University of Texas at Austin

HW Assignment 5

European call options.

Please, provide your **complete solutions** to the free-response problems. Final answers only, without the justification, will earn zero points.

Problem 5.1. (2 points) An agent is **only** allowed to write options on an underlying asset if he/she already owns units of the underlying. *True or false?*

Solution: FALSE
The so-called *naked* option writing is a legal and common practice.

Problem 5.2. (5 points) The initial price of the market index is \$900. After 3 months the market index is priced at \$920. The nominal rate of interest convertible monthly is 4.8%.
The premium on the long call, with a strike price of \$930, is \$2.00. What is the profit or loss at expiration for this long call?

Solution: In our usual notation, the profit is

$$(S_T - K)_+ - C \cdot (1+j)^3$$

with C denoting the price of the call and j the effective monthly interest rate. We get

$$(920 - 930)_+ - 2 \cdot 1.004^3 \approx -2.02.$$

Problem 5.3. (5 points) The current price of stock a certain type of stock is \$50. The premium for a 3−month, at-the-money call option is \$2.74. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) \$50
(b) \$52.71
(c) \$52.77
(d) \$52.85
(e) None of the above.

Solution: (c)
The break-even point is

$$50 + 2.74e^{0.04/4} = 52.7675.$$

Problem 5.4. (8 points) *Source: FM(DM) sample problem #42.*
An investor purchases one share of a non-dividend-paying stock and writes an at-the-money, T-year, European call option in this stock. The call premium is denoted by C. Assume that there are no transaction costs. The continuously compounded, risk-free interest rate is denoted by r. Let the argument s represent the stock price at time T.

(i) (6 points) Determine an algebraic expression for the investor's profit at expiration T in terms of C, r, T and the strike K.
(ii) (2 points) In particular, how does the expression you obtained in (i) simplify if the call is in-the-money on the exercise date?

Solution:

$$s-(s-K)_+-(S(0)-C)e^{rT}=s-(s-K)_+-(K-C)e^{rT}.$$

For $s>K$,

$$s-(s-K)-(K-C)e^{rT}=K(1-e^{rT})+Ce^{rT}.$$

Problem 5.5. (15 points) The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for $1,000.

The jeweler models he market price of gold in one year as follows:

Gold price in one year	Probability
750 per ounce	0.2
850 per ounce	0.5
950 per ounce	0.3

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of $900 per ounce. The option costs $100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jeweley produced.

Solution:

With $S(T)$ denoting the market price of gold at time $T=1$, the jeweler's **hedged** profit per piece of jewelry can be expressed as

$$1,000-\min(S(T),900)-100e^{0.05}=894.873-\min(S(T),900).$$

So, her expected **hedged** profit equals

$$894.873-\mathbb{E}[\min(S(T),900)]=894.873-(0.2\cdot 750+0.5\cdot 850+0.3\cdot 900)=49.873.$$

Problem 5.6. (15 points) *Source: Sample MFE (Intro) Problem #15.*

The current price of a non-dividend paying stock is $40 and the continuously compounded risk-free interest rate is 8%. You enter into a short position on 3 call options, each with 3 months to expiry, a strike price of $35, and an option premium of $6.13. Simultaneously, you enter into a long position on 5 call options, each with 3 months to expiry, a strike price of $40, and an option premium of $2.78. All 8 options are held until maturity. Calculate the range of the profit for the entire option portfolio.

(a) $[-4.58, 3.42]$

(b) $[-10.42, 4.58]$

(c) $[-10.42, \infty)$

(d) $(-\infty, 4.58]$

(e) None of the above.

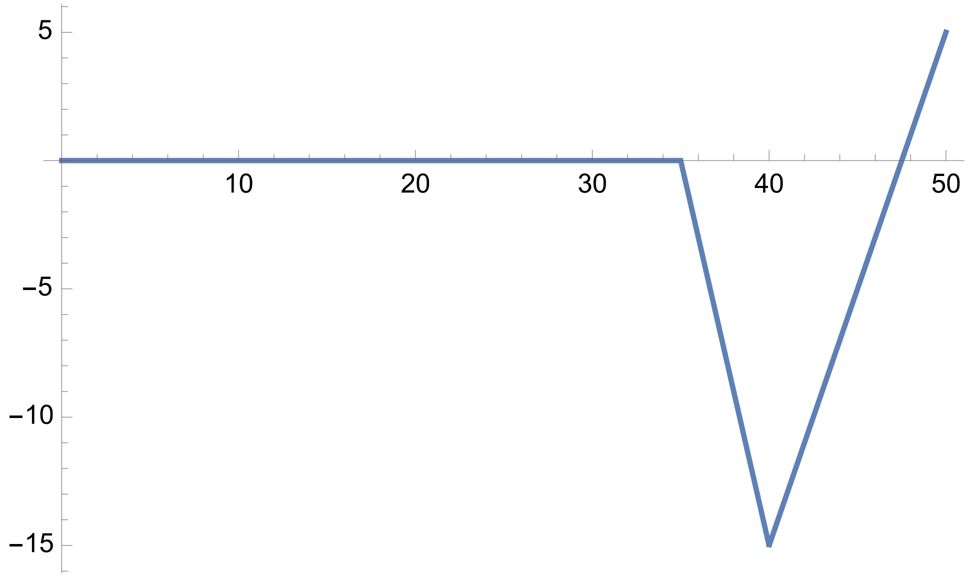
Solution: (c)

The initial cost is $-3(6.13)+5(2.78)=-4.49$.

In our usual notation, the expression for the payoff is

$$-3(S(T)-35)_+ + 5(S(T)-40)_+$$

So, the payoff function is $v(s)=-3(s-35)_+ + 5(s-40)_+$. Its graph looks like this:



We see that the minimum payoff is attained at $s = 40$ and that it equals -15. There is unlimited growth potential. Hence, the range of the profit is

$$[-15 - (-4.49)e^{0.08(0.25)}, \infty) = [-10.4193, \infty).$$