

## Put-Call Parity [cont'd].

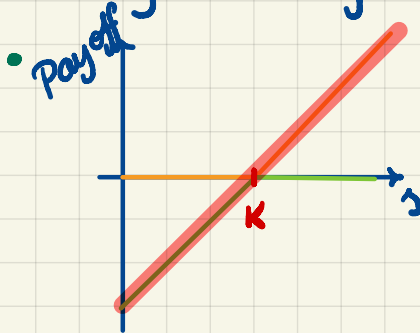
$$V_C(0) - V_P(0) = S(0) - PV_{0,T}(K)$$

More generally: for any  $t \in [0, T]$ :

$$V_C(t) - V_P(t) = S(t) - PV_{t,T}(K)$$

Remarks: • The **no-arbitrage** assumption is sufficient.

• Only works for European options.



With portfolio A, we constructed a replicating portfolio for an "off-market forward" aka a "synthetic forward".

Special Case:

strike = forward price on the stock

$\Leftrightarrow$

$$K = F_{0,T}(S) = S(0)e^{rT}$$

$\Leftrightarrow$

$$PV_{0,T}(K) = S(0)$$

$\Leftrightarrow$

$$V_C(0) - V_P(0) = 0 = S(0) - PV_{0,T}(K)$$

$\Leftrightarrow$

$$V_C(0) = V_P(0)$$

By put-call Parity

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## Advanced Derivatives Questions

1. Consider a European call option and a European put option on a nondividend-paying stock. You are given:

- (i) The current price of the stock is 60.
- (ii) The call option currently sells for 0.15 more than the put option.  $V_c(0) - V_p(0) = 0.15$
- (iii) Both the call option and put option will expire in 4 years.
- (iv) Both the call option and put option have a strike price of 70.

Calculate the continuously compounded risk-free interest rate.

(A) 0.039

(B) 0.049

(C) 0.059

(D) 0.069

(E) 0.079

$r = ?$

Put-Call Parity:

$$V_c(0) - V_p(0) = S(0) - PV_{0,T}(K)$$

$$V_c(0) - V_p(0) = S(0) - Ke^{-rT}$$

$$Ke^{-rT} = S(0) - V_c(0) + V_p(0)$$

$$\ln \mid e^{-rT} = \frac{1}{K} (S(0) - V_c(0) + V_p(0))$$

$$-rT = \ln \left( \frac{S(0) - V_c(0) + V_p(0)}{K} \right)$$

$$r = -\frac{1}{T} \ln \left( \frac{S(0) - V_c(0) + V_p(0)}{K} \right)$$

In this problem:

$$r = -\frac{1}{4} \ln \left( \frac{60 - 0.15}{70} \right) = 0.03916$$



77. You are given:

- i) The current price to buy one share of XYZ stock is 500.
- ii) The stock does not pay dividends.
- iii) The continuously compounded risk-free interest rate is 6%.
- iv) A European call option on one share of XYZ stock with a strike price of  $K$  that expires in one year costs 66.59.
- v) A European put option on one share of XYZ stock with a strike price of  $K$  that expires in one year costs 18.64.

$$T=1$$

Using put-call parity, calculate the strike price,  $K$ .

- :  $V_c(0) - V_p(0) = S(0) - PV_0(T)(K)$
- (A) 449  $66.59 - 18.64 = 500 - Ke^{-0.06}$
- (B) 452  $Ke^{-0.06} = 500 - 66.59 + 18.64$
- (C) 480  $452.05$
- (D) 559  $K = 452.05e^{0.06} = 480.0032$
- (E) 582



78. The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. You are given that the price of a 35-strike call option is 3.35 higher than the price of a 40-strike call option, where both options expire in 3 months.

Calculate the amount by which the price of an otherwise equivalent 40-strike put option exceeds the price of an otherwise equivalent 35-strike put option.

- :
- (A) 1.55  $V_c(0, K_1=35) - V_p(0, K_1=35) = S(0) - 35e^{-0.08(0.25)}$
- (B) 1.65  $V_c(0, K_2=40) - V_p(0, K_2=40) = S(0) - 40e^{-0.02}$
- (C) 1.75  $3.35 + (V_p(0, K_2=40) - V_p(0, K_1=35)) = 5e^{-0.02}$
- (D) 3.25
- (E) 3.35
- answer =  $5e^{-0.02} - 3.35 = 1.55$

