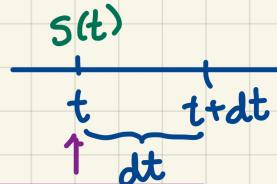


M339D: January 28th, 2022.

Continuous dividend paying stocks [cont'd].

- Notation:
- $S(t)$, $t \geq 0$... the time t stock price
 - δ ... dividend yield



$$\delta S(t) dt$$

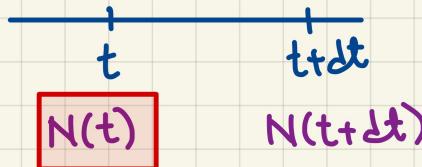
is received by the shareholder per share (for the interval $(t, t+dt)$).

Convention: ALL the dividend are immediately and continuously reinvested in the same stock.

Notation: $N(t)$, $t \geq 0$... the # of shares owned @ time t

At time $t=0$: $N(0) = n_0$ initial condition ✓

Focus on the rate of change.



★ $\Rightarrow dN(t) = N(t+dt) - N(t)$... # of shares purchased @ time t

- The total amount of dividend for the interval $(t, t+dt)$:

$$\underline{N(t) \cdot \delta S(t) dt}$$

- The number of shares of stock a shareholder can buy for that amount is:

$$\frac{N(t) \cancel{\delta S(t) dt}}{\cancel{S(t)}} = \underline{\delta N(t) dt}$$



Combining quantity, we get &  , which represent the same

$$dN(t) = \gamma N(t) dt$$

$$\frac{dN(t)}{N(t)} = \gamma dt$$

$$\ln(N(t)) = St + \text{const}$$

$$N(t) = e^{St + \text{const}}$$

Remember the initial condition:

$$N(0) = n_0$$

$$\Rightarrow N(0) = e^{S \cdot 0 + \text{const}} = n_0 \Rightarrow$$

$$e^{\text{const}} = n_0$$

$$\Rightarrow N(t) = n_0 e^{St} \quad \checkmark$$

It's convenient that the number of shares owned @ time t ends up being deterministic.

\Rightarrow the wealth @ time T equals:

$$N(T) \cdot S(T) = n_0 e^{ST} \cdot S(T)$$

UNIVERSITY OF TEXAS AT AUSTIN

Problem set 3

Continuous-dividend-paying stocks.

Problem 3.1. You have \$300 to invest in a market index worth \$100 per unit. The market index pays dividends continuously with the dividend yield equal to 0.02. How many units of the market index will you own in six months?

$$\rightarrow: n_0 = \frac{300}{100} = 3$$

$$N(1/2) = n_0 \cdot e^{(0.02)(1/2)} = 3 \cdot e^{0.01} (= 3.03015)$$

→

Problem 3.2. Consider a certain stock which pays dividends continuously with the dividend yield of 0.03. How many shares would you need to purchase today to ensure that you own exactly one share in a quarter year? Moreover, let today's price of this stock be \$80 per share. How much does your investment cost you?

$$\rightarrow: n_0 = ?$$

$$N(1/4) = 1 = n_0 e^{0.03(1/4)} \Rightarrow$$

$$n_0 = e^{-0.03(1/4)}$$

$$n_0 = e^{-0.0075}$$

$$\text{initial cost: } n_0 \cdot S(0) = e^{-0.0075} (80) = 79.40$$

B

Financial Portfolios.

Static Portfolios.

Dynamic Portfolios.