

UNIVERSITY OF TEXAS AT AUSTIN

Quiz #11

Log-normal stock prices: Conditional expectation.

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Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

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**Problem 11.1.** (5 pts)

A non-dividend-paying stock is currently valued at \$100 per share. Its annual mean rate of return is given to be 12% while its volatility is given to be 30%.

Assuming the lognormal stock-price model, find

$$\mathbb{E}[S(2) \mid S(2) > 95].$$

- (a) \$86.55
- (b) \$101.60
- (c) \$152.35
- (d) \$159.07
- (e) None of the above.

**Solution: (c)**

In our usual notation,

$$\mathbb{E}[S(T) \mid S(T) > K] = \frac{S(0)e^{(\alpha-\delta)T}N(\hat{d}_1)}{N(\hat{d}_2)}.$$

with

$$\begin{aligned}\hat{d}_1 &= \frac{1}{\sigma\sqrt{T}} \left[ \ln \left( \frac{S(0)}{K} \right) + \left( \alpha - \delta + \frac{\sigma^2}{2} \right) T \right], \\ \hat{d}_2 &= \hat{d}_1 - \sigma\sqrt{T}.\end{aligned}$$

In the present problem,

$$\begin{aligned}\hat{d}_1 &= \frac{1}{0.3\sqrt{2}} \left[ \ln \left( \frac{100}{95} \right) + \left( 0.12 + \frac{0.09}{2} \right) \times 2 \right] = 0.8987, \\ \hat{d}_2 &= 0.8987 - 0.3\sqrt{2} = 0.4745.\end{aligned}$$

So, our answer is

$$\mathbb{E}[S(2) \mid S(2) > 95] = \frac{100e^{(0.12) \times 2}N(0.8987)}{N(0.4745)} = \frac{100e^{0.24} \times 0.8159}{0.6808} = 152.35.$$

**Problem 11.2.** (5 pts) A non-dividend-paying stock is currently valued at \$100 per share. Its annual mean rate of return is given to be 8% while its volatility is given to be 20%.

Assuming the lognormal stock-price model, find

$$\mathbb{E}[S(4) \mid S(4) > 90].$$

- (a) \$96.55
- (b) \$101.60
- (c) \$153.30
- (d) \$159.07
- (e) None of the above.

**Solution: (c)**

In our usual notation,

$$\mathbb{E}[S(T) \mid S(T) > K] = \frac{S(0)e^{(\alpha-\delta)T}N(\hat{d}_1)}{N(\hat{d}_2)}.$$

with

$$\begin{aligned}\hat{d}_1 &= \frac{1}{\sigma\sqrt{T}} \left[ \ln \left( \frac{S(0)}{K} \right) + \left( \alpha - \delta + \frac{\sigma^2}{2} \right) T \right], \\ \hat{d}_2 &= \hat{d}_1 - \sigma\sqrt{T}.\end{aligned}$$

In the present problem,

$$\begin{aligned}\hat{d}_1 &= \frac{1}{0.2\sqrt{4}} \left[ \ln \left( \frac{100}{90} \right) + \left( 0.08 + \frac{0.04}{2} \right) \times 4 \right] = 1.2634, \\ \hat{d}_2 &= 1.2634 - 0.2\sqrt{4} = 0.8634.\end{aligned}$$

So, our answer is

$$\mathbb{E}[S(2) \mid S(2) > 95] = \frac{100e^{(0.08) \times 4}N(1.26)}{N(0.86)} = \frac{100e^{0.32} \times 0.8962}{0.8051} = 153.295.$$

**Problem 11.3.** (5 points) Let  $S(T)$  stand for the lognormally distributed time- $T$  stock price. Then, for every  $K > 0$ , we have that

$$\mathbb{E}[S(T) \mid S(T) > K] + \mathbb{E}[S(T) \mid S(T) < K]$$

equals ...

- (a)  $\mathbb{E}[S(T)]$
- (b)  $K$
- (c)  $\mathbb{E}[S(T)] - K$
- (d)  $\mathbb{E}[S(T)] + K$
- (e) None of the above.

**Solution: (e)**