M339J: February 22^{nd}, 2023.

Problem 5.5. An insurance policy on a ground-up loss X has:

- no deductible; $d=0$
- a coinsurance of 50%; and
- a maximum policy payment per loss of 5000

Let X be modeled using a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 10000$. What is the expected payment per loss for the insurer?

$\longrightarrow$: $X \sim Pareto(\alpha = 2, \theta = 10000)$

By our thm,

$$\mathbb{E}[Y^L] = \alpha(\mathbb{E}[X \wedge u] - \mathbb{E}[X \wedge d])$$

In our problem, $d=0$, and so

$$\mathbb{E}[Y^L] = \alpha \cdot \mathbb{E}[X \wedge u]$$

Coinsurance

Q: How much is u?

In general: maximum policy pmt $= \alpha \cdot (u-d)$

In this problem $\alpha \cdot u = 5000$

$0.5 \cdot u = 5000$

$\boxed{u = 10000}$

$$\mathbb{E}[Y^L] = 0.5 \cdot \boxed{\mathbb{E}[X \wedge 10000]}$$

$$\mathbb{E}[X \wedge 10000] = \frac{10000}{2-1}\left[1 - \left(\frac{10000}{10K + 10000}\right)^{2-1}\right] =$$

10K

$$= 10000\left(1 - \frac{1}{2}\right) = 5000$$

$$\mathbb{E}[Y^L] = 0.5(5000) = 2500$$

□

Problem 5.6. *Source: Sample STAM Problem #279.*
Loss amounts have the distribution function

$$\boxed{F_X(x) = \begin{cases} \left(\frac{x}{100}\right)^2, & 0 \leq x \leq 100 \\ 1, & x > 100 \end{cases}}$$

$d=20$ ✓

An insurance pays 80% of the amount of the loss in excess of an ordinary deductible of 20, subject to a maximum payment of 60 per loss. Calculate the conditional expected claim payment, given that a payment has been made, i.e., the expectation of the per payment random variable.

$\rightarrow$: $\mathbb{E}[Y^P] = ?$

$$\mathbb{E}[Y^P] = \mathbb{E}[Y^L \mid X > d] = \frac{\boxed{\mathbb{E}[Y^L]}}{\boxed{S_X(d)}}$$

$$S_X(d) = 1 - \left(\frac{20}{100}\right)^2 = \boxed{0.96}$$

by our Thm

$$\mathbb{E}[Y^L] = \alpha\left(\mathbb{E}[X \wedge u] - \mathbb{E}[X \wedge d]\right)$$

$u = ?$

$$60 = 0.80(u - 20) \Rightarrow \boxed{u = 95} \checkmark$$

For a constant $c \in (0,100)$:

$$\mathbb{E}[X \wedge c] = \int_0^c S_X(x)\,dx = \int_0^c \left(1 - \frac{x^2}{100^2}\right)dx$$

$$= \left[x - \frac{1}{10^4}\cdot\frac{x^3}{3}\right]_0^c = c - \frac{c^3}{3\cdot 10^4}$$

$$\mathbb{E}[Y^L] = 0.8\left(95 - \frac{95^3}{3\cdot 10^4} - \left(20 - \frac{20^3}{3\cdot 10^4}\right)\right) = \underline{37.35}$$

$$\mathbb{E}[Y^P] = \frac{37.35}{0.96} = \underline{38.906} \quad \square$$

74. A primary insurance company has a 100,000 retention limit. The company purchases a catastrophe reinsurance treaty, which provides the following coverage

		Retain
Layer 1:	85% of 100,000 excess of 100,000	0.15
Layer 2:	90% of 100,000 excess of 200,000	0.10
Layer 3:	95% of 300,000 excess of 300,000	0.05

The primary insurance company experiences a catastrophe loss of $\boxed{450{,}000}$.

Calculate the total loss retained by the primary insurance company.

Prior to Layer 1

within Layer 1

$$\boxed{100000} + \boxed{0.15(100000)}$$

within Layer 2

$$+ \boxed{0.10(100000)}$$

within Layer 3

$$+ \boxed{0.05(450000 - 300000)}$$

$$= \underline{132{,}500}$$

□

(A) 100,000

(B) 112,500

(C) 125,000

(D) 132,500

(E) 150,000

$X_1 \sim \text{Pareto}(\alpha_1 = 2, \theta_1 = 3000)$

50. In Year 1 a risk has a Pareto distribution with $\alpha = 2$ and $\theta = 3000$. In Year 2 losses inflate by 20%.

$X_2 \sim \text{Pareto}(\alpha_2 = 2, \theta_2 = 3600)$

An insurance on the risk has a deductible of 600 in each year. P_i, the premium in year i, equals 1.2 times the expected claims.

$d = 600$: $P_i = 1.2\,\mathbb{E}[(X_i - d)_+]$ $i = 1,2$

d_R

The risk is reinsured with a deductible that stays the same in each year. R_i, the reinsurance premium in year i, equals 1.1 times the expected reinsured claims.

$\tilde{d} = d_R + 600$ $R_i = 1.1\,\mathbb{E}[(X_i - \tilde{d})_+]$ $i = 1,2$.

$\boxed{\dfrac{R_1}{P_1} = 0.55}$ ✓

Calculate $\boxed{\dfrac{R_2}{P_2}}$.

(A) 0.46

(B) 0.52

(C) 0.55

(D) 0.58

(E) 0.66

$X \sim \text{Pareto}(\alpha, \theta)$

$d > 0$

$$\mathbb{E}[(X-d)_+] = \mathbb{E}[X] - \mathbb{E}[X \wedge d]$$

$$= \frac{\theta}{\cancel{\alpha - 1}} - \frac{\theta}{\alpha - 1}\left(\cancel{1} - \left(\frac{\theta}{d+\theta}\right)^{\alpha - 1}\right)$$

$$= \frac{\theta}{\alpha - 1} \cdot \left(\frac{\theta}{d + \theta}\right)^{\alpha - 1}$$

$i = 1,2$ $\dfrac{R_i}{R_i} = ?$