Percentiles.

Def'n. The $100p^{th}$ percentile/quantile of a random variable X is any π_p such that

$$\boxed{F_X(\pi_p^-) \leq p \leq F_X(\pi_p)}$$

In particular, the 50^{th} percentile is called the median.

Special Case: Continuous distributions w/ a strictly positive density.

$$\boxed{\pi_p = F_X^{-1}(p)}$$

Problem. Find the ratio of the 90^{th} percentile to the median of an exponential dist'n w/ mean θ.

$\longrightarrow$: $X \sim \text{Exponential}(\theta)$

Let $p \in (0,1)$. We will find an expression for π_p of an exponential.

$$F_X(\pi_p) = p$$

$$1 - e^{-\frac{\pi_p}{\theta}} = p$$

$$\ln \Big\backslash \; 1-p = e^{-\frac{\pi_p}{\theta}}$$

$$\ln(1-p) = -\frac{\pi_p}{\theta}$$

$$\boxed{\pi_p = -\theta \cdot \ln(1-p)}$$

$$\frac{\pi_{0.9}}{\pi_{0.5}} = \frac{-\cancel{\theta}\ln(1-0.9)}{-\cancel{\theta}\ln(1-0.5)} = \frac{\ln(0.1)}{\ln(0.5)} = 3.3219 \;\blacksquare$$