

M3399D: February 10<sup>th</sup>, 2023.

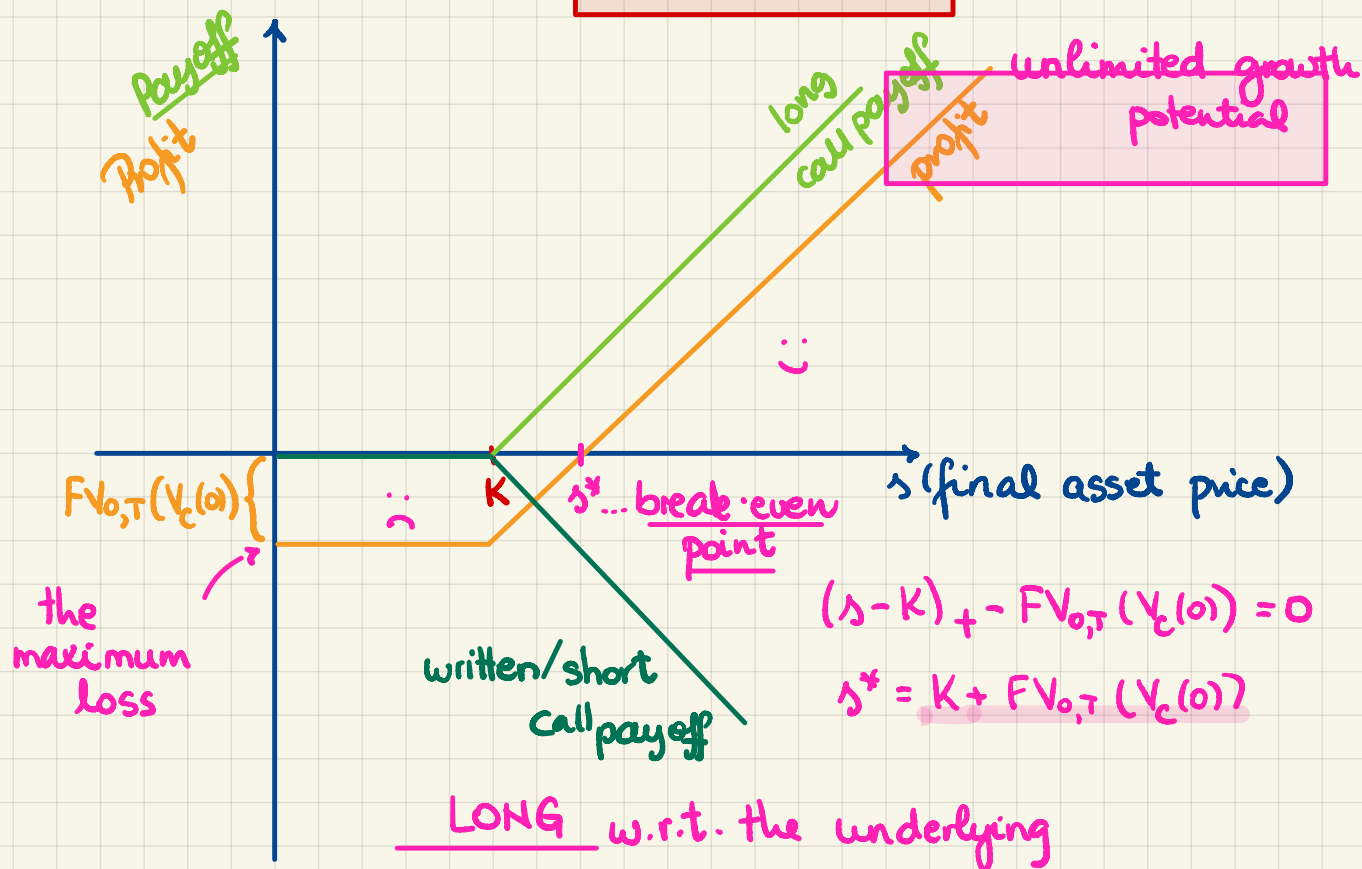
## More on European Call Options.

Payoff:

$$V_c(T) = (S(T) - K)_+$$

the payoff f'n:

$$v_c(s) = (s - K)_+$$



$$\mathbb{E}[V_c(T)] = \mathbb{E}[(S(T) - K)_+]$$

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Problem Set #5

European call options.

**Problem 5.1.** The initial price of a non-dividend-paying asset is \$100. A six-month, \$95 strike European call option is available at a \$8 premium. The continuously compounded risk-free interest rate equals 0.04. What is the break-even point for this call option?

- (a) 86.84
- (b) 87
- (c) 103
- (d) 103.16
- (e) None of the above.

$$\begin{aligned} J^* &= 95 + FV_{0,T}(8) = 95 + 8e^{0.04(0.5)} \\ &= 95 + 8e^{0.02} \\ &= \underline{103.16} \quad \square \end{aligned}$$

**Problem 5.2.** (5 points) A stock's price today is \$1000 and the annual effective interest rate is given to be 5%. You write a one-year \$1,050-strike call option for a premium of \$10 while you simultaneously buy the stock. What is your profit if the stock's spot price in one year equals \$1,200?

- (a) \$150.00
- (b) \$139.90
- (c) \$10.50
- (d) - \$39.00
- (e) None of the above.

→: Payoff of option:  $(1200 - 1050)_+ = 150$

$$V_c(0) = 10$$

$$FV_{0,1}(V_c(0)) = 10(1.05) = 10.5$$

$$\text{Profit of option: } 150 - 10.5 = 139.5$$

$$\text{Profit of stock: } 1200 - 1000(1.05) = 150$$

$$150 - 139.5 = 10.5$$

↑  
call written

Alternatively:

$$\text{Payoff} = S(T) - (S(T) - K)_+ = \begin{cases} K, & \text{if } S(T) \geq K \\ S(T), & \text{if } S(T) < K \end{cases}$$

Covered Call

$$= \min(S(T), K)$$

$$\left. \begin{array}{l} \text{Payoff} = \min(1200, 1050) = 1050 \\ \text{Initial Cost} = 1000 - 10 = 990 \end{array} \right\} \text{Profit} = 1050 - 990(1.05) = 10.50$$

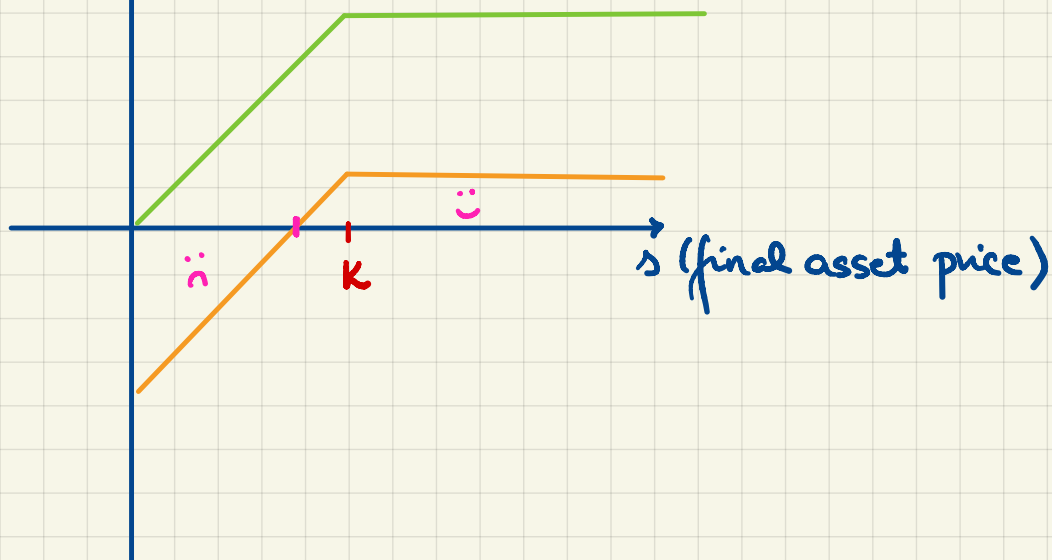
In this problem



Payoff  
Profit

Payoff f'n of Covered  
call:

$$v(s) = \min(s, K)$$



**Problem 5.3.** (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will be able to sell every piece for \$1,000.

The jeweler models the market price of gold in one year as follows:

Gold price in one year	Probability	
750 per ounce	0.2	→ <del>750</del> <sup>min</sup> 750
850 per ounce	0.5	→ 850
950 per ounce	0.3	→ 900

The jeweler hedges the price of gold by buying a 1-year call option with an exercise price of \$900 per ounce. The option costs \$100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the hedged portfolio per piece of jewelry produced.

→ :

$$\text{Payoff: } -S(T) + (S(T) - K)_+ =$$

$$= \begin{cases} -K, & \text{if } S(T) \geq K \\ -S(T), & \text{if } S(T) < K \end{cases}$$

$$= \max(-K, -S(T)) = -\min(S(T), K)$$

CAP

$$\text{Profit} = 1000 - \min(S(T), K) - 100e^{0.05}$$

$$\mathbb{E}[\text{Profit}] = 894.87 - \mathbb{E}[\min(S(T), 900)] = 49.87$$

□