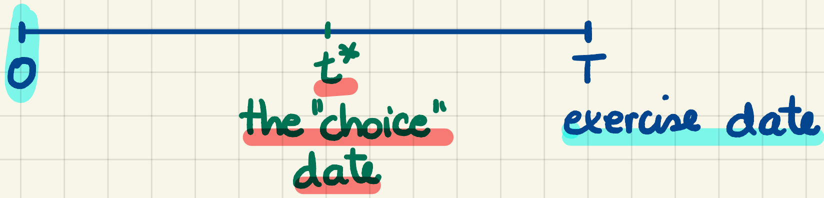
M339D: March 1st, 2023.

Chooser Options (aka "as·you·like·it" options)



K... strike price

At time t^*, the chooser option's owner decides whether the option becomes a call or a put (either w/ strike K & exercise date T).

Assume the owner is rational.

Q: What criterion for the choice between a call or a put does the owner use @ time t^*?

→: The owner can see the stock price, the call & the put prices in the market.

The owner will compare the market price of the K-strike, T-exercise date European put to the market price of the K-strike, T-exercise date European call, and pick the one w/ the higher price.

Notation:

- $V_{CH}(\underset{\text{valuation date}}{t}, \underset{\text{choice date}}{t^*}, \underset{\text{exercise date}}{T})$
- $V_{C/P}(t$, exercise date, strike price$)$

$$\Rightarrow V_{CH}(t^*, t^*, T) = \max\left(V_C(t^*, T, K), V_P(t^*, T, K)\right)$$

$$\max(a,b) = a + \max(0, b-a) = a + (b-a)_+$$
$$= b + \max(a-b, 0) = b + (a-b)_+$$

$$\Rightarrow V_{CH}(t^*, t^*, T) = V_c(t^*, T, K) + \Big(\underbrace{V_P(t^*, T, K) - V_C(t^*, T, K)}_{\overbrace{PV_{t^*,T}(K) - S(t^*)}^{\text{Put-Call Parity}}}\Big)_+$$

$$V_{CH}(t^*, t^*, T) = \underline{V_c(t^*, T, K)} + \boxed{\underline{\left(Ke^{-r(T-t^*)} - S(t^*)\right)_+}}$$

Payoff of a European put w/ strike $K^* = Ke^{-r(T-t^*)}$ and exercise date t^*.

$\Rightarrow$ A replicating portfolio for the chooser option:

- a long call w/ strike K and exercise date T
- a long put w/ strike K^* and exercise date t^*

$$\Rightarrow \boxed{\begin{aligned} V_{CH}(0, t^*, T) &= V_c(0, T, K) + V_P(0, t^*, K^*) \checkmark \\ &= V_P(0, T, K) + V_c(0, t^*, K^*) \checkmark \end{aligned}}$$

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

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t^*

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of $\boxed{\$100}$. ← K ← T

The chooser option price is \$20 at time $t = 0$. $\boxed{V_{CH}(0,1,3) = 20}$

The stock price is \$95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of $\boxed{\$100}$. K

You are given:

(i) The risk-free interest rate is 0.

(ii) $\boxed{C(1) = \$4}$. $V_c(0,1,K=100) = 4$

Determine $C(3)$.

$\underline{V_c(0,3,K=100) = ?}$

(A) \$ 9

(B) \$11

(C) \$13

(D) \$15

(E) \$17

$$V_c(0,3,K) = \underbrace{\boxed{V_{CH}(0,1,3)}}_{=20} - \underbrace{\boxed{V_p(0,1,K=100=K^*)}}_{=?}$$

<u>Put-Call Parity</u>

$$V_p(0,1,100) = V_c(0,1,100) + PV_{0,1}(100) - S(0)$$
$$= 4 + 100 - 95 = 9$$

$$\boxed{V_c(0,3,K) = 20 - 9 = 11}$$ □