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Log-normal stock prices: Tail probabilities.

Problem 6.1. You are considering an investment in a non-dividend-paying stock versus an investment in a savings account. According to your belief, the stock's mean rate of return is α and its volatility is σ.

The continuously compounded interest rate is equal to r.

What is the probability that the stock outperforms the savings account at time$-T$? You should leave your final answer in terms of the function N.

Solution: With $Z \sim N(0,1)$, we are looking for the probability

$$\begin{aligned}\mathbb{P}[S(T) > S(0)e^{rT}] &= \mathbb{P}\left[S(0)e^{(\alpha-\sigma^2/2)T+\sigma\sqrt{T}Z > S(0)e^{rT}}\right] \\ &= \mathbb{P}[(\alpha-\sigma^2/2)T+\sigma\sqrt{T}Z > rT] \\ &= \mathbb{P}\left[Z > \frac{\sqrt{T}}{\sigma}(r-\alpha+\sigma^2/2)\right] \\ &= N\left(\frac{(\alpha-r-\sigma^2/2)\sqrt{T}}{\sigma}\right).\end{aligned}$$

Problem 6.2. Assume the Black-Scholes framework. You are given the following information for a stock that pays dividends continuously at a rate proportional to its price:

(i) The current stock price is \$250.
(ii) The stock's volatility is 0.3.
(iii) The continuously compounded expected rate of stock-price appreciation is 15%.

Find the value s^* such that

$$\mathbb{P}[S(4) > s^*] = 0.05.$$

(a) \$861.65
(b) \$874.18
(c) \$889.94
(d) \$905.48
(e) None of the above.

Solution: (e)

$$s^* = 250e^{(0.15-0.045)(4)+0.3(2)(1.645)} = 1020.92$$