

UNIVERSITY OF TEXAS AT AUSTIN

Quiz #14

Expected returns. Volatility.

Provide your complete solution to the following problems:

Problem 14.1. (10 points) Your model for the economy at the end of your period has three different states *good*, *so-so* and *bad*. You think that the probability that the economy will be in the *so-so* state is twice the probability that it will be in the *good* state. You also think that the probability that the economy will be in the *good* state is twice the probability that it will be in the *bad* state.

There are two assets in your market model called *S* and *Q*. Their returns, depending on the state of the economy are modeled as follows:

Asset	<i>good</i>	<i>so-so</i>	<i>bad</i>
<i>S</i>	10%	2%	-5%
<i>Q</i>	8%	-1%	-4%

Your portfolio is equally weighted between assets *S* and *Q*. What is the volatility of this total portfolio?

Solution: The probability of the *good* economy is $2/7$, the probability of the *so-so* economy is $4/7$, and the probability of the *bad* economy is $1/7$. So, the return R_T of the total portfolio has the following distribution

$$R_T = \begin{cases} 0.09 & \text{with probability } 2/7, \\ 0.005 & \text{with probability } 4/7, \\ -0.045 & \text{with probability } 1/7. \end{cases}$$

We get that the mean return of this portfolio equals

$$\mathbb{E}[R_T] = 0.09 \left(\frac{2}{7}\right) + 0.005 \left(\frac{4}{7}\right) - 0.045 \left(\frac{1}{7}\right) = 0.0221429.$$

The second moment of the portfolio's return is

$$\mathbb{E}[R_T^2] = (0.09)^2 \left(\frac{2}{7}\right) + (0.005)^2 \left(\frac{4}{7}\right) + (-0.045)^2 \left(\frac{1}{7}\right) = 0.00261786.$$

Therefore, the variance of the return equals

$$\text{Var}[R_T] = \mathbb{E}[R_T^2] - (\mathbb{E}[R_T])^2 = 0.00261786 - (0.0221429)^2 = 0.00212755.$$

Finally, the volatility of the portfolio is $\sigma_T = 0.0461254$.

Problem 14.2. (5 points) According to your model, the economy over the next year could be *good* or *bad*. You believe that *bad* and *good* are equally likely.

Consider two assets, *X* and *Y*, existing in this market. If the economy is *good* the return on asset *X* is 0.12, and the return on asset *Y* is 0.08. If the economy is *bad* the return on asset *X* is -0.04 and the return on asset *Y* is -0.02 .

You construct a portfolio *P* using assets *X* and *Y* so that the portfolio's expected return equals 0.0325.

Calculate the volatility of this portfolio's return.

Solution: First, we need to figure out the weight w_X the asset *X* is given in portfolio *P*. The expected returns of assets *X* and *Y* are

$$\begin{aligned} \mathbb{E}[R_X] &= \frac{1}{2}(0.12) + \frac{1}{2}(-0.04) = 0.06 - 0.02 = 0.04, \\ \mathbb{E}[R_Y] &= \frac{1}{2}(0.08) + \frac{1}{2}(-0.02) = 0.04 - 0.01 = 0.03. \end{aligned}$$

So, the portfolio P must be such that the weight given to the asset X equals $w_X = 0.25$. Hence, the distribution of the portfolio's return can be described as

$$R_P \sim \begin{cases} 0.09, & \text{with probability } 1/3 \\ -0.025, & \text{with probability } 2/3 \end{cases}$$

The second moment of the portfolio's return is

$$\mathbb{E}[R_P^2] = (0.09)^2 \times \frac{1}{3} + (-0.025)^2 \times \frac{2}{3} = 0.0043625.$$

The variance of the portfolio's return is

$$\text{Var}[R_P] = 0.0043625 - 0.0325^2 = 0.00330625.$$

Finally, its volatility equals $\sigma = \sqrt{0.00330625} = 0.0575$.