UNIVERSITY OF TEXAS AT AUSTIN

Quiz #23

American-option prices.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 23.1. (2 points)
It is never optimal to exercise an American call option on a non-dividend-paying stock early. *True or false?*

Solution: TRUE
For substantiation, see class notes.

Problem 23.2. (2 points)
American-style options are at least as valuable as otherwise identical European-style options. *True or false?*

Solution: TRUE
For substantiation, see class notes.

Problem 23.3. (2 pts) Let $V^A(0,T)$ denote the price at time 0 of an American option with expiration date T. Then, we always have

$$V^A(0,T) \leq V^A(0,2T).$$

True or false?

Solution: TRUE
For substantiation, see class notes.

Problem 23.4. (9 points) The current price of a non-dividend-paying stock is \$100 per share. A two-period binomial stock-price tree is used to model the movements of the stock price during the following year. The up and down factors are given to be $u = 1.2$ and $d = 0.9$.

The continuously compounded, risk-free interest rate equals 0.06.

Consider a \$110-strike, one-year American put on the above stock. Use the two-period binomial stock-price tree to calculate the current price of the American put.

(a) \$20.03
(b) \$15.41
(c) \$13.38
(d) \$11.43
(e) None of the above.

Solution: (d)
With the given up and down factors, the stock-prices tree looks like this:

$$\begin{array}{ccc} & & S_{uu} > 110 \\ & S_u = 120 > 100 & \\ S_0 = 100 & & S_{ud} = 108 \\ & S_d = 90 & \\ & & S_{dd} = 81 \end{array}$$

The risk-neutral probability of the stock price going up in a single period equals

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{0.03} - 0.9}{1.2 - 0.9} = 0.4348.$$

Should the American option not be exercised early the possible payoffs would be

$$V_{uu} = 0, \quad V_{ud} = 110 - 108 = 2, \quad V_{dd} = 110 - 81 = 29.$$

It is not sensible to exercise the American put at the up node, so the value of the American put equals the continuation value at the up node. We get

$$V_u^A = CV_u = e^{-0.03}(1 - 0.4348) \times 2 = 1.09699.$$

At the down node, the value of immediate exercise is

$$IE_d = 110 - 90 = 20.$$

On the other hand, the continuation value at the down node equals

$$CV_d = e^{-0.03}[0.4348 \times 2 + (1 - 0.4348) \times 29] = 16.7503.$$

We conclude that the American put's value at the down node equals the value of immediate exercise, i.e., $V_d^A = 20$.

Should the option be exercised at time-0, the payoff would be 10. The continuation value at the root node is

$$CV_0 = e^{-0.03}[0.4348 \times 1.09699 + (1 - 0.4348) \times 20] = 11.4328.$$

So, the price we were looking for is \$11.34.