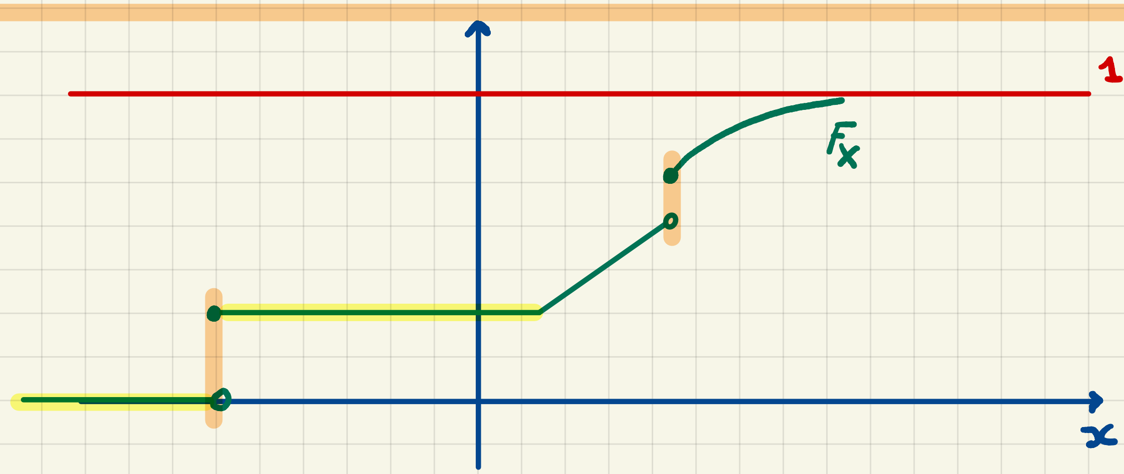
M358K: August 28^{th}, 2023.

Probability Review.

Def'n. For any random variable X the **cumulative distribution function (cdf)** of X is a function $F_X : \underline{\mathbb{R}} \longrightarrow \underline{[0,1]}$ given by

$$\boxed{F_X(x) = \underline{\mathbb{P}[X \leq x]} \quad \text{for all } x \in \mathbb{R}}$$



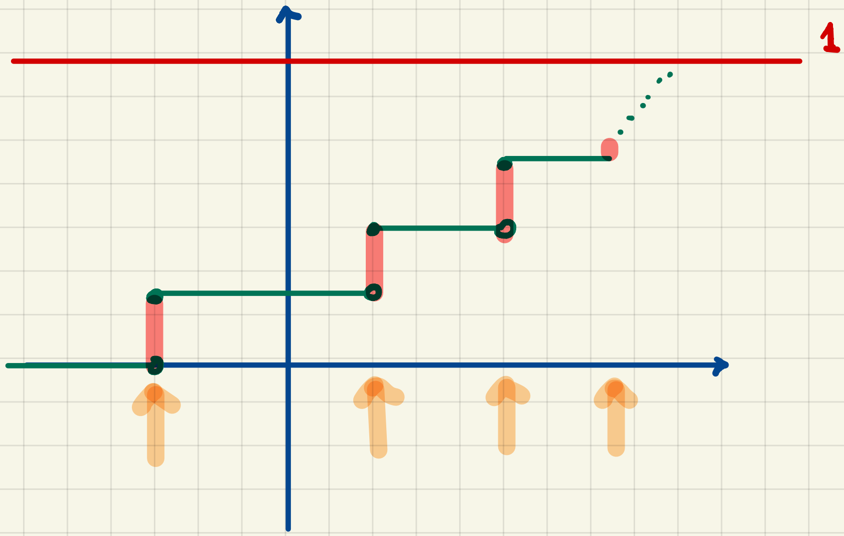
The cdf gives us complete information about the distribution of a random variable.

Q: $\lim\limits_{x \to -\infty} F_X(x) = \underline{\ 0\ }$

Q: $\lim\limits_{x \to +\infty} F_X(x) = \underline{\ 1\ }$

Note: $\boxed{\text{Nondecreasing!}}$

Q: What if your cdf is a step function?



Then, your r.v. is discrete, i.e., it can take @ most countably many values.

It's usually more convenient to express the dist'n of a discrete r.v. using its probability (mass) f'tion (pmf).

In general, the support of a r.v. is (vaguely speaking) the set of all the values that a r.v. can take.

For discrete r.v. the support is the set of all the points @ which the cdf jumps.

For those points, the pmf is

$$p_X(x) = \mathbb{P}[X=x] = \text{size of the jump}$$
$$= F_X(x) - \underbrace{F_X(x-)}_{\text{left limit}}$$

Bernoulli.

The support of an X w/ the Bernoulli dist'n is $\{0,1\}$.

We usually interpret "1" as "success"
and "0" as "failure".

We denote the probability of success in a single Bernoulli trial by p.

Notation:

- $X \sim \begin{cases} 1 & \text{w/ probab. } p \\ 0 & \text{w/ probab. } 1-p \end{cases}$
- $X \sim \text{Bernoulli}(p)$
- $p_X(0) = 1-p$
  $p_X(1) = p$