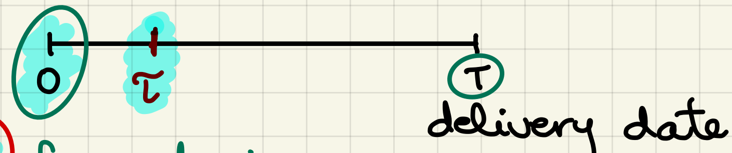
M339W: January 29th, 2021.

Review: Forward contracts



F... forward price

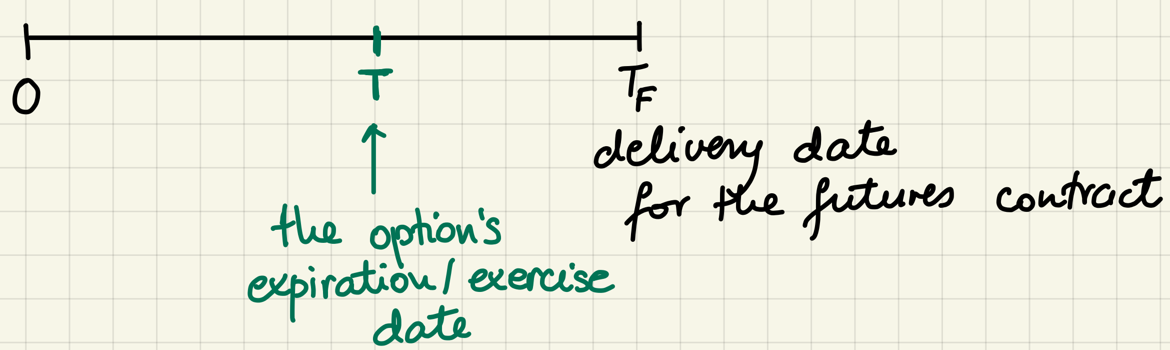
IRL:
- Student @ UT; graduating in May; owns a car
  A friend graduates a year later; wants a car

  → They agree on a price to be paid in May for the car. Binding!
- Ordering Pizza

Futures Contracts.

- tradable versions of forward contracts
- liquid counterparts to forward contracts w/ obseweable prices.

⇒ we can write/buy options on futures contracts as the underlying.



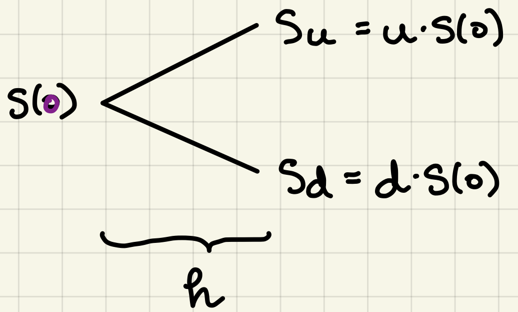
Analogy: Continuous-dividend stocks $\longleftrightarrow$ Futures Contracts

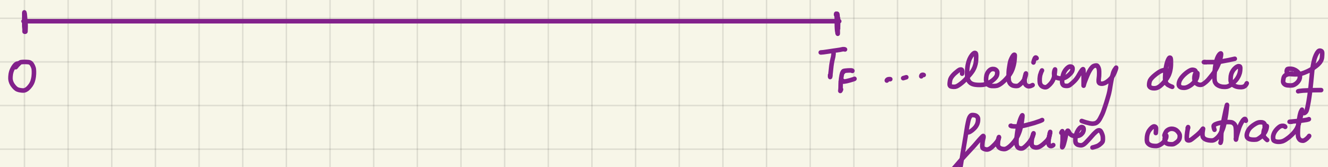
δ ... dividend yield $\longleftrightarrow$ $\cancel{\delta}$ r

Binomial Option Pricing for Futures Options.

Temporarily: Focus on futures on a market index w/ a dividend yield δ

Stock-Price Tree





In our case: since the interest rate is constant,

$\boxed{\text{futures prices} = \text{forward prices.}}$

Also: the forward prices can be calculated as:

$$F_{t,T_F}(S) = S(t) \cdot e^{(r-\delta)(T_F - t)}$$

$\Rightarrow$ If I want to construct a futures/forward price tree:

@ the root node:

$$F_0 := F_{0,T_F}(S) = S(0)\, e^{(r-\delta) \cdot T_F}$$

@ the up node:

$$F_u := S_u e^{(r-\delta)(T_F-h)}$$

$$= u \cdot \underbrace{S(0)\, e^{(r-\delta)T_F}}_{F_0} \cdot e^{-(r-\delta)\cdot h}$$

$$= F_0 \cdot \underbrace{u e^{-(r-\delta)\cdot h}}_{=:u_F}$$

$=: u_F$ ... the up factor for the futures tree

@ the down node:

$$F_d = S_d \cdot e^{(r-\delta)\cdot(T_F-h)}$$

$$= d \cdot \underbrace{S(0)\, e^{(r-\delta)\cdot T_F}}_{F_0} \cdot e^{-(r-\delta)\cdot h}$$

$$= F_0 \cdot \underbrace{d \cdot e^{-(r-\delta)h}}_{=:d_F}$$

$=: d_F$ ... the down factor for the futures tree

The futures price tree

$$\begin{tikzcd}[column sep=large]
 & F_u = F_0 \cdot u_F \\
F_0 \arrow[ur, no head] \arrow[dr, no head] & \\
 & F_d = F_0 \cdot d_F
\end{tikzcd}$$

$$\underbrace{\hspace{3cm}}_{h}$$

We can generalize this model structure to any underlying asset of the futures contract.

The risk·neutral probability

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{\cancel{e^{(r-\delta)h}}^{\,1} - d_F \cdot \cancel{e^{(r-\delta)h}}}{u_F \cdot \cancel{e^{(r-\delta)h}} - d_F \cdot \cancel{e^{(r-\delta)h}}}$$

$$\boxed{p^* = \frac{1-d_F}{u_F - d_F}}$$

Read: Investopedia on Real Options.

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i) The risk-free interest rate is constant.

(ii)

	Several months ago	Now
Stock price	\$40.00	\$50.00
Call option price	\$ 8.88	\$14.42
Put option price	\$ 1.63	\$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.