Subjective Probabilities.

Our agents form models about the relative likelihoods of the price of a particular asset at a later date. Formally, they create a probabilistic model for the distribution of the time-T asset price $S(T)$.
At least, the expected value $\boxed{E[S(T)]}$ is worth paying attention to.

Assume: Among the admissible portfolios in the market model, agents invest in the portfolio which maximizes the profit (according to their model).

Note: Our agents always have the possibility to invest @ the risk-free interest rate.

$\Rightarrow$ Our agents will, at least, require a

strictly positive expected profit.

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Subjective expectations.

Problem 4.1. IFM Sample (Introductory) Problem #6.
The following relates to one share of XYZ stock:
- The current price is 100.
- The forward price for delivery in one year is 105.
- An investor who decides to long the forward contract denotes by P the expected stock price in one year.

Determine which of the following statements about P is **TRUE**.
- (A) $P < 100$
- (B) $P = 100$
- (C) $100 < P < 105$
- (D) $P = 105$
- (E) $P > 105$

$$\mathbb{E}\left[\text{Profit}[\text{Long forward}]\right] > 0$$

$$\mathbb{E}\left[S(1) - F_{0,1}\right] > 0$$

$$P := \mathbb{E}[S(1)] > F_{0,1} = 105$$

$$S(0) = 75 \checkmark$$

$$\mathbb{E}[S(3)] = 90 \checkmark$$

Problem 4.2. IFM Sample (Introductory) Problem #38.
The current price of a medical company's stock is 75. The expected value of the stock price in three years is 90 per share. The stock pays no dividends. You are also given:
- The risk-free interest rate is positive.
- There are no transaction costs.
- Investors require compensation for risk. $\Rightarrow \mathbb{E}[\text{Profit}] > 0$

The price of a three-year forward on a share of this stock is X and at this price an investor is willing to enter into the forward. Determine what can be concluded about X.
- (A) $X < 75$
- (B) $X = 75$
- (C) $75 < X < 90$
- ✗(D) $X = 90$
- ✗(E) $X > 90$

- By the same reasoning as in the previous problem.

$$\mathbb{E}[S(3)] - F_{0,3} > 0$$

$$90 > F_{0,3} = X$$

- Using the formula for the forward price:

$$X = F_{0,3} = S(0)e^{(r-\delta)(3)} \underset{\uparrow}{=} S(0)e^{3r} = 75e^{3r} \geq 75$$

$$\text{no div}$$

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Problem 4.3. IFM Sample (Introductory) Problem #70.

$\boxed{S(0) = 100}$

Investors in a certain stock demand to be compensated for risk. The current stock price is 100. The stock pays dividends at a rate proportional to its price. The dividend yield is 2%. The continuously compounded risk-free interest rate is 5%. Assume there are no transaction costs.

$r = 0.05$

$\delta = 0.02$

Let X represent the expected value of the stock price 2 years from today. Assume it is known that X is a whole number. Determine which of the following statements is true about X.

(A) The only possible value of X is 105.
(B) The largest possible value of X is 106.
(C) The smallest possible value of X is 107.
(D) The largest possible value of X is 110.
(E) The smallest possible value of X is 111.

$$\mathbb{E}[\text{Profit}] > 0$$

$$\mathbb{E}\left[S(2) - F_{0,2}\right] > 0$$

$$X := \mathbb{E}[S(2)] > F_{0,2} = S(0)e^{(r-\delta)(2)} = 100\, e^{(0.05-0.02)(2)}$$

$$= 100\, e^{0.06}$$

$$\boxed{e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots}$$