M358K : October 8^{th}, 2021.

Problem 7.3. Travel time by sled between Whoville and Whoburgh takes on average 36 minutes with a standard deviation of 6 minutes. Over a particular weekend, 64 sled trips take place. What is the (approximate) probability that the average sled trip took more than 38 minutes?

$\longrightarrow$: $n = 64$: $\bar{X}_n$ "$\approx$" Normal $\left(\text{mean} = 36, \text{ sd} = \frac{6}{\sqrt{64}} = \frac{3}{4}\right)$

CLT

$$\mathbb{P}[\bar{X}_n > 38] = \mathbb{P}\left[\underbrace{\frac{\bar{X}_n - 36}{0.75}}_{\approx N(0,1)} > \frac{38-36}{0.75} = \frac{8}{3}\right]$$

$$= 1 - \Phi\left(\frac{8}{3}\right)$$ 1-pnorm(8/3) = 0.00383

Alternatively: 1-pnorm(38,36,0.75) = 0.00383

Problem 7.4. The amount of time your friendly taquero at *Torchy's Tacos* spends to assemble any one tasty taco is a random variable with mean 3 minutes and 15 seconds and standard deviation of thirty seconds. You and your 31 friends from *Applied Statistics* celebrate by ordering two tacos each. What is the probability that the average taco-assembly time is:

- less than 2 minutes and 30 seconds;
- more than 3 minutes and 15 seconds; 1/2
- at least 3 minutes but at most 3 minutes and 30 seconds?

$n = 64$: $\bar{X}_n \approx$ Normal $\left(\text{mean} = 3.25, \text{ sd} = \frac{0.5}{\sqrt{64}} = 0.0625\right)$

$\mathbb{P}[\bar{X}_n < 2.5] = ?$

pnorm (2.5, 3.25, 0.0625) = $1.776482 \cdot 10^{-33}$

$\mathbb{P}[3 < \bar{X}_n < 3.5] = ?$

pnorm(3.5, 3.25, 0.0625) - pnorm(3, 3.25, 0.0625) =

= 0.9999367

The Normal Sample.

Sample Mean.

Linear Combinations of Normal Random Variables.

For any X and Y normally distributed and any constants α and β, the linear combination $\alpha \cdot X + \beta \cdot Y$

is also a normally distributed random variable.

$\Rightarrow$ Any linear combination of normal random variables is also normally distributed.

If we have a random sample $X_1, X_2, \dots, X_n$ from the normal distribution w/ $\mu_X = \mathbb{E}[X_1]$

and $\sigma_X = SD[X_1]$,

then the sample mean has the following dist'n:

$$\bar{X}_n \sim \text{Normal}\left(\text{mean} = \mu_X, \ \text{sd} = \sigma_X / \sqrt{n}\right)$$

Not an approximation;
Not a limit theorem.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 8

Sample mean: The normal sample.

Problem 8.1. The scores of individual students on the Advanced Dark Arts Exam are modeled as normally distributed with a mean of 19.6 and a standard deviation of 5.0. At Voldemort High, 64 seniors take the test. Assume the individual scores at this school are modeled using the same distribution as national scores. What is the sampling distribution of the sample average score for this random sample of 64 students?

State the **name** and the **parameter value(s)** of this distribution.

$$\bar{X}_{64} \sim \text{Normal}\left(\text{mean} = 19.6, \ \text{var} = \frac{25}{64}\right)$$

$$\text{var} \rightarrow \frac{\sigma_X^2}{n} = \text{Var}[\bar{X}_n]$$

Problem 8.2. The *"Aristocratic Hog"* chocolate bars are all labeled to weigh 4.0 ounces. The distribution of the actual weights of these chocolate bars is modeled as normal with a mean of 4.0 ounces and a standard deviation of 0.1 ounces. Bernard, the quality control manager and principal taster, initially plans to take (and weigh) a simple random sample of size n from the production line. Then he reconsiders and decides that a sample twice as large is needed. By what factor does the standard deviation of the sampling distribution of the sample average change?

The sd decreases by factor $\sqrt{2}$.

$$\bar{X}_n \sim N\left(\text{mean} = \mu_X, \ sd = \frac{\sigma_X}{\sqrt{n}}\right)$$

$$\bar{X}_{2n} \sim N\left(\text{mean} = \mu_X, \ sd = \frac{\sigma_X}{\sqrt{2n}}\right)$$

Problem 8.3. The individual students' scores in the ACT exam are modeled using the normal distribution with an unknown mean (say, it varies from year to year) and with the **known** standard deviation of 6.

You take a SRS of students who took the ACT this year. The intention is to use their sample average to estimate (infer) the population mean.

You want the standard deviation of your statistic $\bar{X}_n$ to be at most 0.10. What is the least number of students you need to sample?