

M339D: April 9th, 2021.

Bounds on call/put prices.

Calls. Lower bound:

$$V_c(t) \geq \max(F_{t,T}^P(S) - PV_{t,T}(K), 0)$$

Q: What if the above inequality is violated?

→: Case #1. $V_c(t) < 0 \Rightarrow$ buy the call

Case #2. $V_c(t) < F_{t,T}^P(S) - PV_{t,T}(K)$

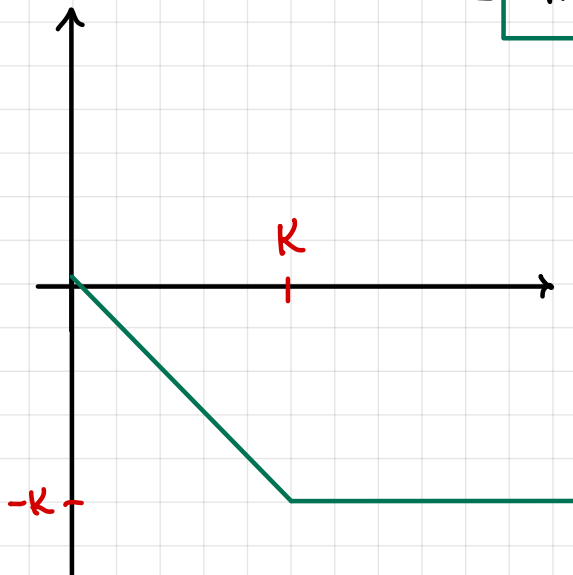
We propose to:

- long the call
- short the prepaid forward

\Rightarrow Cost @ time t : $V_c(t) - F_{t,T}^P(S) < -PV_{t,T}(K)$

Payoff @ time T : $(S(T) - K)_+ + (-S(T))$

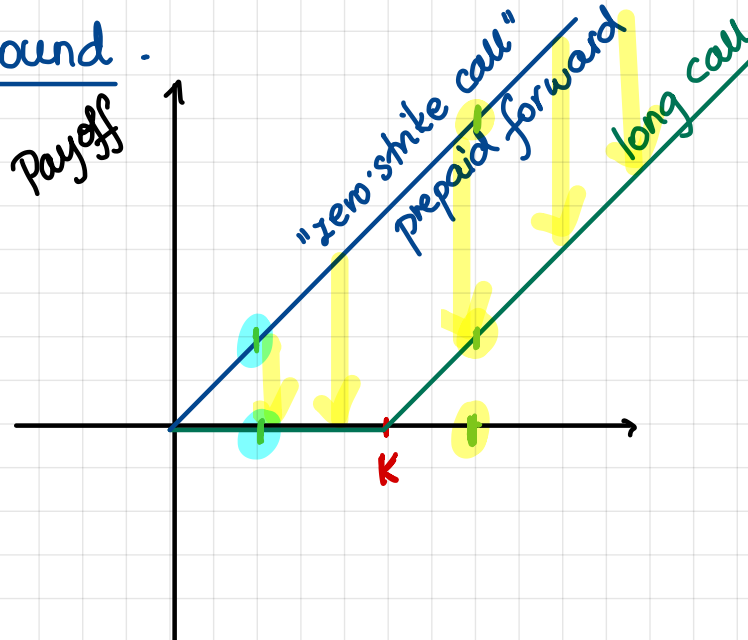
$$= -\min(S(T), K) \geq -K$$



Profit $= \underbrace{-\min(S(T), K)}_{\geq -K} - FV_{t,T}(\text{Cost @ time } t)$

$$\geq -K + FV_{t,T}(PV_{t,T}(K)) = 0$$

Upper bound .



The payoff of the prepaid forward DOMINATES the payoff of a call option.



$$F_{t,T}^P(S) \geq V_c(t)$$

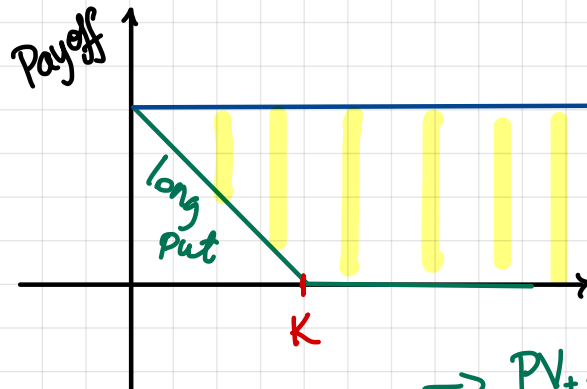
Puts.

Lower bound: • $V_p(t) \geq 0$

Put-call parity: • $V_p(t) \geq -F_{t,T}^P(S) + PV_{t,T}(K)$

$$\Rightarrow V_p(t) \geq \max(PV_{t,T}(K) - F_{t,T}^P(S), 0)$$

Upper bound:



K { zero-coupon bond w/ maturity @ T and redemption amt K

$$\Rightarrow PV_{t,T}(K) \geq V_p(t)$$

for ccrfir r : $Ke^{-r(T-t)} \geq V_p(t)$

Q: Let $T \rightarrow \infty$. What happens to the European call/put prices in the limit? Assume $r > 0$.

→:

$$Ke^{-r(T-t)} \geq V_p(t) \geq 0$$

↓
0

$$\Rightarrow V_p(t) \xrightarrow{T \rightarrow \infty} 0$$

Assume $\delta > 0$:

$$S(t)e^{-\delta(\tau-t)} \geq V_c(t) \geq 0$$

\downarrow
0

$$\Rightarrow V_c(t) \xrightarrow[\tau \rightarrow \infty]{} 0$$

Call/Put prices as functions of the strike.

Assume that all other inputs are fixed.

Fix the valuation date @ $t=0$; suppress it from the notation.

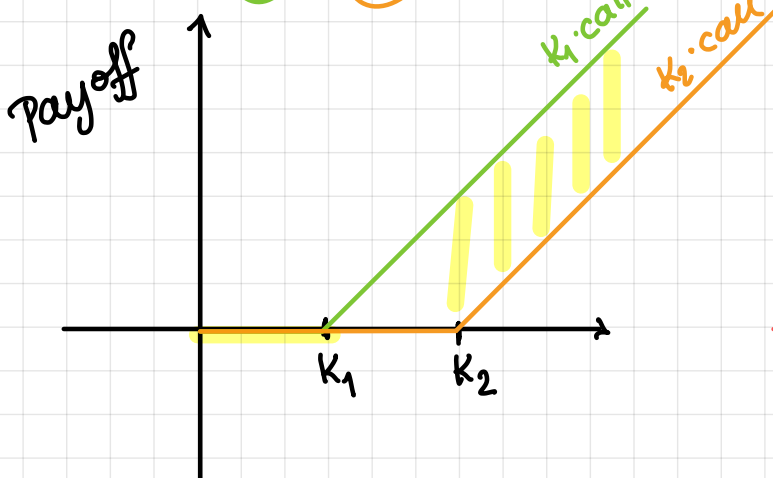
We vary the strike K , so it makes sense to write

$$\left. \begin{array}{l} V_c(K) \dots \text{call} \\ V_p(K) \dots \text{put} \end{array} \right\} \text{price w/ strike } K$$

Monotonicity.

Calls

Let $K_1 < K_2$. We want to try to compare $V_c(K_1)$ and $V_c(K_2)$.



The payoff of the K_1 -strike call dominates the payoff of the K_2 -strike call.



$$V_c(K_1) \geq V_c(K_2)$$

European call prices are DECREASING w/ respect to the strike price.

Q: What if this inequality is violated?
What would you do?