M339D: September 18th, 2023.

UNIVERSITY OF TEXAS AT AUSTIN

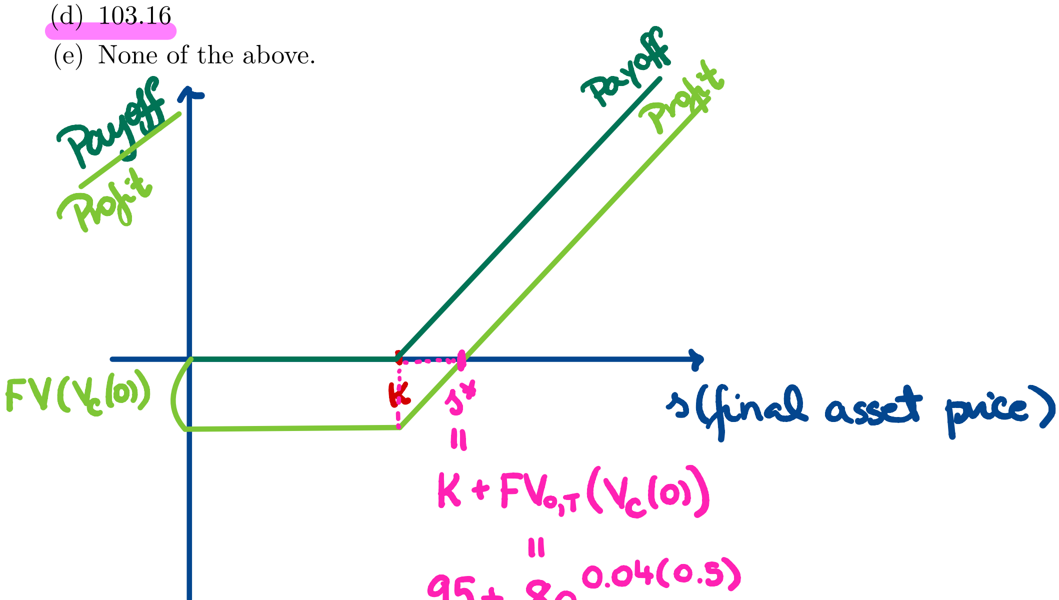
Problem Set #5

European call options.

Problem 5.1. The initial price of a non-dividend-paying asset is $100. A six-month, $95-strike European call option is available at a $8 premium. The continuously compounded risk-free interest rate equals 0.04. What is the break-even point for this call option?

K

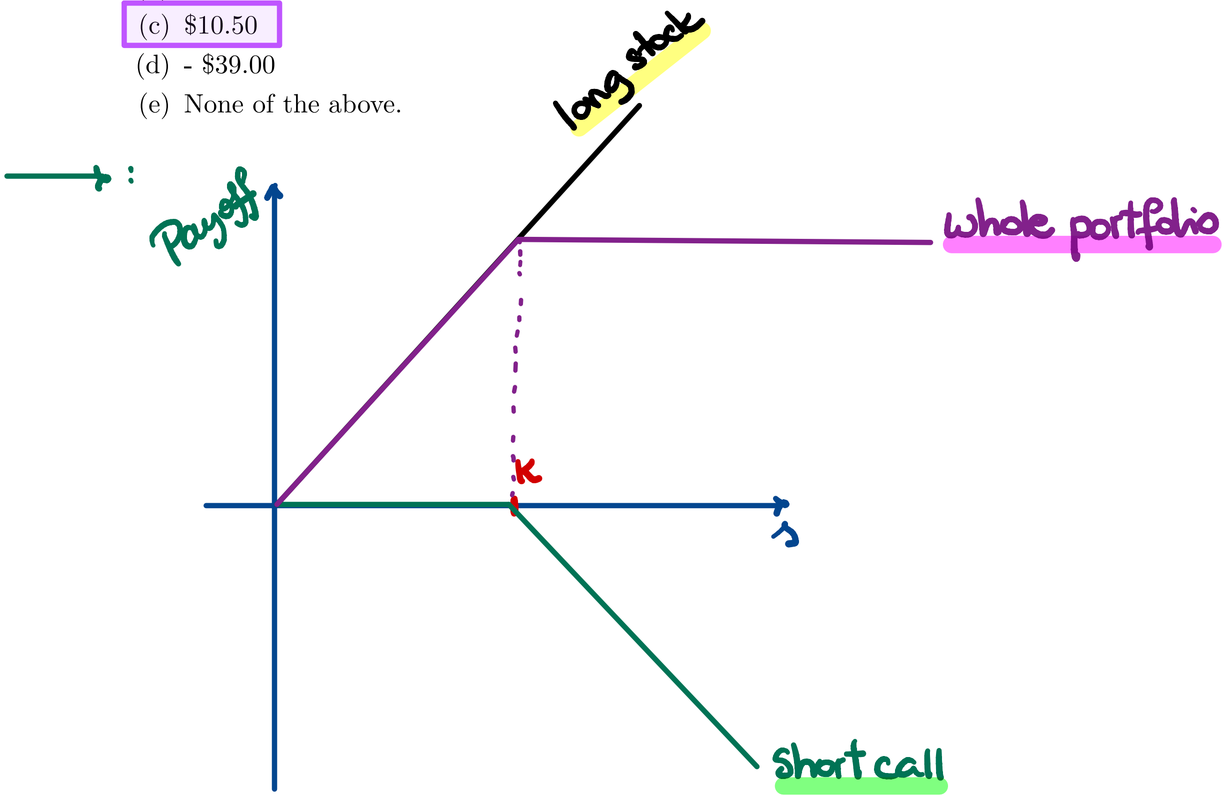
(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.



$$s^* = K + FV_{0,T}(V_c(0)) = 95 + 8e^{0.04(0.5)} = 103.16$$

Problem 5.2. (5 points) A stock's price today is $1000 and the annual effective interest rate is given to be 5%. You write a one-year $1,050-strike call option for a premium of $10 while you simultaneously buy the stock. What is your **profit** if the stock's spot price in one year equals $1,200?

(a) $150.00
(b) $139.90
(c) $10.50
(d) - $39.00
(e) None of the above.



Algebraically,

$$\text{Payoff} = \boxed{S(T) - (S(T)-K)_+} = \begin{cases} K & \text{if } S(T) \geq K \\ S(T) & \text{if } S(T) < K \end{cases}$$

Covered call

$$= \min(S(T), K)$$

In this problem,

$$\text{Payoff} = \min(1200, 1050) = 1050$$

$$\text{Initial Cost} = 1000 - 10 = 990$$

$$\text{Profit} = 1050 - 990(1.05) = 10.50$$

Problem 5.3. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for $1,000.

The jeweler models the market price of gold in one year as follows:

$S(T)$

Gold price in one year	Probability
750 per ounce	0.2
850 per ounce	0.5
950 per ounce	0.3

min
750
850
900

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of $900 per ounce. The option costs $100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelry produced.

→ : Algebraically:

$$\text{Payoff(Total)} = \text{Payoff(Gold)} + \text{Payoff(Call)}$$
$$= -S(T) + (S(T)-K)_+$$
$$= \begin{cases} -K & \text{if } S(T) \geq K \\ -S(T) & \text{if } S(T) < K \end{cases}$$
$$= \boxed{-\min(S(T), K)}$$

CAP

$$\text{Profit} = 1000 - \min(S(T), K) - 100e^{0.05}$$

$$\mathbb{E}[\text{Profit}] = 1000 - \boxed{\mathbb{E}[\min(S(T),K)]} - 100e^{0.05} = \underline{49.87}$$

$\square$

$$\mathbb{E}[\min(S(T),K)] \stackrel{?}{=} 750\cdot\left(\frac{1}{5}\right) + 850\cdot\left(\frac{1}{2}\right) + 900\cdot\left(\frac{3}{10}\right)$$
$$= 150 + 425 + 270 = \underline{845}$$

Problem 5.4. The current price of stock a certain type of stock is \$80. The premium for a 6−month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) \$80
(b) \$85.72
(c) \$85.84
(d) \$85.96
(e) None of the above.

$S^* = 80 + 5.84e^{0.04(0.5)}$

$S(0) = K$

Problem 5.5. The price of gold in half a year is modeled to be equally likely to equal any of the following prices

$$\$1000, \quad \$1100, \quad \text{and} \quad \$1240.$$

Consider a half-year, \$1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

→ : Payoff: $(S(T)-K)_+$ 0 50 190

w/ probab. 1/3 1/3 1/3

answer: $\frac{1}{3}\cdot 50 + \frac{1}{3}\cdot 190 = 80$ $\square$

Problem 5.6. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at \$10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total \$500. They decide to buy six-month, \$5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded risk-free interest rate equals 0.04.

In six months, the price of goat milk equals \$6 per gallon. What is the profit of the company's hedged position?

(a) 395.92

(b) 397.98

(c) 400

(d) 897.98

(e) None of the above.

$$200(10) - 200(5) - 500 - 200(0.5)e^{0.04(0.5)} = \underline{\qquad\qquad}$$