M339D : March 8th, 2021.

European Call Options.

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option written | exercise date

Initial Cost Paid to the writer of the call option.

- The call's owner has the RIGHT, but NOT an obligation to BUY one unit of the underlying asset for the strike price K which was agreed upon @ time $\cdot$ 0
- The call's writer MUST do what the call's owner says.

EXAM

$\boxed{\text{Payoff} = ?}$ We focus on the payoff of the LONG CALL, i.e., the call's owner's payoff.

The call owner's rationale for whether to exercise: their criterion is:

PAYOFF:

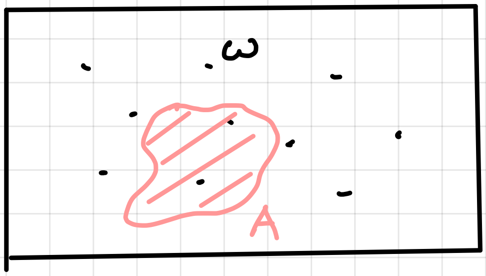
IF $S(T) \geq K$, THEN EXERCISE $\Rightarrow$ $S(T) - K$

IF $S(T) < K$, THEN DO NOT EXERCISE $\Rightarrow$ 0

$V_C(T)$ ... random variable denoting the payoff of the long call

$$\Rightarrow \quad V_C(T) = \begin{cases} S(T) - K, & \text{IF } S(T) \geq K \\ 0, & \text{IF } S(T) < K \end{cases}$$

Indicator Random Variables.



ω... elementary outcomes

Ω (probability space)

"Any" subset of Ω is called an event.

We define:

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}, \text{ i.e.,}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

Now, the payoff of a long call can be rewritten as:

$$\boxed{\begin{aligned} V_C(T) &= (S(T)-K)\cdot\mathbb{I}_{[S(T)\geq K]} \\ &= \max(S(T)-K, 0) \end{aligned}}$$

Classical Insurance.

- X ... random variable which stands for (ground-up) loss aka severity
- d ... (ordinary) deductible the amount which modifies the loss r.v. so that the insurer pays the excess above d if any.

$\Rightarrow$ The insurer pays: $Y^L = (X-d)\cdot\mathbb{I}_{[X\geq d]}$

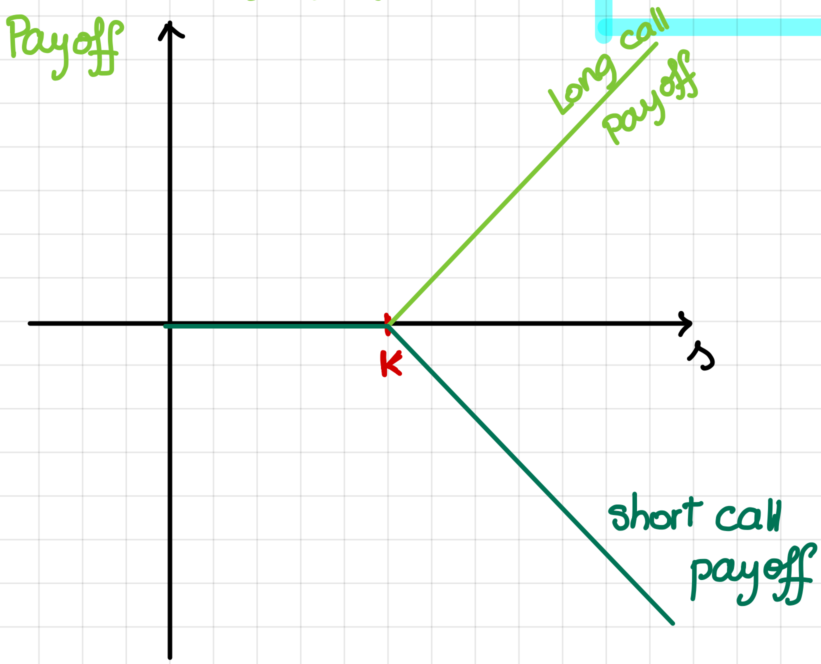
Introduce the positive-part function:

$$x \longmapsto (x)_+ := \max(x, 0).$$

$$\Rightarrow \boxed{V_c(T) = (S(T) - K)_+}$$

$\Rightarrow$ The payoff f'tion : $\boxed{v_c(s) = (s-K)_+}$

"hockey-stick function"

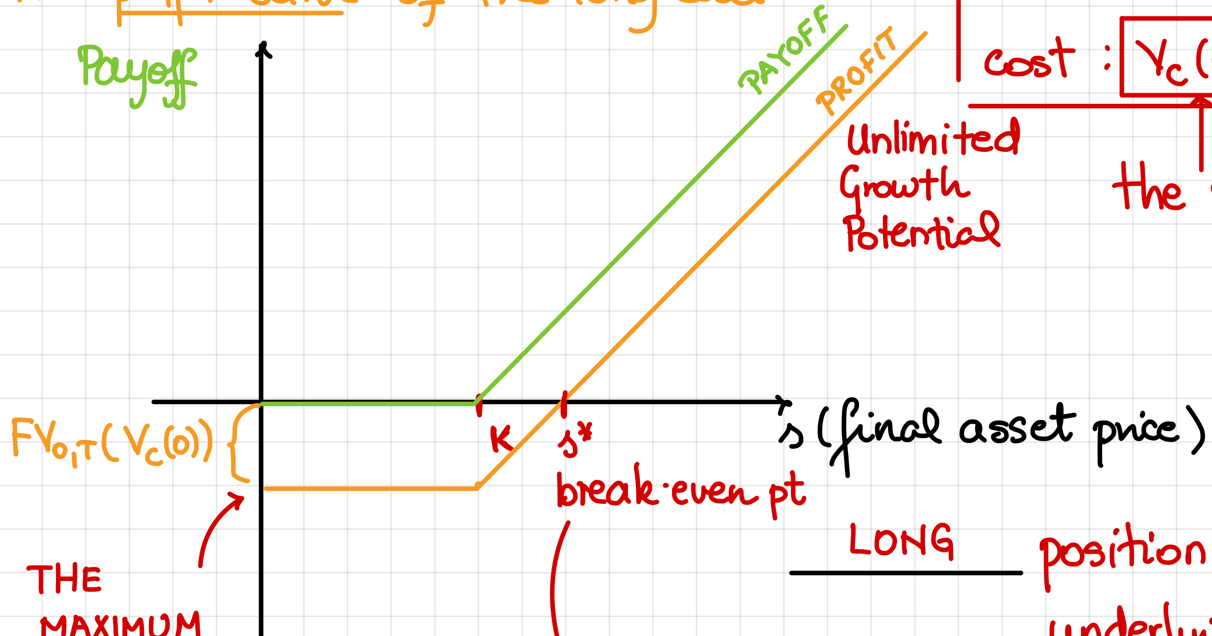


Note: Payoff of the short call is never positive and sometimes it's strictly negative

$\Rightarrow$ There is the initial cost : $\boxed{V_c(0)}$.

the premium

The profit curve of the long call:



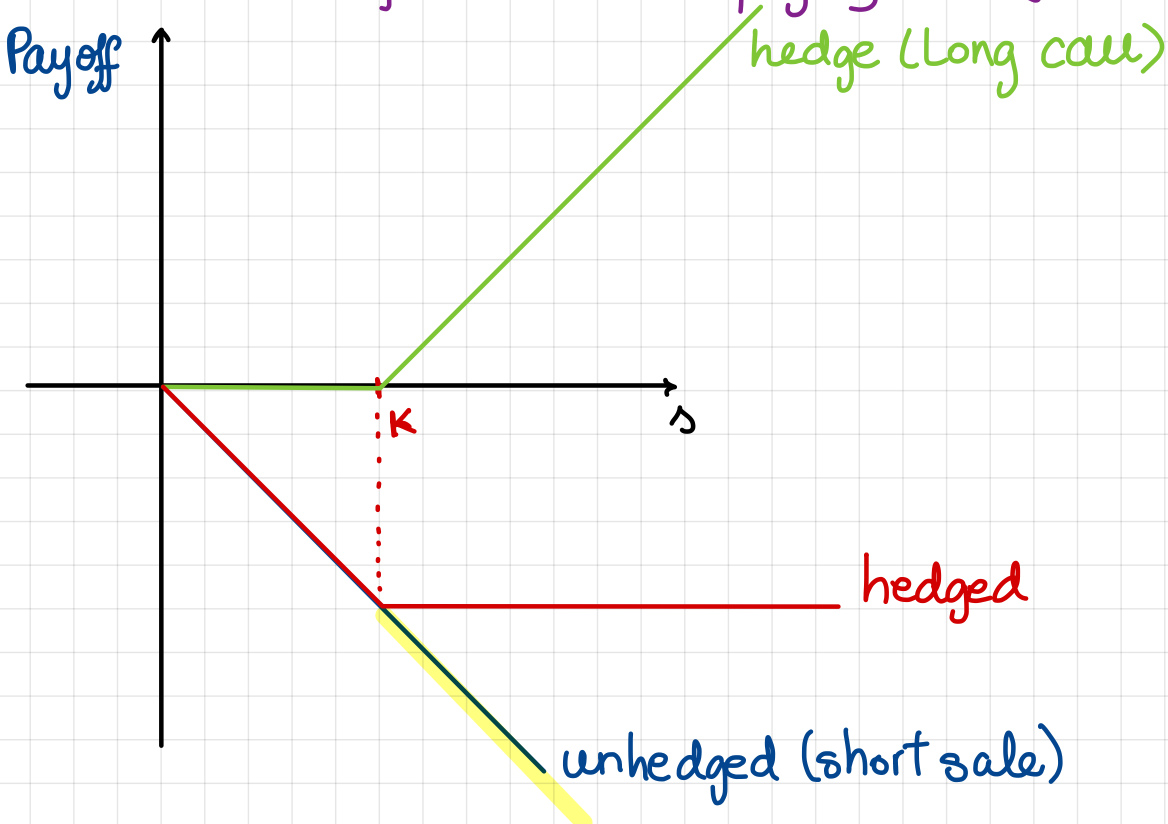
LONG position w.r.t. the underlying

$$(s-K)_+ - FV_{0,T}(V_c(0)) = 0$$

Evidently, $s > K$. So, $s - K - FV_{0,T}(V_c(0)) = 0$

$$\boxed{s^* = K + FV_{0,T}(V_c(0))}$$

Example. [A short sale of non-dividend paying stock]



- short sale
- Long call

} CAP

Think about: How to write the expression for the payoff of a cap as succinctly as possible?