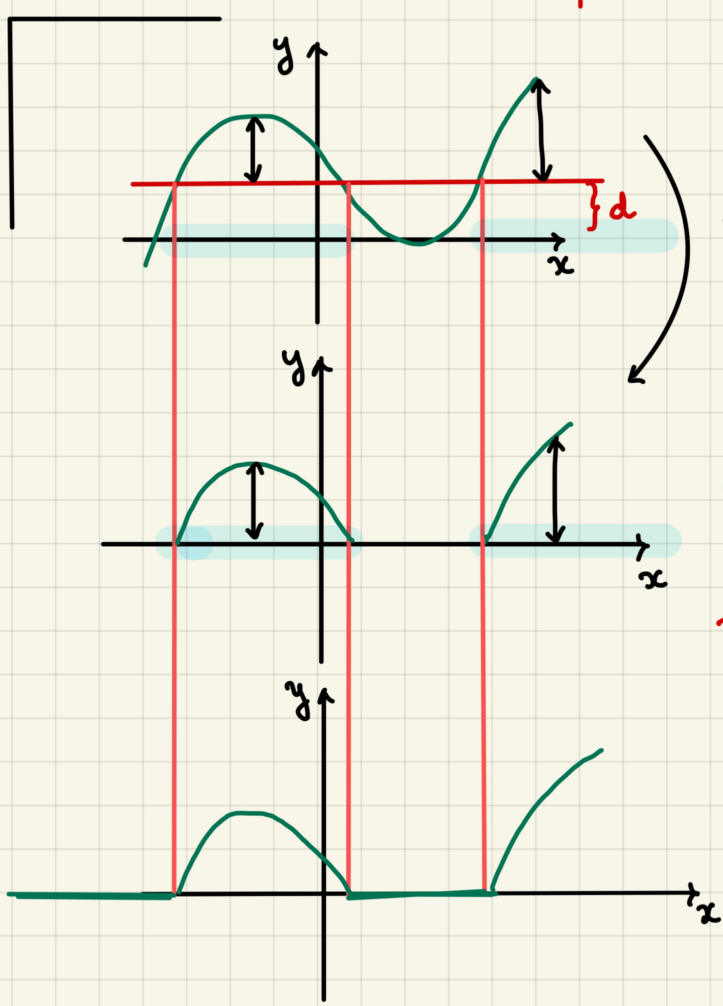
M339J: February 10th, 2021.

Review:

- X ... (ground-up) loss; severity
- d ... deductible: the policyholder is responsible for the losses below d and the insurer is responsible for the losses in excess of the deductible (this is the ordinary case)
- per payment r.v.

$$Y^P := X - d \mid X > d$$



I've subracted d from the whole curve.

$\longleftrightarrow Y^P$

The domains are different!

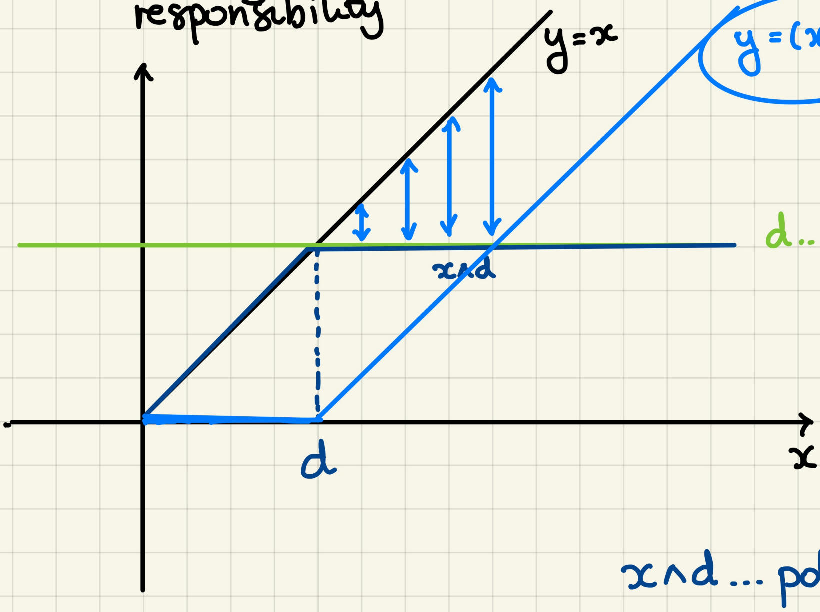
$\longleftrightarrow Y^L$

- per loss random variable:

$$Y^L = (X-d)_+ = (X-d)\cdot \mathbb{I}_{[X>d]}$$

The limited loss (random) variable.

$$\underbrace{X \wedge d}_{\text{the policyholder's responsibility}} := \min(X, d) = \begin{cases} X & \text{if } X < d \\ d & \text{if } d \leq X \end{cases}$$



"hockey-stick" functions

d... deductible

$x \wedge d$ ... policyholder responsibility

$(x-d)_+$ ... the insurer's respons.

Note: $x \wedge d + (x-d)_+ = x$

$\Rightarrow$ For X being our severity r.v., we have

$$X \wedge d + (X-d)_+ = X$$

Now, apply the expectation on both sides:

$$\mathbb{E}[X \wedge d] + \mathbb{E}[(X-d)_+] = \mathbb{E}[X]$$

$$\Rightarrow \underbrace{\mathbb{E}[(X-d)_+]}_{\mathbb{E}[Y^L]} = \mathbb{E}[X] - \mathbb{E}[X \wedge d]$$

$$\mathbb{E}[(X-d)_+] \overset{?}{=} \boxed{\mathbb{E}[X-d \mid X>d]} \cdot \mathbb{P}[X>d]$$

$$\mathbb{E}[X-d \mid X>d] \underset{\text{by def'n}}{=} \frac{\mathbb{E}[(X-d) \cdot \mathbb{I}_{[X>d]}]}{\mathbb{P}[X>d]} = \frac{\mathbb{E}[(X-d)_+]}{\mathbb{P}[X>d]}$$

$/ \cdot \mathbb{P}[X>d]$

$$\mathbb{E}[\underbrace{X-d \mid X>d}_{= Y^P}] \cdot \mathbb{P}[X>d] = \mathbb{E}[(X-d)_+]$$

$$\Rightarrow \boxed{\underbrace{\mathbb{E}[Y^P]}_{= e_X(d)} = \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge d]}{\underbrace{\mathbb{P}[X>d]}_{S_X(d)}}}$$

mean excess loss function

100. The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution:

$$F(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \quad x \geq 0$$

The insurance policy pays amounts up to a limit of 1000 per claim.

Calculate the expected payment under this policy for one claim.

(A) 57

(B) 108

(C) 166

(D) 205

(E) 240

101. The random variable for a loss, X, has the following characteristics:

x	$F(x)$	$E(X \wedge x)$
0	0.0	0
100	0.2	91
200	0.6	153
1000	1.0	331

cdf

Calculate the mean excess loss for a deductible of 100.

$$e_X(100) = \mathbb{E}[X - 100 \mid X > 100]$$

$$= \frac{\mathbb{E}[X] - \mathbb{E}[X \wedge 100]}{1 - F_X(100)} = ?$$

(A) 250

(B) 300

(C) 350

(D) 400

(E) 450

The maximum value that X can take is 1000 since we're given $F_X(1000) = 1$.

$$\Rightarrow \mathbb{E}[X] = \mathbb{E}[X \wedge 1000]$$

$$\Rightarrow e_X(100) = \frac{331 - 91}{1 - 0.2} = \frac{240}{0.8} = 300$$