

UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 21

Multifactor models.

Please, provide your **final answers only** to the following problems:

Problem 21.1. You are given the following information about the return of a security, using a three-factor model:

Factor	Beta	Expected Return
T	0.10	25%
U	0.12	20%
V	0.15	10%

The annual effective risk-free rate of return is 4%.

Calculate the expected return of this security using the given three-factor model.

- (a) About 0.075
- (b) About 0.08
- (c) About 0.085
- (d) About 0.09
- (e) None of the above.

Solution: (d)

$$\begin{aligned}\mathbb{E}[R_S] &= r_f + \beta^T(\mathbb{E}[R_T] - r_f) + \beta^U(\mathbb{E}[R_U] - r_f) + \beta^V(\mathbb{E}[R_V] - r_f) \\ &= 0.04 + 0.10(0.25 - 0.04) + 0.12(0.20 - 0.04) + 0.15(0.10 - 0.04) = 0.0892.\end{aligned}\quad (21.1)$$

Problem 21.2. You are given the following information about the return of a security, using a three-factor model:

Factor	Beta	Expected Return
T	0.12	12%
U	0.16	15%
V	0.20	10%

The expected return of this security using the given three-factor model is equal to 0.08. What is the annual effective risk-free rate of return?

- (a) About 0.375
- (b) About 0.415
- (c) About 0.485
- (d) About 0.595
- (e) None of the above.

Solution: (b) and (e)

By our three-factor model, we have that the expected return of our security S satisfies

$$\begin{aligned}\mathbb{E}[R_S] &= r_f + \beta^T(\mathbb{E}[R_T] - r_f) + \beta^U(\mathbb{E}[R_U] - r_f) + \beta^V(\mathbb{E}[R_V] - r_f) \\ &= \beta^T\mathbb{E}[R_T] + \beta^U\mathbb{E}[R_U] + \beta^V\mathbb{E}[R_V] + r_f(1 - \beta^T - \beta^U - \beta^V).\end{aligned}\quad (21.2)$$

So,

$$\begin{aligned}r_f &= \frac{\mathbb{E}[R_S] - \beta^T\mathbb{E}[R_T] - \beta^U\mathbb{E}[R_U] - \beta^V\mathbb{E}[R_V]}{1 - \beta^T - \beta^U - \beta^V} \\ &= \frac{0.08 - 0.12(0.12) - 0.16(0.15) - 0.2(0.1)}{1 - 0.12 - 0.16 - 0.2} = 0.0415385.\end{aligned}$$

Problem 21.3. You are given the following information about the return of a security, using a three-factor model:

Factor	Beta	Expected Return
T	0.12	16%
U	0.15	18%
V	0.18	20%

The effective risk-free interest rate is 0.04. What is the risk premium of the security?

- (a) About 0.0442
- (b) About 0.0642
- (c) About 0.0842
- (d) About 0.1042
- (e) None of the above.

Solution: (b)

By our three-factor model, we have that the risk premium of our security S equals

$$\begin{aligned}\mathbb{E}[R_S] - r_f &= \beta^T(\mathbb{E}[R_T] - r_f) + \beta^U(\mathbb{E}[R_U] - r_f) + \beta^V(\mathbb{E}[R_V] - r_f) \\ &= 0.12(0.16 - 0.04) + 0.15(0.18 - 0.04) + 0.18(0.20 - 0.04) = 0.1042\end{aligned}\tag{21.3}$$