University of Texas at Austin

Subjective probabilities.

Problem 2.1. MFE Sample (Introductory) Problem #6.
The following relates to one share of XYZ stock:

- The current price is 100.
- The forward price for delivery in one year is 105.
- An investor who decides to long the forward contract denotes by P the expected stock price in one year.

Determine which of the following statements about P is **TRUE**.

(A) $P < 100$
(B) $P = 100$
(C) $100 < P < 105$
(D) $P = 105$
(E) $P > 105$

Problem 2.2. MFE Sample (Introductory) Problem #38.
The current price of a medical companys stock is 75. The expected value of the stock price in three years is 90 per share. The stock pays no dividends. You are also given:

- The risk-free interest rate is positive.
- There are no transaction costs.
- Investors require compensation for risk.

The price of a three-year forward on a share of this stock is X, and at this price an investor is willing to enter into the forward. Determine what can be concluded about X.

(A) $X < 75$
(B) $X = 75$
(C) $75 < X < 90$
(D) $X = 90$
(E) $X > 90$

Problem 2.3. MFE Sample (Introductory) Problem #70.
Investors in a certain stock demand to be compensated for risk. The current stock price is 100. The stock pays dividends at a rate proportional to its price. The dividend yield is 2%. The continuously compounded risk-free interest rate is 5%. Assume there are no transaction costs.

Let X represent the expected value of the stock price 2 years from today. Assume it is known that X is a whole number. Determine which of the following statements is true about X.

(A) The only possible value of X is 105.
(B) The largest possible value of X is 106.
(C) The smallest possible value of X is 107.
(D) The largest possible value of X is 110.
(E) The smallest possible value of X is 111.

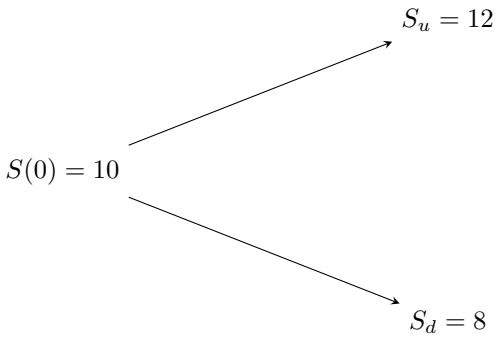
Problem 2.4. MFE Spring 2007: Problem #2
For a one-period binomial model for the price of a stock, you are given:

(i) The period is one year.
(ii) The stock pays no dividends.
(iii) $u = 1.433$, where u is one plus the rate of capital gain on the stock if the price goes up.
(iv) $d = 0.756$, where d is one plus the rate of capital loss on the stock if the price goes down.
(v) Calculate the true probability of the stock price going up.

The continuously compounded annual expected return on the stock is 10%.

(A) 0.52
(B) 0.57
(C) 0.62
(D) 0.67
(E) 0.72

Problem 2.5. MFE Spring 2009: Problem #7.
The following one-period binomial stock price model was used to calculate the price of a one-year, 10−strike call option on the stock.



You are given:

(i) The period is one year.
(ii) The true probability of an up-move is 0.75.
(iii) The stock pays no dividends.
(iv) The price of the one-year, 10−strike call is \$1.13.

Upon review, the analyst realizes that there was an error in the model construction and that S_d, the value of the stock on a down-move, should have been 6 rather than 8. The true probability of an up-move does not change in the new model, and all other assumptions were correct.

Recalculate the price of the call option.

(A) \$1.13
(B) \$1.20
(C) \$1.33
(D) \$1.40
(E) \$1.53

Problem 2.6. There are two possible states of the world in one year: the *bad* and the *good*. Our subjective opinion is that the probability of the *bad* state of the world is 1/3.

In our market-model there are two assets: one risky and one riskless. The risky asset will be worth \$50 in the *bad* state state of the world or \$80 in the *good* state of the world. This asset pays no dividends and its continuously compounded rate of return consistent with our subjective probability equals 8%. The riskless asset has a single \$100 payment in one year regardless of the state of the world. Its price today is \$92.

Find the price today of a \$70-strike, one-year European cash call option on the risky asset.

Problem 2.7. A non-dividend-paying stock is currently trading at \$100 per share. The price of this stock in a year is modeled using a one-period forward binomial tree with the stock's volatility specified as 0.30. The investor believes that the "true" probability of the stock price going up equals 0.46.

The continuously compounded risk-free interest rate equals 0.04.

- What is the continuously compounded rate of return on the above stock under the "true" probability the investor uses to model?
- Consider a \$95-strike, one-year European call option on the above stock. What is the continuously compounded rate of return of this call option under the above "true" probability?
- What is the compounded rate of return of the otherwise identical put option under the above "true" probability?