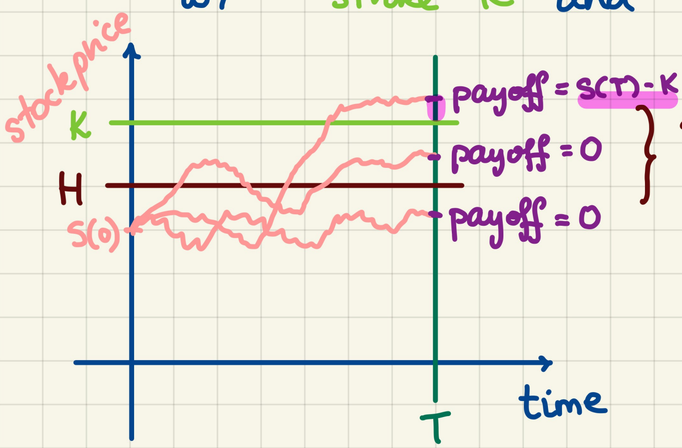
M339D: May 2nd, 2022.

Barrier Options [cont'd].

Family of options: ______ and ______ ______
up/down in/out call/put

$\Rightarrow$ 8 specific type

Example. An up-and-in call w/ strike K and barrier H. Say, $\boxed{S(0) < H}$



since $\boxed{H < K}$, this option is equivalent to a vanilla call w/ strike K.

In general: the payoff is

$$\boxed{V(T) = (S(T)-K)_+ \cdot \mathbb{I}_{[M(T) \geq H]}}$$

Example. An up-and-out put will have the payoff:

$$\boxed{V(T) = (K-S(T))_+ \cdot \mathbb{I}_{[M(T) < H]}}$$

Q: Consider an up-and-out call w/ $K > H$.
What can you say about it?

PAYOFF $\equiv 0$ $\Rightarrow$ PRICE $\equiv 0$

Q: Consider this portfolio:

- up-and-in option
- up-and-out option

otherwise identical

What's the payoff of this portfolio?

$\rightarrow$: Say, it's a call. Denote the "vanilla" payoff of the call by $V_c(T)$.

Then, the payoff of the portfolio will be:

$$V_c(T) \cdot \mathbb{I}_{[M(T) \geq H]} + V_c(T) \cdot \mathbb{I}_{[M(T) < H]} =$$

$$= V_c(T) \Big(\underbrace{\mathbb{I}_{[M(T) \geq H]} + \mathbb{I}_{[M(T) < H]}}_{1} \Big) = V_c(T)$$

2. You have observed the following monthly closing prices for stock XYZ:

Date	Stock Price
January 31, 2008	105
February 29, 2008	120
March 31, 2008	115
April 30, 2008	110
May 31, 2008	115
June 30, 2008	110
July 31, 2008	100
August 31, 2008	90
September 30, 2008	105
October 31, 2008	125
November 30, 2008	110
December 31, 2008	115

The following are one-year European options on stock XYZ. The options were issued on December 31, 2007.

(i) An arithmetic average Asian call option (the average is calculated based on monthly closing stock prices) with a strike of 100. $K = 100 \Rightarrow$ Payoff: $(A(T)-K)_+ = 10$

(ii) An up-and-out call option with a barrier of 125 and a strike of 120. $H_1 = 125$ $K_1 = 120$ Payoff: 0

(iii) An up-and-in call option with a barrier of 120 and a strike of 110. $H_2 = 120$ knocked-in $K_2 = 110$ Payoff: $115-110 = 5$

Calculate the difference in payoffs between the option with the largest payoff and the option with the smallest payoff.

(A) 5

(B) 10

(C) 15

(D) 20

(E) 25

(i) $A(T) = 100 + \frac{1}{12}\left(5+20+15+10+15+\cancel{10}+0+\cancel{(-10)}+5+25+10+15\right)$

$A(T) = 100 + \frac{1}{12}(120) = 100+10$

$\mathbb{I}_{[M(T)<H]}$

42. Prices for 6-month 60-strike European up-and-out call options on a stock S are available. Below is a table of option prices with respect to various H, the level of the barrier. Here, $S(0) = 50$.

H	Price of up-and-out call
60	0
70	$\boxed{0.1294}$
80	$\boxed{0.7583}$
90	1.6616
∞	$\boxed{4.0861}$

Q: Why is this price exactly 0?
H=60, K=60
⇒ PAYOFF=0

Q: What does H=∞ mean?
A "regular" call option.

Q: Why are the option prices increasing w/ H?

It's less likely to get KNOCKED OUT.

Consider a special 6-month 60-strike European "knock-in, partial knock-out" call option that knocks in at $H_1 = 70$, and "partially" knocks out at $H_2 = 80$. The strike price of the option is 60. The following table summarizes the payoff at the exercise date:

H_1 Not Hit	H_1 Hit	
	H_2 Not Hit	H_2 Hit
0	$2 \times \max[S(0.5) - 60, 0]$	$\max[S(0.5) - 60, 0]$

Calculate the price of the option. Our only viable method is to construct a REPLICATING PORTFOLIO consisting of barrier options whose prices are given.

(A) 0.6289

(B) 1.3872

(C) 2.1455

(D) 4.5856

(E) It cannot be determined from the information given above.

43. DELETED

$\longrightarrow$: $V_{SO}(T)$... the payoff of the special option

$$V_{SO}(T) = \mathbb{I}_{[M(T) \geq H_1]} \left(V_c(T) + V_c(T) \cdot \mathbb{I}_{[M(T) < H_2]} \right)$$

$$V_{SO}(T) = \left(1 - \mathbb{I}_{[M(T) < H_1]}\right)\left(V_c(T) + V_c(T) \cdot \mathbb{I}_{[M(T) < H_2]} \right)$$

$$\begin{aligned} &= V_c(T) \\ &\quad + V_c(T) \cdot \mathbb{I}_{[M(T) < H_2]} \\ &\quad - V_c(T) \cdot \boxed{\mathbb{I}_{[M(T) < H_1]} \cdot \mathbb{I}_{[M(T) < H_2]}} = \mathbb{I}_{[M(T) < H_1]} \\ &\quad - V_c(T) \cdot \mathbb{I}_{[M(T) < H_1]} \end{aligned}$$

$$= V_c(T) + V_c(T) \cdot \mathbb{I}_{[M(T) < H_2]} - 2 V_c(T) \cdot \mathbb{I}_{[M(T) < H_1]}$$

$\Rightarrow$ The replicating portfolio is:

- a long vanilla call
- a long barrier call with $H_2 = 80$
- two short barrier calls with $H_1 = 70$

$$\Rightarrow V_{SO}(0) = 4.0861 + 0.7583 - 2(0.1294) = \underline{4.5856}$$