M339D: November 17th, 2023.

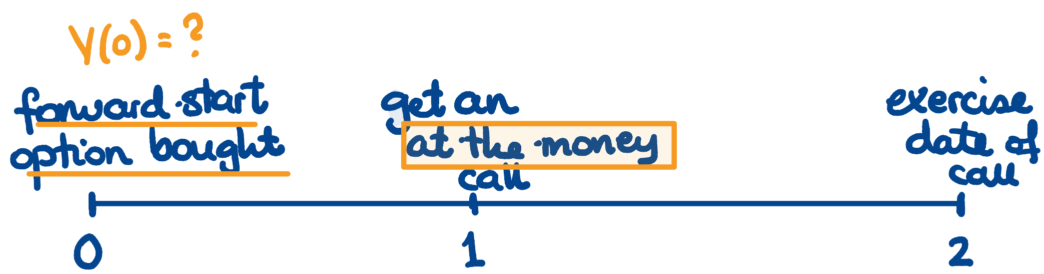
(A) 586

(B) 594

(C) 684

(D) 692

(E) 797



19. Consider a *forward start option* which, 1 year from today, will give its owner a $\boxed{\text{1-year}}$ European call option with a strike price equal to the stock price at that time.

You are given:

(i) The European call option is on a stock that pays no dividends.

(ii) The stock's volatility is 30%. $\boxed{\sigma = 0.3}$

(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100.

$F_{0,1}(S) = 100 = S(0)e^{r \cdot 1}$

(iv) The continuously compounded risk-free interest rate is 8%. $r = 0.08$

Under the Black-Scholes framework, determine the price today of the forward start option.

(A) 11.90

(B) 13.10

(C) 14.50

(D) 15.70

(E) 16.80

At $t < T$:

$$V_c(t) = S(t)N(d_1(t)) - Ke^{-r(T-t)} \cdot N(d_2(t))$$

w/

$$d_1(t) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S(t)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$

and

$$d_2(t) = d_1(t) - \sigma\sqrt{T-t}$$

In this problem: $\boxed{t = 1}$

$$V_c(1) = S(1) \cdot N(d_1(1)) - Ke^{-r(2-1)} \cdot N(d_2(1))$$

$$V_c(1) = S(1) \cdot N(d_1(1)) - S(1)e^{-r} \cdot N(d_2(1))$$

$$V_c(1) = S(1)\boxed{\left[N(d_1(1)) - e^{-r} \cdot N(d_2(1))\right]}$$

w/ $d_1(1) = \frac{1}{0.3\sqrt{2-1}}\left[\cancel{\ln\left(\frac{S(1)}{S(1)}\right)} + \left(0.08 + \frac{0.09}{2}\right)(2-1)\right]$

@ the money

$$d_1(1) = \frac{0.08+0.045}{0.3} = \underline{0.4167}$$

$$d_2(1) = d_1(1) - 0.3\sqrt{2-1} = \underline{0.1167}$$

$$N(d_1(1)) = \text{pnorm}(0.4167) = 0.6615511$$

$$N(d_2(1)) = \text{pnorm}(0.1167) = 0.5464511$$

constant

$$V_c(1) = S(1)\left(0.6615511 - e^{-0.08}\cdot 0.5464511\right) = S(1)\cdot \underline{0.15711}$$

At time 0, our forward start option is worth:

$$\boxed{0.15711 \cdot S(0)}$$

Our answer: $0.15711 \cdot e^{-0.08} \cdot \underbrace{F_{0,1}(S)}_{=100} = \underline{14.50308}$ $\square$

33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

You are given:

(i) The continuously compounded risk-free interest rate is 8%.

(ii) The stock's volatility is 30%.

(iii) The current stock price is $\boxed{45}$.

(iv) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

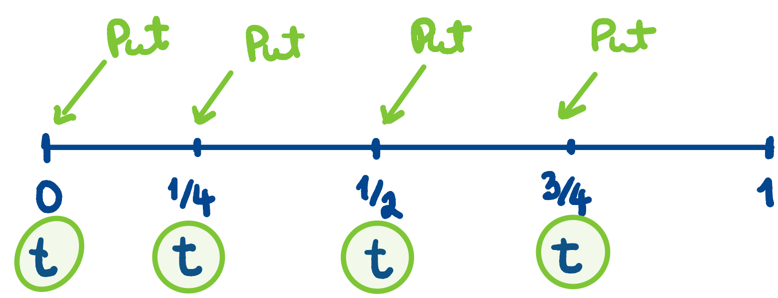
(A) 1.59

(B) 2.24

(C) 2.86

(D) .48

(E) 3.61



t... represents the valuation dates

34-39. DELETED

For each of the four puts in the rolling insurance strategy:

- one quarter to exercise
- $K_t = 0.9 \cdot S(t)$

For every t @ which a put option is received:

$$d_1(t) = \frac{1}{\sigma\sqrt{1/4}}\left[\ln\left(\frac{\cancel{S(t)}}{0.9\cancel{S(t)}}\right) + \left(r+\frac{\sigma^2}{2}\right)\cdot\left(\frac{1}{4}\right)\right]$$

$$d_1(\cancel{t}) = \frac{1}{0.3(1/2)}\left[-\ln(0.9) + \left(0.08+\frac{0.09}{2}\right)(1/4)\right] = 0.9107$$

$$d_2(\cancel{t}) = 0.7607$$

$$N(-0.9107) = 0.1812267$$
$$N(-0.7607) = 0.2234181$$

$$V_P(t) = 0.9 \cdot S(t)\, e^{-0.08(0.25)} \cdot 0.2234181 - S(t) \cdot 0.1812267$$
$$V_P(t) = S(t)\left(0.9 \cdot e^{-0.02} \cdot 0.2234181 - 0.1812267\right) = S(t) \cdot \underline{0.01586801}$$

⇒ Note that for every "put delivery" date, i.e., $t = 0, 1/4, 1/2, 3/4$, the value of the put @ that time is $\boxed{0.01586801 \cdot S(t)}$

Altogether: # of puts

$$4 \cdot \boxed{\overset{\mathcal{X}}{0.01586801}} \cdot \underbrace{S(0)}^{45} = \boxed{\underline{2.856242}}$$

is the total price of the rolling insurance strategy □

$4 \cdot \mathcal{X} \cdot 45$ spent today: sell $\mathcal{X}$ shares, get 1 put @ time 0
sell $\mathcal{X}$ shares, get 1 put @ time (1/4)
sell $\mathcal{X}$ shares, get 1 put @ time (1/2)
sell $\mathcal{X}$ shares, get 1 put @ time (3/4)