

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set 4Profit. Forward contracts. European call and put options.

Problem 4.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time-1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time-0 and liquidate your investment at time-1?

Problem 4.2. Derivative securities can reduce the risk of both the buyer and the writer of the security. *True or false?*

Problem 4.3. A short forward contract has an unlimited loss potential. *True or false?*

Problem 4.4. A farmer produces one million bushels of corn. The total cost of production is \$1.3 million. The farmer entered a forward contract to hedge at a forward price of \$2.50 per bushel on one million bushels. What is the farmer's profit?

Problem 4.5. Assume that farmer Brown is uncertain about his crop yield. Based on past experience, he thinks the following is a good model:

- 100,000 bushels with probability 1/4;
- 80,000 bushels with probability 3/4.

How many forward contracts do you think farmer Brown should short to hedge against fluctuations in corn prices at harvest time? Explain your way of thinking ...

Problem 4.6. Pancakes, Inc. produces chocolate chip pancakes. It longed a forward contract on 100 lbs of chocolate chips at \$3.00 per pound. Total fixed revenue is \$2,000 for the pancakes produced with the above chocolate chips. Other costs total \$1200. Find the company's profit.

- (a) 2,000
- (b) 1,700
- (c) 800
- (d) 500
- (e) None of the above.

Problem 4.7. The **Extra-Healty Cereal (EHC)** company longed 20,000 forward contracts on corn at \$2.80 per bushel. The revenue from cereal made with the above corn is \$200,000 while the other (non-corn) aggregate fixed and variable costs amount to \$120,000. What is the EHC's profit?

Problem 4.8. The current price of stock a certain type of stock is \$80. The premium for a 6-month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

- (a) \$80
- (b) \$85.72
- (c) \$85.84
- (d) \$85.96
- (e) None of the above.

Problem 4.9. The price of gold in half a year is modeled to be equally likely to equal any of the following prices

\$1000, \$1100, and \$1240.

Consider a half-year, \$1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

Problem 4.10. (5 points) The “Very tasty goat cheese Co” sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded, risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

- (a) 395.92
- (b) 397.98
- (c) 400
- (d) 897.98
- (e) None of the above.

Problem 4.11. For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date T in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date T . Assume that the call's premium equals \$10 and that the annual effective interest rate equals 10%.

Express your answer as an interval.

Problem 4.12. *Source: Sample IFM (Derivatives - Intro), Problem#11*

The current stock price is \$40, and the effective annual interest rate is 8%.

You observe the following option prices:

- (1) The premium for a \$35-strike, 1-year European call option is \$9.12.
- (2) The premium for a \$40-strike, 1-year European call option is \$6.22.
- (3) The premium for a \$45-strike, 1-year European call option is \$4.08.

Assuming that all call positions being compared are **long**, at what 1-year stock price range does the \$45-strike call produce a higher profit than the \$40-strike call, but a lower profit than the \$35-strike call?

Express your answer as an interval.

Problem 4.13. (2 points) In which of the following option positions is the investor exposed to an unlimited loss?

- (a) Long put option
- (b) Short put option
- (c) Long call option
- (d) Short call option
- (e) None of the above.

Problem 4.14. (3 points) The initial price of the market index is \$1000. After 3 months the market index is priced at \$950. The nominal rate of interest convertible quarterly is 4.0%.

The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

- (a) \$12.00 loss
- (b) \$14.90 loss
- (c) \$12.00 gain
- (d) \$14.90 gain
- (e) None of the above.

Problem 4.15. (3 points) *Source: Sample FM(DM) Problem #62.*

The stock price today equals \$100 and its price in one year is modeled by the following distribution:

$$S(1) \sim \begin{cases} 125 & \text{with probability } 1/2 \\ 60 & \text{with probability } 1/2 \end{cases}$$

The annual effective interest rate equals 3%.

Consider an at-the-money, one-year European put option on the above stock whose initial premium is equal to \$7.

What is the expected profit of this put option?

Problem 4.16. Aunt Dahlia simultaneously purchased

- one share of a market index at the current spot price of \$1,000;
 - one one-year, \$1,050-strike put option on the above market index for the premium of \$20.
- (i) (5 points) Is the above portfolio's payoff bounded from above? If you believe it is not, substantiate your claim. If you believe that it is, provide the upper bound.
 - (ii) (5 points) Is the above portfolio's payoff bounded from below? If you believe it is not, substantiate your claim. If you believe that it is, provide the lower bound.

Problem 4.17. (5 points) Sample FM(DM) #13.

Suppose that you short one share of a stock index for 50, and that you also buy a 60-strike European call option that expires in 2 years for 10. Assume the effective annual interest rate is 3%. If the stock index increases to 75 after 2 years, what is the profit on your combined position?