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M3398): December 4th, 2023.
Delta Hedging [cont'd].
Example. An agent writes a put option @ time.0.
            At time t, the agent's unhadged position is:
                          - vp(s,t)
        => They must maintain N(s,t) = \Delta_{p}(s,t) = BN(-d_1(s,t))
                                        in the D. hedge.
        => The agent must short a portion of a share.
        At time 0, their total position will be:
         v_{\text{Ret}}(S(0), 0) = -v_{\text{P}}(S(0), 0) + \Delta_{\text{P}}(S(0), 0) \cdot S(0)
 In the Black Scholos model:
    v_{\text{pot}}(s(0),0) = -\left(Ke^{-r^{T}}N(-d_{2}(s(0),0)) - s(0)N(-d_{4}(s(0),0))\right) + (-N(-d_{4}(s(0),0))) \cdot s(0)
                   = - Ke-rTN(-d2(S(0),0))
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- **46.** You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:
 - (i) Each period is 6 months.
 - (ii) u/d = 4/3, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.
 - (iii) The risk-neutral probability of an up move is 1/3.
 - (iv) The initial futures price is 80.
 - (v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_{I}$.

- (A) 0
- (B) 0.022
- (C) 0.044
- (D) 0.066
- (E) 0.088
- 47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

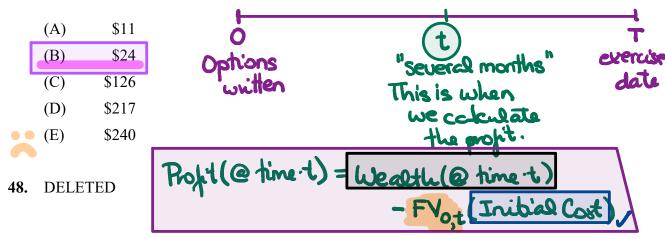
- (i) The risk-free interest rate is constant.
- (ii)

	Several months ago	Now
Stock price	\$40.00	\$50.00
Call option price	\$ 8.88	\$14.42
Put option price	\$ 1.63	\$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

Profit = Payoff-FV(Initial Cost)

Calculate her profit.



- **49.** You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).
 - (i) The period is 3 months.
 - (ii) The initial stock price is \$100.
 - (iii) The stock's volatility is 30%.
 - (iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

- (A) 114
- (B) 115
- (C) 116
- (D) 117
- (E) 118

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• Initial Cost: -100·42(S(0),0) +100 · Δ2(S(0),0)·S(0)
              = 100( - 8.88 + 0.794 · 40) = <u>2.288</u>
• Wealth @ time·t: -100·ν<sub>c</sub>(s(t), t) +100·Δ<sub>c</sub>(s(0), 0)·s(t)
              =100( - 14.42 + 0.794 - 50)
              = 2,528
 Rofit (@ time .t) = 2,528 - 2,288(ert)
 Use put call parity:
 At time · O: vc(S(0),0) - vp(S(0),0) = S(0) - Ke-rT
            8.88 - 1.63 = 40 - Ke-rt
           Ke^{-rT} = 40 - 7.25 = 32.75
 At time t: vc(s(t), t) - vp(s(t), t) = s(t) - Ke-r(T-t)
             14.42 - 0.26 = 50 - Ke-(T-t)
            Ke-((T-t) = 50-14.16 = 35.84 ×
     Profit (@ time·t) = 2,528 - 2,288. (1.09435) = 24.12
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