M339J: April 14th, 2021.

The Impact of Deductibles on Claim Frequency.

On compounding.

INDEPENDENT

In general, for an $\mathbb{N}_0$-valued random variable N w/ the pgf P_N,
and
a sequence of independent, identically dist'd random variables $\{M_1, M_2, \dots\}$ w/ a common pgf P_M,
we set

$$S = M_1 + M_2 + \dots + M_N = \sum_{j=1}^{N} M_j$$

(If $N=0$, then we set $S=0$).

Q: What is the dist'n of S?

If N is independent from the sequence $\{M_1, M_2, \dots\}$, then

$$\boxed{P_S(z) = P_N(P_M(z))}$$ ✓

Example.

- N is the total # of accidents in a year.
  $\{M_j, j \geq 1\}$ ... the # of claims per accident (# of people in the car, e.g.).

- N... the total # of losses
  $\{M_j, j \geq 1\}$ ... indicate whether a loss exceeded the deductible
  S... the number of claims.

The Set-Up.

Independent

Let $\{X_j, j = 1, 2, \dots\}$ be the severity random variables corresponding to the ground up losses.

Let N^L be an $\mathbb{N}_0$-valued random variable representing the number of losses (in a particular time frame).

Introduce a coverage modification such that

$$\mathbb{P}[X_j \text{ results in a claim}] =: v \quad \text{for all } j$$

In most cases, we have a deductible d and we get

$$v = \mathbb{P}[X_j > d] \qquad \text{for all } j$$

For every j, we can introduce

$$I_j := \begin{cases} 1 & \text{if } X_j > d \\ 0 & \text{if } X_j \leq d \end{cases}$$

$\Rightarrow$ $\boxed{I_j \sim \text{Bernoulli}(v)}$ for all j

and $\{I_j, j = 1, 2, \dots\}$ are independent

Let N^P denote the number of payments.

Then,

$$N^P = I_1 + I_2 + \dots + I_{N^L}$$

$$\Rightarrow \quad P_{N^P}(z) = P_{N^L}\left(P_{I_1}(z)\right)$$

$$= P_{N^L}\left((1-v) + v \cdot z\right)$$

$$\Rightarrow \boxed{P_{N^P}(z) = P_{N^L}\left(1 + v(z-1)\right)}$$

In the $(a,b,0)$ class, i.e., when N^L is such that for a parameter θ we can write its pgf as

$$\boxed{P_{N^L}(z) = P_{N^L}(z;\theta) = B(\theta(z-1))}$$

w/ B a real function independent of θ,

we get

$$P_{N^P}(z) = B\left(\theta\left(\cancel{1} + v(z-1) - \cancel{1}\right)\right) = B(\theta \cdot v(z-1))$$

$\Rightarrow$ We conclude that N^P comes from the same dist'n as N^L w/ the value of the parameter $\theta' = \theta \cdot v$.

Note that Poisson thinning is a special case.

39. You are given:

Independent

(i) The frequency distribution for the number of losses for a policy with no deductible is negative binomial with $r = 3$ and $\beta = 5$.

$N^L \sim \text{NegBin}(r=3, \beta=5)$

(ii) Loss amounts for this policy follow the Weibull distribution with $\theta = 1000$ and $\tau = 0.3$.

$X \sim \text{Weibull}(\theta = 1000, \tau = 0.3)$

Determine the expected number of payments when a deductible of 200 is applied.

(A) Less than 5

(B) At least 5, but less than 7

(C) At least 7, but less than 9

(D) At least 9, but less than 11

(E) At least 11

N^P ... # of pmts after the deductible is applied

$\mathbb{E}[N^P] = ?$

$N^P \sim \text{NegBin}(\underbrace{r=3}_{\text{stays unchanged}}, \beta' = \beta \cdot v)$

$$\text{w/ } v := \mathbb{P}[X > d] = e^{-\left(\frac{d}{\theta}\right)^{\tau}} = e^{-\left(\frac{200}{1000}\right)^{0.3}} = e^{-\left(\frac{1}{5}\right)^{0.3}}$$

$$\mathbb{E}[N^P] = r \cdot \beta' = r \cdot \beta \cdot v = 3 \cdot 5 \cdot e^{-\left(\frac{1}{5}\right)^{0.3}}$$

$$\approx 8.093$$

86. Aggregate losses for a portfolio of policies are modeled as follows:

(i) The number of losses before any coverage modifications follows a Poisson distribution with mean λ.

$N^L \sim \mathbb{P}\text{oisson}(\lambda)$

(ii) The severity of each loss before any coverage modifications is uniformly distributed between 0 and b.

$X \sim U(0, b)$

The insurer would like to model the effect of imposing an ordinary deductible, d $(0 < d < b)$, on each loss and reimbursing only a percentage, c $(0 < c \le 1)$, of each loss in excess of the deductible.

It is assumed that the coverage modifications will not affect the loss distribution.

The insurer models its claims with modified frequency and severity distributions. The modified claim amount is uniformly distributed on the interval $[0, c(b-d)]$.

Determine the mean of the modified frequency distribution.

$N^P \sim \mathbb{P}\text{oisson}(\text{mean} = \lambda' = \lambda \cdot v)$

w/ $v = \mathbb{P}[X > d] = \frac{b-d}{b}$

(A) λ

(B) λc

(C) $\lambda \dfrac{d}{b}$

$\mathbb{E}[N^P] = \lambda \cdot \frac{b-d}{b}$

(D) $\lambda \dfrac{b-d}{b}$

(E) $\lambda c \dfrac{b-d}{b}$

Think about how you would simulate the random variables from the compounding set up!