M362K Probability
University of Texas at Austin
Practice Problems for In-Term Exam II
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Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points. **There are many ways in which any single problem can be solved. The solutions herein are just one possible way to tackle the given problems.**

All written work handed in by the student is considered to be
their own work, prepared without unauthorized assistance.

The University Code of Conduct

"The core values of The University of Texas at Austin are learning, discovery, freedom, leadership, individual opportunity, and responsibility. Each member of the university is expected to uphold these values through integrity, honesty, trust, fairness, and respect toward peers and community. As a student of The University of Texas at Austin, I shall abide by the core values of the University and uphold academic integrity."

"I agree that I have complied with the UT Honor Code during my completion of this exam."

Signature:

2.1. **DEFINITIONS.**

Problem 2.1. (5 points) Complete the definition of a *random variable* on a finite outcome space below:

Let Ω be a finite outcome space. A *random variable* on Ω is ...

2.2. **DEFINITIONS.**

Problem 2.2. (5 points) Write down the expression for the *standard normal density.*

2.3. **TRUE/FALSE QUESTIONS.**

Problem 2.3. (2 pts) Let A, B and C be *pairwise independent events.* Then, they are necessarily *independent. True or false?*

Problem 2.4. (2 pts) Two dice are rolled, the probability that the maximum (and **not** necessarily a strict maximum) of the upturned faces is achieved on the second die equals 1/2. *True or false?*

2.4. **FREE RESPONSE PROBLEMS.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 2.5. (10 points) Lady Eleanor Ward-Beaumont is learning to cook pasta. She tosses strands of (possibly) cooked spaghetti at the ceiling until one sticks. She is not a good cook (yet!), so we can assume that her tests are independent and that the probability of the pasta sticking to the ceiling is 0.2 in every trial. What is the probability that the strand of spaghetti sticks to the ceiling at exactly the 8^{th} try? Display your answer as a fully reduced fraction, please.

Problem 2.6. (25 points) A biased coin with the probability of "Heads" equal to 0.7 is tossed 1000 times. The number of "Heads" in the 1000 tosses is represented by the random variable X.

- (i) (5 points) What is the distribution of the random variable X? You need to write down its name and the numerical values of all of its parameters.
- (ii) (10 points) Write down the **exact** expression for the probability that more than 750 "Heads" have been observed.
- (iii) (10 points) Use the normal approximation to estimate the above probability.

Problem 2.7. (25 points) Let the random variable X represent the number of odd numbers obtained in four rolls of a fair die.

- (i) (5 points) What is the support of the random variable X?
- (ii) (5 points) What is the distribution of the random variable X? State its **name** and **parameter values**.
- (iii) (5 points) What is the probability mass function of the random variable X?
- (iv) (5 points) Define the random variable $Y = |X - 1|$. What is the support of Y?
- (v) (5 points) What is the probability mass function of the random variable Y?

Problem 2.8. (25 points) Seven Easter eggs are hidden in a backyard. Three of the seven eggs contain a toy train from the "Thomas the Tank Engine Series". A toddler is on an Easter egg hunt and only really cares about the "train eggs". He continues the egg hunt until he finds the first "train egg". Let the random variable X represent the number of regular eggs the toddler finds **before** discovering the first "train egg".

(i) (5 points) What is the support of the random variable X?

(ii) (10 points) What is the probability mass function of the random variable X?

(iii) (10 points) Now, imagine that the toddler wants to collect **all** of the "train eggs" and stops hunting eggs only after collecting all three. The eggs which remain in the backyard are the parents' responsibility to collect. Let Y denote the number of eggs collected by the parents. What is the support and the pmf of Y?

2.5. MULTIPLE CHOICE QUESTIONS.

Problem 2.9. (5 pts) You are given a TRUE/FALSE exam with 30 questions. Suppose that you need to answer 21 questions correctly in order to pass. You have no idea what the class is about and decide to toss a fair coin to answer all the questions; you circle TRUE if the outcome is tails and you circle FALSE if the outcome is heads. What is your estimate of the probability p that you manage to pass the exam using this strategy?

Hint: It is best to use the Normal Approximation to get the approximate probability.

(a) $p \leq 0.0005$

(b) $0.0005 < p \leq 0.006$

(c) $0.006 < p \leq 0.04$

(d) $0.04 < p$

(e) None of the above

Problem 2.10. (5 pts) An urn contains 20 balls, of which 19 are blue and one is red. If 7 of these balls are drawn, one at a time, with each selection being equally likely to be any of the balls that remain in the urn at the time, what is the probability that the red ball is chosen among those 7?

(a) 1/20

(b) 1/19

(c) 7/20

(d) 1/7

(e) None of the above

Problem 2.11. (5 pts) Find the probability of obtaining exactly two fives in six rolls of a fair die.

(a) $5^5/(2^3 \cdot 3^6)$

(b) $5^5/(2^6 \cdot 3^6)$

(c) $5^5/(2^6 \cdot 3^5)$

(d) $1/5$

(e) None of the above