UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 5

CONTENTS

5.1. Arbitrage.

Problem 5.1. (5 points) Provide the definition of an *arbitrage portfolio.*

Solution:
A portfolio is called an *arbitrage portfolio* if its **profit** is

- **non-negative** in all states of the world, and
- **strictly positive** in at least one state of the world.

Problem 5.2. (5 points) Provide the definition of a *replicating portfolio* of a European-style derivative security.

Solution: A portfolio is called a *replicating portfolio* of a European-style derivative security if their payoffs are equal.

Problem 5.3. (5 points) Consider a non-dividend-paying stock whose current price equals \$54 per share. A pair of one-year European calls on this stock with strikes of \$40 and \$50 is available in the market for the observed prices of \$4 and \$2, respectively.

The continuously compounded, risk-free interest rate is given to be 10%.

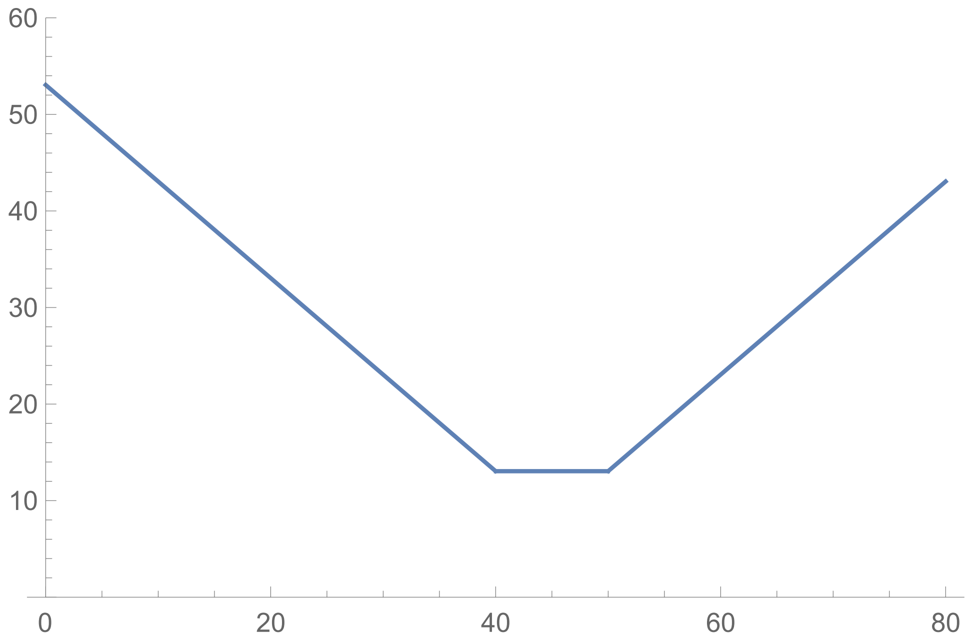
George suspects that there exists an arbitrage portfolio in the above market consisting of the following components:

- **short-sale** of one share of stock,
- **buy** the \$40-strike call,
- **buy** the \$50-strike call.

What is the minimum **gain** from this suspected arbitrage portfolio?

(a) The above is **not** an arbitrage portfolio.
(b) \$0.84
(c) \$8.00
(d) \$13.05
(e) None of the above.

Solution: (d)
The profit curve is given below:



The lower bound on the gain is, hence,

$$48e^{0.1} - 40 = 13.0482.$$

5.2. **Put-call parity.**

Problem 5.4. (5 points) A certain common stock is priced at \$42.00 per share. Assume that the continuously compounded interest rate is $r = 10.00\%$ per annum. Consider a \$50−strike European call, maturing in 3 years which currently sells for \$10.80. What is the price of the corresponding 3−year, \$50−strike European put option?

(a) \$5.20
(b) \$5.69
(c) \$5.04
(d) \$5.84
(e) None of the above.

Solution: (d)
Due to put-call parity, we must have

$$\begin{aligned} V_P(0) &= V_C(0) + e^{-rT}K - S_0 + PV_{0,T}(Div) \\ &= 10.80 + e^{-0.30} \cdot 50 - 42.00 \approx 5.84. \end{aligned}$$

Problem 5.5. (5 points) The initial price of a non-dividend-paying stock is \$55 per share. A 6−month, at-the-money call option is trading for \$1.89. Let the interest rate be $r = 0.065$. Find the price of the European put with the same strike, expiration and the underlying asset.

(a) \$0.05
(b) \$0.13
(c) \$0.56
(d) \$0.88
(e) None of the above

Solution: (b)
Using put-call parity, we get

$$V_P(0) = V_C(0) + Ke^{-rT} - F^P_{0,T}(S) = 1.89 + 55(e^{-0.065\times 0.5} - 1) = 0.1312.$$

Problem 5.6. (5 points) *Source: Problem #2 from the Sample IFM (Derivatives: Introductory) questions.* You are given the following information:

(1) The current price to buy one share of XYZ stock is 500.
(2) The stock does not pay dividends.
(3) The risk-free interest rate, compounded continuously, is 6%.
(4) A European call option on one share of XYZ stock with a strike price of K that expires in one year costs \$66.59.
(5) A European put option on one share of XYZ stock with a strike price of K that expires in one year costs \$18.64.

Determine the strike price K.

(a) \$449
(b) \$452
(c) \$480
(d) \$559
(e) None of the above.

Solution: (c)

Problem 5.7. (5 points) Consider a European call option and a European put option on a non-dividend-paying stock. Assume:

(1) The current price of the stock is $55.
(2) The call option currently sells for $0.15 more than the put option.
(3) Both options expire in 4 years.
(4) Both options have a strike price of $70.

Calculate the continuously compounded risk-free interest rate r.

(a) 0.044
(b) 0.052
(c) 0.06
(d) 0.065
(e) None of the above.

Solution: (c)
In our usual notation,

$$S(0) = 55, \quad V_C(0) - V_P(0) = 0.15, \quad T = 4, \quad K = 70.$$

We employed a *no-arbitrage* argument to get the **put-call parity**:

$$V_C(0) - V_P(0) = S(0) - K^{-rT} \quad \Rightarrow \quad r = \frac{1}{T}\ln\left(\frac{K}{S(0) - V_C(0) + V_P(0)}\right).$$

Using in the data provided, we get $r = 0.06$.

Problem 5.8. (5 points) Consider a European call option and a European put option on a non-dividend paying stock **S**. You are given the following information:

(1) $r = 0.04$
(2) The current price of the call option $V_C(0)$ is by 0.15 greater than the current price of the put option $V_P(0)$.
(3) Both the put and the call expire in 4 years.
(4) The put and the call have the same strikes equal to 70.

Find the spot price $S(0)$ of the underying asset.

(a) 48.90
(b) 59.80
(c) 69.70
(d) 79.60
(e) None of the above.

Solution: (b)
Using put-call parity, we get

$$0.15 = V_C(0) - V_P(0) = S(0) - Ke^{-rT} = S(0) - 70e^{-0.04\times 4} \quad \Rightarrow \quad S(0) = 0.15 + 70e^{-0.16} = 59.80.$$

5.3. **The binomial asset pricing model.**

Problem 5.9. (10 points) Assume that one of the no-arbitrage conditions in the binomial model for pricing options on a non-dividend paying stock S is violated. Namely, let

$$e^{r\cdot h} \leq d < u.$$

Illustrate that the above inequalities indeed violate the no-arbitrage requirement. In other words, construct an arbitrage portfolio and show that your proposed arbitrage portfolio is, indeed, and arbitrage portfolio.

Solution: There are multiple ways to illustrate arbitrage opportunities in the above set-up. We provide just one simple example.

Let today's stock-price be denoted by $S(0)$. We simply borrow $S(0)$ from the money market and buy one share of stock. After one period, according to the binomial model, the stock-price either rises to $S_u = uS(0)$ or drops to $S_d = dS(0)$.

Let us denote the value of our portfolio on the second day by X_u in the case the stock price went up and by X_d if the stock price went down. The values of our portfolio in those two cases are

$$X_u = -e^{rh} \cdot S(0) + uS(0) > 0$$
$$X_d = -e^{rh} \cdot S(0) + dS(0) \geq 0$$

We have non-negative payoffs in both cases and a strictly positive payoff in one of the cases. Hence, the above strategy constitutes arbitrage.