

## M378K Introduction to Mathematical Statistics

### Homework assignment #10

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Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

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**Problem 10.1.** (10 points) Let  $\hat{\theta}_1$  and  $\hat{\theta}_2$  be independent unbiased estimators of  $\theta$ . You are given that  $Var[\hat{\theta}_1] = 4$  and  $Var[\hat{\theta}_2] = 9$ .

- (i) (8 pts) Consider the class of estimators of the form  $\hat{\theta} = \alpha\hat{\theta}_1 + \beta\hat{\theta}_2$ . Find the UMVUE in this class of estimators.
- (ii) (2 pts) Calculate the MSE of the estimator  $\hat{\theta}$  you found in part (i) as an estimator of  $\theta$ .

**Problem 10.2.** (5 points) For a fixed confidence level, a broader confidence interval is preferred since it is more likely to contain the true value of the parameter of interest. True or false? Why?

**Problem 10.3.** (5 points) You model the weights of individual boxes of Turkish delight as normally distributed with an unknown mean  $\mu$  and a known standard deviation of 10 grams. You gather a random sample of 16 boxes of Turkish delight and carefully weigh them. You obtain the following values

97.14, 107.05, 103.17, 106.27, 98.63, 90.66, 105.29, 87.12,  
112.40, 95.61, 95.19, 114.14, 94.47, 112.89, 105.80, 92.96.

Provide the 80% confidence interval for  $\mu$  based on the above data.

**Problem 10.4.** (5 points) A pollster is trying to estimate the proportion of the population in favor of candidate A (in a two-way race between A and B). The quality of her sample is such that it can be safely assumed to be a random sample from the Bernoulli distribution with the unknown parameter  $p$ . She is interested in the smallest sample size she will need in order to be able to pinpoint the value of  $p$  with  $\pm 1\%$  accuracy, with 95% confidence.

Basing your analysis on the estimator  $\hat{p} = Y/n$ , where  $Y$  is the number of supporters of candidate A in the sample, find the smallest such  $n$  under the assumption that the sampling distribution of  $\hat{p}$  is well approximated by a normal distribution (of appropriate mean and variance left for you to figure out).

**Problem 10.5.** (25 points)

Source: "Mathematical Statistics with Applications" by Wackerley, Mendenhall, and Sheaffer.

Let the random variable  $Y$  be gamma distributed with parameters  $k = 4$  and  $\tau$  unknown.

(i) (10 points) Show, using moment generating functions, that

$$U := \frac{2Y}{\tau} \sim \chi^2(df = 8).$$

(ii) (15 points) Using  $U = 2Y/\tau$  as a pivotal quantity, construct a 90%-confidence interval for  $\tau$ .

**Problem 10.6.** (10 points)

Let the random variable  $Y$  be normally distributed with mean zero and the standard deviation  $\sigma$  unknown. Using  $Y^2/\sigma^2$  as a pivotal quantity, construct a 95%-confidence interval for  $\sigma^2$ .