The University of Texas at Austin
Homework Assignment 5

Introduction to Financial Mathematics

February 28, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

European Call Options.

Problem 5.1. (2 points) An agent is f only allowed to write options on an underlying asset if he/she already owns units of the underlying. *True or false?*

Solution. FALSE
The so-called *naked* option writing is a legal and common practice.

Problem 5.2. (5 points) The initial price of the market index is \$900. After 3 months the market index is priced at \$920. The nominal rate of interest convertible monthly is 4.8%.
The premium on the long call, with a strike price of \$930, is \$2.00. What is the profit or loss at expiration for this long call?

Solution. In our usual notation, the profit is

$$(S_T - K)_+ - C \times (1+j)^3$$

with C denoting the price of the call and j the effective monthly interest rate. We get

$$(920 - 930)_+ - 2 \times 1.004^3 \approx -2.02.$$

Problem 5.3. (5 points) The current price of stock a certain type of stock is \$50. The premium for a $3-$month, at-the-money call option is \$2.74. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

a. \$50.
b. \$52.71.
c. \$52.77.
d. \$52.85.
e. None of the above.

Solution. (c)
The break-even point is

$$50 + 2.74e^{0.04/4} = 52.7675.$$

Problem 5.4. (8 points) *Source: FM(DM) sample problem #42.*
An investor purchases one share of a non-dividend-paying stock and writes an at-the-money, T-year, European call option in this stock. The call premium is denoted by C. Assume that there are no transaction

costs. The continuously compounded, risk-free interest rate is denoted by r. Let the argument s represent the stock price at time T.

- (6 points) Determine an algebraic expression for the investor's profit at expiration T in terms of C, r, T and the strike K.
- (2 points) In particular, how does the expression you obtained in (i) simplify if the call is in-the-money on the exercise date?

Solution.

$$s - (s-K)_+ - (S(0) - C)e^{rT} = s - (s-K)_+ - (K - C)e^{rT}.$$

For $s > K$,

$$s - (s-K) - (K-C)e^{rT} = K(1 - e^{rT}) + Ce^{rT}.$$

Problem 5.5. (15 points) The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for \$1,000.

The jeweler models he market price of gold in one year as follows:

Gold price in one year	Probability
750 per ounce	0.2
850 per ounce	0.5
950 per ounce	0.3

The jeweler hedges the price of gold by buying a 1 —year call option with an exercise price of \$900 per ounce. The option costs \$100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelry produced.

Solution. With $S(T)$ denoting the market price of gold at time $T = 1$, the jeweler's **hedged** profit per piece of jewelry can be expressed as

$$1,000 - \min(S(T), 900) - 100e^{0.05} = 894.873 - \min(S(T), 900).$$

So, her expected **hedged** profit equals

$$894.873 - \mathbb{E}[\min(S(T), 900)] = 894.873 - (0.2 \cdot 750 + 0.5 \cdot 850 + 0.3 \cdot 900) = 49.873.$$

Problem 5.6. (15 points) *Source: Sample MFE (Intro) Problem #15.*
The current price of a non-dividend paying stock is \$40 and the continuously compounded risk-free interest rate is 8%. You enter into a short position on 3 call options, each with 3 months to expiry, a strike price of \$35, and an option premium of \$6.13. Simultaneously, you enter into a long position on 5 call options, each with 3 months to expiry, a strike price of \$40, and an option premium of \$2.78. All 8 options are held until maturity. Calculate the range of the profit for the entire option portfolio.

a. $[-4.58, 3.42]$.

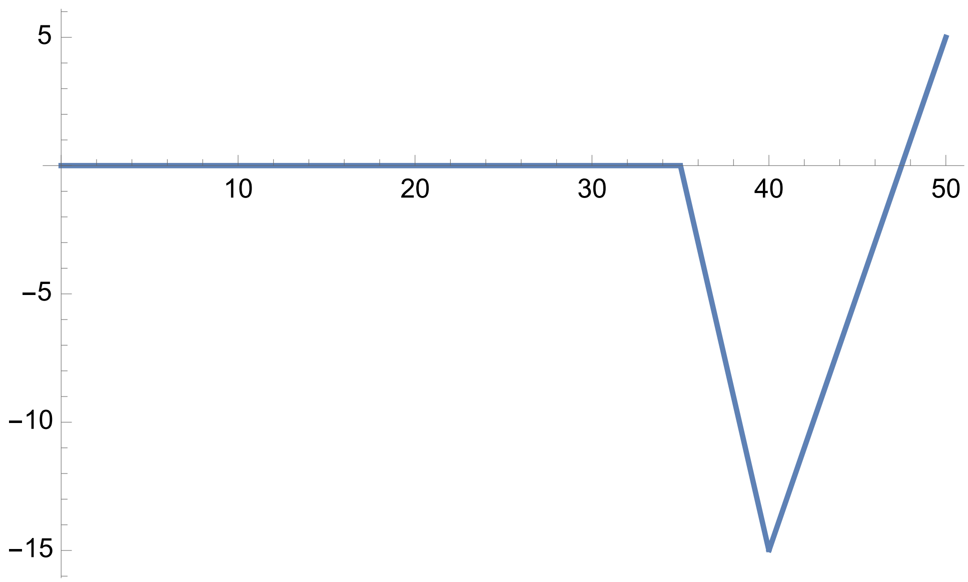
b. $[-10.42, 4.58]$.
c. $[-10.42, \infty)$.
d. $(-\infty, 4.58]$.
e. None of the above.

Solution. (c)
The initial cost is $-3(6.13) + 5(2.78) = -4.49$.

In our usual notation, the expression for the payoff is

$$-3(S(T) - 35)_+ + 5(S(T) - 40)_+$$

So, the payoff function is $v(s) = -3(s - 35)_+ + 5(s - 40)_+$. Its graph looks like this:



We see that the minimum payoff is attained at $s = 40$ and that it equals -15. There is unlimited growth potential. Hence, the range of the profit is

$$\left[-15 - (-4.49)e^{0.08(0.25)}, \infty\right) = [-10.4193, \infty).$$