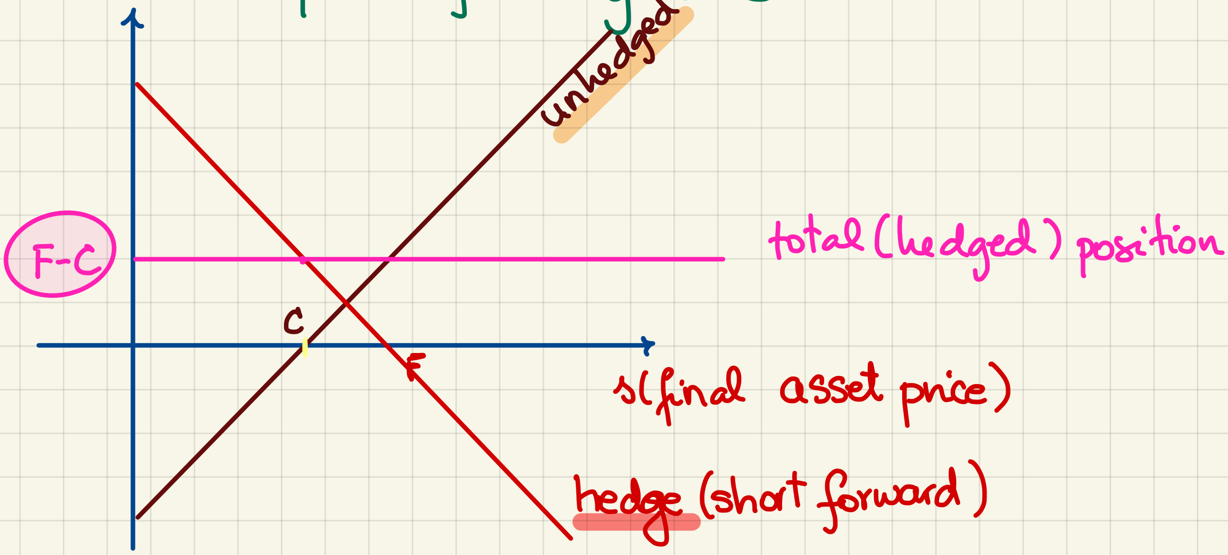
M339D: October 5th, 2022.

Hedging Using Forward Contracts.

Focus on a producer of goods.

C... total aggregate costs of production valued @ time $\cdot T$

$S(T)$... the market price of the good @ time $\cdot T$



Algebraically:

$$\text{Profit(Unhedged)} + \text{Profit(Hedge)} = \text{Profit(total hedged)}$$

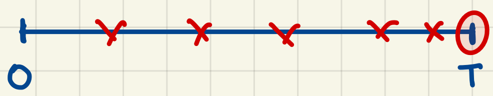
$$\cancel{S(T)} - C \quad + \quad F - \cancel{S(T)} \quad = \quad F - C$$

European Call Options.

The option can be EXERCISED, i.e., the cashflow can be collected only on the exercise date.

Usually, this means a right to buy the underlying asset.

Usually, the option's owner has the RIGHT, but NOT AN OBLIGATION to exercise the option.



Option written.   EXERCISE DATE

At time 0:

- The writer of the option writes/shorts the call.
- The buyer of the option is said to long the call. They are referred to as the option's owner.
- They agree on:
    - the underlying asset: $S(t), t \geq 0$
    - the exercise date T
    - K... the strike/exercise price
- The premium for the call is paid by the buyer to the writer.

At time T:

- The call's owner has a right, but not an obligation to buy one unit of the underlying asset for the strike price K.
- The call's writer is obligated to do what the owner opts for.

Payoff = ? We focus on the long call, i.e., the payoff for the call's owner.

The call owner's rationale for whether to exercise the call is based on "maximizing money in". The criterion for exercise is

$$\begin{cases} \text{IF } S(T) \geq K, \text{ then EXERCISE} & \Rightarrow \text{Payoff} = S(T) - K \\ \text{IF } S(T) < K, \text{ then do NOT exercise.} & \Rightarrow \text{Payoff} = 0 \end{cases}$$

We introduce:

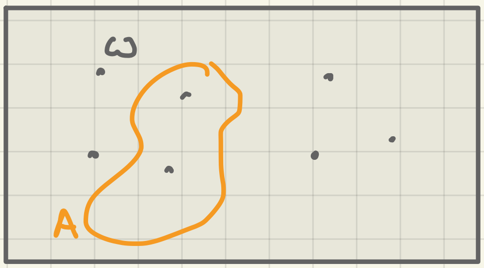
$V_c(T)$ ... the random variable which denotes the payoff of the long call

$\Rightarrow$

$$\boxed{V_c(T) = \max(S(T)-K, 0) = (S(T)-K) \vee 0}$$

$$\boxed{V_c(T) = \begin{cases} S(T)-K & \text{if } S(T) \geq K \\ 0 & \text{if } S(T) < K \end{cases}}$$

Indicator Random Variables:



ω ... elementary outcome

Ω (probability space)

"Any nice" subset of Ω is called an event.

We define:

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

$$\boxed{V_c(T) = (S(T)-K) \cdot \mathbb{I}_{[S(T) \geq K]}}$$

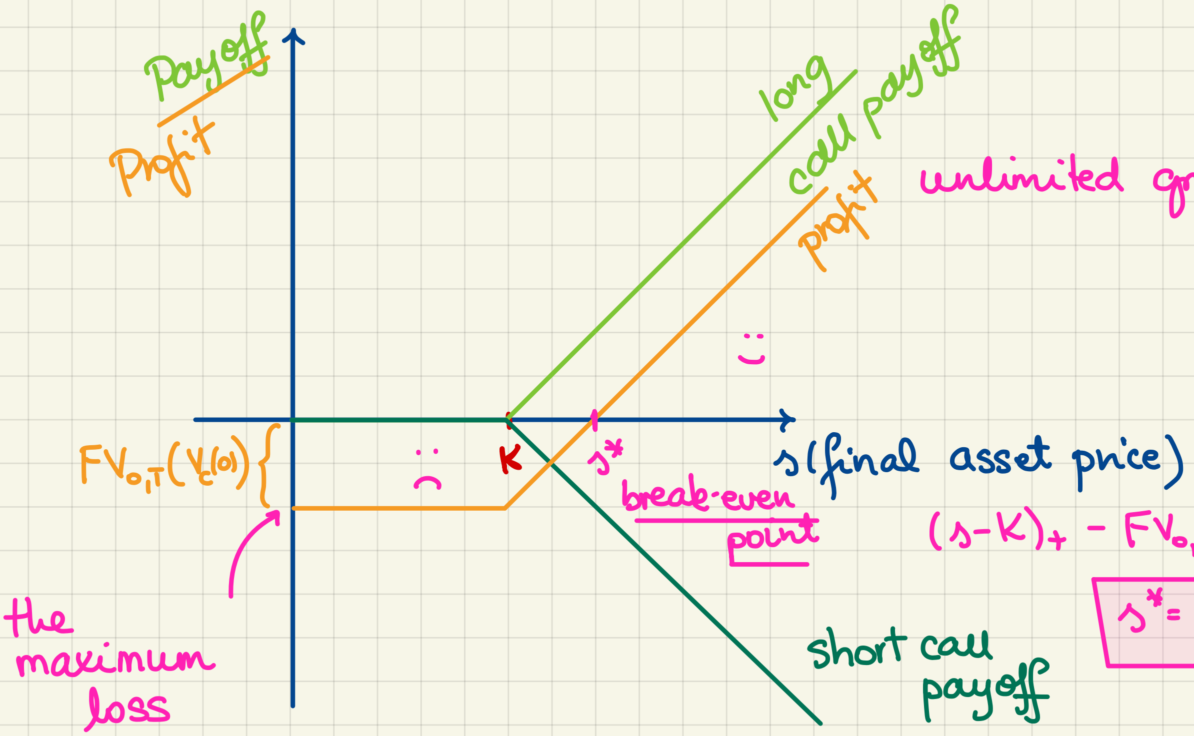
Introduce: the positive part function:

$$x \longmapsto (x)_+ =: \max(x, 0)$$

$$\boxed{V_c(T) = (S(T)-K)_+}$$

$\Rightarrow$ the payoff f'tion: $\boxed{v_c(s) = (s-K)_+}$

There must be an initial cost, i.e., the call premium $\boxed{V_c(0)}$



$$(s-K)_+ - FV_{0,T}(V_c(0)) = 0$$

$$\boxed{s^* = K + FV_{0,T}(V_c(0))}$$

<u>LONG</u> w.r.t. the underlying