The University of Texas at Austin
Homework Assignment 4
Introduction to Financial Mathematics

February 22, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Probability.

Problem 4.1. (10 points) Complete the following definition:
Two random variables X and Y are said to be **equal** if ...

Solution. ...they are defined on the same probability space and

$$\mathbb{P}[X = Y] = 1,$$

i.e., X equals Y almost surely (with probability 1).

Problem 4.2. (10 points) Complete the following definition:
The two random variables X and Y are said to be **identically distributed** if ...

Solution. ...they have the same cumulative distribution functions, i.e.,

$$F_X(x) = F_Y(x), \text{for all } \ x$$

with F_X being the cumulative distribution function of the random variable X and F_Y being the cumulative distribution function of the random variable Y.

Problem 4.3. (10 points) Provide an example of a pair of random variables which are identically distributed, but **not** equal.

Solution. Answers will vary. One example is:

- X is defined as:

$$X = \begin{cases} 1 \text{ if the result of a fair cointoss is heads} \\ 0 \text{ if the result of a fair cointoss is tails} \end{cases}$$

- Y is defined as:

$$Y = \begin{cases} 1 \text{ if the result of a roll of a fair die is a prime number} \\ 0 \text{ if the result of a roll of a fair die is not a prime number} \end{cases}$$

Derivative Securities.

Problem 4.4. (2 points) Derivative securities can only be used for hedging, i.e., they can only be bought and written by agents who already have a position in the underlying asset. *True or false?*

Solution. FALSE
There is no such requirement due to the possibility of cash settlement.

Problem 4.5. (2 points) The profit diagram and the payoff diagram for long positions in a forward contract are identical. *True or false? Why?*

Solution. TRUE
The initial cost is equal to zero.

Problem 4.6. (2 points) In our usual notation, the difference between the profit of a long forward contract and a long investment in one unit of the non-dividend-paying underlying asset equals the forward price. *True or false? Why?*

Solution. FALSE

$$S(T) - F - S(T) + S(0)e^{rT} = S(0)e^{rT} - F.$$

Of course, if the available forward price is the no-arbitrage forward price, the profits are equal.

Problem 4.7. (4 points) We all enter derivative-security-like contracts on a daily basis. For instance, ordering a pizza can be understood as a forward contract on the pizza. Ignore time-limits on when the pizza should be delivered. Imagine that the pizza is to be delivered in 30 minutes **exactly**. Explain why the pizza ordering is similar to a forward contract.

Solution. Solutions will vary.

Problem 4.8. (5 points) Maryam bakes batches of cupcakes for a cupcake convention. She buys forward 21 pounds of raspberries from a local farmer at the forward price of \$5.60 per pound. She projects to bake 336 cupcakes and sell each for \$3. The total and aggregate non-raspberry costs of baking the cupcakes are \$200. If the market price of raspberries on the day of the cupcake convention is \$5.40, what is Maryam's profit?

Solution.

$$336 \times 3 - 21 \times 5.60 - 200 = 690.40.$$

Problem 4.9. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option. The continuously compounded risk-free interest rate equals 0.04. In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

Solution.

$$200 \times 10 - 200 \times 5 - 500 - 200 \times 0.50e^{0.02} = 397.98$$