M362K: February 23rd, 2024.

Standard Normal Distribution.

The standard normal density function is

$$\boxed{\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad \text{for } z \in \mathbb{R}}$$

The standard normal cumulative distribution function is

$$\boxed{\Phi(z) = \int_{-\infty}^{z} \varphi(u)\,du \quad \text{for all } z \in \mathbb{R}}$$

Usage: Under the std normal distribution, the probability of the event $(a,b]$ will be

$$\boxed{\mathbb{P}[(a,b]] = \Phi(b) - \Phi(a)}$$

Note: There is no "simple exact formula" for Φ !

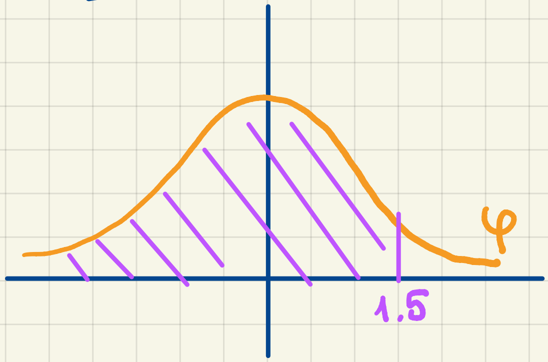
But,
- we have standard normal tables (Appendix 5).
- we also have pnorm in R

Problem. Consider the standard normal distribution.

Q: What is the probability of the event $(-\infty, 1.5]$?

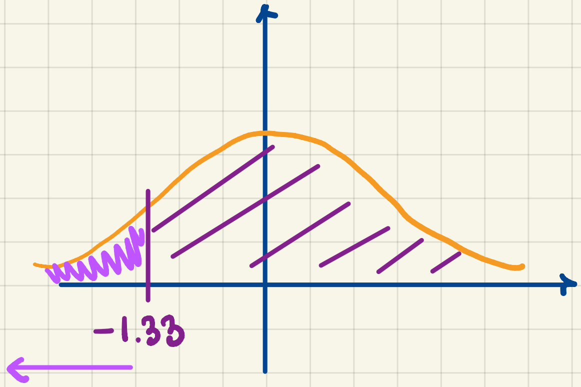
$\longrightarrow$: Using the std normal tables: $\Phi(1.5) = 0.9332$

Using R: pnorm(1.5) = 0.93312928



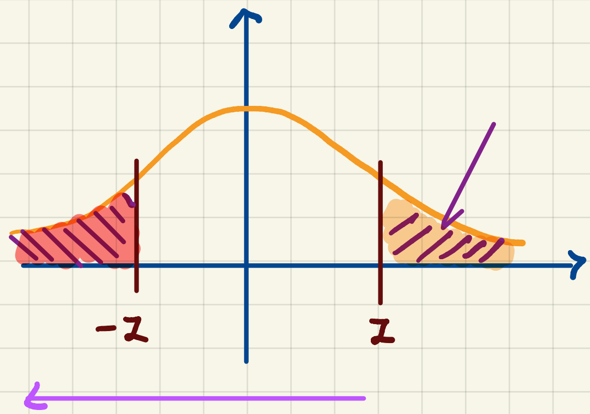
Q: What is the probability of the event $\boxed{(-1.33, +\infty)}$?

→:



$1-\Phi(-1.33) = 1-0.0918 = 0.9082$

`1-pnorm(-1.33)= 0.9082409`



$1-\Phi(z)=\Phi(-z)$ for all z

We could have done:

$\Phi(1.33)=0.9082$

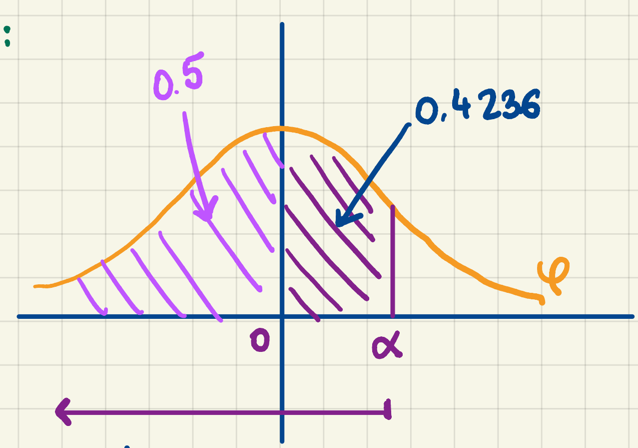
Q: What is the probability of the event $[-2.56, 1.73]$?

→: $\Phi(1.73)-\Phi(-2.56) = 0.9582-0.0052 = 0.9530$

`pnorm(1.73)-pnorm(-2.56)= 0.9529513` □

Problem. Assume that the number α is such that the standard normal distribution assigns the probability of $\boxed{0.4236}$ to the interval $[0, \alpha]$. Find α.

→:



$0.5 + 0.4236 = \Phi(\alpha)$

$\Phi(\alpha) = 0.9236$

$\alpha = \Phi^{-1}(0.9236)$

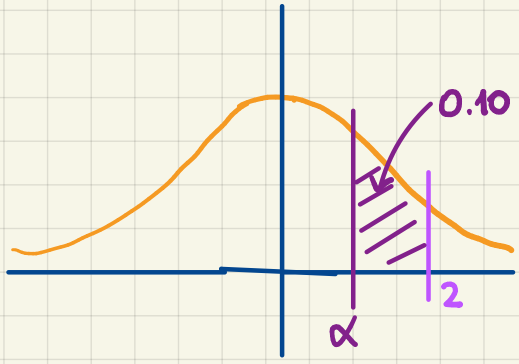
$\Phi^{-1}(0.9236) = 1.43$

Usually, the inverse of the cdf is called the quantile function.

`qnorm(0.9236)= 1.429711` □

Problem. Find α such that the area underneath φ over the interval $[\alpha, 2]$ equals 10%.

$\longrightarrow$:



$$\Phi(2) - \Phi(\alpha) = 0.1$$

$$\Phi(\alpha) = \Phi(2) - 0.1$$

$$= 0.9772 - 0.1 = 0.8772$$

$$\alpha = \Phi^{-1}(0.8772) \approx 1.16$$

Find the closest probability IN THE TABLES.
In this case, it was 0.8770.

qnorm (pnorm (2) - 0.1) = 1.161348

Normal Distributions.

In general, we can change the center of the bell curve and its spread by introducing parameters:

μ ... mean (measure of center)
and
σ ... standard deviation (measure of spread)

x raw units $\longrightarrow$ z standard units

$\boxed{x = \mu + \sigma \cdot z}$ $\longleftrightarrow$ $z = \dfrac{x - \mu}{\sigma}$

linear transforms