Homework assignment #3

Milica Cudina

2021-09-06

More probability review

Problem #1.

A piggy bank contains coins of three different types: T_1, T_2 and T_3. There are twice as many type T_1 coins as type T_2 coins, and twice as % many type T_2 coins as type T_3 coins. The coins are indistinguishable to touch.

a. (5 points)

A coin is extracted from the piggy bank at random. Let the probability that the coin is of type T_i be denoted by p_i for $i = 1, 2, 3$. Find p_1, p_2 and p_3.

b. (10 points)

Coins of type T_1 are fair, coins of type T_2 come up heads (H) when tossed with probability $3/10$, and coins of type T_3 come up heads (H) when tossed with probability $1/10$.

A coin is drawn from the piggy bank at random and tossed. What is the probability that the result of the coin toss was heads?

Problem 2. (15 points)

There are three variants of a genetic marker for *goosepox*: **immune, middling,** and **susceptible**. In the population, 10% are **immune**, 70% are **middling**, and 20% are **susceptible**. Within each category, here are the chances of contracting *goosepox*:

- for **immune** it is 0%,

- for **middling** it is 50%, and

- for **susceptible** it is 90%.

Say that you learn that a randomly chosen individual contracted *goosepox*. What is the probability that this individual was **susceptible**?

The binomial distribution

Problem 3. (5 points)

Using both `R` and analytic methods, find the probability that three independent tosses of a fair coin have exactly two successes.

Problem 4. (10 points)

Using both `R` and analytic methods, find the probability that four independent tosses of a fair coin have at most two successes.

Problem 5. (5 points)

Consider a coin whose probability of landing on *heads* is $1/5$. You encode *heads* as "success". Using both `R` and analytic methods, find the probability that five independent tosses of this coin have exactly four successes.