M339J: January 27th, 2021.

Review^2:

Def'n. The cumulative distribution function (cdf), $F_X : \mathbb{R} \to [0,1]$, of a random variable X is given by $F_X(x) = \mathbb{P}[X \leq x]$ for all $x \in \mathbb{R}$.

Def'n. The survival function $S_X : \mathbb{R} \to [0,1]$ of a random variable X is defined as

$$S_X(x) = \mathbb{P}[X > x] = 1 - F_X(x)$$

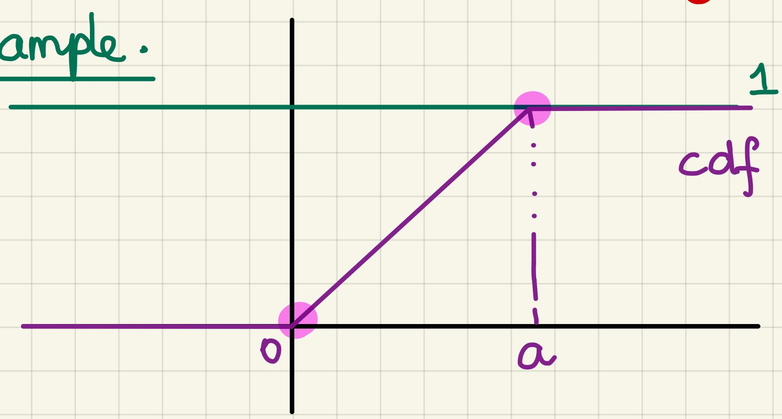
for all $x \in \mathbb{R}$

Def'n. A random variable X is continuous if its cumulative dist'n f'tion F_X is:

(a) continuous, and

(b) differentiable everywhere except @ at most countably many points.

Example.



cdf of a random variable

Uniform : $(0, a)$

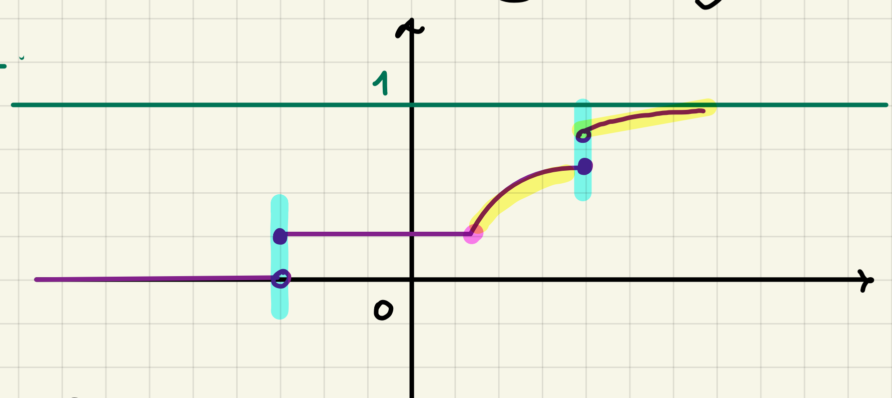
Def'n. A random variable X is mixed if

(a) it's not discrete;

(b) its cdf is continuous everywhere except @ at least one and @ most countably many points;

(c) its cdf is differentiable everywhere except @ at most countably many points.

Example.



Example. Construct a mixed random variable.

Use uniform [0,1] and Bernoulli$(q=\frac{1}{2})$.

→ : Toss a fair coin

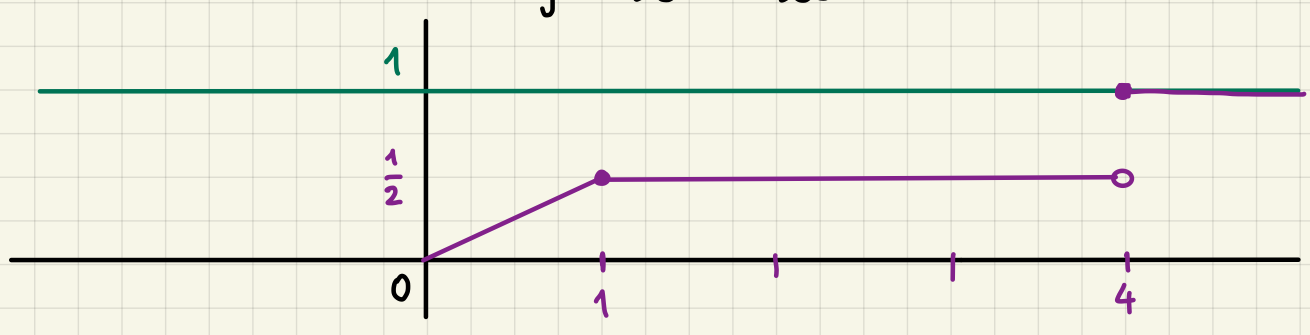
If the coin comes out heads:

draw a U(0,1) and that's the value of your r.v.

If the coin comes out tails:

set the value of your r.v. to be 4.

Q: What does the cdf look like?



Def'n. The probability density function (pdf)

$$f_X : \mathbb{R} \to \mathbb{R}_+$$

of a random variable X is defined as

$$f_X(x) = F_X'(x) = -S_X'(x)$$

wherever the derivative exists.

Note: When the r.v. X is continuous, then by the Fundamental Theorem of Calculus:

$$\mathbb{P}[a < X \leq b] = F_X(b) - F_X(a)$$
$$= \int_a^b f_X(x)\,dx$$

Def'n. The probability mass function (pmf)

$$p_X : \mathbb{R} \to [0,1]$$

of a random variable X is defined as

$$p_X(x) = \mathbb{P}[X = x].$$

Until Friday: Take a glance @ the STAM Tables.