UNIVERSITY OF TEXAS AT AUSTIN

Quiz #25

Barrier options.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 25.1. (2 points) The price of a **up-and-in** option is increasing as a function of its barrier (with every other input held fixed). *True or false?*

Solution: FALSE
This was discussed in connection with the sample IFM problem #42.

Problem 25.2. (5 points) The current price of a certain stock is denoted by $S(0)$. Let the price of an **up-and-out** call option on that stock with barrier H be denoted by $V_C(0,H)$. Let the price of an otherwise identical vanilla call option be denoted by $V_C(0)$. With the limit is taken while all the other parameters are held the same, what is

$$\lim_{H\to\infty} V_C(0,H)$$

equal to?

(a) 0
(b) $V_C(0)$
(c) $S(0)$
(d) ∞
(e) None of the above.

Solution: (b)
This was discussed in connection with the sample IFM problem #42.

Problem 25.3. (8 points) Consider a non-dividend paying stock whose current price is \$100 per share. Its volatility is given to be 0.30. You model the evolution of the stock price over the following year using a two-period forward binomial tree.

The continuously-compounded, risk-free interest rate is 0.05.

Consider a \$100-strike, one-year **knock-in** call option with a barrier of \$110 on the above stock. What is the price of this option consistent with the above stock-price model?

(a) About 11.55
(b) About 12.75
(c) About 13.96
(d) About 66.05
(e) None of the above.

Solution: (b)
The up and down factors in the above forward binomial tree are

$$u = e^{0.025+0.3/\sqrt{2}} = 1.2676, \quad d = 0.8293.$$

The option is knocked-in only if the stock price goes up in the first step. So, the payoff of the option will be

$$\begin{aligned} V_{uu} &= 60.7, && \text{if the path } \textit{up-up} \text{ is taken,} \\ V_{ud} &= 5.14, && \text{if the path } \textit{up-down} \text{ is taken,} \\ V_{du} = V_{dd} &= 0, && \text{otherwise.} \end{aligned}$$

The risk-neutral probability of a single step up in the tree equals

$$p^* = \frac{1}{1+e^{0.3/\sqrt{2}}} = 0.4472.$$

So, the option price is

$$V(0) = e^{-rT}[(p^*)^2 V_{uu} + p^*(1-p^*)V_{ud}] = 12.75$$