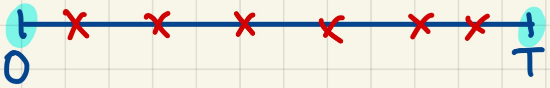
M339D: February 2nd, 2022.

Static Portfolios [cont'd].



Initial Cost → Payoff

$$\boxed{\text{PROFIT} := \text{PAYOFF} - FV_{0,T}(\text{INITIAL COST})}$$

↑ calculated using the ccrfir r

For any portfolio whose payoff depends on the final asset price, we introduce:

s... independent argument which denotes the final asset price
(we can understand it as a placeholder for $S(T)$)

We define the **payoff function** which describes the dependence of the payoff on the final asset price s.

$$\boxed{s \longmapsto v(s)}$$

When we draw the graph of the payoff function, we get the **payoff curve**.

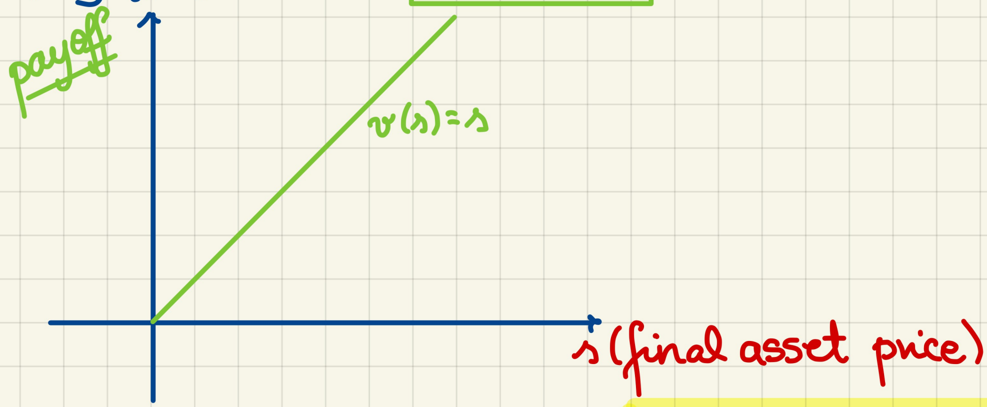
Example. [Outright Purchase of a Non-Dividend-Paying Stock]
(cont'd)

Initial Cost: $S(0)$

Payoff: $S(T)$

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= S(T) - S(0)e^{rT}$$

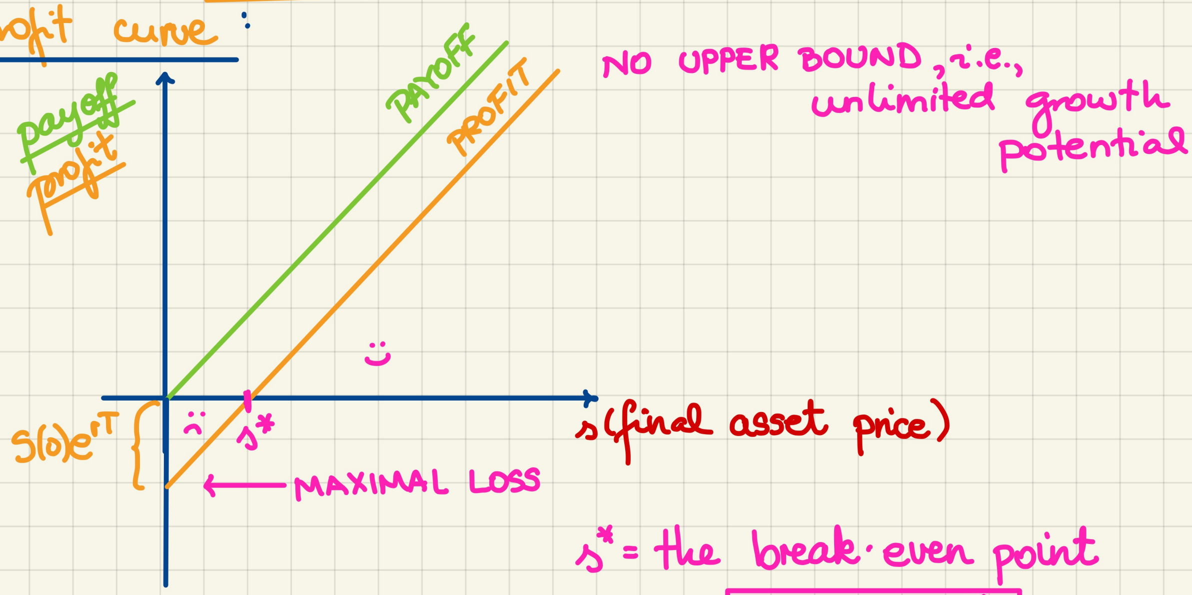
The payoff f'tion: $\boxed{v(s) = s}$



In general, the profit function is: $\boxed{v(s) - FV_{0,T}(\text{Init. Cost})}$

In this example, $\boxed{s - S(0)e^{rT}}$

$\Rightarrow$ The profit curve:



s^* = the break-even point

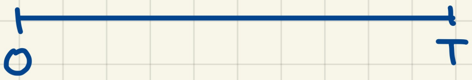
Here: $\boxed{s^* = S(0)e^{rT}}$

The payoff and profit curves are increasing.

Terminology. If the payoff/profit curve is increasing (not necessarily strictly) as a function of the final asset price s, we say that the portfolio is long with respect to the underlying asset.

Example. [Outright Purchase of a Continuous·Dividend·Paying Stock]

δ... dividend yield



Bought 1 share. Own $\underline{e^{\delta T}}$ shares.

Initial Cost: $S(0)$

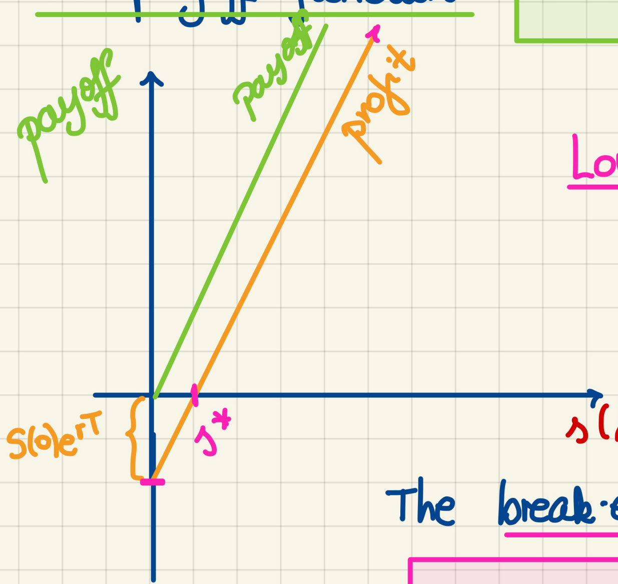
Payoff: $\boxed{e^{\delta T} \cdot S(T)}$

Profit = Payoff − $FV_{0,T}$ (Initial Cost)

$$= e^{\delta T} \cdot S(T) - S(0)e^{rT}$$

The payoff function: $\boxed{v(s) = e^{\delta \cdot T} s}$

Profit function: $\boxed{s \cdot e^{\delta \cdot T} - S(0)e^{rT}}$



Long w.r.t the underlying

The break-even point:

$$\boxed{s^* = S(0)e^{(r-\delta)T}}$$

Try to remember this expression!

Solve for s in

$$s e^{\delta T} - S(0)e^{rT} = 0$$

$$s e^{\delta T} = S(0)e^{rT}$$

$$s = S(0)e^{(r-\delta)T}$$

Example. [A Short Sale of a Non-Dividend-Paying Asset]

0	T
short sale initiated.	short sale closed.

At time 0: The short seller receives $S(0)$.

$\Rightarrow$ Initial Cost: $-S(0)$

At time T: The short seller spends $S(T)$.

$\Rightarrow$ Payoff: $-S(T)$

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= -S(T) + e^{rT} \cdot (+S(0))$$
$$= -S(T) + S(0)e^{rT}$$

- Payoff f'tion
- Profit f'tion : · bounds
  · monotonicity
  · break-even