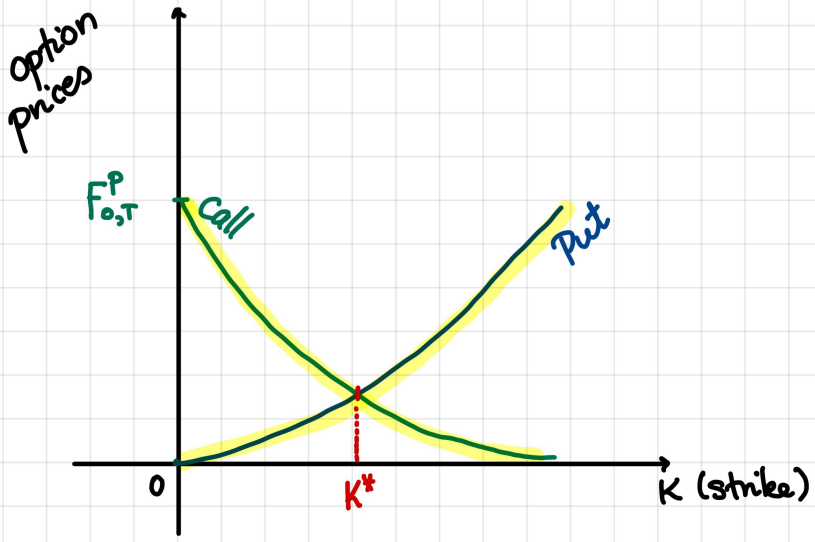
What do we know so far?



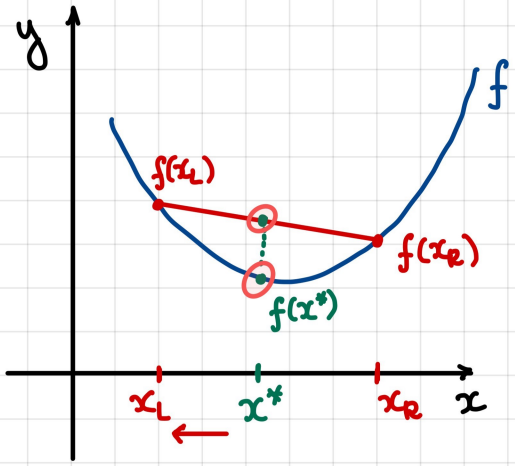
Call Price equals the Put Price @ strike K^*

Put-call parity: $\underbrace{V_C(K^*) - V_P(K^*)}_{=0} = F^P_{0,T}(S) - PV_{0,T}(K^*)$

$$\Rightarrow \quad F^P_{0,T}(S) = PV_{0,T}(K^*)$$

$$\Rightarrow \quad \boxed{K^* = F_{0,T}(S)}$$

Convex Function.



There is a constant λ such that $\lambda \in (0,1)$

$$x^* = \lambda \cdot x_L + (1-\lambda) \cdot x_R$$

In fact: $x^* = \lambda \cdot x_L + x_R - \lambda \cdot x_R$

$$\lambda (x_R - x_L) = x_R - x^*$$

$$\lambda = \frac{x_R - x^*}{x_R - x_L} \quad \text{and} \quad 1-\lambda = \frac{x^* - x_L}{x_R - x_L}$$

Our function f is convex if

$$f(x^*) \leq \lambda \cdot f(x_L) + (1-\lambda) \cdot f(x_R)$$