

On portfolios

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Problem #1

Your initial wealth is exactly \$100. You are allowed to invest in shares of a particular stock. You are also allowed to both lend and borrow at the continuously compounded, risk-free interest rate of 0.05. Keeping your money uninvested is **not allowed**.

You can rebalance your portfolio every morning, once you have observed the opening stock price. This means that you can change the number of shares you own (if you decide to do so) and accordingly adapt your risk-free investment.

You proceed to create a “rule” according to which you will be rebalancing your portfolio. Here is a possible rule you can use:

Start by purchasing half a share of stock. Thereafter, if the stock had gone up overnight, you invest an extra \$10 in the stock (you might need to borrow money); if the stock had gone down, you take out \$10 worth of investment from the stock and put that in the savings account (you might need to short the stock); if the stock price remains the same, change nothing.

Over the following 10 days, you observe the following stock prices for a non-dividend-paying stock:

Day	0	1	2	3	4	5	6	7	8	9	10
Stock price	100	80	64	80	64	80	100	80	64	80	100

As the time passes you follow investment rules above to rebalance your portfolio. Complete the following table describing your portfolio **just before and just after** the rebalancing is done. Even more precisely, for the 10 days, both for “before” and “after” the rebalancing, print out:

- the number of shares of stock in the portfolio,
- the balance of the risk-free investment,
- the wealth in the stock,
- the total wealth.

Solution: First, define the ccrfir:

```
r=0.05
```

Second, make a vector containing the stock prices:

```
s=c(100, 80, 64, 80, 64, 80, 100, 80, 64, 80, 100)
```

Create vectors to contain my quantities of interest:

```
#this is going to contain the number of shares in the portfolio  
#at any time  
pi.v=numeric(length(s))
```

```
#this is the cash at any time
cash=numeric(length(s))
#the total wealth at any time
wealth=numeric(length(s))
```

Now, we implement the rule. We start with the initial condition.

```
pi.v[1]=1/2
wealth[1]=100
cash[1]=wealth[1]-pi.v[1]*s[1]
```

A rebalancing function we will use every day.

```
rebalance<-function(s.beg, s.end, shares){
  if (s.beg==s.end){
    return(shares)
  } else {
    if (s.beg>s.end) {
      return(shares-10/s.end)
    } else {
      return(shares+10/s.end)
    }
  }
}
```

Now we cycle through the ten days.

```
library(tibble)
library(dplyr)
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
## 
##     filter, lag
## The following objects are masked from 'package:base':
## 
##     intersect, setdiff, setequal, union
rows <- vector("list", 2 * 10) # 2 rows per day: before/after
k <- 1

for (i in 1:10) {
  cash[i+1] <- cash[i] * exp(r/365)
  wealth[i+1] <- s[i+1] * pi.v[i] + cash[i+1]

  # BEFORE rebalance
  rows[[k]] <- tibble(
    day = i,
    when = "before",
    price = s[i+1],
    shares = pi.v[i],
    cash = cash[i+1],
    stock_value = pi.v[i] * s[i+1],
    total_wealth = wealth[i+1]
  )
  k <- k + 1
}
```

```

# rebalance
pi.v[i+1] <- rebalance(s[i], s[i+1], pi.v[i])
cash[i+1] <- wealth[i+1] - s[i+1] * pi.v[i+1]

# AFTER rebalance
rows[[k]] <- tibble(
  day = i,
  when = "after",
  price = s[i+1],
  shares = pi.v[i+1],
  cash = cash[i+1],
  stock_value = pi.v[i+1] * s[i+1],
  total_wealth = wealth[i+1]
)
k <- k + 1
}

balances <- bind_rows(rows)
#balances

library(dplyr)
library(gt)
## Warning: package 'gt' was built under R version 4.5.2
library(dplyr)
library(knitr)

balances_pretty <- balances %>%
  mutate(across(where(is.numeric), ~ round(.x, 2))) %>%
  arrange(day, when) %>%
  select(day, when, price, shares, stock_value, cash, total_wealth)

knitr::kable(
  balances_pretty,
  caption = "Portfolio evolution before and after daily rebalancing"
)

```

Table 1: Portfolio evolution before and after daily rebalancing

day	when	price	shares	stock_value	cash	total_wealth
1	after	80	0.38	30.00	60.01	90.01
1	before	80	0.50	40.00	50.01	90.01
2	after	64	0.22	14.00	70.02	84.02
2	before	64	0.38	24.00	60.02	84.02
3	after	80	0.34	27.50	60.02	87.52
3	before	80	0.22	17.50	70.02	87.52
4	after	64	0.19	12.00	70.03	82.03
4	before	64	0.34	22.00	60.03	82.03
5	after	80	0.31	25.00	60.04	85.04
5	before	80	0.19	15.00	70.04	85.04
6	after	100	0.41	41.25	50.05	91.30
6	before	100	0.31	31.25	60.05	91.30
7	after	80	0.29	23.00	60.06	83.06
7	before	80	0.41	33.00	50.06	83.06

day	when	price	shares	stock_value	cash	total_wealth
8	after	64	0.13	8.40	70.07	78.47
8	before	64	0.29	18.40	60.07	78.47
9	after	80	0.26	20.50	60.08	80.58
9	before	80	0.13	10.50	70.08	80.58
10	after	100	0.36	35.62	50.08	85.71
10	before	100	0.26	25.62	60.08	85.71