

F-Distribution.

Def'n. Let U and V be chi-squared random variables w/ ν_1 and ν_2 degrees of freedom, resp. Then, with U and V **independent**, the random variable

$$F = \frac{U/\nu_1}{V/\nu_2}$$

is said to have the **F distribution** w/ numerator degrees of freedom ν_1 and denominator degrees of freedom ν_2 .

We write

$$F \sim F(\nu_1, \nu_2) \sim F_{\nu_1, \nu_2}$$

Theorem. Let two **independent** random samples of size n_1 and n_2 , resp., be drawn from two normal population w/ variances σ_1^2 and σ_2^2 , resp.

Say that the sample variances are denoted by S_1^2 and S_2^2 , resp.

Then, the statistic

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1-1, n_2-1)$$

Corollary. If $\sigma_1 = \sigma_2$, then

$$F = \frac{S_1^2}{S_2^2} \sim F(n_1-1, n_2-1)$$