Name:

M339J: Probability models
University of Texas at Austin
Practice Problems for In-Term One
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is ?? points.
Time: 50 minutes

1.1. **TRUE/FALSE QUESTIONS.** *Please, note your final answer on the front page of this exam.*

Problem 1.1. Let X denote the outcome of a roll of a fair, regular icosahedron (a polyhedron with 20 faces) with numbers $1, 2, \cdots, 20$ written on its sides. Then $\mathbb{E}[X] = 15/2$. *True or false? Why?*

Problem 1.2. (2 pts) Let X be an exponential random variable. Then, its mean and its standard deviation are equal. *True or false?*

Problem 1.3. (2 points) For a random variable X and for a positive constant d, in our usual notation, we have

$$\mathbb{E}[X] = e_X(d)S_X(d) + \mathbb{E}[X \wedge d]\,. \tag{1.1}$$

True or false?

1.2. **Free-response problems.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.4. Let the random variable X have a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 10$. What is the 75^{th} percentile of this distribution?

Problem 1.5. (10 points) A population of insureds consists of three types of people: α, β and γ. There is an equal number of Type α and Type β people in the population. The number of Type γ people is equal to the total number of the remaining two types of people. The probability that a Type α person makes at least one claim in a year is $1/5$. The probability that a Type β person makes at least one claim in a year is $2/5$. The probability that a Type γ person makes at least one claim in a year is $3/5$.

At the end of the year, a person is chosen at random from the population. It is observed that that person had at least one claim. What is the probability that the person was of Type β?

Problem 1.6. (15 points) Losses X follow a Pareto distribution with parameters $\alpha > 1$ and θ unspecified. For a positive constant c, determine the ratio of the mean excess loss function evaluated at $c\theta$ to the mean excess loss function evaluated at θ.

Problem 1.7. Let $\{X_n, n \geq 1\}$ be a sequence of independent random variables. Assume that all the variables in the sequence have the two-parameter Pareto distribution with $\theta = 10$ and $\alpha = 3$. For each n, define the random variable

$$Y_n = \frac{X_1^2 + X_2^2 + \cdots + X_n^2}{n}.$$

Does the limit of the sequence $\{Y_n, n \geq 1\}$ as $n \to \infty$ exist? If so, how much is it? If not, why not?

Problem 1.8. (10 points) Let $X \sim Pareto(\alpha = 3, \theta = 3000)$. Assume that there is a deductible of $d = 5000$. Find the loss elimination ratio.

Problem 1.9. (10 points) Let the ground-up loss X be exponentially distributed with mean \$800.
An insurance policy has an ordinary deductible of \$100 and the maximum amount payable per loss of \$2500.
Find the expected value of the amount paid (by the insurance company) per positive payment.

Problem 1.10. (10 points) Losses in year y follow a two-parameter Pareto distribution with parameters $\alpha = 4$ and $\theta = 10$.
Losses in year $y + 1$ are uniformly 10% higher than those in year y.
An insurance covers each loss subject to a deductible $d = 20$. Calculate the **loss elimination ratio** for year $y + 1$.

Problem 1.11. (10 points) Assume that the severity random variable X is uniform on the interval $(0, 1000)$. There is an insurance policy to cover this loss. The insurance policy has a deductible of 200 per loss and the maximum amount payable by the insurer per loss of 700.
Calculate the expected value of the per payment random variable Y^P under this policy.

Problem 1.12. Assume that the severity random variable X is exponentially distributed with mean 1400. There is an insurance policy to cover this loss. This insurance policy has
(1) a deductible of 200 per loss,
(2) the coinsurance factor of $\alpha = 0.25$, and
(3) the maximum amount payable by the insurer per loss of 700.
Calculate the expected value of the per payment random variable Y^P under this policy.

1.3. **MULTIPLE CHOICE QUESTIONS.** *Please, note your final answers on the front page of this exam.*

Problem 1.13. (5 pts) Let X be exponential with variance 225. Let $a = \mathbb{E}[|20 - X|]$. Then,

(a) $0 \leq a < 50$
(b) $50 \leq a < 150$
(c) $150 \leq a < 325$
(d) $325 \leq a < 550$
(e) None of the above.

Problem 1.14. Let E and F be two events on the same probability space. You know that $\mathbb{P}[E \cup F] = 0.75$ and $\mathbb{P}[E \cup F^c] = 0.85$. What is the probability of the event E?

(a) 0.5
(b) 0.6
(c) 0.65
(d) 0.7
(e) None of the above.

Problem 1.15. The time until the next bus arrives is a continuous random variable T with the density

$$f_T(t) = \begin{cases} \kappa(10 - t) & 0 < t < 10 \\ 0 & \text{otherwise} \end{cases}$$

for some constant κ. **Given** that you have already waited for 4 minutes, what is the probability that you will wait for at least another 4 minutes?

(a) 1/25
(b) 1/9
(c) 1/8
(d) 1/3
(e) None of the above.

Problem 1.16. Let X_1, X_2, and X_3 be independent, identically distributed random variables with the probability mass function

$$p_X(x) = \begin{cases} \frac{1}{4} & x = 0 \\ \frac{1}{2} & x = 1 \\ \frac{1}{4} & x = 2 \end{cases}$$

Find $\mathbb{P}[X_1 X_2 X_3 = 0]$.

(a) 27/64
(b) 1/8
(c) 31/64
(d) 37/64
(e) None of the above.

Problem 1.17. A recent study indicates that the annual cost of fertilizing a Japanese plum tree in Austin has a mean 100 with a variance of 20. A tax of 10% is introduced on fertilizer, i.e., fertilizer is made 10% more expensive. What is the variance of the new annual cost of fertilizing a Japanese plum tree in Austin after the tax is introduced?

(a) 20
(b) 22
(c) 23.1
(d) 24.2
(e) None of the above.

Problem 1.18. Let the random variable X have the cumulative distribution function given by

$$F_X(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{2}(x^2 - 2x + 2) & \text{for } 1 \leq x < 2 \\ 1 & \text{for } 2 \leq x \end{cases}$$

What is the expectation of X?

(a) 2/3
(b) 5/6
(c) 7/6
(d) 4/3
(e) None of the above.

Problem 1.19. Let the independent random variables X and Y have the same mean. You are given that coefficient of variation of X equals 2 and the coefficient of variation of Y equals 4. What is the coefficient of variation of the average of X and Y?

(a) $3/2$
(b) $\frac{\sqrt{13}}{2}$
(c) $5/2$
(d) There is not enough information to answer this problem.
(e) None of the above.

Problem 1.20. (5 points) Let X be the ground-up loss random variable. Assume that X has the two-parameter Pareto distribution with $\theta = 4,000$ and $\alpha = 3$.

Let B denote the expected payment per loss on behalf of an insurer which wrote a policy with no deductible and with a policy limit of $5,000$. Then,

(a) $B \approx 1,000$
(b) $B \approx 1,200$
(c) $B \approx 1,400$
(d) $B \approx 1,600$
(e) None of the above

Problem 1.21. (5 points) Let X be the ground-up loss random variable. Assume that X has the exponential distribution with mean $1,000$.
Let B denote the expected payment per loss on behalf of an insurer who wrote a policy with a deductible of $1,500$ and with the **maximum payment by the insurer** equal to $2,500$. Then,

(a) $B \approx 714$
(b) $B \approx 816$
(c) $B \approx 918$
(d) $B \approx 1020$
(e) None of the above

Problem 1.22. (5 points) The ground-up loss X is modeled by a two-parameter Pareto distribution with parameters $\alpha = 2$ and $\theta = 200$. For an insurance policy on the above loss, there is a **franchise** deductible of 200. Find the expected value of the *per payment* random variable.

(a) 200
(b) 400
(c) 600
(d) 800
(e) None of the above.