UNIVERSITY OF TEXAS AT AUSTIN

Extra Credit Homework Assignment 0

Singular value decomposition.

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 0.1. (10 points) How many *eigenvalues* does an $n \times n$ matrix with real values have? *Caveat: The question is not about* **distinct** *eigenvalues!* Substantiate your answer!

Problem 0.2. (10 points) What are the similarities and differences between the *eigendecomposition* and the *singular value decomposition*?

Problem 0.3. (5 points) What is the geometric interpretation of the action of an orthonormal matrix?

Problem 0.4. (5 points) What is the geometric interpretation of the action of a diagonal matrix?

Problem 0.5. (20 points) Figure out the singular value decomposition of the following matrix

$$A = \begin{bmatrix} 15 & 30 \\ 22 & 4 \\ 4 & 28 \end{bmatrix}$$

using the following steps:

1. (5 points) Calculate $A^T A$. What useful properties does $A^T A$ have?
2. (5 points) Find the *eigenvalues* of $A^T A$. What is Σ, in the usual SVD notation?
3. (5 points) Find the *eigenvectors* of $A^T A$. What is V in the usual SVD notation?
4. (5 points) Construct the matrix denoted by U in the usual SVD notation.