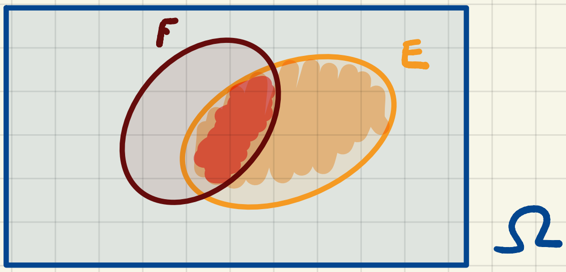
M378K: August 30th, 2024.

More review.

Def'n.



Let E and F be two events on the same Ω.

Let E be such that $\mathbb{P}[E] > 0$.

Then, the conditional probability of F given E is

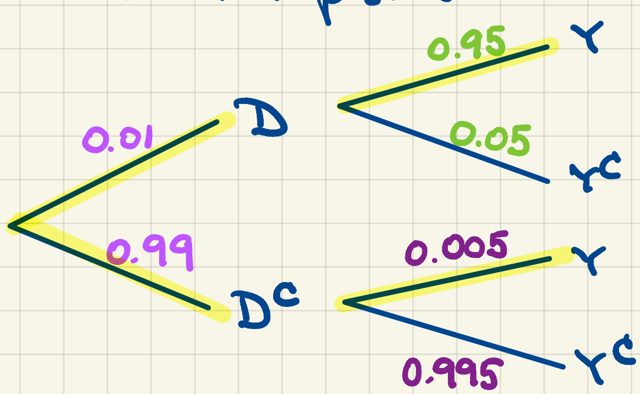
$$\boxed{\mathbb{P}[F \mid E] = \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[E]}}$$

Problem. A blood test indicates the presence of a particular disease $\boxed{95\%}$ of the time when the disease is actually present.
The same test indicates the presence of the disease $\boxed{0.5\%}$ of the time when the disease is actually not present.
$\boxed{\text{One percent}}$ of the population actually has the disease.

Calculate the probability that a person actually has the disease given that they tested positive.

→: D... disease actually present

Y... test positive



$$\mathbb{P}[D \mid Y] = \frac{\mathbb{P}[D]\cdot\mathbb{P}[Y \mid D]}{\mathbb{P}[D]\cdot\mathbb{P}[Y \mid D] + \mathbb{P}[D^c]\cdot\mathbb{P}[Y \mid D^c]}$$

$$= \frac{0.01\cdot 0.95}{0.01\cdot 0.95 + 0.99\cdot 0.005} = \dots = \approx 0.66$$

$\square$

Def'n. We say that events E and F are **independent** if

$$\mathbb{P}[F \mid E] = \mathbb{P}[F], \text{ i.e.,}$$

$$\frac{\mathbb{P}[E\cap F]}{\mathbb{P}[E]} = \mathbb{P}[F], \text{ i.e.,}$$

$$\boxed{\mathbb{P}[E\cap F] = \mathbb{P}[E]\cdot\mathbb{P}[F]}$$

Example. A roll of a die Y $\{1,\dots,6\}$ is the support
If it's a fair die, then Y has the pmf

$$p_Y(k) = \frac{1}{6} \quad \text{for } k=1,\dots,6$$

Example. A fair coin is tossed repeatedly and independently until first $\boxed{\text{heads}}$.

Y... the total # of $\boxed{\text{tails}}$

$$S_Y = \mathbb{N}_0 = \{0, 1, \dots\}$$

Its pmf: for $k \in \mathbb{N}_0$

$$p_Y(k) = \left(\frac{1}{2}\right)^{k+1}$$

Problem. The number of events of interest is modeled by an $\mathbb{N}_0$-valued random variable N w/ pmf

$$\boxed{p_N(n) = \frac{1}{(n+1)(n+2)}} \quad \text{for all } n \in \mathbb{N}_0$$

$$p_N(n) \overset{?}{\geq} 0 \quad \checkmark$$

$$\sum_{n=0}^{\infty} p_N(n) \overset{?}{=} 1$$

$$\sum_{n=0}^{\infty} \boxed{\frac{1}{(n+1)(n+2)}} \checkmark \overset{?}{=} 1$$

$$\frac{1}{n+1} - \frac{1}{n+2} = \frac{\cancel{n}+2-(\cancel{n}+1)}{(n+1)(n+2)} = \frac{1}{(n+1)(n+2)}$$

$$\sum_{n=0}^{+\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right) \overset{?}{=} 1$$

$$1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} + \cdots = 1 \quad \checkmark\checkmark$$

Calculate the probability that at least one event happened given that @ most four events happened.

$\longrightarrow$:

$$\mathbb{P}[N \geq 1 \mid N \leq 4] = \frac{\mathbb{P}[1 \leq N \leq 4]}{\mathbb{P}[N \leq 4]}$$

$$= \frac{p_1 + p_2 + p_3 + p_4}{p_0 + p_1 + p_2 + p_3 + p_4} =$$

$$= \frac{\frac{1}{1+1} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \frac{1}{6}}{1 \cdots\cdots - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \frac{1}{6}} = \frac{\frac{1}{2} - \frac{1}{6}}{5/6} = \frac{\frac{2}{6}}{\frac{5}{6}} = \frac{2}{5} \quad \square$$