UNIVERSITY OF TEXAS AT AUSTIN

Homework Assignment #1

Prerequisite material.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 1.1. (5 points) Roger initially deposits \$4,000 in an investment fund which pays him \$2,000 at time 1 and \$4,000 at time 2.

Sally gets \$2,000 at time 0 and \$4,000 at time 1, and deposits \$5,460 at time 2 in return.

Both investments are governed by compound interest with the same annual effective interest rate i and they have the same net present values.

Find i.

(a) About 9%

(b) About 10.0%

(c) About 11.5%

(d) About 12%

(e) None of the above

Problem 1.2. (5 pts) Find the total amount of interest that would be paid on a \$1,000 loan over a 10−year period, if the effective interest rate is 0.09 per annum under the following repayment method:

The entire loan plus entire accumulated interest is paid as one lump-sum at the end of the loan term.

Problem 1.3. (5 points) Let $\Omega = \{a_1, a_2, a_3, a_4\}$ be an outcome space, and let $\mathbb{P}$ be a probability distribution on Ω. Assume that $\mathbb{P}[\{a_1, a_2\}] = 1/3, \mathbb{P}[\{a_2, a_3\}] = 1/4$ and $\mathbb{P}[\{a_1, a_3\}] = 1/9$. How much is $\mathbb{P}[\{a_4\}]$?

Problem 1.4. (10 points) Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ be a probability space. We denote by p_k the probability of the elementary outcome ω_k, i.e., $p_k = \mathbb{P}[\{\omega_k\}]$ for $k = 1, \ldots, 5$. You are given that p_k/p_{k-1} is constant for $k = 2, 3, 4, 5$. You are also given that $p_1 = 16/31$. Find p_5.

(a) 1/31

(b) 2/31

(c) 4/31

(d) Not enough information is given.

(e) None of the above.

Problem 1.5. (5 points) Emmanuel entered an extra special kind of game with his friend Fischer. First, they toss a fair coin. If the coin comes up heads, Emmanuel gives \$5,000 to Fischer. If the coin comes up tails, Fischer gives \$2,000 to Emmanuel. Then, regardless of the outcome of the first cointoss, they toss the same fair coin again. If it comes up heads, Emmanuel gives Fischer \$4,000. If the coin comes up tails, Fischer gives \$3,000 to Emmanuel. What is the expected cashflow, i.e., what is the expected amount of money that changes hands and who gives it to whom?

Problem 1.6. (5 points) The random variables (R_1, R_2) have the following moments:

$$\mathbb{E}[R_1] = 0.08, \quad SD[R_1] = 0.2,$$
$$\mathbb{E}[R_2] = 0.10, \quad SD[R_2] = 0.25.$$

The correlation coefficient between R_1 and R_2 is given to be 0.2. What is the standard deviation of the random variable $R = \frac{1}{2}(R_1 + R_2)$?
