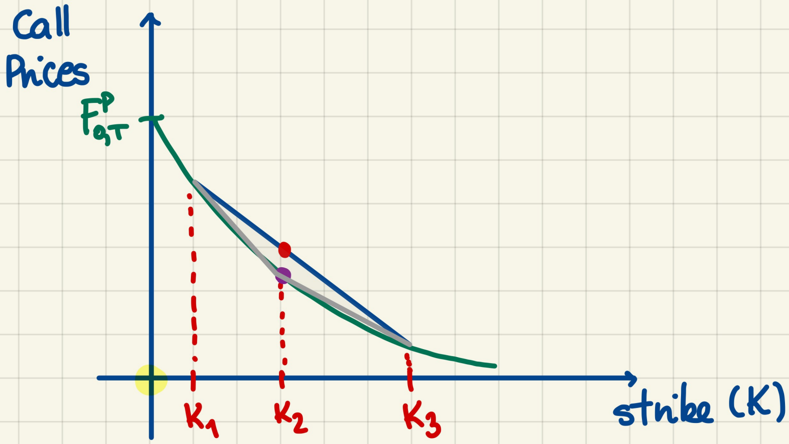
Call Price Convexity.



Claim. $\lambda = \dfrac{K_3 - K_2}{K_3 - K_1}$ (note: $K_2 = \lambda \cdot K_1 + (1-\lambda) K_3$)

we have that

$$\boxed{V_C(K_2) \leq \lambda \cdot V_C(K_1) + (1-\lambda) V_C(K_3)} \quad (CC)$$

$$\iff$$

$$\frac{V_C(K_1) - V_C(K_2)}{K_2 - K_1} \geq \frac{V_C(K_2) - V_C(K_3)}{K_3 - K_2}$$

→: Assume, to the contrary, there are $K_1 < K_2 < K_3$ such that

$$V_C(K_2) > \lambda \cdot V_C(K_1) + (1-\lambda) V_C(K_3) \quad \text{w/} \quad \lambda = \frac{K_3 - K_2}{K_3 - K_1}$$

I. Suspect an arbitrage opportunity. ✓

II. Propose an arbitrage portfolio:

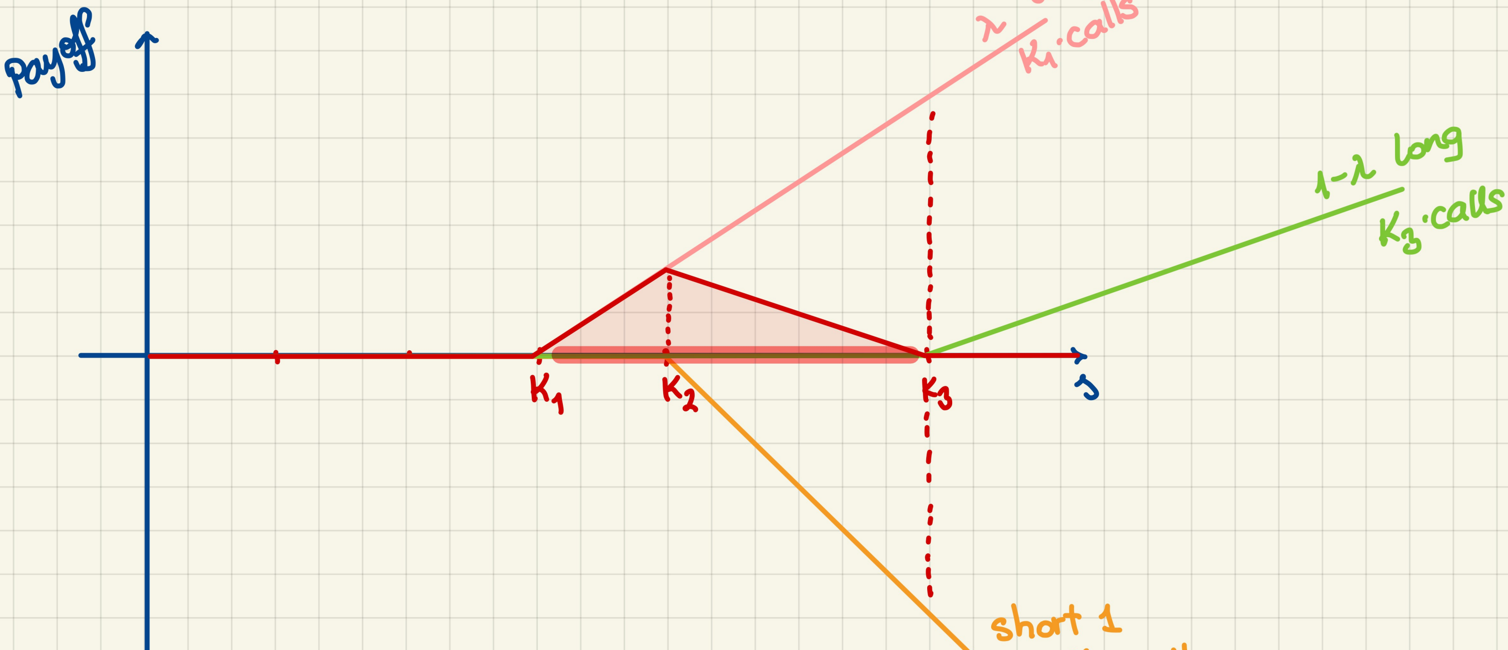
- Long λ K_1 · calls
- short 1 K_2 · calls
- Long $1-\lambda$ K_3 · calls

} Call Butterfly Spread

III. Verification.

Init. Cost: $\lambda \cdot V_C(K_1) + (1-\lambda) V_C(K_3) - V_C(K_2) < 0$ Initial inflow of money :)

It's sufficient to show $\boxed{\text{Payoff} \geq 0.}$



Payoff ≥ 0 ✓

It's an arbitrage portfolio!

- If $K_2 = \frac{K_1+K_3}{2}$, then it's a symmetric butterfly spread.
- Otherwise, it's asymmetric.
- Lack directionality.

However, it is used to speculate on low volatility.