M339G : August 28th, 2024.

Review.

Confidence Intervals.

The Normal Case.

We are in the normal model.

Let $X_1, X_2, \ldots, X_n$ be a normal random sample, i.e.,

$\{X_i, i = 1..n\}$ are all independent, and

$$X_i \sim Normal(mean = \mu, sd = \sigma)$$

?

We know exactly the sampling distribution of the sample mean:

$$\bar{X}_n \sim Normal\left(mean = \mu,\ sd = \frac{\sigma}{\sqrt{n}}\right)$$

We know that $\bar{X}_n$ is a "good" estimator for the population mean μ.

$$\mathbb{P}\left[\bar{X}_n - z^* \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{X}_n + z^* \cdot \frac{\sigma}{\sqrt{n}}\right] = C$$

$$z^* = \Phi^{-1}\left(\frac{1+C}{2}\right) = q((1+C)/2)$$