M339J: February 24th, 2023.

50. In Year 1 a risk has a Pareto distribution with $\alpha = 2$ and $\theta = 3000$. In Year 2 losses inflate by 20%.

$X_1 \sim Pareto(\alpha_1 = 2, \theta_1 = 3000)$

$X_2 \sim Pareto(\alpha_2 = 2, \theta_2 = 3600)$

An insurance on the risk has a deductible of 600 in each year. P_i, the premium in year i, equals 1.2 times the expected claims.

$d = 600$: $P_i = 1.2\,\mathbb{E}[(X_i - d)_+]$ $i = 1,2$

The risk is reinsured with a deductible that stays the same in each year. R_i, the reinsurance premium in year i, equals 1.1 times the expected reinsured claims.

d_R

$\tilde{d} = d_R + 600$ $R_i = 1.1\,\mathbb{E}[(X_i - \tilde{d})_+]$ $i = 1,2.$

$$\boxed{\frac{R_1}{P_1} = 0.55}$$

Calculate $\boxed{\frac{R_2}{P_2}}$.

(A) 0.46

(B) 0.52

(C) 0.55

(D) 0.58

(E) 0.66

$X \sim Pareto(\alpha, \theta)$

$d > 0$

$$\mathbb{E}[(X-d)_+] = \mathbb{E}[X] - \mathbb{E}[X \wedge d]$$

$$= \frac{\theta}{\alpha - 1} - \frac{\theta}{\alpha - 1}\left(\cancel{1} - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1}\right)$$

$$= \frac{\theta}{\alpha - 1} \cdot \left(\frac{\theta}{d+\theta}\right)^{\alpha - 1}$$

$i = 1,2$ $\frac{R_i}{P_i} = ?$

$$\frac{R_i}{P_i} = \frac{1.1 \cdot \mathbb{E}[(X_i - \tilde{d})_+]}{1.2 \cdot \mathbb{E}[(X_i - d)_+]} = \frac{1.1 \cdot \cancel{\left(\frac{\theta_i}{\alpha_i - 1}\right)} \cdot \left(\frac{\cancel{\theta_i}}{d_R + 600 + \theta_i}\right)^{\alpha_i - 1}}{1.2 \cdot \cancel{\left(\frac{\theta_i}{\alpha_i - 1}\right)}\left(\frac{\cancel{\theta_i}}{600 + \theta_i}\right)^{\alpha_i - 1}}$$

$$= \frac{11}{12}\left(\frac{600 + \theta_i}{d_R + 600 + \theta_i}\right)^{\alpha_i - 1}$$

In Year 1:

$$\cancel{0.55}^{0.05} = \frac{R_1}{P_1} = \frac{11}{12}\left(\frac{600+\theta_1}{d_R+600+\theta_1}\right)^{\alpha_1-1} = \frac{\cancel{11}^{1}}{12}\cdot\frac{600+3000}{d_R+3600}$$

$$0.05(12)(d_R+3600) = 3600$$

$$\cancel{0.6}^{0.1}(d_R+3600) = \cancel{3600}^{600}$$

$$d_R = 600\cdot 10 - 3600 = \underline{2400}$$

In Year 2:

$$\frac{R_2}{P_2} = \frac{11}{12}\left(\frac{600+\theta_2}{2400+600+\theta_2}\right)^{\alpha_2-1}$$

$$= \frac{11}{12}\cdot\frac{600+3600}{2400+600+3600} = \frac{\cancel{11}}{12}\cdot\frac{\cancel{4200}^{7}}{\cancel{6600}_{\cancel{11}}} = \frac{7}{12} = 0.58$$

Terminology.

The increased limits factor (ILF) is the ratio of the expected loss at the limit a to the expected loss @ the basic level d, i.e.,

$$ILF(a) = \frac{\mathbb{E}[X\wedge a]}{\mathbb{E}[X\wedge d]}$$

Note: Let $a < b$,

$$\frac{ILF(b)-ILF(a)}{ILF(\infty)} = \frac{\frac{\mathbb{E}[X\wedge b]}{\cancel{\mathbb{E}[X\wedge d]}} - \frac{\mathbb{E}[X\wedge a]}{\cancel{\mathbb{E}[X\wedge d]}}}{\frac{\mathbb{E}[X]}{\cancel{\mathbb{E}[X\wedge d]}}}$$

$$= \frac{\mathbb{E}[X\wedge b]-\mathbb{E}[X\wedge a]}{\mathbb{E}[X]} = \text{proportion of losses within the layer } (a,b)$$

75. A primary liability insurer has a book of business with the following characteristics:

- All policies have a policy limit of 500,000
- The expected loss ratio is 60% on premiums of 4,000,000 $E[X] = 2400000$

A reinsurer provides an excess of loss treaty for the layer 300,000 in excess of 100,000.

The following table of increased limits factors is available:

Limit	ILF
100,000	1.00
200,000	1.25
300,000	1.45
400,000	1.60
500,000	1.70

$\underbrace{\left(\text{proportion w/in } (100K, 400K)\right) \times}_{\text{answer}}$

Calculate the reinsurer's expected losses for this coverage (answer to the nearest 000s).

(A) 840,000

(B) 847,000

(C) 850,000

(D) 862,000

(E) 871,000

$$\text{proportion} = \frac{ILF(400K) - ILF(100K)}{ILF(500K)} = \frac{1.6-1}{1.7} = \frac{6}{17}$$

answer: 847K

Poisson Distribution.

Usually: $N \sim \mathrm{Poisson}(\lambda)$

Support: $\mathbb{N}_0 = \{0, 1, 2, \dots\}$

We say that any r.v. w/ this support is $\mathbb{N}_0$-valued.

The probability mass function:

$$p_N(k) := p_k = \boxed{e^{-\lambda} \cdot \frac{\lambda^k}{k!}} \quad \text{for all } k$$

The probability generating function:

$$P_N(z) := \mathbb{E}\left[z^N\right] = e^{\lambda(z-1)}$$

$$\boxed{\mathbb{E}[N] = \lambda \qquad \text{and} \qquad \mathrm{Var}[N] = \lambda}$$