University of Texas at Austin

Quiz #12

Delta hedger's profit.

Problem 12.1. (15 points) An investor buys a time$-T$ European call option on this stock at time-0 and creates a delta-neutral, fully-leveraged portfolio by trading in the shares of the underlying stock and borrowing/lending at the continuously compounded risk-free interest rate r.

The current price of the underlying stock is \$50 and its dividend yield is equal to the continuously compounded risk-free interest rate . The continuous dividends are assumed to be continuously and immediately reinvested in the same stock.

The time-0 premium of the above call option is \$7.50 and its delta is 0.56. The premium for the otherwise identical put option is \$5.60.

At time$-t$ (prior to the call's exercise date T), the investor decides to liquidate her portfolio. She sees that the current stock price is the same as it was at time-0, the above call premium is \$4.50 and the above put premium is \$2.40.

What is the investor's profit after liquidation?

Solution: The investor's portfolio consists of the following:

- one long call option,
- short-selling $\Delta_C = 0.56$ shares of the underlying stock,
- **investing** $0.56 \times 50 - 7.50 = 28 - 7.50 = 20.50$ at the continuously compounded risk-free interest rate .

So, the investor's wealth at time$-t$ equals

$$V_C(t) - \Delta_C e^{\delta t} S(t) + 20.50 e^{rt} = 4.50 - 28 e^{rt} + 20.50 e^{rt} = 4.50 - 7.50 e^{rt}\,.$$

Note that there was We will get e^{rt} from put-call parity applied at time-0 and time$-t$ with the fiven call and put prices.

$$7.5 - 5.6 = S(0)e^{-\delta T} - Ke^{-rT} = (S(0) - K)e^{-rT},$$
$$4.5 - 2.4 = S(t)e^{-\delta(T-t)} - Ke^{-r(T-t)} = (S(0) - K)e^{-r(T-t)}.$$

We get

$$e^{rt} = \frac{2.1}{1.9} = 1.10526.$$

Our answer is

$$4.50 - 7.50(1.10526) = -3.78945.$$