University of Texas at Austin

# Problem set 4

## Short sales.

**Problem 4.1.** Bertram Wooster short sells two shares of stock whose initial price is \$80 per share. The stock does not pay any dividends.

The continuously compounded, risk-free interest rate is 0.03.

In three months, Bertram closes the short sale. At that time, the stock price is \$78. What is Bertram's loss/gain?

**Solution:** The initial cost equals -\$80. Since the final asset price is revealed, we know the exact value of the payoff Bertram "received". It's -\$78. Hence, the profit equals

$$-78 + 80e^{0.03/4} = 2.602256$$

So, Bertram has a **gain** of about \$2.60.

**Problem 4.2.** Nick Mallory dabbles in the stock market in his spare time. He short-sells one share of continuous-dividend-paying stock whose current price is \$100 per share. The stock's dividend yields is 0.02.

Upon the short sale, Nick invests the proceeds at the risk-free interest rate of 0.04. What is the expression for the profit Nick will have at time$-1$ when he closes his short-sale as a function of the final stock price $s$? What is the maximum loss? What is the maximum gain? What is the break-even point?

**Solution:** The initial cost of this investment is

$$-S(0) = -100.$$

In our usual notation, the payoff function is

$$v(s) = -se^{\delta T}$$

with $\delta = 0.02$ and $T = 1$. So, taking into account that the continuously compounded, risk-free interest rate equals 0.04, the profit function can be written as

$$-se^{0.02} + 100e^{0.04}$$

There is an **unlimited loss potential** for this investment. The **maximal gain** occurs for $s = 0$ and equals $100e^{0.04}$. The **break-even point** is the solution to

$$-se^{0.02} + 100e^{0.04} \quad \Rightarrow \quad s^* = 100e^{0.04-0.02} = 100e^{0.02}.$$