M339W: January 21st, 2022.

The Inverse Transform Method.

Proposition.

(1) Let X be a continuous random variable, i.e., let X have the density function f_X.

Assume that $\boxed{f_X(x) > 0 \text{ for all } x.}$

Denote the cumulative dist'n function of X by F_X.

Set $\boxed{\tilde{X} := F_X(X)}$ ✓

Then, $\tilde{X} \sim \underline{U(0,1)}$

→: Support of $\tilde{X}$ will be contained in $[0,1]$.

Let $\underline{u \in [0,1]}$.

$$F_{\tilde{X}}(u) = \mathbb{P}[\tilde{X} \leq u]$$
$$= \mathbb{P}[F_X(X) \leq u] \quad ✓$$

$\boxed{f_X(x) > 0}$ for all x

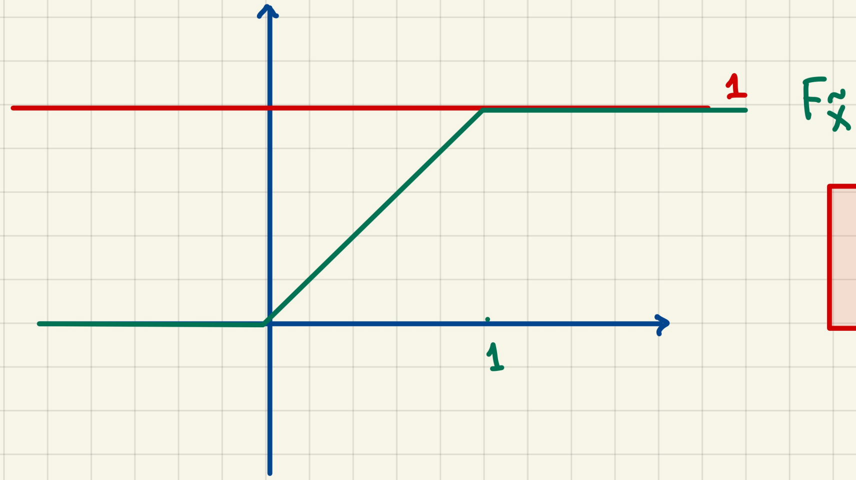
Recall: $F_X(a) = \int_{-\infty}^{a} f_X(x)\,dx$

⇒ the cdf F_X is strictly increasing

⇒ F_X is one-to-one

⇒ F_X^{-1} exists and is increasing

$$\underline{F_{\tilde{X}}(u)} = \mathbb{P}[X \leq F_X^{-1}(u)]$$
$$= \cancel{F_X}(\cancel{F_X^{-1}}(u)) = \underline{u} \quad \Rightarrow \quad \tilde{X} \sim \ ?$$



$\boxed{\tilde{X} \sim U(0,1)}$

∎

(2) Let $U \sim U(0,1)$.

Let F be a cumulative dist'n function.

Set $\boxed{Y := F^{-1}(U)}$

Then, the cumulative dist'n f'tion of Y is F.

Implementation:

1. F... the cdf of the dist'n you want to draw the simulated values from
2. Find an "expression" for F^{-1}
3. Draw: $u_1, u_2, \dots, u_n \sim U(0,1)$ from your rng
4. Set $\boxed{x_i = F^{-1}(u_i) \quad , \quad i = 1, \dots, n}$

These are your simulated values from your desired dist'n.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 1

The inverse transform method.

Problem 1.1. *Source: Course 3, November 1985, Problem #19.*
Your goal is to simulate four draws from a binomial distribution with two trials and the probability of success in every trial equal to 0.30. You intend to use the inverse transform method. Here are the four values produced by the random number generator:

0.90 0.21 0.72 0.48.

Which values of the binomial were obtained from the above outputs of the random number generator?

$\rightarrow$: $X \sim Binomial(n=2, p=0.30)$

The pmf is

$$p_X(0) = (0.70)^2 = 0.49,$$

$$p_X(1) = 2(0.7)(0.3) = 0.42,$$

$$p_X(2) = (0.30)^2 = 0.09.$$

The cdf of X is:

$$F_X(x) = \begin{cases} 0, & \text{for } x<0 \\ 0.49, & \text{for } 0 \leq x < 1 \\ 0.91, & \text{for } 1 \leq x < 2 \\ 1, & \text{for } 2 \leq x \end{cases}$$

- 0.21 and 0.48 are both $< 0.49 \Rightarrow$ they map into 0
- 0.72 and 0.90 both map into 1.