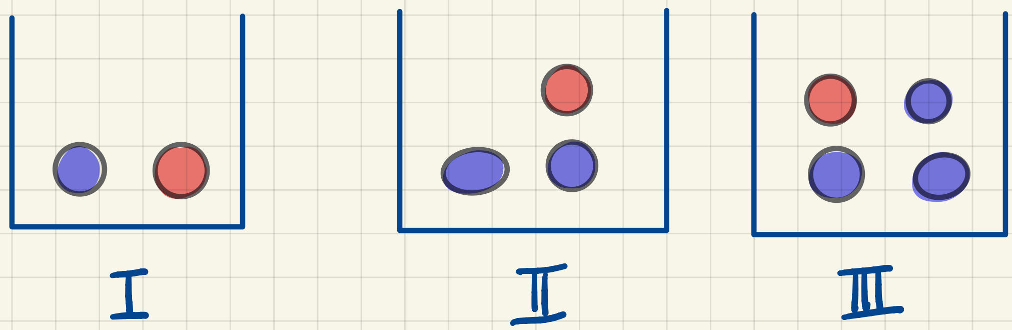
M378K: December 9th, 2024.

Bayesian Stats [cont'd].

Example.



Someone takes out a ball from a box.
You must try to guess its "box of origin"
The priors would be

$$\boxed{\hat{\pi}_I = \frac{2}{9}}, \quad \boxed{\hat{\pi}_{II} = \frac{3}{9}}, \quad \boxed{\hat{\pi}_{III} = \frac{4}{9}}$$

Now, what if we're told the ball is BLUE?

Then,

$$\boxed{\mathbb{P}[\Theta = \overset{I}{1} \mid Y=B]} = \frac{\mathbb{P}[\Theta=1]\cdot\mathbb{P}[Y=B \mid \Theta=1]}{\sum_{i=1}^{3}\mathbb{P}[\Theta=i]\cdot\mathbb{P}[Y=B \mid \Theta=i]}$$

Posterior Probability!

$$= \frac{\frac{\cancel{2}}{9}\cdot\frac{1}{\cancel{2}}}{\frac{\cancel{2}}{9}\cdot\frac{1}{\cancel{2}} + \frac{\cancel{3}}{9}\cdot\frac{2}{\cancel{3}} + \frac{\cancel{4}}{9}\cdot\frac{3}{\cancel{4}}} = \frac{\frac{1}{9}}{\frac{6}{9}} = \frac{1}{6}$$

$$\mathbb{P}[\Theta = \overset{II}{2} \mid Y=B] = \frac{\mathbb{P}[\Theta=2]\cdot\mathbb{P}[Y=B \mid \Theta=2]}{\frac{6}{9}} = \frac{\frac{\cancel{3}}{\cancel{9}}\cdot\frac{2}{\cancel{3}}}{\frac{6}{\cancel{9}}} = \frac{1}{3}$$

$$\mathbb{P}[\Theta = \overset{III}{3} \mid Y=B] = \frac{1}{2}$$

The Continuous Case.

Here, we assume that Θ admits a pdf denoted by $p(\theta)$.

We denote the posterior density by

$$p(\theta \mid y_1, y_2, \ldots, y_n)$$

As usual, $L(\theta; y_1, \ldots, y_n)$ is the likelihood function

$$p(\theta \mid y_1, \ldots, y_n) = \frac{p(\theta)\, L(\theta; y_1, \ldots, y_n)}{\int p(\tilde{\theta})\, L(\tilde{\theta}; y_1, \ldots, y_n)\, d\tilde{\theta}}$$

Note:
- By default the integral is $-\infty$ to ∞
- $\tilde{\theta}$ is the "dummy" variable of integration.

Task: Ex in the lecture notes w/ normal · normal