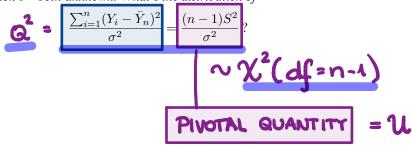
M378K: November 8th, 2024.

Problem 14.3. What is the unbiased estimator for σ^2 .

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\gamma_i - \overline{\gamma})^2$$

Problem 14.4. Assume a random sample Y_1, Y_2, \dots, Y_n from a normal distribution with mean μ and standard deviation σ - both unknown. What's the distribution of



Problem 14.5. Assume that you are assigned a confidence level $1 - \alpha$. What does it mean to find a confidence interval for S^2 ?

$$P\left[\underline{\alpha} \leq U \leq b\right] = 1-\alpha$$

$$P\left[\chi_{L}^{2} \leq \frac{(n-1)S^{2}}{\sigma^{2}} \leq \chi_{R}^{2}\right] = 1-\alpha$$

$$Q^{2}$$

Problem 14.6. Are $\hat{\chi}_L^2$ and $\hat{\chi}_U^2$ as above uniquely defined?

We choose:

$$\frac{\chi^2}{L} = 4 \text{chisq}(\frac{6}{2})$$
, $\frac{\chi^2}{R} = 4 \text{chisq}(1 - \frac{6}{2})$

Problem 14.7. What's the form of the confidence interval, then?

$$\begin{aligned}
\mathbb{P}\left[\chi_{L}^{2} \leq \frac{(n-4)S^{2}}{\sigma^{2}} \leq \chi_{R}^{2} \right] &= 1-\alpha \\
\mathbb{P}\left[\frac{1}{\chi_{L}^{2}} > \frac{\sigma^{2}}{(n-4)S^{2}} > \frac{1}{\chi_{R}^{2}} \right] &= 1-\alpha \\
\mathbb{P}\left[\frac{(n-4)S^{2}}{\chi_{R}^{2}} \leq \sigma^{2} \leq \frac{(n-4)S^{2}}{\chi_{L}^{2}} \right] &= 1-\alpha \\
\mathbb{O}\left[\frac{(n-4)S^{2}}{\chi_{R}^{2}} \leq \sigma^{2} \leq \frac{(n-4)S^{2}}{\chi_{L}^{2}} \right] &= 1-\alpha
\end{aligned}$$

Problem 14.8. Assume the above setting. Let the random sample be of size n=9. You do the arithmetic and arrive at the estimate $s^2=7.93$ (based on the data set). Using the above procedure, find the 90%-confidence interval for σ^2 .

t Distribution.

Def'n. A Student todistribution w/k degrees of freedom is the dist'n of random variable given by

w/.Z~N(0,1)

- · @2 ~ x2(k)
- · Z and Q² are independent

t. Procedures.

Consider a random sample $Y_1, ..., Y_n$ from $N(\mu, \sigma)$ w/ both parameters unknown.

Goal: Confidence intervals for M

$$\frac{\overline{Y} - H}{\overline{Y} - H}$$
 as a pivotal quantity.

 $\frac{\overline{Y} - H}{\overline{Y} - H}$
 $\frac{\overline{Y} - H}{\overline{Y} - H}$

To construct a confidence interval w/ the confidence level 1-a

$$t_{R}^{*} = qt(\frac{0}{2})$$
 $t_{R}^{*} = qt(\frac{1-\frac{0}{2}}{2})$

$$-t_{L}^{*}=t_{R}^{*}=t^{*}$$