M339D: March 24th, 2021.

Derivative Securities.

... have the value which is contingent on the value of another traded asset.

Examples.

- Forward contract / Futures contract

  @ time·0 : the two parties agree on, most importantly, the delivery date T and the forward price F

  $\Rightarrow$ the payoff of the long forward:

  $$\boxed{S(T) - F}$$

- European call options:

  T... expiration date ; K... strike price

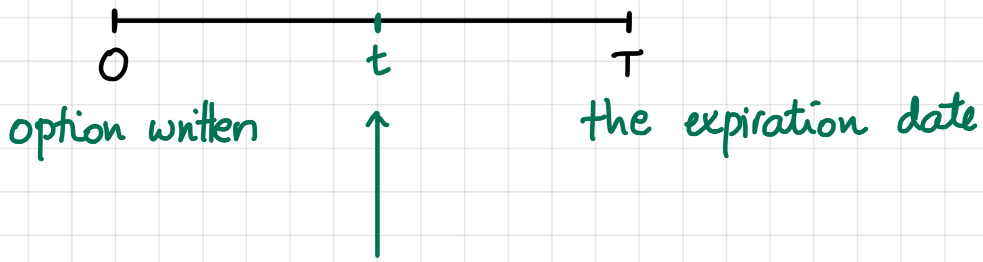
  $\Rightarrow$ the payoff of the long call:

  $$\boxed{V_C(T) = (S(T) - K)_+}$$

- European put options:

  the payoff of the long put:

  $$\boxed{V_P(T) = (K - S(T))_+}$$



We can observe the stock price $S(t)$, $t \geq 0$ during the life of the option.

$$\underline{\{S(t),\ 0 \leq t \leq T\}} \longmapsto \text{PAYOFF}$$

<u>Example.</u>

- Base your payoff on, say, the average behavior

e.g.,
$$\left(\frac{1}{n}\sum_{k=1}^{n} S(t_k) - K\right)_+$$

$\Rightarrow$ this is a type of an Asian option.

- <u>Introduce</u>:
  - the minimum observed stock price:
  $$m(T) := \min_{0 \leq t \leq T} S(t)$$
  - the maximum observed stock price:
  $$M(T) := \max_{0 \leq t \leq T} S(t)$$

e.g., an option which pays a "reward" if the minimum is below a certain threshold or if the maximum rises above a threshold

$\Rightarrow$ these are called rebate options e.g., the payoff is constructed as:

$$M(T) - S(T)$$

or

$$S(T) - m(T)$$

$\Rightarrow$ these are called **Lookback options**

Dynamic Portfolios.

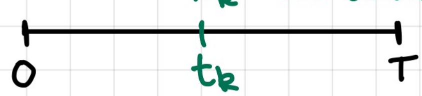
C_k ... intermediate cashflows $\begin{cases} >0 & \text{inflow} \\ <0 & \text{outflow} \end{cases}$

$$\text{PROFIT} := \text{Payoff} - FV_{0,T}(\text{Initial Cost}) + \sum_k FV_{t_k,T}(C_k)$$

↑ our generalized profit

Example. [DISCRETE DIVIDEND PAYING STOCKS]

D_k ... dividend amounts



$k = 1..n$, $t_n \leq T$

Consider an outright purchase of this stock:

At time 0: buy 1 share, i.e., spend $S(0)$

At time t_k: get D_k $\quad k = 1..n$

At time T: own one share : worth $S(T)$

$$\text{Profit} = S(T) - FV_{0,T}(S(0)) + \sum_{k=1}^{n} FV_{t_k,T}(D_k)$$

$$= S(T) - S(0)e^{rT} + \sum_{k=1}^{n} D_k e^{r(T-t_k)} \quad ∎$$

↑ r ccrfir