

M378K Introduction to Mathematical Statistics

Homework assignment #5

Please, provide your **final answer only** to the following problems.

Problem 5.1. (5 points) Let $Z_1 \sim N(1, 1)$, $Z_2 \sim N(2, 2)$ and $Z_3 \sim N(3, 3)$ be independent random variables. The distribution of the random variable $W = Z_1 + \frac{1}{2}Z_2 + \frac{1}{3}Z_3$ is ...

- (a) $N(5/3, 7/6)$
- (b) $N(3, 3)$
- (c) $N(3, \sqrt{3})$
- (d) $N(3, \sqrt{5/3})$
- (e) None of the above

(Note: In our notation $N(\mu, \sigma)$ means normal with mean μ and standard deviation σ .)

Problem 5.2. (5 points) Let Y_1, \dots, Y_{100} be independent random variables with the Bernoulli $B(p)$ distribution, with $p = 0.2$. The best approximation to $\bar{Y} = \frac{1}{n}(Y_1 + \dots + Y_n)$ (among the offered answers) is

- (a) $N(0, 1)$
- (b) $N(100, 20)$
- (c) $N(0.2, 0.04)$
- (d) $N(20, 4)$
- (e) $N(20, 20)$

(Note: In our notation $N(\mu, \sigma)$ means normal with mean μ and standard deviation σ .)

Problem 5.3. (5 points) Use the uniqueness of moment-generating functions to give the distribution of a random variable Y with moment-generating function $m_Y(t) = (0.7e^t + 0.3)^3$.

- (a) $Y \sim b(3, 0.7)$
- (b) $Y \sim b(3, 0.3)$
- (c) $Y \sim B(0.7)$
- (d) $Y \sim P(0.7)$
- (e) None of the above.

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 5.4. (10 points) The moment generating function of a certain random variable Y is given to be equal to

$$m_Y(t) = (1 - 2500t)^{-4}.$$

Calculate the standard deviation of the random variable Y .

Problem 5.5. (10 points) Let Y be a geometric random variable with parameter p . What is its moment generating function m_Y ? Do not forget to explicitly state the domain of m_Y !

Problem 5.6. (15 points) Let $Y \sim E(\tau)$. Find the moment generating function on Y not forgetting to explicitly state the domain. Using the moment generating function, recalculate the mean and the variance of the random variable Y .