

M358K: October 15th, 2021.

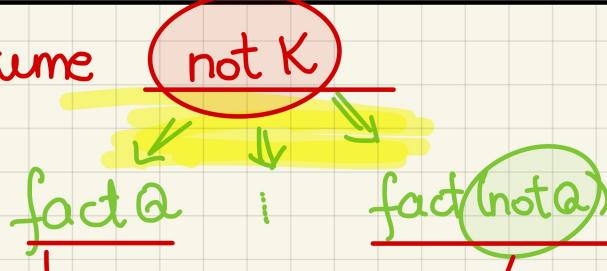
Hypothesis Testing.

Proof by Contradiction.

K... claim that I'm trying to prove to be true

Q: What if K were not true?

Assume



They cannot coexist!

We say we have reached a contradiction!

$\Rightarrow \Leftarrow$



Our assumption of not K was wrong!

Hypothesis Testing.

Claim we're trying to substantiate.

μ ... the population parameter representing the mean cholesterol level after treatment

μ_0 ... the mean cholesterol level before treatment (a number)

$$\mu < \mu_0$$

ALTERNATIVE HYPOTHESIS

Assume

$$\mu = \mu_0$$

←
NULL HYPOTHESIS

collect data
statistical analysis

Figure out the probability of seeing the data that we saw (or something more extreme) if

$$\mu = \mu_0$$

If this probability is "small", you have evidence to the contrary of $\mu = \mu_0$

The smaller this probability, the stronger the evidence.

The Normal Case.

Population model : $X \sim \text{Normal}(\text{mean} = \mu, \text{sd} = \sigma)$

unknown
and of interest

known

Hypothesis-testing procedure.

First: Set the hypotheses.

2nd

Null Hypothesis :

$$H_0: \mu = \mu_0 \quad \checkmark$$

1st

Alternative Hypothesis.

$$H_a: \begin{cases} \mu < \mu_0 & (\text{lower- or left-sided}) \\ \mu \neq \mu_0 & (\text{two-sided}) \\ \mu > \mu_0 & (\text{upper- or right-sided}) \end{cases}$$

Second: Figure out the appropriate test statistic (TS).

Natural choice:

$$\bar{X} \sim \text{Normal}(\text{mean} = \mu, \text{sd} = \frac{\sigma}{\sqrt{n}})$$

Under the null hypothesis, i.e., if $\mu = \mu_0$,

$$\frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

Third: Consider the observed value of the test statistic.

In this case, it's \bar{x} , i.e., the observed sample average.

Q: What's the probability of observing \bar{x} or something more extreme under the null?

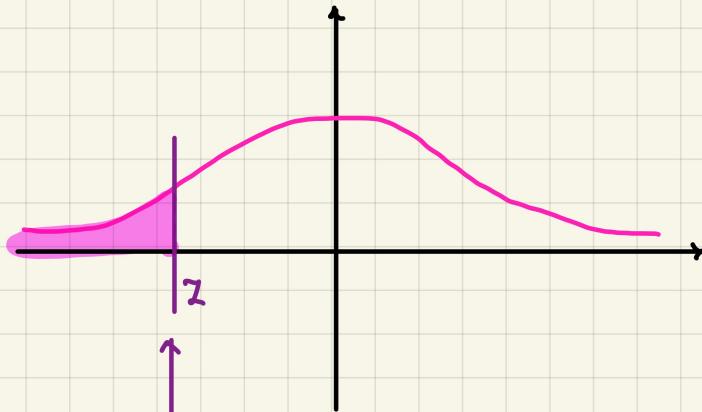
The interpretation depends on the structure of the ALTERNATIVE HYPOTHESIS!

It's always useful to calculate the observed **z-score**, i.e.,

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

- Lower-sided alternative:

$$H_a : \mu < \mu_0$$



$P[Z \leq z]$... [the p-value]
the probability of seeing the observed value of the TS or something more extreme

Def'n.