M339D: March 9th, 2022.

Options on Currencies.

- domestic currency (DC): r_D ... ccrfir for DC
- foreign currency (FC): r_F ... ccrfir for FC

- underlying asset: foreign currency (FC)
- the price of the underlying asset:

the exchange rate; denoted by;

$$\boxed{x(t),\ t \geq 0}$$

is the worth of 1 unit of the FC in terms of the DC

(Prepaid) forward on an FC.

0 ———————— T ... delivery date

Prepaid forward: pay $F^P_{0,T}(x)$ receive 1 unit of FC

no arbitrage ← ||

Outright purchase: buy $e^{-r_F T}$ of FC own 1 unit of FC

cost: $e^{-r_F T} \cdot x(0)$

$$\boxed{F^P_{0,T}(x) = x(0)e^{-r_F \cdot T}}$$ ... DC-denominated

$$\Rightarrow \quad F_{0,T}(x) = FV_{0,T}(F^P_{0,T}(x)) = e^{r_D \cdot T} \cdot x(0)e^{-r_F \cdot T}$$

$$\boxed{F_{0,T}(x) = x(0)\, e^{(r_D - r_F)\cdot T}}$$

Recall: For continuous dividend paying stocks:

$$\boxed{\begin{aligned} F^P_{0,T}(S) &= S(0)e^{-\delta T} \\ F_{0,T}(S) &= S(0)e^{(r-\delta)T} \end{aligned}}$$

$\Rightarrow$ Analogy:

exchange rate $\longleftrightarrow$ price of cont·div·paying stock

$r_F \longleftrightarrow \delta$

$r_D \longleftrightarrow r$

European calls & puts

w/ strike price K and exercise date T

$\uparrow$ DC·denominated

Payoffs:

$$\boxed{\begin{aligned} V_C(T) &= (x(T)-K)_+ \\ V_P(T) &= (K - x(T))_+ \end{aligned}}$$

Goal: Formulate the put·call parity for currency options!

Portfolio A:
- long call
- write put

} both on the FC and otherwise identical

Portfolio B:
- long a prepaid forward on the FC w/ delivery date T
- borrow the $PV_{0,T}(K)$ @ the risk·free rate r_D to be repaid @ time·T

$$\boxed{\begin{aligned} V_A(T) &= x(T) - K \\ &\parallel \qquad\quad \parallel \\ V_B(T) &= x(T) - K \end{aligned}}$$

$\Rightarrow$ no arbitrage

$$\boxed{\begin{aligned} V_C(0) - V_P(0) &= F^P_{0,T}(x) - PV_{0,T}(K) \\ &= x(0)e^{-r_F \cdot T} - Ke^{-r_D \cdot T} \end{aligned}}$$

Put·Call Parity

$V_A(T) = V_C(T) - V_P(T) = (x(T)-K)_+ - (K - x(T))_+$

$$= \begin{cases} (x(T)-K) - \cancel{0} & \text{if } x(T) \geq K \\ \cancel{0} - (K - x(T)) & \text{if } x(T) < K \end{cases}$$

$$= \begin{cases} \boxed{\begin{matrix} x(T)-K \\ x(T)-K \end{matrix}} & \begin{matrix} \text{if } x(T) \geq K \\ \text{if } x(T) < K \end{matrix} \end{cases}$$

$$= x(T) - K$$

stock options

(vanilla calls/puts)

shares $\downarrow$ $\uparrow$ cash

$\Rightarrow$ Call: $V_c(T) = (S(T) - K)_+$

Put: $V_p(T) = (K - S(T))_+$

currency options

FC $\downarrow$ $\uparrow$ DC

$\Rightarrow$ Call: $V_c(T) = (x(T) - K)_+$

Put: $V_p(T) = (K - x(T))_+$

Introduce a "new" option type:

$$\boxed{\text{stock \#1} \downarrow \quad \uparrow \text{stock \#2}}$$