UNIVERSITY OF TEXAS AT AUSTIN

Lecture 6

The Central Limit Theorem

Let $\{X_k; k = 1, 2, \dots\}$ be a sequence of random variables. For every $n \in \mathbb{N}$, we set

$$S_n = X_1 + X_2 + \cdots + X_n$$

If all the expectations exist, then

$$\mathbb{E}[S_n] = \mathbb{E}[X_1] + \mathbb{E}[X_2] + \cdots + \mathbb{E}[X_n] \text{ for every } n \in \mathbb{N}$$

Moreover, if $\{X_k; k = 1, 2, \dots\}$ are independent and their variances exist, then

$$Var[S_n] = Var[X_1] + Var[X_2] + \cdots + Var[X_n] \text{ for every } n \in \mathbb{N}$$

Theorem 6.1. *If the first and second moments are finite for all $X_k, k = 1, 2, \dots,$ then*

$$\frac{S_n - \mathbb{E}[S_n]}{\sqrt{Var[S_n]}} \Rightarrow N(0,1)$$

Usually, we operate in the special case where $X_k, k = 1, 2, \dots$ are identically distributed. In that case, we usually set

$$\mu_X = \mathbb{E}[X_1] \quad \text{and} \quad \sigma_X^2 = Var[X_1]$$

With this notation, we have

$$\frac{S_n - n\mu}{\sigma\sqrt{n}} \Rightarrow N(0,1)$$

Remark 6.2. One relies on the above result when one deals with confidence intervals for the mean parameter μ.