UNIVERSITY OF TEXAS AT AUSTIN

Log-normal stock prices: Tail probabilities.

Problem 4.1. (15 points)

You are considering an investment in a non-dividend-paying stock versus an investment in a savings account. According to your belief, the stock's mean rate of return is α and its volatility is σ.

The continuously compounded interest rate is equal to r.

What is the probability that the stock outperforms the savings account at time$-T$? You should leave your final answer in terms of the function N.

Solution: With $Z \sim N(0,1)$, we are looking for the probability

$$\mathbb{P}[S(T) > S(0)e^{rT}] = \mathbb{P}\left[S(0)e^{(\alpha - \sigma^2/2)T + \sigma\sqrt{T}Z} > S(0)e^{rT}\right]$$
$$= \mathbb{P}[(\alpha - \sigma^2/2)T + \sigma\sqrt{T}Z > rT]$$
$$= \mathbb{P}\left[Z > \frac{\sqrt{T}}{\sigma}(r - \alpha + \sigma^2/2)\right]$$
$$= N\left(\frac{(\alpha - r - \sigma^2/2)\sqrt{T}}{\sigma}\right).$$

Problem 4.2. Assume the Black-Scholes framework. You are given the following information for a stock that pays dividends continuously at a rate proportional to its price:

(i) The current stock price is $250.
(ii) The stock's volatility is 0.3.
(iii) The continuously compounded expected rate of stock-price appreciation is 15%.

Find the value s^* such that

$$\mathbb{P}[S(4) > s^*] = 0.05.$$

(a) $861.65

(b) $874.18

(c) $889.94

(d) $905.48

(e) None of the above.

Solution: (e)

$$s^* = 250e^{(0.15 - 0.045)(4) + 0.3(2)(1.645)} = 1020.92$$