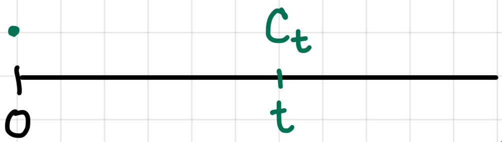
M339W: October 27, 2021.

"Corporate Finance (4th Ed)" by Berk/DeMarzo

Analyzing a Project

Our criterion (w/out considering risk for now!)

Recall: Interest Theory   Maximizing the Net Present Value



Now: Estimates of Incremental Cashflows.

$$NPV = \sum_t PV_{0,t}(C_t)$$

e.g., expectation under a probability measure

Depends on the cost of capital

Notation: r ... effective annual

Break-even analysis: keep all but one of the inputs fixed and then figure out the value of the remaining input for which the NPV is zero.

e.g., break-even points of options in M339D;

e.g., with all the cashflows fixed, we can look for the interest rate @ which the NPV is zero; this interest rate is called the internal rate of return (or the yield rate)

27) Consider a two-year project, where the cost of capital is 10%. $r = 0.10$ (EFFECTIVE?)

There are only three cash flows for this project.

- The first occurs at $t = 0$, and is -100.
- The second occurs at $t = 1$, and is 66.
- The third occurs at $t = 2$, and is X.

$\Rightarrow$ NPV = 0

Determine X, the level of the cash flow at $t = 2$, that leads to the project breaking even.

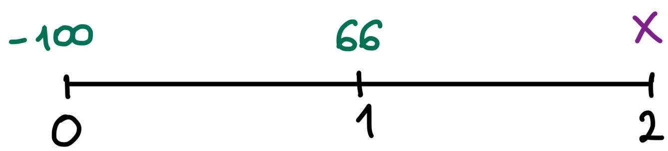
(A) 34.0

(B) 38.4

(C) 44.0

(D) 48.4

(E) 54.0



$$-100 + 66(1.1)^{-1} + X(1.1)^{-2} = 0$$

$$\Rightarrow \quad X = 100(1.1)^2 - 66(1.1) =$$

$$= 121 - 72.6 = 48.4 \Rightarrow \text{(D)}$$

The expected return of a portfolio

Say that your portfolio has n different securities in it,

$i = 1..n$ the indices of the investment components in your portfolio

For every i : R_i .. the realized (simple) return of the ith component over a particular time period (say, a year)

R_P... the realized return of the entire portfolio

beg end

$$R_P := \frac{P_P^{end} - P_P^{beg}}{P_P^{beg}}$$

Compare to the notion of the effective interest rate in interest theory.

w/ P_P... the price of the total portfolio, i.e.,

$$P_P = \sum_{i=1}^{n} \underbrace{P_i}_{\text{value of the component } i}$$

$$R_P = \frac{\sum_{i=1}^{n} P_i^{end} - \sum_{i=1}^{n} P_i^{beg}}{P_P^{beg}} = \sum_{i=1}^{n} \frac{P_i^{end} - P_i^{beg}}{P_P^{beg}} \cdot \frac{P_i^{beg}}{P_i^{beg}}$$

$$= \sum_{i=1}^{n} \underbrace{\frac{P_i^{beg}}{P_P^{beg}}}_{= w_i} \cdot \underbrace{\frac{P_i^{end} - P_i^{beg}}{P_i^{beg}}}_{= R_i}$$

w_i... portfolio weight of investment i (deterministic)

$$R_P = \sum_{i=1}^{n} w_i \cdot R_i$$

$\Rightarrow$ the expected return: $\boxed{\mathbb{E}[R_P] = \sum_{i=1}^{n} w_i \mathbb{E}[R_i]}$ ✓

4) You are given the following information about a portfolio consisting of stocks X, Y, and Z:

Stock	Investment	Expected Return
X	10,000	8%
Y	15,000	12%
Z	25,000	16%

$\Sigma = 50K$

Calculate the expected return of the portfolio.

$w_X = 0.2$, $w_Y = 0.3$, $w_Z = 0.5$

$\mathbb{E}[R_P] = 0.2(0.08) + 0.3(0.12) + 0.5(0.16)$

$= 0.132.$

(A) 10.8%

(B) 11.4%

(C) 12.0%

(D) 12.6%

(E) 13.2%

The Volatility of a Two-Stock Portfolio

We index the securities in this portfolio w/ $i = 1, 2$.

R_i, $i = 1, 2$ ... the (simple) realized return of security i

w_i, $i = 1, 2$ ... the weight of security i in the portfolio

$$\boxed{w_i = \frac{\text{Value of component } i}{\text{Value of the portfolio}}}$$

R_P ... return of the entire portfolio

$$\boxed{R_P = w_1 \cdot R_1 + w_2 \cdot R_2}$$

$$\Rightarrow \mathbb{E}[R_P] = w_1 \cdot \mathbb{E}[R_1] + w_2 \cdot \mathbb{E}[R_2]$$

Goal: Focus on the volatility of a portfolio, i.e., the standard deviation of the return.

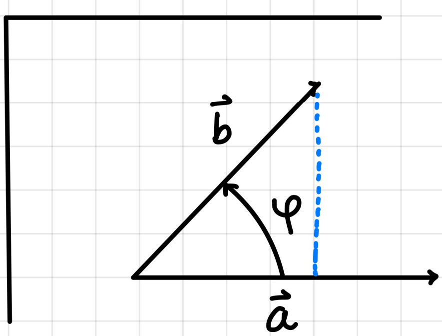
$$\longrightarrow: \quad \mathrm{Var}[R_P] = \mathrm{Var}[w_1 \cdot R_1 + w_2 \cdot R_2]$$

$$= w_1^2 \cdot \mathrm{Var}[R_1] + w_2^2 \cdot \mathrm{Var}[R_2]$$

$$+ 2 w_1 \cdot w_2 \cdot \mathrm{Cov}[R_1, R_2]$$

By def'n. $\mathrm{Cov}[R_1, R_2] = \sigma_1 \cdot \sigma_2 \cdot \rho_{1,2}$

$$\boxed{\rho_{1,2} = \mathrm{corr}[R_1, R_2]}$$



scalar product:

$$\langle \vec{a}, \vec{b} \rangle \overset{\mathrm{Cov}}{=} \vec{a} \cdot \vec{b}$$

$$= \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos(\varphi)$$

sd sd corr

The volatility of the portfolio: $\boxed{\sigma_P = \sqrt{\mathrm{Var}[R_P]}}$

3) You are given the following information about the annual returns of two stocks, X and Y:

i) The expected returns of X and Y are $E[R_X] = 10\%$ and $E[R_Y] = 15\%$.

ii) The volatilities of the returns are $\cancel{V}_X^{\sigma_X} = 18\%$ and $\cancel{V}_Y^{\sigma_Y} = 20\%$.

iii) The correlation coefficient of the returns for these two stocks is 0.25. $\rho_{X,Y} = 0.25$

iv) The expected return for a certain portfolio, consisting only of stocks X and Y, is 12%.

$$R_P = w_X \cdot R_X + w_Y \cdot R_Y$$

Calculate the volatility of the portfolio return.

$$Var[R_P] = \underline{w_X^2} \cdot \sigma_X^2 + \underline{w_Y^2} \cdot \sigma_Y^2 + 2 \cdot \underline{w_X \cdot w_Y} \cdot \sigma_X \cdot \sigma_Y \cdot \rho_{X,Y}$$

(A) 10.88%

(B) 12.56%

(C) 13.55%

(D) 14.96%

(E) 16.91%

$$\underline{0.12} = w_X \cdot E[R_X] + w_Y \cdot E[R_Y]$$

$$= w_X \cdot \underline{0.10} + (1 - w_X) \cdot \underline{0.15}$$

$$w_X = \frac{0.15 - 0.12}{0.15 - 0.10} = 0.6 \Rightarrow w_Y = 0.4$$

$$Var[R_P] = (0.6)^2 \cdot (0.18)^2 + (0.4)^2 \cdot (0.20)^2 + 2 \cdot 0.6 \cdot 0.4 \cdot 0.18 \cdot 0.20 \cdot 0.25$$

$$= 0.022384 \Rightarrow \sigma_P = 0.1496$$