University of Texas at Austin, Department of Mathematics
M358K - Applied Statistics

In-term exam II

Definitions.

Problem 1.1. (10 points) Provide the definition of the *probability density function* of a **continuous** random variable.

Solution: The *probability density function* of a continuous random variable X is defined as

$$f_X(x) = F_X'(x) \quad \text{for all } x \text{ where the derivative exists}$$

where F_X stands for the cumulative distribution function.

Problem 1.2. (10 points) Provide the expression for the *probability density function* of a **standard normal** random variable.

Solution:

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad x \in \mathbb{R}.$$

True/False Questions.

Problem 1.3. (2 points) The margin of error for a confidence interval for the population mean μ, based on a fixed specified sample size n, increases as the confidence level decreases. *True or false?*

Solution: FALSE

Problem 1.4. (3 points) The Midsomer Worthy Middle School has calculated a 95% confidence interval for the population mean height μ of 11−year-old boys at their school. They found it to be 57 ± 2 inches.

This means that there is a 95% probability that the population mean μ is between 55 and 59. *True or false?*

Solution: FALSE

Free-response problems.

Problem 1.5. (15 points) An Airbus a330 can hold 277 passengers. The *Marginaire* airline knows from past experience that only about 90% of the passengers make it to their flights. So, they sell 300 tickets for every Airbus a330 flight.

Assume that passengers travel independently, i.e., the events that individual passengers make it to a flight are independent events. What is the approximate probability that a particular flight gets overbooked? *Note: Don't forget to use the continuity correction.*

Solution: The number of passengers X who do show up for a particular flight is binomial with 300 being the number of trials and 0.90 being the probability of success in every single trial. By the normal approximation to the binomial distribution, we have

$$\mathbb{P}[X > 277] = 1 - \mathbb{P}[X \leq 277.5] \approx 1 - \Phi\left(\frac{277.5 - 300 * 0.9}{\sqrt{300(0.9)(0.1)}}\right) = 1 - \Phi(1.44) = 1 - 0.9251 = 0.0749.$$

Problem 1.6. (15 points) Walter Hingel, the enterprising preschooler, is planning for Halloween. Based on industry standards, the strap on his gigantic plastic pumpkin will snap once the pumpkin is loaded with more than 3lbs of candy. The weight of each piece of candy is assumed to have the mean of 1oz and the standard deviation of 0.1oz. Walter will go to 49 houses politely collecting one piece of candy at each. What is the approximate probability that Walter's pumpkin strap stays intact?

Solution: Let the weight of each piece of candy be denoted by X_i for $i = 1, \dots, 49$. The total weight in Walter's pumpkin at the end of the evening will be $S = X_1 + X_2 + \cdots + X_{49}$. By the Central Limit Theorem, the distribution of S is approximately

$$S \approx Normal(mean = 1(49) = 49, sd = 0.1\sqrt{49} = 0.7).$$

We need to find the probability that S is below $3(16) = 48$. We can use **R** to get

$$pnorm(48, 49, 0.1 * sqrt(49)) = 0.07656373.$$

If I were Walter, I would carry a pillow case.

Problem 1.7. (10 points) A particular type of wool for clothes manufacturing has to have a specific tensile strength in order to be used in weaving machines without breaking. We model its tensile strength as normally distributed with standard deviation 0.4 MPa. How is the variance of the sample mean changed when the sample size increases from 64 to 196?

Solution: The variance of the sample mean $\bar{X}_{64}$ for the sample of size 64 is

$$Var[\bar{X}_{64}] = \frac{(0.4)^2}{64} = 0.0025.$$

The variance of the sample mean $\bar{X}_{196}$ for the sample of size 196 is

$$Var[\bar{X}_{196}] = \frac{(0.4)^2}{196} = 0.0008.$$