M378K Introduction to Mathematical Statistics
Homework assignment #10

Please, provide your **final answer only** to the following problems.

Problem 10.1. *(5 points) Let $Y_1, \dots, Y_5$ be a random sample from the normal distribution $N(\mu, 2)$, with an unknown mean μ and the known standard deviation $\sigma = 2$. The collected data turn out to be*

$$y_1 = 2, y_2 = 5, y_3 = 1, y_4 = 4, y_5 = 3.$$

The right end-point of the one-sided 90%-confidence interval $(-\infty, \hat{\mu}_R]$ for μ is

(a) $3 + \frac{2}{\sqrt{5}} qnorm(0.9, 0, 1)$.

(b) $3 + \frac{2}{5} qnorm(0.9, 0, 1)$.

(c) $3 + \frac{1}{\sqrt{5}} qt(0.9, 4)$.

(d) $3 + \frac{1}{5} qnorm(0.9, 5)$.

(e) **None of the above.**

Problem 10.2. *(5 points) A random sample of size $n = 5$ from the normal distribution with unknown mean μ and an unknown standard deviation σ yielded the values $y_1, \dots, y_5$ such that*

$$\sum y_i = 10 \text{ and } \sum_{i=1}^{5} (y_i - 2)^2 = 4.$$

The value of $\hat{\mu}_L$ such that $(\hat{\mu}_L, \infty)$ is (an asymmetric) 95%-confidence interval for μ is

(a) $2 - \frac{1}{\sqrt{5}}$ `qt(0.95,4)`

(b) $2 - \frac{1}{5}$ `qnorm(0.95)`

(c) $2 - \frac{1}{5}$ `qt(0.95,4)`

(d) $2 - \frac{1}{\sqrt{5}}$ `qnorm(0.95)`

(e) $2 - \frac{1}{\sqrt{5}}$ `qt(0.975,4)`

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 10.3. *(10 points) 5 astronomy teams from across the world measured the distance to Proxima Centauri using a new method. It is reasonable to assume that the error of this method is normally distributed, but, since it is new, there is no information about its standard variation. Find a 95%-confidence interval for the distance if the obtained measurements are (in light years)*

$$4.20, 4.16, 4.04, 4.01, 4.20.$$

What would your confidence interval look like if they used an established method whose standard deviation of the measurement error is 0.1?

Problem 10.4. *(30 points) Let $Y_1, \ldots, Y_n$ be a random sample from $U(0, \theta)$ with θ unknown. Consider the following two estimators:*

$$\hat{\theta}_1 = 2\bar{Y} \quad \text{and} \quad \hat{\theta}_2 = \left(\frac{n+1}{n}\right) Y_{(n)}$$

(i) (5 points) Prove that $\hat{\theta}_1$ is unbiased.

(ii) (10 points) Prove that $\hat{\theta}_2$ is unbiased.

(iii) (15 points) Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$.