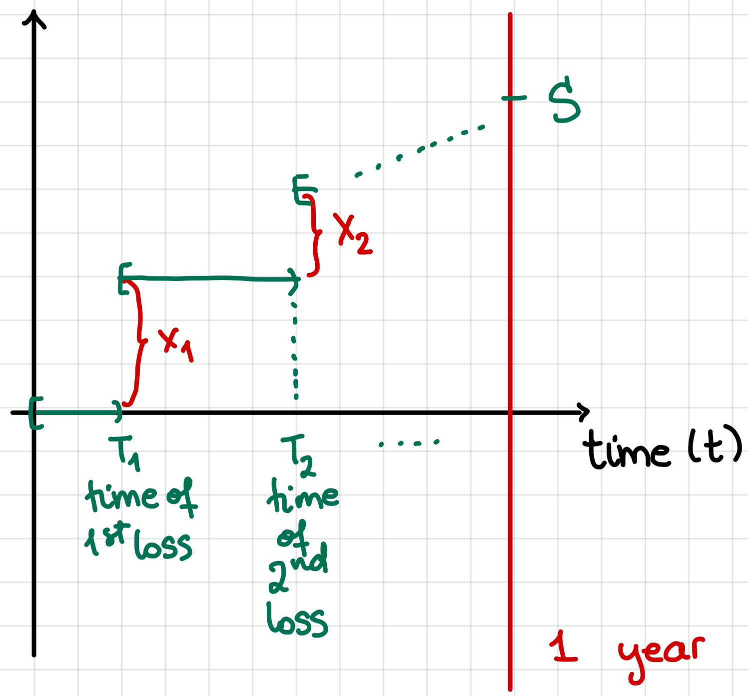
M339J: April 26th, 2021.

Compound Poisson.

independent $\begin{cases} \text{frequency} \ldots N \sim \underline{\text{Dist'n } 1} \\ \text{severity} \ldots X \sim \underline{\text{Dist'n } 2} \end{cases}$ $\Rightarrow$ total aggregate losses $S = X_1 + X_2 + \cdots + X_N$

$S \sim$ Dist'n 3

independent $\begin{cases} \text{frequency} \ldots N \sim \text{Poisson}(\lambda) \\ \text{severity} \ldots\ldots X \sim \text{Dist'n} \end{cases}$ } S is called **compound Poisson**



$\underbrace{\hspace{6em}}_{N \text{ is the total \# of losses}}$

Note:

- $\mathbb{E}[S] = \mathbb{E}[X] \cdot \mathbb{E}[N] = \mathbb{E}[X] \cdot \lambda$
- $$\begin{aligned} Var[S] &= \mathbb{E}[N] \cdot Var[X] + Var[N] \cdot (\mathbb{E}[X])^2 \\ &= \lambda \cdot Var[X] + \lambda \cdot (\mathbb{E}[X])^2 \\ &= \lambda \underbrace{\left(Var[X] + (\mathbb{E}[X])^2 \right)} \\ &= \lambda \cdot \mathbb{E}[X^2] \end{aligned}$$

280. A compound Poisson claim distribution has $\lambda = 5$ and individual claim amounts distributed as follows:

x	$f_X(x)$	
5	0.6	
k	0.4	Where $k > 5$

} pmf of the severity X

The expected cost of an aggregate stop-loss insurance subject to a deductible of 5 is 28.03.

Calculate k.

$$\mathbb{E}[(S-5)_+] = \underline{\mathbb{E}[S]} - \mathbb{E}[S \wedge 5]$$

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

- $\mathbb{E}[S] = \lambda \cdot \mathbb{E}[X] = 5 \cdot (5 \cdot 0.6 + k \cdot 0.4) = \underline{15 + 2k}$ ✓

- Support of S: $\{0, 5, \ldots\}$

  Support of $S \wedge 5$: $\{0, 5\}$

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$$S \wedge 5 \sim \begin{cases} 0 & \text{w/ probab. } p_N(0) = e^{-5} \\ 5 & \text{w/ probab. } 1 - e^{-5} \end{cases}$$

$$\mathbb{E}[S \wedge 5] = 0 \cdot e^{-5} + 5(1 - e^{-5}) = \underline{5(1 - e^{-5})}$$ ✓

$$28.03 = 15 + 2k - 5(1 - e^{-5}) \Rightarrow k = \frac{18.03 - 5e^{-5}}{2} = 8.998$$

$N \sim \text{Poisson}(\lambda=5)$

289. A compound Poisson distribution has $\lambda = 5$ and claim amount distribution as follows:

x	$p(x)$
100	0.80
500	0.16
1000	0.04

pmf of severity

Calculate the probability that aggregate claims will be exactly 600.

500+100 100+500 6 × 100

(A) 0.022

(B) 0.038

(C) 0.049

(D) 0.060

(E) 0.070

$\mathbb{P}[6 \text{ claims of } 100] = p_N(6) \cdot (p_X(100))^6$
$= e^{-5} \cdot \frac{5^6}{6!} \cdot (0.8)^6 = \dots$
$= 0.03833$

$\mathbb{P}[\text{two claims w/ } 1^{st} \text{ claim is } 500, 2^{nd} \text{ is } 100] =$
$= p_N(2) \cdot p_X(500) \cdot p_X(100)$
$= e^{-5} \cdot \frac{5^2}{2!} (0.16)(0.80) = 0.01078$

$\mathbb{P}[\text{two claims w/ } 1^{st} \text{ claim } 100, 2^{nd} \text{ claim } 500] = 0.01078$

$\mathbb{P}[S = 600] = 0.05989$

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A Few Compound Poissons.

Suppose: We have n different streams of losses, each of them a compound Poisson and they are all independent.

$\{S_j, j = 1..n\}$ are independent compound Poissons, i.e.,

for every j:

$N_j \sim \text{Poisson}(\lambda_j)$

and

$X^j \sim \text{cdf } F_j$.

More precisely, for stream j, the severity r.v.s are $\{X_1^j, X_2^j, \ldots, X_k^j, \ldots\}$

$\Rightarrow \quad S_j = X_1^j + X_2^j + \cdots + X_{N_j}^j$

Set: $\boxed{S = S_1 + S_2 + \cdots + S_n}$

Then, S is a compound Poisson w/

$N \sim \text{Poisson}(\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_n)$

and with the severity dist'n w/ cdf

$$F_X(x) = \sum_{j=1}^{n} \frac{\lambda_j}{\lambda} F_j(x)$$

n-point mixture of X_j's