

UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 13

Black-Scholes Delta.

Please, provide your **complete solution** to the following problems. Final answers without shown reasoning will get zero points.

Problem 13.1. (5 points) Assume the Black-Scholes framework. For an at-the-money, T -year European call option on a non-dividend-paying stock you are given that its delta equals 0.5832. What is the delta of an otherwise identical option with exercise date at time $2T$?

Solution: The time-0 delta of a call on a non-dividend-paying stock has the form

$$\Delta_C(S(0), 0) = N(d_1(S(0), 0))$$

with

$$d_1(S(0), 0) = \frac{1}{\sigma\sqrt{T}} \left[\ln \left(\frac{S(0)}{K} \right) + \left(r + \frac{\sigma^2}{2} \right) T \right].$$

Since the option is at-the-money, the above expression simplifies to

$$d_1(S(0), 0) = \frac{r + \frac{\sigma^2}{2}}{\sigma} \sqrt{T}.$$

So, we are given that

$$N \left(\frac{r + \frac{\sigma^2}{2}}{\sigma} \sqrt{T} \right) = 0.5832 \quad \Rightarrow \quad \frac{r + \frac{\sigma^2}{2}}{\sigma} \sqrt{T} = 0.21.$$

Similarly, we obtain that the delta of the otherwise identical call option with twice as long to expiration equals

$$N \left(\frac{r + \frac{\sigma^2}{2}}{\sigma} \sqrt{2T} \right) = N \left(\left(\frac{r + \frac{\sigma^2}{2}}{\sigma} \sqrt{T} \right) \sqrt{2} \right) = N(0.21\sqrt{2}) \approx 0.6179.$$

Problem 13.2. (5 points) Assume the Black-Scholes framework as model for the price of a non-dividend-paying stock. What is the difference between the delta of a European call option and the delta of the otherwise identical put option?

Solution: The difference is 1 by put-call parity.

Problem 13.3. (5 points) Assume the Black-Scholes model. Let the current stock price of a continuous-dividend-paying stock be equal to \$80. The stock's dividend yield is 0.01 and its volatility is 0.30.

The continuously compounded risk-free interest rate is 0.04.

Consider a \$82-strike, six-month European put option on the above stock. What is the put option's delta?

Solution: The expression for the time-0 put delta is

$$\Delta_P(S(0), 0) = -e^{-\delta T} N(-d_1(S(0), 0))$$

with

$$\begin{aligned} d_1(S(0), 0) &= \frac{1}{\sigma\sqrt{T}} \left[\ln \left(\frac{S(0)}{K} \right) + \left(r - \delta + \frac{\sigma^2}{2} \right) T \right] \\ &= \frac{1}{0.3\sqrt{\frac{1}{2}}} \left[\ln \left(\frac{80}{82} \right) + \left(0.04 - 0.01 + \frac{(0.3)^2}{2} \right) \left(\frac{1}{2} \right) \right] = 0.06037. \end{aligned}$$

Consulting the standard normal tables, we get that

$$\Delta_P(S(0), 0) = -e^{-0.01(0.5)}(0.4761) = -0.4737254.$$