Problem set #8: Binomial Monte Carlo: Solutions

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Let the **volatility** of a stock be the standard deviation of its (continuously compounded) realized return on an annual basis. Then, we can define the up and down factors in the so-called *forward binomial tree* for a **non-dividend-paying** stock as

$$\begin{aligned} u &= e^{rh+\sigma\sqrt{h}} \\ d &= e^{rh-\sigma\sqrt{h}} \end{aligned} \tag{1}$$

Let the continuously compounded, risk-free interest rate be 0.04.

```
r=0.04
```

Consider a non-dividend-paying stock whose current price is \$100 and whose volatility is 0.25. We will be pricing a one-year, at-the-money call option in a variety of ways here.

```
s0=100
sigma=0.25
T=1
K=s0
```

Problem #1: Analytic one period

Price the option above using a one period binomial tree.

Solution:

```
periods=1
h=T/periods
u=exp(r*h+sigma*sqrt(h))
d=exp(r*h-sigma*sqrt(h))
u
```

```
## [1] 1.336427
```

```
d
```

```
## [1] 0.8105842
```

```
#the payoff function
v.c<-function(x){
  max(x-K,0)
}

p.star=(exp(r*h)-d)/(u-d)
p.star
```

```
## [1] 0.4378235
```

```r
#the possible stock prices
s.u=s0*u
s.d=s0*d

#the possible payoffs
v.u=v.c(s.u)
v.d=v.c(s.d)

v.0=exp(-r*T)*(p.star*v.u+(1-p.star)*v.d)
v.0
```

```
## [1] 14.15203
```

Problem #2: Monte Carlo one period

Price the option above using Monte Carlo a one period binomial tree. Use 10000 simulations.

Solution:

```r
n.sims=10000
probs=c(p.star, 1-p.star)
factors=c(u,d)
final.prices=s0*sample(factors, size=n.sims, prob=probs, replace=TRUE)
#final.prices
payoffs<-pmax(final.prices-K,0)
#payoffs
v.bar=mean(payoffs)
#v.bar
v.0.mc<-exp(-r*T)*v.bar
v.0.mc
```

```
## [1] 14.10279
```

Problem #3: Analytic two periods

Price the above option using a two-period binomial tree.

Solution:

```r
periods=2
h=T/periods
#h
u=exp(r*h+sigma*sqrt(h))
d=exp(r*h-sigma*sqrt(h))
#u
#d

p.star=(exp(r*h)-d)/(u-d)
#p.star

s.T=s0*c(u^2, u*d, d^2)
#s.T

v.T=pmax(s.T-K,0)
#v.T

probs=c(p.star^2, 2*p.star*(1-p.star), (1-p.star)^2)
```

```r
v.c=exp(-r*T)*sum(v.T*probs)
v.c
```

```
## [1] 11.57622
```

Problem #4: Monte Carlo two periods

Price the option above using Monte Carlo using a two period binomial tree. Use 10000 simulations.

Solution:

```r
#get the simulated values of the final stock price
#it's sufficient to simulate the number of upsteps
x=rbinom(n.sims, size=periods, prob=p.star)
x
```

```
##   [1] 1 1 0 2 1 1 2 2 1 1 2 1 1 1 0 1 1 1 0 1 2 1 1 1 2 1 1 1 1 1 1 0 1 2 1 2 1
##  [38] 1 1 1 0 1 1 1 0 2 1 0 0 2 1 0 0 0 1 2 1 1 2 0 2 1 1 1 1 0 1 2 0 1 1 0 1 2
##  [75] 2 0 1 2 1 1 0 0 1 0 2 0 0 2 1 0 0 1 1 1 0 1 1 0 1 0
##  [ reached getOption("max.print") -- omitted 9900 entries ]
```

```r
s.T=s0*u^x*d^(periods-x)
s.T
```

```
##   [1] 104.08108 104.08108  73.08454 148.22384 104.08108 104.08108 148.22384
##   [8] 148.22384 104.08108 104.08108 148.22384 104.08108 104.08108 104.08108
##  [15]  73.08454 104.08108 104.08108 104.08108  73.08454 104.08108 148.22384
##  [22] 104.08108 104.08108 104.08108 148.22384 104.08108 104.08108 104.08108
##  [29] 104.08108 104.08108 104.08108  73.08454 104.08108 148.22384 104.08108
##  [36] 148.22384 104.08108 104.08108 104.08108 104.08108  73.08454 104.08108
##  [43] 104.08108 104.08108  73.08454 148.22384 104.08108  73.08454  73.08454
##  [50] 148.22384 104.08108  73.08454  73.08454  73.08454 104.08108 148.22384
##  [57] 104.08108 104.08108 148.22384  73.08454 148.22384 104.08108 104.08108
##  [64] 104.08108 104.08108  73.08454 104.08108 148.22384  73.08454 104.08108
##  [71] 104.08108  73.08454 104.08108 148.22384 148.22384  73.08454 104.08108
##  [78] 148.22384 104.08108 104.08108  73.08454  73.08454 104.08108  73.08454
##  [85] 148.22384  73.08454  73.08454 148.22384 104.08108  73.08454  73.08454
##  [92] 104.08108 104.08108 104.08108  73.08454 104.08108 104.08108  73.08454
##  [99] 104.08108  73.08454
##  [ reached getOption("max.print") -- omitted 9900 entries ]
```

```r
#get the payoffs
v.T=pmax(s.T-K,0)

v.bar=mean(v.T)
v.c.mc=exp(-r*T)*v.bar
v.c.mc
```

```
## [1] 11.87218
```

Problem #5: Analytic one hundred periods

Price the above option using a 100−period binomial tree.

Problem #6: Monte Carlo with one hundred periods

Price the option above using Monte Carlo with a hundred period binomial tree. Use 10000 simulations.