M339D : March 4th, 2022.

5.

The PS index has the following characteristics:

- One share of the PS index currently sells for 1,000. $\boxed{S(0) = 1,000}$
- The PS index does not pay dividends.

Sam wants to lock in the ability to buy this index in one year for a price of 1,025. He can do this by buying or selling European put and call options with a strike price of 1,025.

$K = 1,025$

The annual effective risk-free interest rate is 5%.

Determine which of the following gives the hedging strategy that will achieve Sam's objective and also gives the cost today of establishing this position.

(A) Buy the put and sell the call, receive 23.81

(B) Buy the put and sell the call, spend 23.81

(C) Buy the put and sell the call, no cost

(D) Buy the call and sell the put, receive 23.81

(E) Buy the call and sell the put, spend 23.81

Init. Cost > 0

6.

The following relates to one share of XYZ stock:

- The current price is 100.
- The forward price for delivery in one year is 105.
- P is the expected price in one year

Determine which of the following statements about P is TRUE.

(A) $P < 100$

(B) $P = 100$

(C) $100 < P < 105$

(D) $P = 105$

(E) $P > 105$

We buy the call & sell the put:

Initial Cost: $\boxed{V_C(0) - V_P(0)}$

$\parallel$ put-call parity

$$F^P_{0,T}(S) - PV_{0,T}(K)$$

$\boxed{\text{no div.}}$ $\parallel$ $\qquad\qquad$ $\parallel$ effective 5%

$$1000 - 1025(1.05)^{-1} = 23.81 > 0$$

■

Chooser Options (aka "as·you·like·it" option).

0 ——— t^* ——— T

t^*: the "choice" date

T: exercise date

K... strike price

At time t^*, the chooser option's owner decides whether the option becomes a call or a put (either w/ strike K & exercise date T).

Assume the owner is rational.

Q: What criterion for the choice between a call or a put does the owner use @ time t^*?

$\longrightarrow$: The owner can see the stock price, the call & put prices in the market.
The owner will compare the market price of the K·strike, T·exercise date European put to the market price of the K·strike, T·exercise date European call and pick the one w/ the higher price.

Notation:

- $V_{CH}(t, t^*, T)$ — t: valuation date; t^*: choice date; T: exercise date
- $V_{C/P}(t$ (valuation date), exercise date, strike price$)$

$$\Rightarrow \quad V_{CH}(t^*, t^*, T) = \max\left(V_C(t^*, T, K), V_P(t^*, T, K)\right)$$

$$\max(a,b) = a + \max(0, b-a) = a + (b-a)_+$$

$$= b + \max(a-b, 0) = b + (a-b)_+$$

$$\Rightarrow \quad V_{CH}(t^*, t^*, T) = V_C(t^*, T, K) + \Big(\underbrace{V_P(t^*, T, K) - V_C(t^*, T, K)}_{\overset{\text{Put-Call Parity}}{=}\; PV_{t^*,T}(K) - \boxed{F^P_{t^*,T}(S)}}\Big)_+$$

For simplicity: no dividends

$$V_{CH}(t^*, t^*, T) = V_C(t^*, T, K) + \underbrace{\left(Ke^{-r(T-t^*)} - S(t^*)\right)_+}_{\substack{\text{Payoff of a European put} \\ \text{w/ strike } K^* = Ke^{-r(T-t^*)} \\ \text{and exercise date } t^*}}$$

$\Rightarrow$ A replicating portfolio for the chooser option:

- a long call w/ strike K and exercise date T
- a long put w/ strike $K^* = Ke^{-r(T-t^*)}$ and exercise date t^*

$$\boxed{\begin{aligned} \Rightarrow V_{CH}(0, t^*, T) &= V_C(0, T, K) + V_P(0, t^*, \boxed{K^* = Ke^{-r(T-t^*)}}) \\ &= V_P(0, T, K) + V_C(0, t^*, K^* = Ke^{-r(T-t^*)}) \end{aligned}}$$

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

21-24. DELETED

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of $100.

The chooser option price is $20 at time $t = 0$. $\boxed{V_{CH}(0,1,3) = 20}$

The stock price is $95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of $100.

You are given:

(i) ~~The risk-free interest rate is 0.~~

(ii) $C(1) = \$4$. $V_c(0,1,K=100)=4$

Determine $C(3)$.

$V_c(0,3,K=100) = ?$

$$V_c(0,3,K) = \underbrace{\boxed{V_{CH}(0,1,3)}}_{=20} - \underbrace{\boxed{V_P(0,1,100)}}_{=?}$$

(A) $ 9

(B) $11

(C) $13

(D) $15

(E) $17

Put-Call Parity:

$$V_P(0,1,K=100) = V_c(0,1,K=100) + PV_{0,1}(100) - S(0) =$$
$$= 4 + 100 - 95 = 9$$

$$\Rightarrow V_c(0,3,K) = 20 - 9 = 11$$