M339J: March 29th, 2021.

Policy Modifications [review].

- the ordinary deductible d
- the policy limit $\boxed{\alpha(u-d)}$
- coinsurance α
- inflation rate r

Per loss

$$Y^L = \begin{cases} 0 & \text{if } (1+r)X < d \\ \alpha((1+r)X-d) & \text{if } d \leq (1+r)X < u \\ \alpha(u-d) & \text{if } (1+r)X \geq u \end{cases}$$

Per payment

$$\boxed{Y^P = \begin{cases} \text{undefined} & \text{if } (1+r)X < d \\ Y^L & \text{otherwise} \end{cases}}$$

Problem. An insurance policy on a ground-up loss X has:

- no deductible
- a coinsurance of 50%, and
- a maximum policy pmt per loss of 5,000.

What is the expected pmt per loss for the insurer in terms of X?

$\longrightarrow$: $\mathbb{E}[Y^L] = ?$

By our Thm, we have

$$\mathbb{E}[Y^L] = \alpha(1+r)\left(\boxed{\mathbb{E}\left[X \wedge \frac{u}{1+r}\right]} - \mathbb{E}\left[X \wedge \frac{d}{1+r}\right]\right)$$

In this problem, $r=0$ and $d=0$.

$$\mathbb{E}[Y^L] = \alpha \cdot \mathbb{E}[X \wedge u].$$

We're given $\alpha = 0.5$.

Q: How much is u?

Since we know that the maximum policy pmt is 5000, we have

$$\alpha \cdot u = 5000$$

$$u = 10{,}000$$

$$\mathbb{E}[Y^L] = 0.5\, \mathbb{E}[X \wedge 10{,}000].$$

$\blacksquare$

277. You are given:

(i) Loss payments for a group health policy follow an exponential distribution with unknown mean.

(ii) A sample of losses is:

100 200 400 800 1400 3100

Using the delta method, calculate the approximation of the variance of the maximum likelihood estimator of $S(1500)$.

(A) 0.019

(B) 0.025

(C) 0.032

(D) 0.039

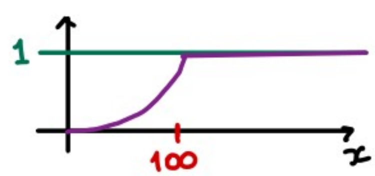
(E) 0.045

278. DELETED

Sample STAM.

279. Loss amounts have the distribution function

$$F(x) = \begin{cases} (x/100)^2, & 0 \le x \le 100 \\ 1, & x > 100 \end{cases}$$



$\alpha = 0.80$

An insurance pays 80% of the amount of the loss in excess of an ordinary deductible of 20, subject to a maximum payment of 60 per loss. $d = 20$

$\alpha(u-20) = 60 \Rightarrow \boxed{u = 95}$

Calculate the conditional expected claim payment, given that a payment has been made, i.e., the expected value of the per payment r.v.

(A) 37

(B) 39

(C) 43

(D) 47

(E) 49

$$E[Y^P] = E[Y^L \mid X > 20] = \frac{\boxed{E[Y^L]}}{S_X(20)}$$

$$\mathbb{E}[Y^L] = \alpha\left(\mathbb{E}[X\wedge u] - \mathbb{E}[X\wedge d]\right)$$
$$= 0.8\left(\mathbb{E}[X\wedge 95] - \mathbb{E}[X\wedge 20]\right)$$

For a constant $c \in (0,100)$:

$$\mathbb{E}[X\wedge c] = \int_0^c S_X(x)\,dx = \int_0^c \left(1 - \frac{x^2}{10^4}\right)dx$$
$$= \left[x - \frac{1}{10^4}\cdot\frac{x^3}{3}\right]_{x=0}^{c}$$
$$= c - \frac{c^3}{3\cdot 10^4}$$

the tail formula for expectation

$$\mathbb{E}[Y^L] = 0.8\left(95 - \frac{95^3}{3\cdot 10^4} - \left(20 - \frac{20^3}{3\cdot 10^4}\right)\right) = 37.35$$

$$\Rightarrow \mathbb{E}[Y^P] = \frac{37.35}{1-\left(\frac{20}{100}\right)^2} = \frac{37.35}{0.96} = 38.91 \Rightarrow (B).$$

Poisson Distribution.

Usually, we write: $N \sim \text{Poisson}(\lambda)$

Q: What's the support?

$$\mathbb{N}_0 = \{0, 1, 2, \dots\}$$

We say that any r.v. w/ this support is $\mathbb{N}_0$-valued.

The probability mass f'tion:

$$p_N(k) := p_k = e^{-\lambda}\cdot\frac{\lambda^k}{k!}, \quad k = 0,1,2,\dots$$

The probability generating f'tion: $P_N(z) = \mathbb{E}[z^N] = e^{\lambda(z-1)}$

$\mathbb{E}[N] = \lambda$ and $Var[N] =$

Sample STAM

170. In a certain town the number of common colds an individual will get in a year follows a Poisson distribution that depends on the individual's age and smoking status. The distribution of the population and the mean number of colds are as follows:

	Proportion of population	Mean number of colds
Children	0.30	3 $\lambda_c = 3$
Adult Non-Smokers	0.60	1 $\lambda_{ANS} = 1$
Adult Smokers	0.10	4 $\lambda_{AS} = 4$

Calculate the conditional probability that a person with exactly 3 common colds in a year is an adult smoker.

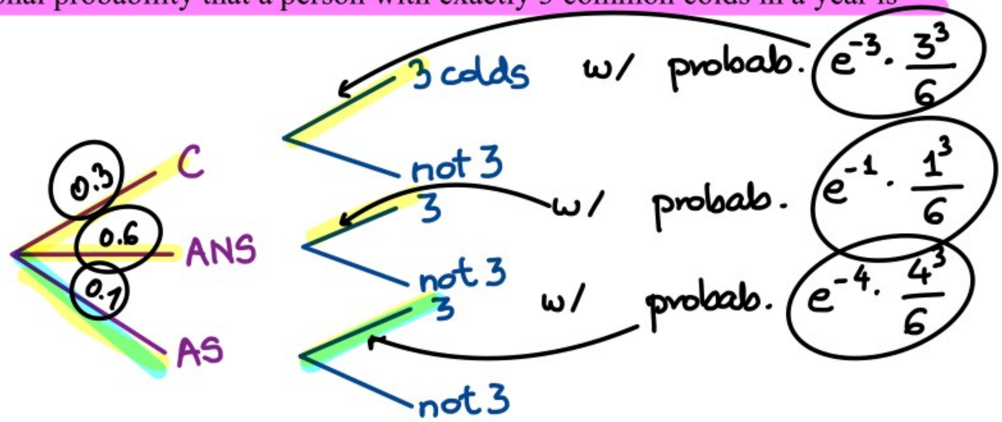
(A) 0.12

(B) 0.16

(C) 0.20

(D) 0.24

(E) 0.28



Q: $\mathbb{P}[AS \mid 3 \text{ colds}] = ?$

171. For aggregate losses, S:

(i) The number of losses has a negative binomial distribution with mean 3 and variance 3.6.

(ii) The common distribution of the independent individual loss amounts is uniform from 0 to 20.

Calculate the 95th percentile of the distribution of S as approximated by the normal distribution.

(A) 61

(B) 63

(C) 65

(D) 67

(E) 69

Bayes' Theorem.

$$\mathbb{P}[AS \mid 3 \text{ colds}] = \frac{\mathbb{P}[AS \text{ and } 3 \text{colds}]}{\mathbb{P}[3 \text{ colds}]}$$

$$= \frac{0.1\, e^{-4} \cdot \frac{4^3}{\cancel{6}}}{0.3 e^{-3} \cdot \frac{3^3}{\cancel{6}} + 0.6 \cdot e^{-1} \cdot \frac{1^3}{\cancel{6}} + 0.1 e^{-4} \cdot \frac{4^3}{\cancel{6}}}$$

$$= 0.1581 \qquad \blacksquare$$