UNIVERSITY OF TEXAS AT AUSTIN

Quiz #1

Basics of probability.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 1.1. (2 points) Let E and F be any two events. Then, $\mathbb{P}[E \cup F] \leq \mathbb{P}[E] + \mathbb{P}[F]$. *True or false? Why?*

Solution: TRUE
By the inclusion-exclusion formula, we know that

$$\mathbb{P}[E \cup F] = \mathbb{P}[E] + \mathbb{P}[F] - \mathbb{P}[E \cap F].$$

Now, remember that $\mathbb{P}[E \cap F] \geq 0$.

Problem 1.2. (2 points) Let E and F be any two events. If $\mathbb{P}[E] = \mathbb{P}[F] = \frac{2}{3}$, then E and F cannot be mutually exclusive. *True or false? Why?*

Solution: TRUE
If E and F were mutually exclusive, we would have that

$$\mathbb{P}[E \cup F] = \mathbb{P}[E] + \mathbb{P}[F] = \frac{4}{3} > 1.$$

Contradiction!

Problem 1.3. (4 points) Let E and F be any two events with positive probability. If $\mathbb{P}[E|F] < \mathbb{P}[E]$, then $\mathbb{P}[F|E] < \mathbb{P}[F]$. *True or false? Why?*

Solution: TRUE
By the definition of conditiona probability, the given inequality can be rewritten as follows

$$\begin{aligned} \mathbb{P}[E|F] < \mathbb{P}[E] \quad &\Leftrightarrow \quad \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[F]} < \mathbb{P}[E] \quad \Leftrightarrow \quad \mathbb{P}[E \cap F] < \mathbb{P}[E]\mathbb{P}[F] \\ &\Leftrightarrow \quad \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[E]} < \mathbb{P}[F] \quad \Leftrightarrow \quad \mathbb{P}[F|E] < \mathbb{P}[F] \end{aligned}$$

Problem 1.4. (2 points) If events E and F are independent and events F and G are independent, then E and G are independent as well. *True or false? Why?*

Solution: FALSE
What if $E = G$?

Problem 1.5. (5 points) The four standard blood types are distributed in a populations as follows:

$$\begin{array}{ll} A - 42\% & O - 33\% \\ B - 18\% & AB - 7\% \end{array}$$

Assuming that people choose their mates independently of their blood type, find the probability that the people in a randomly chosen couple from this population have different blood types.

Solution: Let E denote the event that the people in a randomly chosen couple have different blood types. Then, we have

$$\mathbb{P}[E^c] = (0.42)^2 + (0.33)^2 + (0.18)^2 + (0.07)^2 = 0.3226.$$

So, $\mathbb{P}[E] = 1 - 0.3226 = 0.6774$.
