M339D: April 26th, 2024.

Focus on the Delta.

value f'tion: $v(\boxed{s,t}, r, \sigma)$

Def'n. *The Delta*

$$\boxed{\Delta(s,t) := \frac{\partial}{\partial s} v(s,t)}$$

Example. Outright Purchase of a Non-Dividend-Paying Stock.

$v(s,t) = s$

stands for the time-t stock price

$$\Rightarrow \boxed{\Delta(s,t) = 1}$$

Example. European Call

Δ B

$$v_c(s,t) = s\cdot N(d_1(s,t)) - Ke^{-r(T-t)} N(d_2(s,t))$$

w/
$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$

and $\quad d_2 = d_1 - \sigma\sqrt{T-t}$

By def'n:
$$\boxed{\Delta_c(s,t) = \frac{\partial}{\partial s} v_c(s,t)}$$

After the chain rule and product rule

$$\boxed{\Delta_c(s,t) = N(d_1(s,t))} > 0$$

The positivity makes sense since the call is long w.r.t. the underlying.

Example. European Put.

Put-Call Parity

$$v_c(s,t) - v_p(s,t) = s - Ke^{-r(T-t)}$$

$$\frac{\partial}{\partial s}\Big| \quad \Delta_c(s,t) - \Delta_p(s,t) = 1$$

$$\underline{\Delta_p(s,t)} = \Delta_c(s,t) - 1 = N(d_1(s,t)) - 1 = \underline{-N(-d_1(s,t))} < 0$$

(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

$T = 1/4$

8. You are considering the purchase of a 3-month 41.5-strike European ~~American~~ call option on a nondividend-paying stock.

$K = 41.5$

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40. $S(0) = 40$

(iii) The stock's volatility is 30%. $\sigma = 0.3$

(iv) The current call option delta is 0.5. $\Delta_c(S(0),0) = \boxed{0.5 = N(d_1(S(0),0))}$

Determine the current price of the option. $\boxed{v_c(S(0),0) = ?}$

(A) $20 - 20.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(B) $20 - 16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(C) $20 - 40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(D) $16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

(E) $40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

$$\boxed{d_1(S(0),0) = 0}$$

$$d_2(S(0),0) = d_1(S(0),0) - \sigma\sqrt{T}$$
$$= 0 - 0.3\sqrt{0.25}$$
$$\boxed{d_2(S(0),0) = -0.15}$$

$$v_c(S(0),0) = S(0)\boxed{\cdot N(d_1(S(0),0))} - Ke^{-rT}\cdot N(d_2(S(0),0))$$
$$= \underbrace{40 \cdot (0.5)}_{20} - Ke^{-0T}\cdot N(-0.15)$$

$$d_1(S(0),0) = \frac{1}{\sigma\sqrt{T}}\boxed{\left[\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T\right]} = 0$$

$$\left(r+\frac{\sigma^2}{2}\right)\cdot T = -\ln\left(\frac{S(0)}{K}\right)$$

$$r = -\frac{1}{T}\ln\left(\frac{S(0)}{K}\right) - \frac{\sigma^2}{2} = \underline{0.1032}$$

$$v_c(S(0),0) = 20 - \underbrace{41.5\, e^{-0.1032(0.25)}}_{=\,40.453} \cdot \underbrace{N(-0.15)}_{1-N(0.15)}$$

$$v_c(S(0),0) = 20 - 40.453\,(1-N(0.15))$$

$$= 40.453 \cdot \underbrace{N(0.15)}_{\int_{-\infty}^{0.15} f_Z(z)\,dz} - 20.453$$

$$= \int_{-\infty}^{0.15} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\,dz \qquad \square$$