

**NAME:**

M339W/389W Financial Mathematics for Actuarial Applications  
University of Texas at Austin  
**In-Term Exam III**  
Instructor: Milica Čudina

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The maximum number of points on this exam is 100.

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**Problem 3.1.** (5 points) Assume that the risk-free interest rate equals 0.04. The Sharpe ratio of asset  $S$  is given to be  $1/4$  while the Sharpe ratio of asset  $Q$  equals  $1/3$ . You know that the volatility of  $S$  is three times the volatility of  $Q$ . If you build an equally weighted portfolio with assets  $S$  and  $Q$  as its two components, the expected return of this portfolio will be 0.10. What is the expected return of  $Q$ ?

**Problem 3.2.** (5 points) Assume the **Capital Asset Pricing Model** holds.

You are given the following information about stock X, stock Y, and the market:

- The expected return and volatility for the market portfolio are 0.12 and 0.2, respectively.
- The required return and volatility for the stock  $X$  are 0.0404 and 0.4, respectively.
- The correlation between the returns of stock X and the market is  $-0.25$ .
- The volatility of stock  $Y$  is 0.3.
- The correlation between the returns of stock Y and the market is 0.1.

Calculate the required return for stock Y.

**Problem 3.3.** (5 points) Assume the **CAPM** holds.

Let the risk-free interest rate be 0.025 and let the expected return of the market portfolio be equal to 0.08.

Suppose that stock  $X$  has  $\beta_X = 1.4$  and that stock  $Y$  has  $\beta_Y = 0.8$ . Using the risk-free asset, stock  $X$ , and stock  $Y$ , you create a portfolio such that you invest twice as much in asset  $X$  as in asset  $Y$  while the weight of the risk-free asset is 0.4. What is the expected return of this portfolio?

**Problem 3.4.** (5 points) You are given the following information about stock  $X$  and a portfolio  $P$ :

- The annual effective risk-free rate is 4%.
- The portfolio's expected return is 0.10 and its volatility is 0.2.
- The expected return of stock  $X$  is 0.03 and its volatility is 0.25.
- The correlation between the returns of stock  $X$  and the portfolio  $P$  is  $-0.2$ .

Then:

- (a) The required return of stock  $X$  is 0.025 and the investor holding portfolio  $P$  should invest in stock  $X$ .
- (b) The required return of stock  $X$  is 0.025 and the investor holding portfolio  $P$  should not invest in stock  $X$ .
- (c) The required return of stock  $X$  is 0.055 and the investor holding portfolio  $P$  should invest in stock  $X$ .
- (d) The required return of stock  $X$  is 0.055 and the investor holding portfolio  $P$  should not invest in stock  $X$ .
- (e) None of the above.

*You have to show your work. The final answer without a justification will earn zero points (even if it's correct).*

**Problem 3.5.** (5 points) You are a pessimist and you model the state of the economy to be twice as likely to be *bad* as it is to be *good*. There are no other states of the economy in your model.

According to your model, if the economy is *good*, the return of stock  $S$  will be 0.09 and the return of stock  $Q$  will be 0.15. Also, if the economy is *bad*, the return of stock  $S$  will be  $-0.03$  and the return of stock  $Q$  will be  $-0.045$ .

You build a portfolio out of the stocks  $S$  and  $Q$ . The expected return of this portfolio is 0.014. What is the volatility of your portfolio?

**Problem 3.6.** (5 points) Consider two assets  $X$  and  $Y$  such that:

- their expected returns are  $\mathbb{E}[R_X] = 0.10$  and  $\mathbb{E}[R_Y] = 0.08$ ;
- their volatilities are  $\sigma_X = 0.2$  and  $\sigma_Y = 0.25$ ;
- the correlation coefficient of their returns is  $\rho_{X,Y} = -1$ .

You construct a portfolio consisting of shares of  $X$  and  $Y$  with a risk-free return. What is the value of the risk-free interest rate?

**Problem 3.7.** (5 points) Which one of the following statements is **not** correct?

- (a) Any equally weighted portfolio contains only systematic risk.
- (b) The volatility of an equally weighted portfolio is at most as large as the average of the volatilities of its components.
- (c) Full diversification of an investment portfolio leaves only market risk.
- (d) Adding another investment into your portfolio may increase the volatility of the portfolio.
- (e) The Sharpe ratio reflects the reward-to-risk ratio of an investment.



**Problem 3.8.** (5 points) You are given the following information about the return of a security, using a three-factor model:

Factor	Beta	Expected Return
T	0.32	12%
U	0.40	16%
V	0.50	10%

The expected return of this security using the given three-factor model is equal to 14.7%. What is the annual effective risk-free rate of return?

**Problem 3.9.** (5 points) A market-maker sells option  $I$  for \$8. This option's delta is 0.6557 and its gamma is 0.01. The market maker proceeds to delta-gamma hedge this commitment by trading in the underlying and also in option  $II$  on the same stock. The latter option's price is \$2.37, its delta is 0.5794 and its gamma is 0.04.

What is the market-maker's resulting position in option  $II$ ?

**Problem 3.10.** (15 points) Assume the Black-Scholes framework for the pair of stocks **S** and **Q**.

For the stock **S**, you are given that

- the current stock price is \$50 per share;
- the stock pays dividends in the amount  $0.02S(t) dt$  during the time period  $(t, t + dt)$ ;
- the stock's volatility is 0.20.

For the stock **Q**, you are given that

- the current stock price is \$40 per share;
- the stock pays no dividends;
- the stock's volatility is 0.30.

The correlation between the returns of **S** and **Q** is  $-0.25$ .

The continuously compounded, risk-free interest rate is 0.055.

What is the price of the exchange call option on **S** with the strike asset **Q** with exercise date in a quarter year?

**Problem 3.11.** (5 points) Consider a two-year project. There are only three cash flows for this project:

- The first occurs at  $t = 0$ , and is  $-50$ .
- The second occurs at  $t = 1$ , and is  $40$ .
- The third occurs at  $t = 2$ , and is  $11.50$ .

Determine  $r$ , the cost of capital, that leads to the project breaking even.

**Problem 3.12.** (10 points) Assume that the continuously-compounded, risk-free interest rate equals 0.04

Consider a fund which is modeled using the Black-Scholes framework. The fund pays no dividends and its volatility is 0.20.

A variable annuity is linked to this fund. It has a *guaranteed minimum accumulation benefit (GMAB)*. More precisely, it is guaranteed that the amount paid to the policyholder (if they are still living in five years) will be at least 5% greater than the originally invested amount.

According to your tables, the policyholder will still be alive in five years with probability 0.95.

Assume that there are no lapses.

If the initial investment the policyholder makes is equal to \$10,000, what is the value of the guarantee?

**Problem 3.13.** (5 points) Which of the following exotic options would be useful for hedging an inflation-indexed pension?

- (a) Standard lookback call.
- (b) Extrema lookback put.
- (c) Rainbow option.
- (d) Shout option.
- (e) Chooser option.

**Problem 3.14.** (5 points) Consider a company whose current stock price is \$50 per share. The company has 100,000 shares outstanding. The company also has \$5,000,000 in debt. Let the equity beta be equal to 1 and let the debt beta be equal to 0.4.

The company sells some shares and uses the proceeds to repay some of the debt. This lowers the equity beta to 0.8.

Assuming that the debt beta remains the same, how much of the debt was repaid?

**Problem 3.15.** (5 points) A company has  $X$  in revenue this year. The revenue growth is projected to be 2% annually. The company maintains a unit debt-to-equity ratio at all times. The cost of equity is 12% and the cost of debt is 4%.

The corporate tax rate is 35%.

What is the ratio of the pre-tax value of the company to the post-tax value of the company?



**Problem 3.16.** (5 points) For stock  $S_1$ , you are given that its expected return equals 0.16 and its  $\beta$  is 1.2. For stock  $S_2$ , you are given that its expected return equals 0.08 and its  $\beta$  is 0.32. Both of these stocks lie on the *Security Market Line*. For stock  $S_3$ , you are given that its expected return equals 0.12 and its  $\beta$  is 0.8. What is the  $\alpha$  of stock  $S_3$ ?

**Problem 3.17.** (5 points) Which of the following statements is not correct?

- (a) Familiarity bias generally does not result in a systematic trading bias.
- (b) The disposition effect results in longing the stocks with positive returns and shorting the stocks with negative returns.
- (c) According to the weak formulation of the efficient market hypothesis, one cannot consistently make gains by trading based on the information contained in past prices.
- (d) In the semi-strong form of the efficient market theory, one cannot consistently make gains by trading based on public information.
- (e) Herd behavior results in a systematic trading bias.