University of Texas at Austin

Quiz # 11

Prerequisite material.

Please, provide a **complete** solution to the following problem(s):

Problem 11.1. (5 points) Complete the following definition:
The two random variables X and Y are said to be **equal** if:

Solution: They are on the same probability space and

$$\mathbb{P}[X = Y] = 1,$$

i.e., X equals Y almost surely (with probability 1).

Problem 11.2. (5 points) Complete the following definition:
The two random variables X and Y are said to be **identically distributed** if:

Solution: They have the same cumulative distribution functions, i.e.,

$$F_X(x) = F_Y(x), \text{for all } x$$

with F_X being the cumulative distribution function of the random variable X and F_Y being the cumulative distribution function of the random variable Y.

Problem 11.3. (5 points) Provide an example of a pair of random variables which are identically distributed, but **not** equal.

Solution: Answers will vary. One example is:

- X is defined as:

$$X = \begin{cases} 1 & \text{if the result of a fair cointoss is heads} \\ 0 & \text{if the result of a fair cointoss is tails} \end{cases}$$

- Y is defined as:

$$Y = \begin{cases} 1 & \text{if the result of a roll of a fair die is a prime number} \\ 0 & \text{if the result of a roll of a fair die is not a prime number} \end{cases}$$