Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
In-Term Exam III
Instructor: Milica Čudina

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Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.
Time: 50 minutes

Problem 3.1. (5 points) Let the continuously compounded risk-free interest rate be 0.05. The current price of a continuous-dividend-paying stock is 80. Its dividend yield is 0.02. You model the price of this stock in half a year using a one-period binomial tree with the up factor of 1.2 and the down factor of 0.8. Consider a half-year, 90-strike European put option on the above stock. What is the investment in the shares of stock in the replicating portfolio for the put?

Problem 3.2. (5 points) Assume a positive interest rate. For any strike K, let $V_C(K)$ denote the price of a European call on stock S with expiration date T and strike price K and let $V_P(K)$ denote the price of a European put on stock S with expiration date T and strike price K.

Let $K_1 < K_2 < K_3$.

Which one of the following statements is FALSE?

(a) $V_P(K_1) \leq V_P(K_2)$

(b) $V_C(K_1) \geq V_C(K_2)$

(c) $V_C(K_2) - V_C(K_1) \leq K_2 - K_1$

(d) $V_P(K_1) - V_P(K_2) \leq K_1 - K_2$

(e) $\frac{V_P(K_2) - V_P(K_1)}{K_2 - K_1} \leq \frac{V_P(K_3) - V_P(K_2)}{K_3 - K_2}$

Problem 3.3. (5 points) Let the continuously compounded, risk-free interest rate be 0.04. Your co-worker Psmith considers a non-dividend-paying stock whose price in one year is modeled using a one-period binomial tree. Psmith tells you that a one-year, 93-strike European call option has the following properties:

- the payoff of the option at the up node equals 39.20;
- the number of shares of stock in the replicating portfolio for the call option equals 0.80.

What is the amount borrowed in the replicating portfolio for the call option?

Problem 3.4. (5 points) You are given that the price of:

- a \$50-strike, one-year European call equals \$9,
- a \$65-strike, one-year European call equals \$3.

Both options have the same underlying asset. What is the maximal price of a \$56-strike, one-year European call such that there is no arbitrage in our market model?

Problem 3.5. (5 points) A long strangle position ...

(a) is equivalent to a short butterfly spread.
(b) can be replicated with a short call and a long put with the same strike, underlying asset and exercise date.
(c) is always strictly more expensive than the straddle on the same underlying asset and with the same exercise date.
(d) is a speculation on the stock's volatility.
(e) None of the above.

Problem 3.6. (5 points) Which one of the following positions always has an infinite upward potential in the sense that the payoff diverges to positive infinity as the argument s (standing for the final stock price) tends to positive infinity?

(a) A long call option.

(b) A short straddle.

(c) A long bull spread.

(d) A long butterfly spread.

(e) None of the above.

Problem 3.7. (10 points) Consider a non-dividend-paying stock whose current price is \$90 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$100, or \$85 in one year.

The continuously compounded, risk-free interest rate is 0.05.

The price of a $K-$strike, one-year European straddle on the above stock, consistent with the above stock-price model, is 6.50. How much is K?

Problem 3.8. (2 points) It is never optimal to exercise an American call option on a non-dividend paying stock early. *True or false? Why?*

Problem 3.9. (2 points) The payoff curve of a call bear spread is never positive. *True or false? Why?*

Problem 3.10. (2 points) Consider a binomial asset-pricing model in which the length of every period equals a quarter-year. Let $i^{(4)}$ denote the nominal interest rate compounded quarterly. Then, in our usual notation, to avoid arbitrage, we must have

$$e^{\delta/4}d < 1 + \frac{i^{(4)}}{4} < e^{\delta/4}u. \tag{3.1}$$

True or false?

Problem 3.11. (2 points) Arithmetic-average-price Asian put options are always worth more than the geometric-average-price Asian put options with the same strike price. *True or false? Why?*

Problem 3.12. (2 points) A call-on-put option is worth at least as much as an otherwise identical put-on-put option. *True or false? Why?*

Problem 3.13. (5 points) You are required to price a one-year, yen-denominated currency option on the USD. The exchange rate over the next year is modeled using a forward binomial tree with the number of periods equal to 4. Assume that the volatility of the exchange rate equals 0.1.

The continuously compounded risk-free interest rate for the yen equals 0.05, while the continuously compounded risk-free interest rate for the USD equals 0.02. What is the value of the so-called up factor u in the resulting forward binomial tree?

Problem 3.14. (10 points) Let the continuously-compounded, risk-free interest rate be equal to 0.04. The current price of a non-dividend-paying stock is \$100. Its volatility is 0.2. The evolution of this stock over the following year is modeled using a four-period forward binomial tree. What is the price of a one-year, \$95-strike European put on the above stock consistent with the above tree?

Problem 3.15. (10 points) Consider a one-period forward binomial model for the stock-price movement over the following year. The current stock price is $S(0) = 100$, its dividend yield is 0.05 and its volatility is 0.3 The continuously compounded risk-free interest rate is given to be 0.05.

Consider American call options on this stock with the expiration date at the end of the period/year. What is the maximal strike price K for which there is early exercise?

Problem 3.16. (10 points) Consider a non-dividend paying stock whose current price is \$100 per share. You model the evolution of the stock price over the following year using a two-period binomial tree with $u = 1.12$ and $d = 0.90$.

The continuously-compounded, risk-free interest rate is 0.04.

Consider a \$100-strike, one-year **down-and-out** call option with a barrier of \$95 on the above stock. What is the price of this option consistent with the above stock-price model?

Problem 3.17. (15 points) Your goal is to price a put option on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is three months.
(ii) $u_F/d_F = 5/4$, where u_F is one plus the rate of gain on the futures price if it goes up, and d_F is one plus the rate of loss if it goes down.
(iii) The risk-neutral probability of an up move is $2/3$.
(iv) The initial futures price is 70.
(v) The continuously compounded risk-free interest rate is 4%.

Find the price of a half-year, 70-strike American put option on the futures contract.