Project #1

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```r
library(nimble)
```

Problem #1 (55 points)

Download the historical stock prices of *Walmart* for the last 252 (or so) trading days, i.e., for a one-year period from our course website. Do the same for *IBM*.

(5 points) Draw the time-plot of the evolution of the closing stock prices (not the adjusted) for both of the stocks on the **same coordinate system**. You do **not** need to put the calendar days on the horizontal axis, but you **do** need to label your axes and give your time-plot a title indicating the dates. Make sure that you plot the two trajectories in **different** colors indicating in the text which color corresponds to which company. Include the legend in your plot.

The **simple daily return** of the stock over a day indexed by t is defined as

$$\frac{\text{price at end of day } t - \text{price at end of day } (t-1)}{\text{price at end of day } (t-1)}$$

(5 points) Construct the vector of simple daily returns over the last year for both stocks. With R denoting the daily simple return, the *daily volatility* of the stock is defined as the standard deviation of R. **Assuming independent, identically distributed** daily returns, what is the **daily volatility** point estimate for the daily volatility of both stocks?

(10 points) If you wanted to study the *relationship* between the returns of the two stocks, which plot would you create? What (if anything) can you say after looking at the plot?

(5 points) If you wanted to provide one single, unitless quantity which provided you a measure of association between the returns of *Walmart* and *IBM*, which value would you report?

(5 points) You create a portfolio in which half your wealth is maintained in the *Walmart* stock and the remaining half of your wealth is kept in the *IBM* stock. What is your estimate of the daily volatility of this portfolio?

(25 points) You create a portfolio in which the weight w of your wealth is maintained in the *Walmart* stock and the remainder of your wealth is kept in the *IBM* stock. Create a function in `R` which calculates the daily volatility of this portfolio as it depends on w? Plot the portfolio's volatility as a function of w for w ranging from -2 to 2. **Yes, it's perfectly acceptable to have negative weights; if you want to know why, come ask in office hours.** In the same plot, add two horizontal lines at the values of the daily volatilities of *Walmart* and *IBM*. Now, review the optimal two-stock portfolio work we did in the first week of classes. Add the horizontal line corresponding to the optimal portfolio whose *Walmart* weight is $\hat{w} = \hat{\alpha}$ given by Hastie and Tibshirani. Add the vertical line corresponding to the optimal $\hat{w}$.

Problem #2 (45 points)

Download the historical prices for the *NASDAQ Composite* index for the **maximum** time period and at the **monthly** frequency available on our course website.

(5 points) Use the above definitions of returns, changing the period length to a month (rather than a day). Assuming, as usual **independent, identically distributed** monthly returns, what is your *point estimate* for the mean monthly return of *NASDAQ*?

(5 points) Assuming, as usual **independent, identically distributed** monthly returns, what is your *point estimate* for the monthly volatility of *NASDAQ*?

(20 points) You know that there is uncertainty in point estimation due to sampling variability. So, you want to provide a **confidence interval**.

For more about confidence intervals in general, please watch:

Confidence intervals

A follow up video for the mean parameter is

Inference for the mean

Let n stand for the number of observations. Let $\hat{\mu}$ denote our point estimate for the mean parameter. Let s denote the sample standard deviation. At a confidence level C, the structure of a **confidence interval** is

$$(\hat{\mu} - \frac{s}{\sqrt{n}}z^*, \hat{\mu} + \frac{s}{\sqrt{n}}z^*)$$

where z^* is a critical value of the standard normal distribution such that $\mathbb{P}[-z^* < Z < z^*] = C$ with $Z \sim N(0,1)$.

You should input `?qnorm` into the console in `R` to learn more about the different functions which have to do with the normal distribution in `R`.

Provide confidence intervals for the monthly mean return at the confidence level $C = 0.95$.

(5 points) There is a "shortcut" for constructing confidence intervals in this case which uses the `t.test` function. You should input `?t.test` into the console in `R` to learn more about it. Create the 95%−confidence interval using this command and compare to your result from the "pedestrian" implementation above.

(10 points) Finally, create a **bootstrap** $2SE$ confidence interval and a 95% percentile interval for the monthly mean return. Please, set your seed to 1 for comparison. Plot the histogram of the bootstrap point estimates for $N = 10^5$ draws. Indicate your confidence intervals on the histogram. Compare your confidence intervals to what you obtained above without resampling.