Multifactor Models.

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Even if the market portfolio is not efficient, that does not imply that an efficient portfolio does not exist (remember the tangent portfolio).

If an efficient portfolio exists, then

$$\mathbb{E}[R_S] = r_S = r_f + \beta_S^{eff}\left(\mathbb{E}[R_{eff}] - r_f\right)$$

Note: An efficient portfolio should be well-diversified. We "construct" an efficient portfolio from a collection of well-diversified portfolios.

1st Assume that we have N portfolios from which we can build an efficient portfolio. We call them factor portfolios and we denote them by $F1, F2, \dots, FN$.

2nd How do we express $\mathbb{E}[R_S]$ in terms of $R_{F1}, R_{F2}, \dots, R_{FN}$ (this includes the interactions between S and Fi).

Say that there are two factors F1 and F2. The return of the efficient portfolio we can construct using them is:

$$R_{eff} = w_1 \cdot R_{F1} + w_2 \cdot R_{F2}$$

Run a multiple linear regression:

$$R_S - r_f = \alpha_S + \beta_S^{F1}\left(R_{F1} - r_f\right) + \beta_S^{F2}\left(R_{F2} - r_f\right) + \varepsilon_S$$

$\varepsilon_S \sim N(0, var)$ uncorrelated w/ R_{F1} and R_{F2}

Through an economical argument:

$$\alpha_S = 0$$

$\mathbb{E}$

$$\mathbb{E}[R_S] = r_f + \beta_S^{F1}\left(\mathbb{E}[R_{F1}] - r_f\right) + \beta_S^{F2}\left(\mathbb{E}[R_{F2}] - r_f\right)$$

S

18) You are given the following information about the return of a security, using a two-factor model.

Factors	Beta	Expected Return
T	0.10	25%
U	0.15	20%

The annual effective risk-free rate of return is 5%.

Calculate the expected return of this security using the given two–factor model.

$$E[R_S] = 0.05 + 0.10(0.25 - 0.05) + 0.15(0.20 - 0.05)$$

$$= 0.05 + (0.10)(0.2) + 0.15(0.15)$$

$$= 0.05 + 0.02 + 0.0225$$

$$= 0.0925$$

(A) 6.52%

(B) 8.33%

(C) 9.25%

(D) 11.33%

(E) 13.32%

We can generalize this model to N factors:

$$\mathbb{E}[R_S] = r_f + \sum_{i=1}^{n} \beta_S^{Fi} (\mathbb{E}[R_{Fi}] - r_f)$$

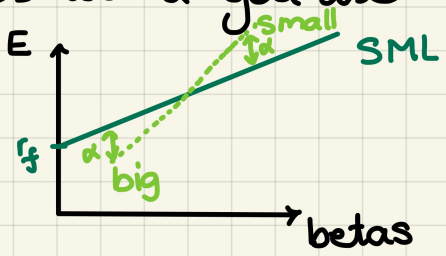
Factor selection in a Multifactor Model

(aka Arbitrage Pricing Theory)

- Include: The Market Portfolio (financed by risk-free borrowing).

Q: What other portfolios would you include?
More precisely, which investment strategies would you use to construct the other portfolios?

- Book-to-Market Ratio
- Market Capitalization
- Momentum Strategy



Market Capitalization.

- Order the stocks by their MV.
- Find the median.
- Create S as the equally weighted portfolio w/ stocks below the median.
- Create B as the equally weighted portfolio w/ stocks above the median.
- Long S and short B to obtain Small-minus-Big (SMB).

Book-to-Market Ratio.

- Order the stocks by their $\frac{BV}{MV}$.
- Construct L as the equally weighted portfolio w/ the lowest 30% of stocks.
- Construct H as the equally weighted portfolio w/ the highest 70% of stocks.
- Long H and short L to obtain the High-minus-Low (HML).

Momentum Strategy.

- Order the stocks according to their returns over the last year.
- Create a portfolio so that you: Long the top 30% and Short the bottom 30%
- The portfolio you obtain: PR1YR

Fama-French-Carhart Factor Specification.

$$\mathbb{E}[R_S] - r_f = \beta_S^{Mkt}\left(\mathbb{E}[R_{Mkt}] - r_f\right) + \beta_S^{SMB}\,\mathbb{E}[R_{SMB}] + \beta_S^{HML}\,\mathbb{E}[R_{HML}] + \beta_S^{PR1YR}\,\mathbb{E}[R_{PR1YR}]$$