NAME:

M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
In-Term Exam II
Instructor: Milica Čudina

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Signature:

The maximum number of points on this exam is 100.

Problem 1.1. (10 points) Assume the Black-Scholes model. Let the current price of a continuous-dividend-paying stock be \$40. The stock's volatility is 0.20 and its dividend yield is 0.02.

The continuously compounded, risk-free interest rate is 0.05.

Find the price of a \$42-strike, half-year European put option on the above stock.

Problem 1.2. (5 points) Assume the Black-Scholes model. Let the current stock price be equal to \$90 per share. Its dividend yield is 0.02 and its volatility is 0.20.

The continuously compounded, risk-free interest rate is 0.03.

Consider a one-year European call option on the above stock. The delta of this call option is 0.49. What is the strike price of the call?

Problem 1.3. (10 points) The current price of a continuous-dividend-paying stock is given to be \$92. The stock's volatility is 0.35 and its dividend yield is 0.02.

The continuously-compounded, risk-free interest rate is 0.05.

Consider a \$90-strike European call option on the above stock with exercise date in a quarter-year. What is the Black-Scholes price of this call option?

Problem 1.4. (5 points) Assume the Black-Scholes model. For a particular stock option, you are given that its price today equals \$11.84. You are given the following values of its Greeks today:

- the option's delta is 0.6122;
- the option's gamma is 0.0153;
- the option's theta is -0.0188 **per day**.

Approximately, what will this option's value be in a day should the stock price increase by 0.50?

Problem 1.5. (10 points) Assume the Black-Scholes framework.

The goal is to delta-hedge a written one-year, $(40, 60)$-strangle on a non-dividend-paying stock whose current price is \$50. The stock's volatility is 0.20.

The continuously compounded, risk-free interest rate is 0.10.

What is the cost of delta-hedging the strangle using shares of the underlying stock?

Problem 1.6. (5 points) Which of the following statements is always TRUE?

(a) The call rho is greater than the put rho.

(b) The put theta is always negative.

(c) The call vega is the negative of the vega of the otherwise identical put.

(d) The call psi is always positive.

(e) None of the above.

Problem 1.7. (15 points) Assume the Black-Scholes model. The current stock price is \$60 per share. The stock's dividend yield is 0.02 and its volatility is 0.3.

Consider a \$70-strike, half-year European call option on this stock. Its price is \$2.40, and its delta is 0.3.

The continuously compounded, risk-free interest rate is 0.04.

What is the volatility of the otherwise identical put option?

Problem 1.8. (15 points) Assume that the Black-Scholes setting holds. Let $\mathbf{S} = \{S(t), t \geq 0\}$ denote the price of a continuous-dividend-paying stock. The stock price today equals \$80 and its dividend yield is 0.02.

Let $r = 0.04$ be the continuously compounded risk-free interest rate.

Consider a European call option with exercise in three months and strike price $K = 80e^{0.005}$. You are given that its price today equals \$3.80.

What is the implied volatility of the stock S?

Problem 1.9. (5 points) The current price of a non-dividend-paying stock is \$40 per share. A market-maker writes a one-year European put option on this stock and proceeds to delta-hedge it.

The put premium is \$5.96, its delta is -0.5753, its gamma is 0.0392, and its theta is 0.01 per day.

The continuously-compounded, risk-free interest rate is 0.04.

Assuming that the stock price does not change, what is the **approximate** overnight profit for the market-maker?

Problem 1.10. (10 points) Assume the Black-Scholes model. The current price of a continuous-dividend-paying stock is denoted by $S(0)$. Its dividend yield is denoted by δ and its volatility is $\sigma = 0.20$.

The continuously compounded, risk-free interest rate is equal to δ.

Consider a one-year, at-the-money European put on the above stock. What is the elasticity of this put?

Problem 1.11. (10 points) Assume the Black-Scholes model. Let the current price of a non-dividend-paying stock be \$100. Its volatility is given to be 0.25.

The continuously compounded, risk-free interest rate is 0.02.

Under the risk-neutral probability, the probability that a one-year, European call option is in the money at expiration is 0.484.

What is the current gamma of this call option?