

# The University of Texas at Austin

## IN-CLASS WORK 7

M378K Introduction to Mathematical Statistics

February 09, 2026

### Cumulative distribution function: Named continuous distributions.

**Problem 7.1.** *Source: Sample P exam, Problem #126.*

A company's annual profit is normally distributed with mean 100 and variance 400. Then, we can express the probability that the company's profit in a year is at most 60, given that the profit in the year is positive in terms of the standard normal cumulative distribution function denoted by  $\Phi$  as

$$1 - \frac{\Phi(2)}{\Phi(5)}$$

*True or false? Why?*

**Solution.** Let's denote the company's profit by a random variable  $Y$ . We are given that

$$Y \sim N(\mu = 100, \sigma^2 = 400).$$

We know that when we express  $Y$  in standard units, we obtain a standard normal distribution, i.e.,

$$Z = \frac{Y - 100}{20} \sim N(0, 1)$$

So, we can calculate our conditional probability as

$$\begin{aligned} \mathbb{P}[Y \leq 60 \mid Y > 0] &= \frac{\mathbb{P}[0 < Y \leq 60]}{\mathbb{P}[Y > 0]} = \frac{\mathbb{P}\left[\frac{0-100}{20} < \frac{Y-100}{20} \leq \frac{60-100}{20}\right]}{\mathbb{P}\left[\frac{Y-100}{20} > \frac{0-100}{20}\right]} \\ &= \frac{\mathbb{P}[-5 < Z \leq -2]}{\mathbb{P}[-5 < Z]} = \frac{\mathbb{P}[Z \leq -2] - \mathbb{P}[Z \leq -5]}{1 - \mathbb{P}[Z \leq -5]} \\ &= \frac{\Phi(-2) - \Phi(-5)}{1 - \Phi(-5)} = \frac{(1 - \Phi(2)) - (1 - \Phi(5))}{\Phi(5)} = \frac{\Phi(5) - \Phi(2)}{\Phi(5)} = 1 - \frac{\Phi(2)}{\Phi(5)}. \end{aligned}$$

**Problem 7.2.** *Source: Sample P exam, Problem #267.*

The lifetime of a certain electronic device has an exponential distribution with mean 0.50. Calculate the probability that the lifetime of the device is greater than 0.70, **given** that it is greater than 0.40.

**Solution.** This problem is an excellent opportunity to study a unique property of the exponential distribution. Consider  $T \sim E(\tau)$  and two positive constants,  $s$  and  $t$ . Then, we could be interested in finding the probability that  $T$  exceeds  $t + s$  given that it exceeds  $s$ .

$$\mathbb{P}[T > t + s \mid T > s] = \frac{\mathbb{P}[T > t + s, T > s]}{\mathbb{P}[T > s]} = \frac{\mathbb{P}[T > t + s]}{\mathbb{P}[T > s]} = \frac{e^{-\frac{t+s}{\tau}}}{e^{-\frac{s}{\tau}}} = e^{-\frac{t}{\tau}} = \mathbb{P}[T > t].$$

In a sense, the random variable  $T$  "forgets" that it already waited for  $s$  time units. This property is, hence, called the **memoryless property**. In the present problem, we have  $T \sim E(\tau = 0.5)$ . So,

$$\mathbb{P}[T > 0.70 \mid T > 0.40] = \mathbb{P}[T > 0.3] = e^{-\frac{0.3}{0.5}} = e^{-0.6}.$$