Name:

UTeid:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
In-Term One
Instructor: Milica Čudina

All written work handed in by the student is considered to be
their own work, prepared without unauthorized assistance.

The University Code of Conduct

"The core values of The University of Texas at Austin are learning, discovery, freedom, leadership, individual opportunity, and responsibility. Each member of the university is expected to uphold these values through integrity, honesty, trust, fairness, and respect toward peers and community. As a student of The University of Texas at Austin, I shall abide by the core values of the University and uphold academic integrity."

"I agree that I have complied with the UT Honor Code during my completion of this exam."

Signature:

The maximum number of points on this exam is 65.

Problem 1.1. (10 points) Write the definition of an **arbitrage portfolio**.

Solution: Check your notes.

Problem 1.2. (5 points) Write the definition of a **replicating portfolio** of a European option.

Problem 1.3. (5 pts) Consider a portfolio consisting of the following four European options with the same expiration date T on the underlying asset S:
long one call with strike 50,
short two calls with strike 55,
long one call with strike 65.

Let $S(T) = 58$. What is the payoff from the above position at time T?

Solution: The payoff is

$$(58 - 50)_+ - 2(58 - 55)_+ + (58 - 65)_+ = 8 - 2(3) + 0 = 2.$$

Problem 1.4. (5 points) The initial price of the market index is \$900. After 3 months the market index is priced at \$960. The **effective** monthly rate of interest is 1.0%.

The premium on the long put, with a strike price of \$975, is \$10.00. What is the break-even point for this put option?

Solution: The break-even point for a put is, in our usual notation,

$$s^* = K - FV_{0,T}(V_P(0)) = 975 - (1.01)^3(10) = 964.697 = 964.70.$$

Problem 1.5. (10 points) *Source: Prof. Jim Daniel (personal communication).*
A stock's price today is \$1000 and the annual **effective** interest rate is given to be 10%. You write a one-year, \$1,050-strike call option for a premium of \$10 while you simultaneously buy the stock. What is your profit if the stock's spot price in one year equals \$1,200?

Solution:

$$S(T) - 1000(1.10) - (S(T) - K)_{+} + 10(1.10) = 1050 - 1010(1.10) = -39.$$

Problem 1.6. (10 points) Let the random variable X have the following density function:

$$f_X(x) = 4x^{-5}, \quad x > 1$$

You use the *inverse transform method* to simulate values from X. Let the simulated value of the unit uniform be equal to 0.25. What is the corresponding value of X?

Solution: The cumulative distribution function of X is

$$F_X(x) = \begin{cases} 0, & \text{for } x \leq 1, \\ 1 - x^{-4}, & \text{for } x > 1. \end{cases}$$

Now, we find the inverse of the cumulative distribution function.

$$y = 1 - x^{-4} \quad \Leftrightarrow \quad x^{-4} = 1 - y \quad \Leftrightarrow \quad x^4 = \frac{1}{1-y} \quad \Leftrightarrow \quad x = \frac{1}{\sqrt[4]{1-y}}$$

So, our answer is

$$x = \frac{1}{\sqrt[4]{1 - 0.25}} = 1.07457.$$

Problem 1.7. (10 points) Let the continuously compounded risk-free interest rate equal 2%. The current price of an index equals \$1,000. The current premium on a six-month, \$97-strike 6-month call on this index is \$109.20. What is the price of an otherwise identical put option?

Solution: This question is a direct application of the put-call parity. In our usual notation, we have

$$V_C(0) - V_P(0) = S(0) - PV_{0,T}(K).$$

So,

$$V_P(0) = V_C(0) - S(0) + PV_{0,T}(K) = 109.20 - 1000 + 970e^{-0.02(0.5)} = 69.54834 \approx 69.55.$$

Problem 1.8. (10 points) The continuously compounded risk-free interest rate equals 0.08. For which stock prices does the profit of a short forward with forward price \$100 and delivery date in one year exceed the profit of a long European put with strike \$100, exercise date in one year and premium equal to \$10? Express your answer as an interval, please.

Solution: The payoff/profit of the short forward is $v_F(s) = 100 - s$ while the profit of the long put equals

$$(100 - s)_+ - 10e^{0.08}.$$

So, we have to solve the inequality

$$100 - s > (100 - s)_+ - 10e^{0.08}. \tag{1.1}$$

For $0 \leq s \leq 100$, the above inequality becomes So, we have to solve the inequality

$$100 - s > 100 - s - 10e^{0.08} \quad \Leftrightarrow \quad 0 > -10e^{0.08}.$$

This inequality is evidently always true, so all $s \in [0, 100]$ satisfy the condition.

For $s > 100$, the inequality (1.2) becomes

$$100 - s > -10e^{0.08} \quad \Leftrightarrow \quad s < 100 + 10e^{0.08} = 110.83287. \tag{1.2}$$

The final answer is the interval $[0, 110.83287)$.