M339G: February 20^{th}, 2026.

Logistic Regression: Motivation.

X... predictor (say, numerical for simplicity)

Y... response; categorical w/ two classes

$$Y = \begin{cases} 1 & \text{if category \#1} \\ 0 & \text{if category \#2} \end{cases}$$

Idea #1: $X \longmapsto Y = \underbrace{\beta_0 + \beta_1 X + \varepsilon}_{\in \mathbb{R}}$ ✗

Idea #2: $X \longmapsto \boxed{p(X) = \mathbb{P}[Y=1 \mid X]}$

$$= \underbrace{\beta_0 + \beta_1 X}_{\mathbb{R}} + \varepsilon \quad ✗$$

Def'n. $\boxed{odds = \frac{p(x)}{1-p(x)}} \in \underline{(0,\infty)}$

$$X \longmapsto odd = \underbrace{\beta_0 + \beta_1 X}_{\mathbb{R}} + \varepsilon$$

Def'n.

$$logodds = \ln(odds) = \ln\left(\frac{p(x)}{1-p(x)}\right) \in \mathbb{R}$$

$$logodds = \beta_0 + \beta_1 X + \varepsilon$$

$\vdots$

$$\ln\left(\frac{p(x)}{1-p(x)}\right) = \beta_0 + \beta_1 x + \varepsilon$$

$$\frac{p(x)}{1-p(x)} = e^{\beta_0+\beta_1 x+\varepsilon}$$

$$p(x) = (1-p(x))\, e^{\beta_0+\beta_1 x+\varepsilon}$$

$$= e^{\beta_0+\beta_1 x+\varepsilon} - p(x) e^{\beta_0+\beta_1 x+\varepsilon}$$

$$\boxed{\hat{p}(x) = \frac{e^{\hat{\beta}_0+\hat{\beta}_1 x}}{1+e^{\hat{\beta}_0+\hat{\beta}_1 x}}}$$

Q: $\mathbb{P}[Y=0 \mid X] = \dfrac{1}{1+e^{\hat{\beta}_0+\hat{\beta}_1 x}}$