M358K: November 30th, 2020.

Problem 7.4.

Assume the two-tailed test. $\boxed{\alpha = 0.01}$

(a) $n = 26$, $t = 2.485$

$$TS \sim t(df = 25)$$

$$\mathbb{P}[TS > 2.485] = \boxed{0.01}$$

$\Rightarrow$ p-value (since it's a two-tailed test) is $\boxed{0.02}$

$\Rightarrow$ @ the 0.01 significance level we fail to reject

(b) $n = 18$, $t = 0.5$

$$TS \sim t(df = 17)$$

$$\mathbb{P}[TS > 0.5] > 0.25$$

$\Rightarrow$ p-value is > 0.5 $\Rightarrow$ fail to reject

Using R: 2*pt(-0.5, df=17) = 0.6234852

Problem 7.12.

(a) $H_0: \mu = 35$ vs. $H_a: \mu \neq 35$

Statistical Inference for Two Means.

Inspiration.

Think about an experiment for testing whether a new drug works better than an existing drug.

"Old Drug" vs. "New Drug"

$$\begin{array}{ccc} & \boxed{\text{Sample}} & \\ \swarrow & \underline{\text{Randomized}} & \searrow \\ \boxed{\text{Control}} & & \boxed{\text{Treatment}} \\ \vdots & & \vdots \\ \mu_1 \ldots \text{mean for (sub)population \#1} & & \mu_2 \ldots \text{mean for (sub)population \#2} \end{array}$$

Goals:
- a confidence interval for $\mu_1 - \mu_2$
- hypothesis testing

Interested in $\mu_1 - \mu_2$

$\Rightarrow$ We must take a closer look @ $\boxed{\bar{X}_1 - \bar{X}_2}$ a random variable

Assumptions:
- both population dist'ns are normal
- the two samples are independent

$\Rightarrow$ for both $i = 1, 2$:

$$\bar{X}_i \sim \text{Normal}\left(\text{mean} = \mu_i, \text{var} = \frac{\sigma_i^2}{n_i}\right) \quad \text{w/ } n_i \ldots \text{sample size}$$

$$\bar{X}_1 - \bar{X}_2 \sim \text{Normal}\left(\text{mean} = \mu_1 - \mu_2,\ \text{var} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

Goals:

- confidence intervals with the sample standard deviations instead of the σ's
  w/ df $= \min(n_1, n_2) - 1$
- For hypothesis testing, our null is always
  $$\boxed{H_0 : \mu_1 = \mu_2}$$

Under the null, the observed value of the TS is

$$\boxed{t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$