M339J *Probability models*
University of Texas at Austin
**HW #6**
Instructor: Milica Čudina

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*Note:* You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

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**Problem 6.1.** (5 points)Let $X$ be the ground-up loss random variable. Assume that $X$ has the exponential distribution with mean $5,000$. Let $B$ denote the expected payment per loss on behalf of an insurer which wrote a policy with a deductible of $1,500$ and with no maximum policy payment. What is the value of $B$?

**Problem 6.2.** (5 points) The ground-up loss random variable is denoted by $X$. An insurance policy on this loss has a **franchise** deductible of $d$ and no policy limit. Then, the expected **policyholder** payment per loss equals ...

(a) $\mathbb{E}[X\mathbb{I}_{[X\leq d]}]$
(b) $\mathbb{E}[X \wedge d]$
(c) $\mathbb{E}[(X-d)_+]$
(d) $\mathbb{E}[X \wedge d] - d$
(e) None of the above.

**Problem 6.3.** (5 points) Let $X \sim Pareto(\alpha = 3, \theta = 3000)$. Assume that there is a deductible of $d = 5000$. Find the loss elimination ratio.

**Problem 6.4.** (15 points) Assume that the severity random variable $X$ is exponentially distributed with mean 1400. There is an insurance policy to cover this loss. This insurance policy has

(1) a deductible of 200 per loss,
(2) the coinsurance factor of $\alpha = 0.25$, and
(3) the maximum amount payable by the insurer per loss of 700.

Calculate the expected value of the per payment random variable $Y^P$ under this policy.

**Problem 6.5.** (10 points) Let $Y$ be lognormal with parameters $\mu = 1$ and $\sigma = 2$.
Define $\tilde{Y} = 3Y$.
Find the median of $\tilde{Y}$, i.e., find the value $m$ such that $\mathbb{P}[\tilde{Y} \leq m_Y] = 1/2$.

**Problem 6.6.** (10 points) In the notation of our tables, let $X$ be a Weibull random variable with parameters $\theta = 20$ and $\tau = 2$.
Define $Y = 5X$ and denote the coefficient of variation of $Y$ by $CV_Y$. Find $CV_Y$.

*Hint:* The following facts you may have forgotten from probability could be useful:

$$\Gamma(1/2) = \sqrt{\pi},$$
$$\Gamma(1) = 1,$$
$$\Gamma(\alpha + 1) = \alpha\Gamma(\alpha), \quad \text{for all } \alpha.$$