Logistic Regression.

$$X \longmapsto Y = \underbrace{\beta_0 + \beta_1 X}_{\in \mathbb{R}} + \varepsilon \qquad \times$$

$$X \longmapsto p(X) = \mathbb{P}[Y=1 \mid X]$$
$$= \underbrace{\beta_0 + \beta_1 X}_{\in \mathbb{R}} + \varepsilon \qquad \times$$

Set $p(X) = \mathbb{P}[Y=1 \mid X]$

Def'n.

$$\boxed{\text{odds} = \frac{p(X)}{1-p(X)}} \in (0, \infty)$$

$$X \longmapsto \text{odds} = \underbrace{\beta_0 + \beta_1 X}_{\in \mathbb{R}} + \varepsilon$$

$$\text{logodds} = \ln(\text{odds}) = \ln\left(\frac{p(X)}{1-p(X)}\right)$$

$$\text{logodds} = \underbrace{\beta_0 + \beta_1 X} + \varepsilon$$

$$\ln\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X$$

$$\frac{p(X)}{1-p(X)} = e^{\beta_0 + \beta_1 X}$$

$$p(X) = (1-p(X)) \cdot e^{\beta_0 + \beta_1 X}$$
$$= e^{\beta_0 + \beta_1 X} - p(X) e^{\beta_0 + \beta_1 X}$$

$$\boxed{p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}}$$