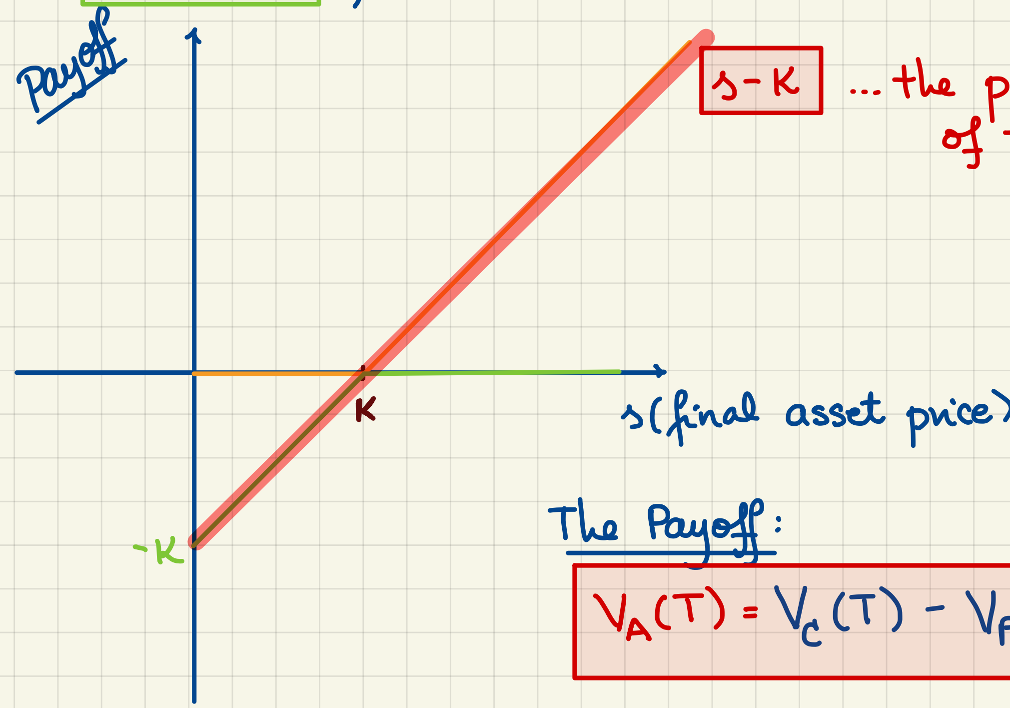
# M339D: October 17th, 2022.

## Put-Call Parity.

### Portfolio A:

- $\boxed{\text{long call}}$
- $\boxed{\text{short put}}$

both European & otherwise identical




... the payoff f'tion of the Portfolio A

**The Payoff:**

$$\boxed{V_A(T) = V_C(T) - V_P(T) = S(T) - K}$$

### Portfolio B:

- long non-dividend-paying stock
- borrow $PV_{0,T}(K)$ @ the risk-free interest rate $r$ to be repaid @ time $T$

$$\Rightarrow \boxed{V_B(T) = \underbrace{S(T)}_{\text{long stock}} - \underbrace{K}_{\text{loan repaid}}}$$

**Note:** $\boxed{V_A(T) = S(T) - K = V_B(T)}$

$\Rightarrow$ NO ARBITRAGE!

$V_A(0) = V_B(0)$

$$\boxed{V_C(0) - V_P(0) = S(0) - PV_{0,T}(K)}$$ Put-Call Parity

More generally: for any $t \in [0, T]$:

$$\boxed{V_c(t) - V_p(t) = S(t) - PV_{t,T}(K)}$$

Note:

- The no-arbitrage assumption is sufficient to get put-call parity.
- Only works for European options.
- We obtained a replicating portfolio for a forward, aka a synthetic forward.

## Advanced Derivatives Questions

**1.** Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is 60.

(ii) The call option currently sells for 0.15 more than the put option.

(iii) Both the call option and put option will expire in $\boxed{4}$ years.

(iv) Both the call option and put option have a strike price of $\boxed{70}$.

Calculate the continuously compounded risk-free interest rate. $r = ?$

(A) 0.039

(B) 0.049

(C) 0.059

(D) 0.069

(E) 0.079

Put-Call Parity:

$$\underbrace{V_c(0) - V_p(0)}_{\overset{(ii)}{=}\ 0.15} = \underbrace{S(0)}_{=\ 60} - \underbrace{PV_{0,T}(K)}_{=\ 70e^{-4r}}$$

$$70e^{-4r} = 60 - 0.15 = 59.85$$

$$e^{-4r} = \frac{59.85}{70}$$

$$-4r = \ln\left(\frac{59.85}{70}\right)$$

$$r = \frac{1}{4}\ln\left(\frac{70}{59.85}\right) = 0.25 * \log(70/59.85)$$

$$= 0.03916$$

**77.** You are given:

i) The current price to buy one share of XYZ stock is $\boxed{500}$.

ii) The stock does not pay dividends.

iii) The continuously compounded risk-free interest rate is 6%. $\boxed{r=0.06}$

iv) A European call option on one share of XYZ stock with a strike price of *K* that expires in one year costs $\boxed{66.59}$.

v) A European put option on one share of XYZ stock with a strike price of *K* that expires in one year costs $\boxed{18.64}$.

Using put-call parity, calculate the strike price, *K*.

(A) 449

(B) 452

(C) 480

(D) 559

(E) 582

$$V_c(0) - V_p(0) = S(0) - Ke^{-rT}$$

$$66.59 - 18.64 = 500 - Ke^{-0.06}$$

$$Ke^{-0.06} = 500 - 66.59 + 18.64$$

$$K = e^{0.06}(500 - 66.59 + 18.64) = 480$$

**78.** The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. You are given that the price of a 35-strike call option is 3.35 higher than the price of a 40-strike call option, where both options expire in 3 months.

$S(0)=40$ $r=0.08$

Calculate the amount by which the price of an otherwise equivalent 40-strike put option exceeds the price of an otherwise equivalent 35-strike put option.

(A) 1.55

(B) 1.65

(C) 1.75

(D) 3.25

(E) 3.35

$$+: \; V_c(0, K_1=35) - V_p(0, K_1=35) = \cancel{S(0)} - 35e^{-0.08(0.25)}$$

$$-: \; V_c(0, K_2=40) - V_p(0, K_2=40) = \cancel{S(0)} - 40e^{-0.02}$$

$$\underbrace{\boxed{V_c(0, K_1=35) - V_c(0, K_2=40)}}_{3.35}$$

$$-(V_p(0, K_1=35) - V_p(0, K_2=40)) = 5e^{-0.02}$$

answer: $5e^{-0.02} - 3.35 = 1.55$

# The Strong Law of Large Numbers (SLLN).

Let $\{X_k, k = 1, 2, \ldots\}$ be a sequence of independent, identically distributed random variables.

Assume: $\mu_X := \mathbb{E}[X_1] < \infty$

Then,

$$\boxed{\frac{X_1 + \cdots + X_n}{n} \xrightarrow[n \to \infty]{} \mu_X}$$