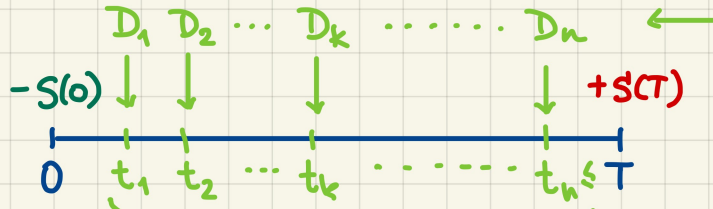
M339D: January 26th, 2022.

Outright Purchase of One share of stock.

Case#1. No Dividends.

Case#2. Discrete Dividends.



Dividend Amounts.
- expressed per share
- deterministic

One would be interested in:

$$PV(Div) = \sum_{k=1}^{n} D_k e^{-r t_k}$$

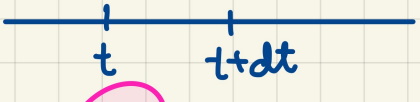
r... ccrfir

Case#3. Continuous Dividends.

δ ... dividend yield

The dividend amount paid to a shareholder during the time interval $(t, t+dt)$ is given by

$$\delta S(t)dt \quad \text{per share owned.}$$



observe: $S(t)$

Q: How does on calculate the total nominal amount of divided paid during $[0,T]$?

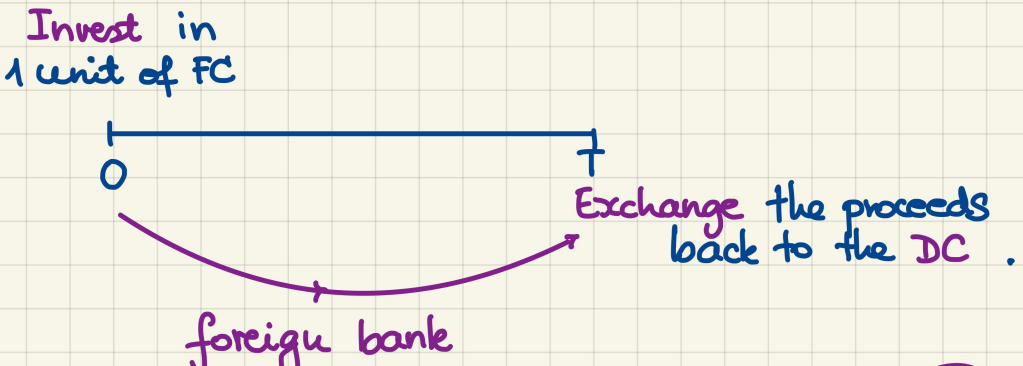
$\longrightarrow$: $\int_0^T \delta S(t)\, dt$ ← stochastic process

Foreign Currencies.

- **Domestic Currency (DC).** ... its continuously compounded, risk-free interest rate is denoted by r_D
- **Foreign Currency (FC).** ... its ccrfir is denoted by r_F

$x(t)$, $t \geq 0$ ... the exchange rate from FC to DC,
i.e., $x(t)$ is the number of units of the DC we have to pay @ time t to acquire one unit of FC

Investment time-line:



At time 0:
- Buy 1 unit of FC. => Spend $x(0)$.
- We deposit that 1 unit of the FC to earn interest @ the ccrfir r_F.

At time T:
- The balance in the account is $e^{r_F \cdot T}$ in the FC.
- We exchange the balance back to the DC
  => Get $\boxed{x(T)e^{r_F \cdot T}}$.

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Problem set 2

Foreign currencies.

Problem 2.1. Paige lives in Great Britain and receives her salary in GBP. She decides to spend 680 GBP and let the proceeds of the exchange accrue interest at the USD continuously compounded risk-free interest rate .

(i) Given that the initial exchange rate is 0.68 GBP per USD, how much (in USD) does Paige receive initially?

(ii) Given that the USD continuously compounded risk-free interest rate is equal to $r_{\$} = 0.02$, what is the balance in Paige's account six months after the initial transaction? Assume that there were no intermediate deposits or withdrawals.

(iii) Paige decides to withdraw the balance in her account at that time (still six months from the initial exchange) and exchange it back to GBP. Given that the exchange rate at that time equals 0.71 GBP per USD, how much (in GBP) does Paige receive?

(iv) Given that the GBP continuously compounded risk-free interest rate equals $r_{£} = 0.03$, what would have Paige's balance have been had she decided to simply deposit her initial investment in a GBP savings account?

$= 0.68\ £/\$$

(i) $680\ £ \left(\frac{1}{0.68\ £/\$}\right) = 1000\ \$$

(ii) $1000\, e^{0.02(0.5)} = 1000 e^{0.01} = 1010.05$

(iii) $1000 e^{0.01}(0.71) = 710 e^{0.01} \approx 717.14$

(iv) $680\, e^{0.03(0.5)} = 680 e^{0.015} \approx 690.28$

Problem 2.2. You are given the following information:

- the current exchange rate is 8.71 Swedish Kronor (SEK) per USD;
- the SEK continuously compounded risk-free interest rate equals 0.04;
- the USD continuously compounded risk-free interest rate equals 0.02.

Niklas wants to have 10,000 USD exactly one year from now. He is going to buy USD today and deposit the proceeds in a USD saving account. He does not intend to make any withdrawals or deposits prior to the end of the one-year period. How many Swedish Kronor does Niklas need to spend in SEK today in order to purchase just enough USD so that the final balance in his USD savings account equals $10,000$?

→: Q: How much in USD should he deposit today?

$10000\,e^{-0.02}$

Q: How much in SEK does that cost?

$8.71 \cdot 10000\,e^{-0.02} =$ _____