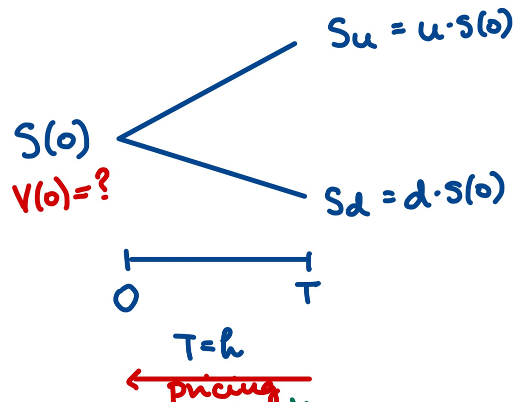
University of Texas at Austin

Binomial option pricing (review).

Problem 3.1. Let the continuously compounded risk-free interest rate be denoted by r. You are building a model for the price of a stock which pays dividends continuously with the dividend yield δ. Consider a binomial tree modeling the evolution of the stock price. Let the length of each period be h and let the up factor be denoted by u, and the down factor by d. What is the **no-arbitrage** condition for the binomial tree you are building?

$$\boxed{d < e^{(r-\delta)h} < u}$$

Problem 3.2. Set up the framework for pricing by replication in a one-period binomial tree! What is the *risk-neutral pricing* formula?



Payoff | Replicating Portfolio

$$V_u = v(S_u) = \Delta e^{\delta h}\cdot S_u + Be^{rh}$$

$$V_d = v(S_d) = \Delta e^{\delta h}\cdot S_d + Be^{rh}$$

$v(\cdot)$ ... payoff f'tion

Replicating Portfolio

- Δ ... # of shares of stock
- B ... risk-free investment

$$\boxed{\Delta = e^{-\delta\cdot h}\frac{V_u - V_d}{S_u - S_d}}$$

$$\boxed{B = e^{-rh}\frac{u V_d - d\cdot V_u}{u-d}}$$

Pricing by Replication: $\boxed{V(0) = \Delta\cdot S(0) + B}$

algebra: $\boxed{V(0) = e^{-rT}\left[p^*\cdot V_u + (1-p^*)V_d\right]}$

$$\boxed{p^* = \frac{e^{(r-\delta)h} - d}{u-d}}$$

... the risk-neutral probability

We generalize: $\boxed{V(0) = e^{-rT}\mathbb{E}^*[V(T)]}$

$\delta = 0$

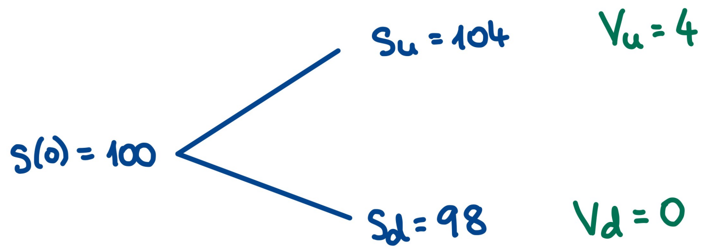
Problem 3.3. The current price of a certain non-dividend-paying stock is \$100 per share. You are modeling the price of this stock at the end of a quarter year using a one-period binomial tree under the assumption that the stock price can either increase by $\boxed{4\%}$, or decrease by $\boxed{2\%}$.

The continuously compounded risk-free interest rate is 3%. $h = 0.25$

What is the price of a three-month, at-the-money European call option on the above stock consistent with the above binomial tree?

$\longrightarrow$:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{0.03(0.25)} - 0.98}{1.04 - 0.98} = 0.4588$$



$$v_c(s) = (s-K)_+ \underset{\uparrow}{=} (s-100)_+$$

@ · money

$$V_c(0) = e^{-0.0075} \cdot 4 \cdot p^* = 1.8215$$

Problem 3.4. Let the continuously compounded risk-free interest rate be equal to 0.04.

The current price of a continuous-dividend-paying stock is \$80 and its dividend yield is 0.02. The stock's volatility is 0.25. You model the evolution of the stock price over the following half year using a two-period **forward** binomial tree.

$h = 0.25$

What is the price of a six-month, \$82-strike European put option on the above stock consistent with the given binomial tree?

$$u = e^{(r-\delta)h + \sigma\sqrt{h}}$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}}$$

$$\boxed{p^* = \frac{1}{1+e^{\sigma\sqrt{h}}}}$$

Do this first; store it!

In this problem:

$$p^* = \frac{1}{1+e^{0.25\sqrt{0.25}}} = \frac{1}{1+e^{0.125}} = 0.4688$$

$$u = e^{(0.04-0.02)(0.25)+0.125} = e^{0.005+0.125} = e^{0.13}$$

$$d = e^{(0.04-0.02)(0.25)-0.125} = e^{0.005-0.125} = e^{-0.12}$$

Final possible stock prices:

$$S_{uu} = S(0)u^2 = 80\cdot e^{0.26} > 82 = K \Rightarrow V_{uu} = 0$$

$$S_{ud} = S(0)\cdot u\cdot d = 80e^{0.01}$$

$$S_{ud} = S(0)\cdot u \cdot d = S(0)e^{(r-\delta)(2h)} = \text{Forward Price}$$

... finish the problem set, please.