M339G: February 11th, 2026.

F Distribution.

Motivation.

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 \qquad \text{unbiasedness}$$

$$\frac{S^2}{\sigma^2} = \frac{1}{n-1}\boxed{\sum_{i=1}^{n}\frac{(X_i-\bar{X})^2}{\sigma^2}}$$

$\sim \chi^2(df = n-1)$

Def'n. Let U and V be chi-squared r.v.s w/ ν_1 and ν_2 df, resp. and independent.

Then, the r.v.

$$F = \frac{U/\nu_1}{V/\nu_2}$$

is said to be F-distributed w/ ν_1 numerator and ν_2 denominator df.

We write $\boxed{F \sim F(\nu_1, \nu_2) \sim F_{\nu_1,\nu_2}}$

Theorem. Let two independent random samples of sizes n_1 and n_2, resp., be drawn from two normal populations w/ variances σ_1^2 and σ_2^2, resp.

Say, we denote the two sample variances by

S_1^2 and S_2^2, resp.

Then, the statistic

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1-1, n_2-1)$$

Corollary. If $\sigma_1 = \sigma_2$,

then $$\boxed{F = \frac{S_1^2}{S_2^2} \sim F(n_1-1, n_2-1)}$$