M339J: April 19th, 2021.

Aggregate Loss Models.

The Individual Risk Model.

Let $\{X_j, j=1,2,\dots,n\}$ be independent (not necessarily identically dist'd) r.v.s.

Set $S = X_1 + \dots + X_n$.
Then, S represents aggregate losses.

The Collective Risk Model.

Let $\{X_j, j=1,2,\dots\}$ be a sequence of independent, identically dist'd r.v.s

Let N be an $\mathbb{N}_0$-valued r.v. independent from $\{X_j, j=1,\dots\}$.

Define $S = X_1 + X_2 + \dots + X_N = \sum_{j=1}^{N} X_j$

w/ the convention that $S=0$ when $N=0$.
Then, S represents aggregate losses.

Q: $F_S = ?$ $\ddot{\frown}$ Convolution.

It is convenient to use the pgfs & mgfs.

If X is discrete, then $\boxed{P_S(z) = P_N(P_X(z))}$

If X is continuous, then $\boxed{M_S(z) = P_N(M_X(z))}$

Facts:

- $\mathbb{E}[S] = \mathbb{E}[N]\cdot\mathbb{E}[X]$ Wald's Identity
- $Var[S] = \mathbb{E}[N]\cdot Var[X] + Var[N]\cdot(\mathbb{E}[X])^2$

8. The number of claims, N, made on an insurance portfolio follows the following distribution:

n	$\Pr(N=n)$
0 ✓	0.7
2 ✓	0.2
3 ✓	0.1

N... frequency.

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2, respectively.

X...severity

The number of claims and the benefit for each claim are independent.

S $\qquad$ μ_S

Calculate the probability that aggregate benefits will exceed expected benefits by more than 2 standard deviations.

σ_S

(A) 0.02

(B) 0.05

(C) 0.07

(D) 0.09

(E) 0.12

$$\mathbb{P}[\,S > \mu_S + 2\sigma_S\,] = ?$$

- $\mu_S = \mathbb{E}[S] = \mathbb{E}[N]\cdot\mathbb{E}[X] = 1.4$

$$\mathbb{E}[N] = 0\cdot 0.7 + 2\cdot 0.2 + 3\cdot 0.1 = 0.7 \ ✓$$

$$\mathbb{E}[X] = 0\cdot 0.8 + 10\cdot 0.2 = 2 \ ✓$$

- $\sigma_S^2 = Var[S] = \mathbb{E}[N]\cdot Var[X] + Var[N]\,(\mathbb{E}[X])^2$

w/ $Var[N] = \mathbb{E}[N^2] - (\mathbb{E}[N])^2$

$$= 2^2\cdot 0.2 + 3^2\cdot 0.1 - (0.7)^2$$

$$= 1.7 - 0.49 = 1.21$$

$$Var[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

$$= 10^2\cdot(0.2) - 2^2 = 20 - 4 = 16$$

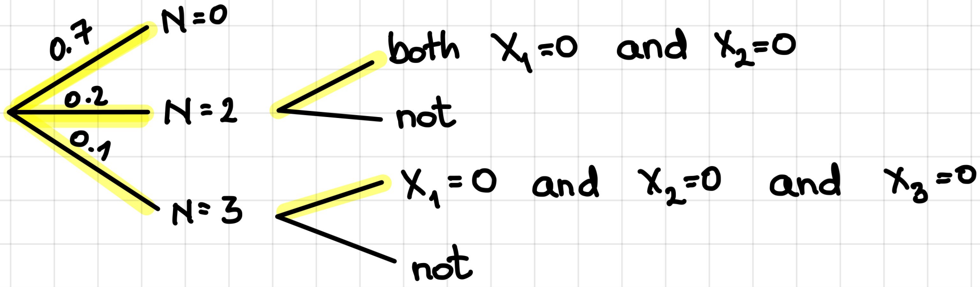
$\Rightarrow Var[S] = 0.7\cdot 16 + 1.21\cdot 2^2 = \ldots = 16.04$

$\Rightarrow \sigma_S = \sqrt{16.04} = 4.005$ ✓

$$\mathbb{P}[S > 1.4 + 2\cdot(4.005)] = \mathbb{P}[S > 9.41] = \mathbb{P}[S \geq 10]$$

$$= 1 - \boxed{\mathbb{P}[S=0]}$$

$\mathbb{P}[S=0] = ?$



$$\mathbb{P}[S=0] = 0.7 + 0.2\cdot 0.8\cdot 0.8 + 0.1\cdot 0.8\cdot 0.8\cdot 0.8 = \cdots = 0.8792$$

$\Rightarrow$ answer $= 1 - 0.8792 = 0.1208$ $\blacksquare$

16. You are given:

		Mean	Standard Deviation
N	Number of Claims	8	3
X	Individual Losses	10,000	3,937

independent

Using the normal approximation, determine the probability that the aggregate loss S will exceed 150% of the expected loss. μ_S

$S = X_1 + X_2 + \cdots + X_N$

(A) $\Phi(1.25)$

(B) $\Phi(1.5)$

(C) $1-\Phi(1.25)$

(D) $1-\Phi(1.5)$

(E) $1.5\,\Phi(1)$

$\mathbb{P}[S > 1.5 \cdot \mu_S] = ?$

- $\mu_S = \mathbb{E}[N] \cdot \mathbb{E}[X] = 8(10{,}000) = 80{,}000$
- $\sigma_S^2 = Var[S] = \mathbb{E}[N] \cdot Var[X] + Var[N](\mathbb{E}[X])^2$

$= 8 \cdot (3937)^2 + 9 \cdot (10^4)^2$

$= 1{,}023{,}999{,}752$

$\Rightarrow \quad \sigma_S = 32\,000$

$\mathbb{P}[S > 1.5 \cdot \mu_S] = \mathbb{P}\left[\frac{S-\mu_S}{\sigma_S} > \frac{1.5\mu_S - \mu_S}{\sigma_S}\right] = \mathbb{P}\left[Z > \frac{0.5\mu_S}{\sigma_S}\right]$

$\overset{??}{} N(0,1) \sim Z$

$= \mathbb{P}\left[Z > \frac{0.5(80000)}{32{,}000}\right] = \mathbb{P}[Z > 1.25] =$

$= 1 - \mathbb{P}[Z \leq 1.25] = 1 - \Phi(1.25)$