M339J: April 13th, 2022.

11. A company provides insurance to a concert hall for losses due to power failure. You are given:

(i) The number of power failures in a year has a Poisson distribution with mean 1. $N \sim Poisson(\lambda = 1)$

(ii) The distribution of ground up losses due to a single power failure is:

x	Probability of x
10	0.3
20	0.3
50	0.4

pmf of X: $p_X(\cdot)$

(iii) The number of power failures and the amounts of losses are independent.

(iv) There is an annual deductible of 30. $\boxed{d=30}$

Calculate the expected amount of claims paid by the insurer in one year.

(A) 5

(B) 8

(C) 10

(D) 12

(E) 14

$\rightarrow$: $S = X_1 + X_2 + \cdots + X_N$

$$\mathbb{E}[(S-d)_+] = \cancel{X} = \boxed{\mathbb{E}[S]} - \boxed{\mathbb{E}[S \wedge d]}$$

- $\mathbb{E}[S] = \mathbb{E}[N] \cdot \boxed{\mathbb{E}[X]} = 1 \cdot 29 = 29$

w/ $\mathbb{E}[X] = 0.3(10) + 0.3(20) + 0.4(50)$
$= 3 + 6 + 20 = 29$

- $\mathbb{E}[S \wedge 30] = ?$

Q: What is the support of S?
$\{0, 10, 20, 30, \ldots\}$

Q: What is the support of $S \wedge 30$?
$\{0, 10, 20, 30\}$

- $p_{S\wedge 30}(0) = p_S(0) = p_N(0) = e^{-1}$
- $p_{S\wedge 30}(10) = p_S(10) = \mathbb{P}[N=1,\ X_1=10] \overset{\text{independence}}{=} p_N(1)\cdot p_X(10) = \underline{e^{-1}\cdot 0.3}$
- $p_{S\wedge 30}(20) = p_S(20) = \mathbb{P}[N=1,\ X_1=20] + \mathbb{P}[N=2,\ X_1=X_2=10]$

$$\overset{\text{independence}}{=} e^{-1}\cdot 0.3 + e^{-1}\cdot\frac{1}{2}\cdot(0.3)^2$$

$$= e^{-1}\left(0.3 + 0.5(0.09)\right) = \underline{e^{-1}\cdot 0.345}$$

- $p_{S\wedge 30}(30) \;\cancel{=?}\; 1 - p_{S\wedge 30}(0) - p_{S\wedge 30}(10) - p_{S\wedge 30}(20)$

$$= 1 - e^{-1} - 0.3e^{-1} - 0.345e^{-1} = \underline{1 - 1.645e^{-1}}$$

$$\mathbb{E}[S\wedge 30] = 10\left(e^{-1}\cdot 0.3\right) + 20\cdot\left(e^{-1}\cdot 0.345\right) + 30\left(1 - e^{-1}\cdot 1.645\right) = \underline{15.487}$$

final answer: $\mathbb{E}[(S-30)_+] = 29 - 15.487 = 13.513$ $\square$

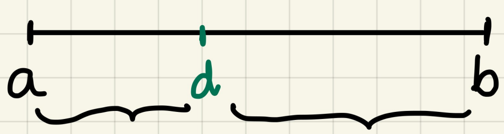
Interpolation Theorem.

Assume that S represents aggregate losses such that for some $a<b$ we have

$$\boxed{\mathbb{P}[a < S < b] = 0}$$

Then: If $a \le d \le b$:

$$\mathbb{E}[(S-d)_+] = \frac{b-d}{b-a}\,\mathbb{E}[(S-a)_+] + \frac{d-a}{b-a}\,\mathbb{E}[(S-b)_+]$$



91. The number of auto vandalism claims reported per month at Sunny Daze Insurance Company (SDIC) has mean 110 and variance 750. Individual losses have mean 1101 and standard deviation 70. The number of claims and the amounts of individual losses are independent.

Using the normal approximation, calculate the probability that SDIC's aggregate auto vandalism losses reported for a month will be less than 100,000.

(A) 0.24

(B) 0.31

(C) 0.36

(D) 0.39

(E) 0.49

92. Prescription drug losses, S, are modeled assuming the number of claims has a geometric distribution with mean 4, and the amount of each prescription is 40.

$\beta = 4$

Calculate $E[(S-100)_+]$.

(A) 60

(B) 82

(C) 92

(D) 114

(E) 146

S... aggregate losses

$S = X_1 + X_2 + \cdots + X_N = 40 \cdot N$

$N \sim g(\text{mean} = 4)$

$\mathbb{E}[(S-100)_+] = ?$

$d = 100$

The support of S is $\{0, 40, 80, 120, \ldots\}$

We use the interpolation thm:

$a = 80 < d = 100 < b = 120$

$\mathbb{E}[(S-100)_+] = \frac{1}{2}\left(\boxed{\mathbb{E}[(S-80)_+]} + \mathbb{E}[(S-120)_+]\right)$ ✓

✓ $$\mathbb{E}[(S-80)_+] = \mathbb{E}[(40N-80)_+] = \underline{40}\,\boxed{\mathbb{E}[(N-2)_+]} = 102.40$$

$$\mathbb{E}[(N-2)_+] = \mathbb{E}[N] - \mathbb{E}[N\wedge 2] = 4 - \boxed{\mathbb{E}[N\wedge 2]} = 2.56$$

$$\mathbb{E}[N\wedge 2] = S_N(0) + S_N(1) = \frac{4}{5} + \left(\frac{4}{5}\right)^2 = \frac{20+16}{25} = \frac{36}{25}$$

$$\mathbb{E}[(S-120)_+] = \mathbb{E}[(40N-120)_+] = 40\,\mathbb{E}[(N-3)_+] = 81.92$$

$$\mathbb{E}[(N-3)_+] = \mathbb{E}[N] - \mathbb{E}[N\wedge 3] = 4 - 1.952 = 2.048$$

$$\mathbb{E}[N\wedge 3] = S_N(0) + S_N(1) + S_N(2)$$
$$= \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3$$

$$\mathbb{E}[(S-100)_+] = 0.5\,(102.40 + 81.92) = 92.16. \quad \square$$