The normal sample.

1.1. **Linear combinations.** We will accept the following fact without a proof:

Let (X, Y) be **jointly normally distributed** and let α and β be two constants. Then, the *linear combination*

$$\alpha X + \beta Y$$

is also normally distributed.

Now, we have the following useful consequence:

Any linear combination of jointly normal random variables is normally distributed.

1.2. **The sample mean.** Let $\{X_1, X_2, \dots, X_n\}$ be a random sample from the normal distribution, i.e., a collection of independent, identically distributed random variables whose distribution is normal. Let their common mean be denoted by μ_X and let their common standard deviation be denoted by σ_X.

Define the **sample mean** as

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

Then,

$$\boxed{\bar{X} \sim Normal(mean = \mu_X, variance = \frac{\sigma_X^2}{n})}$$

Note that the above is an **exact** distribution, <u>not</u> an approximation as in a limit theorem.