UNIVERSITY OF TEXAS AT AUSTIN

Problem Set 4

Profit. Forward contracts. European call options.

Problem 4.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time−1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time−0 and liquidate your investment at time−1?

Solution: The initial cost is $S(0)$ and the payoff is $S(T)$ with $T = 1$. So, the profit equals

$$S(T) - S(0)e^{rT}.$$

Thus, the expected profit equals

$$\mathbb{E}[S(T)] - S(0)e^{rT}.$$

According to the given model for the stock price, we have

$$\mathbb{E}[S(T)] = 45\left(\frac{1}{4}\right) + 42\left(\frac{1}{2}\right) + 38\left(\frac{1}{4}\right) = 41.75.$$

Finally, the expected profit is

$$41.75 - 40e^{0.04} = 0.117569.$$

Problem 4.2. Derivative securities can reduce the risk of both the buyer and the writer of the security. *True or false?*

Solution: TRUE
Forward contracts are an example of this situation.

Problem 4.3. A short forward contract has an unlimited loss potential. *True or false?*

Solution: TRUE

Problem 4.4. A farmer produces one million bushels of corn. The total cost of production is \$1.3 million. The farmer entered a forward contract to hedge at a forward price of \$2.50 per bushel on one million bushels. What is the farmer's profit?

Solution:

$$10^6(2.50 - 1.30) = 1.2 \times 10^6.$$

Problem 4.5. Assume that farmer Brown is uncertain about his crop yield. Based on past experience, he thinks the following is a good model:

- 100,000 bushels with probability 1/4;
- 80,000 bushels with probability 3/4.

How many forward contracts do you think farmer Brow should short to hedge against fluctuations in corn prices at harvest time? Explain your way of thinking ...

Solution: This is an open-ended problem of sorts. Depending on the farmer's attitude toward risk, he might hedge using any number of contracts between 80,000 and 100,000.

However, the most common way of thinking is to say that farmer Brown wants to address what happens on average, i.e., to hedge the expected number of bushels. In this problem we get

$$\frac{1}{4} \times 100000 + \frac{3}{4} \times 80000 = 25000 + 60000 = 85000.$$

Problem 4.6. Pancakes, Inc. produces chocolate chip pancakes. It longed a forward contract on 100 lbs of chocolate chips at \$3.00 per pound. Total fixed revenue is $\$2,000$ for the pancakes produced with the above chocolate chips. Other costs total \$1200. Find the company's profit.

(a) $2,000$
(b) $1,700$
(c) 800
(d) 500
(e) None of the above.

Solution: (d)

$$2000 - 1200 - 300 = 500$$

Problem 4.7. The **Extra-Healty Cereal (EHC)** company longed 20,000 forward contracts on corn at \$2.80 per bushel. The revenue from cereal made with the above corn is \$200,000 while the other (non-corn) aggregate fixed and variable costs amount to \$120,000. What is the EHC's profit?

Solution:

$$200000 - 120000 - 20000 \times 2.80 = 24,000.$$

Problem 4.8. The current price of stock a certain type of stock is \$80. The premium for a $6-$month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) \$80
(b) \$85.72
(c) \$85.84
(d) \$85.96
(e) None of the above.

Solution: (d)
The break-even point is

$$80 + 5.84e^{0.04/2} = 85.958$$

Problem 4.9. The price of gold in half a year is modeled to be equally likely to equal any of the following prices

$$\$1000, \quad \$1100, \quad \text{and} \quad \$1240.$$

Consider a half-year, \$1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

Solution:

$$50 \times \frac{1}{3} + 190 \times \frac{1}{3} = \frac{240}{3} = 80.$$

Problem 4.10. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded, risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

(a) 395.92
(b) 397.98
(c) 400
(d) 897.98
(e) None of the above.

Solution: (b)

$$200 \times 10 - 200 \times 5 - 500 - 200 \times 0.50e^{0.02} = 397.98$$

Problem 4.11. For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date T in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date T. Assume that the call's premium equals \$10 and that the annual effective interest rate equals 10%.

Express your asnwer as an interval.

Solution: The profit function of the forward contract is $v_F(s) = s - 100$. The profit function of the call is

$$v_C(s) - 10 \times 1.10 = (s - 100)_+ - 11.$$

For $s \geq 100$, the call's profit is smaller than the forward contract's profit. So, we focus on $s < 100$. Here we have to solve for s^* in

$$s^* - 100 = -11 \quad \Rightarrow \quad s^* = 89.$$

The answer is $[0, 89)$.

Problem 4.12. *Source: Sample IFM (Derivatives - Intro), Problem#11*
The current stock price is \$40, and the effective annual interest rate is 8%.

You observe the following option prices:

(1) The premium for a \$35-strike, 1-year European call option is \$9.12.
(2) The premium for a \$40-strike, 1-year European call option is \$6.22.
(3) The premium for a \$45-strike, 1-year European call option is \$4.08.

Assuming that all call positions being compared are **long**, at what 1-year stock price range does the \$45-strike call produce a higher profit than the \$40-strike call, but a lower profit than the \$35-strike call?

Express your answer as an interval.

Solution: The profit curve for a long European call option with strike K and exercise date T has the following form:

$$(s - K)_+ - FV_{0,T}(V_C(0, K)),$$

where $V_C(0, K)$ denotes the time-0 premium of the call with strike K. So, in the present problem, we have the following three profit curves:

$$(s - 35)_+ - 9.12(1.08) = (s - 35)_+ - 9.85,$$
$$(s - 40)_+ - 6.22(1.08) = (s - 40)_+ - 6.72,$$
$$(s - 45)_+ - 4.08(1.08) = (s - 45)_+ - 4.41.$$

In order to figure out the region in which the \$45-strike call to has a higher profit than the \$40-strike call, we need to solve the following inequality:

$$(s-40)_+ - 6.72 < (s-45)_+ - 4.41. \tag{4.1}$$

If $0 \leq s \leq 40$, this inequality becomes

$$-6.72 < -4.41. \tag{4.2}$$

We conclude that all values $s \leq 40$ satisfy inequality (4.1). If $40 < s < 45$, the above inequality (4.1) becomes

$$s - 40 - 6.72 < -4.41 \quad \Rightarrow \quad s < 42.31.$$

So, all $s \in [0, 42.31)$ satisfies (4.1). If $s \geq 45$, inequality (4.1) is trivially wrong for all such s.

In order to figure out the region in which the \$45-strike call to has a lower profit than the \$35-strike call, we need to solve the following inequality:

$$(s-45)_+ - 4.41 < (s-35)_+ - 9.85. \tag{4.3}$$

If $s \leq 35$, we get no solutions to the inequality. If $35 < s < 45$, the above inequality (4.3) becomes

$$-4.41 < s - 35 - 9.85 \quad \Rightarrow \quad 40.44 < s.$$

So, any $s \in (40.44, 45)$ satisfies (4.3). Finally, if $s \geq 45$, we have that (4.3) becomes

$$s - 45 - 4.41 < s - 35 - 9.85 \quad \Rightarrow \quad -49.41 < -44.85.$$

So, any $s \geq 45$ satisfies (4.3).

Pooling all of our conclusions together, we get the final answer $s \in (40.44, 42.31)$

Problem 4.13. (2 points) In which of the following option positions is the investor exposed to an unlimited loss?

(a) Long put option
(b) Short put option
(c) Long call option
(d) Short call option
(e) None of the above.

Solution: (d)
Just draw the payoff diagrams to convince yourselves.

Problem 4.14. (3 points) The initial price of the market index is \$1000. After 3 months the market index is priced at \$950. The nominal rate of interest convertible quarterly is 4.0%.
The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

(a) \$12.00 loss
(b) \$14.90 loss
(c) \$12.00 gain
(d) \$14.90 gain
(e) None of the above.

Solution: (d)
The profit is

$$(K - S(T))_+ - FV_{0,T}[V_P(0)] = (975 - 950)_+ - 10\left(1 + \frac{0.04}{4}\right) = 25 - 10.10 = 14.90.$$

Provide your **complete solution** to the following problem:

Problem 4.15. (3 points) *Source: Sample FM(DM) Problem #62.*
The stock price today equals \$100 and its price in one year is modeled by the following distribution:

$$S(1) \sim \begin{cases} 125 & \text{with probability } 1/2 \\ 60 & \text{with probability } 1/2 \end{cases}$$

The annual effective interest rate equals 3%.

Consider an at-the-money, one-year European put option on the above stock whose initial premium is equal to \$7.

What is the expected profit of this put option?

Solution:

$$\frac{1}{2}(100-60) - 7(1.03) = 20 - 7.21 = 12.79.$$

Problem 4.16. Aunt Dahlia simultaneously purchased

· one share of a market index at the current spot price of \$1,000;
· one one-year, \$1,050-strike put option on the above market index for the premium of \$20.

(i) (5 points) Is the above portfolio's payoff bounded from above? If you believe it is not, substantiate your claim. If you believe that it is, provide the upper bound.
(ii) (5 points) Is the above portfolio's payoff bounded from below? If you believe it is not, substantiate your claim. If you believe that it is, provide the lower bound.

Solution: The payoff of the portfolio expressed in terms of the final asset price $S(T)$ is

$$V(T) = S(T) + (K - S(T))_+ = \min[K, S(T)] = K \wedge S(T).$$

(i) The payoff is **not** bounded from above since the stock price $S(T)$ may be arbitrarily large.
(ii) The payoff is bounded from below by the put option's exercise price K. This means that there is a guarantee of the minimum price the owner of the portfolio can fetch for the underlying asset. That's why this type of a portfolio is referred to as the *floor*.

Problem 4.17. (5 points) **Sample FM(DM) #13.**
Suppose that you short one share of a stock index for 50, and that you also buy a 60−strike European call option that expires in 2 years for 10. Assume the effective annual interest rate is 3%. If the stock index increases to 75 after 2 years, what is the profit on your combined position?

Solution: The payoff is simply:

$$-75 + (75-60)_+ = 60.$$

In words, as a short seller, you have to purchase the stock index back and you are going to take advantage of owning the call option on that index (as opposed to paying the higher market price). The initial cost is $-50+10=-40$. So, the value of this initial cost in 2 years equals $-40\cdot(1.03)^2 = -42.436$. The profit is

$$40 - 42.436 = 17.564.$$