UNIVERSITY OF TEXAS AT AUSTIN

Lecture 1

On financial mathematics.

1.1. **Atoms and dynamics.** Before diving into the description of elementary ingredients and relationships in *financial mathematics*, let us turn to the structure of other (older?) disciplines.

Discipline	*What*	*How*	*Where*
Physics	particles	forces	physical space
Chemistry	atoms	chemical bonds	chemical space
Sociology	individuals	social bonds	society
Linear Algebra	vectors	linear maps	linear spaces
Music	notes	composition	time-frequency space
Biology	organism	co-evolution	environment
Traffic science	vehicles	routing	traffic networks
Economics	economic agents	rational decisions	markets

How does **financial mathematics** fit into this structure?

1.2. **Role and history of financial markets.** Plainly speaking, *financial markets* were originally designed to bring together **lenders** and **borrowers**. Lenders might come under the guise of traditional lenders you encountered in *theory of interest*, but stockholders in a company might also be understood as lenders – they are offering monetary funds in return for a share in a company (and the right to its potential profits). The entitities traded in a financial market are referred to as **financial assets**. One can read more about the particulars in Articles 19 and 20 of [1].

However, as we will see, financial markets evolved to also alleviate individual investor's financial risks stemming from changes in prices of goods and services. As the structure of our communities became more sophisticated, financial markets followed suit.

For more on the history and structure of financial markets go to

`http://en.wikipedia.org/wiki/Stock_market`

Note that – even though its aim is to spread financial risk among a number of individuals (policy holders) – *classical insurance* applies to types of risk not directly associated with movement of asset prices in financial markets. It focuses on specific types of financial risks (for example, property, casualty, health, dental, personal umbrella, …). We will revisit the parallels between classical insurance and financial instruments at a later time.

For now, the point of view of classical insurance will help us understand the difference between *diversifiable* and *non-diversifiable* risks in financial markets. Consider homeowner's insurance – an insurance policy which compensates the property owner for the reduction of the value of his/her property due to a specific set of circumstances stipulated in the insurance policy (e.g., fire, theft, hurricane, earthquake, …). The insurance company will address the problem of offsetting its possible liabilities in case of fire or theft in a different way that it would handle the liabilities due to a hurricane or earthquake.

One might guess that the reason is simply the amount of loss in either case. However, there is a more profound difference. While it is unlikely that a group of homeowners is going

to suffer a loss due to theft in the same insurance cycle and in the same area, a natural disaster creates simultaneous losses across a specific area. In the former case, the premiums accumulated from the policy holders who did not experience a loss during an insurance cycle will contribute to the benefit payments to the policy holders who did. In the latter case, this effect is not present, since the overwhelming majority of policy holder's suffered a loss. One might say that the insurance company was able to *diversify* the first (individual) type of risk by insurance a whole pool of people. This was not a possibility for the second (system-wide) type of risk.

Problem 1.1. Try to come up with your own example of diversifiable and non-diversifiable risks in classical insurance.

In finance and economics, the notion of diversifiable risk is associated with the financial risk attributable to a particular asset. One can alleviate that risk by investing in multiple (diverse) financial instruments at the same time. A set of investments made by a particular agent is referred to as a *(financial) portfolio*. The other type of risk is the one to which the entire market is exposed so that the diversification of an agent's portfolio would not reduce the risk-exposure. This type of risk is called non-diversifiable.

Problem 1.2. Try to come up with your own example of diversifiable and non-diversifiable risks in a financial market.

Frequently, though, the two arenas of protection against financial loss do overlap. One example are the so-called *catastrophe bonds*. Read more about them at

`http://en.wikipedia.org/wiki/Catastrophe_bond`

Problem 1.3. **Old SoA Sample FM(DM) Problem #25** Determine which of the following statements concerning risk sharing, in the context of financial risk management, is LEAST accurate.

(a) In an insurance market, individuals that do not incur losses have shared risk with individuals that do incur losses.
(b) Insurance companies can share risk by ceding some of the excess risk from large claims to reinsurers.
(c) Reinsurance companies can further share risk by investing in catastrophe bonds.
(d) Risk sharing reduces diversifiable risk, more so than reducing non-diversifiable risk.
(e) Ideally, any risk-sharing mechanism should benefit all parties sharing the risk.

For more on the role of financial markets, read section 1.2 from [2].

1.3. **Points of view on financial markets.** We can loosely divide the interested parties into three different categories:

(1) participants (individual investors, institutional investors, corporations),
(2) facilitators (market makers, traders), and
(3) researchers (us!).

Of course, we must not forget the **regulators** – finance is more than just Wall Street …

"These new regulations will fundamentally change the way we get around them."

1.4. **Structure of financial mathematics.**
What? *Financial risk* within portfolios of traded entities (aka *assets* or *securities*) – such as stocks and bonds.
How? Nondeterministic change over time due to *market forces*.
Where? Financial markets.

REFERENCES

[1] J. DE ROOY, *Economic literacy: What everyone needs to know about money and markets* (1995)
[2] R. L. MCDONALD, *Derovatives markets (2nd Ed)* (2006)