04/27/2018.

11) You are given the following information about a portfolio that has two equally-weighted stocks, P and Q.

(i) The economy over the next year could be good or bad with equal probability.

(ii) The returns of the stocks can vary as shown in the table below:

Stock	Return when economy is good	Return when economy is bad
P	10%	–2%
Q	18%	–5%

Calculate the volatility of the portfolio return.

(A) 1.80%

(B) 6.90%

(C) 7.66%

(D) 8.75%

(E) 13.42%

R_P... P stock's return

R_Q... Q stock's return

$$\mathbb{E}[R_P] = 0.10 \cdot 0.5 + (-0.02) \cdot 0.5 = 0.04$$

$$\mathbb{E}[R_Q] = 0.18 \cdot 0.5 + (-0.05) \cdot 0.5 = 0.065$$

$$Var[R_P] = \underbrace{\mathbb{E}[R_P^2]}_{} - (\mathbb{E}[R_P])^2$$

$$= 0.5 \cdot (0.1)^2 + 0.5(-0.02)^2 = 0.0052$$

$$\Rightarrow Var[R_P] = 0.0052 - (0.04)^2 = 0.0036$$

$$Var[R_Q] = \mathbb{E}[R_Q^2] - (\mathbb{E}[R_Q])^2 = \dots = 0.013225$$

$$Cov[R_P, R_Q] = \mathbb{E}[R_P \cdot R_Q] - \mathbb{E}[R_P] \cdot \mathbb{E}[R_Q]$$

$$= 0.10 \cdot 0.18 \cdot 0.5 + (+0.02)(+0.05) \cdot 0.5$$

$$- 0.04 \cdot 0.065 = 0.0069$$

1.

$$Var\left[\frac{1}{2}(R_P) + \frac{1}{2}R_Q\right] = \frac{1}{4}\left[Var[R_P + R_Q]\right]$$

$$= \frac{1}{4}\left(Var[R_P] + 2Cov[R_P, R_Q] + Var[R_Q]\right)$$

$$= \frac{1}{4}\left(0.0036 + 2 \cdot 0.0069 + 0.013225\right)$$

$$= 0.00765$$

$$\Rightarrow SD[\text{Portfolio's Return}] = \sqrt{0.00765} = 8.75\% \quad \blacksquare$$

Diversification w/ Equally Weighted Portfolios.

x_i ... weight of the i^{th} investment ; $i = 1..n$

$x_i = \frac{1}{n}$

Then, $\underbrace{R_P}_{\text{Return of portfolio}} = \frac{1}{n}(R_1 + \cdots + R_n)$

$$\Rightarrow Var[R_P] = Var\left[\frac{1}{n}(R_1 + \cdots + R_n)\right]$$

$$= \frac{1}{n^2} \cdot \underline{Var[R_1 + \cdots + R_n]}$$

\# of terms in this sum is $n(n-1)$

$$= \frac{1}{n^2} \cdot \sum_{i=1}^{n} Var[R_i] + \frac{1}{n^2} \cdot \overbrace{\sum_{i \neq j} Cov[R_i, R_j]}$$

$$= \frac{1}{n} \cdot \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n} Var[R_i]\right)} + \underbrace{\frac{1}{n^2} \cdot n(n-1)} \cdot \underbrace{\left(\frac{1}{n(n-1)} \sum_{i \neq j} Cov[R_i, R_j]\right)}$$

$$= \frac{1}{n}\left(\text{Average variance of individual components}\right) + \left(1 - \frac{1}{n}\right)\left(\text{Average covariance between stocks}\right)$$

9) You are given the following information about an equally-weighted portfolio of n stocks:

(i) The average variance of stocks in the portfolio is 0.20.

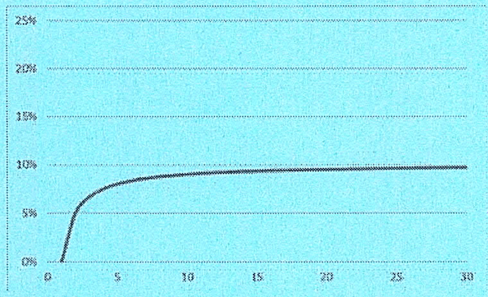
(ii) The average covariance of stocks in the portfolio is 0.10.

(iii) The average variance of stocks in the entire market is 0.15.

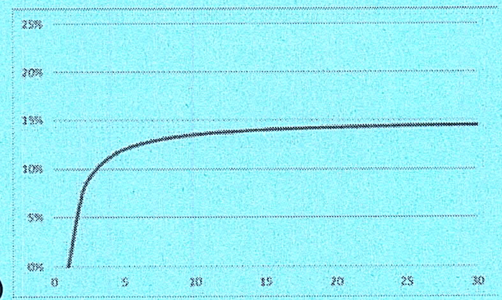
(iv) The average covariance of stocks in the entire market is 0.05.

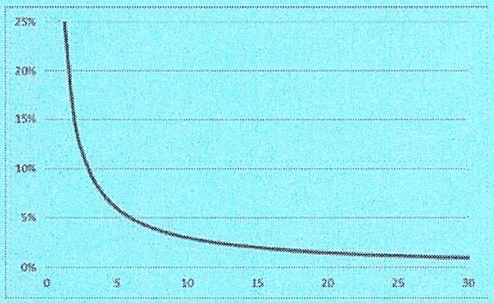
Determine which graph displays the variance of the portfolio as a function of n.

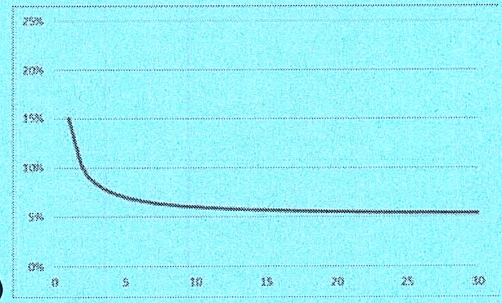
X

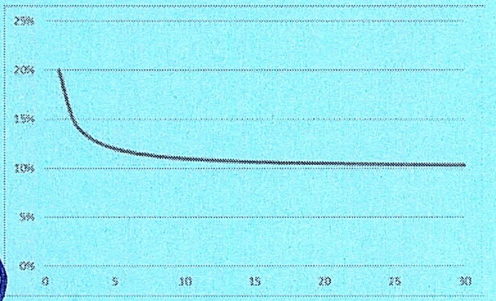
(A)


X

(B)


(C)


(D)


(E)


At $n=1 \Rightarrow$ variance $=0.2$

3.

Example. Volatility when risks are independent.

Independence $\Rightarrow$ <u>uncorrelation</u>

$$\Rightarrow \quad Var[R_P] = \frac{1}{n}(\text{Avg Variance})$$

$$\Rightarrow \quad SD[R_P] = \frac{\sqrt{\text{Avg Variance}}}{\sqrt{n}}$$

Q: What if the risks have identical volatilities?

$$SD[R_P] = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n} Var[R_i]}}{\sqrt{n}} = \frac{\sqrt{n \cdot \sigma_i^2}}{n} = \frac{\sigma_i}{\sqrt{n}} \xrightarrow[n\to\infty]{} 0$$

Diversification w/ a General Portfolio

Recall: $$Var[R_P] = \sum_{i=1}^{n} x_i \cdot Cov[R_i, R_P]$$

$$= \sum_{i=1}^{n} x_i \cdot SD[R_i] \cdot SD[R_P] \cdot corr[R_i, R_P]$$

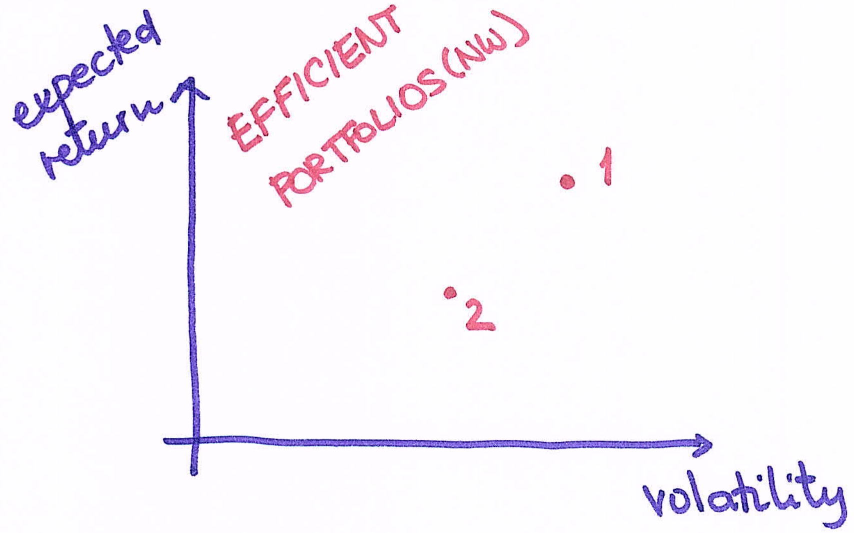
$$\Rightarrow \quad SD[R_P] = \sum_{i=1}^{n} x_i \cdot SD[R_i] \cdot \underbrace{corr[R_i, R_P]}_{\leq 1}$$

$$SD[R_P] \leq \sum_{i=1}^{n} x_i \cdot SD[R_i]$$

Equality only if all investments in the portfolio are perfectly correlated w/ the portfolio (and so w/ one another).

11.4. Risk vs. Return: Choosing an Efficient Portfolio

<u>Efficient Portfolio</u>: Contains only systematic risk.



Every portfolio built out of two stocks "1" and "2" is completely determined by the weight given to asset "1": w_1

→ See the Mathematica Demonstration!