

M339W/389W Financial Mathematics for Actuarial Applications
 University of Texas at Austin
Practice Problems for In-Term Exam 2
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Notes: This is a closed book and closed notes exam.
Time: 50 minutes

TRUE/FALSE			MULTIPLE CHOICE					
			1 (5)	a	b	c	d	e
1 (2)	TRUE	FALSE	2 (5)	a	b	c	d	e
2 (2)	TRUE	FALSE	3 (5)	a	b	c	d	e
3 (2)	TRUE	FALSE	4 (5)	a	b	c	d	e
4 (2)	TRUE	FALSE	5 (5)	a	b	c	d	e
5 (2)	TRUE	FALSE	6 (5)	a	b	c	d	e

FOR GRADER'S USE ONLY:

T/F	1.	2.	M.C.	Σ

2.1. **TRUE/FALSE QUESTIONS.** *Please note your answers on the front page.*

Problem 2.1. Gamma of a call bull spread is always positive. *True or false?*

Problem 2.2. Assume the Black-Scholes model. The elasticity of a European put option is always nonpositive. *True or false?*

Problem 2.3. (2 points) In order to **both** delta hedge and gamma hedge a position in a certain option, the market-maker must trade in another type of option (i.e., not only in the money-market and the underlying risky asset). *True or false?*

Problem 2.4. (2 points) Market makers usually do not need to rebalance their portfolios after the initial hedge is established. *True or false?*

Problem 2.5. (2 points) A market maker who delta-hedges **completely** insures himself against losses. *True or false?*

2.2. **FREE-RESPONSE PROBLEMS.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 2.6. (15 points) Consider a non-dividend-paying stock whose current price is \$45 per share. Its volatility is given to be 0.20.

The continuously compounded risk-free interest rate is 0.04.

A market maker sells a European, 91-day, \$50-strike call option on the above stock for \$0.42 and delta-hedges the commitment using shares of stock. The call's delta at time-0 is 0.1841. The market-maker does not update the delta-hedge for a week. Then, she realizes that the call option is at-the-money and decides to liquidate the entire portfolio. What is the market maker's profit?

Problem 2.7. (10 points) Let $S(t)$ denote the time- t price of a continuous-dividend-paying stock with dividend yield δ and volatility σ .

The continuously compounded risk-free interest rate is denoted by r .

You write a special option which pays $\min(S(T), K)$ for a positive monetary amount K at time- T . You want to delta-hedge this commitment. What is the time-0 delta of the special option, expressed using the notation given above?

Problem 2.8. (10 points) Assume the Black-Scholes framework for a non-dividend-paying stock whose current price is \$51.

A market-maker writes a European call option and sells it for \$9.25. Then, the market-maker delta-hedges by trading in the shares of the underlying stock. You are given the following current values of the greeks of the call option:

- the Δ is 0.66;
- the Γ is 0.02;
- the Θ is -0.01 per day.

The continuously compounded risk-free interest rate is 0.04.

Using the delta-gamma-theta approximation, calculate the approximate profit for the market-maker after one day if the stock price drops to \$50.

2.3. MULTIPLE CHOICE QUESTIONS.

Problem 2.9. (5 points) Assume the Black-Scholes framework.

The goal is to delta-hedge a one-year, at-the-money straddle on a non-dividend-paying stock whose current price is \$50. The stock's volatility is 0.20.

The continuously compounded risk-free interest rate is 0.10.

What is the cost of delta-hedging the straddle using shares of the underlying stock?

- (a) \$22.58
- (b) \$23.23
- (c) \$24.33
- (d) \$25.19
- (e) None of the above.

Problem 2.10. (5 points) Assume the Black-Scholes framework. The current stock price is \$50 per share. Its dividend yield is 0.01 and its volatility is 0.25.

The continuously compounded risk-free interest rate is 0.05.

Consider a one-year, \$55-strike European put option on the above stock. What is the volatility of the put option?

- (a) 1.013
- (b) -0.534
- (c) 6.6
- (d) 0.978
- (e) None of the above.

Problem 2.11. Which of the following greeks is usually negative?

- (a) Call delta.
- (b) Call gamma.
- (c) Call theta.
- (d) Call vega.
- (e) None of the above.

Problem 2.12. Consider the following portfolio:

- 5 long options of type *I*,
- 4 long options of type *II*,
- 1 written option of type *III*.

The prices of the three options are 0.75, 1.00, and 1.50, respectively, while the option elasticities are 10, 7, and 2, respectively. What is the elasticity of the above portfolio?

- (a) 5
- (b) 7
- (c) 10
- (d) 12
- (e) None of the above.

Problem 2.13. (5 points) Assume the Black-Scholes model is used. The current price of a continuous-dividend-paying stock is \$50. Its dividend yield is given to be 0.03.

The continuously compounded, risk-free interest rate equals 0.03.

You observe the price of an at-the-money, one-year European put option on the stock as equal to \$6.93. What is the implied volatility of the stock?

- (a) 0.18
- (b) 0.24
- (c) 0.36
- (d) 0.42
- (e) None of the above.

Problem 2.14. (5 points) The current stock price is equal to \$50. Consider a European call option whose current price is \$3.43. The call's current Δ is 0.60 and its Γ is 0.02. What is the approximate call price if the stock price increases to \$52 in a short time interval?

- (a) 4.03
- (b) 4.27
- (c) 4.41
- (d) 4.67
- (e) None of the above.

Problem 2.15. (5 points) The current price of a continuous-dividend-paying stock is given to be \$92. The stock's volatility is 0.35 and its dividend yield is 0.02.

The continuously compounded risk-free interest rate is 0.05.

Consider a \$90-strike European call option on the above stock with exercise date in a quarter-year.

What is the Black-Scholes price of this call option?

- (a) 5.05
- (b) 7.66
- (c) 7.71
- (d) 7.89
- (e) None of the above.

Problem 2.16. (5 pts) Assume the Black-Scholes framework. Let the current price of a non-dividend-paying stock be equal to $S(0) = 95$ and let its volatility be equal to 0.35. Consider a European call on that stock with strike 100 and exercise date in 9 months. Let the risk-free continuously compounded interest rate be 6% per annum.

Denote the price of the call by $V_C(0)$. Then,

- (a) $V_C(0) < \$5.20$
- (b) $\$5.20 \leq V_C(0) < \7.69
- (c) $\$7.69 \leq V_C(0) < \9.04
- (d) $9.04 \leq V_C(0) < \$11.25$
- (e) None of the above.

Problem 2.17. Assume the Black-Scholes setting. Let $S(0) = \$63.75$, $\sigma = 0.20$, $r = 0.055$. The stock pays no dividend and the option expires in 50 days (simplify the number of days in a year to 360).

What is the price of a \$60-strike European put?

- (a) 0.66
- (b) 0.55
- (c) 0.44
- (d) 0.37
- (e) None of the above.

Problem 2.18. Assume the Black-Scholes setting.

Assume $S(0) = \$28.50$, $\sigma = 0.32$, $r = 0.04$. The stock pays a 1.0% continuous dividend and the option expires in 110 days (simplify the number of days in a year to 360).

What is the price of a \$30-strike put?

- (a) 2.75
- (b) 2.10
- (c) 1.80
- (d) 1.20
- (e) None of the above.