

Name:

M339D=M389D Introduction to Actuarial Financial Mathematics

University of Texas at Austin

**Practice Problems for In-Term One**

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**Notes:** This is a closed book and closed notes exam. This exam is graded out of 100 points.

**Time:** 50 minutes

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1.1. TRUE/FALSE QUESTIONS.

**Problem 1.1.** (2 pts) A (long) put is a short position with respect to the underlying asset price.

**Problem 1.2.** It is possible for the buyer and the writer of the same option to end up having the same profit on the exercise date.

**Problem 1.3.** (2 points) Consider a one-year, \$45-strike European call option and a one-year, \$45-strike European put option on the same underlying asset. You observe that the time-0 stock price equals \$40 while the time-1 stock price equals \$50. Then, both of the options are out-of-the-money at expiration. *True or false?*

**Problem 1.4.** (2 points) An agent is **only** allowed to long a forward contract if he/she is willing to take physical delivery of the underlying asset.

**Problem 1.5.** (2 points) Denote the continuously compounded, risk-free interest rate by  $r$  and denote the equivalent annual effective interest rate by  $i$ . Then,  $\ln(1 + i) = r$ . *True or false?*

**Problem 1.6.** (2 pts) Two dice are rolled, the single most probable sum of the numbers of the upturned faces is 7. *True or false?*

**Problem 1.7.** (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date  $T$  on the underlying asset  $S$ :

- one long call with strike 40,
- two long calls with strike 50,
- one short call with strike 65.

Let  $S(T) = 69$ . Then, the payoff from the above position at time  $T$  is less than 60.

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### 1.2. MULTIPLE CHOICE QUESTIONS.

**Problem 1.8.** (5 pts) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be two functions given by

$$f(x) = 2x - 10$$

and

$$g(x) = \begin{cases} \min(x, 7) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then,  $g(f(7))$  equals ...

- (a) -4
- (b) 0
- (c) 4
- (d) 7
- (e) None of the above

**Problem 1.9.** *Source: Sample P exam, Problem #176.*

In a group of health insurance policyholders, 20% have high blood pressure and 30% have high cholesterol. Of the policyholders with high blood pressure, 25% have high cholesterol. A policyholder is randomly selected from the group. Calculate the probability that a policyholder has high blood pressure, **given** that the policyholder has high cholesterol.

- (a) 1/6
- (b) 1/5
- (c) 1/4
- (d) 2/3
- (e) 5/6

**Problem 1.10.** Harry plays a simple lottery in which the winnings are distributed as follows:

- \$5 with probability 0.2,
- \$10 with probability 0.4,
- \$20 with probability 0.4.

It turns out that Harry has to pay a fee to collect his winnings. If the actual amount he wins is smaller than \$9, then the fee is defined to equal the amount that Harry won – thus, he walks away with nothing. If the actual amount he wins is between \$9 and \$15, he does not have to pay anything in fees and gets a bonus of \$4. If the actual amount he wins is larger than \$15, then he pays the \$15-fee and pockets the remainder. What is the expected value of the net amount Harry collects?

- (a) 3
- (b) 6.4
- (c) 7.6
- (d) 15
- (e) None of the above.

**Problem 1.11.** Hermione sells short one share of a non-dividend-paying stock. The stock is currently valued at \$80 per share. The continuously compounded risk-free interest rate is 0.04. Hermione intends to close the short sale in one year. What is the final stock price for which Hermione will break even?

**Problem 1.12.** The current market price of widgets is \$4 per widget. The widget factory plans to sell their next batch of 100 widgets in half a year. The total aggregate costs of production of widgets will be equal to \$350.

The factory enters 100 short forward contracts on widgets for delivery in half a year. The forward price is \$4.20 per widget.

What is the factory's profit if the final price of widgets in half a year ends up being \$4.40?

- (a) 30
- (b) 50
- (c) 70
- (d) 90
- (e) None of the above.

**Problem 1.13.** Maryam bakes batches of cupcakes for a cupcake convention. She buys forward 21 pounds of raspberries from a local farmer at the forward price of \$5.60 per pound.

She projects to bake 336 cupcakes and sell each for \$3. The total and aggregate non-raspberry costs of baking the cupcakes are \$200.

If the market price of raspberries on the day of the cupcake convention is \$5.40, what is Maryam's profit?

- (a) \$690.40
- (b) \$694.60
- (c) \$890.40
- (d) \$894.60
- (e) None of the above.

**Problem 1.14.** The **writer** of a call option has ...

- (a) an obligation to sell the underlying asset at the strike price.
- (b) a right, but **not** an obligation, to sell the underlying asset at the strike price.
- (c) an obligation to buy the underlying asset at the strike price.
- (d) a right, but **not** an obligation, to buy the underlying asset at the strike price.
- (e) None of the above.

**Problem 1.15.** (5 points) Assume the **Capital Asset Pricing Model** holds.

You are given the following information about stock X, stock Y, and the market:

- The required return and volatility for the market portfolio are 0.10 and 0.25, respectively.
- The required return and volatility for the stock X are 0.08 and 0.4, respectively.
- The correlation between the returns of stock X and the market is  $-0.2$ .
- The volatility of stock Y is 0.25.
- The correlation between the returns of stock Y and the market is 0.4.

Calculate the required return for stock Y.

- (a) About 0.075.
- (b) About 0.08.
- (c) About 0.085.
- (d) About 0.09.
- (e) None of the above.

**Problem 1.16.** (5 points) For a certain stock, you are given that its expected return equals 0.0944 and that its  $\beta$  equals 1.24. For another stock, you are given that its expected return equals 0.068 and that its  $\beta$  equals 0.8. Both stocks lie on the **Security Market Line (SML)**. What is the risk-free interest rate  $r_f$ ?

- (a) About 0.02
- (b) About 0.025
- (c) About 0.03
- (d) About 0.035
- (e) None of the above.

**Problem 1.17.** (5 points) In a market, the risk-free interest rate is given to be 0.04.

Consider an investment  $I$  in this market, whose Sharpe ratio is 0.42. You construct an equally weighted portfolio consisting of the investment  $I$  and the risk-free asset. The expected return of this portfolio is 0.10.

You decide to rebalance your portfolio so that one quarter of your wealth gets invested in the investment  $I$  and the remainder is invested in the risk-free asset. What is the volatility of this new portfolio?

- (a) 0.0625
- (b) 0.0714
- (c) 0.1225
- (d) 0.1625
- (e) None of the above.

**Problem 1.18.** (5 points) According to your model, the economy over the next year could be *good* or *bad*. You are a pessimist and believe that the economy is twice as likely to be *bad* than *good*.

Consider two assets,  $X$  and  $Y$ , existing in this market. If the economy is *good* the return on asset  $X$  is 0.12, and the return on asset  $Y$  is 0.11. If the economy is *bad* the return on asset  $X$  is  $-0.03$  and the return on asset  $Y$  is  $-0.01$ .

You construct a portfolio  $P$  using assets  $X$  and  $Y$  so that the portfolio's expected return equals 0.025.

Calculate the volatility of this portfolio's return.

- (a) 0.0458
- (b) 0.0512
- (c) 0.0584
- (d) 0.0637
- (e) None of the above.

**Problem 1.19.** (5 points) Consider two assets  $X$  and  $Y$  such that:

- their expected returns are  $\mathbb{E}[R_X] = 0.10$  and  $\mathbb{E}[R_Y] = 0.08$ ;
- their volatilities are  $\sigma_X = 0.25$  and  $\sigma_Y = 0.35$ ;
- the correlation coefficient of their returns is  $\rho_{X,Y} = -1$ .

You are tasked with constructing a portfolio consisting of shares of  $X$  and  $Y$  with a risk-free return. What should the weight  $w_Y$  given to asset  $Y$  be?

- (a)  $5/12$
- (b)  $1/2$
- (c)  $7/12$
- (d) Such a weight does not exist.
- (e) None of the above.

**Problem 1.20.** (5 points) For stock  $S_1$ , you are given that its expected return equals 0.08 and its  $\beta$  is 1.22. For stock  $S_2$ , you are given that its expected return equals 0.05 and its  $\beta$  is 0.56. Both of these stocks lie on the *Security Market Line*. For stock  $S_3$ , you are given that its expected return equals 0.07 and its  $\beta$  is 0.7. What is the  $\alpha$  of stock  $S_3$ ?

- (a) 0
- (b) 0.0137
- (c) 0.0245
- (d) 0.0455
- (e) None of the above.

**Problem 1.21.** A market index is currently trading at \$1,000. Which of the following options is/are **in the money**? More than one answer can be true. You get the credit if you circled **all** acceptable answers and **none** of the incorrect ones.

- (a) \$1,500-strike put
- (b) \$900-strike put
- (c) \$1,250 strike call
- (d) \$950 strike call
- (e) None of the above.

**Problem 1.22.** Let the current price of a non-dividend-paying stock equal 100. The forward price for delivery of this stock in 3 months equals \$101.26

Consider a \$90-strike, six-month put option on this stock whose premium today equals \$2.22. What will the profit of this long put option be if the stock price at expiration equals \$96?

- (a) About \$2.28 loss.
- (b) About \$2.22 loss.
- (c) About \$2.28 gain.
- (d) About \$2.22 gain.
- (e) None of the above.

**Problem 1.23.** (5 points) You are tasked with buying oranges in the market in grove A, transporting the oranges to a juice factory in the market B, and selling the oranges to the juice factory in the market B. You want to hedge. Which of the following would be a satisfactory hedge?

- (a) Long a call in market A and long a put in market B
- (b) Short a call in market A and long a put in market B
- (c) Long a call in market A and short a put in market B
- (d) Short a call in market A and short a put in market B
- (e) None of the above.