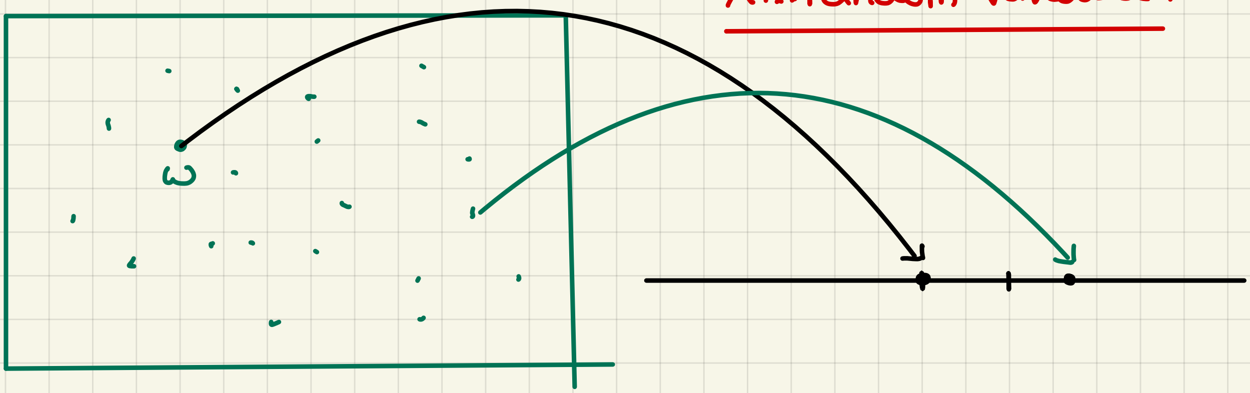
# M339J: January 25th, 2021.

## Probability Review

X... random variable.




Probability Space $\Omega$

(Outcome space; Sample space $\mathcal{S}$)

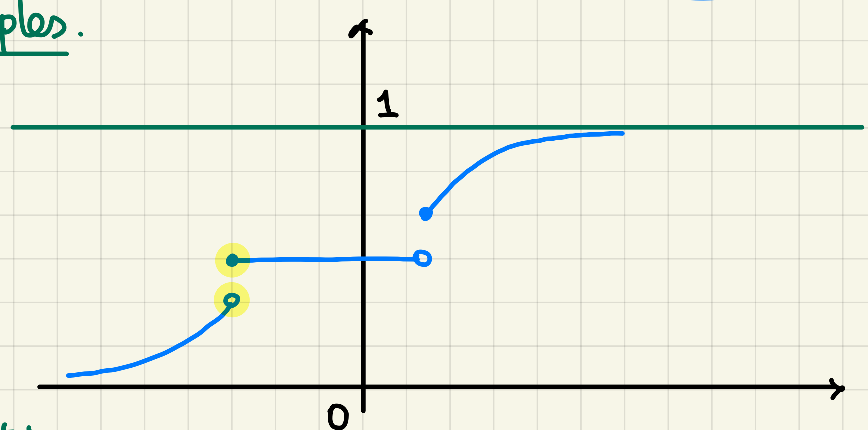
$\omega$ ... elementary outcomes

**Def'n.** The cumulative distribution function (cdf) of a random variable $X$ is a function

$$F_X : \mathbb{R} \to [0,1]$$

given by $F_X(x) = \mathbb{P}[X \le x]$ for all $x \in \mathbb{R}$

**Examples.**




**Properties:**

- Nondecreasing, i.e.,

$$x_1 < x_2 \Rightarrow F_X(x_1) \le F_X(x_2)$$

- Right · continuous

- $\lim_{x \to -\infty} F_X(x) = 0$
- $\lim_{x \to +\infty} F_X(x) = 1$

"Def'n." The support of a random variable $X$ is the set of all of the values it can take.

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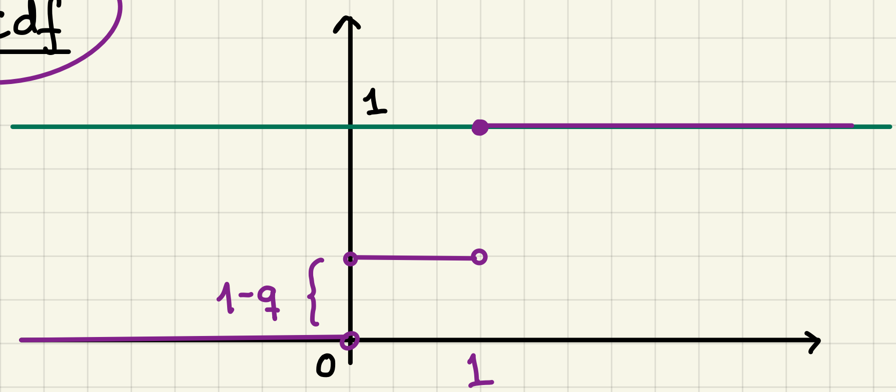
Def'n. A random variable is called discrete its support has @ most countably many values.

Example. • Bernoulli : Support $= \{0, 1\}$

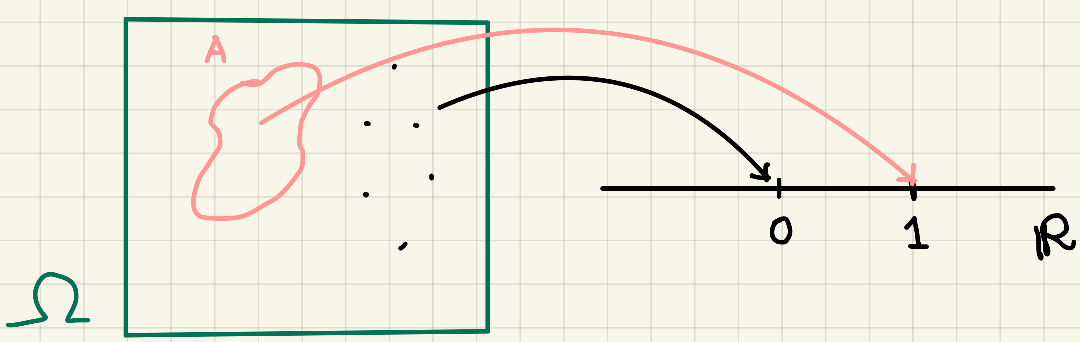
$$X \sim \begin{cases} 0 & \text{w/ } 1-q \\ 1 & \text{w/ } q \end{cases}$$

cdf




Step function

- Indicator Random Variable



$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

$$\mathbb{I}_A \sim \begin{cases} 1 & \text{w/ pr. } \mathbb{P}[A] \\ 0 & \text{w/ pr. } 1-\mathbb{P}[A] \end{cases}$$

- Binomial

Consider $m$ independent Bernoulli trials w/ the same probability of success denoted by $q$.
The number of successes is the outcome of a binomial random variable.
Say that $I_1, I_2, \dots, I_m$ are the independent, identically distributed Bernoulli random variables.

$$X = I_1 + I_2 + \dots + I_m$$

will be the corresponding BINOMIAL random variable.

$$\text{Support} = \{0, 1, \dots, m\}$$

- Poisson, geometric : $\text{Support} = \mathbb{N}_0 = \{0, 1, 2, \dots\}$

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Review from probability:

- probability mass function (pmf)
- probability density function (pdf)