Problem set #10: Binomial Monte Carlo

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2023-10-23

Let the **volatility** of a stock be the standard deviation of its (continuously compounded) realized return on an annual basis. Then, we can define the up and down factors in the so-called *forward binomial tree* for a **non-dividend-paying** stock as

$$\begin{aligned} u &= e^{rh+\sigma\sqrt{h}} \\ d &= e^{rh-\sigma\sqrt{h}} \end{aligned} \tag{1}$$

Let the continuously compounded, risk-free interest rate be 0.04.

```
r=0.04
```

Consider a stock whose current price is \$100 and whose volatility is 0.25. We will be pricing a one-year, at-the-money call option in a variety of ways here.

```
#about the stock
s0=100
sigma=0.25
#about the call
T=1
K=s0
```

Problem #1: Analytic one period

Price the option above using a one period binomial tree.

```
h=1
u.1=exp(r*h+sigma*sqrt(h))
#u.1
d.1=exp(r*h-sigma*sqrt(h))
#d.1

s.u=s0*u.1
s.d=s0*d.1

v.c<-function(x){
  max(x-K,0)
}

p.star=1/(1+exp(sigma*sqrt(h)))

v.0=exp(-r*T)*(v.c(s.u)*p.star+v.c(s.d)*(1-p.star))
v.0
```

```
## [1] 14.15203
```

Problem #2: Monte Carlo one period

Price the option above using Monte Carlo a one period binomial tree. Use 10000 simulations.

```
nsims=10000

probs=c(p.star, 1-p.star)
factors=c(u.1, d.1)

s.T=s0*sample(factors, size=nsims, prob=probs, replace=TRUE)
#s.T

v.T=pmax(s.T-K, 0)
#v.T

v.bar=mean(v.T)
#v.bar

v.0.mc=exp(-r*T)*v.bar
v.0.mc
```

```
## [1] 14.03167
```

Problem #3: Analytic two periods

Price the above option using a two-period binomial tree.

Problem #4: Monte Carlo two periods

Price the option above using Monte Carlo a two period binomial tree. Use 10000 simulations.

Problem #5: Analytic one hundred periods

Price the above option using a $100-$period binomial tree.

Problem #6: Monte Carlo with one hundred periods

Price the option above using Monte Carlo with a hundred period binomial tree. Use 10000 simulations.