UNIVERSITY OF TEXAS AT AUSTIN, DEPARTMENT OF MATHEMATICS
M358K - Applied Statistics

MOCK IN-TERM III

Problem 1.1. (5 points)

Lord Clarence Emsworth keeps meticulous records of the feeding patterns of his prized pet hog *the Empress of Blandings.* He observes that the average weight of slops consumed per diem during the month of November equals 19 lbs with a sample standard deviation of 3lbs. As the incessantly perused tome "The Care of the Pig" states that the daily intake of slops should amount to 20 lbs, Lord Emsworth gets worried.

Luckily, his secretary, the efficient Baxter, knows a bit of statistics and can test whether the Empress is underfed. Which $p-$ value will the efficient Baxter report?

a. Between 0 and 0.005.
b. Between 0.005 and 0.01.
c. Between 0.01 and 0.025.
d. Between 0.025 and 0.05.
e. None of the above.

Solution: d.
Let μ denote the mean daily intake of slops. We are testing

$$H_0 : \mu = \mu_0 = 20 \quad vs. \quad H_a : \mu < \mu_0 = 20.$$

According to the given information, the observed value of the $t-$statistic (under the null) is, in our usual notation, equal to

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{19 - 20}{\frac{3}{\sqrt{30}}} = -1.8257$$

Consulting the tables with the number of degrees of freedom equal to $30-1 = 29$ (since there are 30 days in November), we see that the $p-$value is between 0.025 and 0.05.

Note: Using **R**, we get the $p-$value of 0.0391.

Problem 1.2. (5 points)

Bertie and Tuppy are playing a game. Bertie simulates 5 draws from a normal distribution without telling the parameter values to Tuppy. Then, Tuppy calculates the 95% confidence interval for the mean parameter.

Bertie simulated the values which resulted in a sample average of 5.88 and the sample standard deviation of 1.96. Which confidence interval should Tuppy get?

a. 5.88 ± 2.4333
b. 5.88 ± 2.7209
c. 5.88 ± 2.7764
d. 5.88 ± 2.2532
e. None of the above.

Solution: a.
The point estimate is given to be $\bar{x} = 5.88$. With the given 95% confidence level, and $5-1 = 4$ degrees of freedom, the critical value of the $t-$distribution is $t^* = 2.776$. The sample standard deviation is given to be 1.96. So, the standard error equals $1.96/\sqrt{5}$. Altogether, the 95%-confidence interval is

$$5.88 \pm 2.776 \left(\frac{1.96}{\sqrt{5}}\right) = 5.88 \pm 2.4333.$$

Problem 1.3. (5 points) *Source: "Mathematical Statistics with Applications in R" by Ramachandran and Tsokos.* A cross is hypothesized to result in a 3 : 1 phenotypic ratio of red-flowered to white-flowered plants. You set up a hypothesis test to test this claim. Suppose your cross actually produces 170 red- and 30 white-flowered plants. What is the $p-$value you obtain?

a. Less than 0.005.
b. Between 0.005 and 0.01.
c. Between 0.01 and 0.025.
d. Between 0.025 and 0.05.
e. None of the above.

Solution: a.
Let p denote the population proportion of red-flowered plants. We are testing

$$H_0 : p = 3/4 \quad \text{vs.} \quad H_a : p \neq 3/4.$$

Under the null, the observed value of the $z-$statistic is

$$z = \frac{\frac{170}{200} - \frac{3}{4}}{\sqrt{\frac{(3/4)(1/4)}{200}}} = 3.266$$

The $p-$value is approximately than $2(1 - \Phi(3.27)) = 2\Phi(-3.27) = 2(0.0005) = 0.001$.

Problem 1.4. (5 points) A die is rolled 60 times and the face values are recorded. The results are as follows:

Up face	1	2	3	4	5	6
Number of occurrences	8	11	5	12	15	9

You test the hypothesis that the die is fair. What can you say about the $p-$value?

(a) Less that 0.05.

(b) Between 0.05 and 0.10.

(c) Between 0.10 and 0.20.

(d) Between 0.20 and 0.30.

(e) It's greater than 0.30.

Solution: (e)
Let $p_i, i = 1, \dots, 6$ be the probability that the die falls on i. We are testing

$$\begin{aligned} H_0 : &\quad p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = \frac{1}{6} \\ &\quad vs. \\ H_a : &\quad \text{At least one of the probabilities } p_i \text{ is different from } \frac{1}{6}. \end{aligned}$$

The observed value of the χ^2-statistics is

$$q^2 = \frac{1}{10}((8-10)^2 + (11-10)^2 + (5-10)^2 + (12-10)^2 + (15-10)^2 + (9-10)^2) = 6.$$

With $6 - 1 = 5$ degrees of freedom, using the χ^2-table, we see that the $p-$value is more than 0.30.

Problem 1.5. The final exam in a particular course has 100 multiple-choice questions: for each question there are five offered answers exactly one of which is correct. Out of the 100 questions, 36 questions come from a public problem bank. A student diligently memorizes the correct answers to all of those questions. However, since the student learned by rote, they are not able to do any work on the remaining questions. So, in the exam, they are able to answer exactly 36 questions correctly. For the remaining questions, the student guesses completely at random and independently between problems. Approximately, what is the probability that the student achieves a passing score of 65?

(a) Closest to zero.

(b) Closest to 0.001.

(c) Closest to 0.01

(d) Closest to 0.1.

(e) Closest to 0.5.

Solution: (a)
The number of problems that the student guesses on at random is 64. The probability of guessing correctly for a single problem is 1/5. So, the total number of problems that the student guesses correctly is, in our usual notation,

$$X \sim Binomial(n = 64, p = 0.2).$$

Out of the problems that the student guesses on at random, they need to guess correctly on at least $65 - 36 = 29$. The probability of passing is $\mathbb{P}[X \geq 29]$. The mean of the random variable X is $np = 12.8$ and its standard deviation is $\sqrt{np(1-p)} = 3.2$ Using the normal approximation to the binomial, we get

$$\mathbb{P}[X \geq 29] = \mathbb{P}[X > 28.5] = \mathbb{P}\left[\frac{X - 12.8}{3.2} > \frac{28.5 - 12.8}{3.2}\right] = 1 - \Phi(4.90625) \approx 0.$$

Problem 1.6. (5 points) An improvement is a process for manufacturing is being considered. Samples are taken both from parts manufactured using the existing method and the new method. It is found that 50 out of a sample of 1000 items produced using the existing method are defective. It is also found that 40 out of a sample of 1600 items produced using the new method are defective. The two samples are independent.

Find the 80%-confidence interval for the true difference in the proportions of defectives produced using the existing and the new method. *Note: Round your point estimate and the margin of error to four places after the decimal point.*

(a) $(0.0149, 0.0351)$

(b) $(0.0171, 0.0329)$

(c) $(0.0120, 0.0380)$

(d) $(0.0095, 0.0405)$

(e) None of the above.

Solution: (a)
Let p_1 denote the proportion of defectives resulting from the existing method and let p_2 denote the proportion of defectives resulting from the new method. We are supposed to find the 80%-confidence interval for $p_1 - p_2$.

The sample proportion of defectives for the existing method is $\hat{p}_1 = 50/1000 = 0.05$ and the sample proportion of defectives for the new method is $\hat{p}_2 = 40/1600 = 0.025$. So, the standard error equals

$$\sqrt{\frac{0.05(0.95)}{1000} + \frac{0.025(0.975)}{1600}} = 0.0079.$$

So, with the critical value corresponding to the 80%-confidence being $z^* = 1.28$, we get that the margin of error is

$$1.28(0.0079) = 0.0101.$$

Hence, the confidence interval is

$$0.025 \pm 0.0101 = (0.0149, 0.0351).$$

Problem 1.7. (5 points) Let the random sample $X_1, \dots, X_{10}$ be drawn from a normal distribution with mean 2 and variance 1. Define

$$Y = \sum_{i=1}^{10} (X_i - 2)^2.$$

Find the constant q such that

$$\mathbb{P}[Y \leq q] = 0.975.$$

(a) 18.31

(b) 19.02

(c) 20.48

(d) 21.92

(e) None of the above.

Solution: (c)
The random variable Y has the χ^2-distribution with 10 degrees of freedom. In the χ^2-tables, we find that $q = 20.48$.

Problem 1.8. (5 points) In a simple random sample of 1000 Austinites owning televisions, it is found that 480 do not have cable (but do have Netflix or some such or just game on the big screen). Find an 92% confidence interval for the true proportion of Austinites with television who do not have cable. *Note: Round the margin of error to four places after the decimal point.*

(a) 0.48 ± 0.0277
(b) 0.48 ± 0.0260
(c) 0.48 ± 0.0310
(d) 0.48 ± 0.0158
(e) None of the above.

Solution: (a)
The observed proportion is $\hat{p} = 480/1000 = 0.48$. So, the standard error equals

$$\sqrt{\frac{0.48(0.52)}{1000}} = 0.0158.$$

With the critical value associated with the 92%-confidence level equal to 1.75, we get the margin of error equal to

$$1.75(0.0158) = 0.02765 = 0.0277.$$

Hence, our confidence interval is 0.48 ± 0.0277.

Problem 1.9. Let p_m and p_f be the population proportions of male and female warblers who return to their hatching site. You want to test whether the two proportions are different. The observed number of males who returned is 135 out of 900, while the observed number of females who returned is 84 out of 700. What is your decision for this hypothesis test? *Note: Keep four significant places after the decimal point for all your point estimates.*

(a) Reject at the 0.01 significance level.

(b) Fail to reject at the 0.01 significance level; reject at the 0.025 significance level.

(c) Fail to reject at the 0.025 significance level; reject at the 0.05 significance level.

(d) Fail to reject at the 0.05 significance level; reject at the 0.10 significance level.

(e) Fail to reject at the 0.10 significance level.

Solution: (d)
We are testing

$$H_0 : p_m = p_f \quad vs. \quad H_a : p_m \neq p_f.$$

The observed proportions are

$$\hat{p}_m = \frac{135}{900} = 0.15 \quad \text{and} \quad p_f = \frac{84}{700} = 0.12.$$

The pooled proportion estimate is

$$\hat{p} = \frac{135 + 84}{900 + 700} = 0.136875 = 0.1369.$$

The observed value of the $z-$statistic is

$$z = \frac{\hat{p}_m - \hat{p}_f}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.15 - 0.12}{\sqrt{0.1369(1 - 0.1369)\left(\frac{1}{900} + \frac{1}{700}\right)}} \approx 1.7318.$$

Since this is a two-tailed test, we have that the $p-$value equals about $2\Phi(-1.7318)$. This value is between $2\Phi(-1.74)$ and $2\Phi(-1.73)$. Using the standard normal tables, we conclude that the $p-$value is between $2(0.0409)$ and $2(0.0418)$, i.e., between 0.0818 and 0.0836.

Problem 1.10. (5 points) *Source: "Probability and Statistical Inference" by Hogg, Tanis, and Zimmerman.*
A study was conducted to determine whether there is an association between the choice of the most credible media source for reporting news and the education level. The results are displayed in the following table:

	Newspaper	Television	Radio	Total
Grade School	45	22	6	73
High School	94	115	30	239
College	49	52	13	114
Total	188	189	49	426

Your goal is to test whether the choice of the most credible medium is independent from the education level. The observed value of the relevant test statistic is 11.399. What is your decision?

(a) Reject at the 0.01 significance level.

(b) Fail to reject at the 0.01 significance level; reject at the 0.025 significance level.

(c) Fail to reject at the 0.025 significance level; reject at the 0.05 significance level.

(d) Fail to reject at the 0.05 significance level; reject at the 0.10 significance level.

(e) Fail to reject at the 0.10 significance level.

Solution: (b)
The distribution of the test statistic is approximately χ^2 with $(3-1)(3-1) = 4$ degrees of freedom. Consulting the χ^2-table, we see that the given observed value of the test statistic is between the critical values $\chi^2_{0.01}(df = 4)$ and $\chi^2_{0.025}(df = 4)$. So, the $p-$value is between 0.01 and 0.025.

Problem 1.11. An experiment was designed to test whether people's mean reaction times to an auditory stimulus are different depending on whether the room temperature is 68F or 85F. Upon hearing a sound, the subjects would hit a button and the reaction time (in seconds) would be recorded. The reaction time (in seconds) of 25 subjects was recorded. The average reaction time for the cool setting was 0.1825 seconds, and the average reaction time for the warm setting was 0.1485 seconds. The sample standard deviation of the differences between reaction times was 0.0465. What is the 80%-confidence interval for the difference in mean reaction times? Assume the normal model for the reaction times.

(a) 0.034 ± 0.0163

(b) 0.034 ± 0.0465

(c) 0.034 ± 0.0093

(d) 0.034 ± 0.0128

(e) None of the above.

Solution: (d) or (e)
The point estimate for the mean difference in reaction times is $\bar{d} = 0.1825 - 0.1485 = 0.0340$. Since the sample size is 25, the number of degrees of freedom in our $t-$distribution is $25 - 1 = 24$. The critical value for the 80% confidence level, according to our tables, equals $t^* = 1.318$. So, the confidence interval is

$$0.034 \pm (1.318)\left(\frac{0.0465}{\sqrt{25}}\right) = 0.034 \pm 0.0123.$$

Problem 1.12. Sixteen bright middle schoolers were put on an intense sudoku regimen in the hope of improving their scores in the *Canvas Algebra Test (CAT).* Assume that the distribution of the differences in the scores is normal. Let μ_d be the mean difference between the "pre-test" (before the regimen) and "post-test" (after the regimen). We want to test

$$H_0 : \mu_d = 0 \quad vs. \quad H_a : \mu_d > 0.$$

The observed average difference in the scores was $\bar{x}_d = 1$ while the sample standard deviation was $s_d = 3$. What can you say about the $p-$value for this hypothesis test?

(a) It's below 0.05.

(b) It's between 0.05 and 0.10.

(c) It's between 0.10 and 0.15.

(d) It's between 0.15 and 0.20.

(e) It's over 0.20.

Solution: (c)
The observed value of the $t-$statistic, under the null hypothesis, is

$$t = \frac{1-0}{\frac{3}{\sqrt{16}}} = 1.3333.$$

The number of degrees of freedom of the $t-$distribution of our test statistic is $16 - 1 = 15$. Consulting our $t-$tables, we conclude that the $p-$value is between 0.10 and 0.15. *Note: If one uses* **R**, *one gets* 0.1011593.