

M339W: November 1<sup>st</sup>, 2021.

## Efficient Portfolios.

A portfolio is said to be efficient if there is no other portfolio w/ a higher expected return and a smaller or equal volatility.

A portfolio is not efficient if there exists another portfolio w/ a higher expected return @ a smaller or equal volatility.

- 6) You are given the following information about the four distinct portfolios:

Portfolio	Expected Return	Volatility
P	3%	10%
Q	5%	10%
R	5%	15%
S	7%	20%

Determine which two of the four given portfolios are NOT efficient.

(A) P and Q

$$R \prec Q$$

since they have the same return, but R has a higher volatility

(B) P and R

$$P \prec Q$$

since they have the same volatility, but Q has a higher return

(C) P and S

(D) Q and R

(E) Q and S

## The Effect of Correlation.

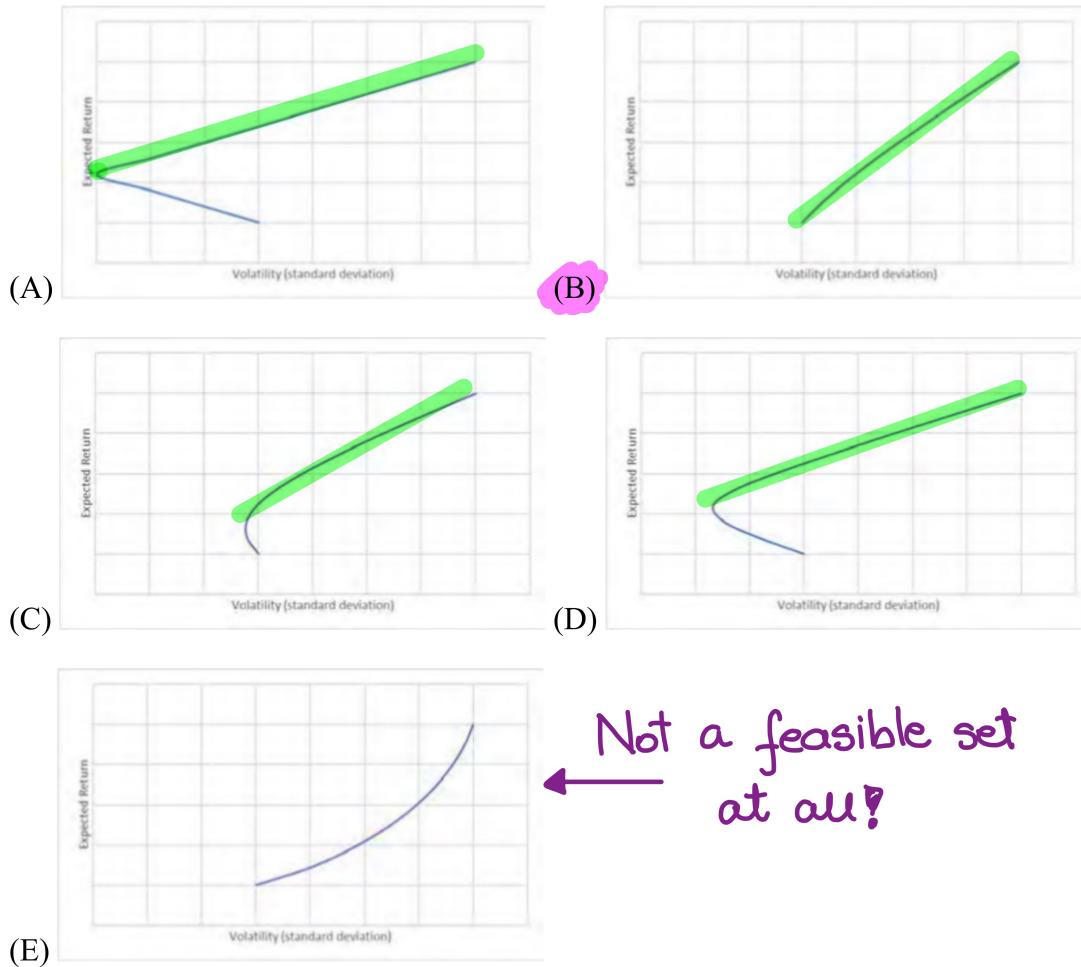
- If  $\rho = 1$ , then the feasible set is a **straight line** between the two assets.
- The higher the correlation, the smaller the curvature of the feasible set.
- If  $\rho = -1$ , then ...

Claim: There is a weight  $w$  of asset #1 such that the resulting portfolio is **risk-free**, i.e., its volatility is zero.

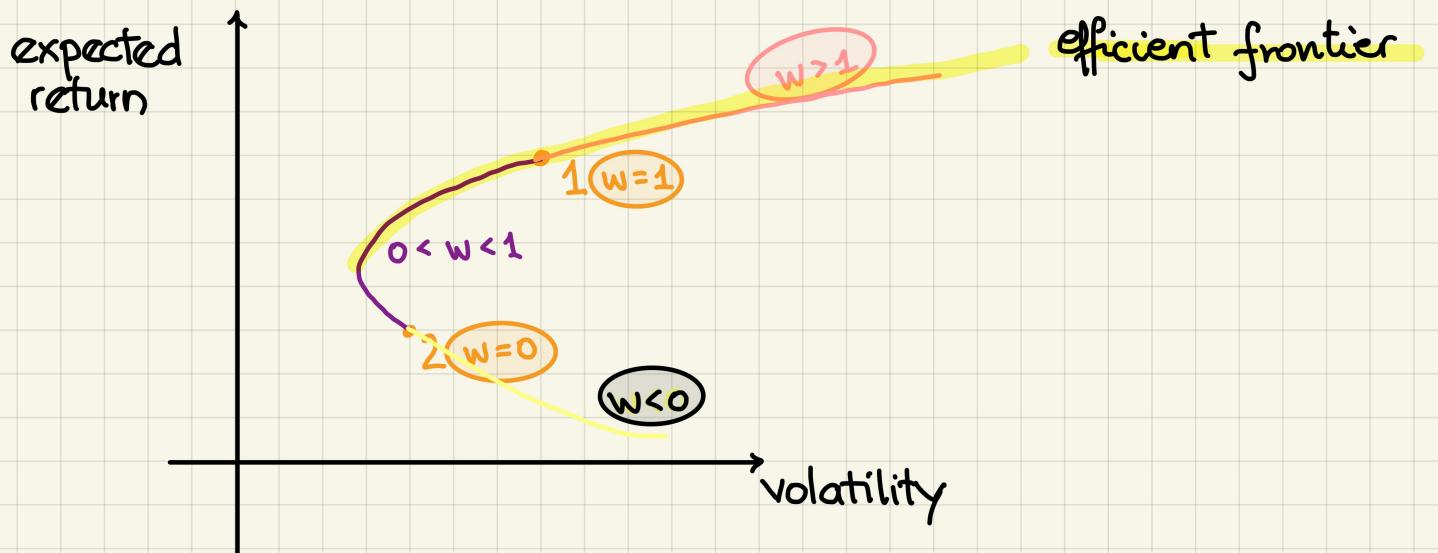
$$\begin{aligned} \rightarrow: \quad & \text{Var}[w \cdot R_1 + (1-w) \cdot R_2] = 0 \\ & w^2 \cdot \text{Var}[R_1] + (1-w)^2 \cdot \text{Var}[R_2] \\ & \quad + 2w(1-w) \cdot \text{Cov}[R_1, R_2] = 0 \\ & w^2 \cdot \sigma_1^2 + (1-w)^2 \cdot \sigma_2^2 + 2w(1-w) \cdot \sigma_1 \cdot \sigma_2 \cdot \rho = 0 \\ & \underline{w^2 \cdot \sigma_1^2 - 2w(1-w)\sigma_1 \cdot \sigma_2 + (1-w)^2 \cdot \sigma_2^2 = 0} \\ & (w \cdot \sigma_1)^2 - 2(w\sigma_1)((1-w)\sigma_2) + ((1-w)\sigma_2)^2 = 0 \\ & (w \cdot \sigma_1 - (1-w) \cdot \sigma_2)^2 = 0 \\ & w \cdot \sigma_1 - (1-w) \cdot \sigma_2 = 0 \\ & w(\sigma_1 + \sigma_2) = \sigma_2 \\ & \boxed{w = \frac{\sigma_2}{\sigma_1 + \sigma_2}} \end{aligned}$$

- 5) You are given the following set of diagrams for a two-stock portfolio, with expected return on the vertical axis and volatility on the horizontal axis. These diagrams are meant to help investors identify the set of efficient portfolios.

Identify the diagram demonstrating the highest correlation between the two stocks.



## Short sales w/ two assets.



Three or more risky assets  $\rightarrow$  Mathematica Demo

## Adding Risk-Free Saving & Borrowing.

Start w/ portfolio  $P$  which consists of risky investments.

Let's denote its return by  $R_P$ .

Let the risk-free interest rate be denoted  $r_f$ .

Now, we construct a new portfolio  $xP$  so that

- the weight  $x$  is given to the risky portfolio  $P$  and
- the weight  $(1-x)$  is given to the risk-free investment.

Let  $R_{xP}$  denote the return of this new portfolio.

Think about  $E[R_{xP}]$  and  $\sigma_{xP} = \sqrt{\text{Var}[R_{xP}]}$ .