M339 J: April 5th, 2021.

Qui2ff2: Problem #3.

$$X' = \frac{1}{X}$$

$$F_{x'}(y) = P[x' \leq y] = P[\frac{1}{x} \leq y] = P[1 \leq x \leq y]$$

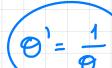
$$= \mathbb{P}\left[\frac{1}{9} \leq X\right] = 1 - \mathbb{F}_{X}\left(\frac{1}{9}\right)$$

$$= 1 - \frac{(\sqrt{9})^{8}}{1 + (\sqrt{9})^{8}}$$

$$= \frac{1}{1 + (\frac{1}{99})^8} = \frac{(99)^8}{(99)^8 + 1}$$

$$= \frac{\left(\frac{9}{9}\right)^{n}}{1 + \left(\frac{9}{9}\right)^{n}} = \times \times \sim \text{loglogistic} \left(\theta = \frac{1}{9}\right).$$

STAM TABLES $\left(\frac{x}{\Phi}\right)^g$ $: F_X(x) = \frac{\left(\frac{x}{\Phi}\right)^g}{1 + \left(\frac{x}{\Phi}\right)^g}$



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Poisson-Gamma Mixing.
     Let N has a mixture distribution w/
                 the mixing parameter A.
     More precisely, let
           N A ~ Roisson (mean = A)
             12 ~ Gamma (d, 0)
   @ : What is the support of N?
            : Support (N) = No = {0,1,2,...}
    Focus on the pmf of N:
       for k = 0,1,2,...
            PN(k) = P[N=k] = FN(k) - FN(k-1)
                           = \int_{N/\Lambda} F_{N/\Lambda}(k|\lambda) f_{\Lambda}(\lambda) d\lambda
mixing
- \int_{N/\Lambda} F_{N/\Lambda}(k-1|\lambda) f_{\Lambda}(\lambda) d\lambda
                                  = (FN/(k/2)-FN/(k-1/2))f/(2)d2
                                  = \int \mathbb{P}[N = k \mid \Lambda = \lambda] f_{\Lambda}(\lambda) d\lambda
       In general:
           P_N(k) = \int P_{N|A}(k|\lambda) f_A(\lambda) d\lambda
       In this case:
           this case
p_{N}(k) = \int_{0}^{\infty} e^{-\lambda \cdot \frac{\lambda^{k}}{k!}} 
                        conditional pmf [ density
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$$=\frac{1}{k! \cdot \Theta^{\alpha} \cdot \Gamma(\alpha)} \int_{e^{-\lambda}}^{e^{-\lambda}} \frac{1+\frac{d}{d}}{\lambda^{k+\alpha-1}} d\lambda$$

$$=\frac{1}{\int_{V}(\lambda) d\lambda}$$

$$=\frac{1}{\int_{V}(\lambda) d\lambda} \int_{v_{0}}^{v_{0}} \frac{1}{\int_{v_{0}}^{v_{0}} \frac{1}{\int_{v_{0}^{v_{0}}} \frac{1}{\int_{v_{0}^{v_{0}}} \frac{1}{\int_{v_{0}^{v_{0}} \frac{1}{\int_{v_{0}^{v_{0}} \frac{1}{\int_{v_{0}^{v_{0}} \frac{1}{\int_{v_{0}^{v_{0}} \frac{1}{\int_$$

Problem. N | 1 ~ Proisson (mean=1) ($\Lambda \sim Gamma (\alpha = \frac{5}{2}, \theta = 4)$ What is the (unconditional) probability TP[N=3]?

$$\rightarrow$$
: N ~ Neg Binomial $(r = \frac{5}{2}, \beta = 4)$

$$P[N=3] = \frac{\Gamma(3+\frac{5}{2})}{\Gamma(3+1)\cdot\Gamma(\frac{5}{2})} \cdot \frac{4^3}{(4+4)^{3+\frac{5}{2}}}$$

Recall:
$$\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$$

$$\Gamma(k) = (k-1)!$$

$$\Gamma\left(\frac{5}{2}\right) = \frac{3}{2} \cdot \Gamma\left(\frac{3}{2}\right) = \frac{3}{2} \cdot \frac{1}{2} \cdot \Gamma\left(\frac{1}{2}\right) = \frac{3}{4} \sqrt{\pi}$$

$$\Gamma(3+\frac{5}{2}) = (2+\frac{5}{2}) \cdot \Gamma(2+\frac{5}{2}) = (2+\frac{5}{2})(1+\frac{5}{2})\Gamma(1+\frac{5}{2})$$

$$= (2+\frac{5}{2})(1+\frac{5}{2}) \cdot \frac{5}{2} \cdot \Gamma(\frac{5}{2}) = \frac{9\cdot 7\cdot 5\cdot 3}{3} \sqrt{\pi}$$

k integer

$$= \left(2 + \frac{5}{2}\right) \left(1 + \frac{5}{2}\right) \cdot \frac{5}{2} \cdot \Gamma\left(\frac{5}{2}\right) = \frac{9 \cdot 7 \cdot 5 \cdot 3}{2^{3} \cdot 4} = 0.0601$$

$$= D P[N=3] = \frac{2^{5}}{3! \cdot \frac{3}{4} \cdot \sqrt{n}!} \cdot \frac{4^{3}}{5^{\frac{1}{2}}} \approx 0.0601$$