

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 4F–distribution. F–statistic.

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Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

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**Problem 4.1.** (5 points) *Source: Ramachandran-Tsokos.*

Let  $S_1^2$  denote the sample variance for a random sample of size 10 from a normal population I and let  $S_2^2$  denote the sample variance for a random sample of size 8 from a normal population II. The variance of population I is assumed to be three times the variance of population II. Assume that the two samples are **independent**. Find two numbers  $a$  and  $b$  such that

$$\mathbb{P}[S_1^2/S_2^2 \leq a] = 0.05 \quad \text{and} \quad \mathbb{P}[S_1^2/S_2^2 \geq b] = 0.05$$

**Problem 4.2.** (10 points) *Source: An old CAS exam problem.*

A sample of size 20 is fitted to a linear regression model of the form

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \varepsilon_i.$$

The resulting  $F$ –ratio used to test the hypothesis

$$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$$

is equal to 21. Determine  $R^2$ .

**Problem 4.3.** In a simple linear regression fit on 16 observations, you obtain the point estimate of the slope parameter to be  $\hat{\beta}_1 = 3$ . The standard error of  $\hat{\beta}_1$  is estimated at 1.5.

(i) (10 points) Show that, in our usual notation,

$$TSS - RSS = \frac{(\sum (x_i - \bar{x})(y_i - \bar{y}))^2}{\sum (x_i - \bar{x})^2}.$$

(ii) (10 points) Prove that for the simple linear regression, the  $F$ –statistic can be obtained as the square of the  $t$ –statistic for the slope.

(iii) (5 points) Provide the value of the  $F$ –statistic.

(iv) (10 points) Provide the value of the coefficient of determination  $R^2$ .