The Set-up

- A compound option is an option to buy an option
- Let us draw a timeline
- Consider a call on a call option, i.e., an option to buy a call option with maturity T and strike price K at some exercise time $T_1 < T$, for some strike price K_1
- This call on a call should be exercise at time T_1 only if the strike price K_1 is lower than the price of the underlying call option at time T_1
- So, the payoff of this option at time T_1 is

$$(C(T_1) - K_1)^+ = (C(S(T_1), K, T - T_1) - K_1)^+$$

where $C(T_1) = C(S(T_1), K, T - T_1)$ is the current price of the underlying call option

Parity

- Let *CallOnCall* denote the price of the compound call on an underlying call option with maturity T_1
- Let *PutOnCall* denote the price of the compound put on an underlying call option (the exact analogue of the above call-on-call)
- Let *Call* denote the price of the underlying call option
- Then the parity for compound options reads as

$$CallOnCall - PutOnCall = Call - x^{-rT_1}$$