M339G : February 25th, 2026.

Logistic Regression w/ 2 Categories in the Response.

We can represent one category by 0,
and the other by 1.

$$\mathbb{P}[Y=1 \mid X=x] = \cancel{?}\ p(x)$$

$$\ln\left(\frac{p(x)}{1-p(x)}\right) = \beta_0 + \beta \cdot X$$

$$\Updownarrow$$

$$p(x) = \frac{e^{\beta_0 + \beta \cdot X}}{1 + e^{\beta_0 + \beta \cdot X}}$$

$$\boxed{\begin{aligned} \mathbb{P}[Y=1 \mid X=x] &= \frac{e^{\beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p}}{1 + e^{\beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p}} \\ \mathbb{P}[Y=0 \mid X=x] &= \frac{1}{1 + e^{\beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p}} \end{aligned}}$$

Logistic Regression w/ K categories in the Response

The labels of the categories: $C = \{1, 2, \dots, K\}$

<u>In the book:</u>

$$\mathbb{P}[Y = K \mid X = x] = \frac{1}{1 + \sum_{k=1}^{K-1} e^{\beta_{0k} + \beta_{1k} x_1 + \dots + \beta_{pk} x_p}}$$

<u>For all other categories:</u>

$$\mathbb{P}[Y = \ell \mid X = x] = \frac{e^{\beta_{0\ell} + \beta_{1\ell} x_1 + \dots + \beta_{p\ell} x_p}}{1 + \sum_{k=1}^{K-1} e^{\beta_{0k} + \beta_{1k} x_1 + \dots + \beta_{pk} x_p}}$$