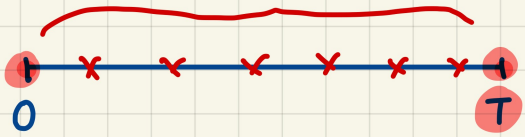
M339D: January 31st, 2022.

Static Portfolios.

Step #1. Decide who your protagonist is!

Step #2. Set up the timeline (mentally or "on paper")!

Static: no intermediate cashflows!



"today"
Cashflow from the perspective of your agent.
!!
Initial Cost

"time horizon"
Cashflow/Wealth from the perspective of the agent.
!!
Payoff

$$\text{Profit} := \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$

- If Profit >0, we call it a **gain**.
- If Profit <0, we call it a **loss**.
- If Profit $=0$, we say that we **broke even**.

Example. [Investing in a zero-coupon bond]

C... redemption amount

T... maturity date

r... continuously compounded, risk-free interest rate

Initial Cost: the bond's price $\quad Ce^{-r\cdot T}$

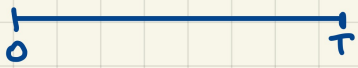
Payoff: $\quad C$

Profit $= \text{Payoff} - FV_{0,T}(\text{Initial Cost})$

$= C - e^{rT} \cdot Ce^{-rT} = 0$

Example. [Taking a simple loan]

r... ccrfir



L... loan amount, i.e., the amount borrowed @ time-0
T... the loan's term, i.e., the time @ which the loan must be repaid in full

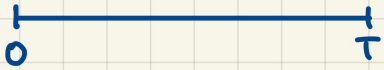
Initial Cost: $-L$ (the negative sign is because the agent is RECEIVING L @ time-0)

Payoff: $-Le^{rT}$ (the negative sign is because the agent is GIVING UP Le^{rT} @ time-T)

Profit = Payoff − $FV_{0,T}$(Initial Cost)

$= -Le^{rT} - e^{rT} \cdot (-L) = 0$ ■

Example. [The Outright Purchase of a Non-Dividend-Paying Stock]



Initial Cost: $S(0)$

Payoff: $S(T)$ a random variable

Goal: Study the payoff and the profit as a function of the final stock price.

Introduce: s... an independent argument taking values in $[0,+\infty)$; it stands for the FINAL ASSET PRICE (a "placeholder" for the random variable S(T))

Now, we can define the PAYOFF FUNCTION which describes the dependence of the payoff on the independent argument s.

Notation: v... payoff function

Typically, $v:[0,+\infty) \longrightarrow \mathbb{R}$

$\Rightarrow$ $v(s)$ is the investor's payoff if the final asset price is s

=> In the example w/ the outright purchase of a non-dividend-paying stock, we have

$$v(s) = s$$ The Identity Function.

When we graph the payoff f'tion, we get the payoff curve (aka payoff diagram).