UNIVERSITY OF TEXAS AT AUSTIN

Extra-credit homework assignment #1

Prepaid forward contracts. Forward contracts.

Please, provide **your complete solution** to the following problems. Only the final answer without justification will receive zero credit.

Problem 1.1. (2 points) An agent is **only** allowed to long a forward contract if he/she is willing to take physical delivery of the underlying asset. *True or false? Why?*

Solution: FALSE
It is possible to have *cash settlement* on the delivery date if the forward contract stipulates so.

Problem 1.2. (2 points) A short forward contract has an unlimites loss potential. *True or false? Why?*

Solution: TRUE
The profit curve is unbounded from below.

Problem 1.3. (2 points)
Assume that the continuously compounded, risk-free interest rate is strictly positive. The forward price of a non-dividend-paying stock is always strictly increasing with respect to the delivery date. *True or false? Why?*

Solution: TRUE
The no-arbitrage forward price is

$$F_{0,T}(S) = S(0)e^{rT}.$$

This is a strictly increasing fuction in T.

Problem 1.4. (2 points) A portfolio consisting of a long forward contract and a bond can replicate a long prepaid forward contract. *True or false? Why?*

Solution: TRUE
Construct the portfolio in this way:

- one long forward contract with delivery date T and with the forward price $F_{0,T}$;
- one long zero-coupon bond with maturity date T and redemption amount $F_{0,T}$.

There are no intermediate cashflows for this portfolio. Its payoff is

$$S(T) - F_{0,T} + F_{0,T} = S(T).$$

Since the payoff is equal to the payoff of the long prepaid forward, this is, indeed, its replicating portfolio.

Problem 1.5. (2 pts) A non-dividend-paying stock sells today for \$100 per share. The yearly effective interest rate is 0.21. Then, $F_{0,1/2}(S) > 110$. *True or false? Why?*

Solution: FALSE
In fact, $F_{0,1/2}(S) = 100(1.21)^{1/2} = 100 \times 1.1 = 110$.

Problem 1.6. (2 points) The profit diagram and the payoff diagram for long positions in a forward contract are identical. *True or false? Why?*

Solution: TRUE
The initial cost of a forward contract is zero, so its payoff and profit functions are identical.

Problem 1.7. If the stock pays discrete dividends, there is a comparative advantage to an outright purchase of the stock as compared to the prepaid forward contract on that stock. *True or false? Why?*

Solution: FALSE
The prepaid forward price includes the "compensation" for forfeited dividends payments.

Problem 1.8. In our usual notation, the difference between the **profit** of a long forward contract and a long investment in one unit of the non-dividend-paying underlying asset equals the forward price. *True or false? Why?*

Solution: FALSE

$$S(T) - F - S(T) + S(0)e^{rT} = S(0)e^{rT} - F.$$

Of course, if the available forward price is the no-arbitrage forward price, the profits are equal.

Problem 1.9. (2 points) The prepaid forward price of a non-dividend-paying stock is strictly decreasing with respect to the delivery date. *True or false? Why?*

Solution: FALSE
For a non-dividend-paying stock, its prepaid-forward price is $F^P_{0,T}(S) = S(0)$. It is a **constant** with respect to the time T. So, it is not strictly decreasing.

Problem 1.10. (2 points) The forward price of a continuous-dividend-paying stock is always strictly increasing with respect to the delivery date. *True or false? Why?*

Solution: FALSE
The forward price equals $F_{0,T}(S) = S(0)e^{(r-\delta)T}$. The monotonicity with respect to T depends on the values of r and δ. For instance, if $r = \delta$, the forward price is constant and equal to $S(0)$.

Problem 1.11. Nick Mallory dabbles in the stock market in his spare time. He short-sells one share of stock whose current price is \$100 per share. At this time he also notices that the prepaid forward price of this continuous-dividend-paying stock for delivery in one year equals \$98.02 per share.

Upon the short sale, Nick invests the proceeds at the risk-free interest rate of 0.04. What is the expression for the profit Nick will have at time-1 when he closes his short-sale as a function of the final stock price s?

Solution: There are multiple correct ways to write down Nick's profit. They are all variants of

$$-(100/98.02)s + 100e^{0.04}$$

Problem 1.12. For a continuous-dividend-paying stock, the current stock price is observed to be \$80. The forward price for delivery in two years is \$82.44. What is the forward price for delivery in three years?

Solution: In our usual notation, we are given that

$$F_{0,2}(S) = S(0)e^{2(r-\delta)} = 80e^{2(r-\delta)} = 82.44.$$

So,

$$F_{0,3}(S) = S(0)e^{3(r-\delta)} = 80 \times \left(\frac{82.44}{80}\right)^{3/2} = 83.67.$$

Problem 1.13. Maryam bakes batches of cupcakes for a cupcake convention. She buys forward 21 pounds of raspberries from a local farmer at the forward price of \$5.60 per pound.

She projects to bake 336 cupcakes and sell each for \$3. The total and aggregate non-raspberry costs of baking the cupcakes are \$200.

If the market price of raspberries on the day of the cupcake convention is \$5.40, what is Maryam's profit?

Solution:

$$336 \times 3 - 21 \times 5.60 - 200 = 690.40.$$

Problem 1.14. The current price of a discrete-dividend-paying stock is \$90 per share. The company projects to pay quarterly dividends starting three months from today to perpetuity. The first dividend amout is \$2 and the dividends are scheduled to increase by a factor of 0.01 every time a dividend is paid.

The continuously compounded, risk-free interest rate is 0.06. What is the prepaid forward price of the above stock for delivery in eight months?

Solution:

$$F^P_{0,T}(S) = 90 - 2e^{-0.015} - 2.02e^{-0.03} = 86.0695.$$

Problem 1.15. (5 points) A non-dividend-paying stock sells for \$100 per share today. The continuously compounded risk-free interest rate equals 0.05. You are the seller in a one-year forward contract. Find your profit if the stock's spot price in one year equals \$130 per share.

Solution: The forward contract has no initial cost, so its profit equals its payoff.

$$F_{0,1}(S) - S(1) = S(0)e^r - S(1) = 100e^{0.05} - 130 = -24.87$$

Problem 1.16. (5 points) A soy-bean farmer shorts forward contracts on soy in an amount matching his crop volume and with delivery at harvest time. Then, he is considered:

(a) an arbitrageur.

(b) a broker.

(c) a speculator.

(d) a hedger.

(e) None of the above.

Why?

Solution: (d)
Because the farmer has an inherent long position in the soy beans, by shorting the forward contract, (s)he is selling his crop forward at a pre-determined price.