

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set 3Payoff. Profit.

Problem 3.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time-1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time-0 and liquidate your investment at time-1?

→: $\text{Profit} = \text{Payoff} - \underbrace{FV_{0,1}(\text{Init Cost})}_{40e^{0.04}}$

E\

$$\mathbb{E}[\text{Profit}] = \mathbb{E}[\text{Payoff}] - 40e^{0.04}$$

||

$$\mathbb{E}[S(1)]$$

$$45 \cdot \frac{1}{4} + 42 \cdot \frac{1}{2} + 38 \cdot \frac{1}{4} = 41.75$$

answer: $41.75 - 40e^{0.04} = 0.18$

□

Problem 3.2. To plant and harvest 20,000 bushels of corn, Farmer Jayne incurs total aggregate costs totaling \$33,000. The current spot price of corn is \$1.80 per bushel. What is the profit if the spot price is \$1.90 per bushel when she harvests and sells her corn?

- (a) About \$3,000 gain
- (b) About \$3,000 loss
- (c) About \$5,000 loss
- (d) About \$5,000 gain
- (e) None of the above

deterministic, valued @ time $\cdot T$ (harvest time)

→ :

$$\underline{20000(1.9)} - \underline{33000} = \underline{5000}$$

□

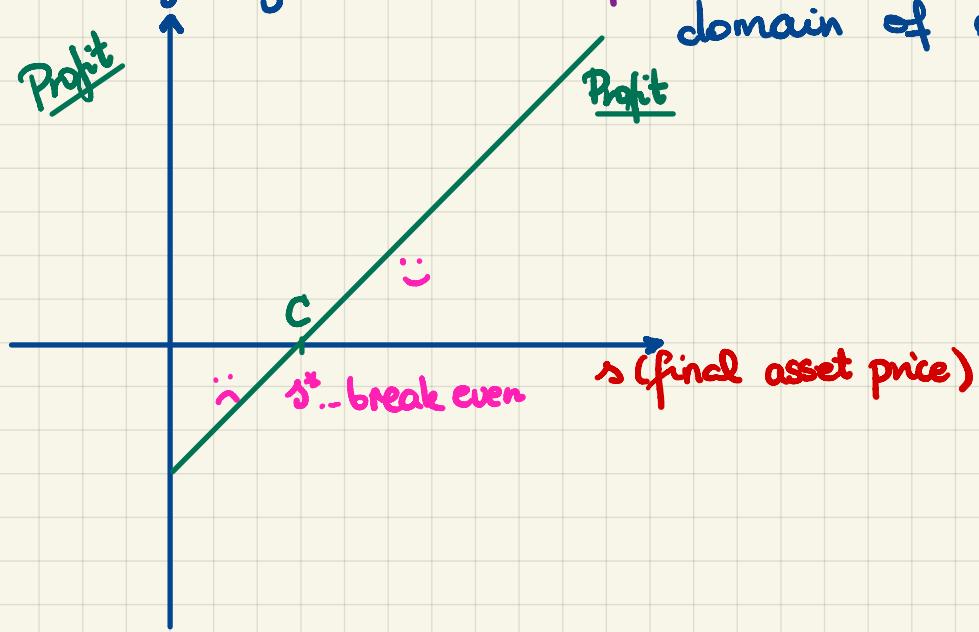
Hedging Motivation.

Example. [Producer of Goods]

- farmers producing corn, soy beans, apples
- factories producing "widgets"
- mining ore
- energy manufacture

C... deterministic total aggregate fixed and variable costs of production valued @ the time of sale of produced good, i.e., time T

If the producer sells their goods in the market, they get the market price. This is outside of their domain of influence.



Forward Contract

* A BINDING CONTRACT ON BOTH SIDES ! *

NO CASHFLOW

Handshake!

0

T

Delivery date

(when the cash is "exchanged" for the asset)

An agreement:

- the underlying asset
- the quantity (for us: 1unit)
- the type of settlement: physical or cash
- T... delivery date
- F... forward price

Initial Cost = 0



Payoff = Profit

Long Forward : Buy Forward

S(T)

1 unit of asset

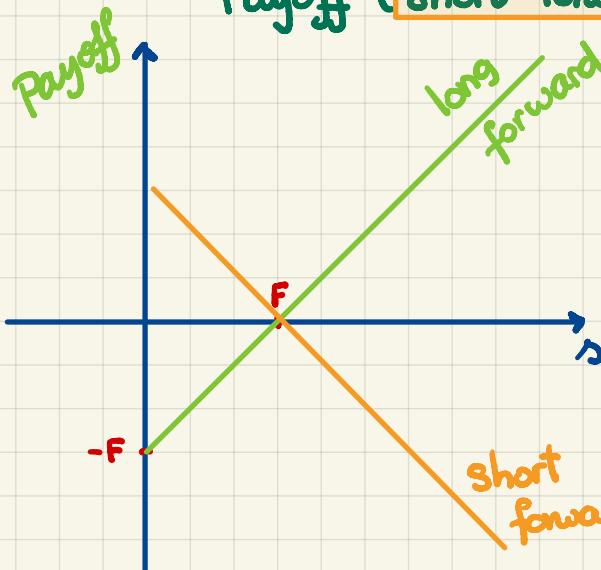
↑
Forward Price: F

Short Forward : Selling Forward

$$\text{Payoff (Long Forward)} = S(T) - F \quad \}$$

$$\text{Payoff (Short Forward)} = F - S(T) \quad \}$$

Payoff function: $v(s) = s - F$ ✓



Problem. [Sample SOA Problem]

Determine which of the following portfolios have the same cashflows as a short sale of a non-dividend-paying stock.

$$0 \quad T$$

Initial Cost: $-S(0)$

Payoff: $-S(T)$

\times (i) long forward and a long zero-coupon bond

Init. Cost: Price of Bond > 0

\times (ii) long forward and a short forward

Init. Cost: 0

(iii) long forward and a short zero-coupon bond

	<u>Init. Cost</u>	<u>Payoff</u>
Long Forward	0	$S(T) - F$
Short Bond	$-P$	$-Pe^{rT}$
Total	$-P$	$S(T) - F - Pe^{rT}$

