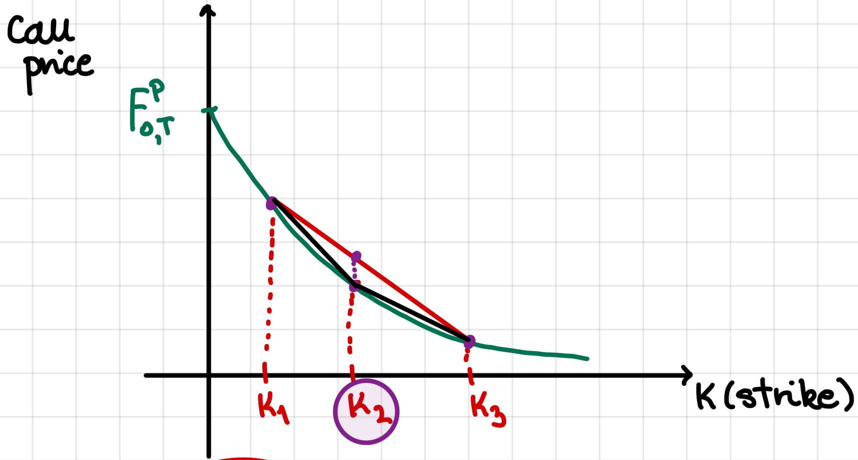
# M339D: April 21st, 2021.

## Call-price Convexity.




**Claim:** $\lambda = \frac{K_3 - K_2}{K_3 - K_1}$ (note: $K_2 = \lambda \cdot K_1 + (1-\lambda) K_3$)

we have that

$$V_c(K_2) \leq \lambda \cdot V_c(K_1) + (1-\lambda) \cdot V_c(K_3) \quad (CC) \longleftarrow$$

$$\iff$$

$$\frac{V_c(K_1) - V_c(K_2)}{K_2 - K_1} \geq \frac{V_c(K_2) - V_c(K_3)}{K_3 - K_2}$$

$\longrightarrow$: Assume, to the contrary, that there exist $K_1 < K_2 < K_3$ such that

$$\underline{V_c(K_2) > \lambda \cdot V_c(K_1) + (1-\lambda) V_c(K_3)} \quad w/ \ \lambda = \frac{K_3 - K_2}{K_3 - K_1}$$

I. Suspect an arbitrage opportunity. ✓

II. Propose an arbitrage portfolio:

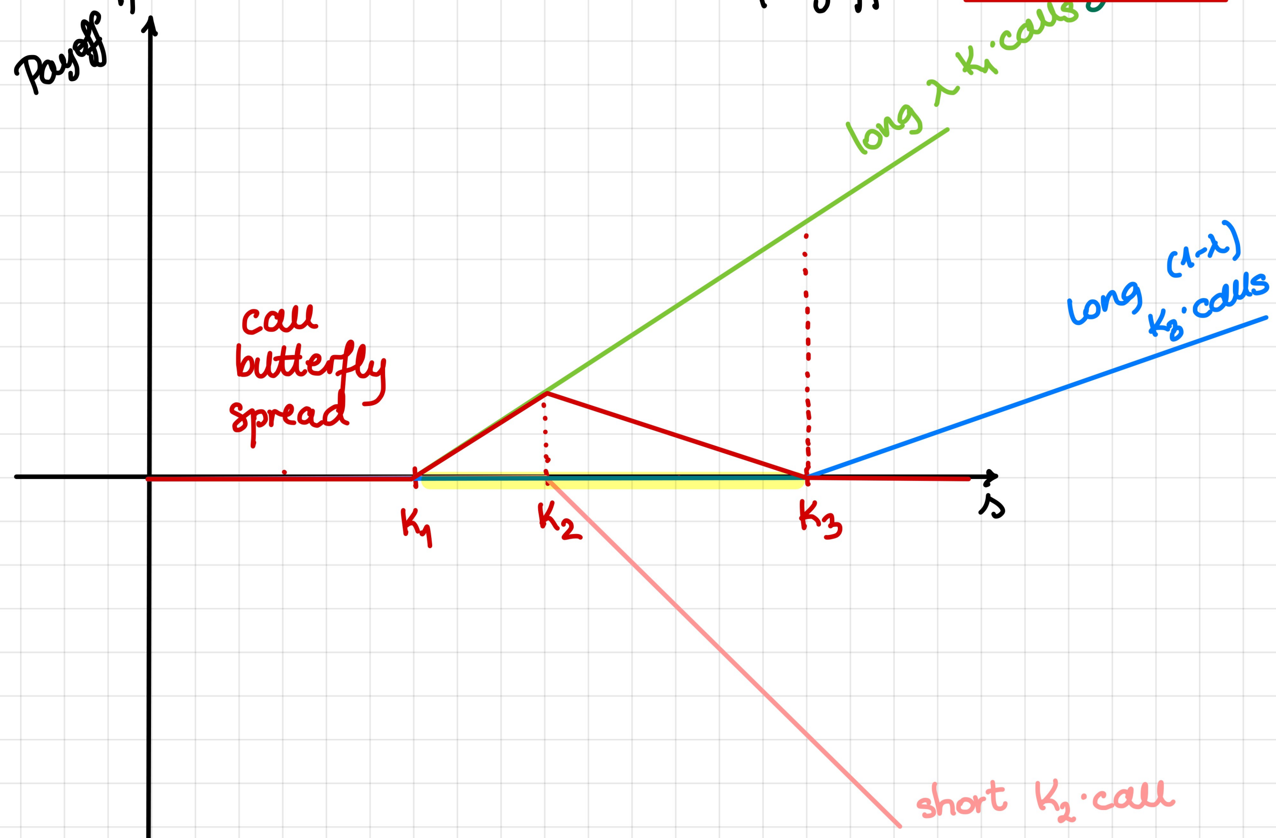
- Long $\lambda$ $K_1$-calls
- Short $1$ $K_2$-calls
- Long $1-\lambda$ $K_3$-calls

} Call BUTTERFLY Spread

III. Verification.

Init. Cost: $\lambda \cdot V_c(K_1) + (1-\lambda) \cdot V_c(K_3) - V_c(K_2) < 0$

It's sufficient to show that the payoff is non-negative.

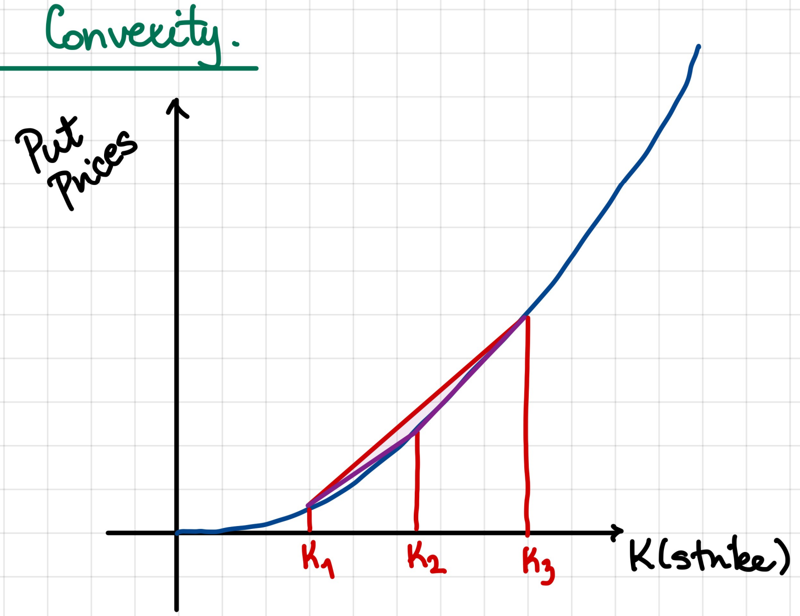



Indeed, the payoff is $\geq 0$

$\Rightarrow$ Profit $> 0$ $\Rightarrow$ It's an arbitrage portfolio!

- If $K_2 = \frac{1}{2}(K_1 + K_3)$, then it's a symmetric butterfly spread.
- Otherwise, it's asymmetric.
- Lack directionality.
  However, it can be used to speculate on low volatility.

## Put·Price Convexity.



**Claim.** $\lambda = \dfrac{K_3 - K_2}{K_3 - K_1}$

we have

$$V_p(K_2) \leq \lambda \cdot V_p(K_1) + (1-\lambda)\cdot V_p(K_3) \qquad \text{(PC)}$$

$$\Longleftrightarrow$$

$$\frac{V_p(K_2) - V_p(K_1)}{K_2 - K_1} \leq \frac{V_p(K_3) - V_p(K_2)}{K_3 - K_2}$$

Q: What would you do if you observed $K_1 < K_2 < K_3$ such that

$$V_p(K_2) > \lambda \cdot V_p(K_1) + (1-\lambda)\, V_p(K_3) \; ?$$

$\longrightarrow$ :

$$\left.\begin{array}{lll} \bullet\ \underline{\text{Long}} & \underline{\lambda} & K_1 \cdot \text{puts} \\ \bullet\ \underline{\text{short}} & \underline{1} & K_2 \cdot \text{put} \\ \bullet\ \underline{\text{Long}} & \underline{1-\lambda} & K_3 \cdot \text{puts} \end{array}\right\} \text{Put Butterfly Spread}$$

ToDo: • Put and call butterfly spreads have the same payoffs. ✓

67.

Consider the following investment strategy involving put options on a stock with the same expiration date.

i) Buy one 25-strike put

ii) Sell two 30-strike puts

iii) Buy one 35-strike put

Symmetric Put Butterfly Spread

$K_2 = \frac{1}{2}(K_1+K_3)$

$\lambda = \frac{K_3-K_2}{K_3-K_1} = \frac{K_3 - \frac{K_1}{2} - \frac{K_3}{2}}{K_3-K_1} = \frac{\frac{1}{2}(K_3-K_1)}{K_3-K_1} = \frac{1}{2} = 1-\lambda$

Calculate the payoffs of this strategy assuming stock prices (i.e., at the time the put options expire) of 27 and 37, respectively.

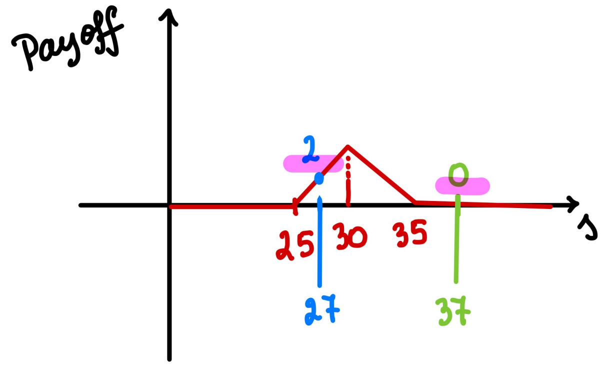
(A) –2 and 2

(B) 0 and 0

(C) 2 and 0

(D) 2 and 2

(E) 14 and 0




68.

For a non-dividend-paying stock index, the current price is 1100 and the 6-month forward price is 1150. Assume the price of the stock index in 6 months will be 1210.

Which of the following is true regarding forward positions in the stock index?

(A) Long position gains 50

(B) Long position gains 60

(C) Long position gains 110

(D) Short position gains 60

(E) Short position gains 110

9.

Stock ABC has the following characteristics:

- The current price to buy one share is 100.
- The stock does not pay dividends.
- European options on one share expiring in one year have the following prices:

| Strike Price | Call option price | Put option price |
|---|---|---|
| 90 | 14.63 | 0.24 |
| 100 | 6.80 | 1.93 |
| 110 | 2.17 | 6.81 |

A butterfly spread on this stock has the following profit diagram.


The continuously compounded risk-free interest rate is 5%.

Determine which of the following will NOT produce this profit diagram.

(A) Buy a 90 put, buy a 110 put, sell two 100 puts

(B) Buy a 90 call, buy a 110 call, sell two 100 calls

(C) Buy a 90 put, sell a 100 put, sell a 100 call, buy a 110 call

(D) Buy one share of the stock, buy a 90 call, buy a 110 put, sell two 100 puts

(E) Buy one share of the stock, buy a 90 put, buy a 110 call, sell two 100 calls.