UNIVERSITY OF TEXAS AT AUSTIN

Homework assignment 2
The covariance formula.

Please, provide your **complete solutions** to the following problems:

Problem 2.1. (5 points) The random vector (X_1, X_2) is jointly normal. Its marginal distributions are:
$$X_1 \sim N(\text{mean} = 0, \text{variance} = 4), \quad X_2 \sim N(\text{mean} = 1, \text{variance} = 1).$$
The correlation coefficient is given to be
$$corr[X_1, X_2] = -0.2.$$
What is the variance of the random variable $X = 3X_1 - 2X_2$?

Problem 2.2. (5 points) The random vector (X_1, X_2) is jointly normal. Its marginal distributions are:
$$X_1 \sim N(\text{mean} = 1, \text{sd} = 1), \quad X_2 \sim N(\text{mean} = 2, \text{sd} = 2).$$
The correlation coefficient is given to be
$$corr[X_1, X_2] = 0.2.$$
What is the standard deviation of the random variable $X = X_1 - 2X_2$?

Problem 2.3. (10 points) An actuary is analyzing total claims for the company. The number of claims of Type #1 has expectation 100 and variance 100. Each such claim has the exact amount of 40. The number of claims of Type #2 has expectation 100 and variance 144. Each such claim has the exact amount of 10. The correlation between the number of claims of Type #1 and those of Type #2 is 0.25. What is the variance of the total **amount** of all the claims?

Problem 2.4. (15 points) The annual profits that Company A and Company B earn follow a bivariate normal distribution. Company A's annual profit has mean 2000 and standard deviation 1000. Company B's annual profit has mean 1000 and standard deviation 500. The correlation coefficient between these annual profits is 0.80. Calculate the probability that Company B's annual profit is less than Company A's annual profit.

Problem 2.5. (15 points) The random vector (X_1, X_2, X_3) is jointly normal. Its marginal distributions are:
$$X_1 \sim N(\text{mean} = 0, \text{variance} = 4), X_2 \sim N(\text{mean} = 1, \text{variance} = 1), X_3 \sim N(\text{mean} = -1, \text{variance} = 9).$$
The correlation coefficients are given to be
$$corr[X_1, X_2] = 0.3, corr[X_2, X_3] = 0.4, corr[X_1, X_3] = -0.3.$$
What is the distribution of the random variable $X = X_1 - X_2 + 2X_3$? Please, provide the **name** of the distribution, as well as the **values** of its parameters.