M339D: April 25^{th}, 2022.

$n=2$

4. For a two-period binomial model, you are given:

(i) Each period is one year. $h=1$

(ii) The current price for a nondividend-paying stock is 20. $S(0)=20$, $\delta=0$

(iii) $u = 1.2840$, where u is one plus the rate of capital gain on the stock per period if the stock price goes up.

(iv) $d = 0.8607$, where d is one plus the rate of capital loss on the stock per period if the stock price goes down.

(v) The continuously compounded risk-free interest rate is 5%. $r=0.05$

Calculate the price of an American call option on the stock with a strike price of 22. $K=22$

Implicitly: $T=n\cdot h=2$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

5. Consider a 9-month dollar-denominated American put option on British pounds. You are given that:

(i) The current exchange rate is 1.43 US dollars per pound.

(ii) The strike price of the put is 1.56 US dollars per pound.

(iii) The volatility of the exchange rate is $\sigma = 0.3$.

(iv) The US dollar continuously compounded risk-free interest rate is 8%.

(v) The British pound continuously compounded risk-free interest rate is 9%.

Using a three-period binomial model, calculate the price of the put.

(A) 0.23

(B) 0.25

(C) 0.27

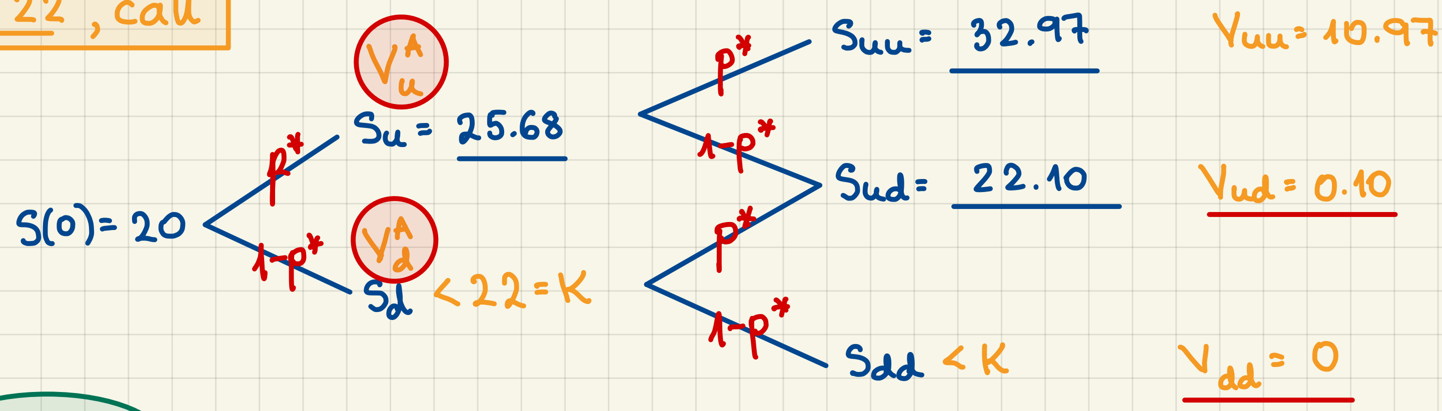
(D) 0.29

(E) 0.31

The risk-neutral probability:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{0.05} - 0.8607}{1.284 - 0.8607} = \underline{0.4502}$$

$\boxed{\underline{K=22}\text{, call}}$



up node:

- $IE_u = 25.68 - 22 = 3.68$
- $CV_u = e^{-0.05}\left(p^* \cdot (10.97) + (1-p^*) \cdot (0.10)\right) = \underline{4.753}$

$\underline{V_u^A = 4.753}$ ⇒ No Early Exercise!

down node: out-o-money ⇒ $V_d^A = CV_d = e^{-0.05} \cdot p^* \cdot 0.10 = \underline{0.0428}$

ROOT: out-o-money ⇒ $V_C^A(0) = CV_0 = e^{-0.05}\left(p^* \cdot 4.753 + (1-p^*)\, 0.0428\right)$

⇒ $\boxed{V_C^A(0) = \underline{2.06}}$ □

BEGINNING OF EXAMINATION

1. You use the usual method in McDonald and the following information to construct a binomial tree for modeling the price movements of a stock. (This tree is sometimes called a forward tree.)

(i) The length of each period is one year. $h=1$

(ii) The current stock price is 100. $\boxed{S(0)=100}$

(iii) The stock's volatility is 30%. $\sigma=0.30$

(iv) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 5%. $\delta=0.05$

(v) The continuously compounded risk-free interest rate is 5%. $r=0.05$

Calculate the price of a two-year $\boxed{\text{100-strike}}$ American call option on the stock.

(A) 11.40

(B) 12.09

(C) 12.78

(D) 13.47

(E) 14.16

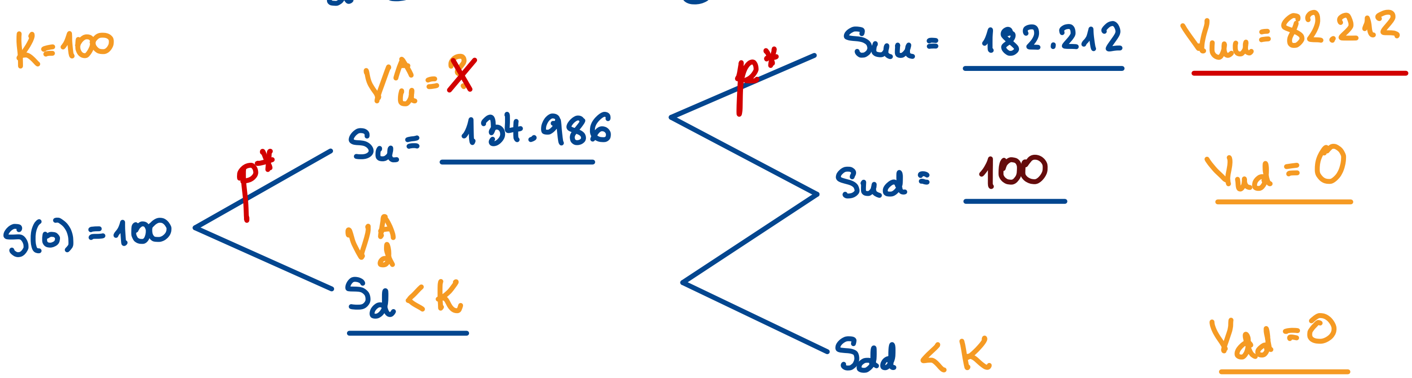
$\longrightarrow$: Forward tree $\Rightarrow$ a shortcut for p^* :

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.3}} = 0.42556 \checkmark$$

Forward tree:

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.05-0.05)\cdot 1+0.3\sqrt{1}} = e^{0.3}$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.05-0.05)\cdot 1-0.3\sqrt{1}} = e^{-0.3}$$



up node:

- $IE_u = \underline{34.986}$
- $CV_u = e^{-0.05} \cdot p^* \cdot 82.212 = \underline{33.2796}$

$\boxed{V_u^A = 34.986}$ => Early Exercise is optimal!

down node: out·o·money => $V_d^A = CV_d = \underline{\ 0\ }$

ROOT: at·the·money => $V_C^A(0) = CV_0 = e^{-0.05} \cdot p^* \cdot 34.986 = \underline{14.16}$ $\square$

$n=2$

11. For a two-period binomial model for stock prices, you are given:

(i) Each period is 6 months. $h = 1/2$

(ii) The current price for a nondividend-paying stock is \$70.00. $S(0)=70$, $\delta=0$

(iii) $u = 1.181$, where u is one plus the rate of capital gain on the stock per period if the price goes up.

(iv) $d = 0.890$, where d is one plus the rate of capital loss on the stock per period if the price goes down.

(v) The continuously compounded risk-free interest rate is 5%. $r=0.05$

Calculate the current price of a one-year American put option on the stock with a strike price of \$80.00.

$\boxed{K=80}$

$\rightarrow$:

$$p^* = \frac{e^{(r-\delta)h}-d}{u-d} = \frac{e^{0.05(0.5)} - 0.89}{1.181-0.89} = \underline{0.465}$$

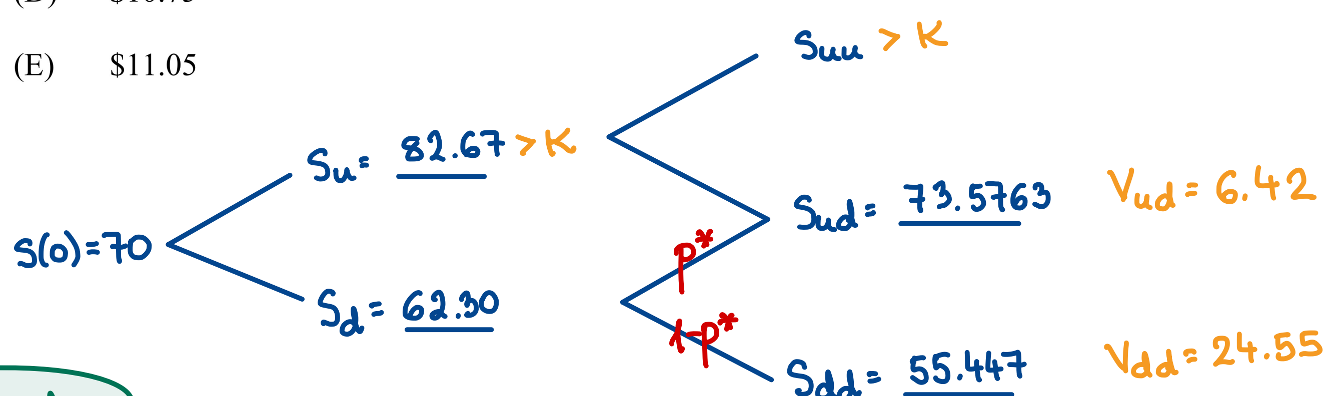
(A) \$9.75

(B) \$10.15

(C) \$10.35

(D) \$10.75

(E) \$11.05



down node:

- $IE_d = 80 - 62.30 = 17.70$
- $CV_d = e^{-0.05(0.5)}\left(p^* \cdot 6.42 + (1-p^*) \cdot 24.55\right) = \underline{15.72}$

$V_d^A = 17.70 \Rightarrow$ Early Exercise Optimal!