The University of Texas at Austin
Homework Assignment 4
Predictive Analytics

February 08, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

F −distribution

Problem 4.1. (5 points) *Source: Ramachandran and Tsokos: Mathematical Statistics with Applications in R.* Let S_1^2 denote the sample variance for a random sample of size 10 from a normal population I and let S_2^2 denote the sample variance for a random sample of size 8 from a normal population II. The variance of population I is assumed to be three times the variance of population II. Assume that the two samples are **independent**. Find two numbers a and b such that

$$\mathbb{P}[S_1^2/S_2^2 \leq a] = 0.05 \quad \text{and} \quad \mathbb{P}[S_1^2/S_2^2 \geq b] = 0.05$$

Problem 4.2. (10 points) *Source: An old CAS exam problem.* A sample of size 20 is fitted to a linear regression model of the form

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \varepsilon_i.$$

The resulting F −ratio used to test the hypothesis

$$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$$

is equal to 21. Determine R^2.

Problem 4.3. In a simple linear regression fit on 16 observations, you obtain the point estimate of the slope parameter to be $\hat{\beta}_1 = 3$. The standard error of $\hat{\beta}_1$ is estimated at 1.5.

- (10 points) Show that, in our usual notation,

$$TSS - RSS = \frac{\left(\sum (x_i - \bar{x})(y_i - \bar{y})\right)^2}{\sum (x_i - \bar{x})^2}.$$

- (10 points) Prove that for the simple linear regression, the F −statistic can be obtained as the square of the t −statistic for the slope.
- (5 points) Provide the value of the F −statistic.
- (10 points) Provide the value of the coefficient of determination R^2.