

## UNIVERSITY OF TEXAS AT AUSTIN

Quiz #1

## Random variables.

Please, provide your **final answer only** to the following questions:

**Problem 1.1.** (2 points) The cumulative distribution function is always strictly increasing. *True or false?*

**Solution: FALSE**

**Problem 1.2.** (2 points) The probability density function is always less than or equal to 1. *True or false?*

**Solution: FALSE**

**Problem 1.3.** (2 points) The probability mass function is always less than or equal to 1. *True or false?*

**Solution: TRUE**

**Problem 1.4.** (2 points) For any random variable  $X$ , we have that

$$\mathbb{E}[|X|] = |\mathbb{E}[X]|.$$

*True or false?*

**Solution: FALSE**

Try the random variable with values  $-1$  and  $1$  being equally likely.

**Problem 1.5.** (2 points) Let the random variable  $X$  have the survival function  $S_X(x) = e^{-x/80}$ , for  $x > 0$ . Then the mean of that random variable equals  $1/80$ . *True or false?*

**Solution: FALSE**

The mean is actually 80. You can recognize the distribution as exponential with mean 80. Or, you can calculate the expected value outright.

**Problem 1.6.** (5 points) *Source: Sample P exam #199.*

A car is new at the beginning of a calendar year. The time, in years, before the car experiences its first failure is exponentially distributed with mean 2. Calculate the probability that the car experiences its first failure in the last quarter of some calendar year.

- (a) 0.081
- (b) 0.088
- (c) 0.102
- (d) 0.205
- (e) None of the above.

**Solution: (d)**

The random variable  $T$  has the cdf given by

$$F_T(x) = 1 - e^{-x/2} \quad \text{for } x \geq 0.$$

So, for any year  $n = 1, 2, 3, \dots$ , the probability of the car breaking down in the last quarter is

$$\mathbb{P}[n - 0.25 < X \leq n] = e^{-\frac{n-0.25}{2}} - e^{-\frac{n}{2}} = e^{-\frac{n}{2}}(e^{0.125} - 1).$$

Thus, the probability that the car breaks down at the end of any year equals

$$\begin{aligned}\sum_{n=1}^{\infty} \mathbb{P}[n - 0.25 < X \leq n] &= \sum_{n=1}^{\infty} e^{-\frac{n}{2}} (e^{0.125} - 1) = (e^{0.125} - 1) e^{-1/2} \sum_{n=0}^{\infty} e^{-\frac{n}{2}} \\ &= (e^{0.125} - 1) e^{-1/2} \frac{1}{1 - e^{-1/2}} = \frac{e^{0.125} - 1}{e^{0.5} - 1} = 0.20525.\end{aligned}$$