M339D: October 28th, 2022.

Homework

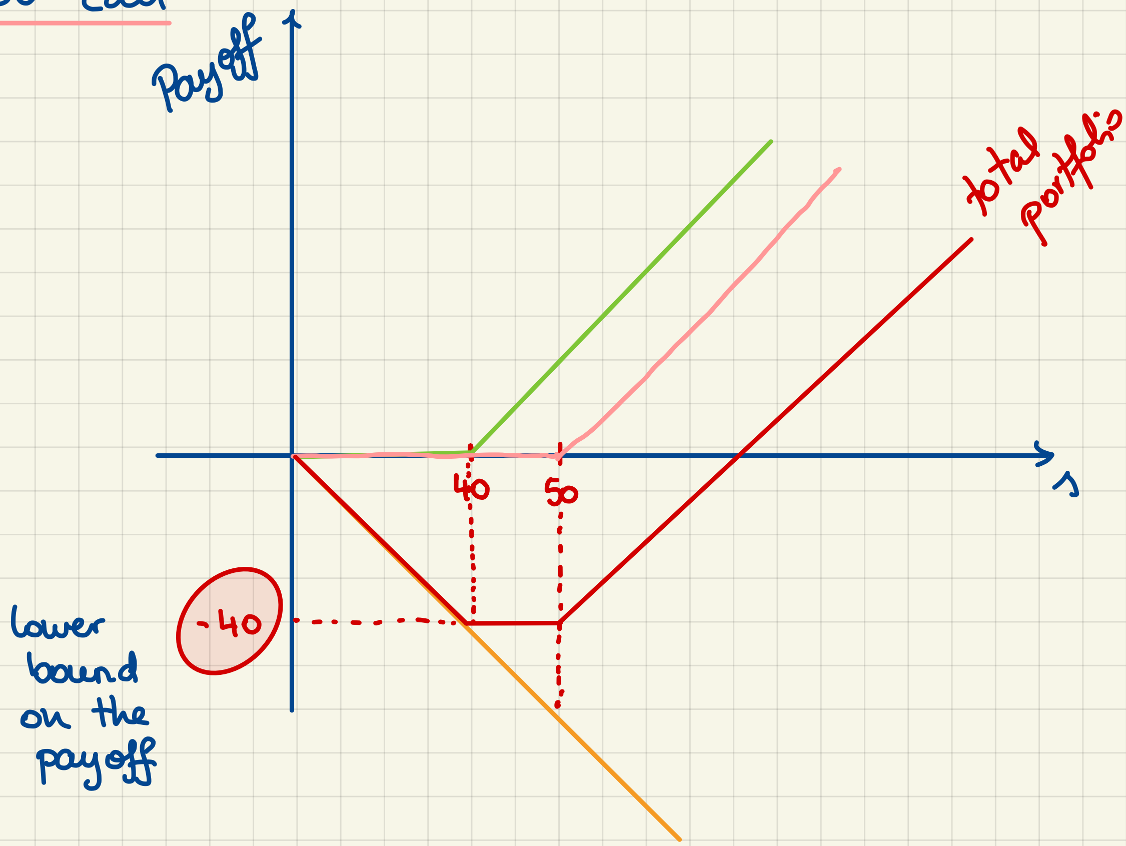
$\boxed{6.3.}$ $S(0) = 54$ $\quad T = 1$

$K_1 = 40 \quad V_c(0, K_1 = 40) = 4$

$K_2 = 50 \quad V_c(0, K_2 = 50) = 2$

$\boxed{r = 0.10}$

- short stock
- long 40 · call
- long 50 · call



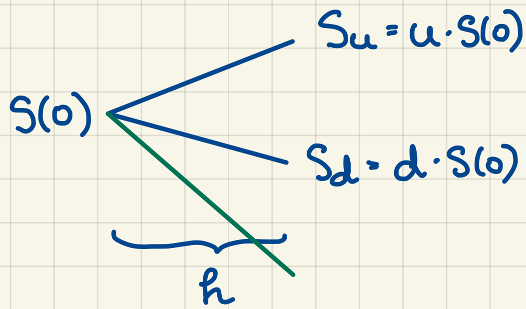
Initial Cost: $-54 + 4 + 2 = -48$

$FV_{0,1}(\text{Init. Cost}) = -48e^{0.10}$

Lower bd on the profit: $-40 - (-48e^{0.10}) = \ldots$

6.9.

$e^{rh} \leq d < u$



$S_u - S(0)e^{rh} = S(0)(u - e^{rh}) > 0$

$S_d - S(0)e^{rh} = S(0)(d - e^{rh}) \geq 0$

Arbitrage Portfolio!

- borrow $S(0)$
- buy one share of stock

Verify that this is an arbitrage portfolio.

Initial Cost: $-S(0) + S(0) = 0$

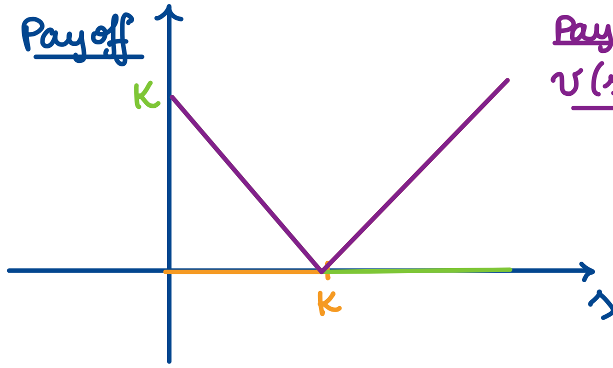
Payoff: $V(T) = -S(0)e^{rh} + S(T)$

Problem 6.5. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

A **straddle** consists of a long call and a long otherwise identical put. Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

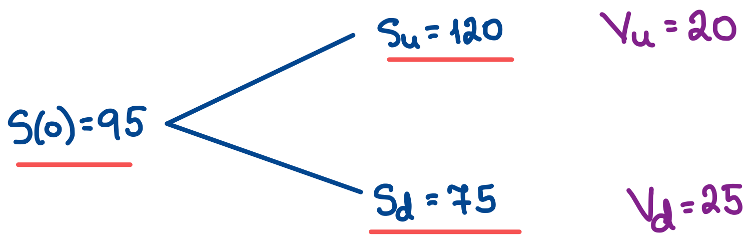
(a) About \$10
(b) About \$10.83
(c) About \$15.45
(d) About \$20.84
(e) None of the above.



Payoff f'tion of straddle

$v(s) = |s-K|$

$\boxed{K=100}$



$$\Delta = \frac{V_u - V_d}{S_u - S_d} = \frac{20-25}{120-75} = -\frac{5}{45} = -\frac{1}{9}$$

$$B = e^{-rh}\frac{u\cdot V_d - d\cdot V_u}{u-d} = e^{-0.06(1)}\cdot\frac{\frac{120}{95}\cdot 25 - \frac{75}{95}\cdot 20}{\frac{120}{95} - \frac{75}{95}}$$

$$= e^{-0.06}\,\frac{120\cdot 25 - 75\cdot 20}{45} = 31.39$$

$$V(0) = \Delta\cdot S(0) + B = -\frac{1}{9}\cdot 95 + 31.39 = 20.83 \checkmark$$

$\square$

Start w/ $V(0) = \Delta \cdot S(0) + B$

$$V(0) = \frac{V_u - V_d}{S_u - S_d} \cdot S(0) + e^{-rh} \frac{u \cdot V_d - d \cdot V_u}{u-d}$$

⋮ algebra

$$V(0) = e^{-r \cdot h}\left[V_u \cdot \frac{e^{rh} - d}{u-d} + V_d \cdot \frac{u - e^{rh}}{u-d} \right]$$

Both positive! Due to the no-arbitrage condition ☺

Add up to 1!

We choose to interpret these two quantities as probabilities!

We define the risk-neutral probability of the stock price going up in a single period as:

$$\boxed{p^* := \frac{e^{rh} - d}{u-d}}$$

=> The risk-neutral pricing formula

$$\boxed{V(0) = e^{-rT}\left[V_u \cdot p^* + V_d (1-p^*) \right]}$$

We generalize this principle:

$$\boxed{V(0) = e^{-rT} \cdot \mathbb{E}^*[V(T)]}$$

$\boxed{6.5.}$ w/ this approach:

$$p^* = \frac{e^{rh} - d}{u-d} = \frac{S(0)e^{rh} - S_d}{S_u - S_d} = \frac{95e^{0.06} - 75}{120 - 75} = \underline{0.577}$$

$$V(0) = e^{-0.06}\left(20(0.575) + 25(1-0.575)\right) = \underline{20.84} \checkmark \quad \square$$