# M339D: April 14th, 2021.

## Call Price Monotonicity [cont'd].

**Recall:** European call prices are decreasing as functions of the strike, i.e.,
for $K_1 < K_2$, we have

$$\boxed{V_C(K_1) \geq V_C(K_2)}.$$

→ Assume, to the contrary, that there exist $K_1 < K_2$ such that

$$V_C(K_1) < V_C(K_2).$$

I. Suspicion. ✓

II. Propose an arbitrage portfolio:
- long the $K_1$-strike call
- write the $K_2$-strike call

CALL BULL SPREAD

III. Verification.

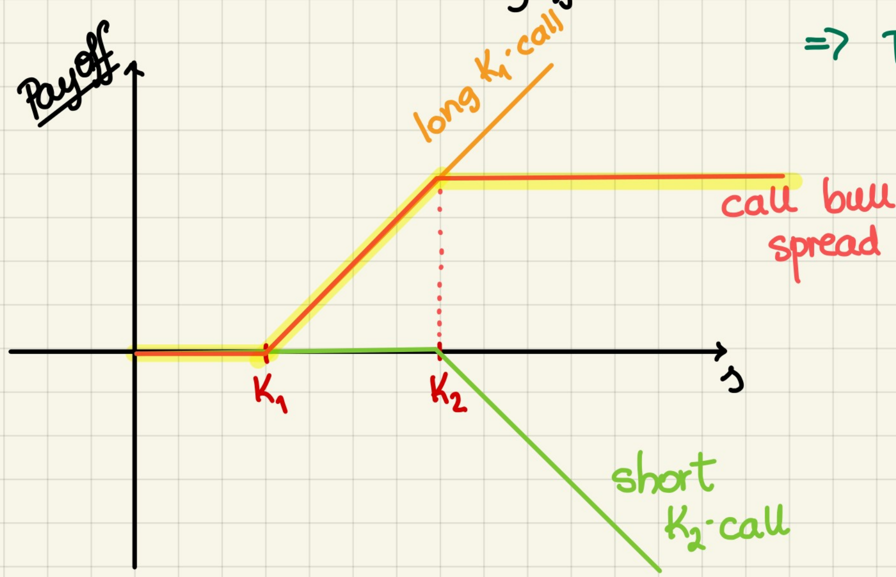
Initial cost: $V_C(K_1) - V_C(K_2) < 0$

Payoff: $(S(T)-K_1)_+ - (S(T)-K_2)_+ =$

$$= \begin{cases} 0, & \text{if } S(T) < K_1 \\ S(T)-K_1, & \text{if } K_1 \leq S(T) < K_2 \\ \cancel{S(T)}-K_1-\cancel{S(T)}+K_2 = K_2-K_1, & \text{if } K_2 \leq S(T) \end{cases}$$

$\Rightarrow$ Payoff $\geq 0$ $\Rightarrow$ Profit $> 0$

$\Rightarrow$ This is, indeed, an arbitrage portfolio!

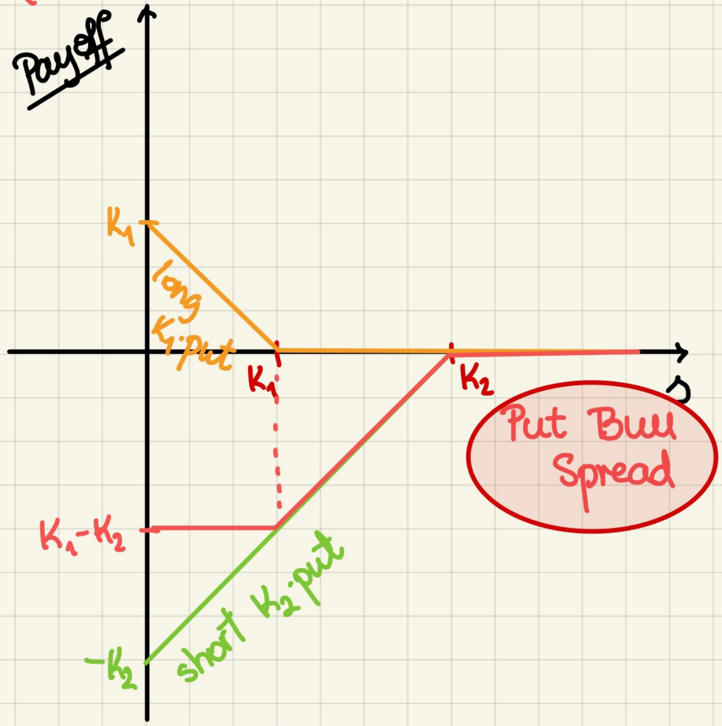



Long w/ respect to the underlying

$\Rightarrow$ a suitable hedge for a short position.

Q: How would you construct a PUT bull spread, i.e., a financial position consisting of puts w/ the same payoff shape?

An idea:
- Long the $K_1$·put  w/ $K_1 < K_2$
- Write the $K_2$·put




Q: What is the difference between the profits of the call bull spread and the put bull spread (both w/ same $K_1$ & $K_2$)?

Put·option monotonicity.

Claim. Put prices are increasing as functions of the strike, i.e., for $K_1 < K_2$, we have $V_P(K_1) \leq V_P(K_2)$

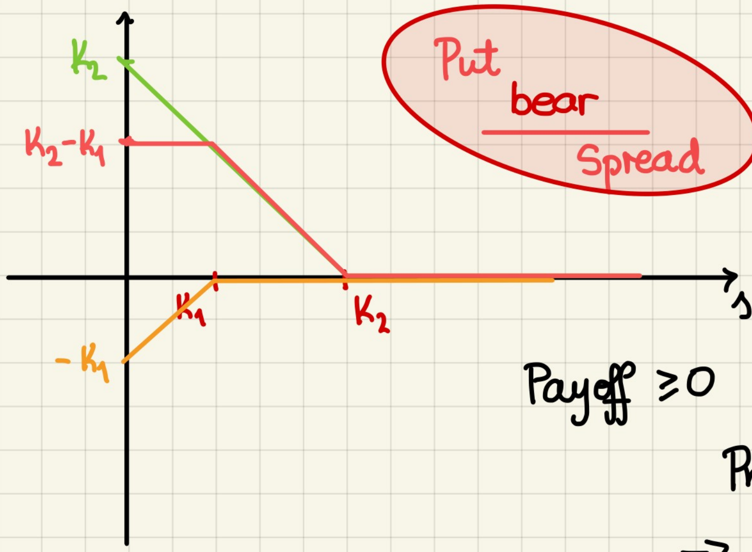
→: Assume, to the contrary, that there exist $K_1 < K_2$ such that

$$V_P(K_1) > V_P(K_2)$$

Propose this arbitrage portfolio:
- short the $K_1$·put
- long the $K_2$·put

Initial cost: $-V_P(K_1) + V_P(K_2) < 0$ ✓

Payoff:




Put bear spread

Payoff $\geq 0$ ✓

Profit $>0$

$\Rightarrow$ This is really an arbitrage portfolio❢

(Call) $(K_1, K_2)$-bull spread

## "Cord-slope" bounds.

Let $K_1 < K_2$:

$$0 \leq \left\{ \begin{array}{l} V_c(K_1) - V_c(K_2) \\ V_p(K_2) - V_p(K_1) \end{array} \right\} \leq PV_{0,T}(K_2 - K_1)$$

monotonicity

Claim❢

## Calls.

Assume, to the contrary, there exist $K_1 < K_2$ such that

$$V_c(K_1) - V_c(K_2) > PV_{0,T}(K_2 - K_1)$$

$$\Leftrightarrow \quad V_c(K_1) > V_c(K_2) + PV_{0,T}(K_2 - K_1)$$

Think about an arbitrage portfolio you would propose :)