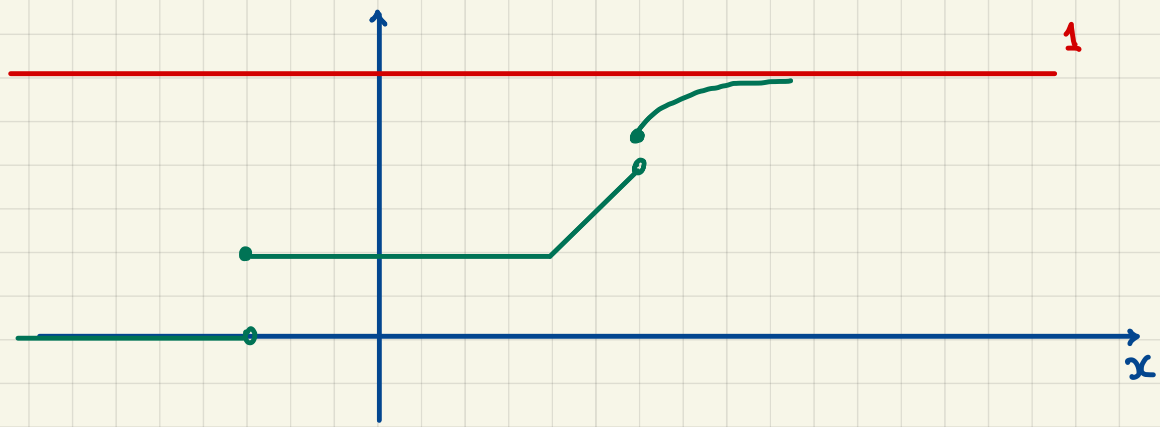
M358K: August 31st, 2022.

Probability Review.

Def'n. For any random variable X the **cumulative distribution function (cdf)** of X is a function $F_X : \mathbb{R} \longrightarrow$ [0,1] given by

$$\boxed{F_X(x) = \mathbb{P}[X \leq x] \quad \text{for all } x \in \mathbb{R}}$$



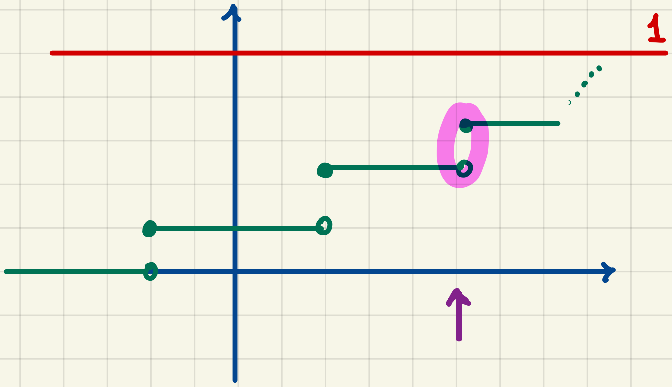
The cdf gives us complete information about the distribution of a random variable.

Q: $\lim_{x \to -\infty} F_X(x) = \underline{\ 0\ }$

Q: $\lim_{x \to +\infty} F_X(x) = \underline{\ 1\ }$

Note: Nondecreasing !

Q: What if your cdf is a step function?



Then, your r.v. is discrete, i.e., it can take up to countably many values.

It's usually more convenient to express the distribution of a discrete r.v. using its probability (mass) f'tion (pmf).

In general, the support of a random variable is (vaguely) the set of all the values it can take

For discrete r.v.s the support is the set of all the points @ which the cdf jumps.

For those points, i.e., for every x in the support of the discrete r.v. X,

the pmf is

$$
\begin{aligned}
p_X(x) &= \mathbb{P}[X = x] \\
&= \text{size of the jump} \\
&= F_X(x) - F_X(x-)
\end{aligned}
$$

left limit

Bernoulli.

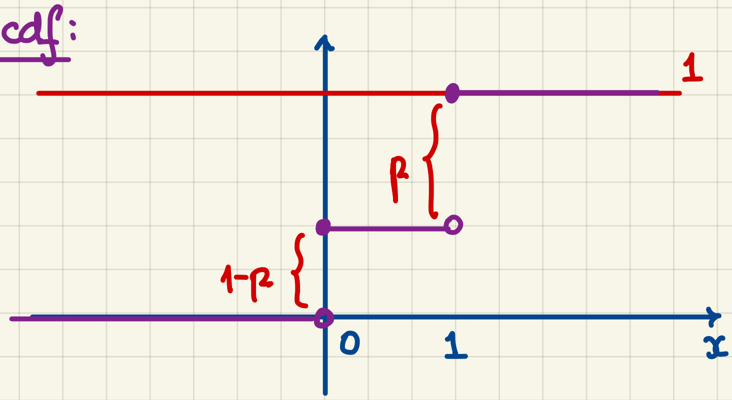
The support of an X w/ the Bernoulli dist'n is $\{0, 1\}$.
We usually interpret "1" as "success"
and "0" as "failure".

We denote the probability of success in a single Bernoulli trial by p.

$\boxed{\text{Notation:}}$

- $X \sim \begin{cases} 1 & \text{w/ probab. } p \\ 0 & \text{w/ probab. } 1-p \end{cases}$
- $X \sim \text{Bernoulli}(p)$
- $p_X(1) = p$
  $p_X(0) = 1-p$

Bernoulli cdf:



Binomial.

Models the number of successes in a set of independent identically distributed Bernoulli dist'n.

p... the probability of success in a single trial
n... the number of trials

$$\boxed{Y \sim \text{Binomial}(n, p)}$$

$\text{Support}(Y) = \{0, 1, \dots, n\}$

the pmf of Y: $\boxed{p_Y(k) = \binom{n}{k} p^k (1-p)^{n-k}}$