Splines

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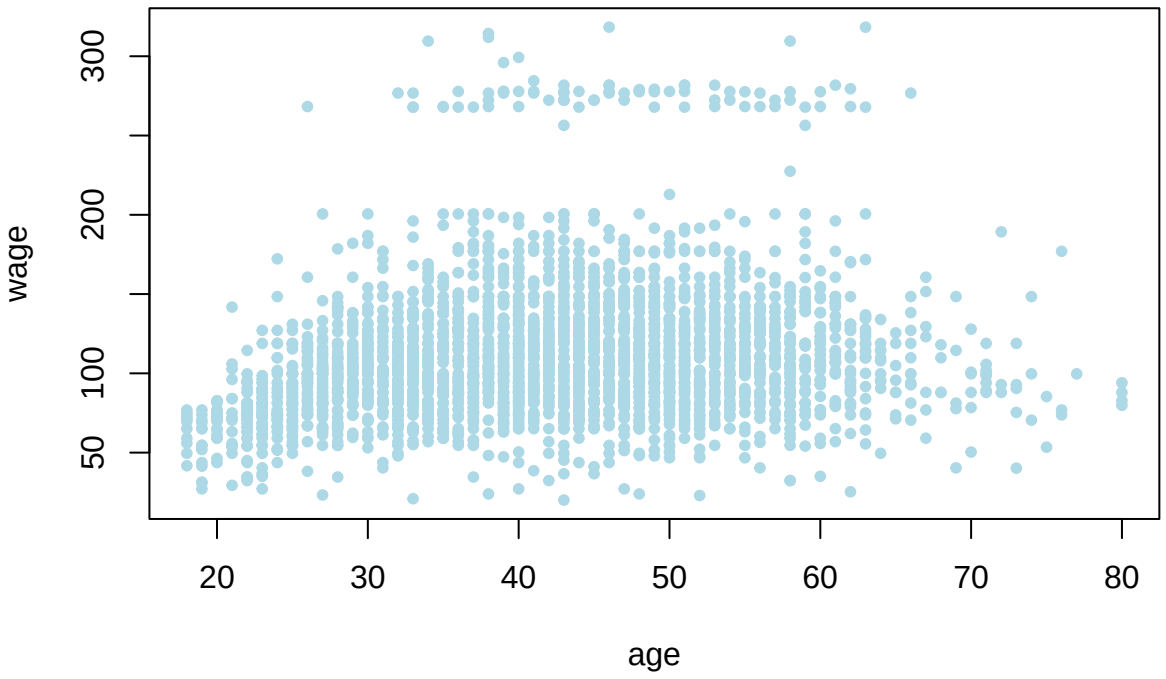
Here, I am adapting part of the lab associated with Chapter 7 of the textbook.

In this lab, we analyze the `Wage` data considered in the examples throughout this chapter, in order to illustrate the fact that many of the complex non-linear fitting procedures discussed can be easily implemented in `R`. We begin by loading the `ISLR2` library, which contains the data.

```
library(ISLR2)
attach(Wage)
dim(Wage)
```

```
## [1] 3000   11
```

```
plot(age, wage,
     pch=20, col="lightblue")
```



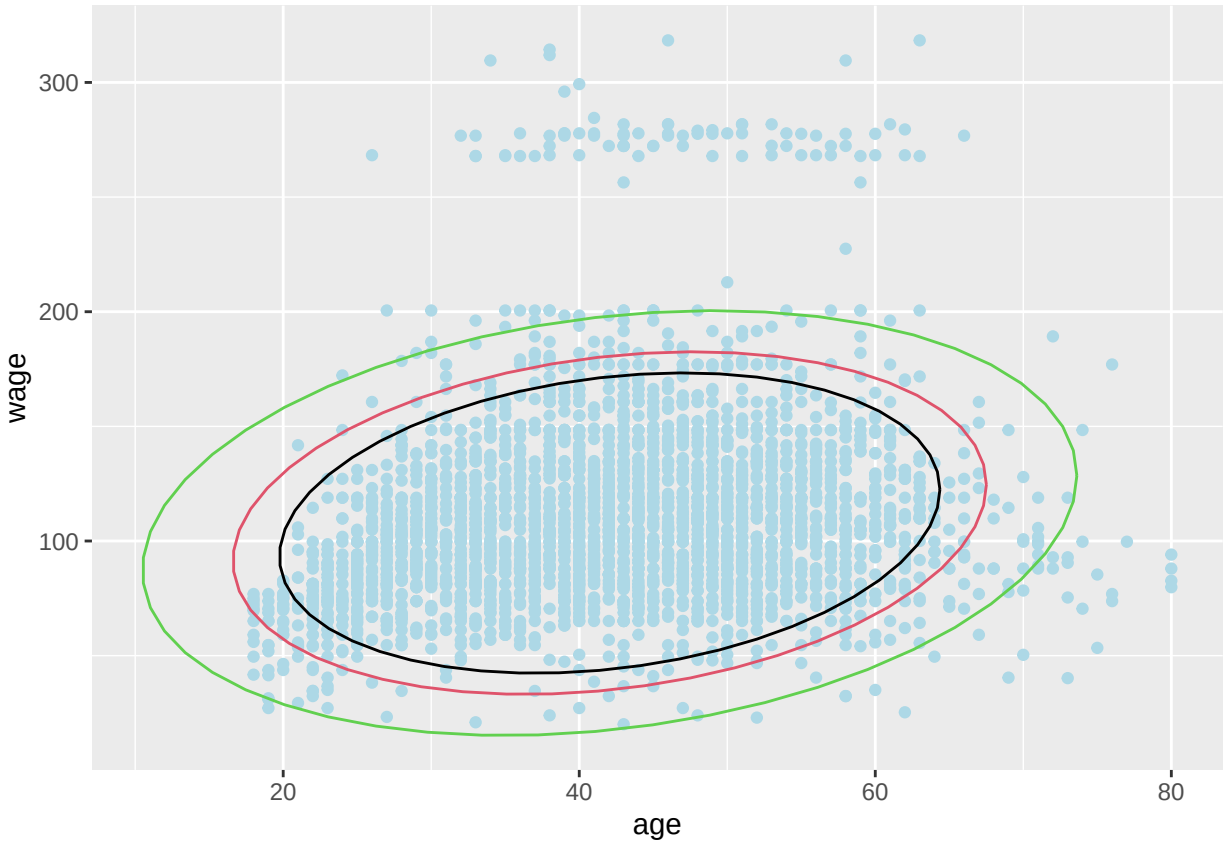
I am inserting some more exploratory data analysis here based on student feedback.

```
library(ggplot2)

data=data.frame(age,wage)

ggplot(data, aes(x = age, y = wage)) +
  geom_point(color="lightblue") +
  stat_ellipse(level = 0.9) +
  stat_ellipse(level = 0.95, color = 2) +
```

```
  stat_ellipse(level = 0.99, color = 3)
```



Cubic splines

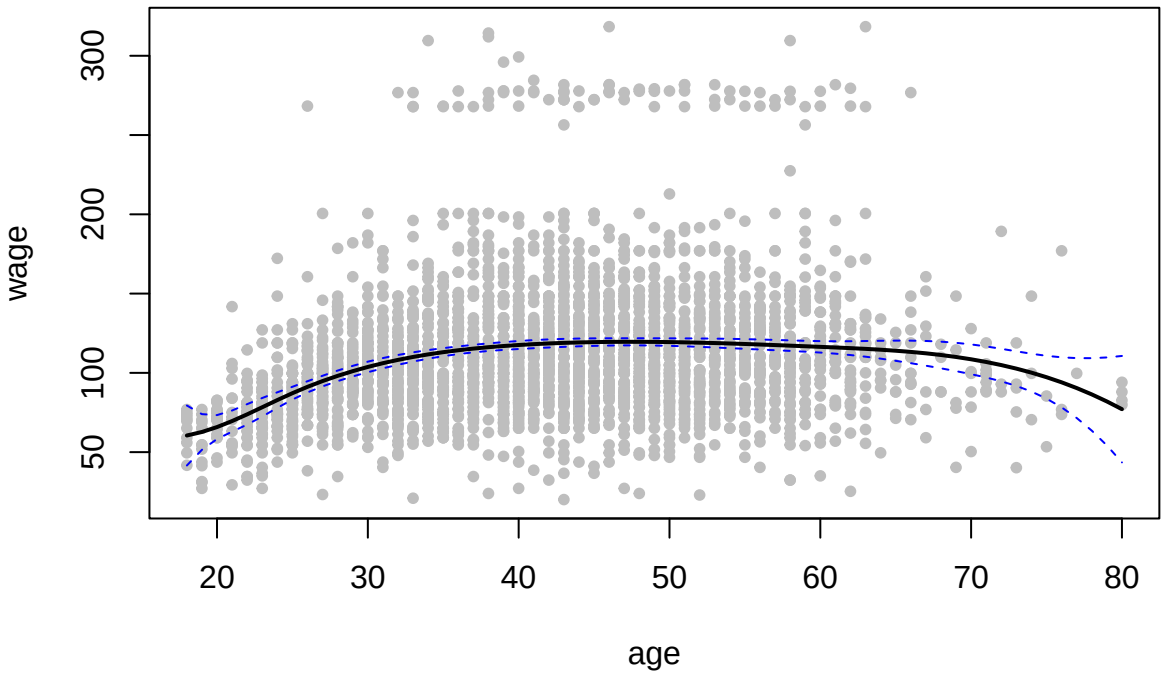
In order to fit regression splines in R, we use the `splines` library. In Section 7.4, we saw that regression splines can be fit by constructing an appropriate matrix of basis functions. The `bs()` function generates the entire matrix of basis functions for splines with the specified set of knots. By default, cubic splines are produced. Fitting `wage` to `age` using a regression spline is simple:

```
library(splines)
fit <- lm(wage ~ bs(age, knots = c(25, 40, 60)), data = Wage)

agelims <- range(age)
age.grid <- seq(from = agelims[1], to = agelims[2])
preds <- predict(fit, newdata = list(age = age.grid),
    se = TRUE)
se.bands <- cbind(preds$fit + 2 * preds$se.fit,
    preds$fit - 2 * preds$se.fit)

pred <- predict(fit, newdata = list(age = age.grid), se = T)

plot(age, wage, col = "gray", pch=20)
lines(age.grid, pred$fit, lwd = 2)
lines(age.grid, pred$fit + 2 * pred$se, lty = "dashed", col="blue")
lines(age.grid, pred$fit - 2 * pred$se, lty = "dashed", col="blue")
```



Here we have prespecified knots at ages 25, 40, and 60. This produces a spline with six basis functions. (Recall that a cubic spline with three knots has seven degrees of freedom; these degrees of freedom are used up by an intercept, plus six basis functions.) We could also use the `df` option to produce a spline with knots at uniform quantiles of the data.

```
dim(bs(age, knots = c(25, 40, 60)))
```

```
## [1] 3000    6
```

```
dim(bs(age, df = 6))
```

```
## [1] 3000    6
```

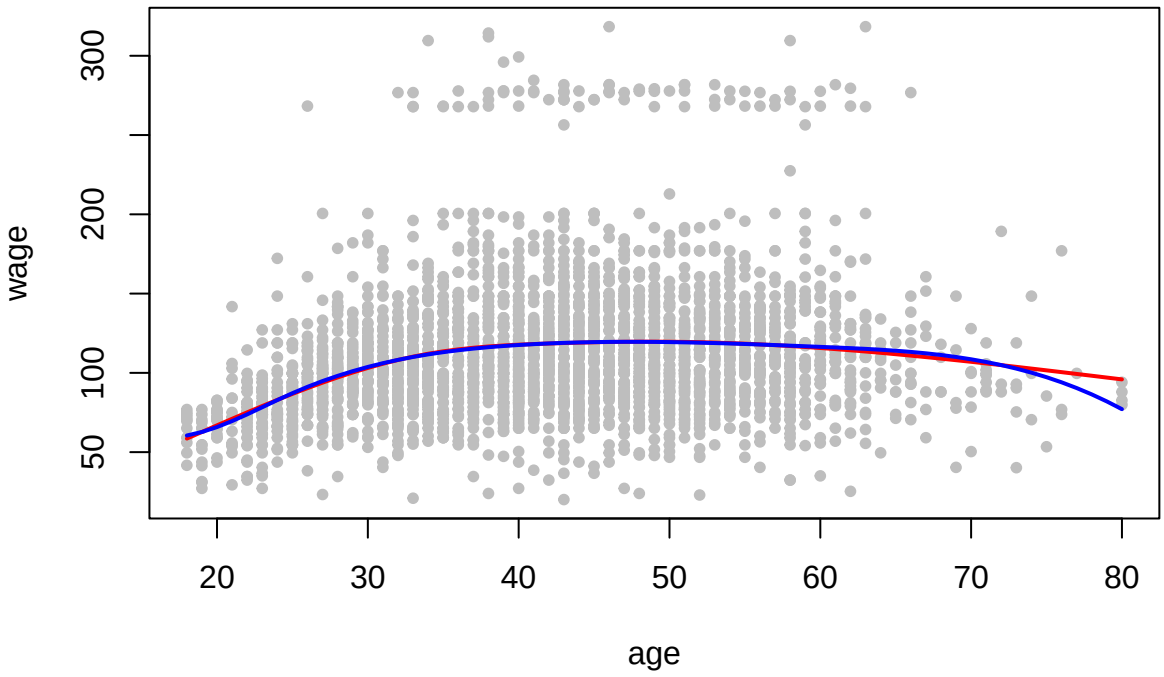
```
attr(bs(age, df = 6), "knots")
```

```
## [1] 33.75 42.00 51.00
```

In this case `R` chooses knots at ages 33.8, 42.0, and 51.0, which correspond to the 25th, 50th, and 75th percentiles of `age`. The function `bs()` also has a `degree` argument, so we can fit splines of any degree, rather than the default degree of 3 (which yields a cubic spline).

In order to instead fit a natural spline, we use the `ns()` function. Here we fit a natural spline with four degrees of freedom.

```
fit2 <- lm(wage ~ ns(age, df = 4), data = Wage)
pred2 <- predict(fit2, newdata = list(age = age.grid),
     se = T)
plot(age, wage, col = "gray", pch=20)
lines(age.grid, pred2$fit, col = "red", lwd = 2)
lines(age.grid, pred$fit, lwd = 2, col="blue")
```



As with the `bs()` function, we could instead specify the knots directly using the `knots` option.

In order to fit a smoothing spline, we use the `smooth.spline()` function. Figure 7.8 was produced with the following code:

```
plot(age, wage, xlim = agelims, cex = .5, col = "darkgrey",
     pch=20)
title("Smoothing Spline")
fit <- smooth.spline(age, wage, df = 16)
fit2 <- smooth.spline(age, wage, cv = TRUE)
```

```
## Warning in smooth.spline(age, wage, cv = TRUE): cross-validation with
## non-unique 'x' values seems doubtful
```

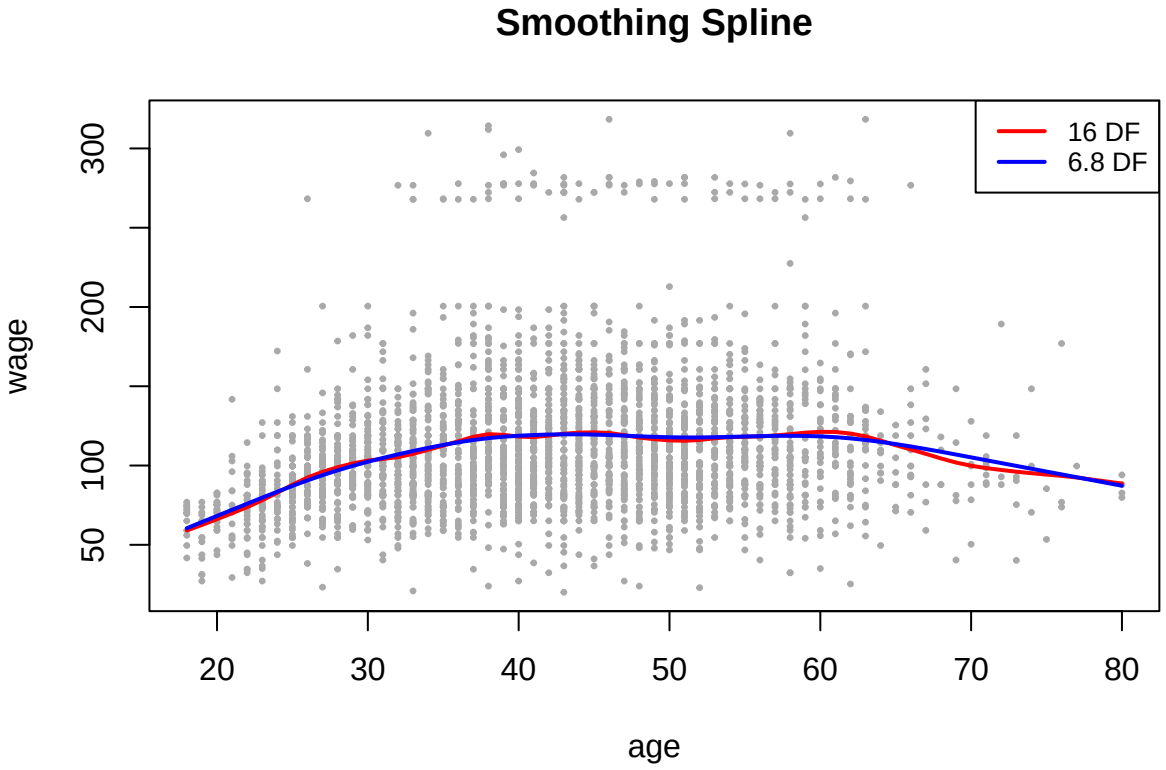
```
fit2
```

```
## Call:
## smooth.spline(x = age, y = wage, cv = TRUE)
##
## Smoothing Parameter  spar= 0.6988943  lambda= 0.02792303 (12 iterations)
## Equivalent Degrees of Freedom (Df): 6.794596
## Penalized Criterion (RSS): 75215.9
## PRESS(l.o.o. CV): 1593.383
```

```
fit2$df
```

```
## [1] 6.794596
```

```
lines(fit, col = "red", lwd = 2)
lines(fit2, col = "blue", lwd = 2)
legend("topright", legend = c("16 DF", "6.8 DF"),
    col = c("red", "blue"), lty = 1, lwd = 2, cex = .8)
```



Notice that in the first call to `smooth.spline()`, we specified `df = 16`. The function then determines which value of λ leads to 16 degrees of freedom. In the second call to `smooth.spline()`, we select the smoothness level by cross-validation; this results in a value of λ that yields 6.8 degrees of freedom.

How do we create confidence intervals here? In this case, the preferred technique is to generate Bayesian "confidence intervals" using bootstrap. Here is the last spline in `ggplot2` with "confidence intervals".

```
library(ggformula)
```

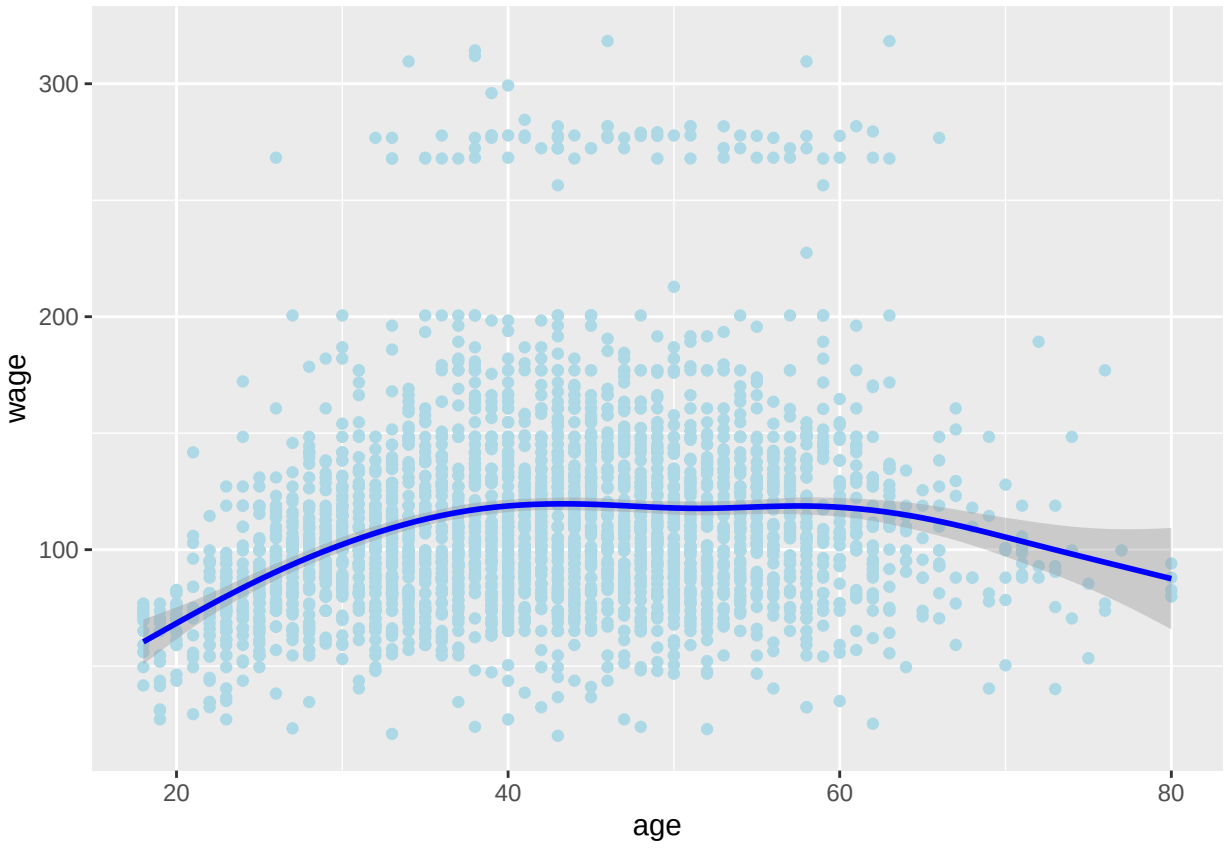
```
## Loading required package: scales
```

```
## Loading required package: ggridges
```

```
##
## New to ggformula?  Try the tutorials:
##  learnr::run_tutorial("introduction", package = "ggformula")
##  learnr::run_tutorial("refining", package = "ggformula")
```

```
ggplot(data, aes(x = age, y = wage)) +
  geom_point(color="lightblue") +
  stat_smooth(color="blue")
```

```
## `geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'
```


300
200
100
wage
20
40
60
80
age