UNIVERSITY OF TEXAS AT AUSTIN

Extra-credit homework assignment 2

Call and put options.

Please, provide **your complete solution** to the following problems. Only the final answer without justification will receive zero credit.

Problem 2.1. (5 points) An investor short sells one share of a non-dividend-paying stock and buys an at-the-money, T-year, European call option on this stock. The call premium is denoted by $V_C(0)$. Assume that there are no transaction costs. The continuously compounded, risk-free interest rate is denoted by r. Let the argument s represent the stock price at time T.

(i) (3 points) Determine an algebraic expression for the investor's profit at expiration T in terms of $V_C(0), r, T$ and the strike K.

(ii) (2 points) In particular, how does the expression you obtained in (i) simplify if the call is in-the-money on the exercise date?

Solution:

$$-s + (s-K)_+ + (S(0) - V_C(0))e^{rT} = -s + (s-K)_+ + (K - V_C(0))e^{rT}.$$

For the option to be in-the-money at expiration, we must have $s < K$. So, the profit simplifies to

$$-s + (s-K) + (K - V_C(0))e^{rT} = -K + (K - V_C(0))e^{rT}.$$

Problem 2.2. (5 points) An investor short sells one share of a non-dividend-paying stock and writes an at-the-money, T-year, European put option on this stock. The put premium is denoted by $V_P(0)$. Assume that there are no transaction costs. The continuously compounded, risk-free interest rate is denoted by r. Let the argument s represent the stock price at time T.

(i) (3 points) Determine an algebraic expression for the investor's profit at expiration T in terms of $V_P(0), r, T$ and the strike K.

(ii) (2 points) In particular, how does the expression you obtained in (i) simplify if the put is in-the-money on the exercise date?

Solution:

$$-s - (K-s)_+ + (S(0) + V_P(0))e^{rT} = -s - (K-s)_+ + (K + V_P(0))e^{rT}.$$

For the option to be in-the-money at expiration, we must have $s < K$. So, the profit simplifies to

$$-s - (K-s) + (K + V_P(0))e^{rT} = -K + (K + V_P(0))e^{rT}.$$

Problem 2.3. (5 points) The current price of a non-dividend-paying stock is \$50 per share. You observe that the price of a three-month, at-the-money American call option on this stock equals \$3.50.

The continuously compounded, risk-free interest rate is 0.04.

Find the premium of the European three-month, at-the-money put option on the same underlying asset.

Solution: Recall that the price of an American call on a non-dividend-paying stock equals the price of the otherwise identical European call option. So, put-call parity yields

$$V_P(0) = V_C(0) + Ke^{-rT} - S(0) = 3.50 - 50(e^{-0.01} - 1) = 3.0025.$$

Problem 2.4. (5 points) The initial price of the market index is \$900. After 3 months the market index is priced at \$920. The nominal rate of interest convertible monthly is 4.8%.
The premium on the long call, with a strike price of \$930, is \$2.00. What is the profit or loss at expiration for this long call?

Solution: In our usual notation, the profit is

$$(S_T - K)_+ - C \cdot (1+j)^3$$

with C denoting the price of the call and j the effective monthly interest rate. We get

$$(920 - 930)_+ - 2 \cdot 1.04^3 \approx -2.02.$$

Problem 2.5. (5 points) An investor wishes to use a put option to hedge a **long** position in an underlying asset S. He is attempting to decide among otherwise identical European put options with different strikes (and all, of course, on the same underlying asset S). Which of the following statements is **correct**?

(a) Put options with higher strikes have a higher price and provide a higher **floor**.
(b) Put options with higher strikes have a lower price and provide a higher **floor**.
(c) Put options with higher strikes have a lower price and provide a lower **floor**.
(d) Put options with higher strikes have a higher price and provide a lower **floor**.
(e) None of the above.

Solution: (a)
The put prices are increasing as functions of the strike, so puts with a higher strike have a higher price. On the other hand, the *floor* is a position consiting of a long position in the underlying combined with a long put. The payoff function of the floor is

$$(K-s)_+ + s = \begin{cases} K & \text{if } K > s \\ s & \text{if } K \leq s \end{cases} = max(K, s)$$

Obviously, the higher the strike price K, the higher the lower bound on the above payoff, i.e., the floor.

Problem 2.6. (5 points) A customer buys a six-month at-the-money put on an index when the market price of the index is 50. The premium for the put is 2.

The continuously compounded, risk-free interest rate equals 0.06.

The price of the index at expiration is modeled as follows

$$\begin{aligned} 45, &\quad \text{with probability } 0.6, \\ 50, &\quad \text{with probability } 0.3, \\ 55, &\quad \text{with probability } 0.1. \end{aligned}$$

What is the expected value of the profit of the long put?

Solution:

$$(50 - 45) * 0.6 - 2e^{0.03} = 0.939.$$

Problem 2.7. (5 points) The price of gold in half a year is modeled to be equally likely to be any of the following prices

$$\$1000, \quad \$1100, \quad \text{and} \quad \$1240.$$

Consider a half-year, \$1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

Solution:

$$50 \times \frac{1}{3} + 190 \times \frac{1}{3} = \frac{240}{3} = 80.$$

Problem 2.8. (5 points) The initial price of a non-dividend-paying asset is \$100. A six-month, \$95-strike European call option is available at a \$8 premium.

The continuously compounded, risk-free interest rate equals 0.04.

What is the break-even point for this call option?

Solution: We need to solve for s in

$$(s-95)_+ = 8e^{0.02} \quad \Rightarrow \quad s = 95 + 8e^{0.02} = 103.16$$

Problem 2.9. (5 points) Let the current price of a non-dividend-paying stock equal 50. The forward price for delivery of this stock in 2 months equals \$50.42

Consider a \$45-strike, six-month put option on this stock whose premium today equals \$1.11.

What will the profit of this long put option be if the stock price at expiration equals \$48?

Solution: The option is out-of-the money at expiration, so its owner suffers a loss of the future value of its premium

$$1.11 \times \left(\frac{50.42}{50}\right)^3 = 1.14.$$

Problem 2.10. (5 points) The current price of a certain non-dividend-paying stock is \$40 per share.

A one-year, \$42-strike European call option on this stock is priced at \$4. An otherwise identicall put option is priced at \$3.95.

What is the continuously compounded, risk-free interest rate?

Solution: Using put-call parity, we get

$$4 - 3.95 = 40 - 42e^{-r} \quad \Rightarrow \quad e^{-r} = \frac{39.95}{42} \quad \Rightarrow \quad r \approx 0.05.$$