

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #4

Forward contracts.

**Problem 4.1.** (5 points) A soy-bean farmer shorts forward contracts on soy in an amount matching his crop volume and with delivery at harvest time. Then, he is considered:

- (a) an arbitrageur.
- (b) a broker.
- (c) a speculator.
- (d) a hedger.
- (e) None of the above.

**Problem 4.2.** Derivative securities can reduce the risk of both the buyer and the writer of the security. *True or false?*

**Problem 4.3.** A short forward contract has an unlimited loss potential. *True or false?*

**Problem 4.4.** A farmer produces one million bushels of corn. The total cost of production is \$1.3 million. The farmer entered a forward contract to hedge at a forward price of \$2.50 per bushel on one million bushels. What is the farmer's profit?

$$2.5 \cdot 10^6 - 1.3 \cdot 10^6 = 1.2 \cdot 10^6$$

**Problem 4.5.** Assume that farmer Brown is uncertain about his crop yield. Based on past experience, he thinks the following is a good model:

- 100,000 bushels with probability 1/4;
- 80,000 bushels with probability 3/4.

How many forward contracts do you think farmer Brown should short to hedge against fluctuations in corn prices at harvest time? Explain your way of thinking ...

**Problem 4.6.** Pancakes, Inc. produces chocolate chip pancakes. It longed a forward contract on 100 lbs of chocolate chips at \$3.00 per pound. Total fixed revenue is \$2,000 for the pancakes produced with the above chocolate chips. Other costs total \$1200. Find the company's profit.

- (a) 2,000
- (b) 1,700
- (c) 800
- (d) 500
- (e) None of the above.

$$2000 - 1200 - 3 \cdot 100 = 500$$

**Problem 4.7.** The Extra-Healty Cereal (EHC) company longed 20,000 forward contracts on corn at \$2.80 per bushel. The revenue from cereal made with the above corn is \$200,000 while the other (non-corn) aggregate fixed and variable costs amount to \$120,000. What is the EHC's profit?

$$200000 - 20000 \cdot 2.80 - 120000 = 24000$$



## European

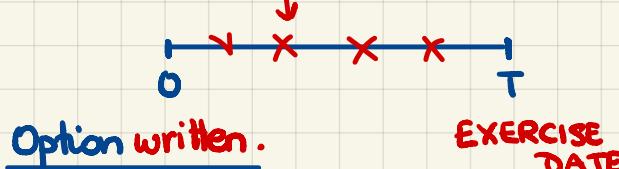
The option can be EXERCISED, i.e., the cashflow can be collected only on the exercise date.

## Call

Usually, this means a right to buy the underlying asset.

## Options.

Usually, the option's owner has the right, but not an obligation to exercise the option.



At time  $t=0$ : • The writer of option writes/shorts the call.

- The buyer of the option is said to long the call. They are referred to as the option's owner.
- They agree on:
  - the underlying asset:  $S(t), t \geq 0$
  - the exercise date  $T$
  - $K$  ... the strike/exercise price
- The buyer pays the premium  $V_c(0)$  to the writer.

At time  $t=T$ : • The call's owner has a right, but not an obligation to buy one unit of the underlying asset for the strike price  $K$ .  
• The call's writer is obligated to do what the owner opts for.

## Payoff = ?

We focus on the long call, i.e., the payoff for the call's owner.  
The call owner's rationale for whether to exercise the call is based on "maximizing money in".

The criterion for exercise is

IF  $S(T) \geq K$ , then EXERCISE.  $\Rightarrow$  Payoff =  $S(T) - K$

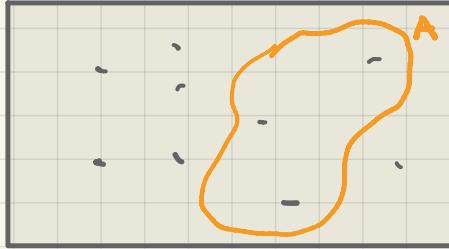
IF  $S(T) < K$ , then do NOT exercise.  $\Rightarrow$  Payoff = 0

We introduce:

$V_C(T)$  ... the random variable denoting the payoff of the long call

$$\Rightarrow V_C(T) = \begin{cases} S(T) - K & \text{if } S(T) \geq K \\ 0 & \text{if } S(T) < K \end{cases}$$

Indicator Random Variables:



$\omega$  ... elementary outcome

$\Omega$  (probability space)

Any "nice" subset of  $\Omega$  is called an EVENT.

We define:  $\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

$$V_C(T) = (S(T) - K) \cdot \mathbb{I}_{[S(T) \geq K]}$$

Also:

$$V_C(T) = (S(T) - K) \vee 0 = \max(S(T) - K, 0)$$

Introduce: the positive part function:

$$x \mapsto (x)_+ =: \max(x, 0)$$

$$\Rightarrow v_c(T) = (S(T) - K)_+$$

$$\Rightarrow \text{the payoff function: } v_c(s) = (s - K)_+$$