M339G: September 13th, 2024.

F-distribution.

Def'n. Let U and V be chi-squared random variable w/ ν_1 and ν_2 degrees of freedom, respectively.

Then, w/ U and V independent, the random variable

$$F = \frac{U/\nu_1}{V/\nu_2}$$

is said to have the F-distribution w/ numerator degrees of freedom ν_1 and denominator degrees of freedom ν_2.

We write $\boxed{F \sim F(\nu_1, \nu_2)}$

Theorem. Let two independent random samples of size n_1 and n_2, resp., be drawn from two normal populations w/ variances σ_1^2 and σ_2^2, resp.

If the variances of the random samples are given by

S_1^2 and S_2^2, resp.,

then, the statistic

$$\boxed{F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1 - 1, n_2 - 1)}$$

Corollary. If $\sigma_1 = \sigma_2$, then,

$$\boxed{F = \frac{S_1^2}{S_2^2} \sim F(n_1 - 1, n_2 - 1)}$$