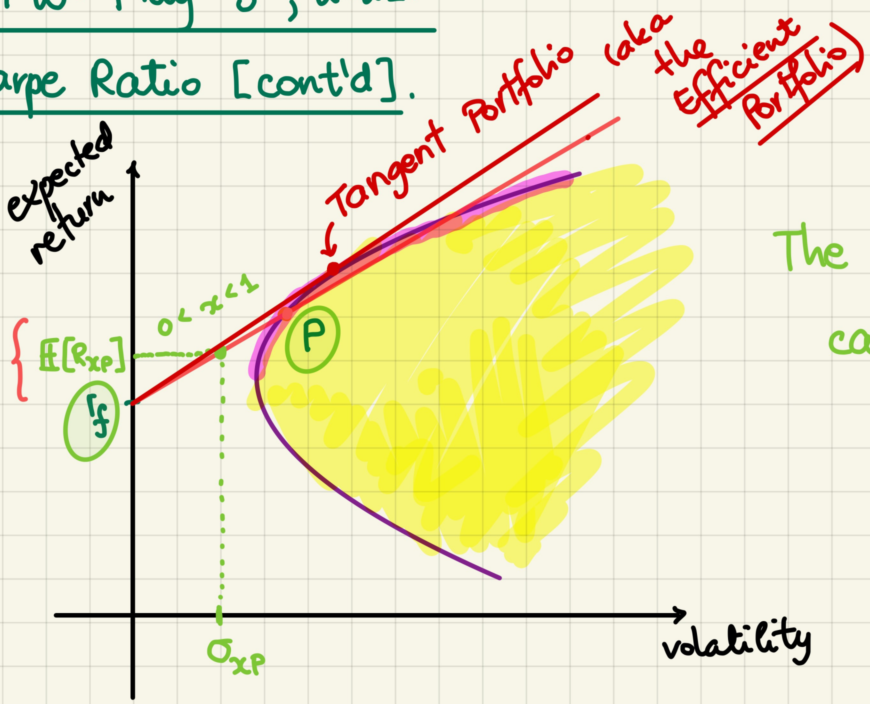
M339 W: May 3rd, 2021.

Sharpe Ratio [cont'd].



The slope of this line is called the Sharpe Ratio.

Reward-to-Risk Ratio

$$= \frac{\text{P's excess return}}{\text{P's volatility}} = \frac{\text{Risk premium}}{\text{Volatility}} = \boxed{\frac{\mathbb{E}[R_P] - r_f}{\sigma_P}}$$

✓

8) You are given the following information about a two-asset portfolio:

(i) The Sharpe ratio of the portfolio is 0.3667. $\Rightarrow$ Sharpe ratio of X is

(ii) The annual effective risk-free rate is 4%.

(iii) If the portfolio were 50% invested in a risk-free asset and 50% invested in a risky asset X, its expected return would be 9.50%. P

Now, assume that the weights were revised so that the portfolio were 20% invested in a risk-free asset and 80% invested in risky asset X. P'

Calculate the standard deviation of the portfolio return with the revised weights.

$\sigma_{p'} = ?$

$R_{p'} = 0.2 \cdot r_f + 0.8\, R_X$

$\Rightarrow \quad \sigma_{p'} = \underbrace{\sigma_{0.8X}}_{\text{In our previously used notation}} = 0.8\, \sigma_X$

(A) 6.0%

(B) 6.2%

(C) 12.8%

(D) 15.0%

(E) 24.0%

We know from (i) that

$$\frac{\mathbb{E}[R_X] - r_f}{\sigma_X} = 0.3667$$

(iii) $\Rightarrow \frac{1}{2} r_f + \frac{1}{2} \mathbb{E}[R_X] = 0.095$

$\frac{1}{2}(0.04) + \frac{1}{2} \mathbb{E}[R_X] = 0.095 \quad / -0.04$

$\frac{1}{2}\left(\mathbb{E}[R_X] - r_f\right) = 0.095 - 0.04 = 0.055$

$\mathbb{E}[R_X] - r_f = 0.11$

$\sigma_X = \frac{0.11}{0.3667} = 0.3 \quad \Longrightarrow \sigma_{p'} = 0.8 \cdot 0.3$

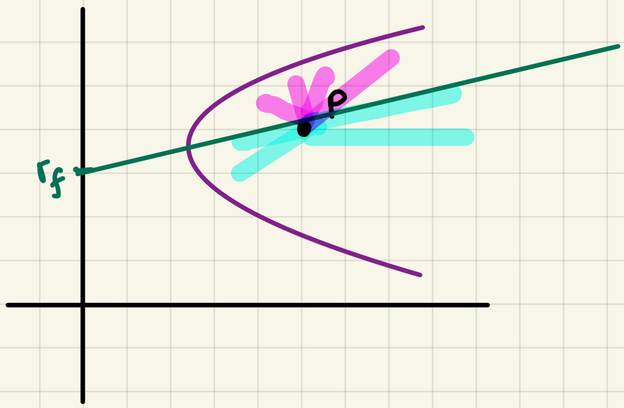
$\sigma_{p'} = 0.24$

Required Return.

Goal: To figure out if we can improve a portfolio by "adding" (more) of a particular security.

Q: What is the condition for the improvement, i.e., what is the new investment's REQUIRED RETURN?

→ : Start w/ a portfolio P.



slope = sharpe ratio of P

$$\eta_P = \frac{\mathbb{E}[R_P] - r_f}{\sigma_P}$$

Consider an investment I.

Construct P':

- keep P (as is);
- borrow $x \cdot$ (Value of P) at the rate r_f;
- invest x (Value of P) in the investment I.

Assume the weight x is small!

The new return:

$$R_{P'} = R_P - x \cdot r_f + x R_I$$

⇒ The excess return of P':

$$\mathbb{E}[R_{P'}] - r_f = \mathbb{E}[R_P] - x\, r_f + x\, \mathbb{E}[R_I] - r_f$$

$$= \underbrace{(\mathbb{E}[R_P] - r_f)}_{\text{the excess return of P}} + x \underbrace{(\mathbb{E}[R_I] - r_f)}_{\text{the excess return of investment I}}$$

The volatility of P':

$$\mathrm{Var}[R_{P'}] = \mathrm{Var}[R_P - \underbrace{x \cdot r_f}_{\text{deterministic}} + x \cdot R_I]$$

$$= \mathrm{Var}[R_P + x \cdot R_I]$$

$$= \mathrm{Var}[R_P] + 2 \cdot x \cdot \mathrm{Cov}[R_P, R_I] + \underbrace{x^2 \cdot \mathrm{Var}[R_I]}_{\text{to be diversified since } x \text{ is small}}$$

$$f(y) = \sqrt{y} = y^{1/2}$$

$$f'(y) = \frac{1}{2} \cdot y^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{y}}$$

$$f(y_0 + dy) = f(y_0) + f'(y_0)\, dy + \text{lower order terms}$$

$$= f(y_0) + \frac{1}{2} \cdot \frac{1}{\sqrt{y_0}}\, dy + \text{—}\parallel\text{—}$$

$$\sqrt{\mathrm{Var}[R_{P'}]} \approx \sqrt{\mathrm{Var}[R_P]} + \frac{1}{\cancel{2}} \cdot \frac{1}{\cancel{\sqrt{\mathrm{Var}[R_P]}}} \cdot \cancel{2} \cdot x \cdot \overbrace{\mathrm{Cov}[R_P, R_I]}^{\cancel{\sigma_P}\, \sigma_I \cdot \mathrm{corr}[R_P, R_I]}$$

$$\underline{SD[R_{P'}]} = \underline{SD[R_P]} + \underbrace{x \cdot SD[R_I] \cdot \mathrm{corr}[R_P, R_I]}_{\text{"incremental" risk added to the portfolio by the introduction of } I}$$

$$\cancel{x}(\mathbb{E}[R_I] - r_f) > \cancel{x} \cdot SD[R_I] \cdot \mathrm{corr}[R_P, R_I] \cdot \eta_P$$

⋮

The required return?

The linear regression ☺