M339D: September 8th, 2023.

Static Portfolios [cont'd].

Example. [Investing in a Zero-Coupon Bond]

$$\underset{0}{|}\!\!\text{—————}\!\!\overset{+C}{\underset{T}{|}}$$

0 ... bond bought

C ... redemption amount

T ... maturity date

r ... continuously compounded, risk-free interest rate

✓ Initial cost: $\boxed{Ce^{-rT}}$

✓ Payoff: $\boxed{C}$

✓ Profit = Payoff − $FV_{0,T}$(Initial Cost)

$$= C - \cancel{e^{rT}}\left(C\cancel{e^{-rT}}\right) = 0$$

Example. [Taking a Simple Loan] r ... ccrfir

L ... the loan amount, i.e., the amt borrowed @ time 0

$$\underset{0}{|}\!\!\text{—————}\!\!\underset{T}{|}$$

T ... the loan's term, i.e., the time @ which the loan must be repaid in full

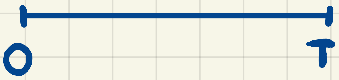
Initial Cost: $-L$ (the negative sign is because the agent RECEIVES L @ time 0)

Payoff: $\boxed{-Le^{rT}}$ (the negative sign is because the agent GIVES UP Le^{rT} @ time T)

Profit = Payoff − $FV_{0,T}$(Init. Cost)

$$= -Le^{rT} + e^{rT}(+L) = 0 \qquad \square$$

Example. [Outright Purchase of Stock]



"stock bought" "stock sold"

Initial Cost: $S(0)$

Payoff: $S(T)$ a random variable

Inspiration

Payoff and Profit Curves.

Goal. To study the payoff and the profit as functions of the final asset price.

Introduce: s... an independent argument taking values in $[0,+\infty)$; it stands for the FINAL ASSET PRICE, i.e., it's a placeholder for the random variable $S(T)$.

Now, we can define the PAYOFF FUNCTION which describes the dependence of the payoff on the independent argument s.

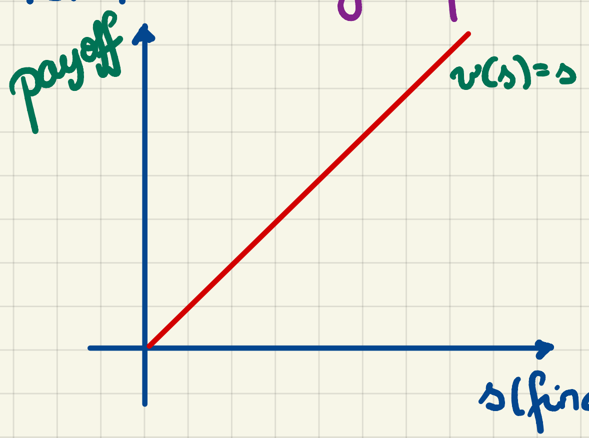
Notation: v... payoff f'tion

$v : [0,+\infty) \longrightarrow \mathbb{R}$

$v(s)$ the agent's payoff if the final asset price is s

Example [cont'd]

For the outright purchase: $\boxed{v(s) = s}$ identity function

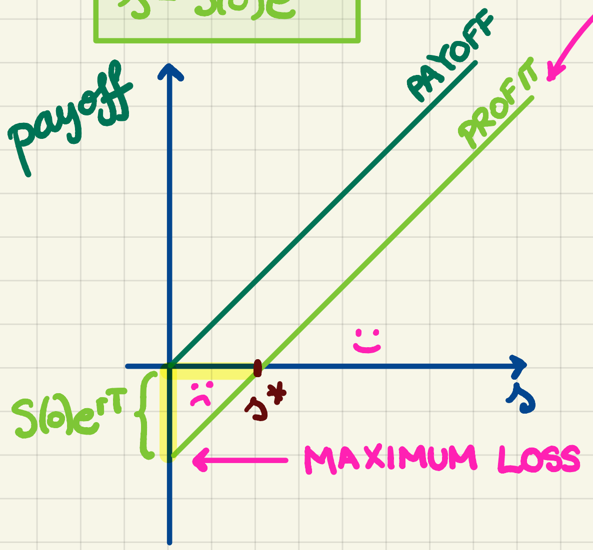


When we plot the payoff function, we get the payoff curve or the payoff diagram.

In general, the profit function is: $\boxed{v(s) - FV_{0,T}(\text{Init. Cost})}$

Example [cont'd].

$\boxed{s - S(0)e^{rT}}$



NO UPPER BOUND, i.e., unlimited growth potential

s^*... break-even point

Here: $\boxed{s^* = S(0)e^{rT}}$

The payoff and profit curves are increasing.

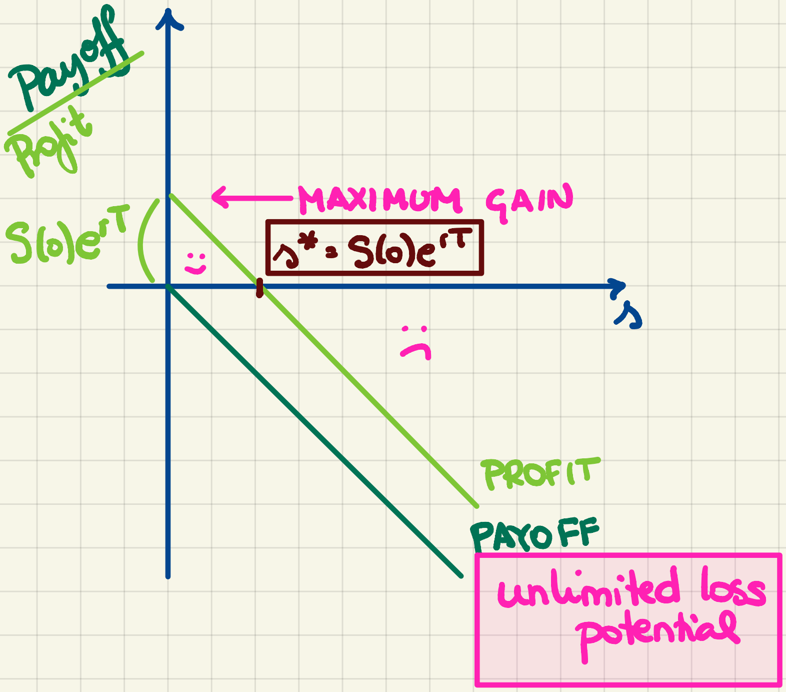
Terminology. If the payoff/profit is increasing (not necessarily strictly), as a function of the final asset price s, we say that the portfolio is long with respect to the underlying asset.

Example. [A Short Sale]

Initial Cost: $-S(0)$

Payoff: $-S(T)$ $\Rightarrow$ payoff f'tion: $\boxed{v(s) = -s}$

$$\text{Profit} = -S(T) - FV_{0,T}(\text{Init. Cost})$$
$$= -S(T) + e^{rT}(+S(0))$$
$$= -S(T) + e^{rT}S(0)$$



The profit/payoff curve is decreasing.

$\Downarrow$

The short sale is <u>short w.r.t. the underlying</u>.