Project #5: Lognormal stock prices. Black-Scholes.

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Problem #1 (25 points)

Assume the **Black-Scholes model**. The continuously compounded, risk-free interest rate is 0.06. Let the current stock price of a non-dividend-paying stock be \$100. Let its volatility be 0.25.

(5 points) Simulate 1000 values of the stock price at time $t = 3$. Draw a histogram of the obtained set of values.

(6 points) Superimpose the graph of the density of a lognormal distribution of $S(3)$ with the appropriate parameters onto the histogram obtained above. The command `dlnorm` should come in handy.

(2 points) Find the median of the set of simulated values you obtained above.

(2 points) Find the mean of the set of simulated values you obtained above.

(2 points) What is the proportion of stock prices you created in this way that above \$105?

(5 points) What is the theoretical risk-neutral probability that the final asset price exceeds \$105?

(3 points) Comment on the comparison between the proportion and the risk-neutral probability you obtained above. Which theorem from probability is useful here?

Problem #2 (25 points)

Let the continuously compounded, risk-free interest rate be 0.05.

Consider a stock whose current price is \$120 and whose volatility is 0.20. We will be pricing a three-month, \$115-strike call option.

Part a: Analytic Black-Scholes (5 points)

Price the option above using the Black-Scholes pricing formula.

Part b: Black-Scholes Monte Carlo with Z (10 points)

Provide the *Monte Carlo* estimate of the price using the simulated draws from the standard normal distribution with 10000 simulations.

Part c: Black-Scholes Monte Carlo with 'rlnorm" (10 points)

Provide the *Monte Carlo* estimate of the price using the simulated draws from the lognormal distribution with 10000 simulations.

Problem #3 (25 points)

Let the continuously compounded, risk-free interest rate be 0.03.

Consider a stock whose current price is \$90 and whose volatility is 0.30. We will be pricing a half-year, \$95-strike put option.

Part a: Analytic Black-Scholes (5 points)

Price the option above using the Black-Scholes pricing formula.

Part b: Black-Scholes Monte Carlo with Z (10 points)

Provide the *Monte Carlo* estimate of the price using the simulated draws from the standard normal distribution with 10000 simulations.

Part c: Black-Scholes Monte Carlo with 'rlnorm" (10 points)

Provide the *Monte Carlo* estimate of the price using the simulated draws from the lognormal distribution with 10000 simulations.

Problem #4 (25 points)

Let the continuously compounded, risk-free interest rate be 0.04.

Consider a stock whose volatility is 0.25.

(5 points) First, we will be focusing on a one-year, \$100-strike European call option. Define a function which calculates the analytic Black-Scholes price of this call as a function of the initial asset price denoted by s. Then, plot the graph of that function. Let the domain of your plot be $[0, 200]$.

(10 points) Now, plot the graph of the above Black-Scholes price (as a function of the underlying stock price) one year, half a year, quarter year and a week prior to exercise date. Add the plot of the payoff function. Let the domain of your plot be $[50, 150]$. Make sure that all the plots are in different colors. What do you notice as the exercise date approaches? What can you say about the call delta?

(10 points) Repeat the previous problem for an otherwise identical put option.