M339D: February 19th, 2025.

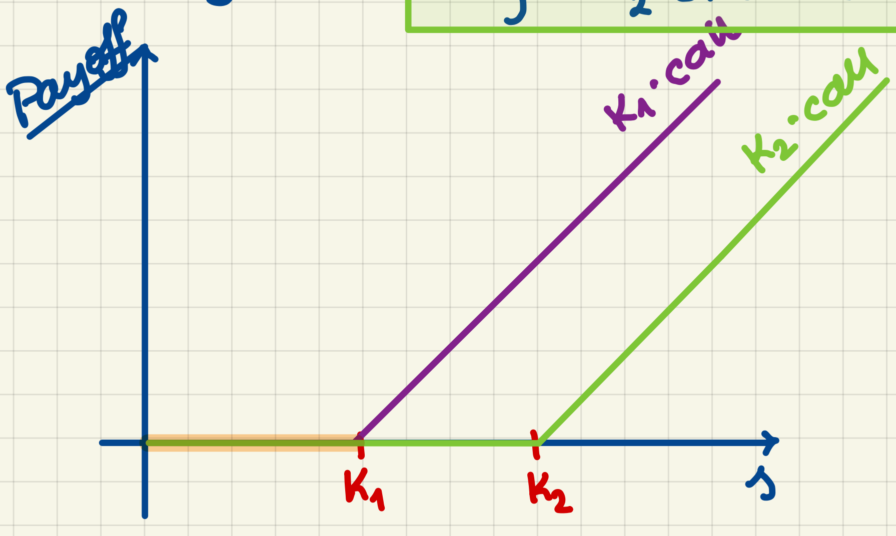
More on Arbitrage Portfolios.

Example. $K_1 < K_2$

A: one $\boxed{\text{long } K_1\text{-strike call}}$

B: one $\boxed{\text{long } K_2\text{-strike call}}$

w/ the same underlying asset and exercise date and European



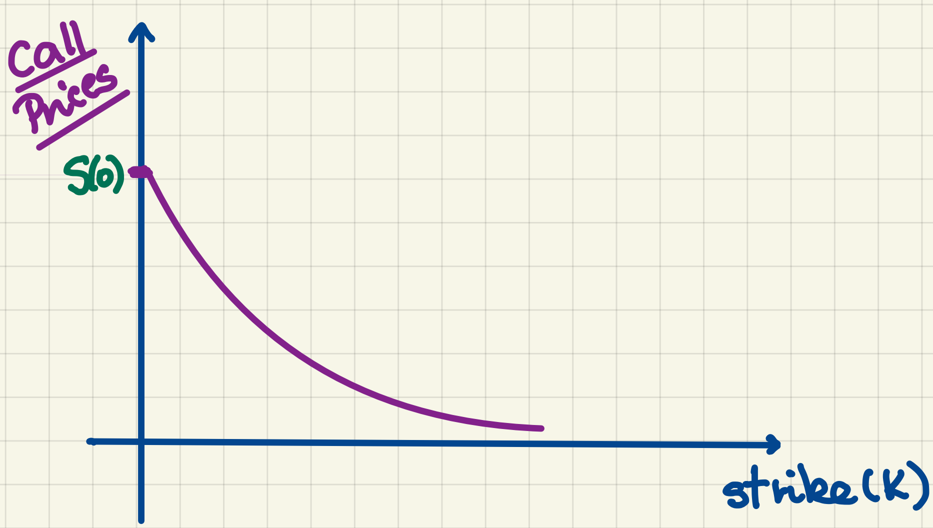
The payoff of the K_1-strike call **dominates** the payoff of the K_2-strike call

$\Downarrow$

$\boxed{\text{The } K_1\text{-call costs @ least as much as the } K_2\text{-call.}}$

$\boxed{\text{In Math:}}$ $\boxed{K_1 < K_2 \Rightarrow V_C(0,K_1) \geq V_C(0,K_2)}$

As a function of the strike price, call prices are <u>decreasing</u>.



Replicating Portfolios.

Def'n. Consider a European-style derivative security. A static portfolio w/ the same payoff as that of the derivative security is called its replicating portfolio.

Note: The initial price of the derivative security is equal to the initial price of its replicating portfolio.

Example. Consider a forward contract on a non-dividend-paying stock/index.

$$0 \longrightarrow T$$

Forward Contract: $S(T) - F$

Replicating Portfolio:
- long one share of stock
- issue a bond w/ redemption amount F and maturity date T

Payoff (Portfolio) $= S(T) - F$

$\Rightarrow$ The forward contract and its replicating portfolio must have the same initial cost, i.e.,

$$0 = \underbrace{S(0)}_{\text{long stock}} \underbrace{- PV_{0,T}(F)}_{\text{short bond}}$$

$$\Rightarrow \quad PV_{0,T}(F) = S(0)$$

$$\Rightarrow \quad \boxed{F = FV_{0,T}(S(0)) = S(0)e^{rT}}$$

$\square$

3. An insurance company sells single premium deferred annuity contracts with return linked to a stock index, the time-t value of one unit of which is denoted by $S(t)$. The contracts offer a minimum guarantee return rate of $g\%$. At time 0, a single premium of amount π is paid by the policyholder, and $\pi \times y\%$ is deducted by the insurance company. Thus, at the contract maturity date, T, the insurance company will pay the policyholder

$$\pi \times (1 - y\%) \times \text{Max}[S(T)/S(0), (1 + g\%)^T].$$

You are given the following information:

(i) The contract will mature in one year.

(ii) The minimum guarantee rate of return, $g\%$, is 3%.

(iii) Dividends are incorporated in the stock index. That is, the stock index is constructed with all stock dividends reinvested. NO DIVIDENDS!

(iv) $S(0) = 100$.

(v) The price of a one-year European put option, with strike price of \$103, on the stock index is \$15.21.

Determine $y\%$, so that the insurance company does not make or lose money on this contract.

(A) 12.8%.

(B) 13.0%

(C) 13.2%

(D) 13.4%

(E) 13.6%.

The Synopsis:

1. Focus on the insurance company's liability

2. Use our $\boxed{\text{data}}$

3. Algebraically simplify w/ an eye on the $\boxed{\text{data}}$