M339W: May 7th, 2021.

14) You are given the following information about Stock X, Stock Y, and the market:

(i) The annual effective risk-free rate is 4%. $r_f = 0.04$

(ii) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	Expected Return	Volatility
Stock X	5.5% > 3.20%	40%
Stock Y	4.5% < 4.84%	35%
Market	6.0%	25%

(iii) The correlation between the returns of stock X and the market is –0.25.

(iv) The correlation between the returns of stock Y and the market is 0.30.

Assume the Capital Asset Pricing Model holds. Calculate the required returns for Stock X and Stock Y, and determine which of the two stocks an investor should choose.

(A) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

(B) The required return for Stock X is 3.20%, the required return for Stock Y is 4.84%, and the investor should choose Stock Y.

✗ (C) The required return for Stock X is 4.80%, the required return for Stock Y is 4.84%, and the investor should choose Stock X.

✗ (D) The required return for Stock X is 6.40%, the required return for Stock Y is 3.16%, and the investor should choose Stock Y.

✗ (E) The required return for Stock X is 3.50%, the required return for Stock Y is 3.16%, and the investor should choose both Stock X and Stock Y.

→ : $r_X = r_f + \beta_X \left(\mathbb{E}[R_{Mkt}] - r_f\right)$

$$\beta_X = \frac{\sigma_X}{\sigma_{Mkt}} \cdot \rho_{Mkt,X} = \frac{0.4}{0.25} \cdot (-0.25) = -0.4$$

$$\Rightarrow r_X = 0.04 + (-0.4)(0.06 - 0.04) = 0.04 - 0.4(0.02) = 0.032$$

$$r_Y = r_f + \beta_Y \left(\mathbb{E}[R_{Mkt}] - r_f \right)$$

$$\beta_Y = \frac{0.35}{0.25}(0.3) = \frac{7}{5} \cdot 0.3 = 0.42$$

$$\Rightarrow r_Y = 0.04 + 0.42\,(0.06 - 0.04) = 0.04 + 0.0084 = 0.0484$$

15) You are given the following information about Stock X, Stock Y, and the market:

(i) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	Required Return	Volatility
Stock X	3.0%	50%
Stock Y	?	35%
Market	6.0%	25%

(ii) The correlation between the returns of stock X and the market is –0.25.

(iii) The correlation between the returns of stock Y and the market is 0.30.

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock Y.

$$r_Y = r_f + \beta_Y (\mathbb{E}[R_{Mkt}] - r_f)$$

(A) 1.48%

(B) 2.52%

(C) 3.16%

(D) 4.84%

(E) 6.52%

From $r_X = r_f + \beta_X (\mathbb{E}[R_{Mkt}] - r_f)$

$$\beta_X = \frac{0.50}{0.25}(-0.25) = -\frac{1}{2}$$

$$0.03 = r_f + (-0.5)(0.06 - r_f)$$

$$0.03 = r_f - 0.03 + 0.5 r_f$$

$$1.5 r_f = 0.06 \qquad r_f = \frac{0.06}{1.5} = 0.04$$

$$\beta_Y = \frac{0.35}{0.25}(0.3) = \frac{7}{5} \cdot 0.3 = 0.42$$

$$\Rightarrow \quad r_Y = 0.04 + 0.42(0.06 - 0.04) = 0.04 + 0.42(0.02)$$

$$= 0.0484$$

Beta of a Portfolio.

Start w/ a portfolio P.

Let $R_P = w_1 \cdot R_1 + w_2 \cdot R_2 + \dots + w_n \cdot R_n$

$$\beta_P = \frac{\sigma_P}{\sigma_{Mkt}} \cdot \rho_{Mkt,P} = \frac{Cov[R_P, R_{Mkt}]}{Var[R_{Mkt}]} =$$

$$= \frac{Cov[w_1 \cdot R_1 + w_2 \cdot R_2 + \dots + w_n \cdot R_n, R_{Mkt}]}{Var[R_{Mkt}]}$$

$$= \frac{\sum_{i=1}^{n} w_i \cdot Cov[R_i, R_{Mkt}]}{Var[R_{Mkt}]} = \sum_{i=1}^{n} w_i \cdot \beta_i$$

7) Consider a portfolio of four stocks as displayed in the following table:

Stock	Weight	Beta
1	0.1	1.3
2	0.2	-0.6
3	0.3	β_3
4	0.4	1.1

Assume the expected return of the portfolio is 0.12, the annual effective risk-free rate is 0.05 and the market risk premium is 0.08.

$\mathbb{E}[R_{Mkt}] - r_f = 0.08$

Assuming the Capital Asset Pricing Model holds, calculate β_3.

A) 0.80

B) 1.06

C) 1.42

D) 1.83

E) 2.17

$$\beta_p = w_1 \cdot \beta_1 + w_2 \cdot \beta_2 + w_3 \cdot \beta_3 + w_4 \cdot \beta_4$$

$$0.12 = 0.05 + \beta_p \cdot (0.08)$$

$$\beta_p = \frac{0.07}{0.08} = 0.875$$

$$0.875 = 0.1 \cdot 1.3 + 0.2 \cdot (-0.6) + 0.3 \cdot \beta_3 + 0.4(1.1)$$

$$\beta_3 = 1.42$$

Compare to the "official" solution :)