

UNIVERSITY OF TEXAS AT AUSTIN

## Quiz # 15

The tangent portfolio. Sharpe ratio.

Please, provide your complete solutions to the following problems. A graphical argument is acceptable.

**Problem 15.1.** (2 points) The tangent portfolio has the highest Sharpe ratio of all the portfolios in the feasible set. *True or false?*

**Solution: TRUE**

**Problem 15.2.** (2 points) Consider our usual coordinate system of portfolios with the volatility on the horizontal axis and the expected return on the vertical axis. Consider a portfolio  $P$  in that plane and look at the line through that portfolio and the point corresponding to the risk-free asset  $(0, r_f)$ . Then, the slope of this line is exactly the Sharpe ratio of the portfolio  $P$ . *True or false?*

**Solution: TRUE**

**Problem 15.3.** (2 points) Consider a portfolio  $P$  consisting of a collection of risky assets. You construct a new portfolio by investing a proportion  $\phi$  of your wealth in portfolio  $P$  and the remainder of your wealth in the risk-free asset. Then, the excess return of the new portfolio is the same proportion  $\phi$  of the excess return of the portfolio  $P$ . *True or false?*

**Solution: TRUE**

**Problem 15.4.** (9 points) Assume that the risk-free interest rate equals 0.04. The Sharpe ratio of asset  $S$  is given to be  $1/4$  while the Sharpe ratio of asset  $Q$  equals  $1/3$ . You know that the volatility of  $S$  is twice the volatility of  $Q$ . If you build an equally weighted portfolio with assets  $S$  and  $Q$  as its two components, the expected return of this portfolio will be 0.10. What is the expected return of  $S$  and what is the expected return of  $Q$ ?

**Solution:** From the condition on the Sharpe ratio of  $S$ , we get

$$\frac{\mathbb{E}[R_S] - r_f}{\sigma_S} = \frac{1}{4} \Rightarrow \sigma_S = 4(\mathbb{E}[R_S] - 0.04).$$

From the condition on the Sharpe ratio of  $Q$ , we obtain

$$\frac{\mathbb{E}[R_Q] - r_f}{\sigma_Q} = \frac{1}{3} \Rightarrow \sigma_Q = 3(\mathbb{E}[R_Q] - 0.04).$$

Since  $\sigma_S = 2\sigma_Q$ , we have

$$\begin{aligned} 4(\mathbb{E}[R_S] - 0.04) &= 2(3)(\mathbb{E}[R_Q] - 0.04) \Rightarrow 2(\mathbb{E}[R_S] - 0.04) = 3(\mathbb{E}[R_Q] - 0.04) \\ &\Rightarrow 2\mathbb{E}[R_S] - 3\mathbb{E}[R_Q] = 0.08 - 0.12 = -0.04. \end{aligned}$$

On the other hand, from the given expected return on the equally-weighted portfolio, we know that

$$\frac{1}{2}(\mathbb{E}[R_S] + \mathbb{E}[R_Q]) = 0.10 \Rightarrow \mathbb{E}[R_S] + \mathbb{E}[R_Q] = 0.20.$$

Solving the above system of two equations with two unknowns, we get

$$\mathbb{E}[R_S] = 0.112 \quad \text{and} \quad \mathbb{E}[R_Q] = 0.088.$$