M339 J: April 12th, 2021.

Binomial Dist'n [cont'd].

Example. $\boxed{N \sim \text{Poisson}(\lambda)}$

- Category 1 w/ probab. p_1
- Category 2 w/ probab. p_2

$\Big\}$ $p_1 + p_2 = 1$

N_i ... # of losses from Category i, $i = 1, 2$.

Thinning:
- $\boxed{N_i \sim \text{Poisson}(\lambda_i = p_i \cdot \lambda)}$, $i = 1, 2$
- N_1 & N_2 are independent

Q: Given $N = m$, find the probab. that $N_1 = k$, for $k = 0, 1, \ldots, m$.

$\rightarrow$: $\mathbb{P}[N_1 = k \mid N = m] = \ldots$

$$\ldots = \frac{\mathbb{P}[N_1 = k] \cdot \mathbb{P}[N_2 = m-k]}{\mathbb{P}[N = m]} = \frac{\cancel{e^{-\lambda_1}} \cdot \frac{\lambda_1^k}{k!} \cdot \cancel{e^{-\lambda_2}} \cdot \frac{\lambda_2^{m-k}}{(m-k)!}}{\cancel{e^{-\lambda}} \cdot \frac{\lambda^m}{m!}}$$

$$= \frac{m!}{k!\,(m-k)!} \cdot \frac{\lambda_1^k \cdot \lambda_2^{m-k}}{\lambda^m}$$

$$= \binom{m}{k} \frac{p_1^k \cdot \cancel{\lambda^k} \cdot p_2^{m-k} \cdot \cancel{\lambda^{m-k}}}{\cancel{\lambda^m}}$$

$$\mathbb{P}[N_1 = k \mid N = m] = \binom{m}{k} p_1^k \cdot p_2^{m-k}$$

$$N_1 \mid N = m \sim \text{Binomial}(m, q = p_1) \quad \blacksquare$$

The (a,b,0) class.

If an $\mathbb{N}_0$-valued distribution has a pmf which satisfies this recursion:

$$p_k = p_{k-1}\left(a + \frac{b}{k}\right) \quad \text{for } k = 1, 2, \dots,$$

then, we say that it is an (a,b,0) distribution.

As it turns, the only representatives in this class are the Poisson, the negative binomial, and the binomial dist'n.

14. In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $\boxed{p(n+1) = 0.2\,p(n)}$ where $p(n)$ represents the probability that the policyholder files n claims during the period.

Under this assumption, calculate the probability that a policyholder files more than one claim during the period.

(A) 0.04
(B) 0.16
(C) 0.20
(D) 0.80
(E) 0.96

15. An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are 1/4, 1/3, and 5/12 respectively.

Calculate the probability that a randomly chosen employee will choose no supplementary coverage.

(A) 0
(B) 47/144
(C) 1/2
(D) 97/144
(E) 7/9

16. An insurance company determines that N, the number of claims received in a week, is a random variable with $P[N = n] = \frac{1}{2^{n+1}}$ where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week.

Calculate the probability that exactly seven claims will be received during a given two-week period.

(A) 1/256
(B) 1/128
(C) 7/512
(D) 1/64
(E) 1/32

$\longrightarrow$: N ... # of claims

answer: $\mathbb{P}[N>1] = 1 - \mathbb{P}[N=0] - \mathbb{P}[N=1] = 1 - p_0 - p_1$

p_0 ... probab of zero claims

$p_1 = \frac{1}{5} p_0$

$\longrightarrow$ $p_2 = \left(\frac{1}{5}\right)^2 \cdot p_0$

$\vdots$

$$\boxed{p_k = \left(\frac{1}{5}\right)^k \cdot p_0}$$

$\vdots$

Since my pmf must sum up to 1:

$$1 = \sum_{k=0}^{+\infty} p_k = \sum_{k=0}^{+\infty} \left(\left(\frac{1}{5}\right)^k p_0\right) = p_0 \sum_{k=0}^{+\infty} \left(\frac{1}{5}\right)^k$$

geometric series :)

$$= p_0 \cdot \frac{1}{1-\frac{1}{5}} = p_0 \cdot \frac{5}{4}$$

$$\Rightarrow \quad p_0 = \frac{4}{5} \qquad \Rightarrow \quad p_1 = \frac{1}{5} \cdot \frac{4}{5} = \frac{4}{25}$$

$$\mathbb{P}[N>1] = 1 - p_0 - p_1 = 1 - \frac{4}{5} - \frac{4}{25} = \frac{1}{25} = 0.04 \qquad \blacksquare$$

Q: What is the dist'n of N?

$\longrightarrow$: $N \sim \text{geometric}(\beta = 4)$ ✓

93. At the beginning of each round of a game of chance the player pays 12.5. The player then rolls one die with outcome N. The player then rolls N dice and wins an amount equal to the total of the numbers showing on the N dice. All dice have 6 sides and are fair.

Using the normal approximation, calculate the probability that a player starting with 15,000 will have at least 15,000 after 1000 rounds.

(A) 0.01

(B) 0.04

(C) 0.06

(D) 0.09

(E) 0.12

Sample STAM.

94. X is a discrete random variable with a probability function that is a member of the $(a,b,0)$ class of distributions.

You are given:

(i) $\Pr(X=0)=\Pr(X=1)=0.25$

(ii) $\Pr(X=2)=0.1875$

Calculate $\Pr(X=3)$.

(a,b,0) class:

$\Rightarrow p_k = p_{k-1}\left(a+\frac{b}{k}\right)$ $k=1,2,\ldots$

for some constants a & b

(i) $\Rightarrow$ $p_0 = p_1 = \frac{1}{4}$ $\Rightarrow$ $p_1 = p_0\left(a+\frac{b}{1}\right)$

$\frac{1}{4} = \frac{1}{4}(a+b)$

$\Rightarrow a+b=1$ ✓

(ii) $\Rightarrow$ $p_2 = 0.1875$ $\Rightarrow$ $p_2 = p_1\left(a+\frac{b}{2}\right)$

$0.1875 = 0.25\left(a+\frac{b}{2}\right)$

$\Rightarrow a+\frac{b}{2} = 4(0.1875) = 0.75$ ✓

(A) 0.120

(B) 0.125

(C) 0.130

(D) 0.135

(E) 0.140

$\Rightarrow \quad \frac{b}{2} = 1 - 0.75 = 0.25 \quad \Rightarrow \quad b = 0.5 \quad \Rightarrow \quad a = 0.5$

Using our recursion:

$$p_3 = p_2\left(a + \frac{b}{3}\right) = 0.1875\left(0.5 + \frac{0.5}{3}\right) = 0.125 = \frac{1}{8}$$

Q: What's the dist'n of X?

$\longrightarrow$: $X \sim \text{NegBinomial}\left(r = \underline{2},\ \beta = \underline{1}\right)$