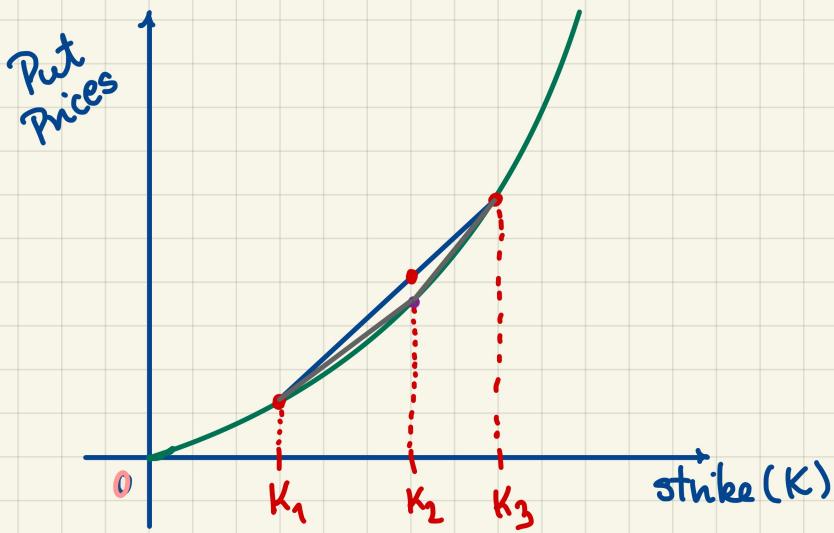


M339D : April 1st, 2022.

Put price Convexity.

$$V_p(T) = (K - S(T))^+ = 0$$



Claim. Let

$$\lambda = \frac{K_3 - K_2}{K_3 - K_1}$$

Then,

$$V_p(K_2) \leq \lambda \cdot V_p(K_1) + (1-\lambda) V_p(K_3) \quad (\text{PC})$$

\Leftrightarrow

$$\frac{V_p(K_2) - V_p(K_1)}{K_2 - K_1} \leq \frac{V_p(K_3) - V_p(K_2)}{K_3 - K_2}$$

Q: What would you do if there exist $K_1 < K_2 < K_3$ such that

$$V_p(K_2) > \lambda V_p(K_1) + (1-\lambda) V_p(K_3) ?$$

→ :

- | | | | |
|----------------|-------------------------------|-------------------------|-----|
| • <u>long</u> | <u>λ</u> | $K_1 \cdot \text{puts}$ | Put |
| • <u>short</u> | <u>1</u> | $K_2 \cdot \text{put}$ | |
| • <u>long</u> | <u>$1-\lambda$</u> | $K_3 \cdot \text{puts}$ | |
- Butterfly
Spread
graph.

Task: Put and call butterfly spreads have the same payoff. ← parity.

67.

Consider the following investment strategy involving put options on a stock with the same expiration date.

- i) Buy one 25-strike put
- ii) Sell two 30-strike puts
- iii) Buy one 35-strike put

SYMMETRIC
Put
Butterfly
Spread

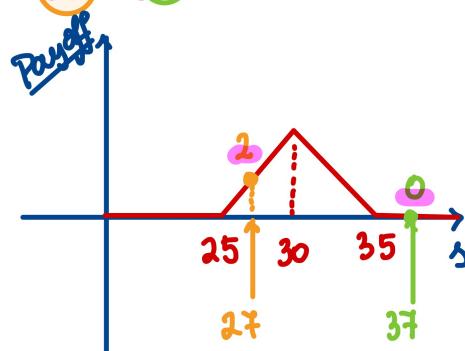
$$K_1 = 25, K_2 = 30, K_3 = 35$$

$$K_2 = \frac{1}{2} K_1 + \frac{1}{2} \cdot K_3$$

$$K_2 = \frac{K_1 + K_3}{2}$$

Calculate the payoffs of this strategy assuming stock prices (i.e., at the time the put options expire) of 27 and 37, respectively.

- (A) -2 and 2
- (B) 0 and 0
- (C) 2 and 0
- (D) 2 and 2
- (E) 14 and 0



68.

For a non-dividend-paying stock index, the current price is 1100 and the 6-month forward price is 1150. Assume the price of the stock index in 6 months will be 1210.

Which of the following is true regarding forward positions in the stock index?

- (A) Long position gains 50
- (B) Long position gains 60
- (C) Long position gains 110
- (D) Short position gains 60
- (E) Short position gains 110

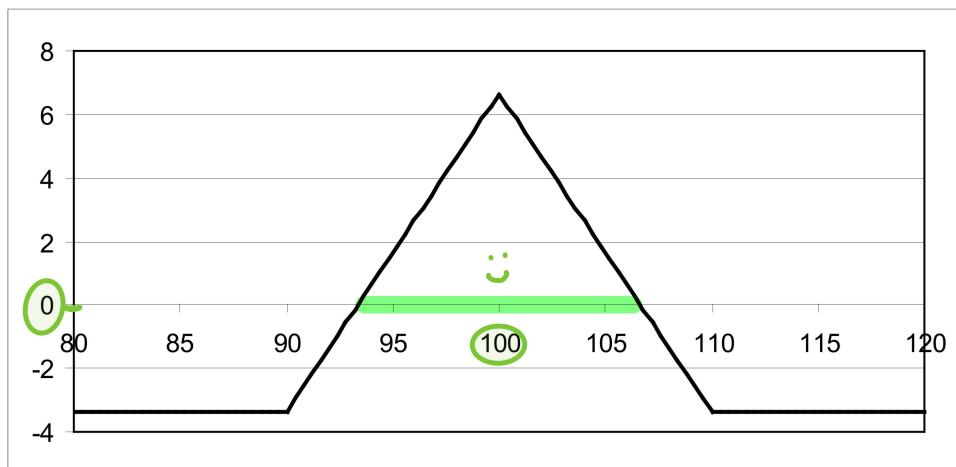
9.

Stock ABC has the following characteristics:

- The current price to buy one share is 100.
- The stock does not pay dividends.
- European options on one share expiring in one year have the following prices:

Strike Price	Call option price	Put option price
90	14.63	0.24
100	6.80	1.93
110	2.17	6.81

A butterfly spread on this stock has the following profit diagram.

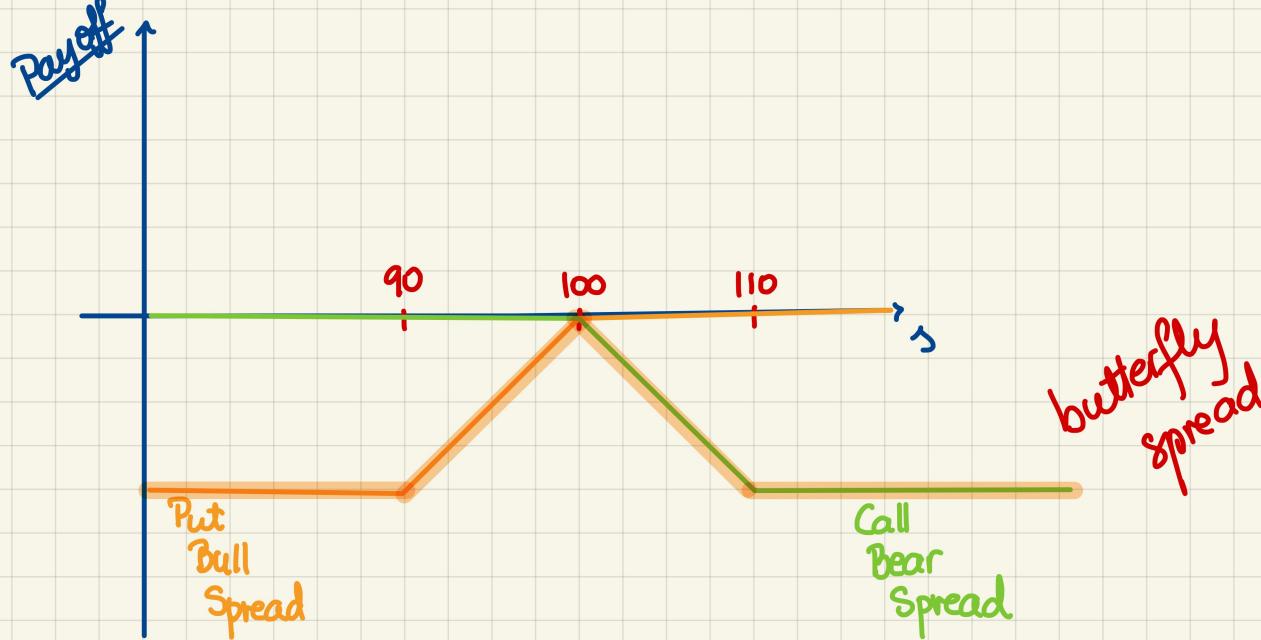


The continuously compounded risk-free interest rate is 5%.

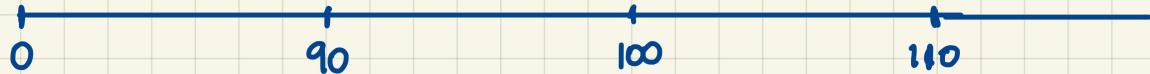
Determine which of the following will NOT produce this profit diagram.

- {
- (A) Buy a 90 put, buy a 110 put, sell two 100 puts
 - (B) Buy a 90 call, buy a 110 call, sell two 100 calls
 - (C) Buy a 90 put, sell a 100 put, sell a 100 call, buy a 110 call
 - (D) Buy one share of the stock, buy a 90 call, buy a 110 put, sell two 100 puts
 - (E) Buy one share of the stock, buy a 90 put, buy a 110 call, sell two 100 calls.
- Floor
"looks like" a long call

- (c)
- buy a 90 put
 - sell a 100 put
 - sell a 100 call
 - buy a 110 call
- } Put Bull Spread } } Call Bear Spread }
- } Butterfly Spread }



(d) slope of the payoff curve:



long stock:

+1 +1 +1 +1 +1

long 90·call:

0 +1 +1 +1 +1

2 short 100·puts:

+2 +2 0 0

long 110·put:

-1 -1 -1 0

Total Portfolio:

+2

0

=> Does not replicate a butterfly spread!

(e) Do (e) "algebraically"; use put-call parity!

