M339W: October 20th, 2021.

Delta-hedging.

Market Makers.

- immediacy
- inventory

$\Big\}$ $\Rightarrow$ exposure to risk $\Rightarrow$ hedge

Say, a market maker <u>writes</u> an option whose value f'tion is:

$$v(s,t)$$

At time 0, they wrote the option. So, they get $v(S(0),0)$.

At time t, the value of the market maker's position is

$$\boxed{-v(s,t)} \quad \checkmark$$

To (partially) hedge this exposure to risk, they construct a portfolio which has zero sensitivity to small perturbations in the stock price. Formally speaking, their goal is to create a

delta-neutral portfolio,

i.e., a portfolio for which $\boxed{\Delta_{Port}(s,t)=0}$.

Theoretically, we continuous rebalancing w/ no transaction costs it's possible.
Practically, continuous rebalancing is impossible and there are transaction costs.

In particular, @ time 0, we want to trade so that

$$\Delta_{Port}(S(0),0)=0.$$

The most straightforward strategy is to trade in the shares of the underlying asset.

At any time t, let $N(s,t)$ denote the required number of shares in the portfolio to maintain Δ-neutrality.

The total value of the portfolio is:

$$v_{Port}(s,t) = -v(s,t) + N(s,t)\cdot s$$

$$\frac{\partial}{\partial s}\Big| \qquad \Delta_{Port}(s,t) = -\Delta(s,t) + N(s,t) = 0$$

$\uparrow$ Δ-neutrality

$$\Rightarrow \boxed{N(s,t) = \Delta(s,t)}$$

Example. A market maker writes a call option @ time 0.

$\Rightarrow$ At any time t, the market maker's position is

$$-v_C(s,t)$$

$\Rightarrow$ They have to maintain $N(s,t) = \Delta_C(s,t)$ in the Δ-hedge.

$\Rightarrow$ In particular, @ time 0:

$$N(S(0),0) = e^{-\delta\cdot T}\cdot N(d_1(S(0),0)) > 0,$$

i.e., the marketmaker should long this much of a share.

Example. A market maker writes a put option @ time 0.

$\Rightarrow$ At any time t, the value of the unhedged portfolio is $-v_P(s,t)$

$\Rightarrow$ In the Δ-hedge, they have to maintain

$$N(s,t) = \Delta_P(s,t)$$

$\Rightarrow$ In particular, @ time 0:

$$N(s,t) = -e^{-\delta\cdot T}\cdot N(-d_1(S(0),0)) < 0,$$

i.e., the market maker should short this much of a share.

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i) The risk-free interest rate is constant.

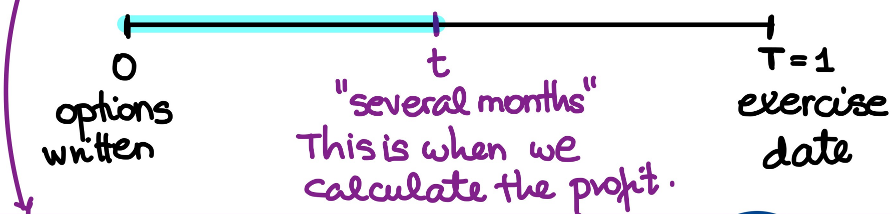
(ii) when the option was written — when the positions are closed out ✓

	Several months ago @ time·0	Now time·t
Stock price	$40.00	$50.00
Call option price	$ 8.88	$14.42
Put option price	$ 1.63	$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

Calculate her profit.

$\boxed{\text{Profit} = \text{Payoff} - FV(\text{Init.Cost})}$



(A) $11

(B) $24

(C) $126

(D) $217

(E) $240

$\boxed{\text{Profit}(@\,\text{time}\cdot t) = \text{Wealth}(@\,\text{time}\cdot t) - FV_{0,t}(\text{Init.Cost})}$

48. DELETED

49. You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).

(i) The period is 3 months.

(ii) The initial stock price is $100.

(iii) The stock's volatility is 30%.

(iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

(A) 114

(B) 115

(C) 116

(D) 117

(E) 118

- Initial cost: $-100\, v_c(S(0),0) + 100 \cdot \Delta_c(S(0),0) \cdot S(0)$

$$= 100(-8.88 + 0.794 \cdot 40) = 2{,}288.$$

- Wealth @ time t: $-100\, v_c(S(t),t) + 100 \cdot \boxed{\Delta_c(S(0),0)} \cdot S(t)$

$$= 100(-14.42 + 0.794 \cdot 50) = 2{,}528.$$

$$\boxed{\text{Profit}(@\ \text{time } t) = 2{,}528 - e^{rt}\, 2{,}288}$$

Use put-call parity:

At time 0:

$$v_c(S(0),0) - v_p(S(0),0) = \underbrace{F^P_{0,T}(S)}_{=S(0)} - Ke^{-r \cdot T}$$

$$8.88 - 1.63 = 40 - Ke^{-rT}$$

$$Ke^{-rT} = 40 - 7.25 = 32.75 \quad \checkmark$$

At time t:

$$v_c(S(t),t) - v_p(S(t),t) = \underbrace{F^P_{t,T}(S)}_{=S(t)} - Ke^{-r(T-t)}$$

$$14.42 - 0.26 = 50 - Ke^{-r(T-t)}$$

$$Ke^{-r(T-t)} = 50 - 14.16 = 35.84 \quad \checkmark\!\!\checkmark$$

$$\frac{\checkmark\!\!\checkmark}{\checkmark} \Rightarrow \frac{\cancel{K}e^{\cancel{-rT}+rt}}{\cancel{K}\cancel{e^{-rT}}} = \frac{35.84}{32.75}$$

$$\Rightarrow \boxed{e^{rt} = 1.09435}$$

$$\text{Profit} = 2{,}528 - (1.09435) \cdot 2{,}288 = 24.1245$$