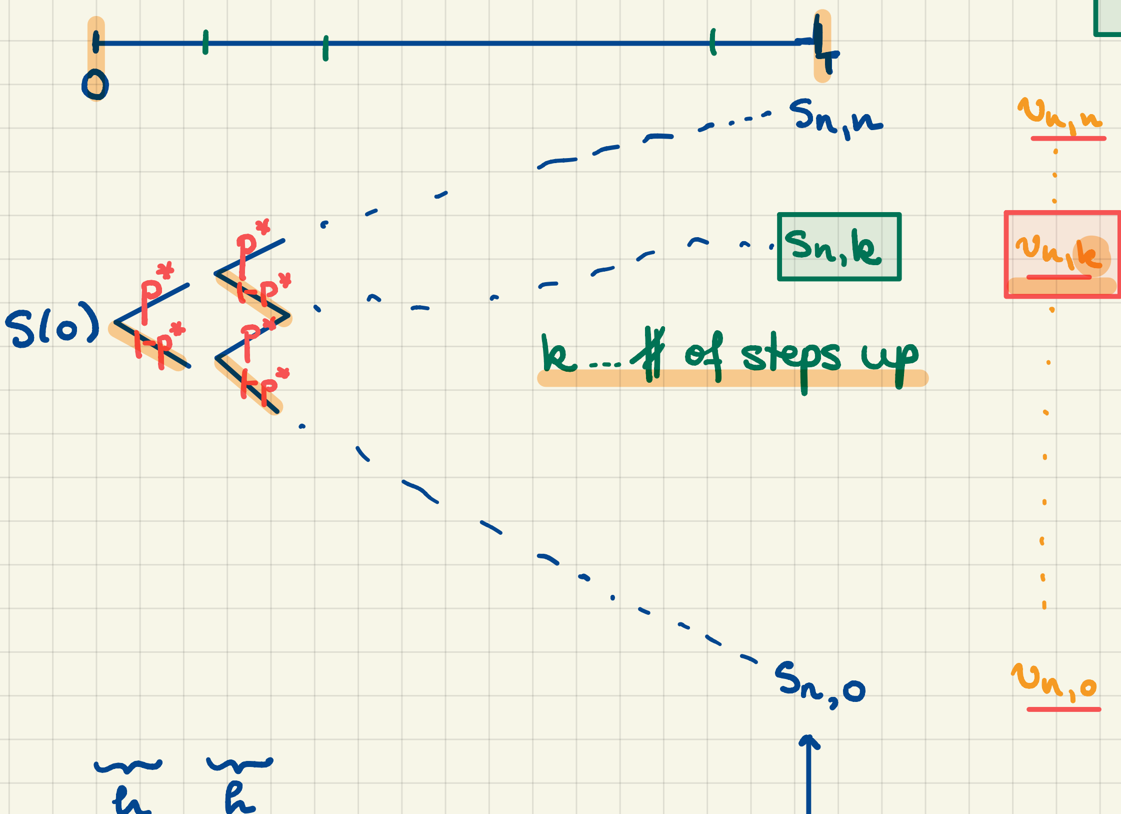
M339D: March 27th, 2023.

Multiple Binomial Periods.

T ... exercise date of a European option
n ... # of periods

} the length of each period $\boxed{h = \frac{T}{n}}$



These $(n+1)$ values are the support of $S(T)$.

$\Rightarrow$ for every $k = 0, 1, \dots, n$:

$$S_{n,k} = S(0)\cdot u^k \cdot d^{n-k} = S(0)\cdot\left(\frac{u}{d}\right)^{k}\cdot d^n$$

# of upsteps

Consider a European option w/ payoff f'tion $v(\cdot)$.
Then, the possible payoff values will be

$$\boxed{v_{n,k} = v(S_{n,k})}$$

Recall: Risk-Neutral Pricing: $\boxed{V(0) = e^{-rT}\mathbb{E}^*[V(T)]}$

p^*... the risk-neutral probability of an upstep, i.e.,

$$\boxed{p^* = \frac{e^{rh} - d}{u - d}}$$

=> The risk-neutral probability of attaining the payoff $v_{n,k}$:

$$\boxed{\binom{n}{k}(p^*)^k (1-p^*)^{n-k}}$$

The risk-neutral option price:

$$\boxed{V(0) = e^{-rT} \sum_{k=0}^{n} \left(\binom{n}{k} (p^*)^k (1-p^*)^{n-k} \cdot v_{n,k} \right)}$$

Problem 9.2. Let the continuously compounded risk-free interest rate be 0.10. Let the initial price of a non-dividend-paying stock be $\boxed{\$100}$ per share. You use a five-period binomial tree to model the stock price over the next year. Let $u = 1.04$ and $d = 0.96$. $K=100$

What is the price of a $\boxed{\text{one-year}}$, $\boxed{\text{at-the-money}}$ European call option on the above stock?

$\boxed{T=1}$ $\Downarrow$ $h=\frac{1}{5}$

→: The Risk-Neutral Probability:

$$p^* = \frac{e^{rh}-d}{u-d} = \frac{e^{0.10(0.2)}-0.96}{1.04-0.96} \approx \underline{0.7525}$$

The relevant stock prices in our tree:

$S_{5,5} = S(0)u^5 = 100\cdot(1.04)^5 = \underline{121.67}$ ⇒ $v_{5,5} = 21.67$

$S_{5,4} = S(0)u^4\cdot d = 100(1.04)^4(0.96) = \underline{112.31}$ ⇒ $v_{5,4} = 12.31$

$S_{5,3} = S(0)u^3\cdot d^2 = 100(1.04)^3(0.96)^2 = \underline{103.67}$ ⇒ $v_{5,3} = 3.67$

The remaining terminal nodes are all out-o-money.

$$\Rightarrow V_c(0) = e^{-0.10}\Big(21.67(p^*)^5 + 12.31\cdot 5(p^*)^4(1-p^*) + 3.67\cdot 10\cdot(p^*)^3(1-p^*)^2\Big) = \underline{10.02}$$

$e^{-rT} = e^{-0.10(1)}$

$\square$