M358K: October 16th, 2020.

Hypothesis Testing: The Normal Case (cont'd).

The population model:

$$X \sim N(\text{mean} = \mu, \text{variance} = \sigma^2)$$

μ: unknown

σ^2: given, known

Hypotheses:

Null hypothesis: $\boxed{H_0 : \mu = \mu_0}$

Alternative hypothesis: $H_a : \begin{cases} \mu < \mu_0 \\ \mu \neq \mu_0 \\ \mu > \mu_0 \end{cases}$

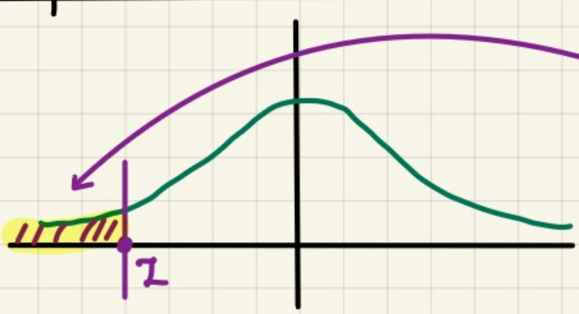
Under the null hypothesis, i.e., if $\mu = \mu_0$:

$$\boxed{\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)}$$

With the observed value of sample average being $\bar{x}$, we always calculate its z-score:

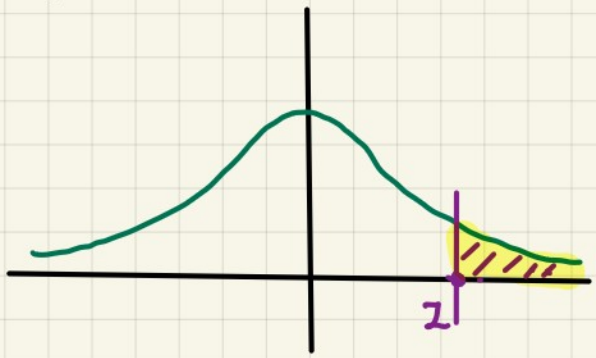
$$\boxed{z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}}$$

Left-sided alternative: $H_a : \mu < \mu_0$



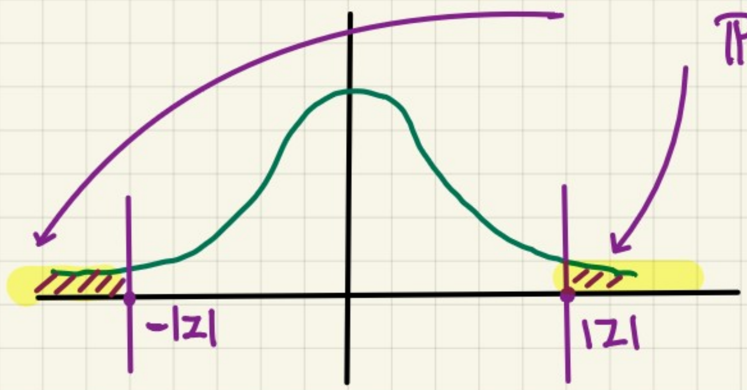
$\mathbb{P}[Z \leq z]$ = p-value

Right-sided alternative: $H_a: \mu > \mu_0$



$\mathbb{P}[Z \geq z]$ = p-value

Two-sided alternative: $H_a: \mu \neq \mu_0$



$\mathbb{P}[Z > |z|] + \mathbb{P}[Z < -|z|] =$

$= 2 \cdot \mathbb{P}[Z > |z|]$ = p-value

Set α... significance level

Typically: $\alpha = \boxed{0.05}$, 0.01, 0.10

Decision process:

If p-value $\leq \alpha$, we REJECT the null hypothesis.

If NOT, we FAIL TO REJECT the null hypothesis.

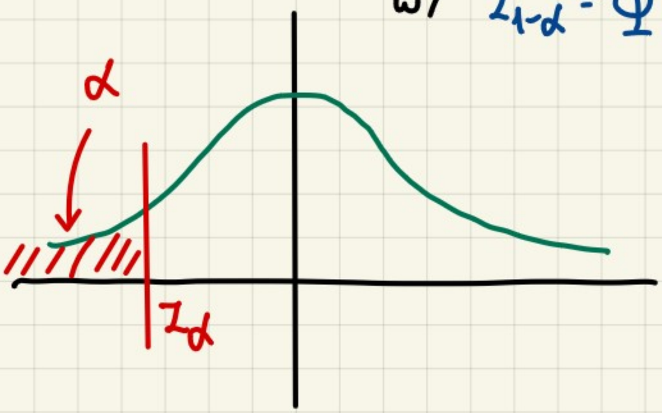
In other words:

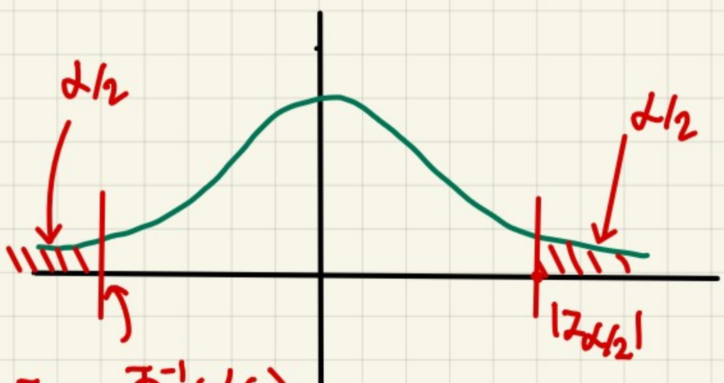
The p-value corresponding to an observed value of the test statistic is the lowest significance level @ which the null hypothesis would be rejected.

Given a significance level α, we can construct (ahead of data gathering) a REJECTION REGION (RR) for our test.

	STD UNITS	RAW UNITS
$\mu < \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq z_\alpha$ w/ $z_\alpha = \Phi^{-1}(\alpha)$	$\bar{x} \leq \underbrace{\mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}}$ i.e., $RR = (-\infty, \;\;]$
$\mu \neq \mu_0$		
$\mu > \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq z_{1-\alpha}$ w/ $z_{1-\alpha} = \Phi^{-1}(1-\alpha)$	$\bar{x} \geq \underbrace{\mu_0 + z_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}}}$ i.e. $RR = [\;\; , +\infty)$

H_a:





$z_{\alpha/2} = \Phi^{-1}(\alpha/2)$

$z \leq z_{\alpha/2}$ OR $z \geq |z_{\alpha/2}|$

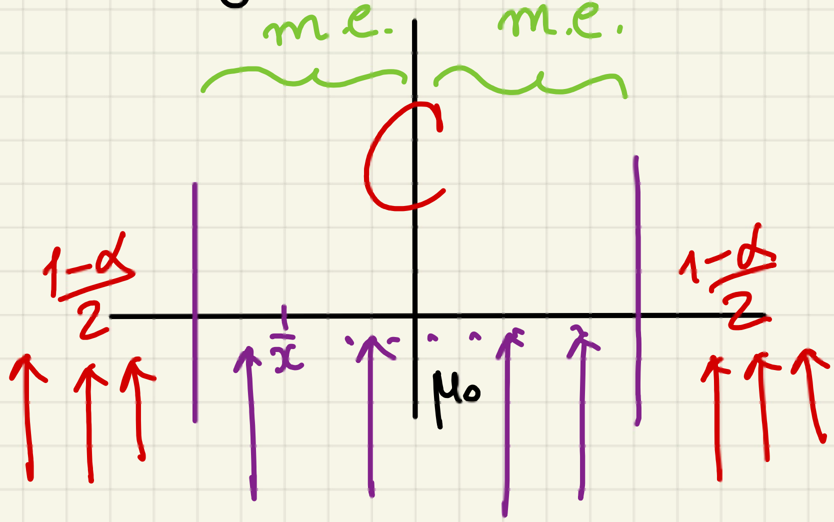
$\bar{x} < \mu + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

or

$\bar{x} > \mu - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$(-\infty, \;\;] \cup [\;\; , +\infty)$

Relationship between a C confidence interval & the two-sided hypothesis test w/ significance level $\alpha = 1 - C$



m.e. = margin of error

University of Texas at Austin

Problem Set # 7

Hypothesis testing.

Provide your **final answer only** for the following problems.

Problem 7.1. A test of significance can be used to test differences in categorical data. *True or false?*

Problem 7.2. The null hypothesis is a statement about the population parameter. *True or false?*

$H_0 : \mu = \mu_0$

Problem 7.3. The null and alternative hypotheses are stated in terms of the statistics obtained from the random sample. *True or false?*

functions of the sample.

parameter $\rightarrow$ $\boxed{\begin{array}{l} H_0 : \mu = \mu_0 \\ H_a : \mu < \mu_0 \text{ or } \mu > \mu_0 \text{ or } \mu \neq \mu_0 \end{array}}$

Problem 7.4. Confidence intervals and two-sided significance tests are linked in the sense that a two-sided test at a significance level α can be carried out in the form of a confidence interval with confidence level $1 - \alpha$. *True or false?*

Problem 7.5. In a test of statistical hypotheses, what does the $p-$value tell us?

a. If the null hypothesis is true.
b. If the alternative hypothesis is true.
c. The largest level of significance at which the null hypothesis can be rejected.
d. The smallest level of significance at which the null hypothesis can be rejected

---

Complete the following statements:

Problem 7.6. When we state the alternative hypothesis to look for a difference in a parameter in any direction, we are doing a ____________________-sided test.

Problem 7.7. When choosing between a one-sided alternative hypothesis and a two-sided alternative hypothesis, you should base the decision on ____________________.

Problem 7.8. When computing $p-$values, if the $p-$value is smaller than the chosen significance level α, we say that the results are ____________________.

Problem 7.9. The ____________________ the $p-$value, the **stronger** the evidence against the null hypothesis provided by the data.

Provide your **complete solution** for the following problems.

Problem 7.10. You perform 2000 significance tests using a significance level 0.10. Under the assumption that all of the null hypotheses for the 2000 significance tests are true, how many of the 2000 significance tests would you expect to be statistically significant?

a. 200
b. 1800
c. 2000
d. 0
e. None of the above.

Problem 7.11. The square footage of several thousand apartments in a new development is advertised to be 1250 square feet, on average. A tenant group thinks that the apartments are smaller than advertised. They hire an engineer to measure a sample of apartments to test their suspicions. Let μ represent the "true" mean area (in square feet) of these apartments. What are the appropriate null and alternative hypotheses?

Problem 7.12. Is the mean height for all adult American males between the ages of 18 and 21 now over 6 feet? Let μ denote the population mean height of all adult American males between the ages of 18 and 21. What are the appropriate null and alternative hypotheses?

Problem 7.13. The hypotheses are $H_0 : \mu = 10$ versus $H_a : \mu > 10$. The test statistic for a significance test for the population mean is $z = -2.12$. What is the corresponding $p-$value?

Problem 7.14. The test statistic for a **two-sided** significance test for a population mean is $z = -2.12$. What is the corresponding $p-$value?