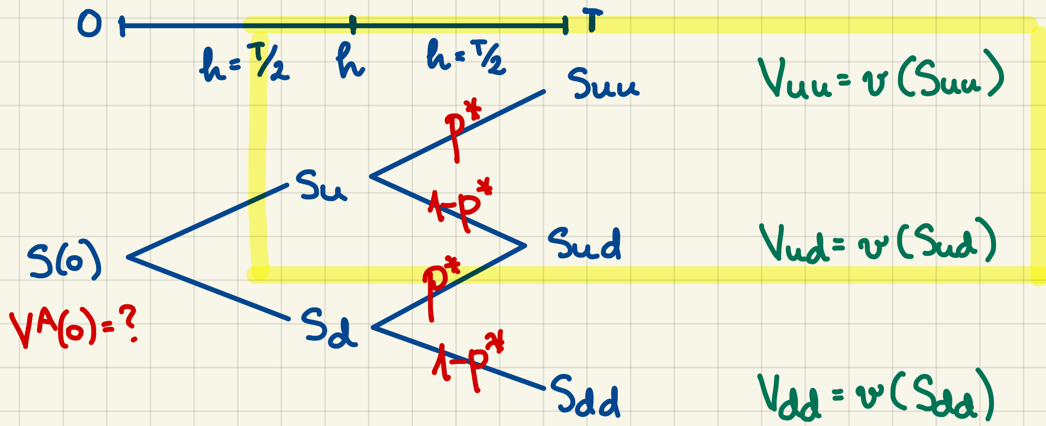
Binomial Pricing of American Options.



Possible payoffs should the option <u>not</u> be exercised early.

population $\longrightarrow$

$\longleftarrow$ pricing

Consider an American option w/ the payoff f'tion $v(\cdot)$

<u>up node</u>:
- IE_u = value of immediate exercise
- CV_u ... continuation value
  $\rightarrow$ If we don't exercise now, then the option "becomes" a European option (since there are no early exercise opportunities left).

$$CV_u = e^{-rh}\left[p^* \cdot V_{uu} + (1-p^*)\, V_{ud}\right]$$

$$\Rightarrow \boxed{V_u^A = \max(CV_u, IE_u)}$$

<u>and</u> the option's owner decides whether to exercise early accordingly

<u>down node</u>:
- IE_d
- $CV_d = e^{-rh}\left[p^* \cdot V_{ud} + (1-p^*) \cdot V_{dd}\right]$

$$\boxed{V_d^A = \max(CV_d, IE_d)}$$

e.g. call: $IE_0 = S(0) - K$

put: $IE_0 = K - S(0)$

ROOT node:

$$\begin{cases} \bullet\ IE_0 \\ \bullet\ CV_0 = e^{-rh}\left[p^* \cdot V_u^A + (1-p^*) \cdot V_d^A\right] \end{cases}$$

$$\boxed{V^A(0) = \max(IE_0, CV_0)}$$

Note:
- We can dynamically replicate American options until we reach the nodes where early exercise is optimal.
- The procedure is analogous for multiperiod trees.

Problem 3.7. The current stock price is observed to be \$100 per share. The stock is projected to pay dividends continuously at the rate proportional to its price with the dividend yield of 0.03. The stock's volatility is given to be 0.23. You model the evolution of the stock price using a two-period forward binomial tree with each period of length one year. $h=1$

The continuously compounded risk-free interest rate is given to be 0.04.

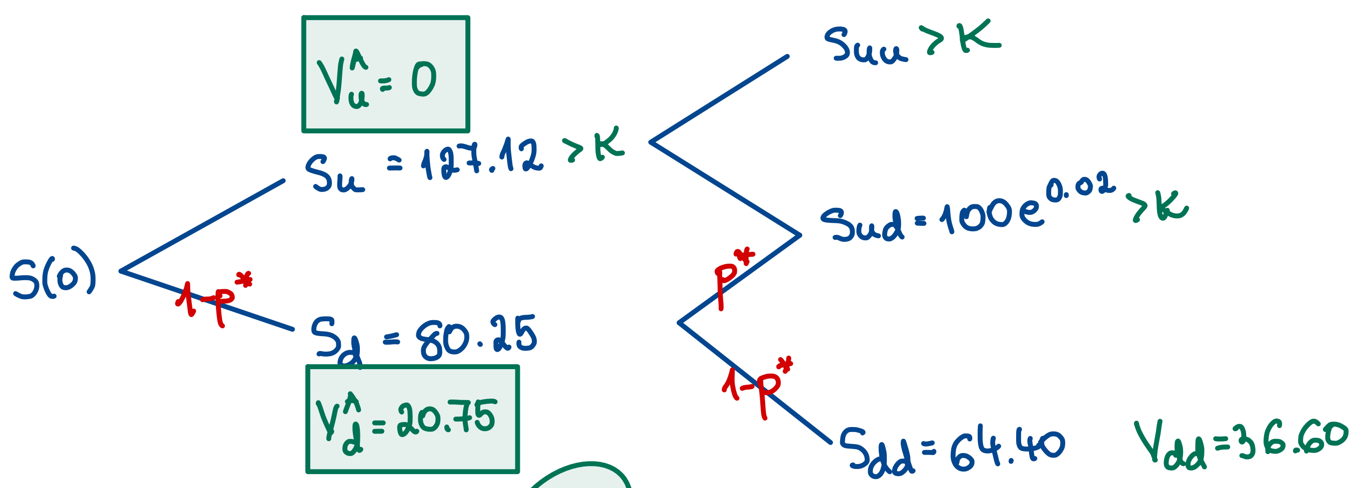
What is the price of a two-year, \$101-strike **American** put option on the above stock consistent with the above stock-price tree?

(a) About \$6.62
(b) About \$8.34
(c) About \$8.83
(d) About \$11.11
(e) None of the above.

$\longrightarrow$: $p^* = \dfrac{1}{1+e^{\sigma\sqrt{h}}} = \dfrac{1}{1+e^{0.23}} = \underline{0.44275}$

$$\begin{cases} u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.04-0.03)+0.23} = e^{0.24} = \underline{1.2712} \\ d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{0.01-0.23} = e^{-0.22} = \underline{0.8025} \end{cases}$$

$\boxed{K=101}$



down node: $\begin{cases} \bullet\ IE_d = 101 - S_d = 20.75 \\ \bullet\ CV_d = e^{-0.04}(1-p^*)\cdot 36.60 = \underline{19.5956} \end{cases}$

$\boxed{V_d^A = 20.75}$

ROOT node:

- $IE_0 = 101 - 100 = 1$
- $CV_0 = e^{-0.04} \cdot (1 - p^*) \cdot (20.75) = \underline{11.11} = y^A(0)$ $\square$