Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Solution: Practice Problems for In-Term One
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. This exam is graded out of 100 points.
Time: 50 minutes

1.1. TRUE/FALSE QUESTIONS.

Problem 1.1. (2 points) An agent is **only** allowed to long a forward contract if he/she is willing to take physical delivery of the underlying asset.

Solution: FALSE
It is possible to have *cash settlement* on the delivery date if the forward contract stipulates so.

Problem 1.2. (2 points) Denote the continuously compounded, risk-free interest rate by r and denote the equivalent annual effective interest rate by i. Then, $\ln(1+i) = r$. *True or false?*

Solution: TRUE

Problem 1.3. (2 pts) Two dice are rolled, the single most probable sum of the numbers of the upturned faces is 7. *True or false?*

Solution: TRUE

Problem 1.4. (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date T on the underlying asset S:

- one long call with strike 40,
- two long calls with strike 50,
- one short call with strike 65.

Let $S(T) = 69$. Then, the payoff from the above position at time T is less than 60.

Solution: FALSE
The payoff is

$$(69-40) + 2(69-50) - (69-65) = 63.$$

1.2. MULTIPLE CHOICE QUESTIONS.

Problem 1.5. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions given by

$$f(x) = 2x - 10$$

and

$$g(x) = \begin{cases} \min(x, 7) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then, $g(f(7))$ equals …

(a) -4
(b) 0
(c) 4
(d) 7
(e) None of the above

Solution: (c)

Problem 1.6. *Source: Sample P exam, Problem #176.*
In a group of health insurance policyholders, 20% have high blood pressure and 30% have high cholesterol. Of the policyholders with high blood pressure, 25% have high cholesterol. A policyholder is randomly selected from the group. Calculate the probability that a policyholder has high blood pressure, **given** that the policyholder has high cholesterol.

(a) 1/6
(b) 1/5
(c) 1/4
(d) 2/3
(e) 5/6

Solution: (a)
Let E be the event containing all the policyholders with a high blood preasure and let F be the event which contains all the policyholders with high cholesterol. We are given the following

$$\mathbb{P}[E] = 0.2, \quad \mathbb{P}[F] = 0.3, \quad \mathbb{P}[F|E] = 0.25.$$

Then, the conditional probability we are looking for equals

$$\mathbb{P}[E|F] = \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[F]} = \frac{\mathbb{P}[F|E]\mathbb{P}[E]}{\mathbb{P}[F]} = \frac{(0.25)(0.2)}{0.3} = \frac{1}{6}\,.$$

Problem 1.7. Harry plays a simple lottery in which the winnings are distributed as follows:

- \$5 with probability 0.2,
- \$10 with probability 0.4,
- \$20 with probability 0.4.

It turns out that Harry has to pay a fee to collect his winnings. If the actual amount he wins is smaller than \$9, then the fee is defined to equal the amount that Harry won – thus, he walks away with nothing. If the actual amount he wins is between \$9 and \$15, he does not have to pay anything in fees and gets a bonus of \$4. If the actual amount he wins is larger than \$15, then he pays the \$15-fee and pockets the remainder. What is the expected value of the net amount Harry collects?

(a) 3

(b) 6.4

(c) 7.6

(d) 15

(e) None of the above.

Solution: (c)
The actual amount that Harry gets is

- \$0 with probability 0.2,
- \$14 with probability 0.4,
- \$5 with probability 0.4.

So, his expected winnings are

$$14(0.4) + 5(0.4) = 7.6$$

Problem 1.8. The current exchange rate is such that one Swiss Franc equals 1.07 USD. The continuously compounded risk-free interest rate for the Swiss Franc is 0.02. The continuously compounded risk-free interest rate for the USD is 0.05. You want to purchase such an amount of Swiss Francs today so that once that amount in deposited in a savings account it will grow to exactly 10,000 Swiss Francs in two years. How much will this purchase cost you today in USD?

(a) 8979.341

(b) 9607.894

(c) 9681.76

(d) 10280.45

(e) None of the above.

Solution: (d)
We need to purchase

$$10000e^{-(0.02)(2)} = 9607.894$$

Swiss Francs today. This will cost us

$$9607.894(1.07) = 10280.45$$

in USD.

Problem 1.9. You buy one share of discrete-dividend-paying stock today with the intention of holding onto this investment for at least one year. The stock price today is \$60 per share. The stock will pay \$2 in dividends per share in one quarter. Thereafter, the dividends increase by 2% and continue to be

paid quarterly. Let the continuously compounded risk-free interest rate be equal to 0.04. What is the present value of all of the dividends you are entitled to over the next year?

(a) 7.80
(b) 7.94
(c) 8.04
(d) 8.35
(e) None of the above.

Solution: (c)
The present value of the dividends is

$$\begin{aligned} PV(Div) &= 2e^{-0.04/4} + 2.04e^{-0.04/2} + 2.0808e^{-(0.04)(3/4)} + 2.122416e^{-0.04} \\ &= 2e^{-0.01} + 2.04e^{-0.02} + 2.0808e^{-0.03} + 2.122416e^{-0.04} = 8.038203 \end{aligned}$$

Problem 1.10. Hermione sells short one share of continuous-dividend-paying stock. The stock is currently valued at \$80 per share. The stock's dividend yield is 0.02. The continuously compounded risk-free interest rate is 0.04. Hermione intends to close the short sale in one year. What is the final stock price for which Hermione will break even?

(a) 80
(b) 80.28
(c) 81.62
(d) 83.26
(e) None of the above.

Solution: (c)
In our usual notation, the break-even point is

$$S(0)e^{(r-\delta)T} = 80e^{(0.04-0.02)(1)} = 80e^{0.02} = 81.61611.$$

Problem 1.11. The current market price of widgets is \$4 per widget. The widget factory plans to sell their next batch of 100 widgets in half a year. The total aggregate costs of production of widgets will be equal to \$350.

The factory enters 100 short forward contracts on widgets for delivery in half a year. The forward price is \$4.20 per widget.

What is the factory's profit if the final price of widgets in half a year ends up being \$4.40?

(a) 30
(b) 50
(c) 70
(d) 90
(e) None of the above.

Solution: (c)
The factory will sell the widgets per the forward contract for $420 total. The total aggregate costs are given to be $350. Hence, the profit is $70.

Problem 1.12. Maryam bakes batches of cupcakes for a cupcake convention. She buys forward 21 pounds of raspberries from a local farmer at the forward price of $5.60 per pound.

She projects to bake 336 cupcakes and sell each for $3. The total and aggregate non-raspberry costs of baking the cupcakes are $200.

If the market price of raspberries on the day of the cupcake convention is $5.40, what is Maryam's profit?

(a) $690.40
(b) $694.60
(c) $890.40
(d) $894.60
(e) None of the above.

Solution: (a)

$$336 \times 3 - 21 \times 5.60 - 200 = 690.40.$$

Problem 1.13. The **writer** of a call option has …

(a) an obligation to sell the underlying asset at the strike price.
(b) a right, but **not** an obligation, to sell the underlying asset at the strike price.
(c) an obligation to buy the underlying asset at the strike price.
(d) a right, but **not** an obligation, to buy the underlying asset at the strike price.
(e) None of the above.

Solution: (a)