Problem 2.5. An insurance policy on a ground-up loss X has:

- no deductible;
- a coinsurance of 50%, and $\alpha = 0.5$
- a maximum policy payment per loss of 5000

Let X be modeled using a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 10000$. What is the expected payment per loss for the insurer?

$X \sim Pareto(\alpha = 2, \theta = 10000)$

$\rightarrow$: $\mathbb{E}[Y^L] = \alpha \cdot \mathbb{E}[X \wedge u]$

by our Theorem

Q: How much is u?

$\alpha \cdot u = 5000$

$0.5\, u = 5000 \Rightarrow \boxed{u = 10000}$

$\mathbb{E}[Y^L] = 0.5\, \mathbb{E}[X \wedge 10000]$

$$\mathbb{E}[X \wedge 10000] = \frac{10000}{2-1}\left(1 - \left(\frac{10000}{10000+10000}\right)^{2-1}\right)$$

$$= 10000\left(1 - \frac{1}{2}\right) = 5000$$

$\mathbb{E}[Y^L] = 0.5 \cdot 5000 = 2500$ ∎

Problem 2.6. *Source: Sample STAM Problem #279.*
Loss amounts have the distribution function

$$\boxed{F_X(x) = \begin{cases} \left(\frac{x}{100}\right)^2, & 0 \le x \le 100 \\ 1, & x > 100 \end{cases}}$$

$d=20$

An insurance pays 80% of the amount of the loss in excess of an ordinary deductible of 20, subject to a maximum payment of 60 per loss. Calculate the conditional expected claim payment, given that a payment has been made, i.e., the expectation of the per payment random variable.

$\longrightarrow$: $\mathbb{E}[Y^P] = ?$

$$\mathbb{E}[Y^P] = \mathbb{E}[Y^L \mid X > 20] = \frac{\boxed{\mathbb{E}[Y^L]}}{\boxed{S_X(20)}}$$

$$S_X(20) = 1 - \left(\frac{20}{100}\right)^2 = 0.96$$

$$\mathbb{E}[Y^L] = \alpha\left(\mathbb{E}[X \wedge u] - \mathbb{E}[X \wedge d]\right)$$

$u = ?$

$$0.8(u-20) = 60 \quad \Rightarrow \quad u = 95$$

For a constant $c \in (0,100)$:

$$\underline{\mathbb{E}[X \wedge c]} = \int_0^c S_X(x)\,dx = \int_0^c \left(1 - \frac{x^2}{10^4}\right) dx$$

$$= \left[x - \frac{1}{10^4} \cdot \frac{x^3}{3}\right]_{x=0}^{c}$$

$$= c - \frac{c^3}{3 \cdot 10^4}$$

$$\mathbb{E}[Y^L] = 0.8\left(95 - \frac{95^3}{3 \cdot 10^4} - \left(20 - \frac{20^3}{3 \cdot 10^4}\right)\right)$$

$$= 37.35$$

$$\Longrightarrow \mathbb{E}[Y^P] = \frac{37.35}{0.96} = 38.90625$$

Poisson Distribution.

Usually : $N \sim \text{Poisson}(\lambda)$

Support : $\mathbb{N}_0 = \{0, 1, 2, \dots\}$

We say that any r.v. w/ this support is $\mathbb{N}_0$-valued,

The probability mass function: $p_N(k) := p_k = \boxed{e^{-\lambda} \cdot \frac{\lambda^k}{k!}}$ for all $k \in \mathbb{N}_0$

The probability generating function:

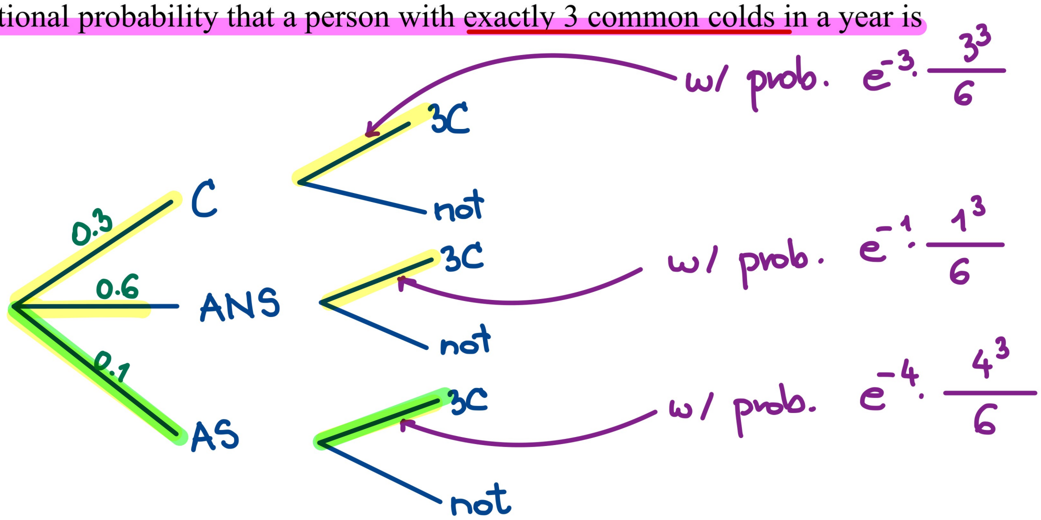
$$P_N(z) := \mathbb{E}[z^N] = e^{\lambda(z-1)}$$

$$\boxed{\mathbb{E}[N] = \lambda \quad \text{and} \quad \text{Var}[N] = \lambda}$$

170. In a certain town the number of common colds an individual will get in a year follows a Poisson distribution that depends on the individual's age and smoking status. The distribution of the population and the mean number of colds are as follows:

	Proportion of population	Mean number of colds
Children	0.30	3 $\lambda_C = 3$
Adult Non-Smokers	0.60	1 $\lambda_{ANS} = 1$
Adult Smokers	0.10	4 $\lambda_{AS} = 4$

Calculate the conditional probability that a person with exactly 3 common colds in a year is an adult smoker.



(A) 0.12

(B) 0.16

(C) 0.20

(D) 0.24

(E) 0.28

171. For aggregate losses, S:

$\mathbb{P}[AS \mid 3C] = ?$

(i) The number of losses has a negative binomial distribution with mean 3 and variance 3.6.

(ii) The common distribution of the independent individual loss amounts is uniform from 0 to 20.

Calculate the 95th percentile of the distribution of S as approximated by the normal distribution.

(A) 61

(B) 63

(C) 65

(D) 67

(E) 69

Bayes' Theorem:

$$\mathbb{P}[AS \mid 3C] = \frac{\mathbb{P}[AS \cap 3C]}{\mathbb{P}[3C]} =$$

$$= \frac{0.1 e^{-4} \cdot \frac{4^3}{\cancel{6}}}{0.3 \cdot e^{-3} \cdot \frac{27}{\cancel{6}} + 0.6 \cdot e^{-1} \cdot \frac{1}{\cancel{6}} + 0.1 \cdot e^{-4} \cdot \frac{4^3}{\cancel{6}}}$$

$$= 0.1581 . \quad \blacksquare$$