4. For a stock, you are given:.

(i) The current stock price is $50.00.

(ii) $\boxed{\delta = 0.08}$

(iii) The continuously compounded risk-free interest rate is $r = 0.04$.

(iv) The prices for one-year European calls (C) under various strike prices (K) are shown below:

K	C
$40	$ 9.12
$50	$ 4.91
$60	$ 0.71
$70	$ 0.00

You own four special put options each with one of the strike prices listed in (iv). Each of these put options can only be exercised immediately or one year from now.

Determine the lowest strike price for which it is optimal to exercise these special put option(s) immediately.

(A) $40

(B) $50

(C) $60

(D) $70

(E) It is not optimal to exercise any of these put options.

The special put owner can

exercise the option NOW

⇒ value of immediate exercise: K−50 ✓

hold onto the option; it becomes a regular European put

⇓

value of European put in the market

Compare the current value of the immediate exercise to the current value of the European put option.
The optimal choice is the one w/ the higher value.

We use put·call parity to find put prices based on the provided call prices:

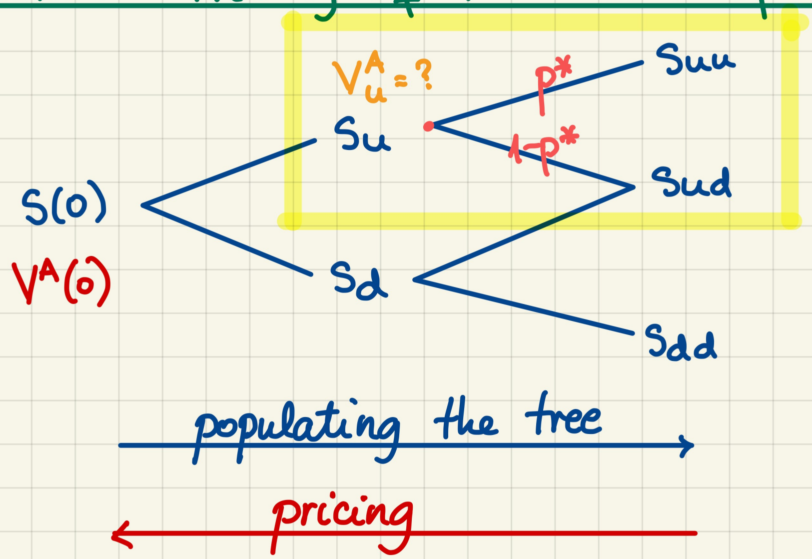
IMM. EX.

- $V_P(K=\underline{40}) = \underbrace{V_C(K=40)}_{9.12} + 40e^{-0.04} - 50e^{-0.08} = 1.39 > -10$
- $V_P(K=50) = \underbrace{V_C(K=50)}_{4.91} + 50e^{-0.04} - 50e^{-0.08} = 6.79 > 0$
- $V_P(K=60) = \underbrace{V_C(K=60)}_{0.71} + 60e^{-0.04} - 50e^{-0.08} = 12.19 > 10$
- $V_P(K=70) = \underbrace{V_C(K=70)}_{0.00} + 70e^{-0.04} - 50e^{-0.08} = 21.10 > 20$

$\Downarrow$

(E) $\square$

Binomial Pricing of American Options.



$\underline{V_{uu}} = v(S_{uu})$

$\underline{V_{ud}} = v(S_{ud})$

$V_{dd} = v(S_{dd})$

Possible payoff values imagining that there is no early exercise.

Consider an American option w/ payoff f'tion $v(\cdot)$.

$\boxed{\text{up node:}}$

- IE_u .... the value of immediate exercise
- CV_u .... the continuation value

if we don't exercise now, the option "becomes" a European option (since there are no more admissible early·exercise dates left!)

$$CV_u = e^{-rh}\left(p^* \cdot V_{uu} + (1-p^*) \cdot V_{ud}\right)$$

$\Rightarrow$ $V_u^A = \max(IE_u, CV_u)$

and the option's owner decides whether to exercise early accordingly!

down node:

- IE_d
- $CV_d = e^{-rh}(p^* \cdot V_{ud} + (1-p^*) \cdot V_{dd})$

$V_d^A = \max(IE_d, CV_d)$

ROOT node:

- IE_0
- $CV_0 = e^{-rh}(p^* \cdot V_u^A + (1-p^*) V_d^A)$

$$\boxed{V^A(0) = \max(IE_0, CV_0)}$$

Note:
- The procedure is analogous in the multi-period tree.
- We can still dynamically replicate the American option until the nodes where early exercise is optimal are reached.