$N \sim g(\text{mean} = \beta = 4)$ $\quad a = \frac{\beta}{1+\beta}$, $\; b = 0$

95. The number of claims in a period has a geometric distribution with mean 4. The amount of each claim X follows $\boxed{\Pr(X = x) = 0.25}$ $\quad x = 1, 2, 3, 4$, The number of claims and the claim amounts are independent. S is the aggregate claim amount in the period.

Calculate $F_S(3)$. $= f_S(0) + f_S(1) + f_S(2) + f_S(3)$

$$f_S(x) = a \sum_{y=1}^{x \wedge m} f_X(y) \cdot f_S(x-y)$$

(A) 0.27

(B) 0.29

(C) 0.31

(D) 0.33

(E) 0.35

- $f_S(0) = p_N(0) = \dfrac{1}{1+\beta} = \dfrac{1}{5} = 0.2$
- $f_S(1) = a \cdot f_X(1) \cdot f_S(1-1) = \dfrac{\cancel{4}}{5} \cdot \dfrac{1}{\cancel{4}} \cdot (0.2) = \underline{0.04}$
- $f_S(2) = a \cdot \left(f_X(1) \cdot f_S(2-1) + f_X(2) \cdot f_S(2-2) \right)$

$$= \frac{\cancel{4}}{5} \cdot \frac{1}{\cancel{4}} \cdot (0.04 + 0.2) = 0.2(0.24) = \underline{0.048}$$

96. Insurance agent Hunt N. Quotum will receive no annual bonus if the ratio of incurred losses to earned premiums for his book of business is 60% or more for the year. If the ratio is less than 60%, Hunt's bonus will be a percentage of his earned premium equal to 15% of the difference between his ratio and 60%. Hunt's annual earned premium is 800,000.

Incurred losses are distributed according to the Pareto distribution, with $\theta = 500{,}000$ and $\alpha = 2$.

Calculate the expected value of Hunt's bonus.

(A) 13,000

(B) 17,000

(C) 24,000

(D) 29,000

(E) 35,000

- $f_S(3) = a\left(f_X(1)\cdot f_S(3-1) + f_X(2)\cdot f_S(3-2) + f_X(3)\cdot f_S(3-3)\right)$

$$= \frac{\cancel{4}}{5}\cdot\frac{1}{\cancel{4}}\cdot\left(\underline{0.048} + \underline{0.04} + \underline{0.2}\right) = 0.2(0.288) = \underline{0.0576}$$

$$\Rightarrow F_S(3) = 0.2 + 0.04 + 0.048 + 0.0576 = \underline{0.3456} \quad \square$$

Per Payment & Per Loss Random Variables.

Goal: Look @ aggregate payments in case that individual losses are modified by an ordinary deductible.

$$\begin{cases} \text{Per Loss: } & Y^L = (X-d)_+ \\ \text{Per Payment: } & Y^P = Y^L \mid Y^L > 0 = X - d \mid X > d \end{cases}$$

In our earlier notation: $v = \mathbb{P}[Y^L > 0] = \mathbb{P}[X > d]$

$$Y^L = \begin{cases} Y^P & \text{if pmt} \quad \text{w/ probab. } v \\ 0 & \text{if no pmt} \quad \text{w/ probab. } 1-v \end{cases}$$

$$\boxed{M_{Y^L}(t) = v\cdot M_{Y^P}(t) + (1-v)\cdot \underbrace{t^0}_{=1} = v\cdot M_{Y^P}(t) + (1-v)}$$

N^L ... # of losses

N^P ... # of payment

$$P_{N^P}(z) = P_{N^L}\left((1-v) + v\cdot z\right)$$

pgf is applied to ...

In aggregate:

On the per·loss basis: $S = Y_1^L + Y_2^L + \dots + Y_{N^L}^L$

On the per·pmt basis: $S = Y_1^P + Y_2^P + \dots + Y_{N^P}^P$

What is the difference?