UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 1

The inverse transform method.

Problem 1.1. *Source: Course 3, November 1985, Problem #19.*
Your goal is to simulate four draws from a binomial distribution with two trials and the probability of success in every trial equal to 0.30. You intend to use the inverse transform method. Here are the four values produced by the random number generator:

$$0.90 \quad 0.21 \quad 0.72 \quad 0.48.$$

Which values of the binomial were obtained from the above outputs of the random number generator?

Problem 1.2. Let the random variable X have the following density function:

$$f_X(x) = 3x^{-4}, \quad x > 1$$

You use the *inverse transform method* to simulate values from X. Let the simulated value of the unit uniform be equal to 0.25. What is the corresponding value of X?

Problem 1.3. *Source: Course 3, November 1980, Problem #33.*
The probability density function of the random variable X is

$$f_X(x) = \begin{cases} 0, & \text{for } x < -1, \\ \frac{3}{2}x^2, & \text{for } -1 \le x \le 1, \\ 0, & \text{for } x > 1. \end{cases}$$

Let u denote the simulated value from a unit uniform random number generator. Which transformation would you apply to u to generate simulated values of x?

Problem 1.4. Let the random variable X have the cumulative distribution function given by
$$F_X(x) = \begin{cases} 0, & \text{for } x < 0, \\ x^4, & \text{for } 0 \le x \le 1, \\ 1, & \text{for } x > 1. \end{cases}$$

The inverse transform method was applied to generate the following three simulated values of X:

$$0.09 \quad 0.64 \quad 0.81.$$

Which values of the random number generator where mapped into the above three draws from X?

Problem 1.5. Let the cumulative distribution function F_X of a random variable X satisfy the following conditions:

- $F_X(0) = 0$;
- $F_X(1) = 0.2$;
- $F_X(2) = 1$;
- F_X is linear on $(0,1)$ and $(1,2)$;
- F_X is continuous.

You use the inverse transform method to simulate values of X. The values given by the random number generator are

$$0.1 \quad 0.6 \quad 0.9.$$

Which simulated values from X were drawn based on the above three values?