
UNIVERSITY OF TEXAS AT AUSTINQuiz #3

SLLN. Monte Carlo.

Problem 3.1. (10 points) Let $\{Y_n, n \in \mathbb{N}\}$ be a sequence of independent, identically distributed random variables. Assume that $Y_1 = e^X$ where X is a standard normal random variable. Use the Strong Law of Large Numbers to find the following limit

$$\lim_{n \rightarrow \infty} \left(\prod_{i=1}^n Y_i \right)^{1/n} = \lim_{n \rightarrow \infty} (Y_1 \cdot Y_2 \cdots Y_n)^{1/n}.$$

Hint: Note that for every n , $Y_n = e^{X_n}$ where $\{X_n, n \in \mathbb{N}\}$ is a sequence of independent identically distributed standard normal random variables. Then, it helps to modify the product in the limit above and use the continuity of the exponential function.

Problem 3.2. (5 points) You use *Monte Carlo* to simulate values from a normal distribution with mean 0 and variance 4. Your plan is to use 10000 simulations. What is the variance of the *Monte Carlo* simulations?