University of Texas at Austin

Problem Set # 13

Mean and median of the log-normal stock prices.

Problem 13.1. The current price of a non-dividend-paying stock is \$80 per share. Under the risk-neutral probability measure, its mean rate of return is 12% and its volatility is 30%.

Let $R(0,t)$ denote the realized return of this stock over the time period $[0,t]$ for any $t > 0$. Calculate $\mathbb{E}^*[R(0,2)]$.

Solution:

$$(0.12 - 0.045)(2) = 0.15.$$

Problem 13.2. A stock is valued at \$75.00. The continuously compounded, risk-free interest rate is 10.0% and the standard deviation of annualized returns is 25.0%. If the stock is lognormally distributed, what is the expected stock price after 2 years under the risk-neutral probability measure?

Solution: Let us denote the stock price today by $S(0)$ and that in three years by $S(2)$. According to the work we did in class, we need to calculate

$$\mathbb{E}[S(2)] = S(0)e^{2r}$$

with r equal to the continuously compounded risk-free interest rate. We are given in the problem that $r = 0.10$. So, the answer is $75e^{0.20} \approx 91.605$.

Problem 13.3. A non-dividend-paying stock is valued at \$55.00 per share. Its standard deviation of annualized returns is given to be 22.0%. The continuously compounded risk-free interest rate is 12%. If the stock price is modeled using the lognormal distribution (as discussed in class), what is the median of the stock price in 3 years under the risk-neutral probability measure?

Solution: In our usual notation, it is given that $S(0) = 55, r = 0.12$ and $\sigma = 0.22$. As we have learned in class, under the risk-neutral probability measure, the median of the random variable $S(3)$ can be expressed as

$$S(0)e^{(r-\frac{\sigma^2}{2})\times 3} = 73.31.$$

Problem 13.4. Assume that the stock price is modeled using the lognormal distribution. The stock pays no dividends. Under $\mathbb{P}^*$, the annual mean rate of return on the stock is given to be 12%. Also under $\mathbb{P}^*$, the median time$-t$ stock price is evaluated to be $S(0)e^{0.1t}$. What is the volatility parameter of this stock price?

Solution:

$$r - \frac{\sigma^2}{2} = 0.12 - \frac{\sigma^2}{2} = 0.1 \quad \Rightarrow \quad \sigma = 0.2.$$

Problem 13.5. The current stock price is \$100 per share. The stock price at any time $t > 0$ is modeled using the lognormal distribution. Assume that the continuously compounded risk-free interest rate equals 8%. Let its volatility equal 20%.

Find the value t^* at which the median stock price equals \$120, under the risk-neutral probability measure.

Solution: We know that the median stock price at time$-t$ equals

$$S(0)e^{(r-\frac{\sigma^2}{2})t}$$

under $\mathbb{P}^*$. So, t^* must satisfy

$$\ln\left(\frac{120}{S(0)}\right) = (r - \frac{\sigma^2}{2})t \quad \Rightarrow \quad t = \frac{\ln(120/100)}{0.12 - 0.04 - 0.02} = 3.0387.$$

Problem 13.6. The volatility of the price of a non-dividend-paying stock is 0.2. The stock price is modeled using a log-normal distribution. Under $\mathbb{P}^*$, the expected time-2 stock price is \$120. What is the median of the time-2 stock price under $\mathbb{P}^*$?

Solution: (d)
The median of the time-2 stock price is

$$\mathbb{E}^*[S(2)]e^{-\frac{2\sigma^2}{2}} = 120e^{-0.04} \approx 115.295.$$