University of Texas at Austin

Quiz #4

Prerequisite material.

Problem 4.1. (5 points) Consider a non-dividend-paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

Solution: The risk-neutral probability of an up movement is

$$p^* = \frac{95e^{0.06} - 75}{120 - 75} = 0.575.$$

So, the price of our straddle is

$$V(0) = e^{-0.06}[0.575 \times (120 - 100) + (1 - 0.575) * (100 - 75)] = 20.8366.$$

Problem 4.2. (10 points) Consider a one-period forward binomial model for the stock-price movement over the following year. The current stock price is $S(0) = 100$, its dividend yield is 0.05 and its volatility is 0.3 The continuously compounded risk-free interest rate is given to be 0.05.

Consider American call options on this stock with the expiration date at the end of the period/year.

For what values of the strike price K for which is there early exercise?

Solution: In our usual notation, $u = e^{0.3} = 1.35$ and $d = 0.74$. The risk-neutral probability is

$$p^* = \frac{1}{1 + e^{\sigma}} = 0.425.$$

Then, the continuation value at the root node is

$$V_C(0) = e^{-0.05}[0.425(135 - K)_+ + 0.575(74 - K)_+]$$

as a function of K. The early-exercise condition is

$$100 - K > V_C(0).$$

It is evident that in order for early exercise to occur it must be that $K < 100$. So, let us focus on the possible solutions to the above inequality in the interval $(74, 100)$ first. For such K, the above inequality becomes

$$100 - K > e^{-0.05} \times 0.425(135 - K).$$

Note the absence of the "positive part"in the last expression. The K which satisfy this inequality are such that

$$100 - 54.57 > 0.596K \quad \Rightarrow \quad 76.22483 > K.$$