M339J/M389J *Probability Models*
University of Texas at Austin
HW #11
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Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 11.1. (5 points) You want to fit to the observed values

$$4, 5, 7$$

a two-parameter Pareto distribution with parameters $\alpha = 4$ and θ unknown using maximum likelihood estimation. Write down **clearly** an **explicit** expression for the loglikelihood function (of course, as a function of θ).

Problem 11.2. (10 points) Consider the following individual observed values:

$$5, 8, 10$$

of a random variable Y such that $Y = X^{-1}$ with $X \sim Gamma(\alpha = 2, \theta)$.

Calculate $\hat{\theta}_{MLE}$, the Maximum Likelihood Estimate of θ based on the above observed values.

Problem 11.3. (10 points) A sample of n independent observations

$$x_1, x_2, \ldots, x_n$$

came from a distribution with the probability density function $f_x(x) = 2\theta e^{-\theta x^2}, x > 0$. Determine the maximum likelihood estimator of θ.

Problem 11.4. (10 points) Consider a random variable Y such that $Y = e^X$ with $X \sim Gamma(\alpha = 2, \theta)$. Your colleague was playing with the collected data and the only things you still know about the observations from Y are:

(i) There was a total of 20 observations;
(ii) The product of all observations was $5,000$.

Find $\hat{\theta}_{MLE}$, i.e., the Maximum Likelihood Estimate of θ based on the observed values.

Problem 11.5. (5 points) Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with an unknown mean θ. Consider these as individual, unmodified data. What is the expression for the maximum likelihood estimator of θ denoted by $\hat{\theta}_{MLE}$?

Problem 11.6. (10 points) *Source: Sample STAM problem #179.* The time to an accident follows an exponential distribution. A random sample of size two has a mean time of 4. Let Y denote the average of a new sample of size two from the same distribution. Calculate the maximum likelihood estimate of $\mathbb{P}[Y > 8]$.

Hint: Remember that the sum of independent, identically distributed exponential random variables has a gamma distribution. You can convince yourselves of this fact using moment generating functions.