UNIVERSITY OF TEXAS AT AUSTIN

Quiz #10

Per loss. Per payment. Limited loss.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 10.1. (2 pts) The ground-up loss random variable is denoted by X. An insurance policy on this loss has an ordinary deductible of d. Then, the expected **policyholder** payment per loss equals

$$\mathbb{E}[X \wedge d].$$

True or false? Why?

Solution: TRUE
This was discussed in class.

Problem 10.2. (6 points) *Source: Sample P exam, Problem #46.* A device that continuously measures and records seismic activity is placed in a remote region. The time T to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Calculate the expected time until discovery of failure.

Solution: We have to calculate $\mathbb{E}[X] = \mathbb{E}[T \vee 2]$ with $T \sim \mathit{Exponential}(\theta = 3)$. In general, for any constants η and ζ, we have

$$\eta \wedge \zeta + \eta \vee \zeta = \eta + \zeta.$$

Now, we use our STAM tables to get

$$\mathbb{E}[T \vee 2] = 2 + \theta - \mathbb{E}[T \wedge 2] = 2 + \theta - \theta(1 - e^{-\frac{2}{\theta}}) = 2 + 3e^{-\frac{2}{3}}$$

Problem 10.3. (7 points) *Source: Sample P Exam, Problem #147.* The severity random variable covered by a car insurance company follows an exponential distribution. By imposing a deductible of d, the insurance company reduces the expected claim payment by 10%. In other words, the expected value of the per loss random variable is by 10% lower than the expected value of the severity. Calculate the percentage reduction on the variance of the claim payment.

Solution: Let X denote the loss random variable. Then, $X \sim \mathit{Exponential}(\theta)$. As usual, we will denote the per loss random variable by Y^L. We need to calculate

$$\frac{Var[X] - Var[Y^L]}{Var[X]} = 1 - \frac{Var[Y^L]}{Var[X]}$$

We are given that $\mathbb{E}[Y^L] = 0.9\mathbb{E}[X] = 0.9\theta$. On the other hand, we know that

$$\mathbb{E}[Y^L] = \mathbb{E}[(X - d)_+] = \mathbb{E}[X] - \mathbb{E}[X \wedge d] = \theta - \theta(1 - e^{-\frac{d}{\theta}}) = \theta S_X(d).$$

We can conclude that $S_X(d) = 0.9$.

From our STAM tables, we can get the variance of the exponential random variable X as follows:

$$Var[X] = \mathbb{E}[X^2] - (\mathbb{E}[X^2]) = 2\theta^2 - \theta^2 = \theta^2.$$

Now, it's time to calculate $Var[Y^L]$. We have

$$\mathbb{E}[(Y^L)^2] = \mathbb{E}[(X - d)_+^2] = \mathbb{E}[(X - d)^2 \mid X > d]S_X(d) + 0F_X(d).$$

By the memoryless property,

$$X - d \mid X > d \sim \mathit{Exponential}(mean = \theta).$$

So, $\mathbb{E}[(Y^L)^2] = 2\theta^2 S_X(d) = 2(0.9)\theta^2 = 1.8\theta^2$. Therefore,

$$Var[Y^L] = 1.8\theta^2 - (0.9\theta)^2 = 0.99\theta^2.$$

Finally, we get

$$1 - \frac{Var[Y^L]}{Var[X]} = 1 - \frac{0.99\theta^2}{\theta^2} = 1 - 0.99 = 0.01. \tag{10.1}$$