M358K: October 26th, 2020.



University of Texas at Austin

# Problem Set # 11

Bernoulli. Binomial.

Provide your **complete solutions** for the following problems.

**Problem 11.1.** Based on the traveling salesman's experience, he makes a same on any visit with probability of 15%. We assume that the individual customer's decisions are independent.

If he makes 10 visits in a certain day, what is the chance that he makes at least five sales?

**Problem 11.2.** **Expected frequency**

Suppose you are going to roll a fair die 60 times and record the proportion of times that a 1 or a 2 is showing. The sampling distribution of the said proportion should be centered about which value?

1/3

**$\mathbb{P}$ 11.1.** $X$ ... the number of sales he makes

$X \sim \text{Binomial}(\text{size} = n = 10,\ \text{prob} = p = 0.15)$

$$\mathbb{P}[X \geq 5] = ?$$

$$\mathbb{P}[X \geq 5] = 1 - \mathbb{P}[X \leq 4]$$

$$\mathbb{P}[X \leq 4] = \mathbb{P}[X=0] + \mathbb{P}[X=1] + \mathbb{P}[X=2] + \mathbb{P}[X=3] + \mathbb{P}[X=4]$$

$$= \binom{10}{0}(0.15)^0(0.85)^{10} + \binom{10}{1}(0.15)^1(0.85)^9 + \binom{10}{2}(0.15)^2(0.85)^8 + \binom{10}{3}(0.15)^3(0.85)^7 + \binom{10}{4}(0.15)^4(0.85)^6 = \dots$$

pbinom(4, size = 10, prob = 0.15) = 0.9901

$$\mathbb{P}[X \geq 5] = 0.0099$$

■

## Binomial Dist'n [two facts].

$X \sim$ Binomial ($n$ = # of trials, $p$ = prob. of success)

- $\mathbb{E}[X] = n \cdot p$
- $Var[X] = np(1-p) \Rightarrow SD[X] = \sqrt{n \cdot p \cdot (1-p)}$

## Normal Approximation to the Binomial Dist'n (4.3.2)

Consider a sequence of binomial random variable

$$S_n \sim \text{Binomial}(\text{size} = n, \text{prob} = p)$$

Let the # of trials become "large".

Keep this fixed!

Q: What happens to the dist'n of $S_n$ as $n$ becomes large?

→: By the de Moivre-Laplace Thm:

$$\boxed{\frac{S_n - np}{\sqrt{n \cdot p(1-p)}} \xrightarrow{\mathcal{D}} N(0,1) \text{ ... standard normal}}$$

Z

Usage: For "large enough" $n$ (rule of thumb: $n \cdot p \geq 10$ and $n \cdot (1-p) \geq 10$):

$$S_n \text{ "}\sim\text{" Normal}(\text{mean} = n \cdot p, \text{sd} = \sqrt{n \cdot p(1-p)})$$

... counts

$\Rightarrow$ For Proportions:

$$\hat{P}_n = \frac{S_n}{n} \text{ "}\sim\text{" Normal}\left(\text{mean} = \underline{\;p\;},\ \text{sd} = \underline{\sqrt{\frac{p(1-p)}{n}}}\right)$$

$$\begin{aligned}
\text{Var}[\hat{P}_n] &= \text{Var}\left[\frac{S_n}{n}\right] \\
&= \frac{1}{n^2} \cdot \text{Var}[S_n] = \frac{1}{n^2} \cdot n \cdot p(1-p) \\
&= \frac{p(1-p)}{n}
\end{aligned}$$

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**Problem.** $\boxed{\text{"True/False"}}$

A student answers a set of 100 T/F questions. She answers 36 questions correctly. She guesses at random the answers to the remaining questions. If the passing mark is 70 questions, what's this student's chance of passing?