M339G: September 9th, 2024.

Bias.

Def'n. Let $\hat{\theta}$ be a point estimator for the parameter θ. The bias of the estimator $\hat{\theta}$ is defined as

$$\boxed{\text{bias}(\hat{\theta}) := \mathbb{E}_{\theta}[\hat{\theta}] - \theta}$$

Def'n. The mean squared error of $\hat{\theta}$ is defined as

$$MSE[\hat{\theta}] := \mathbb{E}_{\theta}[(\hat{\theta}-\theta)^2]$$

Assessing Model Accuracy.

Say, we have the "usual" model: $\boxed{Y = f(X) + \varepsilon}$

Say, we fit our model to some training data:

$$\boxed{Tr = \{(x_i, y_i) : i = 1, \dots, N\}}$$

Let $\hat{f}$ be the fit of the model to our Tr

$$MSE_{Tr} := \underset{i \in \{1,\dots,N\}}{\text{Ave}} \underline{(y_i - \hat{f}(x_i))^2} = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{f}(x_i))^2$$

We propose to look @ other data

$$\boxed{Te = \{(x_i, y_i) : i = 1, \dots, M\}}$$

These are our testing data.

We calculate:

$$\boxed{MSE_{Te} := \underset{i \in \{1,\dots,M\}}{\text{Ave}} (y_i - \hat{f}(x_i))^2 = \frac{1}{M} \sum_{i=1}^{M} (y_i - \hat{f}(x_i))^2}$$