

**Name:**

M339D=M389D Introduction to Actuarial Financial Mathematics

University of Texas at Austin

**Mock In-Term Exam II**

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**Notes:** This is a closed book and closed notes exam. The maximal score on this exam is 50 points.

**Time:** 50 minutes

**Problem 1.1.** (5 points) The current price of stock a certain type of stock is \$80. The premium for a 6-month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

- (a) \$80
- (b) \$85.72
- (c) \$85.84
- (d) \$85.96
- (e) None of the above.

**Problem 1.2.** Let the current price of a non-dividend-paying stock equal 100. The forward price for delivery of this stock in 3 months equals \$101.26

Consider a \$90-strike, six-month put option on this stock whose premium today equals \$2.22.

What will the profit of this long put option be if the stock price at expiration equals \$96?

- (a) About \$2.28 loss.
- (b) About \$2.22 loss.
- (c) About \$2.28 gain.
- (d) About \$2.22 gain.
- (e) None of the above.

**Problem 1.3.** The current price of a discrete-dividend-paying stock is \$90 per share. The company projects to pay quarterly dividends starting three months from today to perpetuity. The first dividend amount is \$2 and the dividends are scheduled to increase by a factor of 0.01 every time a dividend is paid.

The continuously compounded risk-free interest rate is 0.06. What is the prepaid forward price of the above stock for delivery in eight months?

- (a) \$84.24
- (b) \$86.07
- (c) \$88.70
- (d) \$90.00
- (e) None of the above.

**Problem 1.4.** (5 points) A stock's price today is \$1000 and the annual effective interest rate is given to be 5%. You write a one-year, \$1,050-strike call option for a premium of \$10 while you simultaneously buy the stock. What is your **profit** if the stock's spot price in one year equals \$1,200?

- (a) \$150.00 gain
- (b) \$139.90 loss

- (c) \$10.50 gain
- (d) \$39.00 loss
- (e) None of the above.

**Problem 1.5.** (5 points) *Source: Sample MFE (Intro) Problem #20.*

The current price of a stock is 200, and the continuously compounded risk-free interest rate is 4%. A dividend will be paid every quarter for the next 3 years, with the first dividend occurring 3 months from now. The amount of the first dividend is 1.50, but each subsequent dividend will be 1% higher than the one previously paid. Calculate the fair price of a 3-year forward contract on this stock.

- (a) \$200.41
- (b) \$205.41
- (c) \$210.41
- (d) \$215.41
- (e) None of the above.

**Problem 1.6.** (5 points) The initial price of a continuous-dividend-paying market index equals \$1,000. The dividend yield equals 0.03.

An investor simultaneously purchases one unit of the index and a one-year, 975-strike European put option on the index for a premium of \$10.

In one year, the spot price of the index is observed to be \$950.

Given that the continuously compounded risk-free interest rate equals 0.03, what is the profit of the investor's portfolio? *Caveat: Be careful with continuous reinvestment of dividend in the index.*

- (a) 36.83 loss
- (b) 61.82 loss
- (c) 61.82 gain
- (d) 36.83 gain
- (e) None of the above.

**Problem 1.7.** (5 points) Consider a European call option and a European put option on a non-dividend paying stock **S**. You are given the following information:

- (1)  $r = 0.04$
- (2) The current price of the call option  $V_C(0)$  is by 0.15 greater than the current price of the put option  $V_P(0)$ .
- (3) Both the put and the call expire in 4 years.
- (4) The put and the call have the same strikes equal to 70.

Find the spot price  $S(0)$  of the underlying asset.

- (a) 48.90
- (b) 59.80
- (c) 69.70
- (d) 79.60
- (e) None of the above.

**Problem 1.8.** (5 points) Consider a one-year, \$55-strike European call option and a one-year, \$45-strike European put option on the same underlying asset. You observe that the time-0 stock price equals \$40 while the time-1 stock price equals \$50. Then,

- (a) both of the options are out-of-the-money at expiration.
- (b) both of the options are in-the-money at expiration.
- (c) the call is out-of-the-money and the put is in-the-money at expiration.
- (d) the put is out-of-the-money and the call is in-the-money at expiration.
- (e) both options are at-the-money at expiration.

**Problem 1.9.** Consider a non-dividend-paying stock whose current price equals \$54 per share. A pair of one-year European calls on this stock with strikes of \$40 and \$50 is available in the market for the observed prices of \$4 and \$2, respectively.

The continuously compounded risk-free interest rate is given to be 10%.

George suspects that there exists an arbitrage portfolio in the above market consisting of the following components:

- **short-sale** of one share of stock,
- **buy** the \$40-strike call,
- **buy** the \$50-strike call.

What is the minimum **gain** from this suspected arbitrage portfolio?

- (a) The above is **not** an arbitrage portfolio.
- (b) \$0.84
- (c) \$8.00
- (d) \$13.05
- (e) None of the above.

**Problem 1.10.** (5 points) A derivative security has the payoff function given by

$$v(s) = (s^2 - 100)_+$$

Its exercise date is in one year. You model the time-1 price of the underlying asset as

$$S(1) \sim \begin{cases} 9.5 & \text{with probability } 1/4 \\ 10 & \text{with probability } 1/2 \\ 11 & \text{with probability } 1/4 \end{cases}$$

The continuously compounded, risk-free interest rate is 10%. What is the expected **payoff** of the above derivative security?

- (a) 5.25
- (b) 2.81
- (c) 0.31
- (d) 1.42
- (e) None of the above.

**Problem 1.11.** (5 points) An investor wants to hold 100 euros two years from today. The spot exchange rate is \$1.37 per euro. If the euro-denominated continuously compounded annual interest rate equals 3.0% what is the price of a currency prepaid forward (rounded to the nearest dollar)?

- (a) 129
- (b) 176
- (c) 200
- (d) 247
- (e) None of the above.

**Problem 1.12.** (5 points) The current price of stock **S** is \$40. Stock **S** is scheduled to pay a \$2-dividend in two months.

The current price of stock **Q** is \$50. Stock **Q** is scheduled to pay dividends continuously with the dividend yield 0.02.

A six-month European exchange call option with the underlying asset **S** and the strike asset **Q** is sold for \$2.50.

The continuously compounded risk-free interest rate is given to be 0.10.

What is the price of the six-month European exchange put option with the underlying asset **S** and the strike asset **Q**?

- (a) About \$8.58
- (b) About \$9.04
- (c) About \$13.97
- (d) About \$14.54
- (e) None of the above.