University of Texas at Austin

Quiz #5

Please, provide your **complete solutions** to the following questions:

Problem 5.1. (10 points) Aggregate losses S under an insurance policy follow a compound Poisson process with mean equal to 1, and a severity random variable X. The support of the random variable X is $\{10, 20\}$. Moreover, we are given that

$$\mathbb{P}[X = 10] = 3\mathbb{P}[X = 20].$$

The premium for this policy equals 22.

If the insurance company makes a profit, i.e., if the premium exceeds the aggregate losses, it pays a dividend to the policyholder equal to one-third of the profit (the excess of the premium over the aggregate losses). Find the expected dividend.

Solution: From the given conditions, we conclude that the p.m.f. of the severity random variable is

$$p_X(10) = 3/4, \quad p_X(20) = 1/4.$$

Using the notation introduced in the problem, the dividend has the form $\frac{1}{3}\mathbb{E}[(22-S)_+]$. The possible values, i.e., the support of the random variable $D = (22 - S)_+$ is $\{0, 2, 12, 22\}$. Its probability mass function at values greater than zero is

$$\begin{aligned}
p_D(2) &= \mathbb{P}[S = 20] = e^{-1}p_X(20) + \frac{e^{-1}}{2} \cdot (p_X(10))^2 = \frac{17}{32e},\\
p_D(12) &= \mathbb{P}[S = 10] = e^{-1}p_X(10) = \frac{3}{4e},\\
p_D(22) &= \mathbb{P}[S = 0] = e^{-1} = \frac{1}{e}.
\end{aligned}$$

Thus, the dividend is equal to

$$\frac{1}{3}\mathbb{E}[D] = \frac{1}{3}\left(2 \cdot \frac{17}{32e} + 12 \cdot \frac{3}{4e} + 22 \cdot \frac{1}{e}\right) \approx 3.9317.$$

Problem 5.2. (5 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 3. Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be the two-parameter Pareto with parameters $\alpha = 3$ and $\theta = 5$. Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
How much is $\mathrm{Var}[S]$?

Solution:

$$\begin{aligned}
Var[S] &= \mathbb{E}[N]Var[X] + Var[N]\mathbb{E}[X]^2\\
&= 3(Var[X] + \mathbb{E}[X]^2)\\
&= 3\mathbb{E}[X^2]\\
&= 3\frac{\theta^2 \cdot 2!}{(\alpha-1)(\alpha-2)}\\
&= 3\frac{5^2 \cdot 2}{(3-1)(3-2)} = 75.
\end{aligned}$$