M339D : February 21st, 2022.

Dynamic Portfolios.

C_k ... intermediate cashflow $\begin{cases} >0 & \text{inflow} \\ <0 & \text{outflow} \end{cases}$

Initial Cost

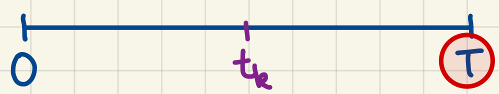
Payoff

$$\boxed{\text{Profit} := \text{Payoff} - FV(\text{Init. Cost}) + \sum_k FV_{t_k,T}(C_k)}$$

This is how we generalize profit!

Example. [Discrete·Dividend·Paying Stocks]

D_k ... dividend amounts



$k = 1..n$, $\boxed{t_n \leq T}$

Consider an outright purchase of this stock!

At time·0 : buy 1 share , i.e., spend $S(0)$, i.e., Init. Cost $= S(0)$

At all times·t_k : get D_k $\quad k=1..n$

At time·T: sell the 1 share , i.e. get $S(T)$, i.e., Payoff $= S(T)$

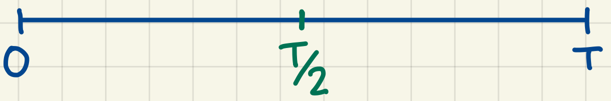
$$\text{Profit} = S(T) - FV_{0,T}(S(0)) + \boxed{\sum_k FV_{t_k,T}(D_k)}$$

r = ccrfir

$$= S(T) - e^{rT}\cdot S(0) + \sum_k \left(e^{r(T-t_k)}\cdot D_k\right)$$

■

Example.



You buy 1 share of a nondividendpaying stock @ time·0.

You plan to rebalance @ time·T/2 after you see the stock price @ that time.

Recipes:

- Sell is $S(T/2)$ is above a particular threshold; maybe depending on $S(0)$.
- Buy if the price falls below a threshold.

$$\text{Profit} = \begin{cases} S(T/2)e^{r(T/2)} - S(0)e^{rT}, & \text{if } S(T/2) > S(0) \\ S(T) - S(0)e^{rT}, & \text{if } S(T/2) \leq S(0) \end{cases}$$

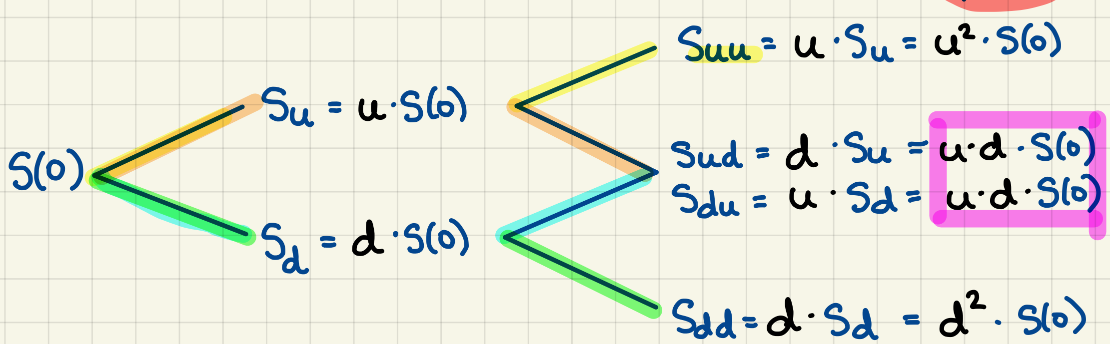
Finite Probability Spaces.

... serve as environments for possible paths that the price of our asset can take.

e.g.,

$$S(T) = \begin{cases} 120 & \text{w/ prob. } 1/6 \\ 100 & \text{w/ prob. } 1/3 \\ 110 & \text{w/ prob. } 1/2 \end{cases}$$

e.g.,



All of the finitely many possible scenarios are called states·of·the·world.

We assume that:

- each can happen, i.e., its probab > 0

and

- they exhaust all possibilities, i.e., $\sum \text{probab} = 1$ ∎