M358K: October 19^{th}, 2020. $\boxed{\text{PART II}}$

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 8

Hypothesis testing: The normal case.

Problem 8.1. *Source: Ramachandran, Tsokos.*
The management of the local health club claims that its members lose on average 15 pounds or more within the first three months of their membership. A consumer agency took a simple random sample of 45 members and found the sample average of 13.8 in pounds lost. Assume that we model the weight loss as normal with an **unknown** mean μ and the **known** standard deviation of 4.2 pounds. What is the $p-$value corresponding to the gathered data? What would your decision be at the 0.05 significance level?

→: μ...population mean; $\sigma = 4.2$.

$H_0: \mu = 15 = \mu_0$ vs. $H_a: \mu < 15 = \mu_0$

SRS: $n = 45$; $\bar{x} = 13.8$.

Our test statistic, i.e., our z-score is:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{13.8 - 15}{4.2/\sqrt{45}} = -1.9166$$

$$\mathbb{P}[Z < z] = \Phi(-1.9166) = \text{pnorm}(-1.9166) = 0.02764$$

If the significance level α is 0.05, we REJECT THE NULL. ∎

Problem 8.2. *Source: Ramachandran, Tsokos.*
It is claimed that sports-car owners drive on the average 20,000 miles per year. A consumer firm believes that the mean annual mileage is actually lower. To check, the consumer firm decided to test this hypothesis.

The modeling assumptions are that the annual mileage is normally distributed with an unknown mean μ and with the standard deviation of 1200.

The consumer firm obtained information from 36 randomly selected sports-car owners that resulted in a sample average of 19,530 miles. What is the decision of this hypothesis test at the signficance level of 0.01?

→: μ... population mean ; $\sigma = 1200$

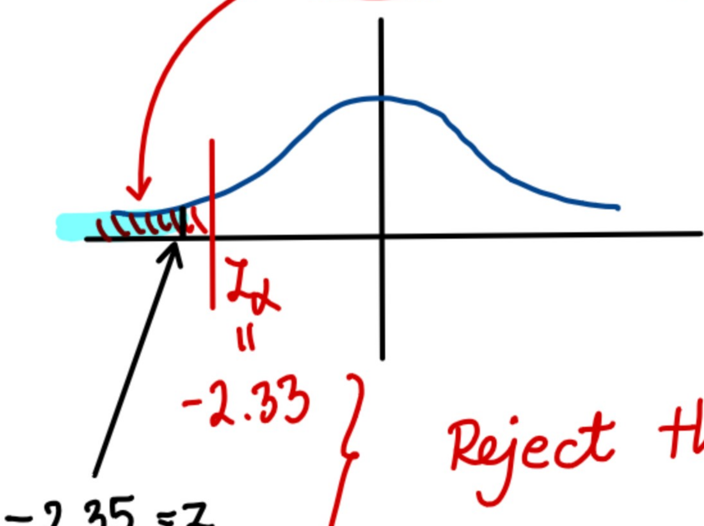
$H_0: \mu = \mu_0 = 20{,}000$ vs. $H_a: \mu < \mu_0 = 20{,}000$

SRS: $n = 36$; $\bar{x} = 19{,}530$

Our observed value of the test statistic is:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{19{,}530 - 20{,}000}{1200/\sqrt{36}} = \dots = -2.35$$

With $\alpha = 0.01$, the rejection region, in std units:



Reject the null! ∎