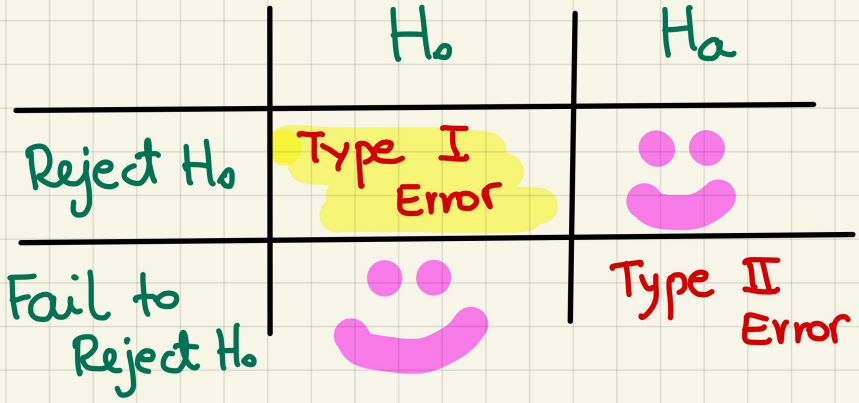
M358: October 23rd, 2020.
Power of Test [practice].



$\mathbb{P}[\text{Type I Error}] = \alpha$ ... significance level

$\mathbb{P}_{\mu_a}[\text{Fail to Reject}] = \beta$

μ_a from the alternative

For a particular μ_a:

$1-\beta$ ... Power of the Test @ $\mu = \mu_a$

Problem. $X \sim \text{Normal}(\text{mean} = \mu, \text{variance} = 9)$

μ: unknown

Test:

$H_0: \mu = 15$ vs. $H_a: \mu > 15$

SRS: $n = 36$; sample average: $\bar{x} = 17$

Significance level $\alpha = 0.05$.

Find the power of this test @ $\mu_a = 16$!

→: ✓ First, figure out the RR:

$RR = [15.8225, +\infty)$

Second, my power of the test is the probability that $\bar{X}$ lands in the RR under the particular given alternative.

$\bar{X} \sim \text{Normal}(\text{mean} = \mu_a = 16, \text{variance} = \frac{9}{36})$

$$\mathbb{P}_{\mu_a}[\bar{X} \geq 15.8225] =$$

$$= \mathbb{P}_{\mu_a}\left[\frac{\bar{X} - 16}{1/2} \geq \frac{15.8225 - 16}{1/2}\right]$$

$\frac{\bar{X} - 16}{1/2} \sim N(0,1)$

$$= \mathbb{P}[Z \geq -0.355] \cong 1 - \Phi(-0.36)$$

$$= \Phi(0.36) = 0.6406 \blacksquare$$

Problem 10.4. The time needed for college students to complete a certain mirror-symmetry puzzle is modeled using a normal distribution with a mean of 30 seconds and a standard deviation of 3 seconds. You wish to see if the population mean time μ is changed by vigorous exercise, so you have a group of nine college students exercise vigorously for 30 minutes and then complete the puzzle.

i. What are your null and alternative hypotheses?

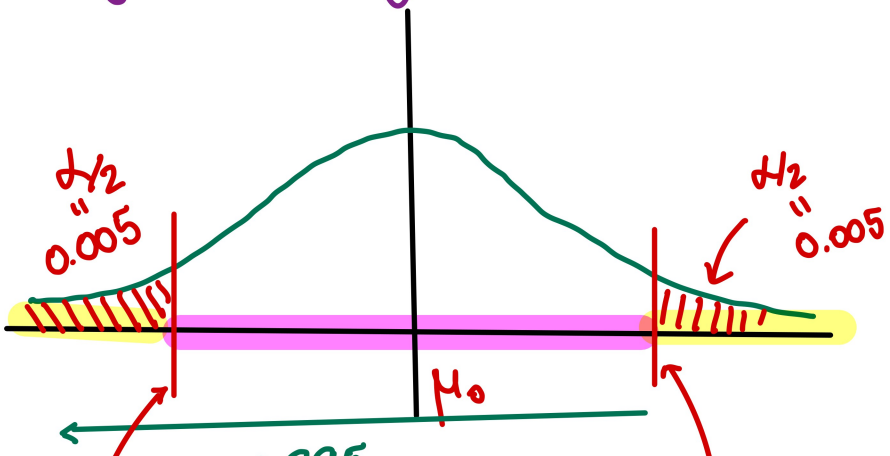
ii. What is the rejection region at the significance level 0.01?

iii. What is the power of your test at $\mu = 28$ seconds?

i. $H_0: \mu = 30$ vs. $H_a: \mu \neq 30$

ii. $\alpha = 0.01$

The Rejection Region (in raw units):



$z_{\alpha/2} = -2.576$

$30 - 2.576 \cdot \frac{3}{\sqrt{9}}$ $30 + 2.576 \cdot \frac{3}{\sqrt{9}}$

=

$RR = (-\infty, 27.424] \cup [32.576, +\infty)$

My "fail-to-reject" region is:

$RR^c = (27.424, 32.576)$

iii. Figure out the power of the test @ $\mu_a = 28$.

Under this alternative:

$\bar{X} \sim \text{Normal}(\text{mean} = 28, \text{var} = 1)$

$$\mathbb{P}_{\mu_a}[27.424 < \bar{X} < 32.576] = \beta$$

$$\mathbb{P}_{\mu_a}[27.424 - 28 < \bar{X} - 28 < 32.576 - 28] = \beta \quad (\bar{X} - 28 \sim N(0,1))$$

$$\mathbb{P}[-0.576 < Z < 4.576] = \beta$$

$$\mathbb{P}[Z < 4.576] - \mathbb{P}[Z < -0.576] = \beta$$

$$1 - \mathbb{P}[Z < -0.576] = \beta$$

$$\mathbb{P}[Z < 0.576] = \beta$$

$$0.7176924$$

$\Rightarrow$ Power of test = 0.2823.