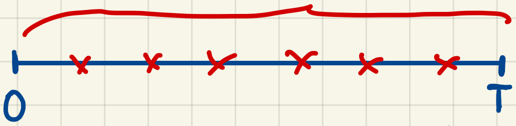
Static Portfolios.

Step #1. Decide who your protagonist is!

Step #2. Set up the time-line (mentally or "on paper")!

Static: no intermediate cashflows!



"today" — Cashflow from the perspective of your agent. ‼ **Initial Cost**

"time horizon" — Cashflow/Wealth from the perspective of the agent. ‼ **Payoff**

$$\boxed{\text{Profit} := \text{Payoff} - \boxed{FV_{0,T}}(\text{Initial Cost})}$$

- If Profit > 0, we call it a **gain**.
- If Profit < 0, we call it a **loss**.
- If Profit $= 0$, we say that we **broke even**.

Example. [Investing in a zero-coupon bond]

C... redemption amount

T... maturity date

r... continuously compounded, risk-free interest rate

Initial Cost: the bond's price Ce^{-rT}

Payoff: C

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= C - \cancel{e^{rT}}(C\cancel{e^{-rT}}) = 0$$

Example. [Taking a simple loan]

r... ccrfir

0 ———— T

T... the loan's term, i.e., the time @ which the loan must be repaid in full

L... loan amount, i.e., the amount borrowed @ time 0

Initial Cost: $-L$ (the negative sign is because the agent is RECEIVING L @ time 0)

Payoff: $-Le^{rT}$ (the negative sign is because the agent is GIVING UP Le^{rT} @ time T)

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= \cancel{-Le^{rT}} + \cancel{e^{rT}\cdot(+L)} = 0$$

$\square$

Example. [The Outright Purchase of a Non-Dividend-Paying Stock]

0 ———— T

Initial Cost: $S(0)$

Payoff: $S(T)$ a random variable

Goal. Study the payoff and the profit as a function of the final stock price.

$\boxed{\text{Introduce:}}$ s... an independent argument taking values in $[0,+\infty)$; it stands for the FINAL ASSET PRICE, i.e., it's a "placeholder" for the random variable $S(T)$

Now, we can define the PAYOFF FUNCTION which describes the dependence of the payoff on the independent argument s.

$\boxed{\text{Notation:}}$ v... payoff function $\quad v: [0,+\infty) \longrightarrow \mathbb{R}$

$v(s)$ is the investor's payoff if the final asset price is s