M378K: December 1st, 2025.

Hypothesis Testing.

Proof by Contradiction.

K... the claim we're trying to PROVE to be true

Q: What if K were not true?

Assume $\boxed{\text{not } K}$

fact Q .... fact (not Q)

These cannot coexist!

We say that we reached a contradiction!

$\Rightarrow\!\Leftarrow$

Our assumption of not K was wrong!

Hypothesis Testing.

Claim we're trying to SUBSTANTIATE.

μ... the population mean parameter (say, the mean cholesterol level w/ pills)

μ_0... the null population mean (A NUMBER) (say, a healthy benchmark)

$\boxed{\mu < \mu_0}$ ← Alternative Hypothesis

Assume $\boxed{\mu = \mu_0}$ ← Null Hypothesis

collect data
statistical analysis

p-value

Figure out the probability of seeing our data (or something more extreme) if $\mu = \mu_0$

If this probability is "small", we have evidence against $\mu = \mu_0$.

The smaller the probability, the STRONGER THE EVIDENCE.

The Normal Case w/ σ known.

Population Model: $Y \sim N(\boxed{\text{mean}=\mu}, sd=\sigma)$

unknown
and of interest

Hypothesis Testing Procedure.

$\boxed{\text{First:}}$ Set the hypotheses.

2nd Null Hypothesis: $\boxed{H_0: \mu=\mu_0}$

1st Alternative Hypothesis:

$$H_a: \begin{cases} \mu<\mu_0 & \text{(lower or left-sided)} \\ \mu\neq\mu_0 & \text{(two-sided)} \\ \mu>\mu_0 & \text{(upper or right-sided)} \end{cases}$$

$\boxed{\text{Second:}}$ Figure out the appropriate TEST STATISTIC (TS).

Natural choice:

$$\bar{Y} \sim Normal\left(mean=\mu, sd=\frac{\sigma}{\sqrt{n}}\right)$$

Under the null hypothesis, i.e., for $\mu=\mu_0$,

$$Z = \frac{\bar{Y}-\mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

$\boxed{\text{Third:}}$ Consider the observed value of the TS.
In this case, it's $\bar{y}$, i.e., the observed sample average.

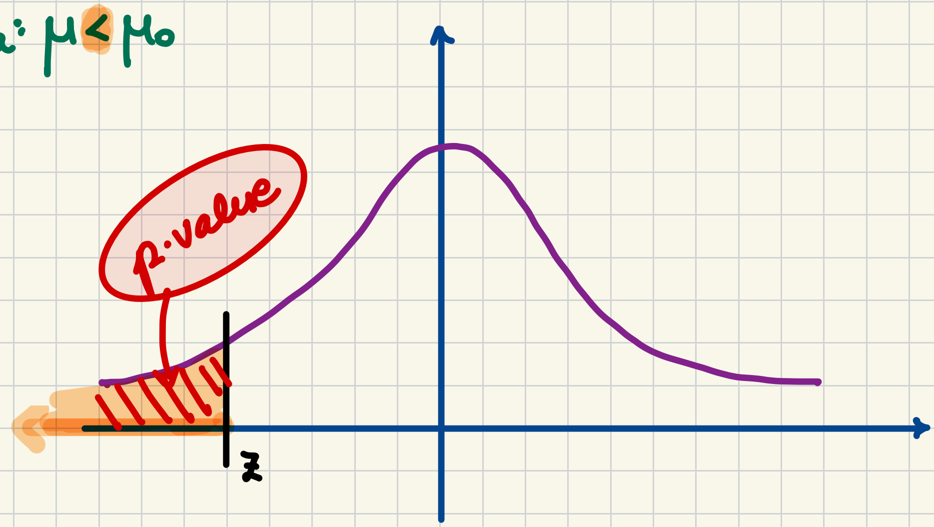
Q: What is the probability of observing $\bar{y}$ or something $\boxed{\text{more extreme}}$ under the null?

Exact interpretation depends on the structure of the alternative hypothesis.

Regardless: $z = \dfrac{\bar{y} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

Left-Sided Alternative:

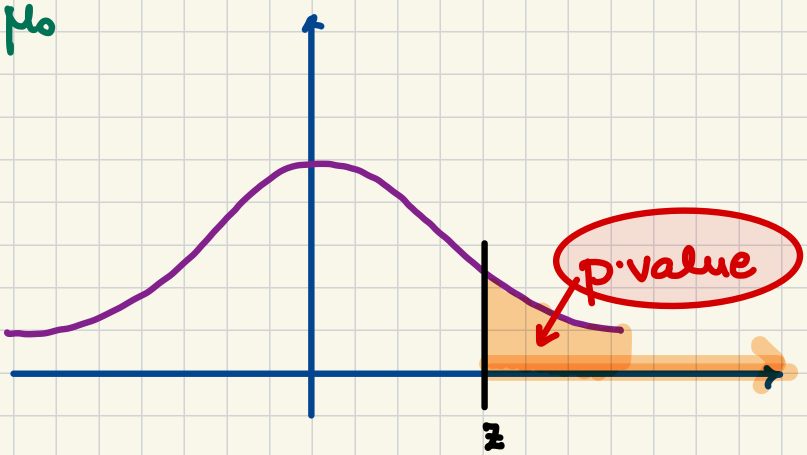
$H_a: \mu < \mu_0$



$\boxed{\text{p-value} = \mathbb{P}[Z \leq z]}$ $= \text{pnorm}(z)$

Right-Sided Alternative:

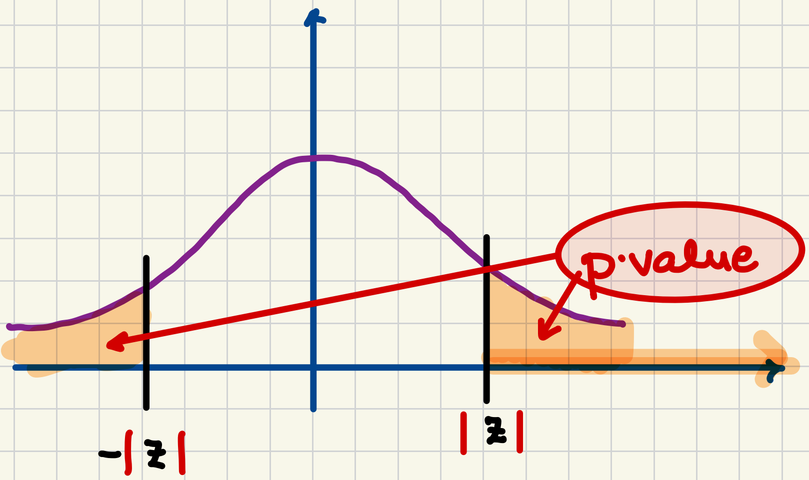
$H_a: \mu > \mu_0$



$\boxed{\text{p-value} = \mathbb{P}[Z \geq z]}$ $= 1 - \text{pnorm}(z)$

Two-Sided Alternative.

$H_a: \mu \neq \mu_0$



$$p\text{-value} = \mathbb{P}[Z \leq -|z|] + \mathbb{P}[Z \geq |z|]$$
$$= 2 \cdot \mathbb{P}[Z \leq -|z|]$$
$$= 2 \cdot \mathbb{P}[Z \geq |z|] \quad = 2 * \text{pnorm}(-\text{abs}(z))$$