

M339 D: March 29th, 2021.

The Relationship between prepaid forwards & forwards.



Forward Contract:

$$\text{Init. Cost} = 0$$

- pmt: $F_{0,T}$
- delivery of stock

$$\boxed{\text{Payoff} = S(T) - F_{0,T}}$$

Prepaid Forward:

- pmt: $F_{0,T}^P$

$$\text{Init. Cost} = F_{0,T}^P$$

- delivery of stock

$$\boxed{\text{Payoff} = S(T)}$$

I propose the following replicating portfolio for the prepaid forward contract:

PORT $\left\{ \begin{array}{l} \bullet \text{forward contract} \\ \bullet \text{invest } PV_{0,T}(F_{0,T}) \text{ @ the ccrfir } r \\ \text{to be withdrawn @ time } T \end{array} \right.$

This is a static portfolio, so I just need to check whether the payoff's match.

$$\begin{aligned} \text{Payoff(PORT)} &= \underbrace{S(T) - F_{0,T}}_{\text{forward contract}} + \underbrace{FV_{0,T}(PV_{0,T}(F_{0,T}))}_{\text{risk-free investment}} \\ &= S(T) - F_{0,T} + F_{0,T} = \underline{\underline{S(T)}} \end{aligned}$$

=> This is, indeed, a replicating portfolio for the prepaid forward

$$\Rightarrow F_{0,T}^P = PV_{0,T}(F_{0,T})$$

\uparrow
NO ARBITRAGE!

\Leftrightarrow

$$F_{0,T} = FV_{0,T}(F_{0,T}^P)$$



Focus on (prepaid) forwards on stocks.

Goal: To figure out the expressions for the prepaid forward price and the forward price in terms of more primitive "ingredients".

Case #1: NO DIVIDENDS.



Prepaid Forward:

$$\text{I.C.} = \boxed{F_{0,T}^P}$$

\Leftarrow
NO ARBITRAGE!

Payoff: $S(T)$

\Downarrow

Payoff: $S(T)$

Outright Purchase:

$$\text{I.C.} = \boxed{S(0)}$$

$$\Rightarrow \boxed{F_{0,T}^P(S) = S(0)}$$

Q: What is the dependence on the delivery date?

NONE!



$$\Rightarrow \boxed{F_{0,T}(S) = FV_{0,T}(F_{0,T}^P(S))}$$

$$= FV_{0,T}(S(0)) = S(0)e^{rT}$$

$r \dots \text{ccfrir} > 0$

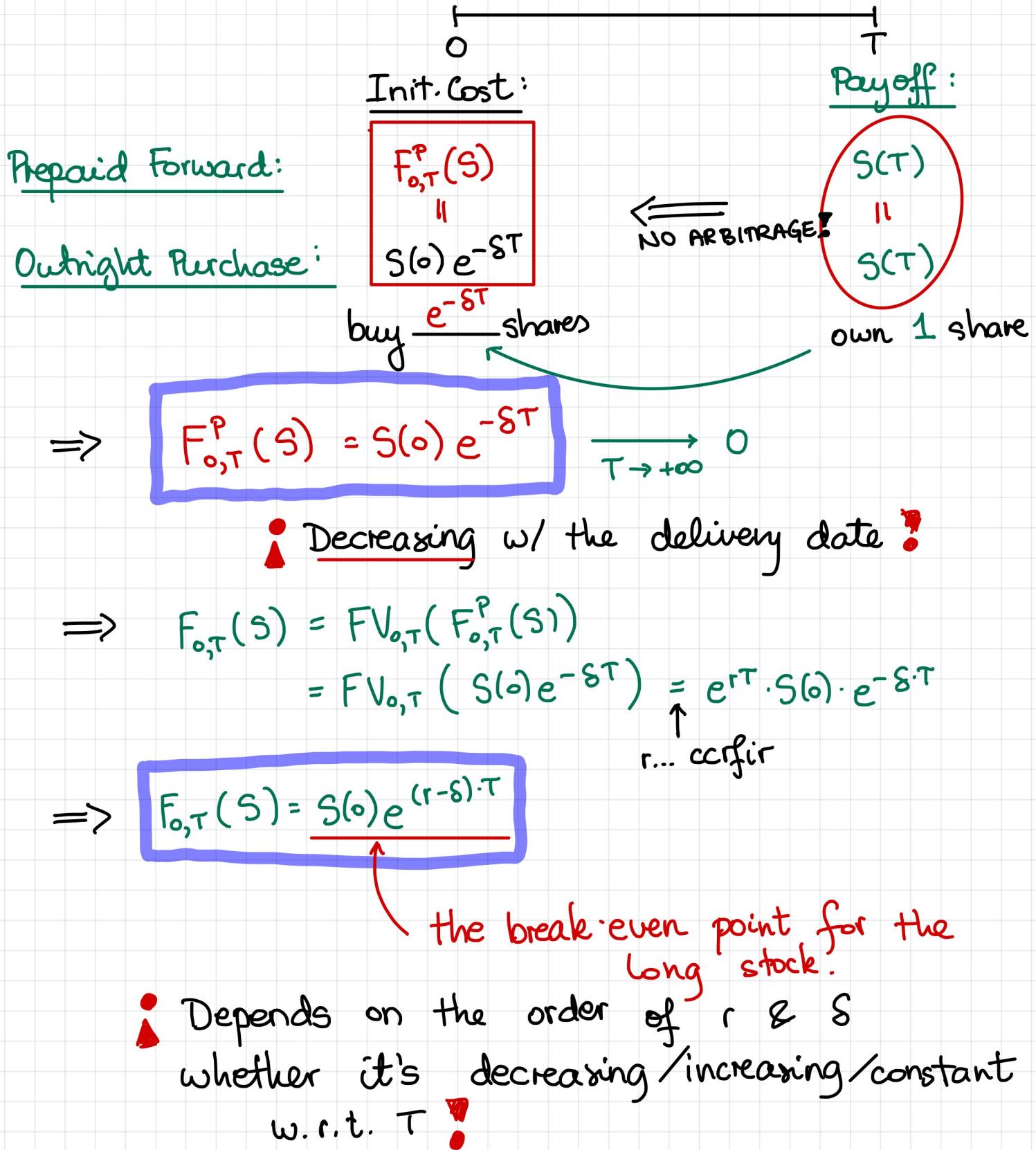
Increasing with the delivery date!

Q: Propose a replicating portfolio for a forward contract? A Fully Leveraged Purchase is a synthetic forward.

Case #2.

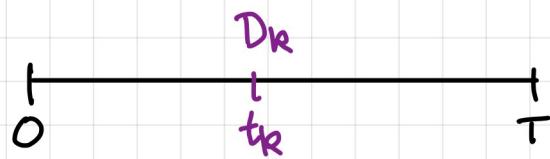
CONTINUOUS DIVIDENDS.

S... dividend yield



Case #3.

DISCRETE DIVIDENDS.



$$k=1 \dots n; \quad t_n \leq T$$

by convention, if $t_n = T$,
the dividend is paid
just prior to the delivery

$$F_{0,T}^P(S) = S(0) - \sum_{k=1}^n PV_{0,t_k}(D_k)$$

In words, the investor must be compensated for the "loss" of dividends through the prepaid forward price.

$$F_{0,T}(S) = S(0)e^{rT} - \sum_{k=1}^n D_k e^{r(T-t_k)}$$

\uparrow
r...ccifir