UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 7

Binomial option pricing.

7.1. **The forward binomial tree.** Please, provide your *final answer only* to the following problem.

Problem 7.1. (5 points) Assume that the a stock price is modeled using a one-period forward binomial tree with the length of a single period equal to three months. According to this model, the stock price can take either the value of \$50, or the value of \$40 in exactly three months. Calculate the volatility of the stock price.

(a) About 0.22
(b) About 0.28
(c) About 0.30
(d) About 0.32
(e) None of the above.

7.2. **Alternative binomial trees.** Please, provide your **complete** solutions to the following problem(s):

Problem 7.2. Cox-Ross-Rubinstein (CRR)
The Cox-Ross-Rubinstein model is a binomial tree in which the up and down factors are given as

$$u = e^{\sigma\sqrt{h}}, \quad d = e^{-\sigma\sqrt{h}},$$

where σ denotes the volatility parameter and h stands for the length of a single period in a tree.

a. (2 points) What is the ratio S_u/S_d?
b. (2 points) What is the (as simplified as possible) expression for the risk-neutral probability of the stock price going up in a single step?
c. (2 points) Express S_{ud} in terms of $S(0), \sigma$ and h in a CRR tree.
d. (5 points) As was the case with the forward tree, the *no-arbitrage* condition for the binomial asset-pricing model is satisfied for the CRR tree regardless of the specific values of σ, r and h. *True or false?*

Problem 7.3. The Jarrow-Rudd model.
The **Jarrow-Rudd** model (aka, the lognormal binomial tree) is a binomial tree in which the up and down factors are defined as follows

$$u = e^{\left(r-\frac{\sigma^2}{2}\right)h+\sigma\sqrt{h}}, \quad d = e^{\left(r-\frac{\sigma^2}{2}\right)h-\sigma\sqrt{h}},$$

where

- r stands for the continuously-compounded, risk-free interest rate,
- δ is the stock's dividend yield,
- σ denotes the volatility parameter, and
- h stands for the length of a single period in a tree.

Answer the following questions:

a. (2 points) What is the ratio S_u/S_d?
b. (2 points) What is the (as simplified as possible) expression for the risk-neutral probability of the stock price going up in a single step?
c. (5 points) As was the case with the forward tree, the *no-arbitrage* condition for the binomial asset-pricing model is satisfied for the Jarrow-Rudd tree regardless of the specific values of σ, δ, r and h. *True or false?*

7.3. **Multi-period binomial option pricing: European options.** Please, provide your **complete** solutions to the following problem:

Problem 7.4. (10 points) The current price of a non-dividend-paying stock is \$100 per share. Its volatility is given to be 0.2.

The continuously compounded risk-free interest rate equals 0.04.

Consider a \$95-strike European put option on the above stock with nine months to expiration. Using a three-period forward binomial tree, find the price of this put option.

7.4. **Strong Law of Large Numbers. Monte Carlo.**

Problem 7.5. (10 points) Let $\{Y_n, n \in \mathbb{N}\}$ be a sequence of independent, identically distributed random variables. Assume that $Y_1 = e^X$ where X is a standard normal random variable. Use the Strong Law of Large Numbers to find the following limit

$$\lim_{n\to\infty}\left(\prod_{i=1}^{n} Y_i\right)^{1/n} = \lim_{n\to\infty}\left(Y_1 \cdot Y_2 \cdots Y_n\right)^{1/n}.$$

Hint: Note that for every n, $Y_n = e^{X_n}$ where $\{X_n, n \in \mathbb{N}\}$ is a sequence of independent identically distributed standard normal random variables. Then, it helps to modify the product in the limit above and use the continuity of the exponential function.

Problem 7.6. (5 points) You use *Monte Carlo* to simulate values from a normal distribution with mean 0 and variance 4. Your plan is to use 10000 simulations. What is the variance of the *Monte Carlo* simulations?