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Binomial option pricing (review).

Problem 3.1. Let the continuously compounded risk-free interest rate be denoted by r. You are building a model for the price of a stock which pays dividends continuously with the dividend yield δ. Consider a binomial tree modeling the evolution of the stock price. Let the length of each period be h and let the up factor be denoted by u, and the down factor by d. What is the **no-arbitrage** condition for the binomial tree you are building?

Problem 3.2. Set up the framework for pricing by replication in a one-period binomial tree! What is the *risk-neutral pricing* formula?

Problem 3.3. The current price of a certain non-dividend-paying stock is \$100 per share. You are modeling the price of this stock at the end of a quarter year using a one-period binomial tree under the assumption that the stock price can either increase by 4%, or decrease by 2%.

The continuously compounded risk-free interest rate is 3%.

What is the price of a three-month, at-the-money European call option on the above stock consistent with the above binomial tree?

Problem 3.4. Let the continuously compounded risk-free interest rate be equal to 0.04.

The current price of a continuous-dividend-paying stock is \$80 and its dividend yield is 0.02. The stock's volatility is 0.25. You model the evolution of the stock price over the following half year using a two-period **forward** binomial tree.

What is the price of a six-month, \$82-strike European put option on the above stock consistent with the given binomial tree?

Problem 3.5. The current price of a continuous-dividend-paying stock is \$100 per share. Its volatility is given to be 0.2 and its dividend yield is 0.03.

The continuously compounded risk-free interest rate equals 0.06.

Consider a \$95-strike European put option on the above stock with nine months to expiration. Using a three-period forward binomial tree, find the price of this put option.

(a) \$2.97
(b) \$3.06
(c) \$3.59
(d) \$3.70
(e) None of the above.

Problem 3.6. Consider a non-dividend-paying stock whose current price is \$100 per share. Its volatility is given to be 0.25. You model the evolution of the stock price over the following year using a two-period forward binomial tree.

The continuously compounded risk-free interest rate is 0.04.

Consider a \$110-strike, one-year **down-and-in** put option with a barrier of \$90 on the above stock. What is the price of this option consistent with the above stock-price model?

(a) About \$10.23
(b) About \$11.55
(c) About \$11.78
(d) About \$11.90
(e) None of the above.

Problem 3.7. The current stock price is observed to be \$100 per share. The stock is projected to pay dividends continuously at the rate proportional to its price with the dividend yield of 0.03. The stock's volatility is given to be 0.23. You model the evolution of the stock price using a two-period forward binomial tree with each period of length one year.

The continuously compounded risk-free interest rate is given to be 0.04.

What is the price of a two-year, \$101-strike **American** put option on the above stock consistent with the above stock-price tree?

(a) About \$6.62
(b) About \$8.34
(c) About \$8.83
(d) About \$11.11
(e) None of the above.