University of Texas at Austin

Quiz #2

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Please, provide your **complete solutions** to the following questions:

Problem 2.1. (2 pts) Let $N_1, N_2, \dots, N_\ell$ be independent, Poisson random variables with respecive parameters $\lambda_1, \lambda_2, \dots, \lambda_e ll$. Then, the random variable $N := N_1 + N_2 + \cdots + N_\ell$ is also Poisson with the parameter $\lambda = \max(\lambda_1, \lambda_2, \dots, \lambda_\ell)$. *True or false? Why?*

Solution: FALSE
The parameter of the Poisson random variable N is actually $\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_\ell$.

Problem 2.2. (2 points) For a random variable X have the exponential distribution. Then, for a constant $\tau > 0$, the random variable $X^{1/\tau}$ has the Weibull distribution. *True or false? Why?*

Solution: TRUE
The justification is in the class notes.

Problem 2.3. (2 pts) Let X have the loglogistic distribution. Then, the random variable $X' = 1/X$ also has the loglogistic distribution. *True or false? Why?*

Solution: TRUE
From the tables, the cdf of X can be written as

$$F_X(x) = \frac{(x/\theta)^\gamma}{1 + (x/\theta)^\gamma}, \qquad x > 0,$$

for parameters γ and θ.

In class, we learned that for $y > 0$,

$$\begin{aligned} F_{X'}(y) &= 1 - F_X(1/y) \\ &= 1 - \frac{(1/y\theta)^\gamma}{1 + (1/y\theta)^\gamma} \\ &= \frac{1}{1 + (1/y\theta)^\gamma} \\ &= \frac{(y/\theta^*)^\gamma}{1 + (y/\theta^*)^\gamma} \end{aligned}$$

with $\theta^* = 1/\theta$.

Please, provide your *final answer only* to the following questions:

Problem 2.4. (2 pts) The ground-up loss random variable is denoted by X. An insurance policy on this loss has an ordinary deductible of d. Then, the expected **policyholder** payment per loss equals

$$\mathbb{E}[X \wedge d].$$

True or false?

Solution: TRUE

Problem 2.5. (2 points) The ground-up loss random variable is denoted by X. An insurance policy on this loss has a **franchise** deductible of d and no policy limit. Then, the expected **policyholder** payment per loss equals

$$\mathbb{E}[X\mathbb{I}_{[X<d]}].$$

True or false?

Solution: TRUE

Problem 2.6. (5 pts) Let the loss random variable X be Pareto with $\alpha = 3$ and $\theta = 5000$. There is a franchise deductible of $d = 1000$.

Then, in our usual notation,

(a) $3,500 \leq \mathbb{E}[Y^P] < 4,500$

(b) $4,500 \leq \mathbb{E}[Y^P] < 5,500$

(c) $5,500 \leq \mathbb{E}[Y^P] < 6,500$

(d) $6,500 \leq \mathbb{E}[Y^P] < 7,500$

(e) None of the above

Solution: (a)

$$\mathbb{E}[Y^P] = e_X(d) + d = \frac{d+\theta}{\alpha-1} + d = \frac{1000+5000}{2} + 1000 = 4000.$$