UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #5

Black-Scholes: Gap options.

Problem 5.1. (15 points) The price of a non-dividend-paying stock is modeled using the Black-Scholes framework. Today's stock price is equal to $100 and its volatility is 0.2.

The continuously compounded risk-free interest rate equals 0.04.

You are constructing a **zero-cost** gap put option. The option is supposed to pay $K - S(1/4)$ in three months if the condition $S(1/4) < 110$ is satisfied. Find the strike price K of your gap put option such that the gap put is free.

Solution: The Black-Scholes price of a gap put with strike price K_s and trigger price K_t is

$$V_{GP}(0) = K_s e^{-rT} N(-d_2) - F^P_{o,T}(S) N(-d_1)$$

with

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K_t}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T\right] \\ &= \frac{1}{0.2\sqrt{0.25}}\left[\ln\left(\frac{100}{110}\right) + \left(0.04 + \frac{0.2^2}{2}\right)(0.25)\right] = -0.80 \\ d_2 &= d_1 - \sigma\sqrt{T} = -0.9. \end{aligned}$$

So,

$$N(-d_1) = N(0.8) = 0.7881, \quad \text{and} \quad N(-d_2) = N(0.9) = 0.8159.$$

Equating the Black-Scholes price of the gap put to zero, we get

$$K_s = \frac{100(0.7881)e^{0.04(0.25)}}{0.8159} = 97.5635.$$