University of Texas at Austin

Quiz #17

Delta-gamma-theta approximation. Market making and delta hedging.

Problem 17.1. (5 points) Let the stock price $S = \{S(t); \geq 0\}$ satisfy the assumptions of the Black-Scholes model.

Consider a European put option on S whose current price is \$2.50. You are given that the current put delta equals -0.60, its gamma is 0.08, and its theta is -0.02 per day.

Assume that the continuously compounded risk-free interest rate is 0.06 per annum.

What is the delta-gamma-theta approximation for the put premium after three days if the stock price increases by \$2?

Solution: Let us denote the three-day time span by dt and let $ds = S(dt) - S(0)$. By the delta-gamma-theta approximation, we get

$$\begin{aligned} v_P(S(dt), dt) &= v_P(S(0), 0) + \Delta_P(S(0), 0)ds + \frac{1}{2}\Gamma_P(S(0), 0)(ds)^2 + \Theta_P(S(0), 0)dt \\ &= 2.50 + (-0.6)(2) + \frac{1}{2}(0.08)(2)^2 + (-0.02)(3) = 1.40 \end{aligned}$$

Problem 17.2. (10 points) Consider a non-dividend-paying stock whose current price is \$100. A market-maker writes a one-year call option on this stock and sells it for \$4.00. He then proceeds to delta-hedge his commitment by trading in the shares of the underlying stock.

The call option's delta is 0.75, its gamma is 0.08 and its theta is -0.02 per day.

The continuously compounded, risk-free interest rate is 0.04.

The stock price has risen to \$101 after one day. Use the delta-gamma-theta approximation to find the market maker's profit after one day.

Solution: The initial cost of the market-maker's portfolio is

$$-4 + 0.75(100) = -4 + 75 = 71.$$

After one day, by the delta-gamma-theta approximation, the call price is approximately

$$4 + 0.75(1) + \frac{1}{2}(0.08)(1)^2 - 0.02 = 4.77.$$

So, the market-maker's payoff is

$$-4.77 + 0.75(101) = 70.98.$$

Finally, the market-maker's profit is

$$70.98 - 71e^{0.04/365} = -0.0278.$$