The University of Texas at Austin

Homework assignment 5

Introduction to Mathematical Statistics

February 22, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Transformations of random variables

Problem 5.1. (10 points) Let X be a continuous random variable with the cumulative distribution function denoted by F_X and the probability density function denoted by f_X.

Express the cumulative distribution function and the density of the random variable $\tilde{X} = X^2$ in terms of F_X and f_X.

Problem 5.2. (10 points) Let Y be lognormal with parameters $\mu = 1$ and $\sigma = 2$, i.e., let $Y \stackrel{(d)}{=} e^X$ with $X \sim N(\mu, \sigma)$.

Define $\tilde{Y} = 3Y$.

Find the median of $\tilde{Y}$, i.e., find the value m such that $\mathbb{P}\big[\tilde{Y} \leq m\big] = 1/2$.

Problem 5.3. (10 points) Let T denote the time for a call center employee to respond to any single telephone call. We model the random variable T by uniform distribution on the interval $(48, 72)$ with the time being measured in seconds. Let R denote the f rate at which the call center employee responds to queries expressed in the number of customers *per minute*.

Does the random variable R have a density? If so, find the density of R.

Problem 5.4. (30 points) Let X, Y and Z be independent and uniformly distributed on $(0, 1)$. Find the density function of $W = X + Y + Z$.