University of Texas at Austin

HW Assignment 7

Implied volatility. Hedging.

Provide your complete solution. Final answers only, even if correct, will receive zero credit. Thank you!

7.1. Implied volatility.

Problem 7.1. (10 points) Let $\mathbf{S} = \{S(t),\, t \geq 0\}$ denote the price of a non-dividend-paying stock. The stock price today equals \$100. Assume that the Black-Scholes setting holds.

Let r denote the continuously compounded risk-free interest rate.

Consider a European call option with exercise date $T = 10$ and strike price $K = S(0)e^{rT}$. You are given that its price today equals $V_C(0) = \$68.26$.

The goal of this problem is to obtain the implied volatility of the stock S.

(i) (5 pts) Write down the expression for the Black-Scholes price of the European call.
(ii) (3 pts) Simplify the expression you obtained in part (i) so that the call price depends only on the volatility σ.
(iii) (2 pts) Using the properties of the standard normal cumulative distribution function N, the standard normal table, the European call price given in the problem and your answer to part (ii), solve for σ.

Problem 7.2. (10 points) Let $\mathbf{S} = \{S(t),\, t \geq 0\}$ denote the price of a continuous-dividend-paying stock. The prepaid forward price for delivery of one share of this stock in one year equals \$98.02. Assume that the Black-Scholes model is used for the evolution of the stock price.

Consider a European call and a European put option both with exercise date in one year. They have the same strike price and the same Black-Scholes price equal to \$9.37. What is the implied volatility of the underlying stock?

Problem 7.3. (10 points) Let $\mathbf{S} = \{S(t),\, t \geq 0\}$ denote the price of a non-dividend-paying stock. The current stock price is \$50. Assume that the Black-Scholes model is used for the evolution of the stock price.

Let the continuously compounded, risk-free interest rate be equal to 0.05.

Consider a European call option on this stock with exercise date in one quarter-year and with the strike price equal to $K = 50e^{0.0125}$. The price of this option is observed to be \$3.98. What is the stock's implied volatility?

Problem 7.4. (10 points) Let $\mathbf{S} = \{S(t),\, t \geq 0\}$ denote the price of a continuous-dividend-paying stock. The current stock price is \$100 and its dividend yield is 0.01. Assume that the Black-Scholes model is used for the evolution of the stock price.

Let the continuously compounded, risk-free interest rate be equal to 0.025.

Consider a European call option on this stock with exercise date in nine months and with the strike price equal to $K = 100e^{0.01125}$. The price of this option is observed to be \$10.26. What is the stock's implied volatility?

7.2. **Delta-hedging.**

Problem 7.5. (2 points) An investor wants to delta-hedge a bull spread she bought. Then, she should short-sell shares of the underlying asset. *True or false? Why?*

Problem 7.6. (2 points) A market-maker writes a call option on a stock. To decrease the delta of this position, (s)he can **write** a call on the underlying stock. *True or false?*

Problem 7.7. (2 points) Market makers usually do not need to rebalance their portfolios after the initial hedge is established. *True or false? Why?*

Problem 7.8. (2 points) In order to **both** delta hedge and gamma hedge a position in a certain option, the market-maker must trade in another type of option (i.e., not only in the money-market and the underlying risky asset). *True or false? Why?*

Problem 7.9. (2 points) Consider an option whose payoff function is given by $v(s,T) = \min(s, 50)$. If a market-maker **writes** this option, they need to short sell shares of stock to create a delta-neutral portfolio. *True or false? Why?*