Quiz #7: Solutions

Milica Cudina

2021-10-02

Problem 1. (1 point)

The mean and median of any normal distribution are equal. *True or false? Why?*

Solution: **TRUE** since both the mean and the median are equal to the parameter μ.

Problem 2. (3 points)

Let the population distribution be normal with mean μ and standard deviation σ. Let $\bar{X}$ denote the sample mean of a sample of size $n$from this population. Then, we know the following about the distribution of $\bar{X}$:

a. $\bar{X} \sim Normal(mean = \mu, variance = \sigma^2)$

b. $\bar{X} \sim Normal(mean = \mu, variance = \frac{\sigma^2}{n})$

c. $\bar{X} \sim Normal(mean = \mu, variance = \frac{\sigma^2}{\sqrt{n}})$

d. $\bar{X} \sim Normal(mean = \frac{\mu}{n}, variance = \frac{\sigma^2}{n})$

e. None of the above are correct.

Solution: The correct statement is **b.** For the verification, see class notes.

Problem 3. (1 point)

Suppose a poll suggested the US President's approval rating is 45%. We would consider 45% to be ...

a. the population mean.

b. the point estimate.

c. the statistic.

Solution: **b.**

Problem #4 (5 points)

Let $Z \sim N(0, 1)$. Given that $Z > 0$, find the probability that $Z < 2$.

Solution: We are looking for the probability

$$\mathbb{P}[Z < 2 \,|\, Z > 0].$$

By the definition of conditional probability,

$$\mathbb{P}[Z < 2 \,|\, Z > 0] = \frac{\mathbb{P}[0 < Z < 2]}{\mathbb{P}[Z > 0]}\,.$$

By the symmetry of the bell curve, $\mathbb{P}[Z > 0] = 0.5$. As for the numerator in the above expression, we have

$$\mathbb{P}[0 < Z < 2] = \mathbb{P}[Z < 2] - \mathbb{P}[Z \leq 0].$$

Again, by the symmetry of the bell curve, $\mathbb{P}[Z \leq 0] = 0.5$. On the other hand, we get $\mathbb{P}[Z < 2] = \Phi(2)$ and

```
pnorm(2)
## [1] 0.9772499
```

Finally, our answer for $\mathbb{P}[Z < 2 \,|\, Z > 0]$ is

```
(pnorm(2)-0.5)/0.5
## [1] 0.9544997
```

Problem #5 (5 points)

Let the monthly profit of a local cupcakery be normally distributed with mean $20,000 and standard deviation of $4,000. What is the probability that the combined profit in the months of October and November exceeds $36,000 (assuming that profits over different months are independent)?

Solution: Let X_1 be the profit for October and let X_2 be the profit for November. Then

$$X_1 + X_2 \sim Normal(mean = 40000, variance = 32,000,000).$$

The probability we are looking for is $\mathbb{P}[X_1 + X_2 > 36000]$. Our answer is

```
1-pnorm(36000,40000,sqrt(32000000))
## [1] 0.7602499
```