M339W: November 3rd, 2021.

Adding the risk-free saving and borrowing.

Start w/ a portfolio P consisting of risky investments.
Let R_P denote its return.
Let r_f denote the risk-free interest rate.

Now, we construct a new portfolio xP so that:

- the weight x is given to the (risky) portfolio P

and

- the weight $1-x$ is given to the risk-free investment.

Let R_{xP} denote the return of this new portfolio.

We know: $\boxed{R_{xP} = x \cdot R_P + (1-x) \cdot r_f}$

- $$\mathbb{E}[R_{xP}] = x \cdot \mathbb{E}[R_P] + (1-x) \cdot r_f$$
$$= r_f + x\left(\mathbb{E}[R_P] - r_f\right)$$

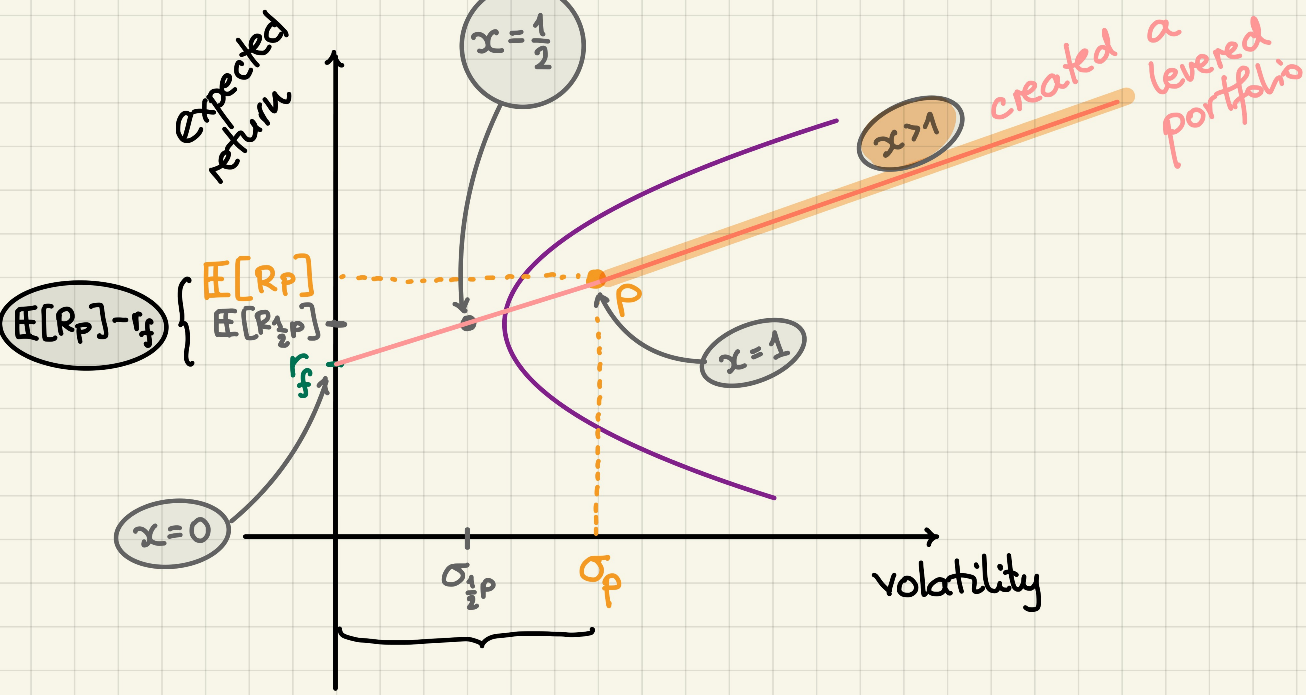
$$\Rightarrow \boxed{\mathbb{E}[R_{xP}] - r_f = x \cdot \underbrace{\left(\mathbb{E}[R_P] - r_f\right)}_{\text{(expected) excess return or risk premium}}}$$

✓

- $$\mathrm{Var}[R_{xP}] = \mathrm{Var}[x \cdot R_P + \underbrace{(1-x) \cdot r_f}_{\text{deterministic}}]$$
$$= \mathrm{Var}[x\, R_P]$$
$$= x^2 \cdot \mathrm{Var}[R_P]$$

$$\Rightarrow \boxed{SD[R_{xP}] = x \cdot SD[R_P]}$$

✓



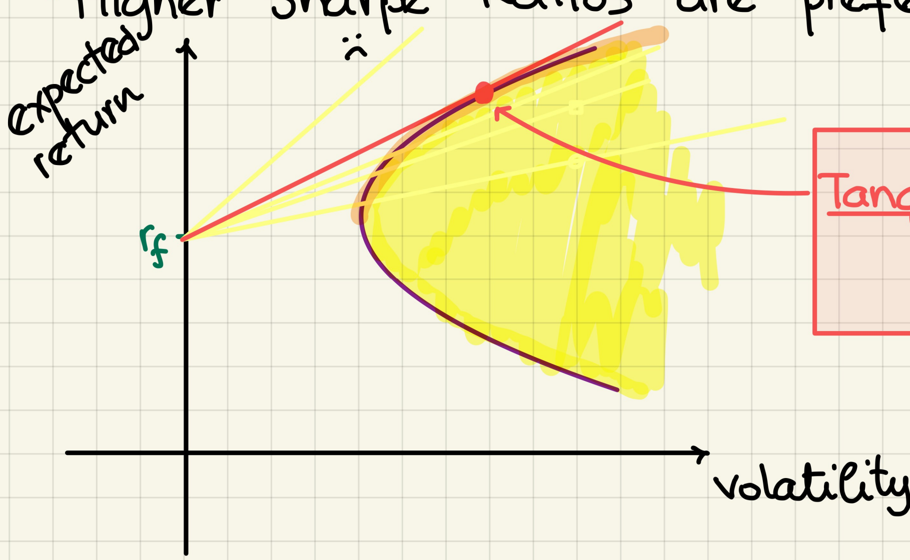
Q: What is the slope of the line through $(0, r_f)$ and $(\sigma_P, \mathbb{E}[R_P])$?

$\longrightarrow$: $$\text{slope} = \frac{\text{rise}}{\text{run}} = \boxed{\frac{\mathbb{E}[R_P]-r_f}{\sigma_P}}$$ Reward-to-Risk Ratio

THE SHARPE RATIO

Note:
- All the portfolios on the line above have the same Sharpe ratio.
- Higher Sharpe Ratios are preferable.

$\therefore$



Tangent Portfolio
(aka the Efficient Portfolio)

8) You are given the following information about a two-asset portfolio:

(i) The Sharpe ratio of the portfolio is $\boxed{0.3667}$.

(ii) The annual effective risk-free rate is 4%.

(iii) If the portfolio were 50% invested in a risk-free asset and 50% invested in a risky asset X, its expected return would be 9.50%. } P

Now, assume that the weights were revised so that the portfolio were 20% invested in a risk-free asset and 80% invested in risky asset X. } P'

Calculate the standard deviation of the portfolio return with the revised weights.

$\sigma_{p'} = ?$

$R_{p'} = 0.2 r_f + 0.8 \cdot R_X$

$\Rightarrow \boxed{\sigma_{p'} = 0.8 \sigma_X}$

(A) 6.0%

(B) 6.2%

(C) 12.8%

(D) 15.0%

(E) 24.0%

(i) $\Rightarrow$ Sharpe Ratio of X is 0.3667.

By def'n, $\dfrac{\boxed{\mathbb{E}[R_X] - r_f}}{\sigma_X} = 0.3667$

σ_X ?

(iii) $\frac{1}{2} r_f + \frac{1}{2} \mathbb{E}[R_X] = 0.095 \quad / \cdot 2$

$\mathbb{E}[R_X] + r_f = 0.19 \quad / -2r_f$

$\mathbb{E}[R_X] - r_f = 0.19 - 2r_f = 0.19 - 0.08 = \boxed{0.11}$

Finally:

$\sigma_{p'} = 0.8 \cdot \dfrac{0.11}{0.3667} = 0.24$