

UNIVERSITY OF TEXAS AT AUSTIN

Quiz #21

Binomial option pricing.

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Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

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**Problem 21.1.** (15 points) The current price of a continuous-dividend-paying stock is \$50 per share. Its dividend yield is 0.01 and its volatility is 0.2.

The continuously compounded, risk-free interest rate is 0.04.

An analyst models the movement of the stock price over the following quarter-year using a one-period binomial tree. He constructs the tree so that the risk-neutral probability equals  $1/2$  and so that the ratio of the higher possible time-1 stock price to the lower possible time-1 stock price equals  $e^{2\sigma\sqrt{h}}$  where  $\sigma$  stands for the stock's volatility and  $h$  stands for the length of the period.

What is the price of a three-month, at-the money put option on the above stock you obtain using the analyst's binomial tree?

**Solution:** We are given that the risk-neutral probability equals  $1/2$  and that the ratio of the up-factor to the down-factor in the tree equals  $e^{0.2}$ . So, by the definition of the risk-neutral probability

$$\begin{aligned}\frac{1}{2} &= \frac{e^{(0.04-0.01)(0.25)} - d}{de^{0.2} - d} \quad \Rightarrow \quad de^{0.3} - d = 2(e^{0.0075} - d) \quad \Rightarrow \quad d(e^{0.2} + 1) = 2e^{0.0075} \\ &\Rightarrow \quad d = 0.90711.\end{aligned}$$

The up factor is  $u = 1.10795$ . So, our option is in-the-money only at the down node and its payoff at that node equals  $50(1 - 0.90711) = 4.6445$ . The time-0 put price is

$$V_P(0) = e^{-0.04(0.25)} \times \frac{1}{2} \times 4.6445 = 2.29914.$$