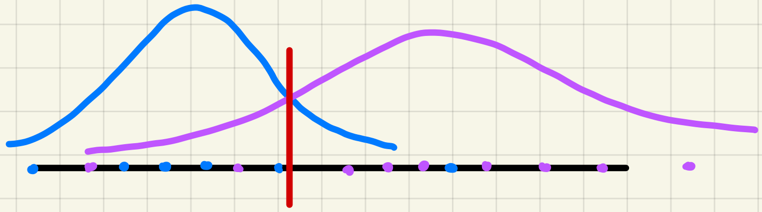
M339G: March 24th, 2025.



Bivariate Normal in Matrix Notation.

Consider a bivariate normal pair (U,V).

In 2D, we can place the means into a vector

$$\mu := \begin{pmatrix} \mu_U \\ \mu_V \end{pmatrix}$$

anything in $\mathbb{R}^2$

The variances/covariances are placed into a matrix

$$\Sigma = \begin{bmatrix} \sigma_U^2 & \sigma_U \cdot \sigma_V \rho \\ \sigma_U \cdot \sigma_V \cdot \rho & \sigma_V^2 \end{bmatrix}$$

(positive definite)

In 1D

$$f_U(u) = \frac{1}{\sigma_U \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{u-\mu_U}{\sigma_U}\right)^2}$$

Then, the joint density of (U,V) can be written as:

$$f_{U,V}(u,v) = \frac{1}{2\pi} \cdot \frac{1}{(\det(\Sigma))^{1/2}} \exp\left(-\frac{1}{2} \underbrace{\begin{pmatrix} u-\mu_U \\ v-\mu_V \end{pmatrix}^T}_{1\times 2} \underbrace{\Sigma^{-1}}_{2\times 2} \underbrace{\begin{pmatrix} u-\mu_U \\ v-\mu_V \end{pmatrix}}_{2\times 1}\right)$$

Multivariate Normal Density.

Let $\mathbb{X} = (X_1, X_2, \dots, X_p)^T$ be

$$\text{Normal}\left(\text{mean} = \mu = (\mu_1, \mu_2, \dots, \mu_p)^T,\ \Sigma = \begin{bmatrix} \sigma_1^2 & \text{Cov} & \text{Cov} & \text{Cov} \\ \text{Cov} & \ddots & & \vdots \\ \text{Cov} & & \ddots & \\ \text{Cov} & \cdots & & \sigma_n^2 \end{bmatrix}\right)$$

w/ Σ positive definite.

Then,

$$\boxed{f_{\mathbb{X}}(\underbrace{x_1, \dots, x_p}_{x}) = \frac{1}{(2\pi)^{p/2}} \cdot \frac{1}{(\det(\Sigma))^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right) \quad \text{for all } x \in \mathbb{R}^p}$$