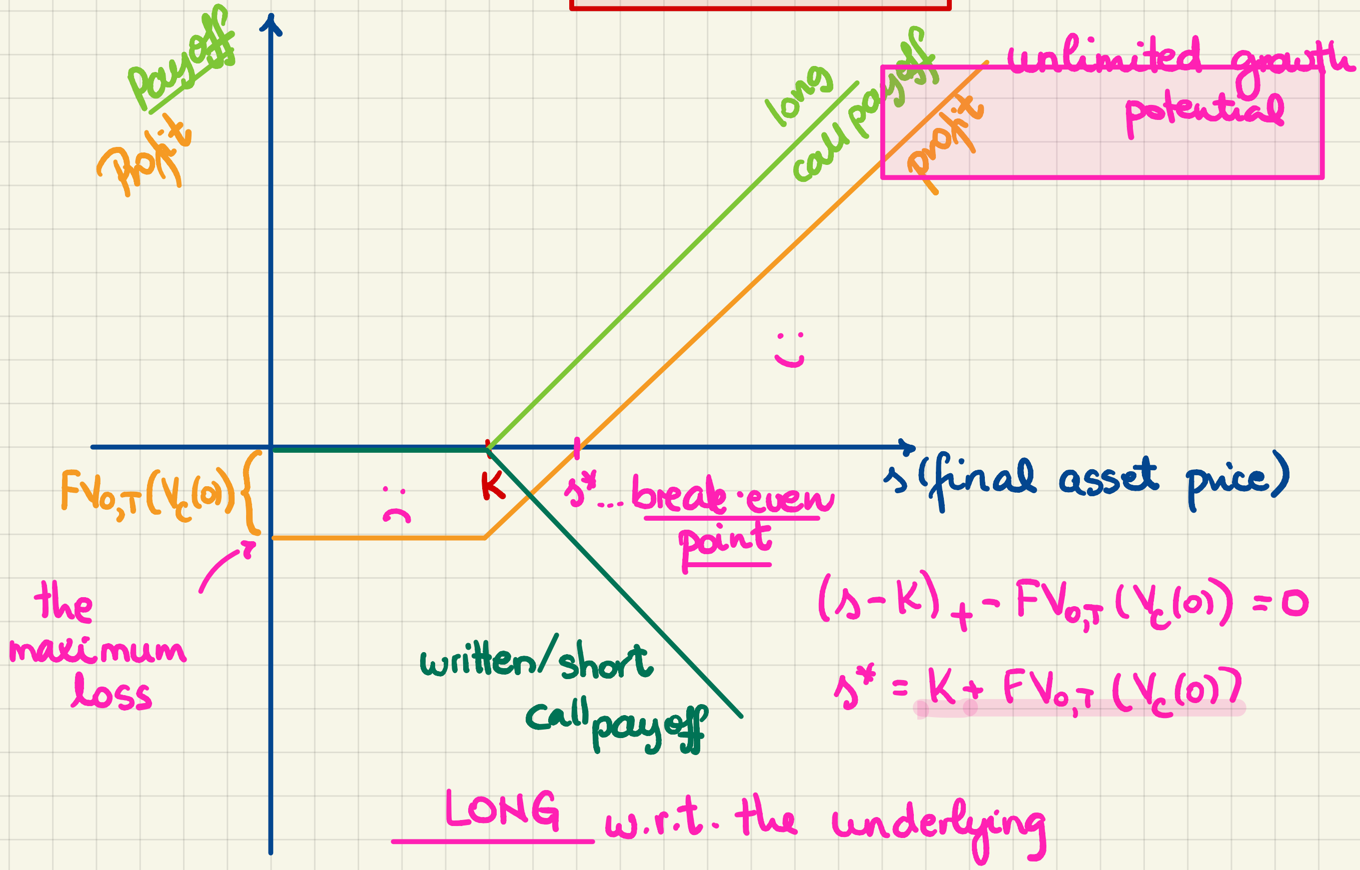
M339D: February 10^{th}, 2023.

More on European Call Options.

Payoff: $\boxed{V_C(T) = (S(T)-K)_+}$

the payoff f'tion: $\boxed{v_C(s) = (s-K)_+}$



$$(s-K)_+ - FV_{0,T}(V_C(0)) = 0$$

$$s^* = K + FV_{0,T}(V_C(0))$$

LONG w.r.t. the underlying

$$\mathbb{E}[V_C(T)] = \mathbb{E}[(S(T)-K)_+]$$

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #5

European call options.

Problem 5.1. The initial price of a non-dividend-paying asset is \$100. A six-month, \$95-strike European call option is available at a \$8 premium. The continuously compounded risk-free interest rate equals 0.04. What is the break-even point for this call option?

(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

$$s^* = 95 + FV_{0,T}(8) = 95 + 8e^{0.04(0.5)}$$
$$= 95 + 8e^{0.02}$$
$$= 103.16$$

Problem 5.2. (5 points) A stock's price today is $1000 and the annual effective interest rate is given to be 5%. You write a one-year $1,050-strike call option for a premium of $10 while you simultaneously buy the stock. What is your **profit** if the stock's spot price in one year equals $1,200?

(a) $150.00

(b) $139.90

(c) $10.50

(d) - $39.00

(e) None of the above.

→: Payoff of option: $(1200-1050)_+ = 150$

$$\boxed{V_c(0) = 10}$$

$$FV_{0,1}(V_c(0)) = 10(1.05) = 10.5$$

Profit of option: $150 - 10.5 = 139.5$

Profit of stock: $1200 - 1000(1.05) = 150$

$150 - 139.5 = 10.5$

↑ call written

Alternatively:

$$\text{Payoff} = S(T) - (S(T)-K)_+ = \begin{cases} K, & \text{if } S(T) \geq K \\ S(T), & \text{if } S(T) < K \end{cases}$$

$\boxed{\text{Covered Call}}$

$$= \min(S(T), K)$$

In this problem

Payoff $= \min(1200, 1050) = 1050$

Initial Cost $= 1000 - 10 = 990$

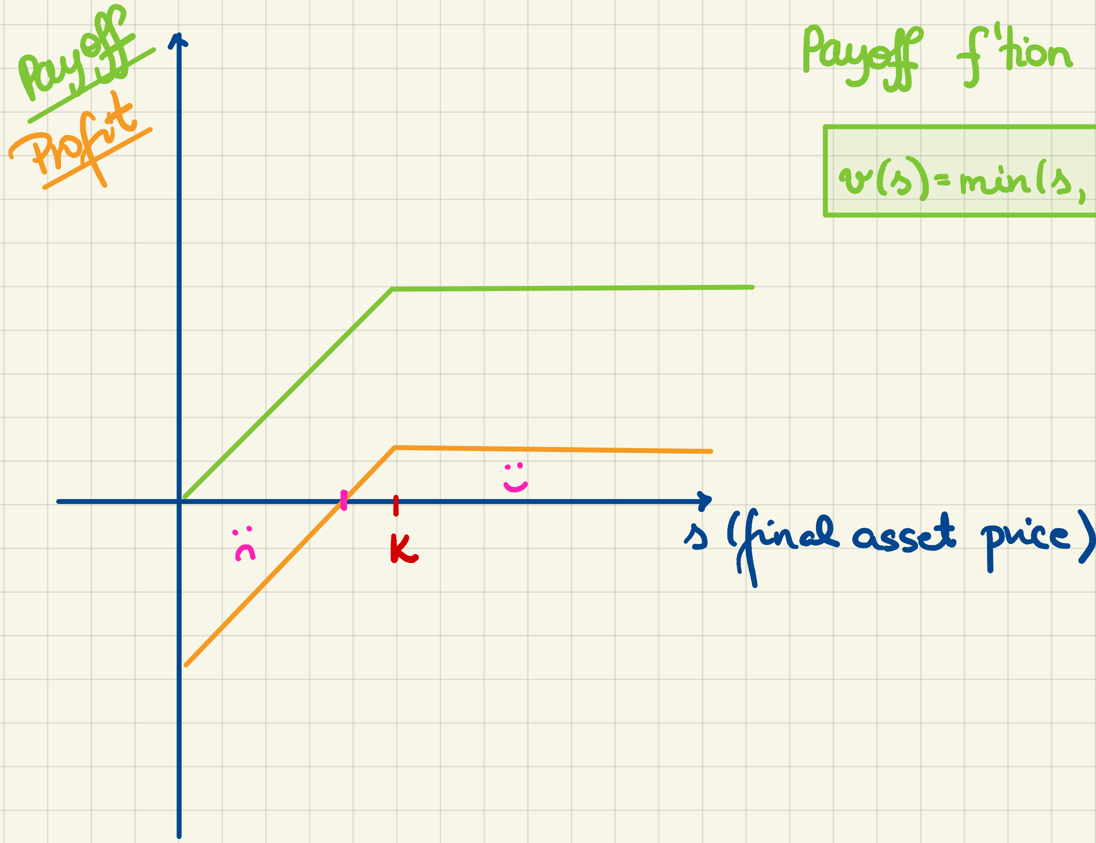
Profit $= 1050 - 990(1.05)$

$= 10.50$

$\square$

Payoff f'tion of Covered call:

$$\boxed{v(s) = \min(s, K)}$$



Problem 5.3. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for \$1,000.

The jeweler models the market price of gold in one year as follows:

Gold price in one year	Probability	min
750 per ounce	0.2	→ 750
850 per ounce	0.5	→ 850
950 per ounce	0.3	→ 900

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of \$900 per ounce. The option costs \$100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelry produced.

→ :

Payoff: $-S(T) + (S(T)-K)_+ =$

$$= \begin{cases} -K, & \text{if } S(T) \geq K \\ -S(T), & \text{if } S(T) < K \end{cases}$$

$$= \max(-K, -S(T)) = -\min(S(T), K)$$

$\boxed{\text{CAP}}$

$$\text{Profit} = 1000 - \min(S(T), K) - 100e^{0.05}$$

$$\mathbb{E}[\text{Profit}] = 894.87 - \mathbb{E}[\min(S(T), 900)] = 49.87$$

$\square$