M358K: October 27th, 2021.

Power of Test [cont'd].

$$\mathbb{P}_{\mu_a}[\text{Fail to Reject } H_0] = \mathbb{P}[\text{Type II Error}] =: \beta$$

Note: The smaller the value of β, the better the test!

Def'n. The power of the test @ $\mu = \mu_a$ is defined as $\boxed{1-\beta}$.

Example. A simple random sample of size 36 is gathered from a normal population w/ an unknown mean μ and the standard deviation of 3.

We are testing:

$$H_0: \mu = 15 \quad \text{vs.} \quad \boxed{H_a: \mu > 15}$$

The significance level is $\alpha = 0.05$.

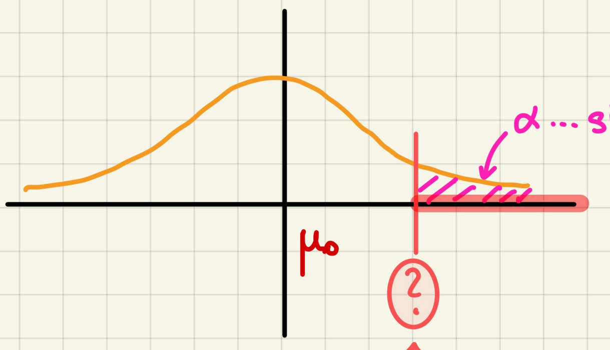
Find the power of the test @ $\mu = 16$.

→: First, find the RR.

Second, calculate the probability that the sample mean falls into the RR if $\mu = \mu_a = 16$.

Right-sided alternative $\Rightarrow$ RR in raw units is the form:

$[\,\underline{?}\,, +\infty)$



$$\boxed{\mu_0 + z_{1-\alpha}\left(\frac{\sigma}{\sqrt{n}}\right)}$$

w/ $z_{1-\alpha} = \Phi^{-1}(1-\alpha) = \text{qnorm}(1-\alpha)$

In this problem, the lower bound of the RR is:

$$15+\underbrace{1.645\left(\frac{3}{\sqrt{36}}\right)}_{0.8225} = 15.8225$$

$\Rightarrow$ $\boxed{RR: [15.8225, +\infty)}$

Second, $\mathbb{P}_{\mu_a}[\bar{X} \geq 15.8225] = ?$ w/ $\mu = \mu_a = 16$

Under the particular given alternative $\mu_a = 16$, the distribution of the sample mean $\bar{X}$ is:

$$\bar{X} \sim \text{Normal}\left(\text{mean} = \mu_a = 16, \ sd = \frac{3}{\sqrt{36}} = \frac{1}{2}\right)$$

$$\mathbb{P}_{\mu_a}[\bar{X} \geq 15.8225] = ?$$

Method #1. Standardize:

$$\mathbb{P}_{\mu_a}\left[\underset{\underset{Z}{\shortparallel}}{\frac{\bar{X}-16}{0.5}} \overset{\sim N(0,1)}{\geq} \overbrace{\frac{15.8225-16}{0.5}}\right] = \mathbb{P}[Z \geq -0.355]$$

$$= 1 - \text{pnorm}(-0.355)$$

$$= 0.6387052$$

Method #2.

$$1-\text{pnorm}(15.8225, 16, 0.5) = 0.6387052$$

Michael's no rounding method:

$$1-\text{pnorm}(15 + \text{qnorm}(0.95)*0.5, 16, 0.5) = 0.63876$$