M378K: November 22nd, 2024.

Hypothesis Testing [cont'd].

The normal case w/ σ known.

Population model: $Y \sim N(\boxed{mean = \mu}, sd = \sigma)$

unknown and of interest

Hypothesis Testing Procedure.

$\boxed{\text{First:}}$ Set the hypotheses.

2nd Null Hypothesis: $\boxed{H_0: \mu = \mu_0}$

1st Alternative Hypothesis:

$$H_a: \begin{cases} \mu < \mu_0 & \text{(lower or left side)} \\ \mu \neq \mu_0 & \text{(two sided)} \\ \mu > \mu_0 & \text{(upper or right side)} \end{cases}$$

$\boxed{\text{Second:}}$ Figure out the appropriate TEST STATISTIC (TS)

Natural choice:

$$\bar{Y} \sim Normal\left(mean = \mu, sd = \frac{\sigma}{\sqrt{n}}\right)$$

Under the null hypothesis, i.e., $\mu = \mu_0$

$$\boxed{\frac{\bar{Y} - \mu_0}{\frac{\sigma}{\sqrt{n}}}} \sim N(0,1)$$

$\boxed{\text{Third:}}$ Consider the observed value of the test statistic. In this case, it's $\bar{y}$, the observed sample average.

Q: What is the probability of observing $\bar{x}$ or something $\boxed{\text{more extreme}}$ under the null?

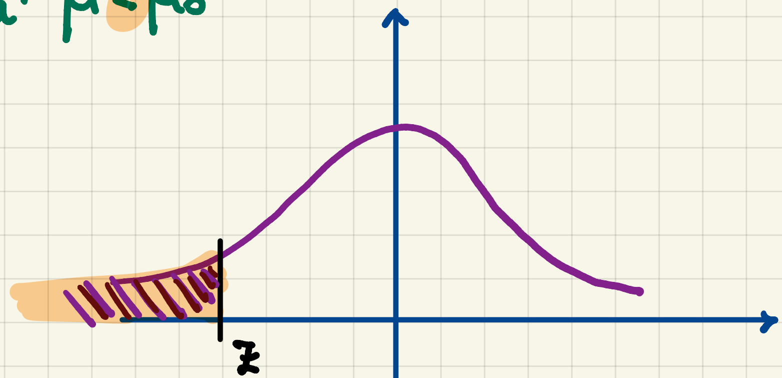
Exact interpretation depends on the structure of the alternative hypothesis.

Regardless:

$$\boxed{z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}}$$

Left-Sided Alternative.

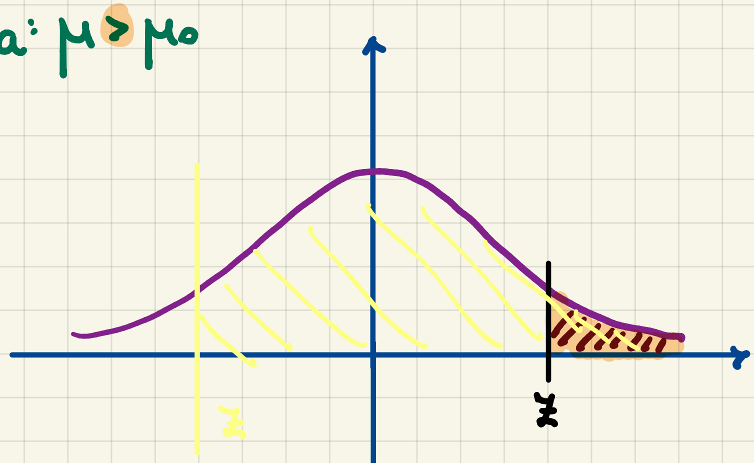
$H_a: \mu < \mu_0$



$$\boxed{\mathbb{P}[Z \leq z] = p\text{-value}}$$

Right-Sided Alternative.

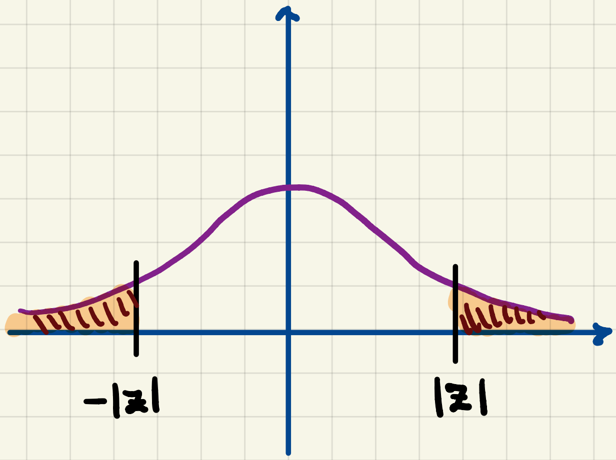
$H_a: \mu > \mu_0$



$$\boxed{\mathbb{P}[Z \geq z] = p\text{-value}}$$

Two-Sided Alternative.

$H_a: \mu \neq \mu_0$



$$\boxed{\mathbb{P}[Z \geq |z|] + \mathbb{P}[Z \leq -|z|] = 2 \cdot \mathbb{P}[Z \leq -|z|] = \text{p-value}}$$

Test of Significance.

Set α ... significance level

Typically: $\alpha = \boxed{0.05}, 0.01, 0.10$

Decision Process.

If p-value $\leq \alpha$, we REJECT the null hypothesis.

If p-value $> \alpha$, we FAIL TO REJECT the null hypothesis.

Note: The p-value corresponding to an observed value of the test statistic is the LOWEST significance level @ which the null hypothesis would still be REJECTED.

Given a significance level α, we construct (ahead of data gathering a REJECTION REGION (RR) for our test.

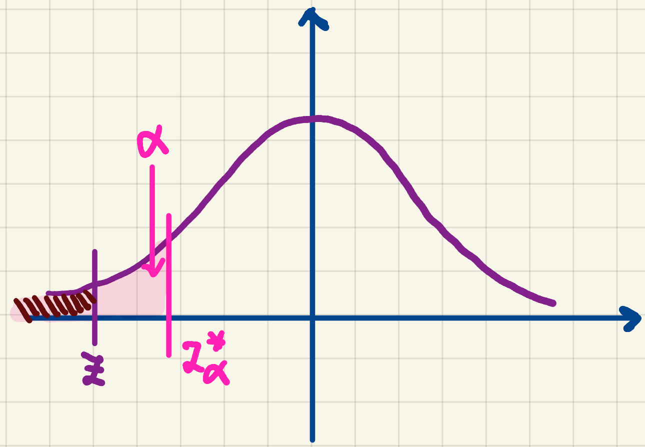
The Null Hypothesis: $H_0: \mu = \mu_0$

The Test statistic: $\bar{Y} \sim N(\text{mean} = \mu, \text{sd} = \frac{\sigma}{\sqrt{n}})$

The Left-Sided Alternative:

$H_a: \mu < \mu_0$

In standard units:



$$\boxed{z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}}$$

$$z^*_\alpha = \Phi^{-1}(\alpha) = \text{qnorm}(\alpha)$$

$$\boxed{RR = (-\infty, z^*_\alpha]}$$

In raw units:

$$z \leq z^*_\alpha$$

$$\frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \leq z^*_\alpha$$

$$\boxed{\bar{x} \leq \mu_0 + z^*_\alpha \cdot \frac{\sigma}{\sqrt{n}}}$$

upper bound of the RR in raw units

$$\boxed{RR = \left(-\infty, \mu_0 + z^*_\alpha \cdot \frac{\sigma}{\sqrt{n}}\right]}$$