Homework assignment #9: Solutions

Milica Cudina

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Problem #1 (3 points)

The mean area of the several thousand new apartments is advertised to be at least 1350 square feet. A tenant group thinks that the apartments are smaller than advertised. They hire an engineer to measure a sample of apartments, they should test

a. $H_0 : \mu = 1350$ against $H_a : \mu > 1350$.

b. $H_0 : \mu = 1350$ against $H_a : \mu < 1350$.

c. $H_0 : \mu < 1350$ against $H_a : \mu = 1350$.

d. $H_0 : \mu = 1350$ against $H_a : \mu \neq 1350$.

e. $H_0 : \mu < 1350$ against $H_a : \mu > 1350$.

Solution: The correct solution is **b.**

Problem #2 (5 points)

In a hypothesis testing problem, $p-$value $= 3\%$ means that ...

a. Null hypothesis has a 3% chance to be wrong.

b. If the null hypothesis is true, the probability of observing as extreme or more extreme than what have been observed is 3%.

c. Alternative hypothesis has a 3% chance to be wrong.

d. If we repeat the procedure a lot times, approximately 3% of the tests will be significant.

e. None of the above.

Solution: The correct answer is **b.**

Problem #2 (5 points)

Freddie Threepwood conducts a hypothesis test. He calculates the observed value of the $z-$statistic to be 0.018 (under the null). At the significance level of 0.05, he should reject the null hypothesis. *True or false? Why?*

Solution: **FALSE** since, in `R`, we have

```
1 - pnorm(0.018)
## [1] 0.4928194
```

Problem #3 (15 points)

Suppose that the thumb sizes of the US males follow a normal distribution with an unknown mean μ and known standard deviation $\sigma = 20$ on the GPI - scale (*Grey's Pollex Index - GPI - from* 50 *to* 280). The US Department of Thumbs and Toes (DTT) reports that the mean thumb size in the country is $\mu = 150$.

Being the chairman of the Faculty of Thumbs of the local university you see an excellent opportunity here and decide to conduct your own study of the size of the average American thumb. After collecting a SRS of 100 American thumbs you obtain the following sample average $\bar{x} = 153$.

- **i** (5 pts) Construct a 95%-confidence interval for the unknown parameter μ based on your study.
- **ii.** (8 pts) Assess the strength of evidence your study carries against the DTT findings. In other words: state the hypotheses and report the $p-$value.
- **iii.** (2 pts) You dream of achieving fame and fortune by being the first person ever to estimate the mean thumb size up to the margin of error equal to ± 0.1. How large a sample size do you need for that?

Solution:

i.

$$\bar{x} \pm \frac{z^*\sigma}{\sqrt{n}} \text{ i.e. } 153 \pm 3.92 \text{ i.e. } (149.08, 156.92).$$

ii. (2 points) The hypotheses are

$$\begin{cases} H_0: & \mu = 150 \\ H_a: & \mu \neq 150 \end{cases}$$

(6 points) To get the p-value we calculate $2\mathbb{P}[\bar{X} > 153]$. A simple z-score calculation gives us that the p-value is $2(0.0668) = 0.1336$.

iii.

$$n \geq \left(\frac{z^*\sigma}{0.1}\right)^2 = (1.96(200))^2 = 153664.$$

Problem #4 (5 points)

Source: "Probability and Statistics for Engineers and Scientists" by Walpole, Myers, Myers, and Ye.

Assume that the compressive strength for a certain type of cement is normal with a known standard deviation of 120 kilograms and an **unknown** mean μ. You test the hypotheses

$$H_0: \mu = 5000 \quad vs. \quad H_a: \mu < 5000.$$

For a planned sample of size 50, your colleague obtains the rejection region $RR = (-\infty, 4970]$. What is the significance level he used?

Solution: For the left-sided alternative, the upper bound rejection region is of the form

$$\mu_0 + z_\alpha \left(\frac{\sigma}{\sqrt{n}}\right).$$

So, in this problem, we have that

$$5000 + z_\alpha \left(\frac{120}{\sqrt{50}}\right) = 4970 \quad \Rightarrow \quad z_\alpha = \frac{4970 - 5000}{\frac{120}{\sqrt{50}}} = -1.767767.$$

We can use `R` or the standard normal tables at this point. I decided to use `R` to get α:

```
pnorm(-1.767767)
## [1] 0.03854993
```

Problem #5 (7 points)

The *Cheesecake Manufacture and Dinery* claims that their famous cheesecake has at most 2000 per slice. You suspect the contrary and plan a study. You model the calorie content per slice using the normal distribution with un unknown mean μ and with a **known** standard deviation of 300.

You diligently study a random sample of 25 slices of cheesecake. The sample average turns out to be 2100.

What is the $p-$value corresponding to these data?

Solution: We are testing

$$H_0 : \mu = 2000 \quad vs. \quad H_a : \mu > 2000.$$

Under the null hypothesis, the $z-$score corresponding to the observed sample average is

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{2100 - 2000}{\frac{300}{\sqrt{25}}} = \frac{100}{60} \approx 1.67.$$

This test has a right-tailed alternative. So, from the standard normal tables, we get that the $p-$value equals

$$\mathbb{P}[Z > z] = 1 - \Phi(1.67) = 1 - 0.9525 = 0.0475.$$

Problem #6 (10 points)

The manufacturer of the *Slim Steakburger* brand claims that the mean fat content of this grade of steakburger is at most 18%.

The *Fat Fighters* consumer group, concerned about the mean fat content of this grade of steakburger submits to an independent laboratory a random sample of 12 steakburgers for analysis.

Assuming the percentage fat content being normally distributed with a variance of 3, they carry out an appropriate hypothesis test in order to advise the consumer group as to the validity of the manufacturer's claim.

The rejection region for the significance level of 0.05 is $[18.8225, \infty)$. With the above significance level of 0.05, find the power of the test at the alternative population mean of 20.

Solution: We need to find the probability that the random variable

$$\bar{X}_{12} \sim N(mean = 20, variance = 0.25)$$

falls above the value 18.8225. We get

$$\mathbb{P}[\bar{X}_{12} > 18.8225] = \mathbb{P}\left[\frac{\bar{X}_{20} - 20}{0.5} > \frac{18.8225 - 20}{0.5}\right] = 1 - \Phi\left(-2.355\right) = \Phi(2.355) \approx 0.99.$$