UNIVERSITY OF TEXAS AT AUSTIN

Quiz #2

The cumulative distribution function. Types of random variables.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 2.1. (5 points) Write down the definition of the *cumulative distribution function* of a random variable.

Solution: Denote the random variable by X. Then, its *cumulative distribution function* $F_X : \mathbb{R} \to [0,1]$ is defined by

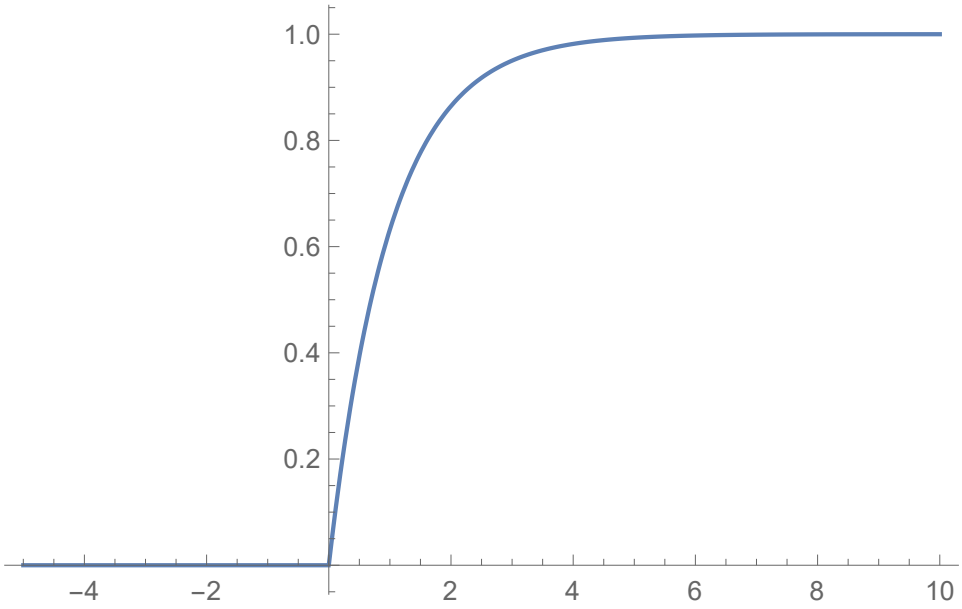
$$F_X(x) = \mathbb{P}[X \leq x] \quad \text{for every } x \in \mathbb{R}. \tag{2.1}$$

Problem 2.2. (2 points) The cumulative distribution function is always strictly increasing. *True or false?*

Solution: FALSE
The cdfs of discrete random variables are step functions. So, they are not strictly increasing.

Problem 2.3. (2 points) Please, sketch an example of a graph of the cumulative distribution function of a *continuous random variable*.

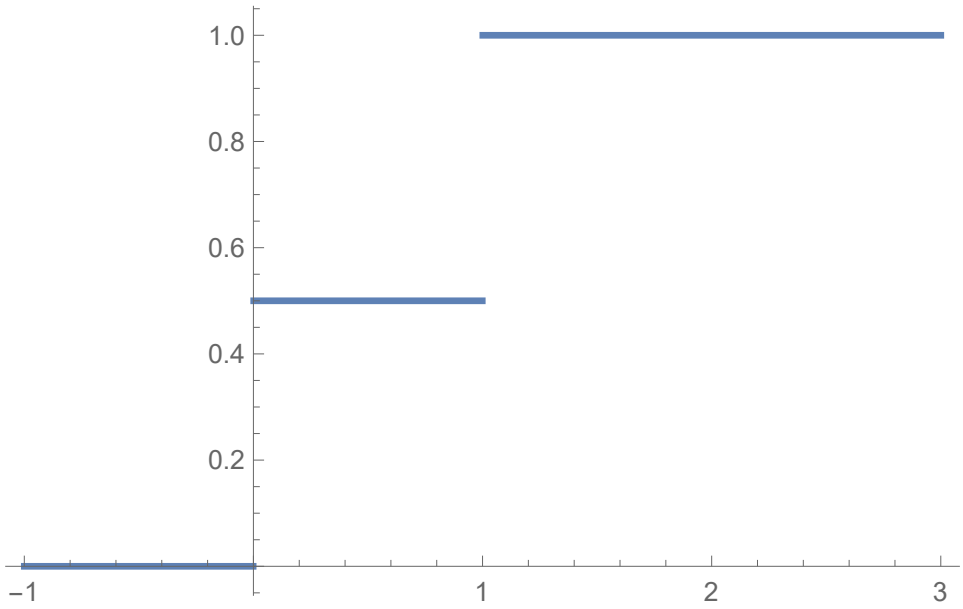
Solution: Solutions will vary. Here is one example:



Those students who remember their probability will notice that the above is the graph of the cdf of an exponential distribution.

Problem 2.4. (2 points) Please, sketch an example of a graph of the cumulative distribution function of a *discrete random variable*.

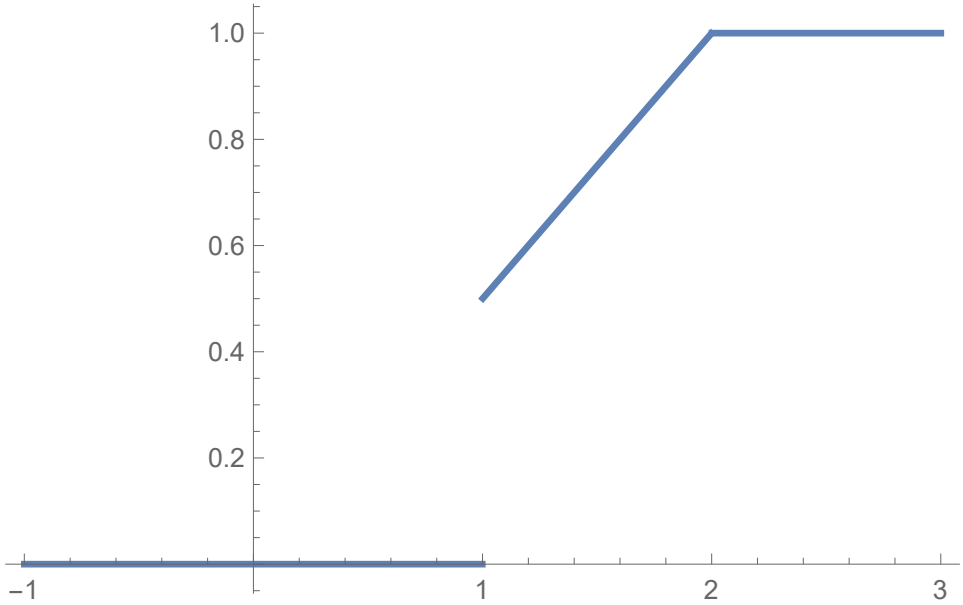
Solution: Solutions will vary. Here is an example:



Again, the students who remember their probability will notice that the above is the graph of the cdf of a single Bernoulli trial with probability of success equal to 1/2.

Problem 2.5. (4 points) Please, sketch an example of a graph of the cumulative distribution function of a *mixed random variable*.

Solution: Solutions will vary. Here is an example:



Those students with coding experience should try to think about how they would simulate the draws from a random variable with the above cdf.