Two-Stock Portfolio.

X and Y stand for returns of two stocks

$$\text{returns} = \frac{\text{Price}_{END} - \text{Price}_{BEG}}{\text{Price}_{BEG}}$$

α ... the proportion of our wealth invested in stock X

R... the return of the total portfolio

$$\boxed{R = \alpha \cdot X + (1-\alpha)Y}$$

Optimization Problem.

Minimize Var[R] across all α

$$\text{Var}[R] \xrightarrow[\alpha]{} \min$$

$$\text{Var}[\alpha X + (1-\alpha)Y] \xrightarrow[\alpha]{} \min$$

$$\alpha^2 \cdot \text{Var}[X] + (1-\alpha)^2 \cdot \text{Var}[Y] + 2\alpha(1-\alpha) \cdot \text{Cov}[X,Y] \xrightarrow[\alpha]{} \min$$

$$\alpha^2 \sigma_X^2 + (1-\alpha)^2 \sigma_Y^2 + 2\overbrace{\alpha(1-\alpha)}^{(\alpha-\alpha^2)} \cdot \sigma_{XY} \xrightarrow[\alpha]{} \min$$

$$\cancel{2}\alpha \sigma_X^2 - \cancel{2}(1-\alpha)\sigma_Y^2 + \cancel{2}(1-2\alpha)\sigma_{XY} = 0$$

$$\alpha \sigma_X^2 + (\alpha - 1)\sigma_Y^2 + (1-2\alpha)\sigma_{XY} = 0$$

$$\alpha(\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}) = \sigma_Y^2 - \sigma_{XY}$$

$$\boxed{\alpha^* = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}}$$