Due on April 19^{th}, 2020
M339W/389W Financial Mathematics for Actuarial Applications
Spring 2021, University of Texas at Austin
Graduate Homework
Instructor: Milica Čudina

The following problems are meant to be the extra special homework assignment for graduate students. If you are taking this course as an undergraduate course, your score on will count as extra credit. Details are below.
A part of the homework involves simulations while the rest involves calculations (and a bit of simulations). You are required to use **R** to complete this part of the assignment. Please, do not include a list of your simulated values in your final submission.
If you are a graduate student, this HW will be taken into account when your HW average is calculated (see the First-Day Handout). It will have the weight of **two** regular homework assignments. If you are taking this course as an undergraduate course and you decide to attempt this assignment, then your score will serve as extra credit toward your final score in the course. More precisely, a full score in this homework will be worth 2 points in the final score in this course.

Problem 1.1. (30 points) Let the current stock price of a continuous-dividend-paying stock be \$100. Let its dividend yield be 0.02 and let its volatility be 0.25. Under your model, the mean rate of return on this stock is 0.06.

(i) (10 pts) Simulate 1000 values of the stock price at time $t = 3$. Draw a histogram of the obtained set of values.
(ii) (2 pts) Find the median of the set of simulated values you obtained in part (i).
(iii) (2 pts) Find the average of the set of simulated values you obtained in part (i).
(iv) (4 pts) Draw the graph of the density of a lognormal distribution of S_3 with the appropriate parameters and compare it to the histogram from part (i).
(v) (12 pts) If you did not do so in the first place, simulate the values from (i) using the step-by-step method, calculating S_1 and S_2 as auxiliary simulated values. What is the proportion of paths that you created in this way that crosses above \$105 at any time? What is the proportion of paths such that $S_1 >$ \$100?

Problem 1.2. Your initial wealth is exactly \$100. You are allowed to invest in shares of a particular stock. You are also allowed to both lend and borrow at the continuously compounded risk-free interest rate of 0.05. Keeping your money uninvested is **not allowed**.

You can rebalance your portfolio every morning, once you have observed the opening stock price. This means that you can change the number of shares you own (if you decide to do so) and accordingly adapt your risk-free investment.

You proceed to create a "rule" according to which you will be rebalancing your portfolio. Here are some possible rules you can use:

- **i.** Always own exactly one share and keep the rest of my wealth invested at the risk-free rate. If needed, borrow at the continuously compounded risk-free interest rate.
- **ii.** Always keep exactly \$100 in the stock and the rest of my wealth invested at the risk-free rate. If needed, borrow at the continuously compounded risk-free interest rate.
- **iii.** Always keep exactly one half of my wealth invested in the stock and the rest of my wealth invested at the risk-free rate. If needed, borrow at the continuously compounded risk-free interest rate.
- **iv.** If the stock price drops overnight (regardless of the extent of the drop), sell half of the stock investment. If the stock price rises overnight (regardless of the extent of the increase), double the amount of the stock investment. If the stock price does not change, do nothing. In all three cases, the rest of my wealth is to be invested at the risk-free rate (if needed, I will borrow at the continuously compounded risk-free interest rate).
- **v.** Every day I will flip a fair coin. If the outcome is *heads*, I will buy one share of stock. If the outcome is *tails*, I will short one share. The balance of my wealth is to be invested at the risk-free rate (if needed, I will borrow at the continuously compounded risk-free interest rate).

(10 points) Create an investment "rule" of your own. You can use the above for inspiration, but do not just merely make simple (trivial!) adjustments. Call this rule **vi.**

Over the following 10 days, you observe the following stock prices for a non-dividend-paying stock:

Day	0	1	2	3	4	5	6	7	8	9	10
Stock price	100	80	64	80	64	80	100	80	64	80	100

(60 points) As the time passes you follow an investment "rules" **i.-vi.** above to rebalance your portfolio. Complete the following table describing your portfolio **just after** the rebalancing is done in the morning for every one of the investment "rules" **i.-iv.**. *Use software to construct and fill out a table of the same format as the one below.*

Day	0	1	2	3	4	5	6	7	8	9	10
Stock price	100	80	64	80	64	80	100	80	64	80	100
Number of shares owned											
Total wealth											