Project #4: Put-call parity. More Monte Carlo. The normal approximation to the binomial.

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Problem #1 (25 points)

Put-call parity

(5 points) Using put-call parity, find the formula for the interest rate if all other ingredients are known.

(5 points) Based on the above, what can you say about the interest rates r you expect to obtain for varying values of the strike price?

(10 points) Based on the data set "apple-parity.csv", calculate the continuously compounded, risk-free interest rate for all the strike prices given. Be careful about how you use the *bid* and *ask/offer* prices of the options. Set $T = 0.25$. Plot the values of the interest rate you obtain as they depend on the strike prices.

(5 points) What do you notice about the above plot? Does it or does it not agree with your prediction from the second question in the problem? Substantiate your answer.

Problem #2 (25 points)

To what do you attribute the discrepancies you observed in the previous problem? Think about it a bit. Then, if you don't have any ideas look at the paper by *Brenner* and *Galai* uploaded into Canvas (under *Files* in the folder *articles*).

Problem #3 (15 points)

Let the continuously compounded, risk-free interest rate be 0.05.

Consider a stock whose current price is \$80 and whose volatility is 0.2. We will be pricing a variety of options using a *forward binomial tree*.

(5 points) Price a one-year, \$85-strike European call option analytically using a $100-$period binomial tree.

(5 points) Price a one-year, \$85-strike European call option using *Monte Carlo* with 10000 simulations with a $100-$period binomial tree.

(5 points) Price a half-year, \$78-strike European put option analytically using a $100-$period binomial tree.

(5 points) Price a half-year, \$78-strike European put option using *Monte Carlo* with 10000 simulations with a $100-$period binomial tree.

(5 points) Comment on the accuracy of the *Monte Carlo* method. Which theorem from probability is useful here?

Problem #4 (25 points)

Let $\{X_n, n = 1, 2, \dots\}$ be a sequence of random variables such that

$$X_n \sim Binomial(n, p)$$

where p is a constant between 0 and 1.

(5 points) State the *DeMoivre-Laplace Theorem* (aka the *normal approximation to the binomial*) in the context of the above sequence of random variables.

(5 points) Let $p = 0.78$. For $n = 1000$, plot the **theoretical** histogram of X_n. Superimpose the appropriate density of the normal distribution on that histogram (according to the theorem referenced above).

(5 points) Let $p = 0.42$. For $n = 1000$, draw 10000 simulated values of X_n and plot the histogram of the draws. Superimpose the appropriate density of the normal distribution on that histogram (according to the theorem referenced above).

Problem #5 (10 points)

Let $\{Y_n, n = 1, 2, \dots\}$ be a sequence of random variables such that

$$Y_n \sim Binomial(n, p_n)$$

where p_n is given by

$$p_n = \frac{1}{1 + e^{0.25\sqrt{1/n}}}$$

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For $n = 100, 1000, 5000, 10000$, draw 10000 simulated values of Y_n and plot the histogram of the draws. Does the theorem referenced in the previous problem apply to this situation or not? Substantiate your answer.