UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 14

Option volatility.

Please, provide your **complete solution** to the following problem. Final answers without shown reasoning will get zero points.

Problem 14.1. (15 points) Assume the Black-Scholes framework for the evolution of a stock price. The stock pays no dividends. Consider a one-year European call on this stock. You are given the following:

- the call's delta is 0.6591,
- under the risk-neutral probability measure, the probability that the option is in-the-money at expiration is 0.3409.

What is the volatility of this call option?

Solution: The volatility of the call option is

$$\sigma_C = \Omega_c \sigma.$$

We are given that

$$N(d_1) = 0.6591 \quad \text{and} \quad N(d_2) = 0.3409.$$

So, using the standard normal table, we get

$$d_1 = -d_2 = 0.41$$

Hence,

$$\sigma = d_1 - d_2 = 0.82.$$

Next, we need to calculate the elasticity of the call option. So,

$$\Omega_C = \frac{S(0)\Delta_C}{V_C(0)} = \frac{S(0)N(d_1)}{S(0)N(d_1) - Ke^{-r}N(d_2)} = \frac{1}{1 - \frac{Ke^{-r}}{S(0)} \times \frac{N(d_2)}{N(d_1)}}$$

Reusing the given value of the delta, we get

$$d_1 = 0.41 = \frac{1}{0.82}\left[\ln\left(\frac{S(0)}{Ke^{-r}}\right) + \frac{(0.82)^2}{2}\right] \quad \Rightarrow \quad \ln\left(\frac{S(0)}{Ke^{-r}}\right) = 0.$$

So,

$$\Omega_C = \frac{1}{1 - \left(\frac{0.3409}{0.6591}\right)} = 2.07134.$$

Finally, $\sigma_C = 2.07134(0.82) = 1.6985$.