M339J: April 30th, 2021.

The Recursive Method.

N... frequency from $(a,b,0)$ class: $p_k = \left(a+\frac{b}{k}\right)\cdot p_{k-1}$

X... severity on the support $\{0,1,\dots,m\}$ (m could be $+\infty$)

use $f_X(x)$ for the pmf of X for $x\in\{0,1,\dots,m\}$

a recursion for $f_S(x)$, for $x\in\mathbb{N}_0$, $S = X_1+X_2+\dots+X_N$

- $f_S(0) = \mathbb{P}[S=0]$

$$= \mathbb{P}[S=0\mid N=0]\cdot\mathbb{P}[N=0] + \mathbb{P}[S=0\mid N=1]\cdot\mathbb{P}[N=1] + \dots + \mathbb{P}[S=0\mid N=k]\cdot\mathbb{P}[N=k] + \dots$$

$$= 1\cdot p_N(0) + f_X(0)\cdot p_N(1) + (f_X(0))^2\cdot p_N(2) + \dots + (f_X(0))^k\cdot p_N(k) + \dots$$

$$= \mathbb{E}\left[(f_X(0))^N\right] = P_N(f_X(0))$$

- $$f_S(x) = \frac{1}{1-a\cdot f_X(0)}\sum_{y=1}^{x\wedge m}\left(a+\frac{by}{x}\right)f_X(y)f_S(x-y) \quad \text{for } x=1,2,3,\dots$$

Special Case: If $f_X(0)=0$, then

- $f_S(0) = \mathbb{P}[N=0] = p_N(0)$
- $$f_S(x) = \sum_{y=1}^{x\wedge m}\left(a+\frac{by}{x}\right)f_X(y)f_S(x-y)$$

Special Case: $\boxed{\text{Compound Poisson}}$ w/ $f_X(0) = 0$

$a = 0$, $b = \lambda$

- $f_S(0) = p_N(0) = e^{-\lambda}$
- $f_S(x) = \frac{\lambda}{x} \sum_{y=1}^{x \wedge m} y \, f_X(y) \, f_S(x-y)$

8. Annual aggregate losses for a dental policy follow the compound Poisson distribution with $\lambda = 3$. The distribution of individual losses is:

$f_X(0)=0$

Loss	Probability
1	0.4
2	0.3
3	0.2
4	0.1

Calculate the probability that aggregate losses in one year do not exceed 3.

(A) Less than 0.20

(B) At least 0.20, but less than 0.40

(C) At least 0.40, but less than 0.60

(D) At least 0.60, but less than 0.80

(E) At least 0.80

$$\mathbb{P}[S \leq 3] = ? = f_S(0) + f_S(1) + f_S(2) + f_S(3)$$

$f_X(0)=0$ and $N \sim \text{Poisson}(\lambda=3)$ $\Rightarrow$ we can use the simplest recursive formula

Calculations:

- $f_S(0) = p_N(0) = e^{-3}$
- $f_S(1) = \frac{3}{1} \cdot 1 \cdot f_X(1) \cdot f_S(1-1) = 3 \cdot 0.4 \cdot e^{-3} = 1.2e^{-3}$
- $f_S(2) = \frac{3}{2} \cdot \left(1 \cdot f_X(1) \cdot f_S(2-1) + 2 \cdot f_X(2) \cdot f_S(2-2)\right)$

$$= \frac{3}{2}\left(0.4 \cdot 1.2e^{-3} + 2 \cdot 0.3 \cdot e^{-3}\right) = 1.62 \cdot e^{-3}$$

- $f_S(3) = \frac{3}{3}\left(1 \cdot f_X(1) f_S(3-1) + 2 \cdot f_X(2) f_S(3-2) + 3 \cdot f_X(3) \cdot f_S(3-3)\right)$

$$f_S(3) = 0.4 \cdot 1.62e^{-3} + 2 \cdot 0.3 \cdot 1.2e^{-3} + 3 \cdot 0.2 \cdot e^{-3}$$

$$= \underline{1.968 \quad e^{-3}}$$

$$\Rightarrow \underline{P[S \leq 3] = 0.28817}$$

$$\overset{\shortparallel}{F_S(3)}$$

95. The number of claims in a period has a geometric distribution with mean 4. The amount of each claim X follows $\Pr(X = x) = 0.25$ $x = 1, 2, 3, 4$. The number of claims and the claim amounts are independent. S is the aggregate claim amount in the period.

Calculate $F_S(3)$. $= \mathbb{P}[S \leq 3] = f_S(0) + f_S(1) + f_S(2) + f_S(3)$

$N \sim g(\beta = 4)$

$p_N(0) = \frac{1}{5} = 0.2$

$p_N(1) = \frac{1}{5} \cdot \frac{4}{5} = \frac{4}{25} = 0.16$

$p_N(2) = \frac{1}{5} \cdot \left(\frac{4}{5}\right)^2 = \frac{16}{125} = 0.128$

$p_N(3) = \frac{1}{5} \left(\frac{4}{5}\right)^3 = \frac{64}{625} = 0.1024$

$f_S(0) = p_N(0) = 0.2$

$a = \frac{4}{5}, b = 0$

(A) 0.27

(B) 0.29

(C) 0.31

(D) 0.33

(E) 0.35

96. Insurance agent Hunt N. Quotum will receive no annual bonus if the ratio of incurred losses to earned premiums for his book of business is 60% or more for the year. If the ratio is less than 60%, Hunt's bonus will be a percentage of his earned premium equal to 15% of the difference between his ratio and 60%. Hunt's annual earned premium is 800,000.

Incurred losses are distributed according to the Pareto distribution, with $\theta = 500,000$ and $\alpha = 2$.

Calculate the expected value of Hunt's bonus.

(A) 13,000

(B) 17,000

(C) 24,000

(D) 29,000

(E) 35,000

$$f_S(1) = \frac{4}{5} \cdot f_X(1) \cdot f_S(1-1) = \frac{4}{5} \cdot \frac{1}{4} \cdot 0.2 = 0.04$$

without recursion: $P_N(1) \cdot f_X(1) = 0.16 \cdot \frac{1}{4} = 0.04$

$\vdots$