M339W: April 23rd, 2021.

The Volatility of a Two-Stock Portfolio

We index the securities in this portfolio w/ $i=1,2$.

R_i, $i=1,2$... the (simple) realized return of security i

w_i, $i=1,2$... the weight of security i in the portfolio

$$\boxed{w_i = \frac{\text{Value of component } i}{\text{Value of the portfolio}}}$$

R_P... return of the entire portfolio

$$\boxed{R_P = w_1 \cdot R_1 + w_2 \cdot R_2}$$

$$\Rightarrow \mathbb{E}[R_P] = w_1 \cdot \mathbb{E}[R_1] + w_2 \cdot \mathbb{E}[R_2]$$

Goal: Focus on the volatility of a portfolio, i.e., the standard deviation of the return.

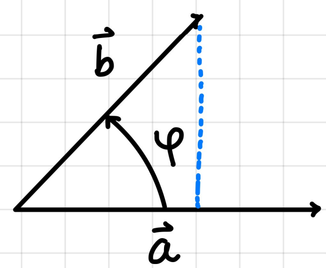
$$\longrightarrow: \quad Var[R_P] = Var[w_1 \cdot R_1 + w_2 \cdot R_2]$$

$$= w_1^2 \cdot Var[R_1] + w_2^2 \cdot Var[R_2]$$

$$+ 2 w_1 \cdot w_2 \cdot Cov[R_1, R_2]$$

By def'n. $\quad Cov[R_1, R_2] = \sigma_1 \cdot \sigma_2 \cdot \rho_{1,2}$

$$\boxed{\rho_{1,2} = corr[R_1, R_2]}$$



scalar product:

Cov

$$\langle \vec{a}, \vec{b} \rangle = \vec{a} \cdot \vec{b}$$

$$= \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos(\varphi)$$

sd   sd   corr

The volatility of the portfolio:

$$\boxed{\sigma_p = \sqrt{Var[R_p]}}$$

3) You are given the following information about the annual returns of two stocks, X and Y:

i) The expected returns of X and Y are $E[R_X] = 10\%$ and $E[R_Y] = 15\%$.

ii) The volatilities of the returns are $\cancel{V_X}^{\sigma_X} = 18\%$ and $\cancel{V_Y}^{\sigma_Y} = 20\%$.

iii) The correlation coefficient of the returns for these two stocks is 0.25. $\rho_{X,Y} = 0.25$

iv) The expected return for a certain portfolio, consisting only of stocks X and Y, is 12%.

$$R_P = w_X \cdot R_X + w_Y \cdot R_Y$$

Calculate the volatility of the portfolio return.

$$Var[R_P] = w_X^2 \cdot \sigma_X^2 + w_Y^2 \cdot \sigma_Y^2 + 2 \cdot w_X \cdot w_Y \cdot \sigma_X \cdot \sigma_Y \cdot \rho_{X,Y}$$

(A) 10.88%

(B) 12.56%

(C) 13.55%

(D) 14.96%

(E) 16.91%

$$0.12 = w_X \cdot E[R_X] + w_Y \cdot E[R_Y]$$
$$= w_X \cdot 0.10 + (1 - w_X) \cdot 0.15$$
$$w_X = \frac{0.15 - 0.12}{0.15 - 0.10} = 0.6 \Rightarrow w_Y = 0.4$$

$$Var[R_P] = (0.6)^2 \cdot (0.18)^2 + (0.4)^2 \cdot (0.20)^2 + 2 \cdot 0.6 \cdot 0.4 \cdot 0.18 \cdot 0.20 \cdot 0.25$$
$$= 0.022384 \Rightarrow \sigma_P = 0.1496$$

The Volatility of a Large Portfolio.

$$R_P = \underbrace{w_1 \cdot R_1 + w_2 \cdot R_2 + \cdots + w_n \cdot R_n}$$

$$\Rightarrow \underline{Var[R_P]} = Cov[R_P, R_P]$$

$$= Cov[w_1 \cdot R_1 + w_2 \cdot R_2 + \cdots + w_n \cdot R_n, R_P]$$

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$$= w_1 \cdot Cov[R_1, R_P] + w_2 \cdot Cov[R_2, R_P] + \cdots + w_n \cdot Cov[R_n, R_P]$$

$$= \underline{\sum_{i=1}^{n} w_i \cdot Cov[R_i, R_P]}$$

=> We can interpret this equality as:

the variance of the entire portfolio is a weighted average of the covariances of the individual returns w/ the entire portfolio.

2) You are given the following information about a portfolio with four assets.

Asset	Market Value of Asset	Covariance of asset's return with the portfolio return
I	40,000	0.15
II	20,000	-0.10
III	10,000	0.20
IV	30,000	-0.05

$\Sigma = 100$

Calculate the standard deviation of the portfolio return.

$w_{I} = 0.4$; $w_{II} = 0.2$; $w_{III} = 0.1$; $w_{IV} = 0.3$

(A) 4.50%

$Var[R_p] = 0.4(0.15) + 0.2(-0.1) + 0.1(0.2) + 0.3(-0.05)$

(B) 13.2%

$= 0.045$

(C) 20.0%

$\Rightarrow \sigma_p = \sqrt{0.045} = 0.212$

(D) 21.2%

(E) 44.7%

11) You are given the following information about a portfolio that has two equally-weighted stocks, P and Q. $w_P = w_Q = \frac{1}{2}$

(i) The economy over the next year could be good or bad with equal probability. $p_g = p_b = \frac{1}{2}$

(ii) The returns of the stocks can vary as shown in the table below:

Stock	Return when economy is good	Return when economy is bad
P	10%	–2%
Q	18%	–5%

Calculate the volatility of the portfolio return.

R_T... return of the total portfolio

$R_T = \frac{1}{2}(R_P + R_Q)$

Q: What is the distribution of R_T?

(A) 1.80%

(B) 6.90%

(C) 7.66%

(D) 8.75%

(E) 13.42%