

## M378K Introduction to Mathematical Statistics

### Homework assignment #7

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Please, provide your **final answer only** to the following problems.

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**Problem 7.1.** ( $3 \times 7 = 21$  points) Identify the distributions with the following mgfs:

- $\frac{2}{2-t}$ .
- $e^{2e^t-2}$ ,
- $e^{t(t-2)}$ ,
- $(3 - 2e^t)^{-1}$
- $\frac{1}{9} + \frac{4}{9}e^t + \frac{4}{9}e^{2t}$ .
- $\frac{1}{t}(1 - e^{-t})$ .
- $\frac{1}{4}(e^{4t} + 3e^{-t})$

If the distribution has a name, give the name and the parameters. If it does not, give the pdf or the pmf (table).

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Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

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**Problem 7.2.** (30 points) Solve **Problem 7.6.11** from the Lecture notes.

**Problem 7.3.** (18 points) Source: "Mathematical Statistics with Applications" by Wackerly, Mendenhall, Scheaffer.

Suppose that a random variable  $Y$  has a probability density function given by

$$f_Y(y) = \kappa y^3 e^{-y/2} \mathbf{1}_{(0,\infty)}(y)$$

- (i) (5 points) Find the value of  $\kappa$  that makes  $f_Y(y)$  a density function.
- (ii) (3 points) Does  $Y$  have a  $\chi^2$ -distribution? If so, how many degrees of freedom?
- (iii) (5 points) What are the mean and standard deviation of  $Y$ ?
- (iv) (5 points) (Extra credit) Using **R**, find the probability that  $Y$  lies within 2 standard deviations of its mean?