University of Texas at Austin

Problem Set # 8

Hypothesis testing: The normal case.

Problem 8.1. *Source: Ramachandran, Tsokos.*
The management of the local health club claims that its members lose on average 15 pounds or more within the first three months of their membership. A consumer agency took a simple random sample of 45 members and found the sample average of 13.8 in pounds lost. Assume that we model the weight loss as normal with an **unknown** mean μ and the **known** standard deviation of 4.2 pounds. What is the $p-$value corresponding to the gathered data? What would your decision be at the 0.05 significance level?

Solution: The population model is

$$X \sim Normal(mean = \mu, sd = \sigma = 4.2).$$

We are testing

$$H_0 : \mu = \mu_0 = 15 \quad vs. \quad H_a : \mu < \mu_0 = 15.$$

Our simple random sample is of size 45 and the observed sample mean is 13.8. So, the z-score under the null equals

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{13.8 - 15}{4.2/\sqrt{45}} = -1.9166.$$

The $p-$value is, with $Z \sim N(0, 1)$,

$$\mathbb{P}[Z < z] = \Phi(-1.9166) = 0.02764.$$

Since this $p-$value is below the given significance level of $\alpha = 0.05$, we reject the null hypothesis.

Problem 8.2. *Source: Ramachandran, Tsokos.*
It is claimed that sports-car owners drive on the average $20,000$ miles per year. A consumer firm believes that the mean annual mileage is actually lower. To check, the consumer firm decided to test this hypothesis.

The modeling assumptions are that the annual mileage is normally distributed with an unknown mean μ and with the standard deviation of 1200.

The consumer firm obtained information from 36 randomly selected sports-car owners that resulted in a sample average of $19,530$ miles. What is the decision of this hypothesis test at the signficance level of 0.01?

Solution:

The null and the alternative hypotheses are

$$H_0 : \mu = 20000 \quad vs. \quad H_a : \mu < 20000.$$

The observed value of the $z-$statistic is (under the null hypothesis), in our usual notation,

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{19530 - 20000}{1200/\sqrt{36}} = -2.35.$$

On the other hand, the $z-$score corresponding to the left-sided test at the 0.01 significance level is -2.33. Because the observed value of the z statistic -2.35 is less than -2.33, the null hypothesis is **rejected** at the significance level of 0.01. There is sufficient evidence to conclude that the mean mileage on sport cars is less than 20,000 miles per year.