UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 7

Option Greeks.

Provide your *final answer only* to the following problem(s):

Problem 7.1. (2 pts) Call *theta* may also be called time decay. *True or false?*

Solution: TRUE

Problem 7.2. (2 points) *Rho* measures the sensitivity of a portfolio to the changes in the applicable risk-free interest rate. *True or false?*

Solution: TRUE

Problem 7.3. (5 pts) Which of the following gives the correct values for the delta and gamma of a single share of non-dividend-paying stock?

(a) $\Delta = 1, \Gamma = 1$
(b) $\Delta = 1, \Gamma = 0$
(c) $\Delta = 0, \Gamma = 1$
(d) $\Delta = 0, \Gamma = 0$
(e) None of the above.

Solution: (b)

Please, provide your **complete solutions** to the following problems. Final answers without correct justification will earn zero points.

Problem 7.4. (2 points) The Black-Scholes delta of a European call option is always between 0 and 1. *True or false? Why?*

Solution: TRUE
The expression for the call delta is

$$\Delta_C(s,t) = e^{-\delta(T-t)}N(d_1(s,t)).$$

Both terms in the product on the right-hand side are non-negative and smaller than 1.

Problem 7.5. (2 points) The Black-Scholes delta of a European put option is always between -1 and 0. *True or false? Why?*

Solution: TRUE
The expression for the call delta is

$$\Delta_P(s,t) = -e^{-\delta(T-t)}N(-d_1(s,t)).$$

Both terms in the product on the right-hand side are non-negative and smaller than 1. However, their product is preceded by a -1 which makes the entire expression always have a value between -1 and 0.

Problem 7.6. (2 points) Consider a European call and an otherwise identical put. Then, the call rho is greater than the put rho. *True or false? Why?*

Solution: TRUE
The call rho is positive and the put rho is negative.

Problem 7.7. (2 points) In the Black-Scholes model, Ψ is the first-order sensitivity with respect to the volatility parameter. *True or false? Why?*

Solution: FALSE
Is should be the first-order sensitivity with respect to the dividend yield.

Problem 7.8. (2 points) In the Black-Scholes model, *volga* is the first-order sensitivity with respect to the volatility parameter. *True or false? Why?*

Solution: FALSE
It is actually called **vega**.

Problem 7.9. (2 points) Consider a European call and an otherwise identical put. Then, the call vega is strictly greater than the put vega. *True or false? Why?*

Solution: FALSE
They are equal by put-call parity.

Problem 7.10. (2 points) In the Black-Scholes model, the put theta is **always** positive. *True or false? Why?*

Solution: FALSE
It can be either positive, or negative, depending on the moneyness.

Problem 7.11. (2 points) The call volatility is greater than or equal to the volatility of the underlying asset. *True or false?*

Solution: TRUE

Problem 7.12. (15 points) Assume the Black-Scholes framework. The current stock price is \$50 per share. Its dividend yield is 0.01 and its volatility is 0.25.

The continuously compounded, risk-free interest rate is 0.05.

Consider a one-year, \$55-strike European put option on the above stock. What is the volatility of the put option?

Solution: In our usual notation,

$$\begin{aligned} d_1 &= \frac{1}{0.25}\left[\ln\left(\frac{50}{55}\right) + 0.05 - 0.01 + \frac{(0.25)^2}{2}\right] = -0.0962 \approx -0.10, \\ d_2 &= -0.10 - 0.25 = -0.35. \end{aligned}$$

So, the put-option delta is

$$\Delta_P = -e^{-0.01}N(0.1) = -e^{-0.01}(0.5398) = -0.5344.$$

The put price is

$$V_P(0) = 55e^{-0.05}N(0.35) - 50(0.5344) = 55e^{-0.05}(0.6368) - 50(0.5344) = 6.59586.$$

The option's elasticity is

$$\Omega_P = -\frac{0.5344(50)}{6.59586} = -4.05103.$$

So, the put's volatility is $\sigma_P = \sigma\Omega_P = 1.01276$.

Problem 7.13. (5 points) *Source: Sample MFE Problem #8.*

Consider a non-dividend-paying stock whose price $\mathbf{S} = \{S(t), t \geq 0\}$ is modeled using the Black-Scholes model. Suppose that the current stock price equals \$40 and that its volatility is given to be 0.30.

Consider a three-month, \$41.5-strike European call option on the above stock. You learn that the current call delta equals 0.5.

What is the Black-Scholes price of this call option?

Solution: 2.18521.

Problem 7.14. (5 points) Consider the following portfolio:

- 5 long options of type I,
- 4 long options of type II,
- 1 written option of type III.

The prices of the three options are 0.75, 1.00, and 1.50, respectively, while the option elasticities are 10, 7, and 2, respectively. What is the elasticity of the above portoflio?

Solution: Let $S(0)$ denote the current stock price. The deltas of the three options are

$$\Delta_I = \frac{10 \times 0.75}{S(0)} = \frac{7.5}{S(0)},$$
$$\Delta_{II} = \frac{7 \times 1}{S(0)} = \frac{7}{S(0)},$$
$$\Delta_{III} = \frac{2 \times 1.5}{S(0)} = \frac{3}{S(0)}.$$

So, the delta of the portfolio is

$$\Delta = 5 \times \frac{7.5}{S(0)} + 4 \times \frac{7}{S(0)} - \frac{3}{S(0)} = \frac{62.5}{S(0)}\,.$$

The portfolio's price is

$$V(0) = 5 \times 0.75 + 4 \times 1 - 1.50 = 6.25.$$

So, the portfolio elasticity is

$$\Omega = \frac{\Delta S(0)}{V(0)} = \frac{62.5}{6.25} = 10.$$