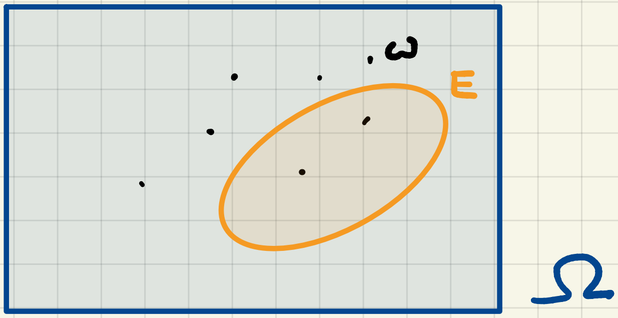
M378K : August 28th, 2024.

Probability.

Def'n.



ω... elementary outcomes

$E \subseteq \Omega$ ... events

Def'n. If E and F are events on the same Ω such that $\boxed{E \cap F = \emptyset}$,
we say that E and F are mutually exclusive (or disjoint).

Def'n. $\mathbb{P}$ is a probability (distribution) on Ω if:

(i) $\mathbb{P}[\Omega] = 1$

(ii) $\mathbb{P}[E] \geq 0$ for all E

(iii) For $\{E_j : j = 1, \dots\}$ pairwise disjoint events

$$\mathbb{P}\left[\bigcup_{j=1}^{\infty} E_j\right] = \sum_{j=1}^{\infty} \mathbb{P}[E_j]$$

Problem. You are given that

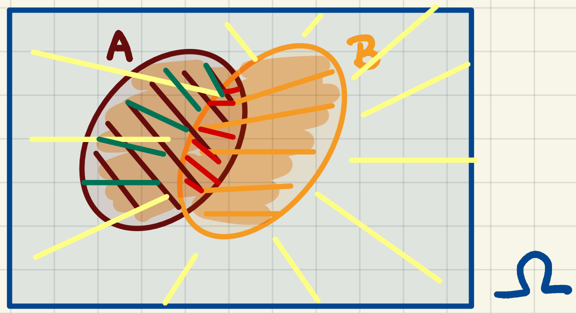
$$\mathbb{P}[A \cup B] = 0.7$$

and

$$\mathbb{P}[A \cup B^c] = 0.9$$

Calculate $\mathbb{P}[A]$.

$\longrightarrow$:



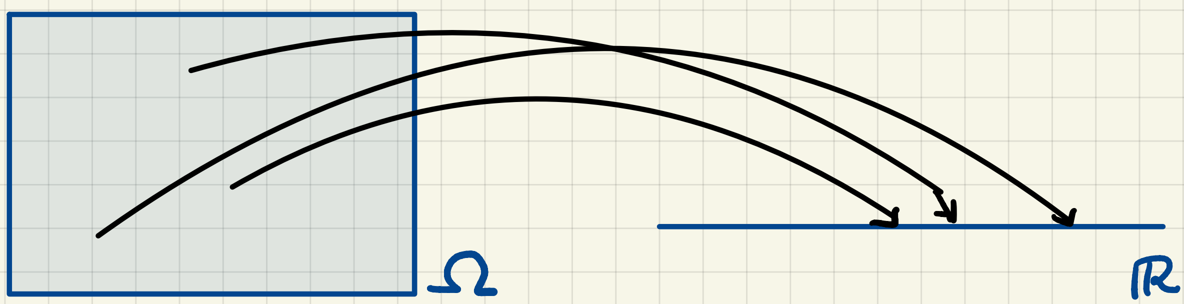
$$\left.\begin{aligned} \mathbb{P}[A\cup B] &= \boxed{\mathbb{P}[A]} + \boxed{\mathbb{P}[B]} - \boxed{\mathbb{P}[A\cap B]} = 0.7 \\ \mathbb{P}[A\cup B^c] &= \boxed{\mathbb{P}[A]} + \boxed{\mathbb{P}[B^c]} - \boxed{\mathbb{P}[A\cap B^c]} = 0.9 \end{aligned}\right\} +$$

$$2\mathbb{P}[A] + 1 - \mathbb{P}[A] = 1.6$$

$$\boxed{\mathbb{P}[A] = 0.6}$$

$\square$

Def'n. $\boxed{\text{Random Variable}}$



Discrete Random Variables.

Def'n. Given a set B, we say that the random variable Y is B-valued if $\mathbb{P}[Y \in B] = 1$.

Def'n. A r.v. Y is said to be discrete if there exists a set S such tha S is either finite or countable and Y is S-valued.

Def'n. The support S_Y of the discrete r.v. Y is the smallest set S such that Y is Y is S-valued.

Note: Say that you know

$$\mathbb{P}[Y=y] \quad \text{for all} \quad y \in S_Y$$

We are interested in the probability that Y hits some $B \subseteq S_Y$.

$$\mathbb{P}[Y \in B] = \sum_{y \in B} \mathbb{P}[Y=y]$$

Def'n. The probability mass function (pmf) of a discrete random variable Y is a function

$$\boxed{p_Y : S_Y \longrightarrow [0,1]}$$

given by $\quad p_Y(y) = \mathbb{P}[Y=y] \quad$ for all $y \in S_Y$

We display p_Y frequently as a distribution table:

$Y \sim$

y	y_1	y_2	y_3	....	y_k	....
$p_Y(y)$	p_1	p_2	p_3	....	p_k	....

Properties: (i) $p_Y(y) \in [0,1]$

(ii) $\displaystyle\sum_{y \in S_Y} p_Y(y) = \underline{1}$