M339D: February 3rd, 2025.

Problem 3.2. To plant and harvest $\boxed{20{,}000}$ bushels of corn, Farmer Jayne incurs total aggregate costs totaling \$33,000. The current spot price of corn is \$1.80 per bushel. What is the profit if the spot price is $\boxed{\$1.90}$ per bushel when she harvests and sells her corn?

IRRELEVANT

(a) About \$3,000 gain
(b) About \$3,000 loss
(c) About \$5,000 loss
(d) About \$5,000 gain
(e) None of the above

$\longrightarrow$: $20{,}000 \cdot 1.90 - 33{,}000 = \underline{5{,}000}$ $\square$

deterministic and valued @ time T (e.g., harvest time)

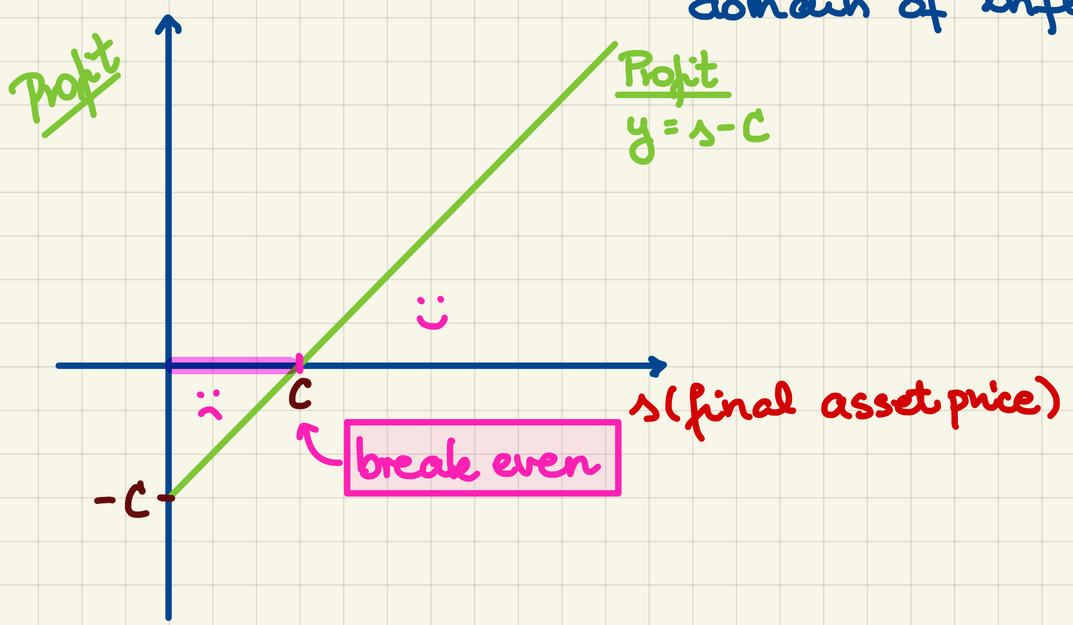
Hedging Motivation.

Example. Producer of Goods

- farmer producing corn, soy beans, peaches, ...
- crude oil
- ore mining
- "widgets"

C ... deterministic, total aggregate fixed and variable costs of production valued @ the time of sale, i.e., time T

If the producer sells their goods in the market, they get the market price. This is outside of their domain of influence.



Forward Contract.

*** A BINDING CONTRACT ON BOTH SIDES! ***

NO CASHFLOW!

0 ——— T

Handshake!

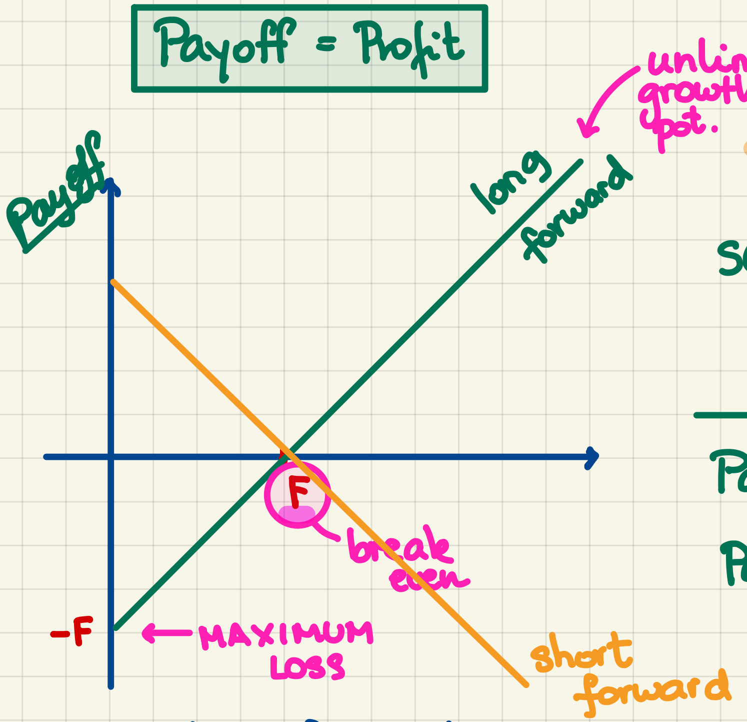
DELIVERY DATE (when the cash is exchanged for the asset)

An Agreement:

- the underlying asset : $S(t)$
- the quantity (for us: 1 unit)
- the type of settlement: physical or cash
- T... the delivery date
- $\boxed{\text{F... forward price}}$

Initial Cost = 0

$\boxed{\text{Payoff = Profit}}$



Long Forward: Buy Forward

S(T) | 1 unit of asset | ⇅ | FORWARD PRICE | F

Short Forward: Sell Forward

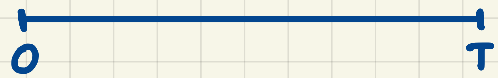
$$\left.\begin{aligned} \text{Payoff(Long Forward)} &= S(T) - F \\ \text{Payoff(Short Forward)} &= F - S(T) \end{aligned}\right\}$$

Payoff f'tion: $\boxed{v(s) = s - F}$

A long forward is a LONG position wrt the underlying.

Problem. Sample SOA Problem.

Determine which of the following portfolios have the same cashflows as a SHORT SALE of a non-dividend-paying stock.



Initial cost: $-S(0)$ Payoff: $-S(T)$

✗ (i) long forward and a zero-coupon bond

Initial Cost: Price of Bond >0

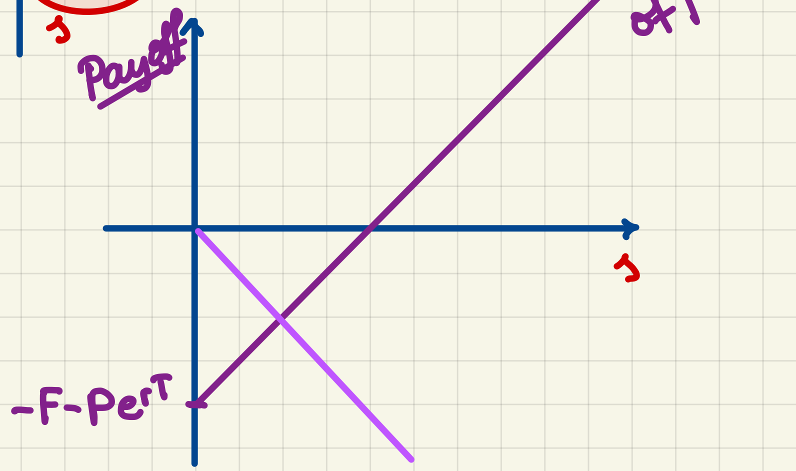
✗ (ii) long forward and a short forward

Init. Cost: 0

✗ (iii) long forward and a short zero-coupon bond

	Initial Cost	Payoff
Long Forward	0	$S(T)-F$
Short Bond	$-P$	$-Pe^{rT}$
Total	$-P$	$S(T)-F-Pe^{rT}$

s



✗ (iv) short forward and a long zero-coupon bond

Initial Cost: Price of Bond >0

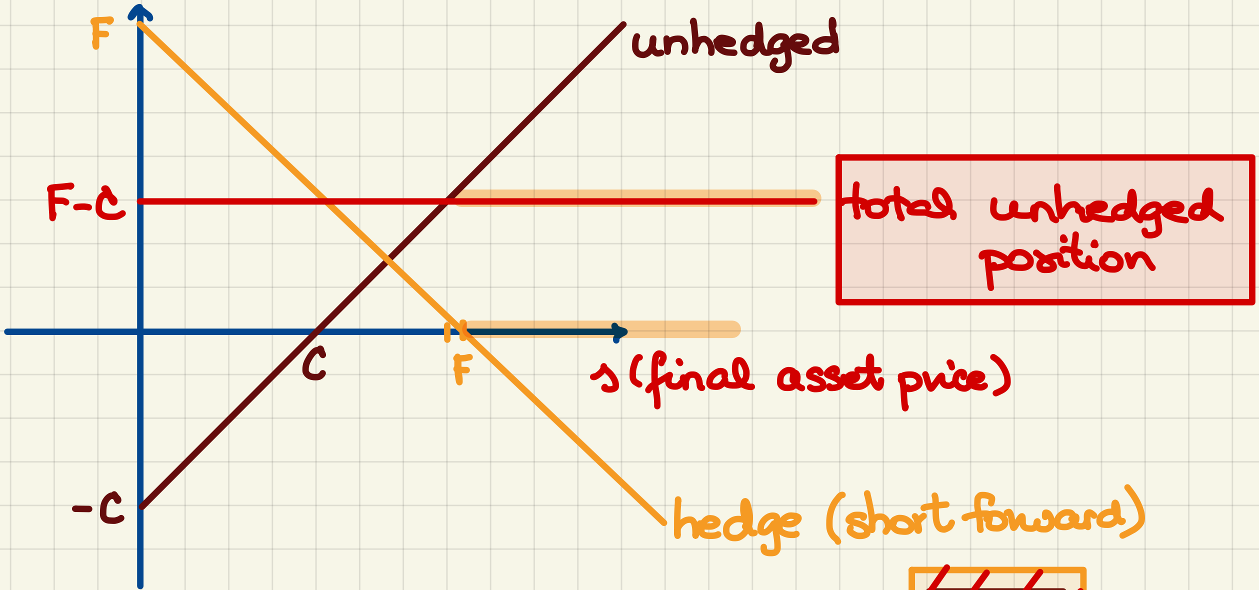
(v) short forward and a short zero-coupon bond

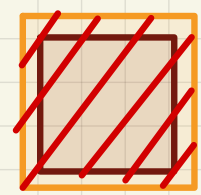
	Initial Cost	Payoff
Short Forward	0	$F - S(T)$
Short Bond	$-P$	$-Pe^{rT}$
Total	$-P$	$F - S(T) - Pe^{rT}$
Short Sale:	$-S(0)$	$-S(T)$
$\boxed{\text{Match}}$	$\boxed{S(0) = P}$	$\boxed{F = Pe^{rT} = S(0)e^{rT}}$

Important!

Hedging Using Forward Contracts.

Focus on a Producer of Goods.





Algebraically:

$$\text{Profit (unhedged)} + \text{Profit (hedge)} = \text{Profit (total hedged)}$$

$$\cancel{S(T)} - C \quad + \quad F - \cancel{S(T)} \quad = \quad F - C$$