M378K Introduction to Mathematical Statistics

Homework assignment #3

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 3.1. *(5 points) Let the density function of a random variable X be given as*

$$f_X(x) = cxI_{[0,1]}(x),$$

for some constant c. Find $\mathbb{E}[X^3]$.

Problem 3.2. *(5 points) Let X denote the outcome of a roll of a fair, regular dodecahedron (a polyhedron with 12 faces) with numbers $1, 2, \cdots, 12$ written on its sides. Find $\mathbb{E}[X]$.*

Problem 3.3. *(5 points) Let X be a random variable with mean $\mu = 2$ and standard deviation equal to $\sigma = 1$. Find $\mathbb{E}[X^2]$.*

Problem 3.4. *(5 points) Let X denote the number of $1's$ in 100 throws of a fair die. Find $\mathbb{E}[X^2]$.*

Problem 3.5. *(10 points) Let the random variable Y have the following cumulative distribution function*

$$F_Y(y) = \begin{cases} 0 & \text{for } y < 0 \\ \frac{y}{2} & \text{for } 0 \le y < 1 \\ \frac{y^2}{\alpha} & \text{for } 1 \le y < \beta \\ 1 & \text{for } \beta \le y \end{cases}$$

(i) *(3 points) Find the constants α and β such that the random variable Y is continuous.*

(ii) *(7 points) Calculate the expectation of the random variable Y for the α and β you obtained in the previous part of the problem.*

Problem 3.6. *(20 points) Let X be a discrete random variable with the support $\mathcal{S}_X = \mathbb{N}$, such that $\mathbb{P}[X = n] = C\frac{1}{n^2}$, for $n \in \mathbb{N}$, where C is a constant chosen so that $\sum_n \mathbb{P}[X = n] = 1$. The distribution table of X is, therefore, given by*

1	2	3	...
$C\frac{1}{1^2}$	$C\frac{1}{2^2}$	$C\frac{1}{3^2}$	...

1. *(10 points) Show that $\mathbb{E}[X]$ does not exist.*
2. *(10 points) Construct a distribution of a similar random variable whose expectation does exist, but the variance does not. (*Hint: *Use the same support $\mathbb{N}$, but tweak the probabilities so that the sum for $\mathbb{E}[X]$ converges, while the sum for $\mathbb{E}[X^2]$ does not.)*