Option Elasticity

W: March 13th, 2019.

Def'n.

$$\Omega(s,t) := \frac{\Delta(s,t)\cdot s}{v(s,t)}$$

Use: We can consider the option volatility:

$$\sigma_{opt} = \underset{\substack{\uparrow \\ \text{volatility} \\ \text{of stock}}}{\sigma_S} \cdot |\Omega|$$

Careful: In our model σ_S is always constant.

$\Omega(s,t)$ is a **true** function of (s,t)

$\Rightarrow$ $\sigma_{opt}(s,t)$ is a function of (s,t)

e.g., European call

$$v_c(s,t) = \underbrace{s e^{-\delta(T-t)} \cdot N(d_1(s,t))}_{\Delta_c(s,t)} - Ke^{-r(T-t)} \cdot N(d_2(s,t))$$

$$\Rightarrow \Omega_c(s,t) = \frac{s\Delta_c(s,t)}{s\Delta_c(s,t) - Ke^{-r(T-t)}\cdot N(d_2(s,t))} \geq 1$$

$\Rightarrow$ $\boxed{\sigma_C \geq \sigma_S}$

e.g.) European put

$$v_p(s,t) = Ke^{-r(T-t)} \cdot N(-d_2(s,t)) - se^{-\delta(T-t)} \cdot N(-d_1(s,t))$$

$$\Delta_p(s,t) = -e^{-\delta(T-t)} N(-d_1(s,t))$$

$$\Rightarrow \Omega_p(s,t) = \frac{-e^{-\delta(T-t)} N(-d_1(s,t)) \cdot s}{Ke^{-r(T-t)} N(-d_2(s,t)) - se^{-\delta(T-t)} \cdot N(-d_1(s,t))}$$

< 0

20. Assume the Black-Scholes framework. Consider a stock and a European call option and a European put option on the stock. The curr nt stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4. $\Omega_A(S(0),0) = 5$ $\Delta_B(S(0),0) = 3.4$

Calculate the current put-option elasticity. $\Omega_P(S(0),0) = ?$

$S(0) = 45$

$v_C(S(0),0) = 4.45$

$v_P(S(0),0) = 1.90$

! (A) −0.55

(B) −1.15

(C) −8.64

(D) −13.03

(E) −27.24

21-24. DELETED

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of $100.

The chooser option price is $20 at time $t = 0$.

The stock price is $95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of $100.

You are given:

(i) The risk-free interest rate is 0.

(ii) $C(1) = \$4$.

Determine $C(3)$.

(A) $ 9

(B) $11

(C) $13

(D) $15

(E) $17

3.

* <u>Investor A</u>: $v_A(s,t) = 2\cdot v_c(s,t) + v_p(s,t)$

$$\Delta_A(s,t) = 2\cdot\Delta_c(s,t) + \Delta_p(s,t)$$

By <u>def'n</u>: $\Omega_A(s,t) = \dfrac{\Delta_A(s,t)\cdot s}{v_A(s,t)}$

$\Rightarrow$ At time 0:

$$5 = \frac{(2\Delta_c(S(0)=45,0) + \Delta_p(S(0)=45,0))\cdot 45}{\underbrace{2\cdot 4.45 + 1.90}_{=10.80}} \quad /:5$$

$$10.80 = 9(2\Delta_c(45,0) + \Delta_p(45,0)) \quad /:9$$

$$2\Delta_c(45,0) + \Delta_p(45,0) = 1.2 \qquad (\mathrm{I})$$

* <u>Investor B</u>: $v_B(s,t) = 2\cdot v_c(s,t) - 3v_p(s,t)$

$$\Rightarrow \Delta_B(s,t) = 2\cdot\Delta_c(s,t) - 3\cdot\Delta_p(s,t)$$

$\Rightarrow$ At time 0:

$$3.4 = 2\cdot\Delta_c(45,0) - 3\Delta_p(45,0) \qquad (\mathrm{II})$$

$\Rightarrow$ (I) − (II) : $4\Delta_p(45,0) = -2.2$

$$\Rightarrow \Delta_p(45,0) = -0.55$$

$$\Rightarrow \Omega_p(45,0) = \frac{\Delta_p(45,0)\cdot 45}{1.9} = -\frac{0.55\cdot 45}{1.9}$$

$$= -13.03 \Rightarrow (\mathrm{D}) \blacksquare$$

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$T=1$

41. Assume the Black-Scholes framework. Consider a 1-year European contingent claim on a stock.

You are given:

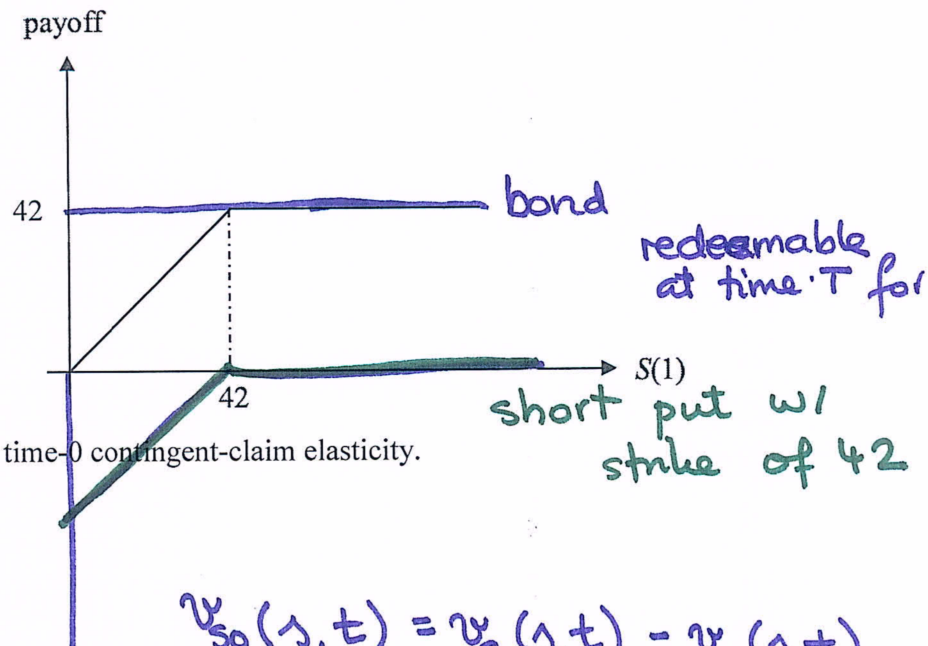
(i) The time-0 stock price is 45. $S(0)=45$

(ii) The stock's volatility is 25%. $\sigma = 0.25$

(iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%. $\delta = 0.03$

(iv) The continuously compounded risk-free interest rate is 7%. $r = 0.07$

(v) The time-1 payoff of the contingent claim is as follows:



redeamable at time $\cdot T$ for 42

Calculate the time-0 contingent-claim elasticity.

(A) 0.24

(B) 0.29

(C) 0.34

(D) 0.39

(E) 0.44

$$v_{SO}(s,t) = v_B(s,t) - v_P(s,t)$$

$$= 42e^{-r(T-t)} - v_P(s,t)$$

$$\boxed{\Omega_{SO}(S(0)=45, 0) = ?}$$

By def'n: $$\Omega_{SO}(s,t) = \frac{\Delta_{SO}(s,t)\cdot s}{v_{SO}(s,t)}$$

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$$* \; v_{SO}(s,t) = 42e^{-r(T-t)} - v_p(s,t)$$

$$= 42e^{-r(T-t)} - \left(42e^{-r(T-t)} \cdot N(-d_2(s,t)) - se^{-\delta(T-t)} \cdot N(-d_1(s,t))\right)$$

B·S model

$$= 42e^{-r(T-t)} \cdot N(d_2(s,t)) + se^{-\delta(T-t)} \cdot N(-d_1(s,t))$$

$\Rightarrow$ At time 0:

$$d_1(45,0) = \frac{1}{0.25\sqrt{1}}\left[\ln\left(\frac{45}{42}\right) + \left(0.07 - 0.03 + \frac{(0.25)^2}{2}\right)\cdot 1\right]$$

$$= 0.56$$

$$d_2(45,0) = 0.56 - 0.25 = 0.31$$

$$\Rightarrow N(-d_1(45,0)) = 1 - N(0.56) = 1 - 0.7123 = 0.2877$$

$$N(d_2(45,0)) = 0.6217$$

$$\Rightarrow v_{SO}(45,0) = 42e^{-0.07} \cdot 0.6217 + 45e^{-0.03} \cdot 0.2877$$

$$= 36.80 \approx 36.90$$

$$* \; \Delta_{SO}(s,t) = \overbrace{\Delta_B(s,t)}^{=0} - \Delta_p(s,t) = e^{-\delta(T-t)} \cdot N(-d_1(s,t))$$

$\Rightarrow$ At time 0:

$$\Delta_{SO}(45,0) = e^{-0.03} \cdot 0.2877$$

$$\Rightarrow \Omega_{SO}(45,0) = \frac{45 \cdot e^{-0.03} \cdot 0.2877}{36.9} \approx 0.34 \Rightarrow (C) \; \blacksquare$$