Problem. Let the current stock price be \$100.

Assume the Black-Scholes model.

According to your model:

- $\boxed{\mathbb{P}[S(1/4) < 95] = 0.2358}$
- $\boxed{\mathbb{P}[S(1/2) < 110] = 0.6026}$

M339W: March 12th, 2021.

What's the expected value of the time-1 stock price?

→: By the B-S model:

$$S(T) = S(0)e^{\overbrace{(\alpha - \delta - \frac{\sigma^2}{2})}^{=:\mu} T + \sigma\sqrt{T}\cdot Z} \quad \text{w/ } Z \sim N(0,1)$$

Random variable

rate of appreciation

Also: $\mathbb{E}[S(T)] = S(0)e^{(\alpha-\delta)\cdot T}$

Expected Value of R.V.

In particular: $\mathbb{E}[S(1)] = S(0)e^{\alpha-\delta} = S(0)e^{\mu + \frac{\sigma^2}{2}}$

Write out either a solution, or a plan for how you would solve this problem.

- $\mathbb{P}[S(1/4) < 95] = 0.2358$

Is 23.58th percentile of $S(1/4)$.

The critical value of the $N(0,1)$ is

$$z^*_{0.2358} = -0.72$$

$$95 = 100e^{\mu(1/4) + \sigma\sqrt{1/4}\cdot(-0.72)} \quad /:100$$

$\ln(\cdot)$

$$\ln(0.95) = \frac{1}{4}\mu + \sigma\left(\frac{1}{2}\right)(-0.72) = \frac{1}{4}\mu - 0.36\sigma \quad \text{(I)}$$

- $\mathbb{P}[S(1/2) < 110] = 0.6026$

110 is the 60.26^{th} percentile $S(1/2)$.

The critical value of the $N(0,1)$ is

$z^*_{0.6026} = 0.26$

$$110 = 100\, e^{\mu(1/2) + \sigma\sqrt{1/2}\,(0.26)} \quad / :100$$

$\ln(\cdot)$

$$\ln(1.1) = \frac{1}{2}\cdot\mu + \sigma\sqrt{1/2}\,(0.26) \quad (\mathrm{II})$$

We solve the system of (I) and (II), and we get

$\mu = \underline{0.1101}$ and $\sigma = \underline{0.2189}$.

$\Rightarrow$ Finally, $\mathbb{E}[S(1)] = S(0)e^{\mu + \frac{\sigma^2}{2}} = 114.35$ ■

Value@Risk (VaR) [Review].

Start w/ a (small) probability p.

- If your r.v. R corresponds to your return or profit or wealth, then the adverse event are all the small values of R.
  In this context, the $VaR_p(R)$ satisfies

$$\boxed{\mathbb{P}[R \leq VaR_p(R)] = p}$$

- If X corresponds to a loss, we set $VaR_p(X)$ to be such that

$$\mathbb{P}[X > VaR_p(X)] = p.$$

Problem [Sample IFM (Part II): P# 35]

You own a share of non-dividend-paying stock. You intend to hold it for a period of time. You want to set aside an amount of capital (a PERCENTAGE of the initial stock price) to reduce the risk of loss @ the end of holding period.

(i) • The Black-Scholes framework applies.

(ii) • The mean rate of return on the stock in your model is $\boxed{\alpha = 0.15}$.

(iii) • The stock's volatility is $\boxed{\sigma = 0.40}$.

(iv) • The investment period is 4 years, i.e., $\boxed{T = 4}$.

(v) • The VaR @ the 3rd percentile for the whole portfolio equals the initial stock price.

Q: Find the amount of capital put in reserve as a percentage of the initial stock price.

→: (v) ⟹ $VaR_{0.03}(S(T) + C) = S(0)$

the amount of capital set aside

$C = \varphi \cdot S(0)$

by the def'n of VaR

$$\Rightarrow \mathbb{P}[S(T) + \varphi \cdot S(0) < S(0)] = 0.03$$

↓(i)

$$S(T) = S(0)\, e^{(\alpha - \frac{\sigma^2}{2}) \cdot T + \sigma\sqrt{T} \cdot Z}$$

w/ $Z \sim N(0,1)$

$$\Rightarrow \mathbb{P}[\cancel{S(0)} e^{(0.15 - \frac{0.16}{2}) \cdot 4 + 0.4\sqrt{4} \cdot Z} + \varphi \cdot \cancel{S(0)} < \cancel{S(0)}] = 0.03$$

$$\Rightarrow \mathbb{P}[\underbrace{e^{(0.15-0.08)\cdot 4 + 0.4 \cdot 2 \cdot Z}}_{e^R} < 1 - \varphi] = 0.03$$

$1 - \varphi$ ↓ The 3rd percentile of e^R