M339D: March 7th, 2022.

3. An insurance company sells single premium deferred annuity contracts with return linked to a stock index, the time-t value of one unit of which is denoted by $S(t)$. The contracts offer a minimum guarantee return rate of $g\cancel{\%}$. At time 0, a single premium of amount π is paid by the policyholder, and $\pi \times y\cancel{\%}$ is deducted by the insurance company. Thus, at the contract maturity date, T, the insurance company will pay the policyholder

$$\boxed{\pi \times (1 - y\cancel{\%})} \times \mathrm{Max}[\boxed{S(T)/S(0)}, (1 + g\cancel{\%})^T].$$

You are given the following information:

(i) The contract will mature in one year. $\boxed{T=1}$

(ii) The minimum guarantee rate of return, ~~g%, is 3%~~. $\boxed{g=0.03}$

(iii) Dividends are incorporated in the stock index. That is, the stock index is constructed with all stock dividends reinvested. $\delta=0$

(iv) $\boxed{S(0) = 100.}$

(v) The price of a one-year European put option, with strike price of \$103, on the stock index is \$15.21. $\boxed{V_P(0, T=1, K=103) = 15.21}$

Determine $y\cancel{\%}$, so that the insurance company does not make or lose money on this contract.

(A) 12.8%.

(B) 13.0%

(C) 13.2%

(D) 13.4%

(E) 13.6%.

$\longrightarrow$:

The insurance company's liability:

$$\underbrace{\pi \times (1-y)}_{\text{constant}} \times \underbrace{\max\left(\frac{S(T)}{S(0)}, (1+g)^T\right)}$$

$$\underbrace{\frac{1}{S(0)}}_{\text{constant}} \underbrace{\max\left(S(T), \underbrace{S(0)}_{100}\underbrace{(1+g)^T}_{(1.03)^1}\right)}_{} \qquad \underbrace{100 \cdot (1.03)^1}_{103}$$

$$\underbrace{S(T) + \max(0, 103 - S(T))}$$

$$= \left(S(T) + \boxed{(103 - S(T))_+}\right)$$

$S(T)$: The payoff of a prepaid forward.

$(103-S(T))_+$: The payoff of a put w/ strike 103 and exercise date @ time T.

The insurance company can perfectly hedge by:

- entering $\frac{\pi(1-y)}{S(0)}$ prepaid forwards,
- buying $\frac{\pi(1-y)}{S(0)}$ European puts (as in (v) of the problem).

=> The condition for the insurance company to break even is that the money received @ time 0 equals the cost of hedging, i.e.,

$$\cancel{\pi} = \frac{\cancel{\pi}(1-y)}{S(0)}\left(\underbrace{F^P_{0,1}(S)}_{\substack{100 \\ \text{(no div)}}} + V_P(0, T=1, K=103)\right)$$

$$\Rightarrow \quad 1-y = \frac{100}{100+15.21}$$

$$\Rightarrow \quad y = \frac{15.21}{115.21} = 0.132$$

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Lecture 12

Gap options.

12.1. **Gap calls.** A European *gap call option* is a derivative security on an underlying asset (with price denoted by $\mathbf{S} = \{S(t), t \geq 0\}$) which given:

- an exercise date T;
- a **strike** price K_s;
- a **trigger** price K_t

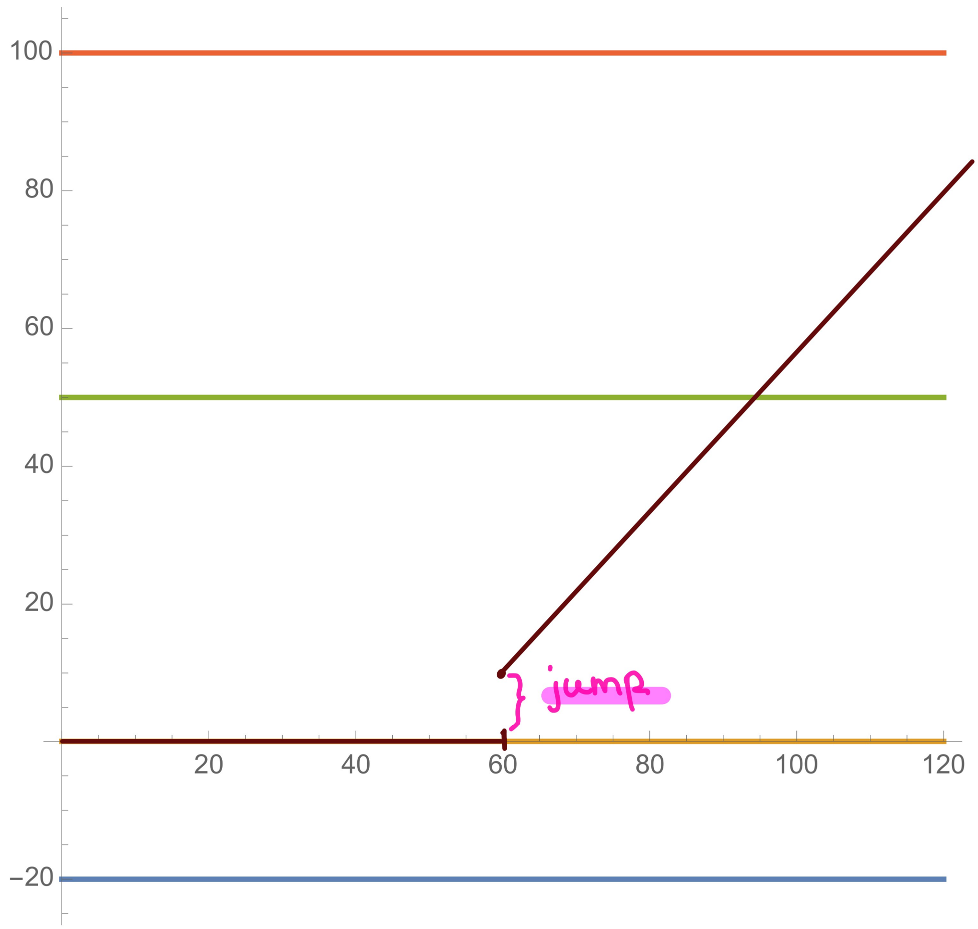
provides the payoff

$$V_{GC}(T) = (S(T) - K_s)\mathbb{I}_{[S(T) \geq K_t]}$$

to its owner.

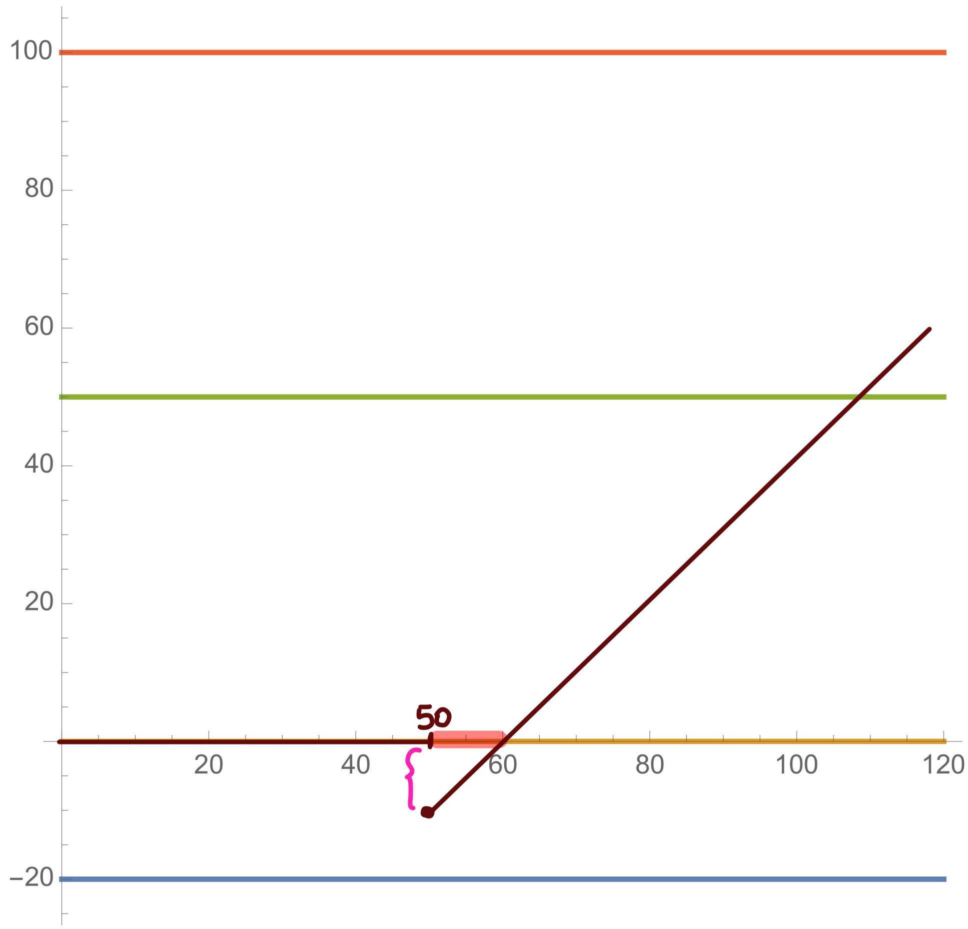
Problem 12.1. Consider a gap call option with $K_s \leq K_t$. $K_s = 50, \ K_t = 60$

- Draw its payoff curve.
- Is a long gap call a long or a short position with respect to the underlying asset for the above ordering of the strike price and the trigger price?



Problem 12.2. Consider a gap call option with $K_t < K_s$. $K_t = 50$, $K_s = 60$

- Draw its payoff curve.
- Do you think that the word "option" is entirely appropriate in this case?



12.2. **Gap puts.** A European *gap put option* is a derivative security on an underlying asset (with price denoted by $\mathbf{S} = \{S(t), t \geq 0\}$) which given:

- an exercise date T;
- a **strike** price K_s;
- a **trigger** price K_t

provides the payoff

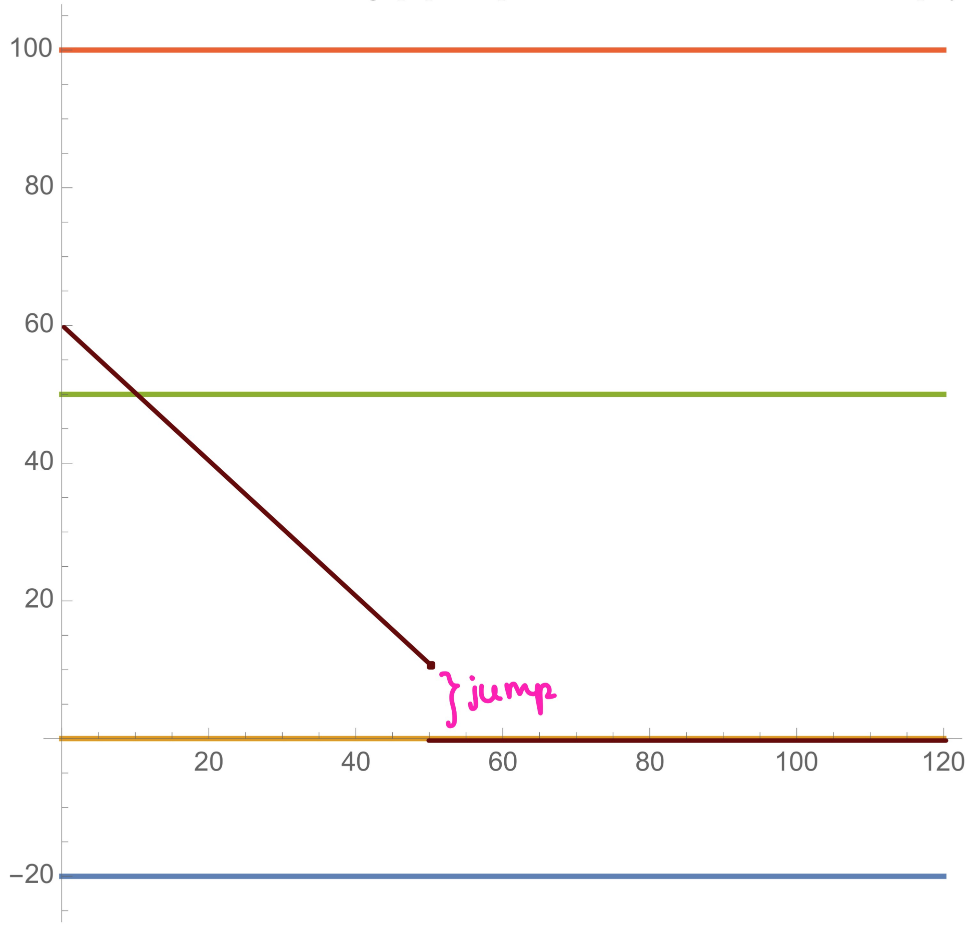
$$V_{GP}(T) = (K_s - S(T))\mathbb{I}_{[S(T)<K_t]}$$

to its owner.

Problem 12.3. Consider a gap put option with $\overset{50}{K_s} \leq \overset{60}{K_t}$. Draw its payoff curve.

60 50

Problem 12.4. Consider a gap put option with $K_s > K_t$. Draw its payoff curve.



short w.r.t. the underlying

12.3. **Put-call parity for gap options.**

Problem 12.5. Consider the following portfolio:

- one **long** gap call option with trigger price K_t and the strike price K_s,
- one **short** otherwise identical gap put option.

(i) What is the initial cost of the above portfolio expressed in terms of the price of the gap call $V_{GC}(0)$ and the price of the gap put $V_{GP}(0)$?
(ii) What is the payoff of the above portfolio?
(iii) Based on your answers to the above two questions, what is **put-call parity** for gap options?

$$V_{GC}(0) - V_{GP}(0)$$

$$(S(T) - K_s)\cdot \mathbb{I}_{[S(T) \geq K_t]} - (K_s - S(T))\cdot \mathbb{I}_{[S(T) < K_t]} = ?$$

$$\text{Payoff} = \left\{ \begin{array}{ll} S(T) - K_S, & \text{if } S(T) \geq K_t \\ S(T) - K_S, & \text{if } S(T) < K_t \end{array} \right\} = S(T) - K_S$$

$$\boxed{V_{GC}(0) - V_{GP}(0) = F^P_{0,T}(S) - PV_{0,T}(K_S)}$$

Put-Call Parity.