

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 4

Prerequisite material. Realized returns.

Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 4.1. (5 points) *Source: Open Course Intro to Statistics.*

Cholesterol levels for women aged 20 to 34 follow an approximately normal distribution with mean 185 milligrams per deciliter (mg/dl). Women with cholesterol levels above 220 mg/dl are considered to have high cholesterol and about 18.5% of women fall into this category. What is the standard deviation of the distribution of cholesterol levels for women aged 20 to 34?

Problem 4.2. (5 points) A fair tetrahedron (a four-sided symmetric Platonic solid) with sides dyed pink, purple, mauve, and fuchsia will be rolled 2000 times. You intend to record the color of the side the tetrahedron fell on after every roll. What is the approximate probability that the outcome is mauve at most 510 times? There is no need to use the continuity correction.

Problem 4.3. (5 points) You roll a fair tetrahedron whose sides are labeled by 1, 2, 3, and 4 a total of 4000 times. What is the approximate probability that you see a 1 strictly more than 1025 times? There is no need to use the continuity correction.

Problem 4.4. (5 pts) For a stock price that was initially \$55.00, what is the price after 4 years if the continuously compounded returns for these 4 years are 4.5%, 6.2%, 8.9%, and -3.2% ?

Problem 4.5. (5 pts) A non-dividend-paying stock is valued at \$55.00. The annual expected return is 12.0% and the standard deviation of annualized returns is 22.0%. If the stock is lognormally distributed, what is the expected stock price after 3 years?

Problem 4.6. (5 pts) For a stock price that was initially \$55.00, what is the price after 4 years if the observed continuously compounded returns for these 4 years are 4.5%, 6.2%, 8.9%, and 3.2%?

Problem 4.7. (5 points) A continuous-dividend-paying stock is valued at \$100.00 per share. Its dividend yield is 0.02. The time- t realized return is modeled as

$$R(0, t) \sim N(\text{mean} = 0.065t, \text{variance} = 0.16t)$$

The continuously-compounded, risk-free interest rate is 0.04.

Find the probability that the realized return in the first year exceeds the continuously compounded, risk-free interest rate.

Problem 4.8. (10 points) Assume the Black-Scholes model. Let $S(t), t \geq 0$, denote the stock price. You are given that the initial stock price equals \$36, the mean rate of appreciation equals 0.08, and the volatility is 0.20. Find the expected value of $\sqrt{S(2)}$.

Problem 4.9. (5 points) A certain common stock is priced at \$36.50 per share. The company just paid its \$0.50 quarterly dividend. The dividends will continue to be paid quarterly in the same amount. Assume that the continuously-compounded, risk-free interest rate equals $r = 6.0\%$. Consider a \$35 strike European call, maturing in 6 months which currently sells for \$3.20. What is the price of the corresponding 6-month, \$35 strike put option? Assume that any dividends due are paid just prior to the options' exercise.