
UNIVERSITY OF TEXAS AT AUSTINProblem Set #9

Binomial option pricing.

Problem 9.1. (2 points) In the setting of the one-period binomial model, denote by i the effective interest rate **per period**. Let u denote the “up factor” and let d denote the “down factor” in the stock-price model. If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage.

Problem 9.2. In our usual notation, which of the parameter choices below creates a binomial model with an arbitrage opportunity?

- (a) $u = 1.18$, $d = 0.87$, $r = 0.05$, $\delta = 0$, $h = 1/4$
- (b) $u = 1.23$, $d = 0.80$, $r = 0.05$, $\delta = 0.06$, $h = 1/2$
- (c) $u = 1.08$, $d = 1$, $r = 0.05$, $\delta = 0.04$, $h = 1$
- (d) $u = 1.28$, $d = 0.78$, $r = \delta$, $h = 2$
- (e) None of the above.

Problem 9.3. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a stock whose current price is \$80 per share. The stock pays dividends continuously with a dividend yield of 0.02. In the model, it is assumed that the stock price can either go up by \$5 or down by \$4.

You use the binomial tree to construct a replicating portfolio for a $(78, 82)$ –strangle on the above stock. What is the stock investment in the replicating portfolio?

- (a) Long 0.1089 shares.
- (b) Long 0.33 shares.
- (c) Short 0.1089 shares.
- (d) Short 0.33 shares.
- (e) None of the above.

Problem 9.4. Let the continuously compounded risk-free interest rate be equal to 0.04. Consider a one-period binomial tree with every period of length one year used to model the stock price of a non-dividend-paying stock whose current price is \$50 per share. In the model, it is assumed that the stock price can either go up by 5% or down by 10%.

You use the binomial tree to construct a replicating portfolio for a $(45, 55)$ –call bull spread on the above stock. What is the risk-free investment in the replicating portfolio?

- (a) Borrow \$45
- (b) Borrow \$43.24
- (c) Lend \$45
- (d) Lend \$43.24
- (e) None of the above.

Problem 9.5. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

- (a) About \$10
- (b) About \$10.83
- (c) About \$15.45
- (d) About \$20.84
- (e) None of the above.

Problem 9.6. (5 points) Consider the one-period binomial option pricing model. Let $V_C(0) > 0$ denote the price of a European call on a stock which pays continuous dividends. What is the impact on the value of European call option prices if the company decides to increase the dividend yield paid to the shareholders?

- (a) The call option price will drop.
- (b) The call option price will increase.
- (c) The call option price will always remain constant.
- (d) The impact on the price of the call cannot be determined using the binomial option pricing model.
- (e) There is not enough information provided.