M339W: March 26th, 2021.

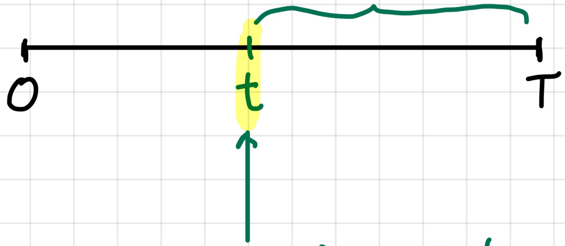
Option Greeks.

GOAL: To study the dependence of the value of a portfolio on the set of independent arguments: t, s, r, δ, σ

s ... asset price @ time t

t ... valuation date

$T-t$ ... the time to exercise



T ... time horizon (say, the exercise date of an option)

valuation date

The underlying stock's price will be modelled in the Black-Scholes framework.

Remember: $S(t), t \geq 0$ ... the stock price

Under the risk-neutral measure $\mathbb{P}^*$:

$$S(T) = S(t)e^{(r-\delta-\frac{\sigma^2}{2})(T-t) + \sigma\sqrt{T-t}\cdot Z} \quad \text{w/ } Z \sim N(0,1)$$

Q: Temporarily focus on a K-strike European call w/ exercise date T. What is its time-t BS Price?

$\longrightarrow$:

$$V_C(t) = S(t)e^{-\delta(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2)$$

w/ $d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S(t)}{K}\right) + \left(r-\delta+\frac{\sigma^2}{2}\right)(T-t)\right]$

and $d_2 = d_1 - \sigma\sqrt{T-t}$

Let's reintroduce s... independent argument which stands for the CURRENT (time t) asset price

In our pricing formula, we would have that the call is worth this much @ time t if the stock price is s:

$$v_c(\overbrace{s, t, r, \delta, \sigma}^{\cdots}) = se^{-\delta(T-t)}N(d_1(\cdots)) - Ke^{-r(T-t)}N(d_2(\cdots))$$

w/ $d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{s}{K}\right) + \left(r-\delta+\frac{\sigma^2}{2}\right)(T-t)\right]$

and $d_2 = d_1 - \sigma\sqrt{T-t}$

Our portfolios consist of:
- the riskless asset
- the risky asset
- European options on our risky asset

$\Rightarrow$ we will always be able to represent our wealth @ time t as a value function of our portfolio, i.e.,

$$v(s, t, r, \delta, \sigma)$$

$\uparrow$

value f'tion

Example. [An outright purchase of a non-dividend-paying stock]

$\Rightarrow \quad v(s, t, r, \delta, \sigma) = s$

stands for the time-t stock price

$\Rightarrow \quad \Delta(s,t) = 1, \qquad \Gamma(s,t) = 0;$

other Greeks $= 0$

skipping ψ

On Monday: Focus on the Δ:
- the prepaid forward
- ☆ European call
- ☆ European put