# M378K Introduction to Mathematical Statistics
## Problem Set #17
## Relative efficiency.

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**Definition 17.1.** *Given two unbiased estimators $\hat{\theta}_1$ and $\hat{\theta}_2$, the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$ is defined as*

$$\mathit{eff}(\hat{\theta}_1, \hat{\theta}_2) = \frac{\mathrm{Var}[\hat{\theta}_2]}{\mathrm{Var}[\hat{\theta}_1]}\,.$$

**Problem 17.1.** *Let $Y_1, Y_2$ be a random sample from the exponential distribution with the unknown parameter $\theta$.*

*(i) The estimator $\hat{\theta}_1 = (Y_1 + Y_2)/2$ for $\theta$ is proposed. What is its variance?*

*(ii) The estimator $\hat{\theta}_2 = cY_{(1)}$ for $\theta$ is proposed. Find the constant $c$ such that $\hat{\theta}_2$ is an unbiased estimator of $\theta$. What is its variance?*

*(iii) Calculate the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$.*