$$p_{S\wedge 30}(20) = p_S(20) = \mathbb{P}[N=1, X_1=20] + \mathbb{P}[N=2, X_1=X_2=10]$$

$$= p_N(1)\cdot p_X(20) + p_N(2)\cdot (p_X(10))^2$$

independence

$$= e^{-1}\cdot 0.3 + e^{-1}\cdot \frac{1}{2}\cdot (0.3)^2$$

$$= e^{-1}\cdot 0.3\cdot 1.15 = e^{-1}\cdot 0.345$$

Easiest way to get $p_{S\wedge 30}(30) = ?$

M339J: April 28th, 2021.

$$p_{S\wedge 30}(30) = 1 - p_{S\wedge 30}(0) - p_{S\wedge 30}(10) - p_{S\wedge 30}(20)$$

$$= 1 - e^{-1} - e^{-1}\cdot 0.3 - e^{-1}\cdot 0.345 = 1 - e^{-1}\cdot 1.645$$

$$\mathbb{E}[S\wedge 30] = 10\cdot p_{S\wedge 30}(10) + 20\, p_{S\wedge 30}(20) + 30\, p_{S\wedge 30}(30)$$

$$= 10\cdot e^{-1}\cdot 0.3 + 20\cdot e^{-1}\cdot 0.345 + 30(1 - e^{-1}\cdot 1.645)$$

$$= 15.487$$

final answer: $29 - 15.487 = 13.513$ ■

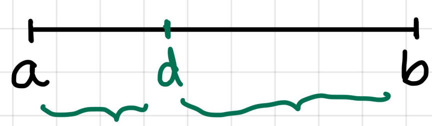
Interpolation Theorem.

Assume that S represents aggregate losses such that for some $a < b$ we have

$$\boxed{\mathbb{P}[a < S < b] = 0.}$$

Then: If $a \le d \le b$:

$$\mathbb{E}[(S-d)_+] = \frac{b-d}{b-a}\cdot \mathbb{E}[(S-a)_+] + \frac{d-a}{b-a}\,\mathbb{E}[(S-b)_+]$$



91. The number of auto vandalism claims reported per month at Sunny Daze Insurance Company (SDIC) has mean 110 and variance 750. Individual losses have mean 1101 and standard deviation 70. The number of claims and the amounts of individual losses are independent.

Using the normal approximation, calculate the probability that SDIC's aggregate auto vandalism losses reported for a month will be less than 100,000.

(A) 0.24

(B) 0.31

(C) 0.36

(D) 0.39

(E) 0.49

92. Prescription drug losses, S, are modeled assuming the number of claims has a geometric distribution with mean 4, and the amount of each prescription is 40.

Calculate $E[(S-100)_+]$.

(A) 60

(B) 82

(C) 92

(D) 114

(E) 146

S... aggregate losses

$S = X_1 + X_2 + \cdots + X_N = 40 \cdot N$

$\begin{cases} N \sim g(\text{mean}=4) \\ X_j \equiv 40 \text{ for all } j \end{cases}$

$\mathbb{E}[(S-100)_+] = ?$

The support of S is $\{0, 40, 80, \ldots\}$

Use the Interpolation thm:

$a=80 < d=100 < b=120$

$\Rightarrow \mathbb{E}[(S-100)_+] = \frac{1}{2} \cdot \mathbb{E}[(S-80)_+] + \frac{1}{2} \cdot \mathbb{E}[(S-120)_+]$

✓ ?

$\mathbb{E}[(S-80)_+] = \mathbb{E}[(40 \cdot N - 80)_+] =$

$= 40 \mathbb{E}[(N-2)_+]$

$$\mathbb{E}[(N-2)_+] = \underbrace{\mathbb{E}[N]}_{4} - \mathbb{E}[N \wedge 2]$$

↙ alternative : ~~memoryless property~~

probab. 1st two trials are failures $\left(\frac{4}{5}\right)^2$

$$\mathbb{E}[(N-2)_+] = \underbrace{\mathbb{E}[N-2 \mid N>2]}_{\text{memoryless}\ =\ 4\ =\ \beta} \cdot \overbrace{\mathbb{P}[N>2]}$$

$\beta \longrightarrow$ probab. of success : $\frac{1}{1+\beta} = \frac{1}{5}$

$\Rightarrow$ probab. of failure : $\frac{4}{5}$

$= 2.56$

- $\mathbb{E}[N \wedge 2] = 0 \cdot p_N(0) + 1 \cdot p_N(1) + 2 \cdot \mathbb{P}[N \geq 2]$
- $\mathbb{E}[N \wedge 2] = S_N(0) + S_N(1) = \frac{4}{5} + \left(\frac{4}{5}\right)^2$

$$\mathbb{E}[(S-80)_+] = 40 \cdot 2.56 = 102.40$$

$$\mathbb{E}[(S-120)_+] = 40 \cdot \underbrace{\mathbb{E}[(N-3)_+]}_{=\ \underbrace{\mathbb{E}[N]}_{4} \cdot \underbrace{\mathbb{P}[N>3]}_{\left(\frac{4}{5}\right)^3}}$$

$$= 40 \cdot 4 \cdot \left(\frac{4}{5}\right)^3 = 81.92$$

$\Rightarrow$ answer: $\frac{1}{2}(102.40 + 81.92) = 92.16$