University of Texas at Austin, Department of Mathematics
M358K - Applied Statistics

The Prerequisite In-term exam

1.1. TRUE/FALSE QUESTIONS.

Problem 1.1. (2 pts)
Let E and F be any two events. If

$$\mathbb{P}[E] = \mathbb{P}[F] = \frac{2}{3},$$

then E and F cannot be mutually exclusive. *True or false?*

Problem 1.2. (2 pts) If events A and B are mutually exclusive, they are necessarily independent. *True or false?*

Problem 1.3. (2 points) If $Var[X] = 0$, then $\mathbb{P}[X = \mathbb{E}[X]] = 0$. *True or false?*

Problem 1.4. (2 points)
Let X denote the outcome of a roll of a fair, regular dodecahedron (a polyhedron with 12 faces) with numbers $1, 2, \cdots, 12$ written on its sides. Then $\mathbb{E}[X] = 13/2$. *True or false?*

1.2. MULTIPLE CHOICE QUESTIONS.

Problem 1.5. (5 pts) A class contains 20 men and 10 women. You know that half the men and half the women have brown eyes. What is the probability that a person chosen at random from this class is a woman or has brown eyes?

  (a) 1/3
  (b) 2/3
  (c) 7/18
  (d) 7/9
  (e) None of the above

Problem 1.6. (5 pts) A pair of dice is thrown. Find the probability that the sum of the outcomes is 10 or greater if a 5 appears on the first die.

  (a) 1/6
  (b) 1/4
  (c) 1/3
  (d) 1/2
  (e) None of the above

Problem 1.7. (5 points) What is the **R** output of the following command:
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>dbinom(2,3,0.5)
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  (a) 0.375
  (b) 0.5
  (c) 0.725

(d) 0.75

(e) None of the above.

Problem 1.8. (5 points) A test is used to determine whether people exhibiting green spots have the *duckpox* or not. It is believed that at any given time 4% of people exhibiting green spots actually have the *duckpox*. The test is 99% accurate if a person actually has the *duckpox*. The test is 96% accurate if a person does **not** have the *duckpox*. What is the probability that a randomly selected person who tests positive for the *duckpox* actually has the *duckpox*?
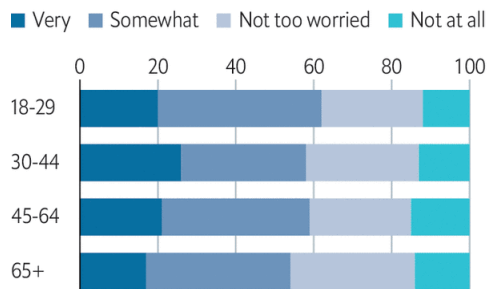
(a) About 4%.

(b) About 8%

(c) About 51%

(d) About 72%.

(e) None of the above.
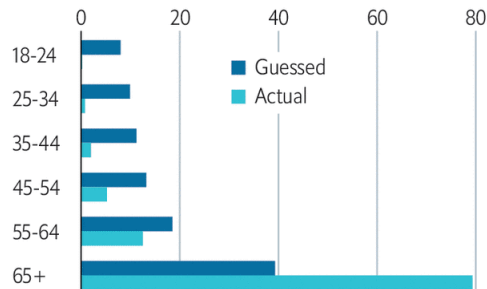
Problem 1.9. (5 points) Consider the following charts:

Youthful pessimism
United States, covid-19 by age group

Worry about contracting covid-19, %

■ Very   ■ Somewhat   ▢ Not too worried   ■ Not at all

Actual and guessed* shares of deaths, %

■ Guessed
■ Actual

Sources: YouGov; Franklin Templeton-Gallup Economics of Recovery Study; Center for Disease Control; *The Economist*   *By a poll of all ages

The Economist

Which of the following statements is **not** correct?

(a) More than three in five 18- to 29-year-old Americans are "very" or "somewhat" worried about contracting COVID-19.

(b) Young people do not fall ill with the virus as often as older people.

(c) The findings **suggest** that many people underestimate the age of a typical COVID-19 victim.

(d) Young people are **underestimating** the number of COVID-19 victims in their age group.

(e) People aged 65 and older account for more than a half of the COVID-19 victims.

Problem 1.10. (5 points) Consider the following histogram:
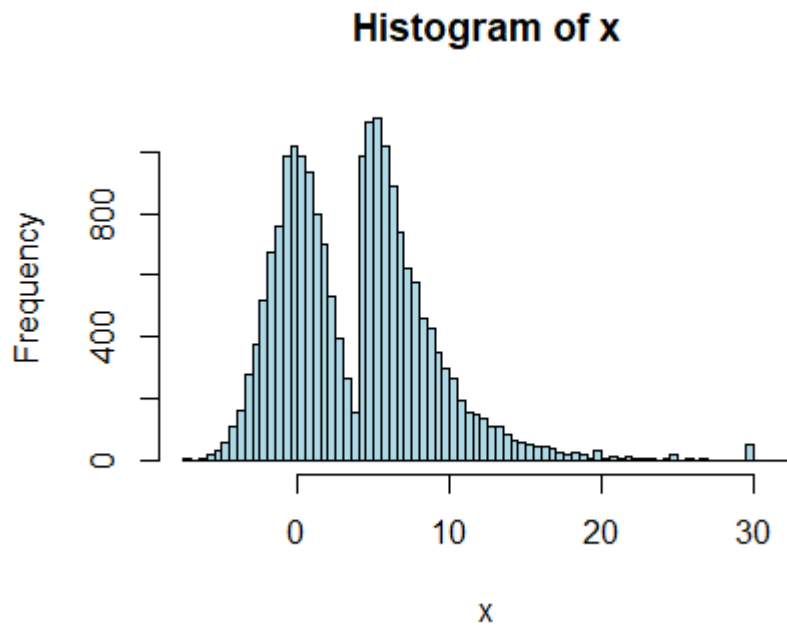
Histogram of x

The histogram is ...

(a) ...unimodal.

(b) ...bimodal, symmetric.

(c) ...bimodal, asymmetric.

(d) ...trimodal.

(e) None of the above.

Problem 1.11. (5 points) *Source: AMC8, 2019.* The diagram shows the number of students at soccer practice each weekday during last week. After computing the mean and median values, Coach discovers that there were actually 21 participants on Wednesday. Which of the following statements describes the change in the mean and median after the correction is made?

Number of students at soccer practice
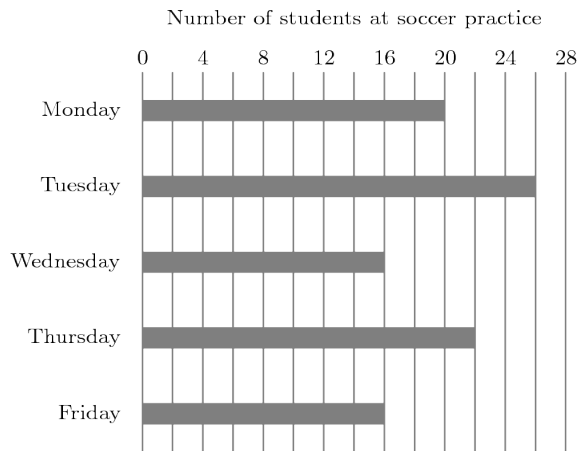
(a) The mean increases by 1 and the median does not change.

(b) The mean increases by 1 and the median increases by 1.

(c) The mean increases by 1 and the median increases by 5.

(d) The mean increases by 5 and the median increases by 1.

(e) The mean increases by 5 and the median increases by 5.

Problem 1.12. (5 points) Your sample consists of 50 sixth-graders from Kealing Middle School (KMS) and 60 sixth-graders from Murchison Middle school (MMS). The measures of center and spread of the students' heights are:

KMS : the mean of 60 inches with the standard deviation of 1 inch, and

MMS : the mean of 60 inches with the standard deviation of 2 inches.

What are the measures of center and spread for the pooled sample of 100 sixth-graders?

(a) The mean is 60; the standard deviation is 1.62.

(b) The mean is 60; the standard deviation is 1.5.

(c) The mean is 59.5; the standard deviation is 1.5.

(d) Not enough information is given.

(e) None of the above.

Problem 1.13. (5 points) Consider the following box plot:
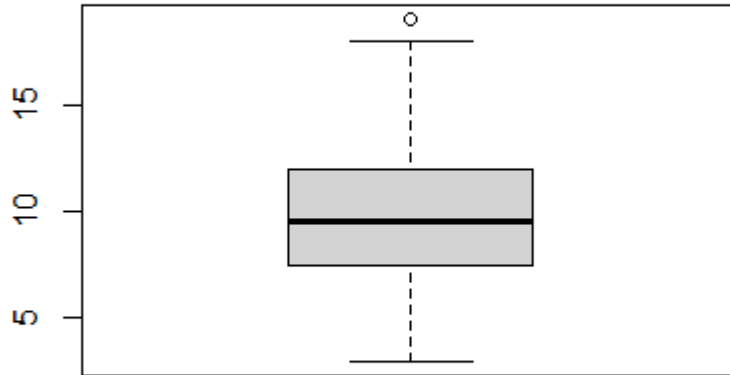
Which summary statistics does it correspond to?

(a)

Min.	Q1	Median	Mean	Q3	Max.
3.00	7.75	9.50	9.95	12.00	19.00

(b)

Min.	Q1	Median	Mean	Q3	Max.
3.00	7.75	9.50	9.95	12.00	17.00

(c)

Min.	Q1	Median	Mean	Q3	Max.
0.00	7.75	8.00	9.95	12.00	17.00

(d)

Min.	Q1	Median	Mean	Q3	Max.
3.00	7.75	9.50	9.95	10.00	17.00

(e) None of the above.

Problem 1.14. (5 points) Which of the following claims in **not** correct?

(a) Conclusions based on data obtained from **volunteer samples** can me misleading.

(b) It is important to be clear about the **target population** of a study.

(c) **Random choice** is the generally accepted method of drawing samples from a population.

(d) The **stratification method** only looks at one particular subset of the population of interest.

(e) Different random samples are **likely** to yield different values of the point estimates of the same parameter.

1.3. **FREE-RESPONSE PROBLEMS.**

Problem 1.15. <u>Don't mess with Texas!</u>

The *Anti-Littering League* wishes to gauge the success of the ingenious <u>*Don't mess with Texas!*</u> campaign.

Realizing the obvious problems with conducting a survey which outright asks the questions: "Are you or have you ever been a litterer?", they resort to the randomized-response method.

They prompt a computer to display the question

"Have you ever littered?"

with probability 0.6. The rest of the time, a virtual fair coin is flipped on the screen and the subject is asked

"Is the outcome heads?"

In both cases, the subject is prompted to click the button with **Yes** or **No**. The interviewer did not know the actual question asked, just the ultimate response. So, there was no real reason for the subject to lie, and we assume that the subjects responded truthfully.

i. (5 points)

It turned out that 50% of the subjects answered "yes". Give an estimate of the proportion of *litterers* in this population.

ii. (5 points) What percentage of "yes" answers would you have obtained in an ideal world in which nobody ever litters?