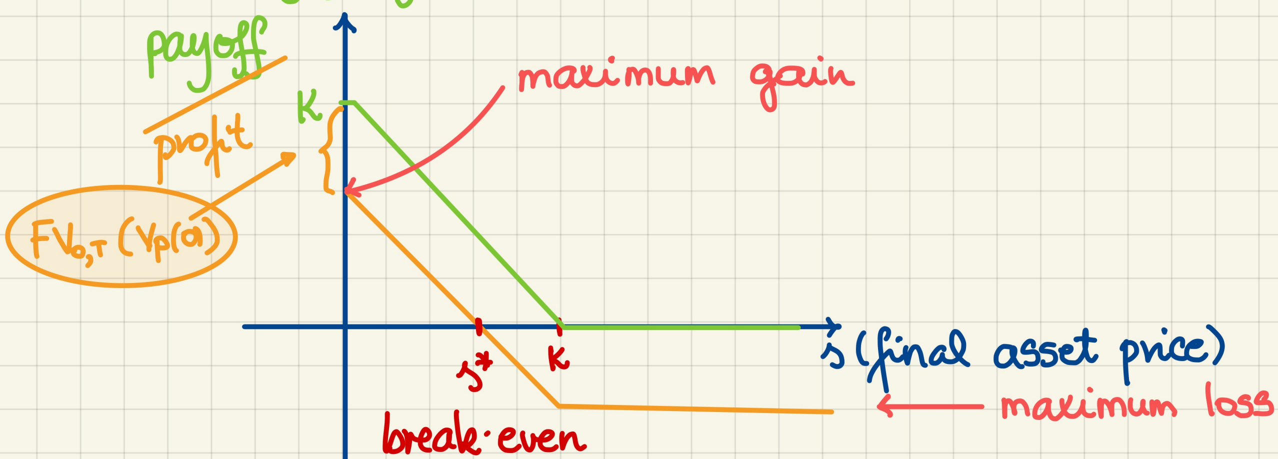
M339D: February 16th, 2022.

European put options [cont'd].

$\boxed{V_P(0)}$ ... put premium paid @ time·0 ⟵ Initial cost

Payoff: $V_P(T) = (K - S(T))_+$

The payoff function: $\boxed{v_P(s) = (K-s)_+}$



A LONG PUT IS A SHORT POSITION w.r.t. the underlying.

Solve for s in:

$$(K-s)_+ - FV_{0,T}(V_P(0)) = 0$$

Evidently, $\boxed{K > s}$

$$K - s - FV_{0,T}(V_P(0)) = 0$$

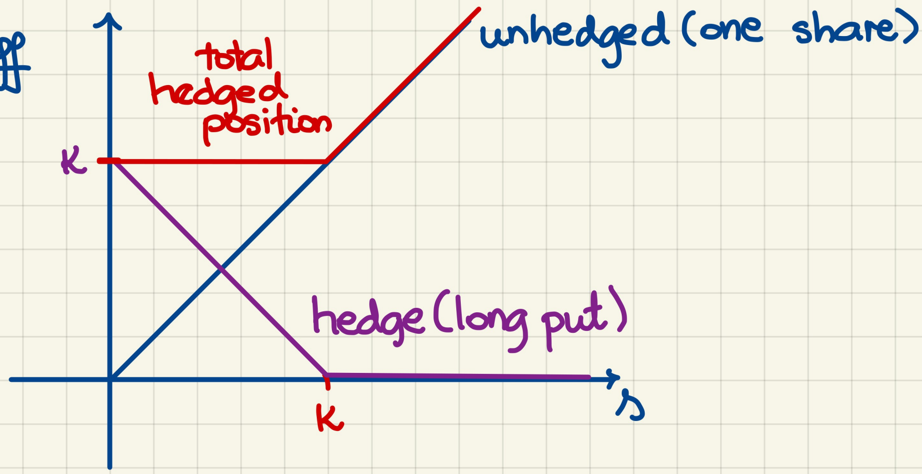
$$\boxed{s^* = K - FV_{0,T}(V_P(0))}$$

Floor.

FLOOR.
- Start w/ a long position in a non·dividend·paying stock.
- To hedge, you long a put option on this stock.

$$\text{Payoff (total)} = S(T) + (K - S(T))_+$$

$$= \begin{cases} \cancel{S(T)} + K - \cancel{S(T)} & \text{if } S(T) < K \\ S(T) & \text{if } S(T) \geq K \end{cases} = \max(S(T), K)$$



Covered Put.

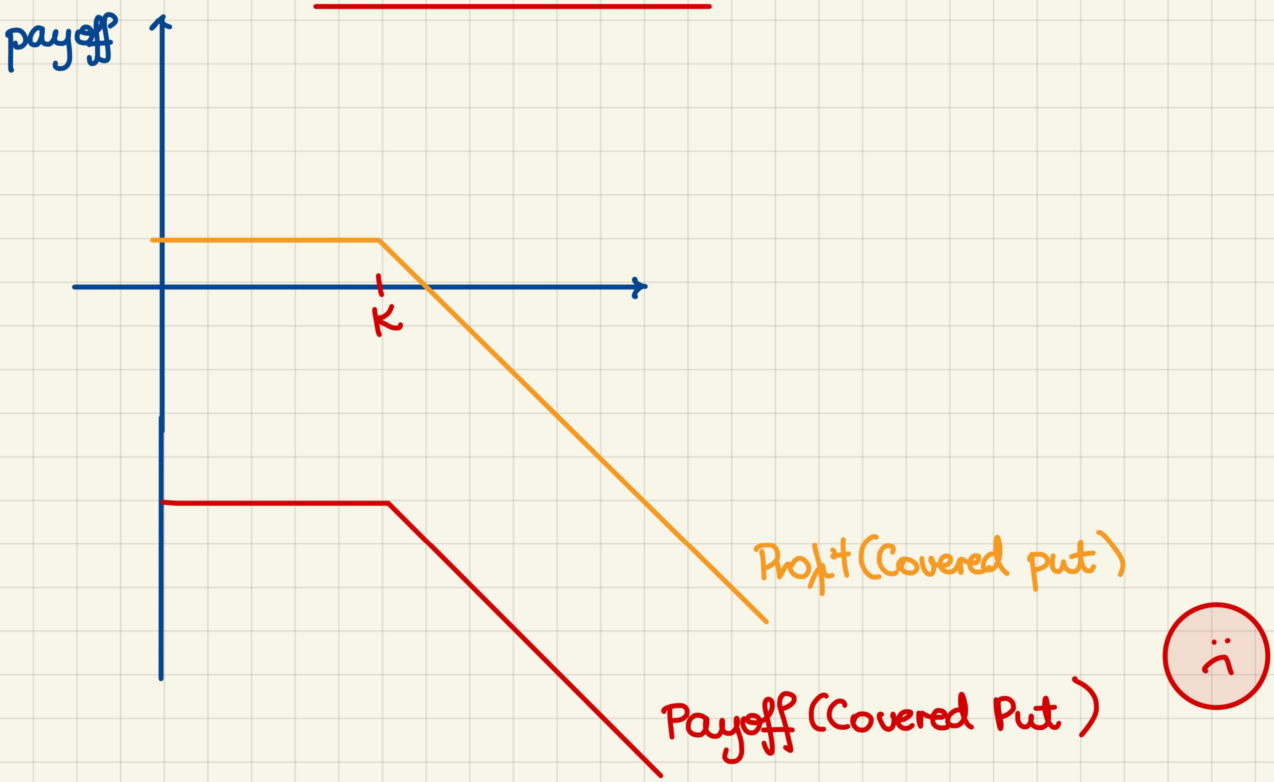
- Start w/ a written put.
- Add a short sale of the underlying asset.

(again: think non-dividend-paying stocks).

$$\text{Payoff (Total)} = -(K-S(T))_+ - S(T)$$

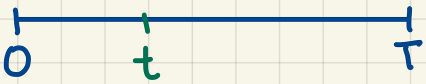
$$= \begin{cases} -(K-\cancel{S(T)}) - \cancel{S(T)} & \text{if } K > S(T) \\ -S(T) & \text{if } K \le S(T) \end{cases}$$

$$= \underline{-\max(S(T), K)}$$



Moneyness.

Consider an option written @ time 0 w/ an exercise/expiration date T.



$\boxed{0 \le t \le T}$

Imagine the cashflow which would happen to the option's owner should they exercise @ time t.

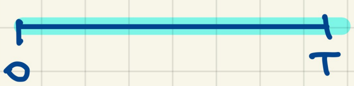
If cashflow is $\begin{cases} > 0 & \text{we say the option is in-the-money} \\ = 0 & \text{we say the option is at-the-money} \\ < 0 & \text{we say the option is out-of-the-money} \end{cases}$

The usual usage:

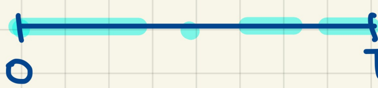
- We can specify the strike for a call/put by stating it's at-the-money, i.e., $\boxed{K = S(0)}$
- To be considered when options have the possibility to be exercised early.

For European options: exercise if and only if the option is in-the-money on the exercise date.

For American options:



For Bermudan options:



For a rational owner of the option, it only makes sense to exercise an option early if it's in-the-money.
This is a necessary condition, but it's not sufficient.