University of Texas at Austin

HW Assignment 5

Contents

Please, provide your **complete solutions** to the free-response problems. Final answers only, even if correct will earn zero points for those problems. As for the True/False and multiple-choice questions, it is sufficient to provided the **final answer only**. No partial credit will be give for those problems.

5.1. European calls.

Problem 5.1. (10 points) *Source: Sample IFM (Derivatives - Intro), Problem#11*
The current stock price is \$40, and the effective annual interest rate is 8%.

You observe the following option prices:

(1) The premium for a \$35-strike, 1-year European call option is \$9.12.
(2) The premium for a \$40-strike, 1-year European call option is \$6.22.
(3) The premium for a \$45-strike, 1-year European call option is \$4.08.

Assuming that all call positions being compared are **long**, at what 1-year stock price range does the \$45-strike call produce a higher profit than the \$40-strike call, but a lower profit than the \$35-strike call?

Express your answer as an interval.

Solution: The profit curve for a long European call option with strike K and exercise date T has the following form:

$$(s-K)_+ - FV_{0,T}(V_C(0,K)),$$

where $V_C(0,K)$ denotes the time-0 premium of the call with strike K. So, in the present problem, we have the following three profit curves:

$$\begin{aligned}(s-35)_+ - 9.12(1.08) &= (s-35)_+ - 9.85,\\ (s-40)_+ - 6.22(1.08) &= (s-40)_+ - 6.72,\\ (s-45)_+ - 4.08(1.08) &= (s-45)_+ - 4.41.\end{aligned}$$

In order to figure out the region in which the \$45-strike call to has a higher profit than the \$40-strike call, we need to solve the following inequality:

$$(s-40)_+ - 6.72 < (s-45)_+ - 4.41. \tag{5.1}$$

If $0 \leq s \leq 40$, this inequality becomes

$$-6.72 < -4.41. \tag{5.2}$$

We conclude that all values $s \leq 40$ satisfy inequality (5.1). If $40 < s < 45$, the above inequality (5.1) becomes

$$s - 40 - 6.72 < -4.41 \quad \Rightarrow \quad s < 42.31.$$

So, all $s \in [0, 42.31)$ satisfies (5.1). If $s \geq 45$, inequality (5.1) is trivially wrong for all such s.

In order to figure out the region in which the \$45-strike call to has a lower profit than the \$35-strike call, we need to solve the following inequality:

$$(s-45)_+ - 4.41 < (s-35)_+ - 9.85. \tag{5.3}$$

If $s \leq 35$, we get no solutions to the inequality. If $35 < s < 45$, the above inequality (5.3) becomes

$$-4.41 < s - 35 - 9.85 \quad \Rightarrow \quad 40.44 < s.$$

So, any $s \in (40.44, 45)$ satisfies (5.3). Finally, if $s \geq 45$, we have that (5.3) becomes

$$s - 45 - 4.41 < s - 35 - 9.85 \quad \Rightarrow \quad -49.41 < -44.85.$$

So, any $s \geq 45$ satisfies (5.3).

Pooling all of our conclusions together, we get the final answer $s \in (40.44, 42.31)$

5.2. **European puts.** Provide your **final answer** only for the following problems:

Problem 5.2. (2 points) In which of the following option positions is the investor exposed to an unlimited loss?

(a) Long put option
(b) Short put option
(c) Long call option
(d) Short call option
(e) None of the above.

Solution: (d)
Just draw the payoff diagrams to convince yourselves.

Problem 5.3. (3 points) The initial price of the market index is \$1000. After 3 months the market index is priced at \$950. The nominal rate of interest convertible quarterly is 4.0%.
The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

(a) \$12.00 loss
(b) \$14.90 loss
(c) \$12.00 gain
(d) \$14.90 gain
(e) None of the above.

Solution: (d)
The profit is

$$(K - S(T))_+ - FV_{0,T}[V_P(0)] = (975 - 950)_+ - 10\left(1 + \frac{0.04}{4}\right) = 25 - 10.10 = 14.90.$$

Provide your **complete solution** to the following problem:

Problem 5.4. (3 points) *Source: Sample FM(DM) Problem #62.*
The stock price today equals \$100 and its price in one year is modeled by the following distribution:

$$S(1) \sim \begin{cases} 125 & \text{with probability } 1/2 \\ 60 & \text{with probability } 1/2 \end{cases}$$

The annual effective interest rate equals 3%.

Consider an at-the-money, one-year European put option on the above stock whose initial premium is equal to \$7.

What is the expected profit of this put option?

Solution:

$$\frac{1}{2}(100-60) - 7(1.03) = 20 - 7.21 = 12.79.$$

Floors. The portfolio consisting of

- the **long** risky asset, and
- a **long** put on that asset

is commonly referred to as the *floor*. It arises naturally when the producer of a commodity or an owner of a risky asset (shares of stock, e.g.) uses puts to hedge his/her exposure to risk.

Provide your final answer only for the following problem.

Problem 5.5. (5 points) **Sample FM(DM) #13.**
Suppose that you short one share of a stock index for 50, and that you also buy a 60−strike European call option that expires in 2 years for 10. Assume the effective annual interest rate is 3%. If the stock index increases to 75 after 2 years, what is the profit on your combined position, and what is an alternative name for the call in this context?

	Profit	**Name**
A.	−22.64	Floor
B.	−17.56	Floor
C.	−22.64	Cap
D.	−17.56	Cap
E.	−22.64	"Written" Covered Call

Solution: First, about the qualitative portion of the answer. A combination of a short asset and a long call is called a **cap**. So, one can eliminate all of the offered answers except for C. and D.

Next, the quantitative component of the problem. The payoff is simply:

$$-75 + (75-60)_{+} = 60.$$

In words, as a short seller, you have to purchase the stock index back and you are going to take advantage of owning the call option on that index (as opposed to paying the higher market price). The initial cost is $-50+10=-40$. So, the value of this initial cost in 2 years equals $-40 \cdot (1.03)^2 = -42.436$. The profit is

$$40 - 42.436 = 17.564.$$

We choose $\boxed{\text{D.}}$ as the correct answer.

Provide your complete solution to the following problem.

Problem 5.6. Aunt Dahlia simultaneously purchased

- one share of a market index at the current spot price of \$1,000;
- one one-year, \$1,050-strike put option on the above market index for the premium of \$20.

(i) (5 points) Is the above portfolio's payoff bounded from above? If you believe it is not, substantiate your claim. If you believe that it is, provide the upper bound.

(ii) (5 points) Is the above portfolio's payoff bounded from below? If you believe it is not, substantiate your claim. If you believe that it is, provide the lower bound.

Solution: The payoff of the portfolio, i.e., the floor, expressed in terms of the final asset price $S(T)$ is

$$V(T) = S(T) + (K - S(T))_+ = \min[K, S(T)] = K \wedge S(T).$$

(i) The floor's payoff is **not** bounded from above since the stock price $S(T)$ may be arbitrarily large.

(ii) The floor's payoff is bounded from below by the put option's exercise price K. This means that there is a guarantee of the minimum price the owner of the floor can fetch for the underlying asset. One might conclude this is the reasoning for using the term *floor*.

Covered puts. The writer of a put option might want to hedge his/her exposure to risk by shorting the underlying asset. The position consisting of

- the **short** risky asset, and
- a **written** put on that asset

is commonly referred to as the *covered put.*

Provide your final answer only for the following problem.

Problem 5.7. (2 points) *Source: Dr. Jim Daniel (personal communication).*
Which of the following constitutes a one-year, \$100-strike **covered put**?

(a) Write a one-year, \$100-strike call and buy the underlying.
(b) Write a one-year, \$100-strike put and short the underlying.
(c) Write a one-year, \$100-strike put and buy the underlying.
(d) Write a one-year, \$100-strike put and write a one-year, \$100-strike call.
(e) None of the above.

Solution: (b)

5.3. **Parallels between put options and classical insurance.** Consider *homeowner's insurance.* An insurance policy is there to compensate the homeowner in case there is a financial loss due to physical damage to the home (fire, e.g.). At the time the insurance policy is issued the home is apraised and its **initial value** becomes part of the insurance policy. If the property is damaged, the insurance company is liable to make a benefit payment to the policyholder in the amount needed to bring the home back to its original state. In order for this to happen, however, the policyholder needs to initiate a claim. The homeowner is not required to file a claim, but should the claim be filed, the insurance company is required to proceed according to the contract and make the benefit payment.

So far, we have discussed the use of derivative securities (forward contracts, call options and put options) for hedging. If we draw parallels between classical insurance and use of options, we get the following correspondence:

Classical homeowner's insurance	**Hedging with derivative securities**
Home	Risky asset
Value of home	Market price of the risky asset
Insurance company	Option writer
Policyholder	Option buyer
Benefit payment	Payoff

If we specify the features of the insurance policy, we can see even more precise connections. Most insurance policies include a type of cost-sharing between the insurer and the insured. Most commonly, homeowner's insurance includes a *deductible.* The deductible d is the monetary amount up to which the policyholder pays for the damages. Once the loss exceeds d, the insurer pays for the excess of the loss over the deductible. So, if we denote the loss amount by the random variable X, the amount paid by the insurer and received by the policyholder is $(X-d)_+$.

The loss X can be understood as the reduction in the home's value due to physical damage. If we denote the home's value at time t by $S(t)$, we see that $X = S(0) - S(T)$ with T denoting the end of the insurance period, say. Having observed this, we see that the amount received by the policyholder is

$$(X-d)_+ = (S(0) - S(T) - d)_+ = ((S(0) - d) - S(T))_+$$

The expression above is exactly the payoff of a put option with strike price $S(0) - d$. With this observation, our analogy is complete.

Please, provide the **final solution only** to the following problem(s):

Problem 5.8. (2 points) *Source: Sample FM(DM) Problem #27.*
The position consisting of *one long homeowner's insurance contract* benefits from falling prices in the underlying asset. *True or false?*

Solution: TRUE
Recall our comparison of the homeowner's insurance policy to the put option. The payoff of the put option is decreasing in the price of the underlying asset.

Problem 5.9. (2 points) The owner of a house worth $\$180,000$ purchases an insurance policy at the beginning of the year for a price of $\$1,000$. The deductible on the policy is $\$5,000$.
If after 6 months the homeowner experiences a casualty loss valued at $\$50,000$, what is the homeowner's net loss? Assume that the continuously compounded interest rate equals 4.0%.

(a) $6,020
(b) $11,020
(c) $50,000
(d) $51,020
(e) None of the above.

Solution: (a)
The homeowner had to give up the insurance premium of $\$1,000$, and six months later he/she also had to pay the $\$5,000$ deductible. Taking into account the interest that the initial premium would have earned over the six-month period, the homeowner's total loss is

$$1,000(1.04)^{1/2} + 5,000 \approx 6,020.$$

Problem 5.10. (8 points) Draw the profit diagram for the homeowner's **complete** position consisting of both the propery and the insurance policy.

Solution: There are two variations of the graph which I am accepting as correct. The first one goes along with a homeowner who has had the property for a while, so that one can choose his/her initial investment in the house to be $0. It is shown in Figure 1. The second one is the version in which the house is purchased at a price whose time-value at the end of the insurance term equals $200,000.

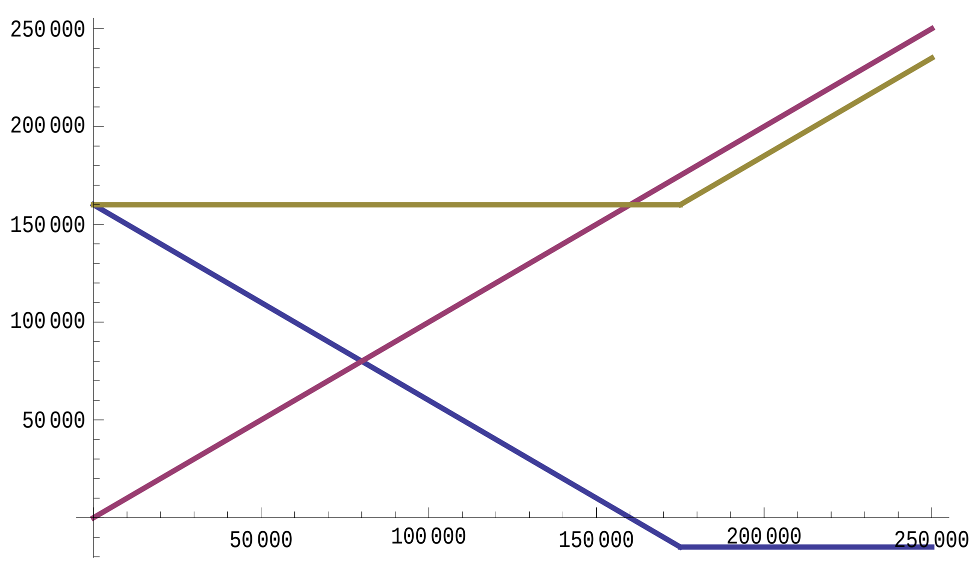

FIGURE 1. Inherited house

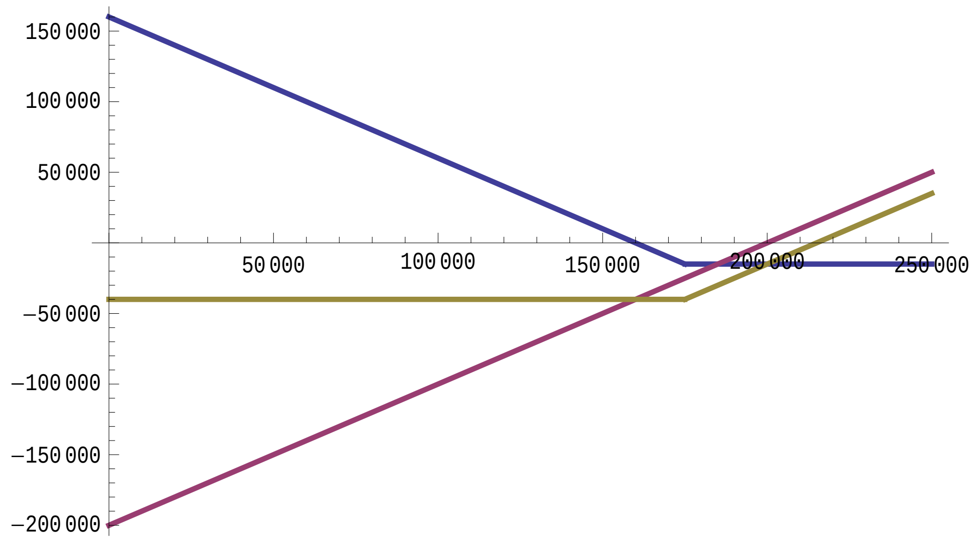

FIGURE 2. House purchased in the beginning of the insurance period

5.4. **Arbitrage.**

Problem 5.11. (5 points) Provide the definition of an *arbitrage portfolio.*

Solution:
A portfolio is called an *arbitrage portfolio* if its **profit** is
- **non-negative** in all states of the world, and
- **strictly positive** in at least one state of the world.

Problem 5.12. Samantha plans to travel to Japan and acquires 100,000 Japanese yen. The exchange rate at time of purchase is 0.0055873 GBP per yen.

(i) (2 points) How many GBP does Samantha have to spend?
(ii) (2 points) Samantha keeps the 100,000 yen safe in a drawer. In two weeks, Samantha decides against going to Japan after all and proceeds to exchange her 100,000 yen back to GBP. At that time, the exchange rate is 0.0062 GBP per yen. How many GBP does Samantha receive?
(iii) (2 points) Based on your responses above, do you think that Samantha inadvertently discovered an arbitrage oportunity?

Solution:

(i) $100,000 \times 0.0055873 = 558.73$
(ii) $100,000 \times 0.0062 = 620.00$
(iii) Of course not! She could not have known that the exchange rate would move in her favor (unless she is a soothsayer, but we do not allow for that).