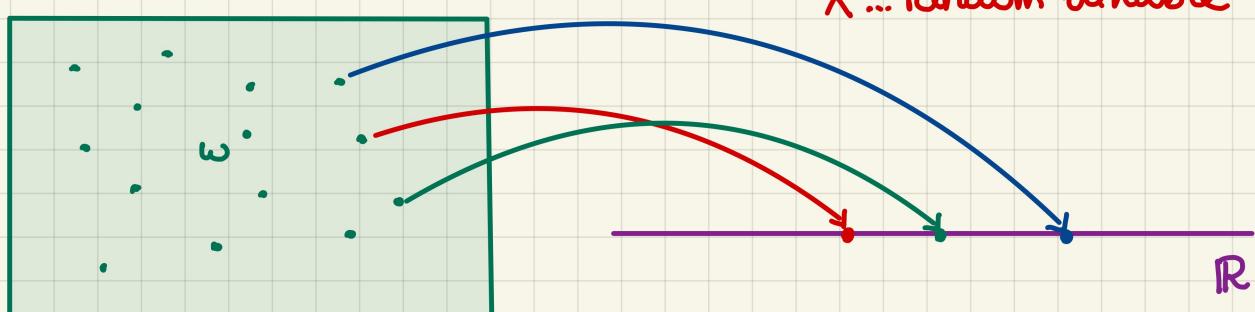


M339J: January 21st, 2022.

Probability Review.



Probability Space Ω
(aka outcome space, sample space \mathcal{S})

ω ... elementary outcomes

E ... event, i.e., a "nice" subset of the probability space

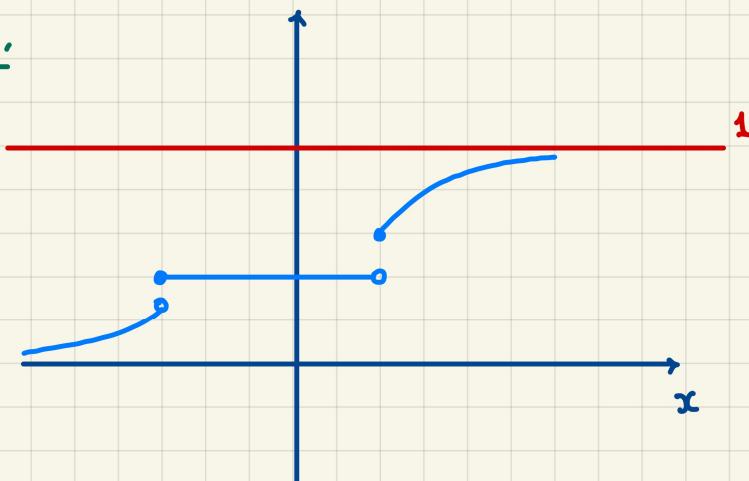
Def'n. The cumulative distribution function (cdf) of a random variable X is a function F_X such that

$$F_X: \mathbb{R} \longrightarrow [0, 1]$$

given by

$$F_X(x) = P[X \leq x] \quad \text{for all } x \in \mathbb{R}.$$

Example.



Properties:

- Non-decreasing, i.e.,

$$x_1 < x_2 \Rightarrow F_X(x_1) \leq F_X(x_2)$$

- Right continuous w/ left limits
- $\lim_{x \rightarrow -\infty} F_X(x) = 0$
- $\lim_{x \rightarrow +\infty} F_X(x) = 1$

Def'n. The **survival function** of a random variable X is the function $S_X : \mathbb{R} \rightarrow [0, 1]$ given by

$$S_X(x) = 1 - F_X(x) = \mathbb{P}[X > x] \quad x \in \mathbb{R}$$

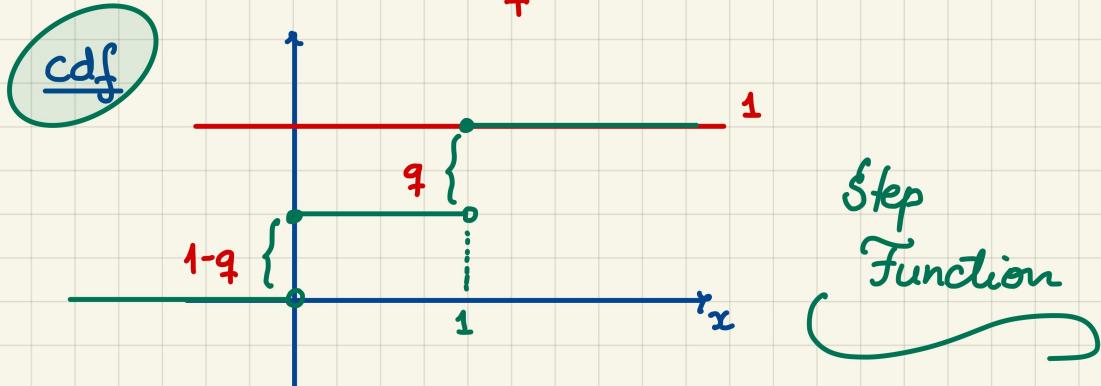
"Def'n." The **support** of a random variable X is the set of all the values it can take.

Def'n. A random variable is called **discrete** if its support has @ most countably many values.

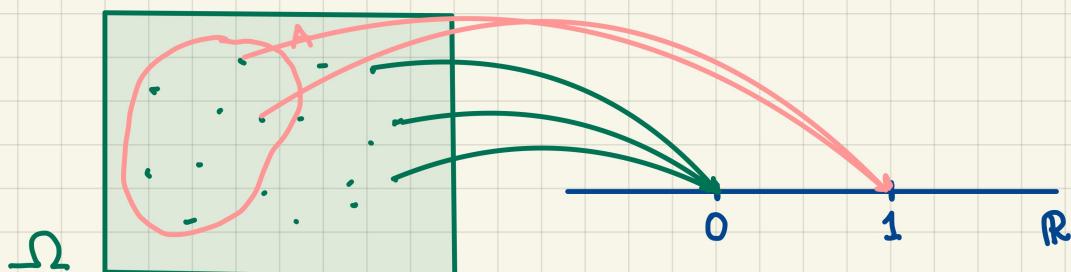
Example. • Bernoulli .

$$\text{Support} = \{0, 1\}$$

$$X \sim \begin{cases} 0 & \text{w/ } 1-q \\ 1 & \text{w/ } q \end{cases}$$



• Indicator Random Variable .



$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

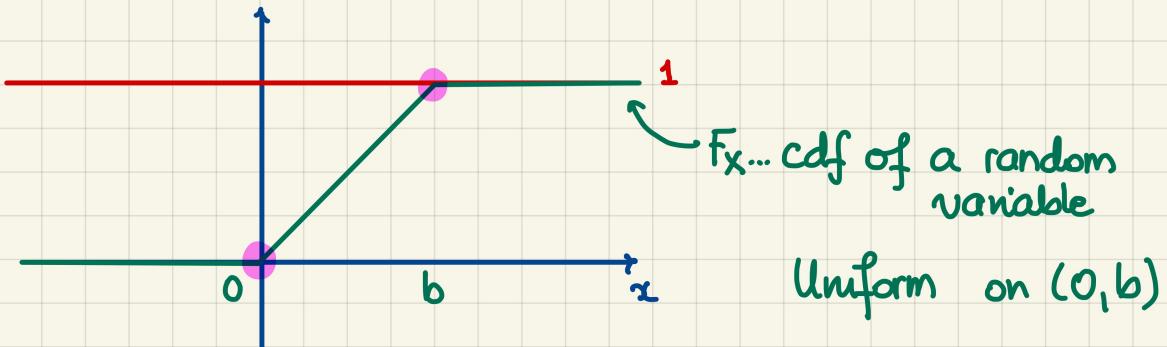
$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A \approx \begin{cases} 1 & \text{w/ probability } P[A] \\ 0 & \text{w/ - } 1 - P[A] \end{cases}$$

Def'n. A random variable X is continuous if its cdf is:

- (a) continuous, and
- (b) differentiable everywhere except @ at most countably many points.

Example.



Def'n. A random variable X is said to be mixed if:

- (a) it's not discrete;
- (b) its cdf is continuous everywhere except @ at least one point and at most countably many points;
- (c) its cdf is differentiable everywhere except @ at most countably many points.

Example.

