UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 13

Mean and median of the log-normal stock prices.

Problem 13.1. The current price of a non-dividend-paying stock is $80 per share. Under the risk-neutral probability measure, its mean rate of return is 12% and its volatility is 30%.

Let $R(0,t)$ denote the realized return of this stock over the time period $[0,t]$ for any $t > 0$. Calculate $\mathbb{E}^*[R(0,2)]$.

$$\mathbb{E}^*[R(0,T)] = \left(r - \frac{\sigma^2}{2}\right)\cdot T = \left(0.12 - \frac{0.09}{2}\right)\cdot 2$$

$$= (0.12 - 0.045)\cdot 2$$

$$= 0.15 \quad \square$$

Problem 13.2. A stock is valued at $\boxed{\$75.00}$. The continuously compounded, risk-free interest rate is $\boxed{10.0\%}$ and the standard deviation of annualized returns is 25.0%. If the stock is lognormally distributed, what is the expected stock price after 2 years under the risk-neutral probability measure?

$\rightarrow$:
$$\mathbb{E}^*[S(2)] = S(0)e^{r\cdot 2} = 75\cdot e^{0.1\cdot 2} = 75e^{0.2} = \dots$$

Problem 13.3. A non-dividend-paying stock is valued at $55.00 per share. Its standard deviation of annualized returns is given to be 22.0%. The continuously compounded risk-free interest rate is $\boxed{12\%}$. If the stock price is modeled using the lognormal distribution (as discussed in class), what is the median of the stock price in $\boxed{3}$ years under the risk-neutral probability measure?

$\rightarrow$:
$$\text{median} = S(0)e^{\left(r - \frac{\sigma^2}{2}\right)\cdot T}$$

$$= 55\, e^{\left(0.12 - \frac{0.22^2}{2}\right)\cdot 3} = \dots$$

Problem 13.4. Assume that the stock price is modeled using the lognormal distribution. The stock pays no dividends. Under $\mathbb{P}^*$, the annual mean rate of return on the stock is given to be 12%. Also under $\mathbb{P}^*$, the median time$-t$ stock price is evaluated to be $S(0)e^{0.1t}$. What is the volatility parameter of this stock price?

$\rightarrow$:

$$\cancel{S(0)}e^{(r-\frac{\sigma^2}{2})\cdot \cancel{t}} = \cancel{S(0)}e^{0.1\cancel{t}}$$

$$r-\frac{\sigma^2}{2}=0.1$$

$$0.12-\frac{\sigma^2}{2}=0.1 \rightarrow \frac{\sigma^2}{2}=0.02$$

$$\sigma^2=0.04$$

$$\boxed{\sigma=0.2}$$

Problem 13.5. The current stock price is $\boxed{\$100}$ per share. The stock price at any time $t>0$ is modeled using the lognormal distribution. Assume that the continuously compounded risk-free interest rate equals 8%. Let its volatility equal 20%.

Find the value t^* at which the median stock price equals \$120, under the risk-neutral probability measure.

$\rightarrow$:

$$\text{median time}\cdot t\text{ stock price} = S(0)e^{(r-\frac{\sigma^2}{2})\cdot t}$$

$$12\cancel{0} = 10\cancel{0}\, e^{(0.08-\frac{0.04}{2})\cdot t^*}$$

$$\ln(1.2)=0.06t^*$$

$$t^*=\frac{\ln(1.2)}{0.06}=\underline{3.039}$$

Problem 13.6. The volatility of the price of a non-dividend-paying stock is 0.2. The stock price is modeled using a log-normal distribution. Under $\mathbb{P}^*$, the expected time-2 stock price is $\boxed{\$120}$. What is the median of the time-2 stock price under $\mathbb{P}^*$?

$\rightarrow$:

$$\text{median} = S(0)e^{(r-\frac{\sigma^2}{2})\cdot T} = \underbrace{\boxed{S(0)e^{rT}}}_{120}\cdot e^{-\frac{\sigma^2}{2}\cdot T}$$

$$=120\,e^{-\frac{0.04}{2}\cdot 2}$$

$$=120\,e^{-0.04}=\underline{115.29}$$