University of Texas at Austin

Quiz #16

Delta hedging.

Please, provide your **final answer only** to the following question(s):

Problem 16.1. (2 points) In practice, a market maker who delta-hedges **completely** insures himself against losses. *True or false?*

Solution: FALSE

Problem 16.2. (2 points) Market makers usually do not need to rebalance their portfolios after the initial hedge is established. *True or false?*

Solution: FALSE

Problem 16.3. (2 points) A market-maker writes a put option on a stock. To delta-hedge, (s)he needs to **buy** shares of the underlying stock. *True or false?*

Solution: FALSE

Problem 16.4. (2 points) A market-maker writes a call option on a stock. To decrease the delta of this position, (s)he can **write** a call on the underlying stock. *True or false?*

Solution: TRUE

Problem 16.5. (2 points) Consider an option whose payoff function is given by $v(s,T) = \min(s, 50)$. If a market-maker **writes** this option, they need to short sell shares of stock to create a delta-neutral portfolio. *True or false?*

Solution: FALSE
The "special" option in the probem can be replicated using a put and a bond. More precisely,

$$v(s,T) = \min(s, 50) = 50 - \max(50 - s, 0) = 50 - v_P(s,T).$$

So, the value of the "special" option is equal to the value of the bond minus the put price at any point in time. Hence, the delta of the "special" option is

$$\Delta(s,t) = -\Delta_P(s,t) > 0.$$

Since the market maker writes the option, the original delta of their position is $\Delta_P(S(0),0)$ which is a negative number. They need to **purchase** shares of stock to create a delta-neutral portfolio.

Problem 16.6. (5 points) Assume the Black-Scholes framework. The goal is to delta-hedge a written one-year, at-the-money straddle on a non-dividend-paying stock whose current price is \$50. The stock's volatility is 0.20.

The continuously compounded risk-free interest rate is 0.10.

What is the cost of delta-hedging the straddle using shares of the underlying stock?

(a) \$21.33

(b) \$22.58

(c) \$24.33

(d) \$25.19

(e) None of the above.

Solution: (b)
The Δ of the straddle equals

$$2\Delta_C - 1 = 2N(d_1) - 1$$

with

$$d_1 = \frac{1}{0.2}(0.10 + 0.02) = 0.6.$$

Our answer is

$$50(2N(0.60) - 1) = 50(2(0.7257) - 1) = 50(0.4515) = 22.575.$$