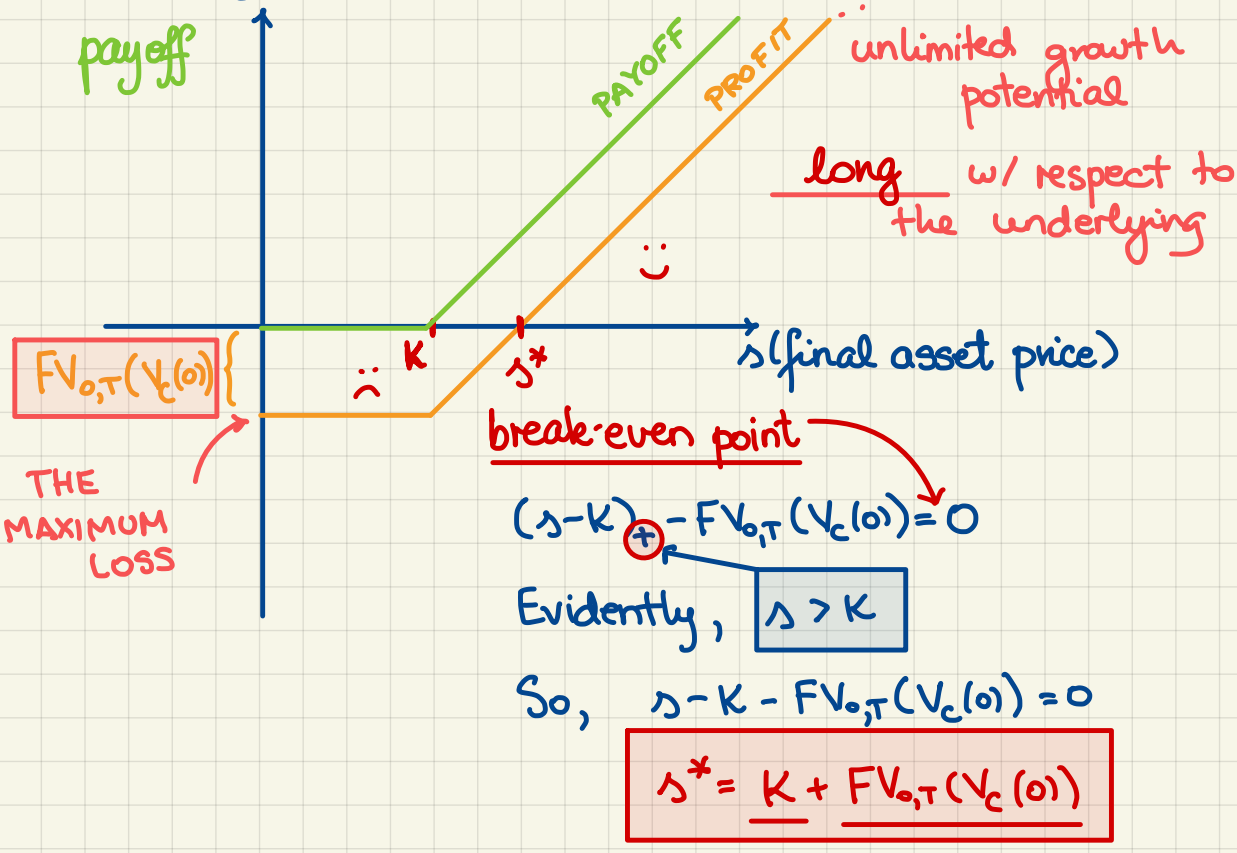
M339D : February 14th, 2022.

European call options [cont'd].

Initial Cost: $V_c(0)$ ... the call's premium ✓

Payoff: $V_c(T) = (S(T)-K)_+$

$\Rightarrow$ **the payoff function:** $v_c(s) = (s-K)_+$



break-even point

$(s-K)_+ - FV_{0,T}(V_c(0)) = 0$

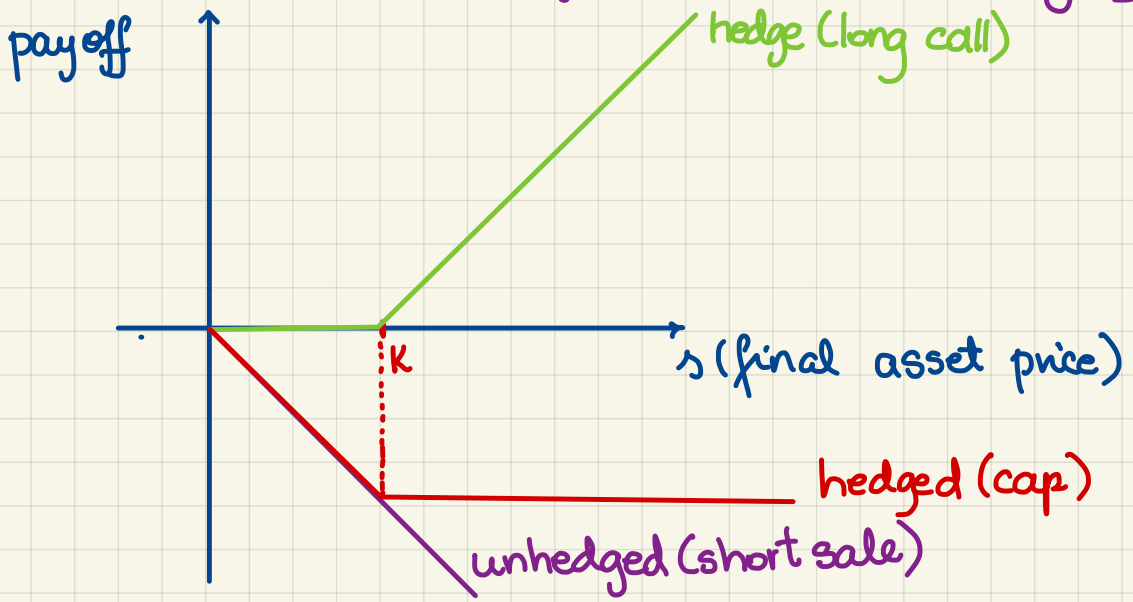
Evidently, $s > K$

So, $s - K - FV_{0,T}(V_c(0)) = 0$

$$s^* = K + FV_{0,T}(V_c(0))$$

Caps

Start w/ a short sale of a non-dividend-paying stock.



- short sale
- long call

} CAP.

The payoff of the cap:

$$-S(T) + (S(T)-K)_+ = \begin{cases} -S(T) + (S(T)-K), & \text{if } S(T) \geq K \\ -S(T), & \text{if } S(T) < K \end{cases}$$

$$= \begin{cases} -K & \text{if } S(T) \geq K \\ -S(T) & \text{if } S(T) < K \end{cases}$$

$$= -\min(S(T), K)$$

Covered / Naked Option Writing.

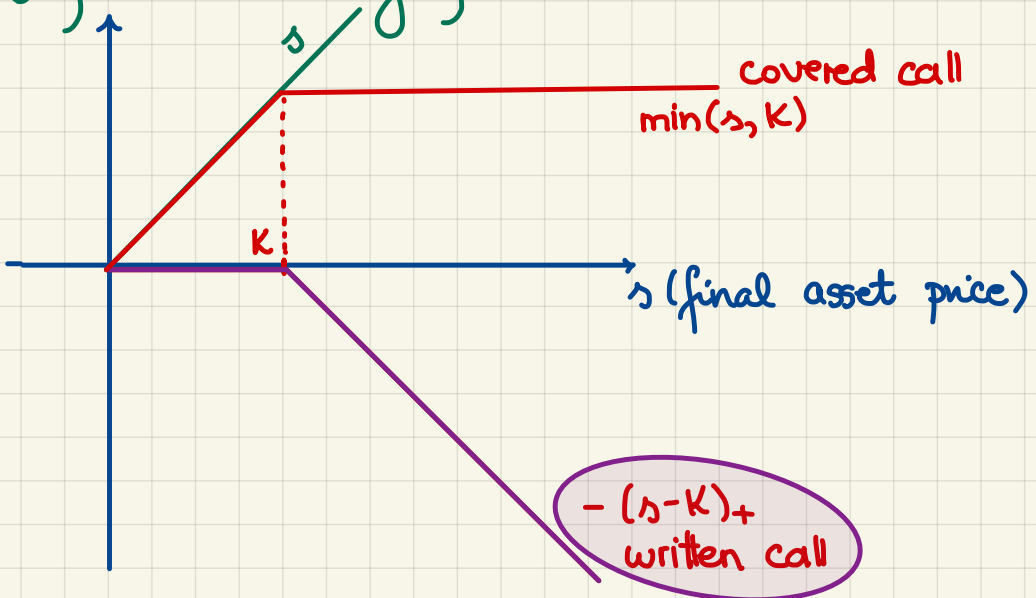
- covered ... if the writer of option has a simultaneous appropriate position in the underlying asset
- naked ... if the writer of option does not have the opposite position in the underlying.

Example. A Covered Call

Assume, for simplicity, that the underlying asset pays no dividends.

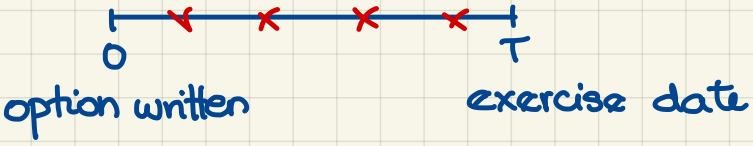
Covered call

- a written call option w/ strike K and exercise date T
- long/buy the underlying asset



European Put Options.

Usually, a right but not an obligation to sell.



At time-0: The writer & the buyer of the put agree on:

- the underlying asset: $S(t)$, $t \geq 0$
- the strike/exercise price K
- the exercise date T

The put premium $V_p(0)$ is paid by the put's buyer to the put's writer.

At time-T:
- The put's owner has a right, but not an obligation to sell one unit of the underlying for the strike price K.
- The put's writer is obligated to do what the put's owner decides.

Q: What is the put owner's optimal behavior @ time-T? What is the criterion for exercise?

⟶: The criterion $K > S(T)$.

Then,

long put (owner of put)

1 unit of asset $S(T)$ ↓ ↑ strike price K

short put (writer of put)

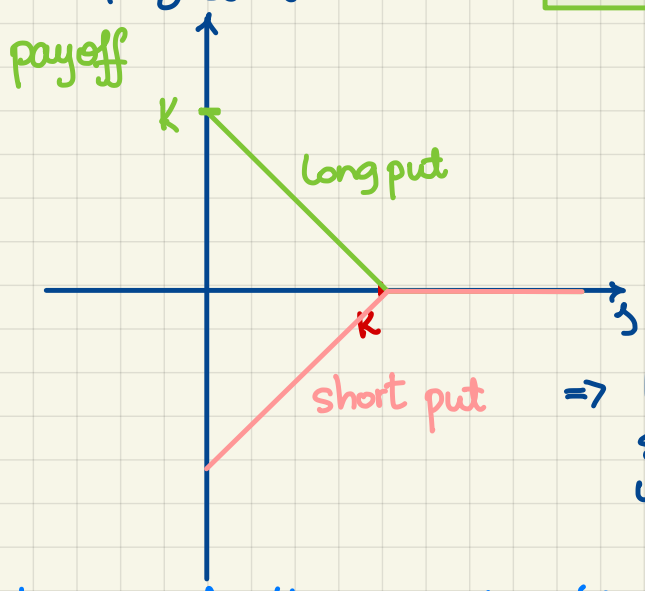
$\Rightarrow$ **The payoff:**

$$V_p(T) = \begin{cases} K - S(T) & \text{if } K > S(T) \\ 0 & \text{if } K \leq S(T) \end{cases}$$

$\Rightarrow$

$$\boxed{V_p(T) = \max(K - S(T), 0) = (K - S(T))_+}$$

=> The payoff function: $v_p(s) = (K-s)_+$



=> Indeed, a premium $V_p(0)$ should be paid to the writer of the put @ time 0.

Task: Think about the construction of the profit curve of the long put!