# The Strong Law of Large Numbers (SLLN).

Let $\{X_k, k = 1, 2, \ldots\}$ be a sequence of independent, identically distributed random variables.

Assume: $\mu_X := \mathbb{E}[X_1] < \infty$

Then,

$$\boxed{\frac{X_1 + \cdots + X_n}{n} \xrightarrow[n \to \infty]{} \mu_X}$$

If a function $g$ is such that

$g(X_1)$ is well-defined,

and

$\mathbb{E}[g(X_1)] < \infty$,

then

$$\boxed{\frac{g(X_1) + g(X_2) + \cdots + g(X_n)}{n} \xrightarrow[n \to \infty]{} \mathbb{E}[g(X_1)]}$$

## Monte Carlo.

Recipe:

- Draw simulated values of a random variable from a particular distribution.
- Apply a function to the simulated values.
- Calculate the arithmetic average of the obtained quantities.

We get the value which is "close to" the theoretical expected value.

About precision:

- $\mathrm{Var}\left[\frac{X_1 + X_2 + \cdots + X_n}{n}\right] = \frac{1}{n^2} \mathrm{Var}[X_1 + X_2 + \cdots + X_n]$ (independence)

$= \frac{1}{n^2}\left(\mathrm{Var}[X_1] + \mathrm{Var}[X_2] + \cdots + \mathrm{Var}[X_n]\right)$ (identically dist'd)

$= \frac{1}{n^{\cancel{2}}} \cdot \cancel{n} \cdot \mathrm{Var}[X_1] = \frac{\mathrm{Var}[X_1]}{n}$

- $\mathrm{SD}\left[\frac{X_1 + X_2 + \cdots + X_n}{n}\right] = \frac{\mathrm{SD}[X_1]}{\sqrt{n}}$

To increase precision by a factor $\eta$, the number of variates must increase by a factor of $\eta^2$.