M362K Probability
University of Texas at Austin
Practice Problems for In-Term Exam III
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Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points. **There are many ways in which any single problem can be solved. The solutions herein are just one possible way to tackle the given problems.**

All written work handed in by the student is considered to be
their own work, prepared without unauthorized assistance.

The University Code of Conduct

"The core values of The University of Texas at Austin are learning, discovery, freedom, leadership, individual opportunity, and responsibility. Each member of the university is expected to uphold these values through integrity, honesty, trust, fairness, and respect toward peers and community. As a student of The University of Texas at Austin, I shall abide by the core values of the University and uphold academic integrity."

"I agree that I have complied with the UT Honor Code during my completion of this exam."

Signature:

3.1. **DEFINITIONS.**

Problem 3.1. (5 points) Complete the definition of a *cumulative distribution function* below:

Let X be a random variable on an outcome space Ω. The *cumulative distribution function* of X is ...

Problem 3.2. (5 points) Complete the definition of the *probability mass function* of a random variable X on a finite outcome space Ω.

Let X be a random variable on a finite outcome space Ω. The *probability mass function* of X is ...

3.2. **TRUE/FALSE QUESTIONS.**

Problem 3.3. (3 points) Let X denote the outcome of a roll of a fair, regular dodecahedron (a polyhedron with 12 faces) with numbers $1, 2, \cdots, 12$ written on its sides. Then $\mathbb{E}[X] = 13/2$. *True or false? Why?*

Problem 3.4. (3 points) If $Var[X] = 0$, then $\mathbb{P}[X = \mathbb{E}[X]] = 0$. *True or false? Why?*

3.3. **FREE RESPONSE PROBLEMS.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 3.5. (10 points) Assume that the number of claims filed by a policyholder in a given year is modeled by a Poisson random variable with parameter λ.

An actuary has discovered that policyholders are three times as likely to file two claims in a single year as they are to file four claims.

(i) (6 points) Find the unknown parameter λ.
(ii) (4 points) Find the variance of the number of claims filed in a given year.

Problem 3.6. (20 points) Let X and Y be independent random variables with distribution functions F_X and F_Y. Define $U = \min(X, Y)$ and $V = \max(X, Y)$ and denote their distribution functions by F_U and F_V, respectively. Express F_U and F_V in terms of F_X and F_Y.

Problem 3.7. If 20 fair octahedra whose sides are labeled with numbers 1 through 8 are rolled, find the approximate probability p that the sum of the obtained numbers falls between 30 and 40 inclusive.

Note: You will be working on this problem at home with calculators and such readily accessible; if a problem such as this one appears on the actual exams, the numbers will be chosen so that it is possible to perform the calculations by hand and in a reasonable amount of time.

3.4. **MULTIPLE CHOICE QUESTIONS.**

Problem 3.8. (5 points) Let X denote the number of $1's$ in 100 throws of a fair die. Find $\mathbb{E}[X^2]$.

(a) 125/9
(b) 50/3
(c) 875/3
(d) 1585/9
(e) None of the above