M339G: January 29th, 2024.

Bias.

Def'n. Let $\hat{\theta}$ be a point estimator for the parameter θ. The bias of the estimator is defined as

$$\text{bias}(\hat{\theta}) := \mathbb{E}_\theta[\hat{\theta}] - \theta.$$

Def'n. We say that an estimator is unbiased if

$$\text{bias}(\hat{\theta}) = 0$$

Def'n. The mean squared error of $\hat{\theta}$ is

$$\boxed{MSE[\hat{\theta}] = \mathbb{E}_\theta\left[(\hat{\theta} - \theta)^2\right]}$$

Assessing Model Accuracy.

Say, we have the "usual" model: $\boxed{Y = f(X) + \varepsilon}$

Say, we fit our model to some training data:

$$\boxed{Tr = \{(x_i, y_i);\ i = 1, \dots, N\}}$$

Let $\hat{f}(\cdot)$ be the fit of the model to our Tr

$$MSE_{Tr} := \underset{i \in Tr}{\text{Ave}}\left(y_i - \hat{f}(x_i)\right)^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\left(y_i - \hat{f}(x_i)\right)\right)^2$$

We propose to look @ other data $\boxed{Te = \{(x_i, y_i);\ i = 1, \dots, M\}}$ constituting our test data, and then calculate:

$$\boxed{MSE_{Te} := \underset{i \in Te}{\text{Ave}}\left(y_i - \hat{f}(x_i)\right)^2}$$