The Collective Risk Model.

Let $\{X_j, j = 1, 2, \ldots\}$ be a sequence of independent, identically dist'd r.v.s.

Let N be an $\mathbb{N}_0$-valued r.v. independent from $\{X_j : j = 1, \ldots\}$.

Define : $\boxed{S = X_1 + X_2 + \cdots + X_N = \sum_{j=1}^{N} X_j}$

w/ the convention that $S = 0$ when $N = 0$.

Then, S represents aggregate losses.

Q: What's the distribution of S?

It is "convenient" to use pgf or mgf.

If X are $\mathbb{N}_0$-valued, then $\boxed{P_S(z) = P_N(P_X(z))}$

If X are continous, then $\boxed{M_S(z) = P_N(M_X(z))}$

Facts:

- $\mathbb{E}[S] = \mathbb{E}[N] \cdot \mathbb{E}[X]$ Wald's Identity
- $\mathrm{Var}[S] = \mathbb{E}[N] \cdot \mathrm{Var}[X] + \mathrm{Var}[N] \cdot (\mathbb{E}[X])^2$

8. The number of claims, N, made on an insurance portfolio follows the following distribution:

n	$\Pr(N=n)$
0	0.7
2	0.2
3	0.1

N… frequency

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2, respectively. X… severity

The number of claims and the benefit for each claim are independent.

Calculate the probability that aggregate (S) benefits will exceed expected (μ_S) benefits by more than 2 standard deviations (σ_S) $\rightarrow$:

(A) 0.02

(B) 0.05

(C) 0.07

(D) 0.09

(E) 0.12

$$\mathbb{P}[S > \mu_S + 2\sigma_S] = ?$$

- $\mu_S = \mathbb{E}[N] \cdot \mathbb{E}[X] = 1.4$
  - $\mathbb{E}[N] = 0(0.7) + 2(0.2) + 3(0.1) = 0.7$
  - $\mathbb{E}[X] = 0(0.8) + 10(0.2) = 2$
- $\sigma_S^2 = Var[S] = \mathbb{E}[N] \cdot Var[X] + Var[N](\mathbb{E}[X])^2$

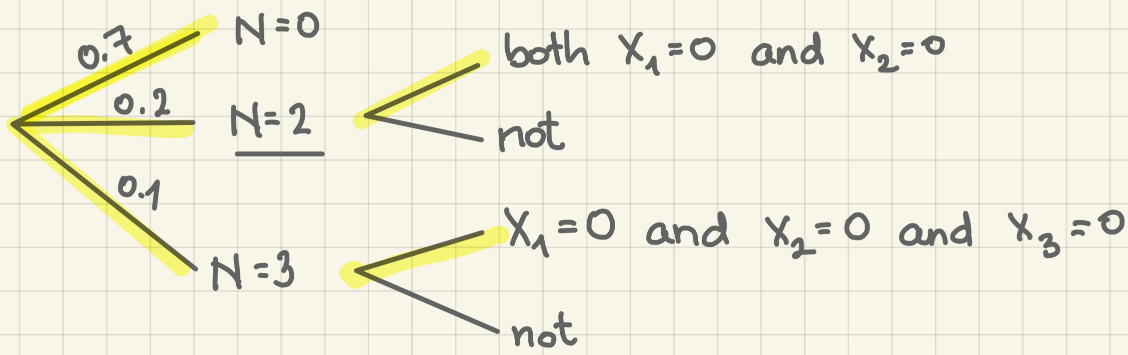
$Var[N] = \mathbb{E}[N^2] - (\mathbb{E}[N])^2$
$= 2^2 \cdot (0.2) + 3^2 \cdot (0.1) - (0.7)^2$
$= 1.7 - 0.49 = 1.21$

$Var[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$
$= 10^2(0.2) - 4 = 16$

$\sigma_S^2 = 0.7(16) + 1.21(2)^2 = \dots = 16.04$

$\sigma_S = \sqrt{16.04} = 4.005$

$$\mathbb{P}[S > 1.4 + 2\cdot(4.005) = 9.41] = \mathbb{P}[S \geq 10] = 1 - \boxed{\mathbb{P}[S=0]}$$



$$\mathbb{P}[S=0] = 0.7 + 0.2\cdot(0.8)(0.8) + 0.1\,(0.8)(0.8)(0.8) = \cdots = 0.8792$$

$\Rightarrow$ answer : $1 - 0.8792 = 0.1208$ $\square$

CLT.

$\{X_j,\ j = 1,2,\dots\}$ i.i.d. w/ $\mu_X = \mathbb{E}[X] < \infty$

and $\sigma_X^2 = \text{Var}[X] < \infty$.

$$\bar{X}_n := \frac{X_1 + X_2 + \cdots + X_n}{n}$$ sample mean

- $\mathbb{E}[\bar{X}_n] = \mu_X$
- $\text{Var}[\bar{X}_n] = \frac{1}{n}\text{Var}[X] = \frac{\sigma_X^2}{n} \Rightarrow SD[\bar{X}_n] = \frac{\sigma_X}{\sqrt{n}}$

$$\boxed{\frac{\bar{X}_n - \mu_X}{\frac{\sigma_X}{\sqrt{n}}}} \xrightarrow[n\to\infty]{\mathcal{D}} N(0,1)$$

Practically: $\bar{X}_n \approx N\left(\mu_X, \overset{sd}{\overset{\shortparallel}{\frac{\sigma_X}{\sqrt{n}}}}\right)$