## Raw Moments.

**Def'n.** The $k^{th}$ raw moment of a random variable $X$ is given by

$$\boxed{\mu_k' := \mathbb{E}[X^k]}$$

**Note:** The $1^{st}$ raw moment is the mean we frequently denote by $\mu_X \equiv \mu$.

## Central Moments.

**Def'n.** The $k^{th}$ central moment of a r.v. $X$ is

$$\mu_k := \mathbb{E}\left[(X-\mu)^k\right]$$

Q: What is the $2^{nd}$ central moment?

$\longrightarrow$: $\mu_2 = Var[X] = \mathbb{E}\left[(X-\mu)^2\right] = \mathbb{E}[X^2] - \mu^2$

$= \mu_2' - \mu^2$

**Problem.** Let $X$ be a two-parameter Pareto w/ $\alpha = 3$ and $\theta = 10$.

Find $Var[X]$.

$\longrightarrow$: $X \sim Pareto(\alpha = 3, \theta = 10)$

Using my STAM tables:

$$\boxed{\mathbb{E}[X^k] = \frac{\theta^k \cdot k!}{(\alpha-1)\cdots\cdots(\alpha-k)}}$$

$k$ integer
$k < \alpha$

$$\mathbb{E}[X] = \frac{\theta^1 \cdot 1!}{\alpha - 1} = \frac{\theta}{\alpha-1} = \frac{10}{3-1} = 5$$

$$\mathbb{E}[X^2] = \frac{\theta^2 \cdot 2!}{(\alpha-1)(\alpha-2)} = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} = \frac{\cancel{2}\cdot 10^2}{\cancel{(3-1)}(3-2)} = 100$$

$$\Rightarrow \quad Var[X] = 100 - 5^2 = 75$$

■

**Task:** The expression for the variance of the exponential & the gamma in terms if it's "simple."

**Def'n.** The **coefficient of variation** of a random variable X is:

$$\boxed{\frac{SD[X]}{\mathbb{E}[X]}}$$

w/ $SD[X] = \sqrt{Var[X]}$ .