UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 8

The Black-Scholes pricing formula.

Please, provide your **complete solution** to the following problem(s):

Problem 8.1. (2 points) Let the stock price be modeled by a lognormal distribution. Assume that the stock's volatility is strictly greater than zero. Then, the mean stock price always exceeds the median stockprice. *True or false? Why?*

Problem 8.2. (2 points) Let the stock price $S(t)$ be modeled using te lognormal distribution. Define $Y(t) = S(t)^3$. Then, the random variable $Y(t)$ is lognormal itself. *True or false? Why?*

Problem 8.3. (2 pts) Let the stochastic process $S = \{S(t), t \geq 0\}$ represent the stock price as in the Black-Scholes model. Let its volatility term be denoted by σ. Then, the volatility parameter of the process $Y(t) = 2S(t)$ is 4σ. *True or false? Why?*

Problem 8.4. (2 points) Under the risk-neutral probability measure, every option on a particular stock has the continuously compounded, risk-free interest rate as its mean rate of return. *True or false? Why?*

Problem 8.5. (8 points) A non-dividend-paying stock is valued at \$75.00 per share. The time$-t$ realized return is modeled as

$$R(0,t) \sim N(\text{mean} = 0.035t, \text{variance} = 0.09t)$$

Find the probability that the time-4 stock price exceeds today's stock price.

Problem 8.6. (10 points) Consider a non-dividend-paying stock whose current price is \$40 per share. The stock's volatility equals 0.20.

The continuously compounded, risk-free interest rate equals 7%.

Using the Black-Scholes pricing formula, calculate the price of a one-year, at-the-money European call option on the above stock.

Problem 8.7. (10 points) Assume the Black-Scholes setting. Today's price of a non-dividend paying stock is \$65, and its volatility is 0.20.

The continuously compounded, risk-free interest rate is 0.055.

What is the price of a three-month, \$60-strike European put option on the above stock?

Problem 8.8. (14 points) Let $S(0) = \$100, K = \$120, \sigma = 0.3$, and $r = 0.08$.

Let $V_C(0,T)$ denote the Black-Scholes European call price for the maturity T. Does the limit of $V_C(0,T)$ as $T \to \infty$ exist? If it does, what is it?