UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #5

European call options.

Problem 5.1. The initial price of a non-dividend-paying asset is \$100. A six-month, \$95-strike European call option is available at a \$8 premium. The continuously compounded risk-free interest rate equals 0.04. What is the break-even point for this call option?

(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

Solution: (d)
We need to solve for s in

$$(s-95)_+ = 8e^{0.02} \quad \Rightarrow \quad s = 95 + 8e^{0.02} = 103.16$$

Problem 5.2. (5 points) A stock's price today is \$1000 and the annual effective interest rate is given to be 5%. You write a one-year, \$1,050-strike call option for a premium of \$10 while you simultaneously buy the stock. What is your **profit** if the stock's spot price in one year equals \$1,200?

(a) \$150.00
(b) \$139.90
(c) \$10.50
(d) - \$39.00
(e) None of the above.

Solution: (c)

$$S(T) - 1000(1.05) - (S(T)-K)_+ + 10(1.05) = 1050 - 990(1.05) = 10.50.$$

Problem 5.3. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for \$1,000.

The jeweler models the market price of gold in one year as follows:

Gold price in one year	Probability
750 per ounce	0.2
850 per ounce	0.5
950 per ounce	0.3

The jeweler hedges the price of gold by buying a $1-$year call option with an exercise price of \$900 per ounce. The option costs \$100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelry produced.

Solution:

With $S(T)$ denoting the market price of gold at time $T=1$, the jeweler's **hedged** profit per piece of jewelry can be expressed as

$$1,000 - \min(S(T), 900) - 100e^{0.05} = 894.873 - \min(S(T), 900).$$

So, her expected **hedged** profit equals

$$894.873 - \mathbb{E}[\min(S(T), 900)] = 894.873 - (0.2 \cdot 750 + 0.5 \cdot 850 + 0.3 \cdot 900) = 49.873.$$

Problem 5.4. The current price of stock a certain type of stock is \$80. The premium for a 6−month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) \$80
(b) \$85.72
(c) \$85.84
(d) \$85.96
(e) None of the above.

Solution: (d)
The break-even point is

$$80 + 5.84e^{0.04/2} = 85.958$$

Problem 5.5. The price of gold in half a year is modeled to be equally likely to equal any of the following prices

$$\$1000, \quad \$1100, \quad \text{and} \quad \$1240.$$

Consider a half-year, \$1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

Solution:

$$50 \times \frac{1}{3} + 190 \times \frac{1}{3} = \frac{240}{3} = 80.$$

Problem 5.6. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

(a) 395.92
(b) 397.98
(c) 400
(d) 897.98
(e) None of the above.

Solution: (b)

$$200 \times 10 - 200 \times 5 - 500 - 200 \times 0.50e^{0.02} = 397.98$$

Problem 5.7. For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date T in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date T. Assume that the call's premium equals \$10 and that the annual effective interest rate equals 10%.

Express your asnwer as an interval.

Solution: The profit function of the forward contract is $v_F(s) = s - 100$. The profit function of the call is

$$v_C(s) - 10 \times 1.10 = (s - 100)_+ - 11.$$

For $s \geq 100$, the call's profit is smaller than the forward contract's profit. So, we focus on $s < 100$. Here we have to solve for s^* in

$$s^* - 100 = -11 \quad \Rightarrow \quad s^* = 89.$$

The answer is $[0, 89)$.

Problem 5.8. *Source: Sample IFM (Derivatives - Intro), Problem#11*
The current stock price is \$40, and the effective annual interest rate is 8%.

You observe the following option prices:

(1) The premium for a \$35-strike, 1-year European call option is \$9.12.
(2) The premium for a \$40-strike, 1-year European call option is \$6.22.
(3) The premium for a \$45-strike, 1-year European call option is \$4.08.

Assuming that all call positions being compared are **long**, at what 1-year stock price range does the \$45-strike call produce a higher profit than the \$40-strike call, but a lower profit than the \$35-strike call?

Express your answer as an interval.

Solution: The profit curve for a long European call option with strike K and exercise date T has the following form:

$$(s - K)_+ - FV_{0,T}(V_C(0, K)),$$

where $V_C(0, K)$ denotes the time-0 premium of the call with strike K. So, in the present problem, we have the following three profit curves:

$$\begin{aligned}(s - 35)_+ - 9.12(1.08) &= (s - 35)_+ - 9.85,\\ (s - 40)_+ - 6.22(1.08) &= (s - 40)_+ - 6.72,\\ (s - 45)_+ - 4.08(1.08) &= (s - 45)_+ - 4.41.\end{aligned}$$

In order to figure out the region in which the \$45-strike call to has a higher profit than the \$40-strike call, we need to solve the following inequality:

$$(s - 40)_+ - 6.72 < (s - 45)_+ - 4.41. \tag{5.1}$$

If $0 \leq s \leq 40$, this inequality becomes

$$-6.72 < -4.41. \tag{5.2}$$

We conclude that all values $s \leq 40$ satisfy inequality (5.1). If $40 < s < 45$, the above inequality (5.1) becomes

$$s - 40 - 6.72 < -4.41 \quad \Rightarrow \quad s < 42.31.$$

So, all $s \in [0, 42.31)$ satisfies (5.1). If $s \geq 45$, inequality (5.1) is trivially wrong for all such s.

In order to figure out the region in which the \$45-strike call to has a lower profit than the \$35-strike call, we need to solve the following inequality:

$$(s - 45)_+ - 4.41 < (s - 35)_+ - 9.85. \tag{5.3}$$

If $s \leq 35$, we get no solutions to the inequality. If $35 < s < 45$, the above inequality (5.3) becomes

$$-4.41 < s - 35 - 9.85 \quad \Rightarrow \quad 40.44 < s.$$

So, any $s \in (40.44, 45)$ satisfies (5.3). Finally, if $s \geq 45$, we have that (5.3) becomes

$$s - 45 - 4.41 < s - 35 - 9.85 \quad \Rightarrow \quad -49.41 < -44.85.$$

So, any $s \geq 45$ satisfies (5.3).

Pooling all of our conclusions together, we get the final answer $s \in (40.44, 42.31)$

Problem 5.9. (5 points) **Sample FM(DM) #13.**
Suppose that you short one share of a stock index for 50, and that you also buy a 60−strike European call option that expires in 2 years for 10. Assume the effective annual interest rate is 3%. If the stock index increases to 75 after 2 years, what is the profit on your combined position?

Solution: The payoff is simply:

$$-75 + (75 - 60)_+ = -60.$$

In words, as a short seller, you have to purchase the stock index back and you are going to take advantage of owning the call option on that index (as opposed to paying the higher market price). The initial cost is $-50 + 10 = -40$. So, the value of this initial cost in 2 years equals $-40 \cdot (1.03)^2 = -42.436$. The profit is

$$-60 + 42.436 = -17.564.$$