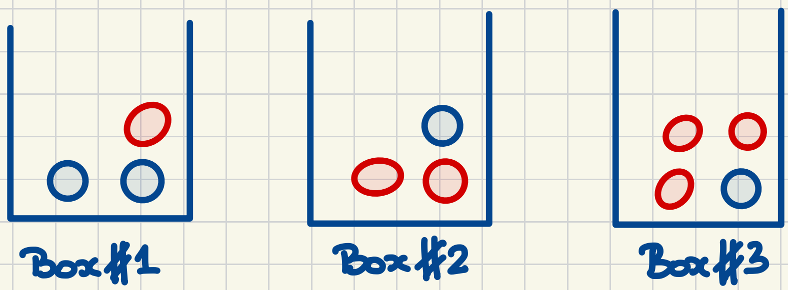
Inspiration.

Example. Which Box?



Q:

$$\mathbb{P}[\text{Box }\#i \mid \text{Red}] \quad \text{for all } i = 1,2,3$$

$$= \frac{\mathbb{P}[\text{Box }\#i \cap \text{Red}]}{\mathbb{P}[\text{Red}]} =$$

$$= \frac{\mathbb{P}[\text{Red} \mid \text{Box }\#i] \cdot \mathbb{P}[\text{Box }\#i]}{\sum_{i=1}^{3} \mathbb{P}[\text{Box }\#i] \cdot \mathbb{P}[\text{Red} \mid \text{Box }\#i]}$$

Making a modeling choice

$$\mathbb{P}[\text{Box }\#i] = \frac{1}{3}$$

$\vdots$

$$\frac{\frac{1}{3} \cdot (\quad)}{\frac{1}{3}(\quad + \quad + \quad)}$$

$$\mathbb{P}[\text{Box }\#i] = \frac{\#\text{ balls in } i}{\text{total }\#\text{ balls}}$$

X... predictor (say, numerical for simplicity).

Y... response; <u>categorical w/ two classes.</u>

$$Y = \begin{cases} 1 & \text{if category \#1} \\ 0 & \text{if category \#2} \end{cases}$$

Idea #1.

$$X \longmapsto Y = \underbrace{\beta_0 + \beta_1 X}_{\mathbb{R}} + \varepsilon \qquad \text{X}$$

Idea #2.

$$X \longmapsto \boxed{p(X) = \mathbb{P}[Y=1 \,/\, X]}$$

$$= \underbrace{\beta_0 + \beta_1 X}_{\mathbb{R}} + \varepsilon \qquad \text{X}$$

Def'n.

$$\boxed{\text{odds} = \frac{p(X)}{1-p(X)}} \in (0,\infty)$$

$$X \longmapsto \text{odds} = \underbrace{\beta_0 + \beta_1 X}_{\in \mathbb{R}} + \varepsilon$$

Def'n.

$$\text{logodds} = \ln(\text{odds}) = \ln\left(\frac{p(X)}{1-p(X)}\right)$$

$$\boxed{\text{logodds} = \beta_0 + \beta_1 X + \varepsilon}$$

⋮

$$\ln\left(\frac{p(x)}{1-p(x)}\right) = \beta_0 + \beta_1 X + \varepsilon$$

$$\frac{p(x)}{1-p(x)} = e^{\beta_0 + \beta_1 X + \varepsilon}$$

$$p(x) = (1-p(x))\, e^{\beta_0 + \beta_1 X + \varepsilon}$$

$$p(x) = e^{\beta_0 + \beta_1 X + \varepsilon} - p(x)\, e^{\beta_0 + \beta_1 X + \varepsilon}$$

$$\boxed{\hat{p}(x) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 X}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 X}}}$$