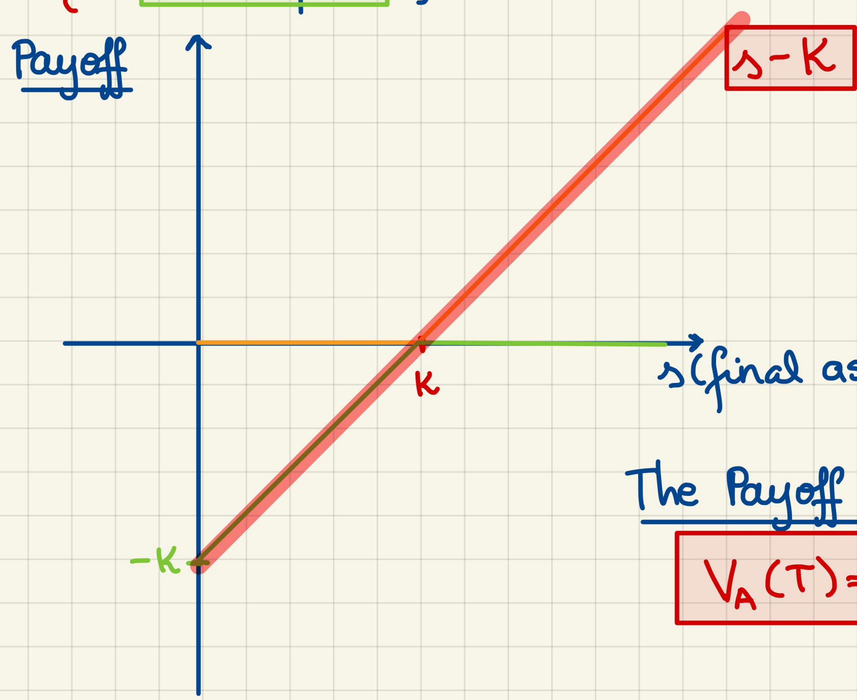
M339D : March 2nd, 2022.

Put·Call Parity.

Portfolio A:

$\begin{cases} \bullet \ \boxed{\text{long call}} \\ \bullet \ \boxed{\text{short put}} \end{cases}$ both European & otherwise identical



... the payoff function of the Portfolio A

The Payoff:

$$\boxed{V_A(T) = V_C(T) - V_P(T) = S(T) - K}$$ ✓

Portfolio B:

- long prepaid forward w/ delivery date T
- borrow $PV_{0,T}(K)$ @ the risk·free interest rate r to be repaid @ time T

$$\Rightarrow \boxed{V_B(T) = \underbrace{S(T)}_{\substack{\text{long}\\\text{prepaid}\\\text{forward}}} - \underbrace{K}_{\substack{\text{loan}\\\text{repaid}}}}$$ ✓

Note: $V_A(T) = S(T) - K = V_B(T)$

$$\Rightarrow \quad V_A(0) = V_B(0)$$

NO ARBITRAGE!

$$\boxed{V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K)}$$

Put·Call Parity

More generally: for any $t \in [0,T]$:

$$\boxed{V_c(t) - V_p(t) = F^P_{t,T}(S) - PV_{t,T}(K)}$$

Note:
- Only works for European options.
- The no-arbitrage assumption is sufficient to get put-call parity.

Advanced Derivatives Questions

1. Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is 60. $\boxed{S(0)=60}$

(ii) The call option currently sells for 0.15 more than the put option.

(iii) Both the call option and put option will expire in 4 years. $\boxed{T=4}$

(iv) Both the call option and put option have a strike price of 70. $\boxed{K=70}$

Calculate the continuously compounded risk-free interest rate.

The interest rate obtained in this way is sometimes called the implied interest rate.

(A) 0.039

(B) 0.049

(C) 0.059

(D) 0.069

(E) 0.079

Put·Call Parity.

$$\underbrace{V_c(0) - V_p(0)}_{\parallel (ii)} = \underbrace{F^P_{0,T}(S)}_{\parallel \text{ no div.}} - PV_{0,T}(K)$$

$$0.15 = 60 - 70 \cdot e^{-r(4)}$$

$$70e^{-4r} = 60 - 0.15 = 59.85$$

$$e^{-4r} = \frac{59.85}{70}$$

$$-4r = \ln\left(\frac{59.85}{70}\right)$$

$$r = -\frac{1}{4}\ln\left(\frac{59.85}{70}\right) = 0.039$$

****BEGINNING OF EXAMINATION****

ACTUARIAL MODELS – FINANCIAL ECONOMICS SEGMENT

time·0 $S(0)=52$

1. On April 30, 2007, a common stock is priced at \$52.00. You are given the following:

(i) Dividends of equal amounts will be paid on June 30, 2007 (t_1) and September 30, 2007 (t_2).

D... dividend amount

(ii) A European call option on the stock with strike price of \$50.00 expiring in six months sells for \$4.50. $\boxed{V_c(0) = 4.50}$ $\boxed{K=50}$ $\boxed{T=\frac{1}{2}}$

(iii) A European put option on the stock with strike price of \$50.00 expiring in six months sells for \$2.45. $\boxed{V_p(0) = 2.45}$

(iv) The continuously compounded risk-free interest rate is 6%. $\boxed{r = 0.06}$

Calculate the amount of each dividend.

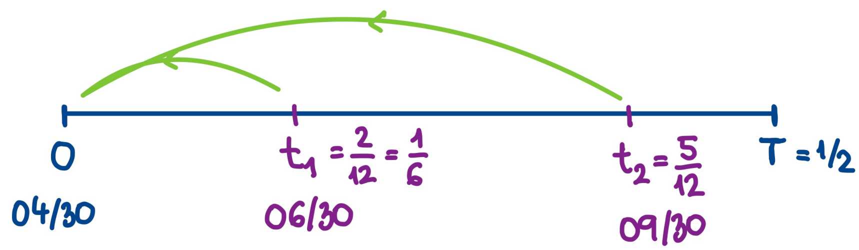
(A) \$0.51

(B) \$0.73

(C) \$1.01

(D) \$1.23

(E) \$1.45



Put·Call Parity.

$$\underbrace{V_c(0) - V_p(0)}_{4.50 - 2.45 \,=\, \boxed{2.05}} = F^P_{0,T}(S) - \underbrace{PV_{0,T}(K)}_{\boxed{50e^{-0.06(0.5)}}}$$

$$F^P_{0,T}(S) = \underbrace{S(0) - PV(Div)}_{52 - D\left(e^{-0.06(1/6)} + e^{-0.06(5/12)}\right)}$$

$$D\left(e^{-0.01} + e^{-0.025}\right) = 52 - 50e^{-0.03} - 2.05$$

$$D = \frac{49.95 - 50e^{-0.03}}{e^{-0.01} + e^{-0.025}} = 0.7264$$

$\blacksquare$

53.

$T=4$ $F_{0,T}=300$

For each ton of a certain type of rice commodity, the four-year forward price is 300. A four-year $\boxed{\text{400-strike}}$ European call option costs 110.

The continuously compounded risk-free interest rate is 6.5%.

Calculate the cost of a four-year 400-strike European put option for this rice commodity.

Put-Call Parity.

$$V_c(0) - V_p(0) = F^P_{0,T} - PV_{0,T}(K)$$
$$= PV_{0,T}(F_{0,T}) - PV_{0,T}(K)$$
$$= PV_{0,T}(F_{0,T} - K)$$

(A) 10.00

(B) 32.89

(C) 118.42

(D) 187.11

(E) 210.00

$$V_p(0) = V_c(0) - PV_{0,T}(F_{0,T} - K) = 110 - (300-400)e^{-0.065(4)}$$

$$V_p(0) = 187.11$$

54.

DELETED

55.

Box spreads are used to guarantee a fixed cash flow in the future. Thus, they are purely a means of borrowing or lending money, and have no stock price risk.

Consider a box spread based on two distinct strike prices (K, L) that is used to lend money, so that there is a positive cost to this transaction up front, but a guaranteed positive payoff at expiration.

Determine which of the following sets of transactions is equivalent to this type of box spread.

(A) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bear spread using puts.

(B) A long position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(C) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bull spread using puts.

(D) A short position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(E) A short position in a (K, L) bull spread using calls and a short position in a(K, L) bull spread using puts.

A synthetic forward is a replicating portfolio of a forward.

If we build Portfolio A from above w/ $K = F_{0,T}$, this is a synthetic forward.

However, we frequently call this type of a portfolio a synthetic forward for other strikes K.