M339J: March 22nd, 2021.

Splicing.

Def'n. We say that a random variable X has a k-component spliced dist'n if its density f_X has the following form:

$$f_X(x) = \begin{cases} a_1 \cdot f_1(x) & , \quad c_0 < x < c_1 \\ a_2 \cdot f_2(x) & , \quad c_1 < x < c_2 \\ \vdots & \qquad \vdots \\ a_k \cdot f_k(x) & \quad c_{k-1} < x < c_k \end{cases}$$

for k a fixed positive integer, where for every $j = 1, 2, \dots, k$ we give a weight $a_j > 0$ to that component w/ $a_1 + \dots + a_k = 1$;

for every $j = 1..k$, the function f_j is a density such that $f_j(x) = 0$ for $x \notin (c_{j-1}, c_j]$.

211. An actuary for a medical device manufacturer initially models the failure time T for a particular device with an exponential distribution with mean 4 years. $\theta = 4$

This distribution is replaced with a spliced model whose density function:

(i) is uniform over [0, 3]

(ii) is proportional to the initial modeled density function after 3 years

(iii) is continuous

Calculate the probability of failure in the first 3 years under the revised distribution.

(A) 0.43

(B) 0.45

(C) 0.47

(D) 0.49

(E) 0.51

212. For an insurance:

(i) The number of losses per year has a Poisson distribution with $\lambda = 10$.

(ii) Loss amounts are uniformly distributed on (0, 10).

(iii) Loss amounts and the number of losses are mutually independent.

(iv) There is an ordinary deductible of 4 per loss.

Calculate the variance of aggregate payments in a year.

(A) 36

(B) 48

(C) 72

(D) 96

(E) 120

$\longrightarrow$: The old model has the density f'tion:

$$f_T(t) = \frac{1}{4} e^{-\frac{t}{4}} \qquad \text{for } t > 0.$$

The new model has the density f'tion:

$$f_{\tilde{T}}(t) = \begin{cases} c & \text{if } 0 < t < 3 \\ \varkappa \cdot e^{-t/4} & \text{if } 3 \le t < +\infty \end{cases}$$

w/ c and $\varkappa$ constants chosen so that

- $f_{\tilde{T}}$ is a density, i.e., it integrates to 1, <u>and</u>
- $f_{\tilde{T}}$ is continuous, i.e., $\boxed{c = \varkappa \cdot e^{-\frac{3}{4}}}$

$$1 = \int_0^{+\infty} f_{\tilde{T}}(t)\,dt = \int_0^3 c\,dt + \int_3^{+\infty} \varkappa e^{-t/4}\,dt$$

$$= 3 \cdot c + \varkappa(-4)e^{-t/4}\Big]_{t=3}^{+\infty}$$

$$= 3c + 4\varkappa\left(0 + e^{-3/4}\right) = 3c + 4\varkappa e^{-\frac{3}{4}}$$

$$\Rightarrow \quad 1 = 3c + 4c = 7c \quad \Rightarrow \quad c = \frac{1}{7}$$

$\Rightarrow$ <u>Our answer</u>: $\mathbb{P}[\tilde{T} \le 3] = 3\left(\frac{1}{7}\right) = 3/7 \approx 0.43$ ■

207. For an insurance:

(i) Losses have density function

$$f(x) = \begin{cases} 0.02x, & 0 < x < 10 \\ 0, & \text{elsewhere} \end{cases}$$

(ii) The insurance has an ordinary deductible of 4 per loss.

(iii) Y^P is the claim payment per payment random variable.

Calculate $E[Y^P]$.

(A) 2.9

(B) 3.0

(C) 3.2

(D) 3.3

(E) 3.4

$$\boxed{Y^P = X - d \mid X > d}$$

Q: What is the pdf of Y^P in terms of f_X, F_X, S_X?

→: Find the cdf first.

$$F_{Y^P}(y) = \mathbb{P}[Y^P \leq y]$$

$$= \mathbb{P}[X - d \leq y \mid X > d]$$

$$= \frac{\mathbb{P}[X \leq d + y,\ X > d]}{\mathbb{P}[X > d]}$$

$$= \frac{\mathbb{P}[d < X \leq d+y]}{\mathbb{P}[X > d]}$$

$$= \frac{F_X(d+y) - F_X(d)}{S_X(d)}$$

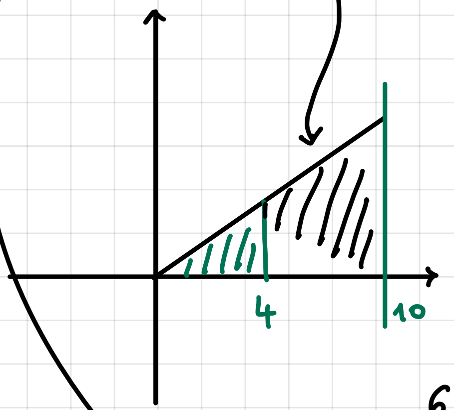
$$f_{Y^P}(y) = F'_{Y^P}(y) = \frac{F'_X(d+y)}{S_X(d)} = \frac{f_X(d+y)}{S_X(d)}$$

208. DELETED

In this problem:

$$\mathbb{E}[Y^P] = \int_0^6 y \cdot f_{Y^P}(y)\,dy$$

$$S_X(d) = S_X(4) = 1 - F_X(4) = 1 - \cancel{\tfrac{1}{2}} \cdot 4 \cdot \underbrace{(\overset{0.01}{\cancel{0.02}}) \cdot 4}_{f_X(d)} = 0.84$$



$$\Rightarrow \quad f_{Y^P}(y) = \frac{0.02(4+y)}{0.84} = \frac{1}{42}(4+y)$$

$$\mathbb{E}[Y^P] = \int_0^6 y \, \frac{1}{42}(4+y)\,dy$$

$$= \frac{1}{42}\int_0^6 (4y + y^2)\,dy = \frac{1}{42}\left(4 \cdot \frac{y^2}{2}\Big|_{y=0}^{6} + \frac{y^3}{3}\Big|_{y=0}^{6}\right)$$

$$= \frac{1}{42}\left(2 \cdot 36 + \frac{1}{\cancel{3}} \cdot \overset{2}{\cancel{6}} \cdot 36\right) = \frac{1}{42}(72 + 72) = \frac{144}{42}$$

$$= \frac{72}{21} = \frac{24}{7} = 3.427 \blacksquare$$

Alternatively:

$$\mathbb{E}[Y^P] = \mathbb{E}[\,X - d \mid X > d\,]$$

$$= \frac{\mathbb{E}[(X-d) \cdot \mathbb{I}_{[X>d]}]}{S_X(d)}$$

.... go down this path ...

Franchise Deductible.

If the loss amount exceeds the deductible d, then the insurer covers the entire loss.

The per payment r.v. is

$$Y^P = \begin{cases} \text{undefined} & \text{if } X \leq d \\ X & \text{if } X > d \end{cases}$$

i.e.,

$$\boxed{Y^P = X \mid X > d}$$

The per loss r.v. is

$$Y^L = \begin{cases} 0 & \text{if } X \leq d \\ X & \text{if } X > d \end{cases}$$

Facts: If X is continuous w/ pdf f_X, then

- Y^P is continuous w/ $f_{Y^P}(y) = \dfrac{f_X(y)}{S_X(d)}$, $y > d$
- Y^L is mixed w/ $p_{Y^L}(d) = F_X(d)$
  $f_{Y^L}(y) = f_X(y)$, $y > d$