

UNIVERSITY OF TEXAS AT AUSTIN

Quiz #1

Prerequisite material.

Provide your complete solution to the following problems:

Problem 1.1. (5 points) Emmanuel entered an extra special kind of game with his friend Fischer. First, they toss a fair coin. If the coin comes up heads, Emmanuel gives \$5,000 to Fischer. If the coin comes up tails, Fischer gives \$2,000 to Emmanuel. Then, regardless of the outcome of the first cointoss, they toss the same fair coin again. If it comes up heads, Emmanuel gives Fischer \$4,000. If the coin comes up tails, Fischer gives \$3,000 to Emmanuel. What is the expected cashflow, i.e., what is the expected amount of money that changes hands and who gives it to whom?

Solution: Let X_i be the cashflow from Emmanuel's perspective after the i^{th} cointoss for $i = 1, 2$. Then, we are looking for

$$\begin{aligned}\mathbb{E}[X_1 + X_2] &= \mathbb{E}[X_1] + \mathbb{E}[X_2] \\ &= \frac{1}{2}(-5,000 + 2,000) + \frac{1}{2}(-4,000 + 3,000) = -2,000.\end{aligned}$$

So, the expected cashflow is \$2,000 from Emmanuel to Fischer.

Problem 1.2. (10 points) Harry plays a simple lottery in which the winnings are distributed as follows:

- \$0 with probability 0.1,
 - \$10 with probability 0.3,
 - \$20 with probability 0.6.
- (i) (3 points) What is the expected value of the amount Harry wins?
- (ii) (7 points) Unfortunately, there is a catch to the lottery Harry plays. It turns out that Harry has to pay a fee to collect his winnings. If the actual amount he wins is smaller than \$15, then the fee is defined to equal the amount that Harry won – thus, he walks away with nothing. If the actual amount he wins is larger than \$15, then he pays the \$15-fee and pockets the remainder. What is the expected value of the net amount Harry collects?

Solution:

(i)

$$10 \cdot 0.3 + 20 \cdot 0.6 = 3 + 12 = 15.$$

(ii)

$$5 \cdot 0.6 = 3$$