Name:

M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
The Prerequisite In-Term Exam
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximum number of points on this exam is 100.
Time: 50 minutes

1.1. **Free-response problems.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.1. (15 points)
Two scales are used to measure the mass m of a precious stone. The first scale makes an error in measurement which we model by a normally distributed random variable with mean $\mu_1 = 0$ and standard deviation $\sigma_1 = 0.04m$. The second scale is more accurate. We model its error by a normal random variable with mean $\mu_2 = 0$ and standard deviation $\sigma_2 = 0.03m$.

We assume that the measurements made using the two different scales are independent.

To get our final estimate of the mass of the stone, we take the average of the two results from the two different scales.

What is the probability that the value we get is within $0.005m$ of the actual mass of the stone?

Problem 1.2. (10 points) Assume that $Y_1 = e^X$ where X is a standard normal random variable.

(i) (2 points) What is the probability that Y_1 exceeds 5?
(ii) $(3 + 5$ points) Find the mean and the variance of Y_1.

Hint: It helps if you use the expression for the moment generating function of a standard normal random variable.

Problem 1.3. (15 points) The random vector (X_1, X_2, X_3) is jointly normal. Its marginal distributions are:

$X_1 \sim N(\text{mean} = 0, \text{variance} = 4), X_2 \sim N(\text{mean} = 1, \text{variance} = 1), X_3 \sim N(\text{mean} = -1, \text{variance} = 9).$

The correlation coefficients are given to be

$$corr[X_1, X_2] = 0.3, corr[X_2, X_3] = 0.4, corr[X_1, X_3] = -0.3.$$

What is the distribution of the random variable $X = X_1 - X_2 + 2X_3$? Please, provide the **name** of the distribution, as well as the **values** of its parameters.

Problem 1.4. (10 pts) For a two-period binomial model, you are given that:

(1) each period is one year;
(2) the current price of a non-dividend-paying stock S is $S(0) = \$20$;
(3) $u = 1.3$, with u as in the standard notation for the binomial model;
(4) $d = 0.9$, with d as in the standard notation for the binomial model;
(5) the continuously compounded risk-free interest rate is $r = 0.05$.

Consider a special call option which pays the excess above the strike price $K = 23$ (if any!) at the end of **every** binomial period.

Find the price of this option.

Problem 1.5. (10 points) Let the stock prices be modeled using the lognormal distribution. The mean stock price at time-1 equals 120 and the median stock price 115. What is the probability that the time-1 stock price exceeds 100?

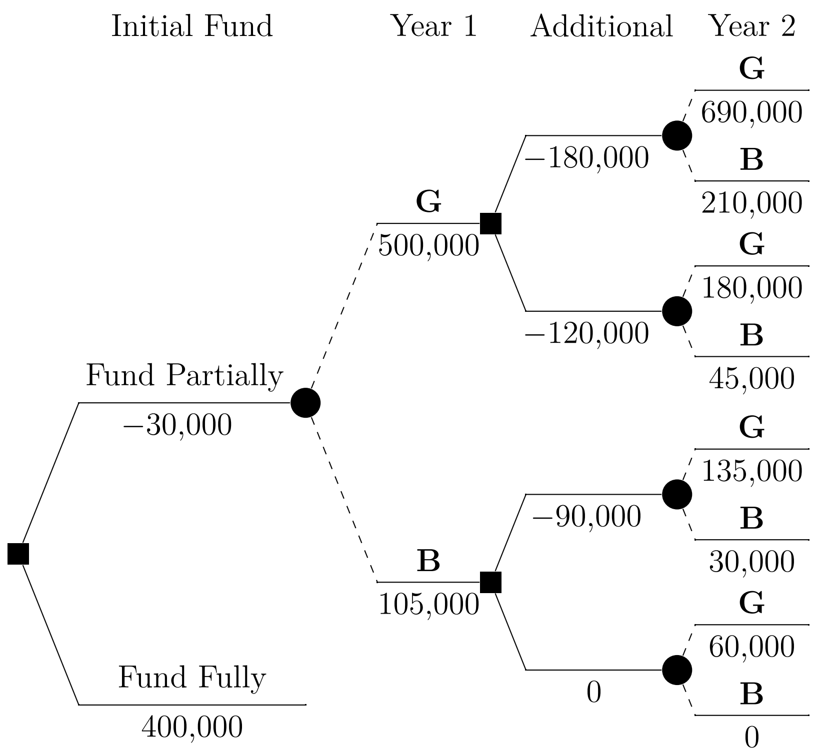
Problem 1.6. (5 points) Netflix is considering a cartoon series. When the production of two seasons is fully funded at time−0 the project has a net present value of 400,000.

The decision tree below shows the cash flows of the series when the promotion at the beginning of the Year 1 (i.e., at $t = 0$) is only partial with an option to provide different amounts of funding at the beginning of Year 2 (i.e., at $t = 1$) depending on how well the first season did.

This tree reflects two possible receptions of the two seasons at each information node (**G** = good, **B** = bad). The probability of the series being a success is given to be 2/3 and the probability of it being merely watchable is 1/3.

Assume the interest rate is 0%.

Find the **initial** (i.e., at $t = 0$) value of the option to fund partially.



Problem 1.7. (5 points) Assume the Black-Scholes model. The initial price of a continuous-dividend-paying stock is \$100. Its dividend yield is 0.03 and its volatility is 0.15. According to your model, the mean rate of return is 0.08.

The continuously compounded risk-free interest rate is 0.04.

Calculate the probability that the realized return for the time period $[0, 2]$ exceeds 0.06.

Problem 1.8. (5 points) Assume the Black-Scholes model. Consider a non-dividend-paying stock with the intial stock price of \$100 and volatility equal to 0.30. According to your model, the stock's mean rate of return is 0.10. Find

$$\mathbb{E}[S(1)\mathbb{I}_{[S(1)\geq 105]}].$$

1.2. **MULTIPLE CHOICE QUESTIONS.** *Please note your answers on the front page.*

Problem 1.9. (5 pts) Consider a non-dividend-paying stock currently priced at \$100 per share.

The price of this stock in one year is modeled using a one-period binomial tree under the assumption that the stock price can either go up to 110 or down to 90.

Let the continuously compounded risk-free interest rate equal 0.04. What is the risk-neutral probability of the stock price going up?

(a) About 0.2969
(b) About 0.3039
(c) About 0.5000
(d) About 0.7041
(e) None of the above.

Problem 1.10. (5 points) The current stock price is 40 per share. The price at the end of a three-month period is modeled with a one-period binomial tree so that the stock price can either increase by \$10, or decrease by \$4. The stock pays dividends continuously with the dividend yield 0.04.

The continuously compounded risk-free interest rate is 0.05.

What is the stock investment in a replicating portfolio for three-month, \$40-strike European **straddle** on the above stock?

(a) Long 0.42 shares
(b) Long 0.71 shares
(c) Short 0.71 shares
(d) Short 0.42 shares
(e) None of the above.

Problem 1.11. The following relates to one share of XYZ stock:

- The current price is 80.
- The forward price for delivery in two years is 88.
- An investor who decides to long the forward contract denotes by P the expected stock price in two years.

Determine which of the following statements about P is **TRUE**.

(a) $P < 80$
(b) $P = 80$
(c) $80 < P < 88$
(d) $P = 88$
(e) $P > 88$

Problem 1.12. The current futures price is given to be \$80. The evolution of this futures price over the following year is modeled using a two-period binomial tree such that the ratio of the up factor to the down factor equals 4/3. Moreover, you are given that the risk-neutral probability of an up movement in the tree in any single step equals 1/3.

The continuously compounded risk-free interest rate is 0.05.

What is the price of a one-year, \$85-strike European put option on the above futures contract consistent with our model?

(a) About \$2.24.
(b) About \$8.12.
(c) About \$8.54.
(d) About \$8.98.
(e) None of the above.

Problem 1.13. The current exchange rate is given to be \$1.11 per Euro and its volatility is given to be 0.16. The continuously compounded risk-free interest rate for the US dollar is 0.02, while the continuously compounded risk-free interest rate for the Euro equals 0.04.

The evolution of the exchange rate over the following nine-months is modeled using a three-period forward binomial tree. What is the value of the so-called up factor in the above tree?

(a) $u \approx 1.0779$
(b) $u \approx 1.0887$
(c) $u \approx 1.1503$
(d) $u \approx 1.1972$
(e) None of the above.

Problem 1.14. (5 points) Let the current price of a continuous-dividend-paying stock be denoted by $S(0)$. We model the time$-T$ stock price as lognormal. The mean rate of return on the stock is 0.10, its dividend yield is 0.01, and its volatility is 0.20. The continuously compounded risk-free interest rate is 0.03. You invest in one share of stock at time-0. and simultaneously deposit an amount $\varphi S(0)$ in a savings account. Assume continuous and immediate reinvestment of all dividends in the same stock. What should the proportion φ be so that the VaR at the level 0.05 of your total wealth at time-1 equals today's stock price $S(0)$?

(a) $\varphi = 0.0573$
(b) $\varphi = 0.1966$
(c) $\varphi = 0.2139$
(d) $\varphi = 0.5$
(e) None of the above.