UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 5

Profit. Forward contracts. European call options.

Please, provide your **complete solutions** to the free-response problems. For those problems, final answers only, without the justification, will earn zero points. For the True/False and Multiple-Choice questions, only your final answer will be graded.

Problem 5.1. (10 points) Let the current price of a continuous-dividend-paying stock be \$40 and let its dividend yield be equal to 0.01. The continuously compounded, risk-free interest rate is 0.04. You model the distribution of the time−1 price of the above stock as follows:

$$S(1) \sim \begin{cases} 45, & \text{with probability } 1/4, \\ 42, & \text{with probability } 1/2, \\ 38, & \text{with probability } 1/4. \end{cases}$$

What is your expected profit under the above model, if you invest in one share of stock at time−0 and liquidate your investment at time−1?

Solution: The initial cost is $S(0)$ and the payoff is $e^{\delta T}S(T)$ with $T = 1$. So, the profit equals

$$e^{\delta T}S(T) - S(0)e^{rT}.$$

Thus, the expected profit equals

$$e^{\delta T}\mathbb{E}[S(T)] - S(0)e^{rT}.$$

According to the given model for the stock price, we have

$$\mathbb{E}[S(T)] = 45\left(\frac{1}{4}\right) + 42\left(\frac{1}{2}\right) + 38\left(\frac{1}{4}\right) = 41.75.$$

Finally, the expected profit is

$$e^{0.01}(41.75) - 40e^{0.04} = 0.537164.$$

Problem 5.2. (2 points) Derivative securities can reduce the risk of both the buyer and the writer of the security. *True or false?*

Solution: TRUE
Forward contracts are an example of this situation.

Problem 5.3. (2 points) An agent is **only** allowed to long a forward contract if he/she is willing to take physical delivery of the underlying asset. *True or false?*

Solution: FALSE
It is possible to have *cash settlement* on the delivery date if the forward contract stipulates so.

Problem 5.4. (2 points) A short forward contract has an unlimites loss potential. *True or false?*

Solution: TRUE

Problem 5.5. (5 points) The current price of stock a certain type of stock is \$80. The premium for a 6−month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) \$80
(b) \$85.72
(c) \$85.84

(d) \$85.96
(e) None of the above.

Solution: (d)
The break-even point is

$$80 + 5.84e^{0.04/2} = 85.958$$

Problem 5.6. (5 points) The price of gold in half a year is modeled to be equally likely to equal any of the following prices

$$\$1000, \quad \$1100, \quad \text{and} \quad \$1240.$$

Consider a half-year, \$1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

Solution:

$$50 \times \frac{1}{3} + 190 \times \frac{1}{3} = \frac{240}{3} = 80.$$

Problem 5.7. (2 points) A cap is a long position with respect to the underlying asset. *True or false?*

Solution: FALSE

Problem 5.8. (2 points) In our usual notation, the payoff of a **cap** is $\min(S(T), K)$ with the underlying asset not paying any dividends. *True or false?*

Solution: FALSE

$$(S(T) - K)_+ - S(T) = -\min(S(T), K).$$

Problem 5.9. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded, risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

(a) 395.92
(b) 397.98
(c) 400
(d) 897.98
(e) None of the above.

Solution: (b)

$$200 \times 10 - 200 \times 5 - 500 - 200 \times 0.50e^{0.02} = 397.98$$

Problem 5.10. (2 points) An agent is **only** allowed to write options on an underlying asset if he/she already owns units of the underlying. *True or false?*

Solution: FALSE
The so-called *naked* option writing is a legal and common practice.

Problem 5.11. (2 points) A covered call is a portfolio consisting of a written call option and the short underlying. *True or false?*

Solution: FALSE

Problem 5.12. (5 points) Today's price of a non-dividend-paying stock is \$1000 and the annual effective interest rate is given to be 5%. You write a one-year, \$1,050-strike Eureopan call option for a premium of \$10 while you simulataneously buy the stock. What is your **profit** if the stock's spot price in one year equals \$1,200?

Solution:

$$S(T) - 1000(1.05) - (S(T) - K)_+ + 10(1.05) = 1050 - 990(1.05) = 10.50.$$

Problem 5.13. (6 points) For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date T in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date T. Assume that the call's premium equals \$10 and that the annual effective interest rate equals 10%.

Express your asnwer as an interval.

Solution: The profit function of the forward contract is $v_F(s) = s - 100$. The profit function of the call is

$$v_C(s) - 10 \times 1.10 = (s - 100)_+ - 11.$$

For $s \geq 100$, the call's profit is smaller than the forward contract's profit. So, we focus on $s < 100$. Here we have to solve for s^* in

$$s^* - 100 = -11 \quad \Rightarrow \quad s^* = 89.$$

The answer is $[0, 89)$.