UNIVERSITY OF TEXAS AT AUSTIN

Quiz 22

Early exercise. Moneyness. Forward binomial trees (multi period).

Please, provide **complete** solution(s) to the following problem(s):

Problem 22.1. (5 points) *Source: Sample MFE Problem Set (Intro); Problem #46.*

Determine which of the following statements about options is **true**:

(A) Naked writing is the practice of buying options without taking an offsetting position in the underlying asset.
(B) A covered call involves taking a long position in an asset together with a written call on the same asset.
(C) An American style option can only be exercised during specified periods, but not for the entire life of the option.
(D) A Bermudan style option allows the buyer the right to exercise at any time during the life of the option.
(E) An in-the-money option is one which would have a positive profit if exercised immediately

Solution: (B)

Problem 22.2. (10 points)

The current stock price of a continuous-dividend-paying stock is 100 per share. Its volatility is 0.25 and its dividend yield is 0.01.

Assume that the continuously compounded risk-free interest rate equals 0.04.

(a) (3 points) Construct a two-period forward binomial tree for the above stock modeling the stock price evolution over the following half-year.
(b) (2 points) Consider a half-year, \$95-strike call option on the stock. What is the moneyness of this option at the *up* and *down* nodes?
(c) (5 points) Assume that the above option is European. What is its price?

Solution:

(a) This is a forward binomial tree, so the up and down factors are

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.04-0.01)(0.25)+0.25(0.5)} = 1.1417,$$
$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.04-0.01)(0.25)-0.25(0.5)} = 0.8891.$$

So, we have

$$S_u = 114.17, \quad S_{uu} = 130.34,$$
$$S_{ud} = 101.51,$$
$$S_d = 88.91, \quad S_{dd} = 79.06.$$

(b) The option is **in-the-money** at the *up* node, and **out-of-the-money** at the *down* node.
(c) The risk-neutral probability of the stock price going up in a single step is

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.25\sqrt{h}(0.5)}} = 0.4688.$$

The price of our call option is

$$V_C(0) = e^{-0.04/2}\left[35.34(0.4688)^2 + 2(0.4688)(1-0.4688)(6.51)\right] = 10.7921.$$