UNIVERSITY OF TEXAS AT AUSTIN

Quiz #8

The log-normal distribution. Log-normal stock prices.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 8.1. (5 points) *Source: Problem 18.6 in McDonald.*
Let $X \sim N(mean = 2, variance = 5)$.

(i) (3 points) Find $\mathbb{E}[e^X]$.
(ii) (2 points) Find the median of e^X.

Solution:

$$\mathbb{E}[e^X] = e^{2+\frac{1}{2}5} \approx 90.017.$$

Let us denote the median by m. Then,

$$\frac{1}{2} = \mathbb{P}[e^X \leq m] = \mathbb{P}[X \leq \ln(m)].$$

So, since the mean of the normal variable X is equal to 2, $m = e^2 = 7.3891$.

Problem 8.2. (2 points) The product of log-normal random variables is normal. *True or false?*

Solution: FALSE
The product of lognormals is itself lognormally distributed.

Problem 8.3. (2 points) The mean of a lognormal stock price is at most as large as its median. *True or false?*

Solution: FALSE
It's the other way around.

Problem 8.4. Let $S(t)$ denote the time$-t$ stock price for $t \geq 0$. Let us use the Black-Scholes framework for the stock price. Then, the random variable

$$\ln\left(\frac{S(t)}{S(0)}\right)$$

has the log-normal distribution for every t. *True or false?*

Solution: FALSE
The realized returns have the normal distribution.

Problem 8.5. (5 points)

Assume the Black-Scholes framework for stock prices, i.e., assume the lognormal distribution of the stock prices. Let the mean rate of appreciation on a stock be 0.05 and let its volatility be equal to 0.25.

The continuously compounded risk-free interest rate is 0.04.

What is the probability that the stock will have a positive return over the period of two years?

(a) 0.5438
(b) 0.7704
(c) 0.8554
(d) 1
(e) None of the above.

Solution: **(a)**
Let us denote the return over the period of two years by $R(0,2)$. Then, in our usual parametrization and notation,

$$\mathbb{P}[R(0,2) > 0] = \mathbb{P}\left[\left(\alpha - \delta - \frac{\sigma^2}{2}\right)2 + \sigma\sqrt{2}Z > 0\right] = \mathbb{P}\left[Z < \frac{\left(\alpha - \delta - \frac{\sigma^2}{2}\right)\sqrt{2}}{\sigma}\right]$$

where $Z \sim N(0,1)$. The answer is

$$N\left(\frac{\left(0.05 - \frac{0.25^2}{2}\right)\sqrt{2}}{0.25}\right) = N(0.11) = 0.5438.$$