M339D: December 1st, 2025.

Focus on the Delta.

Value function: $v(\boxed{s}, \boxed{t}, \boxed{r, \sigma})$

valuation date

omit them from now on

stock price @ time t

$$\boxed{v(s,t)}$$

Def'n.

$$\boxed{\text{The Delta: } \quad \Delta(s,t) := \frac{\partial}{\partial s} v(s,t)}$$

Example. Outright Purchase of a Non-Dividend-Paying Stock.

$$v(s,t) = s \quad \Rightarrow \quad \boxed{\Delta(s,t) = 1}$$

stands for the time-t stock price

Example. European Call.

$$v_C(s,t) = s \cdot \underbrace{N(d_1(s,t))}_{\Delta} \underbrace{- Ke^{-r(T-t)} \cdot N(d_2(s,t))}_{B}$$

w/

$$d_1(s,t) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$

and

$$d_2(s,t) = d_1(s,t) - \sigma\sqrt{T-t}.$$

By def'n:

$$\Delta_C(s,t) = \frac{\partial}{\partial s} v_C(s,t)$$

We need: The product rule and chain rule.

$$\boxed{\Delta_C(s,t) = N(d_1(s,t)) > 0}$$

The positivity makes sense since the call is long w.r.t. the underlying.

Example. European Put.

$$v_p(s,t) = \boxed{Ke^{-r(T-t)} \cdot N(-d_2(s,t))} - s\, N(-d_1(s,t))$$

B $\qquad$ Δ

$$\boxed{\Delta_p(s,t) = -N(-d_1(s,t))} < 0$$

Puts are short w.r.t. the underlying. ✓✓

Also, by put-call parity,

$$v_c(s,t) - v_p(s,t) = s - Ke^{-r(T-t)}$$

$\frac{\partial}{\partial s}\Big|$

$$\Delta_c(s,t) - \Delta_p(s,t) = 1$$

$$\Delta_p(s,t) = \Delta_c(s,t) - 1 = N(d_1(s,t)) - 1 = -N(-d_1(s,t))$$ ✓✓

(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

$T=\frac{1}{4}$ $K=41.5$ European

8. You are considering the purchase of a 3-month 41.5-strike ~~American~~ call option on a nondividend-paying stock.

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40.

(iii) The stock's volatility is 30%.

(iv) The current call option delta is $\boxed{0.5}$.

Determine the current price of the option. $\boxed{v_C(S(0),0)=?}$

(A) $20-20.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(B) $20-16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(C) $20-40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(D) $16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx-20.453$

(E) $40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx-20.453$

$\Delta_C(S(0),0)=0.5$

$N(d_1(S(0),0)) = \boxed{0.5}$

$\Downarrow$

$\boxed{d_1(S(0),0)=0}$

$$\frac{1}{\sigma\sqrt{T}}\left[\underbrace{\ln\left(\frac{40}{41.5}\right)+\left(r+\frac{0.09}{2}\right)\cdot\frac{1}{4}}_{=0}\right]=0$$

$$r + 0.045 = 4\ln\left(\frac{41.5}{40}\right)$$

$$r = 4\ln\left(\frac{41.5}{40}\right) - 0.045 = \underline{0.1032}$$

$$d_1(S(0),0) - \sigma\sqrt{T} = 0 - 0.15$$

$$v_c(S(0),0) = S(0)\cdot N(d_1(S(0),0)) - Ke^{-rT}N\left(\boxed{d_2(S(0),0)}\right)$$

$$= 40 \cdot 0.5 \qquad - 41.5\,e^{-0.1032(1/4)}\cdot N(-0.15)$$

$$= 20 - 40.453\left(1 - N(0.15)\right)$$

$$= 40.454 \cdot \underbrace{N(0.15)}_{\int_{-\infty}^{0.15} f_Z(z)\,dz} - 20.453$$

$$= \int_{-\infty}^{0.15} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\,dz$$

$\square$