

UNIVERSITY OF TEXAS AT AUSTIN

Homework Assignment #1

Prerequisite material. Transaction costs. Continuously compounded interest.

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1.1. **Prerequisite material.** Please, provide your final answer only to the following problems.

**Problem 1.1.** (5 pts) You invest an amount  $A$  into an account at time  $-0$ . The account is governed by a continuously compounded risk-free interest rate equal to 0.04.

At time  $-4$ , you deposit an additional amount  $3A$  into the account and the continuously compounded risk-free interest rate changes to 0.06.

Which of the following best describes your balance at time 8?

- (a)  $A(e^{0.16} + 3e^{0.24})$
- (b)  $A(e^{0.32} + 3e^{0.24})$
- (c)  $A(e^{0.40} + 3e^{0.24})$
- (d)  $A(e^{0.40} + 3e^{0.48})$
- (e) None of the above

**Problem 1.2.** (5 pts) Roger initially deposits \$4,000 in an investment fund which pays him \$2,000 at time 1 and \$4,000 at time 2.

Sally gets \$2,000 at time 0 and \$4,000 at time 1, and deposits \$5,460 at time 2 in return.

Both investments are governed by compound interest with the same annual effective interest rate  $i$  and they have the same net present values.

Find  $i$ .

- (a) About 9%
- (b) About 10.0%
- (c) About 11.5%
- (d) About 12%
- (e) None of the above

**Problem 1.3.** (5 pts) Roger makes an initial deposit of  $K$  into an account governed by the time-varying continuously compounded risk-free interest rate  $r(t) = \frac{9}{10}\sqrt{t}$  (per annum).

At the same time, Harry makes an initial deposit at the same amount into an account governed by the constant annual discount rate  $d$ .

There are no subsequent deposits to or withdrawals from either of the two accounts.

After 4 years, Roger and Harry realize that the balances in their accounts are equal. Which of the following is the closest to  $d$ ?

- (a)  $e^{-6/5}$
- (b)  $e^{-1/5}$
- (c)  $1 - e^{-1/5}$
- (d)  $1 - e^{-6/5}$
- (e) 1

Please provide your **complete solution** to the following problems.

**Problem 1.4.** (10 points) By scenario A there is an offer to pay at the rate of \$10,000 per annum, continuously, for the next 10 years. By scenario B it is offered to pay the amount  $X$  at the end of each of the next 10 years. The force of interest applying to both scenarios is 12%. Find the value of  $X$  such that you are indifferent between these two scenarios in the sense that they have the same present values.

**Problem 1.5.** (5 pts) Find the total amount of interest that would be paid on a \$1,000 loan over a 10-year period, if the effective interest rate is 0.09 per annum under the following repayment method:

The entire loan plus entire accumulated interest is paid as one lump-sum at the end of the loan term.

**Problem 1.6.** (2 points) Assume that the force of interest is constant and denoted by  $r$ . Express the accumulation function  $a(t)$  in terms of  $r$  for  $t \geq 0$ .

**Example 1.1. A warm-up example**

Source: “Calculus” by James Stewart.

“One model of *population growth* is based on the assumption that the population grows at a rate proportional to the size of the population.” Let us denote the proportionality constant by  $k$  and let the function  $P(\cdot)$  stand for the size of the population. Then,  $P$  must satisfy the following (ordinary differential) equation:

$$\frac{dP(t)}{dt} = kP(t)$$

Let the initial population size be  $p_0$ . Then, the population size  $P(t)$  at time  $t \geq 0$  is explicitly given by:

$$P(t) = p_0 e^{kt}$$

Please, provide your **complete solution** to the following problem:

**Problem 1.7.** (8 points) **Continuously compounded interest**

Assume that the balance in a savings account is growing so that its rate of growth is proportional to the current balance at any time. Let us denote the proportionality constant by  $r$  and let the function  $B(\cdot)$  stand for the balance as a function of time. Then,  $B$  must satisfy which (ordinary differential) equation?

If the initial balance in the account is  $b_0$ , then what is the expression for the balance as a function of time  $t \geq 0$ ?

1.2. **Transaction costs.** Please, read the following lecture note prior to attempting the remaining problems:

<https://www.ma.utexas.edu/users/mcudina/m339d-lecture-two-transaction-costs.pdf>

Provide your **final answer** only for the following problems.

**Problem 1.8.** (5 points) What is the cost of purchasing 100 shares of Jiffy, Inc. stock given that the bid-ask prices are \$31.25 – \$32.00 and that there is a \$15.00 commission per transaction?

- (a) \$1,293
- (b) \$3,215
- (c) \$3,504
- (d) \$3,264
- (e) None of the above.

**Problem 1.9.** (5 points) *Source: Prof. Jim Daniel (personal communication).*

The bid-ask spread on a share of stock is \$98-\$102. A 5% commission is paid for either buying or selling. Calculate the round-trip transaction cost.

- (a) \$14
- (b) \$10
- (c) \$6
- (d) \$4
- (e) None of the above.