M378K Introduction to Mathematical Statistics
Problem Set #3
Continuous distributions.

Problem 3.1. Source: Sample P exam, Problem #33.
The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ *with probability density function* f_X*, where*

$$f_X(x) \propto \frac{1}{(10+x)^2}$$

on the interval.

Calculate the probability that the lifetime of the machine part is less than 6.

Problem 3.2. Source: Sample P exam, Problem #419.
A customer purchases a lawnmower with a two-year warranty. The number of years before the lawnmower needs a repair is uniformly distributed on $[0, 5]$. *Calculate the probability that the lawnmower needs no repairs within* 4.5 *years after the purchase, given that the lawnmower needs no repairs within the warranty period*

Problem 3.3. *Consider a continuous random variable Y whose probability density function is given by*

$$f_Y(y) = 2y\mathbf{1}_{[0,1]}(y)$$

What is the expected value of this random variable?