

## UNIVERSITY OF TEXAS AT AUSTIN

Quiz #12

Delta-gamma-theta approximation. Market making and delta hedging.

**Problem 12.1.** (5 points) Let the stock price  $S = \{S(t); \geq 0\}$  satisfy the assumptions of the Black-Scholes model.

Consider a European put option on  $S$  whose current price is \$2.50. You are given that the current put delta equals  $-0.60$ , its gamma is  $0.08$ , and its theta is  $-0.02$  per day.

Assume that the continuously compounded risk-free interest rate is  $0.06$  per annum.

What is the delta-gamma-theta approximation for the put premium after three days if the stock price increases by \$2?

**Solution:** Let us denote the three-day time span by  $dt$  and let  $ds = S(dt) - S(0)$ . By the delta-gamma-theta approximation, we get

$$\begin{aligned} v_P(S(dt), dt) &= v_P(S(0), 0) + \Delta_P(S(0), 0)ds + \frac{1}{2}\Gamma_P(S(0), 0)(ds)^2 + \Theta_P(S(0), 0)dt \\ &= 2.50 + (-0.6)(2) + \frac{1}{2}(0.08)(2)^2 + (-0.02)(3) = 1.40 \end{aligned}$$

**Problem 12.2.** (10 points) Consider a non-dividend-paying stock whose current price is \$100. A market-maker writes a one-year call option on this stock and sells it for \$4.00. He then proceeds to delta-hedge his commitment by trading in the shares of the underlying stock.

The call option's delta is  $0.75$ , its gamma is  $0.08$  and its theta is  $-0.02$  per day.

The continuously compounded, risk-free interest rate is  $0.04$ .

The stock price has risen to \$101 after one day. Use the delta-gamma-theta approximation to find the market maker's profit after one day.

**Solution:** The initial cost of the market-maker's portfolio is

$$-4 + 0.75(100) = -4 + 75 = 71.$$

After one day, by the delta-gamma-theta approximation, the call price is approximately

$$4 + 0.75(1) + \frac{1}{2}(0.08)(1)^2 - 0.02 = 4.77.$$

So, the market-maker's payoff is

$$-4.77 + 0.75(101) = 70.98.$$

Finally, the market-maker's profit is

$$70.98 - 71e^{0.04/365} = -0.0278.$$