University of Texas at Austin

Quiz #3

SLLN. Monte Carlo.

Problem 3.1. (10 points) Let $\{Y_n, n \in \mathbb{N}\}$ be a sequence of independent, identically distributed random variables. Assume that $Y_1 = e^X$ where X is a standard normal random variable. Use the Strong Law of Large Numbers to find the following limit

$$\lim_{n\to\infty}\left(\prod_{i=1}^{n} Y_i\right)^{1/n} = \lim_{n\to\infty}(Y_1 \cdot Y_2 \cdots Y_n)^{1/n}.$$

Hint: Note that for every n, $Y_n = e^{X_n}$ where $\{X_n, n \in \mathbb{N}\}$ is a sequence of independent identically distributed standard normal random variables. Then, it helps to modify the product in the limit above and use the continuity of the exponential function.

Solution: For every $n \in \mathbb{N}$,

$$(\prod_{i=1}^{n} Y_i)^{1/n} = (\prod_{i=1}^{n} e^{X_i})^{1/n} = \exp\{\frac{1}{n}\sum_{i=1}^{n} X_i\}.$$

By the SLLN, with probability 1,

$$\frac{1}{n}\sum_{i=1}^{n} X_i \to \mathbb{E}[X_1] = 0, \text{ as } n \to \infty.$$

So, thanks to the continuity of the exponential function

$$(\prod_{i=1}^{n} Y_i)^{1/n} \to e^0 = 1, \text{ as } n \to \infty$$

with probability 1.

Problem 3.2. (5 points) You use *Monte Carlo* to simulate values from a normal distribution with mean 0 and variance 4. Your plan is to use 10000 simulations. What is the variance of the *Monte Carlo* simulations?

Solution: Let $n = 10000$. Then, every *Monte Carlo* simulation will be of the form

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

where $X_i \sim Normal(mean = 0, var = 4)$ for all $i = 1, \dots, n$. We have

$$Var[\bar{X}] = \frac{1}{n^2}\sum_{i=1}^{n} Var[X_i] = \frac{Var[X_1]}{n} = \frac{4}{10000} = 0.0004.$$