M339J : March 4th, 2022.

168. For an insurance:

(i) Losses can be 100, 200 or 300 with respective probabilities 0.2, 0.2, and 0.6.

(ii) The insurance has an ordinary deductible of 150 per loss.

(iii) Y^P is the claim payment per payment random variable.

Calculate $\mathrm{Var}(Y^P)$.

(A) 1500

(B) 1875

(C) 2250

(D) 2625

(E) 3000

→ : Support (Y^P) = ?

d=150

X losses:	100	200	300
Y^P :	undef.	50	150

Support of Y^P.

169. The distribution of a loss, X, is a two-point mixture:

(i) With probability 0.8, X has a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 100$.

(ii) With probability 0.2, X has a two-parameter Pareto distribution with with $\alpha = 4$ and $\theta = 3000$.

Calculate $\Pr(X \le 200)$.

(A) 0.76

(B) 0.79

(C) 0.82

(D) 0.85

(E) 0.88

p_{Y^P} ... the probability mass function of Y^P

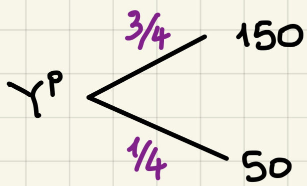
$$\left.\begin{aligned} p_{Y^P}(50) &= \boxed{\frac{p_X(200)}{S_X(150)}} = \frac{0.2}{0.2+0.6} = 0.25 \\ p_{Y^P}(150) &= \boxed{\frac{p_X(300)}{S_X(150)}} = \frac{0.6}{0.8} = 0.75 \end{aligned}\right\}$$

$$\mathbb{E}[Y^P] = 50\left(\frac{1}{4}\right) + 150\left(\frac{3}{4}\right) = 125 \checkmark$$

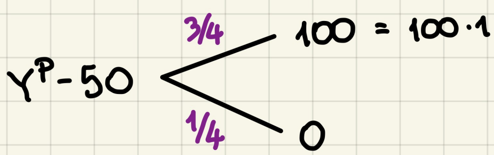
$$\mathbb{E}[(Y^P)^2] = 50^2\left(\frac{1}{4}\right) + (150)^2\left(\frac{3}{4}\right) = 17{,}500$$

$$Var[Y^P] = \mathbb{E}[(Y^P)^2] - (\mathbb{E}[Y^P])^2 = 17{,}500 - (125)^2 = 1875 \blacksquare$$

"A trick":



$$Var[Y^P] = Var[Y^P - 50]$$



$$Y^P - 50 \sim 100 \cdot \text{Bernoulli}\left(\text{prob. of success} = \frac{3}{4}\right)$$

$$\Rightarrow Var[Y^P - 50] = 100^2 \cdot \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = 1875 \blacksquare$$

Franchise Deductible.

If the loss amout exceeds the deductible d, then the insurer covers the entire loss.

The per payment random variable:

$$Y^P = \begin{cases} \text{undefined} & \text{if } X \le d \\ X & \text{if } X > d \end{cases}$$

i.e.,

$$\boxed{Y^P = X \mid X > d}$$

The per loss r.v.:

$$Y^L = \begin{cases} 0 & \text{if } X \le d \\ X & \text{if } X > d \end{cases}$$

i.e.,

$$\boxed{Y^L = X \cdot \mathbb{I}_{[X>d]}}$$

Fact: If X is continuos w/ pdf f_X, then

- Y^P is continuous w/ $f_{Y^P}(y) = \dfrac{f_X(y)}{S_X(d)}$ $\boxed{y > d}$
- Y^L is mixed w/ $p_{Y^L}(d) = F_X(d)$
  and $f_{Y^L}(y) = f_X(y)$ for all $y > d$

Loss Elimination Ratio (LER).

... is the ratio of the decrease in the <u>insurer's</u> expected payment w/ an ordinary deductible d
to the <u>insurer's</u> expected payment w/ no deductible.

<u>As usual</u>: X... loss r.v.

<u>Assume</u>: $\mathbb{E}[X] < +\infty$

$$LER = \frac{\mathbb{E}[X] - \mathbb{E}[(X-d)_+]}{\mathbb{E}[X]} = \boxed{\frac{\mathbb{E}[X \wedge d]}{\mathbb{E}[X]}}$$

<u>Note</u>: $\mathbb{E}[X \wedge d] \leq \mathbb{E}[X] \Rightarrow \boxed{LER \leq 1}$

<u>Example</u>. Let the ground-up loss r.v. X be exponential w/ mean 5000.

Assume that it's insured by an insurance policy w/ an ordinary deductible of 2500.

Find the loss elimination ratio!

$\rightarrow$: $X \sim \text{Exponential}(\text{mean} = \theta = 5000)$

<u>By def'n</u>: $LER = \frac{\mathbb{E}[X \wedge d]}{\mathbb{E}[X]}$

$$LER = \frac{\cancel{\theta}(1 - e^{-\frac{d}{\theta}})}{\cancel{\theta}} = 1 - e^{-\frac{d}{\theta}} = F_X(d)$$

In this problem:

$$LER = 1 - e^{-\frac{2500}{5000}} = 1 - e^{-\frac{1}{2}} = 0.3935$$

89. You are given: $X \sim \text{Exponential}(\text{mean} = \theta)$

(i) Losses follow an exponential distribution with the same mean in all years.

(ii) The loss elimination ratio this year is 70%. $LER = 0.7$

$d = d_{OLD}$

(iii) The ordinary deductible for the coming year is 4/3 of the current deductible. $\tilde{d} = d_{new}$ $\boxed{\tilde{d} = \frac{4}{3} d}$

Calculate the loss elimination ratio for the coming year.

$\widetilde{LER} = ?$

(ii) $0.7 = 1 - e^{-\frac{d}{\theta}} \Rightarrow \boxed{e^{-\frac{d}{\theta}} = 0.3}$

$\widetilde{LER} = 1 - e^{-\frac{\tilde{d}}{\theta}} = 1 - e^{-\frac{\frac{4}{3}d}{\theta}} = 1 - \left(e^{-\frac{d}{\theta}}\right)^{4/3}$

$= 1 - (0.3)^{4/3} = 0.79917$

(A) 70%

(B) 75%

(C) 80%

(D) 85%

(E) 90%

90. Actuaries have modeled auto windshield claim frequencies. They have concluded that the number of windshield claims filed per year per driver follows the Poisson distribution with parameter λ, where λ follows the gamma distribution with mean 3 and variance 3.

Calculate the probability that a driver selected at random will file no more than 1 windshield claim next year.

(A) 0.15

(B) 0.19

(C) 0.20

(D) 0.24

(E) 0.31

Effect of Inflation.

After uniform inflation @ the rate r, the losses are

$$(1+r)\cdot X$$

w/ X being the losses from the previous year.

If the deductible d remains the same:

- the expected cost per loss is:

$$\mathbb{E}[((1+r)X-d)_+] = (1+r)\left(\mathbb{E}[X] - \mathbb{E}\left[X \wedge \frac{d}{1+r}\right]\right)$$

$$\text{if } \boxed{F_X\left(\frac{d}{1+r}\right) < 1}$$

- the expected cost per payment is:

$$\frac{1}{S_X\left(\frac{d}{1+r}\right)}(1+r)\left(\mathbb{E}[X] - \mathbb{E}\left[X \wedge \frac{d}{1+r}\right]\right)$$

Note: Usually, it's easier to use the scaling.

$$(1+r)X > d$$

$$X > \frac{d}{1+r}$$