UNIVERSITY OF TEXAS AT AUSTIN

Quiz #10

Log-normal stock prices: Tail probabilities and VaR.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 10.1. (15 points)Let the current stock price be denoted by $S(0)$. We model the time$-T$ stock price as lognormal. The mean rate of return on the stock is 0.12, its dividend yield is 0.02, and its volatility is 0.20. The continuously compounded risk-free interest rate is 0.04. You invest in one share of stock at time-0 and let all the dividends be continuously and immediately reinvested in the same stock. You simultaneously deposit an amount $\varphi S(0)$ in a savings account. What should the proportions φ be so that the VaR at the level 0.05 of your total wealth at time-1 equals todays stock price?

Solution: The total wealth at time-1 is equal to $S(1)e^{\delta} + \varphi S(0)e^{r}$. So, our condition on the VaR is

$$\mathbb{P}[S(1)e^{\delta} + \varphi S(0)e^{r} < S(0)] = 0.05.$$

The above is equivalent to

$$\mathbb{P}[S(0)e^{\alpha - \frac{\sigma^2}{2} + \sigma Z} + \varphi S(0)e^{r} < S(0)] = 0.05$$

with $Z \sim N(0,1)$. We have that the above is, in turn, equivalent to

$$\mathbb{P}[e^{\alpha - \frac{\sigma^2}{2} + \sigma Z} + \varphi e^{r} < 1] = 0.05.$$

The constant z^* such that $\mathbb{P}[Z < z^*] = 0.05$ equals -1.645. Therefore,

$$e^{\alpha - \frac{\sigma^2}{2} + \sigma(-1.645)} + \varphi e^{r} = 1.$$

So,

$$\varphi = e^{-r}\left(1 - e^{\alpha - \frac{\sigma^2}{2} + \sigma(-1.645)}\right) = e^{-0.04}\left(1 - e^{0.12 - \frac{(0.2)^2}{2} + 0.2(-1.645)}\right) = 0.196646.$$