Delta. Hedging. Market Makers. · immediacy } => exposure to risk => (hedge) Say, a market maker writes an option whose value f'hion is At time 0, they wrote the option. So, they get v(5(0),0). At time t, the value of the market maker's position is - v(s,t) To (partially) hedge this exposure to risk, they construct a portfolio which has zero sensitivity to small perturbations in the stock price. Formally speaking, their goal is to create a delta neutral portfolio, i.e., a portfolie for which $\Delta_{prt}(s,t)=0$ Theoretically, with continuous rebalancing w/no transaction costs it's possible. Practically, continuous rebalancing is impossible and there are transaction costs. In particular, @ time.0, we want to trade so that Δ_{Port} (S(0),0) = 0. The most straightforward strategy is to trade in the shares of the underlying asset. At time t let N(s,t) denote the required number of shares in the portfolio necessary to maintain D. neutrality.

```
The total value of the portfolio is:

v_{Brt}(s,t) = N(s,t) \cdot s - v(s,t)

\Delta v_{Brt}(s,t) = N(s,t) - \Delta(s,t) = 0

\Delta v_{Brt}(s,t) = N(s,t) - \Delta(s,t)

[Example. A market maker units a call option @ time·0.

At time·t, the market maker's position is:

-v_{c}(s,t)

=> They have to maintain N(s,t) = \Delta_{c}(s,t)

in the \Delta·hedge.

=> In particular, @ time·0:
```

N(S(0),0) = N(d,(S(0),0)) >0,

Example.

i.e., the market maker should long this much of a share.

- **46.** You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:
 - (i) Each period is 6 months.
 - (ii) u/d = 4/3, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.
 - (iii) The risk-neutral probability of an up move is 1/3.
 - (iv) The initial futures price is 80.
 - (v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

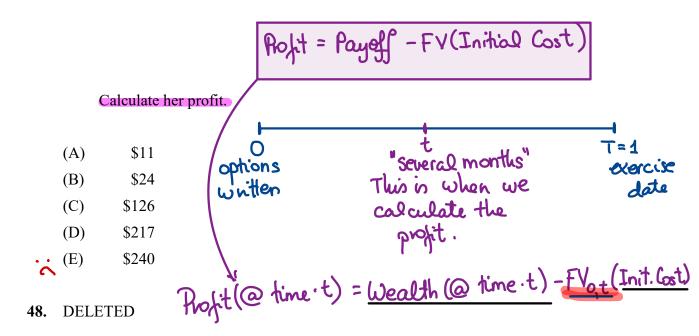
Determine $C_{II} - C_{I}$.

- (A) 0
- (B) 0.022
- (C) 0.044
- (D) 0.066
- (E) 0.088
- 47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i)	The risk-free interest	rate is constant.	. 4 11
(ii)		When the option	when the positions are closed out
		was written	are closed out
		Several months ago	Now
		@ time.0	timet
	Stock price	\$40.00	\$50.00
	Call option price	\$ 8.88	\$14.42
	Put option price	\$ 1.63	\$ 0.26
	Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.



- **49.** You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).
 - (i) The period is 3 months.
 - (ii) The initial stock price is \$100.
 - (iii) The stock's volatility is 30%.
 - (iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

- (A) 114
- (B) 115
- (C) 116
- (D) 117
- (E) 118

```
• Initial Cost: -100·ν<sub>c</sub>(S(0),0) + 100 · Δ<sub>c</sub>(S(0),0)·S(0) = 1,288 + 0.794·40) = 2,288
```

Use put call parity:

At time · 0:

$$v_c(s(0), 0) - v_p(s(0), 0) = s(0) - Ke^{-rT}$$

 $8.88 - 1.63 = 40 - Ke^{-rT}$

$$\frac{V}{V} = \frac{10^{-1}(7-t)}{10^{-1}} = e^{-t} = \frac{35.84}{32.75}$$

Profit (@ time·t) =
$$2528 - \frac{35.84}{32.75} \cdot 2288 = 24.12458$$