M378K Introduction to Mathematical Statistics

Homework assignment #4

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 4.1. *(15 points)Let (Y_1, Y_2) be a random vector with the joint pdf*

$$f_{Y_1,Y_2}(y_1, y_2) = \frac{1}{4} 1_{\{-1 \le y_1 \le 1\}} 1_{\{-1 \le y_2 \le 1\}}.$$

Find $\mathbb{P}\Big[|Y_1| + |Y_2| \le 1/2\Big]$.

Problem 4.2. *(15 points) Two random numbers, Y_1 and Y_2 are chosen independently of each other, according to the uniform distribution $U(-1, 2)$ on $[-1, 2]$. What is the probability that their product is positive?*

Problem 4.3. *(20 points) Three (fair and independent) coins are thrown; let Y_1, Y_2 and Y_3 be the outcomes (encoded as H or T). Player 1 gets $\$1$ if H shows on coin 1 ($Y_1 = H$) and/or $\$2$ if H shows on coin 2 ($Y_2 = H$). Player 2, on the other hand, gets $\$1$ when $Y_2 = H$ and/or $\$2$ when $Y_3 = H$. With W_1 and W_2 denoting the total amount of money given to Player 1 and Player 2, respectively,*

1. *(5 points) Write down the marginal distributions (pmfs) of W_1 and W_2,*
2. *(10 points) Write down the joint distribution table of (W_1, W_2).*
3. *(5 points) Are W_1 and W_2 independent?*