

M339W: December 6th, 2021.

Inflation Indexing.

This is a property of (some) pensions (e.g., Social Security).

Denote by I_t the consumer price index (CPI) @ time t .

Denote by P_t the pension pmt @ time t .

Let the first pmt be @ time 0. That pmt equals P_0 .

Then, for the pmt @ time 1 .

$$\begin{aligned} P_1 &= \max(P_0, P_0 \left(\frac{I_1}{I_0} \right)) \\ \Rightarrow P_1 &= P_0 + \max(0, P_0 \left(\frac{I_1}{I_0} \right) - P_0) \\ &= P_0 + P_0 \underbrace{\max\left(\frac{I_1}{I_0} - 1, 0\right)}_{\text{brace}} \\ &= P_0 \left(\frac{I_1}{I_0} \right) + P_0 \underbrace{\max\left(1 - \frac{I_1}{I_0}, 0\right)}_{\text{brace}} \end{aligned}$$

Recursively, we set

$$\begin{aligned} P_t &= \max(P_{t-1}, P_0 \left(\frac{I_t}{I_0} \right)) \\ \Rightarrow P_t &= \max_{0 \leq s \leq t} \left(P_0 \left(\frac{I_s}{I_0} \right) \right) \\ \Rightarrow P_t &= P_0 \left(\frac{I_t}{I_0} \right) + \underbrace{\max_{0 \leq s \leq t} \left(P_0 \cdot \frac{I_s}{I_0} \right)}_{\text{brace}} - \underbrace{P_0 \left(\frac{I_t}{I_0} \right)}_{\text{brace}} \end{aligned}$$

Lookback Options.

S_t ... account value @ time t

T ... exercise date

$$m_T := \min_{0 \leq s \leq T} S_s$$

$$M_T := \max_{0 \leq s \leq T} S_s$$

Payoff

$$S_T - m_T$$

$$M_T - S_T$$

• Standard lookback call :

• Standard lookback put :

Say, K denotes a strike price.

• Extrema lookback call : $(M_T - K)_+$

• Extrema lookback put : $(K - m_T)_+$

- 40) Several lookback options are written on the same underlying index. They all expire in 3 years.

Let S_t denote the value at time t of the index on which the option is written.

The initial index price, S_0 , is 150.

The index price when the option expires, S_3 , is 200.

The maximum index price over the 3-year period is 210.

The minimum index price over the 3-year period is 120.

Calculate the sum of the payoffs for the following three lookback options:

- Standard lookback call
- Extrema lookback call with a strike price of 100
- Extrema lookback put with a strike price of 100

$$\begin{aligned}S_T - m_T &= 200 - 120 = 80 \\(M_T - K)_+ &= (210 - 100)_+ = 110 \\(K - m_T)_+ &= (100 - 120)_+ = 0\end{aligned}$$

- (A) 180
(B) 190
(C) 200
(D) 210
(E) 220

answer = 190

Shout Options.

The owner of the option has the right to select a date (**shout!**) during the right of the option. Then the price of the underlying on that date is used to calculate the payoff on the exercise date if that provides a higher payoff.

S^* ... the price of the underlying on the shout date

Payoff

Shout Call:

$$\max(S^* - K, S(T) - K, 0)$$

Shout Put:

$$\max(K - S^*, K - S(T), 0)$$

Purpose: Hedging against the policyholder resetting the pmt guarantee once.

If the reset takes place, then the payoff of the annuity equals

$$\max(K, S^*, S(T)) = K + \underbrace{\max(0, S^* - K, S(T) - K)}_{\text{payoff of a shout call}}$$

Chooser Options.

The owner of the option decides whether the option becomes a call or a put on the choice date.

(see Put-Call Parity)

Rainbow Options.

... have the payoff which depends on two or more risky assets,
e.g., $\max(S_T - K, Q_T - K, 0)$ for a rainbow call, or
 $\max(K - S_T, K - Q_T, 0)$ for a rainbow put.

Forward start options.

... a right to buy an option @ a later time w/ the strike dependent on the asset price @ that later time
(see Black-Scholes Pricing).