M378K Introduction to Mathematical Statistics
Problem Set #18
Consistency.

Definition 18.1. *$\hat{\theta}_n$ is said to be a* consistent estimator *of θ if*

$$\hat{\theta}_n \to \theta \quad \textit{in probability as } n \to \infty,$$

i.e., if for any $\varepsilon > 0$,

$$\lim_{n\to\infty} \mathbb{P}\left[|\hat{\theta}_n - \theta| > \varepsilon\right] = 0.$$

Theorem 18.2. *Let $\hat{\theta}_n$ be* unbiased *and such that*

$$\mathrm{Var}\left[\hat{\theta}_n\right] \xrightarrow{n\to\infty} 0.$$

Then, $\hat{\theta}_n$ is a **consistent estimator**.

Problem 18.1. *Let $Y_1, Y_2, \dots, Y_n$ be a random sample from any distribution with finite first and second moments. Propose a consistent estimator for the population mean μ and* **prove** *that it is, indeed, consistent.*

Problem 18.2. *Consider a random sample $Y_1, Y_2, \dots, Y_n$ from a power distribution with the density of the form*

$$f_Y(y) = \theta y^{\theta-1}\mathbf{1}_{(0,1)}(y).$$

What is a consistent estimator for $\frac{\theta}{\theta+1}$? **Prove** *that your choice is indeed consistent.*