Chooser Options (aka "as·you·like·it" options).

0 — chooser option written

t^* — the "choice" date

T — exercise date

K... strike price

At time t^*, the chooser option's owner decides whether the option becomes a call or a put (either way w/ strike K and exercise date T).

Assume that the owner is rational.

Notation:

$V_{CH}(t, t^*, T)$ — t: valuation date; t^*: choice date; T: exercise date

$V_{\substack{c\\p}}(t$, exercise date, strike price$)$ — t: valuation date

Our criterion

$$\Rightarrow V_{CH}(t^*, t^*, T) = \max\left(V_c(t^*, T, K), V_p(t^*, T, K)\right)$$

$$\max(a,b) = a + \max(0, b-a) = a + (b-a)_+$$
$$= b + \max(a-b, 0) = b + (a-b)_+$$

$$V_{CH}(t^*, t^*, T) = V_c(t^*, T, K) + \Big(\underbrace{V_p(t^*, T, K) - V_c(t^*, T, K)}_{\substack{\parallel \text{ Put·Call Parity} \\ PV_{t^*,T}(K) - S(t^*)}}\Big)_+$$

$$= V_c(t^*, T, K) + \Big(\underbrace{Ke^{-r(T-t^*)}}_{\text{"}K^*\text{"}} - S(t^*)\Big)_+$$

Payoff of a European put w/ strike $K^* = Ke^{-r(T-t^*)}$ and exercise date t^*

$\Rightarrow$ A replicating portfolio for the chooser option:

$$\begin{cases} \bullet \text{ a long call w/ strike } K \text{ and exercise date } T \\ \bullet \text{ a long put w/ strike } K^* = Ke^{-r(T-t^*)} \text{ and exercise date } t^* \end{cases}$$

$$\Rightarrow \boxed{\begin{aligned} V_{CH}(0, t^*, T) &= V_C(0, T, K) + V_P(0, t^*, K^*) \\ &= V_P(0, T, K) + V_C(0, t^*, K^*) \end{aligned}}$$

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

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t^*... choice date

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of $100.

T exercise date

K

The chooser option price is $20 at time $t = 0$. $V_{CH}(0,1,3)=20$

The stock price is $95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of $100.

You are given:

(i) The risk-free interest rate is 0. $K^* = K$

(ii) $C(1) = \$4$. $V_c(0,1,K=100)=4$

Determine $C(3)$. $V_c(0,3,K=100)=\ ?$

(A) $ 9

(B) $11

(C) $13

(D) $15

(E) $17

$$V_{CH}(0,1,3)=20=\underbrace{V_c(0,3,K=100)}_{\cancel{?}\ \boxed{11}}+\boxed{V_p(0,1,K^*=100)}$$

$$V_p(0,1,K^*=100) = V_c(0,1,K^*=100) + PV_{0,1}(K^*) - S(0) = 4+100-95=9$$