M339W: April 9th, 2021.

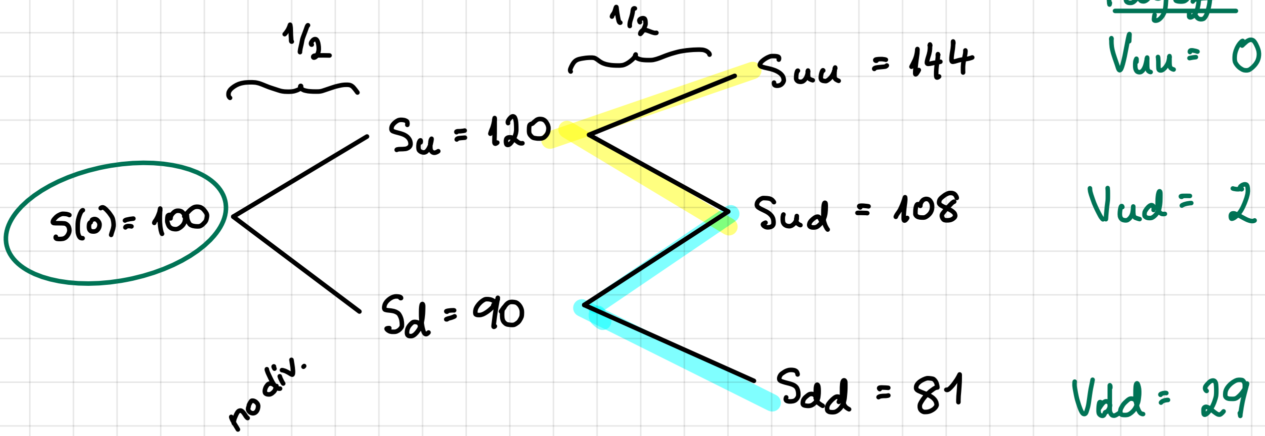
ECHW #6: Problem 6.3.

$S(0) = 100$

$n = 2$; $u = 1.2$ and $d = 0.9$

$r = 0.06$

$K = 110$, $T = 1$ American put



$$p^* = \frac{e^{(r-\cancel{\delta})h} - d}{u-d} = \frac{e^{0.06\left(\frac{1}{2}\right)} - 0.9}{1.2 - 0.9} = \frac{e^{0.03} - 0.9}{0.3} = 0.43485$$

no div.

@ the up node:

$$CV_u = e^{-0.06(1/2)}\left(p^* \cdot V_{uu} + (1-p^*) \cdot V_{ud}\right) = 1.096 = V_u^A$$

$$IE_u = 0$$

@ the down node:

$$CV_d = e^{-0.06(1/2)}\left(p^* \cdot V_{ud} + (1-p^*) \cdot V_{dd}\right) = 16.749$$

$$IE_d = 20 = V_d^A$$

@ the root node:

$$CV_0 = e^{-0.03}\left(p^* \cdot V_u^A + (1-p^*) \cdot V_d^A\right) = 11.4319 = V_C^A(0)$$

$$IE_0 = 10$$

Delta · Gamma · Theta Approximation

In our market model, we have:

- risk-free asset, i.e., borrowing / lending money @ the ccrfir r

and

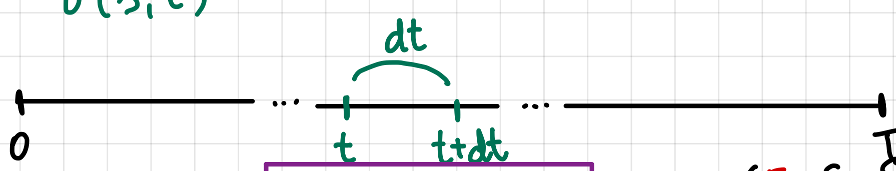
- risky asset, i.e., continuous-dividend-paying stock w/ price denoted by $\underbrace{\{S(t),\ t \geq 0\}}_{\text{stochastic process}}$

Derivative securities w/ S as the underlying are also available (just European for us).

Assume that we model S using the Black-Scholes framework.

For any portfolio in this market model, look @ its value f'tion

$v(s,t)$



$$S(t+dt) = S(t) \cdot e^{(r-\delta-\frac{\sigma^2}{2})dt + \sigma\sqrt{dt}\cdot Z} \qquad Z \sim N(0,1)$$

$S(t) \leftrightarrow s \qquad S(t+dt) \leftrightarrow s+ds$

$$ds = S(t+dt) - S(t)$$

$v(s,t) \qquad v(s+ds,\ t+dt)$

Taylor-like expansion:

$$v(s+ds,\ t+dt) \cong v(s,t) + \underbrace{\frac{\partial}{\partial s} v(s,t)}_{=\Delta(s,t)} ds + \frac{1}{2} \underbrace{\frac{\partial^2}{\partial s^2} v(s,t)}_{=\Gamma(s,t)} (ds)^2 + \underbrace{\frac{\partial}{\partial t} v(s,t)}_{=\Theta(s,t)} dt$$

Delta · Gamma · Theta Approximation

$\delta = 0$

19. Assume that the Black-Scholes framework holds. The price of a nondividend-paying stock is \$30.00. The price of a put option on this stock is \$4.00.

$S(0)=30$

$v_p(S(0),0) = 4$

You are given:

(i) $\Delta = -0.28$

(ii) $\Gamma = 0.10$

Using the delta-gamma approximation, determine the price of the put option if the stock price changes to \$31.50.

$$v_p(S(dt), dt) \cong v_p(S(0),0) + \Delta_p(S(0),0)\,ds + \frac{1}{2}\Gamma_p(S(0),0)\,(ds)^2$$

(A) \$3.40

(B) \$3.50

(C) \$3.60

(D) \$3.70

(E) \$3.80

w/ $ds = 31.50 - 30 = 1.50$

$$v_p(S(dt), dt) \approx 4 + (-0.28)(1.50) + \frac{1}{2}\cdot 0.10 \cdot (1.50)^2 = 3.6925.$$

****END OF EXAMINATION****

20. Assume that the Black-Scholes framework holds. Consider an option on a stock.

You are given the following information at time 0:

(i) $\boxed{\text{The stock price is } S(0) \text{, which is greater than 80.}}$

(ii) The option price is 2.34. $v(S(0),0) = 2.34$

(iii) The option delta is –0.181. $\Delta(S(0),0) = -0.181$

(iv) The option gamma is 0.035. $\Gamma(S(0),0) = 0.035$

The stock price changes to 86.00. Using the delta-gamma approximation, you find that the option price changes to 2.21.

$$v(S(dt), dt) = v(S(0),0) + \Delta(S(0),0)\,ds + \frac{1}{2}\Gamma(S(0),0)\,(ds)^2$$

Determine $S(0)$.

(A) 84.80 : $ds = 1.20$

(B) 85.00 $ds = 1$

(C) 85.20

(D) 85.40

(E) 85.80

$$2.21 = 2.34 - 0.181\,ds + \frac{1}{2}(0.035)(ds)^2$$

$$0.0175(ds)^2 - 0.181\,ds + 0.13 = 0$$

Solve the quadratic.

$\Rightarrow$ the possible sol'ns are : $\begin{cases} 0.7765 \\ 9.5663 \end{cases}$ × since (i)

END OF EXAMINATION