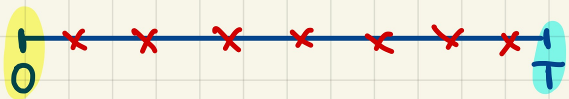
M339D: February 11th, 2022.

European Call Options.

The option can be EXERCISED, i.e., the cashflow can be collected only on a fixed exercise date.

Usually, this means a RIGHT TO BUY the underlying asset.

Usually, the option's owner has a RIGHT, but NOT AN OBLIGATION to exercise the option.



Option written. EXERCISE DATE

At time 0:
- The writer of the option is said to write/short the call.
- The buyer of the call is said to long the call. They will be referred to as the call's owner.
- They agree on:
  - the underlying asset: $S(t), t \geq 0$
  - T... exercise date
  - K... the strike/exercise price
- The premium for the call is paid by the buyer to the writer.

At time T:
- The call's owner has a right, but not an obligation to buy one unit of the underlying asset for the strike price K.
- The call's writer is obligated to do what the owner decides to do.

Payoff = ? We focus on the payoff of the long call, i.e., the payoff of the call's owner.

The owner's rationale for whether to exercise the call is based on "maximizing money in". Thus, their criterion is:

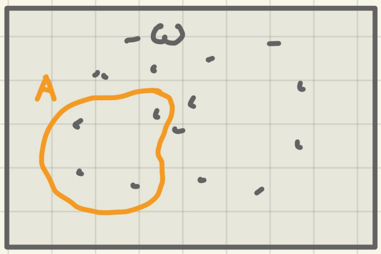
$$\begin{cases} \text{IF } S(T) \geq K, \text{ then exercise} & \Rightarrow \text{Payoff} = S(T) - K \\ \text{IF } S(T) < K, \text{ then do NOT exercise.} & \Rightarrow \text{Payoff} = 0 \end{cases}$$

We introduce: $V_c(T)$... the random variable denoting the payoff the long call

$$\Rightarrow \quad V_c(T) = \begin{cases} S(T) - K & \text{if } S(T) \geq K \\ 0 & \text{if } S(T) < K \end{cases}$$

A

Indicator Random Variables.



Ω (probability space)

ω... elementary outcomes

"Any nice" subset of Ω is called an event.

We define:

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

Now, the payoff of the call can be rewritten as:

$$V_c(T) = (S(T) - K) \cdot \mathbb{I}_{\underbrace{[S(T) \geq K]}_{\Updownarrow \; S(T) - K \geq 0}} = \max(S(T) - K, 0) = (S(T) - K) \vee 0$$

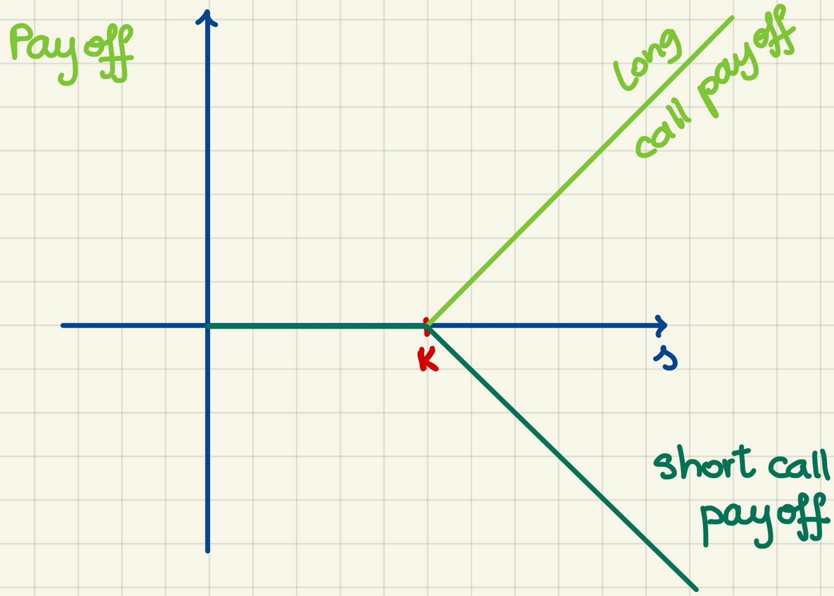
Introduce: the positive·part·function :

$$x \longmapsto (x)_+ =: \max(x, 0)$$

$$\Rightarrow \quad \boxed{V_c(T) = (S(T) - K)_+}$$

$\Rightarrow$ The payoff f'tion: $\boxed{v_c(s) = (s - K)_+}$

Note: There must be an initial cost, i.e., a premium $\boxed{V_c(0)}$ paid.



"hockey-stick" function