M358K: October 31st, 2022.

Power of Test [Practice].

Example. A simple random sample of size 36 gathered from a normal population w/ an unknown mean μ and the standard deviation of 3.

We are testing:

$$\boxed{H_0: \mu = 15} \quad \text{vs.} \quad H_a: \mu > 15$$

The significance level is $\alpha = 0.05$.

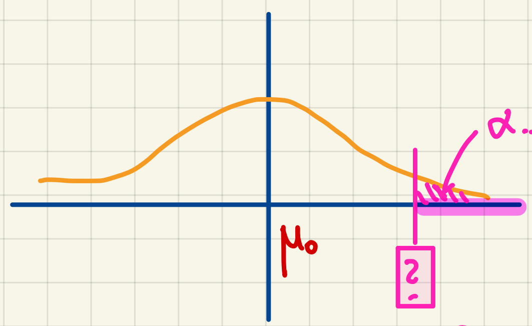
Find the power of the test @ $\mu_a = 16$.

$\longrightarrow$: First, find the RR.

Second, calculate the probability that the sample mean falls into the RR if $\mu = \mu_a = 16$.

Right-sided alternative $\Rightarrow$ RR in raw units is of the form

$[\boxed{?}, +\infty)$



$$\mu_0 + \frac{\sigma}{\sqrt{n}} \cdot z_{1-\alpha}$$

w/ $z_{1-\alpha} = \Phi^{-1}(1-\alpha) = \text{qnorm}(1-\alpha)$

In this problem, the lower bound of the RR is:

$$15 + \underbrace{\frac{3}{\sqrt{36}} \cdot (1.645)}_{0.8225} = 15.8225$$

$$\boxed{RR: [15.8225, +\infty)}$$

Second, $\mathbb{P}_{\mu_a}[\bar{X} \geq 15.8225] = ?$ w/ $\mu_a = 16$

Under the particular given alternative $\mu_a = 16$, the distribution of the sample mean $\bar{X}$ is:

$$\bar{X} \sim \text{Normal}\left(\text{mean} = \mu_a = 16, \ \text{sd} = \frac{3}{\sqrt{36}} = 0.5\right)$$

Method I.

$1 - \text{pnorm}(15.8225, 16, 0.5) = 0.6387052$

Method II.

$1 - \text{pnorm}(15 + \text{qnorm}(0.95) * 0.5, 16, 0.5) = 0.63876$

Method III.

Standardize.

$$\mathbb{P}_{\mu_a}\Big[\underbrace{\frac{\bar{X} - 16}{0.5}}_{= Z \,\sim N(0,1)} \geq \frac{15.8225 - 16}{0.5}\Big] = \mathbb{P}[Z \geq -0.355]$$

$$\cong \mathbb{P}[Z \leq 0.36] = 0.6406$$

std normal tables.

$\square$

Problem 14.4. The time needed for college students to complete a certain mirror-symmetry puzzle is modeled using a normal distribution with a mean of 30 seconds and a standard deviation of 3 seconds. You wish to see if the population mean time μ is changed by vigorous exercise, so you have a group of nine college students exercise vigorously for 30 minutes and then complete the puzzle.

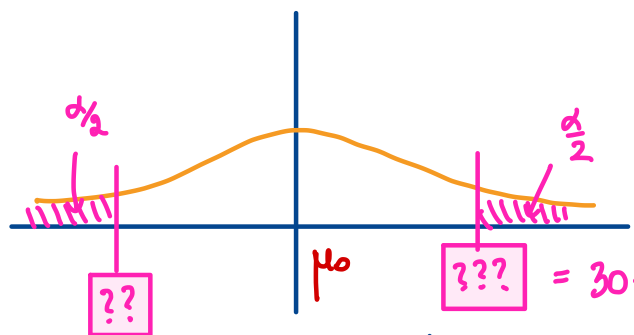
i. What are your null and alternative hypotheses?

ii. What is the rejection region at the significance level 0.01?

iii. What is the power of your test at $\mu = 28$ seconds?

i. $H_0: \mu = 30$ vs. $H_a: \mu \neq 30$

ii. $\alpha = 0.01$



$= 30 + 2.576 = 32.576$

$$\mu_0 + \frac{\sigma}{\sqrt{n}} \cdot z_{\frac{\alpha}{2}} = 30 + \frac{\cancel{3}}{\cancel{\sqrt{9}}} \cdot (-2.575829) = 27.424$$

$z_{\frac{\alpha}{2}} = -2.575829$

RR: $[0, 27.424] \cup [32.576, +\infty)$

Power of test: $\boxed{\mathbb{P}_{\mu_a}[\bar{X} \text{ falls into RR}] = ?}$

$\bar{X} \sim \text{Normal}(\text{mean} = 28, \text{sd} = 1)$

pnorm(27.424, 28, 1) + (1 − pnorm(32.576, 28, 1)) = 0.28231

□

Problem 14.5. (10 points) You believe that the mean pancake consumption at the pancake jamboree is more than 16 per person. So, you decide to test your hypothesis. You model the pancake consumption as normally distributed with an unknown mean and with variance equal to 4. The plan is to collect the information on the number of pancakes consumed from a sample of 64 people. Since you want to have everything ready for the big day, you work out the rejection region right away and you get $\boxed{(16.4375, \infty)}$

Right-sided

✓ (i) (5 points) What is the significance level used to obtain the above rejection region?

The lower bound of RR is:

$16.4375 = 16 + \frac{2}{\sqrt{64}} \cdot z^*$ $\Rightarrow z^* = 1.75$

std Normal Tables.

$\alpha = 0.0401$

✓ (ii) (5 points) What is the power of the above test at the alternative mean of 17?

1- pnorm(16.4375, 17, 0.25) = 0.9877755

(ii) (5 points) What is the power of the above test at the alternative mean of 17?