

M358K: November 30<sup>th</sup>, 2020.

7.10. from book.

radius of conf. interval = margin of error

$$= \boxed{\text{critical value}} \times \underbrace{\text{std error}}_{\text{data \& sample size}}$$

$\updownarrow$   
C... confidence level

data  
& sample size

upper tail probability :  $\frac{1-C}{2}$

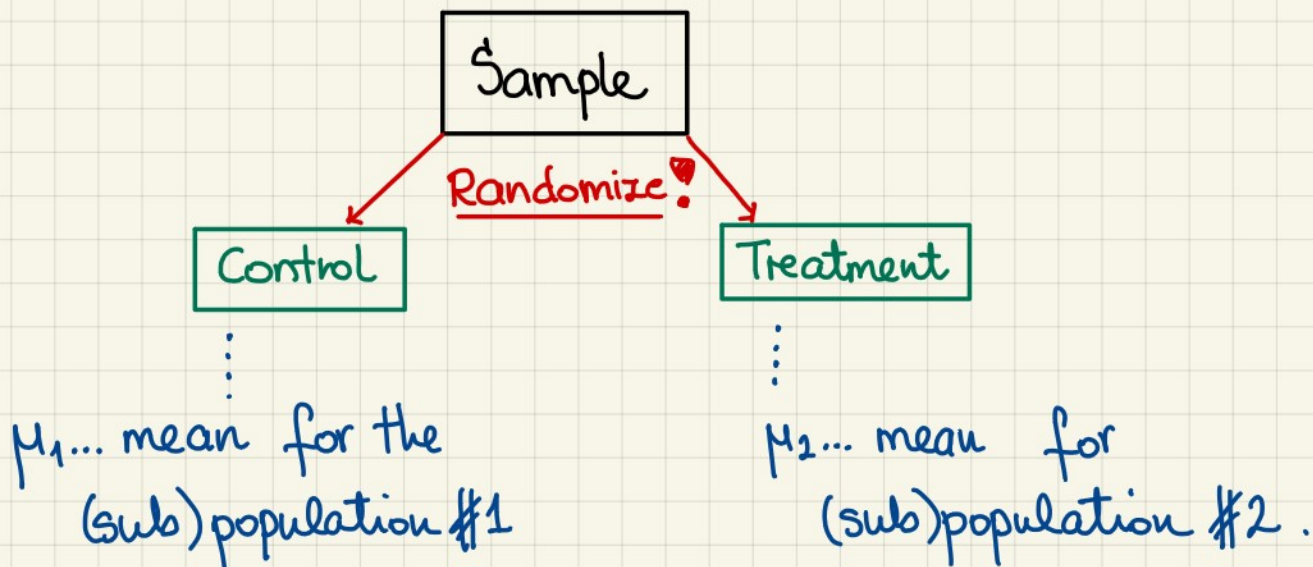
$$t^* > z^*$$

## Statistical Inference for Two Means.

### Inspiration.

Consider an experiment for testing whether a new drug works better than an existing drug.

"OLD DRUG" vs. "NEW DRUG"



### Goals:

- confidence intervals for  $\mu_1 - \mu_2$
- hypothesis testing

The focus is on  $\mu_1 - \mu_2$

$\Rightarrow$  We should look @

$$\bar{X}_1 - \bar{X}_2$$

A RANDOM VARIABLE

where  $\bar{X}_i$  ... sample mean for sample  $i=1,2$   
control treatment

Assumptions:

- both population dist'n's are normal
- the two samples are independent

$\Rightarrow$  for both groups  $i=1,2$ :

$\bar{X}_i \sim \text{Normal}(\text{mean}=\mu_i, \text{var}=\frac{\sigma_i^2}{n_i})$  w/  $n_i$  ... sample size

$$\Rightarrow \bar{X}_1 - \bar{X}_2 \sim \text{Normal}(\text{mean}=\mu_1 - \mu_2, \text{var}=\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$$

Confidence intervals:

- point estimate :  $\bar{x}_1 - \bar{x}_2$
- we use  $s_1$  and  $s_2$ , i.e., the sample standard deviations, instead of the  $\sigma_1$  and  $\sigma_2$

$\Rightarrow$  we must use the t-distribution

w/  $df = \min(n_1, n_2) - 1$

Hypothesis testing:

Our null is always :  $H_0: \mu_1 = \mu_2$  (no effect)

$\Rightarrow$  Under the null, the observed value of the test statistic is :

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$