M339D: April 19th, 2021.

"Cord-Slope" Bounds.

Let $K_1 < K_2$.

$$0 \leq \left\{ \begin{array}{c} V_C(K_1) - V_C(K_2) \\ V_P(K_2) - V_P(K_1) \end{array} \right\} \leq \boxed{PV_{0,T}(K_2 - K_1)}$$

monotonicity

our current claim.

✓ $\boxed{\text{Calls.}}$ Assume, to the contrary, that there exist $K_1 < K_2$ such that

$$V_C(K_1) - V_C(K_2) > PV_{0,T}(K_2 - K_1) \qquad \text{X}$$

$$\Longleftrightarrow$$

$$\boxed{V_C(K_1)} > \boxed{V_C(K_2)} + PV_{0,T}(K_2 - K_1)$$

I. Suspicion. ✓

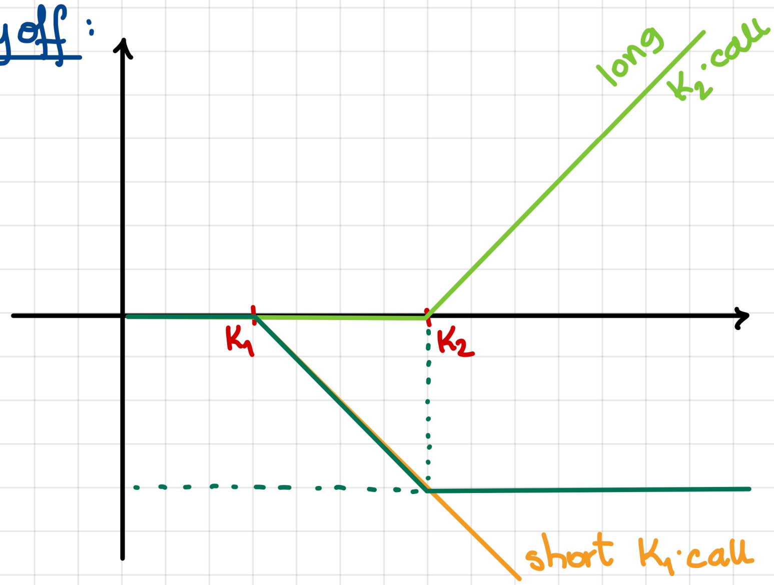
II. Propose an arbitrage portfolio:

- write the K_1-strike call
- long the K_2-strike call

$\Big\}$ call Bear Spread

III. Verification.

$$\left\{ \begin{array}{l} \text{Initial Cost: } V_C(K_2) - V_C(K_1) < -PV_{0,T}(K_2 - K_1) \\ \text{Payoff:} \end{array} \right.$$



lower bound @ $K_1 - K_2$

$$\text{Profit} > \cancel{K_1} - \cancel{K_2} + \cancel{FV_{0,T}}\left(+ \cancel{PV_{0,T}}\,(\cancel{K_2} - \cancel{K_1})\right) = 0$$

$\Rightarrow$ Indeed, we managed to construct an arbitrage portfolio!

$\boxed{\text{Puts.}}$ What is the suspected arbitrage portfolio you would propose if the above inequality for puts is violated?

Assume that there exist $K_1 < K_2$ such that

$$V_P(K_2) - V_P(K_1) > PV_{0,T}(K_2 - K_1)$$

$$\left.\begin{array}{l}\bullet \text{ long } K_1\text{-strike put} \\ \bullet \text{ write } K_2\text{-strike put}\end{array}\right\} \text{Put bull spread}$$

12. You are given:

(i) $C(K, T)$ denotes the current price of a K-strike T-year European call option on a nondividend-paying stock.

(ii) $P(K, T)$ denotes the current price of a K-strike T-year European put option on the same stock.

(iii) S denotes the current price of the stock.

(iv) The continuously compounded risk-free interest rate is r.

Which of the following is (are) correct?

monotonicity $\quad \leq PV_{0,T}(55-50) = PV_{0,T}(5) = 5e^{-rT}$

(I) $\quad 0 \leq C(50, T) - C(55, T) \leq 5e^{-rT}$ ✓

(II) $\quad 50e^{-rT} \leq P(45, T) - C(50, T) + S \leq 55e^{-rT}$

(III) $\quad 45e^{-rT} \leq P(45, T) - C(50, T) + S \leq 50e^{-rT}$

Focus on:

$\underbrace{P(45,T)}_{} - C(50,T) + S$

$\parallel$ Put-Call Parity

$C(45,T) - \underbrace{\cancel{F^P_{0,T}(S)}}_{S} + PV(45) - C(50,T) \cancel{+S}$

$45e^{-rT} \leq \underbrace{C(45,T) - C(50,T)}_{0 \leq \;\cdot\; \leq 5e^{-rT}} + \overbrace{PV(45)}^{45e^{-r\cdot T}} \leq 50e^{-rT}$ ✓

(A) (I) only

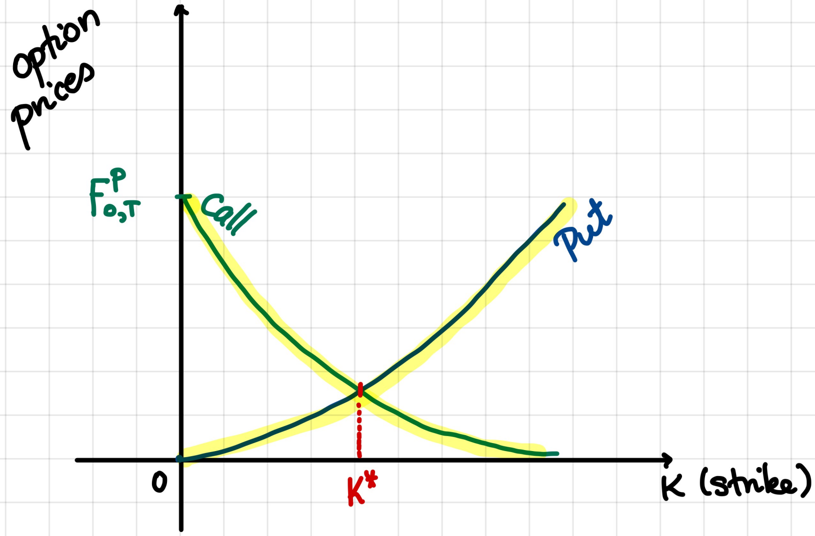
✗ (B) (II) only

✗ (C) (III) only

(D) (I) and (II) only

(E) (I) and (III) only

What do we know so far?



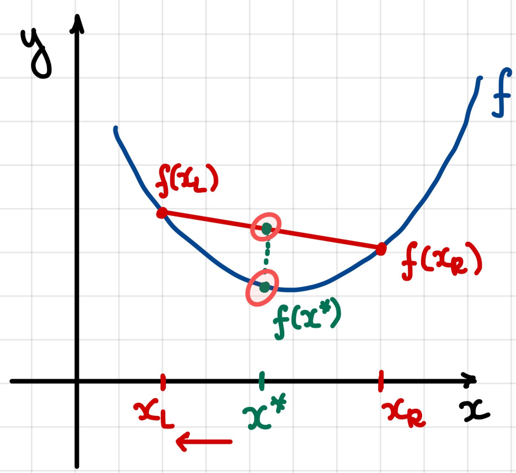
Call Price equals the Put Price @ strike K^*

Put-call parity: $\underbrace{V_c(K^*) - V_p(K^*)}_{= 0} = F^P_{0,T}(S) - PV_{0,T}(K^*)$

$\Rightarrow \quad F^P_{0,T}(S) = PV_{0,T}(K^*)$

$\Rightarrow \quad \boxed{K^* = F_{0,T}(S)}$

Convex Function.



There is a constant λ such that $\lambda \in (0,1)$

$$x^* = \lambda \cdot x_L + (1-\lambda) \cdot x_R$$

In fact: $\quad x^* = \lambda \cdot x_L + x_R - \lambda \cdot x_R$

$$\lambda(x_R - x_L) = x_R - x^*$$

$$\lambda = \frac{x_R - x^*}{x_R - x_L} \quad \text{and} \quad 1-\lambda = \frac{x^* - x_L}{x_R - x_L}$$

Our function f is convex if

$$\boxed{f(x^*) \leq \lambda \cdot f(x_L) + (1-\lambda) \cdot f(x_R)}$$