Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Solution: Practice Problems for In-Term One
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. This exam is graded out of 100 points.
Time: 50 minutes

1.1. TRUE/FALSE QUESTIONS.

Problem 1.1. (2 pts) A (long) put is a short position with respect to the underlying asset price.

Solution: TRUE

Problem 1.2. It is possible for the buyer and the writer of the same option to end up having the same profit on the exercise date.

Solution: TRUE
This happens if they both break even, i.e., if both of their profits equal zero.

Problem 1.3. (2 points) Consider a one-year, \$45-strike European call option and a one-year, \$45-strike European put option on the same underlying asset. You observe that the time−0 stock price equals \$40 while the time−1 stock price equals \$50. Then, both of the options are out-of-the-money at expiration. *True or false?*

Solution: FALSE

Problem 1.4. (2 points) An agent is **only** allowed to long a forward contract if he/she is willing to take physical delivery of the underlying asset.

Solution: FALSE
It is possible to have *cash settlement* on the delivery date if the forward contract stipulates so.

Problem 1.5. (2 points) Denote the continuously compounded, risk-free interest rate by r and denote the equivalent annual effective interest rate by i. Then, $\ln(1+i) = r$. *True or false?*

Solution: TRUE

Problem 1.6. (2 pts) Two dice are rolled, the single most probable sum of the numbers of the upturned faces is 7. *True or false?*

Solution: TRUE

Problem 1.7. (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date T on the underlying asset S:

- one long call with strike 40,

- two long calls with strike 50,
- one short call with strike 65.

Let $S(T) = 69$. Then, the payoff from the above position at time T is less than 60.

Solution: FALSE
The payoff is

$$(69 - 40) + 2(69 - 50) - (69 - 65) = 63.$$

1.2. **MULTIPLE CHOICE QUESTIONS.**

Problem 1.8. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions given by

$$f(x) = 2x - 10$$

and

$$g(x) = \begin{cases} \min(x, 7) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then, $g(f(7))$ equals ...

(a) -4
(b) 0
(c) 4
(d) 7
(e) None of the above

Solution: (c)

Problem 1.9. *Source: Sample P exam, Problem #176.*
In a group of health insurance policyholders, 20% have high blood pressure and 30% have high cholesterol. Of the policyholders with high blood pressure, 25% have high cholesterol. A policyholder is randomly selected from the group. Calculate the probability that a policyholder has high blood pressure, **given** that the policyholder has high cholesterol.

(a) 1/6
(b) 1/5
(c) 1/4
(d) 2/3
(e) 5/6

Solution: (a)
Let E be the event containing all the policyholders with a high blood preasure and let F be the event which contains all the policyholders with high cholesterol. We are given the following

$$\mathbb{P}[E] = 0.2, \quad \mathbb{P}[F] = 0.3, \quad \mathbb{P}[F|E] = 0.25.$$

Then, the conditional probability we are looking for equals

$$\mathbb{P}[E|F] = \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[F]} = \frac{\mathbb{P}[F|E]\mathbb{P}[E]}{\mathbb{P}[F]} = \frac{(0.25)(0.2)}{0.3} = \frac{1}{6}.$$

Problem 1.10. Harry plays a simple lottery in which the winnings are distributed as follows:

- \$5 with probability 0.2,
- \$10 with probability 0.4,
- \$20 with probability 0.4.

It turns out that Harry has to pay a fee to collect his winnings. If the actual amount he wins is smaller than \$9, then the fee is defined to equal the amount that Harry won – thus, he walks away with nothing. If the actual amount he wins is between \$9 and \$15, he does not have to pay anything in fees and gets a bonus of \$4. If the actual amount he wins is larger than \$15, then he pays the \$15-fee and pockets the remainder. What is the expected value of the net amount Harry collects?

(a) 3

(b) 6.4

(c) 7.6

(d) 15

(e) None of the above.

Solution: (c)
The actual amount that Harry gets is

- \$0 with probability 0.2,
- \$14 with probability 0.4,
- \$5 with probability 0.4.

So, his expected winnings are

$$14(0.4) + 5(0.4) = 7.6$$

Problem 1.11. Hermione sells short one share of a non-dividend-paying stock. The stock is currently valued at \$80 per share. The continuously compounded risk-free interest rate is 0.04. Hermione intends to close the short sale in one year. What is the final stock price for which Hermione will break even?

Solution: In our usual notation, the break-even point is

$$S(0)e^{rT} = 80e^{0.04} = 83.26486.$$

Problem 1.12. The current market price of widgets is \$4 per widget. The widget factory plans to sell their next batch of 100 widgets in half a year. The total aggregate costs of production of widgets will be equal to \$350.

The factory enters 100 short forward contracts on widgets for delivery in half a year. The forward price is \$4.20 per widget.

What is the factory's profit if the final price of widgets in half a year ends up being \$4.40?

(a) 30

(b) 50

(c) 70

(d) 90

(e) None of the above.

Solution: (c)
The factory will sell the widgets per the forward contract for \$420 total. The total aggregate costs are given to be \$350. Hence, the profit is \$70.

Problem 1.13. Maryam bakes batches of cupcakes for a cupcake convention. She buys forward 21 pounds of raspberries from a local farmer at the forward price of \$5.60 per pound.

She projects to bake 336 cupcakes and sell each for \$3. The total and aggregate non-raspberry costs of baking the cupcakes are \$200.

If the market price of raspberries on the day of the cupcake convention is \$5.40, what is Maryam's profit?

(a) \$690.40
(b) \$694.60
(c) \$890.40
(d) \$894.60
(e) None of the above.

Solution: (a)

$$336 \times 3 - 21 \times 5.60 - 200 = 690.40.$$

Problem 1.14. The **writer** of a call option has ...

(a) an obligation to sell the underlying asset at the strike price.

(b) a right, but **not** an obligation, to sell the underlying asset at the strike price.

(c) an obligation to buy the underlying asset at the strike price.

(d) a right, but **not** an obligation, to buy the underlying asset at the strike price.

(e) None of the above.

Solution: (a)

Problem 1.15. (5 points) Assume the **Capital Asset Pricing Model** holds.

You are given the following information about stock X, stock Y, and the market:

- The required return and volatility for the market portfolio are 0.10 and 0.25, respectively.
- The required return and volatility for the stock X are 0.08 and 0.4, respectively.
- The correlation between the returns of stock X and the market is -0.2.
- The volatility of stock Y is 0.25.
- The correlation between the returns of stock Y and the market is 0.4.

Calculate the required return for stock Y.

(a) About 0.075.
(b) About 0.08.
(c) About 0.085.
(d) About 0.09.
(e) None of the above.

Solution: (d)
The βs of stocks X and Y are

$$\beta_X = \frac{0.4(-0.2)}{0.25} = -0.32,$$
$$\beta_Y = \frac{0.4(0.25)}{0.25} = 0.4.$$

So, the required return of stock X must satisfy

$$0.08 = r_X = r_f + (-0.32)(0.10 - r_f) \quad \Rightarrow \quad 0.08 = r_f - 0.032 + 0.32 r_f$$
$$\Rightarrow \quad 1.32 r_f = 0.112 \quad \Rightarrow \quad r_f = 0.0848.$$

Finally, the required return of stock Y equals

$$r_Y = 0.0848 + 0.4(0.10 - 0.0848) = 0.09088.$$

Problem 1.16. (5 points) For a certain stock, you are given that its expected return equals 0.0944 and that its β equals 1.24. For another stock, you are given that its expected return equals 0.068 and that its β equals 0.8. Both stocks lie on the **Security Market Line (SML)**. What is the risk-free interest rate r_f?

(a) About 0.02
(b) About 0.025
(c) About 0.03
(d) About 0.035
(e) None of the above.

Solution: (a)
Denote the expected return of the market portfolio by r_m. Then,

$$0.0944 = r_f + 1.24(r_m - r_f),$$
$$0.068 = r_f + 0.8(r_m - r_f).$$

Subtracting the second equation from the first one, we get

$$0.0264 = 0.44(r_m - r_f) \quad \Rightarrow \quad r_m - r_f = \frac{0.0264}{0.44} = 0.06.$$

Substituting the obtained risk premium of the market portfolio into the first equation above, we obtain

$$r_f = 0.0944 - 1.24(0.06) = 0.02.$$

Problem 1.17. (5 points) In a market, the risk-free interest rate is given to be 0.04.

Consider an investment I in this market, whose Sharpe ratio is 0.42. You construct an equally weighted portfolio consisting of the investment I and the risk-free asset. The expected return of this portfolio is 0.10.

You decide to rebalance your portfolio so that one quarter of your wealth gets invested in the investment I and the remainder is invested in the risk-free asset. What is the volatility of this new portfolio?

(a) 0.0625
(b) 0.0714
(c) 0.1225

(d) 0.1625
(e) None of the above.

Solution: (b)
Let's denote the volatility of investment I by σ_I and the volatility of the new portfolio by $\sigma_{P'}$. Then, $\sigma_{P'} = 0.25\sigma_I$.

The expected return of the old portfolio is

$$\mathbb{E}[R_P] = \frac{1}{2}\mathbb{E}[R_I] + \frac{1}{2}r_f\,.$$

So,

$$\mathbb{E}[R_I] = 2\mathbb{E}[R_P] - r_f = 2(0.10) - 0.04 = 0.16.$$

We are given the Sharpe ratio of the investment I, and so we can calculate

$$\sigma_I = \frac{0.16 - 0.04}{0.42} = 0.2857143.$$

Finally, the new portfolio's volatility is

$$\sigma_{P'} = 0.25(0.2857143) = 0.07142857.$$

Problem 1.18. (5 points) According to your model, the economy over the next year could be *good* or *bad*. You are a pessimist and believe that the economy is twice as likely to be *bad* than *good*.

Consider two assets, X and Y, existing in this market. If the economy is *good* the return on asset X is 0.12, and the return on asset Y is 0.11. If the economy is *bad* the return on asset X is -0.03 and the return on asset Y is -0.01.

You construct a portfolio P using assets X and Y so that the portfolio's expected return equals 0.025.

Calculate the volatility of this portfolio's return.

(a) 0.0458

(b) 0.0512

(c) 0.0584

(d) 0.0637

(e) None of the above.

Solution: (d)
First, we need to figure out the weight w_X the asset X is given in portfolio P. The expected returns of assets X and Y are

$$\mathbb{E}[R_X] = \frac{1}{3}(0.12) + \frac{2}{3}(-0.03) = \frac{0.12 - 0.06}{3} = 0.02,$$
$$\mathbb{E}[R_Y] = \frac{1}{3}(0.11) + \frac{2}{3}(-0.01) = \frac{0.11 - 0.02}{3} = 0.03.$$

So, the portfolio P must be equally weighted, i.e., $w_X = 0.5$. Hence, the distribution of the portfolio's return can be described as

$$R_P \sim \begin{cases} 0.115, & \text{with probability } 1/3 \\ -0.02, & \text{with probability } 2/3 \end{cases}$$

The second moment of the portfolio's return is

$$\mathbb{E}[R_P^2] = (0.115) \times \frac{1}{3} + (-0.02)^2 \times \frac{2}{3} = 0.004675.$$

The variance of the portfolio's return is

$$Var[R_P] = 0.004675 - 0.025^2 = 0.00405.$$

Finally, its volatility equals $\sigma = \sqrt{0.00405} = 0.0636396$.

Problem 1.19. (5 points) Consider two assets X and Y such that:

- their expected returns are $\mathbb{E}[R_X] = 0.10$ and $\mathbb{E}[R_Y] = 0.08$;
- their volatilities are $\sigma_X = 0.25$ and $\sigma_Y = 0.35$;
- the correlation coefficient of their returns is $\rho_{X,Y} = -1$.

You are tasked with constructing a portfolio consisting of shares of X and Y with a risk-free return. What should the weight w_Y given to asset Y be?

(a) 5/12

(b) 1/2

(c) 7/12

(d) Such a weight does not exist.

(e) None of the above.

Solution: (a)

$$w_Y = \frac{\sigma_X}{\sigma_X + \sigma_Y} = \frac{0.25}{0.25 + 0.35} = \frac{5}{12}\,.$$

Problem 1.20. (5 points) For stock S_1, you are given that its expected return equals 0.08 and its β is 1.22. For stock S_2, you are given that its expected return equals 0.05 and its β is 0.56. Both of these stocks lie on the *Security Market Line*. For stock S_3, you are given that its expected return equals 0.07 and its β is 0.7. What is the α of stock S_3?

(a) 0
(b) 0.0137
(c) 0.0245
(d) 0.0455
(e) None of the above.

Solution: (b)
Since both S_1 and S_2 are on the **SML**, we know that

$$\begin{aligned} 0.08 &= r_f + 1.22(r_m - r_f), \\ 0.05 &= r_f + 0.56(r_m - r_f), \end{aligned}$$

where r_f stands for the risk-free interest rate and r_m stands for the expected return of the market. Subtracting the second equation from the first one, we get

$$0.03 = 0.66(r_m - r_f) \quad \Rightarrow \quad r_m - r_f = \frac{0.03}{0.66} = 0.0455.$$

The risk-free interest rate r_f can, then, be calculated as

$$r_f = 0.08 - 1.22(0.0455) = 0.02449.$$

Hence, the α of stock S_3 is

$$0.07 - 0.02449 - 0.7(0.0455) = 0.01366.$$

Problem 1.21. A market index is currently trading at $1,000. Which of the following options is/are **in the money**? More than one answer can be true. You get the credit if you circled **all** acceptable answers and **none** of the incorrect ones.

(a) $1,500-strike put
(b) $900-strike put
(c) $1,250 strike call
(d) $950 strike call
(e) None of the above.

Solution: (a) and (d)

Problem 1.22. Let the current price of a non-dividend-paying stock equal 100. The forward price for delivery of this stock in 3 months equals \$101.26

Consider a \$90-strike, six-month put option on this stock whose premium today equals \$2.22.

What will the profit of this long put option be if the stock price at expiration equals \$96?

(a) About \$2.28 loss.
(b) About \$2.22 loss.
(c) About \$2.28 gain.
(d) About \$2.22 gain.
(e) None of the above.

Solution: (a)
The option is out-of-the money at expiration, so its owner suffers a loss of the future value of its premium

$$2.22 \times \left(\frac{101.26}{100}\right)^2 = 2.2763.$$

Problem 1.23. (5 points) You are tasked with buying oranges in the market in grove A, transporting the oranges to a juice factory in the market B, and selling the oranges to the juice factory in the market B. You want to hedge. Which of the following would be a satisfactory hedge?

(a) Long a call in market A and long a put in market B
(b) Short a call in market A and long a put in market B
(c) Long a call in market A and short a put in market B
(d) Short a call in market A and short a put in market B
(e) None of the above.

Solution: (a)