

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 2

Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 2.1. (20 points) Two coins are tossed and a (6-sided) die is rolled.

- (i) (5 points) Describe a sample space (probability space), together with the probability, on which such a situation can be modelled.
- (ii) (15 points) Find the probability mass function of the random variable whose value is the sum of the number on the die and the total number of heads.

Problem 2.2. (10 points) *Source: Sample P exam, Problem #126.* Under an insurance policy, a maximum of five claims may be filed per year by a policyholder. Let p_n be the probability that a policyholder files exactly n claims during a given year, where $n = 0, 1, 2, 3, 4, 5$. An actuary makes the following observations:

- $p_n \geq p_{n+1}$ for $n = 0, 1, 2, 3, 4$.
- The difference between p_n and p_{n+1} is the same for $n = 0, 1, 2, 3, 4$.
- Exactly 40% of policyholders file strictly fewer than two claims during a given year.

Calculate the probability that a random policyholder will file strictly more than three claims during a given year.

Problem 2.3. (10 points) A continuous random variable X has the probability density function f_X given by

$$f_X(x) = A - \frac{x}{50}, \quad 0 \leq x \leq 10.$$

- (a) Find the value of the constant A .
- (b) Find the value of the survival function of X at 7, i.e., calculate $S_X(7)$.

Problem 2.4. (10 points) The lifespan of a certain machine is exponentially distributed. The probability that the lifespan exceeds 4 years is p . Find the expression for the density of the lifespan in terms of p .