M339 J : March 24th, 2021.

Loss Elimination Ratio.

... is the ratio of the decrease in the insurer's expected payment w/ an ordinary deductible d to the insurer's expected payment w/ no deductible.

As usual : X ... (ground up) loss r.v. / severity

Assuming: $\boxed{\mathbb{E}[X] < \infty}$

By def'n:

$$LER = \frac{\mathbb{E}[X] - \mathbb{E}[(X-d)_+]}{\mathbb{E}[X]} = \frac{\mathbb{E}[X \wedge d]}{\mathbb{E}[X]}$$

Note: $\mathbb{E}[X \wedge d] \leq \mathbb{E}[X] \Rightarrow \boxed{LER \leq 1}$

Example. Assume there is policy w/ an ordinary deductible of $d = \$2,500$.

Let the ground-up loss r.v. X be exponential w/ mean $\$5,000$.

Find the loss elimination ratio!

$\rightarrow$:

$X \sim$ Exponential (mean $= \theta = 5,000$)

By def'n : $LER = \frac{\mathbb{E}[X \wedge d]}{\mathbb{E}[X]}$ w/ $d = 2,500$

Use the STAM tables:

$$\mathbb{E}[X \wedge d] = \theta(1 - e^{-\frac{d}{\theta}})$$

$$\Rightarrow LER = \frac{\cancel{\theta}(1 - e^{-\frac{d}{\theta}})}{\cancel{\theta}} = 1 - e^{-\frac{d}{\theta}} = F_X(d)$$

$\Rightarrow$ In this problem:

$$LER = 1 - e^{-\frac{2500}{5000}} = 1 - e^{-\frac{1}{2}} = 0.3935$$

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Sample STAM

89. You are given:

$X \sim \text{Exponential}(\text{mean} = \theta)$

(i) Losses follow an exponential distribution with the same mean in all years.

(ii) The loss elimination ratio this year is 70%. $d_{OLD} = d$ $\quad LER_{OLD} = 0.70$

(iii) The ordinary deductible for the coming year is 4/3 of the current deductible. $d_{new} = \frac{4}{3} d$

Calculate the loss elimination ratio for the coming year.

$LER_{new} = ?$

(A) 70%

(B) 75%

(C) 80%

(D) 85%

(E) 90%

(ii) $\Rightarrow$ $0.7 = 1 - e^{-\frac{d}{\theta}} \Rightarrow \boxed{e^{-\frac{d}{\theta}} = 0.3}$

$$LER_{new} = 1 - e^{-\frac{d_{new}}{\theta}} = 1 - e^{-\frac{\frac{4}{3}d}{\theta}} = 1 - \left(e^{-\frac{d}{\theta}}\right)^{4/3}$$

$$= 1 - (0.3)^{4/3} \approx 0.80$$

90. Actuaries have modeled auto windshield claim frequencies. They have concluded that the number of windshield claims filed per year per driver follows the Poisson distribution with parameter λ, where λ follows the gamma distribution with mean 3 and variance 3.

Calculate the probability that a driver selected at random will file no more than 1 windshield claim next year.

(A) 0.15

(B) 0.19

(C) 0.20

(D) 0.24

(E) 0.31

The Effect of Inflation.

For an ordinary deductible d, after uniform inflation w/ rate r:

- the expected cost per loss is

$$\mathbb{E}\left[((1+r)X-d)_+\right] = (1+r)\left(\mathbb{E}[X]-\mathbb{E}\left[X \wedge \frac{d}{1+r}\right]\right)$$

$$\boxed{\text{if } F_X\left(\frac{d}{1+r}\right) < 1}$$

- the expected cost per payment is

$$\frac{1}{S_X\left(\frac{d}{1+r}\right)}(1+r)\left(\mathbb{E}[X]-\mathbb{E}\left[X \wedge \frac{d}{1+r}\right]\right)$$

if $S_X\left(\frac{d}{1+r}\right) > 0$

Note: Usually, it's easier to use the scaling property of the severity distribution.

126. The number of annual losses has a Poisson distribution with a mean of 5. The size of each loss has a two-parameter Pareto distribution with $\theta = 10$ and $\alpha = 2.5$. An insurance for the losses has an ordinary deductible of 5 per loss.

Calculate the expected value of the aggregate annual payments for this insurance.

(A) 8

(B) 13

(C) 18

(D) 23

(E) 28

SAMPLE STAM

$X_{OLD} \sim \text{Pareto}(\alpha=2, \theta=5)$

127. Losses in 2003 follow a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 5$. Losses in 2004 are uniformly 20% higher than in 2003. An insurance covers each loss subject to an ordinary deductible of 10.

$\boxed{X_{NEW} = 1.2 \cdot X_{OLD}}$

Calculate the Loss Elimination Ratio in 2004.

(A) 5/9

(B) 5/8

(C) 2/3

(D) 3/4

(E) 4/5

$X_{NEW} \sim \text{Pareto}(\boxed{\alpha = 2}, \boxed{\theta_{NEW} = 1.2 \cdot 5 = 6})$

Pareto is a scale distribution w/ the scale parameter θ!

By def'n:

$$LER_{NEW} = \frac{\mathbb{E}[X_{NEW} \wedge d]}{\mathbb{E}[X_{NEW}]} = \frac{\cancel{\frac{\theta_{NEW}}{\alpha-1}} \boxed{\left[1 - \left(\frac{\theta_{NEW}}{d+\theta_{NEW}}\right)^{\alpha-1}\right]}}{\cancel{\frac{\theta_{NEW}}{\alpha-1}}}$$

128. DELETED

129. DELETED

$$\Rightarrow LER_{NEW} = 1 - \left(\frac{6}{10+6}\right)^{2-1} = 1 - \frac{6}{16} = 1 - \frac{3}{8} = \frac{5}{8} \qquad \blacksquare$$

Policy Limits.

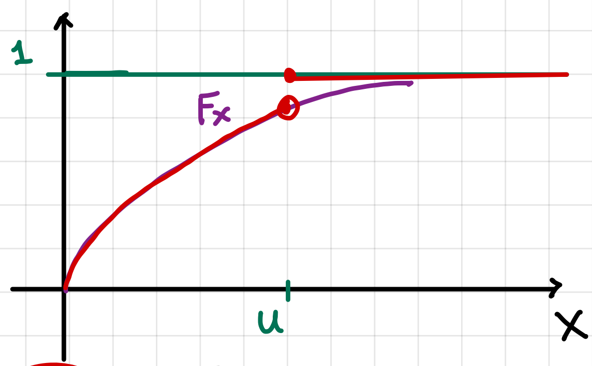
For a policy limit u, with no deductible, the effect on the insurer's payment on the loss r.v. X is:

$$\boxed{Y = X \wedge u}$$

In other words, Y is the RIGHT-CENSORED r.v.

Q: What is the form of the cumulative dist'n f'tion of Y?

→ :



Starting w/ a continuous dist'n for X such that

$$S_X(u) > 0,$$

what kind of dist'n of Y do I get?

$$F_Y(y) = \begin{cases} F_X(y) & \text{if } y < u \\ 1 & \text{if } y \geq u \end{cases}$$

For $y < d$:

$$F_Y(y) = \mathbb{P}[Y \leq y] = \mathbb{P}[X \wedge d \leq y] = \mathbb{P}[X \leq y] = F_X(y)$$

$\Rightarrow$ Y has a mixed dist'n.

$$\Rightarrow \begin{cases} f_Y(y) = f_X(y) & \text{for } y < u \\ p_Y(y) = S_X(u) & \text{for } y = u \end{cases}$$