M358K: November 30th, 2020.

7.10. from book.

radius of conf. interval = margin of error

$= \boxed{\text{critical value}} \times \underbrace{\text{std error}}_{\substack{\text{data}\\ \text{\& sample size}}}$

$\updownarrow$

C... confidence level

upper tail probability : $\frac{1-C}{2}$

$t^* > z^*$

Statistical Inference for Two Means.

Inspiration.

Consider an experiment for testing whether a new drug works better than an existing drug.

"OLD DRUG" vs. "NEW DRUG"

Sample

Randomize!

Control

Treatment

μ_1... mean for the (sub)population #1

μ_2... mean for (sub)population #2.

Goals:

- confidence intervals for $\mu_1 - \mu_2$
- hypothesis testing

The focus is on $\mu_1 - \mu_2$

$\Rightarrow$ We should look @ $\boxed{\bar{X}_1 - \bar{X}_2}$ A RANDOM VARIABLE

where $\bar{X}_i$ ... sample mean for sample $i = 1, 2$

control treatment

Assumptions:
- both population dist'ns are normal
- the two samples are independent

$\Rightarrow$ for both groups $i = 1, 2$:

$\bar{X}_i \sim \text{Normal}\left(\text{mean} = \mu_i, \text{var} = \frac{\sigma_i^2}{n_i}\right)$ w/ n_i... sample size

$$\Rightarrow \boxed{\bar{X}_1 - \bar{X}_2 \sim \text{Normal}\left(\text{mean} = \mu_1 - \mu_2, \text{var} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}$$

Confidence intervals:
- point estimate: $\bar{x}_1 - \bar{x}_2$
- we use s_1 and s_2, i.e., the sample standard deviations, instead of the σ_1 and σ_2

$\Rightarrow$ we must use the t-distribution

w/ $df = \min(n_1, n_2) - 1$

Hypothesis testing:

Our null is always: $H_0: \mu_1 = \mu_2$ (no effect)

$\Rightarrow$ Under the null, the observed value of the test statistic is:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$