University of Texas at Austin

HW Assignment 7

Contents

7.1. **Put-call parity.** Provide your **final solution only** to the following problem(s).

Problem 7.1. (5 points) A company forecasts to pay dividends of $\$0.90, \$1.20$ and $\$1.45$ in $3, 6$ and 9 months from now, respectively. Given that the interest rate is $r = 5.5\%$, how much dollar impact will dividends have on prices of 9−month options? More precisely, what is the present value of the projected dividend payments?

(a) \$3.45
(b) \$3.90
(c) \$4.22
(d) \$4.50
(e) None of the above.

Solution: (a)
The present value of the discrete dividends paid over the next 9 months per the above schedule is

$$PV_{0,3/4}(Div) = e^{-0.055\cdot 0.25}0.90 + e^{-0.055\cdot 0.5}1.20 + e^{-0.055\cdot 0.75}1.45 = 3.45.$$

Problem 7.2. (5 points) A certain common stock is priced at \$42.00 per share. Assume that the continuously compounded interest rate is $r = 10.00\%$ per annum. Consider a \$50−strike European call, maturing in 3 years which currently sells for \$10.80. What is the price of the corresponding 3−year, \$50−strike European put option?

(a) \$5.20
(b) \$5.69
(c) \$5.04
(d) \$5.84
(e) None of the above.

Solution: (d)
Due to put-call parity, we must have

$$\begin{aligned} V_P(0) &= V_C(0) + e^{-rT}K - S_0 + PV_{0,T}(Div) \\ &= 10.80 + e^{-0.30} \cdot 50 - 42.00 \approx 5.84. \end{aligned}$$

Problem 7.3. (5 points) A certain common stock is priced at \$99.00 per share and pays a continuous dividend yield of 2% per annum. Consider a \$100−strike European call and put, maturing in 9 months which currently sell for \$11.71 and \$5.31. Let the continuously compounded risk-free interest rate be denoted by r. Then,

(a) $0 \leq r < 0.05$
(b) $0.05 \leq r < 0.10$

(c) $0.10 \leq r < 0.15$
(d) $0.15 \leq r < 0.20$
(e) None of the above.

Solution: (c)
By the put-call parity, in our usual notation:

$$V_C(0) = V_P(0) + e^{-\delta T}S(0) - e^{-rT}K.$$

So,

$$r = -\frac{1}{T}\ln\left[\frac{1}{K}(V_P(0) + e^{-\delta T}S(0) - V_C(0))\right] = -\frac{1}{0.75}\ln\left[\frac{1}{100}(5.31 + e^{-0.02\cdot 0.75}\cdot 99 - 11.71)\right] \approx 0.124.$$

Problem 7.4. (5 points)
The initial price of a non-dividend-paying stock is \$55 per share. A 6−month, at-the-money call option is trading for \$1.89. Let the interest rate be $r = 0.065$. Find the price of the European put with the same strike, expiration and the underlying asset.

(a) \$0.05
(b) \$0.13
(c) \$0.56
(d) \$0.88
(e) None of the above

Solution: (b)
Using put-call parity, we get

$$V_P(0) = V_C(0) + Ke^{-rT} - F_{0,T}^P(S) = 1.89 + 55(e^{-0.065\times 0.5} - 1) = 0.1312.$$

Problem 7.5. (5 points) A stock currently sells for \$32.00. A 6-month European call option with a strike of \$30.00 has a premium of \$4.29, and the otherwise identical put has a premium of \$2.64. Assume a 4% continuously compounded, risk-free rate. What is the present value of the dividends payable over the next 6 months?

(a) \$0.05
(b) \$0.13
(c) \$0.52
(d) \$0.94
(e) None of the above

Solution: (d)

This problem requires the application of put-call parity. We have:

$$S(0) - V_C(0) + V_P(0) - 30e^{-rT} = PV\,(dividends) \Rightarrow PV\,(dividends) = 32 - 4.29 + 2.64 - 29.406 = \$0.944.$$

Problem 7.6. (5 points) *Source: Problem #2 from the Sample IFM (Derivatives: Introductory) questions.*
You are given the following information:

(1) The current price to buy one share of XYZ stock is 500.
(2) The stock does not pay dividends.
(3) The risk-free interest rate, compounded continuously, is 6%.
(4) A European call option on one share of XYZ stock with a strike price of K that expires in one year costs \$66.59.
(5) A European put option on one share of XYZ stock with a strike price of K that expires in one year costs \$18.64.

Determine the strike price K.

(a) \$449
(b) \$452
(c) \$480
(d) \$559
(e) None of the above.

Solution: (c)

Problem 7.7. (5 points) Consider a European call option and a European put option on a non-dividend-paying stock. Assume:

(1) The current price of the stock is \$55.
(2) The call option currently sells for \$0.15 more than the put option.
(3) Both options expire in 4 years.
(4) Both options have a strike price of \$70.

Calculate the continuously compounded risk-free interest rate r.

(a) 0.044
(b) 0.052
(c) 0.06
(d) 0.065
(e) None of the above.

Solution: (c)
In our usual notation,

$$S(0) = 55, \quad V_C(0) - V_P(0) = 0.15, \quad T = 4, \quad K = 70.$$

We employed a *no-arbitrage* argument to get the **put-call parity**:

$$V_C(0) - V_P(0) = S(0) - K^{-rT} \quad \Rightarrow \quad r = \frac{1}{T}\ln\left(\frac{K}{S(0) - V_C(0) + V_P(0)}\right).$$

Using in the data provided, we get $r = 0.06$.

7.2. **Chooser options (optional material).** Provide your **final answer** only for the following problems.

Problem 7.8. (2 points) The initial price of a chooser option is greater than or equal to the price of a regular European call on the same asset with the same strike and exercise date. *True or false?*

Solution: TRUE

Problem 7.9. (5 points) Consider a chooser option on a stock S whose current price is \$100 per share. Assume that we are using our usual notation, i.e., let

$$V_{CH}(0, t^*, T, K)$$

denote the time-0 price of a chooser option with choice date t^*, exercise date T and strike price K. Also, let $V_C(0, T, K)$ denote the time-0 price of a European call option with strike K and exercise date T. Likewise, let $V_P(0, T, K)$ denote the time-0 price of a European put option with strike K and exercise date T. Then, the following inequality holds:

(a) $V_{CH}(0, t^*, T, K) \leq V_P(0, T, K)$
(b) $V_{CH}(0, t^*, T, K) \leq V_C(0, T, K)$
(c) $\max(V_P(0, T, K), V_C(0, T, K)) \leq V_{CH}(0, t^*, T, K)$
(d) $V_{CH}(0, t^*, T, K) < \max(V_P(0, T, K), V_C(0, T, K))$
(e) None of the above

Solution: **(c)**

7.3. **Exchange options.** Provide your **complete solution** to the following problem:

Problem 7.10. (5 points) **The minimum option**

Let $\mathbf{S} = \{S(t), t \geq 0\}$ and $\mathbf{Q} = \{Q(t), t \geq 0\}$ denote the prices of two risky assets. The payoff of the *minimum option* is given by

$$V_{min}(T) = \min(S(T), Q((T)).$$

Propose a replicating portfolio consisting of prepaid forward contracts on **S** and/or **Q**, and exchange options on **S** and **Q**.

Solution:

$$\begin{aligned} V_{min}(T) = \min(S(T), Q((T)) &= S(T) + \min(0, Q(T) - S(T)) \\ &= S(T) - \max(S(T) - Q(T), 0). \end{aligned}$$

So, an example of a replicating portfolio is

$$\begin{cases} \text{a prepaid forward contract on } \mathbf{S}\text{, and} \\ \text{a } \textbf{short} \text{ exchange call with } \mathbf{S} \text{ as underlying and } \mathbf{Q} \text{ as the strike asset} \end{cases}$$

Other examples are

$$\begin{cases} \text{a prepaid forward contract on } \mathbf{S}\text{, and} \\ \text{a } \textbf{short} \text{ exchange put with } \mathbf{Q} \text{ as underlying and } \mathbf{S} \text{ as the strike asset} \end{cases}$$

$$\begin{cases} \text{a prepaid forward contract on } \mathbf{Q}\text{, and} \\ \text{a } \textbf{short} \text{ exchange call with } \mathbf{Q} \text{ as underlying and } \mathbf{S} \text{ as the strike asset} \end{cases}$$

$$\begin{cases} \text{a prepaid forward contract on } \mathbf{Q}\text{, and} \\ \text{a } \textbf{short} \text{ exchange put with } \mathbf{S} \text{ as underlying and } \mathbf{Q} \text{ as the strike asset} \end{cases}$$

Problem 7.11. (5 points) Let our market model include two continuous-dividend-paying stocks whose time$-t$ prices are denoted by $S(t)$ and $Q(t)$ for $t \geq 0$. The current stock prices are $S(0) = 160$ and $Q(0) = 80$. The dividend yield for the stock S is $\delta_S = 0.06$ and the dividend yield for the stock Q is $\delta_Q = 0.03$.

The price of an exchange option giving its bearer the right to forfeit one share of Q for one share of S in one year is given to be \$11.

Find the price of a maximum option on the above two assets with exercise date in a year. Remember that the payoff of the maximum option is $\max(S(1), Q(1))$.

Solution: As we showed in class

$$V_{\max}(0) = F_{0,1}^P(Q) + V_{EC}(0, S, Q) = Q(0)e^{-\delta_Q} + V_{EC}(0, S, Q) = 80e^{-0.03} + 11 = 88.64.$$

Provide your **final answer only** to the following problems:

Problem 7.12. (2 points)

Exchange options are options where the underlying asset is an exchange rate. *True or false?*

Solution: FALSE

Problem 7.13. (2 points) Consider two European exchange options both with exercise date T, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return. Assume that neither asset pays any dividends.

As usual $V_{EC}(0, X(0), Y(0))$ stands for the time-0 price of the exchange call with the underlying asset X and the strike asset Y.

Then, in our usual notation,

$$V_{EC}(0, \mathbf{S}, \mathbf{Q}) - V_{EC}(0, \mathbf{Q}, \mathbf{S}) = S(0) - Q(0). \tag{7.1}$$

Solution: TRUE
Generalized put-call parity.

Problem 7.14. (5 points) Assume that the continuously compounded interest rate equals 0.10.

Stock S has the current price of $S(0) = 70$ and does not pay dividends. Stock Q has the current price of $Q(0) = 65$ and it pays continuous dividends at the rate of 0.04.

An exchange option gives its holder the right to give up one share of stock Q for a share of stock S in exactly one year. The price of this option is \$11.50.

Another exchange option gives its holder the right to give up one share of stock S for a share of stock Q in exactly one year. Find the price of this option.

(a) About \$3.95
(b) About \$11.10
(c) About \$12.00
(d) About \$14.25
(e) None of the above.

Solution: (a)
By the generalized put-call parity, we get the price we are looking for should be

$$V_{EC}(Q(0), S(0), 0) = V_{EC}(S(0), Q(0), 0) + F^P_{0,T}(Q) - F^P_{0,T}(S) = 11.50 + 65e^{-0.04} - 70 = 3.95.$$

Problem 7.15. (2 points) Consider two exchange options, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return.
Then, the prepaid forward prices of the two assets are the same if and only if the two exchange options have the same price.

Solution: TRUE

Problem 7.16. (2 points) Consider two exchange options, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return. Assume neither of the two stocks pays dividends.
Then, the current spot prices of the two assets are the same if and only if the two exchange options have the same price.

Solution: TRUE

Problem 7.17. (2 pts) Consider two European exchange options both with exercise date T, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return.
On the other hand, consider the maximum option with the payoff

$$V_{max}(T) = \max(S(T), Q(T)),$$

and the minimum option with the payoff

$$V_{min}(T) = \min(S(T), Q(T)).$$

Then, in our usual notation,

$$V_{EC}(0, \mathbf{S}, \mathbf{Q}) + V_{EC}(0, \mathbf{Q}, \mathbf{S}) = V_{max}(0) + V_{min}(0).$$

Solution:
FALSE
If $S(T) \leq Q(T)$, the payoff of a long exchange option allowing you to give up a unit of Q and receive a unit of S is

$$V_{EC}(S(T), Q(T), T) = (S(T) - Q(T))_+ = 0,$$

i.e., the option goes unexercised. On the other hand, the payoff of a long exchange option allowing you to give up a unit of S and receive a unit of Q is

$$V_{EC}(Q(T), S(T), T) = (Q(T) - S(T))_+ = Q(T) - S(T).$$

So, the payoff of the portfolio whose price is on the left-hand side of (7.2) is simply $Q(T) - S(T)$.

The payoff of the portfolio whose initial cost is on the right-hand side of (7.2) is always $S(T) + Q(T)$.

So, it is impossible for the proposed equality in prices to always be true.