University of Texas at Austin

# Quiz # 9

## Black-Scholes Delta.

Please, provide your **complete solution** to the following problems. Final answers without shown reasoning will get zero points.

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**Problem 9.1.** (5 points) Assume the Black-Scholes framework. For an at-the-money, $T-$year European call option on a non-dividend-paying stock you are given that its delta equals 0.5832. What is the delta of an otherwise identical option with exercise date at time $2T$?

**Solution:** The time$-0$ delta of a call on a non-dividend-paying stock has the form

$$\Delta_C(S(0),0) = N(d_1(S(0),0))$$

with

$$d_1(S(0),0) = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right].$$

Since the option is at-the-money, the above expression simplifies to

$$d_1(S(0),0) = \frac{r + \frac{\sigma^2}{2}}{\sigma}\sqrt{T}.$$

So, we are given that

$$N\left(\frac{r + \frac{\sigma^2}{2}}{\sigma}\sqrt{T}\right) = 0.5832 \quad \Rightarrow \quad \frac{r + \frac{\sigma^2}{2}}{\sigma}\sqrt{T} = 0.21.$$

Similarly, we obtain that the delta of the otherwise identical call option with twice as long to expiration equals

$$N\left(\frac{r + \frac{\sigma^2}{2}}{\sigma}\sqrt{2T}\right) = N\left(\left(\frac{r + \frac{\sigma^2}{2}}{\sigma}\sqrt{T}\right)\sqrt{2}\right) = N(0.21\sqrt{2}) \approx 0.6179.$$

**Problem 9.2.** (5 points) Assume the Black-Scholes framework as model for the price of a non-dividend-paying stock. What is the difference between the delta of a European call option and the delta of the otherwise identical put option?

**Solution:** The difference is 1 by put-call parity.

**Problem 9.3.** (5 points) Assume the Black-Scholes model. Let the current stock price of a continuous-dividend-paying stock be equal to \$80. The stock's dividend yield is 0.01 and its volatility is 0.30.

The continuously compounded risk-free interest rate is 0.04.

Consider a \$82-strike, six-month European put option on the above stock. What is the put option's delta?

**Solution:** The expression for the time$-0$ put delta is

$$\Delta_P(S(0),0) = -e^{-\delta T}N(-d_1(S(0),0))$$

with

$$\begin{aligned} d_1(S(0),0) &= \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T\right] \\ &= \frac{1}{0.3\sqrt{\frac{1}{2}}}\left[\ln\left(\frac{80}{82}\right) + \left(0.04 - 0.01 + \frac{(0.3)^2}{2}\right)\left(\frac{1}{2}\right)\right] = 0.06037. \end{aligned}$$

Consulting the standard normal tables, we get that

$$\Delta_P(S(0),0) = -e^{-0.01(0.5)}(0.4761) = -0.4737254.$$

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