UNIVERSITY OF TEXAS AT AUSTIN

Quiz #6

Please, provide your **complete solutions** to the following questions:

Problem 6.1. (5 points) Aggregate losses for a portfolio of policies are modeled as follows:

(i) The number of losses before any coverage modifications follows a geometric distribution with mean β.

(ii) The severity of each loss before any coverage modifications is uniformly distributed between 0 and ω.

The insurer would like to model the effect of imposing an ordinary deductible d such that $0 < d < \omega$ on each loss and reimbursing only a percentage α, such that $0 < \alpha < 1$ of each loss in excess of the deductible.

It is assumed that the coverage modifications will not affect the loss distribution. The insurer models its claims with modified frequency and severity distributions.

What is the mean of the modified frequency distribution?

Solution: The probability of exceeding the deductible is $\frac{\omega-d}{\omega}$, so the mean number of claims is $\frac{\beta(\omega-d)}{\omega}$.

Problem 6.2. (5 points) A group insurance policy has a negative binomial claim count distribution with mean 200 and variance 600.

The severity random variable X has the following probability mass function:

$$p_X(60) = p_X(120) = p_X(160) = p_X(200) = 1/4.$$

There is a per-loss deductible of 100. Calculate the expected total claim payment.

Solution: The per-loss random variable has the following distribution

$$Y^L \sim \begin{cases} 0 & \text{with probability } 1/4 \\ 20 & \text{with probability } 1/4 \\ 60 & \text{with probability } 1/4 \\ 100 & \text{with probability } 1/4 \end{cases}$$

Its expectation is

$$\mathbb{E}[Y^L] = 20\left(\frac{1}{4}\right) + 60\left(\frac{1}{4}\right) + 100\left(\frac{1}{4}\right) = 45.$$

So,

$$\mathbb{E}[S] = 200(45) = 9000.$$

Problem 6.3. (5 points) Let the loss count random variable have the Poisson distribution with parameter λ. The losses are assumed to be uniform on $(0, a)$. The losses are all mutually independent and independent from the loss count random variable.

There is a per-loss deductible of d such that $d < a$. What is the variance of aggregate claim payments? Express your answer in terms of λ, a and d.

Solution: The payment count random variable is Poisson with parameter $\lambda^P = \lambda\left(\frac{a-d}{a}\right)$.

The per payment random variable Y^P is uniform on $(0, a-d)$.

The aggregate claims S are compound Poisson and their variance equals

$$\mathrm{Var}[S] = \lambda^P \mathbb{E}[(Y^P)^2] = \lambda^P(\mathrm{Var}[Y^P] + (\mathbb{E}[Y^P])^2) = \lambda\left(\frac{a-d}{a}\right)\left(\frac{(a-d)^2}{12} + \left(\frac{a-d}{2}\right)^2\right) = \frac{\lambda(a-d)^3}{3a}.$$