M378K Introduction to Mathematical Statistics
Homework assignment #9

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 9.1. (10 *points) Let* $\hat{\theta}_1$ *and* $\hat{\theta}_2$ *be independent unbiased estimators of* θ*. You are given that* $Var[\hat{\theta}_1] = 4$ *and* $Var[\hat{\theta}_2] = 9$.

(i) *(8 pts) Consider the class of estimators of the form* $\hat{\theta} = \alpha\hat{\theta}_1 + \beta\hat{\theta}_2$*. Find the UMVUE in this class of estimators.*

(ii) *(2 pts) Calculate the MSE of the estimator* $\hat{\theta}$ *you found in part (i) as an estimator of* θ*.*

Problem 9.2. *(5 points) For a fixed confidence level, a broader confidence interval is preferred since it is more likely to contain the true value of the parameter of interest.* True or false? Why?

Problem 9.3. *(5 points) You model the weights of individual boxes of Turkish delight as normally distributed with an unknown mean* μ *and a know standard deviation of* 10 *grams. You gather a random sample of* 16 *boxes of Turkish delight and carefully weigh them. You obtain the following values*

$97.14, 107.05, 103.17, 106.27, 98.63, 90.66, 105.29, 87.12, 112.40, 95.61, 95.19, 114.14, 94.47, 112.89, 105.80, 92.96.$

Provide the 80% confidence interval for μ *based on the above data.*

Problem 9.4. *(5 points) A pollster is trying to estimate the proportion of the population in favor of candidate* A *(in a two-way race between* A *and* B*). The quality of her sample is such that it can be safely assumed to be a random sample from the Bernoulli distribution with the unknown parameter* p*. She is interested in the smallest sample size she will need in order to be able to pinpoint the value of* p *with* $\pm 1\%$ *accuracy, with* 95% *confidence.*

Basing your analysis on the estimator $\hat{p} = Y/n$*, where* Y *is the number of supporters of candidate* A *in the sample, find the smallest such* n *under the assumption that the sampling distribution of* $\hat{p}$ *is well approximated by a normal distribution (of appropriate mean and variance left for you to figure out).*

Problem 9.5. *(25 points)* Source: "Mathematical Statistics with Applications" by Wackerley, Mendenhall, and Sheaffer.
Let the random variable Y be gamma distributed with parameters $k = 4$ and τ unknown.

(i) *(10 points) Show, using moment generating functions, that*

$$U := \frac{2Y}{\tau} \sim \chi^2(df = 8).$$

(ii) *(15 points) Using $U = 2Y/\tau$ as a pivotal quantity, construct a 90%-confidence interval for τ.*

Problem 9.6. *(10 points) Let the random variable Y be normally distributed with mean zero and the standard deviation σ unknown.*

Using Y^2/σ^2 as a pivotal quantity, construct a 95%-confidence interval for σ^2.