NAME:

M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
Mock In-Term Exam III
Instructor: Milica Čudina

Problem 3.1. (5 points) Assume the Black-Scholes framework for the pair of stocks **S** and **Q**.

For the stock **S**, you are given that

- the current stock price is \$40 per share;
- the stock pays dividends in the amount $0.05S(t)\,dt$ during the time period $(t, t + dt)$;
- the stock's volatility is 0.2.

For the stock **Q**, you are given that

- the current stock price is \$40 per share;
- the stock pays no dividends;
- the stock's volatility is 0.4.

The correlation between the returns of **S** and **Q** is -0.4.

The continuously compounded, risk-free interest rate is 0.055.

What is the price of the exchange call option on **S** with the strike asset **Q** with exercise date in a quarter year?

(a) 1.14
(b) 9.13
(c) 18.26
(d) 31.17
(e) None of the above.

Solution: (e)
In order to price the exchange call, we first need to find the "relative" volatility between **S** and **Q**. We get

$$\begin{aligned}\sigma^2 &= \sigma_S^2 + \sigma_Q^2 - 2\sigma_S\sigma_Q\rho \\ &= 0.04 + 0.16 - 2(0.2)(0.4)(-0.4) = 0.264 \quad \Rightarrow \quad \sigma = 0.5138.\end{aligned}$$

Next, we calculate the terms in the Black-Scholes price of the exchange call. We obtain

$$\begin{aligned}d_1 &= \frac{1}{0.5138\sqrt{1/4}}\left[\ln\left(\frac{40}{40}\right) + \left(0 - 0.05 + \frac{0.5138^2}{2}\right)\left(\frac{1}{4}\right)\right] = 0.07979614 = 0.08, \\ d_2 &= 0.07979614 - 0.5138\sqrt{\frac{1}{4}} = -0.1771085 = -0.18.\end{aligned}$$

From the standard normal tables, we get

$$N(d_1) = N(0.08) = 0.5319, \quad N(d_2) = 1 - N(0.18) = 1 - 0.5714 = 0.4286.$$

So,

$$V_{EC}(0, \mathbf{S}, \mathbf{Q}) = 40e^{-0.05(1/4)}(0.5319) - 40(0.4286) = 3.87.$$

Problem 3.2. (5 points) Consider a two-year project. There are only three cash flows for this project:

- The first occurs at $t = 0$, and is -50.
- The second occurs at $t = 1$, and is 40.
- The third occurs at $t = 2$, and is 11.50.

Determine r, the cost of capital, that leads to the project breaking even.

(a) 0.0245
(b) 0.0345
(c) 0.045
(d) 0.05
(e) None of the above.

Solution: (a)
The break-even value of the cost of capital must satisfy

$$-50(1+r)^2 + 40(1+r) + 11.50 = 0 \quad \Leftrightarrow \quad (1+r)^2 - 0.8(1+r) - 0.23 = 0.$$

Solving the quadratic equation, we obtain

$$(1+r)_{1,2} = \frac{0.8 \pm \sqrt{0.8^2 + 4(0.23)}}{2} = \frac{0.8 \pm \sqrt{1.56}}{2} = \frac{0.8 \pm 1.249}{2}.$$

Our acceptable solution is $1 + r = 1.0245$, i.e., $r = 0.0245$.

Problem 3.3. (5 points) You are an pessimist and you model the state of the economy to be twice as likely to be *bad* as it is to be *good*. There are no other states of the economy in your model. You build an equally weighted portfolio out of two stocks S and Q. According to your model, if the economy is *good*, the return of stock S will be 0.08 and the return of stock Q will be 0.10. Also, if the economy is *bad*, the return of stock S will be -0.02 and the return of stock Q will be -0.04. What is the volatility of your portfolio?

(a) 2.1%
(b) 5.66%
(c) 7.61%
(d) 10.21%
(e) None of the above.

Solution: (b)
Let R_S and R_Q denote the returns of the two stocks S and Q, respectively. Then, the return of the entire portfolio can be expressed as

$$R_P = \frac{1}{2}R_S + \frac{1}{2}R_Q.$$

So,

$$Var[R_P] = \frac{1}{4} Var[R_S + R_Q]$$

The random variable $R_S + R_Q$ has the value 0.18 if the economy is *good*, i.e., with probability 1/3. It has the value -0.06 if the economy is *bad*, i.e., with probability 2/3. So,

$$\mathbb{E}[R_S + R_Q] = 0.18 \times \frac{1}{3} - 0.06 \times \frac{2}{3} = 0.02,$$
$$\mathbb{E}[(R_S + R_Q)^2] = (0.18)^2 \times \frac{1}{3} + (0.06)^2 \times \frac{2}{3} = 0.0132.$$

Thus,

$$Var[R_S + R_Q] = 0.0132 - 0.02^2 = 0.0128.$$

So, $Var[R_P] = 0.0128/4 = 0.0032$. Our answer is $\sqrt{0.0032} = 0.0565685$.

Problem 3.4. (5 points) Which one of the following statements is correct?

(a) Any equally weighted portfolio contains only systematic risk.
(b) The volatility of an equally weighted portfolio is at most as large as the average of the volatilities of its components.
(c) Full diversification of an investment portfolio completely eliminates market risk.
(d) Adding another investment into your portfolio always reduces the volatility of the portfolio.
(e) None of the above.

Solution: (b)

Problem 3.5. (5 points) Consider two assets X and Y such that:

- their expected returns are $\mathbb{E}[R_X] = 0.10$ and $\mathbb{E}[R_Y] = 0.08$;
- their volatilities are $\sigma_X = 0.4$ and $\sigma_Y = 0.25$;
- the correlation coefficient of their returns is $\rho_{X,Y} = -1$.

You are tasked with constructing a portfolio consisting of shares of X and Y with a risk-free return. What should the weight w_Y given to asset Y be?

(a) 5/13
(b) 1/2
(c) 8/13
(d) Such a weight does not exist.
(e) None of the above.

Solution: (c)

$$w_Y = \frac{\sigma_X}{\sigma_X + \sigma_Y} = \frac{0.4}{0.4 + 0.25} = \frac{8}{13}\,.$$

Problem 3.6. (5 points) Assume that the risk-free interest rate equals 0.04. The Sharpe ratio of asset S is given to be 1/4 while the Sharpe ratio of asset Q equals 1/3. You know that the volatility of S is three times the volatility of Q. If you build an equally weighted portfolio with assets S and Q as its two components, the expected return of this portfolio will be 0.10. What is the expected return of S?

(a) 11.2%
(b) 12.31%
(c) 13.04%
(d) 13.86%
(e) None of the above.

Solution: (b)
From the condition on the Sharpe ratio of S, we get

$$\frac{\mathbb{E}[R_S] - r_f}{\sigma_S} = \frac{1}{4} \quad \Rightarrow \quad \sigma_S = 4(\mathbb{E}[R_S] - 0.04).$$

From the condition on the Sharpe ratio of Q, we obtain

$$\frac{\mathbb{E}[R_Q] - r_f}{\sigma_Q} = \frac{1}{3} \quad \Rightarrow \quad \sigma_Q = 3(\mathbb{E}[R_Q] - 0.04).$$

Since $\sigma_S = 3\sigma_Q$, we have

$$\begin{aligned} 4(\mathbb{E}[R_S] - 0.04) = 3(3)(\mathbb{E}[R_Q] - 0.04) \quad &\Rightarrow \quad \mathbb{E}[R_S] - 0.04 = 2.25(\mathbb{E}[R_Q] - 0.04) \\ &\Rightarrow \quad \mathbb{E}[R_S] - 2.25\mathbb{E}[R_Q] = 0.04 - 0.09 = -0.05. \end{aligned}$$

On the other hand, from the given expected return on the equally-weighted portfolio, we know that

$$\frac{1}{2}(\mathbb{E}[R_S] + \mathbb{E}[R_Q]) = 0.10 \quad \Rightarrow \quad \mathbb{E}[R_S] + \mathbb{E}[R_Q] = 0.20.$$

Solving the above system of two equations with two unknowns, we get

$$\mathbb{E}[R_S] = 0.1231 \quad \text{and} \quad \mathbb{E}[R_Q] = 0.0769.$$

Problem 3.7. (5 points) You are given the following information about stock X and a portfolio P:

- The annual effective risk-free rate is 5%.
- The portfolio's expected return is 0.10 and its volatility is 0.2.
- The expected return of stock X is 0.08 and its volatility is 0.3.
- The correlation between the returns of stock X and the portfolio P is 0.2.

Then:

(a) The required return of stock X is 0.065 and the investor holding portfolio P should invest in stock X.

(b) The required return of stock X is 0.065 and the investor holding portfolio P should not invest in stock X.

(c) The required return of stock X is 0.105 and the investor holding portfolio P should invest in stock X.

(d) The required return of stock X is 0.105 and the investor holding portfolio P should not invest in stock X.

(e) None of the above.

Solution: (a))
The β for the stock X equals

$$\beta_X = \frac{0.3(0.2)}{0.2} = 0.3.$$

So, the stock X has a required return equal to

$$r_X = r_f + \beta_X(\mathbb{E}[R_m] - r_f) = 0.05 + (0.3)(0.10 - 0.05) = 0.05 + 0.015 = 0.065.$$

Since the expected return is smaler than the required return, one should not invest in stock X.

Problem 3.8. (5 points) Assume the **Capital Asset Pricing Model** holds.

You are given the following information about stock X, stock Y, and the market:

- The required return and volatility for the market portfolio are 0.08 and 0.2, respectively.
- The required return and volatility for the stock X are 0.0404 and 0.4, respectively.
- The correlation between the returns of stock X and the market is -0.2.
- The volatility of stock Y is 0.25.
- The correlation between the returns of stock Y and the market is 0.4.

Calculate the required return for stock Y.

(a) About 0.062.
(b) About 0.08.
(c) About 0.085.
(d) About 0.09.
(e) None of the above.

Solution: (a) or (e)
The βs of stocks X and Y are

$$\beta_X = \frac{0.4(-0.2)}{0.2} = -0.4,$$
$$\beta_Y = \frac{0.4(0.25)}{0.2} = 0.5.$$

So, the required return of stock X must satisfy

$$\begin{aligned} 0.0404 = r_X = r_f + (-0.4)(0.08 - r_f) \quad &\Rightarrow \quad 0.0404 = r_f - 0.032 + 0.4r_f \\ &\Rightarrow \quad 1.4r_f = 0.0724 \quad \Rightarrow \quad r_f = 0.0517. \end{aligned}$$

Finally, the required return of stock Y equals

$$r_Y = 0.0517 + 0.5(0.08 - 0.0517) = 0.0659.$$

Problem 3.9. (5 points) Assume the **CAPM** holds.

Let the risk-free interest rate be 0.04 and let the expected return of a market portfolio be equal to 0.15.

Suppose that stock X has $\beta_X = 1.4$ and that stock Y has $\beta_Y = 0.8$. Using the risk-free asset, stock X, and stock Y, you create a portfolio such that the weight given to X equals the weight given to Y while the weight of the risk-free asset is 0.4. What is the expected return of this portfolio?

(a) 0.0830
(b) 0.1126
(c) 0.1268
(d) 0.1610
(e) None of the above.

Solution: (b)
The β of the risk-free asset is zero. Hence, the β of the portfolio is

$$\beta_P = 0.3\beta_X + 0.3\beta_Y = 0.3(2.2) = 0.66.$$

So, realizing that the expected return of the portfolio equals its required return, we get

$$\mathbb{E}[R_P] = r_f + \beta_P(r_m - r_f) = 0.04 + 0.66(0.15 - 0.04) = 0.1126.$$

Problem 3.10. (5 points) For stock S_1, you are given that its expected return equals 0.176 and its β is 1.2. For stock S_2, you are given that its expected return equals 0.0616 and its β is 0.32. Both of these stocks lie on the *Security Market Line*. For stock S_3, you are given that its expected return equals 0.13 and its β is 0.8. What is the α of stock S_3?

(a) 0
(b) 0.0190
(c) 0.0245
(d) 0.0455
(e) None of the above.

Solution: (e)
Since both S_1 and S_2 are on the **SML**, we know that

$$\begin{aligned} 0.176 &= r_f + 1.2(r_m - r_f), \\ 0.0616 &= r_f + 0.32(r_m - r_f), \end{aligned}$$

where r_f stands for the risk-free interest rate and r_m stands for the expected return of the market. Subtracting the second equation from the first one, we get

$$0.1144 = 0.88(r_m - r_f) \quad \Rightarrow \quad r_m - r_f = \frac{0.1144}{0.88} = 0.13.$$

The risk-free interest rate r_f can, then, be calculated as

$$r_f = 0.176 - 1.2(0.13) = 0.02.$$

Hence, the α of stock S_3 is

$$0.13 - 0.02 - 0.8(0.13) = 0.006.$$

Problem 3.11. (5 points) Which of the following statements is not correct?

(a) Familiarity bias generally does not result in a systematic trading bias.

(b) Overconfidence bias can result from uninformed individuals overestimating the precision of their knowledge.

(c) According to the weak formulation of the efficient market hypothesis, one cannot consistently make gains by trading based on the information contained in past prices.

(d) In the strong form of the efficient market theory, prices reflect all private information.

(e) Herd behavior does not result in a systematic trading bias.

Solution: (e)

Problem 3.12. You are given the following information about the return of a security, using a three-factor model:

Factor	Beta	Expected Return
T	0.16	12%
U	0.18	16%
V	0.24	10%

The expected return of this security using the given three-factor model is equal to 8.25%. What is the annual effective risk-free rate of return?

(a) About 0.025
(b) About 0.045
(c) About 0.055
(d) About 0.065
(e) None of the above.

Solution: (a)
By our three-factor model, we have that the expected return of our security S satisfies

$$\begin{aligned} \mathbb{E}[R_S] &= r_f + \beta^T(\mathbb{E}[R_T] - r_f) + \beta^U(\mathbb{E}[R_U] - r_f) + \beta^V(\mathbb{E}[R_V] - r_f) \\ &= \beta_T\mathbb{E}[R_T] + \beta_U\mathbb{E}[R_U] + \beta_V\mathbb{E}[R_V] + r_f(1 - \beta_T - \beta_U - \beta_V). \end{aligned} \tag{3.1}$$

So,

$$\begin{aligned} r_f &= \frac{\mathbb{E}[R_S] - \beta_T\mathbb{E}[R_T] - \beta_U\mathbb{E}[R_U] - \beta_V\mathbb{E}[R_V]}{1 - \beta_T - \beta_U - \beta_V} \\ &= \frac{0.0825 - 0.16(0.12) - 0.18(0.16) - 0.24(0.1)}{1 - 0.16 - 0.18 - 0.24} = 0.025. \end{aligned}$$