M339D : February 12^{th}, 2024.

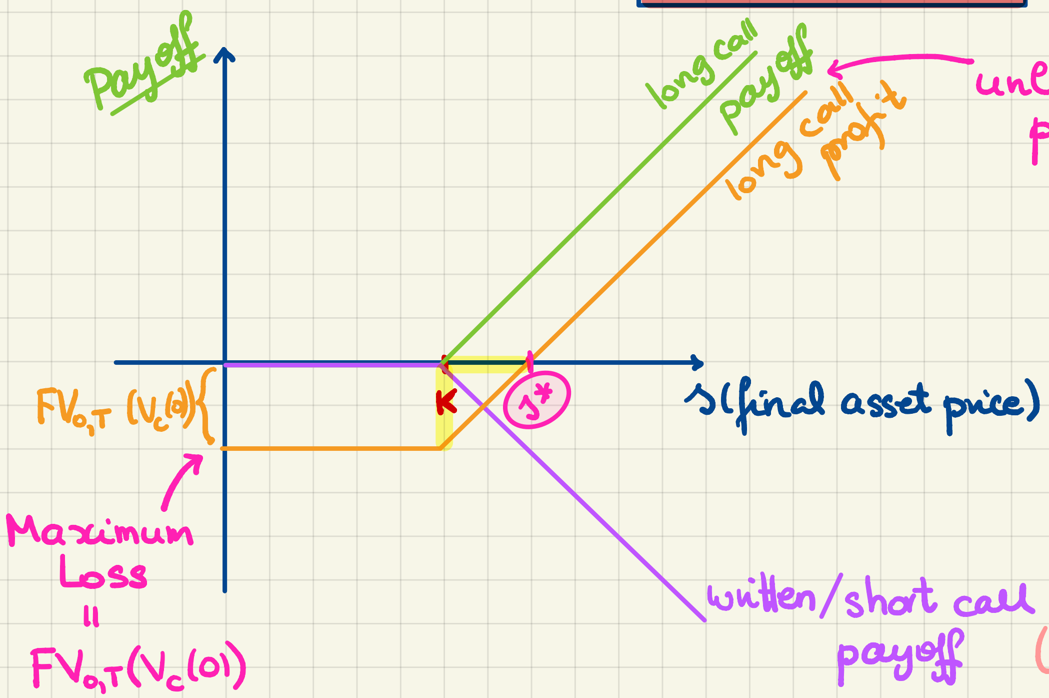
Introduce:

The positive part function

$$x \longmapsto (x)_+ =: \max(x,0) = x \vee 0$$

$\Rightarrow$ $\boxed{V_c(T) = (S(T)-K)_+}$

$\Rightarrow$ the payoff f'tion: $\boxed{v_c(s) = (s-K)_+}$



A long call is a long position w.r.t. the underlying.

(never positive and sometimes strictly negative $\Rightarrow$ premium $V_c(0)$)

s^*... break even point

$$\boxed{s^* = FV_{0,T}(V_c(0)) + K}$$

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set #5

European call options.

Problem 5.1. The initial price of a non-dividend-paying asset is \$100. A $\boxed{\text{six-month}}$ \$95-strike European call option is available at a \$8 premium. The continuously compounded risk-free interest rate equals $\boxed{0.04}$. What is the break-even point for this call option?

(a) 86.84

(b) 87

(c) 103

(d) 103.16

(e) None of the above.

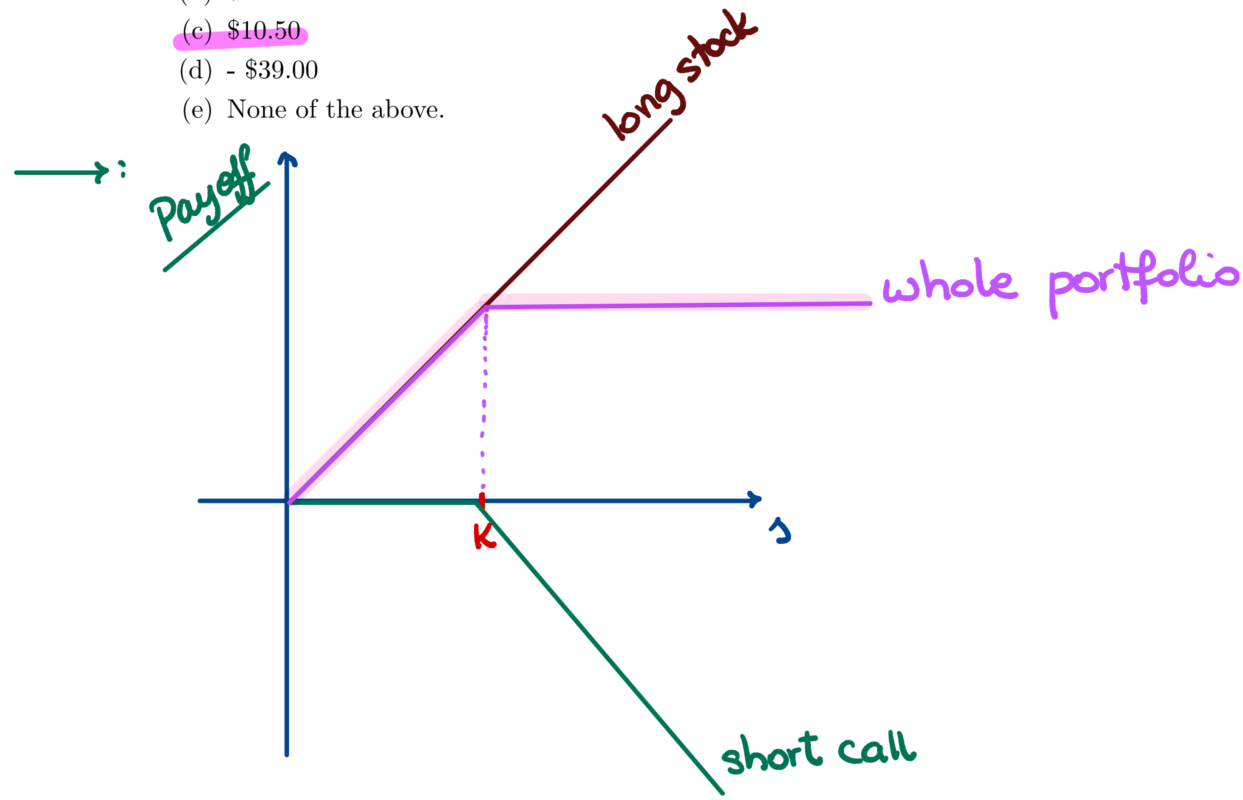
$\longrightarrow$: $s^* = FV_{0,T}(V_C(0)) + K = 8 \cdot e^{0.04(0.5)} + 95 = \underline{103.16}$

$\square$

Problem 5.2. (5 points) A stock's price today is $1000 and the annual effective interest rate is given to be 5%. You write a one-year, $1,050-strike call option for a premium of $10 while you simultaneously buy the stock. What is your **profit** if the stock's spot price in one year equals $1,200?

(a) $150.00
(b) $139.90
(c) $10.50
(d) - $39.00
(e) None of the above.

→ :



Algebraicly :

$$\text{Payoff} = \boxed{S(T) - (S(T)-K)_+} = \begin{cases} K & \text{if } S(T) \geq K \\ S(T) & \text{if } S(T) < K \end{cases}$$

Covered Call

$$= \min(S(T), K)$$

In this problem,

$$\text{Payoff} = \min(1200, 1050) = 1050$$

$$\text{Initial Cost} = 1000 - 10 = 990$$

$$\text{Profit} = 1050 - 990(1.05) = \underline{10.50}$$

Problem 5.3. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for $\boxed{\$1,000}$.

The jeweler models the market price of gold in one year as follows:

Gold price in one year	Probability
750 per ounce	0.2
850 per ounce	0.5
950 per ounce	0.3

$S(T)$

min

$\to 750$

$\to 850$

$\to 900$

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of \$900 per ounce. The option costs \$100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelry produced.

Algebraically:

$$\text{Payoff (Total)} = \text{Payoff (Gold)} + \text{Payoff (Call)}$$

$$= -S(T) \quad + (S(T)-K)_+$$

$$= \begin{cases} -K & \text{if } S(T) \geq K \\ -S(T) & \text{if } S(T) < K \end{cases}$$

$$= \boxed{-\min(S(T), K)}$$

$$\text{Profit} = \underline{1000 - \min(S(T), K)} - 100e^{0.05}$$

$$\mathbb{E}[\text{Profit}] = 1000 - \boxed{\mathbb{E}[\min(S(T), K)]} - 100e^{0.05} = \underline{49.873}$$

$$= 750 \cdot \frac{1}{5} + 850 \cdot \frac{1}{2} + 900 \cdot \frac{3}{10}$$

$$= 150 + 425 + 270$$

$$= 845$$

Problem 5.4. The current price of stock a certain type of stock is $80. The premium for a 6−month $\boxed{\text{at-the-money}}$ call option is $\boxed{\$5.84}$. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) $80
(b) $85.72
(c) $85.84
(d) $85.96
(e) None of the above.

$S(0)=K$

$S^* = K + FV_{0,T}(V_C(0))$

$S^* = 80 + 5.84 \cdot e^{0.04(0.5)}$

□

Problem 5.5. The price of gold in half a year is modeled to be equally likely to equal any of the following prices

$$\$1000, \quad \$1100, \quad \text{and} \quad \$1240.$$

Consider a half-year, \$1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

: Payoff : $(S(T)-K)_+$ 0 50 190

w/ probab. 1/3 1/3 1/3

answer: $\frac{1}{3}\cdot 50 + \frac{1}{3}\cdot 190 = 80$ $\square$

Problem 5.6. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

(a) 395.92

(b) 397.98

(c) 400

(d) 897.98

(e) None of the above.

→ : $200(10) - 200(5) - 500 - 200 \cdot 0.50 \cdot e^{0.04(0.5)} = \underline{397.98}$