Name:

M339D/M389D Introduction to Financial Mathematics for Actuaries
University of Texas at Austin
In-Term Three
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximum number of points on this exam is 65.

Time: 50 minutes

Name:

UTeid:

All written work handed in by the student is considered to be

their own work, prepared without unauthorized assistance.

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"I agree that I have complied with the UT Honor Code during my completion of this exam."

Signature:

1.1. **Free-response problems.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.1. (20 points) Assume the Black-Scholes setting. Let the initial price of a non-dividend-paying stock be 60 and let its volatility be 0.32.

The continuously compounded, risk-free interest rate equals 0.04.

Consider a \$45-strike European put option which expires in four months. What is the price of the put?

Solution: In our usual notation, the price is

$$V_P(0) = Ke^{-r\cdot T}N(-d_2) - S(0)N(-d_1)$$

with

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right] = 1.72167, \\ d_2 &= d_1 - \sigma\sqrt{T} = 1.536918. \end{aligned}$$

So, $V_P(0) = 0.2061258$.

Problem 1.2. (20 points) The current price of a non-dividend-paying stock is given to be \$90. The stock's volatility is 0.2.

The continuously compounded risk-free interest rate is 0.04.

Consider an \$85-strike European call option on the above stock with exercise date in a quarter-year. What is the Black-Scholes price of this call option?

Solution: In our usual notation, the price is

$$V_C(0) = S(0)N(d_1) - Ke^{-r\cdot T}N(d_2)$$

with

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right] = 0.7215841, \\ d_2 &= d_1 - \sigma\sqrt{T} = 0.6215841. \end{aligned}$$

So,

$$V_C(0) = S(0)N(d_1) - Ke^{-rT}N(d_2) = 7.149248.$$

Problem 1.3. (15 points) Assume the Black-Scholes model. A non-dividend-paying stock is currently valued at \$80 per share. Its volatility is given to be 30%. The continuously compounded risk-free interest rate is 0.08. Find

$$\mathbb{E}^*[S(4)\mathbb{I}_{[S(4)>90]}].$$

Solution: In our usual notation,

$$\mathbb{E}^*[S(T)\mathbb{I}_{[S(T)>K]} = S(0)e^{rT}N(d_1)$$

with

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right].$$

In the present problem,

$$d_1 = \frac{1}{0.3\sqrt{4}}\left[\ln\left(\frac{80}{90}\right) + \left(0.08 + \frac{0.09}{2}\right) \times 4\right] = 0.6370283.$$

So, our answer is

$$\mathbb{E}^*[S(4)\mathbb{I}_{[S(4)>90]}] = 80e^{(0.08)\times 4}N(0.6370283) = 81.29976.$$

Problem 1.4. (10 points) The current stock price is given to be $S(0) = 30$ and its volatility is 0.3

The continuously compounded risk-free interest rate is 0.12.

(i) (2 points) What is the expected stock price in three months under the risk-neutral probability measure?

(ii) (3 points) What is the median stock price in three months under the risk-neutral probability measure?

(iii) (5 points) Find the risk-neutral probability that the stock price in three months is less than \$32.

Solution:

(i)

$$\mathbb{E}^*[S(1/4)] = S(0)e^{r/4} = 30.91364$$

(ii)

$$S(0)e^{(r-\frac{\sigma^2}{2})/4} = 30.56781$$

(iii) First, we calculate d_2. We get

$$d_2 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T\right] = \frac{1}{0.3\sqrt{\frac{1}{4}}}\left[\ln\left(\frac{30}{32}\right) + \left(0.12 - \frac{0.09}{2}\right) \times \frac{1}{4}\right] = -0.30525.$$

Then, we use the standard normal tables to obtain

$$\mathbb{P}[S(1/4) < 32] = N(-d_2) \approx N(0.31) = 0.6217 \tag{1.1}$$