M339D: February 10^{th}, 2025.

Problem 5.3. (20 points)

The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for $1,000.

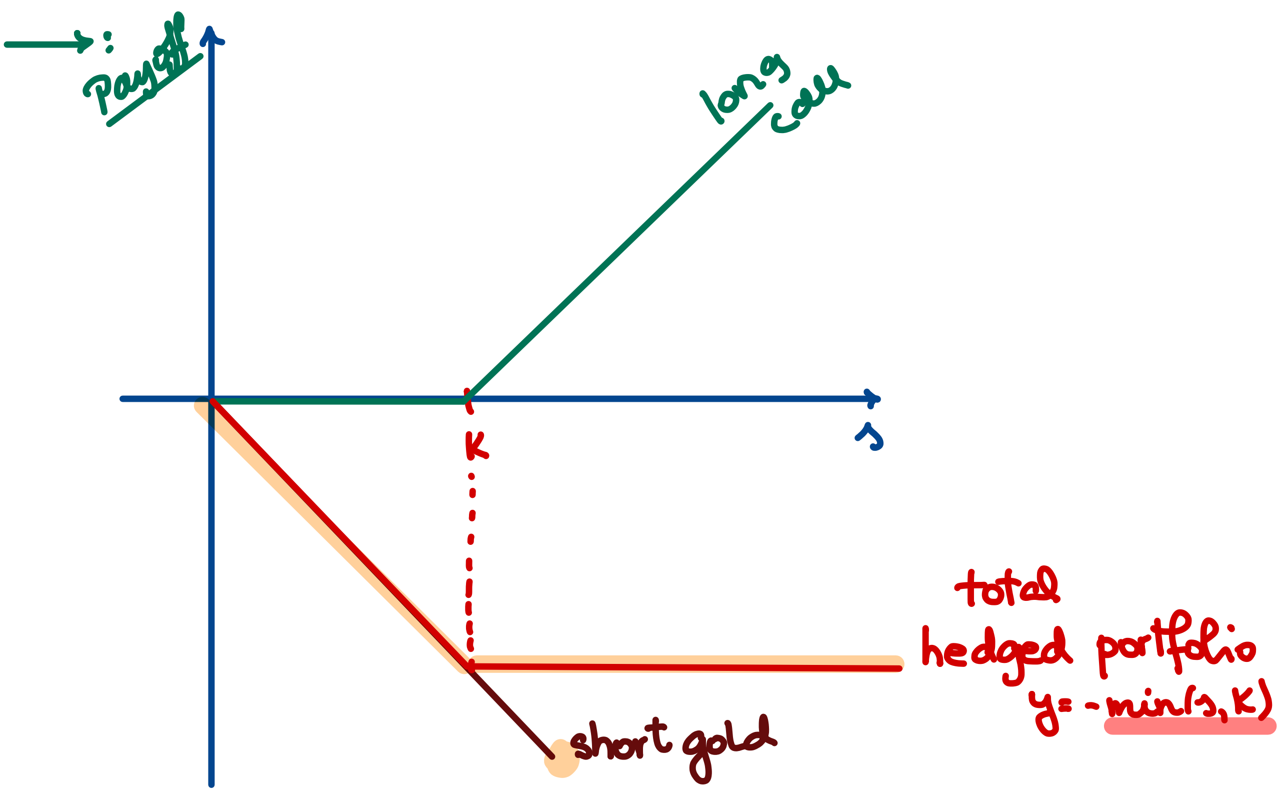
The jeweler models the market price of gold in one year as follows:

Gold price in one year	Probability	$\min(S(T), 900)$
750 per ounce	0.2	750
850 per ounce	0.5	850
950 per ounce	0.3	900

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of $900 per ounce. The option costs $100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the **hedged** portfolio per piece of jewelry produced.



Algebra:

$$
\begin{aligned}
\text{Payoff(total)} &= \text{Payoff(Gold)} + \text{Payoff(Call)} \\
&= -S(T) + (S(T) - K)_+ \\
&= \begin{cases} -K & \text{if } S(T) \geq K \\ -S(T) & \text{if } S(T) < K \end{cases} = -\min(S(T), K)
\end{aligned}
$$

- short underlying
- long call

} CAP

In this problem:

$$\text{Profit} = 1000 - \min(S(T), K) - 100e^{0.05}$$

$\mathbb{E}$

$$\mathbb{E}[\text{Profit}] = 1000 - \boxed{\mathbb{E}[\min(S(T), K)]} - 100e^{0.05}$$

$$= 750 \cdot 0.2 + 850 \cdot 0.5 + 900 \cdot 0.3 = 150 + 425 + 270 = 845$$

$$\mathbb{E}[\text{Profit}] = 1000 - 845 - 100e^{0.05} = 155 - 100e^{0.05} = \underline{49.87}$$

$\square$

Problem 5.4. The current price of stock a certain type of stock is $80. The premium for a 6−month, at-the-money call option is $5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) $80
(b) $85.72
(c) $85.84
(d) $85.96
(e) None of the above.

$\boxed{K=S(0)}$

$$S^* = K + FV_{0,T}(V_C(0))$$
$$= 80 + 5.84e^{0.04(0.5)} = \underline{85.96}$$

Problem 5.5. The price of gold in half a year is modeled to be equally likely to equal any of the following prices

$$\$1000, \quad \$1100, \quad \text{and} \quad \$1240.$$

Consider a half-year, $\boxed{\$1050}$-strike European call option on gold. What is the expected payoff of this option according to the above model?

$\rightarrow$: $\mathbb{E}[(S(T)-K)_+] = ?$

$$(S(T)-1050)_+ \sim \begin{cases} 190 & \text{w/ prob. } 1/3 \\ 50 & \text{w/ prob. } 1/3 \\ 0 & \text{w/ prob. } 1/3 \end{cases}$$

$$\mathbb{E}[(S(T)-K)_+] = 190 \cdot \frac{1}{3} + 50 \cdot \frac{1}{3} = \underline{80}$$

$\square$

Problem 5.6. (5 points) The "Very tasty goat cheese Co" sells artisan goat cheese at $10 per oz. They need to buy 200 gallons of goat milk in six months to make 200 oz of their specialty fall-equinox cheese. Non-goat milk aggregate costs total $500. They decide to buy six-month, $5-strike call options on gallons of goat milk for 0.50 per call option.

The continuously compounded risk-free interest rate equals 0.04.

In six months, the price of goat milk equals $6 per gallon. What is the profit of the company's hedged position?

(a) 395.92

(b) 397.98

(c) 400

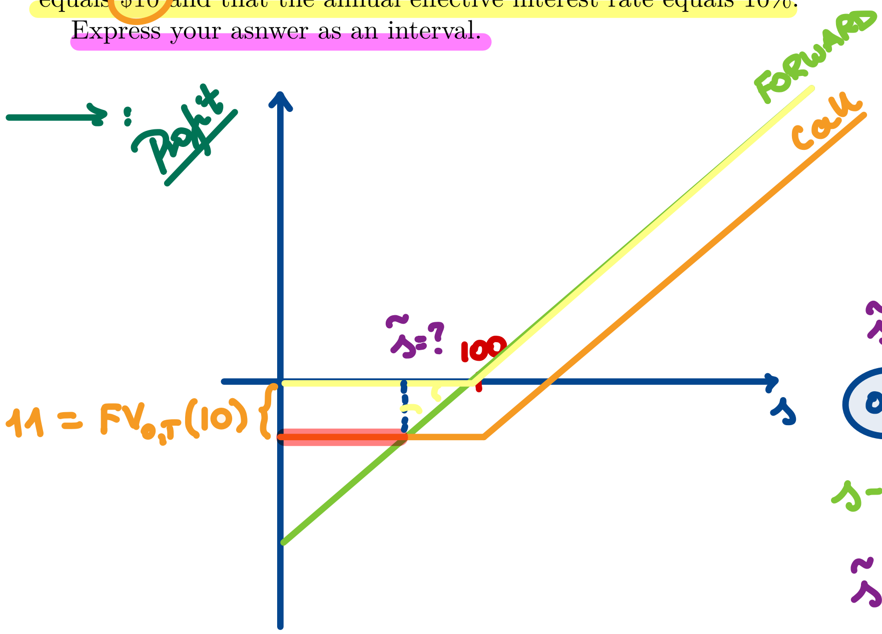
(d) 897.98

(e) None of the above.

$\longrightarrow$: $200(10) - 5(200) - 200(0.5)e^{0.04(0.5)} - 500 = \underline{397.98}$ $\square$

Problem 5.7. For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date T in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date T. Assume that the call's premium equals \$10 and that the annual effective interest rate equals 10%.

Express your asnwer as an interval.



$\tilde{s} = 100 - 11 = 89$

OR

$s - F = \underbrace{\boxed{(s-K)_+}}_{=0} - FV_{0,T}(V_C(0))$

$\tilde{s} = 100 - 11 = 89$

$[0, 89)$ $\square$

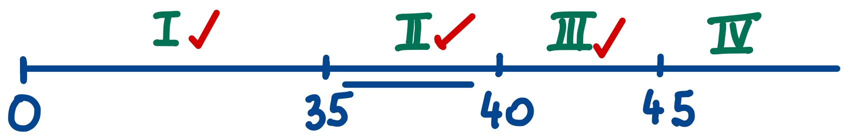
Problem 5.8. *Source: Sample IFM (Derivatives - Intro), Problem#11*
The current stock price is \$40, and the effective annual interest rate is 8%.
You observe the following option prices:

(1) The premium for a \$35-strike, 1-year European call option is \$9.12.
(2) The premium for a \$40-strike, 1-year European call option is \$6.22.
(3) The premium for a \$45-strike, 1-year European call option is \$4.08.

Assuming that all call positions being compared are **long**, at what 1-year stock price range does the \$45-strike call produce a higher profit than the \$40-strike call, but a lower profit than the \$35-strike call? Express your answer as an interval.

$\rightarrow: \quad (s-40)_+ - 6.22(1.08) < (s-45)_+ - 4.08(1.08) < (s-35)_+ - 9.12(1.08)$

$(s-40)_+ - 6.72 < (s-45)_+ - 4.41 < (s-35)_+ - 9.85$



I: $-6.72 < -4.41 < -9.85$ No sol'ns.

II: $0 - 6.72 < 0 - 4.41 < s-35-9.85$

$0 < s-35-9.85+4.41$

$\boxed{40.44 < s}$ Doesn't work!

III: $\underbrace{s-40-6.72 < 0-4.41} < s-35-9.85$

$s < 46.72-4.41$ $\qquad$ $40.44 < s$

$s < 42.31 \rightarrow 40.44 < s < 42.31$

$\boxed{(40.44, 42.31)}$

IV: $\underbrace{s-40-6.72 < s-45-4.41} < s-35-9.85$

$\underbrace{-46.72 < -49.41}_{\emptyset} < -44.85$