M339J: Probability models with actuarial applications
University of Texas at Austin
HW VI
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Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 6.1. (10 points) The aggregate loss random variable S has a compound Poisson claims distribution, i.e., let the frequency random variable N have the Poisson distribution. You are given that

i. Individual claim amounts may only be equal to 1, 2, or 3.
ii. $\mathbb{E}[S] = 56$
iii. $Var[S] = 126$
iv. The rate of the Poisson claim count random variable is $\lambda = 29$.

Determine the probability mass function of the claim amounts.

Problem 6.2. (15 points) In the compound model for aggregate claims, let the frequency random variable N be Negative Binomial with parameters $r = 2$ and $\beta = 4$, and let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the probability (mass) function $p_X(1) = 0.3$ and $p_X(2) = 0.7$.
Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Calculate $\mathbb{P}[S \leq 3]$.

Problem 6.3. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the probability (mass) function

$$p_N(0) = 0.5, p_N(1) = 0.3, p_N(2) = 0.2.$$

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the probability (mass) function $p_X(1) = 0.3$ and $p_X(2) = 0.7$.
Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Calculate $\mathbb{E}[(S - 2)_+]$.

Problem 6.4. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 1.

Let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the following p.m.f.

$$p_X(100) = 1/2, p_X(200) = 3/10, p_X(300) = 1/5.$$

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Find the expected value of the **policyholder's** payment for a stop-loss insurance policy with an ordinary deductible of 200, i.e., calculate $\mathbb{E}[S \wedge 200]$.

Problem 6.5. (5 pts) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 5. Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be the two-parameter Pareto with parameters $\alpha = 3$ and $\theta = 10$. Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
What is the variance of S?