M339D: April 5th, 2021.

Quiz #9, Problem #2:

$S(0) = 52$

at·the·money $\Rightarrow$ $S(0) = K$

$T = 38$ days

$r = 0.06$

$V_c(0) - V_p(0) = ?$

By put·call parity:

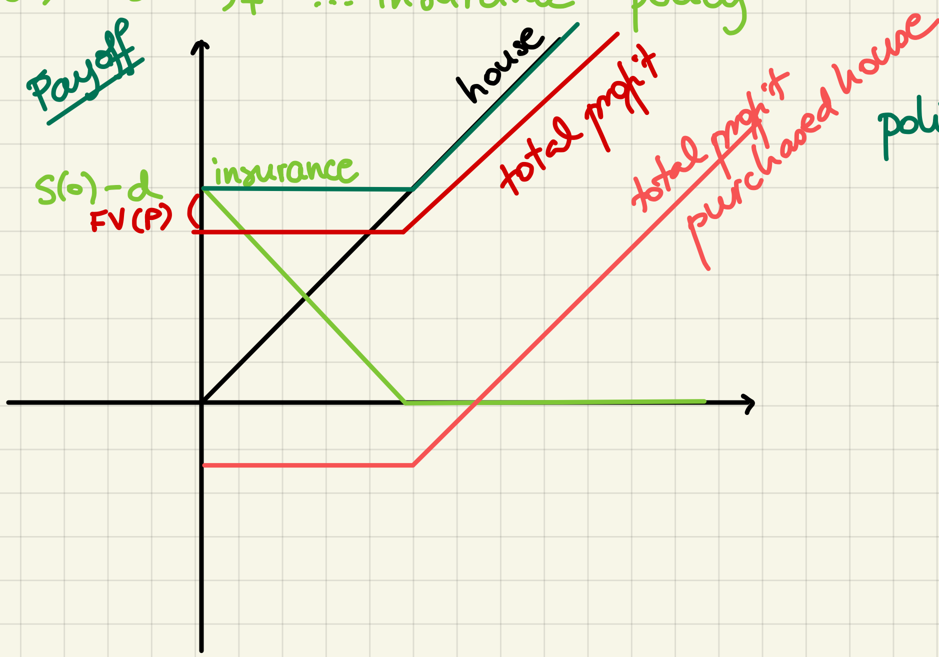
$$V_c(0) - V_p(0) = F^P_{0,T}(S) - PV_{0,T}(K)$$
$$= S(0) \quad - S(0)\, e^{-0.06\left(\frac{38}{365}\right)}$$
$$= 52\left(1 - e^{-0.06\left(\frac{38}{365}\right)}\right)$$
$$= 0.3238$$

$\blacksquare$

HW#6: Problem #10.

$S(T)$ ... price of house ✓

$((S(0)-d) - S(T))_+$ ... insurance policy ✓



policy premium : P

Exchange options [cont'd].

S ... underlying asset
Q ... strike asset

Exchange call: $V_{EC}(T) = (S(T) - Q(T))_+$

Exchange put: $V_{EP}(T) = (Q(T) - S(T))_+$

$\Rightarrow$ A special symmetry: $V_{EC}(T, S, Q) = V_{EP}(T, Q, S)$

$\Rightarrow$ for any $0 \le t \le T$: $V_{EC}(t, S, Q) = V_{EP}(t, Q, S)$

Maximum options.

$\{S(t), t \geq 0\}$, $\{Q(t), t \geq 0\}$ ... risky asset

Set the payoff of the maximum option to be:

$$\boxed{V_{MAX}(T) := \max(S(T), Q(T))}$$

Q: Can you come up w/ a financial story for how to implement this payoff?

→: The owner of the maximum option gets to receive either one share of S or one share of Q.

Q: Bounds on the price of a maximum option?

→: $V_{MAX}(T) \geq \begin{cases} S(T) \\ Q(T) \end{cases}$

$\Rightarrow$ no arbitrage

$$V_{MAX}(0) \geq \begin{cases} F^P_{0,T}(S) \\ F^P_{0,T}(Q) \end{cases}$$

$$\Rightarrow \quad V_{MAX}(0) \geq \max\left(F^P_{0,T}(S), F^P_{0,T}(Q)\right)$$

Q: Can we construct a replicating portfolio for the maximum option?

→: $V_{MAX}(T) = \max(S(T), Q(T))$

$= S(T) + \max(0, Q(T) - S(T))$

$= Q(T) + \boxed{\max(0, S(T) - Q(T))}$

PAYOFF of an EXCHANGE OPTIONS

$\Rightarrow$ Our maximum option can be replicated using:

- one LONG prepaid forward on S

and

- one LONG exchange put w/ underlying S and strike asset Q

$\Rightarrow$

$$V_{MAX}(0) = F^P_{0,T}(S) + V_{EP}(0, S, Q)$$

Similarly: ....

$$= F^P_{0,T}(S) + V_{EC}(0, Q, S)$$

$$= F^P_{0,T}(Q) + V_{EC}(0, S, Q) \quad \checkmark$$

$$= F^P_{0,T}(Q) + V_{EP}(0, Q, S)$$

$\Rightarrow$ $F^P_{0,T}(S) + V_{EP}(0,S,Q) = F^P_{0,T}(Q) + V_{EC}(0,S,Q)$

$\Rightarrow$ $\boxed{V_{EC}(0,S,Q) - V_{EP}(0,S,Q) = F^P_{0,T}(S) - F^P_{0,T}(Q)}$

Generalized Put-Call Parity

6. Consider a model with two stocks. Each stock pays dividends continuously at a rate proportional to its price.

$S_j(t)$ denotes the price of one share of stock *j* at time *t*.

T=3

Consider a claim maturing at time 3. The payoff of the claim is

Maximum $(S_1(3), S_2(3))$ ... precisely our maximum option

You are given:

(i) $S_1(0) = \$100$

(ii) $S_2(0) = \$200$

$\delta_1 = 0.05$

(iii) Stock 1 pays dividends of amount $(0.05)S_1(t)dt$ between time *t* and time $t + dt$.

$\delta_2 = 0.1$

(iv) Stock 2 pays dividends of amount $(0.1)S_2(t)dt$ between time *t* and time $t + dt$.

(v) The price of a European option to exchange Stock 2 for Stock 1 at time 3 is \$10.

An exchange call w/ underlying S_1 and strike asset S_2

Calculate the price of the claim.

(A) \$96

(B) \$145

(C) \$158

(D) \$200

(E) \$234

$$V_{MAX}(0) = F^P_{0,3}(S_2) + V_{EC}(0, S_1, S_2)$$

$$= 200e^{-0.1(3)} + 10 = 158.16$$