Focus on the Delta.

value f'tion: $v(s,t,r,\sigma)$

Def'n. The Delta $\Delta(s,t) := \frac{\partial}{\partial s} v(s,t)$

Example. Outright Purchase of a Non-Dividend-Paying Stock.

$v(s,t) = s$

s stands for the time-t stock price

$\Rightarrow \Delta(s,t) = 1$

Example. European Call

$$v_c(s,t) = s \cdot N(d_1(s,t)) - Ke^{-r(T-t)} N(d_2(s,t))$$

($s \cdot N(d_1(s,t))$: Δ; $Ke^{-r(T-t)} N(d_2(s,t))$: B)

w/

$$d_1 = \frac{1}{\sigma\sqrt{T-t}} \left[\ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right]$$

and $d_2 = d_1 - \sigma\sqrt{T-t}$

By def'n: $\Delta_c(s,t) = \frac{\partial}{\partial s} v_c(s,t)$

After the chain rule and product rule

$$\Delta_c(s,t) = N(d_1(s,t)) > 0$$

The positivity makes sense since the call is long w.r.t. the underlying.