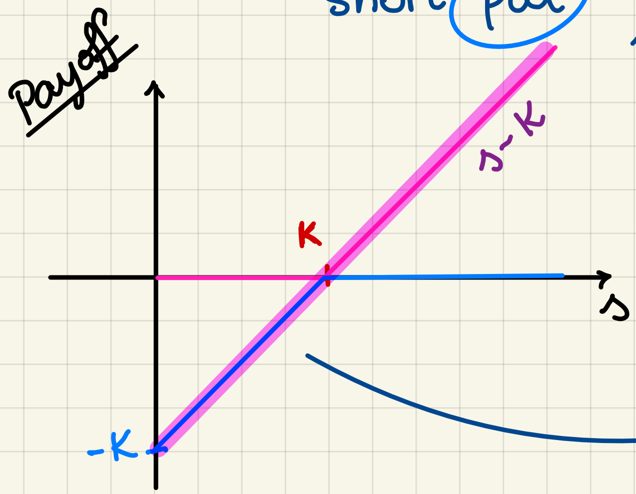
M339D: March 31st, 2021.

Put·Call Parity.

Portfolio A:
- Long call
- short put

} both European & otherwise identical



The payoff:

$$V_A(T) = V_C(T) - V_P(T)$$
$$\boxed{= S(T) - K}$$

Portfolio B:
- Long prepaid forward w/ delivery date T
- borrow $PV_{0,T}(K)$ @ the risk·free rate to be repaid @ time T

$$\Rightarrow \quad V_B(T) = S(T) - K$$

Our Conclusion:

$$\boxed{V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K)}$$

Put·Call Parity ☺

More generally: for any $t \in [0,T]$: $V_A(t) = V_B(t)$

$$V_C(t) - V_P(t) = F^P_{t,T}(S) - PV_{t,T}(K)$$

Note:
- Only works for European options.
- The no·arbitrage assumption is sufficient to get put·call parity.

Sample IFM : Derivatives : Advanced

Advanced Derivatives Questions

1. Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is 60.

(ii) The call option currently sells for 0.15 more than the put option.

(iii) Both the call option and put option will expire in 4 years. $T=4$

(iv) Both the call option and put option have a strike price of 70.

Calculate the continuously compounded risk-free interest rate.

The r we get here is sometimes called the IMPLIED INTEREST RATE.

(A) 0.039

(B) 0.049

(C) 0.059

(D) 0.069

(E) 0.079

Put-Call Parity:

$$\underbrace{V_C(0) - V_P(0)}_{\| \text{ (ii)}} = \underbrace{F^P_{0,T}(S)}_{\| \text{ no div.}} - \underbrace{PV_{0,T}(K)}_{\|}$$

$$0.15 = 60 - 70e^{-r\cdot 4}$$

$$70e^{-4r} = 60 - 0.15 = 59.85 \quad /:70$$

$$\ln(\cdot)/ \quad e^{-4r} = \frac{59.85}{70}$$

$$-4r = \ln\left(\frac{59.85}{70}\right)$$

$$\Rightarrow \quad r = \frac{1}{4}\ln\left(\frac{70}{59.85}\right) = 0.03916 \quad \checkmark$$

BEGINNING OF EXAMINATION
ACTUARIAL MODELS – FINANCIAL ECONOMICS SEGMENT

$S(0)=52$

1. On $\boxed{\text{April 30, 2007}}$, a common stock is priced at \$52.00. You are given the following:

(i) Dividends of equal amounts will be paid on June 30, 2007 and September 30, 2007.

↳ D... dividend amt

(ii) A European call option on the stock with strike price of \$50.00 expiring in six months sells for \$4.50.

$K=50$ $\quad T=1/2$

$V_c(0)=4.50$

(iii) A European put option on the stock with strike price of \$50.00 expiring in six months sells for \$2.45.

$V_p(0)=2.45$

(iv) The continuously compounded risk-free interest rate is 6%. $r=0.06$

Calculate the amount of each dividend. $D=?$

(A) \$0.51

(B) \$0.73

(C) \$1.01

(D) \$1.23

(E) \$1.45

0 (04/30) $\quad t_1 = 2/12 = 1/6$ (06/30) $\quad t_2 = 5/12$ (09/30) $\quad T = 1/2$

Put·Call Parity.

$$\underbrace{V_c(0)}_{4.50} - \underbrace{V_p(0)}_{2.45} = F^P_{0,T}(S) - \underbrace{PV_{0,T}(K)}_{50e^{-0.06(1/2)}}$$

$$F^P_{0,T}(S) = S(0) - PV(\text{Div}) = 52 - D\left(e^{-0.06(1/6)} + e^{-0.06(5/12)}\right)$$

$$D\left(e^{-0.01} + e^{-0.025}\right) = 52 - 50e^{-0.03} - 2.05 = 1.42772$$

$$\Rightarrow D = 0.72644.$$

53. Sample IFM: Derivatives : Introductory

For each ton of a certain type of rice commodity, the four-year forward price is 300. A four-year 400-strike European call option costs 110.

The continuously compounded risk-free interest rate is 6.5%.

Calculate the cost of a four-year 400-strike European put option for this rice commodity.

Put·Call Parity:

$$\underbrace{V_c(0)}_{110} - \underbrace{V_p(0)}_{?} = F^P_{0,T} - PV_{0,T}(K)$$

$$110 - ? = PV_{0,T}(F_{0,T} - K) = -100e^{-0.065(4)}$$

$$V_p(0) = 110 + 100e^{-0.065(4)} = \dots = 187.11$$

(A) 10.00

(B) 32.89

(C) 118.42

(D) 187.11

(E) 210.00

54.

DELETED

55.

Box spreads are used to guarantee a fixed cash flow in the future. Thus, they are purely a means of borrowing or lending money, and have no stock price risk.

Consider a box spread based on two distinct strike prices (K, L) that is used to lend money, so that there is a positive cost to this transaction up front, but a guaranteed positive payoff at expiration.

Determine which of the following sets of transactions is equivalent to this type of box spread.

(A) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bear spread using puts.

(B) A long position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(C) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bull spread using puts.

(D) A short position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(E) A short position in a (K, L) bull spread using calls and a short position in a(K, L) bull spread using puts.

A synthetic forward is a replicating portfolio for a forward contract.

If we create our portfolio A using options w/ strike $K = F_{0,T}$, we get that the payoff of that portfolio equals

$$S(T) - F_{0,T},$$

i.e., it's exactly equal to the payoff of a forward contract.

However, we frequently say that for any strike K, portfolio A is a synthetic forward.

5.

Try to solve this problem ☺

The PS index has the following characteristics:

- One share of the PS index currently sells for 1,000.
- The PS index does not pay dividends.

Sam wants to lock in the ability to buy this index in one year for a price of 1,025. He can do this by buying or selling European put and call options with a strike price of 1,025.

The annual effective risk-free interest rate is 5%.

Determine which of the following gives the hedging strategy that will achieve Sam's objective and also gives the cost today of establishing this position.

(A) Buy the put and sell the call, receive 23.81

(B) Buy the put and sell the call, spend 23.81

(C) Buy the put and sell the call, no cost

(D) Buy the call and sell the put, receive 23.81

(E) Buy the call and sell the put, spend 23.81

6.

The following relates to one share of XYZ stock:

- The current price is 100.
- The forward price for delivery in one year is 105.
- P is the expected price in one year

Determine which of the following statements about P is TRUE.

(A) $P < 100$

(B) $P = 100$

(C) $100 < P < 105$

(D) $P = 105$

(E) $P > 105$