M339D : March 10th, 2021.

Covered / Naked Option Writing.

Covered ... if the writer of the option has a simultaneous position in the underlying asset
↑ appropriate to the option which was written

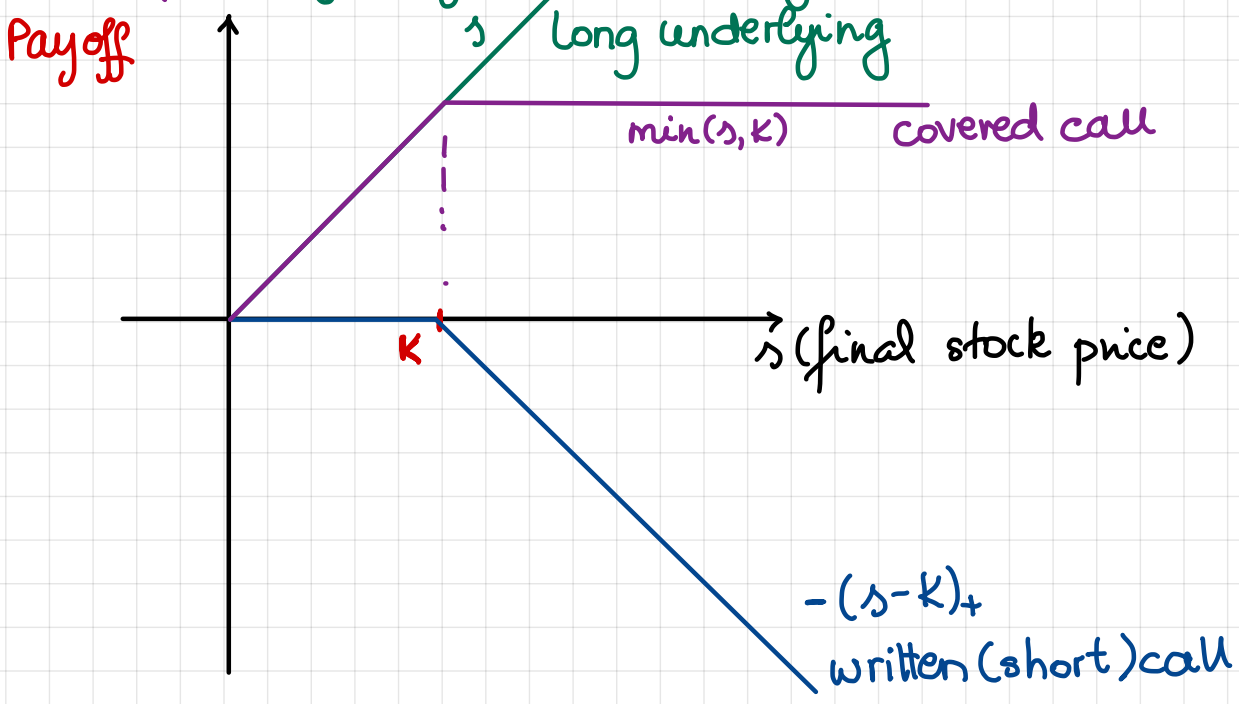
Naked ... if the writer of the option does not cover the option, i.e., they don't take the opposite position in the underlying

Example. A COVERED CALL

Assume, for simplicity, that the underlying stock pays no dividends.

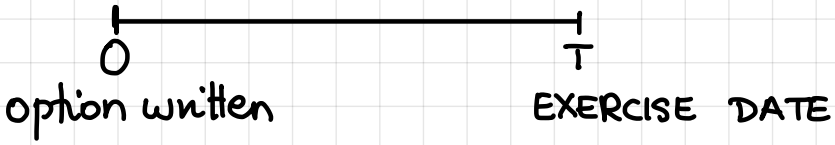
covered call

- written call option w/ strike K and exercise date T
- long/buy the underlying



European Put Option.

↖ Usually, a RIGHT to SELL



At time-0: The writer and the buyer of the put agree on:

- the underlying asset: $S(t), t \geq 0$
- the exercise date T
- the strike/exercise price K

The put premium $V_P(0)$ is paid by the buyer of the put and received by the writer of the put.

At time-T: The put's owner has the right, but NOT an obligation to SELL one unit of the underlying for the strike price K

The put's writer is obligated to do what the put's owner decides.

Q: What is the put's owner's optimal behavior @ time-T, i.e., what is the condition for exercise?

→: The condition is $K > S(T)$; then

LONG PUT (owner of put)

1 unit $S(T)$ ↓ ↑ strike price K

SHORT PUT (writer of put)

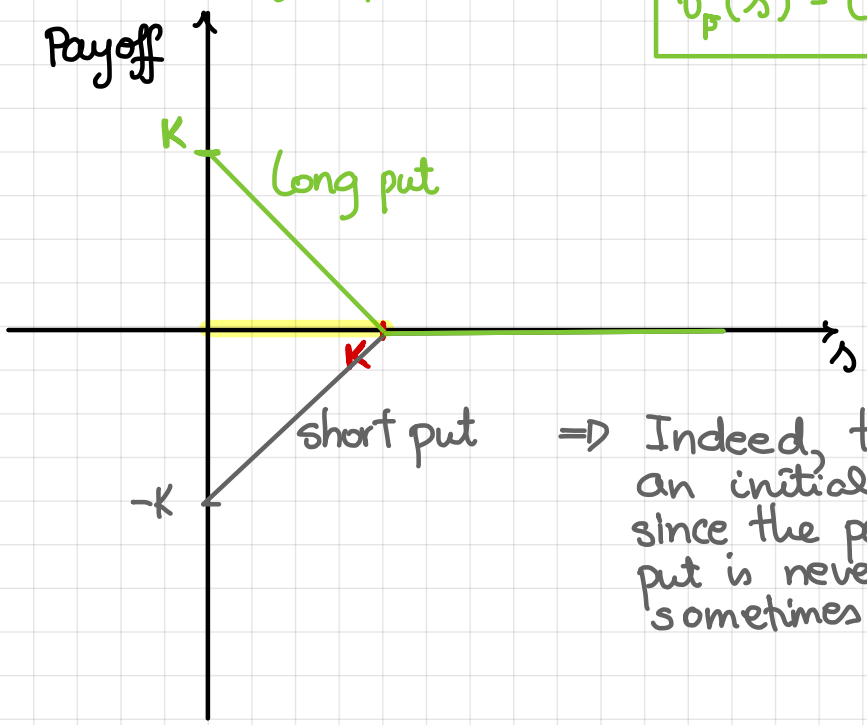
⇒ The payoff is: $V_P(T) = \begin{cases} K - S(T) & \text{if } K > S(T) \\ 0 & \text{if } K \leq S(T) \end{cases}$

$\Rightarrow$ The payoff is:

$$V_p(T) = \max(K - S(T), 0) = (K - S(T))_+$$

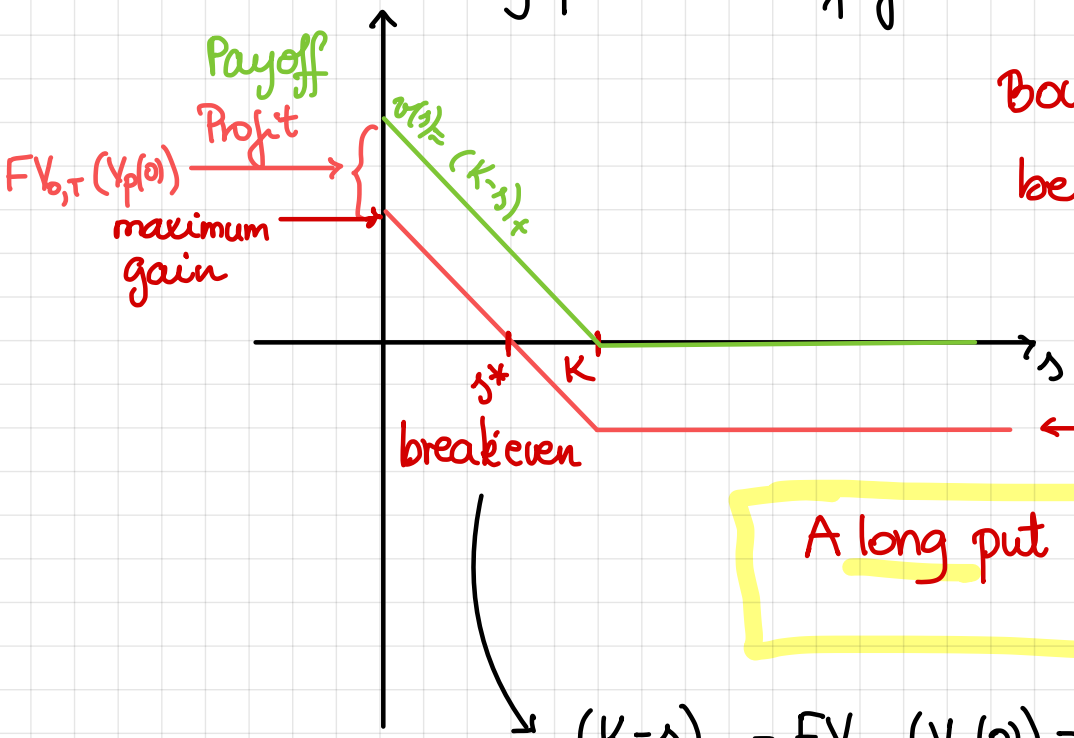
$\Rightarrow$ The payoff function:

$$v_p(s) = (K-s)_+$$



$\Rightarrow$ Indeed, there must be an initial premium $V_p(0)$ since the payoff of the short put is never positive and sometimes strictly negative.

Focus on the long put and figure out the profit curve.



Bounded both from below and from above.

A long put is short w.r.t. the underlying.

$$(K-s)_+ - FV_{0,T}(V_p(0)) = 0$$

It must be that $s^* < K$

$$K - s^* - FV_{0,T}(V_p(0)) = 0$$

$$s^* = K - FV_{0,T}(V_p(0))$$

Q: Tanker transporting crude oil from Port A to Port B. You can buy and write options on oil @ both ports. What do you do?

→:

Port A	Port B
Long CALL	Long PUT
Long Forward	Short Forward