Multiclass logistic regression

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For a similar analysis, look at this tutorial from UCLA. For more about `glmnet`, consult this tutorial by Trevor Hastie.

First, we import the data.

```
data<-read.csv("hsbdemo.csv")
data
```

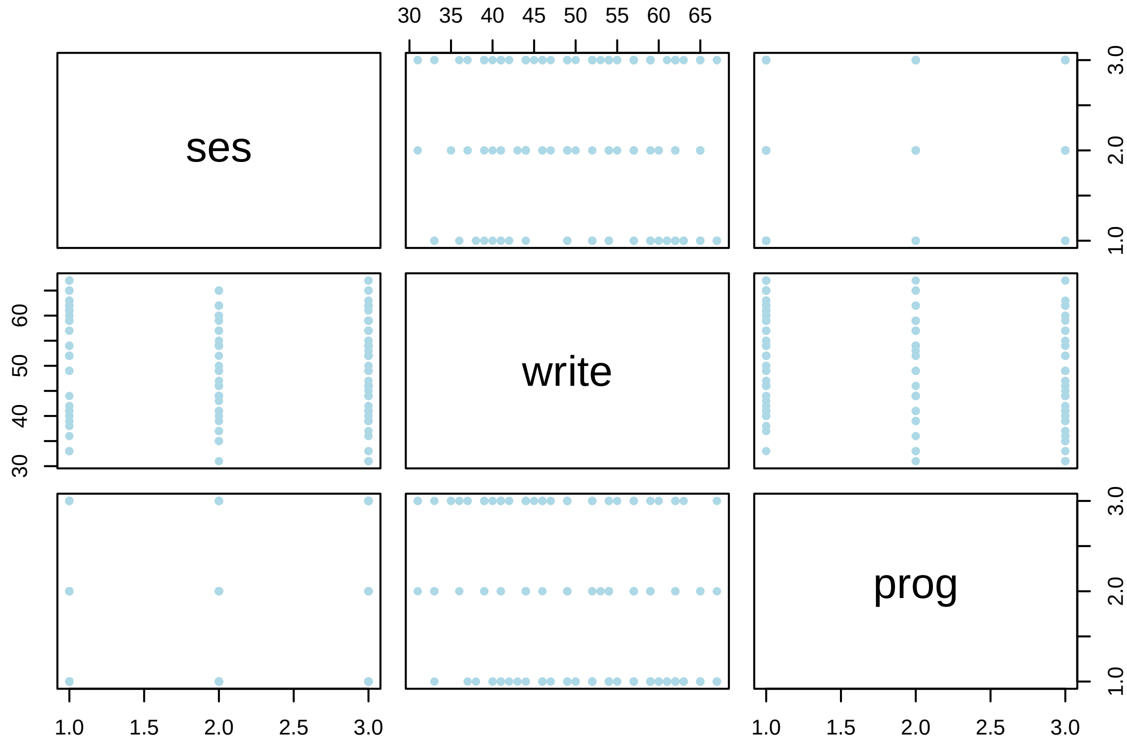
```
##   X  id female    ses schtyp     prog read write math science socst
## 1 1  45 female    low public vocation   34    35   41      29    26
## 2 2 108   male middle public  general   34    33   41      36    36
## 3 3  15   male   high public vocation   39    39   44      26    42
## 4 4  67   male    low public vocation   37    37   42      33    32
## 5 5 153   male middle public vocation   39    31   40      39    51
## 6 6  51 female   high public  general   42    36   42      31    39
## 7 7 164   male middle public vocation   31    36   46      39    46
##         honors awards cid
## 1 not enrolled      0   1
## 2 not enrolled      0   1
## 3 not enrolled      0   1
## 4 not enrolled      0   1
## 5 not enrolled      0   1
## 6 not enrolled      0   1
## 7 not enrolled      0   1
##  [ reached 'max' / getOption("max.print") -- omitted 193 rows ]
```

The data set contains variables on 200 students. We will focus on a small subset. The predictor variables will be social economic status `ses` (a three-level categorical variable) and writing score `write` (a quantitative variable). The outcome variable is program type `prog` (a three-level categorical variable). Since I am interested in just this subset, I will create a smaller data frame to analyze.

```
df=data.frame(data$ses, data$write, data$prog)
colnames(df)<-c("ses", "write", "prog")
attach(df)
```

Some exploratory data analysis is called for. The first idea is, probably to try the `plot` command.

```
plot(df,
     pch=20, col="lightblue")
```



As we can see, this is not too useful. To look at the *association* between `ses` and `prog`, a two-way table might do the trick.

```
tab=table(ses, prog)
tab
```

```
##         prog
## ses       academic general vocation
##   high          42       9        7
##   low           19      16       12
##   middle        44      20       31
```

Which test might we use to test the independence hypothesis?

```
chi2<-chisq.test(tab)
chi2
```

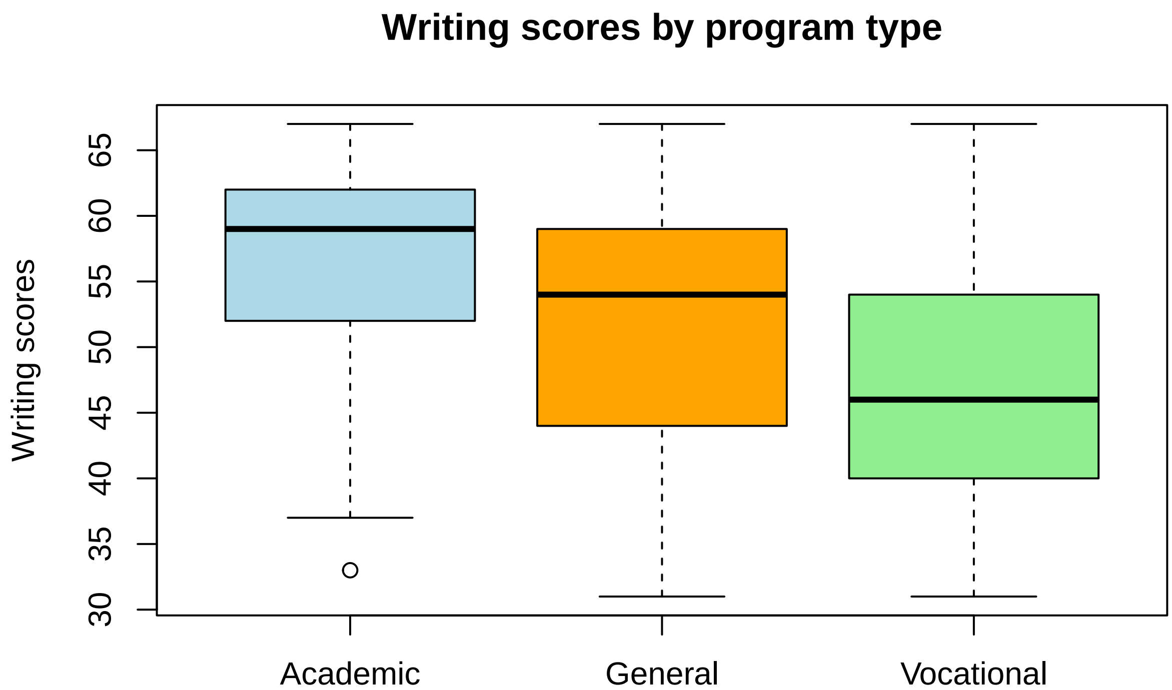
```
##
##  Pearson's Chi-squared test
##
## data:  tab
## X-squared = 16.604, df = 4, p-value = 0.002307
```

What about the association between the writing score and the program type? Side-by-side boxplots might give insight.

```
write.ac=write[which(prog=="academic")]
write.gen=write[which(prog=="general")]
write.voc=write[which(prog=="vocation")]

boxplot(write.ac,write.gen, write.voc,
  main = "Writing scores by program type",
  ylab = "Writing scores",
  names = c("Academic", "General", "Vocational"),
```

```
  col=c("lightblue", "orange", "lightgreen")
)
```



We see that there is an association, but we will learn more about the extent of the effect if we run a multiclass logistic regression.

`multinom`

Out first option is the `multinom` implementation in the `nnet` package.

```
#install.packages("nnet")
library(nnet)
```

This implementation requires of us to set a *baseline* level for the response variable.

```
df$prog<-as.factor(df$prog)
df$prog.base <- relevel(df$prog, ref = "academic")
```

Now, we can run the `multinom` function.

```
mn.fit<-multinom(prog.base ~ ses + write, data=df)
```

```
## # weights:  15 (8 variable)
## initial  value 219.722458
## iter  10 value 179.983731
## final  value 179.981726
## converged
```

Her is what the object `mn.fit` looks like:

```
summary(mn.fit)
```

```
## Call:
## multinom(formula = prog.base ~ ses + write, data = df)
##
## Coefficients:
```

```
##          (Intercept)    seslow sesmiddle       write
## general     1.689478 1.1628411 0.6295638 -0.05793086
## vocation    4.235574 0.9827182 1.2740985 -0.11360389
##
## Std. Errors:
##          (Intercept)    seslow sesmiddle      write
## general     1.226939 0.5142211 0.4650289 0.02141101
## vocation    1.204690 0.5955688 0.5111119 0.02222000
##
## Residual Deviance: 359.9635
## AIC: 375.9635
```

Importantly, the coefficients above are **different** from the coefficients one sees in our textbook. However, let us look at the fitted probabilities for our data set.

```
fitted.ps <- fitted(mn.fit)
fitted.ps
```

```
##      academic    general   vocation
## 1   0.1482721 0.33825094 0.51347695
## 2   0.1201988 0.18063349 0.69916776
## 3   0.4186768 0.23681368 0.34450947
## 4   0.1726839 0.35084330 0.47647284
## 5   0.1001206 0.16894278 0.73093666
## 6   0.3533583 0.23780466 0.40883700
## 7   0.1562526 0.19735515 0.64639223
## 8   0.1001206 0.16894278 0.73093666
## 9   0.2331247 0.22040133 0.54647393
## 10  0.1699365 0.20255764 0.62750590
## 11  0.2777676 0.37620933 0.34602312
## 12  0.2917517 0.23361220 0.47463615
## 13  0.1071652 0.30822610 0.58460873
## 14  0.2888732 0.22953872 0.48158807
## 15  0.1482721 0.33825094 0.51347695
## 16  0.2777676 0.37620933 0.34602312
## 17  0.3126201 0.37709158 0.31028831
## 18  0.3293850 0.23309576 0.43751925
## 19  0.3293850 0.23309576 0.43751925
## 20  0.6324633 0.20043456 0.16710218
## 21  0.1998541 0.21215684 0.58798906
## 22  0.2888732 0.22953872 0.48158807
## 23  0.3306561 0.37639804 0.29294591
## 24  0.2777676 0.37620933 0.34602312
## 25  0.1726839 0.35084330 0.47647284
## 26  0.3966353 0.23772687 0.36563784
## 27  0.3676891 0.37276358 0.25954735
## 28  0.2888732 0.22953872 0.48158807
## 29  0.2291956 0.36934430 0.40146013
## 30  0.3865745 0.36985112 0.24357436
## 31  0.2888732 0.22953872 0.48158807
## 32  0.1996861 0.36131931 0.43899458
## 33  0.2141360 0.36565697 0.42020707
##  [ reached getOption("max.print") -- omitted 167 rows ]
```

Also importantly, we do not have $p-$values reported above. We can figure them out easily, though.

```
z.scores=z <- summary(mn.fit)$coefficients/summary(mn.fit)$standard.errors
p.val=1-pnorm(abs(z.scores))
p.val
```

```
##           (Intercept)     seslow   sesmiddle           write
## general  0.0842581947 0.01186836 0.087897465 0.0034084569434
## vocation 0.0002191301 0.04946638 0.006337052 0.0000001588044
```

How well did we do on the training set?

```
pred.mn.tr=predict(mn.fit, newdata = df, "class")
#pred.mn.tr
ind.tr=(pred.mn.tr==prog)
mean(ind.tr)
```

```
## [1] 0.61
```

How would we predict for a test case? First, we need to define what the inputs for the test case would be.

```
new.data <- data.frame(ses = c("low", "middle", "high"), write = c(quantile(df$write, 0.25), median(df$
```

Now, we can use the `predict` command to see what the predicted probabilities would be.

```
pred.mn=predict(mn.fit, newdata = new.data, "class")
pred.mn
```

```
## [1] general  academic academic
## Levels: academic general vocation
```

glmnet

Here, we need to install and load a package create by Hastie et al.

```
#install.packages("glmnet")
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loaded glmnet 4.1-8
```

We notice that we need to transform our categorical predictor using dummy variables. For that, the following library is useful:

```
library(caret)
```

```
## Loading required package: ggplot2
```

```
## Loading required package: lattice
```

First, I will put all of my predictors into a separate data frame.

```
temp.x=data.frame(data$ses, data$write)
```

Now, I will use the command `dummyVars` to create dummy variables for my one categorical predictor `ses` in my array of predictors.

```
dummy.x <- dummyVars(" ~ .", data = temp.x)
x <- as.matrix(predict(dummy.x, newdata = temp.x))
print(x)
```

```
##     data.seshigh data.seslow data.sesmiddle data.write
## 1              0           1              0         35
```

```
## 2                0           0              1          33
## 3                1           0              0          39
## 4                0           1              0          37
## 5                0           0              1          31
## 6                1           0              0          36
## 7                0           0              1          36
## 8                0           0              1          31
## 9                0           0              1          41
## 10               0           0              1          37
## 11               0           1              0          44
## 12               1           0              0          33
## 13               0           1              0          31
## 14               0           0              1          44
## 15               0           1              0          35
## 16               0           1              0          44
## 17               0           1              0          46
## 18               0           0              1          46
## 19               0           0              1          46
## 20               1           0              0          49
## 21               0           0              1          39
## 22               0           0              1          44
## 23               0           1              0          47
## 24               0           1              0          44
## 25               0           1              0          37
##  [ reached getOption("max.print") -- omitted 175 rows ]
```

Now, we would like to fit the model using `glmnet`.

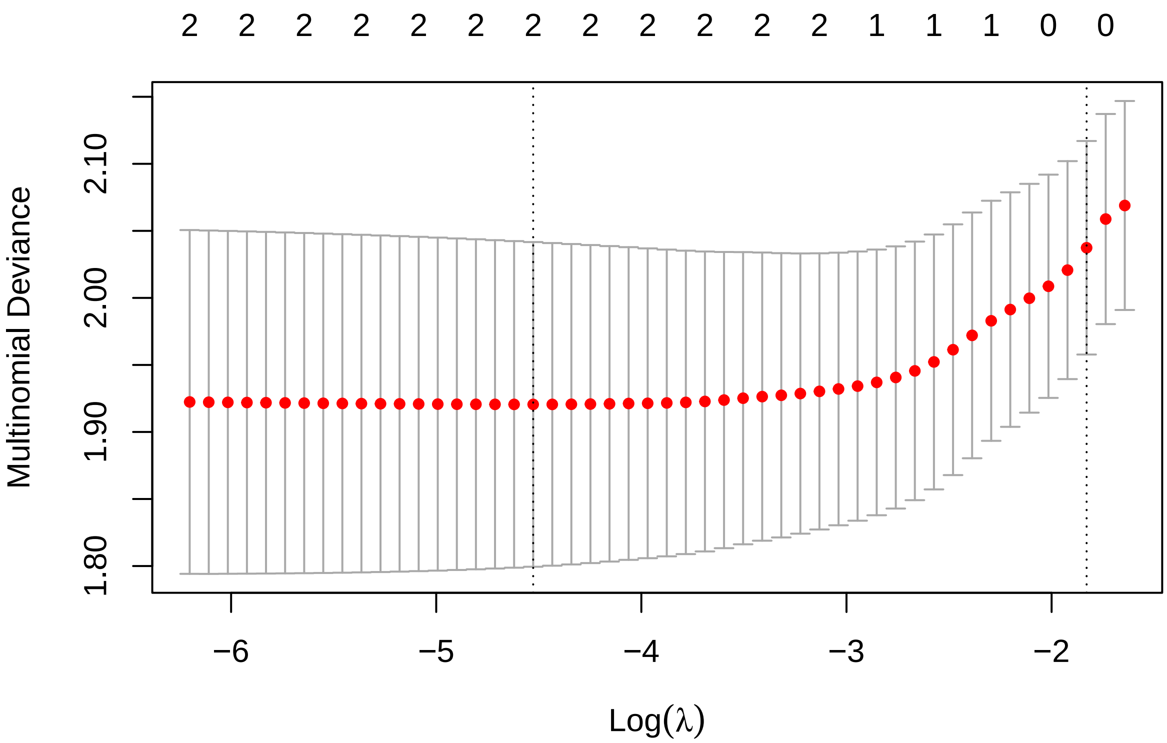
```r
y <- prog
#y
glm.fit <- glmnet(x, y, family = "multinomial")
#summary(glm.fit)
#glm.fit
predict(glm.fit, newx=x, s=0, type="response")
```

```
## , , 1
##
##     academic    general   vocation
## 1   0.1526166 0.33690080 0.51048259
## 2   0.1240333 0.18489797 0.69106876
## 3   0.4172688 0.23461004 0.34812117
## 4   0.1771834 0.34881146 0.47400509
## 5   0.1037572 0.17343832 0.72280443
## 6   0.3528570 0.23557453 0.41156851
## 7   0.1602391 0.20117013 0.63859078
## 8   0.1037572 0.17343832 0.72280443
## 9   0.2368129 0.22328970 0.53989743
## 10  0.1739242 0.20620006 0.61987571
## 11  0.2822784 0.37220402 0.34551754
## 12  0.2921047 0.23156195 0.47633332
## 13  0.1110538 0.30824709 0.58069910
## 14  0.2919886 0.23186238 0.47614904
## 15  0.1526166 0.33690080 0.51048259
## 16  0.2822784 0.37220402 0.34551754
## 17  0.3169693 0.37272452 0.31030614
```

```
## 18  0.3319611 0.23508177 0.43295716
## 19  0.3319611 0.23508177 0.43295716
## 20  0.6288569 0.19944323 0.17169987
## 21  0.2037564 0.21543051 0.58081312
## 22  0.2919886 0.23186238 0.47614904
## 23  0.3348986 0.37189274 0.29320866
## 24  0.2822784 0.37220402 0.34551754
## 25  0.1771834 0.34881146 0.47400509
## 26  0.3955289 0.23549149 0.36897962
## 27  0.3716716 0.36807063 0.26025772
## 28  0.2919886 0.23186238 0.47614904
## 29  0.2338110 0.36607265 0.40011633
## 30  0.3904065 0.36510890 0.24448463
## 31  0.2919886 0.23186238 0.47614904
## 32  0.2042785 0.35863911 0.43708236
## 33  0.2187490 0.36267251 0.41857853
```

What if I try to find, by cross-validation, an optimal tuning parameter?

```
cv.fit <- cv.glmnet(x, y, family = "multinomial")
plot(cv.fit)
```



Now, we can try to predict at the optimal λ:

```
predict(cv.fit, newx = x, s = "lambda.min", type = "response")
```

```
## , , 1
##
##       academic   general   vocation
## 1   0.1718389 0.3298599 0.49830123
## 2   0.1408002 0.2028582 0.65634161
## 3   0.4117018 0.2263582 0.36193997
```

```
## 4   0.1969543 0.3390764 0.46396933
## 5   0.1198378 0.1925126 0.68764961
## 6   0.3511232 0.2272935 0.42158328
## 7   0.1774037 0.2170891 0.60550721
## 8   0.1198378 0.1925126 0.68764961
## 9   0.2523753 0.2352513 0.51237345
## 10  0.1910143 0.2213621 0.58762359
## 11  0.3017076 0.3548559 0.34343650
## 12  0.2939479 0.2240330 0.48201915
## 13  0.1285259 0.3067262 0.56474792
## 14  0.3050421 0.2415077 0.45345017
## 15  0.1718389 0.3298599 0.49830123
## 16  0.3017076 0.3548559 0.34343650
## 17  0.3356184 0.3540263 0.31035534
## 18  0.3427405 0.2433663 0.41389326
## 19  0.3427405 0.2433663 0.41389326
## 20  0.6134691 0.1957158 0.19081511
## 21  0.2203388 0.2290088 0.55065238
## 22  0.3050421 0.2415077 0.45345017
## 23  0.3530527 0.3526884 0.29425890
## 24  0.3017076 0.3548559 0.34343650
## 25  0.1969543 0.3390764 0.46396933
## 26  0.3912435 0.2271426 0.38161393
## 27  0.3886486 0.3482032 0.26314823
## 28  0.3050421 0.2415077 0.45345017
## 29  0.2538445 0.3515181 0.39463732
## 30  0.4067125 0.3450842 0.24820327
## 31  0.3050421 0.2415077 0.45345017
## 32  0.2243287 0.3463704 0.42930094
## 33  0.2388290 0.3492247 0.41194637
```

We might want to see how well we did on the training data.

```r
preds=predict(cv.fit, newx = x, s = "lambda.min", type = "class")
#preds
ind=(preds==y)
mean(ind)
```

```
## [1] 0.595
```