M339D : April 29th, 2024.

Delta Hedging.

Market Maker.

- immediacy
- inventory

$\Big\}$ ⟹ exposure to risk ⟹ hedge

Say, a market maker writes an option whose value f'tion is

$$v(s,t)$$

At time 0, they write the option ⟹ They get $v(S(0),0)$

At time t, the value of the market maker's position

$$\boxed{-v(s,t)}$$

To (partially) hedge this exposure to risk, they construct a portfolio which has zero sensitivity to small perturbations in the stock price. Formally speaking, their goal is to create a

delta-neutral portfolio,

i.e., a portfolio for which $\boxed{\Delta_{Port}(s,t) = 0}$ Theoretically possible; Practically not

In particular, @ time·0, they want to trade so that

$$\boxed{\Delta_{Port}(S(0),0) = 0}$$

The most straightforward strategy is to trade in the shares of the underlying asset.

At time·t, let $N(s,t)$ denote the required number of shares in the portfolio to maintain Δ-neutrality.

The total value of the portfolio:

$$v_{Port}(s,t) = -v(s,t) + N(s,t)\cdot s$$

$$\frac{\partial}{\partial s}\Bigg|$$

$$\Delta_{Port}(s,t) = -\Delta(s,t) + N(s,t) = 0$$

$$\boxed{N(s,t) = \Delta(s,t)}$$

Example. An agent writes a call option @ time·0.

At time·t, the agent's unhedged position is:

$$-v_C(s,t)$$

$\Rightarrow$

$$N(s,t) = \Delta_C(s,t) \text{ in the } \Delta\text{-hedge.}$$

$\Rightarrow$ In particular, @ time·0:

$$N(S(0),0) = N(d_1(S(0),0)) > 0, \text{ i.e.,}$$

the agent must long this much of a share.

$\Rightarrow$ The total position will be:

$$v_{Port}(S(0),0) = -v_C(S(0),0) + \Delta_C(S(0),0)\cdot S(0)$$

Example. An agent $\boxed{\text{writes}}$ a put option @ time 0.

At time t, the agent's unhedged position is:

$$-v_P(s,t)$$

$\Rightarrow$ They must maintain $N(s,t) = \Delta_P(s,t) = -N(-d_1(s,t))$ in the Δ-hedge.

$\Rightarrow$ The agent must short a portion of a share.

At time 0, their total position will be:

$$v_{Port}(S(0),0) = -v_P(S(0),0) + \Delta_P(S(0),0)\cdot S(0)$$

In the Black-Scholes model:

$$v_{Port}(S(0),0) = -\left(Ke^{-rT}N(-d_2(S(0),0)) - \cancel{S(0)N(-d_1(S(0),0))}\right) + \cancel{(-N(-d_1(S(0),0)))\cdot S(0)}$$

$$= -Ke^{-rT}N(-d_2(S(0),0))$$

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i) The risk-free interest rate is constant.

(ii)

	Several months ago	Now
Stock price	\$40.00	\$50.00
Call option price	\$ 8.88	\$14.42
Put option price	\$ 1.63	\$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

Hint! Put-Call Parity :)

Profit = Payoff − FV(Initial Cost)

Calculate her profit.

0 Options written — t "several months" This is when we calculate the profit. — T exercise date

(A) $11

(B) $24

(C) $126

(D) $217

(E) $240

$$\text{Profit(@ time } t) = \boxed{\text{Wealth(@ time } t)} - FV_{0,t}\left(\boxed{\text{Initial Cost}}\right)$$

48. DELETED

49. You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).

(i) The period is 3 months.

(ii) The initial stock price is $100.

(iii) The stock's volatility is 30%.

(iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

(A) 114

(B) 115

(C) 116

(D) 117

(E) 118

- **Initial Cost:** $-100 \cdot v_c(S(0),0) + 100 \cdot \Delta_c(S(0),0) \cdot S(0)$

$$= 100(-8.88 + 0.794 \cdot 40) = \underline{2{,}288}$$

- **Wealth @ time t:** $-100 \cdot v_c(S(t),t) + 100 \cdot \Delta_c(S(0),0) \cdot S(t)$

$$= 100(-14.42 + 0.794 \cdot 50)$$

$$= \underline{2{,}528}$$

Profit (@ time t) $= 2{,}528 - 2{,}288 \cdot e^{rt}$

Use put-call parity:

At time 0: $v_c(S(0),0) - v_p(S(0),0) = S(0) - Ke^{-rT}$

$$8.88 - 1.63 = 40 - Ke^{-rT}$$

$$\underline{Ke^{-rT} = 40 - 7.25 = 32.75} \quad \checkmark$$

At time t: $v_c(S(t),t) - v_p(S(t),t) = S(t) - Ke^{-r(T-t)}$

$$14.42 - 0.26 = 50 - Ke^{-r(T-t)}$$

$$\underline{Ke^{-r(T-t)} = 50 - 14.16 = 35.84} \quad \checkmark\!\!\checkmark$$

$$\frac{\checkmark\!\!\checkmark}{\checkmark} = \frac{\cancel{Ke^{-rT}} \cdot e^{rt}}{\cancel{Ke^{-rT}}} = e^{rt} = \frac{35.84}{32.75} = 1.09435$$

Profit (@ time t) $= 2{,}528 - 2{,}288 \cdot (1.09435) = \underline{24.12}$ $\square$