M339D: April 25th, 2025.

Delta-Hedging.

Market Makers.

- immediacy
- inventory

$\Big\}$ ⇒ exposure to risk ⇒ hedge

Say, our agent writes an option whose value ftion is

$$v(s,t)$$

At time-0, they write the option ⇒ They get $v(S(0),0)$.

At time-t, the value of the agent's position

$$\boxed{-v(s,t)} \leftarrow$$

To (partially) hedge their exposure to risk, they construct a portfolio which has zero sensitivity to small perturbations in the stock price. Formally speaking, their goal is to create a

delta-neutral portfolio,

i.e., $\boxed{\Delta_{Port}(s,t)=0}$ — Theoretically possible but practically not

In particular, @ time-0, they want to trade so that

$$\boxed{\Delta_{Port}(S(0),0)=0}$$

The most straightforward strategy is to trade in the shares of the underlying asset.

At time-t, let $N(s,t)$ denote the required number of shares in the portfolio to maintain Δ-neutrality.

The total value of the portfolio

$$\frac{\partial}{\partial s}\Big| \quad v_{Port}(s,t) = -v(s,t) + N(s,t)\cdot s$$

$$\Delta_{Port}(s,t) = -\Delta(s,t) + N(s,t) = 0$$

$$\boxed{N(s,t)=\Delta(s,t)}$$

Example. An agent writes a call option @ time·0.

At time·t, the agent's unhedged position is:

$$-v(s,t)$$

$\Rightarrow$

$N(s,t) = \Delta_c(s,t)$ in the Δ·hedge.

$\Rightarrow$ In particular, @ time·0:

$N(S(0),0) = N(d_1(S(0),0)) > 0$, i.e.,

the agent $\boxed{\text{longs}}$ this much of a share.

$\Rightarrow$ the total position is

$$\begin{aligned} v_{Port}(S(0),0) &= -v_c(S(0),0) + \Delta_c(S(0),0)\cdot S(0) \\ &= -\left(S(0)\cdot \cancel{\Delta_c(S(0),0)} - Ke^{-rT}\cdot N(d_2(S(0),0))\right) \\ &\qquad + \cancel{\Delta_c(S(0),0)}\cdot S(0) \\ &= Ke^{-rT}N(d_2(S(0),0)) \end{aligned}$$

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i) The risk-free interest rate is constant.

(ii)

	Several months ago	Now
Stock price	\$40.00	\$50.00
Call option price	\$ 8.88	\$14.42
Put option price	\$ 1.63	\$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

Hint!
Put-call
Parity

Profit = Payoff − FV(Initial Cost)

Calculate her profit.

0 Options Written

t "several months" This is when we calculate the profit!

T exercise date

(A) $11

(B) $24

(C) $126

(D) $217

(E) $240

Profit (@ time-t) = $\boxed{\text{Wealth (@ time-t)}}$ $- \boxed{FV_{@t}}(\boxed{\text{Initial Cost}})$

48. DELETED

49. You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).

(i) The period is 3 months.

(ii) The initial stock price is $100.

(iii) The stock's volatility is 30%.

(iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

(A) 114

(B) 115

(C) 116

(D) 117

(E) 118

Initial Cost: $-100 \cdot v_c(S(0),0) + 100 \cdot \Delta_c(S(0),0) \cdot S(0) =$

$= 100(-8.88 + 0.794 \cdot 40) =$

$= 2{,}288$

Wealth @ time t: $-100 \cdot v_c(S(t),t) + 100 \cdot \Delta_c(S(0),0) \cdot S(t) =$

$= 100(-14.42 + 0.794 \cdot 50) =$

$= 2{,}528$

Profit @ time t: $2{,}528 - 2{,}288 \cdot e^{rt}$

Use put-call parity:

At time 0: $v_c(S(0),0) - v_p(S(0),0) = S(0) - Ke^{-rT}$

$8.88 - 1.63 = 40 - Ke^{-rT}$

$Ke^{-rT} = 32.75$ ✓

At time t: $v_c(S(t),t) - v_p(S(t),t) = S(t) - Ke^{-r(T-t)}$

$14.42 - 0.26 = 50 - Ke^{-r(T-t)}$

$Ke^{-r(T-t)} = 50 - 14.16 = 35.84$ ✓

$$\frac{W}{V} = \frac{\cancel{Ke^{-rT}} \cdot e^{rt}}{\cancel{Ke^{-rT}}} = e^{rt} = \frac{35.84}{32.75} = 1.09435$$

Profit @ time t $= 2{,}528 - 2{,}288 \cdot 1.09435 = 24.12$ □

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