University of Texas at Austin

Quiz #3
Prerequisite material.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 3.1. Monotonicity.

    (i) (3 points) Write down the definition of an **increasing** real-valued function whose domain are all nonnegative real numbers.

    (ii) (3 points) Write down the definition of an **decreasing** real-valued function whose domain are all nonnegative real numbers.

Solution:

    (i) A function $f : [0, \infty) \to \mathbb{R}$ is said to be an **increasing** function if
$$x_1 < x_2 \quad \Rightarrow \quad f(x_1) \le f(x_2).$$

    (ii) A function $f : [0, \infty) \to \mathbb{R}$ is said to be a **decreasing** function if
$$x_1 < x_2 \quad \Rightarrow \quad f(x_1) \ge f(x_2).$$

Problem 3.2. (1 point) Draw the graph of an *increasing* function.

Solution: Correct answers vary. $f(x) = x$ works.

Problem 3.3. (1 point) Draw the graph of a *decreasing*.

Solution: Correct answers vary. $f(x) = -x$ works.

Problem 3.4. (2 points) Draw the graph of a function which is neither decreasing nor decreasing.

Solution: Correct answers vary. $f(x) = |x|$ works.

Problem 3.5. (5 points) Let the accumulation function be given by
$$a(t) = (1 + 0.05)^{t^2}$$
Then, we can say the following about the continuously compounded, risk-free interest rate r associated with the above accumulation function:

    (a) $r = 0.05$

    (b) $r = 2\ln(1.05)$

    (c) $r = 0.10$

    (d) The continuously compounded, risk-free interest rate is not constant.

    (e) None of the above

Solution: (d)

$$r = \frac{d}{dt}\ln(a(t)) = \frac{d}{dt}\ln[1.05^{t^2}] = 2t\ln(1.05).$$