M378K Introduction to Mathematical Statistics

Homework assignment #8

Please, provide your **final answer only** to the following problems.

Problem 8.1. *(5 points) Let $Y_1, \ldots, Y_n$ be a random sample from $U(0, \theta)$, with an unknown $\theta > 0$. For what value of the constant c is the estimator $\hat{\theta} = c\sum_{i=1}^{n} Y_i$ unbiased for θ?*

(a) 1

(b) $1/n$

(c) $2/n$

(d) n

(e) **None of the above.**

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 8.2. *(20 points)* Source: "Probability" by Pitman
Four people agree to meet at a cafe at noon. Suppose that each person arrives at a time normally distributed with mean 12noon and standard deviation of 5 minutes, independently of all the others.

1. *(5 points) What is the chance that the first person to get to the cafe arrives before 11:50am?*
2. *(5 points) What is the chance that some of the four have still not arrived at 12:15pm?*
3. *(10 points) Approximately, what is the chance that the second person to arrive gets there within 10 seconds on 12noon?*

Problem 8.3. *(5 points) Consider an estimator $\hat{\theta}$ for a parameter θ. Let's say that*

$$\mathbb{E}[\hat{\theta}] = \kappa_1\theta + \kappa_2$$

for some constants $\kappa_i \neq 0, i = 1, 2$. Is the estimator $\hat{\theta}$ unbiased? If so, justify your answer; if not, how would you transform the estimator $\hat{\theta}$ to obtain an unbiased estimator?

Problem 8.4. *(20 points) Let $Y_1, \dots, Y_n$ be a random sample from the uniform distribution on $[0, \theta]$, where $\theta > 0$ is an unknown parameter. We consider the estimator*

$$\hat{\theta} = c\,\frac{1}{n}\sum_{i=1}^{n} Y_i^2.$$

where c is a constant (not dependent on θ or on $Y_1, \dots, Y_n$).

1. *(10 points) For what value of the constant c will $\hat{\theta}$ be an unbiased estimator for θ^2? Is there such a value if $\hat{\theta}$ is used as an estimator for θ instead of θ^2?*
2. *(10 points) Using the value of c obtained above, compute the mean squared error of $\hat{\theta}$ (when interpreted as an estimator of θ^2).*