M339D : February 28th, 2022.

21.

A market maker in stock index forward contracts observes a 6-month forward price of 112 on the index. The index spot price is 110 and the continuously compounded dividend yield on the index is 2%.

The continuously compounded risk-free interest rate is 5%.

$T = 1/2$ $\quad \boxed{F_{0,T}^{obs}(S) = 112}$ $\quad \boxed{S(0) = 110}$ $\quad \boxed{\delta = 0.02}$ $\quad \boxed{r = 0.05}$

Describe actions the market maker could take to exploit an arbitrage opportunity and calculate the resulting profit (per index unit).

(A) Buy observed forward, sell synthetic forward, Profit = 0.34

(B) Buy observed forward, sell synthetic forward, Profit = 0.78

(C) Buy observed forward, sell synthetic forward, Profit = 1.35

(D) Sell observed forward, buy synthetic forward, Profit = 0.78

(E) Sell observed forward, buy synthetic forward, Profit = 0.34

→ : The no-arbitrage forward price:

$$\boxed{F_{0,T}(S) = S(0)e^{(r-\delta)T}}$$

In this problem:

$$F_{0,T}(S) = 110e^{(0.05-0.02)(0.5)} = 110e^{0.015}$$

$$F_{0,T}(S) = \underline{111.66} < 112 = F_{0,T}^{obs}(S)$$

Diagnosis.

Propose an arbitrage portfolio:

- short the observed forward contract
- long $e^{-\delta \cdot T}$ units of the index

A part of the synthetic forward!

Construction.

Verification.

Initial Cost: $\boxed{e^{-\delta \cdot T} \cdot S(0)}$

Payoff: $\underbrace{F^{obs}_{0,T}(S) - \cancel{S(T)}}_{\text{short observed forward}} + \underbrace{\cancel{S(T)}}_{\text{units of index}} = F^{obs}_{0,T}(S)$ ✓

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= F^{obs}_{0,T}(S) - e^{rT} \cdot e^{-\delta T} \cdot S(0)$$
$$= F^{obs}_{0,T}(S) - F_{0,T}(S)$$

In this problem: $112 - 111.66 = 0.34$

Q: What if $F_{0,T}(S) > F^{obs}_{0,T}(S)$,

i.e., the no arbitrage forward price exceeds the observed forward price?

- Long the observed forward
- short-sell $e^{-\delta T}$ unit of the underlying asset

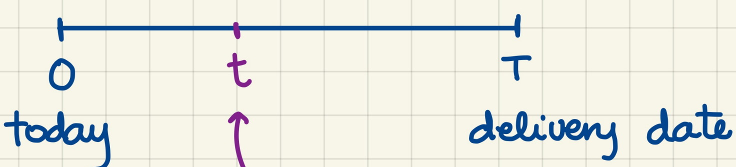
$\Rightarrow$ Profit $= F_{0,T}(S) - F^{obs}_{0,T}(S)$.

Futures Contracts.

... standardized versions of forward contracts w/ credit risk addressed.

$\Rightarrow$ They are traded and liquid.

$F_{0,T}$ ... the futures price @ time 0 for delivery @ time T



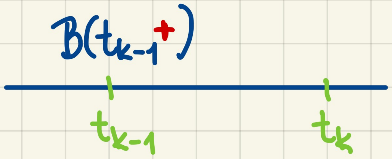
$F_{t,T}$ ... the futures price @ time t for delivery @ time T

Notional value: $N = \#\text{ of contracts} \times \#\text{ of units per contract} \times \text{Price per Unit}$

Margin Account:

$$\underbrace{B^b(0)}_{\substack{\text{the}\\\text{buyer's}\\\text{balance}}} = \underbrace{B^s(0)}_{\substack{\text{the}\\\text{seller's}\\\text{balance}}} = \underbrace{q}_{\text{percentage}} \cdot N$$

Settlement Times:



$$B(t_k-) = B(t_{k-1}+)\, e^{r(t_k - t_{k-1})}$$ effect of accruing interest

marking to market

$$B^b(t_k) = B^b(t_k-) + \#\text{of contracts} \times \text{size} \times \left(F_{t_k,T} - F_{t_{k-1},T}\right)$$

$$B^s(t_k) = B^s(t_k-) - \#\text{of contracts} \times \text{size} \times \left(F_{t_k,T} - F_{t_{k-1},T}\right)$$

Maintenance margin (MM):

If $B(t_k) < MM$, then a margin call is issued

$$B(t_k+) = \max\left(B(t_k), B(0)\right)$$ ✓