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Problem set 2

Loss elimination ratio. Policy modifications.

Problem 2.1. *Source: Sample STAM Exam Problem #87.*
Let X be the ground-up loss random variable whose density is given by

$$f_X(x) = \begin{cases} 0.01, & 0 \le x \le 80, \\ 0.03 - 0.00025x, & 80 < x \le 120. \end{cases}$$

Let there be an ordinary deductible of $d = 20$.
Calculate the **loss elimination ratio.**

$\longrightarrow$: $LER = \dfrac{\boxed{\mathbb{E}[X \wedge d]}}{\boxed{\mathbb{E}[X]}}$

$$\mathbb{E}[X \wedge d] = \int_0^{120} (x \wedge 20) f_X(x)\,dx$$

$$= \int_0^{20} x f_X(x)\,dx + \boxed{\int_{20}^{120} 20 f_X(x)\,dx}$$

$$= 20 \underbrace{\int_{20}^{120} f_X(x)\,dx}_{= S_X(20)}$$

$$S_X(20) = 1 - F_X(20) = 1 - 20(0.01) = 0.8$$

$$\boxed{\mathbb{E}[X \wedge d] = \int_0^d x f_X(x)\,dx + d \cdot S_X(d)}$$

$$\int_0^{20} x(0.01)\,dx = 0.01 \int_0^{20} x\,dx = 0.01 \left(\frac{x^2}{2}\right)_{x=0}^{20}$$

$$= 0.01 (20)^2 \cdot \frac{1}{2} = 2 ✓$$

$$\mathbb{E}[X \wedge 20] = 2 + 20(0.8) = 18 ✓$$

$$\mathbb{E}[X] = \int_0^{120} x f_X(x)\,dx$$

$$= \int_0^{80} x(0.01)\,dx + \int_{80}^{120} x(0.03 - 0.00025x)\,dx$$

$$= 0.01\left(\frac{x^2}{2}\right)_{x=0}^{80} + 0.03\left(\frac{x^2}{2}\right)_{x=80}^{120} - 0.00025\left(\frac{x^3}{3}\right)_{x=80}^{120}$$

$$= 0.005(80)^2 + 0.015\left((120)^2 - (80)^2\right) - \frac{0.00025}{3}\left((120)^3 - (80)^3\right)$$

$$= \boxed{50.6667}$$

$$LER = \frac{18}{50.6667} = 0.35526 \ \blacksquare$$

$X' \sim \text{Pareto}(\alpha=2, \theta=5)$

Problem 2.2. *Source: Sample STAM Exam Problem #127.*
Losses in 2003 follow a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 5$. Losses in 2004 are 20% uniformly higher than in 2003. An insurance covers each loss subject to a deductible of 10. Calculate the loss elimination ratio in 2004.

$\boxed{X'' = 1.2X'}$

$\longrightarrow$: $X \sim \text{Pareto}(\alpha, \theta)$

$$LER = \frac{\mathbb{E}[X \wedge d]}{\mathbb{E}[X]} = \frac{\cancel{\frac{\theta}{\alpha-1}}\left(1-\left(\frac{\theta}{\theta+d}\right)^{\alpha-1}\right)}{\cancel{\frac{\theta}{\alpha-1}}} = 1-\left(\frac{\theta}{\theta+d}\right)^{\alpha-1}$$

$X'' \sim \text{Pareto}(\alpha=2, \theta''=5\cdot(1.2)=6)$

$\uparrow$

Scale dist'n

w/ scale parameter θ.

$$LER = 1-\left(\frac{6}{6+10}\right)^{2-1}$$

$$= 1-\frac{6}{16} = 1-\frac{3}{8} = \frac{5}{8}$$

Policy Limits.

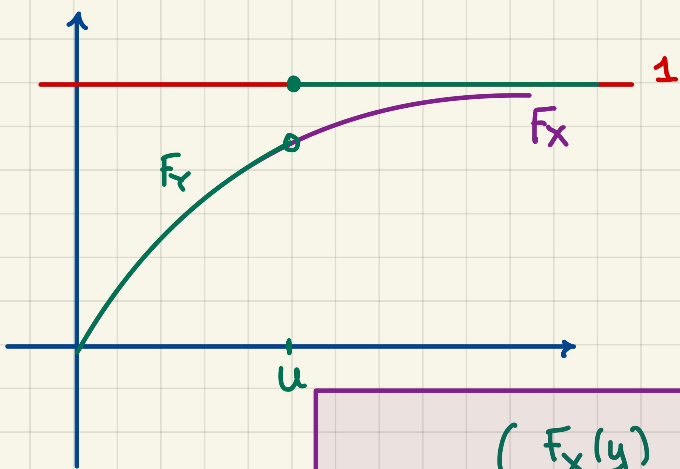
For an insurance policy w/ no deductible and a policy limit u, the insurer's payment will be

$$\boxed{Y = X \wedge u}.$$

In other words, Y is the right-censored random variable (also limited loss).

Q: Start w/ a continuous random variable X such that $\boxed{S_X(u) > 0}$.
What's the cumulative distribution function of Y?

→ :



$$\boxed{F_Y(y) = \begin{cases} F_X(y) & \text{for } y < u \\ 1 & \text{for } y \geq u \end{cases}}$$

A mixed dist'n.

$$\begin{cases} f_Y(y) = f_X(y) & \text{for } y < u \\ p_Y(u) = S_X(u) & \end{cases}$$

Problem 2.3. *Source: Two old exams 3; I forgot to note the years.*
A jewelry store purchases two separate insurance policies that together provide full coverage. You are given:

- The expected ground-up loss is 11,100.
- Policy A has an ordinary deductible of 5,000 and **no** policy limit.
- Under policy A, the expected amount paid per loss is 6,500.
- Under policy A, the expected amount paid per payment is 10,000.
- Policy B has **no** deductible and has a policy limit of 5,000.

i. **Given** that a loss has occurred, find the probability that the payment under policy B equals 5,000.
ii. **Given** that a loss less than or equal to 5,000 has occurred, what is the expected payment under policy B?

Think!

X... the random variable denoting the ground-up loss

Assume that X is continuous.

$$\mathbb{P}[X \geq 5000] = S_X(5000) = ?$$

$d = 5000$

$$\mathbb{E}[Y_A^L] = 6500 = \mathbb{E}[(X-d)_+]$$

$$\mathbb{E}[Y_A^P] = 10000 = \mathbb{E}[X-d \mid X > d] = \frac{\mathbb{E}[(X-d)_+]}{S_X(d)}$$

$$S_X(5000) = \frac{6500}{10\,000} = 0.65 \blacksquare$$