

M378K Introduction to Mathematical Statistics

Problem Set #11

Order Statistics.

Problem 11.1. An insurance company is handling claims from two categories of drivers: the good drivers and the bad drivers. The waiting time for the first claim from a **good** driver is modeled by an exponential random variable T_g with mean 6 (in years). The waiting time for the first claim from a **bad** driver is modeled by an exponential random variable T_b with mean 3 (in years). We assume that the random variables T_g and T_b are independent.

What is the distribution of the waiting time T until the first claim occurs (regardless of the type of driver this claim was filed by)?

Definition 11.1. Let Y_1, \dots, Y_n be a **random sample**. The random sample ordered in an increasing order is called an order statistic and denoted by

$$Y_{(1)}, Y_{(2)}, \dots, Y_{(n)}.$$

Question Write $Y_{(1)}$ as a function of Y_1, Y_2, \dots, Y_n .

Question Write $Y_{(n)}$ as a function of Y_1, Y_2, \dots, Y_n .

Problem 11.2. What is the distribution function of the random variable $Y_{(n)}$?

Problem 11.3. Assume that the random sample comes from a density f_Y . Is the r.v. $Y_{(n)}$ continuous? If so, what is its density $g_{(n)}$?

Problem 11.4. What is the distribution function of the random variable $Y_{(1)}$?

Problem 11.5. Assume that the random sample comes from a density f_Y . Is the r.v. $Y_{(1)}$ continuous? If so, what is its density $g_{(1)}$?

Theorem 11.2. *Let Y_1, \dots, Y_n be independent, identically distributed random variables with the common cumulative distribution function F_Y and the common probability density function f_Y . Let $Y_{(k)}$ denote the k^{th} order statistic and let $g_{(k)}$ denote its probability density function. Then,*

$$g_{(k)}(y) = \frac{n!}{(k-1)!(n-k)!} (F_Y(y))^{k-1} f_Y(y) (1 - F_Y(y))^{n-k} \quad \text{for all } y \in \mathbb{R}.$$