M339D: February 1st, 2021.

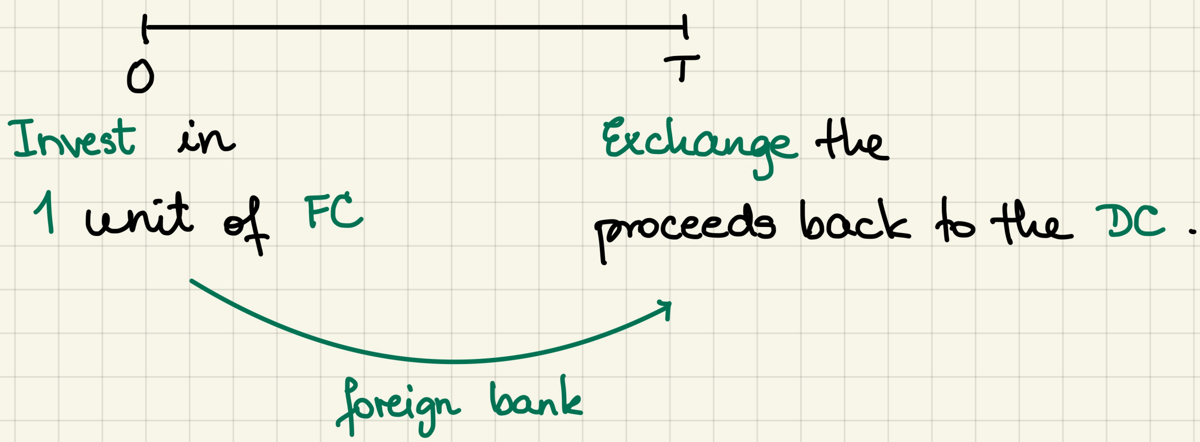
Foreign Currencies [cont'd].

DC ... domestic currency w/ r_D as its ccrfir

FC ... foreign currency w/ r_F as its ccrfir

$x(t), t \geq 0$ ... exchange rate from FC into DC

Investment Schematic



At time 0: • Buy 1 unit of FC.

$\Rightarrow$ We spend $x(0)$.

• We deposit that 1 unit of FC; it will earn interest @ ccrfir r_F.

At time T: • The balance in the account is $\underline{e^{r_F \cdot T}}$ units of FC.

• We exchange this balance to DC; we get $\underline{e^{r_F \cdot T} \cdot x(T)}$.

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Problem set 2

Foreign currencies.

Problem 2.1. Paige lives in Great Britain and receives her salary in GBP. She decides to spend 680 GBP and let the proceeds of the exchange accrue interest at the USD continuously compounded risk-free interest rate .

(i) Given that the initial exchange rate is 0.68 GBP per USD, how much (in USD) does Paige receive initially?

(ii) Given that the USD continuously compounded risk-free interest rate is equal to $r_\$ = 0.02$, what is the balance in Paige's account six months after the initial transaction? Assume that there were no intermediate deposits or withdrawals.

(iii) Paige decides to withdraw the balance in her account at that time (still six months from the initial exchange) and exchange it back to GBP. Given that the exchange rate at that time equals 0.71 GBP per USD, how much (in GBP) does Paige receive?

(iv) Given that the GBP continuously compounded risk-free interest rate equals $r_£ = 0.03$, what would have Paige's balance have been had she decided to simply deposit her initial investment in a GBP savings account?

(i) $\dfrac{680\,£}{0.68\,£/\$} = 1000\,\$$

(ii) $1000e^{0.02/2} = 1000e^{0.01} = 1010.05$

(iii) $1010.05(0.71) = 717.14$

(iv) $680e^{0.03/2} = 680e^{0.015} = 690.28$

Problem 2.2. You are given the following information:

- the current exchange rate is 8.71 Swedish Kronor (SEK) per USD;
- the SEK continuously compounded risk-free interest rate equals 0.04;
- the USD continuously compounded risk-free interest rate equals 0.02.

Niklas wants to have 10,000 USD exactly one year from now. He is going to buy USD today and deposit the proceeds in a USD saving account. He does not intend to make any withdrawals or deposits prior to the end of the one-year period. How many Swedish Kronor does Niklas need to spend in SEK today in order to purchase just enough USD so that the final balance in his USD savings account equals $10,000$?

$\longrightarrow$:

$$10000e^{-0.02} = 9801.99$$

answer: $9801.99 \times 8.71 = 85{,}375.30$ ∎

Continuous dividend paying stocks [revisited].

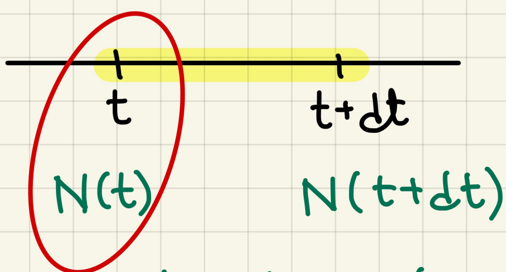
δ ... dividend yield

For any time interval $(t, t+dt)$ the shareholders receive $\delta S(t)\,dt$ per share owned.

Convention: With continuous dividends all the dividends are immediately and continuously invested in the same asset.

Notation: $N(t)$, $t \geq 0$ ... the # of shares owned @ time t

At time 0: Set $N(0) = n_0$ ... the total # of shares purchased initially



$\Rightarrow$ $dN(t) = N(t+dt) - N(t) =$ # of shares bought for $(t, t+dt)$

Q: What is the total dividend amt paid for $(t, t+dt)$?

$$N(t)\cdot \underbrace{\delta S(t)\,dt}_{\text{per share}}$$

Q: How many shares can one buy for that amount?