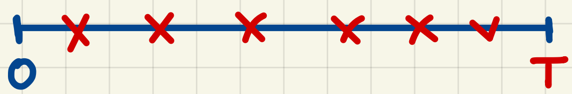
M339D: February 7th, 2025.

European Call Options.

European: The option can only be exercised, i.e., the transactions can take place on the exercise date.

Call: Usually, this means a right to buy the underlying asset.

Options: Usually, the option's owner has the right but not an obligation to exercise the option.



Option written. EXERCISE DATE

At time 0:

- The writer of the option write/shorts the call.
- The buyer of the call is said to long the call. They are referred to as the option's owner.
- The agreement:
    - the underlying asset: $S(t), t \geq 0$
    - the exercise date: T
    - K... the strike/exercise price
- $\boxed{\text{The buyer pays the premium to the writer.}}$ $V_C(0)$

At time T:

- The call's owner has a right but not an obligation to buy one unit of the underlying asset for the strike price K.
- The call's writer is obligated to do what the owner decides.

Payoff = ?

We focus on the payoff of the long call, i.e., the payoff for the call's owner.

The call owner's rationale for whether to exercise is to "maximize money in".

The criterion for exercise:

IF $S(T) \geq K$, then EXERCISE. $\Rightarrow$ Payoff $= S(T)-K$

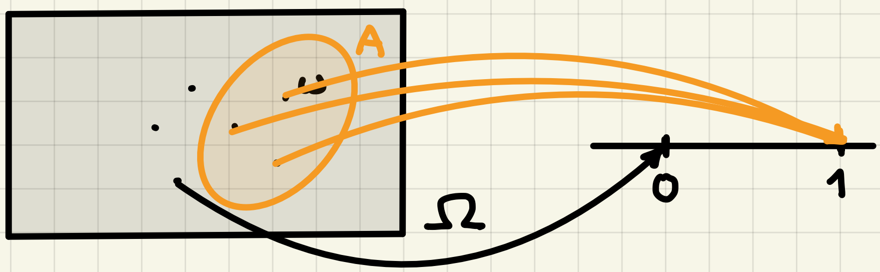
IF $S(T) < K$, then do not exercise. $\Rightarrow$ Payoff $= 0$

We introduce:

$V_c(T)$ ... the r.v. denoting the payoff of a long call

$$\Rightarrow \boxed{V_c(T) = \begin{cases} S(T)-K & \text{if } \boxed{S(T)\geq K} \\ 0 & \text{if } S(T)<K \end{cases}}$$

Indicator Random Variables.



Ω ... outcome space

$\omega \in \Omega$ ... elementary outcomes

A... a "nice" subset of Ω aka an EVENT

We define:

$$\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$$

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

$$\Rightarrow \boxed{V_c(T) = (S(T)-K)\cdot \mathbb{I}_{\boxed{[S(T)\geq K]}}}$$

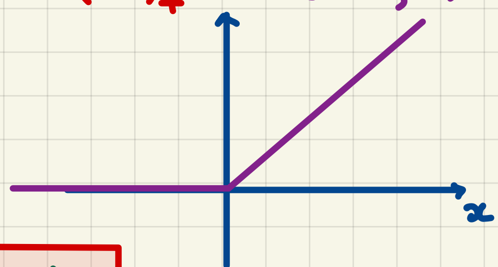
Note: $S(T) \geq K \iff S(T) - K \geq 0$

We could also write

$$\boxed{V_c(T) = \max(S(T)-K, 0) = \underset{\substack{\uparrow \\ \text{MAXIMUM OPERATOR}}}{(S(T)-K) \vee 0}}$$

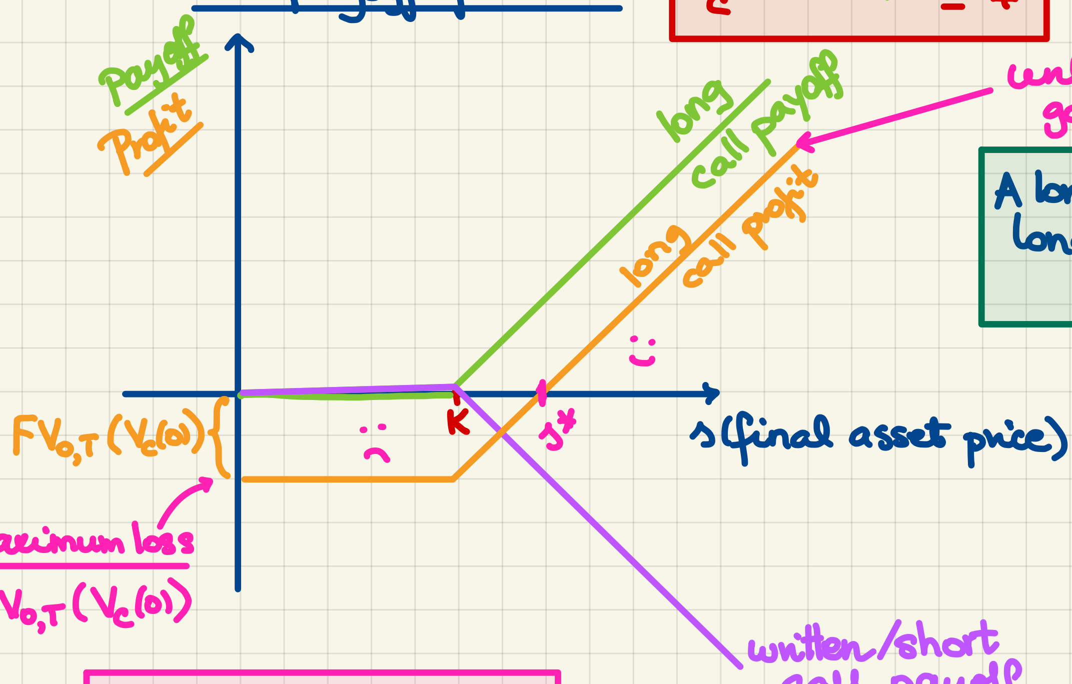
Introduce: The positive part function.

$$x \longmapsto (x)_+ =: \max(x, 0) = x \vee 0$$



$$\Rightarrow \quad \boxed{V_c(T) = (S(T) - K)_+}$$

$\Rightarrow$ the payoff function: $\boxed{v_c(s) = (s - K)_+}$



A long call is a long position w.r.t. the underlying.

Maximum loss
$FV_{0,T}(V_c(0))$

(Never positive and sometimes strictly negative $\Rightarrow \exists$ premium $V_c(0)$)

s^*... break-even point

$s^* = K + FV_{0,T}(V_c(0))$

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Problem Set #5

European call options.

Problem 5.1. The initial price of a non-dividend-paying asset is \$100. A $\boxed{\text{six-month}}$ \$95-strike European call option is available at a \$8 premium. The continuously compounded risk-free interest rate equals $\boxed{0.04}$. What is the break-even point for this call option?

T K r

(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

$$\rightarrow: \quad FV_{0,T}(V_C(0)) = 8 \cdot e^{0.04 \cdot \frac{1}{2}} = 8 \cdot e^{0.02}$$

$$S^* = 8e^{0.02} + 95 = \underline{103.16}$$

$\square$

Problem 5.2. (5 points) A stock's price today is $\boxed{\$1000}$ and the annual effective interest rate is given to be 5%. You $\boxed{\text{write a}}$ one-year, $\boxed{\$1{,}050}$-strike $\boxed{\text{call}}$ option for a premium of $\boxed{\$10}$ while you simultaneously $\boxed{\text{buy}}$ the stock. What is your **profit** if the stock's spot price in one year equals $\boxed{\$1{,}200}$?

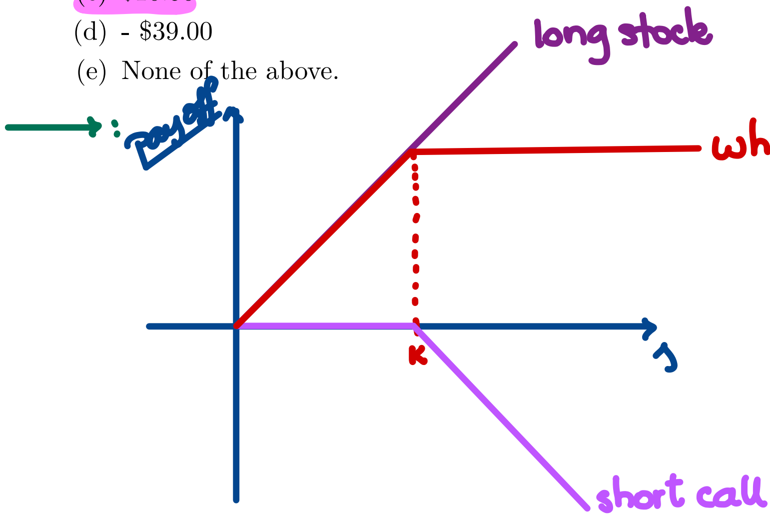
(a) $150.00

(b) $139.90

(c) $10.50

(d) - $39.00

(e) None of the above.



Algebraically:

$$\text{Payoff} = -(S(T)-K)_+ + S(T) = \begin{cases} K & \text{if } S(T) \geq K \\ S(T) & \text{if } S(T) < K \end{cases}$$

$$= \min(S(T), K)$$

$$\boxed{\text{Covered Call} = \text{Short Call} + \text{Long Underlying}}$$

In this problem:

$$\text{Payoff} = \min(1200, 1050) = 1050$$

$$\text{Initial Cost} = -10 + 1000 = 990$$

$$\text{Profit} = 1050 - 990\,(1.05) = \underline{10.50}$$

□