

## M378K Introduction to Mathematical Statistics

### Homework assignment #7

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Please, provide your **final answer only** to the following problems.

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**Problem 7.1.** (5 points) Let  $Y_1, \dots, Y_n$  be a random sample from  $U(0, \theta)$ , with an unknown  $\theta > 0$ . For what value of the constant  $c$  is the estimator  $\hat{\theta} = c \sum_{i=1}^n Y_i$  unbiased for  $\theta$ ?

- (a) 1
- (b)  $1/n$
- (c)  $2/n$
- (d)  $n$
- (e) **None of the above.**

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Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

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**Problem 7.2.** (20 points) Source: “Probability” by Pitman

Four people agree to meet at a cafe at noon. Suppose that each person arrives at a time normally distributed with mean 12noon and standard deviation of 5 minutes, independently of all the others.

1. (5 points) What is the chance that the first person to get to the cafe arrives before 11:50am?
2. (5 points) What is the chance that some of the four have still not arrived at 12:15pm?
3. (10 points) Approximately, what is the chance that the second person to arrive gets there within 10 seconds on 12noon?

**Problem 7.3.** (5 points) Consider an estimator  $\hat{\theta}$  for a parameter  $\theta$ . Let's say that

$$\mathbb{E}[\hat{\theta}] = \kappa_1 \theta + \kappa_2$$

for some constants  $\kappa_i \neq 0, i = 1, 2$ . Is the estimator  $\hat{\theta}$  unbiased? If so, justify your answer; if not, how would you transform the estimator  $\hat{\theta}$  to obtain an unbiased estimator?

**Problem 7.4.** (20 points) Let  $Y_1, \dots, Y_n$  be a random sample from the uniform distribution on  $[0, \theta]$ , where  $\theta > 0$  is an unknown parameter. We consider the estimator

$$\hat{\theta} = c \frac{1}{n} \sum_{i=1}^n Y_i^2.$$

where  $c$  is a constant (not dependent on  $\theta$  or on  $Y_1, \dots, Y_n$ ).

1. (10 points) For what value of the constant  $c$  will  $\hat{\theta}$  be an unbiased estimator for  $\theta^2$ ? Is there such a value if  $\hat{\theta}$  is used as an estimator for  $\theta$  instead of  $\theta^2$ ?
2. (10 points) Using the value of  $c$  obtained above, compute the mean squared error of  $\hat{\theta}$  (when interpreted as an estimator of  $\theta^2$ ).