Two-stock Portfolio.

(X) and (Y) stand for the returns of the stocks.

$$\text{Price}_{END} - \text{Price}_{BEG}$$

$$\text{Price}_{BEG}$$

α ... the proportion of your wealth invested in stock X

$R = \alpha X + (1-\alpha) Y$ ... the total return of such a portfolio.

Optimization Problem.

Minimize $Var[R]$ across all α

$$Var[R] \longrightarrow \min$$

$$Var[\alpha X + (1-\alpha) Y] \longrightarrow \min$$

$$\alpha^2 Var[X] + (1-\alpha)^2 Var[Y] + 2\alpha(1-\alpha) Cov[X,Y] \longrightarrow \min$$

$$\alpha^2 \sigma_X^2 + (1-\alpha)^2 \cdot \sigma_Y^2 + 2\alpha(1-\alpha)\sigma_{XY} \longrightarrow \min$$

$$2\alpha\sigma_X^2 + 2(-1)(1-\alpha)\cdot\sigma_Y^2 + 2(1-2\alpha)\sigma_{XY} = 0$$

$$\alpha\sigma_X^2 + (\alpha-1)\sigma_Y^2 + (1-2\alpha)\sigma_{XY} = 0$$

$$\alpha(\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}) = \sigma_Y^2 - \sigma_{XY}$$

$$\boxed{\alpha^* = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}}$$

Multivariate Data.

Obs	X_1	X_2	....	X_p	Y
1	x_{11}	x_{12}	....	x_{1p}	
2					
⋮					
n	x_{n1}	x_{n2}	...	x_{np}	