6.10

(a) 61% is a sample statistic, it's the observed sample proportion.

(b) A 95% confidence interval can be calculated as follows:

$$\begin{aligned}\hat{p} \pm z^{\star}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.61 \pm 1.96\sqrt{\frac{0.61 \times (1-0.61)}{1578}} \\ &= 0.61 \pm 1.96 \times 0.012 \\ &= 0.61 \pm 0.024 \\ &(0.586, 0.634)\end{aligned}$$

We are 95% confident that approximately 58.6% to 63.4% of Americans think marijuana should be legalized.

(c) 1. Independence: The sample is random, and comprises less than 10% of the American population, therefore we can assume that the individuals in this sample are independent of each other

2. Success-failure: The number of successes (people who said marijuana should be legalized: $1578 \times 0.61 = 962.58$) and failures (people who said it shouldn't be: $1578 \times 0.39 = 615.42$) are both greater than 10, therefore the success-failure condition is met as well.

Therefore the distribution of the sample proportion is expected to be approximately normal.

(d) Yes, the interval is above 50%, suggesting, with 95% confidence, that the true population proportion of Americans who think marijuana should be legalized is greater than 50%.

6.14

(a) We have previoiusly confirmed that the independence condition is satisfied. We need to recheck the success-failure condition using the sample proportion: $331 \times 0.48 = 158.88 > 10$ and $331 \times 0.52 = 172.12 > 10$. An 80% confidence interval can be calculated as follows:

$$\begin{aligned}
\hat{p} \pm z^{\star}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.48 \pm 1.65 \times \sqrt{\frac{0.48 \times 0.52}{331}} \\
&= 0.48 \pm 1.65 \times 0.0275 \\
&= 0.48 \pm 0.045 \\
&= (0.435, 0.525)
\end{aligned}$$

We are 90% confident that the 43.5% to 52.5% of all Americans who decide not to go to college do so because they cannot afford it. This agrees with the conclusion of the earlier hypothesis test since the interval includes 50%.

(b) We are asked to solve for the sample size required to achieve a 1.5% margin of error for a 90% confidence interval and the point estimate is $\hat{p} = 0.48$.

$$\begin{aligned}
ME = z^{\star}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \rightarrow \quad 0.01 &\geq 1.65\sqrt{\frac{0.48 \times 0.52}{n}} \\
0.015^2 &\geq 1.65^2 \frac{0.48 \times 0.52}{n} \\
n &\geq \frac{1.65^2 \times 0.48 \times 0.52}{0.015^2} \\
n &\geq 3020.16 \approx 3121
\end{aligned}$$

The sample size n should be at least 3,121.

6.16 We are asked to solve for the sample size required to achieve a 2% margin of error for a 95% confidence interval and the point estimate is $\hat{p} = 0.61$.

$$\begin{aligned}
ME = z^\star \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \rightarrow \quad 0.02 &\geq 1.96\sqrt{\frac{0.61 \times 0.39}{n}} \\
0.02^2 &\geq 1.96^2 \frac{0.61 \times 0.39}{n} \\
n &\geq \frac{1.96^2 \times 0.61 \times 0.39}{0.02^2} \\
n &\geq 2284.792 \\
n &\geq 2285
\end{aligned}$$

The sample size n should be at least 2,285.