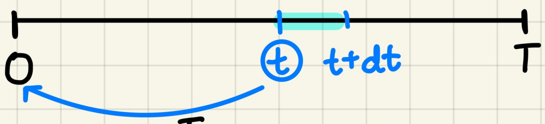
M339D: January 29th, 2021.

HW#1.

Problem 4.

Scenario A: continuous annuity w/ rate R



$$PV(\text{cont.}) = \int_0^T e^{-r\cdot t} R\,dt$$

w/ r the continuously compounded, risk-free interest rate

$$\Rightarrow \quad PV(\text{cont.}) = R\cdot\int_0^T e^{-r\cdot t}dt =$$

$$= R\cdot\left(-\frac{1}{r}\right)\cdot\left[e^{-r\cdot t}\right]_{t=0}^{T}$$

$$= R\cdot\left(-\frac{1}{r}\right)\left(e^{-r\cdot T}-1\right)$$

$$= R\cdot\frac{1-e^{-r\cdot T}}{r} = \bar{a}_{\overline{T}|r}$$

Scenario B: discrete annuity immediate w/ annual pmts equal to X

$$PV(\text{disc.}) = X\cdot a_{\overline{T}|i} = X\cdot\frac{1-v^T}{i}$$

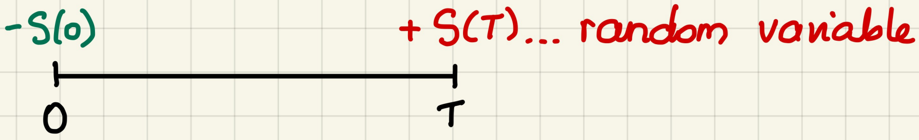
w/ i ... the effective interest rate per period and $v = \frac{1}{1+i}$

In the problem: $\quad R\cdot\frac{\cancel{1-e^{-rT}}}{r} = X\cdot\frac{\cancel{1-v^T}}{\boxed{i}}$

In terms of r, we have $\boxed{i = e^r - 1}$

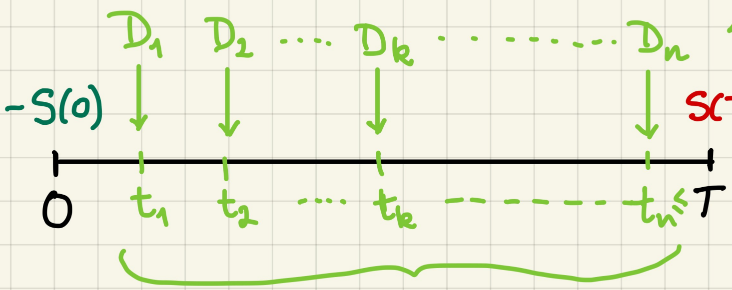
Outright Purchase of one share of stock

Case #1.



Case #2.

Discrete Dividends



dividend amounts

- expressed per share
- deterministic

Fixed deterministic times @ which the dividends are paid.

One should be interested in:

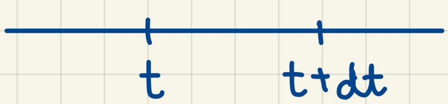
$$PV(\text{dividends}) = \sum_{k=1}^{n} D_k \cdot e^{-r \cdot t_k}$$

Case #3. Continuous dividends.

δ... dividend yield

The dividend amount paid to the shareholders during the time interval $(t, t+dt)$ is given as

$\delta \cdot S(t)\, dt$ per share owned.



observe: $S(t)$

Q: How would one calculate the total nominal amount of dividend paid over $[0,T]$?

STOCHASTIC PROCESS.

$\rightarrow$: $\displaystyle\int_0^T \delta\, S(t)\, dt$

Foreign Currencies.

0 ——— T

Domestic Currency (DC) ... its continuously compounded, risk-free interest rate is r_D

Foreign Currency (FC) ... its c.c. r.f. i.r. is r_F

$x(t),\ t \geq 0$ ... the EXCHANGE RATE from the FC to the DC;

i.e., $x(t)$ is the number of units of the DC we have to pay @ time t to receive one unit of the FC

0 ——— T

Invest in (buy) 1 unit of FC $\xrightarrow{r_F}$ Withdraw the balance from the foreign acct and exchange back to DC

Until next time:

- Work on HW and Quiz.
- Read PS#2.