

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 16

The t -procedure.

Provide your complete solution for the following problems.

Problem 16.1. When examining data to determine if a t -procedure can be used, which of the following are useful?

- (a) Histogram
- (b) $Q - Q$ plot
- (c) Boxplot
- (d) All of the above.
- (e) None of the above.

Problem 16.2. A simple random sample of 20 third-grade children from a certain school district is selected. Each child is given a test to measure his/her reading ability. You are interested in calculating a 95% confidence interval for the population mean score based on this SRS. The sample mean score is 64 points and the sample standard deviation is 12 points. What is the margin of error associated with the 95% confidence interval?

Problem 16.3. The hypotheses $H_0 : \mu = 10$ versus $H_a : \mu \neq 10$ are examined using a sample of size $n = 18$. The one-sample t -statistic has the value of $t = -2.05$. Between what two values does the P-value of this test fall?

- (a) $0.01 < P\text{-value} < 0.02$,
- (b) $0.02 < P\text{-value} < 0.025$,
- (c) $0.025 < P\text{-value} < 0.05$,
- (d) $0.05 < P\text{-value} < 0.10$
- (e) None of the above.

Problem 16.4. The time (in seconds) needed to clear a certain *Super Mario* run is normally distributed with an unknown mean μ . The local *Bowser basher* league reports the mean clearance time of at most 75 seconds. You are incredulous of their statements and decide to test your hypothesis.

- ✓ i. What are your null and alternative hypotheses?
- ✓ ii. Note that the standard deviation of the clearance time is not specified. Which test statistic are you going to use to test your hypotheses?
- ✓ iii. Let the sample size be 20. What is the distribution of your test statistic?
- iv. You gather the data, calculate \bar{x} and s and evaluate the t -statistic. The value you obtain equals 1.68. Is your null hypotheses rejected at the significance level 0.05?

Problem 16.2. margin of error = critical value · standard error

- sample standard deviation : $s = 12$

$$\Rightarrow \text{standard error} = \frac{s}{\sqrt{n}} = \frac{12}{\sqrt{20}}$$

- t distribution w/ $\frac{n-1}{19}$ degrees of freedom

$t^*(df = 19) = 2.093$ (using the t-table)
@ the confidence level of 95%

$$\Rightarrow \text{margin of error} = 2.093 \cdot \frac{12}{\sqrt{20}} = 5.616$$

Just for fun : the 95% confidence interval is

$$64 \pm 5.616$$

Problem 16.3.

Method I: Tables

$$df = 18 - 1 = 17$$

The observed value of the t statistic is -2.05 .
 \Rightarrow The upper-tail probability is between
0.025 and 0.05

\Rightarrow Since this is a two-tailed test, the p-value
is DOUBLE the upper-tail probability, i.e.,
it's between 0.05 and 0.10.

Method I: R

$$pt(-2.05, df=17) * 2 = 0.05611492$$

Problem 16.4.

i. $H_0: \mu = 75 = \mu_0$ vs. $H_a: \mu > 75$

ii. The t-statistic:

Its observed value is calculated as

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

$$w/ \quad s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

iii. $t(df = n-1 = 20-1 = 19)$

iv. $t = 1.68$

significance level: $\alpha = 0.05$.

In R: $1 - pt(1.68, df=19) = 0.05466274$

is the p-value

\Rightarrow We fail to reject!