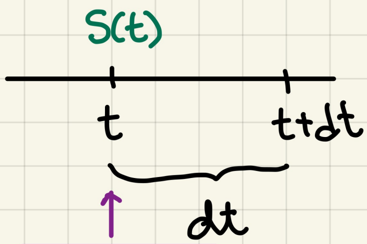
M339D : February 3rd, 2021.

Continuous dividend paying Stocks [cont'd].

Notation:
- $S(t)$, $t \geq 0$ ... stock price at time t
- δ ... dividend yield

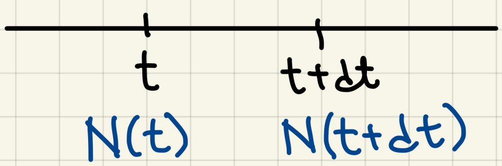


$\boxed{\delta S(t)\,dt}$ received in dividend per share.

Convention. ALL the dividends are immediately & continuously invested in the same stock!

Notation: $N(t)$, $t \geq 0$ ... the # of shares owned @ time t

At time 0: $N(0) = n_0$



$$\Rightarrow \underset{(*)}{dN(t)} = N(t+dt) - N(t) = \text{\# of shares purchased @ time } t$$

- The total amount of dividend for this infinitesimal interval is

$$N(t) \cdot \delta S(t)\,dt$$

- The number of shares one can buy for the above amount is

$$\frac{N(t) \cdot \delta \cdot \cancel{S(t)}\,dt}{\cancel{S(t)}} = \underset{(**)}{\delta N(t)\,dt}$$

Combining * & **, which represent the same quantity, we get

$$\boxed{dN(t) = \delta N(t)\,dt}$$

$$\frac{dN(t)}{N(t)} = \delta\,dt$$

$$\ln(N(t)) = \delta t + \text{const}$$

$$N(t) = e^{\delta t} \cdot e^{\text{const}}$$

Remember the initial condition: $N(0) = n_0$.

We get $N(0) = e^{\delta \cdot 0} \cdot e^{\text{const}} = n_0 \Rightarrow e^{\text{const}} = n_0$

So, in general: $\boxed{N(t) = n_0 e^{\delta \cdot t}}$

It's pretty awesome that the number of shares owned @ any time ends up being a deterministic quantity.