Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
In-Term Exam II
Instructor: Milica Čudina

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The maximum number of points on this exam is 100.

2.1. <u>**DEFINITIONS**</u>.

Problem 2.1. (10 points) Write the definition of an **arbitrage portfolio**.

2.2. **TRUE/FALSE QUESTIONS.**

Problem 2.2. (2 pts)
A cap consists of a long call option and a long asset. *True or false? Why?*

Solution: FALSE
A cap consists of a long call option and a short asset.

Problem 2.3. (4 points) The chooser option with the exercise date T and with the strike K is worth at least as much as a vanilla call with the same underlying, strike and exercise date. *True or false? Why?*

Solution: TRUE
Remember the pricing formula for the chooser option.

Problem 2.4. (2 points) Exchange options are defined as options where the underlying asset is an exchange rate. *True or false? Why?*

Solution: FALSE
The underlying asset is not necessarily a foreign currency.

Problem 2.5. (2 points) Assume that the continuously compounded, risk-free interest rate is strictly positive. The forward price of a non-dividend-paying stock is always strictly increasing with respect to the delivery date. *True or false? Why?*

Solution: TRUE

Problem 2.6. (4 points) If the stock pays discrete dividends, there is a comparative advantage to an outright purchase of the stock as compared to the prepaid forward contract on that stock. *True or false?*

Solution: FALSE
The prepaid forward price includes the “compensation” for forfeited dividends payments.

2.3. FREE-RESPONSE PROBLEMS.

Problem 2.7. (5 points) A portfolio consists of the following:

- one **short** one-year, 50−strike call option with price equal to \$8.50,
- one **long** one-year, 60−strike put option with price equal to \$6.75

All of the options are European and with the same underlying asset.

Assume that the continuously compounded, risk-free interest rate equals 0.04.

What is the portfolio's profit if the final price of the underlying asset equals \$55?

Solution:

$$-(55-50)_+ + (60-55)_+ + (8.50-6.75)e^{0.04} = 1.82$$

Problem 2.8. (5 points) The initial price of a continuous-dividend-paying market index equals \$1,000. The dividend yield equals 0.03.

An investor simultaneosly purchases one unit of the index and a one-year, 975-strike European put option on the index for a premium of \$10.

In one year, the spot price of the index is observed to be \$950.

Given that the continuously-compounded, risk-free interest rate equals 0.03, what is the profit of the investor's portfolio? *Caveat: Be careful with continuous reinvestment of dividend in the index.*

Solution: In our usual notation,

$$e^{\delta T}S(T) + (K - S(T))_+ - (S(0) + V_P(0))e^{rT} = e^{0.03} \times 950 + 25 - 1010e^{0.03} = 25 - 60e^{0.03} = -36.83$$

Problem 2.9. (10 points) A derivative security has the payoff function given by

$$v(s) = \begin{cases} 10 & \text{if } s < 90 \\ 0 & \text{if } 90 \leq s < 100 \\ 20 & \text{if } 100 \leq s \end{cases}$$

Its exercise date is in one year. You model the time−1 price of the underlying asset as

$$S(1) \sim \begin{cases} 85 & \text{with probability } 1/4 \\ 95 & \text{with probability } 1/2 \\ 105 & \text{with probability } 1/4 \end{cases}$$

What is the expected payoff of the above derivative security?

Solution: (a)

$$10\left(\frac{1}{4}\right) + 20\left(\frac{1}{4}\right) = \frac{30}{4} = 7.5$$

Problem 2.10. (6 points) Consider a non-dividend-paying stock whose current price equals \$50 per share. A pair of one-year European calls on this stock with strikes of \$40 and \$50 is available in the market for the observed prices of \$2 and \$4, respectively.

The continuously-compounded, risk-free interest rate is given to be 10%.

George suspects that there exists an arbitrage portfolio in the above market consisting of the following components:

- the **long** \$40-strike call,
- the **written** \$50-strike call.

What is the minimum **gain** from this suspected arbitrage portfolio?

Solution: The initial cost of this portfolio is $2-4=-2$. The lower bound on the payoff is zero. The lower bound on the gain is, hence,

$$2e^{0.1} = 2.21$$

Problem 2.11. (5 points) The current exchange rate is \$1.08 per Swiss Franc. The continuously compounded, risk free interest rate for the USD is 0.05. The continuously compounded, risk free interest rate for the Swiss Franc is 0.02.

The current price of a USD-denominated call option which allows its owner to purchase 100 Swiss Francs for \$100 in one year is equal to \$11.47. What is the price of the otherwise identical put option?

Solution: By put-call parity, the value of the put is, in our usual notation,

$$V_P(0) = V_C(0) - F^P_{0,T}(x) + PV_{0,T}(K) = 11.47 - 108e^{-0.02} + 100e^{-0.05} = 0.7314857.$$

Problem 2.12. (10 points) Let the current price of a market index be \$80. Consider a European six-month, at-the-money call option on this market index.

We model the price of the market index in half a year as follows:

$$S(1/2) \sim \begin{cases} 78 & \text{with probability } 1/6 \\ 82 & \text{with probability } 1/2 \\ 84 & \text{with probability } 1/3 \end{cases}$$

What is the expected payoff of this call option?

Solution: Since the option is at-the-money, the strike price is \$80. We have

$$V_C(1/2) = (S(1/2) - 80)_+ \sim \begin{cases} 0 & \text{with probability } 1/6 \\ 2 & \text{with probability } 1/2 \\ 4 & \text{with probability } 1/3 \end{cases}$$

So,

$$\mathbb{E}[V_C(T)] = 2\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) = \frac{7}{3}.$$

Problem 2.13. (10 points) The current price of the non-dividend-paying stock **S** is \$40.

The current price of stock **Q** is \$50. Stock **Q** is scheduled to pay dividends continuously with the dividend yield δ_Q.

A one-year European exchange call option with the underlying asset **S** and the strike asset **Q** is sold for \$2.50.

A one-year European exchange put option with the underlying asset **S** and the strike asset **Q** is sold for \$10.50.

The continuously-compounded, risk-free interest rate is given to be 0.10.

What is the value of δ_Q?

Solution: In our usual notation, by the generalized put-call parity, we have

$$V_{EC}(0, \mathbf{S}, \mathbf{Q}) - V_{EP}(0, \mathbf{S}, \mathbf{Q}) = F^P_{0,T}(S) - F^P_{0,T}(Q).$$

Using the given information, we get

$$2.50 - 10.50 = 40 - 50e^{-\delta_Q} \quad \Rightarrow \quad 50e^{-\delta_Q} = 40 + 8 = 48 \quad \Rightarrow \quad \delta_Q = \ln\left(\frac{50}{48}\right) = 0.04082199.$$

Problem 2.14. (10 points) Let the current price of a non-dividend-paying stock be \$40. A market maker writes a \$38-strike, three-month call option on this stock. The option's price is \$2.72. The market-maker simultaneously buys one share of the underlying stock.

The continuously compounded, risk-free interest rate is 0.04.

For which final value of the stock price will the market maker break even?

Solution: The initial cost of the portfolio is $40-2.72 = 37.28$. This is a covered call, so the expression for the payoff is, in our usual notation,

$$-(S(T) - K)_+ + S(T) = \min(S(T), K).$$

In this problem, the payoff function for the portfolio is, therefore, $v(s) = min(s, 38)$. We need to solve for s in

$$\min(s, 38) - 37.28e^{0.04/4} = 0 \quad \Rightarrow \quad \min(s, 38) = 37.65467 \quad \Rightarrow \quad s = 37.65467.$$

Problem 2.15. (5 points) A stock is currently priced at \$100 per share. It is scheduled to pay a continuous dividend in the amout proportional to its price with the dividend yield of 1.5%. One-year, \$102-strike European call and put options on this stock have equal prices. What is the continuously-compounded annual risk-free rate of interest?

Solution: By put-call parity, we have

$$K = F_{0,T}(S) = S(0)e^{(r-\delta)T} \quad \Rightarrow \quad r = \delta + \frac{1}{T}\ln\left(\frac{K}{S(0)}\right).$$

So,

$$r = \delta + \frac{1}{T}\ln\left(\frac{K}{S(0)}\right) = 0.015 + \ln(102/100) = 0.0348.$$

2.4. **MULTIPLE-CHOICE QUESTIONS.**

Problem 2.16. (5 points) In which of the following option positions is the investor exposed to an unlimited loss?

(a) Long a put option
(b) Short a put option
(c) Long a call option
(d) Short a call option
(e) None of the above

Solution: **(d)**
Just draw the payoff diagrams to convince yourselves.

Problem 2.17. (5 points) Consider a one-year, \$45-strike European call option and a one-year, \$55-strike European put option on the same underlying asset. You observe that the time−0 stock price equals \$40 while the time−1 stock price equals \$50. Then,

(a) both of the options are out-of-the-money at expiration.

(b) both of the options are in-the-money at expiration.

(c) the call is out-of-the-money and the put is in-the-money at expiration.

(d) the put is out-of-the-money and the call is in-the-money at expiration.

(e) both options are at-the-money at expiration.

Solution: (b)