11) You are given the following information about a portfolio that has two equally-weighted stocks, P and Q.

(i) The economy over the next year could be good or bad with equal probability.

(ii) The returns of the stocks can vary as shown in the table below:

Stock	Return when economy is good	Return when economy is bad
P	10%	–2%
Q	18%	–5%

Calculate the volatility of the portfolio return.

R_T... return of the total portfolio

$R_T = \frac{1}{2}(R_P + R_Q)$

(A) 1.80%

(B) 6.90%

(C) 7.66%

(D) 8.75%

(E) 13.42%

$$R_T \sim \begin{cases} 0.14, & \text{if good} \quad \text{w/ probab. } 1/2 \\ -0.035, & \text{if bad} \quad \text{w/ probab. } 1/2 \end{cases}$$

$$\text{Var}[R_T] = \mathbb{E}[R_T^2] - (\mathbb{E}[R_T])^2$$

$$\mathbb{E}[R_T] = \frac{1}{2} \cdot 0.14 + \frac{1}{2}(-0.035) = 0.0525$$

$$\mathbb{E}[R_T^2] = \frac{1}{2}\left((0.14)^2 + (-0.035)^2\right) = 0.0104$$

$$\Rightarrow \text{Var}[R_T] = 0.0104 - (0.0525)^2 = 0.00766$$

$$\Rightarrow \sigma_T = \sqrt{\text{Var}[R_T]} = 0.087$$

Diversification w/ Equally Weighted Portfolio.

$w_i = \frac{1}{n}$ for all $i = 1..n$

$$\Rightarrow \quad R_p = \frac{1}{n}(R_1 + \cdots + R_n)$$

$$\Rightarrow \quad \text{Var}[R_p] = \text{Var}\left[\frac{1}{n}(R_1 + \cdots + R_n)\right]$$

$$= \frac{1}{n^2}\text{Var}[R_1 + \cdots + R_n]$$

$$= \frac{1}{n^2}\left(\sum_{i=1}^{n}\text{Var}[R_i] + \sum_{i \neq j}\text{Cov}[R_i, R_j]\right)$$

$$= \frac{1}{n} \cdot \underbrace{\frac{1}{n}\cdot\sum_{i=1}^{n}\text{Var}[R_i]}_{\text{Average Variance of the individual components}} + \underbrace{\frac{1}{n^2}\cdot n(n-1)}_{(1-\frac{1}{n})} \underbrace{\left(\frac{1}{n(n-1)}\sum_{i \neq j}\text{Cov}[R_i, R_j]\right)}_{\text{Average Covariance between the stocks in the portfolio}}$$

$\downarrow n \to \infty$

0

w/ latter term bdd

$\downarrow n \to \infty$

Average covariance

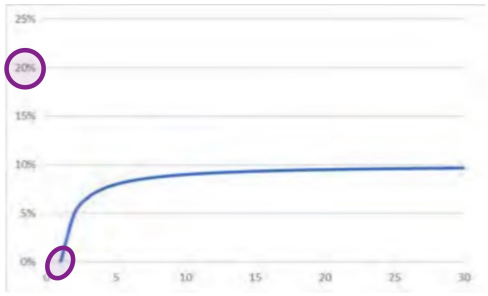
9) You are given the following information about an equally-weighted portfolio of n stocks:

(i) For each individual stock in the portfolio, the variance is 0.20.

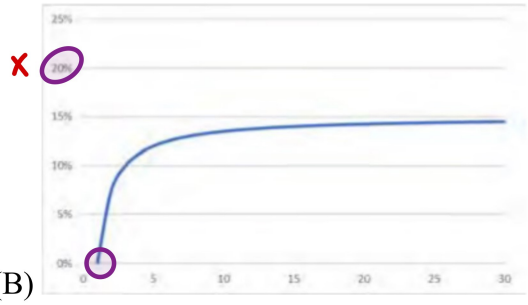
(ii) For each pair of distinct stocks in the portfolio, the covariance is 0.10.

Determine which graph displays the variance of the portfolio as a function of n.

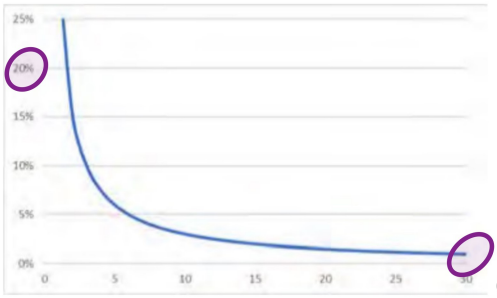
(A)



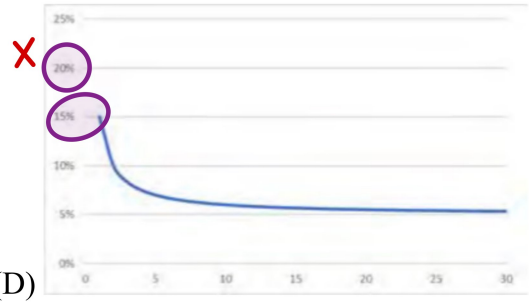
(B)



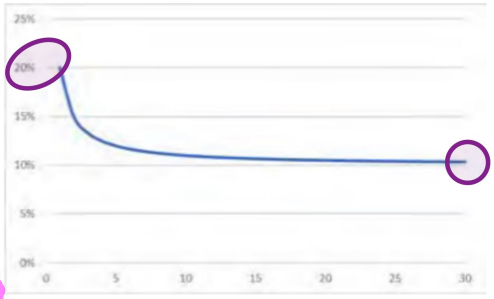
(C)



(D)



(E)



Diversification w/ a General Portfolio.

w_i... weights of the individual components

Assume: $w_i \geq 0$

Recall:

$$\text{Var}[R_P] = \sum_{i=1}^{n} w_i \, \text{Cov}[R_i, R_P]$$

$$(\text{SD}[R_P])^2$$

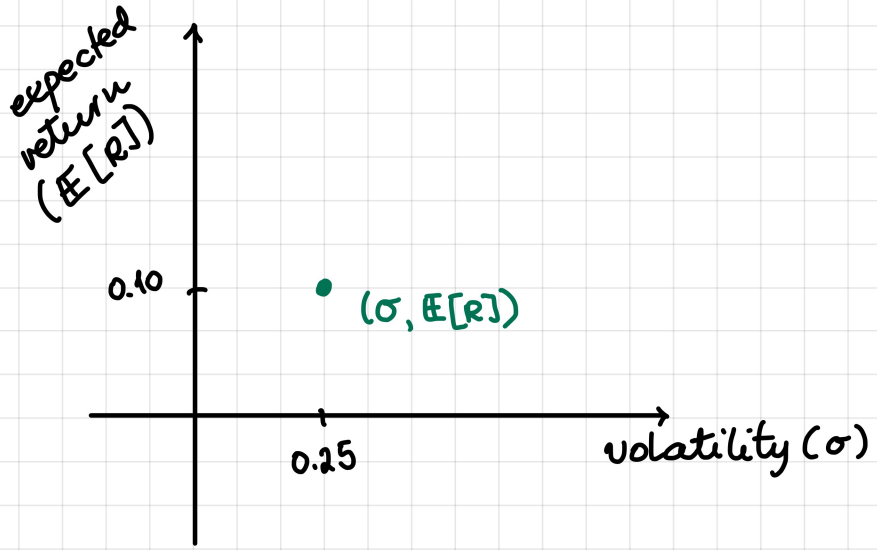
$$= \sum_{i=1}^{n} w_i \cdot \text{SD}[R_i] \cdot \text{SD}[R_P] \cdot \text{corr}[R_i, R_P]$$

$$= \text{SD}[R_P] \cdot \sum_{i=1}^{n} w_i \cdot \text{SD}[R_i] \cdot \text{corr}[R_i, R_P] \quad / : \text{SD}[R_P]$$

$$\Rightarrow \quad \sigma_P = \text{SD}[R_P] = \sum_{i=1}^{n} w_i \cdot \text{SD}[R_i] \cdot \underbrace{\text{corr}[R_i, R_P]}_{\leq 1}$$

$$\sigma_P \leq \sum_{i=1}^{n} w_i \cdot \sigma_i$$

Equality if all the investments are perfectly positively correlated.



6) You are given the following information about the four distinct portfolios:

Portfolio	Expected Return	Volatility
P	3%	10%
Q	5%	10%
R	5%	15%
S	7%	20%

Determine which two of the four given portfolios are NOT efficient.

(A) P and Q

(B) P and R

(C) P and S

(D) Q and R

(E) Q and S