UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 2

Prerequisite material.

Provide your **complete solution** for the following problems.

Problem 2.1. (5 pts) Consider a continuous-dividend-paying stock currently priced at \$100 per share whose dividend yield is projected to equal 0.02. The price of this stock in one year is modeled using a one-period binomial tree. The down factor d is given to be 0.83, while the up factor u is unknown. (If you want to you can construct a cutesy narative about coffee being spilled on the model write-up or some such.)

The continuously compounded, risk-free interest rate is given to equal 0.04.

You observe that the price of a one-year, \$110-strike European call option on this stock consistent with the above model equals \$5.15. Find the price of the otherwise identical put option.

Solution: By put-call parity

$$V_P(0) = V_C(0) + Ke^{-rT} - S(0)e^{-\delta T} = 5.15 + 110e^{-0.04} - 100e^{-0.02} \approx 12.80.$$

Provide your **final answer only** for the following problems:

Problem 2.2. (5 pts) Let X be a normally distributed random variable with mean $\mu = 2$ and standard deviation equal to $\sigma = 1$. Find $\mathbb{E}[X^2]$.

(a) 2
(b) 3
(c) 4
(d) 5
(e) None of the above

Solution: (d)

$$\mathbb{E}[X^2] = Var[X] + \mathbb{E}[X]^2 = 5.$$

Problem 2.3. (5 points) Let X and Y be independent and normally distributed. Assume that X is normal with mean 0 and standard deviation 3 and that Y has mean 1 and standard deviation 5. Find $\mathbb{P}[X - Y > 0]$.

(a) 0.5793
(b) 0.5160
(c) 0.5080
(d) 0.5
(e) None of the above

Solution: (e)
The distribution of $X - Y$ is normal with mean -1. So,

$$\mathbb{P}[X - Y > 0] < \mathbb{P}[X - Y > -1] = 0.5.$$