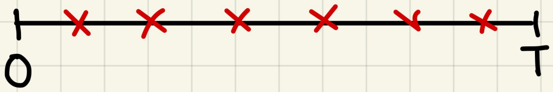
M339D : February 8th, 2021.

Review.

Static Portfolios.



Initial Cost $\qquad\qquad$ Payoff

$$\boxed{\text{PROFIT} := \text{PAYOFF} - FV_{0,T}(\text{INITIAL COST})}$$

$\uparrow$ calculated using the interest rate r

Note: For any portfolio whose payoff depends on the final asset price, we introduce

s ... independent argument which denotes the final asset price (we can understand it as a placeholder for $S(T)$)

$\Rightarrow$ We define the payoff function which describes the dependence of the investor's payoff on the final asset price s.
When we draw the graph of this function, we get the payoff curve.

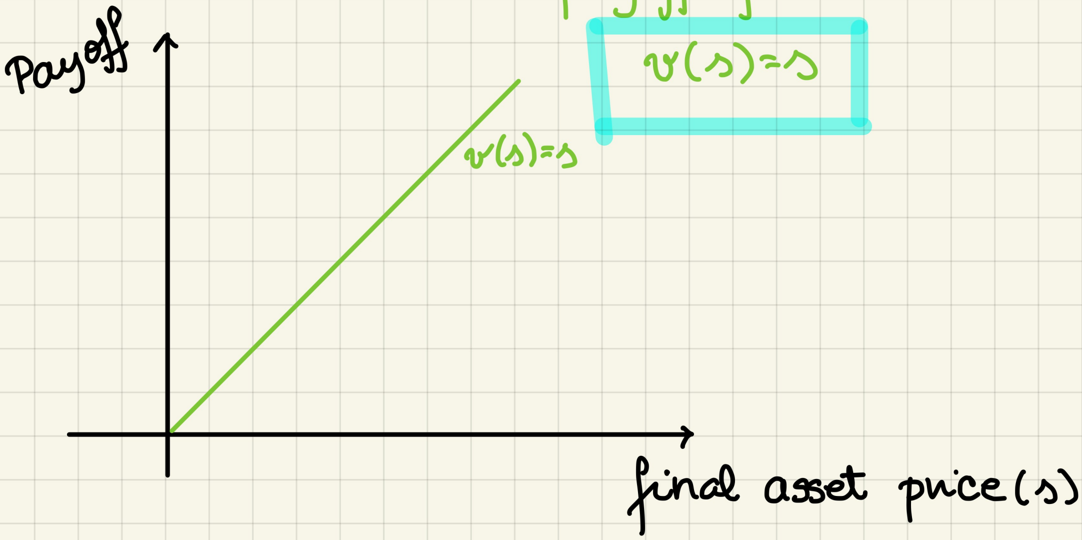
$$s \longmapsto v(s)$$

Example. [Outright purchase of a non-dividend paying stock (again)].

Initial cost: $S(0)$

Payoff: $S(T)$

$\Longrightarrow$ the payoff f'tion is: $\boxed{v(s) = s}$



By def'n: Profit = Payoff $- FV_{0,T}$(Init. Cost)

$$= S(T) - \underbrace{S(0)e^{r \cdot T}}_{\text{constant}}$$

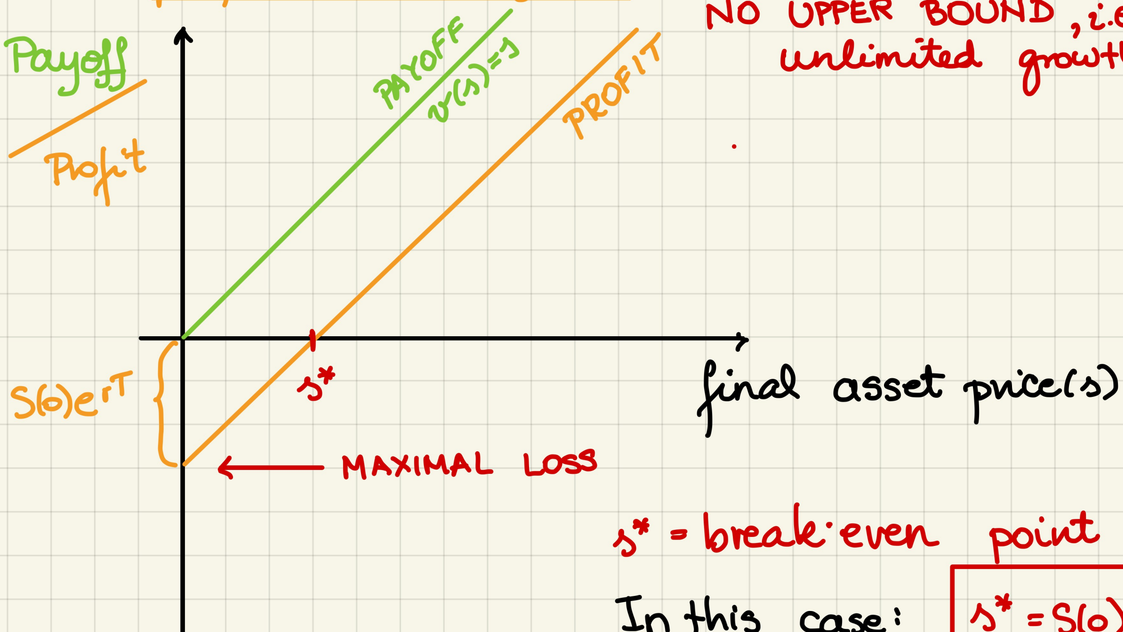
Introduce the profit function: $\boxed{v(s) - FV_{0,T}(\text{Init. Cost})}$

In this example:

$$v(s) - FV_{0,T}(S(0)) = s - S(0)e^{r \cdot T}$$

$\Longrightarrow$ The profit curve/diagram

NO UPPER BOUND, i.e., unlimited growth potential



s^* = break-even point

In this case: $\boxed{s^* = S(0)e^{rT}}$

Note that the payoff/profit curves are increasing.

Terminology. If the payoff/profit curve is increasing (not necessarily strictly) as a function of the final asset price (s), we say that the portfolio is **long with respect to the underlying asset.**

Example. [OUTRIGHT PURCHASE OF ONE SHARE OF CONTINUOUS-DIVIDEND-PAYING STOCK]

δ... dividend yield

$\longrightarrow$: Initial Cost: $S(0)$

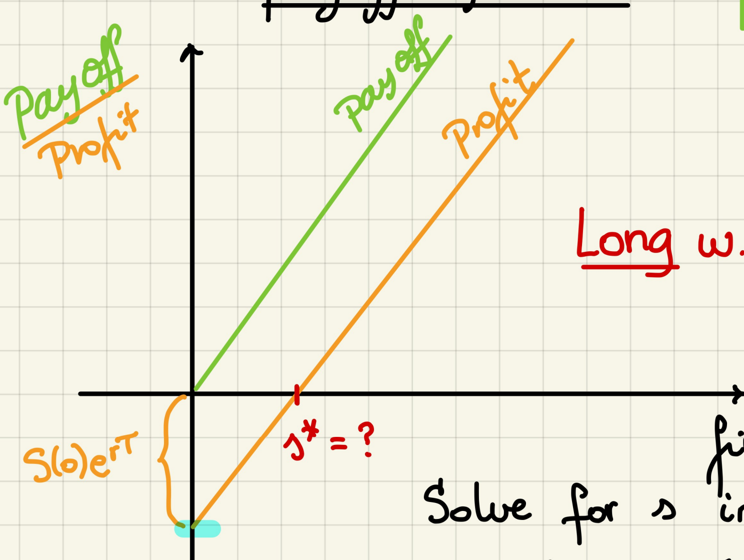
Payoff: $\boxed{e^{\delta \cdot T} \cdot S(T)}$

0 ———————— T

Bought 1 share. Own $e^{\delta \cdot T}$ shares.

$\Rightarrow$ The payoff f'tion is: $\boxed{v(s) = s \cdot e^{\delta \cdot T}}$

$\Rightarrow$ Profit function: $\boxed{s \cdot e^{\delta \cdot T} - S(0)e^{rT}}$



final asset price (s)

Long w.r.t. the underlying

Solve for s in:

$$s e^{\delta \cdot T} - S(0)e^{rT} = 0$$

$$s e^{\delta \cdot T} = S(0)e^{rT} \quad / : e^{\delta T}$$

$$\boxed{s^* = S(0)e^{(r-\delta) \cdot T}}$$

Try to remember this expression :)