The Specifics of the LDA ($p=1$).

The Choice: f_k are normal densities for each $k = 1, \dots, K$, i.e.,

$$f_k(x) = \frac{1}{\sigma_k\sqrt{2\pi}} e^{-\frac{(x-\mu_k)^2}{2\sigma_k^2}} \quad \text{for } k = 1, \dots, K$$

w/ μ_k as the mean and σ_k as the standard deviation for class k

Additional Assumption:

Homogeneity: $\boxed{\sigma_1 = \dots = \sigma_K = \sigma}$

We now return to the posterior probabilities, i.e.,

$$p_k(x) = \frac{\pi_k f_k(x)}{\boxed{\sum_{l=1}^{K} \pi_l f_l(x)}}$$

Remember: We're looking for the k for which the above is MAXIMAL

Since all $p_k(x)$ have the same denominator, it's sufficient to find the k such that

$$\pi_k f_k(x) \xrightarrow{k} \max$$

Because $\ln(\cdot)$ is increasing, the above is equivalent to:

$$\ln(\pi_k) + \ln(f_k(x)) \xrightarrow{k} \max$$

$$\Longleftrightarrow$$

$$\ln(\pi_k) + \ln\left(\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu_k)^2}{2\sigma^2}}\right) \xrightarrow{k} \max$$

$$\Longleftrightarrow$$

$$\ln(\pi_k) - \boxed{\ln(\sigma\sqrt{2\pi})} - \frac{(x-\mu_k)^2}{2\sigma^2} \xrightarrow{k} \max$$

constant in terms of k

$$\Longleftrightarrow$$

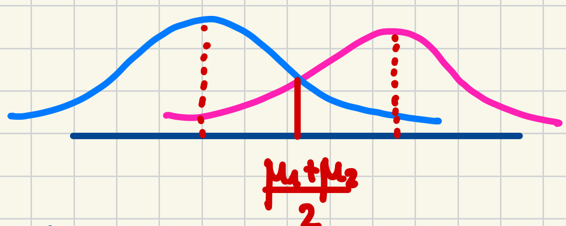
$$\ln(\pi_k) - \frac{x^2}{2\sigma^2} + \frac{\cancel{2}x\mu_k}{\cancel{2}\sigma^2} - \frac{\mu_k^2}{2\sigma^2} \xrightarrow{k} \max$$

$$\delta_k(x) := \ln(\pi_k) + \frac{\mu_k}{\sigma^2}x - \frac{\mu_k^2}{2\sigma^2} \xrightarrow{k} \max$$

These are called DISCRIMINANT (SCORES) and they are LINEAR in x.

Special Case:

$$\boxed{K=2,\ \pi_1=\pi_2=\frac{1}{2}}$$



$$\frac{\mu_k}{\cancel{\sigma^2}}x - \frac{\mu_k^2}{2\cancel{\sigma^2}} \xrightarrow{k} \max$$

$$\mu_k x - \frac{\mu_k^2}{2} \xrightarrow{k} \max$$

IF

$$\mu_1 x - \frac{\mu_1^2}{2} > \mu_2 x - \frac{\mu_2^2}{2}$$

, **THEN** classify as "1".

$$\iff$$

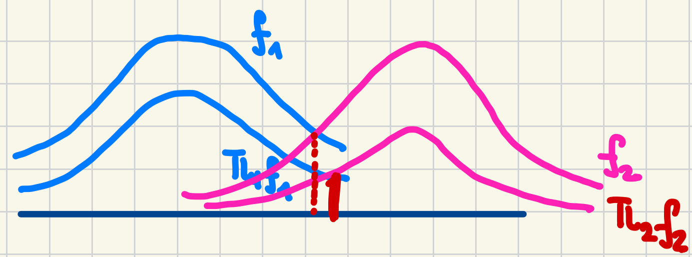
$$\mu_1 x - \mu_2 x > \frac{\mu_1^2 - \mu_2^2}{2}$$

$$\iff$$

$$\boxed{(\mu_1 - \mu_2)}\, x > \frac{1}{2}(\mu_1^2 - \mu_2^2) = \frac{1}{2}\boxed{(\mu_1 - \mu_2)}(\mu_1 + \mu_2)$$

Boundary is always $\frac{\mu_1 + \mu_2}{2}$

$\pi_1 > \pi_2$



Bivariate Normal Random Variables.

(Based on Pitman's "Probability")

Recall: In 1-D, the standard normal density is

$$\boxed{\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad \text{for all } z \in \mathbb{R}}$$

In 2-D, we start w/ X and Y that are independent and both are standard normal, i.e., $N(0,1)$

Then, the joint density of the pair (X,Y) is

$$f(x,y) = \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)} \quad \text{for all } (x,y) \in \mathbb{R}^2.$$

Standard.

Start w/ a pair of independent, standard normal variables.

Say, X and Y.