University of Texas at Austin, Department of Mathematics
M358K - Applied Statistics

The Mock In-term One

Problem 1.1. (5 points) The standard deviation of the zinc concentration in a certain river is given to be 0.45 grams per milliliter. You want to create a 90%-confidence interval for the mean zinc concentration. Your goal is to have the margin of error of at most 0.04. How large should your sample be?

a. 342
b. 343
c. 376
d. 388
e. None of the above.

Problem 1.2. (5 points) A medical researcher thinks that adding calcium to the diet will help reduce blood pressure. She believes that the effect is different for men and women. 20 men and 20 women are willing to participate in the study. The researcher chooses 10 of the men and 10 of the women at random. These chosen 20 men and women take a calcium pill every day. The other 20 men and women take a placebo. This is a ...

a.: stratified random sample design.
b.: simple random sample design.
c.: randomized block experimental design.
d.: completely randomized experimental design.
e.: None of the above is correct.

Problem 1.3. (5 points) To estimate a population mean, our resident statistician Martyn Rivera plans to pick two simple random samples, each of size 100, from the population. He also plans to calculate the confidence interval with level C for each sample. What is the probability that at least one of his confidence intervals will cover the population mean?

a.: C^2
b.: $1 - C^2$
c.: $2C$
d.: $1 - (1 - C)^2$
e.: None of the above

Problem 1.4. (5 points) Let $Z \sim N(0, 1)$. Given that $Z > 0$, find the probability that $Z < 2$.

(a) 0.4772
(b) 0.6800
(c) 0.9544
(d) 0.9772
(e) None of the above.

Problem 1.5. (5 points) Let the monthly profit of a local cupcakery be normally distributed with mean \$20,000 and standard deviation of \$4,000. What is the probability that the combined profit in the months of October and November exceeds \$36,000?

(a) 0.4052

(b) 0.7611

(c) 0.7642

(d) 0.8023

(e) None of the above.

Problem 1.6. (5 points) Your diamond scale's measurement have a normally distributed error with mean 0 and standard deviation of 0.001 carats. Your procedure is to weigh a single diamond using your scale n times, average out the results, and report the average as the mass of the diamond. How many times n do you have to weigh your diamond so that your reported mass is at most 0.001 from the actual mass with probability 99%?

(a) 6

(b) 7

(c) 8

(d) 9

(e) None of the above.

Problem 1.7. (5 points) Your friend Cyril works as a work-study for the statistics department. The chair of the department decides Cyril's weekly pay by spinning a spinner which is equally likely to land on red, yellow, or blue. If the spinner lands on yellow, Cyril gets \$0. If the spinner lands on red, Cyril gets \$400. If the spinner lands on blue, Cyril gets \$500. What is the probability that Cyril's average pay in the following three weeks is \$300?

(a) 0

(b) 1/27

(c) 1/9

(d) 2/9

(e) None of the above.

Problem 1.8. (5 points) Alice performs a $z-$test. The $z-$score she obtains is equal to -1.76. Which decision does she make?

(a) Reject the null hypothesis.

(b) Fail to reject the null hypothesis.

(c) Reject the alternative hypothesis.

(d) Not enough information is given to answer this question.

(e) None of the above.

Problem 1.9. (5 points) A manufacturer of scented candles claims that their luxury candles last at least 12 hours. You suspect that this might not be entirely true and you decide to test their claim. You model the candle burn times as normal with a known standard deviation of 2 hours (based on the last holiday season's study). You purchase and burn 16 candles recording the sample average of 11 hours and 45 minutes. What is your decision?

(a) Reject at the 1% significance level.

(b) Fail to reject at the 1% significance level; reject at the 5% significance level.

(c) Fail to reject at the 5% significance level; reject at the 10% significance level.

(d) Fail to reject at the 10% significance level.

(e) None of the above.

Problem 1.10. (5 points) *Organically Produced* claims that their supplements contain 65 mg of iron per capsule. To be able to continue to maintain their claim, they periodically test the contents of a batch of 100 randomly chosen capsules from their production line. They model the iron content as normally distributed with a known standard deviation of 5 mg. In the last test, the sample average was 64 mg. What is the $p-$value?

(a) 0.0228

(b) 0.0384

(c) 0.0418

(d) 0.0456

(e) None of the above.

Problem 1.11. (5 points) Suppose that we have a random sample of size 25 from a normal population with an unknown mean μ and a standard deviation of 4. Bertram Wooster was in charge of testing the hypotheses

$$H_0 : \mu = 10 \quad vs. \quad H_a : \mu > 10.$$

Before he went on vacation, he obtained the rejection region $[11.2, \infty)$. However, he forgot to tell anyone which significance level α he used. Calculate α.

(a) 0.0401

(b) 0.0495

(c) 0.05

(d) 0.0668

(e) None of the above.

Problem 1.12. (5 points) Suppose that we have a random sample of size 25 from a normal population with an unknown mean μ and a standard deviation of 4. Bertram Wooster was in charge of testing the hypotheses

$$H_0 : \mu = 10 \quad vs. \quad H_a : \mu > 10.$$

Before he went on vacation, he obtained the rejection region $[11.2, \infty)$. What is the power of the above test at the alternative mean $\mu_a = 11$?

(a) 0.4013

(b) 0.4503

(c) 0.5120

(d) 0.6368

(e) None of the above.