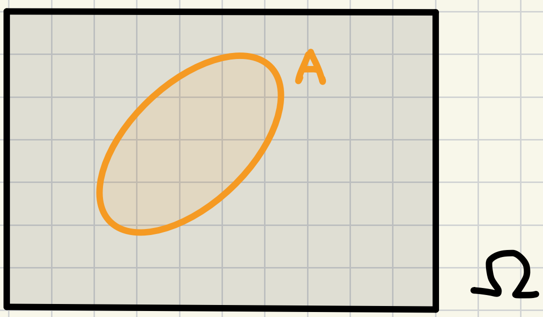
Named Discrete Distributions.

Def'n. **Bernoulli trials** have two possible outcomes.

They are also known as **indicators** (or indicator random variables).

Usually, the outcomes are encoded as

$$\begin{cases} 1 & \text{for "success"} \\ 0 & \text{for "failure"} \end{cases}$$



$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

Example. Y_i, $i = 1, 2 \ldots$ result of a throw of a regular die

$$S_{Y_1} = S_{Y_2} = \{1, 2, 3, 4, 5, 6\}$$

(i) Define $\boxed{W = Y_1 + Y_2}$

$$S_W = \{2, \ldots, 12\}$$

Define
$$I = \begin{cases} 1 & \text{if } W \geq 9 \\ 0 & \text{if } W < 9 \end{cases}$$

(ii) We win if the result on the 1st die is even and the result on the 2nd die is prime.

$$I_1 = \mathbb{I}_{\{Y_1 \in \{2,4,6\}\}}$$

$$I_2 = \mathbb{I}_{\{Y_2 \in \{2,3,5\}\}}$$

Then, our indicator of a win is $\boxed{I_1 \cdot I_2}$ □

Example.
- **Quality Control.** Say, whether a lightbulb is defective or not.
- **Insurance.** An indicator of whether a deductible was met or not.

Example. **Bernoulli Dist'n.** $Y \sim B(p)$ w/ $p \in (0,1)$

y	0	1
$p_Y(y)$	$q := 1-p$	p

Example. Say that we repeat independently the Bernoulli trials w/ the same success probability p a fixed number of times n.
Then, we count the total number of successes Y.
Its dist'n is the binomial distribution.

We write $\boxed{Y \sim b(n,p)}$

$$S_Y = \{0, 1, \dots, n\}$$

Its pmf is for $k = 0, \dots, n$,

$$p_Y(k) = \boxed{\binom{n}{k}} p^k (1-p)^{n-k}$$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

M378K Introduction to Mathematical Statistics
Problem Set #3
Named discrete random variables.

Problem 3.1. Source: Sample P exam, Problem #125.
An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail. Calculate the probability that the system will overheat.

→ : Y... # of components that overheated

$\mathbb{P}[Y \geq 2] = ?$

$\mathbb{P}[Y \geq 2] = 1 - \mathbb{P}[Y \leq 1] = 1 - \mathbb{P}[Y=0] - \mathbb{P}[Y=1]$

$Y \sim b(n=3, p=0.05)$

$$\mathbb{P}[Y \geq 2] = 1 - \binom{3}{0}(0.05)^0(0.95)^3 - \binom{3}{1}(0.05)^1(0.95)^2$$

$$= \binom{3}{2}(0.05)^2(0.95) + \binom{3}{3}(0.05)^3(0.95)^0$$

$$= \dots = 0.00725 \quad \square$$

$$\boxed{3(0.05)^2(0.95) + (0.05)^3}$$