M339 J : April 28th, 2021.

Quiz #5.1.

$N \sim \text{Poisson}(\lambda = 1)$

$$X \sim \begin{cases} 10 & \text{w/ } p_X(10) = 0.75 \\ 20 & \text{w/ } p_X(20) = 0.25 \end{cases}$$

$S = X_1 + X_2 + \dots + X_N$

$\Downarrow$

The support of S is all the multiples of 10.

Premium $= 22$

$$\mathbb{E}\left[\frac{1}{3}(22-S)_+\right] = ?$$

$$\frac{1}{3}\,\mathbb{E}\left[\underbrace{(22-S)_+}_{=D}\right]$$

Support of D : $\{0, 2, 12, 22\}$

$$\mathbb{P}[D=2] = \mathbb{P}[S=20] =$$

$$= \mathbb{P}[N=1, X_1=20] + \mathbb{P}[N=2, X_1=X_2=10]$$

$$= p_N(1)\cdot p_X(20) + p_N(2)\cdot(p_X(10))^2$$

$$= e^{-1} \cdot 0.25 + \frac{e^{-1}}{2}\cdot(0.75)^2$$

$$= e^{-1}\cdot\left(0.25 + \frac{1}{2}(0.75)^2\right) = \underline{0.53125 \cdot e^{-1}}$$

$$\mathbb{P}[D=12] = \mathbb{P}[S=10] = \underline{e^{-1}\cdot 0.75}$$

$$\mathbb{P}[D=22] = \mathbb{P}[S=0] = p_N(0) = \underline{e^{-1}}$$

$$\mathbb{E}[D] = 2\cdot 0.53125\, e^{-1} + 12\cdot 0.75\cdot e^{-1} + 22e^{-1}$$

$$= (2\cdot 0.53125 + 9 + 22)\,e^{-1} = 11.795$$

answer: $\frac{1}{3}\cdot 11.795 = 3.9317$ $\blacksquare$

125. Two types of insurance claims are made to an insurance company. For each type, the number of claims follows a Poisson distribution and the amount of each claim is uniformly distributed as follows:

Type of Claim	Poisson Parameter λ for Number of Claims in one year		Range of Each Claim Amount	
I	12	N_I	(0, 1)	X^I
II	4	N_{II}	(0, 5)	X^{II}

The numbers of claims of the two types are independent and the claim amounts and claim numbers are independent.

Calculate the normal approximation to the probability that the total of claim amounts in one year exceeds 18.

$$S_I = X_1^I + X_2^I + \cdots + X_{N_I}^I$$

$$S_{II} = X_1^{II} + X_2^{II} + \cdots + X_{N_{II}}^{II}$$

(A) 0.37

(B) 0.39

(C) 0.41

(D) 0.43

(E) 0.45

$$S = S_I + S_{II}$$

$$\mathbb{P}[S > 18] = ?$$

The normal approximation $\Rightarrow$ we need to express S in standard units

$\Rightarrow$ $\mathbb{E}[S] = ?$ and $Var[S] = ?$

$$\mathbb{E}[S] = \mathbb{E}[S_I] + \mathbb{E}[S_{II}] = 16$$

$$\left.\begin{aligned} \mathbb{E}[S_I] &= \mathbb{E}[N_I]\cdot\mathbb{E}[X^I] = 12\cdot\tfrac{1}{2} = 6 \\ \mathbb{E}[S_{II}] &= \mathbb{E}[N_{II}]\cdot\mathbb{E}[X^{II}] = 4\cdot\tfrac{5}{2} = 10 \end{aligned}\right\}$$

$$Var[S] = Var[S_I + S_{II}]$$ independence

$$= Var[S_I] + Var[S_{II}]$$

$$\mathrm{Var}[S_I] = \lambda_I \cdot \left(\mathbb{E}[X_I^2]\right)$$

compound Poisson

$$= \lambda_I \left(\mathrm{Var}[X_I] + (\mathbb{E}[X_I])^2 \right)$$

$$= \lambda_I \left(\frac{1}{12} + \left(\frac{1}{2}\right)^2 \right) =$$

$$= 12 \cdot \left(\frac{1}{12} + \frac{1}{4} \right) = 4$$

$$\mathrm{Var}[S_{II}] = \lambda_{II} \left(\mathrm{Var}[X_{II}] + (\mathbb{E}[X_{II}])^2 \right)$$

$$= 4 \left(\frac{25}{12} + \left(\frac{5}{2}\right)^2 \right) = \frac{100}{3}$$

$$\mathrm{Var}[S] = 37.3333$$

$$SD[S] = 6.1101$$

$$\mathbb{P}[S > 18] = \mathbb{P}\left[\underbrace{\frac{S-16}{6.1101}}_{\approx N(0,1) \sim Z} > \frac{18-16}{6.1101} \right] \approx \mathbb{P}[Z > 0.3273]$$

$$= 1 - \Phi(0.33)$$

$$= 0.3717 \quad \blacksquare$$

Problem. Medical and dental claims are assumed to be independent w/ compound Poisson dist'ns:

Claim Type	Claim Amt Dist'n	λ
Medical	$U(0,1000)$	2 $=\lambda_M$
Dental	$U(0,200)$	3 $=\lambda_D$

Let X be the amount of a given claim under a policy which covers both medical & dental claims. Find $\mathbb{E}[(X-100)_+]$.

$\rightarrow$: The combined claim count is Poisson$(\lambda = \underbrace{2+3}_{5})$

For an individual claim amt X, its cdf is

$$F_X(x) = \frac{\lambda_M}{\lambda}\cdot F_{X_M}(x) + \frac{\lambda_D}{\lambda}\cdot F_{X_D}(x)$$

$\Rightarrow$ its pdf is:

$$f_X(x) = \frac{\lambda_M}{\lambda}\cdot f_{X_M}(x) + \frac{\lambda_D}{\lambda}\cdot f_{X_D}(x)$$

$$= \begin{cases} \frac{2}{5}\cdot\frac{1}{1000} + \frac{3}{5}\cdot\frac{1}{200} & x\in(0,200) \\ \frac{2}{5}\cdot\frac{1}{1000} & x\in(200,1000) \end{cases}$$

$$= \begin{cases} \frac{17}{5000} & x\in(0,200) \\ \frac{1}{2500} & x\in(200,1000) \end{cases}$$

$$\mathbb{E}[(X-d)_+] = \int_{100}^{1000} (x-100)\cdot f_X(x)\,dx = \cdots = 177.$$

Check @ home!