

UNIVERSITY OF TEXAS AT AUSTIN

Problem set 6

Problem 6.1. Let the function f be given by

$$f(x) = \begin{cases} 3 - x & \text{for } x \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

Draw the graph of the function g defined as

$$g(x) = f(x) + 1.$$

Problem 6.2. Let the function f be given by

$$f(x) = \begin{cases} 3 - x & \text{for } x \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

Draw the graph of the function g defined as

$$g(x) = f(x) + 2x.$$

Problem 6.3. Let the function f be given by

$$f(x) = \begin{cases} x - 5 & \text{for } x \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

Draw the graph of the function g defined as

$$g(x) = f(x)/2.$$

Problem 6.4. (2 pts)

We define the minimum of two values in the usual way, i.e.,

$$\min(x, y) = \begin{cases} x & \text{if } x \leq y \\ y & \text{if } x \geq y \end{cases}$$

We define the maximum of two values in the usual way, i.e.,

$$\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{if } x \leq y \end{cases}$$

Then, for every x and y we have that

$$\min(x, y) + \min(x - y, 0) = y$$

True or false? Why?

Problem 6.5. (2 pts)

We define the minimum of two values in the usual way, i.e.,

$$\min(x, y) = \begin{cases} x & \text{if } x \leq y \\ y & \text{if } x \geq y \end{cases}$$

We define the maximum of two values in the usual way, i.e.,

$$\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{if } x \leq y \end{cases}$$

Then, for every x and y we have that

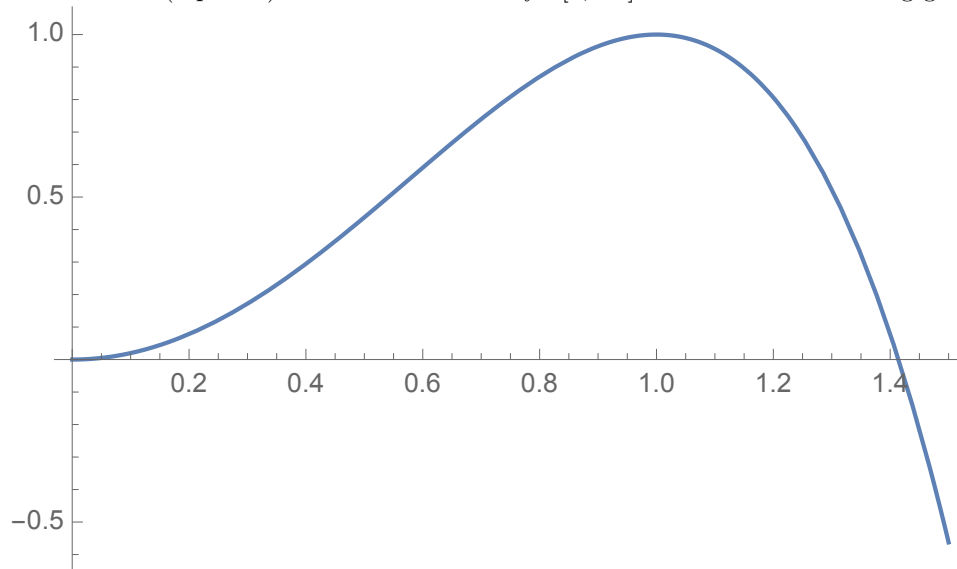
$$\max(x, y) + \min(x, y) = x + y.$$

True or false?

Problem 6.6. (5 pts) Which of the following formulas hold for the exponential function:

- (a) $e^x + e^y = e^{x+y}$
- (b) $e^x e^y = e^x + e^y$
- (c) $e^{x+y} = e^x e^y$
- (d) $e^{x-y} = e^x - e^y$
- (e) none of the above

Problem 6.7. (5 points) Consider a function $f : [0, 1.5] \rightarrow \mathbb{R}$ with the following graph



Then, this function is ...

- (a) ...increasing.
- (b) ...decreasing.
- (c) ...both increasing and decreasing
- (d) ...neither increasing, nor decreasing.
- (e) None of the above.

Problem 6.8. If events E and F are independent, then E^c and F^c are independent as well. *True or false?*

Problem 6.9. (5 pts) Let Y be a random variable such that $\mathbb{P}[Y = 2] = 1/2$, $\mathbb{P}[Y = 3] = 1/3$ and $\mathbb{P}[Y = 6] = 1/6$. Then $\mathbb{E}[Y^2] = \dots$

- (a) 1
- (b) 2
- (c) 3
- (d) 11
- (e) None of the above.

Problem 6.10. Four fair coins are tossed. The probability that at least one of them fell on *heads* is

- (a) $\frac{15}{16}$
- (b) $\frac{1}{16}$
- (c) $\frac{1}{4}$
- (d) $\frac{3}{4}$
- (e) None of the above.

Problem 6.11. Find the total amount of interest that would be paid on a \$1,000 loan over a 10-year period, if the effective interest rate is 0.09 per annum under the following two repayment method:

The entire loan plus entire accumulated interest is paid as one lump-sum at the end of the loan term.

- (a) \$900
- (b) \$990
- (c) \$1,367
- (d) \$1,557
- (e) None of the above

Problem 6.12. (5 pts) *Source: Sample FM Problem #26.*

A 5-year loan for 10,000 is charged a nominal interest rate of 12% compounded semiannually.

The loan is to be repaid so that interest is repaid at the end of every 6 month period as it accrues and the principal is repaid in total at the end of the 5 years.

Denote the total amount of interest paid on this loan by I . Then,

- (a) $I \approx 2,750$
- (b) $I \approx 3,000$
- (c) $I \approx 3,250$
- (d) $I \approx 3,500$
- (e) None of the above

Problem 6.13. Roger deposits opens a savings account at time=0. He does not make any subsequent withdrawals or deposits. The account earns at a continuously compounded, risk-free interest rate r .

After 15 years and 3 months, the balance in his account has doubled. Then,

- (a) $0 \leq r < 0.0150$
- (b) $0.0150 \leq r < 0.0250$
- (c) $0.0250 \leq r < 0.0550$
- (d) $0.0550 \leq r < 0.0650$
- (e) None of the above