M339D: February 9th, 2022.

Forward Contracts [cont'd].

* A binding contract on both sides! *

0 ——————————— T

0: An agreement, but NO cashflows!

$\Downarrow$

Initial Cost = 0

$\Downarrow$

$\boxed{\text{Profit} = \text{Payoff}}$

T: $\boxed{\text{The Delivery Date}}$

Long Forward : Buying Forward

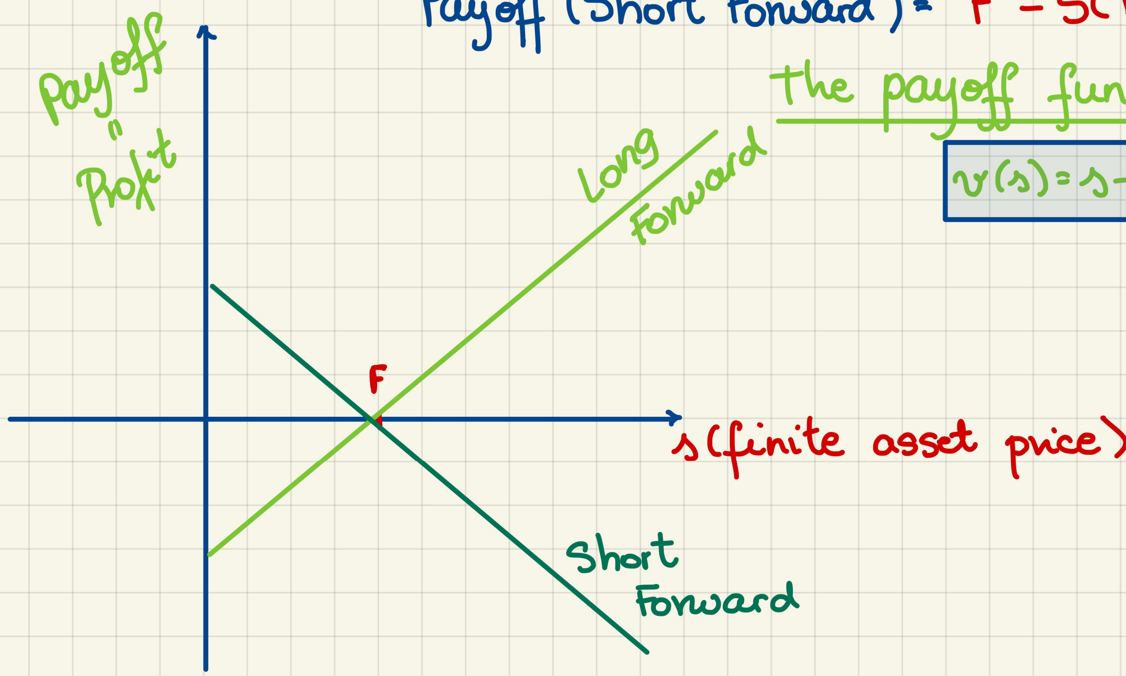
1 unit of asset $S(T)$ $\uparrow$ $\downarrow$ Forward Price F

Short Forward : Selling Forward

$$\left.\begin{aligned} \text{Payoff (Long Forward)} &= S(T) - F \\ \text{Payoff (Short Forward)} &= F - S(T) \end{aligned}\right\}$$

The payoff function:

$\boxed{v(s) = s - F}$



Example. [IFM Sample Problem # 56 from Part I: Intro]

Determine which of the following portfolios has the same cashflows as a short position in a non-dividend-paying stock.

0 ———————————— S(T)

Init. Cost: $-S(0)$ Payoff: $-S(T)$

× (i) long forward and a long zero-coupon bond

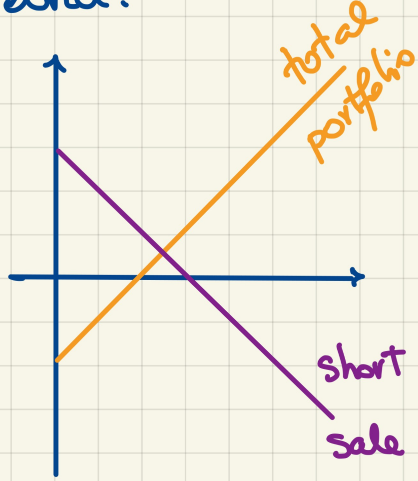
Init. Cost: Price of bond > 0

× (ii) long forward and a short forward

Init. Cost: $= 0$

× (iii) long forward and a short zero-coupon bond.

	Init. Cost.	Payoff.
Long forward	0	$S(T)-F$
short zero-coupon	$-P$	$-Pe^{rT}$
Total	$-P$	$S(T)-F-Pe^{rT}$



× (iv) short forward and long zero-coupon bond

Init. Cost: $P > 0$

↑ price of bond

(v) short forward and a short zero-coupon bond

	Init. Cost	Payoff
short forward	0	$F-S(T)$
short bond	$-P$	$-Pe^{rT}$
total	$-P$	$F-S(T)-Pe^{rT}$

short sale: $-S(0)$ $-S(T)$

If we let the bond have the redemption amount such that

$$\boxed{F = Pe^{rT}}$$

and also we have that the bond's price is

$$\boxed{P = S(0)}$$

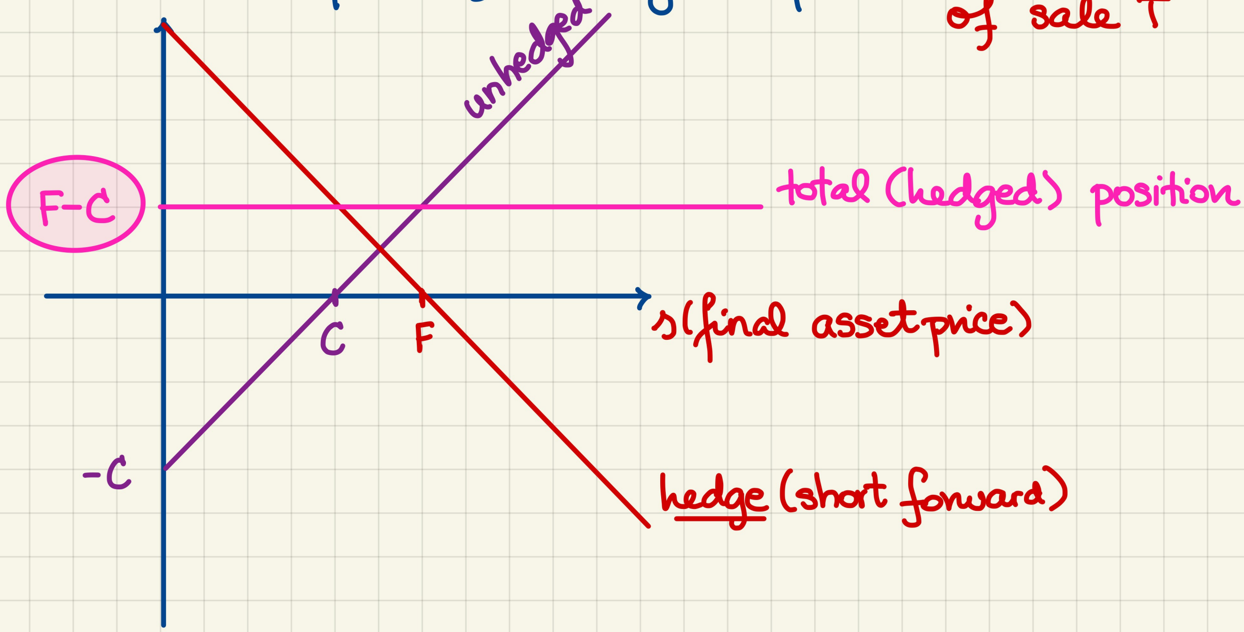
Note: $\boxed{F = S(0)e^{rT}}$ Think on this conclusion!

Hedging Using Forward Contracts.

Example. [Producer of Goods]

C... total aggregate cost of production per unit of good valued @ the time of sale T

S(T)... the market price of the good per unit @ the time of sale T



The appropriate hedge allows the producer to sell forward, i.e., the enter a short forward.

Assume $F > C$.

Algebraically: Profit (unhedged) + Profit (hedge) = Profit (total hedged)

$$S(\cancel{T}) - C + F - S(\cancel{T}) = F - C$$