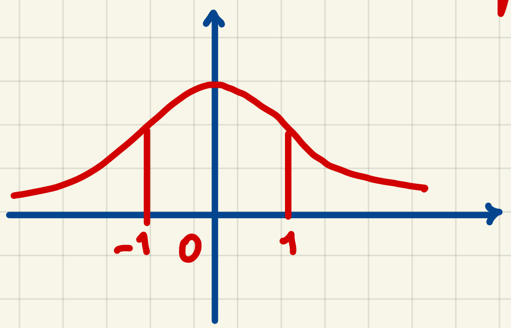
M339D: March 26th, 2025.

Standard Normal Distribution.

We say that a random variable Z has the **standard normal distribution** if its probability density function (pdf) has the form

$$f_Z(z) = \varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad \text{for all } z \in \mathbb{R}$$



- mean/median/mode $= 0$
- symmetric about the vertical axis, i.e., $\boxed{\varphi(z) = \varphi(-z)}$, i.e., $\boxed{\text{even}}$

The cumulative distribution f'tion (cdf) of the standard normal is

$$N(z) = \Phi(z) = \mathbb{P}[Z \leq z] = \int_{-\infty}^{z} f_Z(u)\,du = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$

$\boxed{\text{No analytic form!}}$

There are the standard normal tables!

We can use 'dnorm' and 'pnorm' and 'qnorm' in 'R'.

We write $\boxed{Z \sim N(0,1)}$

$\square$