M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
Practice Problems for In-Term Exam 2
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Notes: This is a closed book and closed notes exam.
Time: 50 minutes

TRUE/FALSE

1 (2)	TRUE	FALSE
2 (2)	TRUE	FALSE
3 (2)	TRUE	FALSE
4 (2)	TRUE	FALSE
5 (2)	TRUE	FALSE

MULTIPLE CHOICE

1 (5)	a	b	c	d	e
2 (5)	a	b	c	d	e
3 (5)	a	b	c	d	e
4 (5)	a	b	c	d	e
5 (5)	a	b	c	d	e
6 (5)	a	b	c	d	e

FOR GRADER'S USE ONLY:

T/F	1.	2.	M.C.	Σ

2.1. **TRUE/FALSE QUESTIONS.** *Please note your answers on the front page.*

2.2. **FREE-RESPONSE PROBLEMS.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 2.1. (10 points) Assume that $Y_1 = e^X$ where X is a standard normal random variable.

(i) (2 points) What is the probability that Y_1 exceeds 5?
(ii) $(3+5$ points) Find the mean and the variance of Y_1.

Hint: It helps if you use the expression for the moment generating function of a standard normal random variable.

Problem 2.2. (10 points) Let the stock prices be modeled using the lognormal distribution. The mean stock price at time-1 equals 120 and the median stock price 115. What is the probability that the time-1 stock price exceeds 100?

Problem 2.3. (5 points) Assume the Black-Scholes model. The initial price of a continuous-dividend-paying stock is \$100. Its dividend yield is 0.03 and its volatility is 0.15. According to your model, the mean rate of return is 0.08.

The continuously compounded risk-free interest rate is 0.04.

Calculate the probability that the realized return for the time period $[0, 2]$ exceeds 0.06.

Problem 2.4. (5 points) Assume the Black-Scholes model. Consider a non-dividend-paying stock with the intial stock price of \$100 and volatility equal to 0.30. According to your model, the stock's mean rate of return is 0.10. Find

$$\mathbb{E}[S(1)\mathbb{I}_{[S(1)\geq 105]}].$$

2.3. **MULTIPLE CHOICE QUESTIONS.**

Problem 2.5. (5 points) The current price of a continuous-dividend-paying stock is given to be \$92. The stock's volatility is 0.35 and its dividend yield is 0.02.

The continuously compounded risk-free interest rate is 0.05.

Consider a \$90-strike European call option on the above stock with exercise date in a quarter-year.

What is the Black-Scholes price of this call option?

(a) 5.05
(b) 7.66
(c) 7.71
(d) 7.89
(e) None of the above.

Problem 2.6. (5 pts) Assume the Black-Scholes framework. Let the current price of a non-dividend-paying stock be equal to $S(0) = 95$ and let its volatility be equal to 0.35. Consider a European call on that stock with strike 100 and exercise date in 9 months. Let the risk-free continuously compounded interest rate be 6% per annum.

Denote the price of the call by $V_C(0)$. Then,

(a) $V_C(0) < \$5.20$
(b) $\$5.20 \leq V_C(0) < \$7.69$
(c) $\$7.69 \leq V_C(0) < \$9.04$
(d) $9.04 \leq V_C(0) < \$11.25$
(e) None of the above.

Problem 2.7. Assume the Black-Scholes setting. Let $S(0) = \$63.75, \sigma = 0.20, r = 0.055$. The stock pays no dividend and the option expires in 50 days (simplify the number of days in a year to 360).

What is the price of a \$60-strike European put?

(a) 0.66
(b) 0.55
(c) 0.44
(d) 0.37
(e) None of the above.

Problem 2.8. Assume the Black-Scholes setting. Assume $S(0) = \$28.50, \sigma = 0.32, r = 0.04$. The stock pays a 1.0% continuous dividend and the option expires in 110 days (simplify the number of days in a year to 360).
What is the price of a \$30-strike put?

(a) 2.75
(b) 2.10
(c) 1.80
(d) 1.20
(e) None of the above.

Problem 2.9. (5 points) Let the current price of a continuous-dividend-paying stock be denoted by $S(0)$. We model the time$-T$ stock price as lognormal. The mean rate of return on the stock is 0.10, its dividend yield is 0.01, and its volatility is 0.20. The continuously compounded risk-free interest rate is 0.03. You invest in one share of stock at time-0. and simultaneously deposit an amount $\varphi S(0)$ in a savings account. Assume continuous and immediate reinvestment of all dividends in the same stock. What should the proportion φ be so that the VaR at the level 0.05 of your total wealth at time-1 equals today's stock price $S(0)$?

(a) $\varphi = 0.0573$
(b) $\varphi = 0.1966$
(c) $\varphi = 0.2139$
(d) $\varphi = 0.5$

(e) None of the above.

Problem 2.10. (5 points) Assume that the stock price of a certain non-dividend-paying stock is modeled using the lognormal distribution, i.e., the Black-Scholes framework.

The time-0 delta of an at-the-money, time$-T$ European call option is 0.5557. What is the time-0 delta of an otherwise identical call option with exercise date $4T$?

(a) 0.3011
(b) 0.4145
(c) 0.5255
(d) 0.6103
(e) None of the above.

Problem 2.11. (5 points) Assume the Black-Scholes framework. The current price of a certain stock is \$50 per share. Its dividend yield is 0.04 and its volatility is 0.14.

The continuously compounded risk-free interest rate is 0.02.

What is the current delta of a European, \$43.75-strike, six-year put on the above stock?

(a) -0.13
(b) -0.23
(c) -0.33
(d) -0.45
(e) None of the above.

Problem 2.12. (5 pts) Let the stochastic process $S = \{S(t); t \geq 0\}$ denote the stock price. The stock's rate of appreciation is 10% while its volatility is 0.30 Then,

(a) $Var[\ln(S(t))] = 0.3t$
(b) $Var[\ln(S(t))] = 0.09t^2$
(c) $Var[\ln(S(t))] = 0.09t$
(d) $Var[\ln(S(t))] = 0.09$
(e) None of the above.