UNIVERSITY OF TEXAS AT AUSTIN

Quiz #14

Splicing. Franchise deductibles.

Please, provide your **complete solutions** to the following questions:

Problem 14.1. (8 points) A model for the arrival time T for a particular event is initially an exponential distribution with mean 2 years. Upon reconsideration, this distribution is replaced with a spliced model whose density function:

(i) is proportional to the initial model's density function over $[0,1]$,
(ii) is uniform over $[1,3]$,
(iii) is continuous at 1,
(iv) is zero on $(3,\infty)$.

Calculate the probability of failure in the first year under the revised distribution.

Solution: For the "old" model, the density function is

$$f_T(x) = \frac{1}{2}e^{-\frac{x}{2}} \quad \text{for } x > 0.$$

Let $\tilde{f}$ be the new density function. Then, it satisfies:

(i) $\tilde{f}(x) = ce^{-\frac{x}{2}}$ for $0 < x < 1$ for some constant c,
(ii) $\tilde{f}(x) = \kappa$ for some constant κ for $x \in [1,3]$,
(iii) $\tilde{f}(1) = \kappa = ce^{-\frac{1}{2}}$,
(iv) otherwise, the density function $\tilde{f}$ is zero.

Since $\tilde{f}$ is a density function, it must integrate to 1. Thus,

$$1 = c\int_0^1 e^{-x/2}\,dx + 2\kappa = 2c\left(1 - e^{-\frac{1}{2}}\right) + 2\kappa.$$

Using (iii) above, we can substitute $\kappa = ce^{-\frac{1}{2}}$ in the last equation to obtain

$$1 = 2c - 2\kappa + 2\kappa \quad \Rightarrow \quad c = \frac{1}{2}.$$

The answer is

$$2c\left(1 - e^{-\frac{1}{2}}\right) = 1 - e^{-\frac{1}{2}}.$$

Problem 14.2. (2 points) The ground-up loss random variable is denoted by X. An insurance policy on this loss has a **franchise** deductible of d and no policy limit. Then, the expected **policyholder** payment per loss equals

$$\mathbb{E}[X\mathbb{I}_{[X<d]}].$$

True or false?

Solution: TRUE

Problem 14.3. (5 pts) Let the loss random variable X be Pareto with $\alpha = 3$ and $\theta = 5000$. There is a franchise deductible of $d = 1000$. What is the expected value of the per payment random variable?

Solution:

$$\mathbb{E}[Y^P] = e_X(d) + d = \frac{d+\theta}{\alpha - 1} + d = \frac{1000 + 5000}{2} + 1000 = 4000.$$