

UNIVERSITY OF TEXAS AT AUSTIN

Problem set 5

Problem 5.1. Which one is greater: 23% of 61 or 61% of 23?

Problem 5.2. (2 pts)

We define the minimum of two values in the usual way, i.e.,

$$\min(x, y) = \begin{cases} x & \text{if } x \leq y \\ y & \text{if } x \geq y \end{cases}$$

Then, for every x and y we have that

$$\min(x, y) = \min(x - y, 0) + y$$

True or false?

Problem 5.3. (2 pts)

We define the maximum of two values in the usual way, i.e.,

$$\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{if } x \leq y \end{cases}$$

Then, for every x and y we have that

$$-\max(x, y) = \max(x - y, 0) - x$$

True or false?

Problem 5.4. (2 pts)

We define the minimum of two values in the usual way, i.e.,

$$\min(x, y) = \begin{cases} x & \text{if } x \leq y \\ y & \text{if } x \geq y \end{cases}$$

We define the maximum of two values in the usual way, i.e.,

$$\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{if } x \leq y \end{cases}$$

Then, for every x and y we have that

$$\max(x, y) + \min(x, y) = x + y.$$

True or false?

Problem 5.5. Twelve people enter the elevator on the ground floor of a ten-story building and start riding up. By the time the elevator reaches the top floor, all the people have exited the elevator. What is the probability that at least two people exited the elevator on the same floor?

Problem 5.6. (5 pts) Let $\Omega = \{a_1, a_2, a_3, a_4\}$ be an outcome space, and let \mathbb{P} be a probability distribution on Ω . Assume that $\mathbb{P}[\{a_1\}] = 1/3$, $\mathbb{P}[\{a_2\}] = 1/6$ and $\mathbb{P}[\{a_3\}] = 1/9$. Then we have that $\mathbb{P}[\{a_4\}]$ equals the following value:

- (a) $1/3$
- (b) $2/3$
- (c) $7/18$
- (d) $7/9$
- (e) None of the above

Problem 5.7. Two dice are rolled, the probability that the sum of the upturned faces equals 7 is $1/6$. *True or false?*

Problem 5.8. (2 pts) Two dice are rolled, the probability that the maximum (and **not** necessarily a strict maximum) of the upturned faces is achieved on the second die equals $1/2$. *True or false?*

Problem 5.9. (5 pts) A class has 12 boys and 4 girls. If three students are selected at random from this class, what is the probability that they are all boys?

- (a) $1/4$
- (b) $5/9$
- (c) $11/28$
- (d) $17/36$
- (e) None of the above

Problem 5.10. A pair of dice is thrown. Find the probability that the sum of the outcomes is 10 or greater if a 5 appears on at least one of the dice.

- (a) $1/6$
- (b) $3/11$
- (c) $1/3$
- (d) $1/2$
- (e) None of the above

Problem 5.11. (5 pts) Find the probability of obtaining exactly two fives in six rolls of a fair die.

- (a) $5^5/(2^3 \cdot 3^6)$
- (b) $5^5/(2^6 \cdot 3^6)$
- (c) $5^5/(2^6 \cdot 3^5)$
- (d) $1/5$
- (e) None of the above

Problem 5.12. Four balls are drawn (without replacement) from a box which contains 4 black and 5 red balls. Given that the four drawn balls were *not* all of the same color, what is the probability that there were exactly two balls of each color among the four.

Problem 5.13. (5 pts) Roger deposits \$100 into an account at time 0.

For the following three years, he does not make any subsequent withdrawals or deposits and the account earns at a constant continuously compounded, risk-free interest rate r .

After 15 years and 6 months, the balance in his account equals \$133. Then,

- (a) $0 \leq r < 0.0150$
- (b) $0.0150 \leq r < 0.0250$
- (c) $0.0250 \leq r < 0.0550$
- (d) $0.0550 \leq r < 0.0650$
- (e) None of the above

Problem 5.14. (5 pts) *Source: Sample FM Problem #26.*

A 5-year loan for 10,000 is charged a nominal interest rate of 12% compounded semiannually.

The loan is to be repaid so that interest is repaid at the end of every 6 month period as it accrues and the principal is repaid in total at the end of the 5 years.

Denote the total amount of interest paid on this loan by I . Then

- (a) $I \approx 2,750$
- (b) $I \approx 3,000$
- (c) $I \approx 3,250$
- (d) $I \approx 3,500$
- (e) None of the above

Problem 5.15. To plant and harvest 20,000 bushels of corn, Farmer Jayne incurs total aggregate costs totaling \$33,000. The current spot price of corn is \$1.80 per bushel. What is the profit if the spot price is \$1.90 per bushel when she harvests and sells her corn?

- (a) About \$3,000 gain
- (b) About \$3,000 loss
- (c) About \$5,000 loss
- (d) About \$5,000 gain
- (e) None of the above

Problem 5.16. Assume that you open a 100–share short position in a common stock S when the bid-ask is \$100.00–\$101.00. When you close your position the bid-ask prices are \$99.50–\$100.00. Assume that you pay a commission rate of 1.00%. Calculate your (roundtrip) gain or loss on this short investment (assume $r = 0$)?

- (a) The investor breaks even; i.e., the gain/loss is 0.
- (b) About \$200 loss
- (c) About \$132.50 loss
- (d) About \$200 gain
- (e) None of the above