Project #2: On volatility and cost of capital

Milica Cudina

2022-09-23

Problem #1 (60 points)

Go to Yahoo!Finance.

Then, download the historical stock prices of *Walmart* for the last 252 (or so) trading days, i.e., for the last year. Do the same for *IBM* for the same time period.

(5 points) Draw the time-plot of the evolution of the closing stock prices (not the adjusted) for both of the stocks on the same coordinate system. You do **not** need to put the calendar days on the horizontal axis, but you **do** need to label your axes and give your time-plot a title indicating the dates. Make sure that you plot the two trajectories in **different** colors indicating in the text which color corresponds to which company.

The **simple daily return** of the stock over a day indexed by t is defined as

$$\frac{\text{price at end of day } t - \text{price at end of day } (t-1)}{\text{price at end of day } (t-1)}$$

(5 points) Construct the vector of simple daily returns over the last year for both stocks. What is the **daily volatility** estimate for both of them?

(10 points) In general, **assuming independent, identically distributed** daily returns, what is the relationship between the daily volatility σ_h and the annual volatility σ? Calculate the estimate of the annual volatility for both stocks.

(10 points) If you wanted to study the *relationship* between the returns of the two stocks, which plot would you create? What (if anything) can you say after looking at the plot?

(5 points) If you wanted to provide one single, unitless quantity which provided you a measure of association between the returns of *Walmart* and *IBM*, which value would you report?

(5 points) You create a portfolio in which half your wealth is maintained in the *Walmart* stock and the remaining half of your wealth is kept in the *IBM* stock. What is your estimate of the annual volatility of this portfolio?

(20 points) You create a portfolio in which the weight w of your wealth is maintained in the *Walmart* stock and the remainder of your wealth is kept in the *IBM* stock. Create a function in `R` which calculates the annual volatility of this portfolio as it depends on w? Plot the portfolio's volatility as a function of w for w ranging from -2 to 2. In the same plot, add two horizontal lines at the values of the annual volatilities of *Walmart* and *IBM*.

Problem #2 (40 points)

Go to Yahoo!Finance.

Then, download the historical prices for the *NASDAQ Composite* index for the **maximum** time period and at the **monthly** frequency.

(5 points) Use the above definitions of returns, changing the period length to a month (rather than a day). What is your *point estimate* for the mean monthly return of *NASDAQ*? Assuming, as usual **independent, identically distributed** monthly returns, what is the mean annual return?

(5 points) What is your *point estimate* for the monthly volatility of *NASDAQ*? Assuming, as usual **independent, identically distributed** monthly returns, what is the annual volatility?

(20 points) You know that there is uncertainty in point estimation due to sampling variability. So, you want to provide a **confidence interval**.

For more about confidence intervals in general, please watch:

Confidence intervals

A follow up video for the mean parameter is

Inference for the mean

Let n stand for the number of observation. Let $\hat{\mu}$ denote our point estimate for the mean parameter. Let s denote the sample standard deviation. At a confidence level C, the structure of a **confidence interval** is

$$(\hat{\mu} - \frac{s}{\sqrt{n}}z^*, \hat{\mu} + \frac{s}{\sqrt{n}}z^*)$$

where z^* is a critical value of the standard normal distribution such that $\mathbb{P}[-z^* < Z < z^*] = C$ with $Z \sim N(0,1)$.

You should input `?qnorm` into the console in `R` to learn more about the different functions which have to do with the normal distribution in `R`.

Provide confidence intervals for the annual mean return at the confidence levels $C = 0.90, 0.95, 0.99$.

(10 points) Focus on the 95%-confidence interval. Draw the three "accumulation functions" of the form $a(t) = (1+r)^t$ for $t \in [0.10]$ for the lower bound, the midpoint, and the upper bound of the 95%-confidence interval. Please, comment on the graphs.Do you find the confidence-interval approach to estimation of the cost of capital entirely satisfactory?