Aggregate Loss Models.

The Individual Risk Model.

Let $\{X_j : j=1,2,\dots,n\}$ be independent (but not necessarily identically dist'd)

$$\boxed{S = X_1 + X_2 + \dots + X_n}$$

Then, S represents aggregate losses.

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The Collective Risk Model.

Let $\{X_j, j=1,2,\dots\}$ be a sequence of independent, identically distributed r.v.

Let N be an $\mathbb{N}_0$-valued r.v. independent from $\{X_j, j=1,2,\dots\}$.

Define: $$\boxed{S = X_1 + X_2 + \dots + X_N = \sum_{j=1}^{N} X_j}$$

w/ the convention that $S=0$ when $N=0$.

Then, S represents aggregate losses.

Facts:

- $\mathbb{E}[S] = \mathbb{E}[N] \cdot \mathbb{E}[X]$ Wald Identity.
- $Var[S] = \mathbb{E}[N] \cdot Var[X] + Var[N] \cdot (\mathbb{E}[X])^2$

Q: What's the dist'n of S?

It is convenient to use pgfs and mgfs.

If X is $\mathbb{N}_0$-valued, then $\boxed{P_S(z) = P_N(P_X(z))}$

If X is continuous, then $\boxed{M_S(z) = P_N(M_X(z))}$

:)

8. The number of claims, N, made on an insurance portfolio follows the following distribution:

n	$\Pr(N=n)$
0	0.7
2	0.2
3	0.1

N... frequency

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2, respectively.

X... severity

The number of claims and the benefit for each claim are independent.

Calculate the probability that aggregate benefits (S) will exceed expected benefits (μ_S) by more than 2 standard deviations (σ_S).

$$\mathbb{P}[S > \mu_S + 2\sigma_S] = ?$$

(A) 0.02

(B) 0.05

(C) 0.07

(D) 0.09

(E) 0.12

- $\mu_S = \mathbb{E}[N]\cdot\mathbb{E}[X] = 0.7\cdot 2 = 1.4$

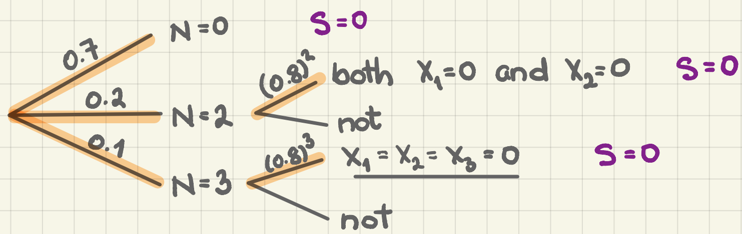
$$\mathbb{E}[N] = 0\cdot 0.7 + 2\cdot 0.2 + 3\cdot 0.1 = 0.7$$
$$\mathbb{E}[X] = 0\cdot 0.8 + 10\cdot 0.2 = 2$$

- $\sigma_S^2 = Var[S] = \mathbb{E}[N]\cdot Var[X] + Var[N]\cdot(\mathbb{E}[X])^2$

$$Var[N] = \mathbb{E}[N^2] - (\mathbb{E}[N])^2$$
$$= 2^2\cdot 0.2 + 3^2\cdot 0.1 - (0.7)^2$$
$$= 1.7 - 0.49 = 1.21$$
$$Var[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$
$$= 10^2\cdot 0.2 - (2)^2 = 16$$

$$\sigma_S^2 = 0.7\cdot 16 + 1.21\cdot 2^2 = 16.04$$
$$\sigma_S = \sqrt{16.04} = 4.005$$

$$\mathbb{P}[S > 1.4 + 2\cdot(4.005) = 9.401] = \mathbb{P}[S \geq \underset{20}{10}] = 1 - \mathbb{P}[S=0]$$



$$\mathbb{P}[S=0] = 0.7 + 0.2\cdot(0.8)^2 + 0.1\cdot(0.8)^3 = \cdots = \underline{0.88}$$

answer : 0.12.

Note: We can adapt the CLT to this setting.

$$S \text{ "}\sim\text{" } \mathrm{Normal}(\text{mean}=\mu_S,\ \text{var}=\sigma_S^2)$$