The University of Texas at Austin
Homework assignment 6

Introduction to Mathematical Statistics

February 28, 2026

Instructions: Provide your complete solution to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Moment-generating functions.

Problem 6.1. (5 points) Let $Z_1 \sim N(1,1)$, $Z_2 \sim N(2,2)$ and $Z_3 \sim N(3,3)$ be independent random variables. The distribution of the random variable $W = Z_1 + \frac{1}{2}Z_2 + \frac{1}{3}Z_3$ is ...

a. $N(5/3, 7/6)$
b. $N(3,3)$
c. $N\left(3, \sqrt{3}\right)$
d. $N\left(3, \sqrt{5/3}\right)$
e. None of the above

(*Note:* In our notation $N(\mu, \sigma)$ means normal with mean μ and *standard deviation* σ.)

Problem 6.2. (5 points) Let $Y_1, \dots, Y_{100}$ be independent random variables with the Bernoulli $B(p)$ distribution, with $p = 0.2$ The best approximation to $\bar{Y} = \frac{1}{n}(Y_1 + \dots + Y_n)$ (among the offered answers) is

a. $N(0, 1)$
b. $N(100, 20)$
c. $N(0.2, 0.04)$
d. $N(20, 4)$
e. $N(20, 20)$

(*Note:* In our notation $N(\mu, \sigma)$ means normal with mean μ and *standard deviation* σ.)

Problem 6.3. (5 points) Use the uniqueness of moment-generating functions to give the distribution of a random variable Y with moment-generating function $m_Y(t) = \left(0.7e^t + 0.3\right)^3$.

a. $Y \sim b(3, 0.7)$
b. $Y \sim b(3, 0.3)$
c. $Y \sim B(0.7)$
d. $Y \sim P(0.7)$

+None of the above.

Problem 6.4. (10 points) The moment generating function of a certain random variable Y is given to be equal to

$$m_Y(t) = (1 - 2500t)^{-4}.$$

Calculate the standard deviation of the random variable Y.

Problem 6.5. (10 points) Let Y be a geometric random variable with parameter p. What is its moment generating function m_Y? Do not forget to explicitly state the domain of m_Y!

Problem 6.6. (15 points) Let $Y \sim E(\tau)$. Find the moment generating function on Y not forgetting to explicitly state the domain. Using the moment generating function, recalculate the mean and the variance of the random variable Y.