UNIVERSITY OF TEXAS AT AUSTIN

Quiz #15

Exchange options.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 15.1. (5 points) **The minimum option**

Let $\mathbf{S} = \{S(t), t \geq 0\}$ and $\mathbf{Q} = \{Q(t), t \geq 0\}$ denote the prices of two risky assets. The payoff of the *minimum option* is given by

$$V_{min}(T) = \min(S(T), Q((T)).$$

Propose a replicating portfolio consisting of prepaid forward contracts on $\mathbf{S}$ and/or $\mathbf{Q}$, and exchange options on $\mathbf{S}$ and $\mathbf{Q}$.

Problem 15.2. (3 points) Let our market model include two continuous-dividend-paying stocks whose time$-t$ prices are denoted by $S(t)$ and $Q(t)$ for $t \geq 0$. The current stock prices are $S(0) = 160$ and $Q(0) = 80$. The dividend yield for the stock S is $\delta_S = 0.06$ and the dividend yield for the stock Q is $\delta_Q = 0.03$.

The price of an exchange option giving its bearer the right to forfeit one share of Q for one share of S in one year is given to be \$11.

Find the price of a maximum option on the above two assets with exercise date in a year. Remember that the payoff of the maximum option is $\max(S(1), Q(1))$.

Problem 15.3. (5 points) Assume that the continuously compounded interest rate equals 0.10.

Stock S has the current price of $S(0) = 70$ and does not pay dividends. Stock Q has the current price of $Q(0) = 65$ and it pays continuous dividends at the rate of 0.04.

An exchange option gives its holder the right to give up one share of stock Q for a share of stock S in exactly one year. The price of this option is \$11.50.

Another exchange option gives its holder the right to give up one share of stock S for a share of stock Q in exactly one year. Find the price of this option.

(a) About \$3.95
(b) About \$11.10
(c) About \$12.00
(d) About \$14.25
(e) None of the above.

Problem 15.4. (2 pts) Consider two European exchange options both with exercise date T, one that allows you to exchange a share of asset S for a share of asset Q, and another one that allows you to forfeit a share of asset Q and obtain a share of asset S in return.
On the other hand, consider the maximum option with the payoff

$$V_{max}(T) = \max(S(T), Q(T)),$$

and the minimum option with the payoff

$$V_{min}(T) = \min(S(T), Q(T)).$$

Then, in our usual notation,

$$V_{EC}(0, \mathbf{S}, \mathbf{Q}) + V_{EC}(0, \mathbf{Q}, \mathbf{S}) = V_{max}(0) + V_{min}(0).$$