M378K: December 6th, 2024.

Hypothesis Testing [cont'd].

Review.

H_0: _____ vs. H_a: _____

α ... significance level.

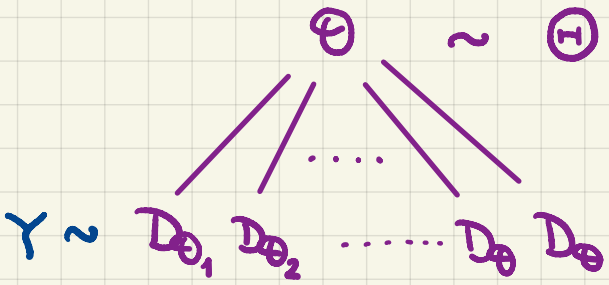
Decision \ "Truth"	H_0 true	H_0 not true
Reject H_0	Type I Error	
Fail to Reject H_0		Type II Error

$$\mathbb{P}[\text{Type I Error}] = \mathbb{P}_0[\text{Reject } H_0] = \alpha \text{ (significance level)}$$

$\mathbb{P}_0$: under the null

The Bayesian Approach.

The Set Up.



The simplest version:

Θ can be any one value from $\Theta_1, \Theta_2, \ldots, \Theta_k$

Then, we see a value y of the random variable Y in question.

By the Bayes rule, we have, for $k = 1, \ldots, K$,

$$\mathbb{P}[\Theta = \Theta_k \mid Y = y] = \frac{\mathbb{P}[\Theta = \Theta_k] \cdot \mathbb{P}[Y = y \mid \Theta = \Theta_k]}{\sum_{i=1}^{K} \mathbb{P}[\Theta = \Theta_i] \cdot \mathbb{P}[Y = y \mid \Theta = \Theta_i]}$$

$\mathbb{P}[\Theta = \Theta_k \mid Y = y]$: posterior probabilities

$\mathbb{P}[\Theta = \Theta_i]$: prior probabilities

This procedure is called Bayesian updating.

When no prior information is available, we use the uninformed prior.