University of Texas at Austin, Department of Mathematics
M358K - Applied Statistics

The Mock In-term Three

Problem 1.1. (5 points) Alice performs a $z-$test. The $z-$score she obtains is equal to -1.76. Which decision does she make?

(a) Reject the null hypothesis.
(b) Fail to reject the null hypothesis.
(c) Reject the alternative hypothesis.
(d) Not enough information is given to answer this question.
(e) None of the above.

Problem 1.2. (5 points) A manufacturer of scented candles claims that their luxury candles last at least 12 hours. You suspect that this might not be entirely true and you decide to test their claim. You model the candle burn times as normal with a known standard deviation of 2 hours (based on the last holiday season's study). You purchase and burn 16 candles recording the sample average of 11 hours and 45 minutes. What is your decision?

(a) Reject at the 1% significance level.
(b) Fail to reject at the 1% significance level; reject at the 5% significance level.
(c) Fail to reject at the 5% significance level; reject at the 10% significance level.
(d) Fail to reject at the 10% significance level.
(e) None of the above.

Problem 1.3. (5 points) *Organically Produced* claims that their supplements contain 65 mg of iron per capsule. To be able to continue to maintain their claim, they periodically test the contents of a batch of 100 randomly chosen capsules from their production line. They model the iron content as normally distributed with a known standard deviation of 5 mg. In the last test, the sample average was 64 mg. What is the $p-$value?

(a) 0.0228
(b) 0.0384
(c) 0.0418
(d) 0.0456
(e) None of the above.

Problem 1.4. (5 points) Suppose that we have a random sample of size 25 from a normal population with an unknown mean μ and a standard deviation of 4. Bertram Wooster was in charge of testing the hypotheses

$$H_0 : \mu = 10 \quad vs. \quad H_a : \mu > 10.$$

Before he went on vacation, he obtained the rejection region $[11.2, \infty)$. However, he forgot to tell anyone which significance level α he used. Calculate α.

(a) 0.0401

(b) 0.0495

(c) 0.05

(d) 0.0668

(e) None of the above.

Problem 1.5. (5 points) *Source: "Mathematical Statistics with Applications in R" by Ramachandran and Tsokos.* A cross is hypothesized to result in a 3 : 1 phenotypic ratio of red-flowered to white-flowered plants. You set up a hypothesis test to test this claim. Suppose your cross actually produces 170 red- and 30 white-flowered plants. What is the $p-$value you obtain?

a. Less than 0.005.
b. Between 0.005 and 0.01.
c. Between 0.01 and 0.025.
d. Between 0.025 and 0.05.
e. None of the above.

Problem 1.6. (5 points) A die is rolled 60 times and the face values are recorded. The results are as follows:

Up face	1	2	3	4	5	6
Number of occurrences	8	11	5	12	15	9

You test the hypothesis that the die is fair. What can you say about the $p-$value?

(a) Less that 0.05.

(b) Between 0.05 and 0.10.

(c) Between 0.10 and 0.20.

(d) Between 0.20 and 0.30.

(e) It's greater than 0.30.

Problem 1.7. (5 points) An improvement is a process for manufacturing is being considered. Samples are taken both from parts manufactured using the existing method and the new method. It is found that 50 out of a sample of 1000 items produced using the existing method are defective. It is also found that 40 out of a sample of 1600 items produced using the new method are defective. The two samples are independent.

Find the 80%-confidence interval for the true difference in the proportions of defectives produced using the existing and the new method. *Note: Round your point estimate and the margin of error to four places after the decimal point.*

(a) $(0.0149, 0.0351)$
(b) $(0.0171, 0.0329)$
(c) $(0.0120, 0.0380)$
(d) $(0.0095, 0.0405)$
(e) None of the above.

Problem 1.8. (5 points) Let the random sample $X_1, \ldots, X_{10}$ be drawn from a normal distribution with mean 2 and variance 1. Define

$$Y = \sum_{i=1}^{10} (X_i - 2)^2.$$

Find the constant q such that

$$\mathbb{P}[Y \leq q] = 0.975.$$

(a) 18.31
(b) 19.02
(c) 20.48
(d) 21.92
(e) None of the above.

Problem 1.9. (5 points) In a simple random sample of 1000 Austinites owning televisions, it is found that 480 do not have cable (but do have Netflix or some such or just game on the big screen). Find an 92% confidence interval for the true proportion of Austinites with television who do not have cable. *Note: Round the margin of error to four places after the decimal point.*

(a) 0.48 ± 0.0277
(b) 0.48 ± 0.0260
(c) 0.48 ± 0.0310
(d) 0.48 ± 0.0158
(e) None of the above.

Problem 1.10. Let p_m and p_f be the population proportions of male and female warblers who return to their hatching site. You want to test whether the two proportions are different. The observed number of males who returned is 135 out of 900, while the observed number of females who returned is 84 out of 700. What is your decision for this hypothesis test? *Note: Keep four significant places after the decimal point for all your point estimates.*

(a) Reject at the 0.01 significance level.

(b) Fail to reject at the 0.01 significance level; reject at the 0.025 significance level.

(c) Fail to reject at the 0.025 significance level; reject at the 0.05 significance level.

(d) Fail to reject at the 0.05 significance level; reject at the 0.10 significance level.

(e) Fail to reject at the 0.10 significance level.

Problem 1.11. (5 points) *Source: "Probability and Statistical Inference" by Hogg, Tanis, and Zimmerman.*
A study was conducted to determine whether there is an association between the choice of the most credible media source for reporting news and the education level. The results are displayed in the following table:

	Newspaper	Television	Radio	Total
Grade School	45	22	6	73
High School	94	115	30	239
College	49	52	13	114
Total	188	189	49	426

Your goal is to test whether the choice of the most credible medium is independent from the education level. The observed value of the relevant test statistic is 11.399. What is your decision?

(a) Reject at the 0.01 significance level.

(b) Fail to reject at the 0.01 significance level; reject at the 0.025 significance level.

(c) Fail to reject at the 0.025 significance level; reject at the 0.05 significance level.

(d) Fail to reject at the 0.05 significance level; reject at the 0.10 significance level.

(e) Fail to reject at the 0.10 significance level.