M378K: September 16th, 2024.

Cumulative Distribution Function [cont'd].

Example. The Exponential Distribution.

$$Y \sim E(\tau)$$

$$f_Y(y) = \frac{1}{\tau} e^{-\frac{y}{\tau}} \mathbb{1}_{[0,\infty)}(y)$$

$$F_Y(y) = ?$$

Evidently, $F_Y(y) = 0$ for $y \leq 0$

for $y > 0$: $$F_Y(y) = \mathbb{P}[Y \leq y] = \int_0^y \frac{1}{\tau} e^{-\frac{u}{\tau}}\, du = \frac{1}{\tau}(-\tau) e^{-\frac{u}{\tau}} \Big|_{u=0}^{y}$$

$$= -\left(e^{-\frac{y}{\tau}} - 1\right) = 1 - e^{-\frac{y}{\tau}}$$

Problem 5.2. Source: Sample P exam, Problem #267.
The lifetime of a certain electronic device has an exponential distribution with mean 0.50. *Calculate the probability that the lifetime of the device is greater than* 0.70*, given that it is greater than* 0.40.

→: T... lifetime

$T \sim E(\underline{\tau = 0.5})$

$$\mathbb{P}[T > 0.7 \mid T > 0.4] = \frac{\mathbb{P}[T > 0.7, T > 0.4]}{\mathbb{P}[T > 0.4]} = \frac{\mathbb{P}[T > 0.7]}{\mathbb{P}[T > 0.4]} = \frac{e^{-\frac{0.7}{\tau}}}{e^{-\frac{0.4}{\tau}}} = e^{-\frac{0.3}{\tau}} = e^{-0.6} \quad \square$$

This is a special case of the memoryless property, i.e.,

$$\boxed{\mathbb{P}[T > t + s \mid T > t] = \mathbb{P}[T > s]}$$

w/ $t = 0.4$ and $s = 0.3$

Quantiles.

Def'n. For $\alpha \in (0,1)$, the α-quantile of the dist'n of a r.v. Y is defined as the number $q_Y(\alpha) \in \mathbb{R}$ with this property:

$$\mathbb{P}[Y \le q_Y(\alpha)] = \alpha$$

$$\Longleftrightarrow$$

$$F_Y(q_Y(\alpha)) = \alpha$$

Note: If F_Y^{-1} exists, then $q_Y(\alpha) = F_Y^{-1}(\alpha)$

This is the case w/ the standard normal.

Random Vectors.

Say, we are interested in two (or more) random variables as a PAIR (or VECTOR), i.e.,

we look @ (Y_1, Y_2).

Then, we must not only look @ their "individual" dist'ns, but also @ how they are associated.

Example. Y_i ... cointoss for $i = 1, 2$ for a fair coin

independence

$\{Y_1 = H, Y_2 = H\}$ $\{Y_1 = T, Y_2 = H\}$

$\{Y_1 = H, Y_2 = T\}$ $\{Y_1 = T, Y_2 = T\}$

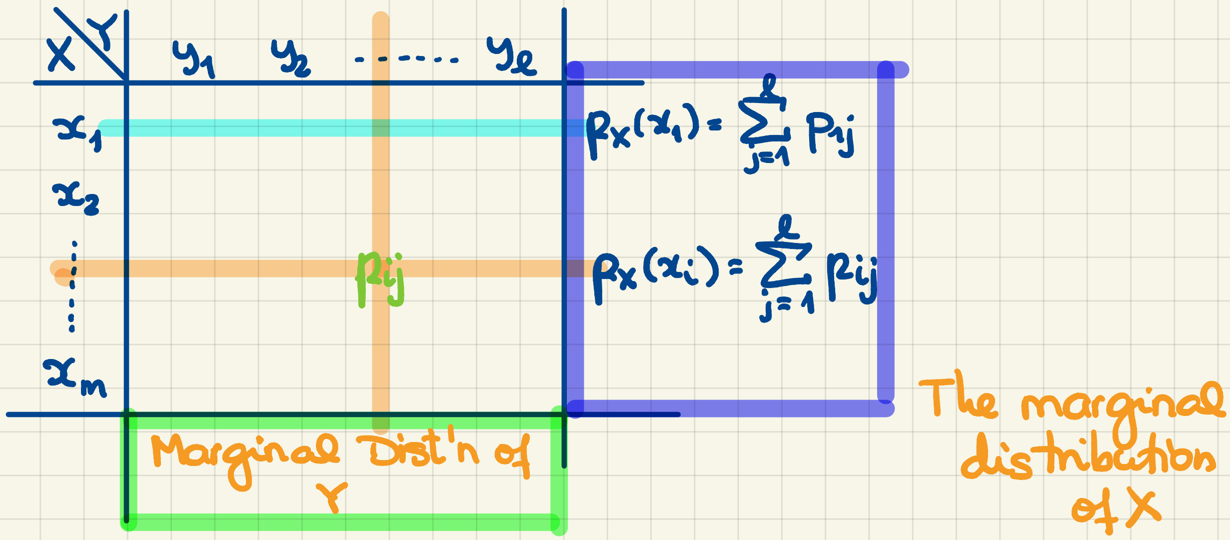
complete dependence

$\{Y_1 = H, Y_2 = H\}$ ✗

✗ $\{Y_1 = T, Y_2 = T\}$

Discrete 2D Environment.

The Joint Dist'n Table.



$p_{ij} = \mathbb{P}[X = x_i, Y = y_j]$, i.e., the joint pmf