A Quick Review of Hypothesis Testing

Hypothesis tests allow us to answer simple “yes-or-no” questions, such as:

- Is the true coefficient β_j in a linear regression equal to zero?
- Does the expected blood pressure among mice in the treatment group equal the expected blood pressure among mice in the control group?

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2. Construct the test statistic
3. Compute the p-value
4. Decide whether to reject the null hypothesis

1. Define the Null and Alternative Hypotheses

- We divide the world into *null* and *alternative* hypotheses.
- The null hypothesis, H_0, is the default state of belief about the world. For instance:
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- The null hypothesis, H_0, is the default state of belief about the world. For instance:
    1. The true coefficient β_j equals zero.
    2. There is no difference in the expected blood pressures.
- The alternative hypothesis, H_a, represents something different and unexpected. For instance:
    1. The true coefficient β_j is non-zero.
    2. There is a difference in the expected blood pressures.

2. Construct the Test Statistic

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- The test statistic summarizes the extent to which our data are consistent with H_0.
- Let $\hat{\mu}_t$ / $\hat{\mu}_c$ respectively denote the average blood pressure for the n_t / n_c mice in the treatment and control groups.
- To test $H_0 : \mu_t = \mu_c$, we use a two-sample t-statistic

$$T = \frac{\hat{\mu}_t - \hat{\mu}_c}{s\sqrt{\frac{1}{n_t} + \frac{1}{n_c}}}$$

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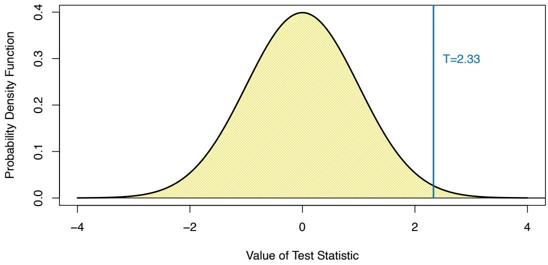
- Under H_0, $T \sim N(0, 1)$ for a two-sample t-statistic.



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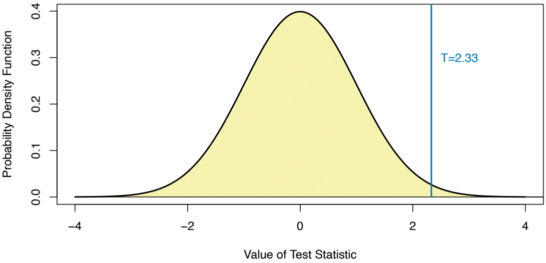

- The p-value is 0.02 because, if H_0 is true, we would only see $|T|$ this large 2% of the time.

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- *But how small is small enough*? To answer this, we need to understand the *Type I error*.

4. Decide Whether to Reject H_0, Part 2

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- We want to ensure a small Type I error rate.
- If we only reject H_0 when the p-value is less than α, then the Type I error rate will be at most α.
- So, *we reject H_0 when the p-value falls below some α*: often we choose α to equal 0.05 or 0.01 or 0.001.