M358K: October 6th, 2023.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 8

Sample mean: The normal sample.

Problem 8.1. The scores of individual students on the Advanced Dark Arts Exam are modeled as normally distributed with a mean of 19.6 and a standard deviation of 5.0. At Voldemort High, 64 seniors take the test. Assume the individual scores at this school are modeled using the same distribution as national scores. What is the sampling distribution of the sample average score for this random sample of 64 students?

State the **name** and the **parameter value(s)** of this distribution.

$$\bar{X} \sim \underline{\text{Normal}}\left(\text{mean} = \underline{19.6},\ \text{sd} = \underline{\frac{5}{8}} = 0.625\right)$$

Problem 8.2. The *"Aristocratic Hog"* chocolate bars are all labeled to weigh 4.0 ounces. The distribution of the actual weights of these chocolate bars is modeled as normal with a mean of 4.0 ounces and a standard deviation of 0.1 ounces. Bernard, the quality control manager and principal taster, initially plans to take (and weigh) a simple random sample of size n from the production line. Then he reconsiders and decides that a sample twice as large is needed. By what factor does the standard deviation of the sampling distribution of the sample average change?

The sd <u>decrease</u> by a factor of $\underline{\sqrt{2}}$.

Problem 8.3. The individual students' scores in the ACT exam are modeled using the normal distribution with an unknown mean (say, it varies from year to year) and with the **known** standard deviation of 6.

You take a SRS of students who took the ACT this year. The intention is to use their sample average to <u>estimate</u> (infer) the <u>population</u> mean.

You want the standard deviation of your statistic $\bar{X}_n$ to be at most 0.10. What is the least number of students you need to sample?

$$SD[\bar{X}_n] = \frac{6}{\sqrt{n}} \leq 0.10$$

$$6 \leq 0.1\sqrt{n} \quad / \cdot 10$$

$$60 \leq \sqrt{n}$$

$$\boxed{3600 \leq n}$$

$\square$

Problem. Your diamond scale's measurements errors are normally dist'd w/ mean 0 and std dev of 0.001 carats.

Your proceduer is to weigh a single diamond using the scale n times, and report the average as the mass of the diamond.

How many times n do you have to weigh your diamond so that your reported mass is @ most 0.001 from the actual mass w/ probability 99%?

→: X_i ... error of the i^{th} measurement

⇒ average error: $\bar{X}_n = \frac{1}{n}(X_1 + \dots + X_n)$

$$\bar{X}_n \sim \underline{\text{Normal}}\left(\text{mean} = 0, \text{sd} = \frac{0.001}{\sqrt{n}}\right)$$

$$\mathbb{P}\left[|\bar{X}_n| < 0.001\right] \geq 0.99$$

$$\mathbb{P}\left[-0.001 < \bar{X}_n < 0.001\right] \geq 0.99$$

$$\mathbb{P}\left[\bar{X}_n < 0.001\right] - \mathbb{P}\left[\bar{X}_n < -0.001\right] \geq 0.99$$

$$\mathbb{P}\left[\underbrace{\frac{\bar{X}_n - 0}{\frac{0.001}{\sqrt{n}}}}_{\sim N(0,1)} < \overset{\sqrt{n}}{\frac{\cancel{0.001} - 0}{\frac{\cancel{0.001}}{\sqrt{n}}}}\right] - \mathbb{P}\left[\underbrace{\frac{\bar{X}_n - 0}{\frac{0.001}{\sqrt{n}}}}_{\sim N(0,1)} < \overset{-\sqrt{n}}{\frac{-\cancel{0.001} - 0}{\frac{\cancel{0.001}}{\sqrt{n}}}}\right] \geq 0.99$$

$$\Phi(\sqrt{n}) - \underbrace{\Phi(-\sqrt{n})}_{1 - \Phi(\sqrt{n})} \geq 0.99$$

$$2\Phi(\sqrt{n}) - 1 \geq 0.99$$

$$\Phi(\sqrt{n}) \geq \frac{1.99}{2} = 0.995$$

$$\sqrt{n} \geq 2.576 \quad /^2$$

$$n \geq 6.63 \quad \Rightarrow \quad \boxed{n \geq 7}$$

□