

Project #4: Elementary Monte Carlo. More on portfolios.

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Problem #1 (25 points)

A weighted spinner has four different regions: blue, red, green, and yellow. The blue is twice as likely as red, the red is twice as likely as green, and the green is twice as likely as yellow. You are invited to play the game with the following outcomes:

- if the spinner lands on blue, you lose \$5;
- if the spinner lands on red, you win \$2;
- if the spinner lands on green, you win \$10;
- if the spinner lands on yellow, you win \$20.

Use *Monte Carlo* simulation to figure out the fair price for entering this game.

Problem #2 (25 points)

Use *Monte Carlo* simulation to estimate the number π .

Problem #3 (25 points)

Use *Monte Carlo* to estimate the expected distance of a randomly chosen point in a square to the center of that square.

Problem #4 (25 points)

Your initial wealth is exactly \$100. You are allowed to invest in shares of a particular stock. You are also allowed to both lend and borrow at the continuously compounded risk-free interest rate of 0.05. Keeping your money uninvested is **not allowed**.

You can rebalance your portfolio every morning, once you have observed the opening stock price. This means that you can change the number of shares you own (if you decide to do so) and accordingly adapt your risk-free investment.

You proceed to create a “rule” according to which you will be rebalancing your portfolio. Here is a possible rule you can use:

If the stock price drops overnight (regardless of the extent of the drop), sell half of the stock investment. If the stock price rises overnight (regardless of the extent of the increase), double the amount of the stock investment. If the stock price does not change, do nothing. In all three cases, the rest of my wealth is to be invested at the risk-free rate (if needed, I will borrow at the continuously compounded risk-free interest rate).

Over the following 10 days, you observe the following stock prices for a non-dividend-paying stock:

Day	0	1	2	3	4	5	6	7	8	9	10
Stock price	100	80	64	80	64	80	100	80	64	80	100

As the time passes you follow investment rules above to rebalance your portfolio. Complete the following table describing your portfolio **just before and just after** the rebalancing is done. Even more precisely, for the 10 days, both for “morning” and “evening” print out:

- the number of shares of stock in the portfolio,
- the balance of the risk-free investment,
- the wealth in the stock,
- the total wealth.