

*Note:* You **must** show all your work for the free-response problems. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

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**Problem 3.1.** (2 pts) Let  $X$  have a Pareto distribution with parameters  $\alpha$  and  $\theta$ . Then,  $\mathbb{E}[X^k]$  exists for every  $k = 1, 2, \dots$ . *True or false? Why?*

**Problem 3.2.** (2 pts) Let the random variable  $X$  represent the outcome of rolling a fair dodecahedron (i.e., a 12 sided “die”) with faces numbered 1 through 12. Then the expectation of  $X$  equals 6. *True or false? Why?*

**Problem 3.3.** (10 points) *Source: Sample P exam, Problem #192.*

Losses covered by a flood insurance policy are uniformly distributed on the interval  $[0, 2]$ . The insurer pays the amount of the loss in excess of a deductible  $d$ . The probability that the insurer pays at least 1.20 on a random loss is 0.30. Calculate the probability that the insurer pays at least 1.44 on a random loss.

**Problem 3.4.** (10 points) *Source: Sample P exam, Problem #206.*

An insurance company issues policies covering damage to automobiles. The amount of damage is modeled by a uniform distribution on  $[0, b]$ . The policy payout is subject to a deductible of  $b/10$ . A policyholder experiences automobile damage. Calculate the ratio of the standard deviation of the policy payout to the standard deviation of the amount of the damage.

**Problem 3.5.** (10 points) *Source: Sample P exam, Problem #209.*

A policy covers a gas furnace for one year. During that year, only one of three problems can occur:

- (i) The igniter switch may need to be replaced at a cost of 60. There is a 0.10 probability of this.
- (ii) The pilot light may need to be replaced at a cost of 200. There is a 0.05 probability of this.
- (iii) The furnace may need to be replaced at a cost of 3000. There is a 0.01 probability of this.

Calculate the deductible that would produce an expected claim payment of 30.

**Problem 3.6.** (10 points) *Source: Sample P exam, Problem #214.*

Losses due to accidents at an amusement park are exponentially distributed. An insurance company offers the park owner two different policies, with different premiums, to insure against losses due to accidents at the park. Policy A has a deductible of 1.44. For a random loss, the probability is 0.640 that under this policy, the insurer will pay some money to the park owner. Policy B has a deductible of  $d$ . For a random loss, the probability is 0.512 that under this policy, the insurer will pay some money to the park owner. Calculate  $d$ .

**Problem 3.7.** (6 points) For two random variables,  $X$  and  $Y$ , you are given that

$$e_Y(30) = e_X(30) + 4.$$

Let  $X$  have a uniform distribution on the interval from 0 to 100, and let  $Y$  have a uniform distribution on the interval from 0 to  $w$ .

Determine  $w$ .