M378K Introduction to Mathematical Statistics Problem Set #11 Order Statistics.

Problem 11.1. An insurance company is handling claims from two categories of drivers: the good drivers and the bad drivers. The waiting time for the first claim from a **good** driver is modeled by an exponential random variable T_g with mean 6 (in years). The waiting time for the first claim from a **bad** driver is modeled by an exponential random variable T_b with mean 3 (in years). We assume that the random variables T_g and T_b are independent.

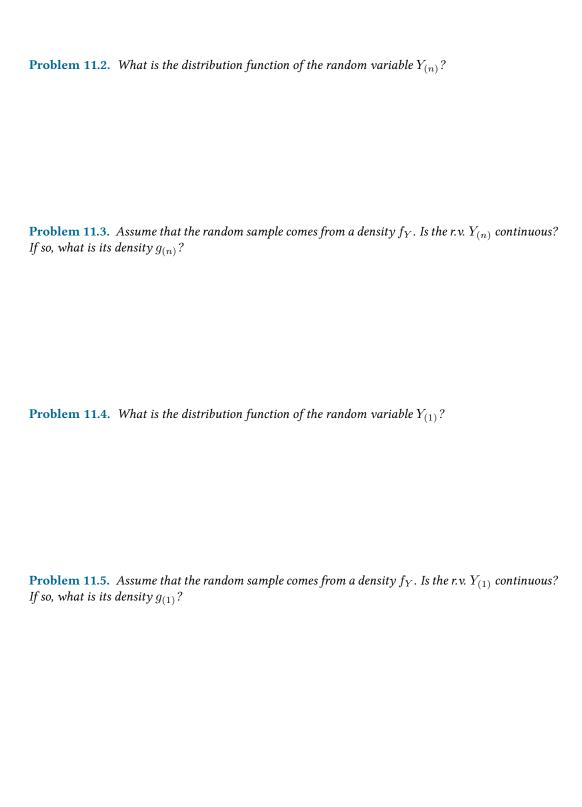
What is the distribution of the waiting time T until the first claim occurs (regardless of the type of driver this claim was filed by)?

Definition 11.1. Let Y_1, \ldots, Y_n be a **random sample**. The random sample ordered in an increasing order is called an order statistic and denoted by

$$Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}.$$

Question Write $Y_{(1)}$ as a function of Y_1, Y_2, \ldots, Y_n .

Question Write $Y_{(n)}$ as a function of Y_1, Y_2, \dots, Y_n .



Theorem 11.2. Lt Y_1, \ldots, Y_n be independent, identically distributed random variables with the common cumulative distribution function F_Y and the common probability density function f_Y . Let $Y_{(k)}$ denote the k^{th} order statistic and let $g_{(k)}$ denote its probability density function. Then,

$$g_{(k)}(y) = \frac{n!}{(k-1)!(n-k)!} (F_Y(y))^{k-1} f_Y(y) (1 - F_Y(y))^{n-k} \quad \text{for all } y \in \mathbb{R}.$$