M358K: November 8^{th}, 2021.

Problem 17.3. A simple random sample of 60 households in Whoville is taken. In the sample, there are 45 households that decorate their houses with lights for the holidays. p_1 $\hat{p}_1 = 45/60 = 0.75$

A simple random sample of 50 households is also taken from the neighboring Whoburgh. In the sample, there are 40 households that decorate their houses. p_2 $\hat{p}_2 = 40/50 = 0.80$

(i) What is a 95% confidence interval for the difference in population proportions of households that decorate their houses with lights for the holidays?

$p_1 - p_2$

pt. estimate ± margin·of·error

z^* = 1.96, se

$$\boxed{\hat{p}_1 - \hat{p}_2}$$

$0.75 - 0.80$

-0.05

$$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$\underbrace{\sqrt{\frac{0.75(0.25)}{60} + \frac{0.80(0.20)}{50}}}_{0.0795}$$

$$p_1 - p_2 = -0.05 \pm 1.96 \cdot 0.0795 = \boxed{-0.05 \pm 0.1558}$$

(ii) If you want to test the hypothesis whether one of the two cities has more festive inhabitants, i.e., whether one of the two cities has a higher proportion of decorated domiciles or not, what p–value would you obtain?

$$H_0: p_1 = p_2 \quad \text{vs} \quad \boxed{H_a: p_1 \neq p_2}$$

Under the null: $\boxed{p_1 = p_2 = p}$

$$\hat{p} = \frac{45+40}{60+50} = \frac{85}{110} = 0.77$$

$$\Rightarrow se_0 = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{0.77(0.23)\left(\frac{1}{60}+\frac{1}{50}\right)}$$

$$se_0 = 0.0806$$

$$\Rightarrow z = \frac{\hat{p}_1 - \hat{p}_2}{se_0} = \frac{0.75-0.80}{0.0806} = -0.6203$$

$\Rightarrow$ p-value:

$$2\left(1-\Phi(|z|)\right) =$$

$$= 2\cdot\Phi(-|z|) =$$

$$= 2 * \text{pnorm}(-0.6203) = 0.5351$$

$\blacksquare$

χ^2-distribution.

The following definition of the χ^2-distributed random variable can be extended.
For our purposes:

Let $Z_1, Z_2, \dots, Z_r$ be independent, standard normal random variables.
Define:

$$X = Z_1^2 + Z_2^2 + \cdots + Z_r^2 .$$

We say that X has the χ^2-distribution w/ r degrees of freedom (parameter).

We write:

$$X \sim \chi^2(\underbrace{df}_{\text{also denoted by } \nu \text{ "nu"}} = r)$$

Example. Let $X \sim \chi^2(df=5)$.

Find $\mathbb{P}[1.145 \le X \le 12.83] = ?$

$\longrightarrow$: $\mathbb{P}[X \le 12.83] - \mathbb{P}[X \le 1.145]$

$= F_X(12.83) - F_X(1.145)$

1st R: pchisq(12.83, df=5)

-pchisq(1.145, df=5) = 0.92502 ✓

2nd Tables: $0.975 - 0.05 = 0.925$ ✓