Classification Trees

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Here, I am adapting part of the lab associated with Chapter 8 of the textbook.

The `tree` library is used to construct classification and regression trees.

```
#install.packages("tree")
library(tree)
library(ISLR2)
```

Fitting Classification Trees

We first use classification trees to analyze the `Carseats` data set. In these data, `Sales` is a continuous variable, and so we begin by recoding it as a binary variable. We use the `ifelse()` function to create a variable, called `High`, which takes on a value of `Yes` if the `Sales` variable exceeds 8, and takes on a value of `No` otherwise.

```
library(ISLR2)
data(Carseats)
attach(Carseats)
High <- factor(ifelse(Sales <= 8, "No", "Yes"))
```

Finally, we use the `data.frame()` function to merge `High` with the rest of the `Carseats` data.

```
Carseats <- data.frame(Carseats, High)
```

We now use the `tree()` function to fit a classification tree in order to predict `High` using all variables but `Sales`. The syntax of the `tree()` function is quite similar to that of the `lm()` function.

```
tree.carseats <- tree(High ~ . - Sales, Carseats)
```

The `summary()` function lists the variables that are used as internal nodes in the tree, the number of terminal nodes, and the (training) error rate.

```
summary(tree.carseats)
```

```
##
## Classification tree:
## tree(formula = High ~ . - Sales, data = Carseats)
## Variables actually used in tree construction:
## [1] "ShelveLoc"   "Price"       "Income"      "CompPrice"   "Population"
## [6] "Advertising" "Age"         "US"
## Number of terminal nodes:  27
## Residual mean deviance:  0.4575 = 170.7 / 373
## Misclassification error rate: 0.09 = 36 / 400
```

We see that the training error rate is 9%. For classification trees, the deviance reported in the output of `summary()` is given by

$$-2\sum_{m}\sum_{k} n_{mk}\log\hat{p}_{mk},$$

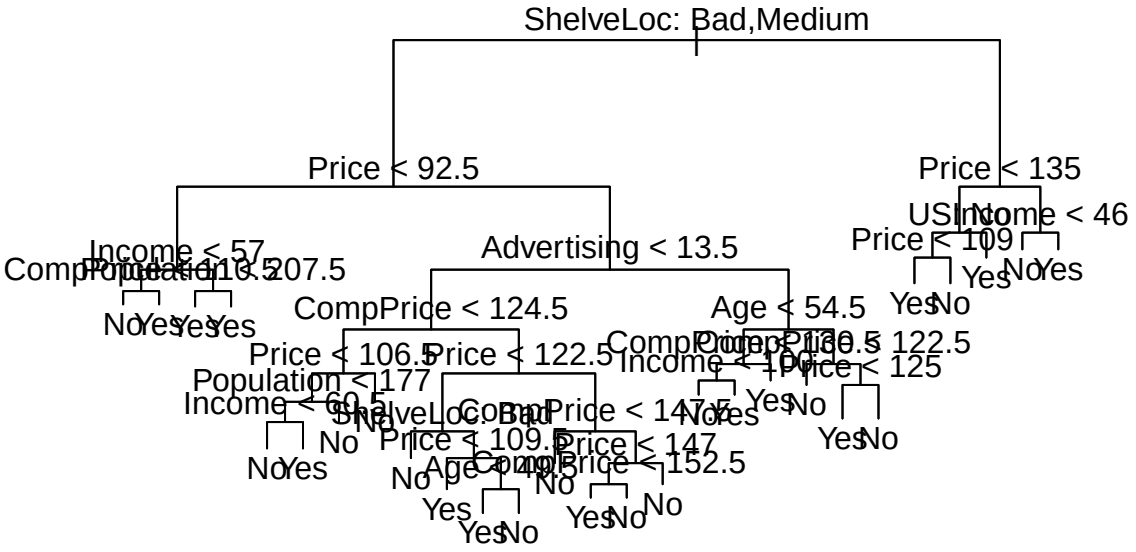
where n_{mk} is the number of observations in the mth terminal node that belong to the kth class. This is closely related to the entropy, defined as

$$-\sum_{k=1}^{K} \hat{p}_{mk} \ln(\hat{p}_{mk})$$

A small deviance indicates a tree that provides a good fit to the (training) data. The *residual mean deviance* reported is simply the deviance divided by $n - |T_0|$ where T_0 is the number of terminal nodes, which in this case is $400 - 27 = 373$.

One of the most attractive properties of trees is that they can be graphically displayed. We use the `plot()` function to display the tree structure, and the `text()` function to display the node labels. The argument `pretty = 0` instructs `R` to include the category names for any qualitative predictors, rather than simply displaying a letter for each category.

```
plot(tree.carseats)
text(tree.carseats, pretty = 0)
```



The most important indicator of `Sales` appears to be shelving location, since the first branch differentiates `Good` locations from `Bad` and `Medium` locations.

If we just type the name of the tree object, `R` prints output corresponding to each branch of the tree. `R` displays the split criterion (e.g. `Price < 92.5`), the number of observations in that branch, the deviance, the overall prediction for the branch (`Yes` or `No`), and the fraction of observations in that branch that take on values of `Yes` and `No`. Branches that lead to terminal nodes are indicated using asterisks.

```
tree.carseats
```

```
## node), split, n, deviance, yval, (yprob)
##       * denotes terminal node
##
##   1) root 400 541.500 No ( 0.59000 0.41000 )
##     2) ShelveLoc: Bad,Medium 315 390.600 No ( 0.68889 0.31111 )
##       4) Price < 92.5 46  56.530 Yes ( 0.30435 0.69565 )
##         8) Income < 57 10  12.220 No ( 0.70000 0.30000 )
##          16) CompPrice < 110.5 5   0.000 No ( 1.00000 0.00000 ) *
##          17) CompPrice > 110.5 5   6.730 Yes ( 0.40000 0.60000 ) *
##         9) Income > 57 36  35.470 Yes ( 0.19444 0.80556 )
##          18) Population < 207.5 16  21.170 Yes ( 0.37500 0.62500 ) *
```

```
##           19) Population > 207.5 20   7.941 Yes ( 0.05000 0.95000 ) *
##         5) Price > 92.5 269 299.800 No ( 0.75465 0.24535 )
##          10) Advertising < 13.5 224 213.200 No ( 0.81696 0.18304 )
##            20) CompPrice < 124.5 96  44.890 No ( 0.93750 0.06250 )
##              40) Price < 106.5 38  33.150 No ( 0.84211 0.15789 )
##                80) Population < 177 12  16.300 No ( 0.58333 0.41667 )
##                 160) Income < 60.5 6   0.000 No ( 1.00000 0.00000 ) *
##                 161) Income > 60.5 6   5.407 Yes ( 0.16667 0.83333 ) *
##                81) Population > 177 26   8.477 No ( 0.96154 0.03846 ) *
##              41) Price > 106.5 58   0.000 No ( 1.00000 0.00000 ) *
##            21) CompPrice > 124.5 128 150.200 No ( 0.72656 0.27344 )
##              42) Price < 122.5 51  70.680 Yes ( 0.49020 0.50980 )
##                84) ShelveLoc: Bad 11   6.702 No ( 0.90909 0.09091 ) *
##                85) ShelveLoc: Medium 40  52.930 Yes ( 0.37500 0.62500 )
##                 170) Price < 109.5 16   7.481 Yes ( 0.06250 0.93750 ) *
##                 171) Price > 109.5 24  32.600 No ( 0.58333 0.41667 )
##                   342) Age < 49.5 13  16.050 Yes ( 0.30769 0.69231 ) *
##                   343) Age > 49.5 11   6.702 No ( 0.90909 0.09091 ) *
##              43) Price > 122.5 77  55.540 No ( 0.88312 0.11688 )
##                86) CompPrice < 147.5 58  17.400 No ( 0.96552 0.03448 ) *
##                87) CompPrice > 147.5 19  25.010 No ( 0.63158 0.36842 )
##                 174) Price < 147 12  16.300 Yes ( 0.41667 0.58333 )
##                   348) CompPrice < 152.5 7   5.742 Yes ( 0.14286 0.85714 ) *
##                   349) CompPrice > 152.5 5   5.004 No ( 0.80000 0.20000 ) *
##                 175) Price > 147 7   0.000 No ( 1.00000 0.00000 ) *
##          11) Advertising > 13.5 45  61.830 Yes ( 0.44444 0.55556 )
##            22) Age < 54.5 25  25.020 Yes ( 0.20000 0.80000 )
##              44) CompPrice < 130.5 14  18.250 Yes ( 0.35714 0.64286 )
##                88) Income < 100 9  12.370 No ( 0.55556 0.44444 ) *
##                89) Income > 100 5   0.000 Yes ( 0.00000 1.00000 ) *
##              45) CompPrice > 130.5 11   0.000 Yes ( 0.00000 1.00000 ) *
##            23) Age > 54.5 20  22.490 No ( 0.75000 0.25000 )
##              46) CompPrice < 122.5 10   0.000 No ( 1.00000 0.00000 ) *
##              47) CompPrice > 122.5 10  13.860 No ( 0.50000 0.50000 )
##                94) Price < 125 5   0.000 Yes ( 0.00000 1.00000 ) *
##                95) Price > 125 5   0.000 No ( 1.00000 0.00000 ) *
##       3) ShelveLoc: Good 85  90.330 Yes ( 0.22353 0.77647 )
##         6) Price < 135 68  49.260 Yes ( 0.11765 0.88235 )
##          12) US: No 17  22.070 Yes ( 0.35294 0.64706 )
##            24) Price < 109 8   0.000 Yes ( 0.00000 1.00000 ) *
##            25) Price > 109 9  11.460 No ( 0.66667 0.33333 ) *
##          13) US: Yes 51  16.880 Yes ( 0.03922 0.96078 ) *
##         7) Price > 135 17  22.070 No ( 0.64706 0.35294 )
##          14) Income < 46 6   0.000 No ( 1.00000 0.00000 ) *
##          15) Income > 46 11  15.160 Yes ( 0.45455 0.54545 ) *
```

In order to properly evaluate the performance of a classification tree on these data, we must estimate the test error rather than simply computing the training error. We split the observations into a training set and a test set, build the tree using the training set, and evaluate its performance on the test data. The `predict()` function can be used for this purpose. In the case of a classification tree, the argument `type = "class"` instructs `R` to return the actual class prediction. This approach leads to correct predictions for 77% of the locations in the test data set.

```
set.seed(2)
train <- sample(1:nrow(Carseats), 200)
```

```
Carseats.test <- Carseats[-train, ]
High.test <- High[-train]
tree.carseats <- tree(High ~ . - Sales, Carseats,
    subset = train)
tree.pred <- predict(tree.carseats, Carseats.test,
    type = "class")
#tree.pred
(tab=table(tree.pred, High.test))
```

```
##          High.test
## tree.pred  No Yes
##       No  104  33
##       Yes  13  50
```

```
(sum(diag(tab))/sum(tab))
```

```
## [1] 0.77
```

(If you re-run the `predict()` function then you might get slightly different results, due to "ties": for instance, this can happen when the training observations corresponding to a terminal node are evenly split between `Yes` and `No` response values.)

Next, we consider whether pruning the tree might lead to improved results. The function `cv.tree()` performs cross-validation in order to determine the optimal level of tree complexity; cost complexity pruning is used in order to select a sequence of trees for consideration. We use the argument `FUN = prune.misclass` in order to indicate that we want the classification error rate to guide the cross-validation and pruning process, rather than the default for the `cv.tree()` function, which is deviance. The `cv.tree()` function reports the number of terminal nodes of each tree considered (`size`) as well as the corresponding error rate and the value of the cost-complexity parameter used. This is `k`, which corresponds to α in the similar condition for regression trees.

```
set.seed(7)
cv.carseats <- cv.tree(tree.carseats, FUN = prune.misclass)
names(cv.carseats)
```

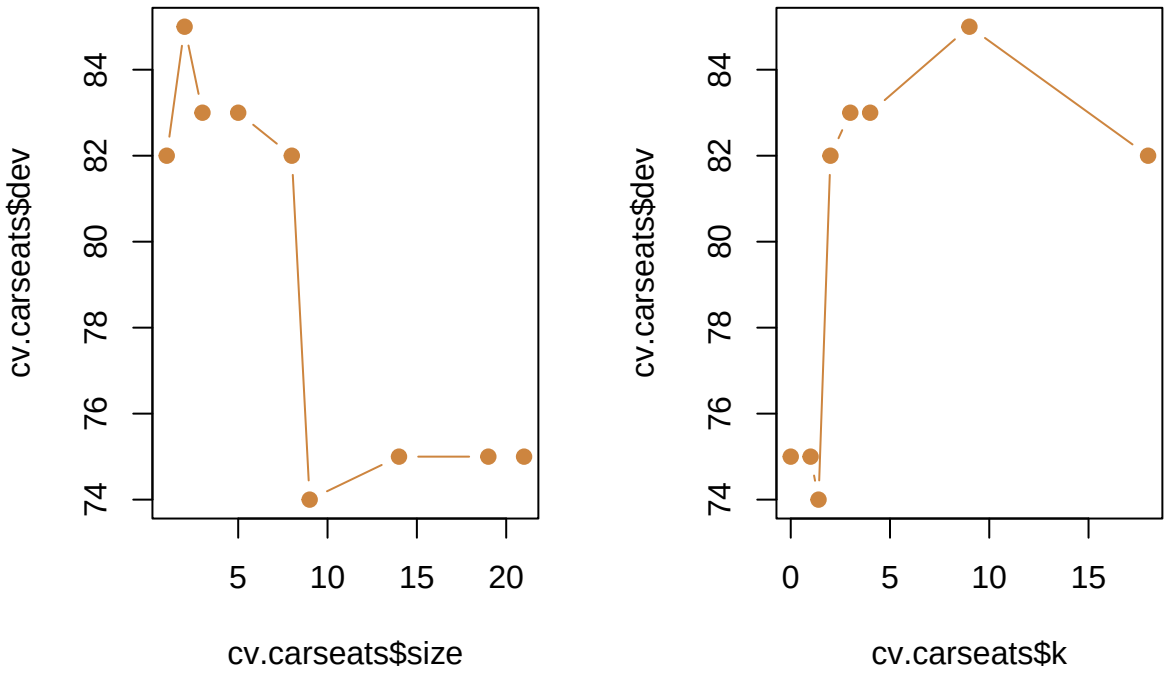
```
## [1] "size"   "dev"    "k"      "method"
```

```
cv.carseats
```

```
## $size
## [1] 21 19 14  9  8  5  3  2  1
##
## $dev
## [1] 75 75 75 74 82 83 83 85 82
##
## $k
## [1] -Inf  0.0  1.0  1.4  2.0  3.0  4.0  9.0 18.0
##
## $method
## [1] "misclass"
##
## attr(,"class")
## [1] "prune"         "tree.sequence"
```

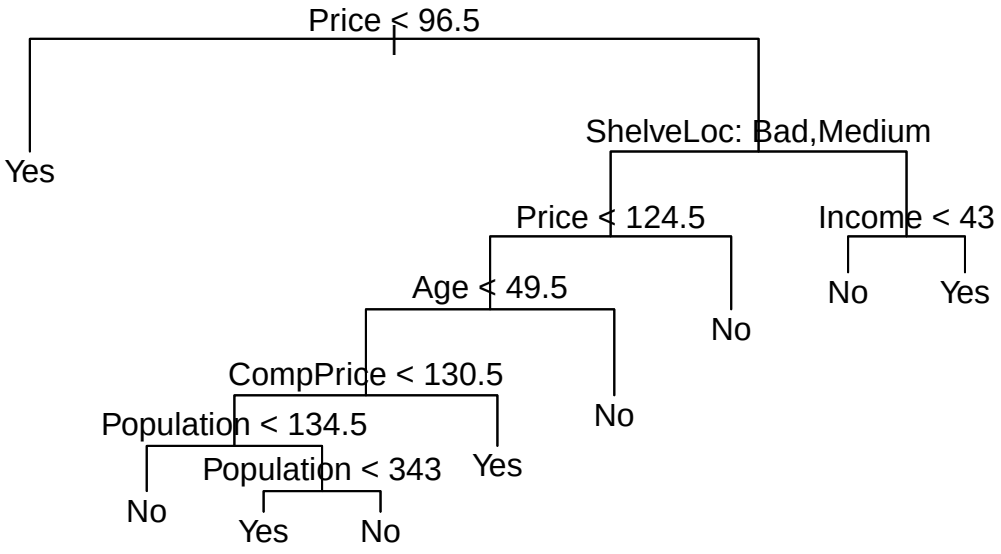
Despite its name, `dev` corresponds to the number of cross-validation errors. The tree with 9 terminal nodes results in only 74 cross-validation errors. We plot the error rate as a function of both `size` and `k`.

```
par(mfrow = c(1, 2))
plot(cv.carseats$size, cv.carseats$dev, type = "b",
     pch=19, col="peru")
plot(cv.carseats$k, cv.carseats$dev, type = "b",
     pch=19, col="peru")
```



We now apply the `prune.misclass()` function in order to prune the tree to obtain the nine-node tree.

```
prune.carseats <- prune.misclass(tree.carseats, best = 9)
plot(prune.carseats)
text(prune.carseats, pretty = 0)
```



How well does this pruned tree perform on the test data set? Once again, we apply the `predict()` function.

```
tree.pred <- predict(prune.carseats, Carseats.test,
    type = "class")
```

```
(tab=table(tree.pred, High.test))
```

```
##          High.test
## tree.pred No Yes
##       No  97  25
##       Yes 20  58
```

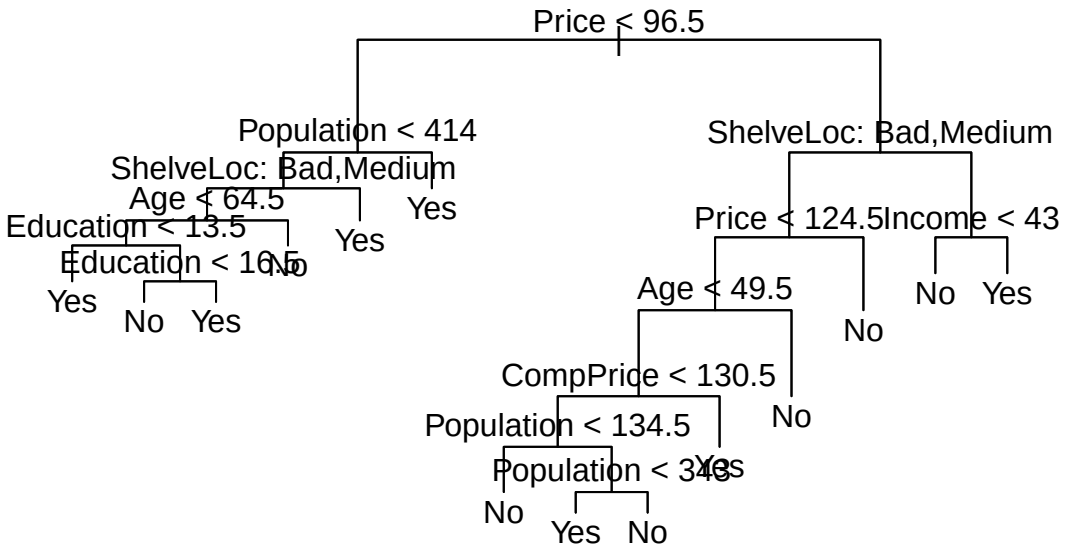
```
(sum(diag(tab))/sum(tab))
```

```
## [1] 0.775
```

Now 77.5% of the test observations are correctly classified, so not only has the pruning process produced a more interpretable tree, but it has also slightly improved the classification accuracy.

If we increase the value of `best`, we obtain a larger pruned tree with lower classification accuracy:

```
prune.carseats <- prune.misclass(tree.carseats, best = 14)
plot(prune.carseats)
text(prune.carseats, pretty = 0)
```




```
tree.pred <- predict(prune.carseats, Carseats.test,
    type = "class")
(tab=table(tree.pred, High.test))
```

```
##          High.test
## tree.pred  No Yes
##       No  102  31
##       Yes  15  52
```

```
(sum(diag(tab))/sum(tab))
```

```
## [1] 0.77
```