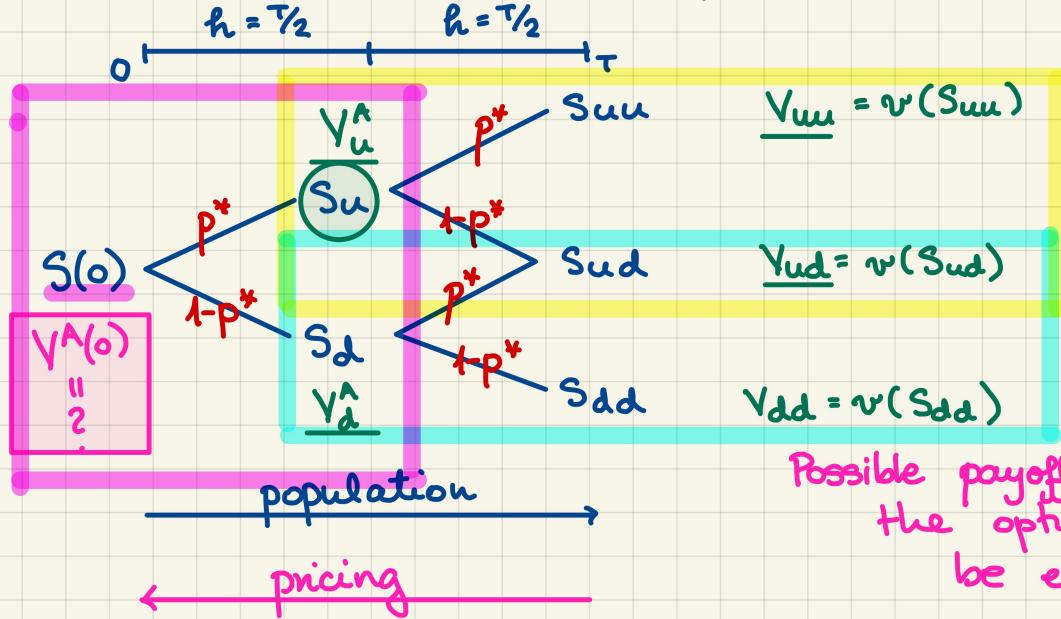


M339W: January 31st, 2022.

Binomial Pricing of American Options.



Consider an American option w/ payoff f'ction $v(\cdot)$.

up node: • I_{E_u} ... value of immediate exercise

• CV_u ... continuation value

If we don't exercise now,
then the option "becomes" a European option
(since there are no early exercise opportunities
left in the tree)

$$\Rightarrow CV_u = e^{-rh} (p^* \cdot V_{uu} + (1-p^*) \cdot V_{ud})$$

$$\Rightarrow V_u^A = \max(I_{E_u}, CV_u)$$

and the option's owner decides whether to
exercise early accordingly

down node:

$$\begin{cases} \cdot I_{E_d} \\ \cdot CV_d = e^{-rh} (p^* \cdot V_{ud} + (1-p^*) \cdot V_{dd}) \end{cases}$$

$$V_d^A = \max(I_{E_d}, CV_d)$$

ROOT node:

$$\left\{ \begin{array}{l} \cdot IE_0 \\ \cdot CV_0 = e^{-rh} \left(p^* \cdot V_u^A + (1-p^*) \cdot V_d^A \right) \end{array} \right.$$

$$V^A(0) = \max (IE_0, CV_0)$$

- Note:
- We can *dynamically* replicate American options until we reach the nodes where early exercise is optimal.
 - The procedure is analogous for multiperiod trees.

Problem 3.7. The current stock price is observed to be \$100 per share. The stock is projected to pay dividends continuously at the rate proportional to its price with the dividend yield of 0.03. The stock's volatility is given to be 0.23. You model the evolution of the stock price using a two-period forward binomial tree with each period of length one year. $\rho=1$

The continuously compounded risk-free interest rate is given to be 0.04. ✓

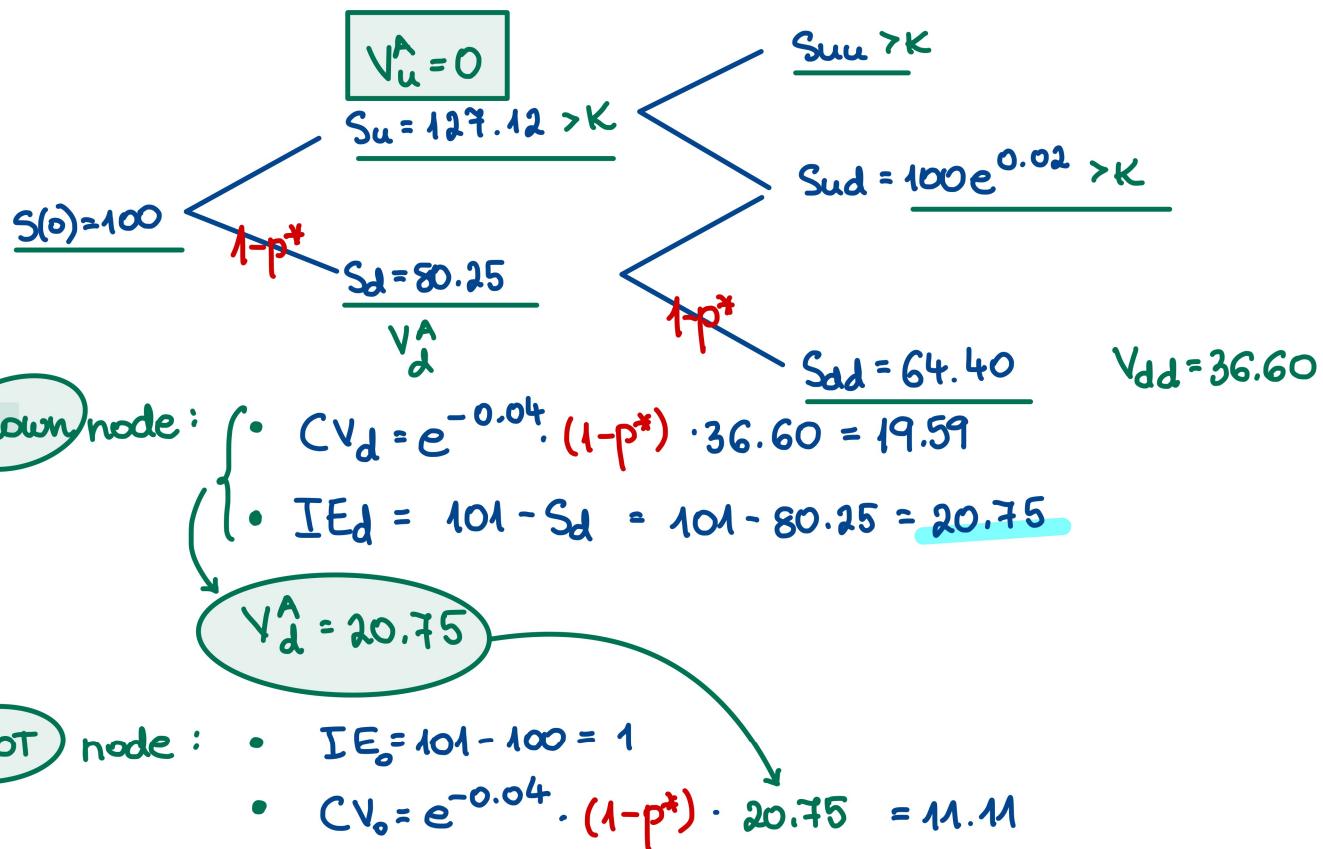
What is the price of a two-year, \$101-strike American put option on the above stock consistent with the above stock-price tree?

- About \$6.62
- About \$8.34
- About \$8.83
- About \$11.11
- None of the above.

$$\rightarrow: p^* = \frac{1}{1+e^{0.04}} = \frac{1}{1+e^{0.23}} = 0.4428$$

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{0.01 + 0.23} = e^{0.24} = 1.2712 \quad \checkmark$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{0.01 - 0.23} = e^{-0.22} = 0.8025 \quad \checkmark$$



Real Options.

A **real option** is a right to make a business decision or to make an investment.

Example.

- Jasper Fforde: options on whether members of a royal family marry a member of another royal family
- options to make a film or a series based on a book.
- renewing a TV show for a season
- land: parking lot downtown: to build or not to build?