# Project #2: More on portfolios. Futures prices. Call prices.

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## Problem #1 (25 points)

Your initial wealth is exactly \$100. You are allowed to invest in shares of a particular stock. You are also allowed to both lend and borrow at the continuously compounded risk-free interest rate of 0.05. Keeping your money uninvested is **not allowed**.

You rebalance your portfolio every morning, once you have observed the opening stock price. This means that you can change the number of shares you own (if you decide to do so) and accordingly adapt your risk-free investment.

You proceed to create a "rule'' according to which you will be rebalancing your portfolio. Here is a rule you will implement in this problem:

*If the stock price drops overnight (regardless of the extent of the drop), sell half of the stock investment. If the stock price rises overnight (regardless of the extent of the increase), double the amount of the stock investment. If the stock price does not change, do nothing. In all three cases, the rest of my wealth is to be invested at the risk-free rate (if needed, I will borrow at the continuously compounded risk-free interest rate).*

Over the following 10 days, you observe the following stock prices for a non-dividend-paying stock:

| **Day** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Stock price** | 100 | 80 | 64 | 80 | 64 | 80 | 100 | 80 | 64 | 80 | 100 |

As the time passes you follow the investment rule above to rebalance your portfolio. Complete the following table describing your portfolio **just before and just after** the rebalancing is done. More precisely, for the 10 days, both for **before** and **after** the rebalancing print out:

- the number of shares of stock in the portfolio,
- the balance of the risk-free investment,
- the wealth in the stock,
- the total wealth.

## Problem #2 (25 points)

### (10 points)

Repeat the above problem with **your own rule** based on *technical analysis*, i.e., the study of past prices. Your rule must not be arbitrary and it must have some financial logic behind it. Moreover, it must not be a simple reversal or multiple of the rule used in the previous problem.

### (15 points)

**Explain** why you designed the rule the way you did and why you believe it would prove to be a successful investment strategy. You must provide at least one credible reference indicating that the rule is used in practice.

## Problem #3 (25 points)

### The Dow index and Dow futures

Please, read the Wikipedia entry about *futures contracts* and listen to the fun and educational episode of *Planet money* about credit risk in cafeteria futures. Both are linked below:

Wikipedia: Futures

Planet money: Delicious cake futures

**(10 points)** Provide the definitions of the forward contract and the futures contract. Then, highlight at least two similarities and at least two differences between the two.

The files "dow.csv" and "dow-futures.csv" contain index prices and the futures prices for delivery on a particular date in the future.

**(5 points)** Import the data into a data frame (or two?). Draw the time-plot of the evolution of the closing index prices and the futures prices on the same coordinate system. You do **not** need to put the calendar days on the horizontal axis, but you **do** need to label your axes and give your time-plot a title indicating the dates. Make sure that you plot the two trajectories in **different** colors indicating in the text which color corresponds to which company.

Here is a link to a straightforward tutorial which will help you accomplish the above neatly:

Timeline plotting

**(10 points)** What do you notice when you compare the timeplots? Which of the **formulae** for calculating futures prices you encountered on the Wikipedia entry come in handy?

## Problem #4 (25 points)

### Apple calls

The file `apple-calls.csv` contains call prices observed on February 14th for European call options on the stocks of Apple Inc with expiry on May 19th and with varying strike prices.

**(5 points)** Import the data into a data frame. Create a line plot of the mid price (defined as the midpoint between the bid and the ask price) as it depends on the strike.

**(5 points)** What do you notice about the monotonicity and convexity of the above curve?

**(15 points) Prove** that the same conclusions regarding the monotonicity and convexity of the call prices with respect to the strike must hold in general **under the no arbitrage assumption**.