Quiz #12: Solutions

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Simulations

Problem 1.

Consider the following two-step experiment. First you draw a simulated value from a Bernoulli(p=1/3). If the drawn value from the Bernoulli equals 0, then you draw a simulated value from $Gamma(\alpha = 2, \theta = 5)$. On the other hand, if the drawn value from the Bernoulli equals 1, then you draw a simulated value from $Gamma(\alpha = 10, \theta = 20)$

(10 points) Repeat the above two-step experiment 10000 times and draw the histogram of the simulated values.

Solution:

```
nsim=10000
sims<-c()
for(i in 1:nsim){
    coin<-rbinom(n=1,size=1,prob=1/3)
    if(coin==0){
        new.sim<-rgamma(1,shape=2,scale=5)
    } else {
        new.sim<-rgamma(1,shape=10,scale=20)
    }
    sims<-c(sims,new.sim)
}
hist(sims,breaks=50)</pre>
```

Histogram of sims 0007 00007 00007 0001 0001 1000 2000 3000 4000

(2 points) Find the theoretical mean of the above distribution and compare it to the mean of the simulated values.

Solution: The theoretical mean is

```
(2/3)*2*5+(1/3)*10*20
## [1] 73.33333
```

The mean of the simulated values is

```
mean(sims)
## [1] 73.20651
```

(3 points) Find the theoretical probability that the above random variable exceeds 100. It's acceptable to use the pgamma command. Compare it to the proportion of the simulated values exceeding 100.

Solution: This is the theoretical probability:

```
(2/3)*(1-pgamma(100, shape=2, scale=5))+(1/3)*(1-pgamma(100, shape=10, scale=20))
## [1] 0.322724
```

Here is the proportion of the simulated values exceeding 100.

```
mean(sims>100)
## [1] 0.3206
```