

- 11) You are given the following information about a portfolio that has two equally-weighted stocks, P and Q.

$$w_P = w_Q = \frac{1}{2}$$

- (i) The economy over the next year could be good or bad with equal probability.

$$P_g = P_b = \frac{1}{2}$$

- (ii) The returns of the stocks can vary as shown in the table below:

Stock	Return when economy is good	Return when economy is bad
P	10%	-2%
Q	18%	-5%

Calculate the volatility of the portfolio return.

→ R_T ... the return of the total portfolio

(A) 1.80%

$$\sigma_T = \sqrt{\text{Var}[R_T]} = ?$$

(B) 6.90%

✗ (C) 7.66%

(D) 8.75%

(E) 13.42%

$$R_T = \frac{1}{2}(R_P + R_Q)$$

$$R_T \sim \begin{cases} \frac{0.14}{\text{if good}} & \text{w/ probab. } P_g = \frac{1}{2} \\ \frac{-0.035}{\text{if bad}} & \text{w/ probab. } P_b = \frac{1}{2} \end{cases}$$

$$\text{Var}[R_T] = \mathbb{E}[R_T^2] - (\mathbb{E}[R_T])^2$$

$$\bullet \mathbb{E}[R_T] = \frac{1}{2}(0.14 + (-0.035)) = 0.0525$$

$$\bullet \mathbb{E}[R_T^2] = \frac{1}{2}((0.14)^2 + (-0.035)^2) = 0.0104125$$

$$\text{Var}[R_T] = 0.0104125 - (0.0525)^2 = 0.0076563$$

$$\sigma_T = \sqrt{0.0076563} = 0.0875$$

□

Diversification of an Equally Weighted Portfolio.

$$w_i = \frac{1}{n} \quad \text{for } i = 1 \dots n$$

$$\Rightarrow R_p = \frac{1}{n} (R_1 + R_2 + \dots + R_n)$$

$$\Rightarrow \text{Var}[R_p] = \text{Var}\left[\frac{1}{n}(R_1 + R_2 + \dots + R_n)\right]$$

$$= \frac{1}{n^2} \cdot \text{Var}[R_1 + R_2 + \dots + R_n]$$

$$= \frac{1}{n^2} \left(\sum_{i=1}^n \text{Var}[R_i] + \sum_{i \neq j} \text{Cov}[R_i, R_j] \right)$$

$$= \frac{1}{n} \cdot \frac{1}{n} \sum_{i=1}^n \text{Var}[R_i] + \frac{1}{n^2} n(n-1) \frac{1}{n(n-1)} \sum_{i \neq j} \text{Cov}[R_i, R_j]$$

Average Variance
of the Individual
Components
(assume bounded)

Average Covariance
Between the Stocks in
the Portfolio

$$\downarrow n \rightarrow \infty$$

$$0$$

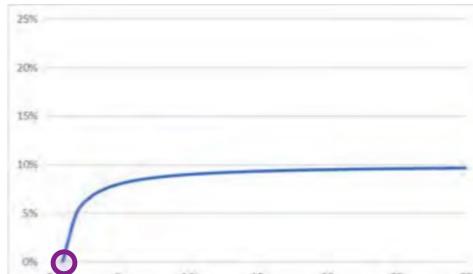
Average Covariance

- 9) You are given the following information about an equally-weighted portfolio of n stocks:

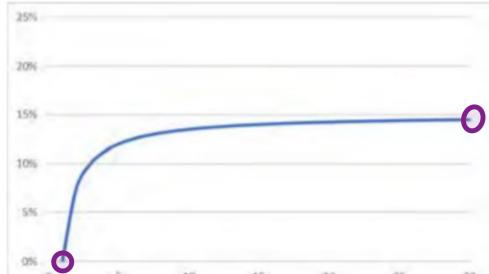
- (i) For each individual stock in the portfolio, the variance is 0.20.
- (ii) For each pair of distinct stocks in the portfolio, the covariance is 0.10.

Determine which graph displays the variance of the portfolio as a function of n .

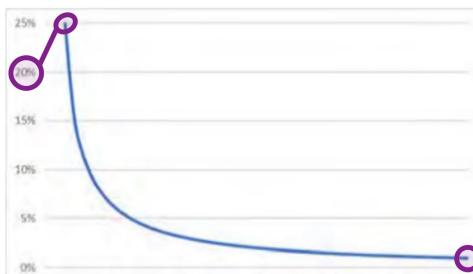
X
(A)



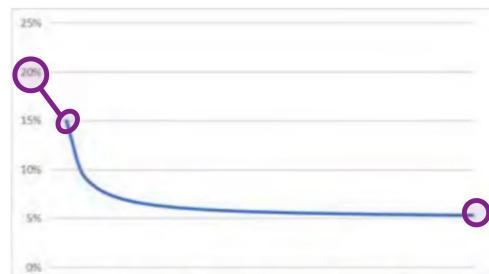
X
(B)



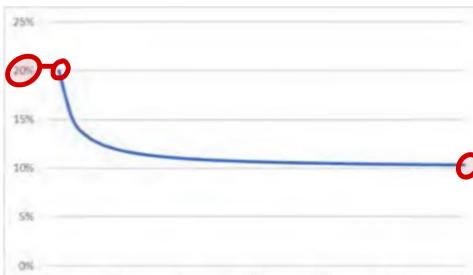
X
(C)



X
(D)



∴
(E)



Diversification for a General Portfolio.

Assume : $w_i > 0$

Recall :

$$\begin{aligned}\sigma_p^2 &= \text{Var}[R_p] = \sum_{i=1}^n w_i \cdot \text{Cov}[R_i, R_p] \\ &= \sum_{i=1}^n w_i \cdot \sigma_i \cdot \underbrace{\sigma_p}_{\sigma_p} \rho_{i,p} \\ &= \underbrace{\sigma_p}_{\sigma_p} \cdot \sum_{i=1}^n w_i \cdot \sigma_i \cdot \rho_{i,p} \quad / : \sigma_p\end{aligned}$$

$$\sigma_p = \sum_{i=1}^n w_i \cdot \underbrace{\sigma_i}_{\leq 1} \underbrace{\rho_{i,p}}_{\leq 1}$$

$$\sigma_p \leq \sum_{i=1}^n w_i \cdot \sigma_i$$

Equality only if all the assets are perfectly positively correlated.