

UNIVERSITY OF TEXAS AT AUSTIN

Quiz #11

Exchange options.

Please, provide your complete solution to the following problem:

Problem 11.1. (15 points) There are two risky assets available in our market model: **S** and **Q**. Assume that the economy in which the two risky assets exist has three possible different states in three months: *sunny*, *overcast* and *rainy*. The *sunny* and *rainy* states of the world are equally likely, while the *overcast* has the same probability as the other two states combined. The risky assets' prices in three months (time- T) have the following possible values within our model:

$$S(T) = \begin{cases} 100, & \text{if } \textit{sunny} \\ 80, & \text{if } \textit{overcast} \\ 50, & \text{if } \textit{rainy} \end{cases} \quad \text{and} \quad Q(T) = \begin{cases} 40, & \text{if } \textit{sunny} \\ 70, & \text{if } \textit{overcast} \\ 60, & \text{if } \textit{rainy} \end{cases}$$

What is the expected payoff of a three-month **exchange call** with underlying asset **S** and strike asset **Q** according to the above model?

Solution: From the given information on the relative probabilities of the three different states-of-the-world, we conclude that the economy is

$$\begin{aligned} \textit{sunny} & \text{ with probability } 1/4 \\ \textit{overcast} & \text{ with probability } 1/2 \\ \textit{rainy} & \text{ with probability } 1/4 \end{aligned}$$

The payoff of the exchange call in the problem is

$$V_{EC}(T, \mathbf{S}, \mathbf{Q}) = (S(T) - Q(T))_+$$

So, the possible values of the payoff (within our model) are

$$V_{EC}(T, \mathbf{S}, \mathbf{Q}) = \begin{cases} 60, & \text{if } \textit{sunny} \\ 10, & \text{if } \textit{overcast} \\ 0, & \text{if } \textit{rainy} \end{cases}$$

Finally, the expected value is

$$60 \times \frac{1}{4} + 10 \times \frac{1}{2} = 15 + 5 = 20.$$