

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 10Binomial option pricing.

10.1. **The forward binomial tree.** Please, provide your *final answer only* to the following problem.

Problem 10.1. (5 points) Assume that the a stock price is modeled using a one-period forward binomial tree with the length of a single period equal to three months. According to this model, the stock price can take either the value of \$50, or the value of \$40 in exactly three months. Calculate the volatility of the stock price.

- (a) About 0.22
- (b) About 0.28
- (c) About 0.30
- (d) About 0.32
- (e) None of the above.

10.2. **Alternative binomial trees.** Please, provide your complete solutions to the following problem(s):

Problem 10.2. Cox-Ross-Rubinstein (CRR)

The Cox-Ross-Rubinstein model is a binomial tree in which the up and down factors are given as

$$u = e^{\sigma\sqrt{h}}, \quad d = e^{-\sigma\sqrt{h}},$$

where σ denotes the volatility parameter and h stands for the length of a single period in a tree.

- a. (2 points) What is the ratio S_u/S_d ?
- b. (2 points) What is the (as simplified as possible) expression for the risk-neutral probability of the stock price going up in a single step?
- c. (2 points) Express S_{ud} in terms of $S(0)$, σ and h in a CRR tree.
- d. (5 points) As was the case with the forward tree, the *no-arbitrage* condition for the binomial asset-pricing model is satisfied for the CRR tree regardless of the specific values of σ, δ, r and h . *True or false?*

Problem 10.3. The Jarrow-Rudd model.

The **Jarrow-Rudd** model (aka, the lognormal binomial tree) is a binomial tree in which the up and down factors are defined as follows

$$u = e^{\left(r - \delta - \frac{\sigma^2}{2}\right)h + \sigma\sqrt{h}}, \quad d = e^{\left(r - \delta - \frac{\sigma^2}{2}\right)h - \sigma\sqrt{h}},$$

where

- r stands for the continuously-compounded, risk-free interest rate,
- δ is the stock's dividend yield,
- σ denotes the volatility parameter, and
- h stands for the length of a single period in a tree.

Answer the following questions:

- a. (2 points) What is the ratio S_u/S_d ?
- b. (2 points) What is the (as simplified as possible) expression for the risk-neutral probability of the stock price going up in a single step?
- c. (5 points) As was the case with the forward tree, the *no-arbitrage* condition for the binomial asset-pricing model is satisfied for the Jarrow-Rudd tree regardless of the specific values of σ, δ, r and h . *True or false?*

10.3. **Multi-period binomial option pricing: European options.** Please, provide your complete solutions to the following problem:

Problem 10.4. (10 points) The current price of a continuous-dividend-paying stock is \$100 per share. Its volatility is given to be 0.2 and its dividend yield is 0.03.

The continuously compounded risk-free interest rate equals 0.04.

Consider a \$95-strike European put option on the above stock with nine months to expiration. Using a three-period forward binomial tree, find the price of this put option.

10.4. **Two binomial periods: American options.** Please, provide your complete solutions to the following problem:

Problem 10.5. (15 points) Find the current price of a one-year, \$110-strike American put option on a non-dividend-paying stock whose current price is $S(0) = \$100$. Assume that the continuously compounded interest rate equals $r = 0.06$.

Use a two-period binomial tree with $u = 1.23$, and $d = 0.86$ to calculate the price $V_P(0)$ of the put option.