M339J: May 7th, 2021.

164. For a collective risk model the number of losses, N, has a Poisson distribution with $\lambda = 20$. The common distribution of the individual losses has the following characteristics:

(i) $E[X] = 70$

(ii) $E[X \wedge 30] = 25$

(iii) $\Pr(X > 30) = 0.75$

(iv) $E[X^2 \mid X > 30] = 9000$

$$\boxed{\lambda^P = 20 \cdot 0.75 = 15}$$

An insurance covers aggregate losses subject to an ordinary deductible of 30 per loss.

Calculate the variance of the aggregate payments of the insurance.

$$Var[S] = \underset{\mathbb{E}[N^P]}{\lambda^P} \cdot \mathbb{E}[(Y^P)^2]$$

(A) 54,000

(B) 67,500

(C) 81,000

(D) 94,500

(E) 108,000

$$Y^P = X - d \mid X > d \qquad (d = 30)$$

$$\mathbb{E}[(X-d)^2 \mid X > d] =$$

$$= \mathbb{E}[X^2 - 60X + 900 \mid X > 30]$$

$$= \underbrace{\mathbb{E}[X^2 \mid X > 30]}_{9000} - 60\,\mathbb{E}[X \mid X > 30] + 900$$

90

$$\mathbb{E}[X \mid X > 30] = \mathbb{E}[X - 30 \mid X > 30] + 30$$

$$= \frac{\boxed{\mathbb{E}[(X-30)\cdot \mathbb{I}_{[X>30]}]}}{\mathbb{P}[X>30]} + 30$$

$$= \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge 30]}{\mathbb{P}[X>30]} + 30$$

$$= \frac{70-25}{0.75} + 30 = \frac{45}{\frac{3}{4}} + 30 = 90 \quad //$$

$$\mathbb{E}[(Y^P)^2] = 9000 - 60 \cdot 90 + 900 = 4500$$

$$Var[S] = 15 \cdot (4500) = 67500 \quad \blacksquare$$

17. You are given:

(i) Aggregate losses follow a compound model.

(ii) The claim count random variable has mean 100 and standard deviation 25.

var = 625

(iii) The single-loss random variable has mean 20,000 and standard deviation 5000.

Determine the normal approximation to the probability that aggregate claims exceed 150% of expected costs.

↓

std units

↓

$\mathbb{E}$, Var

(A) 0.023

(B) 0.056

(C) 0.079

(D) 0.092

(E) 0.159

$$\mathbb{E}[S] = \underline{2 \cdot 10^6} = \mu_S$$

$$\mathrm{Var}[S] = \mathbb{E}[N] \cdot \mathrm{Var}[X] + \mathrm{Var}[N] \cdot (\mathbb{E}[X])^2$$

$$= 100 \cdot 25 \cdot 10^6 + 625 \cdot 4 \cdot 10^8$$

$$= \underline{2525} \cdot 10^8$$

$$\mathbb{P}[S > 1.5 \cdot \mu_S] = \mathbb{P}\left[\underbrace{\frac{S-\mu_S}{\sigma_S}}_{\approx N(0,1) \sim Z} > \frac{1.5 \cdot \mu_S - \mu_S}{\sigma_S}\right]$$

$$\cong \mathbb{P}\left[Z > \frac{0.5 \mu_S}{\sigma_S} = \underbrace{\frac{10^6}{502{,}493.78}}_{=1.99}\right]$$

$$= 1 - \Phi(1.99) = 0.0233$$

26. The random variables $X_1, X_2, \ldots, X_n$ are independent and identically distributed with probability density function

$X \sim Exp(mean = \theta)$

$$f(x) = \frac{e^{-x/\theta}}{\theta}, \quad x \geq 0$$

Determine $E\left[\bar{X}^2\right]$.

(A) $\left(\frac{n+1}{n}\right)\theta^2$

(B) $\left(\frac{n+1}{n^2}\right)\theta^2$

(C) $\frac{\theta^2}{n}$

(D) $\frac{\theta^2}{\sqrt{n}}$

(E) θ^2

$\bar{X} = \frac{1}{n}(X_1 + \cdots + X_n)$

$\mathbb{E}[\bar{X}^2] = Var[\bar{X}] + (\mathbb{E}[\bar{X}])^2 = \frac{\theta^2}{n} + \theta^2 =>$

$\mathbb{E}[\bar{X}] = \mathbb{E}\left[\frac{1}{n}(X_1 + \cdots + X_n)\right] =$

$= \frac{1}{\cancel{n}} \cdot \cancel{n} \cdot \theta = \theta$

$Var[\bar{X}] = Var\left[\frac{1}{n}(X_1 + \cdots + X_n)\right]$

$= \frac{1}{n^2} \cdot Var[X_1 + \cdots + X_n]$ independent

$= \frac{1}{n^2}(Var[X_1] + \cdots + Var[X_n])$

$= \frac{1}{n^{\cancel{2}}} \cdot \cancel{n} \cdot \theta^2 = \frac{\theta^2}{n}$

7. Annual prescription drug costs are modeled by a two-parameter Pareto distribution with $\theta = 2000$ and $\alpha = 2$.

A prescription drug plan pays annual drug costs for an insured member subject to the following provisions:

(i) The insured pays 100% of costs up to the ordinary annual deductible of 250.

(ii) The insured then pays 25% of the costs between 250 and 2250.

(iii) The insured pays 100% of the costs above 2250 until the insured has paid 3600 in total.

(iv) The insured then pays 5% of the remaining costs.

Determine the expected annual plan payment.

(A) 1120

(B) 1140

(C) 1160

(D) 1180

(E) 1200

8. For a tyrannosaur with a taste for scientists:

(i) The number of scientists eaten has a binomial distribution with $q = 0.6$ and $m = 8$.

(ii) The number of calories of a scientist is uniformly distributed on (7000, 9000).

(iii) The numbers of calories of scientists eaten are independent, and are independent of the number of scientists eaten.

Calculate the probability that two or more scientists are eaten and exactly two of those eaten have at least 8000 calories each.

(A) 0.23

(B) 0.25

(C) 0.27

(D) 0.30

(E) 0.35