M339D: March 3rd, 2023.

Strong Law of Large Numbers (SLLN).

Let $\{X_k, k=1,2,\dots\}$ be a sequence of independent, identically distributed r.v.s.

Assume: $\mu_X := \mathbb{E}[X_1] < \infty$

Then,

$$\boxed{\frac{X_1 + X_2 + \dots + X_n}{n} \xrightarrow[n\to\infty]{} \mu_X}$$

If a f'tion g is such that $g(X_1)$ is well-defined, and $\mathbb{E}[g(X_1)] < \infty$,

then

$$\boxed{\frac{g(X_1) + g(X_2) + \dots + g(X_n)}{n} \xrightarrow[n\to\infty]{} \mathbb{E}[g(X_1)]}$$

Monte Carlo.

Recipe:
- Draw simulated values of a random variable from a distribution.
- Apply a function to the simulated values.
- Calculate the arithmetic average of the obtained quantities.

We get the value is "close to" the theoretical expected value.

About Precision:

- $$\mathrm{Var}\left[\frac{X_1 + \dots + X_n}{n}\right] = \frac{1}{n^2}\mathrm{Var}[X_1 + \dots + X_n] \quad \text{(independence)}$$
$$= \frac{1}{n^2}\left(\mathrm{Var}[X_1] + \dots + \mathrm{Var}[X_n]\right) \quad \text{(identically dist'n)}$$
$$= \frac{1}{n^{\cancel{2}}}\cdot \cancel{n}\cdot \mathrm{Var}[X_1] = \frac{\mathrm{Var}[X_1]}{n}$$

- $$SD\left[\frac{X_1 + \dots + X_n}{n}\right] = \frac{SD[X_1]}{\sqrt{n}}$$

To increase the precision by a factor η, the number of variates must increase by a factor of η^2.