M339D: December 1st, 2023.

(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

8. You are considering the purchase of a 3-month 41.5-strike ~~American~~ European call option on a nondividend-paying stock.

$T=1/4$ $K=41.5$

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40. $S(0)=40$

(iii) The stock's volatility is 30%. $\sigma=0.3$

(iv) The current call option delta is 0.5. $\Delta_C(S(0),0)=\boxed{0.5=N(d_1(S(0),0))}$

$\boxed{d_1(S(0),0)=0}$

Determine the current price of the option. $\boxed{v_C(S(0),0)=?}$

(A) $20 - 20.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(B) $20 - 16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(C) $20 - 40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx$

(D) $16.138\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

(E) $40.453\int_{-\infty}^{0.15} e^{-x^2/2}dx - 20.453$

$$d_2(S(0),0) = d_1(S(0),0) - \sigma\sqrt{T}$$
$$= 0 - 0.3\sqrt{0.25}$$
$$\boxed{d_2(S(0),0) = -0.15}$$

$$v_C(S(0),0) = S(0)\boxed{\cdot N(d_1(S(0),0))} - Ke^{-rT}\cdot N(d_2(S(0),0))$$
$$= \underbrace{40\cdot(0.5)}_{20} - Ke^{-0T}\cdot N(-0.15)$$

$$d_1(S(0),0) = \frac{1}{\sigma\sqrt{T}}\boxed{\left[\ln\left(\frac{S(0)}{K}\right) + \left(r+\frac{\sigma^2}{2}\right)\cdot T\right]} = 0$$

$$\left(r+\frac{\sigma^2}{2}\right)\cdot T = -\ln\left(\frac{S(0)}{K}\right)$$

$$r = -\frac{1}{T}\ln\left(\frac{S(0)}{K}\right) - \frac{\sigma^2}{2} = \underline{0.1032}$$

$$v_c(S(0),0) = 20 - \underbrace{41.5\, e^{-0.1032(0.25)}}_{\overset{\shortparallel}{40.453}} \cdot \underbrace{N(-0.15)}_{1-N(0.15)}$$

$$v_c(S(0),0) = 20 - 40.453\left(1-N(0.15)\right)$$

$$= 40.453\, \underbrace{N(0.15)}_{\int_{-\infty}^{0.15} f_Z(z)\,dz} - 20.453$$

$$\shortparallel$$

$$\int_{-\infty}^{0.15} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\,dz \qquad \square$$

Delta Hedging.

Market Maker.

- immediacy
- inventory

$\Big\}$ ⇒ exposure to risk ⇒ hedge

Say, a market maker writes an option whose value f'tion is

$v(s,t)$

At time 0, they write the option ⇒ They get $v(S(0),0)$

At time t, the value of the market maker's position

$$\boxed{-v(s,t)}$$

To (partially) hedge this exposure to risk, they construct a portfolio which has zero sensitivity to small perturbations in the stock price. Formally speaking, their goal is to create a

delta-neutral portfolio,

i.e., a portfolio for which $\boxed{\Delta_{Port}(s,t)=0}$ Theoretically possible

Practically not

In particular, @ time-0, they want to trade so that

$$\boxed{\Delta_{Port}(S(0),0)=0}$$

The most straightforward strategy is to trade in the shares of the underlying asset.

At time-t, let $N(s,t)$ denote the required number of shares in the portfolio to maintain Δ-neutrality.

The total value of the portfolio:

$$v_{Port}(s,t) = -v(s,t) + N(s,t)\cdot s$$

$\frac{\partial}{\partial s}\Big|$

$$\Delta_{Port}(s,t) = -\Delta(s,t) + N(s,t) = 0$$

$$\boxed{N(s,t) = \Delta(s,t)}$$

Example. An agent writes a call option @ time-0.

At time-t, the agent's unhedged position is:

$$-v_C(s,t)$$

$\Rightarrow$

$N(s,t) = \Delta_C(s,t)$ in the Δ-hedge.

$\Rightarrow$ In particular, @ time-0:

$N(S(0),0) = N(d_1(S(0),0)) > 0$, i.e.,

the agent must long this much of a share.

$\Rightarrow$ The total position will be:

$$v_{Port}(S(0),0) = -v_C(S(0),0) + \Delta_C(S(0),0)\cdot S(0)$$