University of Texas at Austin

Quiz 4

The lognormal distribution.

Please, provide your complete solution to the following problems.

Problem 4.1. (5 points) Suppose that the failure time (in seconds) of a certain component is modeled as lognormal random variable $Y = e^X$ such that the mean of X is -0.35 and its variance is 0.04.

What is the failure time t^* such that 95% of the components of the same type would still function after that time?

Solution: We are looking for the value t^* such that

$$\mathbb{P}[Y > t^*] = 0.95 \quad \Leftrightarrow \quad \mathbb{P}[Y \leq t^*] = 0.05.$$

The critical value z^* such that $N(z^*) = 0.05$ is -1.645. So,

$$t^* = e^{-0.35+0.2(-1.645)} = 0.5071.$$

Problem 4.2. (5 points) Suppose that the failure time (in seconds) of a certain component is modeled as lognormal random variable $Y = e^X$ such that the mean of X is -0.4 and its variance is 0.04.

Find the probability that the failure time is less than 0.4 seconds.

Solution: We are looking for

$$\begin{aligned}\mathbb{P}[Y < 0.4] = \mathbb{P}[e^X < 0.4] = \mathbb{P}[X < \ln(0.4)] &= \mathbb{P}\left[\frac{X+0.4}{0.2} < \frac{\ln(0.4)+0.4}{0.2}\right] \\ &= N(-2.58) = 1 - N(2.58) = 1 - 0.9951 = 0.0049.\end{aligned}$$

Problem 4.3. (5 points) The time it takes to answer a call at a call center is lognormal with mean $e^{3/2}$ and variance $e^3(e-1)$. What is the distribution of the **rate** at which the calls get answered? State the **name** of the distribution and the value(s) of its parameter(s).

Solution: Let us denote the time it takes to answer a call by Y. Since Y is modelled as lognormal, we know that it can be rewritten as $Y = e^X$ where X is normal with some mean μ_X and some variance τ_X^2. The **rate** at which the calls get answered R is the reciprocal of Y, i.e., $R = 1/Y$. So, we can immediately see that $R = e^{-X}$. We conclude that R is lognormal with parameters $-\mu_X$ and τ_X^2. We can find the values of these parameters using the given information about the moments of Y. We get

$$e^{3/2} = \mathbb{E}[Y] = e^{\mu_X + \frac{\tau_X^2}{2}}$$

$$e^3(e-1) + (e^{3/2})^2 = \mathbb{E}[Y^2] = e^{2\mu_X + \frac{4\tau_X^2}{2}}$$

Hence,

$$\begin{aligned}\frac{3}{2} &= \mu_X + \frac{\tau_X^2}{2} \\ 4 &= 2\mu_X + 2\tau_X^2.\end{aligned}$$

We get $\mu_X = \tau_X = 1$.