M 339D: January 27th, 2023.

Payoff and profit curves.

Goal: Study the payoff and the profit as functions of the final stock price.

Introduce: s... an independent argument taking values in $[0, \infty)$; it stands for the FINAL ASSET PRICE, i.e., it is a "placeholder" for the random variable $S(T)$.

Now, we can define the PAYOFF FUNCTION which describes the dependence of the payoff on the independent argument s

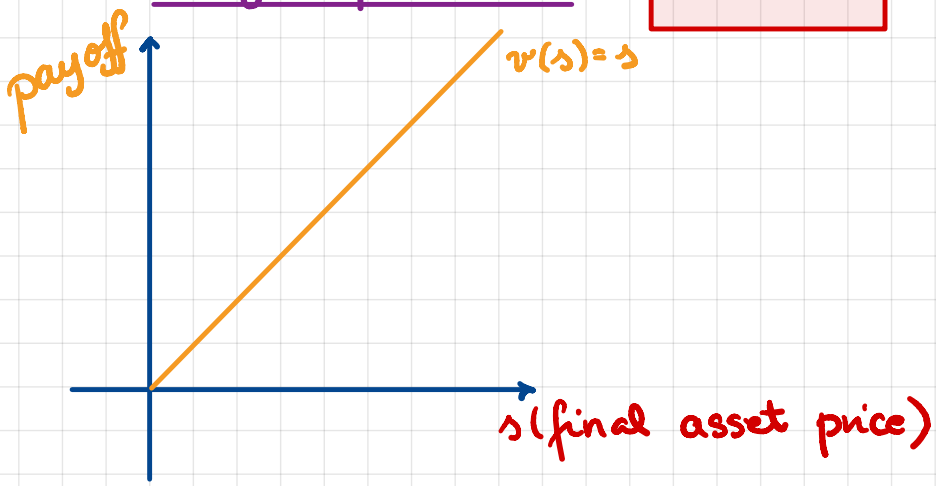
Notation: v... payoff f'tion

$v: [0, \infty) \longrightarrow \mathbb{R}$

$v(s)$ is the investor's payoff if the final asset price is s

Example [cont'd].

For the outright purchase: $\boxed{v(s) = s}$ identity function

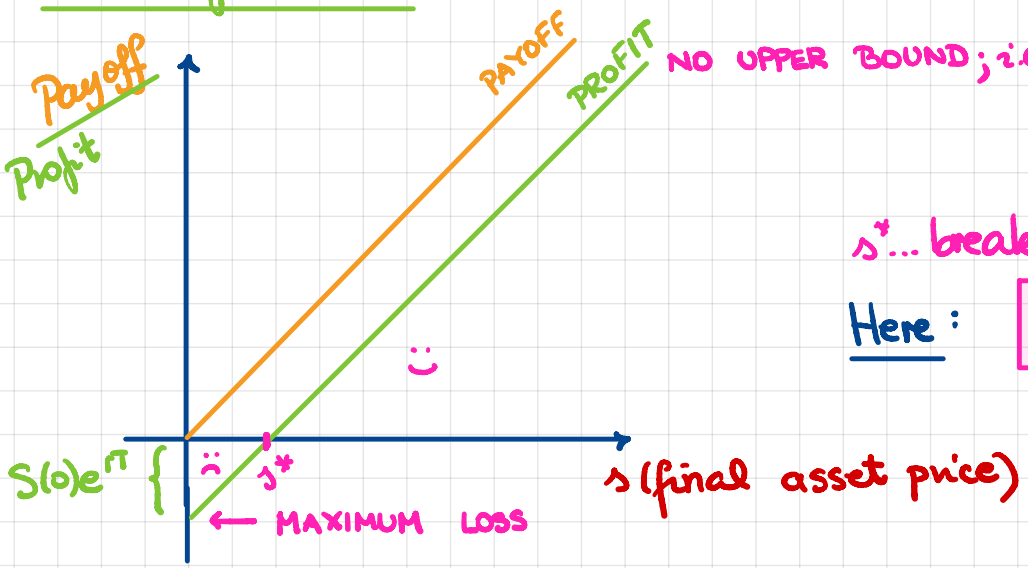


When we plot the payoff f'tion, we get a payoff curve (or a payoff diagram).

In general, the profit f'tion is: $\boxed{v(s) - FV_{0,T}(\text{Initial Cost})}$

Outright Purchase: $s - S(0)e^{rT}$

=> The Profit Curve.



The payoff and profit curves are increasing.

Terminology. If the payoff/profit is increasing (not necessarily strictly) as a function of the final asset price s, we say that the portfolio is long with respect to the underlying asset.

Example. [A Short Sale]

At time 0: The short seller receives $S(0)$.

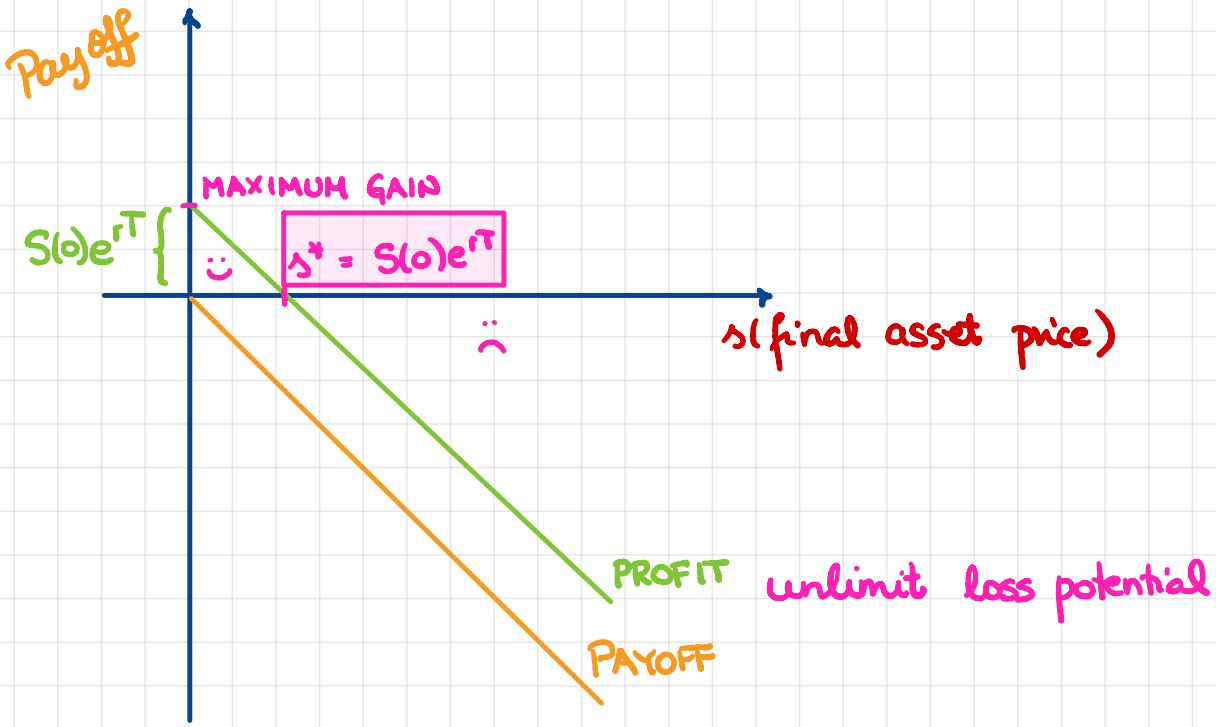
=> Initial Cost: $-S(0)$

At time T: The short seller spends $S(T)$.

=> Payoff: $-S(T)$ => payoff f'tion: $v(s) = -s$

$$\text{Profit} = -S(T) - FV_{0,T}(\text{Init. Cost})$$

$$= -S(T) + e^{rT} \cdot S(0)$$



The profit/payoff curve is decreasing.

⇒ The short sale is short w.r.t. the underlying.

Project Hint: