M339D: April 29th, 2022.

Asian Options [practice].

Problem. Let the current price of a non-dividend-paying stock be $S(0)=100$. Let its volatility be $\sigma=0.3$. Let the ccrfir be equal to 0.04.

We model the stock-price evolution over the next year using a two-period forward binomial tree.

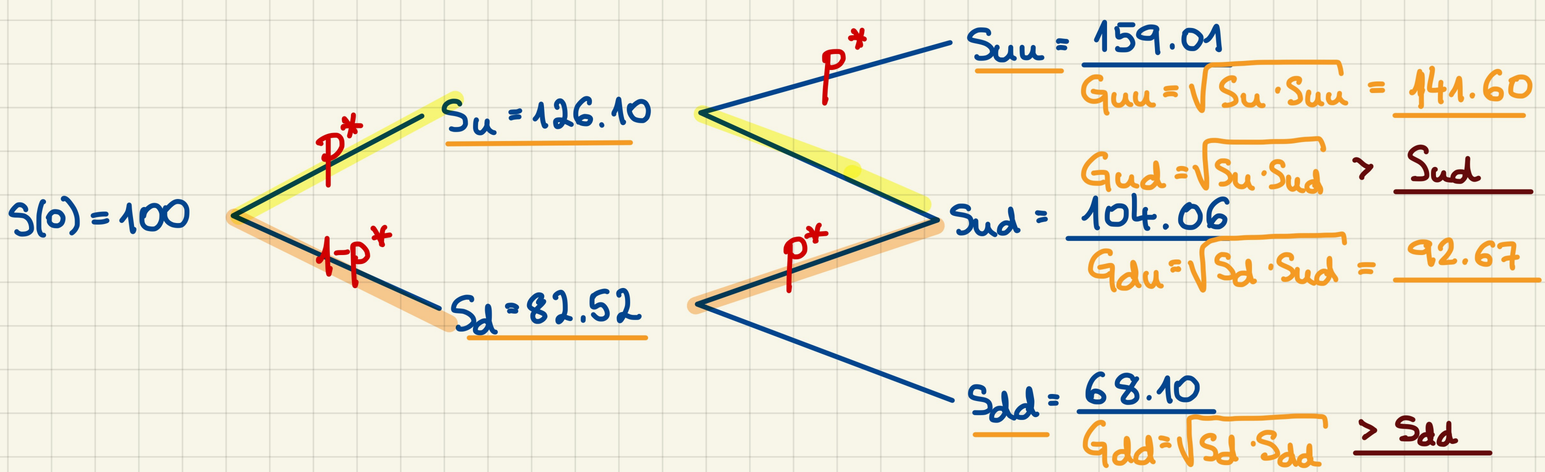
Consider an Asian, geometric-average-strike call w/ exercise date in one year. Price this option using the above tree!

$\rightarrow$:

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} \underset{\substack{\uparrow \\ h=\frac{1}{2}}}{=} \frac{1}{1+e^{0.3\sqrt{0.5}}} = \underline{0.447165}$$

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{0.04\cdot 0.5 + 0.3\sqrt{0.5}} = \underline{1.261}$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{0.02 - 0.3\sqrt{0.5}} = \underline{0.8252}$$



Payoff: $\boxed{(S(T)-G(T))_+}$

up-up: $V_{uu} = 159.01 - 141.60 = 17.41$ w/ prob. $(p^*)^2$

up-down: $V_{ud} = 0$

down-up: $V_{du} = 104.06 - 92.67 = 11.39$ w/ prob. $(1-p^*)\cdot p^*$

down-down: $V_{dd} = 0$

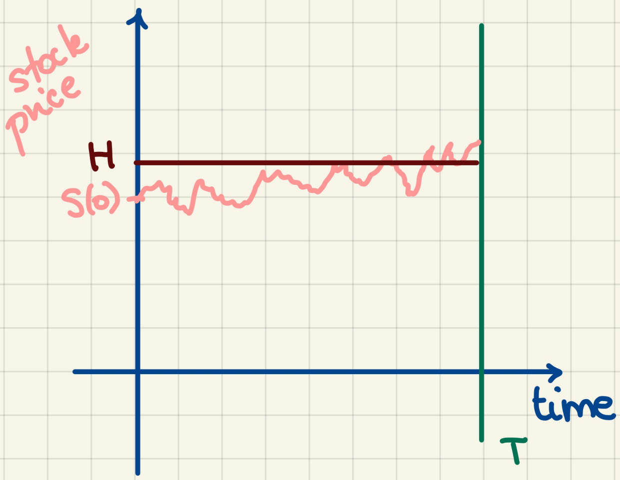
Risk-neutral pricing: $\boxed{V(0) = e^{-rT}\,\mathbb{E}^*[Y(T)]}$

$$V(0) = e^{-0.04}\left((p^*)^2 \cdot 17.41 + (1-p^*)\cdot p^* \cdot 11.39\right) = \underline{6.05}$$

$\square$

Barrier Option.

threshold / hurdle / level (H)



If the stock price touches/crosses the barrier H before or @ the exercise date, then the barrier state.

"Dormant" → "Alive" (RIP → ♡): KNOCK IN ⇓ $\boxed{\text{The PAYOFF is paid on the exercise date.}}$

"Alive" → "Dead" (♡ → RIP): KNOCK OUT ⇓ PAYOFF = 0

Example. $\boxed{\text{Rebate Options}}$

A fixed amount R is paid to the option's owner if the stock price hits the barrier H.

- $M(T) := \max\limits_{0 \le t \le T} S(t)$
- $m(T) := \min\limits_{0 \le t \le T} S(t)$

- If $S(0) < H$, then rebate iff $M(T) \geq H$.
- If $S(0) > H$, then rebate iff $m(T) \leq H$.

PAYOFF:

$\boxed{V(T)} = R \cdot \mathbb{I}_{[M(T) \geq H]}$

$\boxed{V(T)} = R \cdot \mathbb{I}_{[m(T) \leq H]}$

Problem. Let the current price of a non-dividend-paying stock be 100, and let its volatility be 0.3.

Let the ccrfir be equal to 0.08.

We use a two-period forward binomial tree to model the stock price over the following year.

A rebate option pays \$20 @ time-1 if the stock price ever reaches the level of $\boxed{\$110}$ during the following years.

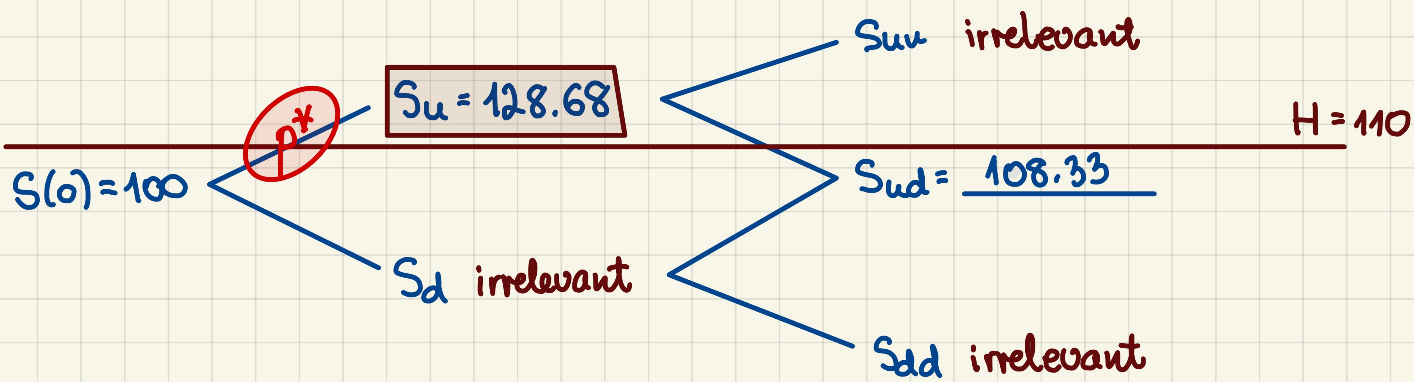
What's the price of the rebate option?

$\longrightarrow$:

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.3\sqrt{0.5}}} = \underline{0.447165}$$

$$u = e^{0.08(0.5)+0.3\sqrt{0.5}} = \underline{1.2868}$$

$$d = e^{0.04 - 0.3\sqrt{0.5}} = \underline{0.8419}$$



$$V_R(0) = e^{-0.08} \cdot 20 \cdot p^* = \underline{8.26}$$ $\square$