

## Advanced Derivatives Questions

1. Consider a European call option and a European put option on a nondividend-paying stock. You are given:

- (i) The current price of the stock is 60.
- (ii) The call option currently sells for 0.15 more than the put option.
- (iii) Both the call option and put option will expire in 4 years.
- (iv) Both the call option and put option have a strike price of 70.

Calculate the continuously compounded risk-free interest rate.

- (A) 0.039
- (B) 0.049
- (C) 0.059
- (D) 0.069
- (E) 0.079

$$r = ?$$

Put-Call Parity

$$V_C(0) - V_P(0) = S(0) - PV_{0,T}(K)$$

$$\underbrace{\quad}_{\text{ii}} \quad 0.15 = 60 - 70e^{-4r}$$

$$70e^{-4r} = 60 - 0.15$$

$$e^{-4r} = \frac{59.85}{70}$$

ln |

$$-4r = \ln\left(\frac{59.85}{70}\right)$$

$$r = -\frac{1}{4} \ln\left(\frac{59.85}{70}\right) = 0.03916$$



77. You are given:

- i) The current price to buy one share of XYZ stock is 500.
- ii) The stock does not pay dividends.
- iii) The continuously compounded risk-free interest rate is 6%.
- iv) A European call option on one share of XYZ stock with a strike price of  $K$  that expires in one year costs 66.59.
- v) A European put option on one share of XYZ stock with a strike price of  $K$  that expires in one year costs 18.64.

Using put-call parity, calculate the strike price,  $K$ .

- :  $V_c(0) - V_p(0) = S(0) - PV_{0,T}(K)$   
 $66.59 - 18.64 = 500 - Ke^{-0.06}$   
 $Ke^{-0.06} = 500 - 66.59 + 18.64$   
 $K = e^{0.06} \cdot 452.05 = 480.0032$
- (A) 449  
 (B) 452  
 (C) 480  
 (D) 559  
 (E) 582



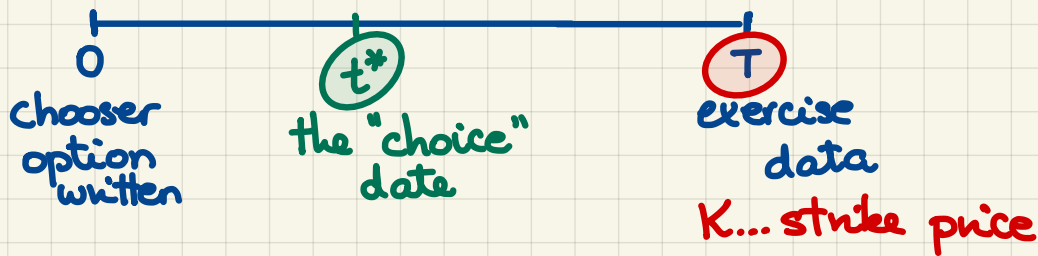
78. The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. You are given that the price of a 35-strike call option is 3.35 higher than the price of a 40-strike call option, where both options expire in 3 months.

Calculate the amount by which the price of an otherwise equivalent 40-strike put option exceeds the price of an otherwise equivalent 35-strike put option.

- :  $V_c(0, K=35) - V_p(0, K=35) = S(0) - 35e^{-0.08(0.25)}$   
 $V_c(0, K=40) - V_p(0, K=40) = S(0) - 40e^{-0.02}$
- 3.35 -  $(V_p(0, K=35) - V_p(0, K=40)) = 5e^{-0.02}$
- answer =  $5e^{-0.02} - 3.35 = 1.55$
- (A) 1.55  
 (B) 1.65  
 (C) 1.75  
 (D) 3.25  
 (E) 3.35



# Chooser Options (aka "as you like it" options).



At time  $t^*$ , the chooser option's owner decides whether the option becomes a call or a put (always w/ strike  $K$  and exercise date  $T$ ).

Assume that the owner is rational.

Q: What is their criterion for choosing between a call and a put @ time  $t^*$ ?

→: Notation.

- $V_{CH}(t, t^*, T)$  (with  $t$  circled in purple,  $t^*$  circled in green, and  $T$  circled in red)
  - valuation date (pointing to  $t$ )
  - choice date (pointing to  $t^*$ )
  - exercise date (pointing to  $T$ )
- $V_{CP}(t, \text{exercise date, strike price})$  (with  $t$  circled in purple)

Our criterion

$$V_{CH}(t^*, t^*, T) = \max \left( \overbrace{V_C(t^*, T, K)}^a, \overbrace{V_P(t^*, T, K)}^b \right)$$

$$= V_C(t^*, T, K) + \left( V_P(t^*, T, K) - V_C(t^*, T, K) \right)_+$$

$$= V_C(t^*, T, K) + \left( \underbrace{K e^{-r(T-t^*)}}_{\text{"} K^* \text{"}} - S(t^*) \right)_+ \quad \text{"PUT-CALL PARITY"}$$

The Pay off a European put w/ strike  $K^*$  and exercise date  $t^*$

A replicating portfolio for the chooser option:

- a long call w/ strike  $K$  and exercise date  $T$
- a long put w/ strike  $K^*$  and exercise date  $t^*$

$$\Rightarrow V_{CH}(0, t^*, T) = V_C(0, T, K) + V_P(0, t^*, K^*) \\ = V_P(0, T, K) + V_C(0, t^*, K^*)$$

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

- (A) -0.55
- (B) -1.15
- (C) -8.64
- (D) -13.03
- (E) -27.24

21-24. DELETED

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of \$100.
- t\*... choice date*  
*exercise date*

The chooser option price is \$20 at time  $t = 0$ .  $V_{CH}(0, 1, 3) = 20$

The stock price is \$95 at time  $t = 0$ . Let  $C(T)$  denote the price of a European call option at time  $t = 0$  on the stock expiring at time  $T$ ,  $T > 0$ , with a strike price of \$100.

You are given:

- (i) The risk-free interest rate is 0.  $K^* = K$

- (ii)  $C(1) = \$4$ .  $V_C(0, 1, K=100) = 4$

Determine  $C(3)$ .  $V_C(0, 3, K=100) = ?$

- (A) \$ 9

- (B) \$11

- (C) \$13

- (D) \$15

- (E) \$17

$$V_{CH}(0, 1, 3) = 20 = \underbrace{V_C(0, 3, K=100)}_{\text{X}} + V_P(0, 1, K^*=100)$$

$$= \underbrace{11}_{\text{X}} + V_C(0, 1, K^*=100) + PV_{0,1}(K^*)$$

$$= 11 + 4 + 100 - 95 = 9$$