M378K Introduction to Mathematical Statistics

Homework assignment #6

Please, provide your **final answer only** to the following problems.

Problem 6.1. *(5 points) Let $Z_1 \sim N(1,1)$, $Z_2 \sim N(2,2)$ and $Z_3 \sim N(3,3)$ be independent random variables. The distribution of the random variable $W = Z_1 + \frac{1}{2}Z_2 + \frac{1}{3}Z_3$ is ...*

(a) $N(5/3, 7/6)$

(b) $N(3,3)$

(c) $N(3, \sqrt{3})$

(d) $N(3, \sqrt{5/3})$

(e) *None of the above*

(Note: *In our notation $N(\mu, \sigma)$ means normal with mean μ and* standard deviation σ.)

Problem 6.2. *(5 points) Let $Y_1, \dots, Y_{100}$ be independent random variables with the Bernoulli $B(p)$ distribution, with $p = 0.2$ The best approximation to $\bar{Y} = \frac{1}{n}(Y_1 + \cdots + Y_n)$ (among the offered answers) is*

(a) $N(0,1)$

(b) $N(100, 20)$

(c) $N(0.2, 0.04)$

(d) $N(20, 4)$

(e) $N(20, 20)$

(Note: *In our notation $N(\mu, \sigma)$ means normal with mean μ and* standard deviation σ.)

Problem 6.3. *(5 points) Use the uniqueness of moment-generating functions to give the distribution of a random variable Y with moment-generating function $m_Y(t) = (0.7e^t + 0.3)^3$.*

(a) $Y \sim b(3, 0.7)$

(b) $Y \sim b(3, 0.3)$

(c) $Y \sim B(0.7)$

(d) $Y \sim P(0.7)$

(e) None of the above.

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 6.4. *(10 points) The moment generating function of a certain random variable Y is given to be equal to*

$$m_Y(t) = (1 - 2500t)^{-4}.$$

Calculate the standard deviation of the random variable Y.

Problem 6.5. *(10 points) Let Y be a geometric random variable with parameter p. What is its moment generating function m_Y? Do not forget to explicitly state the domain of m_Y!*

Problem 6.6. *(15 points) Let $Y \sim E(\tau)$. Find the moment generating function on Y not forgetting to explicitly state the domain. Using the moment generating function, recalculate the mean and the variance of the random variable Y.*