M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
Practice Problems for In-Term Exam 2
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam.
Time: 50 minutes

TRUE/FALSE

1 (2)	TRUE	FALSE
2 (2)	TRUE	FALSE
3 (2)	TRUE	FALSE
4 (2)	TRUE	FALSE
5 (2)	TRUE	FALSE

MULTIPLE CHOICE

1 (5)	a	b	c	d	e
2 (5)	a	b	c	d	e
3 (5)	a	b	c	d	e
4 (5)	a	b	c	d	e
5 (5)	a	b	c	d	e
6 (5)	a	b	c	d	e

FOR GRADER'S USE ONLY:

T/F	1.	2.	M.C.	Σ

2.1. **TRUE/FALSE QUESTIONS.** *Please note your answers on the front page.*

Problem 2.1. Gamma of a call bull spread is always positive. *True or false?*

Solution: FALSE

Problem 2.2. Assume the Black-Scholes model. The elasticity of a European put option is always nonpositive. *True or false?*

Solution: TRUE

Problem 2.3. (2 points) In order to **both** delta hedge and gamma hedge a position in a certain option, the market-maker must trade in another type of option (i.e., not only in the money-market and the underlying risky asset). *True or false?*

Solution: TRUE

Problem 2.4. (2 points) Market makers usually do not need to rebalance their portfolios after the initial hedge is established. *True or false?*

Solution: FALSE

Problem 2.5. (2 points) A market maker who delta-hedges **completely** insures himself against losses. *True or false?*

Solution: FALSE

2.2. **FREE-RESPONSE PROBLEMS.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 2.6. (15 points) Consider a non-dividend-paying stock whose current price is \$45 per share. Its volatility is given to be 0.20.

The continuously compounded risk-free interest rate is 0.04.

A market maker sells a European, $91-$day, \$50-strike call option on the above stock for \$0.42 and delta-hedges the commitment using shares of stock. The call's delta at time-0 is 0.1841. The market-maker does not update the delta-hedge for a week. Then, she realizes that the call option is at-the-money and decides to liquidate the entire portfolio. What is the market maker's profit?

Solution: The initial cost of the portfolio is

$$-0.42 + 0.1841(45) = 7.8645.$$

After one week, the time to expiration of the call option is 84 days and the current stock price is \$50. In our usual notation, we have

$$d_1 = \frac{1}{0.2\sqrt{\frac{84}{365}}}\left[\ln(50/50) + \left(0.04 + \frac{0.04}{2}\right)\left(\frac{84}{365}\right)\right] = 0.143918 \approx 0.14,$$

$$d_2 = 0.14 - 0.2\sqrt{84/365} = 0.0440548 \approx 0.04.$$

From the standard normal tables, we get

$$N(d_1) = 0.5557, \quad N(d_2) = 0.516.$$

So, the call's price one week after it was written is

$$V_C(1 \text{ week}) = 50(0.5557 - e^{-0.04(84/365)}(0.516)) = 2.22141.$$

So, the value of the total portfolio at that time equals

$$-2.22141 + 0.1841(50) = 6.98359.$$

So, the profit over the one-week period is

$$6.98359 - e^{0.04(7/365)}(7.8645) = -0.886945.$$

Problem 2.7. (10 points) Let $S(t)$ denote the time$-t$ price of a continuous-dividend-paying stock with dividend yield δ and volatility σ.

The continuously compounded risk-free interest rate is denoted by r.

You write a special option which pays $\min(S(T), K)$ for a positive monetary amount K at time$-T$. You want to delta-hedge this commitment. What is the time-0 delta of the special option, expressed using the notation given above?

Solution: The value function of the special option at time$-T$, i.e., its payoff function, is

$$v(s,T) = \min(s,K) = K + \min(s-K,0) = K - \max(K-s,0) = K - (K-s)_+.$$

This means that we can replicate the special option with a long zero-coupon bond redeemable at time$-T$ for K and a short European time$-T$, strike$-K$ put option. So, the current delta of the special option is

$$\Delta(S(0),0) = e^{-\delta T}N(-d_1(S(0),0))$$

with

$$d_1(S(0),0) = \frac{1}{\sigma\sqrt{T}}\left[\ln(S(0)/K) + (r - \delta + \sigma^2/2)T\right].$$

Problem 2.8. (10 points) Assume the Black-Scholes framework for a non-dividend-paying stock whose current price is \$51.

A market-maker writes a European call option and sells it for \$9.25. Then, the market-maker delta-hedges by trading in the shares of the underlying stock. You are given the following current values of the greeks of the call option:

- the Δ is 0.66;

- the Γ is 0.02;
- the Θ is -0.01 per day.

The continuously compounded risk-free interest rate is 0.04.

Using the delta-gamma-theta approximation, calculate the approximate profit for the market-maker after one day if the stock price drops to \$50.

Solution: The initial cost for the market maker is

$$-9.25 + 0.66(51) = 24.41.$$

The delta-gamma-theta approximation for the call price after one day is

$$9.25 - 0.66 + \frac{1}{2}(0.02) - 0.01 = 8.59.$$

So, the market-maker's wealth after one day is approximately

$$-8.59 + 0.66(50) = 24.41.$$

So, the market-maker's approximate profit equals

$$24.41 - 24.41e^{0.04/365} = -0.00267522.$$

2.3. MULTIPLE CHOICE QUESTIONS.

Problem 2.9. (5 points) Assume the Black-Scholes framework.

The goal is to delta-hedge a one-year, at-the-money straddle on a non-dividend-paying stock whose current price is \$50. The stock's volatility is 0.20.

The continuously compounded risk-free interest rate is 0.10.

What is the cost of delta-hedging the straddle using shares of the underlying stock?

(a) \$22.58
(b) \$23.23
(c) \$24.33
(d) \$25.19
(e) None of the above.

Solution: **(a)**
The Δ of the straddle equals

$$2\Delta_C - 1 = 2N(d_1) - 1$$

with

$$d_1 = \frac{1}{0.2}(0.10 + 0.02) = 0.6.$$

Our answer is

$$50(2N(0.60) - 1) = 50(2(0.7257) - 1) = 50(0.4515) = 22.575.$$

Problem 2.10. (5 points) Assume the Black-Scholes framework. The current stock price is \$50 per share. Its dividend yield is 0.01 and its volatility is 0.25.

The continuously compounded risk-free interest rate is 0.05.

Consider a one-year, \$55-strike European put option on the above stock. What is the volatility of the put option?

(a) 1.013
(b) -0.534
(c) 6.6
(d) 0.978
(e) None of the above.

Solution: **(a)**
In our usual notation,

$$\begin{aligned} d_1 &= \frac{1}{0.25}\left[\ln\left(\frac{50}{55}\right) + 0.05 - 0.01 + \frac{(0.25)^2}{2}\right] = -0.0962 \approx -0.10, \\ d_2 &= -0.10 - 0.25 = -0.35. \end{aligned}$$

So, the put-option delta is

$$\Delta_P = -e^{-0.01}N(0.1) = -e^{-0.01}(0.5398) = -0.5344.$$

The put price is

$$V_P(0) = 55e^{-0.05}N(0.35) - 50(0.5344) = 55e^{-0.05}(0.6368) - 50(0.5344) = 6.59586.$$

The option's elasticity is

$$\Omega_P = -\frac{0.5344(50)}{6.59586} = -4.05103.$$

So, the put's volatility is $\sigma_P = \sigma\Omega_P = 1.01276$.

Problem 2.11. Which of the following greeks is usually negative?

(a) Call delta.
(b) Call gamma.
(c) Call theta.
(d) Call vega.
(e) None of the above.

Solution: (c)

Problem 2.12. Consider the following portfolio:

- 5 long options of type I,
- 4 long options of type II,
- 1 written option of type III.

The prices of the three options are 0.75, 1.00, and 1.50, respectively, while the option elasticities are 10, 7, and 2, respectively. What is the elasticity of the above portoflio?

(a) 5
(b) 7
(c) 10
(d) 12
(e) None of the above.

Solution: (c)
Let $S(0)$ denote the current stock price. The deltas of the three options are

$$\begin{aligned}\Delta_I &= \frac{10 \times 0.75}{S(0)} = \frac{7.5}{S(0)},\\ \Delta_{II} &= \frac{7 \times 1}{S(0)} = \frac{7}{S(0)},\\ \Delta_{III} &= \frac{2 \times 1.5}{S(0)} = \frac{3}{S(0)}.\end{aligned}$$

So, the delta of the portfolio is

$$\Delta = 5 \times \frac{7.5}{S(0)} + 4 \times \frac{7}{S(0)} - \frac{3}{S(0)} = \frac{62.5}{S(0)}.$$

The portfolio's price is

$$V(0) = 5 \times 0.75 + 4 \times 1 - 1.50 = 6.25.$$

So, the portfolio elasticity is

$$\Omega = \frac{\Delta S(0)}{V(0)} = \frac{62.5}{6.25} = 10.$$

Problem 2.13. (5 points) Assume the Black-Scholes model is used. The current price of a continuous-dividend-paying stock is \$50. Its dividend yield is given to be 0.03.

The continuously compounded, risk-free interest rate equals 0.03.

You observe the price of an at-the-money, one-year European put option on the stock as equal to \$6.93. What is the implied volatility of the stock?

(a) 0.18

(b) 0.24

(c) 0.36

(d) 0.42

(e) None of the above.

Solution: (c)
Let's find the expression for d_1. We have

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T\right] = \frac{1}{\sigma\sqrt{T}}\left[\left(\frac{\sigma^2}{2}\right)T\right] = \frac{\sigma}{2}.$$

Hence, $d_2 = -\frac{\sigma}{2}$. The Black-Scholes put price satisfies

$$6.93 = 50e^{-0.03}\left(N\left(\frac{\sigma}{2}\right) - N\left(-\frac{\sigma}{2}\right)\right) = 50e^{-0.03}\left(2N\left(\frac{\sigma}{2}\right) - 1\right).$$

So, we get

$$N\left(\frac{\sigma}{2}\right) = 0.5714 \quad \Rightarrow \quad \frac{\sigma}{2} = 0.18 \quad \Rightarrow \quad \sigma = 0.36.$$

Problem 2.14. (5 points) The current stock price is equal to \$50. Consider a European call option whose current price is \$3.43. The call's current Δ is 0.60 and its Γ is 0.02. What is the approximate call price if the stock price increases to \$52 in a short time interval?

(a) 4.03
(b) 4.27
(c) 4.41
(d) 4.67
(e) None of the above.

Solution: (d)
According to the delta-gamma approximation, after a short time interval dt, we have that the value of the call is approximately

$$v_C(S(dt), dt) \approx 3.43 + 0.60(2) + \frac{1}{2}(0.02)(2)^2 = 4.67.$$

Problem 2.15. (5 points) The current price of a continuous-dividend-paying stock is given to be \$92. The stock's volatility is 0.35 and its dividend yield is 0.02.
The continuously compounded risk-free interest rate is 0.05.
Consider a \$90-strike European call option on the above stock with exercise date in a quarter-year.
What is the Black-Scholes price of this call option?

(a) 5.05
(b) 7.66
(c) 7.71
(d) 7.89
(e) None of the above.

Solution: (d)

$$d_1 = 0.26, d_2 = 0.08.$$

So,

$$V_C(0) = 92e^{-0.02/4} \times 0.6026 - 90e^{-0.05/4} \times 0.5319 \approx 7.89.$$

Problem 2.16. (5 pts) Assume the Black-Scholes framework. Let the current price of a non-dividend-paying stock be equal to $S(0) = 95$ and let its volatility be equal to 0.35. Consider a European call on that stock with strike 100 and exercise date in 9 months. Let the risk-free continuously compounded interest rate be 6% per annum.

Denote the price of the call by $V_C(0)$. Then,

(a) $V_C(0) < \$5.20$
(b) $\$5.20 \leq V_C(0) < \$7.69$
(c) $\$7.69 \leq V_C(0) < \$9.04$
(d) $9.04 \leq V_C(0) < \$11.25$
(e) None of the above.

Solution: (d)
Using the Black-Scholes formula one gets the price of about 11.06.

Problem 2.17. Assume the Black-Scholes setting. Let $S(0) = \$63.75, \sigma = 0.20, r = 0.055$. The stock pays no dividend and the option expires in 50 days (simplify the number of days in a year to 360).

What is the price of a \$60-strike European put?

(a) 0.66
(b) 0.55
(c) 0.44
(d) 0.37
(e) None of the above.

Solution: (d)
In our usual notation, the price is

$$V_P(0) = Ke^{-r \cdot T} N(-d_2) - S(0)N(-d_1)$$

with

$$d_1 = \frac{1}{0.2\sqrt{5/36}} \left(\ln\left(\frac{63.75}{60}\right) + (0.055 + \frac{1}{2}0.2^2)\left(\frac{5}{36}\right) \right) = 0.95,$$
$$d_2 = d_1 - 0.25\sqrt{0.125} = 0.88.$$

So,

$$V_P(0) = 60e^{-0.055 \cdot \frac{5}{36}}(1 - 0.8106) - 63.75 \cdot (1 - 0.8289) = 0.37.$$

Problem 2.18. Assume the Black-Scholes setting.
Assume $S(0) = \$28.50, \sigma = 0.32, r = 0.04$. The stock pays a 1.0% continuous dividend and the option expires in 110 days (simplify the number of days in a year to 360).
What is the price of a \$30-strike put?

(a) 2.75
(b) 2.10
(c) 1.80
(d) 1.20
(e) None of the above.

Solution: (a)
In our usual notation, the price is

$$V_P(0) = Ke^{-r \cdot T} N(-d_2) - S(0)e^{-\delta \cdot T} N(-d_1)$$

with

$$d_1 = -0.15, \quad d_2 = -0.33.$$

So, $V_P(0) = 2.75$.