

UNIVERSITY OF TEXAS AT AUSTIN

Quiz #3

pmf and pdf. Exponential.

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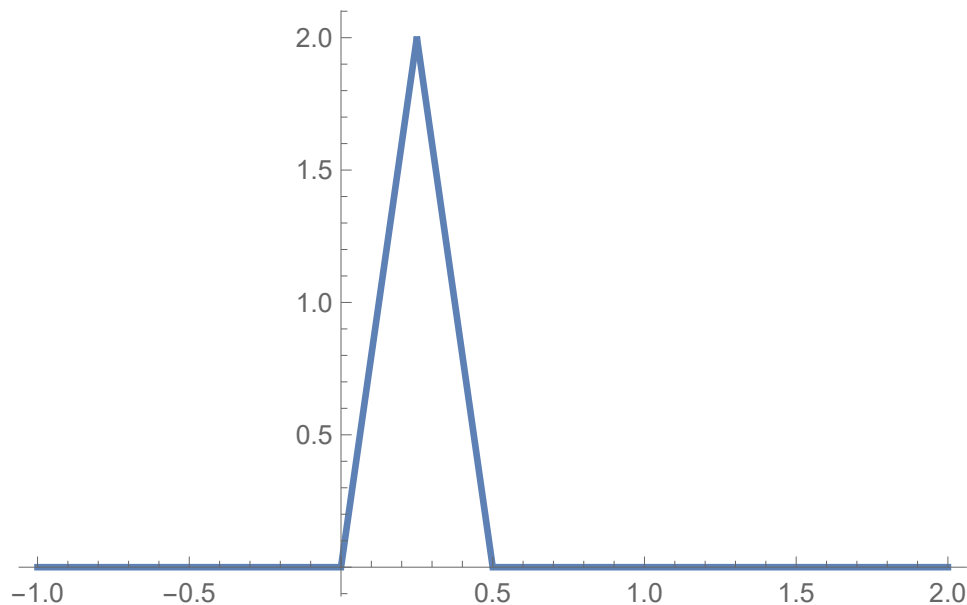
Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

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**Problem 3.1.** (4 points) The probability density function is always less than or equal to 1. *True or false?*

**Solution: FALSE**

Here is a counterexample:



**Problem 3.2.** (2 points) The probability mass function is always less than or equal to 1. *True or false?*

**Solution: TRUE**

By definition, in our usual notation,

$$p_X(x) = \mathbb{P}[X = x]$$

for every  $x$  in the support of the discrete random variable  $X$ . Also by definition, the probability is less than or equal to 1.

**Problem 3.3.** (9 points) *Source: Sample P exam #199.*

A car is new at the beginning of a calendar year. The time, in years, before the car experiences its first failure is exponentially distributed with parameter 2. Calculate the probability that the car experiences its first failure in the last quarter of some calendar year.

- (a) 0.081
- (b) 0.088
- (c) 0.102
- (d) 0.205
- (e) None of the above.

**Solution: (d)**

The random variable  $X$  has the cdf given by

$$F_X(x) = 1 - e^{-x/2} \quad \text{for } x \geq 0.$$

So, for any year  $n = 1, 2, 3, \dots$ , the probability of the car breaking down in the last quarter is

$$\mathbb{P}[n - 0.25 < X \leq n] = e^{-\frac{n-0.25}{2}} - e^{-\frac{n}{2}} = e^{-\frac{n}{2}}(e^{0.125} - 1).$$

Thus, the probability that the car breaks down at the end of any year equals

$$\begin{aligned} \sum_{n=1}^{\infty} \mathbb{P}[n - 0.25 < X \leq n] &= \sum_{n=1}^{\infty} e^{-\frac{n}{2}}(e^{0.125} - 1) = (e^{0.125} - 1)e^{-1/2} \sum_{n=0}^{\infty} e^{-\frac{n}{2}} \\ &= (e^{0.125} - 1)e^{-1/2} \frac{1}{1 - e^{-1/2}} = \frac{e^{0.125} - 1}{e^{0.5} - 1} = 0.20525. \end{aligned}$$