University of Texas at Austin

Quiz #5

Log-normal stock prices: Conditional expectation.

Problem 5.1. (5 pts)
A non-dividend-paying stock is currently valued at \$100 per share. Its annual mean rate of return is given to be 12% while its volatility is given to be 30%.

Assuming the lognormal stock-price model, find

$$\mathbb{E}[S(2) \mid S(2) > 95].$$

(a) \$86.55
(b) \$101.60
(c) \$152.35
(d) \$159.07
(e) None of the above.

Solution: (c)
In our usual notation,

$$\mathbb{E}[S(T) \mid S(T) > K] = \frac{S(0)e^{(\alpha-\delta)T}N(\hat{d}_1)}{N(\hat{d}_2)}.$$

with

$$\hat{d}_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(\alpha - \delta + \frac{\sigma^2}{2}\right)T\right],$$
$$\hat{d}_2 = \hat{d}_1 - \sigma\sqrt{T}.$$

In the present probem,

$$\hat{d}_1 = \frac{1}{0.3\sqrt{2}}\left[\ln\left(\frac{100}{95}\right) + \left(0.12 + \frac{0.09}{2}\right) \times 2\right] = 0.8987,$$
$$\hat{d}_2 = 0.8987 - 0.3\sqrt{2} = 0.4745.$$

So, our answer is

$$\mathbb{E}[S(2) \mid S(2) > 95] = \frac{100e^{(0.12)\times 2}N(0.8987)}{N(0.4745)} = \frac{100e^{0.24} \times 0.8159}{0.6808} = 152.35.$$

Problem 5.2. (5 pts) A non-dividend-paying stock is currently valued at \$100 per share. Its annual mean rate of return is given to be 8% while its volatility is given to be 20%.

Assuming the lognormal stock-price model, find

$$\mathbb{E}[S(4) \mid S(4) > 90].$$

(a) \$96.55
(b) \$101.60
(c) \$153.30
(d) \$159.07
(e) None of the above.

Solution: (c)
In our usual notation,

$$\mathbb{E}[S(T) \mid S(T) > K] = \frac{S(0)e^{(\alpha-\delta)T}N(\hat{d}_1)}{N(\hat{d}_2)}.$$

with

$$\hat{d}_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(\alpha - \delta + \frac{\sigma^2}{2}\right)T\right],$$
$$\hat{d}_2 = \hat{d}_1 - \sigma\sqrt{T}.$$

In the present probem,

$$\hat{d}_1 = \frac{1}{0.2\sqrt{4}}\left[\ln\left(\frac{100}{90}\right) + \left(0.08 + \frac{0.04}{2}\right)\times 4\right] = 1.2634,$$
$$\hat{d}_2 = 1.2634 - 0.2\sqrt{4} = 0.8634.$$

So, our answer is

$$\mathbb{E}[S(2) \mid S(2) > 95] = \frac{100e^{(0.08)\times 4}N(1.26)}{N(0.86)} = \frac{100e^{0.32}\times 0.8962}{0.8051} = 153.295.$$

Problem 5.3. (5 points) Let $S(T)$ stand for the lognormally distributed time$-T$ stock price. Then, for every $K > 0$, we have that

$$\mathbb{E}[S(T) \mid S(T) > K] + \mathbb{E}[S(T) \mid S(T) < K]$$

equals ...

(a) $\mathbb{E}[S(T)]$
(b) K
(c) $\mathbb{E}[S(T)] - K$
(d) $\mathbb{E}[S(T)] + K$
(e) None of the above.

Solution: (e)