UNIVERSITY OF TEXAS AT AUSTIN

Problem Set 3

Payoff. Profit.

3.1. **Static portfolios.**
Step #1. Remember the **bottom-line approach** from *theory of interest*. Decide who your **protagonist** is!
Step #2. Set up the **timeline** (on paper or mentally):

This is how we will talk about **profit**:

- If **Profit**> 0, then we call it a **gain**.
- If **Profit**< 0, then we call it a **loss**.
- If **Profit**$= 0$, then we say that we **break even**.

3.2. **Riskless assets.**

Example 3.1. **Investing in a zero-coupon bond**

Example 3.2. **Taking a loan**

3.3. **Risky assets.**

Example 3.3. **Ouright purchase of a stock**

Problem 3.1. Let the current price of a non-dividend-paying stock be \$40. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time-1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time-0 and liquidate your investment at time-1?

Solution: The initial cost is $S(0)$ and the payoff is $S(T)$ with $T = 1$. So, the profit equals

$$S(T) - S(0)e^{rT}.$$

Thus, the expected profit equals

$$\mathbb{E}[S(T)] - S(0)e^{rT}.$$

According to the given model for the stock price, we have

$$\mathbb{E}[S(T)] = 45\left(\frac{1}{4}\right) + 42\left(\frac{1}{2}\right) + 38\left(\frac{1}{4}\right) = 41.75.$$

Finally, the expected profit is

$$41.75 - 40e^{0.04} = 0.117569.$$

Goal. To study the payoff and the profit as **functions** of the **final asset price**.
Introduce. $s\ldots$ an independent **argument** taking values in $[0, \infty)$ which will stand for the **final asset price**, i.e., it will be a "placeholder" for the random variable $S(T)$

Problem 3.2. To plant and harvest 20,000 bushels of corn, Farmer Jayne incurs total aggregate costs totaling \$33,000. The current spot price of corn is \$1.80 per bushel. What is the profit if the spot price is \$1.90 per bushel when she harvests and sells her corn?

(a) About \$3,000 gain

(b) About \$3,000 loss

(c) About \$5,000 loss

(d) About \$5,000 gain

(e) None of the above

Solution: (d)

$$1.90 \cdot 20,000 - 33,000 = 5,000$$