Name:

M339J/M389J: Probability models for actuarial applications
Spring 2022
University of Texas at Austin
In-Term Exam II
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Signature:

The maximum number of points on this exam is 100.

Problem 2.1. (5 points) Consider a severity random variable X which is modelled as a two-parameter Pareto with $\alpha = 2$ and an unknown value of the parameter θ. You are given that

$$\mathbb{E}[X - 10|X > 10] = \frac{3}{2}\,\mathbb{E}[X - 5|X > 5].$$

Find $\mathbb{E}[X - 15|X > 15]$.

Problem 2.2. (5 points) Let the severity random variable X be modelled as exponential with mean 1000. There is an insurance policy on this loss with the deductible of 400. What is the expected value of the per-loss random variable under this policy?

Problem 2.3. (5 points) You will replace your hair dryer at failure or in two years, whichever occurs first. The dryer's age at failure is a random variable T which is modelled as uniform over the interval $[0, 4]$. What is the expected time at which the dryer is replaced?

Problem 2.4. (5 points) Let T denote the time in minutes for a customer service representative to respond to a telephone inquiry. T is uniformly distributed on the interval with endpoints 8 minutes and 12 minutes. Let R denote the average rate, in customers per minute, at which the representative responds to inquiries. Which one of the following is a density function for R on the interval $(1/12, 1/8)$?

Problem 2.5. (5 points) Let X be a two-point mixture. More precisely, let X be

- a two-parameter Pareto with mean equal to 10 and variance equal to 200 with probability 3/4;
- gamma distributed with parameters $\alpha = 4$ and $\theta = 10$ with probability 1/4.

What is the variance of the random variable X?

Problem 2.6. (10 points) Assume that X has a mixing distribution such that

$$X \mid \Lambda \sim Exponential(mean = \Lambda)$$

with $\Lambda \sim U(100, 200)$. Find the (unconditional) coefficient of variation of X.

Problem 2.7. (5 points) The distribution of the random variable X is a spliced distribution with a continuous probability density function f_X. It is assumed that:

- the pdf is constant on $[0, 100]$, and
- the pdf is proportional to the pdf of an exponential distribution with mean 50 on $(100, \infty)$.

What is the probability that the random variable X is less than 50?

Problem 2.8. (10 points) Let X be the ground-up loss random variable. Assume that X has the two-parameter Pareto distribution with mean 100 and variance $15,000$.

Let B denote the expected payment **per loss** on behalf of an insurer which wrote a policy with an ordinary deductible of 500 and with no policy limit. How much is B?

Problem 2.9. (5 points) Let the ground-up loss X be modeled by a two-parameter Pareto distribution with parameters $\alpha = 2$ and $\theta = 400$. For an insurance policy on the above loss, there is a franchise deductible of 200. Find the expected value of the **per loss** random variable.

Problem 2.10. (5 points) Losses in a particular year follow a two-parameter Pareto distribution with parameters $\alpha = 4$ and $\theta = 11$.An insurance covers each loss subject to a deductible $d = 20$.

Calculate the **loss elimination ratio** for that year.

Problem 2.11. (5 points) Let the ground-up loss X be exponentially distributed with mean \$400. An insurance policy has an ordinary deductible of \$200 and the maximum amount payable per loss of \$2200. Find the expected value of the amount paid (by the insurance company) **per positive payment**.

Problem 2.12. (10 points) Let the loss amounts X have the distribution function given by

$$F_X(x) = \begin{cases} 0, & x < 0 \\ x/100, & 0 \leq x \leq 100 \\ 1, & 100 < x. \end{cases}$$

An insurance policy has the following properties:
(i) there is an ordinary deductible of 20,
(ii) the maximum payment per loss that the insurer pays equals 60,
(iii) the insurance pays 80% of the amount of the loss in excess of the deductible and subject to the above maximum payment.

Let Y^P denote the per-payment random variable, i.e., the amount paid by the insurer given that a payment was made. Find $\mathbb{E}[Y^P]$.

Problem 2.13. (5 points) Let N represent the number of customers arriving during the morning hours and let N' represent the number of customers arriving during the afternoon hours at a diner. You are given:

(i) N and N' are Poisson distributed.
(ii) The first moment of N is less than the first moment of N' by 8.
(iii) The second raw moment of N is 60% of the second raw moment of N'.

Calculate the variance of N'.

Problem 2.14. (5 points) The number of cars one sees passing by the local playground in an afternoon is modeled using a Poisson distribution with mean 25. The proportion of black cars in the stream is 1/5. The color of the cars is independent of the number of cars that drive by. What is the probability that exactly 5 black cars and exactly 10 non-black cars drive by in a particular afternoon?

Problem 2.15. (5 points) The Clampetts conduct a geological study seeking sites for oil wells. Past date indicate that each exploratory oil well should have a 20% chance of striking oil independent of all other wells. The Clampetts are modest folk and they will stop the study when they strike oil for the third time. What is the probability that they drill exactly ten exploratory oil wells total before stopping the study?

Problem 2.16. (5 points) Let the independent random variables X_1, X_2 and X_3 all have the following probability mass function:

$$p_{X_1}(0) = 1/4, \quad p_{X_1}(1) = 1/2, \quad p_{X_1}(2) = 1/4.$$

Let $X = X_1 X_2 X_3$. What is the probability generating function of X?

Problem 2.17. (5 points) Let the random variable X have a Weibull distribution with parameters $\tau = 2$ and $\theta = 10$. What is the 75^{th} percentile of this distribution?