

**The University of Texas at Austin**  
**PRACTICE FOR IN-TERM 1**  
*Introduction to Financial Mathematics*

February 19, 2026

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**Instructions:** Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

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## Practice Problems

### DEFINITIONS

**Problem 1.1.** Provide the definition of the **arbitrage portfolio**.

**Solution.** See your notes.

### TRUE/FALSE QUESTIONS

**Problem 1.2.** (2 points) An agent is **only** allowed to long a forward contract if he/she is willing to take physical delivery of the underlying asset. *True or false? Why?*

**Solution. FALSE**

It is possible to have *cash settlement* on the delivery date if the forward contract stipulates so.

**Problem 1.3.** (2 points) Denote the continuously compounded, risk-free interest rate by  $r$  and denote the equivalent annual effective interest rate by  $i$ . Then,  $\ln(1 + i) = r$ . *True or false?*

**Solution. TRUE**

**Problem 1.4.** (2 pts) Two dice are rolled, the single most probable sum of the numbers of the upturned faces is 7. *True or false?*

**Solution. TRUE**

**Problem 1.5.** (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date  $T$  on the underlying asset  $S$ :

- one long call with strike 40,
- two long calls with strike 50,
- one short call with strike 65.

Let  $S(T) = 69$ . Then, the payoff from the above position at time  $T$  is less than 60. *True or false? Why?*

**Solution. FALSE**

The payoff is

$$(69 - 40) + 2(69 - 50) - (69 - 65) = 63.$$

## MULTIPLE CHOICE QUESTIONS

**Problem 1.6.** (5 pts) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be two functions given by

$$f(x) = 2x - 10$$

and

$$g(x) = \begin{cases} \min(x, 7) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then,  $g(f(7))$  equals ...

- a. -4
- b. 0
- c. 4
- d. 7
- e. None of the above

### Solution. (c)

**Problem 1.7.** Source: Sample P exam, Problem #176.

In a group of health insurance policyholders, 20% have high blood pressure and 30% have high cholesterol. Of the policyholders with high blood pressure, 25% have high cholesterol. A policyholder is randomly selected from the group. Calculate the probability that a policyholder has high blood pressure,  $f$  given that the policyholder has high cholesterol.

- a. 1/6
- b. 1/5
- c. 1/4
- d. 2/3
- e. 5/6

### Solution. (a)

Let  $E$  be the event containing all the policyholders with a high blood pressure and let  $F$  be the event which contains all the policyholders with high cholesterol. We are given the following

$$\mathbb{P}[E] = 0.2, \quad \mathbb{P}[F] = 0.3, \quad \mathbb{P}[F | E] = 0.25.$$

Then, the conditional probability we are looking for equals

$$\mathbb{P}[E | F] = \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[F]} = \frac{\mathbb{P}[F | E]\mathbb{P}[E]}{\mathbb{P}[F]} = \frac{(0.25)(0.2)}{0.3} = \frac{1}{6}.$$

**Problem 1.8.** Harry plays a simple lottery in which the winnings are distributed as follows:

- \$5 with probability 0.2,
- \$10 with probability 0.4,
- \$20 with probability 0.4.

It turns out that Harry has to pay a fee to collect his winnings. If the actual amount he wins is smaller than \$9, then the fee is defined to equal the amount that Harry won – thus, he walks away with nothing.

If the actual amount he wins is between \$9 and \$15, he does not have to pay anything in fees and gets a bonus of \$4. If the actual amount he wins is larger than \$15, then he pays the \$15-fee and pockets the remainder. What is the expected value of the net amount Harry collects?

- a. 3.0
- b. 6.4
- c. 7.6
- d. 15
- e. None of the above.

**Solution. (c)**

The actual amount that Harry gets is

- \$0 with probability 0.2,
- \$14 with probability 0.4,
- \$5 with probability 0.4.

So, his expected winnings are

$$14(0.4) + 5(0.4) = 7.6$$

**Problem 1.9.** Hermione sells short one share of non-dividend-paying stock. The stock is currently valued at \$80 per share. The interest rate is 0.04. Hermione intends to close the short sale in one year. What is the final stock price for which Hermione will break even?

- a. 76.92
- b. 80.00
- c. 83.26
- d. Such a stock price does not exist.
- e. None of the above.

**Solution. (c)**

In our usual notation, the break-even point is

$$S(0)e^{rT} = 80e^{0.04(1)} = 80e^{0.04} = 83.26486.$$

**Problem 1.10.** The current market price of widgets is \$4 per widget. The widget factory plans to sell their next batch of 100 widgets in half a year. The total aggregate costs of production of widgets will be equal to \$350.

The factory enters 100 short forward contracts on widgets for delivery in half a year. The forward price is \$4.20 per widget.

What is the factory's profit if the final price of widgets in half a year ends up being \$4.40?

- a. 30
- b. 50
- c. 70
- d. 90
- e. None of the above.

**Solution. (c)**

The factory will sell the widgets per the forward contract for \$420 total. The total aggregate costs are given to be \$350. Hence, the profit is \$70.

**Problem 1.11.** Maryam bakes batches of cupcakes for a cupcake convention. She buys forward 21 pounds of raspberries from a local farmer at the forward price of \$5.60 per pound.

She projects to bake 336 cupcakes and sell each for \$3. The total and aggregate non-raspberry costs of baking the cupcakes are \$200.

If the market price of raspberries on the day of the cupcake convention is \$5.40, what is Maryam's profit?

- a. \$690.40
- b. \$694.60
- c. \$890.40
- d. \$894.60
- e. None of the above.

**Solution. (a)**

$$336 \times 3 - 21 \times 5.60 - 200 = 690.40.$$

**Problem 1.12.** The **writer** of a call option with physical exercise has ...

- a. an obligation to sell the underlying asset at the strike price.
- b. a right, but **not** an obligation, to sell the underlying asset at the strike price.
- c. an obligation to buy the underlying asset at the strike price.
- d. a right, but **not** an obligation, to buy the underlying asset at the strike price.
- e. None of the above.

**Solution. (a)**

**Problem 1.13.** (5 points) Roger owns a cow named Elsie. Her estimated worth today is \$3,750. Roger enters into a forward agreement with Harry to sell him Elsie the cow in 6 months for \$4,000. On the delivery date, Roger changes his mind and wants cash settlement instead. Harry agrees. They look into the "Bovine Blue Book" and realize that Elsie's worth on that date is \$3,500.

What is the cash flow that has to take place as part of the cash settlement?

- a. \$500 from Roger to Harry
- b. \$500 from Harry to Roger
- c. \$250 from Roger to Harry
- d. \$250 from Harry to Roger
- e. None of the above.

**Solution. (b)**

**Problem 1.14.** (5 points) A farmer produces one thousand crates of apples. The total and aggregate costs of production are \$48,000. The farmer enters a forward contract for the entire harvest to hedge at a forward price of \$69 per crate at harvest time.

The market price of apples at harvest time is \$70 per crate.

What is the farmer's profit?

- a. 1000 loss
- b. 1000 gain
- c. 21000 gain
- d. 22000 gain
- e. None of the above.

**Solution. (c)**

$$69000 - 48000 = 21000$$

**Problem 1.15.** (5 points) Pancakes, Inc. produces strawberry pancakes for the pancake festival. It longed a forward contract on 100 pounds of strawberries at \$2.50 per pound to be delivered to the festival and added to the pancakes. According to the contract with the organizers, the total fixed revenue will be \$6,000 for the pancakes produced with the above strawberries. Costs other than strawberries total \$1200.

On the morning of the pancake festival, the market price of strawberries is \$2.25 per pound.

Find the company's profit.

- a. 2300
- b. 1550
- c. 2550
- d. 1500
- e. None of the above.

**Solution. (e)**

$$6000 - 1200 - 250 = 4550$$

**Problem 1.16.** (5 points) The current price of stock a certain type of stock is \$80. The premium for a 6 –month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

- a. \$80
- b. \$85.72
- c. \$85.84
- d. \$85.96
- e. None of the above.

**Solution. (d)**

The break-even point is

$$80 + 5.84e^{0.04/2} = 85.958$$

**Problem 1.17.** The following nine-month European put options are available in the market:

- a \$120-strike put with the premium of \$12,
- a \$127-strike put with the premium of \$10,

The continuously compounded, risk-free interest rate is 0.04.

You construct a portfolio by buying the \$127-strike put and writing the \$120-strike put. Which of the following statements is correct?

- a. The minimum **profit** of this portfolio is  $-9.06$ .

- b. The minimum **profit** of this portfolio is  $-2.06$ .
- c. The minimum **profit** of this portfolio is  $-7$ .
- d. This is an arbitrage portfolio.
- e. None of the above.

**Solution. (d)**

The initial cost of this portfolio is  $10 - 12 = -2$ . The minimum payoff of this portfolio happens for the final asset price above 127. It is equal to 0. So, the minimal gain of this portfolio is

$$0 - (-2)e^{0.03} = 2.06.$$

**Problem 1.18.** Let the current price of a non-dividend-paying stock equal 100. The forward price for delivery of this stock in 3 months equals \$101.26

Consider a \$90-strike, six-month put option on this stock whose premium today equals \$2.22.

What will the profit of this long put option be if the stock price at expiration equals \$96?

- a. About \$2.28 loss.
- b. About \$2.22 loss.
- c. About \$2.28 gain.
- d. About \$2.22 gain.
- e. None of the above.

**Solution. (a)**

The option is out-of-the money at expiration, so its owner suffers a loss of the future value of its premium

$$2.22 \times \left( \frac{101.26}{100} \right)^2 = 2.2763.$$

**Problem 1.19.** (5 points) A derivative security has the payoff function given by

$$v(s) = (s^2 - 100)_+$$

Its exercise date is in one year. You model the time-1 price of the underlying asset as

$$S(1) \sim \begin{cases} 9.5 & \text{with probability } 1/4 \\ 10 & \text{with probability } 1/2 \\ 11 & \text{with probability } 1/4 \end{cases}$$

The continuously compounded, risk-free interest rate is 10%. What is the expected **payoff** of the above derivative security?

- a. 5.25
- b. 2.81
- c. 0.31
- d. 1.42
- e. None of the above.

**Solution. (a)**

$$(9.5^2 - 100)_+ \left( \frac{1}{4} \right) + (10^2 - 100)_+ \left( \frac{1}{2} \right) + (11^2 - 100)_+ \left( \frac{1}{4} \right) = 21 \left( \frac{1}{4} \right) = 5.25$$