M339D: September 23rd, 2024.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #5

European call options.

$V_c(0)$ $S(0)$ T K

Problem 5.1. The initial price of a non-dividend-paying asset is \$100. A six-month, \$95-strike European call option is available at a \$8 premium. The continuously compounded risk-free interest rate equals 0.04. What is the break-even point for this call option?

r

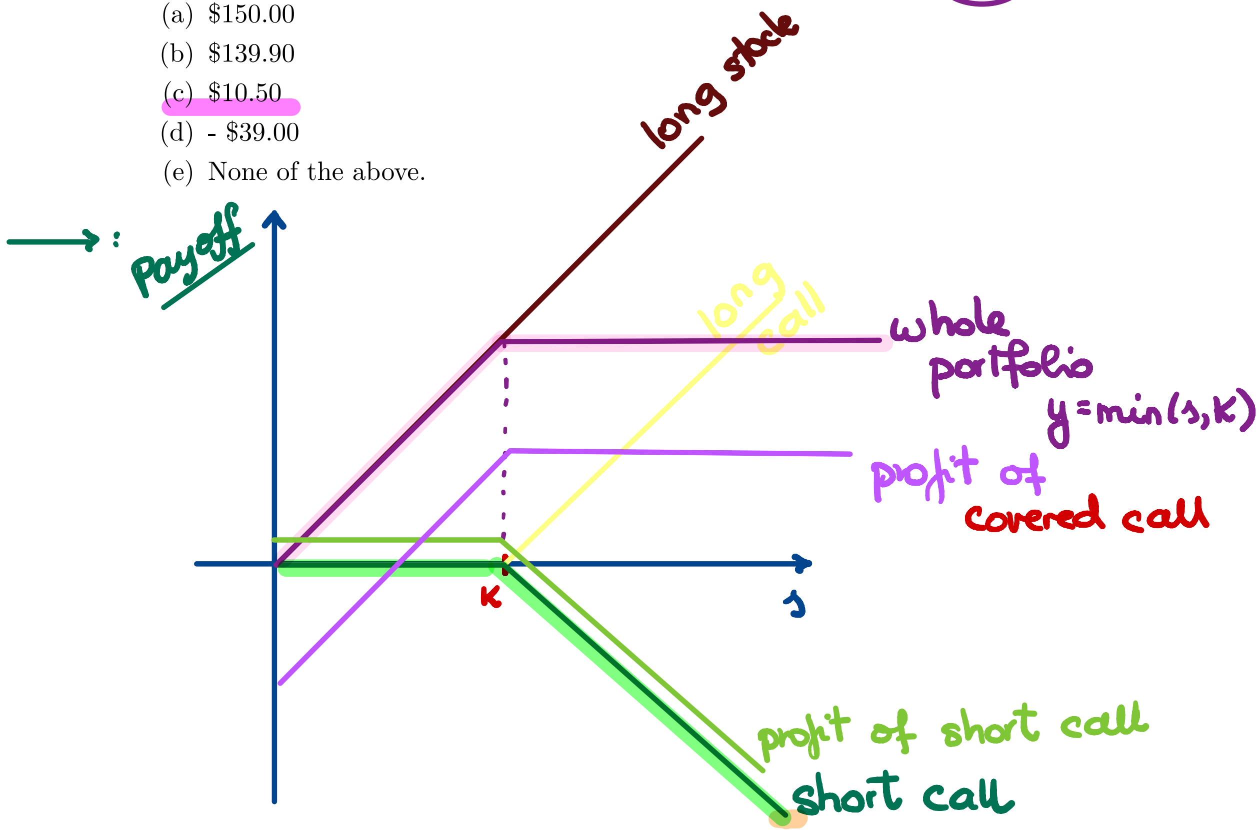
(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

$\longrightarrow$:

$$s^* = K + FV_{0,T}(V_c(0)) = 95 + 8e^{0.04 \cdot (0.5)} = 103.16 \quad \square$$

Problem 5.2. (5 points) A stock's price today is $\boxed{\$1000}$ and the annual effective interest rate is given to be 5%. You write a one-year, $1,050-strike call option for a premium of $10 while you simultaneously buy the stock. What is your **profit** if the stock's spot price in one year equals $1,200?

(a) $150.00
(b) $139.90
(c) $10.50
(d) - $39.00
(e) None of the above.



Algebra:

$$\text{Payoff} = -(S(T)-K)_+ + S(T) = \begin{cases} K & \text{if } S(T)\geq K \\ S(T) & \text{if } S(T)<K \end{cases}$$

$$= \min(S(T), K)$$

Covered call = Short Call + Long Underlying

In this problem:

$$\text{Payoff} = \min(1200, 1050) = 1050$$
$$\text{Initial Cost} = -10 + 1000 = 990$$
$$\text{Profit} = 1050 - 990(1.05) = 10.50$$ $\square$

Problem 5.3. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for $\boxed{\$1{,}000}$.

The jeweler models the market price of gold in one year as follows:

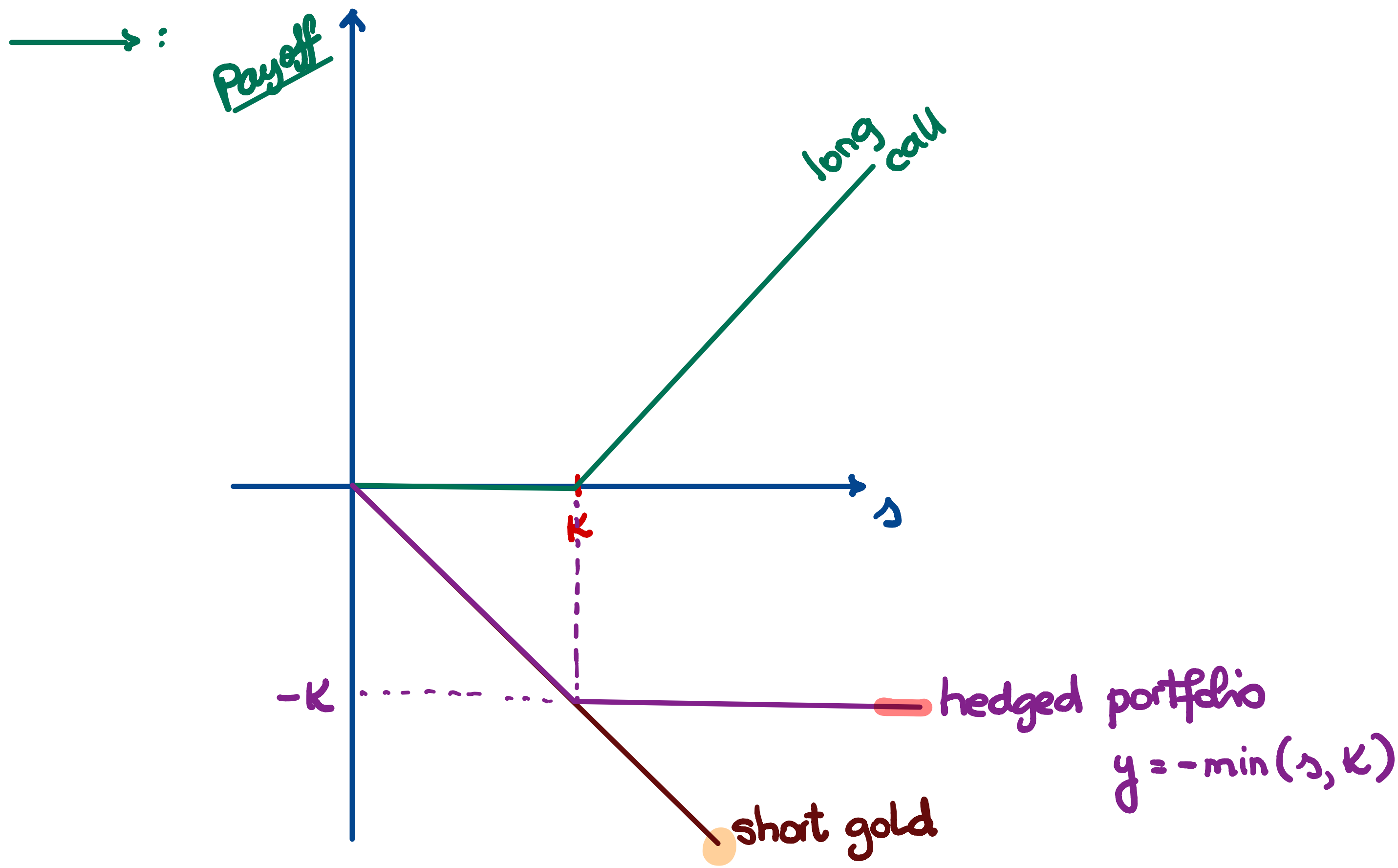
Gold price in one year	Probability	$\underline{\min(S(T), 900)}$
750 per ounce	0.2	750
850 per ounce	0.5	850
950 per ounce	0.3	900

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of \$900 per ounce. The option costs \$100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelry produced.

→ :



- short underlying
- long call

} FLOOR

Algebra

Payoff(Total) = Payoff(Gold) + Payoff(Call)

$= -S(T) + (S(T) - K)_+$

$$= \begin{cases} -K & \text{if } S(T) \geq K \\ -S(T) & \text{if } S(T) < K \end{cases} = -\min(S(T), K)$$

In this problem:

$$\text{Profit} = 1000 - \min(S(T), K) - 100e^{0.05}$$

$\mathbb{E}$

$$\mathbb{E}[\text{Profit}] = 1000 - \boxed{\mathbb{E}[\min(S(T), K)]} - 100e^{0.05}$$

$$\mathbb{E}[\min(S(T), K)] = 750 \cdot 0.2 + 850 \cdot 0.5 + 900 \cdot 0.3 = 150 + 425 + 270 = 845$$

$$\mathbb{E}[\text{Profit}] = 1000 - 845 - 100e^{0.05} = 155 - 100e^{0.05} = \underline{49.873}$$

$\square$

Problem 5.8. *Source: Sample IFM (Derivatives - Intro), Problem#11*
The current stock price is \$40, and the effective annual interest rate is 8%.

You observe the following option prices:

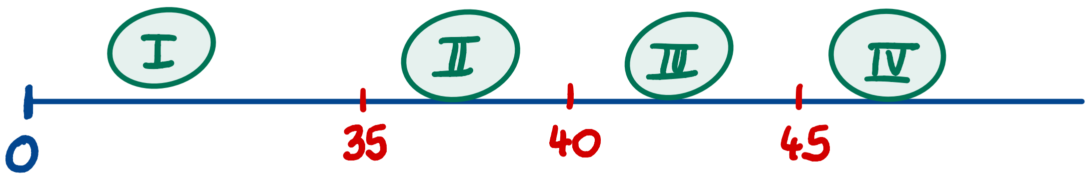
(1) The premium for a \$35-strike, 1-year European call option is \$9.12.
(2) The premium for a \$40-strike, 1-year European call option is \$6.22.
(3) The premium for a \$45-strike, 1-year European call option is \$4.08.

Assuming that all call positions being compared are **long**, at what 1-year stock price range does the \$45-strike call produce a higher profit than the \$40-strike call, but a lower profit than the \$35-strike call? Express your answer as an interval.

→ :

$$(s-40)_+ - 6.22(1.08) \leq (s-45)_+ - 4.08(1.08) \leq (s-35)_+ - 9.12(1.08)$$

$$(s-40)_+ - 6.72 \leq (s-45)_+ - 4.41 \leq \boxed{(s-35)_+} - 9.85$$



I $\quad -6.72 \leq -4.41 \leq -9.85$ No sol'ns here

II $\quad 0 - 6.72 \leq 0 - 4.41 \leq s - 35 - 9.85$