Two-Stock Portfolio.

X and Y stand for returns of the stocks.

α ... the proportion of your wealth invested in stock X

$R = \alpha \cdot X + (1-\alpha) \cdot Y$ ... is the total return of such a portfolio

Optimization Problem. Minimize $\mathrm{Var}[R]$ across all α

$$\mathrm{Var}[R] \longrightarrow \min$$

$$\mathrm{Var}[\alpha X + (1-\alpha) \cdot Y] \longrightarrow \min$$

$$\alpha^2 \sigma_X^2 + (1-\alpha)^2 \cdot \sigma_Y^2 + 2\alpha(1-\alpha) \cdot \sigma_{XY} \longrightarrow \min$$

$$2\alpha \sigma_X^2 + (-1)(2)(1-\alpha)\sigma_Y^2 + 2(1-2\alpha)\sigma_{XY} = 0$$

$$\alpha \sigma_X^2 + (\alpha - 1)\sigma_Y^2 + (1-2\alpha)\sigma_{XY} = 0$$

$$\alpha(\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}) = \sigma_Y^2 - \sigma_{XY}$$

$$\boxed{\alpha^* = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}}$$