Name:

UTeid:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
In-Term One
Instructor: Milica Čudina

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The maximum number of points on this exam is 65.

Problem 1.1. (10 points) Write the definition of an **arbitrage portfolio**.

Solution: Check your notes.

Problem 1.2. (5 points) Write the definition of a **replicating portfolio** of a European option.

Problem 1.3. (5 points) From a manufacturer's perspective, why would they decide to use derivative securities on their product to hedge? Respond in five lines or less.

Solution: Answers may vary, but the bottom line is that the manufacturer can prevent losses only by a limited amount using operations optimisation and other tools within his/her area of expertise. Their profit still depends heavily on market-price fluctuations – well outside of their area of influence and/or expertise. So, derivative securities are a welcome tool to hedge that risk.

Problem 1.4. (5 pts) Consider a portfolio consisting of the following four European options with the same expiration date T on the underlying asset S:
long one call with strike 40,
long two calls with strike 50,
short one call with strike 65.

Let $S(T) = 52$. What is the payoff from the above position at time T?

Solution: The payoff is

$$(52 - 40)_+ + 2(52 - 50)_+ - (52 - 65)_+ = 12 + 2(2) + 0 = 16.$$

Problem 1.5. (5 points) The initial price of the market index is \$900. After 3 months the market index is priced at \$960. The **effective** monthly rate of interest is 1.0%.

The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

Solution: The profit is

$$\begin{aligned}(K - S(T))_+ - FV_{0,T}[V_P(0)] &= (K - S(T))_+ - FV_{0,T}[V_P(0)] \\ &= (975 - 960)_+ - 10(1 + 0.01)^3 \\ &= 4.70.\end{aligned}$$

Problem 1.6. (10 points) *Source: Course 3, November 1980, Problem #33.*
The probability density function of the random variable X is

$$f_X(x) = \begin{cases} 0, & \text{for } x < -1, \\ \frac{3}{2}x^2, & \text{for } -1 \le x \le 1, \\ 0, & \text{for } x > 1. \end{cases}$$

Let u denote the simulated value from a unit uniform random number generator. Which transformation would you apply to u to generate simulated values of x?

Solution: First, we figure out the cumulative distribution function of X.

$$F_X(x) = \frac{3}{2}\int_{-1}^{x} \xi^2 d\xi = \frac{\xi^3}{2}\Big]_{\xi=-1}^{x} = \frac{x^3+1}{2}.$$

Now, we invert the cumulative distribution function of X.

$$y = \frac{x^3+1}{2} \quad \Leftrightarrow \quad 2y-1 = x^3 \quad \Leftrightarrow \quad \sqrt[3]{2y-1} = x.$$

So, we obtain simulated values of X through the following transform $x = \sqrt[3]{2u-1}$.

Problem 1.7. (10 points) *Source: Problem # 3.8 from McDonald.*
Assume that the **effective** 6-month interest rate equals 2%. The current price of an index equals \$1,000. The current premium on a 6-month call on this index is \$109.20, while the current premium on a 6-month put with the same strike price on this index equals \$60.18. Find the strike price.

Solution: This question is a direct application of the put-call parity. In our usual notation, we have

$$V_C(0) - V_P(0) = S(0) - PV_{0,T}(K),$$

i.e.,

$$K(1+i)^{-1} = S(0) - V_C(0) + V_P(0) \quad \Rightarrow \quad K = (1.02)(1000 - 109.20 + 60.18) = 969.9996 \approx 970.$$

Problem 1.8. (15 points) The continuously compounded risk-free interest rate equals 0.08.

Jonathan sells short one share of a non-dividend-paying stock and simultaneously buys a six-month, \$85-strike call option on the same stock. The current stock price is \$88, while the call price equals \$8. What is the break-even price of Jonathan's position?

Solution: Jonathan's initial cost is $8 - 88 = -80$. The expression for his payoff, in terms of the final asset price s is

$$-s + (s-K)_+ = -\min(s, K).$$

Hence, the break-even price s^* satisfies

$$-\min(s^*, K) = FV_{0,1/2}(-80) = -80e^{0.04} \quad \Rightarrow \quad s^* = 83.26486.$$