University of Texas at Austin

Problem Set # 1

Conditional probability. Independence.

Problem 1.1. Let E and F be any two events. If

$$\mathbb{P}[E|F] > \mathbb{P}[E],$$

then

$$\mathbb{P}[F|E] > \mathbb{P}[F].$$

Problem 1.2. Let A and B be events such that $\mathbb{P}[A] = 1/2, \mathbb{P}[B] = 1/3$ and $\mathbb{P}[A \cap B] = 1/4$. Calculate the following probabilities:

(i) $\mathbb{P}[A \cup B]$
(ii) $\mathbb{P}[A|B]$
(iii) $\mathbb{P}[B|A]$
(iv) $\mathbb{P}[A^c|B^c]$

Problem 1.3. Two cards are drawn at random from an ordinary deck of 52 cards. Find the probability that both are spades.

Problem 1.4. If events E and F are independent, then they are necessarily mutually exclusive.

Problem 1.5. The four standard blood types are distributed in a populations as follows:

$$A - 42\%$$
$$O - 33\%$$
$$B - 18\%$$
$$AB - 7\%$$

Assuming that people choose their mates independently of their blood type, find the probability that a randomly chosen couple from this population has the same blood type.

Problem 1.6. *Source: Sample P exam problems.*
An actuary studying the insurance preferences of automobile owners makes the following conclusions:

(i) An automobile owner is twice as likely to purchase collision coverage as disability coverage.
(ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.
(iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15.

Calculate the probability that an automobile owner purchases neither collision nor disability coverage.