

M358K: October 20th, 2021.

Test of Significance.

Set α ... significance level

Typically: $\alpha = 0.05, 0.01, 0.10$

Decision process:

If $p\text{-value} \leq \alpha$, we REJECT the null hypothesis.

If not, we FAIL TO REJECT the null hypothesis.

Note: The p-value corresponding to an observed value of the test statistic is the LOWEST significance level @ which the null hypothesis would still be REJECTED.

Given a significance level α , we can construct (ahead of data gathering) a **REJECTION REGION (RR)** for our test.

The null hypothesis: $H_0: \mu = \mu_0$

The Test statistic: \bar{X} (the sample mean)

The observed value of the TS is \bar{x} .

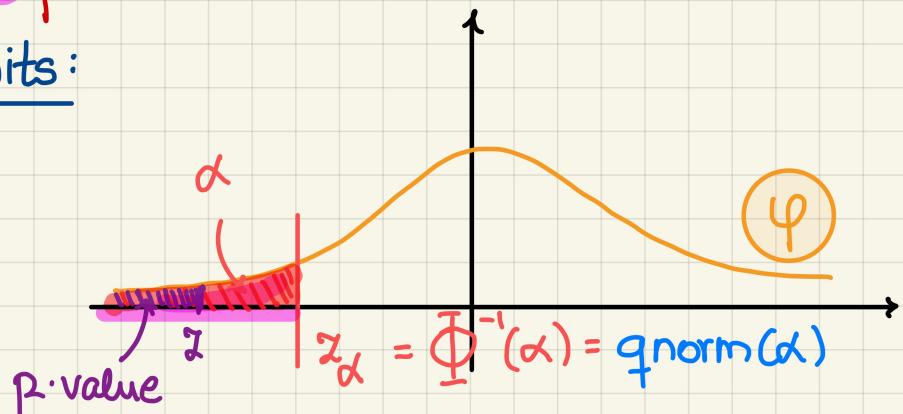
Its z-score (under the null) :

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

✓

The left-sided alternative:

$H_a: \mu < \mu_0$
In standard units:



Rejection Region (RR): $(-\infty, z_\alpha]$

In raw units:

$$z \leq z_\alpha$$

$$\frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \leq z_\alpha$$

$$\bar{x} \leq \mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

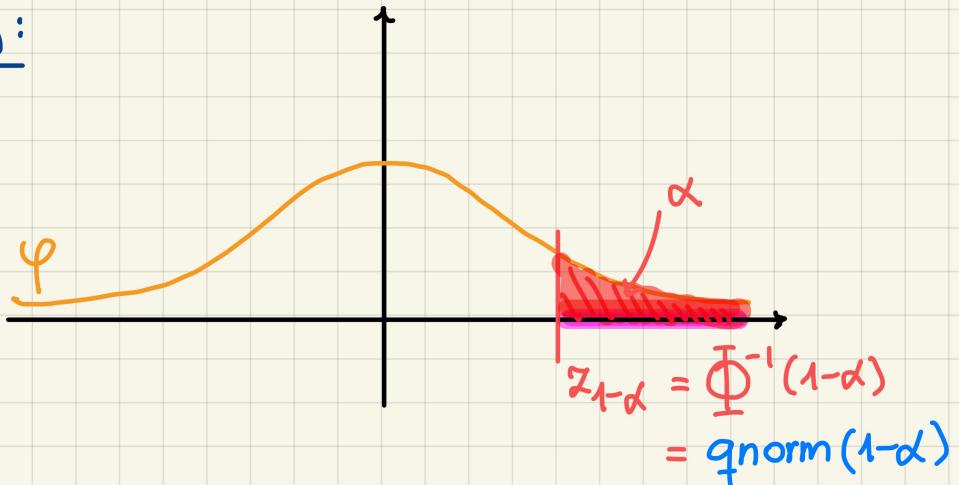
upper bound of the RR
in raw units

$$\text{RR: } (-\infty, \mu_0 + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}]$$

The right-sided alternative.

$$H_a: \mu > \mu_0$$

In standard units:



$$\text{RR: } [z_{1-\alpha}, +\infty)$$

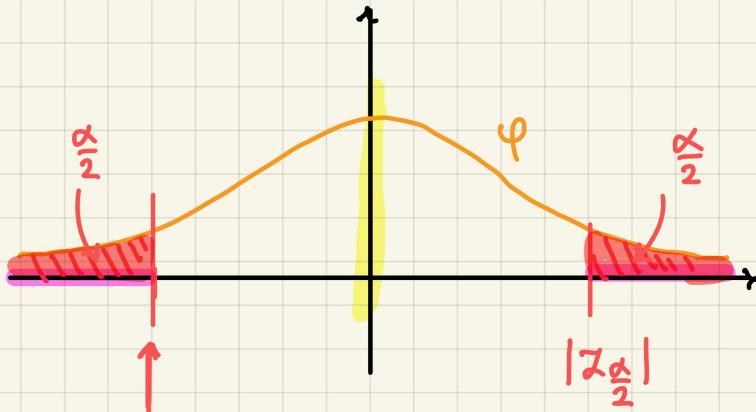
In raw units:

$$\text{RR: } [\mu_0 + z_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}}, +\infty)$$

The two-sided alternative:

$$H_a: \mu \neq \mu_0$$

In standard units:



$$z_{\alpha/2} = \Phi^{-1}\left(\frac{\alpha}{2}\right) = qnorm\left(\frac{\alpha}{2}\right)$$

$$RR: (-\infty, z_{\alpha/2}] \cup [z_{\alpha/2}, +\infty)$$

In raw units:

$$RR: (-\infty, \mu_0 + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}] \cup [\mu_0 + (z_{\alpha/2}) \cdot \frac{\sigma}{\sqrt{n}}, +\infty)$$