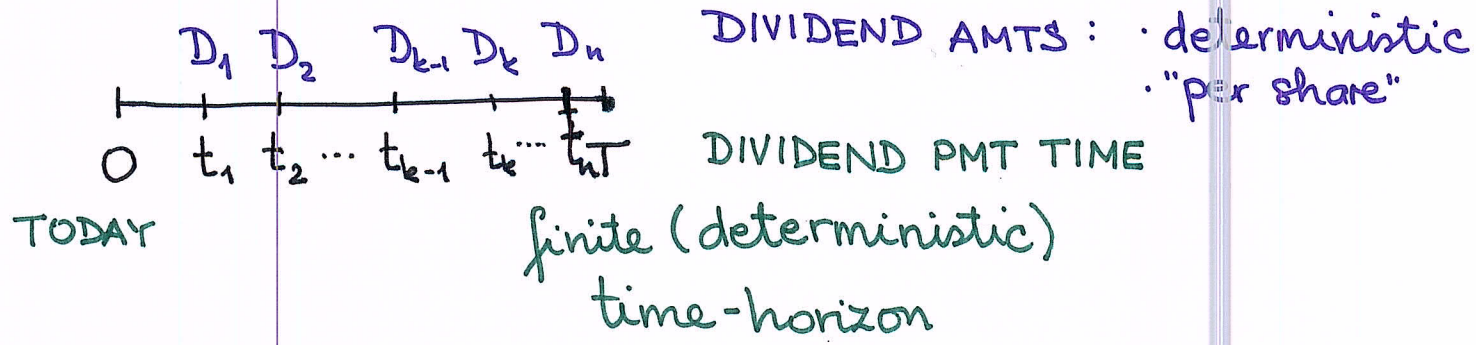


Discrete dividends

January 29TH



Continuously paid dividends

Example 1. Savings account governed by the continuously compounded risk-free interest rate (r)

· $B(t)$... balance in the account @ time t

· $B(0) = b_0$

$$\frac{dB(t)}{dt} = r \cdot B(t) \Rightarrow dB(t) = r B(t) dt$$

$t \quad t+dt$

$B(t) \quad B(t+dt)$

$$B(t+dt) - B(t) = r \cdot B(t) dt$$

Solution:

$$B(t) = b_0 e^{rt}$$

Example 2. (FOREIGN CURRENCY)

Domestic currency DC, e.g., USD (\$)

Foreign currency FC, e.g., GBP (£)

EXCHANGE RATE... value of 1 unit of foreign currency expressed in domestic currency @ time- t .

$x(t)$... the exchange rate from FC to DC

→ a function of time: PROCESS

→ random

: STOCHASTIC

stochastic process

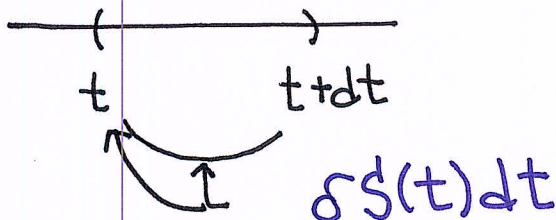
Investment ("buy-and-hold")

1. At time-0 : acquire 1 unit of FC.
Q: What is the DC-denominated cost of doing this? $x(0)$
2. Invest the 1 unit of FC to earn interest at the "foreign" continuously compounded interest rate r_F
3. At time- T , withdraw the balance, i.e., $1 \cdot e^{r_F T}$ units of FC.
4. If the proceeds are exchanged back to DC, we get: $x(T)e^{r_F T}$ random variable

Continuous-dividend-paying stocks

The shareholders receive $\delta \cdot S(t) dt$ during the time interval $(t, t+dt)$ per share owned, with

- $\delta \geq 0$ constant: THE DIVIDEND YIELD
- $S(t)$... the time- t market price of a share of stock



Q: What is the total nominal amt of dividend received per share during $[0, T]$?

DISCRETE

$$\sum_{k=1}^n D_k$$

CONTINUOUS

$$\int_0^T \delta S(t) dt$$

Q: What is the total accumulated amt of dividend?

DISCRETE

$$\sum_{k=1}^n D_k e^{r(T-t_k)}$$

CONTINUOUS

$$\int_0^T e^{r(T-t)} \delta S(t) dt$$

Continuously, immediately
invest the dividend in the stock?