The False Discovery Rate

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- Instead, we can control the *false discovery rate*:

$$\text{FDR} = \text{E}(V/R).$$

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- She wants reassurance that this smaller set is really "promising", i.e. not too many falsely rejected H_0's.
- FWER controls Pr(at least one false rejection).
- FDR controls the fraction of candidates in the smaller set that are really false rejections. This is what she needs!

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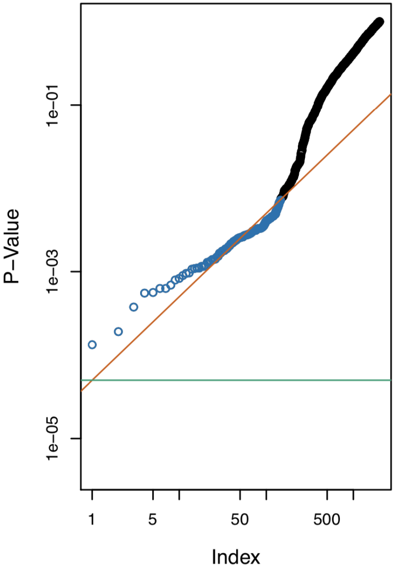
Then, $\mathrm{FDR} \leq q$.

A Comparison of FDR Versus FWER, Part 1



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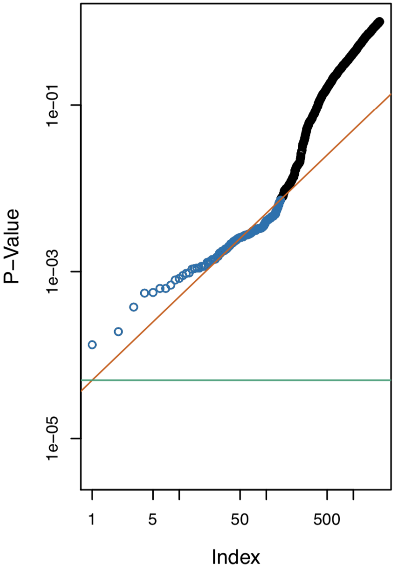

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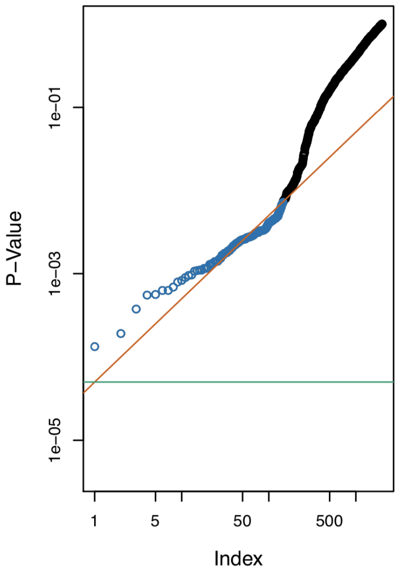

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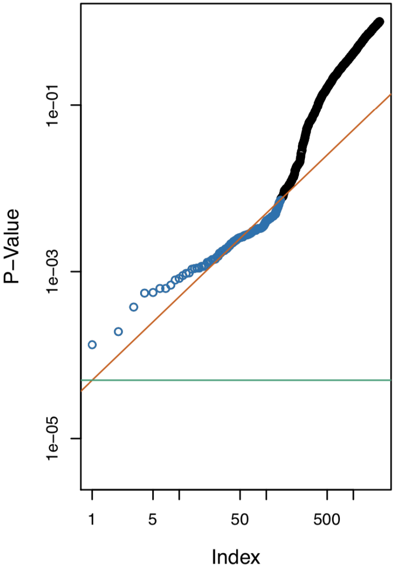

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- To control FDR at level $q = 0.05$ using Benjamini-Hochberg:
  - Notice that $p_{(1)} < 0.05/5$, $p_{(2)} < 2 \times 0.05/5$, $p_{(3)} > 3 \times 0.05/5$, $p_{(4)} > 4 \times 0.05/5$, and $p_{(5)} > 5 \times 0.05/5$.
  - So, we reject H_{01} and H_{03}.

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  - So, we reject H_{01} and H_{03}.
- To control FWER at level $\alpha = 0.05$ using Bonferroni:
  - We reject any null hypothesis for which the p-value is less than $0.05/5$.
  - So, we reject only H_{01}.