

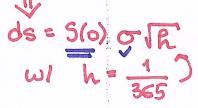
9. Consider the Black-Scholes framework. A market-maker, who delta-hedges, sells a three-month at-the-money European call option on a nondividend-paying stock.

You are given:

- (i) The continuously compounded risk-free interest rate is 10%.
- (ii) The current stock price is 50.
- (iii) The current call option delta is 0.61791.
- (iv) There are 365 days in the year.

If, after one day, the market-maker has zero profit or loss, determine the stock price move over the day.

- (A) 0.41
- (B) 0.52
- (C) 0.63
- (D) 0.75
- (E) 1.11



## **10-17.** DELETED

18. A market-maker sells 1,000 1-year European gap call options, and delta-hedges the position with shares.

You are given:

- (i) Each gap call option is written on 1 share of a nondividend-paying stock.
- (ii) The current price of the stock is 100.
- (iii) The stock's volatility is 100%.
- (iv) Each gap call option has a strike price of 130.
- (v) Each gap call option has a payment trigger of 100.
- (vi) The risk-free interest rate is 0%.

Under the Black-Scholes framework, determine the initial number of shares in the delta-hedge.

Given that  $\Delta_{c}(s(0), 0) = 0.61791 \approx 0.6179.$ In general,  $\Delta_c(s(0),0) = e^{-S.T} \cdot N(d_1(s(0),0)) = 0.6479$ no dividends , tables => d1(S(0),0) = N-1(0.6179) = 0.3 by defin -> 11 at the money 1 [la(36) + (r+ 2).T]  $\frac{1}{\sigma \sqrt{M_1}} \left[ 0.10 + \frac{\sigma^2}{2} \right] \cdot \frac{1}{4} = 0.3 / \sigma$  $(0.40 + \frac{\sigma^2}{3}) \cdot \sqrt{2} = 0.3 \cdot \sigma$ \$ +0.10 = 0.6.0 /.2  $\sigma^2 - 1.2\sigma + 0.2 = 0$ One of the solutions is, immediately, 0, = 1  $(\sigma - 1)(\sigma - 0.2) = 0$ Discord o=1 as a sol'n ; just keep 0=0.2 => onswer:  $50.0.2.\sqrt{\frac{1}{365}} = 10.\frac{1}{\sqrt{1268}} = 0.52^{(2.)}$ 

## Delta Gamma Hedging.

Let your investor start w/ a delta neutral portfolio. So, w/ the value function of the investor's portfolio denoted by v(s,t), (s) he maintains  $\Delta(s,t) = \frac{3}{35} v(s,t) = 0$ 

This can be accomplished by trading in the shares of the underlying continuously.

Then, the investor decides to Phedge as well, i.e., (s)he wants to create a [neutral portfolio. Q: Can they accomplish this by simply trading

in the shares of stock?

→: Remember: the 1 of the stock is 0. So, another option w/ non-zero T is needed.

Let the price of such an option be denoted by  $\tilde{v}(s,t)$ . Then:  $\tilde{\Delta}(s,t) = \frac{2}{3s} \tilde{v}(s,t)$ 

$$\tilde{\Gamma}(s,t) = \frac{\partial^2}{\partial s^2} \tilde{v}(s,t) .$$

Let  $\tilde{n}(s,t)$  be the number of these options to hold @ time.t w/ the stock price s. Then, to have  $\Gamma$ . neutrality, it must satisfy:

 $\Gamma(s,t) + \tilde{n}(s,t) \cdot \tilde{\Gamma}(s,t) = 0$ 

$$\omega / \Gamma(s,t) = \frac{\partial^2}{\partial s^2} v(s,t)$$

$$= > \tilde{n}(s,t) = -\frac{\Gamma(s,t)}{\tilde{\Gamma}(s,t)}$$

Now: the  $\Delta$  of the total portfolio will be  $\Delta(s,t) + \tilde{n}(s,t) \cdot \tilde{\Delta}(s,t)$ 

To re: establish D: neutrality, we need to trade in shares of stock. Let  $M_S(s,t)$  be the required if of shares. It must satisfy:

 $\Delta(s,t) + \tilde{n}(s,t) \cdot \tilde{\Delta}(s,t) + n_s(s,t) = 0$   $\tilde{\tau}$   $\Delta \cdot \text{neutrality}$ 

the original portfolio was D. neutral

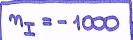
=>  $m_s(s,t) = -\tilde{m}(s,t)\cdot\tilde{\Delta}(s,t)$ 

Again, the  $\Gamma$  of the stack investment is Q. So,  $\Gamma$  neutrality is maintained.

For two European call options, Call-I and Call-II, on a stock, you are given:

Greek	Call-I	Call-II
Delta	0.5825	0.7773
Gamma	0.0651	0.0746
Vega	0.0781	0.0596

Suppose you just sold 1000 units of Call-I.



Determine the numbers of units of Call-II and stock you should buy or sell in order to both delta-hedge and gamma-hedge your position in Call-I.

- buy 95.8 units of stock and sell 872.7 units of Call-II (A)
- sell 95.8 units of stock and buy 872.7 units of Call-II (B)
  - buy 793.1 units of stock and sell 692.2 units of Call-II (C)
  - sell 793.1 units of stock and buy 692.2 units of Call-II (D)
  - sell 11.2 units of stock and buy 763.9 units of Call-II (E)

$$\triangle$$
 neutrality:  $(-1000) \cdot 0.5825 + m_{II} \cdot (0.7773) + m_{S} = 0$   
 $\Gamma$  neutrality:  $(-1000) \cdot (0.0651) + m_{II} \cdot (0.0746) = 0$ 

=> 
$$n_{\text{II}} = \frac{651}{0.0746} = 872.7 \text{ (long)}$$

Exchange Options.

T... exercise date

two risky assets:  $\begin{cases} 5... \text{ underlying asset} \\ Q... \text{ strike asset} \end{cases}$ 

For an exchange call:

the payoff: V(T,S,Q) = (S(T)-Q(T))+

For an exchange put:

 $V_{EP}(T,S,a) = (Q(T)-S(T))_{+}$ 

We have a special symmetry:

=> The prices at time:0 must be equal, i.e.,  $V_{EC}(0,S,Q) = V_{EP}(0,Q,S)$ 

=> It's sufficient to develop the Black. Scholes pricing formula for EXCHANGE CALLS.

• S... underlying; has  $S_s$ ,  $\sigma_s$ Because we're pricing, we look at the its time. T price under the risk neutral measure: In the Black Scholes model:  $S(T) = S(0) e^{(T-S_s-\frac{\sigma_s^2}{2}) \cdot T} + \sqrt[3]{T} \cdot Z_s$ 

• Q... strike asset; has  $\delta_{\alpha}$ ,  $\sigma_{\alpha}$ Q(T) = Q(0) e  $(r - \delta_{\alpha} - \frac{\sigma_{\alpha}^2}{2}) \cdot T + \sigma_{\alpha} \cdot \sqrt{T} \cdot Z_{\alpha}$ 

W/ Zs and Za std normal rnd variables
W/ p... the correlation coefficient between
Zs and Za