Focus on the Delta.

value f'tion: $v(\boxed{s,t}, r, \sigma)$

Def'n. The Delta $\boxed{\Delta(s,t) := \frac{\partial}{\partial s} v(s,t)}$

Example. Outright Purchase of a Non-Dividend-Paying Stock.

$$v(s,t) = s$$

stands for the time-t stock price

$$\Rightarrow \boxed{\Delta(s,t) = 1}$$

Example. European Call.

$$v_c(s,t) = \boxed{s}\, N(d_1(s,t)) - Ke^{-r(T-t)} N(d_2(s,t))$$

w/ $d_1(s,t) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot(T-t)\right]$

and $d_2(s,t) = d_1(s,t) - \sigma\sqrt{T-t}$

By def'n: $\boxed{\Delta_c(s,t) = \frac{\partial}{\partial s} v_c(s,t)}$

After the chain rule and product rule:

$$\boxed{\Delta_c(s,t) = N(d_1(s,t))} > 0$$

The positivity makes sense since the call is long w.r.t. the underlying.

Example. European Put.

Put-Call Parity.

$$v_c(s,t) - v_p(s,t) = s - Ke^{-r(T-t)}$$

$\frac{\partial}{\partial s}\Big|$

$$\Delta_c(s,t) - \Delta_p(s,t) = 1$$

$$\boxed{\Delta_p(s,t) = \Delta_c(s,t) - 1 = N(d_1(s,t)) - 1 = -N(-d_1(s,t))} < 0$$

(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

$T = 1/4$ European

8. You are considering the purchase of a 3-month 41.5-strike ~~American~~ call option on a nondividend-paying stock.

$\boxed{K = 41.5}$

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40. $S(0) = 40$

(iii) The stock's volatility is 30%. $\sigma = 0.30$

(iv) The current call option delta is 0.5.

$$\Delta_c(S(0), 0) = \boxed{0.5 = N(d_1(S(0), 0))}$$

Determine the current price of the option. $\boxed{v_c(S(0), 0) = ?}$

(A) $20 - 20.453 \int_{-\infty}^{0.15} e^{-x^2/2} dx$

(B) $20 - 16.138 \int_{-\infty}^{0.15} e^{-x^2/2} dx$

(C) $20 - 40.453 \int_{-\infty}^{0.15} e^{-x^2/2} dx$

(D) $16.138 \int_{-\infty}^{0.15} e^{-x^2/2} dx - 20.453$

(E) $40.453 \int_{-\infty}^{0.15} e^{-x^2/2} dx - 20.453$

$$\boxed{d_1(S(0), 0) = 0}$$

$$d_2(S(0), 0) = d_1(S(0), 0) - \sigma\sqrt{T}$$

$$d_2(S(0), 0) = 0 - 0.3 \cdot \sqrt{0.25}$$

$$\boxed{d_2(S(0), 0) = -0.15}$$

$N(d_2(S(0), 0)) = \dots =$ the multiple choices are in integral form

$$v_c(S(0), 0) = S(0) \cdot \boxed{N(d_1(S(0), 0))} - Ke^{-rT} \cdot \boxed{N(d_2(S(0), 0))}$$

$$d_1(S(0), 0) = \frac{1}{\sigma\sqrt{T}} \boxed{\Big[\underbrace{\ln\Big(\frac{S(0)}{K}\Big) + \Big(r + \frac{\sigma^2}{2}\Big) \cdot T}_{=0}\Big]} = 0$$

$$\left(r+\frac{0.09}{2}\right)\cdot\frac{1}{4}=-\ln\left(\frac{40}{41.5}\right)=\ln\left(\frac{41.5}{40}\right)$$

$$r=4\cdot\ln\left(\frac{41.5}{40}\right)-0.045=\underline{0.10}$$