UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 3

Monotonicity. Outright purchase. Fully-leveraged portfolios. Long/short positions.

Please, provide your **complete solutions** to the following problems. For the first three problems, draw the required graphs. If your graphs are correct, you will earn the full credit. For the other problems, final answers only, without the justification, will earn zero points.

3.1. **Monotonicity.**

Problem 3.1. (1 point) Draw the graph of an *increasing* function.

Solution: Correct answers vary.

Problem 3.2. (1 point) Draw the graph of a *decreasing* function.

Solution: Correct answers vary. $f(x) = -x$ works.

Problem 3.3. (3 points) Draw the graph of a function which is neither decreasing nor decreasing.

Solution: Correct answers vary.

Problem 3.4. (4 points) Consider the functions $f : [0, \infty) \to \mathbb{R}$ and $g : [0, \infty) \to \mathbb{R}$. Let f be given by

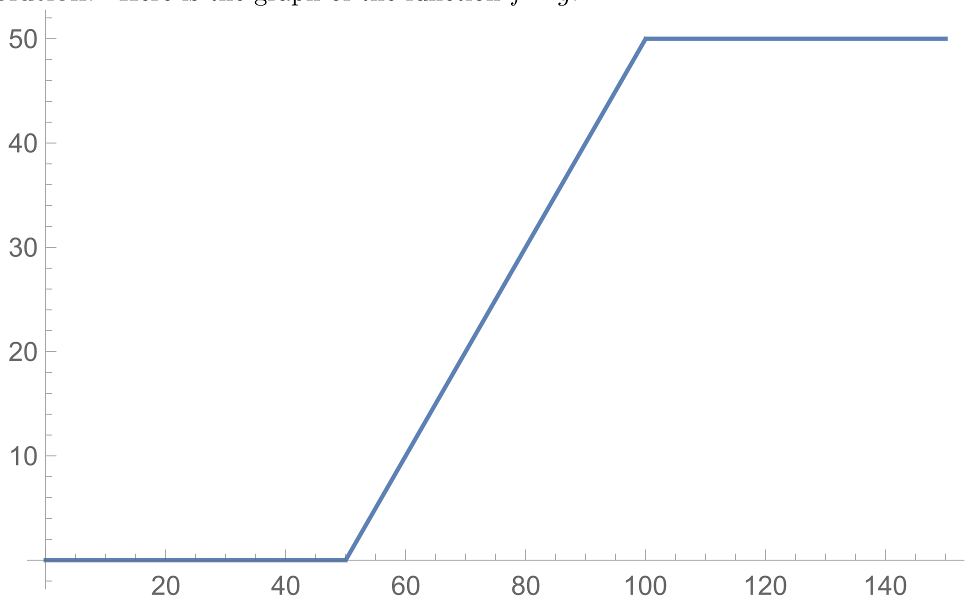
$$f(x) = \max(x - 50, 0).$$

Let g be given by

$$g(x) = \max(x - 100, 0).$$

What can you say about the monotonicity of the function $f - g$? Remember to justify your answer!

Solution: Here is the graph of the function $f - g$:



For any $x_1 < x_2$, we see that $f(x_1) - g(x_1) \leq f(x_2) - g(x_2)$.

3.2. **Fully-leveraged portfolios.**

Problem 3.5. (5 points) Write down the definition of a *fully-leveraged portfolio.*

Solution: We say that a portoflio is *fully leveraged* if its value at time 0 equals zero, i.e., if its initial cost equals zero.

3.3. **Long/short positions.**

Problem 3.6. (5 points) Complete the following definition:

A financial portfolio is said to be *long with respect to an underlying asset* if
Solution:
its payoff/profit function is increasing as a function of the final asset price.

Problem 3.7. (5 points) Complete the following definition:

A financial portfolio is said to be *short with respect to an underlying asset* if
Solution:
its payoff/profit function is decreasing as a function of the final asset price.

Problem 3.8. "Partially-leveraged" purchase.
Consider a continuous-dividend-paying stock with the dividend yield δ whose market price at any time$-t$ is denoted by $S(t)$. You decide to purchase one share of this stock at time-0 and you partially finance your purchase by borrowing a portion φ of the initial stock price at the continuously-compounded, risk-free interest rate r to be repaid in full at time$-T$.

(i) (2 points) What is the initial cost of your portfolio?
Solution:

$$S(0) - \varphi S(0) = (1-\varphi)S(0)$$

(ii) (3 points) What is the payoff of your portflio?
Solution:

$$e^{\delta T} S(T) - \varphi S(0) e^{rT}$$

(iii) (3 points) What is the profit of your portfolio?
Solution:

$$e^{\delta T} S(T) - \varphi S(0) e^{rT} - e^{rT}(1-\varphi)S(0) = e^{\delta T} S(T) - e^{rT} S(0)$$

(iv) (2 points) How does the profit curve of your portfolio compare to the profit curve of an outright purchase of the same asset? How about the fully-leveraged purchase of the same asset?
Solution: They are all identical.

Problem 3.9. (3 points) Consider an outright purchase of a share of continuous-dividend-paying stock whose current price is \$80 per share and whose dividend yield is 0.02. Let the continuously compounded, risk-free interest rate be equal to 0.04. What is the time-2 break-even stock price for this investment?

Solution: In our usual notation, the break-even price is

$$S(0)e^{(r-\delta)T} = 80e^{(0.04-0.02)(2)} = 80e^{0.04} = 83.2649$$

Problem 3.10. (3 points) Bertram sells short 10 shares of a continuous-dividend-paying stock. The time-0 price of this stock is \$100 and its dividend yield is 0.03. Assume that the continuously compounded, risk-free interest rate equals 0.06. If Bertram closes the short sale in six months, what is his break-even final stock price?

Solution: In our usual notation, the break-even price for Bertram's short sale equals

$$S(0)e^{(r-\delta)T} = 100e^{(0.06-0.03)(0.5)} = 100e^{0.015} = 101.511.$$

Problem 3.11. The market in which Inaho trades has three possibilities for investment:

- a risk-free asset with the continuously compounded, risk-free interest rate equal to r;
- a risky asset whose price is denoted by $S(t), t \geq 0$ and whose dividend yield is δ_S;
- a risky asset whose price is denoted by $Q(t), t \geq 0$ and whose dividend yield is δ_Q.

Initially, the market prices of assets S and Q are equal. Inaho opens a one-share short position in the asset S and uses the proceeds of the short sale to purchase a share of the asset Q. At time$-T$, Inaho sells the shares of asset Q she owns and closes the short sale of the asset S.

(i) (2 points) What is the initial cost of this portfolio?
Solution: The initial cost is equal to zero.

(ii) (5 points) What is the profit of this portfolio?
Solution:

$$e^{\delta_Q T}Q(T) - e^{\delta_S T}S(T)$$

(iii) (3 points) What is the condition on the ratio of the final prices of assets S and Q for Inaho to break even?
Solution:

$$\frac{Q(T)}{S(T)} = e^{(\delta_S - \delta_Q)T}$$