M339J: April 25th, 2022.

$N^L \sim \text{Poisson}(\lambda_L = 20)$

109. A company insures a fleet of vehicles. Aggregate losses have a compound Poisson distribution. The expected number of losses is 20. Loss amounts, regardless of vehicle type, have exponential distribution with $\theta = 200$. $X \sim \text{Exponential}(\theta = 200)$

To reduce the cost of the insurance, two modifications are to be made:

(i) a certain type of vehicle will not be insured. It is estimated that this will reduce loss frequency by 20%. Thinning: $\tilde{N}^L \sim \text{Poisson}(\tilde{\lambda}_L = \frac{4}{5} \cdot 20 = 16)$ ← the new loss count

(ii) a deductible of 100 per loss will be imposed. $d = 100$ $\quad Y^L = (X-d)_+$

Calculate the expected aggregate amount paid by the insurer after the modifications.

(A) 1600

(B) 1940

(C) 2520

(D) 3200

(E) 3880

→: $S = Y_1^L + Y_2^L + \dots + Y_{\tilde{N}^L}^L$

$\mathbb{E}[S] = \underbrace{\mathbb{E}[\tilde{N}^L]}_{=16} \cdot \boxed{\mathbb{E}[Y^L]} = 16 \cdot 200 \cdot e^{-\frac{1}{2}} = 1940.90$ □

$\mathbb{E}[Y^L] = \mathbb{E}[(X-100)_+] = \mathbb{E}[X] - \mathbb{E}[X \wedge 100]$

$= 200 - 200 \cdot F_X(100) = 200 \cdot S_X(100) = 200 e^{-\frac{100}{200}}$

110. You are the producer of a television quiz show that gives cash prizes. The number of prizes, N, and prize amounts, X, have the following distributions:

n	$\Pr(N = n)$
1	0.8
2	0.2

x	$\Pr(X = x)$
0	0.2
100	0.7
1000	0.1

Your budget for prizes equals the expected prizes plus the standard deviation of prizes.

Calculate your budget.

(A) 306

(B) 316

(C) 416

(D) 510

(E) 518

$N^L \sim \text{Poisson}(\lambda = 5)$

126. The number of annual losses has a Poisson distribution with a mean of 5. The size of each loss has a two-parameter Pareto distribution with $\theta = 10$ and $\alpha = 2.5$. An insurance for the losses has an ordinary deductible of 5 per loss.

$X \sim \text{Pareto}(\theta = 10, \alpha = 2.5)$

$d = 5 \; : \; Y^L = (X-d)_+$

Calculate the expected value of the aggregate annual payments for this insurance.

$\longrightarrow : \quad S = Y^L_1 + Y^L_2 + \cdots + Y^L_{N^L}$

$$\mathbb{E}[S] = \cancel{X}\ \underbrace{\mathbb{E}[N^L]}_{=5} \cdot \boxed{\mathbb{E}[Y^L]} = 5 \cdot 3.62887 = 18.144$$

$$\mathbb{E}[Y^L] = \mathbb{E}[(X-5)_+] = \mathbb{E}[X] - \mathbb{E}[X \wedge 5]$$

$$= \cancel{\frac{\theta}{\alpha-1}} - \frac{\theta}{\alpha-1}\left[\cancel{1} - \left(\frac{\theta}{5+\theta}\right)^{\alpha-1}\right] = \frac{\theta}{\alpha-1}\left(\frac{\theta}{5+\theta}\right)^{\alpha-1}$$

$$= \frac{\theta^\alpha}{(\alpha-1)(5+\theta)^{\alpha-1}} = \frac{10^{2.5}}{(2.5-1)(15)^{1.5}} = 3.62887$$

(A) 8

(B) 13

(C) 18

(D) 23

(E) 28

127. Losses in 2003 follow a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 5$. Losses in 2004 are uniformly 20% higher than in 2003. An insurance covers each loss subject to an ordinary deductible of 10.

Calculate the Loss Elimination Ratio in 2004.

(A) 5/9

(B) 5/8

(C) 2/3

(D) 3/4

(E) 4/5

128. DELETED

129. DELETED

211. An actuary for a medical device manufacturer initially models the failure time for a particular device with an exponential distribution with mean 4 years.

This distribution is replaced with a spliced model whose density function:

(i) is uniform over [0, 3]

(ii) is proportional to the initial modeled density function after 3 years

(iii) is continuous

Calculate the probability of failure in the first 3 years under the revised distribution.

(A) 0.43

(B) 0.45

(C) 0.47

(D) 0.49

(E) 0.51

212. For an insurance:

$N^L \sim \text{Poisson}(\lambda^L = 10)$

(i) The number of losses per year has a Poisson distribution with $\lambda = 10$.

(ii) Loss amounts are uniformly distributed on (0, 10). $\boxed{X \sim U(0,10)}$

(iii) Loss amounts and the number of losses are mutually independent.

(iv) There is an ordinary deductible of 4 per loss.

Calculate the variance of aggregate payments in a year.

(A) 36

(B) 48

(C) 72

(D) 96

(E) 120

We choose to use the per pmt perspective!

- $N^P \sim \text{Poisson}(\lambda^P = \lambda^L \cdot v = 10(0.6) = 6)$

  w/ $v = \mathbb{P}[X > 4] = \frac{6}{10} = 0.6$

- $Y^P = X - d \mid X > d \sim U(0,6)$

Aggregate pmts: $S = Y_1^P + Y_2^P + \dots + Y_{N^P}^P$

$$\mathrm{Var}[S] = \underbrace{\mathbb{E}[N^P]}_{\lambda} \cdot \mathrm{Var}[Y^P] + \underbrace{\mathrm{Var}[N^P]}_{\lambda} \cdot \left(\mathbb{E}[Y^P]\right)^2$$

$$= \lambda \left(\mathrm{Var}[Y^P] + \left(\mathbb{E}[Y^P]\right)^2 \right)$$

$$= 6 \left(\frac{6^2}{12} + 3^2 \right) = 6 \cdot (3 + 9) = 72 \quad \square$$

164. For a collective risk model the number of losses, $\cancel{N}$, has a Poisson distribution with $\lambda = 20$.

$N^L \sim \text{Poisson}(\lambda^L = 20)$

The common distribution of the individual losses has the following characteristics:

(i) $E[X] = 70$

(ii) $E[X \wedge 30] = 25$

(iii) $\Pr(X > 30) = 0.75$

(iv) $\boxed{E[X^2 \mid X > 30] = 9000}$

An insurance covers aggregate losses subject to an ordinary deductible of 30 per loss.

Calculate the variance of the aggregate payments of the insurance.

(A) 54,000

(B) 67,500

(C) 81,000

(D) 94,500

(E) 108,000

$\longrightarrow$:

$Var[S] = \lambda^P \cdot \boxed{\mathbb{E}[(Y^P)^2]}$ because compound Poisson

$\lambda^P = \lambda^L \cdot \mathbb{P}[X > 30] = 20 \cdot 0.75 = 15$

$Var[S] = 15 \cdot 4500 = 67{,}500$

$\square$

$\boxed{\mathbb{E}[(Y^P)^2]} = \mathbb{E}[(X-d)^2 \mid X > d] = \dots$

$Y^P = X - d \mid X > d$

$\dots = \mathbb{E}[X^2 - 2dX + d^2 \mid X > d]$

linearity

$= \underbrace{\boxed{\mathbb{E}[X^2 \mid X > 30]}}_{= 9000} - 2d \underbrace{\boxed{\mathbb{E}[X \mid X > d]}}_{= 90} + 900 =$

$= 9000 - 2 \cdot 30 \cdot 90 + 900 = \boxed{4500}$

$$\mathbb{E}[X \mid X>30] = \cancel{?} \overset{\text{linearity}}{=} \mathbb{E}[X-30 \mid X>30] + 30$$

$$= \frac{\mathbb{E}[(X-30)_+]}{\mathbb{P}[X>30]} + 30$$

$$= \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge 30]}{\mathbb{P}[X>30]} + 30$$

$$= \frac{70 - 25}{0.75} + 30 = \overset{15}{\cancel{45}} \cdot \frac{4}{\cancel{3}} + 30 = 90$$