(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

$T = 1/4$

8. You are considering the purchase of a 3-month 41.5-strike ~~American~~ European call option on a nondividend-paying stock.

$\boxed{K = 41.5}$

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40. $S(0) = 40$

(iii) The stock's volatility is 30%. $\sigma = 0.30$

(iv) The current call option delta is 0.5.

$$\Delta_c(S(0),0) = \boxed{0.5 = N(d_1(S(0),0))}$$

Determine the current price of the option. $\boxed{v_c(S(0),0) = ?}$

(A) $20 - 20.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(B) $20 - 16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(C) $20 - 40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(D) $16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

(E) ~~40.453~~ $\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

$$\boxed{d_1(S(0),0) = 0}$$

$$d_2(S(0),0) = d_1(S(0),0) - \sigma\sqrt{T}$$

$$d_2(S(0),0) = 0 - 0.3\cdot\sqrt{0.25}$$

$$\boxed{d_2(S(0),0) = -0.15}$$

$N(d_2(S(0),0)) = \dots =$ the multiple choices are in integral form

$$v_c(S(0),0) = S(0)\cdot\boxed{N(d_1(S(0),0))} - Ke^{0T}\cdot\boxed{N(d_2(S(0),0))}$$

$$d_1(S(0),0) = \frac{1}{\sigma\sqrt{T}}\boxed{\Big[\underbrace{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T}_{=0}\Big]} = 0$$

$$\left(r+\frac{0.09}{2}\right)\cdot\frac{1}{4} = -\ln\left(\frac{40}{41.5}\right) = \ln\left(\frac{41.5}{40}\right)$$

$$r = 4\cdot\ln\left(\frac{41.5}{40}\right) - 0.045 = \underline{0.10}$$

☆ $v_c(S(0),0) = 40\cdot(0.5) - 41.5e^{-0.10(1/4)}\cdot N(-0.15)$

$$v_c(S(0),0) = 20 - 40.453\underbrace{N(-0.15)}_{(1-N(0.15))}$$

symmetry of $N(0,1)$

$$v_c(S(0),0) = 40.453\boxed{N(0.15)} - \underline{20.453}$$

$$v_c(S(0),0) = \boxed{40.453\cdot\frac{1}{\sqrt{2\pi}}}\int_{-\infty}^{0.15} e^{-\frac{x^2}{2}}dx - 20.453$$

$\square$

Delta Hedging.

Market Makers.

- immediacy
- inventory

$\Big\}$ $\Rightarrow$ exposure to risk $\Rightarrow$ hedge

Say, a market maker writes an option whose value f'tion is

$$v(s,t)$$

At time 0, they write the option $\Rightarrow$ They get $\boxed{v(S(0),0)}$

At time t, the value of the market maker's position is

$$\boxed{-v(s,t)}$$

To (partially) hedge this exposure to risk, they construct a portfolio which has zero sensitivity to small perturbations in the stock price. Formally speaking, their goal is to create a

delta-neutral portfolio,

Theoretically possible

i.e., a portfolio for which $\boxed{\Delta_{Port}(s,t)=0}$ Practically, not

In particular, @ time 0, we want to trade so that

$$\boxed{\Delta_{Port}(S(0), 0) = 0}$$

The most straightforward strategy is to trade in the shares of the underlying asset.

At time t, let $N(s,t)$ denote the required number of shares in the portfolio necessary to maintain Δ-neutrality.

The total value of the portfolio:

$$v_{Port}(s,t) = -v(s,t) + N(s,t)\cdot s$$

$$\frac{\partial}{\partial s}\Big|$$

$$\Delta_{Port}(s,t) = \boxed{-\Delta(s,t) + N(s,t) = 0}$$

Δ-netrality

$$\boxed{N(s,t) = \Delta(s,t)}$$

Example. An agent $\boxed{\text{writes}}$ a call option @ time 0.

At time t, the agent's unhedged position is:

$$-v_C(s,t)$$

=> They must maintain $N(s,t) = \Delta_C(s,t)$ in the Δ-hedge.

=> In particular, @ time 0.

$$N(S(0),0) = N(d_1(S(0),0)) > 0,$$

i.e., the agent must long this much of a share.

=> Their total position will be:

$$v_{Port}(S(0),0) = -v_C(S(0),0) + \Delta_C(S(0),0)\cdot S(0)$$

Example. An agent $\boxed{\text{writes}}$ a put option @ time 0.

At time t, the agent's unhedged position is:

$$-v_P(s,t)$$

$\Rightarrow$ They must maintain $N(s,t) = \Delta_p(s,t) = -\underline{N(-d_1(s,t))}$

in the Δ-hedge.

$\Rightarrow$ They agent must short a portion of a share.

At time 0, their total position will be:

$$v_{Port}(S(0),0) = -v_p(S(0),0) + \Delta_p(S(0),0) \cdot S(0)$$