M339J *Probability models*
University of Texas at Austin
HW V
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Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 5.1. (5 points) Let the ground-up loss X be exponentially distributed with mean \$800. An insurance policy has an ordinary deductible of \$100 and the maximum amount payable per loss of \$2500.
Find the expected value of the amount paid (by the insurance company) **per positive payment.**

Problem 5.2. (5 pts)Let X be the ground-up loss random variable. Assume that X has the exponential distribution with mean $5,000$. Let B denote the expected payment per loss on behalf of an insurer which wrote a policy with a deductible of $1,500$ and with no maximum policy payment. What is the value of B?

Problem 5.3. (10 points) Let X have a two-point mixture distribution. More precisely, with probability $1/3$, X has the Pareto distribution with parameters $\alpha = 3$ and $\theta = 10$ and with probability $2/3$, X has the Gamma distribution with parameters $\alpha = 2$ and $\theta = 8$.

Find $Var[X]$.

Problem 5.4. (10 points) *Source: Sample C Exam Problem #100.*
Let X have the following cumulative distribution function

$$F_X(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \qquad x \geq 0.$$

Let $u = 1000$.
Find $\mathbb{E}[X \wedge u]$.

Problem 5.5. (10 points) Let Y be lognormal with parameters $\mu = 1$ and $\sigma = 2$.
Define $\tilde{Y} = 3Y$.
Find the median of $\tilde{Y}$, i.e., find the value m such that $\mathbb{P}[\tilde{Y} \leq m_Y] = 1/2$.

Problem 5.6. (10 points) In the notation of our tables, let X be a Weibull random variable with parameters $\theta = 20$ and $\tau = 2$.
Define $Y = 5X$ and denote the coefficient of variation of Y by CV_Y. Find CV_Y.

Hint: The following facts you may have forgotten from probability could be useful:

$$\begin{aligned}\Gamma(1/2) &= \sqrt{\pi},\\ \Gamma(1) &= 1,\\ \Gamma(\alpha+1) &= \alpha\Gamma(\alpha), \quad \text{for all } \alpha.\end{aligned}$$