Name:

M339D/M389D Introduction to Financial Mathematics for Actuaries
University of Texas at Austin
Practice for $\Delta-$hedging.
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximum number of points on this exam is 100.
Time: 50 minutes

1.1. **Free-response problems.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.1. (10 points) Assume the Black-Scholes framework.

The goal is to delta-hedge a written one-year, $(40, 60)$-strangle on a non-dividend-paying stock whose current price is \$50. The stock's volatility is 0.20.

The continuously compounded, risk-free interest rate is 0.10.

What is the cost of delta-hedging the strangle using shares of the underlying stock?

Solution: The Δ of the strangle equals

$$\Delta_P(S(0), 0; K_P = 40) + \Delta_C(S(0), 0; K_C = 60),$$

i.e., it is the sum of the delta of the call with strike 60 and the delta of the put with strike 40. We have

$$d_1(S(0), 0; K_P = 40) = \frac{1}{0.2}\left[\ln\left(\frac{50}{40}\right) + \left(0.10 + \frac{0.04}{2}\right)\right] = 1.715.$$

So, the put's delta is approximately

$$-N(-d_1(S(0), 0; K_P = 40)) = -N(-1.72) = N(1.72) - 1 = -0.0427.$$

Similarly, for the call, we have

$$d_1(S(0), 0; K_C = 60) = \frac{1}{0.2}\left[\ln\left(\frac{50}{60}\right) + \left(0.10 + \frac{0.04}{2}\right)\right] = -0.31.$$

So, the call's delta is approximately

$$N(d_1(S(0), 0; K_C = 60)) = N(-0.31) = 1 - N(0.31) = 0.3783.$$

Our answer is

$$50(0.3783 - 0.0427) = 16.78.$$

Problem 1.2. (2 points) Let $K_1 < K_2$. A call bull spread consists of a long K_1-strike call and a short K_2-strike call. The options are otherwise identical and European.

An investor wants to delta-hedge a bull spread she bought. Then, she should short-sell shares of the underlying asset. *True or false? Why?*

Solution: TRUE
She owns a bull spread. The delta of the bull spread is positive. So, before she hedges, the delta of her portfolio is positive. Therefore, the delta of her stock investment needs to be negative. Hence, she needs to short sell shares of stock in order to create a delta-neutral portfolio.

Problem 1.3. (2 points) A market-maker writes a call option on a stock. To decrease the delta of this position, (s)he can **write** a call on the underlying stock. *True or false?*

Solution: TRUE

Problem 1.4. (2 points) Consider an option whose payoff function is given by $v(s,T) = \min(s, 50)$. If a market-maker **writes** this option, they need to short sell shares of stock to create a delta-neutral portfolio. *True or false? Why?*

Solution: FALSE
The "special" option in the probem can be replicated using a put and a bond. More precisely,

$$v(s,T) = \min(s,50) = 50 - \max(50-s,0) = 50 - v_P(s,T).$$

So, the value of the "special" option is equal to the value of the bond minus the put price at any point in time. Hence, the delta of the "special" option is

$$\Delta(s,t) = -\Delta_P(s,t) > 0.$$

Since the market maker writes the option, the original delta of their position is $\Delta_P(S(0),0)$ which is a negative number. They need to **purchase** shares of stock to create a delta-neutral portfolio.