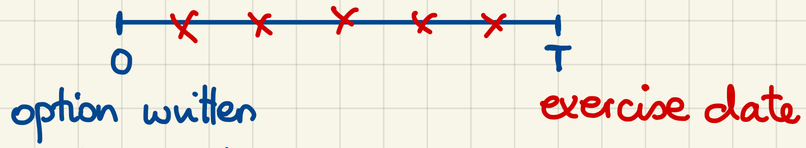
M339D: October 7th, 2022.

European Put Options.

↖ Usually, a right but not an obligation to sell!



At time 0: The writer and buyer of the put agree on:

- the underlying asset: $S(t), t \geq 0$
- the exercise date T
- the strike/exercise price K

The put premium $V_P(0)$ is paid by the put's buyer to the put's writer.

At time T:
- The put's owner has a right, but not an obligation to sell one unit of the underlying for the strike price K.
- The put's writer is obligated to do what the put's owner decides.

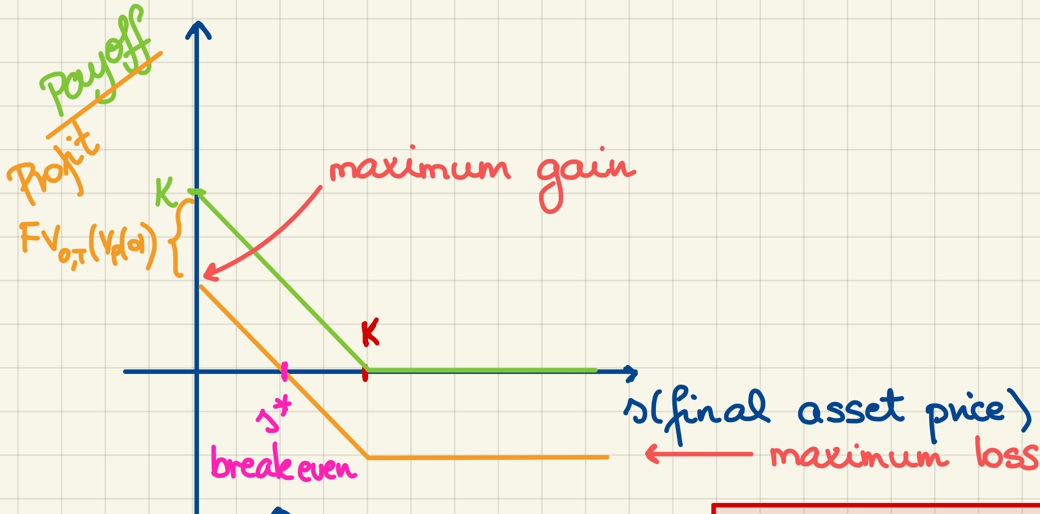
Q: What is the put owner's optimal behavior @ time T? What is the criterion for exercise?

$\longrightarrow$: $\boxed{K > S(T)}$ $\Rightarrow$ The payoff is: $K - S(T)$

Otherwise, the payoff is: 0

The payoff: $$V_P(T) = (K - S(T))_+$$

The payoff f'tion: $v_P(s) = (K - s)_+$

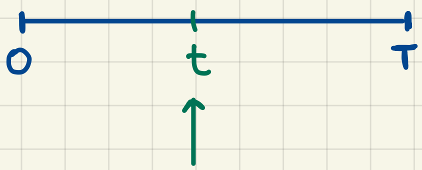


$$\boxed{s^* = K - FV_{0,T}(V_p(0))}$$

A LONG PUT is a <u>SHORT</u> position w.r.t. the underlying.

Moneyness.

Consider an option written @ time 0 w/ exercise date @ time T



Imagine the cashflow that would happen to the option's owner should they exercise the option @ that time t.

- call: $S(t) - K$
- put: $K - S(t)$

If cashflow is

- >0 we say the option is **in-the-money**.
- $=0$ we say the option is **at-the-money**.
- <0 we say the option is **out-of-the-money**.

The usual usage:

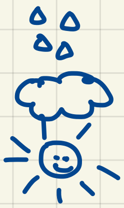
- We can specify the strike for a call/put by saying that it's <u>at-the-money</u>, i.e., $\boxed{K = S(0)}$

Finite Probability Spaces.

... serve as environments for possible paths that the price of the asset can take.

e.g.,

$$S(T) = \begin{cases} 120 & \text{w/ probab. } 1/6 \\ 100 & \text{w/ probab. } 1/2 \\ 85 & \text{w/ probab. } 1/3 \end{cases}$$



Q: What is the expected payoff of a $\boxed{\text{100-strike call}}$ on S?

$\longrightarrow$:

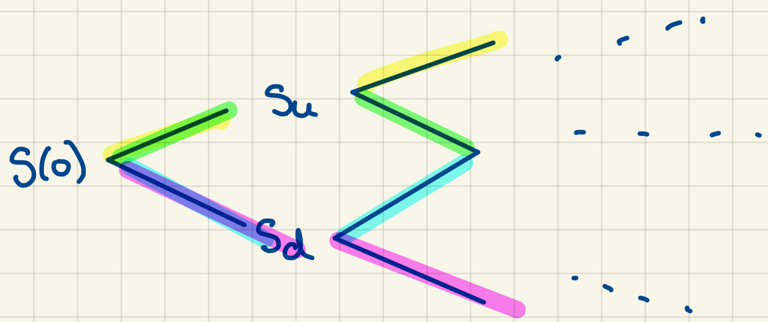
$$V_c(T) \sim \begin{cases} 20 & \text{w/ probab } 1/6 \\ 0 & \text{w/ probab. } 5/6 \end{cases}$$

$$\mathbb{E}[V_c(T)] = 20\left(\tfrac{1}{6}\right) = \frac{10}{3}$$

In general:

$$\boxed{\mathbb{E}[g(X)] \neq g(\mathbb{E}[X])}$$

e.g.,



All the finitely many scenarios are called states of the world.

We assume that:

- each can happen, i.e., its $\boxed{\text{probab} > 0}$

and

- they exhaust all possibilites, i.e., $\boxed{\sum \text{probab} = 1}$