UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 3

The feasible set. Portfolio returns and volatilities. Efficient portfolios.

Provide your final answer only to the following problem(s):

Problem 3.1. (2 points) Consider the feasible set for two stocks. The higher the correlation of the two stocks' returns, the higher the curvature of the feasible set. *True or false?*

Solution: FALSE

Problem 3.2. (2 points) You are considering equally-weighted large portfolios which you construct so that:

- For each individual stock in the portfolio, the variance is 0.20.
- For each pair of distinct stocks in the portfolio, the covariance is 0.10.

Then, as the number of stocks in the portfolio gets larger, the variance of the portfolio's return approaches 0.10. *True or false?*

Solution: TRUE

Problem 3.3. (2 points) We call risk that is perfectly correlated across assets **systematic risk.** *True or false?*

Solution: TRUE
Also known as **non-diversifiable** or **common** risk.

Problem 3.4. (2 points) An efficient portfolio contains only systematic risk. *True or false?*

Solution: TRUE

Problem 3.5. Portfolio P has expected return 0.08 and volatility equal to 12%. Porfolio Q has expected return 0.10 and volatility equal to 12.5%. Then, we can say with certainty that portfolio P is not efficient. *True or false?*

Solution: FALSE

Problem 3.6. (5 points) Consider two assets X and Y such that:

- their expected returns are $\mathbb{E}[R_X] = 0.12$ and $\mathbb{E}[R_Y] = 0.15$;
- their volatilities are $\sigma_X = 0.2$ and $\sigma_Y = 0.25$;
- the correlation coefficient of their returns is $\rho_{X,Y} = -1$.

You are tasked with constructing a portfolio consisting of shares of X and Y with a risk-free return. What should the weight w_X given to asset X be?

(a) 4/9
(b) 1/2
(c) 5/9
(d) Such a weight does not exist.
(e) None of the above.

Solution: (c)

$$w_X = \frac{\sigma_Y}{\sigma_X + \sigma_Y} = \frac{0.25}{0.2 + 0.25} = \frac{5}{9}.$$

Problem 3.7. (5 points) You are given the following information about a portfolio consisting of stocks X, Y, and Z:

Stock	Investment Amount	Expected Return
X	20,000	8%
Y	5,000	12%
Z	25,000	14%

Calculate the expected return of the portfolio.

(a) 11.3%
(b) 11.4%
(c) 11.5%
(d) 11.6%
(e) None of the above.

Solution: (b)

The three weights given to the assets X, Y, and Z are $w_X = 0.4, w_y = 0.1$ and $w_Z = 0.5$. So,

$$\mathbb{E}[R_P] = 0.4(0.08) + 0.1(0.12) + 0.5(0.14) = 0.114.$$

Provide your complete solution(s) to the following problem(s):

Problem 3.8. (10 points) You are given the following information about a portfolio with four assets:

Asset	Market value of asset	Covariance of asset's return with the portfolio return
I	40,000	0.2
II	30,000	-0.10
III	20,000	0.25
IV	10,000	-0.05

Calculate the standard deviation of the portfolio return.

Solution: Let the weights of the four assets in the portfolio be denoted by w_I, w_{II}, w_{III} and w_{IV}. We have established the following in class:

$$\begin{aligned} Var[R_P] &= Cov[R_P, R_P] = Cov[w_I R_I + w_{II} R_{II} + w_{III} R_{III} + w_{IV} R_{IV}, R_P] \\ &= w_I Cov[R_I, R_P] + w_{II} Cov[R_{II}, R_P] + w_{III} Cov[R_{III}, R_P] + w_{IV} Cov[R_{IV}, R_P] \end{aligned}$$

Using the information provided in the table in the problem, we get

$$Var[R_P] = 0.4(0.2) + 0.3(-0.1) + 0.2(0.25) + 0.1(-0.05) = 0.095.$$

So, the portfolio's volatility equals $SD[R_P] = \sqrt{0.095} = 0.3082$.

Problem 3.9. (10 points) You are given the following information about the annual returns of two stocks X and Y:

- The expected returns of X and Y are $\mathbb{E}[R_X] = 0.08$ and $\mathbb{E}[R_Y] = 0.10$.
- The volatilities of the returns are $\sigma_X = 0.20$ and $\sigma_Y = 0.25$.
- The correlation coefficient of the returns for these two stocks is -0.4.
- The expected return for a certain portfolio, consisting only of stocks X and Y, is 0.09

Calculate the volatility of this portfolio.

Solution: Let the weights of the two stocks in the portfolio be denoted by w_X and w_Y. Then,

$$0.09 = \mathbb{E}[R_P] = w_X(0.08) + (1 - w_X)(0.10) \quad \Rightarrow \quad w_x = 0.5.$$

Hence,

$$Var[R_P] = Var[0.5R_X + 0.5R_Y] = Var[0.5(R_X + R_Y)] = 0.25Var[R_X + R_Y].$$

From the given information, we can calculate

$$Var[R_X + R_Y] = \sigma_X^2 + \sigma_Y^2 + 2\sigma_X\sigma_Y\rho_{X,Y} = (0.2)^2 + (0.25)^2 + 2(0.2)(0.25)(-0.4) = 0.0625.$$

Finally,

$$SD[R_P] = 0.5\sqrt{0.0625} = 0.125.$$

Problem 3.10. (10 points) Your model for the economy at the end of your period has two different states *good* and *bad*. You are an optimist and you think that the probability that the economy will be in the *good* state is twice the probability that it will be in the *bad* state.

There are three assets in your market model called S, T and Q. Their returns, depending on the state of the economy are modeled as follows:

Asset	*good*	*bad*
S	10%	-4%
T	6%	-5%
Q	8%	-1%

You put half of your wealth into asset S, a quarter into asset Q and the remainder into asset T. What is the volatility of this total portfolio?

Solution: The probability of the *good* economy is 2/3 and the probability of the *bad* economy is 1/3. So, the return R_W of the total portfolio has the following distribution

$$R_W = \begin{cases} 0.5(0.1) + 0.25(0.06) + 0.25(0.08) & \text{with probability } 2/3, \\ 0.5(-0.04) + 0.25(-0.05) + 0.25(-0.01) & \text{with probability } 1/3. \end{cases} = \begin{cases} 0.085 & \text{with probability } 2/3, \\ -0.035 & \text{with probability } 1/3. \end{cases}$$

We get that the mean return of this portfolio equals

$$\mathbb{E}[R_W] = 0.085\left(\frac{2}{3}\right) + (-0.035)\left(\frac{1}{3}\right) = 0.045.$$

The second moment of the portfolio's return is

$$\mathbb{E}[R_W^2] = (0.085)^2\left(\frac{2}{3}\right) + (-0.035)^2\left(\frac{1}{3}\right) = 0.005225.$$

Therefore, the variance of the return equals

$$Var[R_W] = \mathbb{E}[R_W^2] - (\mathbb{E}[R_W])^2 = 0.005225 - (0.045)^2 = 0.0032.$$

Finally, the volatility of the portfolio is $\sigma_W = 0.0565685$.