M339W: October 3rd, 2022.

Option Elasticity.

Def'n. For any portfolio w/ the value f'tion $v(s,t)$, we define its portfolio elasticity as:

$$\boxed{\Omega(s,t) := \frac{\Delta(s,t)\cdot s}{v(s,t)}}$$

For a single option, the same quantity is called option elasticity.

Example. European Call.

Its B.S Price:

$$v_c(s,t) = \underbrace{\boxed{s e^{-\delta(T-t)} N(d_1(s,t))}}_{\Delta_c(s,t)} - Ke^{-r(T-t)} \cdot N(d_2(s,t))$$

$$\Rightarrow \quad \Omega_c(s,t) = \frac{s \cdot \Delta_c(s,t)}{s \cdot \Delta_c(s,t) - Ke^{-r(T-t)} N(d_2(s,t))} \quad \geq 1$$

Example. European Put.

Its B.S Price:

$$v_p(s,t) = Ke^{-r(T-t)} N(-d_2(s,t)) - s e^{-\delta(T-t)} \cdot N(-d_1(s,t))$$

$$\Delta_p(s,t) = -e^{-\delta(T-t)} N(-d_1(s,t))$$

$$\Rightarrow \quad \Omega_p(s,t) = \frac{s \cdot \Delta_p(s,t)}{Ke^{-r(T-t)} N(-d_2(s,t)) + \Delta_p(s,t)\cdot s} \quad < 0$$

Use of option elasticity:

σ_S ... stock volatility (in the B-S model: deterministic, constant)

We get the option volatility as:

$$\boxed{\sigma_{opt}(s,t) = \sigma_S \cdot \left|\Omega_{opt}(s,t)\right|}$$

not a constant

e.g., for a European call

$$\sigma_c(s,t) = \sigma_S \cdot \underbrace{\left|\Omega_c(s,t)\right|}_{\geq 1} \geq \sigma_S$$

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

$\Omega_P(S(0),0) = ?$

$$\Omega_P(S(0),0) = \frac{\boxed{S(0)} \cdot \boxed{\Delta_P(S(0),0)}}{V_P(S(0),0)}$$

by def'n

?

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

21-24. DELETED

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of \$100.

The chooser option price is \$20 at time $t = 0$.

The stock price is \$95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of \$100.

You are given:

(i) The risk-free interest rate is 0.

(ii) $C(1) = \$4$.

Determine $C(3)$.

(A) \$ 9

(B) \$11

(C) \$13

(D) \$15

(E) \$17

$\longrightarrow$:

A: $v_A(s,t) = \underline{2\cdot v_c(s,t) + v_p(s,t)}$

$\frac{\partial}{\partial s}\Big|$

$\underline{\Delta_A(s,t) = 2\cdot\Delta_c(s,t) + \Delta_p(s,t)}$

<u>At time·0:</u>

$$\Delta_A(S(0),0) = 2\cdot\Delta_c(S(0),0) + \Delta_p(S(0),0)$$

$$\cancel{5} = \frac{(2\cdot\Delta_c(S(0),0) + \Delta_p(S(0),0))\cdot \cancel{45}\,9}{2\cdot(4.45)+1.90}$$

$$2\cdot\Delta_c(S(0),0) + \boxed{\Delta_p(S(0),0)} = \frac{10.80}{9} = 1.20 \qquad (A)$$

B: $v_B(s,t) = 2\cdot v_c(s,t) - 3v_p(s,t)$

$\frac{\partial}{\partial s}\Big|$

$\Delta_B(s,t) = 2\cdot\Delta_c(s,t) - 3\Delta_p(s,t)$

<u>At time·0:</u>

$$\underline{2\cdot\Delta_c(S(0),0) - 3\Delta_p(S(0),0) = 3.4} \qquad (B)$$

(A) − (B): $4\cdot\Delta_p(S(0),0) = 1.2 - 3.4 = -2.2$

$$\underline{\Delta_p(S(0),0) = -0.55}$$

$$\Omega_p(S(0),0) = \frac{\Delta_p(S(0),0)\cdot S(0)}{v_p(S(0),0)} = \frac{-0.55(45)}{1.90} = \underline{-13.027}$$

$\square$

41. Assume the Black-Scholes framework. Consider a 1-year European contingent claim on a stock.

You are given:

(i) The time-0 stock price is 45. $S(0)=45$

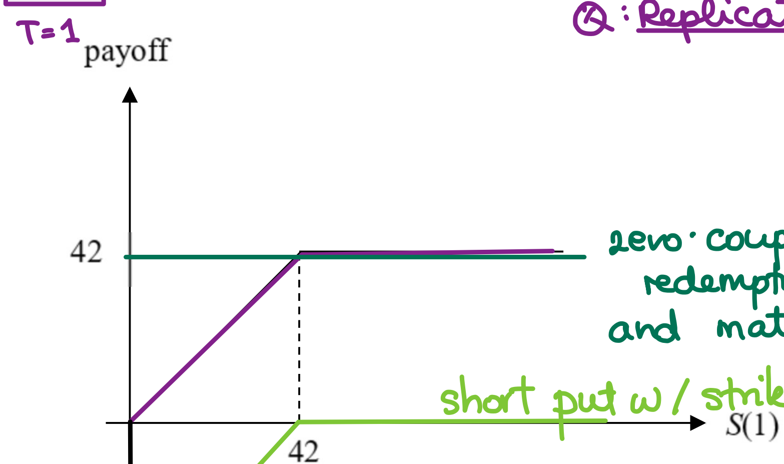
(ii) The stock's volatility is 25%. $\sigma=0.25$

(iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%. $\delta=0.03$

(iv) The continuously compounded risk-free interest rate is 7%. $r=0.07$

(v) The $\boxed{\text{time-1}}$ payoff of the contingent claim is as follows:

Q: Replicating Portfolio.



zero-coupon bond w/ redemption amount of 42 and maturity @ T=1

short put w/ strike K=42 and exercise date T=1

Calculate the time-0 contingent-claim elasticity.

(A) 0.24

(B) 0.29

(C) 0.34

(D) 0.39

(E) 0.44