W: March 23rd, 2019.

5. For a European call option on a stock within the Black-Scholes framework, you are given:

(i) The stock price is \$85. $S(0) = 85$

(ii) The strike price is \$80. $K = 80$

(iii) The call option will expire in one year. $T = 1$

(iv) The continuously compound risk-free interest rate is 5.5%. $r = 0.055$

(v) $\sigma = 0.50$

(vi) The stock pays no dividends. $\delta = 0$

Calculate the volatility of this call option. $\sigma_C = ?$

X (A) 50%

(B) 69%

(C) 123%

(D) 139%

(E) 278%

$$\sigma_C = |\Omega_C| \cdot \sigma_S$$

We know that $\Omega_C \geq 1$.

By def'n:

$$\Omega_C(s,t) = \frac{s \cdot \Delta_C(s,t)}{v_C(s,t)}$$

Today,

$$d_1(S(0)=85,0) = \frac{1}{0.5\sqrt{1}}\left[\ln\left(\frac{85}{80}\right) + \left(0.055 + \frac{0.25}{2}\right)\cdot 1\right]$$

$$\Rightarrow d_1(85,0) \approx 0.48$$

$$\Rightarrow d_2(85,0) = d_1(85,0) - \sigma\sqrt{1} = 0.48 - 0.50 = -0.02$$

$$\Rightarrow N(d_1(85,0)) = 0.6844$$

$$N(d_2(85,0)) = 1 - N(0.02) = 1 - 0.5080 = 0.4920$$

1.

$$\Omega_c(s,t) = \frac{s \cdot \Delta_c(s,t)}{v_c(s,t)}$$

$$= \frac{s \cdot \Delta_c(s,t)}{s \cdot \Delta_c(s,t) - Ke^{-r(T-t)} \cdot N(d_2(s,t))}$$

$$= \frac{1}{1 - \frac{Ke^{-r(T-t)} \cdot N(d_2(s,t))}{s\,\Delta_c(s,t)}}$$

↑

One method: test it @ home!

We can also do this:

$$\Delta_c(s,t) = e^{-\delta(T-t)} \cdot N(d_1(s,t))$$
$$= N(d_1(s,t))$$

↑ no dividends

$\Rightarrow$ <u>In our problem</u>: $\Delta_c(85,0) = 0.6844$

$$v_c(85,0) = 85 \cdot 0.6844 - 80\,e^{-0.055} \cdot 0.492 = 20.92$$

$$\Rightarrow \Omega_c(85,0) = \frac{85 \cdot 0.6844}{20.92} = 2.78$$

2.

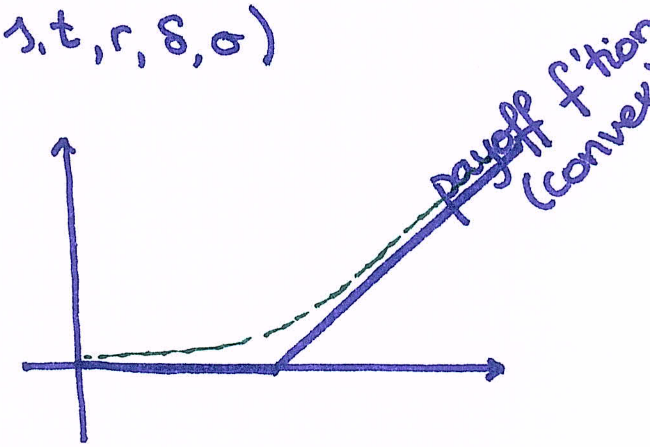
$$\Rightarrow \sigma_c(85,0) = \sigma_S \cdot \Omega_c(85,0) = 0.5 \cdot 2.78 = 1.39 \Rightarrow \mathcal{D} \blacksquare$$

Gamma.

$v(s,t,r,\delta,\sigma)$ ... value f'tion of portfolio

$$\Gamma(s,t,r,\delta,\sigma) = \frac{\partial^2}{\partial s^2} v(s,t,r,\delta,\sigma)$$

* Focus on the call:



Put·call Parity:

$$v_c(s,t) - v_p(s,t) = se^{-\delta(T-t)} - Ke^{-r(T-t)} \quad \Big/ \frac{\partial}{\partial s}$$

$$\Delta_c(s,t) - \Delta_p(s,t) = e^{-\delta(T-t)} \quad \Big/ \frac{\partial}{\partial s}$$

$$\boxed{\Gamma_c(s,t) = \Gamma_p(s,t)}$$

Vega.

$v(s,t,r,\delta,\sigma)$

$$\text{vega}(s,t,r,\delta,\sigma) = \frac{\partial}{\partial\sigma} v(s,t,r,\delta,\sigma)$$

Put-call Parity:

$$\frac{\partial}{\partial\sigma}\Big| \quad v_c(s,t,r,\delta,\sigma) - v_p(s,t,r,\delta,\sigma) = \underbrace{se^{-\delta(T-t)} - Ke^{-r(T-t)}}$$

$$\boxed{\text{vega}_c(s,t,r,\delta,\sigma) = \text{vega}_p(s,t,r,\delta,\sigma)}$$