UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 12

$t-$procedures: Two means.

Provide your **complete solution** to the following problems.

Problem 12.1. (8 points) *Source: Problem #7.74 from Moore-McCabe-Craig.*
Researchers were interested in the long-term psychological effects of being on a high-carbohydrate, low-fat (LF) diet versus a high-fat, low-carbohydrate (LC) diet. A total of 106 overweight and obese participants were randomly assigned to one of these two energy-restricted diets. At 52 weeks, 32 LC dieters and 33 LF dieters remained. Mood was assessed using a total mood disturbance score (TMDS), where a lower score is associated with a less negative mood. The sample average for the LC group was 47.3 with the sample standard deviation of 28.3. The sample avergage for the LF group was 19.3 with the sample standard deviation of 25.8. At the 0.05 significance level, is there a difference in the mean mood between the two kinds of dieters?

Solution: We are testing

$$H_0 : \mu_{LF} = \mu_{LC} \quad \text{vs.} \quad H_a : \mu_{LC} \neq \mu_{LF}$$

The observed value of the $t-$statistic is

$$t = \frac{\bar{x}_{LC} - \bar{x}_{LF}}{\sqrt{\frac{s_{LF}^2}{n_{LF}} + \frac{s_{LC}^2}{n_{LC}}}} = \frac{47.3 - 19.3}{\sqrt{\frac{28.3^2}{32} + \frac{25.8^2}{33}}} = 4.16481$$

The conservative number of degrees of freedom for the $t-$distribution of our test statistic is $\min(32, 33) - 1 = 31$. However, your $t-$table only contains 30 degrees of freedom, so that's what we'll use. The critical value associates with the upper-tail probability of 0.025 is 2.042. Since the observed value of the $t-$statistic is higher than the critical value, we reject the null hypothesis.

Alternatively, you could do the following in **R**:

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2*pt(-4.16481,df=31)
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You obtain the $p-$value of 0.0002305793.

Problem 12.2. (7 points) *Source: Ramachandran-Tsokos.*
A study of two kinds of machine failures shows that 58 failures of the first kind took on the average 79.7 minutes to repair with a sample standard deviation of 18.4 minutes, whereas 71 failures of the second kind took on average 87.3 minutes to repair with a sample standard deviation of 19.5 minutes. Find a 99% confidence interval for the difference between the true mean amounts of time it takes to repair failures of the two kinds of machines. Assume the normal distribution for the repair times.

Solution: Taking into account the obtained data, we get the standard error of

$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{18.4^2}{58} + \frac{19.5^2}{71}} = 3.3456. \tag{12.1}$$

The conservative number of degrees of freedom for the $t-$distribution of the difference in the sample means is $\min(58, 71) - 1 = 57$. In our tables, we will look at the further more conservative choice of 50 degrees of freedom (since we do not have exactly 57). We get the critical value associated with the upper-tail probability of 0.005 to be 2.678. The resulting confidence interval is

$$(79.7 - 87.3) \pm 2.678(3.3456) = -7.6 \pm 8.95952. \tag{12.2}$$

Alternatively, you could use **R** to get the critical value at 57 degrees of freedom:

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qt(0.995, df=57)
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You would get the critical value of $t^* = 2.66487$. Your confidence interval would be

$$(79.7 - 87.3) \pm 2.66487(3.3456) = -7.6 \pm 8.915589. \tag{12.3}$$