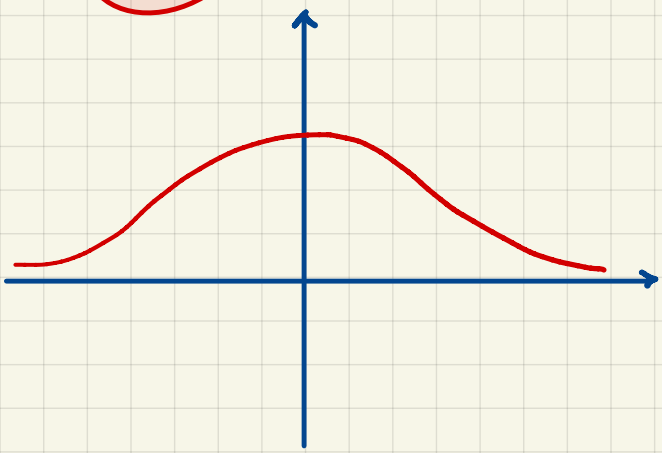
M339D: March 29th, 2023.

Standard Normal Distribution.

We say that a random variable Z has the **standard normal distribution** if its pdf has the following form:

$$f_Z(z) = \varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad \text{for all } z \in \mathbb{R}$$



- symmetric about the vertical axis, i.e., $\varphi(z) = \varphi(-z)$ i.e. **EVEN**
- mean of $Z = 0$
- median of $Z = 0$

The cdf of the standard normal is:

$$N(z) = \Phi(z) = \mathbb{P}[Z \leq z] = \int_{-\infty}^{z} f_Z(u)\,du$$

$$= \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}\,du$$

No analytic form!

There are the standard normal tables!

We can use the built-in commands in R.

We write $Z \sim N(0,1)$