

M339D: April 19th, 2024.

Black-Scholes Practice.

Problem. Assume the Black-Scholes model.

For a European call option, the strike is $S(0)e^{rT}$ ✓
w/ T being the exercise date.

The price of a call option w/ one year to exercise is

$$0.6 \cdot S(0)$$

Find the price of such a call option
w/ three months to exercise in terms of $S(0)$.

→: For any T :

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln \left(\frac{S(0)}{K} \right) + \left(r + \frac{\sigma^2}{2} \right) \cdot T \right]$$

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\underbrace{\ln \left(\frac{S(0)}{S(0)e^{rT}} \right)}_{= -rT} + \left(r + \frac{\sigma^2}{2} \right) \cdot T \right] = \frac{\sigma\sqrt{T}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{\sigma\sqrt{T}}{2} - \sigma\sqrt{T} = -\frac{\sigma\sqrt{T}}{2}$$

Compare to the previous problem 😊

$$V_c(0, T) = S(0) N \left(\frac{\sigma\sqrt{T}}{2} \right) - S(0) e^{-rT} \cdot e^{-\sigma\sqrt{T}} N \left(-\frac{\sigma\sqrt{T}}{2} \right)$$

$$V_c(0, T) = S(0) \left(2N \left(\frac{\sigma\sqrt{T}}{2} \right) - 1 \right)$$

For $T=1$:

$$V_c(0, T=1) = S(0) \left(2N \left(\frac{\sigma}{2} \right) - 1 \right) = 0.6 \cdot S(0)$$

$$\Rightarrow 2N \left(\frac{\sigma}{2} \right) - 1 = 0.6$$

$$\Rightarrow N \left(\frac{\sigma}{2} \right) = 0.8$$

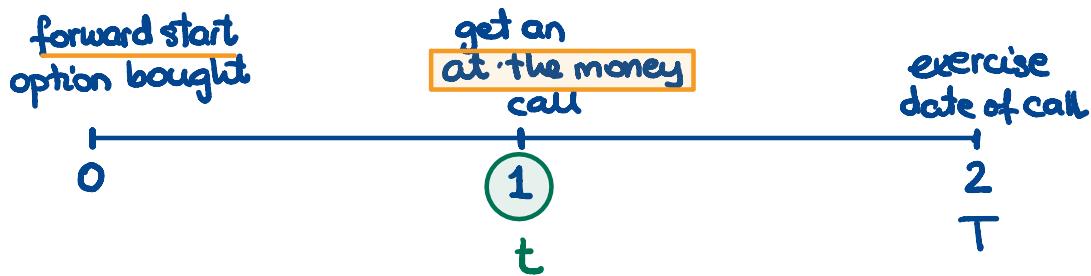
$$\Rightarrow \frac{\sigma}{2} = 0.84 \Rightarrow \sigma = 1.68$$

For $T=\frac{1}{4}$:

$$V_c(0, T=\frac{1}{4}) = S(0) \left(2 \cdot N \left(\frac{\sigma\sqrt{\frac{1}{4}}}{2} \right) - 1 \right)^{0.42} = S(0) \cdot (2 \cdot 0.6628 - 1) = S(0) \cdot 0.3256 \quad \square$$

$V(0) = ?$

- (A) 586
- (B) 594
- (C) 684
- (D) 692
- (E) 797



19. Consider a *forward start option* which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.

You are given:

- (i) The European call option is on a stock that pays no dividends.
- (ii) The stock's volatility is 30%. $\sigma = 0.3$
- (iii) The forward price for delivery of 1 share of the stock 1 year from today is 100. $F_{0,1}(S) = 100 = S(0)e^{r \cdot 1}$
- (iv) The continuously compounded risk-free interest rate is 8%.

Under the Black-Scholes framework, determine the price today of the forward start option.

At $t < T$:

- (A) 11.90
- (B) 13.10
- (C) 14.50
- (D) 15.70
- (E) 16.80

$$V_c(t) = S(t) \cdot N(d_1(t)) - K e^{-r(T-t)} \cdot N(d_2(t))$$

$$\text{w/ } d_1(t) = \frac{1}{\sigma \sqrt{T-t}} \left[\ln \left(\frac{S(t)}{K} \right) + (r + \frac{\sigma^2}{2})(T-t) \right]$$

and

$$d_2(t) = d_1(t) - \sigma \sqrt{T-t}$$

In this problem: $t=1$

$$V_c(1) = S(1) \cdot N(d_1(1)) - K e^{-r(2-1)} \cdot N(d_2(1))$$

$$V_c(1) = S(1) \cdot (N(d_1(1)) - e^{-r} \cdot N(d_2(1))) \quad \leftarrow$$

$$w/ d_1(1) = \frac{1}{0.3\sqrt{2-1}} \left[\ln\left(\frac{S(1)}{S(0)}\right) + \left(0.08 + \frac{0.09}{2}\right)(2-1) \right]$$

@ the money

$$d_1(1) = \frac{0.08 + 0.045}{0.3} = \frac{0.125}{0.3} = 0.42$$

$$d_2(1) = d_1(1) - 0.3\sqrt{2-1} = 0.12$$

$$N(d_1(1)) = N(0.42) = 0.6628$$

$$N(d_2(1)) = N(0.12) = 0.5438$$

$$V_c(1) = S(1) \left(0.6628 - e^{-0.08} \cdot 0.5438 \right) = \boxed{S(1) \cdot 0.1608}$$

At time 0, our forward start option is worth:

$$\boxed{S(0) \cdot 0.1608}$$

Our answer: $0.1608 \cdot F_{0,1}(S) \cdot e^{-0.08} = 0.1608 \cdot 100 \cdot e^{-0.08}$

$$= \boxed{14.84}$$

□

33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

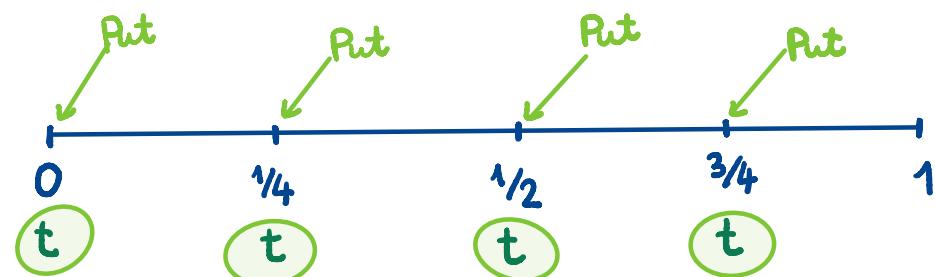
You are given:

- (i) The continuously compounded risk-free interest rate is 8%.
- (ii) The stock's volatility is 30%.
- (iii) The current stock price is 45.
- (iv) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

- (A) 1.59
 (B) 2.24
 (C) 2.86
 (D) .48
 (E) 3.61



t... represents the valuation date

34-39. DELETED

For each of the four puts in the rolling insurance strategy:

- one quarter year to exercise
- $K_t = 0.9 \cdot S(t)$

For every t @ which a put option is received:

$$d_1(\cancel{X}) = \frac{1}{\sigma \sqrt{\frac{1}{4}}} \left[\ln \left(\frac{S(t)}{0.9 \cdot S(t)} \right) + (r + \frac{\sigma^2}{2}) \left(\frac{1}{4} \right) \right]$$

$$d_1(\cancel{X}) = \frac{1}{0.3(\frac{1}{2})} \left[-\ln(0.9) + (0.08 + \frac{0.09}{2}) \left(\frac{1}{4} \right) \right] = \underline{0.9107}$$

$$d_2(\cancel{X}) = \underline{0.7607}$$