M339D: March 3rd, 2021.

Partially Leveraged purchase.

At time 0:

- borrow $\varphi \cdot S(0)$ $\longrightarrow$ get $\varphi \cdot S(0)$
- buy one share of stock $\longrightarrow$ give up $S(0)$

$\Rightarrow$ Initial Cost: $S(0) - \varphi \cdot S(0) = S(0)(1-\varphi)$

At time T:

- pay back $\boxed{\varphi \cdot S(0) \cdot e^{rT}}$ $\longrightarrow$ out
- own $e^{\delta \cdot T}$ shares of stock
  $\Rightarrow$ the shares' worth is $\boxed{e^{\delta \cdot T} \cdot S(T)}$ $\rightarrow$ in

$\Rightarrow$ Payoff: $+e^{\delta \cdot T} \cdot S(T) - \varphi \cdot S(0) e^{rT}$

$\Rightarrow$ Profit = Payoff $- FV_{0,T}$(Initial Cost)

$$= e^{\delta \cdot T} \cdot S(T) - \varphi \cdot S(0) e^{rT} - S(0)(1-\varphi)e^{r \cdot T}$$

$$= e^{\delta \cdot T} \cdot S(T) - \cancel{\varphi \cdot S(0) e^{rT}} - S(0)e^{r \cdot T} + \cancel{S(0)\,\varphi \cdot e^{rT}}$$

Profit $= e^{\delta \cdot T} S(T) - S(0)e^{r \cdot T}$

Forward Contracts. [Review]

* A binding contract for both sides! *

0 ———————————————————— T ... the delivery date

Agreement, but no cashflows!

$\Downarrow$

Initial Cost = 0

$\Downarrow$

$\boxed{\text{Profit} = \text{Payoff}}$

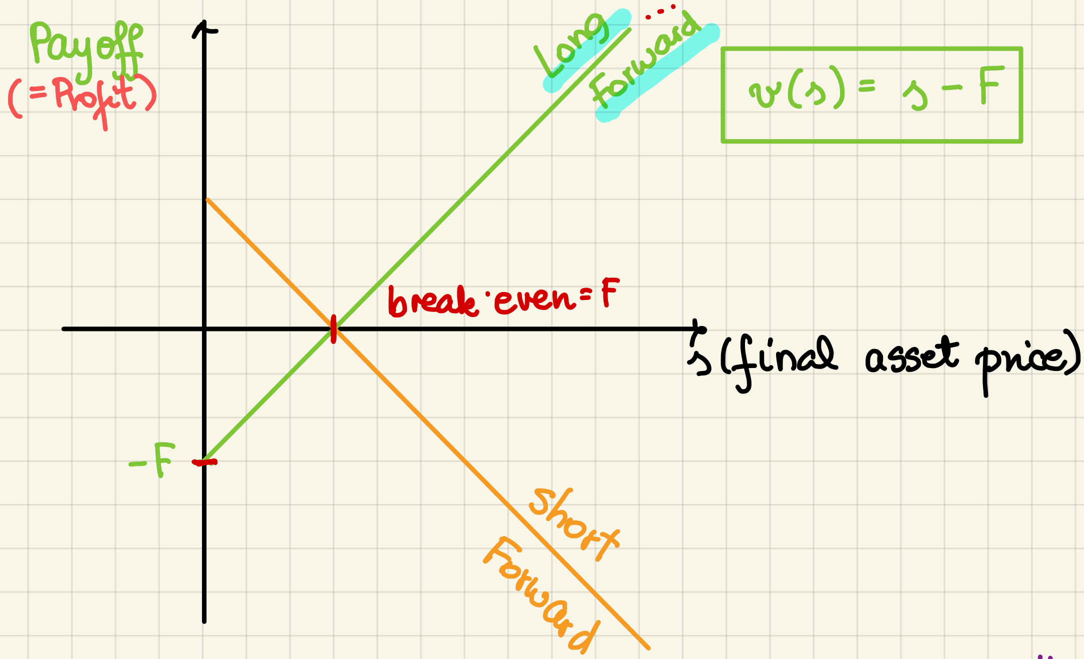
Long Forward : Buying Forward

$+S(T) \uparrow \quad \downarrow F$ ... the forward price

Short Forward : Selling Forward

Payoff (LONG) = $S(T) - F$

Payoff (SHORT) = $F - S(T)$



$\boxed{v(s) = s - F}$ is the payoff function

Problem. [IFM Sample Problems : Part I : Intro : P#56]

Determine which of the following positions has the cashflows as a short position in a non-dividend-paying stock :

(i) Long Forward and Long zero-coupon bond — Init. Cost > 0 ✗

(ii) Long Forward and short forward — Init. Cost = 0 ✗

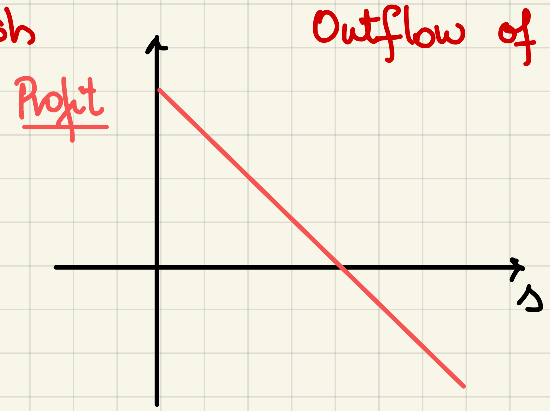
(iii) $\boxed{\text{Long Forward}}$ and short zero-coupon bond ✗ — Long w.r.t. the underlying

(iv) short Forward and Long zero-coupon bond — Init. Cost > 0 ✗

(v) short Forward and short zero-coupon bond

Initial cost : $-S(0)$ $\Downarrow$ Inflow of cash

Payoff : $-S(T)$ $\Downarrow$ Outflow of cash



short w.r.t. the underlying

Focus on: short forward
+
short zero-coupon bond

	Initial Cost	Payoff
short forward	0	$F - S(T)$
short bond	$-P$	$-Pe^{r \cdot T}$
Total	$-P$	$F - S(T) - Pe^{rT}$
Short Sale	$-S(0)$	$-S(T)$

$\left.\begin{matrix} F - S(T) - Pe^{rT} \\ -S(T) \end{matrix}\right\}$ Match them!

$\Downarrow$

The bond needs to have the redemption amount:

$$\boxed{F = Pe^{rT}}$$

Ponder this a bit..

P... bond price

Pe^{rT}... redemption amount