Name:

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

3.1. **TRUE/FALSE QUESTIONS.**

Problem 3.1. (2 pts) Let N have a mixture distribution with the mixing variable Λ. More precisely, let

$$N \,|\, \Lambda = \lambda \sim Poisson(\lambda)$$

with $\Lambda \sim Gamma(\alpha, \theta)$. Then,

$$N \sim NegBin(\alpha, \theta).$$

True or false?

Problem 3.2. (2 points) With our usual assumptions in place, assume that the claim count follows the Poisson distribution with parameter $\lambda = 10$.
The severities of individual claims are such that on average half of the claims are expected to have claim amounts equal to 50 and the other half are expected to have the claim amounts equal to 100.
Then, the total losses have the expected value 750. *True or false?*

Problem 3.3. (2 points) In the compound aggregate loss model, with our usual notation and with the usual assumptions in place, we have that the claim count distribution is binomial with $m = 40$ and $q = 1/4$, and that the p.m.f. of the individual losses is

$$p_X(10) = p_X(20) = 0.3, p_X(50) = 0.4.$$

Then, $\mathbb{E}[S] \leq 290$. *True or false?*

3.2. **Free-response problems.** *Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.*

Problem 3.4. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 1.

Let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \ldots\}$ be given by the following p.m.f.

$$p_X(100) = 1/2, p_X(200) = 3/10, p_X(300) = 1/5.$$

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \ldots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Find the expected value of the **policyholder's** payment for a stop-loss insurance policy with an ordinary deductible of 200, i.e., calculate $\mathbb{E}[S \wedge 200]$.

Problem 3.5. (10 points) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 20.

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \ldots\}$ be the Gamma distribution with parameters $\alpha = 2.5$ and $\theta = 3,000$.

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \ldots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.

Using the normal approximation, find the amount of the premium π such that the total aggregate losses S exceed the premium with the probability of at most 1%.

Problem 3.6. (10 points) The frequency random variable N is assumed to have a Poisson distribution with a mean of 2. Individual claim severity random variable X has the following probability mass function

$$p_X(100) = 0.6, \quad p_X(200) = 0.3, \quad p_X(300) = 0.1.$$

Let the above be the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \ldots\}$, and Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \ldots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$. Calculate the probability that S is exactly equal to 3.

Problem 3.7. (6 points) In the compound model for aggregate claims, let the frequency random variable N have the probability (mass) function

$$p_N(0) = 0.4, p_N(1) = 0.3, p_N(2) = 0.2, p_N(4) = 0.1.$$

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \ldots\}$ be given by the probability (mass) function $p_X(1) = 0.3$ and $p_X(2) = 0.7$.
Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \ldots\}$.

Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Calculate $\mathbb{E}[(S-2)_+]$.

3.3. **MULTIPLE CHOICE QUESTIONS.**

Problem 3.8. (5 pts) Let us denote the claim count r.v. by N. We are given that N is a mixture random variable such that

$$N \,|\, \Lambda = \lambda \sim Poisson(\lambda)$$

while Λ is Gamma distributed with mean 2 and variance equal to 4. Then,

   (a) $F_N(1) \leq 0.1$
   (b) $0.1 < F_N(1) \leq 0.2$
   (c) $0.2 < F_N(1) \leq 0.3$
   (d) $0.3 < F_N(1) \leq 0.5$
   (e) None of the above

Problem 3.9. (5 points) Let the number of typos in the new edition of the *"Lord of the Rings"* trilogy be denoted by N and modeled using the Poisson distribution with mean 80. Some typos involve elves' names and others do not involve the elves' names. The number of typos and the type of the typo are assumed to be independent.

Assume that the probability that an observed typo involves an elf's name equals $1/5$.

Find the probability that the number of typos involving elves' names is 2, given that the **total** number of typos in the trilogy equals 5.

   (a) 0.0406
   (b) 0.0842
   (c) 0.1632
   (d) 0.2048
   (e) None of the above.

Problem 3.10. (8 points) Let N be an $\mathbb{N}_0-$valued random variable from the $(a, b, 0)$ class. You are given that

$$p_0 = 0.049787, \quad p_1 = 0.149361, \quad \text{and} \quad p_2 = 0.224042.$$

Find p_6.
   (a) 0.04
   (b) 0.05
   (c) 0.06
   (d) 0.07

(e) None of the above.

Problem 3.11. (5 points) Let S be the aggregate claims random variable. You are given the following:
(i) $\mathbb{E}[(S - 100)_+] = 15$,
(ii) $\mathbb{E}[(S - 120)_+] = 10$,
(iii) $\mathbb{P}[80 < S \leq 120] = 0$.
  Find $\mathbb{E}[(S - 105)_+]$.

    (a) $\mathbb{E}[(S - 105)_+] \approx 10.75$

    (b) $\mathbb{E}[(S - 105)_+] \approx 12.75$

    (c) $\mathbb{E}[(S - 105)_+] \approx 13.75$

    (d) $\mathbb{E}[(S - 105)_+] \approx 15.75$

    (e) None of the above