# NAME:

M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
**Mock In-Term Exam I**
Instructor: Milica Čudina

**Notes**: This is a closed book and closed notes exam. The maximum number of points on this exam is 50.
**Time**: 50 minutes

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**Problem 1.1.** (5 points) The current price of a continuous-dividend-paying stock is \$80 per share. The stock's dividend yield is 0.02. According to your model, the expected value of the stock price in two years is \$90 per share. You are also given:

The risk-free interest rate exceeds the dividend yield.

The two-year forward price on a share of this stock is denoted by $F$. At this price you are willing to enter into the forward. What is the smallest range of values $F$ can take according to the above information?

(a) $F < 77$
(b) $77 < F < 80$
(c) $80 < F < 90$
(d) $F > 90$
(e) None of the above.

**Solution: (c)**
Using the fact that the investor is willing to enter a forward contract, we conclude that the forward contract's profit is positive. So,

$$\mathbb{E}[S(T)] > F \quad \Rightarrow \quad 90 > F$$

On the other hand, we know that

$$F = S(0)e^{(r-\delta)T} = 80e^{2(r-0.02)} > 80.$$

So, the most we can say about $F$ is that $80 < F < 90$.

**Problem 1.2.** Consider a non-dividend-paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously-compounded, risk-free interest rate is 0.06.

Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

(a) About \$10
(b) About \$10.83
(c) About \$15.45
(d) About \$20.84
(e) None of the above.

**Solution: (d)**
The risk-neutral probability of an up movement is
$$p^* = \frac{95e^{0.06} - 75}{120 - 75} = 0.575.$$
So, the price of our straddle is
$$V_C(0) = e^{-0.06}[0.575 \times (120 - 100) + (1 - 0.575) * (100 - 75)] = 20.8366.$$

**Problem 1.3.** The current exchange rate is given to be \$1.25 per Euro and its volatility is given to be 0.15.

The continuously-compounded, risk-free interest rate for the US dollar is 0.03, while the continuously-compounded, risk-free interest rate for the Euro equals 0.06.

The evolution of the exchange rate over the following nine-month period is modeled using a three-period forward binomial tree.

What is the value of the so-called down factor in the above tree?

(a) $d \approx 0.8586$
(b) $d \approx 0.8982$
(c) $d \approx 0.9208$
(d) $d \approx 0.9347$
(e) None of the above.

**Solution: (c)**
In the forward binomial tree, the up and down factors are given as
$$u = e^{(0.03-0.06)\times 0.25 + 0.15\times\sqrt{0.25}} = 1.0698$$
$$d = e^{(0.03-0.06)\times 0.25 - 0.15\times\sqrt{0.25}} = 0.9208.$$

**Problem 1.4.** The evolution of a market index over the following year is modeled using a four-period binomial tree. We are given that the current value of the market index equals \$144, that its volatility equals 0.25, and that it pays dividends continuously.

You are tasked with constructing a four-period forward tree for the evolution over the following year of the forward price of the above market index with delivery at time-2.

What is the down factor $d_F$ in the forward price tree for the futures prices on the stock?

(a) 0.7788
(b) 0.8825
(c) 0.9914
(d) There is not enough information given.
(e) None of the above.

**Solution: (b)**
In our usual notation,
$$d_F = de^{-(r-\delta)h} = e^{-\sigma\sqrt{h}} = e^{-0.25\sqrt{1/4}} = 0.8825.$$

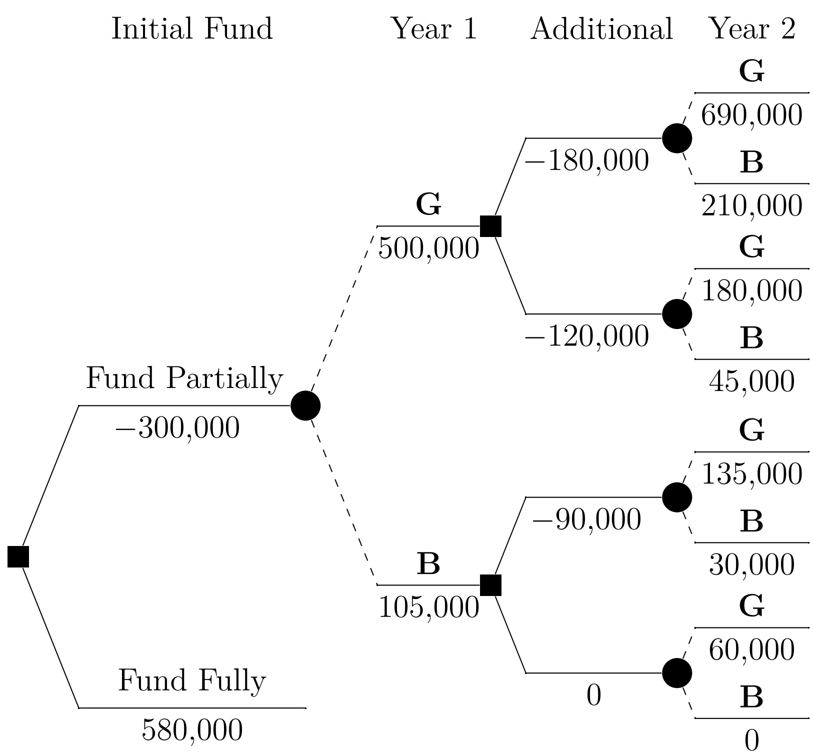
**Problem 1.5.** (5 points) Netflix is considering a cartoon series. When the production of two seasons is fully funded at time−0 the project has a net present value of 580,000.

The decision tree below shows the cash flows of the series when the promotion at the beginning of the Year 1 (i.e., at $t = 0$) is only partial with an option to provide different amounts of funding at the beginning of Year 2 (i.e., at $t = 1$) depending on how well the first season did.

This tree reflects two possible receptions of the two seasons at each information node (**G** = good, **B** = bad). The probability of the series being a success is given to be 1/2 and the probability of it being merely watchable is 1/2.

Assume the interest rate is 0%.

Find the **initial** (i.e., at $t = 0$) value of the option to fund partially.




(a) 15000

(b) 20000

(c) 25000

(d) 30000

(e) None of the above.

**Solution: (e)**

As usual, when pricing options, we are moving backwards through the tree.

- In the *uppermost final* information node, the possible cashflows are 690,000 with probability 1/2 and 210,000 with probability 1/2. So, the value of the project at that node equals

$$690000\left(\frac{1}{2}\right)+210000\left(\frac{1}{2}\right)=450000.$$

- In the *second-by-height final* information node, the possible cashflows are 180,000 with probability 1/2 and 45,000 with probability 1/2. So, the value of the project at that node equals

$$180000\left(\frac{1}{2}\right)+45000\left(\frac{1}{2}\right)=112500.$$

- In the *third-by-height final* information node, the possible cashflows are 135,000 with probability 1/2 and 30,000 with probability 1/2. So, the value of the project at that node equals

$$135000\left(\frac{1}{2}\right)+30000\left(\frac{1}{2}\right)=82500.$$

- In the *lowest final* information node, the possible cashflows are 60,000 with probability 1/2 and 0 with probability 1/2. So, the value of the project at that node equals

$$60000\left(\frac{1}{2}\right)=30000.$$

We continue working backwards, at the **upper decision** node at the end of Year 1, we can go ”up” or ”down” in the tree.

- We go ”up” by investing 180,000; combining this cashflow with the average revenue at the *uppermost final* node, we get the total effect of going ”up” to be

$$450000-180000=270000.$$

- We go ”down” by investing 120,000; combining this cashflow with the average revenue at the *second-by-height final* node, we get the total effect of going ”down” to be

$$112500-120000=-7500.$$

Comparing the two values we obtained, we conclude that the **optimal** decision at this node is to go ”up” and we keep the value of this project at this node to be

$$270000+500000=770000.$$

Here, we took into account that the first season was a success resulting in 500,000 in revenue in Year 1.

Similarly, at the **lower decision** node at the end of Year 1, we can go ”up” or ”down” in the tree.

- We go ”up” by investing 90,000; combining this cashflow with the average revenue at the *third-by-height final* node, we get the total effect of going ”up” to be

$$82500-90000=-7500.$$

- We go "down" by investing nothing; so, the total effect of going "down" is 30000. Comparing the two values we obtained, we conclude that the **optimal** decision at this node is to go "down" and we keep the value of this project at this node to be

  $$30000 + 105000 = 135000.$$

  Here, we took into account that the first season was "meh" resulting in 105,000 in revenue in Year 1.

  Altogether, at the information node corresponding to Year 1, we have that the expected value of the project is

  $$770000\left(\frac{1}{2}\right) + 135000\left(\frac{1}{2}\right) = 425500.$$

Now, we take into account that we funded the series partially with 300,000. So, the total expected present value of the cashflows we get should we decide to fund partially is

$$425500 - 300000 = 125500$$

The total value of the option is

$$125500 - 580000 = -454500.$$

**Problem 1.6.** *Source: Open Course Intro to Statistics.*
Cholesterol levels for women aged 20 to 34 follow an approximately normal distribution with mean 185 milligrams per deciliter (mg/dl). Women with cholesterol levels above 220 mg/dl are considered to have high cholesterol and about 18.5% of women fall into this category. What is the standard deviation of the distribution of cholesterol levels for women aged 20 to 34?

(a) 38.9
(b) 41.3
(c) 43.7
(d) 45.1
(e) None of the above.

**Solution: (a)**
Let $X$ be the random variable denoting the cholesterol level. Then,

$$X \sim N(mean = 185, variance = \sigma^2).$$

We are given that

$$\mathbb{P}[X > 220] = 0.185 \quad \Rightarrow \quad \mathbb{P}[X \leq 220] = 1 - 0.185 = 0.815.$$

So,

$$220 = 185 + \sigma z_*$$

where $z_*$ is the critical value such that $N(z_*) = 0.815$. The closest value in the standard normal tables is $z_* = 0.9$. Hence, our answers is

$$\sigma = \frac{220 - 185}{0.9} = 38.8889$$

**Problem 1.7.** (5 points) A discrete-dividend-paying stock sells today for \$90 per share. The continuously compounded, risk-free interest rate is 0.05. The first dividend will be paid at in three months in the amount of \$2.50. The remaining dividends will be equal to \$2 and continue to be paid out quarterly for three more years. What is the **prepaid forward price** of this stock for delivery in eight months?

(a) \$73.02
(b) \$85.58
(c) \$90
(d) \$99.33
(e) None of the above.

**Solution:** The correct answer is **(b)**.

$$F^P_{0,7/12}(S) = 90 - 2.50e^{-0.0125} - 2e^{-0.025} = 85.58044.$$

**Problem 1.8.** (5 points) *Source: Sample FM(DM) Problem #41.*
The current price of a non-dividend-paying stock is \$100. The **effective** risk-free interest rate equals 0.01. Which of the following portfolios has the highest initial cost?

(a) Long a six-month, \$100-strike European put and short a six-month, \$100-strike European call.
(b) Long a forward contract for the delivery of the above stock in six months.
(c) Long a six-month, \$101-strike European put and short a six-month, \$101-strike European call.
(d) Short a forward contract for the delivery of the above stock in six months.
(e) Long a six-month, \$105-strike European put and short a six-month, \$105-strike European call.

**Solution: (e)**

**Problem 1.9.** The **writer** of a call option has ...

(a) an obligation to sell the underlying asset at the strike price.
(b) a right, but **not** an obligation, to sell the underlying asset at the strike price.
(c) an obligation to buy the underlying asset at the strike price.
(d) a right, but **not** an obligation, to buy the underlying asset at the strike price.
(e) None of the above.

**Solution: (a)**

**Problem 1.10.** Consider a one-period forward binomial model for the stock-price movement over the following year. The current stock price is $S(0) = 100$, its dividend yield is 0.05 and its volatility is 0.3 The continuously compounded risk-free interest rate is given to be 0.05.

Consider American call options on this stock with the expiration date at the end of the period/year.

Which of the following is closest to the maximal (rounded to the nearest dollar) strike price $K$ for which there is early exercise?

(a) 76
(b) 80
(c) 100
(d) 135
(e) 180

**Solution: (a)**
In our usual notation, $u = e^{0.3} = 1.35$ and $d = 0.74$. The risk-neutral probability is

$$p^* = \frac{1}{1+e^{\sigma}} = 0.425.$$

Then, the continuation value at the root node is

$$V_C(0) = e^{-0.05}[0.425(135-K)_+ + 0.575(74-K)_+]$$

as a function of $K$. The early-exercise condition is

$$100 - K > V_C(0).$$

It is evident that in order for early exercise to occur it must be that $K < 100$. So, let us focus on the possible solutions to the above inequality in the interval $(74, 100)$ first. For such $K$, the above inequality becomes

$$100 - K > e^{-0.05} \times 0.425(135-K).$$

Note the absence of the "positive part"in the last expression. The $K$ which satisfy this inequality are such that

$$100 - 54.57 > 0.596K \quad \Rightarrow \quad 76 > K.$$

**Problem 1.11.** In the setting of the one-period binomial model, denote by $i$ the **effective** interest rate **per period**. Let $u$ denote the "up factor" and let $d$ denote the "down factor" in the stock-price model. Which of the following statements is the correct no-arbitrage condition for the binomial asset-pricing model?

(a) $d < 1 + i < u$
(b) $d < 1 < u$
(c) $d < e^i < u$
(d) $d = \frac{i}{1+i}$
(e) None of the above.

**Solution: (a)**

**Problem 1.12.** (5 points) You roll a fair tetrahedron whose sides are labeled by $1, 2, 3$, and $4$ a total of 4000 times. What is the approximate probability that you see a 1 strictly more than 1025 times? There is no need to use the continuity correction.

(a) 0.0446

(b) 0.1287

(c) 0.1456

(d) 0.1814

(e) None of the above.

**Solution: (d)**
The number of heads is $X \sim Binomial(n = 4000, p = 0.25)$. Evidently, we can use the normal approximation to the binomial. We have

$$\mu_X = \mathbb{E}[X] = 1000 \quad \text{and} \quad \sigma_X = 27.38613.$$

The probability we are seeking is

$$\mathbb{P}[X > 1025] \approx 1 - N\left(\frac{1025 - 1000}{27.38613}\right) \approx 1 - N(0.91) = 1 - 0.8186 = 0.1814.$$