M339J: March 30th, 2022.

287. For an aggregate loss distribution S:

(i) The number of claims has a negative binomial distribution with $r = 16$ and $\beta = 6$.

(ii) The claim amounts are uniformly distributed on the interval (0, 8).

(iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium such that the probability that aggregate losses will exceed the premium is 5%.

(A) 500

(B) 520

(C) 540

(D) 560

(E) 580

288. The random variable N has a mixed distribution:

(i) With probability p, N has a binomial distribution with $q = 0.5$ and $m = 2$.

(ii) With probability $1 - p$, N has a binomial distribution with $q = 0.5$ and $m = 4$.

Which of the following is a correct expression for $\Pr(N = 2)$?

(A) $0.125p^2$

(B) $0.375 + 0.125p$

(C) $0.375 + 0.125p^2$

(D) $0.375 - 0.125p^2$

(E) $0.375 - 0.125p$

$$N \sim \begin{cases} N_1 \sim \text{Binomial}(q_1 = 0.5, m_1 = 2) & \text{w/ prob } p \\ N_2 \sim \text{Binomial}(q_2 = 0.5, m_2 = 4) & \text{w/ prob } 1-p \end{cases}$$

We seek:

$\mathbb{P}[N=2] = ?$

$$\mathbb{P}[N=2] = \mathbb{P}[N_1 = 2 \mid \underbrace{\Lambda = 1}_{\text{choice of bin. dist'n}}] \cdot \mathbb{P}[\Lambda = 1] + \mathbb{P}[N_2 = 2 \mid \Lambda = 2] \cdot \mathbb{P}[\Lambda = 2]$$

$$\mathbb{P}[N=2] = p\cdot\left(\tfrac{1}{4}\right) + (1-p)\cdot\underbrace{\mathbb{P}[N_2=2]}_{\binom{4}{2}\left(\frac{1}{2}\right)^4}$$

$$= p\cdot\left(\tfrac{1}{4}\right) + (1-p)\cdot \cancel{6}^{3}\cdot\frac{1}{\cancel{16}_{8}}$$

$$= \frac{p}{4} + \frac{3}{8} - \frac{3}{8}p = \frac{3}{8} - \frac{1}{8}p \qquad \square$$

Note on the "counting" distributions:

	mean		variance.
Poisson(λ)	λ	$=$	λ
Negative Binomial(r,β)	$r\cdot\beta$	$<$	$r\beta\underbrace{(1+\beta)}_{>1}$
Binomial(m,q)	$m\cdot q$	$>$	$mq\underbrace{(1-q)}_{<1}$

Poisson thinning & the binomial.

Let $N\sim \text{Poisson}(\lambda)$ be our frequency random variable.
Every loss is from:

- Category 1 w/ probability p_1
- Category 2 w/ probability p_2 w/ $p_1+p_2=1$.

N_i ... # of events from Category i, $i=1,2$.

From our thinning theorem, we know that

$$N_i \sim \text{Poisson}\,(\lambda_i = p_i\cdot\lambda) \qquad i=1,2$$

We also know that N_1 and N_2 are independent.

Q: $\boxed{\text{Given}}$ that $N=m$, what is the probability that

$N_1=k$ for $k=0,1,\dots m$?

$\longrightarrow$:

$$\mathbb{P}[N_1 = k \mid N = m] = \frac{\mathbb{P}[N_1 = k, N_1 + N_2 = m]}{\mathbb{P}[N=m]}$$ by def'n of conditional probab.

$$= \frac{\mathbb{P}[N_1 = k, N_2 = m-k]}{\mathbb{P}[N=m]}$$ (N_1 and N_2 are independent)

$$= \frac{\boxed{\mathbb{P}[N_1=k]} \cdot \boxed{\mathbb{P}[N_2 = m-k]}}{\boxed{\mathbb{P}[N=m]}} = \text{(Poisson)}$$

$$= \frac{\cancel{e^{-\lambda_1}} \frac{\lambda_1^k}{k!} \cdot \cancel{e^{-\lambda_2}} \cdot \frac{\lambda_2^{m-k}}{(m-k)!}}{\cancel{e^{-\lambda}} \cdot \frac{\lambda^m}{m!}} =$$

$$= \boxed{\frac{m!}{k!(m-k)!}} \cdot \frac{p_1^k \cdot \cancel{\lambda^k} \cdot p_2^{m-k} \cdot \cancel{\lambda^{m-k}}}{\cancel{\lambda^m}}$$

$$= \binom{m}{k} p_1^k \cdot p_2^{m-k}$$

$$\boxed{N_1 \mid N=m \sim \text{Binomial}(m, q = p_1)}$$

The (a,b,0) class.

If an $\mathbb{N}_0$-valued distribution has a pmf which satisfies the following recursion:

$$p_k = p_{k-1}\left(a + \frac{b}{k}\right) \quad \text{for } k = 1, 2, \ldots,$$

then, we say that it is an (a,b,0) distribution.

The Poisson, the negative binomial, and the binomial are the only representatives.