

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set 4

Profit. Forward contracts. European call and put options.

Problem 4.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time-1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

$$\mathbb{E}[S(1)] = 45\left(\frac{1}{4}\right) + 42\left(\frac{1}{2}\right) + 38\left(\frac{1}{4}\right) = 41.75$$

What is your expected profit under the above model, if you invest in one share of stock at time-0 and liquidate your investment at time-1?

$$41.75 - 40e^{0.04} = 0.1176 \quad \square$$

Problem 4.2. Derivative securities can reduce the risk of both the buyer and the writer of the security. *True or false?* (T)

Problem 4.3. A short forward contract has an unlimited loss potential. *True or false?* (T)

Problem 4.4. A farmer produces one million bushels of corn. The total cost of production is \$1.3 million. The farmer entered a forward contract to hedge at a forward price of \$2.50 per bushel on one million bushels. What is the farmer's profit?

$$2.5 \cdot 10^6 - 1.3 \cdot 10^6 = 1.2 \cdot 10^6$$

Problem 4.5. Assume that farmer Brown is uncertain about his crop yield. Based on past experience, he thinks the following is a good model:

- 100,000 bushels with probability 1/4;
- 80,000 bushels with probability 3/4.

ONE APPROACH: $100,000\left(\frac{1}{4}\right) + 80,000\left(\frac{3}{4}\right) = \dots$

How many forward contracts do you think farmer Brown should short to hedge against fluctuations in corn prices at harvest time? Explain your way of thinking ...

Problem 4.6. Pancakes, Inc. produces chocolate chip pancakes. It longed a forward contract on 100 lbs of chocolate chips at \$3.00 per pound. Total fixed revenue is \$2,000 for the pancakes produced with the above chocolate chips. Other costs total \$1200. Find the company's profit.

- (a) 2,000
- (b) 1,700
- (c) 800
- (d) 500
- (e) None of the above.

$$2000 - 1200 - 300 = 500$$

Problem 4.7. The Extra-Healty Cereal (EHC) company longed 20,000 forward contracts on corn at \$2.80 per bushel. The revenue from cereal made with the above corn is \$200,000 while the other (non-corn) aggregate fixed and variable costs amount to \$120,000. What is the EHC's profit?

$$200,000 - 120,000 - 20,000(2.80) = 24,000$$

Problem 4.8. The current price of stock a certain type of stock is \$80. The premium for a 6-month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

- (a) \$80
(b) \$85.72
(c) \$85.84
(d) \$85.96
(e) None of the above.

$$P_{\text{opt}} = 0$$

$$(1-K)_+ - FV_{0,T}(V_C(0)) = 0$$

$$K = K + FV_{0,T}(V_C(0)) = 80 + 5.84e^{0.04(0.5)} = 85.96$$

Problem 4.9. The price of gold in half a year is modeled to be equally likely to equal any of the following prices

\$1000, \$1100, and \$1240.

Consider a half-year, \$1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

Call Payoff:

0 50 190

$$E[V_C(T)] = 50 \cdot \frac{1}{3} + 190 \cdot \frac{1}{3} = 80$$

Problem 4.10. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded, risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

- (a) 395.92
(b) 397.98
(c) 400
(d) 897.98
(e) None of the above.

$$200 \cdot 10 - 500 - 200 \cdot 5 - 200 \cdot (0.5)e^{0.04(0.5)} = 397.98$$

Problem 4.11. For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date T in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date T . Assume that the call's premium equals \$10 and that the annual effective interest rate equals 10%.

Express your answer as an interval.

Problem 4.12. Source: Sample IFM (Derivatives - Intro), Problem #11

The current stock price is \$40, and the effective annual interest rate is 8%.

You observe the following option prices:

- (1) The premium for a \$35-strike, 1-year European call option is \$9.12.
- (2) The premium for a \$40-strike, 1-year European call option is \$6.22.
- (3) The premium for a \$45-strike, 1-year European call option is \$4.08.

Assuming that all call positions being compared are long, at what 1-year stock price range does the \$45-strike call produce a higher profit than the \$40-strike call, but a lower profit than the \$35-strike call?

Express your answer as an interval.

11. The profit f'n of the forward contract:

$$v_F(s) = s - 100$$

The profit f'n of the call option:

$$v_C(s) - 10 \cdot 1.1 = v_C(s) - 11 = (s - 100)_+ - 11$$

Find s such that

$$s - 100 < (s - 100)_+ - 11$$

No sol'ns for $s \geq 100$.

Focus on $s < 100$.

$$s - 100 < -11$$

$$s < 89$$

$$s \in [0, 89)$$