M339D: October 21st, 2022.

The Binomial Asset Pricing Model.

$S(0)$ ... the observable initial asset price

0 ——— T

T ... time horizon (i.e., exercise date of an option)

$$S(0) \quad \begin{cases} \boxed{S_u = u \cdot S(0)} & \text{up node} \\ \boxed{S_d = d \cdot S(0)} & \text{down node} \end{cases}$$

ROOT NODE

h ... length of a single period

By convention: $\boxed{u > d}$ ✓

u ... up factor

d ... down factor

one period $\Rightarrow$ $S(T) = S(h)$ ... a r.v. denoting the time-T stock price w/ two possible values: S_u and S_d

---

Returns:

$$\frac{S_u - S(0)}{S(0)} = \frac{S_u}{S(0)} - 1 = u - 1$$

$$\frac{S_d - S(0)}{S(0)} = \frac{S_d}{S(0)} - 1 = d - 1$$

Market Model.

- riskless asset: @ the ccrfir $\boxed{r}$
- risky asset: non-dividend-paying stock

Imagine investing in one share of stock @ time 0:

$$\boxed{S(0)} \quad \begin{cases} S_u = u \cdot S(0) \\ S(0)e^{rh} \\ S_d = d \cdot S(0) \end{cases}$$

h

Wealth

$S_u = u \cdot S(0)$

$S_d = d \cdot S(0)$

At the risk-free rate $S(0)$ accumulates to $\underline{S(0)e^{rh}}$ @ time T.
" time h.

$$d \cdot \cancel{S(0)} < \cancel{S(0)}e^{rh} < u \cdot \cancel{S(0)}$$

$$\boxed{d < e^{rh} < u}$$ The no-arbitrage condition.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #6

Binomial option pricing.

Problem 6.1. In the setting of the one-period binomial model, denote by i the **effective** interest rate **per period**. Let u denote the "up factor" and let d denote the "down factor" in the stock-price model. If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage.

F

Fixed Statement:

$$\boxed{d < \boxed{1+i} < u}$$

Problem 6.2. In our usual notation, does this parameter choice create a binomial model with an arbitrage opportunity?

$u = 1.18$, $d = 0.87$, $r = 0.05$, $\cancel{\delta = 0}$, $h = 1/4$

No!

$$d < e^{rh} = e^{0.05(0.25)} = 1.013 < u$$

Binomial Option Pricing.

Stock Price Tree.

$$\begin{tikzcd}[row sep=small] & S_u = uS(0) \\ S(0) \arrow[ur] \arrow[dr] & \\ & S_d = dS(0) \end{tikzcd}$$

$$\xrightarrow{\text{populating}} \text{the tree}$$

We want to price a European-style derivative security w/ the exercise date @ the end of the period.

It is completely determined by its payoff function : $v(\cdot)$

e.g., for a call: $v_c(s) = (s-K)_+$

for a put: $v_p(s) = (K-s)_+$

The payoff of the derivative security is a random variable :

$$\boxed{V(T) := v(S(T))}$$

Payoff

$$\begin{tikzcd}[row sep=small] & S_u & V_u := v(S_u) \\ S(0) \arrow[ur] \arrow[dr] & & \\ & S_d & V_d := v(S_d) \end{tikzcd}$$

$V(0) = ?$

$$\xrightarrow{\text{population}}$$

$$\xleftarrow{\text{pricing}}$$