M339W: April 28th, 2021.

The Effect of Correlation.

- If $\rho = 1$, then the feasible set is a straight line between the two assets.
- The higher the correlation, the smaller the curvature of the feasible set.
- If $\boxed{\rho = -1}$, then ...

Claim: There is a weight w of asset #1 such that the resulting portfolio is risk-free, i.e., it has volatility zero.

$\longrightarrow$: $Var[w \cdot R_1 + (1-w) \cdot R_2] = 0$

$$\Leftrightarrow$$

$$w^2 \cdot \underbrace{Var[R_1]}_{\sigma_1^2} + (1-w)^2 \cdot \underbrace{Var[R_2]}_{\sigma_2^2} + 2w(1-w) \underbrace{Cov[R_1, R_2]}_{\sigma_1 \cdot \sigma_2 \cdot \rho} = 0$$

$\rho = -1$

$$\Leftrightarrow$$

$$w^2 \cdot \sigma_1^2 - 2w(1-w)\sigma_1 \cdot \sigma_2 + (1-w)^2 \cdot \sigma_2^2 = 0$$

$$\Leftrightarrow$$

$$(w \cdot \sigma_1)^2 - 2(w \cdot \sigma_1)((1-w) \cdot \sigma_2) + ((1-w) \cdot \sigma_2)^2 = 0$$

$$\Leftrightarrow$$

$$(w \cdot \sigma_1 - (1-w) \cdot \sigma_2)^2 = 0$$

$$\Leftrightarrow$$

$$w \cdot \sigma_1 - (1-w) \cdot \sigma_2 = 0$$

$$\Leftrightarrow$$

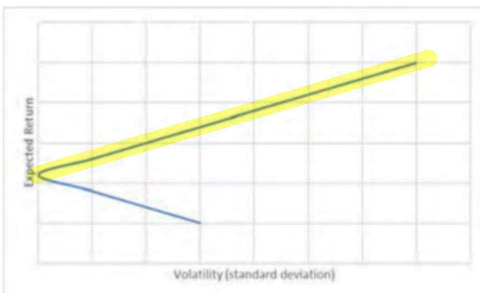
$$w(\sigma_1 + \sigma_2) = \sigma_2$$

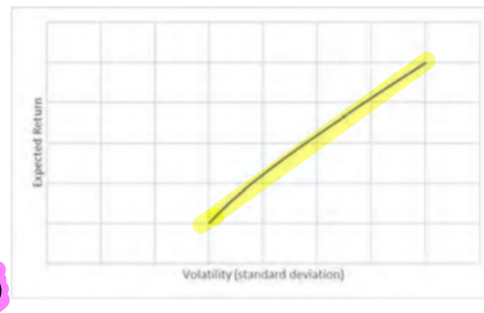
$$\Leftrightarrow$$

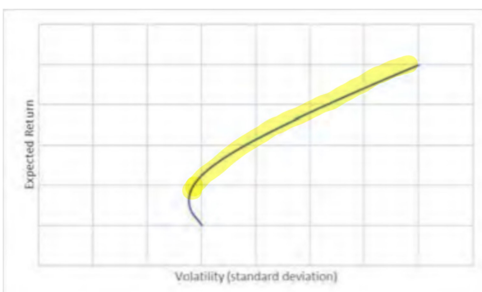
$$\boxed{w = \frac{\sigma_2}{\sigma_1 + \sigma_2}}$$

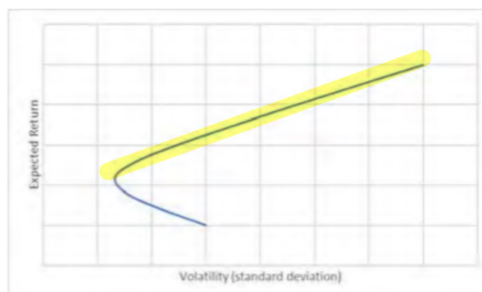
5) You are given the following set of diagrams for a two-stock portfolio, with expected return on the vertical axis and volatility on the horizontal axis. These diagrams are meant to help investors identify the set of efficient portfolios.

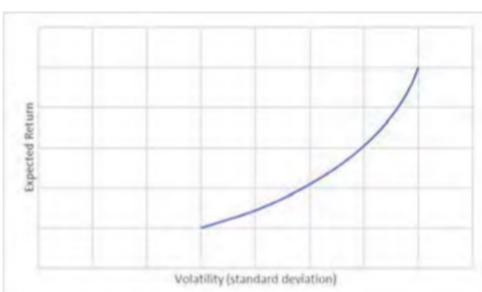
Identify the diagram demonstrating the highest correlation between the two stocks.

(A)


(B)


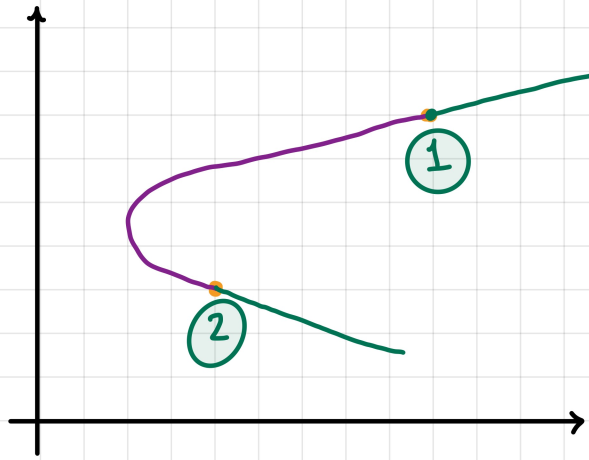
(C)


(D)


(E)


Not a feasible set at all!

Short sales w/ two assets.



Feasible set and the efficient frontier for three or more risky assets

See Mathematica Demo.

Adding the risk-free saving and borrowing.

Start w/ a portfolio P consisting of risky investments. Let its return be denoted by R_P.

Let the risk-free interest rate be denoted by r_f.

Now, we construct a new portfolio such that:

- the weight x is given to (the risky) P,

and

- the weight $(1-x)$ is given to the risk-free investment.

Denote the return of this new portfolio by R_{xP}

$$\Rightarrow \quad \mathbb{E}[R_{xP}] = \mathbb{E}[x \cdot R_P + (1-x) \cdot r_f]$$
$$= x \cdot \mathbb{E}[R_P] + (1-x) \cdot r_f$$
$$= r_f + x \cdot (\mathbb{E}[R_P] - r_f)$$

$$\Rightarrow \quad \boxed{\underline{\mathbb{E}[R_{xP}] - r_f} = x \cdot (\underline{\mathbb{E}[R_P] - r_f})}$$

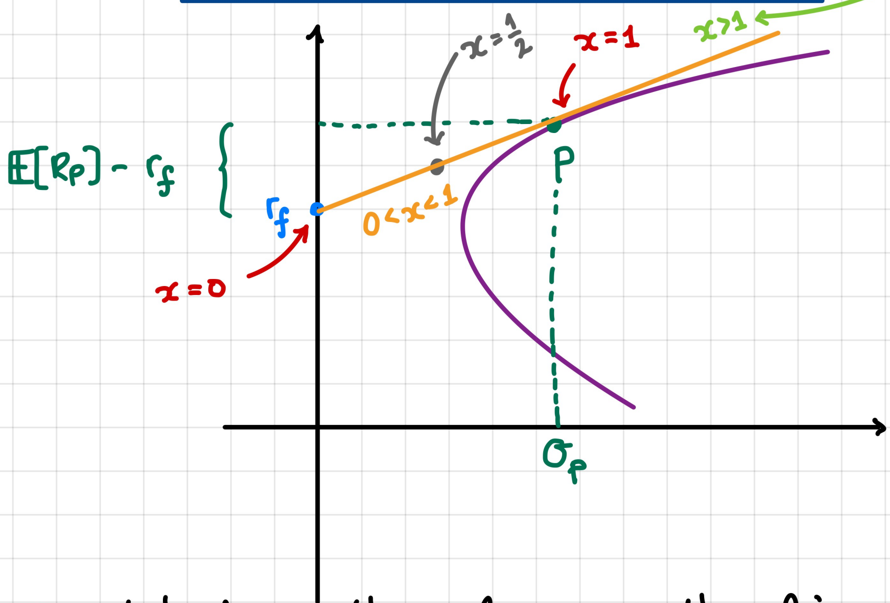
(expected) excess return or risk premium

$$Var[R_{xP}] = Var[x \cdot R_P + \underbrace{(1-x)\cdot r_f}_{\text{deterministic}}]$$

$$= x^2 \cdot Var[R_P]$$

$$\Rightarrow \boxed{SD[R_{xP}] = x \cdot SD[R_P]}$$

creating a levered portfolio



$x = \frac{1}{2}$

Task: The new efficient frontier now that the risk-free asset is "allowed".

Q: What is the slope of the line through $(0, r_f)$ and $(\sigma_P, \mathbb{E}[R_P])$?

$\longrightarrow$: $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\mathbb{E}[R_P] - r_f}{\sigma_P}$

A Reward-to-Risk Ratio

The Sharpe Ratio