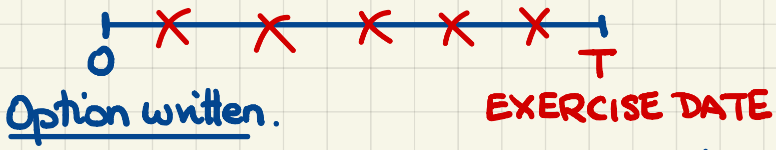
M339D: September 20th, 2024.

European Call Options.

The option can be exercised, i.e., the cashflow can be collected only only on the exercise date.

Usually, this means a right to buy the underlying asset.

Usually, the option's owner has the right but not an obligation to exercise the option.



At time 0:

- The writer of the option writes/shorts the call.
- The buyer of the call is said to long the call. They are referred to as the option's owner.
- The agreement:
  - the underlying asset : $S(t), t \geq 0$
  - the exercise date : T
  - K... the strike/exercise price
- The buyer pays the premium to the writer. $V_c(0)$

At time T:

- The call's owner has a right but not an obligation to buy one unit of the underlying asset for the strike price K.
- The call's writer is obligated to do what the owner decides.

Payoff = ?

We focus on the payoff of the long call, i.e., the payoff for the call's owner.
The call owner's rationale for whether to exercise is "maximum money in".

The criterion for exercise:

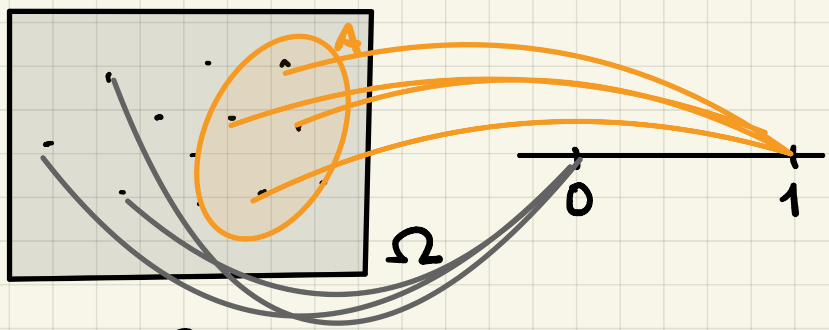
$$\begin{cases} \text{IF } S(T) \geq K, \text{ then EXERCISE.} & \Rightarrow \text{Payoff} = S(T)-K \\ \text{IF } S(T) < K, \text{ then do not exercise.} & \Rightarrow \text{Payoff} = 0 \end{cases}$$

We introduce:

$V_c(T)$... the random variable denoting the payoff of a long call

$$\Rightarrow \boxed{V_c(T) = \begin{cases} S(T)-K & \text{if } S(T) \geq K \\ 0 & \text{if } S(T) < K \end{cases}}$$

Indicator Random Variables:



Ω... outcome space

$\omega \in \Omega$

elementary outcomes

A... a "nice" subset of Ω aka an EVENT

We define:

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

$$\Rightarrow \boxed{V_c(T) = (S(T)-K) \cdot \mathbb{I}_{[S(T) \geq K]}}$$

Also: $\boxed{V_c(T) = \max\left(S(T)-K, 0\right) = (S(T)-K) \vee 0}$

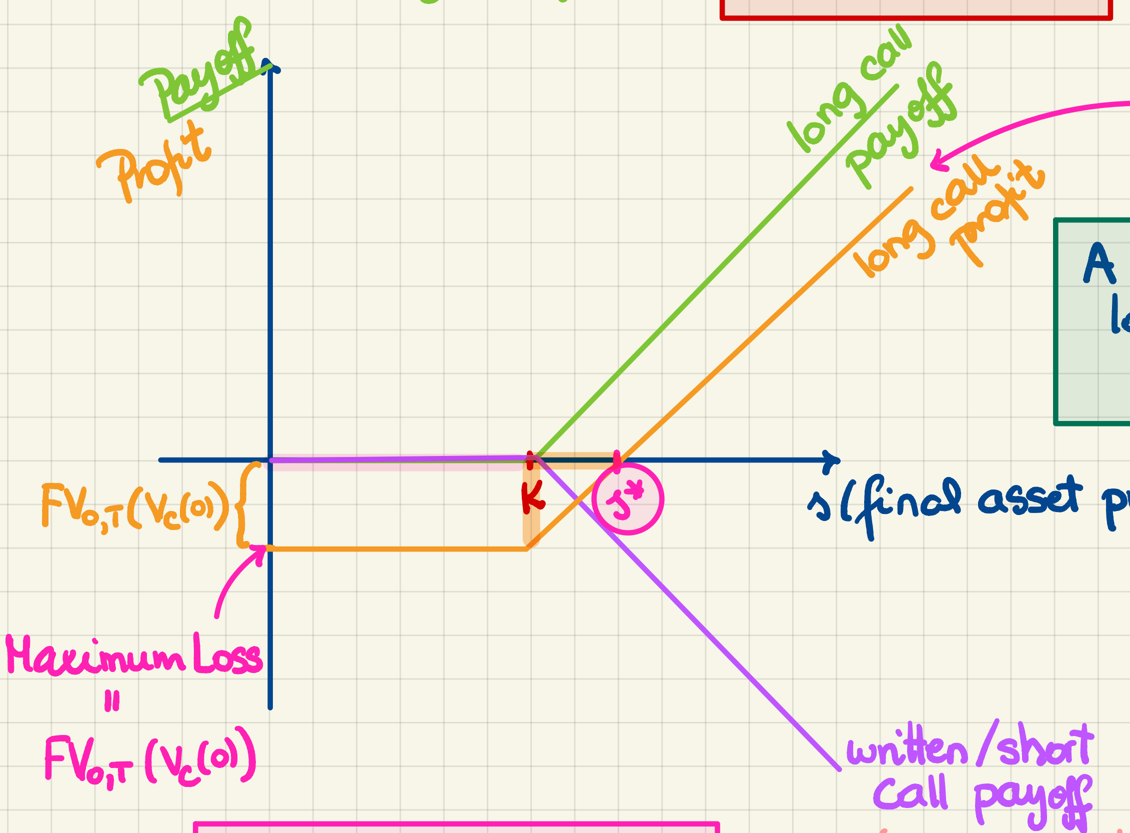
maximum operator

Introduce: The positive part function

$$x \longmapsto (x)_+ =: \max(x, 0) = x \vee 0$$

$\Rightarrow$ $\boxed{V_c(T) = (S(T) - K)_+}$

$\Rightarrow$ the payoff f'tion: $\boxed{v_c(s) = (s-K)_+}$



unlimited gain potential

A long call is a long position w.r.t the underlying.

(Never positive and sometimes strictly negative

$\Rightarrow$ $\exists$ premium $V_c(0)$)

s^* = break even point

$s^* = K + FV_{0,T}(V_c(0))$