M358K: October 20^{th}, 2021.

Test of Significance.

Set α ... significance level

Typically: $\alpha = \boxed{0.05}, 0.01, 0.10$

Decision process:

If p·value $\leq \alpha$, we REJECT the null hypothesis.

If not, we FAIL TO REJECT the null hypothesis.

Note: The p·value corresponding to an observed value of the test statistic is the LOWEST significance level @ which the null hypothesis would still be REJECTED.

Given a significance level α, we can construct (ahead of data gathering) a $\boxed{\text{REJECTION REGION (RR)}}$ for our test.

The null hypothesis: $H_0: \mu = \mu_0$

The Test statistic: $\bar{X}$ (the sample mean)

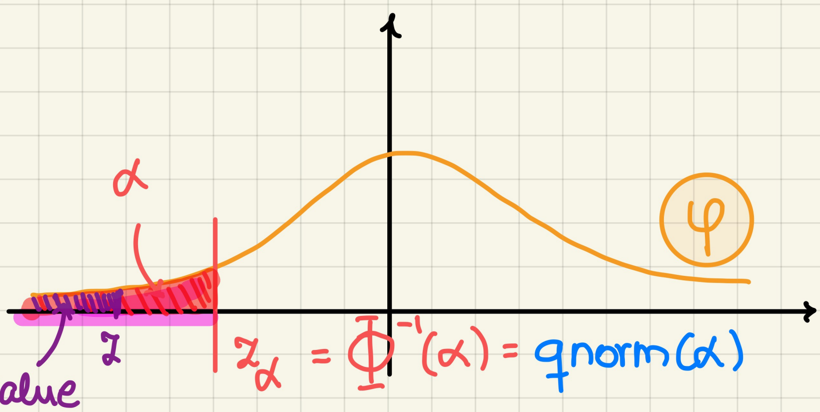
The observed value of the TS is $\bar{x}$.

Its z·score (under the null): $\boxed{z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}}$ ✓

The left·sided alternative:

$$H_a: \mu < \mu_0$$

In standard units:



Rejection Region (RR): $(-\infty, z_\alpha]$

In raw units:

$$z \le z_\alpha$$

$$\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} \le z_\alpha$$

$$\bar{x} \le \boxed{\mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}}$$

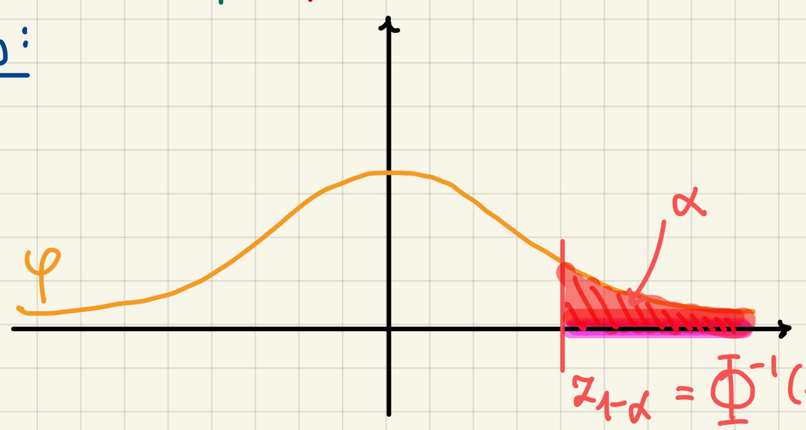
upper bound of the RR in raw units

RR: $\left(-\infty, \mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}\right]$

The right-sided alternative.

$H_a: \mu > \mu_0$

In standard units:



$z_{1-\alpha} = \Phi^{-1}(1-\alpha)$

$= \text{qnorm}(1-\alpha)$

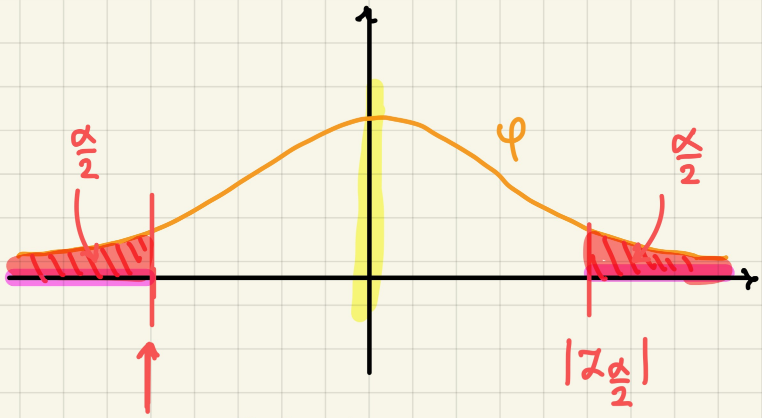
RR: $[z_{1-\alpha}, +\infty)$

In raw units: RR: $\left[\mu_0 + z_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}}, +\infty\right)$

The two-sided alternative:

$H_a: \mu \neq \mu_0$

In standard units:



$z_{\frac{\alpha}{2}} = \Phi^{-1}\left(\frac{\alpha}{2}\right) = \text{qnorm}\left(\frac{\alpha}{2}\right)$

$RR: \left(-\infty, z_{\frac{\alpha}{2}}\right] \cup \left[|z_{\frac{\alpha}{2}}|, +\infty\right)$

In raw units:

$RR: \left(-\infty, \mu_0 + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right] \cup \left[\mu_0 + |z_{\frac{\alpha}{2}}| \cdot \frac{\sigma}{\sqrt{n}}, +\infty\right)$