UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 3

Prerequisite material. Long/short positions. Short sales.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 3.1. (10 points) *Source: Sample P exam, Problem #201.*
A theme park conducts a study of families that visit the park during a year. The fraction of such families of size m is $\frac{8-m}{28}$, for $m = 1, 2, 3, 4, 5, 6$, and 7.

For a family of size m that visits the park, the number of members of the family that ride the roller coaster follows a discrete uniform distribution on the set $\{1, \ldots, m\}$.

Calculate the probability that a family visiting the park has exactly six members, **given** that exactly five members of the family ride the roller coaster.

Problem 3.2. Monotonicity.

(i) (3 points) Write down the definition of an **increasing** real-valued function whose domain are all nonnegative real numbers.
(ii) (3 points) Write down the definition of an **decreasing** real-valued function whose domain are all nonnegative real numbers.

Problem 3.3. (1 point) Draw the graph of an *increasing* function.

Problem 3.4. (1 point) Draw the graph of a *decreasing*.

Problem 3.5. (2 points) Draw the graph of a function which is neither decreasing nor decreasing.

Problem 3.6. (4 points) Consider the functions $f : [0, \infty) \to \mathbb{R}$ and $g : [0, \infty) \to \mathbb{R}$. Let f be given by

$$f(x) = \max(x - 50, 0).$$

Let g be given by

$$g(x) = \max(x - 100, 0).$$

What can you say about the monotonicity of the function $f - g$? Remember to justify your answer!

Problem 3.7. (5 points) Complete the following definition:

A financial portfolio is said to be *long with respect to an underlying asset* if

Problem 3.8. (5 points) Complete the following definition:

A financial portfolio is said to be *short with respect to an underlying asset* if

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Problem 3.9. (3 points) Consider an outright purchase of a share of non-dividend-paying stock whose current price is \$80 per share. Let the continuously compounded, risk-free interest rate be equal to 0.04. What is the time-2 break-even stock price for this investment?

Problem 3.10. (3 points) Bertram sells short 10 shares of a non-dividend-paying stock. The time-0 price of this stock is \$100. Assume that the continuously compounded, risk-free interest rate equals 0.06. If Bertram closes the short sale in six months, what is his break-even final stock price?

Problem 3.11. (5 points) Let the current price of a non-dividend-paying stock be \$40.

You model the distribution of the time-1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 42, & \text{with probability } 1/4, \\ 38, & \text{with probability } 1/2, \\ 36, & \text{with probability } 1/4. \end{cases}$$

The continuously compounded, risk-free interest rate is 0.04.

What is your expected profit under the above model, if you short sell one share of stock at time-0 and intend to close the short sale at time-1?

Problem 3.12. (5 points) The current price of a non-dividend paying stock is \$100 per share. You purchase one share of this stock. You do not intend to make any further trades over the next year. You intend to liquidate your investment at the end of the year.

You model the stock price at the end of the year to be distributed as follows:

$$S(T) \sim \begin{cases} 90 & \text{with probability } 1/10 \\ 100 & \text{with probability } 1/2 \\ 110 & \text{with probability } 2/5 \end{cases}$$

The continuously compounded, risk-free interest rate is 0.01.

What is the expected profit of your investment?