

M339Q : December 4th, 2023.

Delta Hedging [cont'd].

Example. An agent writes a put option @ time 0.

At time t , the agent's unhedged position is:

$$-v_p(s, t)$$

\Rightarrow They must maintain $N(s, t) = \Delta_p(s, t) = \underline{-N(-d_1(s, t))}$
in the Δ -hedge.

\Rightarrow The agent must **short** a portion of a share.

At time 0, their total position will be:

$$\underline{v_{\text{Port}}(S(0), 0) = -v_p(S(0), 0) + \Delta_p(S(0), 0) \cdot S(0)}$$

In the Black-Scholes model:

$$\begin{aligned} \underline{v_{\text{Port}}(S(0), 0)} &= - \left(Ke^{-rT} N(-d_2(S(0), 0)) - \cancel{S(0) N(-d_1(S(0), 0))} \right) \\ &\quad + \cancel{(-N(-d_1(S(0), 0)))} \cdot S(0) \\ &= \underline{-Ke^{-rT} N(-d_2(S(0), 0))} \end{aligned}$$

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:
- (i) Each period is 6 months.
 - (ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.
 - (iii) The risk-neutral probability of an up move is $1/3$.
 - (iv) The initial futures price is 80.
 - (v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

- (A) 0
 - (B) 0.022
 - (C) 0.044
 - (D) 0.066
 - (E) 0.088
47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

- (i) The risk-free interest rate is constant.
- (ii)

	Several months ago	Now
Stock price	\$40.00	\$50.00
Call option price	\$ 8.88	\$14.42
Put option price	\$ 1.63	\$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

Hint!
Put-Call
Parity :-

$$\text{Profit} = \text{Payoff} - \text{FV}(\text{Initial Cost})$$

Calculate her profit.

(A) \$11

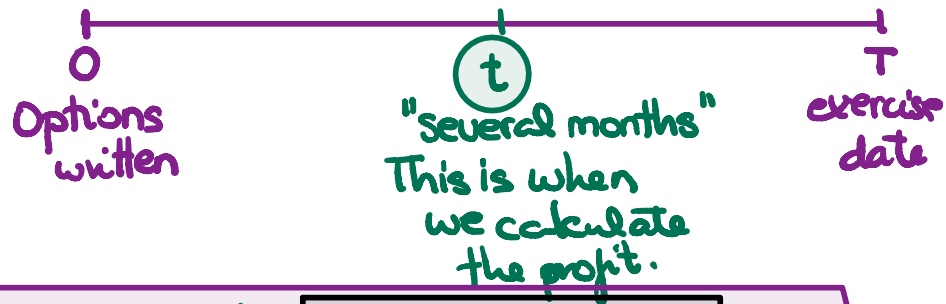
(B) \$24

(C) \$126

(D) \$217

☹️ (E) \$240

48. DELETED



$$\text{Profit}(@ \text{time } t) = \text{Wealth}(@ \text{time } t) - \text{FV}_{0,t}(\text{Initial Cost})$$

49. You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).

- (i) The period is 3 months.
- (ii) The initial stock price is \$100.
- (iii) The stock's volatility is 30%.
- (iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

(A) 114

(B) 115

(C) 116

(D) 117

(E) 118

- Initial Cost: $-100 \cdot v_c(S(0), 0) + 100 \cdot \Delta_c(S(0), 0) \cdot S(0)$
 $= 100(-8.88 + 0.794 \cdot 40) = \underline{2,288}$
- Wealth @ time t : $-100 \cdot v_c(S(t), t) + 100 \cdot \Delta_c(S(0), 0) \cdot S(t)$
 $= 100(-14.42 + 0.794 \cdot 50)$
 $= \underline{2,528}$

Profit (@ time t) = $2,528 - 2,288 \cdot e^{rt}$

Use put-call parity:

At time 0 : $v_c(S(0), 0) - v_p(S(0), 0) = S(0) - Ke^{-rT}$
 $8.88 - 1.63 = 40 - Ke^{-rT}$
 $Ke^{-rT} = 40 - 7.25 = 32.75 \quad \checkmark$

At time t : $v_c(S(t), t) - v_p(S(t), t) = S(t) - Ke^{-r(T-t)}$
 $14.42 - 0.26 = 50 - Ke^{-r(T-t)}$
 $Ke^{-r(T-t)} = 50 - 14.16 = 35.84 \quad \checkmark\checkmark$

$$\frac{\checkmark\checkmark}{\checkmark} = \frac{\cancel{Ke^{-rT}} \cdot e^{rt}}{\cancel{Ke^{-rT}}} = e^{rt} = \frac{35.84}{32.75} = 1.09435$$

Profit (@ time t) = $2,528 - 2,288 \cdot (1.09435) = \underline{24.12} \quad \square$