Option Greeks [cont'd].

W: March 11th, 2019.

Review: For a portfolio (in particular, for a single option), you consider its worth through its value function:

$$v(s,t,r,\delta,\sigma)$$

In the Black-Scholes framework, we can look @ derivatives w/ respect to all of the above arguments. The derivatives are called option Greeks.

For instance: the delta is defined as:

$$\Delta(s,t,r,\delta,\sigma) = \frac{\partial}{\partial s} v(s,t,\dots)$$

For a call:

$$\Delta_C(\dots) = e^{-\delta(T-t)} N(d_1(\dots))$$

For a put

$$\Delta_P(\dots) = -e^{-\delta(T-t)} N(-d_1(\dots))$$

(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

8. You are considering the purchase of a 3-month 41.5-strike American call option on a nondividend-paying stock.

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40.

(iii) The stock's volatility is 30%. $\sigma = 0.30$

(iv) The current call option delta is 0.5. $\Rightarrow d_1 = 0 \Rightarrow d_2 = -0.15$

$\Rightarrow \boxed{r = 0.1023}$

Determine the current price of the option.

(A) $20 - 20.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(B) $20 - 16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(C) $20 - 40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(D) $16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

(E) $40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

$$v_C(S(0),0) = S(0)\cdot N(d_1) - Ke^{-r\cdot T}\cdot N(d_2)$$

$$= 40\cdot(0.5) - 41.5\,e^{-0.1023(1/4)}\cdot \overbrace{N(-0.15)}^{=1-N(0.15)}$$

By def'n:

$$N(x) = \int_{-\infty}^{x}\varphi(z)\,dz = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{z^2}{2}}dz$$

(2.)

$$v_c(S(0),0) = 20 - 40.453\,(1 - N(0.15)) \quad (*)$$

$$= 40.453 \cdot N(0.15) - 20.453$$

$$= \underbrace{40.453 \frac{1}{\sqrt{2\pi}}}_{16.138} \int_{-\infty}^{0.15} e^{-\frac{z^2}{2}} dz - 20.453 \Rightarrow (\circledast)$$

<u>At home</u>: Evaluate the price!

	European Call	American Call	European Put	American Put
(A)	I	I	III	III
(B)	II	I	IV	III
(C)	II	I	III	III
(D)	II	II	IV	III
(E)	II	II	IV	IV

27-30. DELETED

K_1 K_2 → Call Bull Spread

31. You compute the current delta for a 50-60 bull spread with the following information:

(i) The continuously compounded risk-free rate is 5%. $r = 0.05$

(ii) The underlying stock pays no dividends. $\delta = 0$

(iii) The current stock price is \$50 per share. $S(0) = 50$

(iv) The stock's volatility is 20%. $\sigma = 0.20$

(iv) The time to expiration is 3 months. $T = 1/4$

How much does delta change after 1 month, if the stock price does not change?

$t = 1/12$ $S(1/12) = 50$

(A) increases by 0.04

(B) increases by 0.02

(C) does not change, within rounding to 0.01

(D) decreases by 0.02

(E) decreases by 0.04

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4.

Plan: $\boxed{\Delta(S(0),0) = ?}$ $\Delta(S(1/12), 1/12) = ?$

Find the difference!

$\Delta_{Bull}(s,t) = ?$

We know: bull spread $\begin{cases} \cdot \text{ long } K_1\text{-strike call} \\ \cdot \text{ short } K_2\text{-strike call} \end{cases}$

$\Rightarrow$ the value f'tion of the bull spread:

$$v_{Bull}(s,t) = v_{50}(s,t) - v_{60}(s,t) \quad \Big/ \frac{\partial}{\partial s}$$

$$\Delta_{Bull}(s,t) = \Delta_{50}(s,t) - \Delta_{60}(s,t)$$

In particular, @ time 0:

at the money

$$\bullet\ d_1(K_1=50) = \frac{1}{0.2\sqrt{1/4}}\left[\cancel{\ln\left(\frac{50}{50}\right)} + \left(0.05 + \frac{0.04}{2}\right)\cdot\frac{1}{4}\right]$$

$$= 5\cdot\frac{1}{2}\cdot(0.07) = 0.175\ ;$$

$$\bullet\ d_1(K_2=60) = \frac{1}{0.2\sqrt{1/4}}\left[\ln\left(\frac{50}{60}\right) + (0.07)\cdot\frac{1}{4}\right] = -1.648$$

$$\Rightarrow \Delta_{50}(S(0),0) = N(d_1(K_1=50)) \cong N(0.\cancel{18}175) \cong 0.5695$$

$$\Delta_{60}(S(0),0) = N(-1.648) = 1 - N(1.648) = 0.05$$

5.

$$\Rightarrow \Delta_{Bull}(S(0),0) = 0.5695 - 0.05 = 0.5195$$ At the start!

Then, @ time $(t = 1/12)$:

The time to exercise is $T - t = 1/4 - 1/12 = 1/6$.

- $d_1(K_1 = 50) = \frac{1}{0.2\sqrt{1/6}}\left[\cancel{\ln\left(\frac{50}{50}\right)} + 0.07 \cdot \frac{1}{6}\right]$ at the money

$$= \frac{0.07}{0.2}\sqrt{\frac{1}{6}} = 0.14 \checkmark$$

- $d_1(K_1 = 60) = \frac{1}{0.2\sqrt{\frac{1}{6}}}\left[\ln\left(\frac{50}{60}\right) + 0.07 \cdot \frac{1}{6}\right] =$

$$= -2.09 \checkmark$$

$\Rightarrow \Delta_{50}(S(1/12) = 50, t = 1/12) = N(0.14) = 0.5557$

$\Delta_{60}(S(1/12) = 50, t = 1/12) = N(-2.09) = 1 - N(2.09)$

$= 1 - 0.9817 = 0.0183$

$\Rightarrow \Delta_{Bull}(S(1/12) = 50, t = 1/12) = 0.5557 - 0.0183 = 0.5374$

At the end!

$\Rightarrow$ answer: increases by approximately 0.02.

$\Rightarrow$ (B)

Example.

Assume <u>no dividends</u>.

Consider an at-the-money call option w/ an exercise date T.

at the start

$$\Rightarrow d_1(s,t) = d_1(K,0)$$

$$= \frac{1}{\sigma\sqrt{T}}\left[\cancel{\ln\left(\frac{K}{K}\right)} + \left(r+\frac{\sigma^2}{2}\right)\cdot T\right]$$

at-the-money

$$= \frac{r+\frac{\sigma^2}{2}}{\sigma}\sqrt{T}$$

$$\Rightarrow \Delta_c(K,0) = N\left(\frac{r+\frac{\sigma^2}{2}}{\sigma}\sqrt{T}\right)$$

What if we have an otherwise identical call w/ $2T$ to expiry?

$$\tilde{\Delta}_c(K,0) = N\left(\frac{r+\frac{\sigma^2}{2}}{\sigma}\sqrt{2T}\right)$$