UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 5

Mean and median of the log-normal stock prices.

Problem 5.1. The current price of a continuous-dividend-paying stock is \$80 per share. Its rate of appreciation is 12% and its volatility is 30%.

Let $R(0,t)$ denote the realized return of this stock over the time period $[0,t]$ for any $t>0$. Calculate $\mathbb{E}[R(0,2)]$.

Solution:

$$(0.12 - 0.045)(2) = 0.15.$$

Problem 5.2. (5 points)

A stock is valued at \$75.00. The annual expected rate of appreciation is 10.0% and the standard deviation of annualized returns is 25.0%. If the stock is lognormally distributed, what is the expected stock price after 2 years?

(a) About \$71.61
(b) About \$81.63
(c) About \$91.61
(d) About \$108.83
(e) None of the above.

Solution: (c)

Let us denote the stock price today by $S(0)$ and that in three years by $S(2)$. According to the work we did in class, we need to calculate

$$\mathbb{E}[S(2)] = S(0)e^{2(\alpha-\delta)}$$

with α equal to the expected continuously compounded rate of return on the stock S and δ is the dividend yield. We are given in the problem that $\alpha - \delta = 0.10$. So, the answer is $75e^{0.20} \approx 91.605$.

Problem 5.3. (5 pts) A non-dividend-paying stock is valued at \$55.00 per share. The annual expected (rate of) return is 12.0% and the standard deviation of annualized returns is given to be 22.0%. If the stock price is modeled using the lognormal distribution (as discussed in class), what is the median of the stock price in 3 years?

(a) \$57.67
(b) \$67.67
(c) \$73.31
(d) \$87.31
(e) None of the above.

Solution: (c)

In our usual notation, it is given that $S(0) = 55, \alpha = 0.12$ and $\sigma = 0.22$. As we have learned in class, the median of the random variable $S(3)$ can be expressed as

$$S(0)e^{(\alpha-\frac{\sigma^2}{2})\times 3} = 73.31.$$

Problem 5.4. Assume that the stock price is modeled using the lognormal distribution. The annual mean rate of appreciation on the stock is given to be 12%. The median time$-t$ stock price is evaluated to be $S(0)e^{0.1t}$. What is the volatility parameter of this stock price?

Solution:

$$\alpha - \delta - \frac{\sigma^2}{2} = 0.12 - \frac{\sigma^2}{2} = 0.1 \quad \Rightarrow \quad \sigma = 0.2.$$

Problem 5.5. The current stock price is \$100 per share. The stock price at any time $t > 0$ is modeled using the lognormal distribution. Assume that the continuously compounded mean rate of return for the stock equals 12%. Let the stock's dividend yield be 4% and let its volatility equal 20%.

Find the value t^* at which the median stock price equals \$120.

Solution: We know that the median stock price at time$-t$ equals

$$S(0)e^{(\alpha-\delta-\frac{\sigma^2}{2})t}$$

So, t^* must satisfy

$$\ln\left(\frac{120}{S(0)}\right) = (\alpha - \delta - \frac{\sigma^2}{2})t \quad \Rightarrow \quad t = \frac{\ln(120/100)}{0.12 - 0.04 - 0.02} = 3.0387.$$

Problem 5.6. The volatility of the price of a continuous-dividend-paying stock is 0.2. The stock price is modeled using a log-normal distribution. The expected time-2 stock price is \$120.

Then, the median of the time-2 stock price falls within this interval:

(a) $[0, 86)$
(b) $[86, 106)$
(c) $[106, 112)$
(d) $[112, 124)$
(e) None of the above.

Solution: (d)
The median of the time-2 stock price is

$$\mathbb{E}[S(2)]e^{-\frac{2\sigma^2}{2}} = 120e^{-0.04} \approx 115.295.$$