M339J: March 26th, 2021.

By def'n for a two-pt mixture:

$$X \sim \begin{cases} X_1 & \text{w/ probab. } a_1 \\ X_2 & \text{w/ probab. } a_2 \end{cases}$$

$$F_X(x) = a_1 \cdot F_{X_1}(x) + a_2 \cdot F_{X_2}(x)$$

If your X_1 and X_2 are continuous, then we differentiate and get

$$\boxed{f_X(x) = a_1 \cdot f_{X_1}(x) + a_2 \cdot f_{X_2}(x)}$$

Q: What about the moments?

$\longrightarrow$: By def'n of expectation:

$$\mathbb{E}[X^k] = \int x^k \cdot f_X(x)\, dx$$

$$= \int x^k (a_1 \cdot f_{X_1}(x) + a_2 \cdot f_{X_2}(x))\, dx$$

$$= \int (a_1 \cdot x^k \cdot f_{X_1}(x) + a_2 \cdot x^k \cdot f_{X_2}(x))\, dx$$

$$= a_1 \underbrace{\int x^k \cdot f_{X_1}(x)\, dx}_{\mathbb{E}[X_1^k]} + a_2 \underbrace{\int x^k f_{X_2}(x)\, dx}_{\mathbb{E}[X_2^k]}$$

$$\Rightarrow \boxed{\mathbb{E}[X^k] = a_1 \cdot \mathbb{E}[X_1^k] + a_2 \cdot \mathbb{E}[X_2^k]}$$

$$\boxed{Var[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2}$$

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Problem. A jewelry store purchases two separate insurance policies that together provide full coverage.

You are given:

(i) The expected ground-up loss is 11,100.

(ii) Policy A has an ordinary deductible of 5,000 and no policy limit.

(iii) Under policy A, the expected amt paid per loss is 6,500.

(iv) Under policy A, the expected amt paid per payment is 10,000.

(v) Policy B has no deductible and has a policy limit of 5,000.

Q: Given that a loss has occured, find the probability that the payment under Policy B equals 5,000.

→: X... r.v. denoting the ground-up loss

Assume that X is continuous:

$$\mathbb{P}[X \geq 5{,}000] = S_X(5000) = ? \qquad d = 5000$$

$$6{,}500 = \mathbb{E}[Y_A^L] = \mathbb{E}[(X-d)_+] = \mathbb{E}[(X-d)\cdot\mathbb{I}_{[X>d]}]$$

$$10{,}000 = \mathbb{E}[Y_A^P] = \mathbb{E}[X-d \mid X>d] = \frac{\mathbb{E}[(X-d)\cdot\mathbb{I}_{[X>d]}]}{S_X(d)}$$

$$\Rightarrow S_X(5000) = \frac{6500}{10000} = 0.65 \quad ■$$

Q: Given that a loss less than or equal to 5,000 has occurred, what is the expected pmt under policy B?

$$\rightarrow: \quad \mathbb{E}[X \mid X \leq 5000] = \text{by def'n of conditional expectation}$$

$$= \frac{\mathbb{E}[X\cdot\mathbb{I}_{[X\leq 5000]}]}{\mathbb{P}[X \leq 5000]}$$

The policy limit, i.e., the maximum amount payable is $\alpha(u-d)$.

The per-payment random variable is:

$$Y^P = \begin{cases} \text{undefined} & \text{if } (1+r)X < d \\ Y^L & \text{otherwise} \end{cases}$$

Thm.

$$\mathbb{E}[Y^L] = \alpha(1+r)\left(\mathbb{E}\left[X \wedge \frac{u}{1+r}\right] - \mathbb{E}\left[X \wedge \frac{d}{1+r}\right]\right)$$

$$\mathbb{E}[Y^P] = \mathbb{E}\left[Y^L \mid (1+r)X > d\right] = \frac{\mathbb{E}[Y^L]}{S_X\left(\frac{d}{1+r}\right)}$$