M339J *Probability models*
University of Texas at Austin
HW #5
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Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 5.1. (5 pts) The ground-up loss X is modeled by an exponential distribution with mean \$500. There is an ordinary deductible of $d = 200$. What is the expected value of the **per-loss** random variable?

Problem 5.2. (5 points) Let the severity random variable X be modelled using the Pareto distribution with parameters $\theta = 0.5$ and $\alpha = 6$. For a particular value of the ordinary deductible d, the expected value of the per-payment random variable Y^P is 10. What is the value of the deductible?

Problem 5.3. (10 points) For a random variable X and for a positive constant d, in our usual notation, we have

$$\mathbb{E}[X] = e_X(d)S_X(d) + \mathbb{E}[X \wedge d]\,. \tag{5.1}$$

True or false? Why?

Problem 5.4. (5 points) Let $X \sim Pareto(\alpha = 3, \theta = 3000)$. Assume that there is a deductible of $d = 5000$. Find $\mathbb{E}[X \wedge d]$.

Problem 5.5. (10 points) Let X be the ground-up loss random variable whose density is given by

$$f_X(x) = \begin{cases} 0.01, & 0 \leq x \leq 80, \\ 0.03 - 0.00025x, & 80 < x \leq 120. \end{cases}$$

Let there be an ordinary deductible of $d = 20$.
Calculate $\mathbb{E}[X \wedge d]$.

Problem 5.6. (5 points) Let the ground-up loss X be exponentially distributed with mean \$800. An insurance policy has an ordinary deductible of \$100 and the maximum amount payable per loss of \$2500.
Find the expected value of the amount paid (by the insurance company) **per positive payment.**

Problem 5.7. (10 points) *Source: Problem 4.3 from "Loss Models".*
Assume that the claims r.v. X has a Pareto distribution with $\alpha = 2$ and θ unknown. Claims for the following year are denoted by Y and will experience uniform inflation of 6%.

(i) (2 points) Find the expression for the probability $\mathbb{P}[X > d]$ in terms of d and θ.
(ii) (3 points) Find the expression for the probability $\mathbb{P}[Y > d]$ in terms of d and θ.
(iii) (5 points) Find the expression for the ratio $\rho(d) = \frac{\mathbb{P}[X>d]}{\mathbb{P}[Y>d]}$ in terms of d and θ. Find the limit of $\rho(d)$ as $d \to \infty$.