UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #3

Forward contracts. European call options. European put options.

3.1. Forwards.

Problem 3.1. (5 points) A soy-bean farmer shorts forward contracts on soy in an amount matching his crop volume and with delivery at harvest time. Then, he is considered:

(a) an arbitrageur.
(b) a broker.
(c) a speculator.
(d) a hedger.
(e) None of the above.

3.2. Calls.

Problem 3.2. The initial price of a non-dividend-paying asset is $100. A $\boxed{\text{six-month}}$ $95-strike European call option is available at a $8 premium.

The continuously compounded risk-free interest rate equals 0.04.

What is the break-even point for this call option?

(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

We solve for s in:

$$\underbrace{(s-95)_+}_{\text{payoff}} - 8\cdot e^{0.04(0.5)} = 0 \qquad \boxed{s \geq 95}$$

$$s^* = 95 + 8e^{0.02} = 103.16$$

Problem 3.3. (5 points) A stock's price today is $1000 and the annual effective interest rate is given to be 5%. You $\boxed{\text{write}}$ a one-year, $\boxed{\$1{,}050}$-strike call option for a premium of $10 while you simulatanеously buy the stock. What is your **profit** if the stock's spot price in one year equals $1,200?

(a) $150.00
(b) $139.90
(c) $10.50
(d) - $39.00
(e) None of the above.

Portfolio = written call + long stock

Payoff $= -(s-K)_+ + s$

In this problem: $-(1200-1050)_+ + 1200 = 1050$

Initial Cost: $1000 - 10 = 990$ → Profit: $1050 - 990(1.05) = 10.50$

Problem 3.4. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for $1,000.

The jeweler models the market price of gold in one year as follows:

Gold price in one year	Probability
750 per ounce	0.2
850 per ounce	0.5
950 per ounce	0.3

dist'n of S(T)

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of $900 per ounce. The option costs $100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelery produced.

$\longrightarrow$: $S(T)$ ... the market price of gold @ $T=1$.

Profit (hedged portfolio) = ?

Payoff (total hedged portfolio) =

$$= \underbrace{1000 - S(T)}_{\text{unhedged}} + \underbrace{(S(T) - 900)_+}_{\text{hedge (call)}}$$

$$= 1000 - S(T) + (S(T) - 900)_+$$

$$= 1000 - \begin{cases} 900 & \text{if } S(T) \geq 900 \\ S(T) & \text{if } S(T) < 900 \end{cases}$$

$$= 1000 - \min(900, S(T))$$

$$\min(900, S(T)) \sim \begin{cases} 750 & \text{w/ probab. } 0.2 \\ 850 & \text{w/ probab. } 0.5 \\ 900 & \text{w/ probab. } 0.3 \end{cases}$$

$$\mathbb{E}[\text{Payoff}] = 1000 - \mathbb{E}[\min(900, S(T))]$$

$$= 1000 - (750(0.2) + 850(0.5) + 900(0.3))$$

$$= \underline{155}$$

$$\mathbb{E}[\text{Profit}] = 155 - 100e^{0.05} = \underline{49.873}$$

3.3. Puts.

Problem 3.5. The initial price of the market index is \$900. After 3 months the market index is priced at \$915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of $\boxed{\$930}$ is \$8.00. What is the profit at expiration for a **long** put?

- (a) \$15.00 loss
- (b) \$6.90 loss
- (c) \$6.90 gain
- (d) \$15.00 gain
- (e) None of the above.

$$\text{Payoff} = (K-S)_+ = (930-915)_+ = 15$$

$$\text{Profit} = 15 - 8(1.004)^3 = \underline{6.90}$$

Problem 3.6. **<u>Sample FM(DM) #12</u>**
Consider a European put option on a stock index without dividends, with $\boxed{\text{6 months}}$ to expiration, and a strike price of $\boxed{1{,}000.}$ Suppose that the nominal annual risk-free rate is 4% **<u>convertible semiannually</u>**, and that the put costs $\boxed{74.20}$ now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

- A. 922.83
- B. 924.32
- C. 1,000.00
- D. 1,075.68
- E. 1,077.17

In other words, we're looking for the break-even point.

$$(K-S)_+ = 74.20(1.02)$$

$$\boxed{K > S}$$

$$1000 - S = 74.2(1.02)$$

$$S^* = 1000 - 74.2(1.02) = 924.32$$

Problem 3.7. Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000−cartons' worth in six months. His total costs are \$12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of \$13 per carton and another with the strike price of \$15 per carton. Their premiums are \$0.15 and \$0.18, respectvely.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals \$14. What would Farmer Shaun's profit be if he had decided to hedge using the \$13-strike put versus his profit if he had decided to use the \$15-strike put to hedge?