M378K Introduction to Mathematical Statistics
Fall 2025
University of Texas at Austin
**Practice for In-Term Exam II**
Instructor: Milica Čudina

**Notes**: This is a closed book and closed notes exam. The maximal score on the real exam will be 100 points.
**There are many ways in which any single problem can be solved. The solutions herein are just one possible way to tackle the given problems.**
**Time**: 50 minutes

All written work handed in by the student is considered to be
**their own work, prepared without unauthorized assistance.**

**The University Code of Conduct**

"The core values of The University of Texas at Austin are learning, discovery, freedom, leadership, individual opportunity, and responsibility. Each member of the university is expected to uphold these values through integrity, honesty, trust, fairness, and respect toward peers and community. As a student of The University of Texas at Austin, I shall abide by the core values of the University and uphold academic integrity."

"I agree that I have complied with the UT Honor Code during my completion of this exam."

**Signature:**

2.1. **Formulas.** If $Y$ has the binomial distribution with parameters $n$ and $p$, then $p_Y(k) = \mathbb{P}[Y = k] = \binom{n}{k} p^k (1-p)^{n-k}$, for $k = 0, \dots, n$, $\mathbb{E}[Y] = np$, $\mathrm{Var}[Y] = np(1-p)$. The binomial coefficients are defined as follows for integers $0 \le k \le n$, $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. The moment generating function of $Y$ is given by $m_Y(t) = (pe^t + q)^n$.

If $Y$ has a geometric distribution with parameter $p$, then $p_Y(k) = p(1-p)^k$ for $k = 0, 1, \dots$, $\mathbb{E}[Y] = \frac{1-p}{p}$, $\mathrm{Var}[Y] = \frac{1-p}{p^2}$. Its mgf is $m_Y(t) = \frac{p}{1-qe^t}$ for $t$ such that $qe^t < 1$.

If $Y$ has a Poisson distribution with parameter $\lambda$, then $p_Y(k) = e^{-\lambda}\frac{\lambda^k}{k!}$ for $k = 0, 1, \dots$, $\mathbb{E}[Y] = \mathrm{Var}[Y] = \lambda$. Its mgf is $m_Y(t) = e^{\lambda(e^t-1)}$.

If $Y$ has a uniform distribution on $[l, r]$, its density is

$$f_Y(y) = \frac{1}{r-l}\mathbf{1}_{(l,r)}(y),$$

its mean is $\frac{l+r}{2}$, and its variance is $\frac{(r-l)^2}{12}$. Let $U \sim U(0,1)$. The mgf of $U$ is $m_U(t) = \frac{1}{t}(e^t - 1)$.

If $Y$ has the standard normal distribution, then its mean is zero, its variance is one, and its density equals

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad x \in \mathbb{R}.$$

Its mgf is $m_Y(t) = e^{\frac{t^2}{2}}$.

If $Y$ has the exponential distribution with parameter $\tau$, then its cumulative distribution function is $F_Y(y) = 1 - e^{-\frac{y}{\tau}}$ for $y \ge 0$, its probability density function is $f_Y(y) = \frac{1}{\tau}e^{-y/\tau}$ for $y \ge 0$. Also, $\mathbb{E}[Y] = SD[Y] = \tau$. Its mgf is $m_Y(t) = \frac{1}{1-\tau t}$.

The mgf of $Y \sim \Gamma(k, \tau)$ is

$$m_Y(t) = \frac{1}{(1-\tau t)^k} \text{ for } t < 1/\tau.$$

Its expectation is $k\tau$ and its variance is $k\tau^2$. The $\chi^2-$distribution with $n$ degrees of freedom is the special case $\Gamma\left(\frac{n}{2}, 2\right)$

## 2.2. DEFINITIONS.

**Problem 2.1.** (10 points) Write down the definition of the **moment generating function** of a random variable $Y$.

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## 2.3. TRUE/FALSE QUESTIONS.

**Problem 2.2.** (5 points) The random vector $(X, Y)$ is jointly continuous with the joint probability density function given by

$$f_{(X,Y)}(x,y) = \begin{cases} (1/8)xe^{-(x+y)/2}, & x > 0, y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Then, the random variables $X$ and $Y$ are independent. *True or false? Why?*

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## 2.4. Free-response problems.

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Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

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**Problem 2.3.** (15 points) Let $Y_1$ and $Y_2$ be independent exponential random variables with parameters $\tau_1$ and $\tau_2$.

(1) (5 points) What is the joint density of $(Y_1, Y_2)$?
(2) (10 points) Compute $\mathbb{P}[Y_1 \geq Y_2]$.

**Problem 2.4.** (10 points) Let $Y \sim U(l, r)$ What is the moment generating function of $Y$?

**Problem 2.5.** (20 points) Luka owns a mechanical pencil. The lifetime of every piece of lead has mean 4 days and standard deviation of 1 day. The lifetimes of different lead pieces are independent. After one lead piece is exhausted, Luka immediately uses the next one. What is the smallest number of packets (each containing 10 pieces of lead) that Luka should buy in order to have enough lead for the next 360 days with probability at least 0.9987?

*Hint:*

$$\sqrt{(8(360) + 9)^2 - 4(16)(360^2)} \approx 228$$

**Problem 2.6.** (20 points) In Croatia, if you go to the chocolate-factory store, you can buy broken off chunks of rice-puff chocolate. From past experience, we know that the weight of the individual chunks has mean of 40 grams and standard deviation of 5 grams. Assume that the weights of individual pieces of chocolate are independent.

You buy 400 chocolate chunks. What is the probability that the total weight exceeds 16128 grams?

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## 2.5. MULTIPLE CHOICE QUESTIONS.

**Problem 2.7.** (5 points) The pdf of $W = 1/Y^2$, where $Y \sim E(\tau)$, is ...

(a) $\frac{2}{\tau} y^{-3/2} e^{-y/\tau} \mathbf{1}_{\{y>0\}}$

(b) $\frac{1}{2\tau}(-y^{-3/2}) e^{-\sqrt{y}/\tau} \mathbf{1}_{\{y>0\}}$

(c) $\frac{1}{\tau} e^{-1/(y^2\tau)} \mathbf{1}_{\{y>0\}}$

(d) $\frac{1}{2\tau y^{3/2}} e^{-1/(\tau\sqrt{y})} \mathbf{1}_{\{y>0\}}$

(e) none of the above

**Problem 2.8.** (5 points) Let $Y_1, Y_2, \dots, Y_n$ be independent, standard normal random variables. What is the distribution of the random variable $Y$ defined as

$$Y = Y_1^2 + Y_2^2 + \cdots + Y_n^2\,?$$

(a) $N(0, \sqrt{n})$

(b) $\chi^2(n)$

(c) $\chi^2(n-1)$

(d) $N(0, n^2)$

(e) **None of the above.**

(*Note:* In our notation $N(\mu, \sigma)$ means normal with mean $\mu$ and *standard deviation* $\sigma$.)

**Problem 2.9.** (5 points) *Source: Sample P exam, Problem #250.*
A delivery service owns two cars that consume 15 and 20 miles per gallon, respectively. Fuel costs \$3 per gallon. On any given business day, each car travels a number of miles that is independent of the other and is normally distributed with mean 25 miles and standard deviation 3 miles. Calculate the probability that on any given business day, the total fuel cost to the delivery service will be less than 7.

(a) About 0.0013

(b) About 0.0073

(c) About 0.0099

(d) About 0.0138

(e) **None of the above.**

**Problem 2.10.** (5 points) *Source: Sample P exam, Problem #362.*
At a certain airport, $\frac{1}{6}$ of all scheduled flights are delayed. Assume that flight delays are mutually independent events. Use the normal approximation **with continuity correction** to calculate the probability that at least 40 of the next 180 flights are delayed.

(a) About 0.0110
(b) About 0.0143
(c) About 0.0182
(d) About 0.0234
(e) About 0.0287