# M339W: May 5th, 2021.

## Required Returns [cont'd].

- Start w/ an arbitrary portfolio P whose Sharpe ratio is

$$\eta_P = \frac{\mathbb{E}[R_P] - r_f}{\sigma_P}$$

- Consider an investment I.
- Construct the new portfolio P' so that

$$R_{P'} = R_P - x \cdot r_f + x \cdot R_I$$

Assume that $x$ is "small".

$$\mathbb{E}[R_{P'}] - r_f = \mathbb{E}[R_P] - r_f + \underbrace{x \cdot (\mathbb{E}[R_I] - r_f)}$$ (E)

$$SD[R_{P'}] = SD[R_P] + \underbrace{x \cdot SD[R_I] \cdot corr[R_P, R_I]}_{\text{the increment of the risk due to the "introduction" of I}}$$ (σ)

Combining (E) and (σ), we get that in order to have P' be an improvement over P, we must have:

$$\cancel{x}(\mathbb{E}[R_I] - r_f) > \underbrace{\eta_P \cdot \cancel{x} \cdot SD[R_I] \cdot corr[R_P, R_I]}_{\text{the effect of staying on the line through P w/ the same increment in risk.}}$$

$$\mathbb{E}[R_I] - r_f > \frac{\mathbb{E}[R_P] - r_f}{\sigma_P} \cdot SD[R_I] \cdot corr[R_P, R_I]$$

$$\mathbb{E}[R_I] > \boxed{r_f + (\mathbb{E}[R_P] - r_f) \cdot \frac{\sigma_I}{\sigma_P} \cdot \rho_{P,I}}$$

$=: \beta_I^P$ ... the BETA of investment I w/ the portfolio P

**Def'n.** The required return of investment I given portfolio P is

$$r_I := r_f + \beta_I^P \cdot (\mathbb{E}[R_P] - r_f)$$

**Note:** Recall: A portfolio $P^*$ is efficient if no other portfolio outperforms it in the sense of the Sharpe ratio.

Imagine that there is an investment I such that

$$\mathbb{E}[R_I] > r_I = r_f + \beta_I^{P^*}(\mathbb{E}[R_{P^*}] - r_f)$$

=> You can improve portfolio $P^*$ by additionally investing in I.

=> ⇐ Contradicts the fact that $P^*$ is efficient ⍨

=> For any security I :

$$\boxed{\mathbb{E}[R_I] = r_I = r_f + \beta_I^{eff}\big(\overbrace{\mathbb{E}[R_{eff}] - r_f}^{\text{excess return of the efficient portfolio}}\big)}$$

$\beta$ of investment I w/ the efficient portfolio

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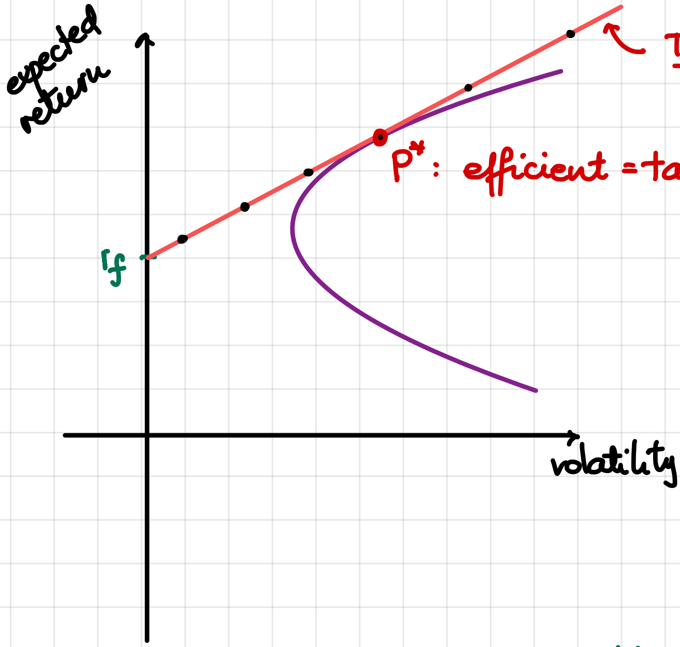
## The Capital Asset Pricing Model (CAPM).

1. **No Friction:** The investors buy/sell all the securities @ competitive market prices w/ no transaction costs (no bid-ask spread; no fees). Both borrowing & lending are @ the same risk-free interest rate.

2. **Rationality:** Investors hold only efficient portfolios of traded securities, i.e., they only hold portfolios which yield the maximum expected return for a particular volatility.

3. **Homogeneous Expectations:** All investors have homogeneous beliefs about:
   - expected returns
   - volatilities
   - correlations




**The Capital Market Line (CML)**

... slope = sharpe ratio of the market portfolio

$P^*$: efficient = tangent = **MARKET PORTFOLIO**

All the assets $i$:

× { # of shares of $i$
market price per share }

→ $MV_i$ ... the market value of $i$

The weight of asset $i$ in the market portfolio:

$$w_i = \frac{MV_i}{\sum MV_i}$$

In CAPM:

$$\mathbb{E}[R_I] = r_I = r_f + \beta_I \left(\mathbb{E}[R_{Mkt}] - r_f\right) \quad \checkmark$$

$\beta_I \leftrightarrow \beta_I^{Mkt}$

w/
$$\beta_I = \frac{SD[R_I]}{SD[R_{Mkt}]} \cdot corr[R_I, R_{Mkt}]$$

or
$$\beta_I = \frac{Cov[R_I, R_{Mkt}]}{Var[R_{Mkt}]}$$

16) You are given the following information about Stock X and the market:

(i) The annual effective risk-free rate is 5%. $r_f = 0.05$ ✓

(ii) The expected return and volatility for Stock X and the market are shown in the table below:

| | Expected Return | Volatility |
|---|---|---|
| Stock X | 5% | 40% |
| Market | $\boxed{8\%}$ ✓ | 25% |

(iii) The correlation between the returns of stock X and the market is –0.25.

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock X and determine if the investor should invest in Stock X.

✗ (A) The required return is 1.8%, and the investor should invest in Stock X.

(B) The required return is 3.8%, and the investor should NOT invest in stock X.

(C) The required return is 3.8%, and the investor should invest in stock X.

✗ (D) The required return is 6.2%, and the investor should NOT invest in Stock X.

✗ (E) The required return is 6.2%, and the investor should invest in stock X.

$\longrightarrow$: $r_I = r_f + \beta_X \left(\mathbb{E}[R_{Mkt}] - r_f\right)$

$$\beta_X = \frac{\sigma_X}{\sigma_{Mkt}} \cdot \rho_{X,Mkt} = \frac{0.4}{\cancel{0.25}} \cdot (-\cancel{0.25}) = -0.4$$

$$r_I = 0.05 + (-0.4)\left(0.08 - 0.05\right) = 0.05 - 0.4 \cdot 0.03$$

$$= 0.038 < 0.05 = \mathbb{E}[R_X]$$

14) You are given the following information about Stock X, Stock Y, and the market:

(i) The annual effective risk-free rate is 4%.

(ii) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

| | Expected Return | Volatility |
|---|---|---|
| Stock X | 5.5% | 40% |
| Stock Y | 4.5% | 35% |
| Market | 6.0% | 25% |

(iii) The correlation between the returns of stock X and the market is –0.25.

(iv) The correlation between the returns of stock Y and the market is 0.30.

Assume the Capital Asset Pricing Model holds. Calculate the required returns for Stock X and Stock Y, and determine which of the two stocks an investor should choose.

(A) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(B) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock Y.

(C) The required return for Stock X is 4.80%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(D) The required return for Stock X is 6.40%, the required return for Stock Y is 3.16%, and the investor should choose Stock Y.

(E) The required return for Stock X is 3.50%, the required return for Stock Y is 3.16%, and the investor should choose both Stock X and Stock Y.

15) You are given the following information about Stock X, Stock Y, and the market:

(i) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

| | Required Return | Volatility |
|---|---|---|
| Stock X | 3.0% | 50% |
| Stock Y | ? | 35% |
| Market | 6.0% | 25% |

(ii) The correlation between the returns of stock X and the market is –0.25.

(iii) The correlation between the returns of stock Y and the market is 0.30.

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock Y.

(A) 1.48%

(B) 2.52%

(C) 3.16%

(D) 4.84%

(E) 6.52%

7) Consider a portfolio of four stocks as displayed in the following table:

| Stock | Weight | Beta |
|---|---|---|
| 1 | 0.1 | 1.3 |
| 2 | 0.2 | -0.6 |
| 3 | 0.3 | $\beta_3$ |
| 4 | 0.4 | 1.1 |

Assume the expected return of the portfolio is 0.12, the annual effective risk-free rate is 0.05, and the market risk premium is 0.08.

Assuming the Capital Asset Pricing Model holds, calculate $\beta_3$ .

A) 0.80

B) 1.06

C) 1.42

D) 1.83

E) 2.17