UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 2

Realized returns. Futures options. Currency options. Subjective expectations.

Problem 2.1. (5 pts) For a stock price that was initially \$55.00, what is the price after 4 years if the continuously compounded returns for these 4 years are $4.5\%, 6.2\%, 8.9\%$, and -3.2%?

Problem 2.2. (5 pts) A non-dividend-paying stock is valued at \$55.00. The annual expected return is 12.0% and the standard deviation of annualized returns is 22.0%. If the stock is lognormally distributed, what is the expected stock price after 3 years?

Problem 2.3. (5 pts) For a stock price that was initially \$55.00, what is the price after 4 years if the observed continuously compounded returns for these 4 years are $4.5\%, 6.2\%, 8.9\%$, and 3.2%?

Problem 2.4. (10 points) Your goal is to price a call option on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is three months.
(ii) $u_F/d_F = 5/4$, where u_F is one plus the rate of gain on the futures price if it goes up, and d_F is one plus the rate of loss if it goes down.
(iii) The risk-neutral probability of an up move is $1/2$.
(iv) The initial futures price is 80.
(v) The continuously compounded risk-free interest rate is 5%.

Find the price of a half-year, 85-strike European call option on the futures contract.

Problem 2.5. (5 points) You are required to price a one-year, yen-denominated currency option on the USD. The exchange rate over the next year is modeled using a forward binomial tree with the number of periods equal to 4. Assume that the volatility of the exchange rate equals 0.1.

The continuously compounded risk-free interest rate for the yen equals 0.05, while the continuously compounded risk-free interest rate for the USD equals 0.02. What is the value of the so-called up factor u in the resulting forward binomial tree?

Problem 2.6. (5 points) The evolution of a market index over the following year is modeled using a four-period binomial tree. We are given that the current value of the market index equals \$144, that its volatility equals 0.25, and that it pays dividends continuously.

You are tasked with constructing a four-period forward tree for the evolution over the following year of the forward price of the above market index with delivery at time-2.

What is the **down** factor d_F in the **forward price tree** for the **futures prices** on the stock?

Problem 2.7. (10 points) The evolution over the following year of futures prices with delivery at time 2 on a certain commodity are modeled using a one-period forward binomial tree. The volatility is given by 0.2. The continuously compounded risk-free interest rate is given to be 0.05.

Let the current futures price equal \$50. What is the price of a one-year, \$45-strike European put on the futures contract described above?

Problem 2.8. (5 points) The current price of a continuous-dividend-paying stock is \$80 per share. The stock's dividend yield is 0.02. According to your model, the expected value of the stock price in two years is \$90 per share. You are also given:

The risk-free interest rate exceeds the dividend yield.

The two-year forward price on a share of this stock is denoted by F. At this price you are willing to enter into the forward. What is the smallest range of values F can take according to the above information?