Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Solution: Practice Problems for In-Term One
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Notes: This is a closed book and closed notes exam. This exam is graded out of 100 points.
Time: 50 minutes

1.1. TRUE/FALSE QUESTIONS.

Problem 1.1. (2 points) An agent is **only** allowed to long a forward contract if he/she is willing to take physical delivery of the underlying asset.

Solution: FALSE
It is possible to have *cash settlement* on the delivery date if the forward contract stipulates so.

Problem 1.2. (2 points) Denote the continuously compounded, risk-free interest rate by r and denote the equivalent annual effective interest rate by i. Then, $\ln(1+i) = r$. *True or false?*

Solution: TRUE

Problem 1.3. (2 pts) Two dice are rolled, the single most probable sum of the numbers of the upturned faces is 7. *True or false?*

Solution: TRUE

Problem 1.4. (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date T on the underlying asset S:

- one long call with strike 40,
- two long calls with strike 50,
- one short call with strike 65.

Let $S(T) = 69$. Then, the payoff from the above position at time T is less than 60.

Solution: FALSE
The payoff is

$$(69 - 40) + 2(69 - 50) - (69 - 65) = 63.$$

1.2. MULTIPLE CHOICE QUESTIONS.

Problem 1.5. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions given by

$$f(x) = 2x - 10$$

and

$$g(x) = \begin{cases} \min(x, 7) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then, $g(f(7))$ equals ...

(a) -4
(b) 0
(c) 4
(d) 7
(e) None of the above

Solution: (c)

Problem 1.6. *Source: Sample P exam, Problem #176.*
In a group of health insurance policyholders, 20% have high blood pressure and 30% have high cholesterol. Of the policyholders with high blood pressure, 25% have high cholesterol. A policyholder is randomly selected from the group. Calculate the probability that a policyholder has high blood pressure, **given** that the policyholder has high cholesterol.

(a) 1/6
(b) 1/5
(c) 1/4
(d) 2/3
(e) 5/6

Solution: (a)
Let E be the event containing all the policyholders with a high blood preasure and let F be the event which contains all the policyholders with high cholesterol. We are given the following

$$\mathbb{P}[E] = 0.2, \quad \mathbb{P}[F] = 0.3, \quad \mathbb{P}[F|E] = 0.25.$$

Then, the conditional probability we are looking for equals

$$\mathbb{P}[E|F] = \frac{\mathbb{P}[E \cap F]}{\mathbb{P}[F]} = \frac{\mathbb{P}[F|E]\mathbb{P}[E]}{\mathbb{P}[F]} = \frac{(0.25)(0.2)}{0.3} = \frac{1}{6}\,.$$

Problem 1.7. Harry plays a simple lottery in which the winnings are distributed as follows:

- \$5 with probability 0.2,
- \$10 with probability 0.4,
- \$20 with probability 0.4.

It turns out that Harry has to pay a fee to collect his winnings. If the actual amount he wins is smaller than \$9, then the fee is defined to equal the amount that Harry won – thus, he walks away with nothing. If the actual amount he wins is between \$9 and \$15, he does not have to pay anything in fees and gets a bonus of \$4. If the actual amount he wins is larger than \$15, then he pays the \$15-fee and pockets the remainder. What is the expected value of the net amount Harry collects?

(a) 3

(b) 6.4

(c) 7.6

(d) 15

(e) None of the above.

Solution: (c)
The actual amount that Harry gets is

- \$0 with probability 0.2,
- \$14 with probability 0.4,
- \$5 with probability 0.4.

So, his expected winnings are

$$14(0.4) + 5(0.4) = 7.6$$

Problem 1.8. Hermione sells short one share of non-dividend-paying stock. The stock is currently valued at \$80 per share.The continuously compounded risk-free interest rate is 0.04. Hermione intends to close the short sale in one year. What is the final stock price for which Hermione will break even?

Solution: In our usual notation, the break-even point is

$$S(0)e^{rT} = 80e^{0.04(1)} = 80e^{0.04} = 83.26486.$$

Problem 1.9. The current market price of widgets is \$4 per widget. The widget factory plans to sell their next batch of 100 widgets in half a year. The total aggregate costs of production of widgets will be equal to \$350.

The factory enters 100 short forward contracts on widgets for delivery in half a year. The forward price is \$4.20 per widget.

What is the factory's profit if the final price of widgets in half a year ends up being \$4.40?

(a) 30

(b) 50

(c) 70

(d) 90

(e) None of the above.

Solution: (c)
The factory will sell the widgets per the forward contract for \$420 total. The total aggregate costs are given to be \$350. Hence, the profit is \$70.

Problem 1.10. Maryam bakes batches of cupcakes for a cupcake convention. She buys forward 21 pounds of raspberries from a local farmer at the forward price of \$5.60 per pound.

She projects to bake 336 cupcakes and sell each for \$3. The total and aggregate non-raspberry costs of baking the cupcakes are \$200.

If the market price of raspberries on the day of the cupcake convention is \$5.40, what is Maryam's profit?

(a) \$690.40
(b) \$694.60
(c) \$890.40
(d) \$894.60
(e) None of the above.

Solution: (a)

$$336 \times 3 - 21 \times 5.60 - 200 = 690.40.$$

Problem 1.11. The **writer** of a call option has ...

(a) an obligation to sell the underlying asset at the strike price.
(b) a right, but **not** an obligation, to sell the underlying asset at the strike price.
(c) an obligation to buy the underlying asset at the strike price.
(d) a right, but **not** an obligation, to buy the underlying asset at the strike price.
(e) None of the above.

Solution: (a)

Problem 1.12. (5 points) Roger owns a cow named Elsie. Her estimated worth today is \$3,750. Roger enters into a forward agreement with Harry to sell him Elsie the cow in 6 months for \$4,000. On the delivery date, Roger changes his mind and wants cash settlement instead. Harry agrees. They look into the "Bovine Blue Book" and realize that Elsie's worth on that date is \$3,500.

What is the cash flow that has to take place as part of the cash settlement?

(a) \$500 from Roger to Harry
(b) \$500 from Harry to Roger
(c) \$250 from Roger to Harry
(d) \$250 from Harry to Roger
(e) None of the above.

Solution: (b)

Problem 1.13. (5 points) A farmer produces one thousand crates of apples. The total and aggregate costs of production are \$48,000. The farmer enters a forward contract for the entire harvest to hedge at a forward price of \$69 per crate at harvest time.

The market price of apples at harvest time is \$70 per crate.

What is the farmer's profit?

(a) 1000 loss

(b) 1000 gain

(c) 21000 gain

(d) 22000 gain

(e) None of the above.

Solution: (c)

$$69000 - 48000 = 21000$$

Problem 1.14. (5 points) **Pancakes, Inc.** produces strawberry pancakes for the pancake festival. It longed a forward contract on 100 pounds of strawberries at \$2.50 per pound to be delivered to the festival and added to the pancakes. According to the contract with the organizers, the total fixed revenue will be \$6,000 for the pancakes produced with the above strawberries. Costs other than strawberries total \$1200.

On the morning of the pancake festival, the market price of strawberries is \$2.25 per pound.

Find the company's profit.

(a) 2300

(b) 1550

(c) 2550

(d) 1500

(e) None of the above.

Solution: (e)

$$6000 - 1200 - 250 = 4550$$

Problem 1.15. (5 points) The current price of stock a certain type of stock is \$80. The premium for a 6−month, at-the-money call option is \$5.84. Let the continuously compounded, risk-free interest rate be 0.04. What is the break-even point of this call option?

(a) \$80
(b) \$85.72
(c) \$85.84
(d) \$85.96
(e) None of the above.

Solution: (d)
The break-even point is

$$80 + 5.84e^{0.04/2} = 85.958$$