M378K Introduction to Mathematical Statistics
Homework assignment #1

Please, provide your **final answer only** to the following problems.

Problem 1.1. *(4 points) Evaluate the limit* $\lim\limits_{n\to\infty}\left(1-\frac{2}{n}\right)^n$.

Problem 1.2. *(2 points) Evaluate the limit* $\lim\limits_{t\to\infty} e^{-t}$.

Problem 1.3. *(4 points) Find the sum* $\sum\limits_{i=0}^{\infty}\frac{4^i}{i!}$.

Problem 1.4. *(5 points) A class has 7 female and 13 male students. It is also known that there are 15 blue-eyed and 5 brown-eyed students in that class. The probability that a student picked at random is a brown-eyed female is*

(a) $\frac{7}{80}$

(b) $\frac{13}{80}$

(c) $\frac{21}{80}$

(d) $\frac{39}{80}$

(e) *Not enough information is given.*

Problem 1.5. *(5 points) Let A and B be two events, and the only thing we know about them is that $\mathbb{P}[A]=\mathbb{P}[B]=\frac{2}{3}$. Then, it is **necessarily** true that*

(a) $A=B$

(b) $A\subseteq B$ *or* $B\subseteq A$

(c) A *and* B *are independent*

(d) A *and* B^c *are mutually exclusive*

(e) *All of the above are possible, but not necessarily true.*

Problem 1.6. *(5 points) Which of the following formulas hold for the exponential function:*

(a) $e^x + e^y = e^{x+y}$

(b) $e^x e^y = e^x + e^y$

(c) $e^{x+y} = e^x e^y$

(d) $e^{x-y} = e^x - e^y$

(e) None of the above.

Problem 1.7. *(5 points) A coin is tossed, and, independently, a 6-sided die is rolled. Let*

$$A = \{4 \text{ is obtained on the die}\} \text{ and}$$
$$B = \{\text{Heads is obtained on the coin and an even number is obtained on the die}\}.$$

Then

(a) A and B are mutually exclusive

(b) A and B are independent

(c) $A \subseteq B$

(d) $A \cap B = B$

(e) None of the above.

Problem 1.8. *(5 points) If $n!$ is the factorial function $n! = n \times (n-1) \times \cdots \times 2 \times 1$, then $\log(\sqrt[n]{n!})$ equals . . .*

(a) $\sum_{i=1}^{n} \log(n/i)$

(b) $\frac{1}{n} \sum_{i=1}^{n} \log(i)$

(c) $\sqrt[n]{\prod_{i=1}^{n} \log(n)}$

(d) $\frac{1}{n} \prod_{i=1}^{n} \log(i)$

(e) None of the above.

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 1.9. *(5 points) Every possible combination of a letter in the English alphabet (i.e., chosen from the 26-element set $\{A, B, C, \ldots, X, Y, Z\}$) and a number from the set $\{1, 2, \ldots, 19, 20\}$ is written on a card. The cards are otherwise identical, and well shuffled in a deck. If a single card is drawn from that deck, what is the probability that the number on it is odd or that the letter is a vowel (i.e., in the set $\{A, E, I, O, U\}$)?*

Problem 1.10. *(5 points) Four fair coins are tossed independently. What is the probability that at least one for them came up heads?*

Problem 1.11. *How much is*

$$\sum_{i=1}^{99} \log_{10}\left(\tfrac{i}{i+1}\right)$$

when simplified completely?