

**The University of Texas at Austin**  
**HOMEWORK ASSIGNMENT 4**  
*Introduction to Mathematical Statistics*

February 18, 2026

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**Instructions:** Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

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**Problem 4.1.** (15 points) Let  $(Y_1, Y_2)$  be a random vector with the joint pdf

$$f_{Y_1, Y_2}(y_1, y_2) = \frac{1}{4} \mathbf{1}_{\{-1 \leq y_1 \leq 1\}} \mathbf{1}_{\{-1 \leq y_2 \leq 1\}}.$$

Find  $\mathbb{P}[|Y_1| + |Y_2| \leq 1/2]$ .

**Problem 4.2.** (15 points) Two random numbers,  $Y_1$  and  $Y_2$  are chosen independently of each other, according to the uniform distribution  $U(-1, 2)$  on  $[-1, 2]$ . What is the probability that their product is positive?

**Problem 4.3.** (20 points) Three (fair and independent) coins are thrown; let  $Y_1, Y_2$  and  $Y_3$  be the outcomes (encoded as  $H$  or  $T$ ). Player 1 gets \$1 if  $H$  shows on coin 1 ( $Y_1 = H$ ) and/or \$2 if  $H$  shows on coin 2 ( $Y_2 = H$ ). Player 2, on the other hand, gets \$1 when  $Y_2 = H$  and/or \$2 when  $Y_3 = H$ . With  $W_1$  and  $W_2$  denoting the total amount of money given to Player 1 and Player 2, respectively,

1. (5 points) Write down the marginal distributions (pmfs) of  $W_1$  and  $W_2$ ,
2. (10 points) Write down the joint distribution table of  $(W_1, W_2)$ .
3. (5 points) Are  $W_1$  and  $W_2$  independent?