M339W: October 4th, 2021.

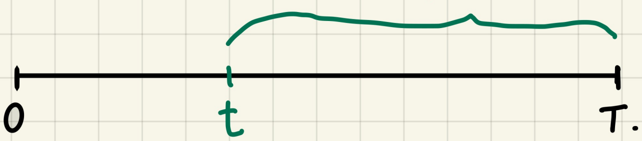
Option Greeks.

Objective: To study the dependence of the value of our portfolio on the set of these independent arguments: s, t, r, δ, σ

s ← asset price @ a particular time $\cdot t$

t ← valuation date

$(T-t)$ ... time to exercise



T ... time horizon (say, the exercise date of an option)

The stock price will be modelled using Black·Scholes.

Recall: $S(t), t \geq 0$ ... time·t stock price

Under the risk·neutral probability measure $\mathbb{P}^*$:

$$S(T) = S(t) e^{(r-\delta-\frac{\sigma^2}{2})(T-t) + \sigma\sqrt{T-t}\cdot Z} \quad \text{w/ } Z \sim N(0,1)$$

Look @ the time·t price of a European call/put w/ strike K and exercise date T.

$$V_C(t) = \underbrace{S(t)e^{-\delta(T-t)}}_{\text{prepaid forward price}} \cdot N(d_1) - \underbrace{Ke^{-r(T-t)}}_{\text{present value of strike}} \cdot N(d_2)$$

and

$$V_P(t) = \underline{Ke^{-r(T-t)}} \cdot N(-d_2) - \underline{S(t)\cdot e^{-\delta(T-t)}} N(-d_1)$$

w/ $$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S(t)}{K}\right) + \left(r-\delta+\frac{\sigma^2}{2}\right)(T-t)\right]$$

and $d_2 = d_1 - \sigma\sqrt{T-t}$

We reintroduce: s... independent argument which stands for the time $\cdot t$ ("current") stock price

In our pricing formula, we would have the following for the call and the put from above:

$$v_C(\underbrace{s,t,r,\delta,\sigma}_{\dots}) = s e^{-\delta(T-t)} N(d_1(\dots)) - Ke^{-r(T-t)} N(d_2(\dots))$$

$$v_P(\overbrace{s,t,r,\delta,\sigma}) = Ke^{-r(T-t)} N(-d_2(\dots)) - se^{-\delta(T-t)} N(-d_1(\dots))$$

w/ $d_1(\dots) = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{s}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)(T-t)\right]$

$$d_2(\dots) = d_1(\dots) - \sigma\sqrt{T-t}$$

For now: Our portfolios consist of:
- the riskless asset
- the risky asset
- European options on our risky asset

$\Rightarrow$ We will be able to represent the value of such a portfolio as a value function of our portfolio, i.e.,

$$\boxed{v(s,t,r,\delta,\sigma)}$$

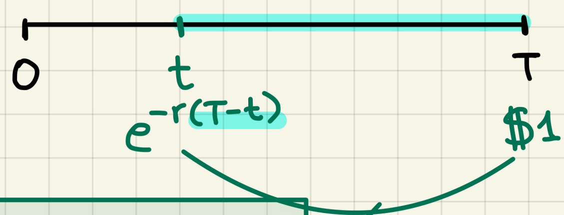
Def'n:
- $\frac{\partial}{\partial s} v(\dots) =: \Delta(\dots)$ DELTA
- $\frac{\partial^2}{\partial s^2} v(\dots) =: \Gamma(\dots)$ GAMMA
- $\frac{\partial}{\partial t} v(\dots) =: \Theta(\dots)$ THETA

Greeks

- $\frac{\partial}{\partial r} v(\cdots) =: \rho(\cdots)$ RHO
- $\frac{\partial}{\partial \delta} v(\cdots) =: \psi(\cdots)$ PSI
- $\frac{\partial}{\partial \sigma} v(\cdots) =: \text{vega}(\cdots)$ VEGA

<u>Example:</u> Consider a zero-coupon bond redeemable @ time T for \$1. Let this be the only component in your portfolio. <u>Then</u>, your value f'tion is:

$$v(\underbrace{s, t, r, \delta, \sigma}_{\cdots}) = \ ?$$



$$\boxed{v(\cdots) = e^{-r(T-t)}}$$

$$\Delta(\cdots) = \frac{\partial}{\partial s} v(\cdots) = \frac{\partial}{\partial s}\left(e^{-r(T-t)}\right) = 0$$

$$\Rightarrow \Gamma(\cdots) = 0$$

$$\Theta(\cdots) = \frac{\partial}{\partial t} v(\cdots) = \frac{\partial}{\partial t}\left(e^{-r(T-t)}\right) = r e^{-r(T-t)} > 0$$

$$\rho(\cdots) = \frac{\partial}{\partial r} v(\cdots) = \frac{\partial}{\partial r}\left(e^{-r(T-t)}\right) = -(T-t) e^{-r(T-t)} < 0$$

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