M339D: September 13th, 2024.

Payoff and Profit Curves.

Outright Purchase of a Stock.

$S(t), t \geq 0$ ... stock price @ time t

stochastic process

0 ——— T

"stock bought" "stock sold"

Initial Cost: $S(0)$

Payoff: $S(T)$ ... a random variable

r... ccrfir

$$\text{Profit} := \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$

$$= S(T) - e^{rT} \cdot S(0)$$

Inspiration

Goal. Studying the payoff and the profit as functions of the final asset price

Introduce: s... an independent argument taking values in $[0, +\infty)$
It stands for the FINAL ASSET PRICE,
i.e., it's a PLACEHOLDER for the random variable $S(T)$.

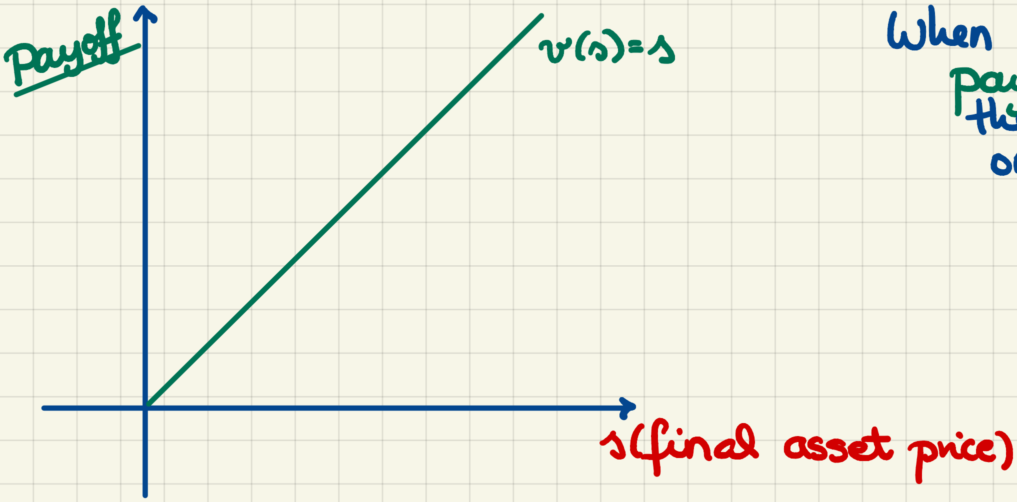
Now, we can define the PAYOFF FUNCTION which describes the dependence of the payoff on the independent argument s.

Notation: v... payoff function

$$v : [0, \infty) \longrightarrow \mathbb{R}$$

$v(s)$... the agent's payoff if the final asset price equals s

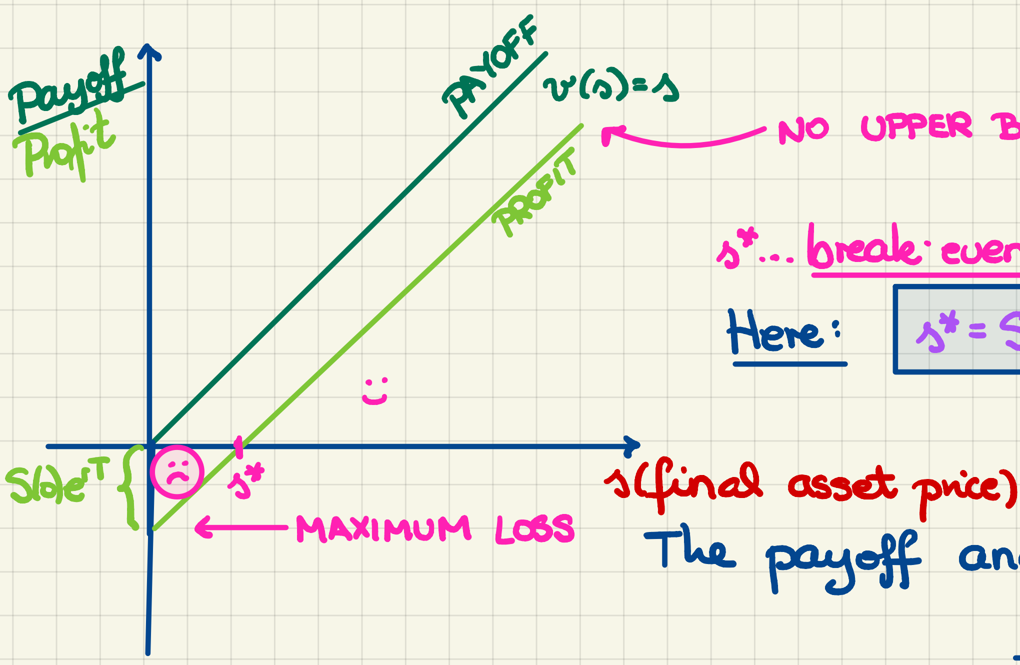
→ For the outright purchase: $\boxed{v(s) = s}$ identity function



When we plot the payoff function, we get the payoff curve or the payoff diagram.

In general, the profit function is: $v(s) - FV_{0,T}(\text{Initial Cost})$

For the outright purchase: $\boxed{s - S(0)e^{rT}}$



NO UPPER BOUND, i.e., unlimited growth potential

s^*... break-even point

Here: $\boxed{s^* = S(0)e^{rT}}$

The payoff and profit curves are increasing

Terminology. If the payoff/profit is increasing (not necessarily strictly) as a function of the final asset price s, we say that the portfolio is

long with respect to the underlying asset

Short Sale.

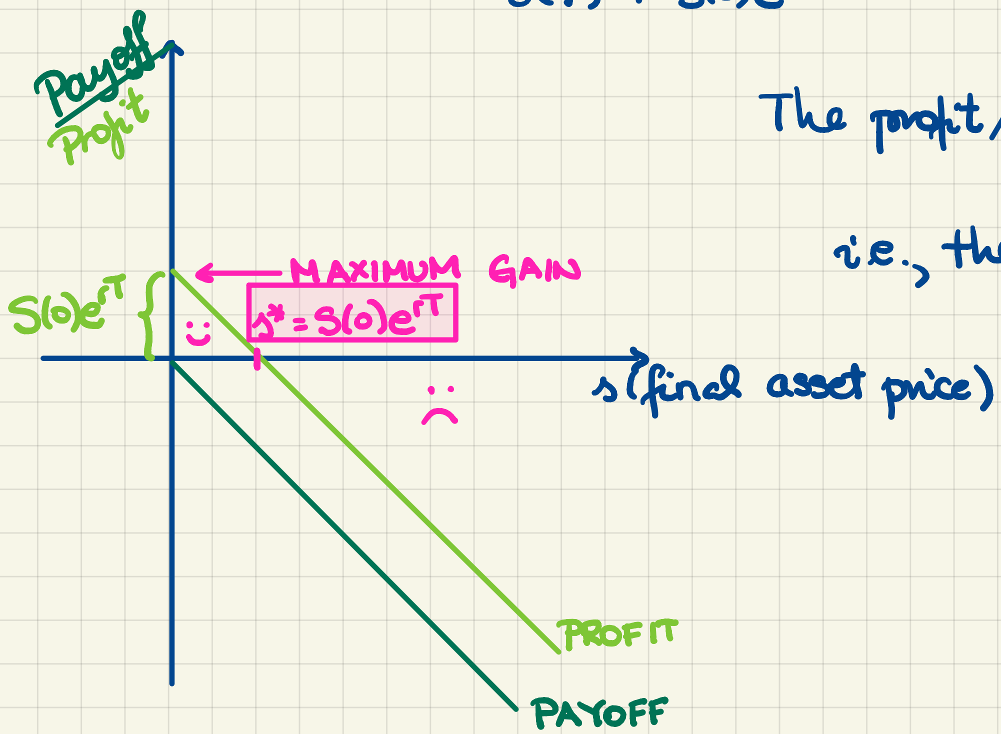
$$\underset{\text{short sale initiated}}{0} \longrightarrow \underset{\text{short sale closed}}{T}$$

Initial Cost: $-S(0)$

Payoff: $-S(T)$ $\Rightarrow$ payoff f'tion: $\boxed{v(s) = -s}$

Profit $= -S(T) + FV_{0,T}(+S(0))$

$= -S(T) + S(0)e^{rT}$



The profit/payoff is decreasing, i.e., the short sale is short w.r.t. the underlying

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Problem Set 3

Payoff. Profit.

Problem 3.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time−1 price of the above stock as follows:

$$\boxed{S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}}$$

What is your expected profit under the above model, if you invest in one share of stock at time−0 and liquidate your investment at time−1?

→ :

$$\text{Profit} = \text{Payoff} - \underbrace{FV_{0,1}(\text{Initial Cost})}_{40\cdot e^{0.04}}$$

$\mathbb{E}\Big|$

$$\mathbb{E}[\text{Profit}] = \boxed{\mathbb{E}[\text{Payoff}]} - 40e^{0.04}$$

$$\mathbb{E}[\text{Payoff}] = \mathbb{E}[S(1)] = 45\cdot\left(\tfrac{1}{4}\right) + 42\cdot\left(\tfrac{1}{2}\right) + 38\cdot\left(\tfrac{1}{4}\right) = \underline{41.75}$$

answer: $41.75 - 40e^{0.04} = \underline{0.1176}$ □