M339J: Probability models with actuarial applications
University of Texas at Austin
HW #9
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Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 9.1. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the probability (mass) function

$$p_N(0) = 0.5, p_N(1) = 0.3, p_N(2) = 0.2.$$

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the probability (mass) function $p_X(1) = 0.3$ and $p_X(2) = 0.7$.
Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Calculate $\mathbb{E}[(S-2)_+]$.

Problem 9.2. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 1.

Let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the following p.m.f.

$$p_X(100) = 1/2, p_X(200) = 3/10, p_X(300) = 1/5.$$

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Find the expected value of the **policyholder's** payment for a stop-loss insurance policy with an ordinary deductible of 200, i.e., calculate $\mathbb{E}[S \wedge 200]$.

Problem 9.3. (15 points) *Source: Based on Problem #165 from sample STAM Exam.*
Consider the following collective risk model:
(i) The claim count random variable N is Poisson with mean 3.
(ii) The severity random variable has the following probability (mass) function:

$$p_X(1) = 0.6,\ p_X(2) = 0.4.$$

(iii) As usual, individual loss random variables are mutually independent and independent of N.

Assume that an insurance covers **aggregate losses** subject to a deductible $d = 3$.
Find the expected value of aggregate payments for this insurance.

Problem 9.4. (10 pts) In the compound model for aggregate claims, let the frequency random variable N have the geometric distribution with mean 4.

Moreover, let the individual losses have the distribution

$$p_X(0) = 1/2, p_X(100) = 1/2.$$

Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$. How much is $\mathbb{E}[(S-100)_+]$?

Problem 9.5. (5 points) An insurer pays aggregate claims in excess of the deductible d. In return, they receive a stop-loss premium $\mathbb{E}[(S-d)_+]$. You model the aggregate losses S using a continuous distribution. Moreover, you are given the following information about the aggregate losses S:

(i) $\mathbb{E}[(S-100)_+] = 15$,
(ii) $\mathbb{E}[(S-120)_+] = 10$,
(iii) $\mathbb{P}[80 < S \leq 120] = 0$.

Find the probability that the aggregate claim amounts are less than or equal to 80.