$\{X_j\,;\, j=1,2,\dots\}$ ... i.i.d. severity r.v.s
N ... $\mathbb{N}_0$-valued frequency r.v.
independent

$$\boxed{S = X_1 + X_2 + \cdots X_N}$$

aggregate losses

We could adapt the use of the CLT:

$$S \sim \text{Normal}(\text{mean} = \mu_S, \text{variance} = \sigma_S^2)$$

w/ $\mu_S := \mathbb{E}[S] = \mathbb{E}[N] \cdot \mathbb{E}[X]$ Wald's Identity

$\sigma_S^2 := \text{Var}[S] = \mathbb{E}[N] \cdot \text{Var}[X] + \text{Var}[N]\,(\mathbb{E}[X])^2$

16. You are given:

		Mean	Standard Deviation
N	Number of Claims	8 ✓	3 ✓
X	Individual Losses	10,000 ✓	3,937

> independent

Using the normal approximation, determine the probability that the aggregate loss will exceed 150% of the expected loss.

(A) $\Phi(1.25)$

(B) $\Phi(1.5)$

(C) $1-\Phi(1.25)$

(D) $1-\Phi(1.5)$

(E) $1.5\,\Phi(1)$

$$S = X_1 + X_2 + \cdots + X_N$$

$$\mathbb{P}[S > 1.5\mu_S] = ?$$

- $\mu_S = \mathbb{E}[N]\cdot\mathbb{E}[X] = 8\cdot(10{,}000) = \underline{80{,}000}$
- $\mathrm{Var}[S] = \mathbb{E}[N]\cdot\mathrm{Var}[X] + \mathrm{Var}[N]\cdot(\mathbb{E}[X])^2$

$$= 8\cdot(3937)^2 + 3^2\cdot(10{,}000)^2$$

$$= 1{,}023{,}999{,}752$$

$$\Rightarrow \sigma_S = 32{,}000$$

$$\mathbb{P}[S > 1.5\mu_S] = \mathbb{P}\left[\underbrace{\frac{S-\mu_S}{\sigma_S}}_{N(0,1)\sim Z} > \frac{1.5\mu_S - \mu_S}{\sigma_S}\right]$$

$$= \mathbb{P}\left[Z > \frac{0.5\mu_S}{\sigma_S}\right] = \mathbb{P}\left[Z > \frac{40{,}000}{32{,}000}\right]$$

$$= \mathbb{P}[Z > 1.25] = 1 - \mathbb{P}[Z \leq 1.25]$$

$$= 1 - \Phi(1.25) \quad \square$$

32. For an individual over 65:

(i) The number of pharmacy claims is a Poisson random variable with mean 25.

$N \sim Poisson(\lambda = 25)$

(ii) The amount of each pharmacy claim is uniformly distributed between 5 and 95.

$X \sim U(5,95)$

(iii) The amounts of the claims and the number of claims are mutually independent.

Determine the probability that aggregate claims for this individual will exceed 2000 using the
normal approximation.

$\mathbb{P}[S > 2000] = ?$

(A) $1 - \Phi(1.33)$

(B) $1 - \Phi(1.66)$

(C) $1 - \Phi(2.33)$

(D) $1 - \Phi(2.66)$

(E) $1 - \Phi(3.33)$

- $\mu_S = \mathbb{E}[N] \cdot \mathbb{E}[X] = 25 \cdot \frac{5+95}{2} = 25(50) = 1250$
- $Var[S] = \mathbb{E}[N] \cdot Var[X] + Var[N] \cdot (\mathbb{E}[X])^2$

$$= 25\left(\frac{(95-5)^2}{12} + 50^2\right)$$

$$= 79{,}735$$

$$\Rightarrow \sigma_S = 281.74$$

$$\mathbb{P}[S > 2000] = \mathbb{P}\left[\underbrace{\frac{S-\mu_S}{\sigma_S}}_{\approx N(0,1) \sim Z} > \boxed{\frac{2000-\mu_S}{\sigma_S}}\right]$$

$$= \mathbb{P}[Z > 2.66] = 1 - \Phi(2.66)$$

$\square$

"Def'n." Insurance on the aggregate losses, subject to an ordinary deductible is called stop-loss insurance.
The expected cost of this type of insurance is called the net stop-loss premium.

$$\boxed{\mathbb{E}\left[(S-d)_+\right]}$$

Note: The following are useful in problems:

- the tail formula: $\mathbb{E}\left[(S-d)_+\right] = \int_d^{+\infty} (1-F_S(x))\,dx$
- $\mathbb{E}\left[(S-d)_+\right] = \mathbb{E}[S] - \mathbb{E}[S \wedge d]$
- combinatorics.