M358K: November 13th, 2020.

Quiz #10: Problem #2.

$$\boxed{\frac{(n-1)\cdot S^2}{\sigma^2} \sim \chi^2(df = n-1)}$$

In the problem: $n = 11$, $\sigma^2 = 2$

$\boxed{\mathbb{P}[S^2 \leq a] = 0.025}$ and $\mathbb{P}[S^2 \leq b] = 0.0975$

$$\frac{(11-1)\cdot S^2}{2} \sim \chi^2(df = 11-1 = 10)$$

$$5S^2 \sim \chi^2(df = 10)$$

$$\mathbb{P}[5S^2 \leq 5a] = 0.025$$

$$\Rightarrow 5a = \chi^2_{0.975}(df = 10) = 3.247$$

$$\Rightarrow \boxed{a = 0.6494}$$

Goodness·of·Fit

Looking @ a MULTINOMIAL EXPERIMENT, possibly w/ CATEGORICAL DESCRIPTIONS of POSSIBLE OUTCOMES.

Say that the possible outcomes of this experiment are always described in categories which are mutually exclusive and exhaustive.

Say that these categories are events

$$A_1, A_2, \dots, A_k .$$

In our probabilistic model, the parameters are:

p_i, $i=1..k$ which stand for $p_i = \mathbb{P}[A_i]$.

<u>Note</u>: $\boxed{p_1 + p_2 + \dots + p_k = 1}$

Repeat the same multinomial experiment n times.

Let X_i ... the # of times that outcome i occured

$i=1..k$

<u>Note</u>: $\boxed{X_1 + X_2 + \dots + X_k = n}$

<u>Define</u>:

$$\boxed{Q^2 = \sum_{i=1}^{k} \frac{(X_i - n \cdot p_i)^2}{n \cdot p_i} \text{ "}\sim\text{" } \chi^2(df = k-1)}$$

Works for $n \cdot p_i \geq 5$ for all $i = 1..k$

Test Summary

Testing:

$H_0: p_1 = p_1^\circ,\ p_2 = p_2^\circ, \dots, p_k = p_k^\circ$

vs.

H_a: At least one of the population probabilities is different from its null value

$$\boxed{\text{This } \chi^2\text{-test is always the upper-tailed one!}}$$

We denote: $\cdot\, O_i \dots$ the observed counts
$\cdot\, E_i \dots$ the expected counts under the null hypothesis

$\Rightarrow$ The TEST STATISTIC is

$$q^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \text{ "}\sim\text{" } \chi^2(df = k-1)$$

With a significance level α:

Find $\chi^2_\alpha(df = k-1)$

IF $q^2 \geq \chi^2_\alpha(df=k-1)$, then REJECT THE NULL.

IF not, then fail to reject.

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Problem Set # 15

Goodness of fit.

Problem 15.1. Gregor Almond, the local horticulturalist, grows 400 progeny from a cross of peas. The cross is hypothesised to have a ratio of 1 green to 7 yellow seeds. Suppose that the cross actually produces 360 yellow and 40 green seeded plants.

(i) Calculate the observed value of the test statistic.
(ii) Using the χ^2–tables, what would your decision be at the significance level α. $\alpha = 0.025$
(iii) Using **R** with the observed value of the test statistic, find the p–value.
(iv) Using the command

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chisq.test()
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perform the χ^2–test and provide the summary.
(v) In this case, you can test the same hypotheses using the z–test. Do this for practice!

Problem 15.2. (8 points) You suspect that a die has been altered so that the outcomes of a roll (the numbers 1 through 6) are not equally likely. You roll the die 600 times and observe the following counts:

Outcome	1	2	3	4	5	6
Count	85	86	120	118	91	100

(i) Using the $\chi 2$–tables, at the significance level of 0.05 perform the goodness-of-fit test and report your conclusions.
(ii) Using **R**, perform the goodness-of-fit test.

Problem 15.3. The early education department of the local community college conducts a survey of 1000 randomly chosen children on their favorite among certain offered holidays. Here are the results of this survey:

Halloween	Thanksgiving	Arbor Day	Other
60%	10%	2%	28%

Then, the children were shown an inspirational movie on ecology and horticulture. After that, they were asked, again, to choose their favorite holiday among those offered. Here are the results of the renewed survey:

Halloween	Thanksgiving	Arbor Day	Other
58%	8%	8%	26%

Using the χ^2–goodness-of-fit test, say whether there is sufficient evidence that the chidrens' opinion was changed by the movie at the 0.01 significance level. Solve this problem both ways, i.e., using the χ^2–tables and usinf **R**.

PR#1. (i) H_0: $p_g = \frac{1}{8}$ and $p_y = \frac{7}{8}$

vs.

H_a: the color dist'n is different from the null

n ... sample size : $n = 400$

k ... # of categories : $k = 2$

the observed counts: $O_g = 40$; $O_y = 360$

The expected counts: $E_g = 50$; $E_y = 350$

$$q^2 = \frac{(40-50)^2}{50} + \frac{(360-350)^2}{350} = \underline{2.285714}$$

(ii) $\alpha = 0.025$

$df = 2-1 = 1$: $\chi^2_{0.025}(df=1) = \underline{5.024}$

$\Rightarrow$ Fail to Reject ■

(iii) 1-pchisq(2.28514, df=1) = 0.13057