Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Solution: Mock In-Term Exam II
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

Problem 1.1. (5 points) The current price of stock a certain type of stock is \$80. The premium for a $6-$month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) \$80
(b) \$85.72
(c) \$85.84
(d) \$85.96
(e) None of the above.

Solution: (d)
The break-even point is

$$80 + 5.84e^{0.04/2} = 85.958$$

Problem 1.2. Let the current price of a non-dividend-paying stock equal 100. The forward price for delivery of this stock in 3 months equals \$101.26

Consider a \$90-strike, six-month put option on this stock whose premium today equals \$2.22.
What will the profit of this long put option be if the stock price at expiration equals \$96?

(a) About \$2.28 loss.
(b) About \$2.22 loss.
(c) About \$2.28 gain.
(d) About \$2.22 gain.
(e) None of the above.

Solution: (a)
The option is out-of-the money at expiration, so its owner suffers a loss of the future value of its premium

$$2.22 \times \left(\frac{101.26}{100}\right)^2 = 2.2763.$$

Problem 1.3. The current price of a discrete-dividend-paying stock is \$90 per share. The company projects to pay quarterly dividends starting three months from today to perpetuity. The first dividend amount is \$2 and the dividends are scheduled to increase by a factor of 0.01 every time a dividend is paid.

The continuously compounded risk-free interest rate is 0.06. What is the prepaid forward price of the above stock for delivery in eight months?

(a) \$84.24
(b) \$86.07

(c) \$88.70
(d) \$90.00
(e) None of the above.

Solution: (b)

$$F_{0,T}^P(S) = 90 - 2e^{-0.015} - 2.02e^{-0.03} = 86.0695.$$

Problem 1.4. (5 points) A stock's price today is \$1000 and the annual effective interest rate is given to be 5%. You write a one-year, \$1,050-strike call option for a premium of \$10 while you simultaneously buy the stock. What is your **profit** if the stock's spot price in one year equals \$1,200?

(a) \$150.00 gain
(b) \$139.90 loss
(c) \$10.50 gain
(d) \$39.00 loss
(e) None of the above.

Solution: (c)

$$S(T) - 1000(1.05) - (S(T) - K)_+ + 10(1.05) = 1050 - 990(1.05) = 10.50.$$

Problem 1.5. (5 points) *Source: Sample MFE (Intro) Problem #20.*
The current price of a stock is 200, and the continuously compounded risk-free interest rate is 4%. A dividend will be paid every quarter for the next 3 years, with the first dividend occurring 3 months from now. The amount of the first dividend is 1.50, but each subsequent dividend will be 1% higher than the one previously paid. Calculate the fair price of a 3-year forward contract on this stock.

(a) \$200.41
(b) \$205.41
(c) \$210.41
(d) \$215.41
(e) None of the above.

Solution: (b)

Problem 1.6. (5 points) The initial price of a continuous-dividend-paying market index equals \$1,000. The dividend yield equals 0.03.

An investor simultaneously purchases one unit of the index and a one-year, 975-strike European put option on the index for a premium of \$10.

In one year, the spot price of the index is observed to be \$950.

Given that the continuously compounded risk-free interest rate equals 0.03, what is the profit of the investor's portfolio? *Caveat: Be careful with continuous reinvestment of dividend in the index.*

(a) 36.83 loss
(b) 61.82 loss
(c) 61.82 gain

(d) 36.83 gain

(e) None of the above.

Solution: (a)
In our usual notation,

$$e^{\delta T}S(T)+(K-S(T))_{+}-(S(0)+V_P(0))e^{rT}=e^{0.03}\times 950+25-1010e^{0.03}=25-60e^{0.03}=-36.83$$

Problem 1.7. (5 points) Consider a European call option and a European put option on a non-dividend paying stock **S**. You are given the following information:

(1) $r = 0.04$
(2) The current price of the call option $V_C(0)$ is by 0.15 greater than the current price of the put option $V_P(0)$.
(3) Both the put and the call expire in 4 years.
(4) The put and the call have the same strikes equal to 70.

Find the spot price $S(0)$ of the underying asset.

(a) 48.90
(b) 59.80
(c) 69.70
(d) 79.60
(e) None of the above.

Solution: (b)
Using put-call parity, we get

$$0.15 = V_C(0) - V_P(0) = S(0) - Ke^{-rT} = S(0) - 70e^{-0.04\times 4} \quad \Rightarrow \quad S(0) = 0.15 + 70e^{-0.16} = 59.80.$$

Problem 1.8. (5 points) Consider a one-year, \$55-strike European call option and a one-year, \$45-strike European put option on the same underlying asset. You observe that the time-0 stock price equals \$40 while the time-1 stock price equals \$50. Then,

(a) both of the options are out-of-the-money at expiration.

(b) both of the options are in-the-money at expiration.

(c) the call is out-of-the-money and the put is in-the-money at expiration.

(d) the put is out-of-the-money and the call is in-the-money at expiration.

(e) both options are at-the-money at expiration.

Solution: (a)

Problem 1.9. Consider a non-dividend-paying stock whose current price equals \$54 per share. A pair of one-year European calls on this stock with strikes of \$40 and \$50 is available in the market for the observed prices of \$4 and \$2, respectively.

The continuously compounded risk-free interest rate is given to be 10%.

George suspects that there exists an arbitrage portfolio in the above market consisting of the following components:

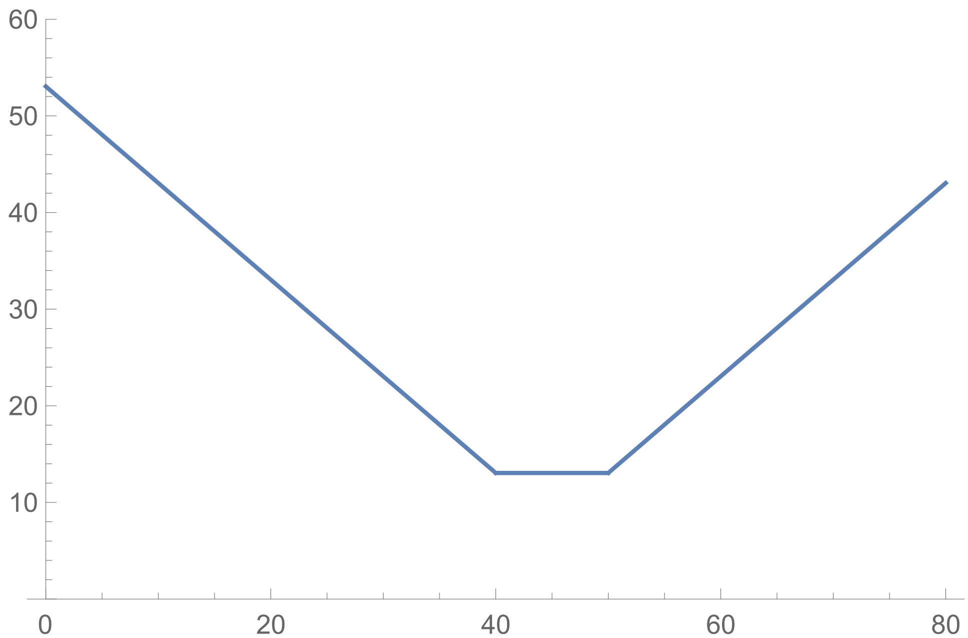
- **short-sale** of one share of stock,

- **buy** the \$40-strike call,
- **buy** the \$50-strike call.

What is the minimum **gain** from this suspected arbitrage portfolio?

(a) The above is **not** an arbitrage portfolio.
(b) \$0.84
(c) \$8.00
(d) \$13.05
(e) None of the above.

Solution: (d)
The profit curve is given below:



The lower bound on the gain is, hence,

$$48e^{0.1} - 40 = 13.0482.$$

Problem 1.10. (5 points) A derivative security has the payoff function given by

$$v(s) = (s^2 - 100)_+$$

Its exercise date is in one year. You model the time-1 price of the underlying asset as

$$S(1) \sim \begin{cases} 9.5 & \text{with probability } 1/4 \\ 10 & \text{with probability } 1/2 \\ 11 & \text{with probability } 1/4 \end{cases}$$

The continuously compounded, risk-free interest rate is 10%. What is the expected **payoff** of the above derivative security?

(a) 5.25

(b) 2.81

(c) 0.31

(d) 1.42

(e) None of the above.

Solution: (a)

$$(9.5^2 - 100)_+ \left(\frac{1}{4}\right) + (10^2 - 100)_+ \left(\frac{1}{2}\right) + (11^2 - 100)_+ \left(\frac{1}{4}\right) = 21 \left(\frac{1}{4}\right) = 5.25$$

Problem 1.11. (5 points) An investor wants to hold 100 euros two years from today. The spot exchange rate is \$1.37 per euro. If the euro-denominated continuously compounded annual interest rate equals 3.0% what is the price of a currency prepaid forward (rounded to the nearest dollar)?

(a) 129

(b) 176

(c) 200

(d) 247

(e) None of the above.

Solution: (a)

$$F^P_{0,T}(x) = 100e^{-0.03 \cdot 2} \cdot 1.37 = 129.02$$

Problem 1.12. (5 points) The current price of stock **S** is \$40. Stock **S** is scheduled to pay a \$2-dividend in two months.

The current price of stock **Q** is \$50. Stock **Q** is scheduled to pay dividends continuously with the dividend yield 0.02.

A six-month European exchange call option with the underlying asset **S** and the strike asset **Q** is sold for \$2.50.

The continuously compounded risk-free interest rate is given to be 0.10.

What is the price of the six-month European exchange put option with the underlying asset **S** and the strike asset **Q**?

(a) About \$8.58

(b) About \$9.04

(c) About \$13.97

(d) About \$14.54

(e) None of the above.

Solution: (c)

$$V_{EP}(0, \mathbf{S}, \mathbf{Q}) = V_{EC}(0, \mathbf{S}, \mathbf{Q}) + F^P_{0,T}(Q) - F^P_{0,T}(S) = 2.50 + 50e^{-0.02 \cdot 0.5} - 40 + 2e^{-0.10/6} = 13.96943.$$