M339J: April 27th, 2022.

A Statistics Note.

Non-Parametric → LTAM Exam (U & V courses)
Parametric → STAM Exam (J & P courses)

} Look into $\boxed{\text{FAM}\cdot\text{I}}$

Parametric → **Focus on the distributions:**

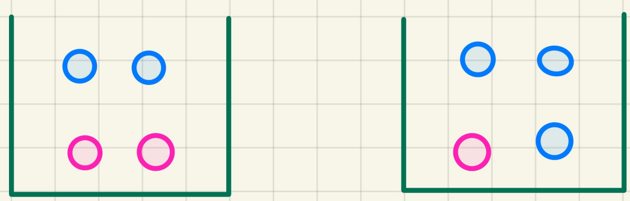
- cdf, pmf, pdf ... involving a parameter θ
- named dist'ns ... in terms of a parameter θ

Caveat: Sometimes, your parameter is a vector. Say, in the lognormal case, we would have:

$$\theta = (\underset{\in\mathbb{R}}{\mu}, \underset{>0}{\sigma})$$

Maximum Likelihood Estimation.

Motivation.



I II

 ⟹ $\underline{\text{II}}$

 ⟹ $\underline{\text{I}}$

Now, w/ censoring ☺

56. You are given the following information about a group of policies:

Claim Payment		Policy Limit		A_j
5	<	50	unmodified	$x_1 = 5$
15	<	50	unmodified	$x_2 = 15$
60	<	100	unmodified	$x_3 = 60$
100	=	100	censoring	$A_4 = (100, +\infty)$
500	=	500	censoring	$A_5 = (500, +\infty)$
500	<	1000	unmodified	$x_6 = 500$

Determine the likelihood function.

(A) $f(50)f(50)f(100)f(100)f(500)f(1000)$

(B) $f(50)f(50)f(100)f(100)f(500)f(1000)/[1-F(1000)]$

(C) $f(5)f(15)f(60)f(100)f(500)f(500)$

(D) $f(5)f(15)f(60)f(100)f(500)f(1000)/[1-F(1000)]$

(E) $f(5)f(15)f(60)[1-F(100)][1-F(500)]f(500)$

Maximum Likelihood for Individual, Unmodified Data.

Let X_j, $j=1..n$, be continuous random variable.
Denote by f_{X_j} the pdf of X_j for all $i=1..n$.
All of the f_{X_j} must depend on the same parameter θ.

Say that our data set consists of singletons, i.e.,

$$x_1, x_2, \dots, x_n$$

The likelihood function is: $L(\theta) = \prod_{j=1}^{n} f_{X_j}(x_j; \theta)$

Goal: Maximize the likelihood function across all θ.

Introduce: $\ell(\theta) = \ln(L(\theta)) = \sum_{j=1}^{n} \ln(f_{X_j}(x_j;\theta))$

Now, we maximize the log-likelihood f'tion; typically, we differentiate.