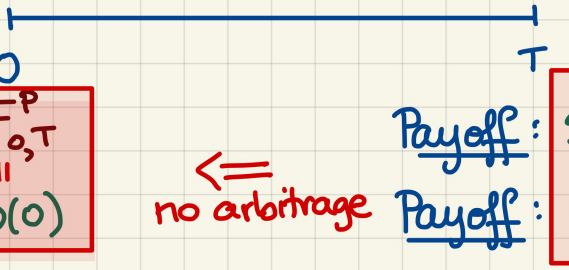
M339D : February 25th, 2022.

Focus on (Prepaid) Forwards on Stocks.

Goal: To figure out the expressions for the prepaid forward price and the forward price in terms of the more primitive "ingredients".

Case #1. NO DIVIDENDS.



Prepaid Forward: $F^P_{0,T}$

Outright Purchase: $S(0)$

$\Leftarrow$ no arbitrage

Payoff: $S(T)$

Payoff: $S(T)$

$$\Rightarrow \boxed{F^P_{0,T}(S) = S(0)}$$

Q: What is the dependence on the delivery date?

NONE!

$$\Rightarrow F_{0,T}(S) = FV_{0,T}(F^P_{0,T}(S)) = FV_{0,T}(S(0)) = S(0)e^{rT}$$

$r > 0$

Q: What is the dependence on the delivery date?

Increasing!

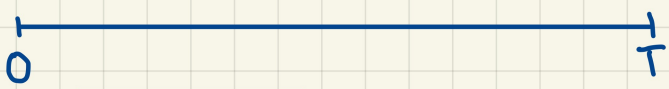
Q: Propose a replicating portfolio for a forward contract?

$\longrightarrow$: A fully leveraged purchase would replicate a forward.

Terminology. Any replicating portfolio of a forward contract is called a synthetic forward.

Case #2. CONTINUOUS DIVIDENDS.

δ... dividend yield



Prepaid Forward: Init. Cost: $F^P_{0,T}$

Outright Purchase: Init. Cost: $e^{-\delta T} \cdot S(0)$

$\Leftarrow$ no arbitrage

Payoff: $S(T)$

Payoff: $S(T)$

buy $e^{-\delta T}$ shares

own 1 share

$$F^P_{0,T}(S) = S(0)e^{-\delta \cdot T} \xrightarrow[T\to\infty]{} 0$$

Decreasing w/ the delivery date!

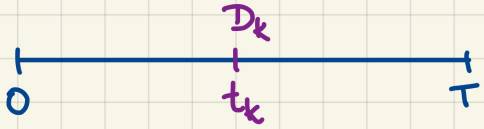
$$\Rightarrow F_{0,T}(S) = FV_{0,T}(S(0)e^{-\delta T}) = e^{r\cdot T}\cdot S(0)e^{-\delta T} = S(0)e^{(r-\delta)\cdot T}$$

↑ r... ccrfir

↑ The break-even point for purchase of stock!

The increase/decrease/contant of the forward price depends on the r & d!

Case #3. DISCRETE DIVIDENDS.



$k = 1..n$

$t_n \le T$

By convention, any dividend payable @ time-T is paid just prior to delivery.

$$F^P_{0,T}(S) = S(0) - PV(Div)$$

$$= S(0) - \sum_{k=1}^{n} PV_{0,t_k}(D_k) = S(0) - \sum_{k=1}^{n} D_k e^{-rt_k}$$

↑ r... ccrfir

The investor must be "compensated" for the loss of dividend until time-T.

$$F_{0,T}(S) = S(0)e^{rT} - \sum_{k=1}^{n} D_k e^{r(T-t_k)}$$

↑ r... ccrfir

27.

DELETED

28.

DELETED

29.

The dividend yield on a stock and the interest rate used to discount the stock's cash flows are both continuously compounded. The dividend yield is less than the interest rate, but both are positive. $r > \delta > 0 \Rightarrow r - \delta > 0$

The following table shows four methods to buy the stock and the total payment needed for each method. The payment amounts are as of the time of payment and have not been discounted to the present date.

METHOD	TOTAL PAYMENT
Outright purchase	A $= S(0)$
Fully leveraged purchase	B $= S(0)e^{rT}$
Prepaid forward contract	C $= S(0)e^{-\delta T}$
Forward contract	D $= S(0)e^{(r-\delta)T}$

Determine which of the following is the correct ranking, from smallest to largest, for the amount of payment needed to acquire the stock.

(A) C < A < D < B $C < A < D < B$

(B) A < C < D < B

(C) D < C < A < B

(D) C < A < B < D

(E) A < C < B < D

18.

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19.

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20.

r = 0.04

The current price of a stock is 200, and the continuously compounded risk-free interest rate is 4%. A dividend will be paid every quarter for the next 3 years, with the first dividend occurring 3 months from now. The amount of the first dividend is 1.50, but each subsequent dividend will be 1% higher than the one previously paid.

Calculate the fair price of a 3-year forward contract on this stock.

Steps:

1. PV(Div) geometric sequence.
2. $F^P_{0,T}(S) = S(0) - PV(Div)$
3. $F_{0,T}(S) = F^P_{0,T}(S) \cdot e^{rT}$

(A) 200
(B) 205
(C) 210
(D) 215
(E) 220

21.

A market maker in stock index forward contracts observes a 6-month forward price of 112 on the index. The index spot price is 110 and the continuously compounded dividend yield on the index is 2%.

The continuously compounded risk-free interest rate is 5%.

Describe actions the market maker could take to exploit an arbitrage opportunity and calculate the resulting profit (per index unit).

(A) Buy observed forward, sell synthetic forward, Profit = 0.34
(B) Buy observed forward, sell synthetic forward, Profit = 0.78
(C) Buy observed forward, sell synthetic forward, Profit = 1.35
(D) Sell observed forward, buy synthetic forward, Profit = 0.78
(E) Sell observed forward, buy synthetic forward, Profit = 0.34