M339W: October 6th, 2021.

Focus on the Delta.

value f'tion: $v(\underbrace{s, t, r, \delta, \sigma}_{\dots})$

Def'n. The delta

$$\Delta(\dots) := \frac{\partial}{\partial s} v(\dots)$$

Example. [Outright Purchase of a Non-Dividend-Paying Stock]

The value f'tion: $v(s, t, r, \delta, \sigma) = s$

stands for the
the time-t stock price

$$\Rightarrow \boxed{\Delta(\dots) = 1}$$

Example. [Prepaid Forward Contract on a Continuous-Dividend-Paying Stock]

T... delivery date

$v(\dots) = \,?$

$$F^{P}_{t,T}(S) = S(t)\, e^{-\delta(T-t)}$$

$$\Rightarrow \quad v(\dots) = s e^{-\delta(T-t)}$$

$$\Rightarrow \boxed{\Delta(\dots) = e^{-\delta(T-t)} \quad > 0}$$

$$\Rightarrow \Gamma(\dots) = 0$$

$$\Theta(\cdots) = \delta \cdot s e^{-\delta(T-t)} > 0$$

$$\Psi(\cdots) = -(T-t)\, s e^{-\delta(T-t)} < 0$$

Example. [EUROPEAN CALL]

K... strike price

T... exercise date

Black-Scholes

$$v_c(\cdots) = s\boxed{e^{-\delta(T-t)} \cdot N(d_1(\cdots))} - Ke^{-r(T-t)} \cdot N(d_2(\cdots))$$

w/

$$d_1(\cdots) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{s}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)(T-t)\right]$$

and

$$d_2(\cdots) = d_1(\cdots) - \sigma\sqrt{T-t}$$

By def'n of Delta:

$$\Delta_c(\cdots) = \frac{\partial}{\partial s} v_c(\cdots)$$

After the chain rule and product rule.

$$\boxed{\Delta_c(\cdots) = e^{-\delta(T-t)} \cdot N(d_1(\cdots))} > 0$$

The positivity of the call Δ makes sense since the call is long w.r.t. the underlying.