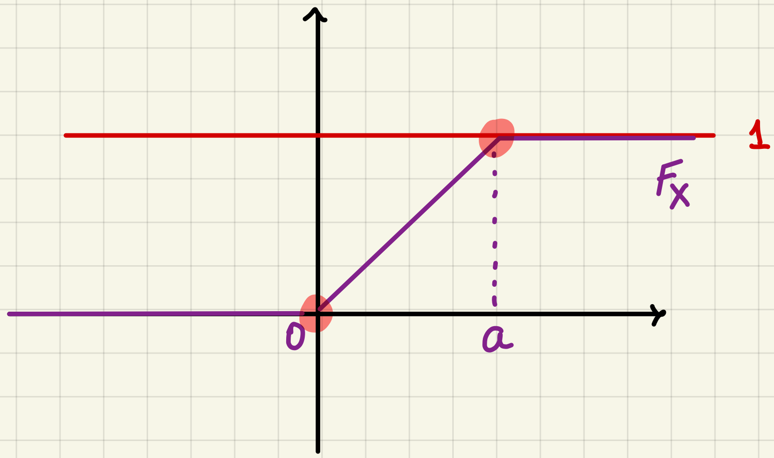
M358K: September 24th, 2021.

Def'n. A random variable X is said to be continuous if its cumulative distribution fction is:

(i) continuous everywhere;

(ii) differentiable everywhere except @ at most countably many points.

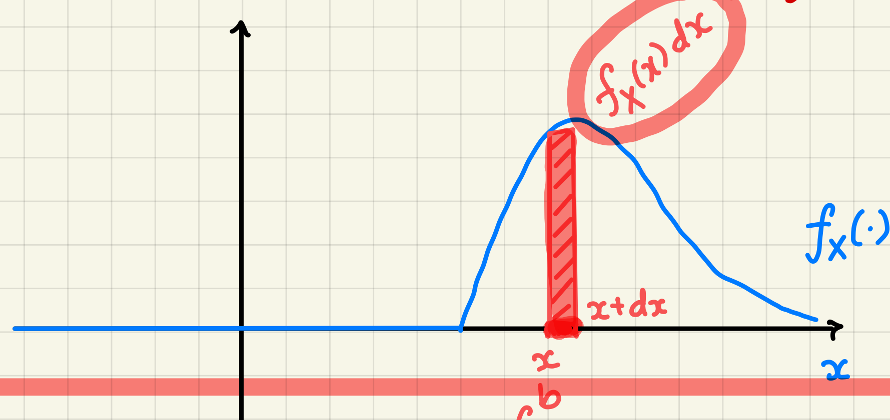


$X \sim \text{Uniform}(0,a)$

Def'n. Any function $f_X : \mathbb{R} \to [0, +\infty)$ such that

$$f_X(x) = F_X'(x)$$ for all x where the derivative exists

is called the probability density function (pdf) of X.



Q: $$\boxed{\mathbb{P}[a < X \leq b] = \int_a^b f_X(x)\,dx = F_X(b) - F_X(a)}$$ ☺

Q: X is continuous $\Rightarrow$ $\boxed{\mathbb{P}[X = x] = 0}$

Q: $\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$

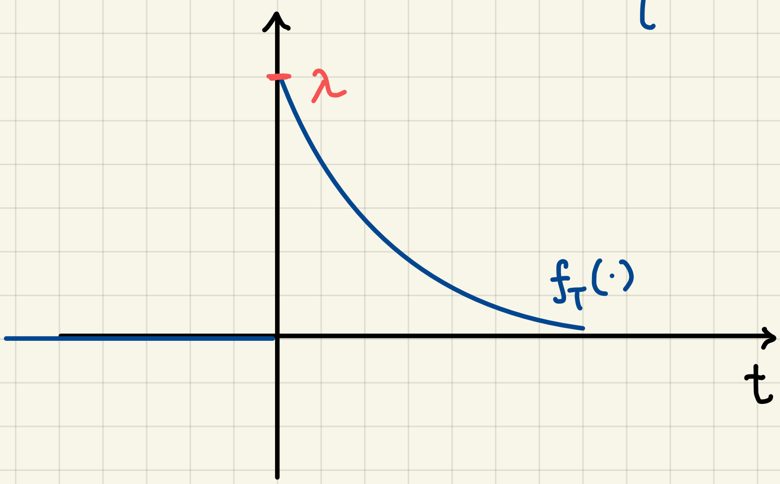
Q: Is it possible that $f_X(x) > 1$ for some x? YES!

Example. **Exponential distribution.**

w/ a positive parameter λ

Its pdf is:

$$f_T(t) = \begin{cases} \lambda e^{-\lambda \cdot t} & \text{for } t > 0 \\ 0 & \text{otherwise} \end{cases}$$



Note: If $\lambda = 25$, $f_T(x) \approx 25$ for x close to zero

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Def'n. For a discrete r.v. X, its expected value is given by:

$$\boxed{\mathbb{E}[X] := \sum_x x\, p_X(x)}$$ when the sum exists

For a continuous r.v. X, its expected value is given by:

$$\boxed{\mathbb{E}[X] := \int_{-\infty}^{+\infty} x\, f_X(x)\,dx}$$ when the integral exists

Example. $T \sim \text{Exponential}(\lambda)$

$$\Rightarrow \quad \mathbb{E}[T] = \frac{1}{\lambda}$$

Def'n. For any r.v. X, its variance is defined as

$$\boxed{\mathrm{Var}[X] := \mathbb{E}\left[\left(X-\mathbb{E}[X]\right)^2\right]}$$

if it exists

Note: Set $\mu_X := \mathbb{E}[X]$

$$\Rightarrow \quad \mathrm{Var}[X] = \mathbb{E}[X^2 - 2\mu_X X + \mu_X^2]$$

linearity of expectation

$$= \mathbb{E}[X^2] - 2\mu_X \mathbb{E}[X] + \mu_X^2$$

$$= \mathbb{E}[X^2] - 2\mu_X^2 + \mu_X^2$$

$$\Rightarrow \boxed{\mathrm{Var}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2}$$

Def'n. The standard deviation of the r.v. X is:

$$\boxed{SD[X] = \sqrt{\mathrm{Var}[X]}}$$