

## UNIVERSITY OF TEXAS AT AUSTIN

## HW Assignment 1

The prerequisite material.

Please, provide just the final answer to the True/False and Multiple-choice questions. For the free-response questions, provide your complete solutions if you expect partial credit. For the free-response questions, even if your final answer is correct and your work is not shown, you should not expect **any** partial credit.

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**Problem 1.1.** (10 points)

Provide the definition of an arbitrage portfolio.

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**Problem 1.2.** (2 points) It is never optimal to exercise an American call option on a non-dividend paying stock early. *True or false?*

**Problem 1.3.** (2 points) Gap put options always have a nonnegative payoff. *True or false?*

**Problem 1.4.** (6 pts) Let  $Z$  be a standard normal random variable. Using the standard normal tables, calculate the following probabilities:

- (i) (2 points)  $\mathbb{P}[-1.23 < Z < 2.37]$
- (ii) (2 points)  $\mathbb{P}[|Z| < 0.5]$
- (iii) (2 points)  $\mathbb{P}[Z^2 > 2.56]$

**Problem 1.5.** (5 points) An investor short sells one share of a non-dividend-paying stock and buys an at-the-money,  $T$ -year, European call option on this stock. The call premium is denoted by  $V_C(0)$ . Assume that there are no transaction costs. The continuously compounded, risk-free interest rate is denoted by  $r$ . Let the argument  $s$  represent the stock price at time  $T$ .

- (i) (3 points) Determine an algebraic expression for the investor's profit at expiration  $T$  in terms of  $V_C(0)$ ,  $r$ ,  $T$  and the strike  $K$ .
- (ii) (2 points) In particular, how does the expression you obtained in (i) simplify if the call is in-the-money on the exercise date?

**Problem 1.6.** (10 pts) For a two-period binomial model, you are given that:

- (1) each period is one year;
- (2) the current price of a non-dividend-paying stock  $S$  is  $S(0) = \$20$ ;
- (3)  $u = 1.2$ , with  $u$  as in the standard notation for the binomial model;
- (4)  $d = 0.8$ , with  $d$  as in the standard notation for the binomial model;
- (5) the continuously compounded risk-free interest rate is  $r = 0.04$ .

Consider a **special** call option which pays the excess above the strike price  $K = 23$  (if any!) at the end of every binomial period.

Find the price of this option.

**Problem 1.7.** (5 points) A discrete-dividend-paying stock sells today for \$100 per share. The continuously compounded, risk-free interest rate is 0.04. The first dividend will be paid at in three months in the amount of \$2. The remaining dividends will be equal to \$1 and continue to be paid out quarterly. What is the **prepaid forward price** of this stock for delivery in seven months?

- (a) \$73.02
- (b) \$97.04
- (c) \$98.02
- (d) \$100
- (e) None of the above.

**Problem 1.8.** (5 points) The random vector  $(X_1, X_2)$  is jointly normal. Its marginal distributions are:

$$X_1 \sim N(\text{mean} = 0, \text{variance} = 4), \quad X_2 \sim N(\text{mean} = 1, \text{variance} = 1).$$

The correlation coefficient is given to be

$$\text{corr}[X_1, X_2] = 0.3.$$

What is the variance of the random variable  $X = 3X_1 - 2X_2$ ?

- (a) 32.8
- (b) 47.2
- (c) 54.4
- (d) 58.2
- (e) None of the above.

**Problem 1.9.** (5 points) A coin is tossed and, independently, a 6-sided die is rolled. Let

$A = \{4 \text{ is obtained on the die}\}$  and

$B = \{Heads \text{ is obtained on the coin and}$   
an even number is obtained on the die}.

Then

- (a)  $A$  and  $B$  are mutually exclusive
- (b)  $A$  and  $B$  are independent
- (c)  $A \subseteq B$
- (d)  $A \cap B = B$
- (e) None of the above.