UNIVERSITY OF TEXAS AT AUSTIN

Quiz #20

Binomial option pricing: Delta and B.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 20.1. The current stock price is 20 per share. The price at the end of a four-month period is modeled with a one-period binomial tree so that the stock price can either increase by \$5, or decrease by \$5. The stock pays dividends continuously with the dividend yield 0.04.

The continuously compounded, risk-free interest rate is 0.05.

What is the stock investment in a replicating portfolio for four-month, \$20-strike European call option on the above stock?

Solution:

$$\Delta = e^{-\delta h}\frac{V_u - V_d}{S_u - S_d} = e^{-0.04/3}\frac{5}{10} \approx 0.4934.$$

Problem 20.2. The current price of a continuous-dividend-paying stock is \$65 per share. Its dividend yield is 0.02. We model the stock price at the end of two years using a binomial tree. It is assumed that the stock price can either go up, or go down by 30%.

The continuously compounded, risk-free interest rate equals 0.05.

Consider a two-year, \$70-strike European call option on the above stock. What is the risk-free component of the replicating portfolio for this option?

Solution: The two possible stock prices at the end of the two years are $S_u = 84.5$ and $S_d = 45.5$. So, the two possible call payoffs are $V_u = 14.5$ and $V_d = 0$. The risk-free component of the replicating porfolio is

$$B = e^{-0.05(2)}\frac{-0.7(14.5)}{1.3 - 0.7} = -15.3068.$$

This means borrowing \$15.31.

Problem 20.3. The current stock price is 40 per share. The price at the end of a three-month period is modeled with a one-period binomial tree so that the stock price can either increase by \$10, or decrease by \$4. The stock pays dividends continuously with the dividend yield 0.04.

The continuously compounded, risk-free interest rate is 0.05.

What is the stock investment in a replicating portfolio for three-month, \$40-strike European **straddle** on the above stock?

Solution: In our usual notation,

$$\Delta = e^{-\delta h}\frac{V_u - V_d}{S_u - S_d} = e^{-0.04/4}\left(\frac{10-4}{14}\right) \approx 0.4243$$