20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

21-24. DELETED

t^* ... choice date

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of $\boxed{\$100}$.

K

T exercise date

The chooser option price is \$20 at time $t = 0$. $V_{CH}(0,1,3)=20$

The stock price is \$95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of \$100.

You are given:

(i) The risk-free interest rate is 0.

(ii) $C(1) = \$4$. $V_C(0,1,K=100)=4$

Determine $C(3)$. $V_C(0,3,K=100)=?$

(A) \$ 9

(B) \$11

(C) \$13

(D) \$15

(E) \$17

$$V_{CH}(0,1,3)=20=\underbrace{V_C(0,1,K=100)}_{4}+V_P(0,3,K=100)$$

$$\boxed{16=V_P(0,3,K=100)}$$

$$V_c(0,3,K=100) = V_p(0,3,K=100) + S(0) - PV_{0,3}(100)$$
$$= 16 + 95 - 100 = 11 \quad \square$$

Strong Law of Large Numbers. (SLLN)

Let $\{X_k, k=1,2,\dots\}$ be a sequence of independent, identically distributed r.v.s.

Assume: $\mu_X := \mathbb{E}[X_1] < \infty$

Then,
$$\boxed{\frac{X_1 + X_2 + \dots + X_n}{n} \xrightarrow[n\to\infty]{} \mu_X}$$

If a f'tion g is such that $g(X_1)$ is well-defined, and $\mathbb{E}[g(X_1)] < \infty$,

then,
$$\boxed{\frac{g(X_1) + g(X_2) + \dots + g(X_n)}{n} \xrightarrow[n\to\infty]{} \mathbb{E}[g(X_1)]} \quad \checkmark$$

Monte Carlo.

Recipe:
- Draw simulated values of a random variable from a distribution.
- Apply a f'tion to the simulated values.
- Calculate the arithmetic average of the obtained quantities.

We get the value which is "close to" the theoretical expected value.

Precision:

- $$\mathrm{Var}\left[\frac{X_1+X_2+\cdots+X_n}{n}\right] = \frac{1}{n^2}\mathrm{Var}[X_1+X_2+\cdots+X_n] \text{ (independence)}$$
$$= \frac{1}{n^2}\left(\mathrm{Var}[X_1]+\mathrm{Var}[X_2]+\cdots+\mathrm{Var}[X_n]\right)$$
(identically distributed)
$$= \frac{1}{n^{\cancel{2}}}\cdot\cancel{n}\cdot\mathrm{Var}[X_1] = \frac{\mathrm{Var}[X_1]}{n}$$

- $$\mathrm{SD}\left[\frac{X_1+\cdots+X_n}{n}\right] = \frac{\mathrm{SD}[X_1]}{\sqrt{n}}$$

To increase the precision by a factor η, the number of <u>variates</u> must increase by a factor of η^2.