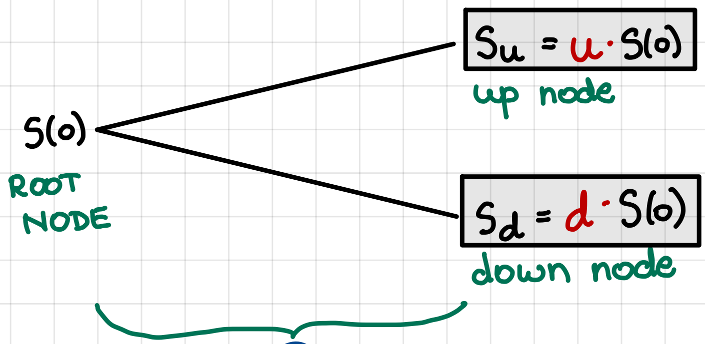
M339 D: April 26th, 2021.

The Binomial Asset-Pricing Model.

$S(0)$ ... the observeable initial stock price

T time horizon (i.e., exercise or expiration date of an option)



h .. length of a single period

By convention: $S_u > S_d$
$\Updownarrow$
$u > d$

$S(T) = S(h)$ ... a random variable denoting the time-T stock price w/ two possible values: S_u and S_d

u and d completely describe our stock-price model.

An interpretation:

$$u = \frac{S_u}{S(0)} = \boxed{\frac{S_u - S(0)}{S(0)}} + 1$$

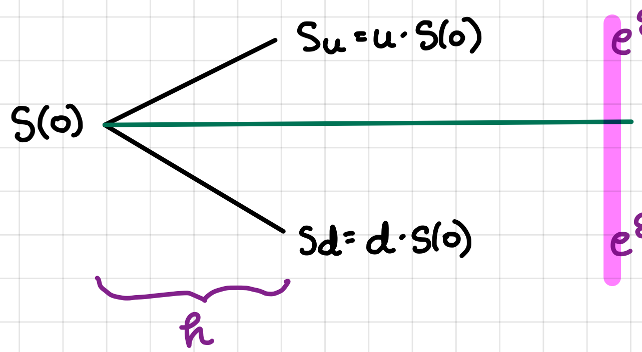
$$d = \frac{S_d}{S(0)} = \boxed{\frac{S_d - S(0)}{S(0)}} + 1$$

simple rate of return

Market Model:

- **riskless asset:** $\boxed{\text{@ the ccrfir } r}$
- **risky asset:** continuous dividend paying stock w/ dividend yield δ.

Imagine investing in one share of stock @ time 0.



Wealth

$e^{\delta \cdot h} \cdot S_u = e^{\delta \cdot h} \cdot u \cdot S(0)$

$S(0)e^{rh}$ (balance of the risk-free investment)

$e^{\delta \cdot h} \cdot S_d = e^{\delta \cdot h} \cdot d \cdot S(0)$

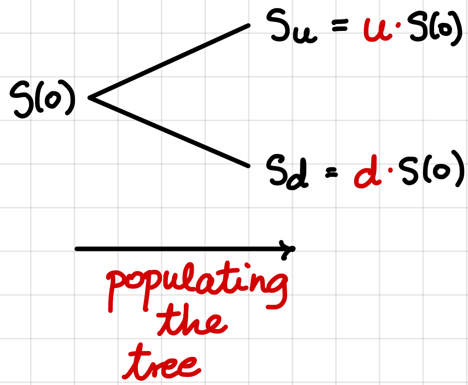
The no-arbitrage condition:

$$e^{\delta h} \cdot d \cdot \cancel{S(0)} < \cancel{S(0)} e^{rh} < e^{\delta h} \cdot u \cdot \cancel{S(0)} \qquad /: e^{\delta h}$$

$$\boxed{d < e^{(r-\delta)h} < u}$$

Binomial Option Pricing.

Stock Price Tree.



We want to price a European style derivative security w/ exercise date @ the end period.

It is completely determined by its payoff function : $v(\cdot)$

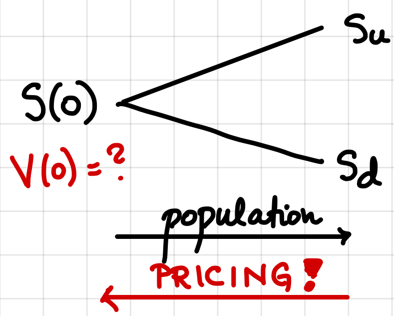
e.g., for a call : $v_c(s) = (s-K)_+$

for a put : $v_p(s) = (K-s)_+$

=> The payoff of this derivative security is a random variable given by:

$$\boxed{V(T) := v(S(T))}$$

using the payoff f'tion



payoff

$V_u = v(S_u)$

$V_d = v(S_d)$

e.g. for a call option

$V_u = (S_u - K)_+$

$V_d = (S_d - K)_+$

In the binomial model, any derivative security can be replicating w/ a portfolio consisting of:

- $\underline{\quad \Delta \quad}$ shares of stock $\begin{cases} \Delta > 0 & \text{buying} \\ \Delta = 0 & \text{nothing} \\ \Delta < 0 & \text{shorting} \end{cases}$
- $\underline{\quad B \quad}$ @ the ccrfir r $\begin{cases} B > 0 & \text{Lending (buying bond)} \\ B = 0 & \text{nothing} \\ B < 0 & \text{borrowing (issuing a bond)} \end{cases}$