50. Assume the Black-Scholes framework.

You are given the following information for a stock that pays dividends continuously at a rate proportional to its price.

(i) The current stock price is 0.25.

(ii) The stock's volatility is 0.35.

(iii) The continuously compounded expected rate of stock-price appreciation is 15%.

Calculate the upper limit of the 90% lognormal confidence interval for the price of the stock in 6 months.

(A) 0.393

(B) 0.425

(C) 0.451

(D) 0.486

(E) 0.529

W: April 10th, 2019.

51-53. DELETED

[Cont'd]

54. Assume the Black-Scholes framework. Consider two nondividend-paying stocks whose time-t prices are denoted by $S_1(t)$ and $S_2(t)$, respectively.

You are given:

(i) $S_1(0) = 10$ and $S_2(0) = 20$.

(ii) Stock 1's volatility is 0.18.

(iii) Stock 2's volatility is 0.25.

(iv) The correlation between the continuously compounded returns of the two stocks is –0.40.

(v) The continuously compounded risk-free interest rate is 5%.

(vi) A one-year European option with payoff $\max\{\min[2S_1(1), S_2(1)] - 17, 0\}$ has a current (time-0) price of 1.632.

Special Put

Consider a European option that gives its holder the right to sell either two shares of Stock 1 or one share of Stock 2 at a price of 17 one year from now.

Calculate the current (time-0) price of this option.

→ : Put-call Parity

$$\underbrace{V_{SC}(0)}_{\substack{\| \\ 1.632}} - \underset{\substack{\| \\ ?}}{V_{SP}(0)} = V_{MIN}(0) - 17e^{-0.05} \quad \text{A}$$

Payoff : $V_{MIN}(1) = MIN(2S_1(1), S_2(1))$

Replicating portfolio for the minimum option:

- prepaid forward on S_2 w/ delivery @ time 1
- SHORT exchange call w/ underlying S_2 and strike asset $2S_1$

⟹ Now, we find the price of the exchange call using the Black-Scholes model:

$$V_{EC}(0) = \underbrace{F^P_{0,1}(S_2)}_{\substack{\| \text{ (no div)} \\ S_2(0)}} \cdot N(d_1) - 2 \cdot \underbrace{F^P_{0,1}(S_1)}_{\substack{\| \text{ (no div)} \\ S_1(0)}} \cdot N(d_2)$$

at the money

w/ $d_1 = \frac{1}{\sigma\sqrt{1}}\left[\cancel{\ln\left(\frac{S_2(0)}{2 \cdot S_1(0)}\right)} + \frac{\sigma^2}{2}\right] = \frac{\sigma}{2}$

and $d_2 = d_1 - \sigma\sqrt{1} = -\frac{\sigma}{2}$

where : $\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1 \cdot \sigma_2$

$\sigma^2 = (0.18)^2 + (0.25)^2 - 2(-0.4) \cdot 0.18 \cdot 0.25$

$\Rightarrow \boxed{\sigma = .3618}$ $\boxed{\sigma = 0.3618}$

2.

$\Rightarrow d_1 = 0.1809 = -d_2$

$\Rightarrow V_{EC}(0) = 20(2 \cdot N(0.18) - 1) = 20(2 \cdot 0.5714 - 1) = \underline{2.856}$

$$Y_{SP}(0) = 1.632 - \left(20 - 2.856\right) + 17e^{-0.05}$$

$$\cong 0.65 \implies \text{(A)}$$ (the difference is due to the std normal tables) ■

Text used by the SoA is

"Corporate Finance (4th Ed)" By Berk/DeMarzo

Analyzing the Project

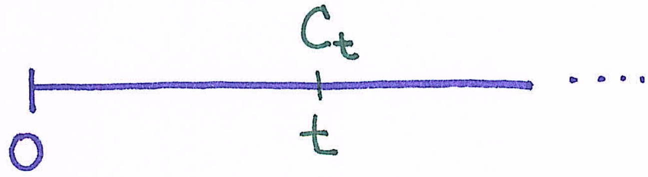
Capital Budgeting... an analysis of investment opportunities and deciding which ones to accept.

$\Rightarrow$ The result is the **CAPITAL BUDGET**.

* for a company, it is a list of all the projects they decide to undertake in the next period.
* for an investor, the analog is that given an initial wealth, an allocation into different investment opportunities is created (risky assets of different kinds, riskless asset).

Our criterion: Maximizing the NPV (Net Present Value)

$\boxed{\text{So far:}}$ In Interest Theory:



$$NPV = \sum_t PV_{0,t}(C_t)$$

Now: Estimates of incremental cashflows

Now: the cost of capital

Notation: r (effective annual)

4.

Break-Even Analysis: keeping all other inputs fixed, find the value(s) of one input @ which the NPV is zero.

e.g., we were looking @ the break-even points of options in M339D.

e.g., with all cashflows fixed, we can evaluate the IRR, i.e., @ most how high the cost of capital can be so that the project breaks even.

We can assume a certain cost of capital, and look @ the required cash amounts.

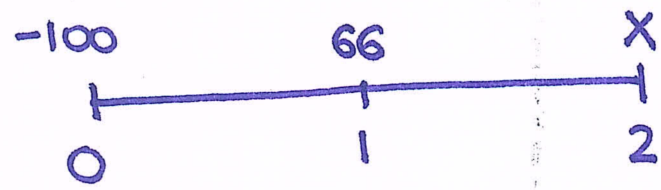
27) Consider a two-year project, where the cost of capital is 10%. $r = 0.10$

There are only three cash flows for this project.

- The first occurs at $t = 0$, and is -100.
- The second occurs at $t = 1$, and is 66.
- The third occurs at $t = 2$, and is X.

Determine X, the level of the cash flow at $t = 2$, that leads to the project breaking even.

$r = 0.10$



(A) 34.0

(B) 38.4

(C) 44.0

(D) 48.4

(E) 54.0

To break even, we need NPV = 0

$$-100 + 66(1.1)^{-1} + X(1.1)^{-2} = 0$$

$$\Rightarrow \quad X = 100(1.1)^2 - 66(1.1) =$$

$$X = 121 - 72.6 = 48.4 \Rightarrow (D)$$