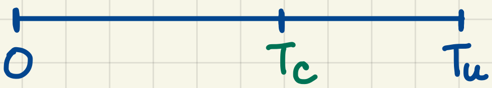
M339D: May 4th, 2022.

Compound Options.



$T_c \leq T_u$

The payoff of the compound option is paid.

$$v\Big(\underbrace{V^u(T_c)}_{\text{value of the underlying option}}\Big)$$

$\boxed{\text{payoff ftion of the compound option}}$

Focus on this family: $\underline{\qquad\qquad}$ on a $\underline{\qquad\qquad}$
call/put call/put

The table of payoffs:

K_c ... strike of the compound option

compound \ underlying	call	put
call	$(V_c^u(T_c)-K_c)_+$	$(V_p^u(T_c)-K_c)_+$
put	$(K_c-V_c^u(T_c))_+$	$(K_c-V_p^u(T_c))_+$

Put-call Parity for Compound Options.

$$\text{Call}\cdot\text{on}\cdot\underline{\text{Call}}(0) - \text{Put}\cdot\text{on}\cdot\underline{\text{Call}}(0) = \text{Call}(0) - K_c e^{-rT_c}$$

$$\text{Call}\cdot\text{on}\cdot\underline{\text{Put}}(0) - \text{Put}\cdot\text{on}\cdot\underline{\text{Put}}(0) = \text{Put}(0) - K_c e^{-rT_c}$$

Problem. Consider a non-dividend-paying stock w/ the current price of \$100. Assume $\boxed{r=0.05}$

There is an at-the-money European put option on the above stock w/ exercise in two years. Its current price is $\boxed{\$11.54}$

A compound call on this put issued. Its exercise date is in in one year and its strike is \$6. The price of this compound call is $\boxed{\$7.18}$

What is the price of the otherwise identical compound put option?

$\longrightarrow$:

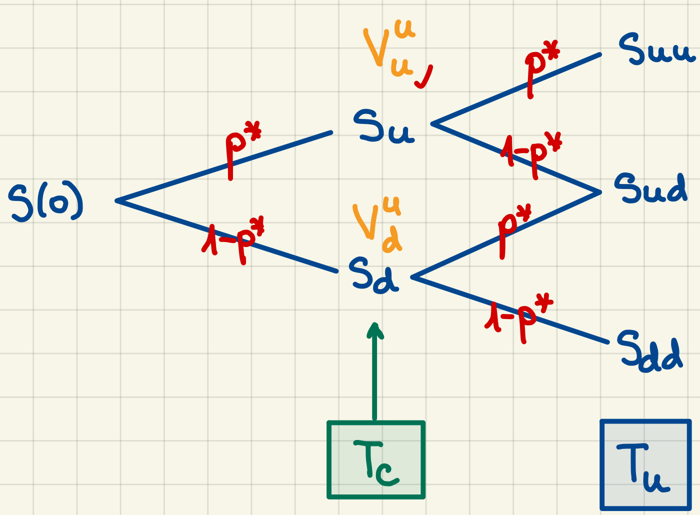
$$\underbrace{Call\cdot on\cdot Put(0)}_{7.18} - \underbrace{Put\cdot on\cdot Put(0)}_{?} = \underbrace{Put(0)}_{11.54} - K_c e^{-rT_c}$$

$$11.54 - 6e^{-0.05(1)}$$

$$\boxed{? = 1.35}$$

Binomial Pricing.

The simplest suitable binomial tree has two periods.



$$V_{uu}^u = (K - S_{uu})_+$$

$$V_{ud}^u = (K - S_{ud})_+$$

$$V_{dd}^u = (K - S_{dd})_+$$

Let's price a $\underbrace{\text{put}}_{\text{strike } K_c}$ on $\underbrace{\text{put}}_{\text{strike } K}$.

- $V_u^u = e^{-r(T_u - T_c)}\left(p^* \cdot V_{uu}^u + (1-p^*) \cdot V_{ud}^u\right)$

- $V_d^u = e^{-r(T_u - T_c)}\left(p^* \cdot V_{ud}^u + (1-p^*) \cdot V_{dd}^u\right)$

$\Rightarrow$ The two possible payoff values of the compound put are:

$$V_u = (K_c - V_u^u)_+$$
and
$$V_d = (K_c - V_d^u)_+$$

$\Rightarrow$ The price of the compound put is

$$\boxed{V(0) = e^{-rT_c}\left(p^* \cdot V_u + (1-p^*) \cdot V_d\right)}$$

This approach can be generalized to:

- T_c is at the end of the k^{th} period for some $k \leq n$;
- T_u is at the end of the tree.

Currency Options.

- Underlying asset ... FOREIGN CURRENCY (FC)

  r_F ... the ccrfir for FC

- DOMESTIC CURRENCY (DC) $\quad r_D$ ... the ccrfir for DC

Analogy: Foreign Currency $\longleftrightarrow$ Continuous-Dividend Stocks

$$r_F \longleftrightarrow \boxed{\delta}$$

One period: $x(\cdot)$ ... the exchange rate

$$x(0) \to x_u = u \cdot x(0), \qquad x(0) \to x_d = d \cdot x(0), \qquad h = T$$

PAYOFF $\qquad$ REPLICATING PORTFOLIO

$$V_u = v(x_u) = \Delta e^{r_F h} \cdot x_u + Be^{r_D \cdot h}$$

$$V_d = v(x_d) = \Delta e^{r_F h} \cdot x_d + Be^{r_D \cdot h}$$

- Δ ... the number of units of the FC bought @ time 0
- B ... the risk-free investment in the DC

$$V(0) = \Delta \cdot x(0) + B = e^{-r_D \cdot T}\left[p^* \cdot V_u + (1-p^*) \cdot V_d\right]$$

$$\text{w/} \quad p^* = \frac{e^{(r_D - r_F)\cdot h} - d}{u-d}$$

Forward Binomial Tree.

$$\begin{cases} u = e^{(r_D - r_F)\cdot h + \sigma\sqrt{h}} \\ d = e^{(r_D - r_F)\cdot h - \sigma\sqrt{h}} \end{cases}$$

$$\boxed{p^* = \frac{1}{1 + e^{\sigma\sqrt{h}}}}$$

4. For a two-period binomial model, you are given:

(i) Each period is one year.

(ii) The current price for a nondividend-paying stock is 20.

(iii) $u = 1.2840$, where u is one plus the rate of capital gain on the stock per period if the stock price goes up.

(iv) $d = 0.8607$, where d is one plus the rate of capital loss on the stock per period if the stock price goes down.

(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of an American call option on the stock with a strike price of 22.

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$T=\frac{3}{4}$ $\$ \ldots DC$ $£ \ldots FC$

5. Consider a 9-month dollar-denominated American put option on British pounds. You are given that:

(i) The current exchange rate is 1.43 US dollars per pound. $x(0)=1.43$

(ii) The strike price of the put is 1.56 US dollars per pound. $K=1.56$

(iii) The volatility of the exchange rate is $\sigma = 0.3$.

(iv) The US dollar continuously compounded risk-free interest rate is 8%. $r_D = 0.08$

(v) The British pound continuously compounded risk-free interest rate is 9%. $r_F = 0.09$

Using a three-period binomial model, calculate the price of the put.

(A) 0.23 — 1st p^*

(B) 0.25 — 2nd $u, d = ?$

(C) 0.27 — 3rd Populate the tree: put w/ $K=1.56$

(D) 0.29 — 4th Find $V_{uuu}, V_{uud}, V_{udd}, V_{ddd}$

(E) 0.31 — 5th Move backwards through the tree one step @ a time!