

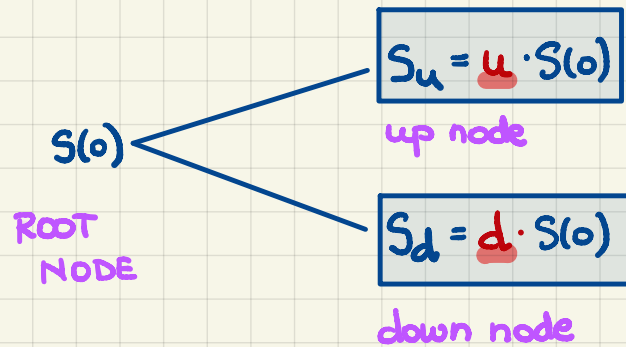
M3398: March 6th, 2023.

The Binomial Asset Pricing Model.

$S(0)$... the observable initial asset price



time horizon (i.e., the exercise date of an option)



By convention:

$$u > d$$

u ... up factor
 d ... down factor

h ... length of a single period

one period \Rightarrow $S(T) = S(h)$... a r.v. denoting the time-T stock price w/ two possible values: S_u and S_d

Returns:

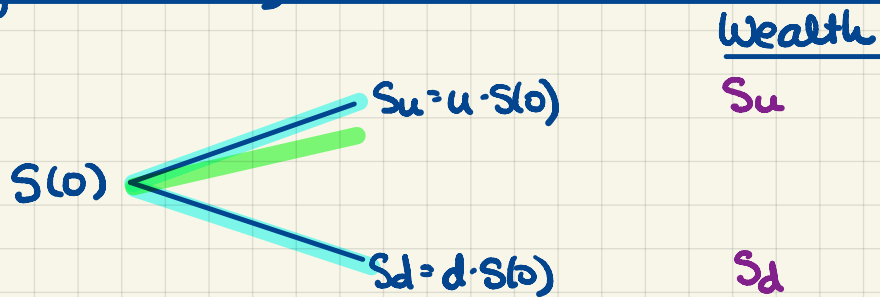
As a random variable: simple return = $\frac{S(T) - S(0)}{S(0)}$

- $\frac{S_u - S(0)}{S(0)} = \frac{u \cdot S(0) - S(0)}{S(0)} = u - 1$
- $\frac{S_d - S(0)}{S(0)} = \frac{d \cdot S(0) - S(0)}{S(0)} = d - 1$

Market Model.

- riskless asset: @ the ccrfir r
- risky asset: non-dividend paying stock

Imagine investing in one share of stock @ time 0.



At the risk-free rate $S(0)$ accumulates to $\underline{S(0)e^{rh}}$ @ time $T=h$

The No-Arbitrage Condition.

$$d \cdot S(0) < S(0)e^{rh} < u \cdot S(0)$$

$$d < e^{rh} < u$$

Half a Proof.

Say, to the contrary, $e^{rh} \leq d < u$

Propose: Long one share of stock.

Verify: Profit = $S(T) - S(0)e^{rh}$

In the up node: $S(0) \cdot u - S(0)e^{rh} > 0$

In the down node: $S(0) \cdot d - S(0)e^{rh} \geq 0$

Indeed,
an arbitrage
portfolio.

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Problem Set #8

Binomial option pricing.

Problem 8.1. In the setting of the one-period binomial model, denote by i the effective interest rate per period. Let u denote the “up factor” and let d denote the “down factor” in the stock-price model. If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage.

(F)

Fixed statement:

$$d < 1 + i < u$$

Problem 8.2. In our usual notation, does this parameter choice create a binomial model with an arbitrage opportunity?

$$u = 1.18, \quad d = 0.87, \quad r = 0.05, \quad \delta = 0, \quad h = 1/4$$

(No)

$$d = 0.87 \stackrel{?}{<} e^{rh} = e^{0.05(0.25)} \stackrel{?}{<} 1.18 = u$$

1.0125

$$\tilde{d} = 1.01$$