Name:

UTeid:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Mock Exam One
Instructor: Milica Čudina

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The maximum number of points on this exam is 60.

Problem 1.1. (10 points) Write the definition of an **arbitrage portfolio**.

Solution: Check your notes.

Problem 1.2. (5 points) From a manufacturer's perspective, why would they decide to use derivative securities on their product to hedge? Respond in five lines or less.

Solution: Answers may vary, but the bottom line is that the manufacturer can prevent losses only by a limited amount using operations optimisation and other tools within his/her area of expertise. Their profit still depends heavily on market-price fluctuations – well outside of their area of influence and/or expertise. So, derivative securities are a welcome tool to hedge that risk.

Problem 1.3. (5 pts) Consider a portfolio consisting of the following four European options with the same expiration date T on the underlying asset S:
long one call with strike 40,
long two calls with strike 50,
short one call with strike 65.

Let $S(T) = 52$. What is the payoff from the above position at time T?

Solution: The payoff is

$$(52 - 40)_+ + 2(52 - 50)_+ - (52 - 65)_+ = 12 + 2(2) + 0 = 16.$$

Problem 1.4. (5 points) The initial price of the market index is \$900. After 3 months the market index is priced at \$960. The **effective** monthly rate of interest is 1.0%.

The premium on the long put, with a strike price of \$975, is \$10.00. What is the profit at expiration for this long put?

Solution: The profit is

$$\begin{aligned}(K - S(T))_+ - FV_{0,T}[V_P(0)] &= (K - S(T))_+ - FV_{0,T}[V_P(0)] \\ &= (975 - 960)_+ - 10(1 + 0.01)^3 \\ &= 4.70.\end{aligned}$$

Problem 1.5. (15 points) The continuously compounded risk-free interest rate equals 0.08.

Jonathan sells short one share of a non-dividend-paying stock and simultaneously buys a six-month, \$85-strike call option on the same stock. The current stock price is \$88, while the call price equals \$8. What is the break-even price of Jonathan's position?

Solution: Jonathan's initial cost is $8 - 88 = -80$. The expression for his payoff, in terms of the final asset price s is

$$-s + (s - K)_+ = -\min(s, K).$$

Hence, the break-even price s^* satisfies

$$-\min(s^*, K) = FV_{0,1/2}(-80) = -80e^{0.04} \quad \Rightarrow \quad s^* = 83.26486.$$

Problem 1.6. (20 points) The future value in one year of the total costs of manufacturing a widget is \$200. You will sell a widget in one year at its market price of $S(1)$.

Assume that the annual effective interest rate equals 5%, and that the current price of the widget equals \$230.

You now purchase a one-year, \$220-strike put on one widget for a premium of \$7. You sell some of the potential gain by writing a one-year, \$250-strike call on one widget for a \$2 premium.

What is the **range** of the profit of your hedged porfolio?

Solution: If you want to write the total payoff as a piecewise function, this is what you get:

$$v(s) = \begin{cases} K_P, & \text{for } 0 \leq s \leq K_P \\ s, & \text{for } K_P \leq s \leq K_C \\ K_C, & \text{for } K_C \leq s \end{cases}$$

where K_P denotes the strike price for the put while K_C denotes the call's strike price. So, the range of the payoff function is $[220, 250]$.

The future value of the total cost of both production and hedging is

$$200 + (7 - 2)(1 + 0.05) = 205.25.$$

So, the range of the profit equals $[14.75, 44.75]$.