

M339D: April 18<sup>th</sup>, 2025.

33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

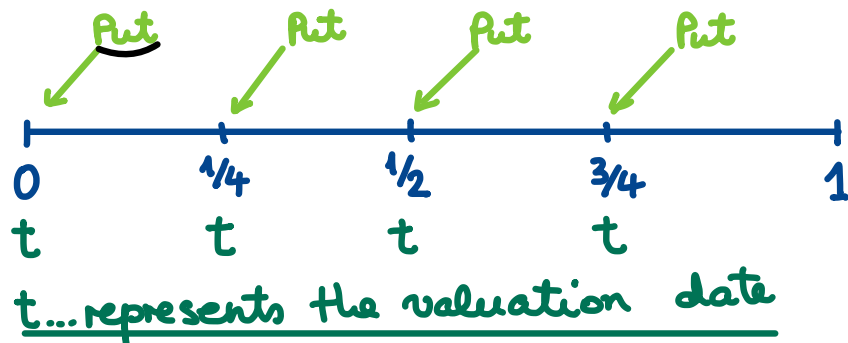
You are given:

- (i) The continuously compounded risk-free interest rate is 8%.
- (ii) The stock's volatility is 30%.
- (iii) The current stock price is 45.
- (iv) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

- (A) 1.59
- (B) 2.24
- (C) 2.86
- (D) .48
- (E) 3.61



For each of the four puts in the *rolling insurance strategy*:

- one quarter-year to exercise
- $K_t = 0.9 \cdot S(t)$

For every  $t$  @ which a put option is received:

$$d_1(\cancel{t}) = \frac{1}{0.3\sqrt{\frac{1}{4}}} \left[ \ln\left(\frac{\cancel{S(t)}}{0.9\cancel{S(t)}}\right) + \left(0.08 + \frac{0.09}{2}\right) \cdot \frac{1}{4} \right]$$

$$d_1(\cancel{t}) = \frac{1}{0.15} \left[ -\ln(0.9) + \frac{0.25}{8} \right] = \underline{0.9107} \approx 0.91$$

$$d_2(t) = 0.9107 - 0.15 = \underline{0.7607} \approx 0.76$$

$$N(-0.91) = \underline{0.1814}, \quad N(-0.76) = \underline{0.2236}$$

In general,

$$V_p(t) = Ke^{-r(T-t)} \cdot N(-d_2(t)) - S(t)N(-d_1(t))$$

$$V_p(t) = 0.9 \cdot S(t) e^{-0.08(1/4)} \cdot 0.2236 - S(t) \cdot 0.1814$$

$$V_p(t) = S(t) \cdot \underline{0.0159}$$

=> Note that EVERY put is worth  $S(t) \cdot 0.0159$  on its delivery date  $t=0, 1/4, 1/2, 3/4$ .

In order to perfectly replicate, we should buy 0.0159 shares of stock today for each put.

With 4 puts, the total cost is:  $4 \cdot \overbrace{S(0)}^{45} \cdot 0.0159 =$

$$= \underline{2.8562}$$

