

Problem 4.6. Pancakes, Inc. produces chocolate chip pancakes. It longed a forward contract on 100 lbs of chocolate chips at \$3.00 per pound. Total fixed revenue is \$2,000 for the pancakes produced with the above chocolate chips. Other costs total \$1200. Find the company's profit.

- (a) 2,000
- (b) 1,700
- (c) 800
- (d) 500
- (e) None of the above.

→:

$$R - C - F = 2000 - 1200 - 3 \cdot 100 = 500$$

□

Problem 4.7. The Extra-Healthy Cereal (EHC) company longed 20,000 forward contracts on corn at \$2.80 per bushel. The revenue from cereal made with the above corn is \$200,000 while the other (non-corn) aggregate fixed and variable costs amount to \$120,000. What is the EHC's profit?

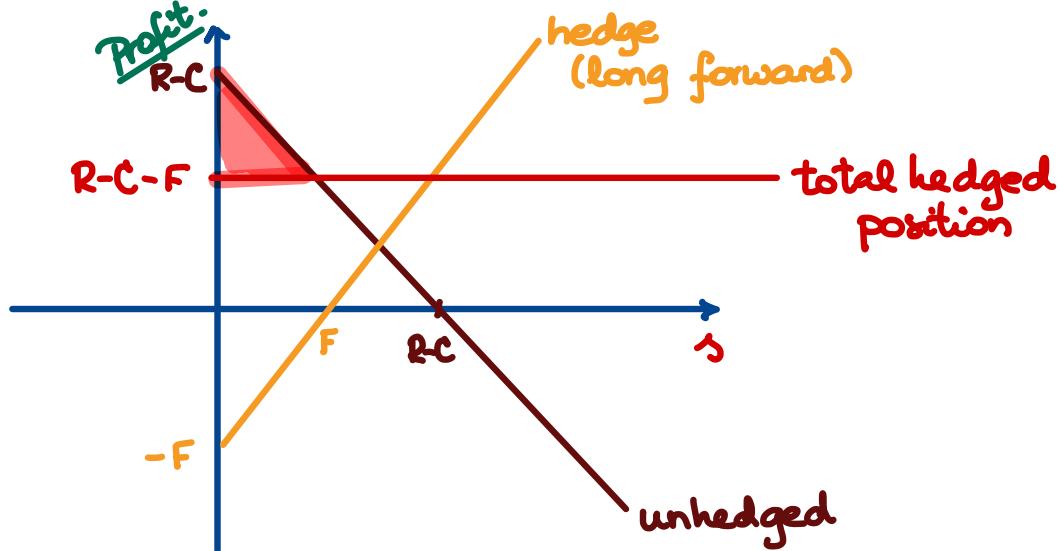
→:

$$\begin{aligned} R - C - F &= 200000 - 120000 - 2.8 \cdot 20000 \\ &= 24000 \end{aligned}$$

□

Q: What if I add : "The price of corn @ time T is \$2.50 per bushel in the market."?

Inspiration. Buyer/User of Goods.



European

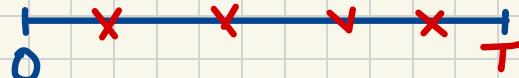
The contract can only be exercised, i.e., its transaction can take place only on the exercise date.

Call

Usually, the right to buy the underlying asset.

Options

Usually, the option's owner has the right, but not an obligation to exercise the option.



Option written.

EXERCISE DATE

- At time 0:
- The writer of the option writes/shorts the call.
 - The buyer of the call is said to long the call. They are referred to as the option's owner.
 - The agreement:
 - the underlying asset: $S(t), t \geq 0$
 - the exercise date: T
 - K ... the strike/exercise price
 -

- At time T :
- The call's owner has a right but not an obligation to buy one unit of the underlying asset for the strike price K .
 - The call's writer is obligated to do what the owner decides.

Payoff = ?

We focus on the payoff of the long call, i.e., the payoff for the call's owner.

The call owner's rationale for whether to exercise is to "maximize money in".

The criterion for exercise:

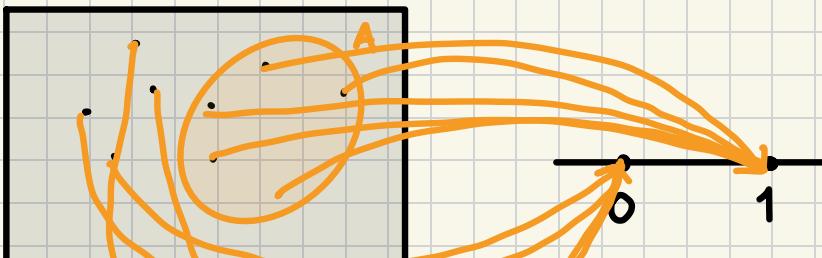
IF $S(T) \geq K$, then EXERCISE. \Rightarrow Payoff = $S(T) - K$
IF $S(T) < K$, then DO NOT EXERCISE. \Rightarrow Payoff = 0

We introduce:

$V_C(T)$... the r.v. denoting the payoff of a long call)

$$\Rightarrow V_C(T) = \begin{cases} S(T) - K & \text{if } S(T) \geq K \\ 0 & \text{if } S(T) < K \end{cases}$$

Indicator Random Variables.



Ω ... outcome space
 $\omega \in \Omega$... elementary outcomes

A ... any "nice" subset of Ω aka an EVENT

We define:

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

\Rightarrow

$$V_C(T) = (S(T) - K) \cdot \mathbb{I}_{[S(T) \geq K]}$$

Note:

$$S(T) \geq K \Leftrightarrow \underline{S(T) - K \geq 0}$$

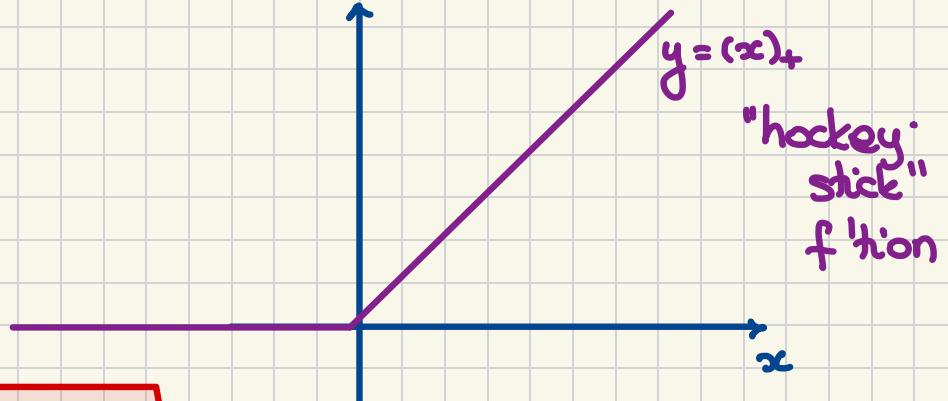
We could also write:

$$V_c(T) = \max(S(T) - K, 0) = (S(T) - K) \vee 0$$

MAXIMUM OPERATOR

Introduce: The Positive Part Function.

$$x \mapsto (x)_+ := \max(x, 0) = x \vee 0$$

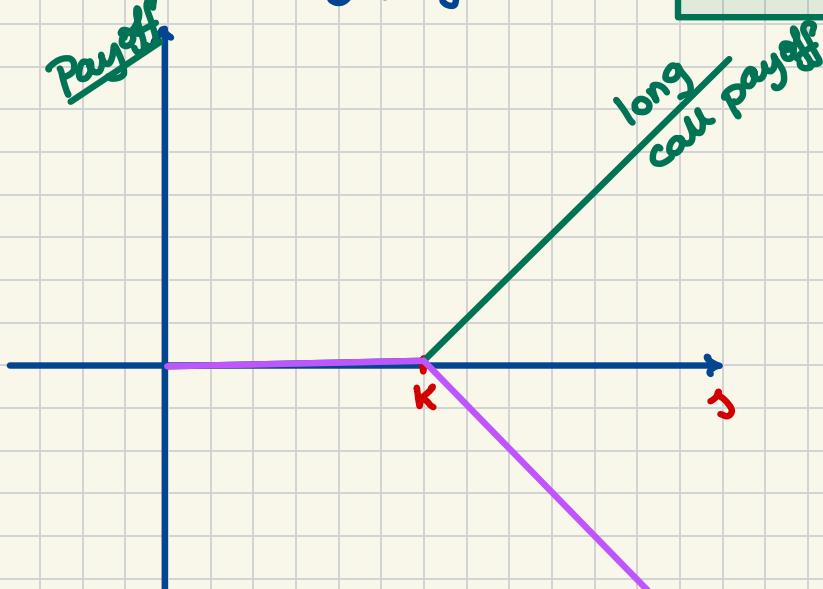


"hockey stick" function

$$\Rightarrow V_c(T) = (\underline{S(T) - K})_+$$

the payoff function:

$$V_c(s) = (s - K)_+$$



written/short call payoff
Never positive and sometimes strictly negative
 $\Rightarrow \exists$ premium