M339W: November 8th, 2021.

14) You are given the following information about Stock X, Stock Y, and the ~~market:~~ P

(i) The annual effective risk-free rate is 4%. $\boxed{r_f = 0.04}$

(ii) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	Expected Return	Volatility
Stock X	5.5%	40%
Stock Y	4.5%	35%
~~Market~~ P	6.0%	25%

(iii) The correlation between the returns of stock X and the ~~market~~ P is –0.25.

(iv) The correlation between the returns of stock Y and the ~~market~~ P is 0.30.

~~Assume the Capital Asset Pricing Model holds.~~ Calculate the required returns for Stock X and Stock Y, and determine which of the two stocks an investor should choose.

(A) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(B) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock Y.

(C) The required return for Stock X is 4.80%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(D) The required return for Stock X is 6.40%, the required return for Stock Y is 3.16%, and the investor should choose Stock Y.

(E) The required return for Stock X is 3.50%, the required return for Stock Y is 3.16%, and the investor should choose both Stock X and Stock Y.

→: The required return for Stock X:

$$r_X = r_f + \beta_X^P \left(\mathbb{E}[R_P] - r_f\right)$$

$$\beta_X^P = \frac{\sigma_X}{\sigma_P} \cdot \rho_{P,X} = \frac{0.4}{\cancel{0.25}}\left(-\cancel{0.25}\right) = -0.4$$

$$\Rightarrow \quad r_X = 0.04 + (-0.4)\left(0.06 - 0.04\right) = 0.04 - 0.008$$
$$= 0.032$$

Since 0.055 > 0.032, we conclude that we should invest in X. ⇒ (A).

Let's just check what happens w/ Y.

$$r_Y = r_f + \beta_Y^P \left(\mathbb{E}[R_P] - r_f\right)$$

$$\beta_Y^P = \frac{\sigma_Y}{\sigma_P} \cdot \rho_{Y,P} = \frac{0.35}{0.25}(0.3) = 0.42$$

$$\Rightarrow r_Y = 0.04 + 0.42\,(0.06 - 0.04) = 0.04 + 0.0084 = 0.0484$$

Since $0.045 < 0.0484$, we do not want to invest in Y.

$\square$

The Capital Asset Pricing Model (CAPM).

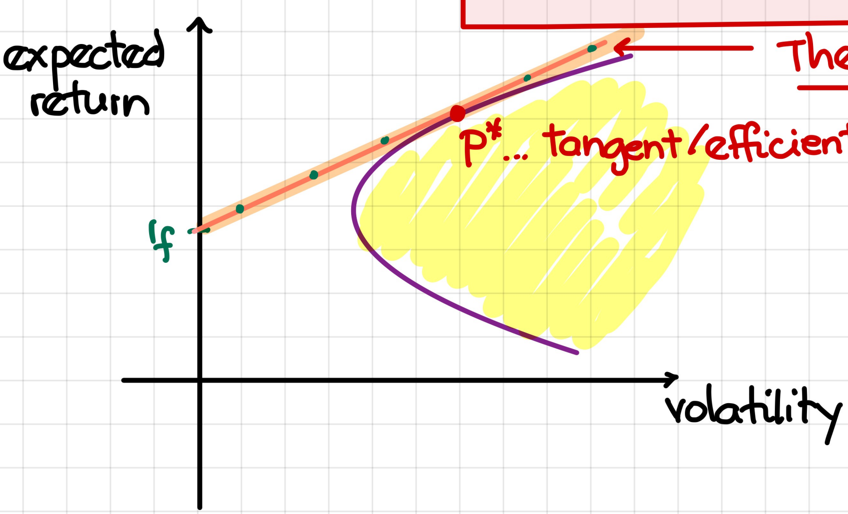
1. **No friction:** The investors buy/sell all the securities @ competitive market prices w/ no transaction costs (no bid-ask spread; no fees). Both borrowing and lending are at the same risk-free interest rate.

2. **Rationality:** Investors only hold efficient portfolios, i.e., they hold only the portfolios which yield the maximum available expected return for a particular volatility.

3. **Homogeneous Expectations:**

All investors have homogeneous beliefs about:

- expected returns
- volatilities
- correlation coefficients



All the assets i in the market:

- # of shares of asset i
- market value of asset i per share

×

MV_i ... market value of i

In the market portfolio, the weight of asset is

$$w_i = \frac{MV_i}{\sum_i MV_i}$$

In CAPM:

$$\mathbb{E}[R_I] = r_I = r_f + \beta_I \left(\mathbb{E}[R_{Mkt}] - r_f\right), \quad \beta_I = \beta_I^{Mkt}$$

w/ $\beta_I = \frac{\sigma_I}{\sigma_{Mkt}} \cdot \rho_{I,Mkt}$

or $\beta_I = \frac{Cov[R_I, R_{Mkt}]}{Var[R_{Mkt}]}$

15) You are given the following information about Stock X, Stock Y, and the market:

(i) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	Required Return	Volatility
Stock X	3.0%	50%
Stock Y	?	35%
Market	6.0%	25%

(ii) The correlation between the returns of stock X and the market is –0.25.

(iii) The correlation between the returns of stock Y and the market is 0.30.

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock Y.

(A) 1.48%

(B) 2.52%

(C) 3.16%

(D) 4.84%

(E) 6.52%

$\rightarrow$: $r_Y = r_f + \beta_Y (\mathbb{E}[R_{Mkt}] - r_f)$

? ✓ ? ✓

From $r_X = r_f + \beta_X (\mathbb{E}[R_{Mkt}] - r_f)$

$$\beta_X = \frac{\sigma_X}{\sigma_{Mkt}} \cdot \rho_{X,Mkt} = \frac{0.5}{\cancel{0.25}} \cdot (-\cancel{0.25}) = -0.5$$

$\Rightarrow$ $0.03 = r_f + (-0.5)(0.06 - r_f) = r_f - 0.03 + 0.5 r_f$

$\Rightarrow$ $1.5 r_f = 0.06$ $\Rightarrow$ $\boxed{r_f = 0.04}$

$r_Y = 0.04 + \beta_Y (0.06 - 0.04)$

w/ $\beta_Y = \frac{\sigma_Y}{\sigma_{Mkt}} \cdot \rho_{Y,Mkt} = \frac{0.35}{0.25}(0.3) = 0.42$

$r_Y = 0.04 + 0.42(0.02) = 0.0484$