Project #3: Elementary Monte Carlo. More on portfolios. Put prices.

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Problem #1 (10 points)

A weighted spinner has four different regions: blue, red, green, and yellow. The blue is twice as likely as red, the red is twice as likely as green, and the green is twice as likely as yellow. You are invited to play the game with the following outcomes:

- if the spinner lands on blue, you lose $5;
- if the spinner lands on red, you win $2;
- if the spinner lands on green, you win $10;
- if the spinner lands on yellow, you win $20.

Use *Monte Carlo* simulation to figure out the fair price for entering this game.

Problem #2 (10 points)

Use *Monte Carlo* simulation to estimate the number π.

Problem #3 (10 points)

Use *Monte Carlo* to estimate the expected distance of a randomly chosen point in a unit square to the center of that square.

Problem #4 (25 points)

Your initial wealth is exactly $100. You are allowed to invest in shares of a particular stock. You are also allowed to both lend and borrow at the continuously compounded risk-free interest rate of 0.05. Keeping your money uninvested is **not allowed**.

You can rebalance your portfolio every morning, once you have observed the opening stock price. This means that you can change the number of shares you own (if you decide to do so) and accordingly adapt your risk-free investment.

Here is a rule you will use to rebalance your portfolio daily:

I start by investing in one share of stock. Then, every day I will flip a fair coin. If the outcome is heads, I will buy one share of stock. If the outcome is tails, I will short one share. The balance of my wealth is to be invested at the risk-free rate (if needed, I will borrow at the continuously compounded risk-free interest rate).

Over the following 10 days, you observe the following stock prices for a non-dividend-paying stock:

Day	0	1	2	3	4	5	6	7	8	9	10
Stock price	100	80	64	80	64	80	100	80	64	80	100

As the time passes you follow investment rules above to rebalance your portfolio. Complete the following table describing your portfolio **just before and just after** the rebalancing is done. Even more precisely, for the 10 days, both for "morning" and "evening" print out:

- the number of shares of stock in the portfolio,
- the balance of the risk-free investment,
- the wealth in the stock,
- the total wealth.

Problem #4 (20 points)

Using *Monte Carlo* and the work you did in the previous problem, estimate the expected final wealth of a "noise trader" who uses the above strategy. Compare your estimate to the expected wealth you get by using the definition of expected value.

Problem #5 (25 points)

Apple puts

The file "apple-puts.csv" contains put prices observed on February 14th for European call options on the stocks of Apple Inc with expiry on May 19th and with varying strike prices.

(5 points) Import the data into a data frame. Create a line plot of the mid price (defined as the midpoint between the bid and the ask price) as it depends on the strike.

(5 points) What do you notice about the monotonicity and convexity of the above curve?

(15 points) Prove that the same conclusions regarding the monotonicity and convexity of the put prices with respect to the strike must hold in general **under the no arbitrage assumption**.