M378K Introduction to Mathematical Statistics

Problem Set #13

Order Statistics.

Problem 13.1. *An insurance company is handling claims from two categories of drivers: the good drivers and the bad drivers. The waiting time for the first claim from a* **good** *driver is modeled by an exponential random variable T_g with mean 6 (in years). The waiting time for the first claim from a* **bad** *driver is modeled by an exponential random variable T_b with mean 3 (in years). We assume that the random variables T_g and T_b are independent.*

What is the distribution of the waiting time T until the first claim occurs (regardless of the type of driver this claim was filed by)?

Definition 13.1. *Let $Y_1, \dots, Y_n$ be a* **random sample**. *The random sample ordered in an increasing order is called an* order statistic *and denoted by*

$$Y_{(1)}, Y_{(2)}, \dots, Y_{(n)}.$$

Question Write $Y_{(1)}$ as a function of $Y_1, Y_2, \dots, Y_n$.

Question Write $Y_{(n)}$ as a function of $Y_1, Y_2, \dots, Y_n$.

Problem 13.2. *What is the distribution function of the random variable $Y_{(n)}$?*

Problem 13.3. *Assume that the random sample comes from a density f_Y. Is the r.v. $Y_{(n)}$ continuous? If so, what is its density $g_{(n)}$?*

Problem 13.4. *What is the distribution function of the random variable* $Y_{(1)}$*?*

Problem 13.5. *Assume that the random sample comes from a density* f_Y. *Is the r.v.* $Y_{(1)}$ *continuous? If so, what is its density* $g_{(1)}$*?*

Theorem 13.2. *Lt* $Y_1, \dots, Y_n$ *be independent, identically distributed random variables with the common cumulative distribution function* F_Y *and the common probability density function* f_Y. *Let* $Y_{(k)}$ *denote the* k^{th} *order statistic and let* $g_{(k)}$ *denote its probability density function. Then,*

$$g_{(k)}(y) = \frac{n!}{(k-1)!(n-k)!}(F_Y(y))^{k-1} f_Y(y)(1 - F_Y(y))^{n-k} \quad \textit{for all } y \in \mathbb{R}.$$