UNIVERSITY OF TEXAS AT AUSTIN

Quiz #1

Prerequisite material.

Problem 1.1. (2 pts) If X and Y are independent random variables, then

$$F_{X+Y}(a) = F_X(a) \cdot F_Y(a).$$

True or false?

Solution: **FALSE**

Problem 1.2. (2 points) Let X be a normal random variable with parameters $(\mu = 2, \sigma^2 = 1)$, and let Y be a normal random variable with parameters $(\mu = -2, \sigma^2 = 1)$. Assume that X and Y are independent. Then, the variance of the random variable $X + Y$ equals 2. *True or false?*

Solution: **TRUE**
See the "Addition rule for variances".

Problem 1.3. (2 points) In our usual notation, let $S(0) = 40, r = 0.08, \sigma = 0.3, \delta = 0$. You need to construct a $2-$period forward binomial tree for the above stock with every period in the tree of length $h = 0.5$. Then, $u > 1.45$. *True or false?*

Solution: **FALSE**

$$u = \exp\{(0.08 - 0) \cdot 0.5 + 0.3\sqrt{0.5}\} \approx 1.29.$$

Problem 1.4. (2 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the Δ in the replicating portfolio of a single call option on that stock never exceeds 1. *True or false?*

Solution: **TRUE**
The call's Δ will always be between 0 and 1.

Problem 1.5. (8 points)

Let X be a continuous random variable with probability density function $f_X(x)$. Let its cumulative distribution function be denoted by $F_X(x) = \mathbb{P}[X \leq x]$. Define the new random variable Y as

$$Y = F_X(X).$$

Find $\mathbb{E}[Y]$.

Solution: The connection between the probability density function and the cumulative distribution function is

$$f_X(x) = F_X'(x).$$

Using the definition of the expected value of a function of a continuous random variable, we get

$$\mathbb{E}[Y] = \mathbb{E}[F_X(X)] = \int_{-\infty}^{\infty} F_X(x) f_X(x)\, dx.$$

We can use a change of variables $u = F_X(x)$ with which $du = f_X(x)\, dx$. So, our result is

$$\mathbb{E}[Y] = \int_0^1 u\, du = 1/2.$$