M339W: April 12th, 2021.

Delta-hedging.

Market Makers.

- immediacy
- inventory

} ⟹ exposure to risk ⟹ hedge

Say, a market maker writes an option whose value ftion is $v(s,t)$.

⟹ At time·0, they wrote the option. So, they get $v(S(0),0)$.

At time·t, the value of the market makers position is

$$\boxed{-v(s,t)} \quad ✓$$

To (partially) hedge this exposure to risk, they construct a portfolio which has zero sensitivity to to the small changes in the stock price.
Formally speaking, their goal is to create a delta·neutral portfolio, i.e., a portfolio for which

$$\Delta_{port}(s,t) = 0$$

(theoretically, w/ continuous rebalancing; in practice, continuous rebalancing is impossible).

In particular, initially, we have the goal to trade so that

$$\Delta_{port}(S(0),0) = 0 .$$

The most straightforward strategy is to trade in shares of the underlying asset.

At any time·t, let $N(s,t)$ denote the required # of shares in the portfolio to maintain Δ-neutrality.

⟹ The total value of the portfolio is:

$$v_{port}(s,t) = -v(s,t) + \boxed{N(s,t)} \cdot s$$

$$\Rightarrow \Delta_{port}(s,t) = -\Delta(s,t) + N(s,t) = 0$$

↑ Δ-neutrality

$$\Rightarrow \quad \boxed{N(s,t) = \Delta(s,t)}$$

Example. A market maker $\boxed{\text{writes}}$ a European $\boxed{\text{call}}$ option.

$\Rightarrow$ At any time t, the market maker's liability is

$$-v_C(s,t)$$

$\Rightarrow$ They have to maintain $\boxed{N(s,t) = \Delta_C(s,t)}$ in the Δ-hedge.

$\Rightarrow$ In particular, @ time 0:

$$N(S(0),0) = e^{-\delta \cdot T} \cdot N(d_1(S(0),0)),$$

i.e., the market maker should long this much of a share.

Example. A market maker $\boxed{\text{writes}}$ a European $\boxed{\text{put}}$ option.

At time t, the value of the unhedged position is

$$-v_P(s,t)$$

$\Rightarrow$ In the Δ-hedge, they want to maintain

$$\boxed{N(s,t) = \Delta_P(s,t)}$$

$\Rightarrow$ At time 0: $\quad N(S(0),0) = - e^{-\delta \cdot T} N(-d_1(S(0),0))$

They need to short the shares of stock to accomplish Δ-neutrality.

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

Sample IFM: Derivatives : Advanced

T = 1

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i) The risk-free interest rate is constant.

(ii)

	Several months ago when to option was written time·0	Now when the positions are closed out time·t
Stock price	$\boxed{\$40.00}$	$\boxed{\$50.00}$
Call option price	$\boxed{\$\ 8.88}$	$\boxed{\$14.42}$
Put option price	\$ 1.63	\$ 0.26
Call option delta	$\boxed{0.794}$	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

Calculate her profit.

$\text{Profit} = \text{Payoff} - FV\,(\text{Init. cost})$

(A) $11

(B) $24

(C) $126

(D) $217

(E) $240

0 — options written

t — "several months" This is when we calculate the profit.

T=1 — exercise date

48. DELETED

$\text{Profit}\,(@\ \text{time } t) = \text{Wealth}\,(@\ \text{time } t) - FV_{0,t}\,(\text{Init. Cost})$

49. You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).

(i) The period is 3 months.

(ii) The initial stock price is $100.

(iii) The stock's volatility is 30%.

(iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

(A) 114

(B) 115

(C) 116

(D) 117

(E) 118

- Initial Cost: $-100 \cdot v_c(S(0),0) + \Delta_c(S(0),0) \cdot S(0) \cdot 100$

$$= 100\left(-8.88 + 0.794 \cdot 40\right) = 2,288.$$

- Wealth @ time t: $-100\, v_c(S(t),t) + \Delta_c(S(0),0) \cdot S(t) \cdot 100$

$$= 100\left(-14.42 + 0.794 \cdot 50\right) = 2,528$$

Let's figure out the e^{rt} using put-call parity w/ our given put & call prices.

At time 0: $v_c(S(0),0) - v_p(S(0),0) = F^P_{0,T}(S) - Ke^{-r \cdot T}$

$$8.88 - 1.63 = 40 - Ke^{-r \cdot T}$$

$$Ke^{-rT} = 40 - 7.25 = 32.75 \qquad (*)$$

At time t: $v_c(S(t),t) - v_p(S(t),t) = F^P_{t,T}(S) - Ke^{-r(T-t)}$

$$14.42 - 0.26 = 50 - Ke^{-r(T-t)}$$

$$Ke^{-r(T-t)} = 50 - 14.16 = 35.84 \qquad (**)$$

$$\frac{(**)}{(*)} \Rightarrow \frac{\cancel{K}e^{-r(T-t)}}{\cancel{K}e^{-rT}} = \frac{35.84}{32.75}$$

$$\Rightarrow e^{rt} = 1.09435$$

$\Rightarrow$ Profit $= 2,528 - 2,288 \cdot 1.09435 = 24.1272$