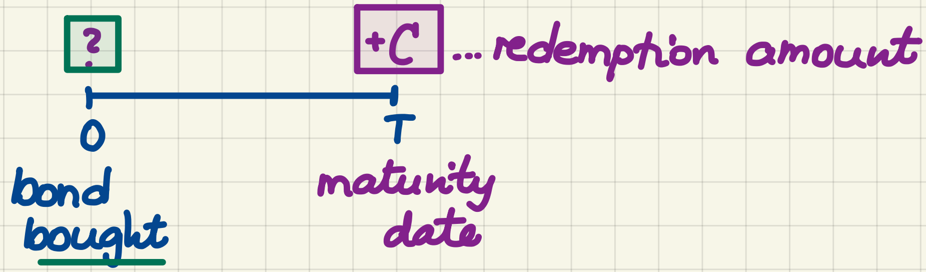
M339D: January 31st, 2025.

More on Payoff and Profit.

Example. Investing in a zero-coupon bond.



r... continuously compounded, risk-free interest rate

Initial Cost: Ce^{-rT}

Payoff: C

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= C - FV_{0,T}(Ce^{-rT}) = 0$$

Example. Taking a Loan.

L... loan amt

Initial Cost: $-L$

Payoff: $-Le^{rT}$

$$\text{Profit} = -Le^{rT} + FV_{0,T}(+L) = 0$$

Payoff and Profit Curves.

Outright Purchase of a Stock.

$S(t), t \geq 0$ .... time$\cdot t$ stock price

Initial Cost: $\underline{S(0)}$

Payoff: $S(T)$ ... a random variable

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$

$$= S(T) - e^{rT} \cdot S(0)$$

Inspiration.

Goal: To study the payoff and the profit as functions of the final asset price

Introduce: s... an independent argument taking values in $[0, +\infty)$; it stands for the FINAL ASSET PRICE, i.e., it's a "placeholder" for the r.v. $S(T)$

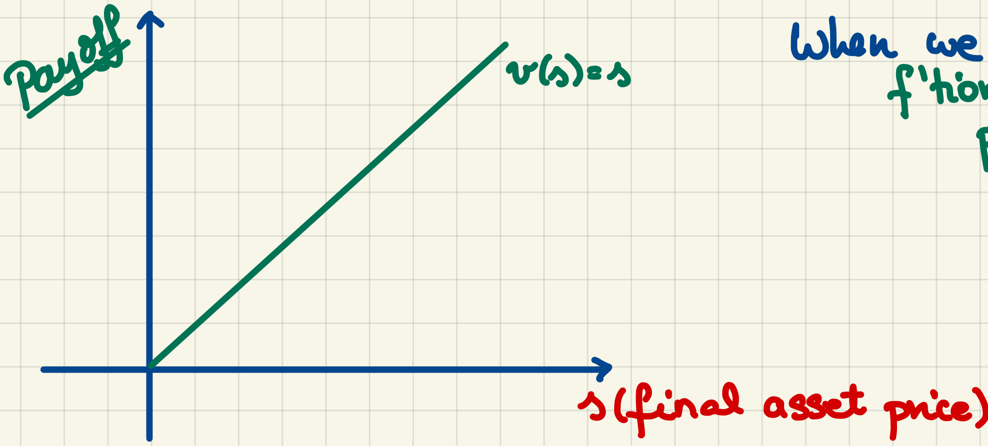
Now, we can define the PAYOFF FUNCTION which describes the dependence of the payoff on the independent argument s.

Notation: v... payoff f'tion

$v : [0, +\infty) \longrightarrow \mathbb{R}$

$v(s)$... the agent's payoff if the final asset price equals s

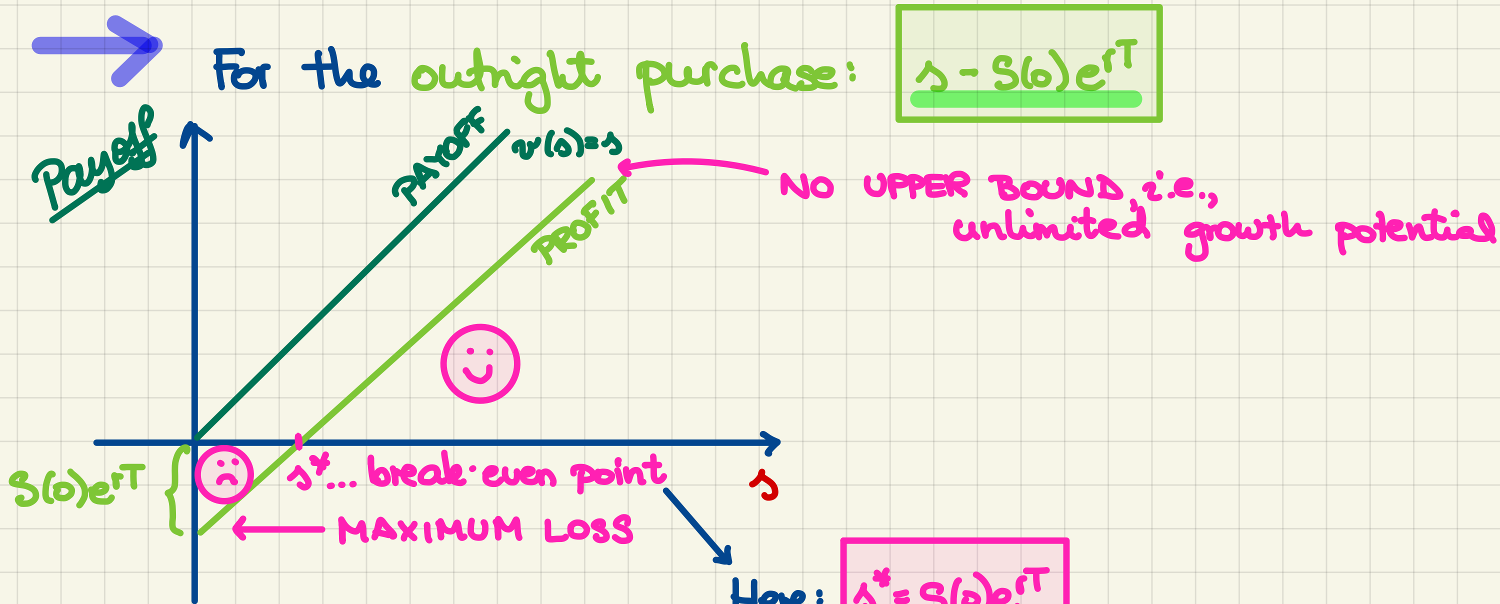
For the outright purchase: $\boxed{v(s) = s}$ identity function



When we plot the payoff f'tion, we get the payoff curve/diagram.

In general, the profit function is: $\boxed{v(s) - FV_{0,T}(\text{Init. Cost})}$

For the outright purchase: $\boxed{s - S(0)e^{rT}}$



Here: $\boxed{s^* = S(0)e^{rT}}$

The payoff and the profit curves are increasing

Terminology.

If the payoff/profit is increasing (not necessarily strictly) as a function of the final asset price s, we say that the portfolio is

long with respect to the underlying asset.

Short Sales.

0 — short sale initiated

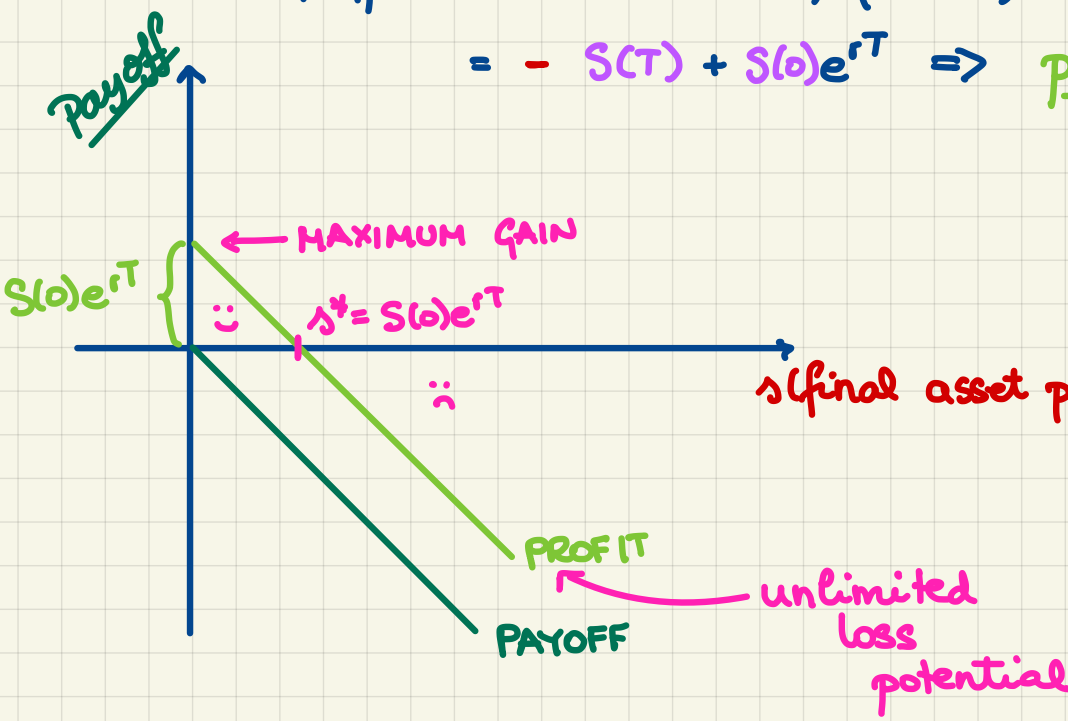
T — short sale closed

Initial Cost: $\underline{-S(0)}$

Payoff: $\underline{-S(T)}$ $\Rightarrow$ payoff f'tion: $\boxed{v(s) = -s}$

Profit $= -S(T) + FV_{0,T}(+S(0))$

$= -S(T) + S(0)e^{rT}$ $\Rightarrow$ profit f'tion: $\boxed{-s + S(0)e^{rT}}$



The payoff/profit is decreasing,

i.e, the short sale is short w.r.t. the underlying.

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Problem Set 3

Payoff. Profit.

Problem 3.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time−1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time−0 and liquidate your investment at $\boxed{\text{time}-1}$?

→:

$$\text{Profit} = \text{Payoff} - \underbrace{FV_{0,1}(\text{Initial Cost})}_{40e^{0.04}}$$

$\mathbb{E}$

$$\mathbb{E}[\text{Profit}] = \boxed{\mathbb{E}[\text{Payoff}]} - 40e^{0.04}$$

$$= \mathbb{E}[S(1)]$$

$$= 45\cdot\left(\frac{1}{4}\right) + 42\cdot\left(\frac{1}{2}\right) + 38\cdot\left(\frac{1}{4}\right) = \underline{41.75}$$

answer: $41.75 - 40e^{0.04} = \underline{0.1176}$

□