UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 6

The Black-Scholes pricing formula.

Please, provide your **complete solution** to the following problem(s):

Problem 6.1. (2 points) Let the stock price $S(t)$ bemodeled using te lognormal distribution. Define $Y(t) = S(t)^3$. Then, the random variable $Y(t)$ is lognormal itself. *True or false? Why?*

Problem 6.2. (2 pts) Let the stochastic process $S = \{S(t), t \geq 0\}$ represent the stock price as in the Black-Scholes model. Let its volatility term be denoted by σ. Then, the volatility parameter of the process $Y(t) = 2S(t)$ is 4σ. *True or false? Why?*

Problem 6.3. (2 points) Under the risk-neutral probability measure, every option on a particular stock has the continuously compounded, risk-free interest rate as its mean rate of return. *True or false? Why?*

Problem 6.4. (2 points) The Black-Scholes option pricing formula can **always** be used for pricing American-type call options on non-dividend-paying assets. *True or false? Why?*

Problem 6.5. (2 points) The Black-Scholes option pricing formula can always be used for pricing American-type options. *True or false?*

Problem 6.6. (20 points) Let $S(0) = \$100, K = \$120, \sigma = 0.3, r = 0.08$ and $\delta = 0$.

a. (8 pts) Let $V_C(0,T)$ denote the Black-Scholes European call price for the maturity T. Does the limit of $V_C(0,T)$ as $T \to \infty$ exist? If it does, what is it?
b. (8 pts) Now, set $\delta = 0.001$ and let $V_C(0,T,\delta)$ denote the Black-Scholes European call price for the maturity T. Again, how does $V_C(0,T,\delta)$ behave as $T \to \infty$?
c. (4 pts) Interpret in a sentence or two the differences, if there are any, between your answers to questions in a. and b.

Problem 6.7. (20 points) Let $S(0) = \$120, K = \$100, \sigma = 0.3, r = 0$ and $\delta = 0.08$.

a. (10 pts) Let $V_C(0,T)$ denote the Black-Scholes European call price for the maturity T. Does the limit of $V_C(0,T)$ as $T \to \infty$ exist? If it does, what is it?
b. (8 pts) Now, set $r = 0.001$ and let $V_C(0,T,r)$ denote the Black-Scholes European call price for the maturity T. Again, how does $V_C(0,T,r)$ behave as $T \to \infty$?
c. (2 pts) Interpret in a sentence or two the differences, if any, between your answers to questions in a. and b.