M339D: September 21st, 2022.

Sharpe Ratio.

Start w/ a portfolio P consisting of risky investments.

Let R_P denote its return.

Let $\boxed{r_f}$ denote the risk-free interest rate.

Now, we construct the portfolio xP so that:

- we give the weight x to portfolio P

and

- we give the weight $\boxed{(1-x)}$ to the risk-free investment.

Let R_{xP} denote the return of this new portfolio.

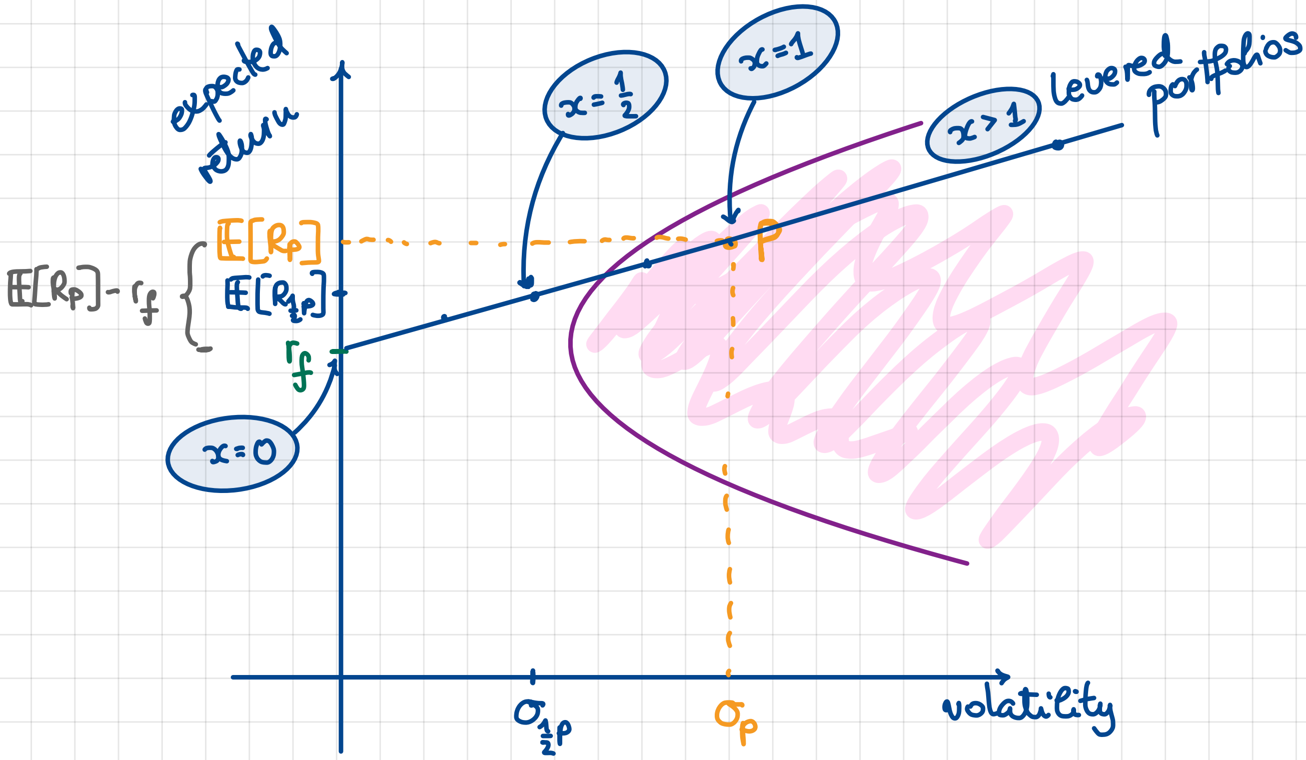
We know: $\boxed{R_{xP} = x \cdot R_P + (1-x) \cdot r_f}$

- $$\mathbb{E}[R_{xP}] = x \cdot \mathbb{E}[R_P] + (1-x) \cdot r_f$$
$$= r_f + x\left(\mathbb{E}[R_P] - r_f\right)$$

$$\boxed{\mathbb{E}[R_{xP}] - r_f = x \cdot \underbrace{\left(\mathbb{E}[R_P] - r_f\right)}_{\text{(expected) excess return or risk premium}}}$$

- $$\mathrm{Var}[R_{xP}] = \mathrm{Var}[x \cdot R_P + \underbrace{(1-x) \cdot r_f}_{\text{deterministic}}] =$$
$$= \mathrm{Var}[x \cdot R_P] = x^2 \cdot \mathrm{Var}[R_P]$$

$$\boxed{SD[R_{xP}] = x \cdot SD[R_P]}$$

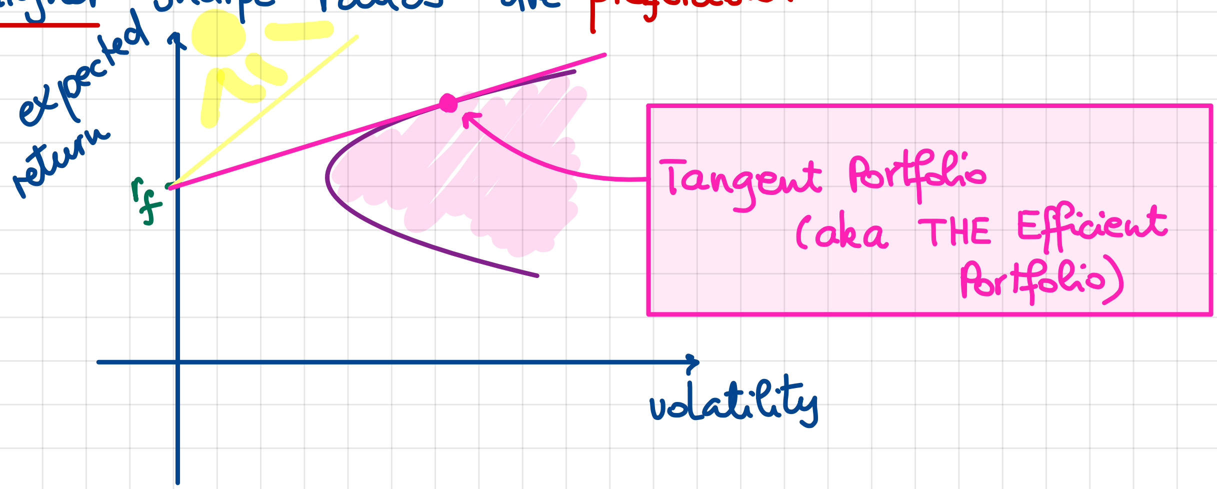


Q: What is the slope of the line $(0, r_f)$ and $(\sigma_P, \mathbb{E}[R_P])$?

$\longrightarrow$: $\boxed{\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\mathbb{E}[R_P] - r_f}{\sigma_P}}$ Reward-to-Risk Ratio

The Sharpe Ratio

Note:
- All the portfolios on the blue line above have the same Sharpe ratio.
- Higher Sharpe ratios are preferable.



8) You are given the following information about a two-asset portfolio:

(i) $\boxed{\text{The Sharpe ratio of the portfolio is 0.3667.}}$

(ii) The annual effective risk-free rate is 4%. $\boxed{r_f = 0.04}$

(iii) If the portfolio were 50% invested in a risk-free asset and 50% invested in a risky asset X, its expected return would be 9.50%. $\}\ 0.5X = P$

Now, assume that the weights were revised so that the portfolio were 20% invested in a risk-free asset and 80% invested in risky asset X. $\}\ 0.8X = P'$

Calculate the standard deviation of the portfolio return with the revised weights.

$\longrightarrow$: $\boxed{\sigma_{p'} = ?}$

(A) 6.0%

(B) 6.2%

(C) 12.8%

(D) 15.0%

(E) 24.0%

$R_{p'} = 0.8 \cdot R_X + 0.2 r_f$

$\Rightarrow \boxed{\sigma_{p'} = 0.8 \cdot \sigma_X}$

(i) $\Rightarrow$ Sharpe ratio of X is 0.3667.

$\Rightarrow \dfrac{\boxed{E[R_X]} - \boxed{r_f}}{\sigma_X} = 0.3667$

By def'n

$$\frac{1}{2}E[R_X] + \frac{1}{2}r_f = 0.095$$

$$E[R_X] + r_f = 0.19 \qquad /(-2r_f)$$

$$E[R_X] - r_f = 0.19 - 2(0.04) = 0.11$$

$$\sigma_X = \frac{0.11}{0.3667} = 0.3 \quad \Rightarrow \quad \boxed{\sigma_{p'} = 0.8 \cdot 0.3 = 0.24}$$

Required Returns

Objective. To figure out whether a portfolio P can be improved by "adding" (more of) a particular security I.

The Criterion.

$$\mathbb{E}[R_I] > r_f + \boxed{\frac{\sigma_I}{\sigma_P} \cdot \rho_{P,I}} \left(\mathbb{E}[R_P] - r_f \right)$$

$\boxed{\frac{\sigma_I}{\sigma_P} \cdot \rho_{P,I}} \;!!\; \beta_I^P$ the beta of the investment I w/ portfolio P

Def'n. The required return of Investment I given portfolio P is:

$$\boxed{r_I := r_f + \beta_I^P \left(\mathbb{E}[R_P] - r_f \right)}$$