

M339 Q: March 9th, 2022.

Options on Currencies.

- domestic currency (DC): r_D ...ccrfir for DC
 - foreign currency (FC): r_F ...ccrfir for FC

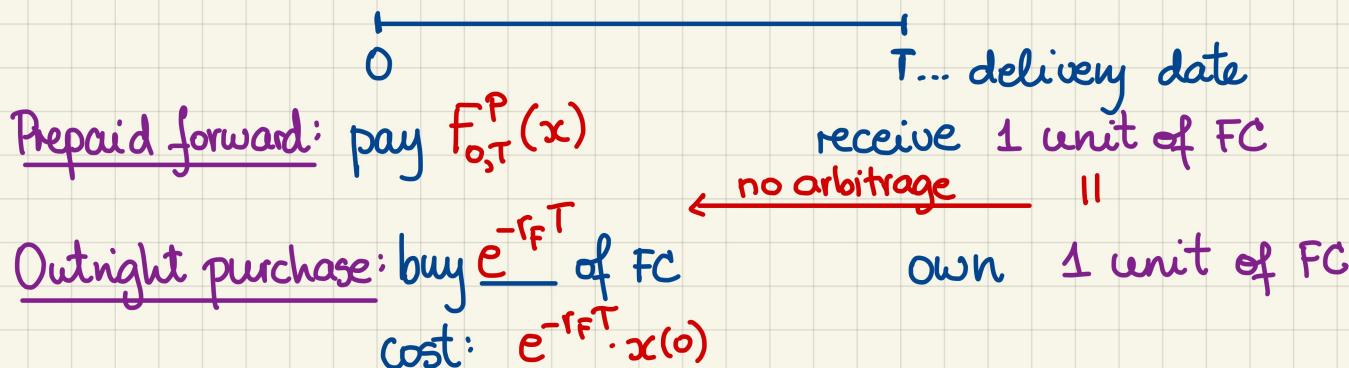
- underlying asset: foreign currency (FC)
 - the price of the underlying asset:

the exchange rate ; denoted by ;

$x(t), t \geq 0$

is the worth of 1 unit of the FC in terms of the DC

(Prepaid) forward on an FC.



$$F_{0,T}^P(x) = \underline{x(0)} e^{-r_F \cdot T}$$

... DC-denominated

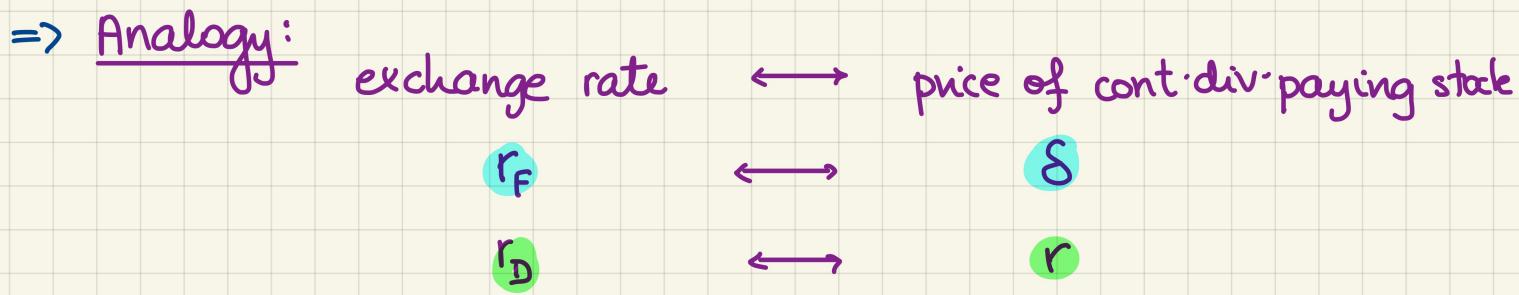
$$\Rightarrow F_{0,T}(x) = FV_{0,T}(F_{0,T}^P(x)) = e^{r_D \cdot T} \cdot x(0) e^{-r_F \cdot T}$$

$$F_{0,T}(x) = x(0) e^{(r_D - r_F) \cdot T}$$

Recall: For continuous dividend paying stocks:

$$F_{0,T}^P(s) = S(0) e^{-sT}$$

$$F_{\sigma,T}(s) = S(0)e^{(r-\delta)T}$$



European calls & puts

w/ strike price K and exercise date T

DC-denominated

Payoffs:

$$V_C(T) = (x(T) - K)_+$$

$$V_P(T) = (K - x(T))_+$$

Goal: Formulate the put-call parity for currency options!

- Portfolio A: • long call } both on the FC and
 • write put } otherwise identical
- Portfolio B: • long a prepaid forward on the FC w/
 delivery date T
 • borrow the $PV_{0,T}(K)$ @ the risk-free rate r_D
 to be repaid @ time T

$$V_A(T) = x(T) - K$$

$$\Downarrow V_B(T) = x(T) - K$$

\Rightarrow

no arbitrage

$$V_C(0) - V_P(0) = F_{0,T}^P(x) - PV_{0,T}(K)$$

$$= x(0) e^{-r_F \cdot T} - K e^{-r_D \cdot T}$$

Put-Call Parity

$$V_A(T) = V_C(T) - V_P(T) = (x(T) - K)_+ - (K - x(T))_+$$

$$= \begin{cases} (x(\tau) - K) & - 0 \\ 0 & - (K - x(\tau)) \end{cases} \quad \begin{array}{l} \text{if } x(\tau) \geq K \\ \text{if } x(\tau) < K \end{array}$$

$$= \begin{cases} x(\tau) - K \\ x(\tau) - K \end{cases} \quad \begin{array}{l} \text{if } x(\tau) \geq K \\ \text{if } x(\tau) < K \end{array}$$
$$= x(\tau) - K$$

stock options

(vanilla calls/puts)

shares ↓ ↑ cash

$$\Rightarrow \underline{\text{Call}}: V_c(\tau) = (S(\tau) - K)_+$$

$$\underline{\text{Put}}: V_p(\tau) = (K - S(\tau))_+$$

currency options

FC ↓ ↑ DC

$$\Rightarrow \underline{\text{Call}}: V_c(\tau) = (x(\tau) - K)_+$$

$$\underline{\text{Put}}: V_p(\tau) = (K - x(\tau))_+$$

Introduce a "new" option type:

stock #1 ↓ ↑ stock #2