M358K: October 3rd, 2022.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 5

Normal distribution.

Problem 5.1. Let Z be a standard normal random variable. Find the following probabilities:

i. $\mathbb{P}[-1.33 < Z \leq 0.24]$
ii. $\mathbb{P}[0.49 < |Z|]$
iii. $\mathbb{P}[Z^4 < 0.0256]$
iv. $\mathbb{P}[e^{2Z} < 2.25]$
v. $\mathbb{P}\left[\frac{1}{Z} < 2\right]$

$\mathbb{P}[-1.33 < Z \leq 0.24] = \mathbb{P}[Z \leq 0.24] - \mathbb{P}[Z \leq -1.33]$

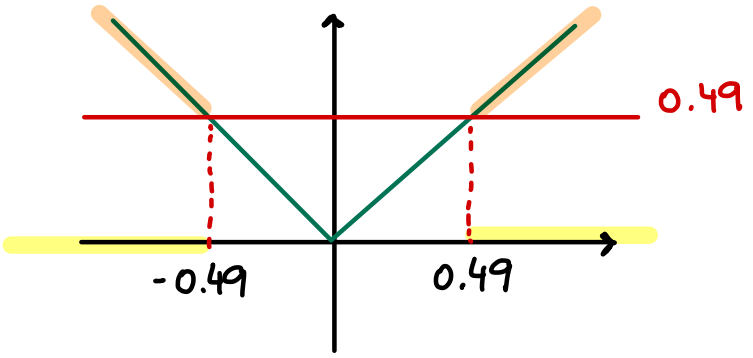
$= \Phi(0.24) - \Phi(-1.33)$ (std normal table)

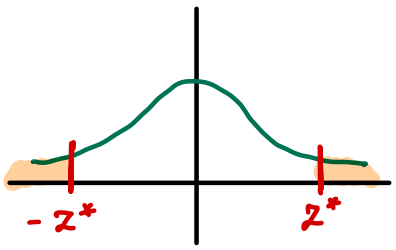
$= 0.5948 - 0.0918$

$= 0.5030$

In R: pnorm(0.24) - pnorm(-1.33) = 0.5030757

$\mathbb{P}[0.49 < |Z|] = ?$





$\mathbb{P}[Z < -0.49] + \mathbb{P}[Z > 0.49] = 2 \cdot \mathbb{P}[Z < -0.49]$

2 * pnorm(-0.49) = 0.6241339

alternatively:

$2 \cdot \mathbb{P}[Z > 0.49] = 2 \cdot (1 - \mathbb{P}[Z \leq 0.49])$

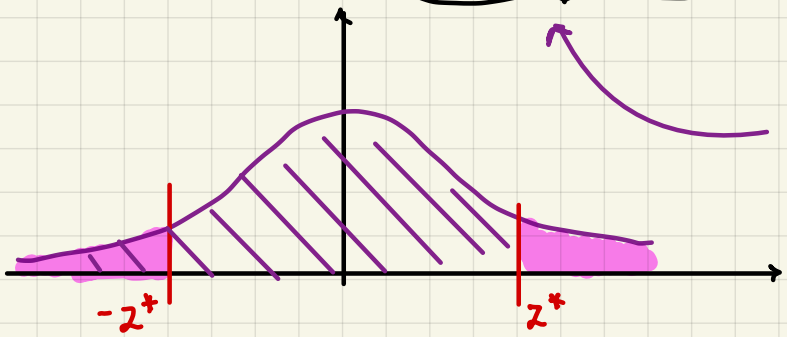
2 * (1 - pnorm(0.49))

= 2 * pnorm(0.49, lefttail = FALSE)

iii. $\mathbb{P}[Z^4 < 0.0256] = \mathbb{P}[|Z| < \sqrt[4]{0.0256} = 0.4]$

$= \mathbb{P}[-0.4 < Z < 0.4]$

$= \mathbb{P}[Z \leq 0.4] - \mathbb{P}[Z \leq -0.4]$



$\mathbb{P}[Z < -z^*] = \mathbb{P}[Z > z^*]$

$= 1 - \mathbb{P}[Z \leq z^*]$

$= \mathbb{P}[Z \leq 0.4] - (1 - \mathbb{P}[Z \leq 0.4])$

$= 2 \cdot \mathbb{P}[Z \leq 0.4] - 1$

2 * pnorm(0.4) - 1 = 0.3108435

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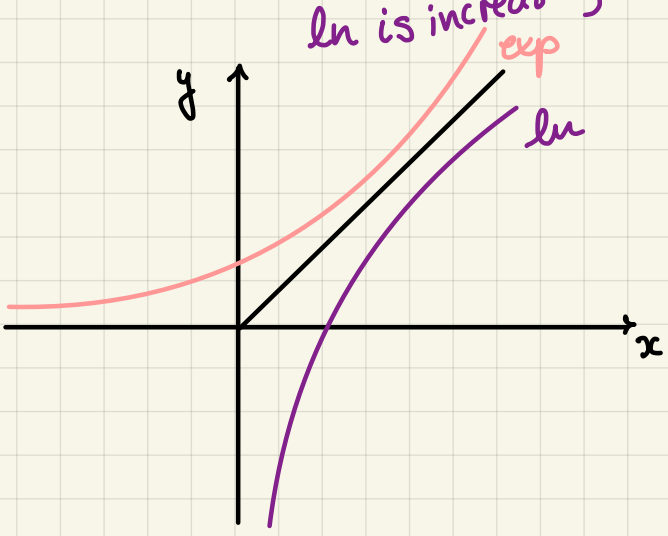
iv. $\mathbb{P}[e^{2Z} < 2.25] = \mathbb{P}[2Z < \ln(2.25)] = \mathbb{P}[Z < 0.5\ln(2.25)]$

ln is increasing



pnorm(0.5 * log(2.25)) =

= 0.6574322

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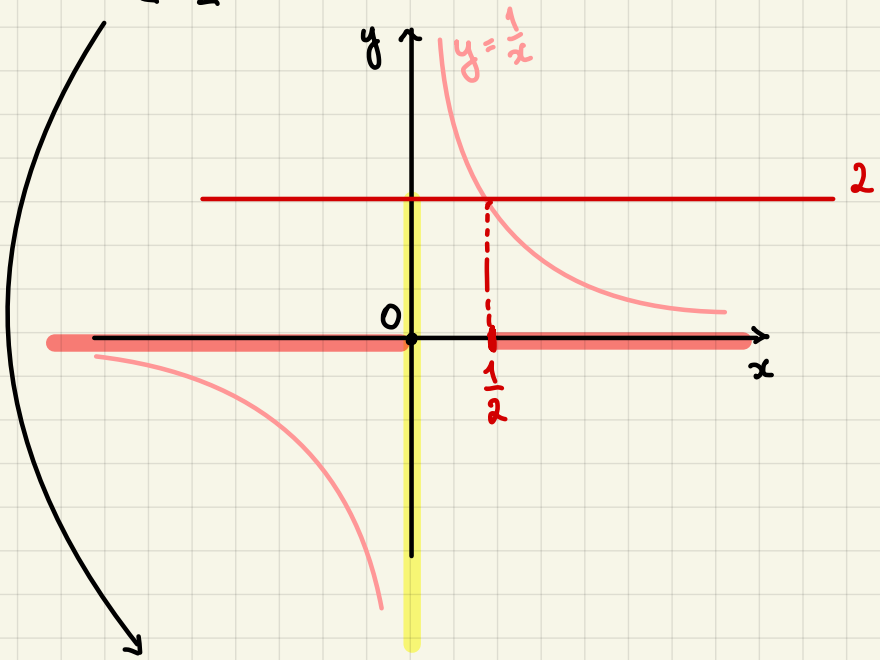
v. $\mathbb{P}\left[\frac{1}{Z} < 2\right] = ?$



$\mathbb{P}[Z<0] + \mathbb{P}[Z > \frac{1}{2}] = 0.5 + (1 - \mathbb{P}[Z \leq 0.5])$

$= 1.5 - \mathbb{P}[Z \leq 0.5]$

1.5 - pnorm(0.5) = 0.8085375