, Problem #125.

An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail. Calculate the probability that the system will overheat.

$$\rightarrow: N \sim b(3, 0.05)$$

$$\begin{aligned} \Pr[N \geq 2] &= \Pr[N = 2] + \Pr[N = 3] \\ &= \left(\frac{3}{2}\right)(0.05)^2(0.95) + \left(\frac{3}{3}\right)(0.05)^3 \\ &= 3(0.05)^2(0.95) + (0.05)^3 \\ &= \dots = 0.00725 \end{aligned}$$

□

Example. Say we repeat **independent** Bernoulli trials w/ the same success probability p until the first success. The random variable Y will denote the total number of failures until the first success.

We write $Y \sim g(p)$

and we call Y geometric.

Set $q = 1-p$

y	0	1	2	...	k
$P_Y(y)$	p	$q \cdot p$	$q^2 \cdot p$...	$q^k \cdot p$

Problem 1.2. Source: Sample P exam, Problem #462.

Each person in a large population independently has probability p of testing positive for diabetes where $0 < p < 1$. People are tested for diabetes, one person at a time, until a test is positive. Individual tests are independent. Determine the probability that m or fewer people are tested, given that n or fewer people are tested, where $1 \leq m \leq n$.

→: N' ... total # of people tested
 ↳ SHIFTED geometric r.v. w/ parameter p

i.e., $N = N' - 1 \sim g(p)$ ✓

$$\begin{aligned} \mathbb{P}[N' \leq m \mid N' \leq n] &= \mathbb{P}[N+1 \leq m \mid N+1 \leq n] \\ &= \mathbb{P}[N \leq m-1 \mid N \leq n-1] \end{aligned}$$

$$= \frac{\mathbb{P}[N \leq m-1, N \leq n-1]}{\mathbb{P}[N \leq n-1]}$$

$$= \frac{\mathbb{P}[N \leq m-1]}{\mathbb{P}[N \leq n-1]}$$

$$= \frac{1 - \mathbb{P}[N > m-1]}{1 - \mathbb{P}[N > n-1]}$$

$$= \frac{1 - q^m}{1 - q^n}$$

□

Example. The Poisson Distribution is No-valued and has the

pmf $P_k = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$ for $k = 0, 1, \dots$

where λ is a positive parameter