

Name:

M339D=M389D Introduction to Actuarial Financial Mathematics

University of Texas at Austin

Solution: Mock In-Term One

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Notes: This is a closed book and closed notes exam. This exam is graded out of 50 points.

Time: 50 minutes

Problem 1.1. (5 points) Let the accumulation function be given by

$$a(t) = (1 + 0.05)^{t^2}$$

Then, we can say the following about the continuously compounded, risk-free interest rate r associated with the above accumulation function:

- (a) $r = 0.05$
- (b) $r = 2 \ln(1.05)$
- (c) $r = 0.10$
- (d) The continuously compounded, risk-free interest rate is not constant.
- (e) None of the above

Solution: (d)

$$r = \frac{d}{dt} \ln(a(t)) = \frac{d}{dt} \ln[1.05^{t^2}] = 2t \ln(1.05).$$

Problem 1.2. (5 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be two functions given by

$$f(x) = |x - 10|$$

and

$$g(x) = \begin{cases} \min(x, 4) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then, $g(f(7))$ equals ...

- (a) 0
- (b) 3
- (c) 4
- (d) 7
- (e) None of the above

Solution: (b)

Problem 1.3. (5 points) Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ be a probability space. We denote by p_k the probability of the elementary outcome ω_k , i.e., $p_k = \mathbb{P}[\{\omega_k\}]$ for $k = 1, \dots, 5$. You are given that p_k/p_{k-1} is constant for $k = 2, 3, 4, 5$. You are also given that $p_1 = 16/31$. Find p_5 .

- (a) $1/31$
- (b) $2/31$
- (c) $4/31$
- (d) Not enough information is given.
- (e) None of the above.

Solution: (a)

Problem 1.4. (5 points) *Source: Sample P exam, Problem #185.*

A student takes an examination consisting of 20 true/false questions. The student knows the correct answer to n of the questions and guesses the answers to the rest at random. The conditional probability that the student knows the answer to a question, given that the student answered it correctly, is 0.824. Calculate n .

- (a) 8
- (b) 10
- (c) 14
- (d) 16
- (e) 18

Solution: (c)

Problem 1.5. (5 points) The probability mass function p_X of a discrete random variable X is given by

$$p_X(x) = \begin{cases} 1/2, & \text{for } x = 1 \\ 1/3, & \text{for } x = 2 \\ 1/6, & \text{for } x = 4 \end{cases}$$

Find $\mathbb{E}[|X - 3|]$.

- (a) $-7/6$
- (b) $1/6$
- (c) $11/6$
- (d) $3/2$
- (e) None of the above.

Solution: (d)

The random variable $|X - 3|$ has the following distribution

$$|X - 3| \sim \begin{cases} 1 & \text{with probability } 1/2 \\ 2 & \text{with probability } 1/2 \end{cases}$$

So, its expectation is $3/2$.

Problem 1.6. (5 points) Maria lives in Austria and receives her salary in Euro. She decides to spend 1000 Euro and let the proceeds of the exchange accrue interest at the USD continuously compounded, risk-free interest rate. She will withdraw the balance in three months and exchange it back to Euros. Assume that there were no intermediate deposits or withdrawals. You know the following:

- The initial exchange rate is 1.19 USD per Euro.
- The USD continuously compounded, risk-free interest rate is equal to $r_{\$} = 0.02$.
- The Euro continuously compounded, risk-free interest rate is equal to $r_e = 0.06$.

Given that the exchange rate at the end of the three months equals 1.23 USD per Euro, how much (in Euro) does Maria receive?

- (a) 972.33
- (b) 982.10
- (c) 987.02
- (d) 1195.97
- (e) None of the above.

Solution: (a)

Maria spends 1000 Euro. So, she receives 1190 USD. The balance at the end of the three-month period is

$$1190e^{0.02/4} = 1195.965.$$

Taking into account the final exchange rate, her payoff is

$$1195.965/1.23 = 972.3293.$$

Problem 1.7. (5 points) The current price of a continuous-dividend paying stock is \$100 per share. Its dividend yield is 0.02. You spend \$20 of your own money to buy this stock. You also borrow \$80 at this time to be used towards the purchase of this stock. You do not intend to make any further trades over the next year. You intend to liquidate your investment at the end of the year.

The continuously compounded, risk-free interest rate is 0.06.

What is the break-even point of your investment?

- (a) 94.18
- (b) 100
- (c) 104.8
- (d) 106.18
- (e) None of the above.

Solution: (c)

As we learned in a homework assignment, the profit for this partially leveraged investment is the same as the profit for the outright purchase. So, the break-even point is, in our usual notation,

$$s^* = S(0)e^{(r-\delta)T} = 100e^{(0.06-0.02)(1)} = 100e^{0.04} = 104.0811.$$

Problem 1.8. (5 points) Let the current price of a non-dividend-paying stock be \$40.

You model the distribution of the time=1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 42, & \text{with probability } 1/4, \\ 38, & \text{with probability } 1/2, \\ 36, & \text{with probability } 1/4. \end{cases}$$

The continuously compounded, risk-free interest rate is 0.04.

What is your expected profit under the above model, if you short sell one share of stock at time=0 and intend to close the short sale at time=1?

- (a) 1.50
- (b) 2.17
- (c) 2.54
- (d) 3.13
- (e) None of the above.

Solution: (d)

The initial cost is $-S(0)$ and the payoff is $-S(1)$. So, with $T = 1$, the profit equals

$$-S(T) + S(0)e^{rT}.$$

Thus, the expected profit equals

$$-\mathbb{E}[S(T)] + S(0)e^{rT}.$$

According to the given model for the stock price, we have

$$\mathbb{E}[S(T)] = 42 \left(\frac{1}{4} \right) + 38 \left(\frac{1}{2} \right) + 36 \left(\frac{1}{4} \right) = 38.5.$$

Finally, the expected profit is

$$-38.5 + 40e^{0.04} = 3.132431.$$

Problem 1.9. (5 points) Roger owns a cow named Elsie. Her estimated worth today is \$3,750. Roger enters into a forward agreement with Harry to sell him Elsie the cow in 6 months for \$4,000. On the delivery date, Roger changes his mind and wants cash settlement instead. Harry agrees. They look into the “Bovine Blue Book” and realize that Elsie’s worth on that date is \$3,500.

What is the cash flow that has to take place as part of the cash settlement?

- (a) \$500 from Roger to Harry
- (b) \$500 from Harry to Roger
- (c) \$250 from Roger to Harry
- (d) \$250 from Harry to Roger
- (e) None of the above.

Solution: (b)

Problem 1.10. (5 points) A farmer produces one thousand crates of apples. The total and aggregate costs of production are \$48,000. The farmer enters a forward contract for the entire harvest to hedge at a forward price of \$69 per crate at harvest time.

The market price of apples at harvest time is \$70 per crate.

What is the farmer's profit?

- (a) 1000 loss
- (b) 1000 gain
- (c) 21000 gain
- (d) 22000 gain
- (e) None of the above.

Solution: (c)

$$69000 - 48000 = 21000$$

Problem 1.11. (5 points) Pancakes, Inc. produces strawberry pancakes for the pancake festival. It longed a forward contract on 100 pounds of strawberries at \$2.50 per pound to be delivered to the festival and added to the pancakes. According to the contract with the organizers, the total fixed revenue will be \$6,000 for the pancakes produced with the above strawberries. Costs other than strawberries total \$1200.

On the morning of the pancake festival, the market price of strawberries is \$2.25 per pound.

Find the company's profit.

- (a) 2300
- (b) 1550
- (c) 2550
- (d) 1500
- (e) None of the above.

Solution: (e)

$$6000 - 1200 - 250 = 4550$$

Problem 1.12. (5 points) Which of the following claims is **not** correct?

- (a) The owner of the European call has the right to exercise the option at any time prior to the expiration date.
- (b) The owner of the European call has the right to buy the underlying at the strike price.
- (c) The writer of the European call has an obligation to buy the underlying at the strike price.
- (d) The strike price is agreed upon when the option is written.
- (e) The writer of the European call receives a premium when the option is written.

Solution: (a) and (c)