M339W: February 14^{th}, 2022.

Value@ Risk.

p ... probability of an adverse event you're still willing to live with (e.g., the probability of experiencing a loss)

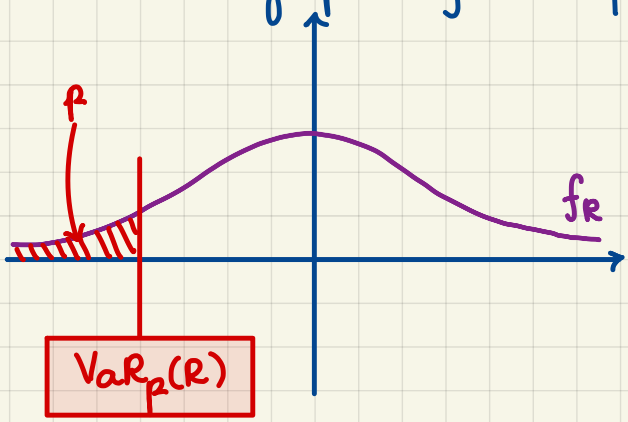
R ... return random variable (i.e., one benefits if the value of R is high, and the values of R being low constitutes the adverse event)

Def'n. $VaR_p(R)$ is the value such that

$$\mathbb{P}[R \leq VaR_p(R)] = p$$

$$\overset{|||}{\pi_p} \text{ ... } 100p^{th} \text{ quantile}$$

Example. Consider an R which is a continuous random variable. Let the graph of its pdf f_R look like this:



Generally, with continuous random variables, we solve for

$VaR_p(R)$ in $\boxed{F_R(VaR_p(R)) = p}$

If $f_R > 0$, then $\boxed{VaR_p(R) = F_R^{-1}(p)}$

In particular, for normal returns we can use the Prometric calculator or the standard normal tables or software (R).

Note: In classical insurance, we worry about the upper tail of the loss r.v. One way to look @ this is: $\boxed{VaR_{1-p}(X)}$

Sample IFM: Part II

34) Let X be the random ~~gain~~ from operations of a company. You are given:

∴ the PROFIT r.v.

(i) X is normally distributed with mean 42 and variance 6400.

(ii) p is the probability that X is negative.

(iii) K is the amount of capital such that the Value-at-Risk (VaR) at the 5th percentile for $X + K$ is zero.

Calculate p and K.

(A) $p = 0.7$; $K = 157$ ✗

(B) $p = 0.7$; $K = 131$ ✗

(C) $p = 0.5$; $K = 115$ ✗

(D) $p = 0.3$; $K = 115$

(E) $p = 0.3$; $K = 90$

(i) $X \sim \text{Normal}(\text{mean} = 42, \text{var} = 80^2)$

(ii) $p = \mathbb{P}[X < 0] =$ (rewrite in std units)

$= \mathbb{P}\left[\frac{X-42}{80} < \frac{0-42}{80}\right] =$

$= \mathbb{P}[Z < -0.525] =$

$= \text{pnorm}(-0.525) \cong 0.30$

$\text{VaR}_{0.05}(X+K) = 0$

By def'n, $\mathbb{P}[X+K < 0] = 0.05$

$\mathbb{P}[X < -K] = 0.05$

$-K$ is the 5th quantile of X

Since X is normal, it can be written as

$\boxed{X = 42 + 80 \cdot Z}$ w/ $Z \sim N(0,1)$

Let $z^*_{0.05}$ be the 5th percentile of $N(0,1)$

Then, $z^*_{0.05} = \text{qnorm}(0.05) = -1.645$

$\Rightarrow -K = 42 + 80 \cdot (-1.645) \cong -90 \Rightarrow \boxed{K = 90}$

$\Rightarrow$ (E)

Tail Probabilities.

Example. You are considering an investment in a continuous-dividend-paying stock.
You want to compare it to the risk-free investment.

Q: What is the probability that the stock outperforms the risk-free account @ time T?

$\longrightarrow$: The invested amount: $S(0)$

- If it's the risk-free investment, the balance @ time T is:

$$S(0)\cdot e^{rT}$$

- If it's the stock investment, the number of shares owned @ time T is:

$$e^{\delta\cdot T}$$

$\Rightarrow$ The wealth @ time T is: $S(T)\cdot e^{\delta\cdot T}$

$$\mathbb{P}[\underbrace{e^{\delta\cdot T}S(T) > S(0)e^{rT}}] = ?$$

This question is equivalent to the question of whether the profit from purchase of stock is positive.

In the Black-Scholes model:

$$S(T) = S(0)e^{(\alpha-\delta-\frac{\sigma^2}{2})\cdot T+\sigma\sqrt{T}\cdot Z} \quad \text{w/} \quad Z\sim N(0,1)$$