Focus on the Delta.

value f'tion: $v(s,t,r,\sigma)$

Def'n. The Delta:

$$\boxed{\Delta(s,t,r,\sigma) := \frac{\partial}{\partial s} v(s,t,r,\sigma)}$$

Example. [Outright Purchase of a Non-Dividend-Paying Stock]

The value f'tion: $v(s,t,r,\sigma) = s$

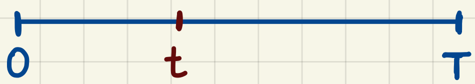
stands for the time-t stock price

$$\Rightarrow \quad \boxed{\Delta(s,t,r,\sigma) = 1}$$

Example. [European Call]

K... strike price

T... exercise date



Black-Scholes.

$$v_c(s,t,r,\sigma) = s \cdot N(d_1(s,t,r,\sigma)) - Ke^{-r(T-t)} \cdot N(d_2(s,t,r,\sigma))$$

valuation date

w/

$$d_1(\dots) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$

and

$$d_2(\dots) = d_1(\dots) - \sigma\sqrt{T-t}$$

By def'n of Delta: $\Delta_c(\dots) = \frac{\partial}{\partial s} v_c(\dots)$

After the chain rule & the product rule:

$$\boxed{\Delta_c(s,t,\sigma,r) = N(d_1(s,t,r,\sigma))} > 0$$

The positivity makes sense since the call is long w.r.t. the underlying.

Example. [European Put]

$$v_p(s,t,r,\sigma) = Ke^{-r(T-t)} \cdot N(-d_2(s,t,r,\sigma)) - s \cdot N(-d_1(s,t,r,\sigma))$$

Put-Call Parity.

$$v_c(\cdots) - v_p(\cdots) = s - Ke^{-r(T-t)}$$

$$\frac{\partial}{\partial s}\Big|$$

$$\Delta_c(\cdots) - \Delta_p(\cdots) = 1$$

$$\Delta_p(\cdots) = -1 + \Delta_c(\cdots) = -1 + N(d_1(\cdots)) =$$

$$= -\big(\underbrace{1 - N(d_1(\cdots))}_{N(-d_1(\cdots))}\big)$$

$$\boxed{\Delta_p(s,t,r,\sigma) = -N(-d_1(s,t,r,\sigma))} < 0$$

Makes sense that it's negative since puts are short w.r.t. the underlying.

(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

$T = \frac{1}{4}$ $K = 41.5$ European

8. You are considering the purchase of a 3-month 41.5-strike ~~American~~ call option on a nondividend-paying stock.

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40. $S(0) = 40$

(iii) The stock's volatility is 30%. $\sigma = 0.30$

(iv) The current call option delta is 0.5. $\Delta_c(S(0), 0) = 0.5$

$\overset{\shortparallel}{N(d_1(S(0),0))}$

Determine the current price of the option. $v_c(S(0),0) = ?$

(A) $20 - 20.453 \int_{-\infty}^{0.15} e^{-x^2/2} dx$

(B) $20 - 16.138 \int_{-\infty}^{0.15} e^{-x^2/2} dx$

(C) $20 - 40.453 \int_{-\infty}^{0.15} e^{-x^2/2} dx$

(D) $16.138 \int_{-\infty}^{0.15} e^{-x^2/2} dx - 20.453$

(E) $40.453 \int_{-\infty}^{0.15} e^{-x^2/2} dx - 20.453$

$$v_c(S(0),0) = S(0) \cdot \boxed{N(d_1(S(0),0))} - Ke^{-rT} \cdot N(d_2(S(0),0))$$

$$N(d_1(S(0),0)) = 0.5$$

$$\Rightarrow \boxed{d_1(S(0),0) = 0} \Rightarrow \cdots \quad r = ?$$