

## UNIVERSITY OF TEXAS AT AUSTIN

Quiz #2

Prerequisite material.

**Problem 2.1.** (2 pts) If  $X$  and  $Y$  are independent random variables, then

$$F_{X+Y}(a) = F_X(a) \cdot F_Y(a).$$

*True or false?***Solution: FALSE****Problem 2.2.** (2 points) Let  $X$  be a normal random variable with parameters  $(\mu = 2, \sigma^2 = 1)$ , and let  $Y$  be a normal random variable with parameters  $(\mu = -2, \sigma^2 = 1)$ . Assume that  $X$  and  $Y$  are independent. Then, the variance of the random variable  $X + Y$  equals 2. *True or false?***Solution: TRUE**

See the “Addition rule for variances”.

**Problem 2.3.** (2 points) In our usual notation, let  $S(0) = 40, r = 0.08, \sigma = 0.3, \delta = 0$ . You need to construct a 2-period forward binomial tree for the above stock with every period in the tree of length  $h = 0.5$ . Then,  $u > 1.45$ . *True or false?***Solution: FALSE**

$$u = \exp\{(0.08 - 0) \cdot 0.5 + 0.3\sqrt{0.5}\} \approx 1.29.$$

**Problem 2.4.** (2 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the  $\Delta$  in the replicating portfolio of a single call option on that stock never exceeds 1. *True or false?***Solution: TRUE**The call's  $\Delta$  will always be between 0 and 1.**Problem 2.5.** (8 points)Let  $X$  be a continuous random variable with probability density function  $f_X(x)$ . Let its cumulative distribution function be denoted by  $F_X(x) = \mathbb{P}[X \leq x]$ . Define the new random variable  $Y$  as

$$Y = F_X(X).$$

Find  $\mathbb{E}[Y]$ .**Solution:** The connection between the probability density function and the cumulative distribution function is

$$f_X(x) = F'_X(x).$$

Using the definition of the expected value of a function of a continuous random variable, we get

$$\mathbb{E}[Y] = \mathbb{E}[F_X(X)] = \int_{-\infty}^{\infty} F_X(x) f_X(x) dx.$$

We can use a change of variables  $u = F_X(x)$  with which  $du = f_X(x) dx$ . So, our result is

$$\mathbb{E}[Y] = \int_0^1 u du = 1/2.$$