University of Texas at Austin

Quiz # 20

The tangent portfolio. Sharpe ratio.

Please, provide your **complete solutions** to the following problems. A graphical argument is acceptable.

Problem 20.1. (2 points) The tangent portfolio has the highest Sharpe ratio of all the portfolios in the feasible set. *True or false?*

Solution: TRUE

Problem 20.2. (2 points) Consider our usual coordinate system of portoflios with the volatility on the horizontal axis and the expected return on the vertical axis. Consider a portfolio P in that plane and look at the line through that portfolio and the point corresponding to the risk-free asset $(0, r_f)$. Then, the slope of this line is exactly the Sharpe ratio of the portfolio P. *True or false?*

Solution: TRUE

Problem 20.3. (2 points) Consider a portfolio P consisting of a collection of risky assets. You construct a new portfolio by investing a proportion ϕ of your wealth in portfolio P and the remainder of your wealth in the risk-free asset. Then, the excess return of the new portoflio is the same proportion ϕ of the excess return of the portfolio P. *True or false?*

Solution: TRUE

Problem 20.4. (9 points) Assume that the risk-free interest rate equals 0.04. The Sharpe ratio of asset S is given to be $1/4$ while the Sharpe ratio of asset Q equals $1/3$. You know that the volatility of S is twice the volatility of Q. If you build an equally weighted portfolio with assets S and Q as its two components, the expected return of this portfolio will be 0.10. What is the expected return of S and what is the expected return of Q?

Solution: From the condition on the Sharpe ratio of S, we get

$$\frac{\mathbb{E}[R_S] - r_f}{\sigma_S} = \frac{1}{4} \quad \Rightarrow \quad \sigma_S = 4(\mathbb{E}[R_S] - 0.04).$$

From the condition on the Sharpe ratio of Q, we obtain

$$\frac{\mathbb{E}[R_Q] - r_f}{\sigma_Q} = \frac{1}{3} \quad \Rightarrow \quad \sigma_Q = 3(\mathbb{E}[R_Q] - 0.04).$$

Since $\sigma_S = 2\sigma_Q$, we have

$$\begin{aligned} 4(\mathbb{E}[R_S] - 0.04) = 2(3)(\mathbb{E}[R_Q] - 0.04) \quad &\Rightarrow \quad 2(\mathbb{E}[R_S] - 0.04) = 3(\mathbb{E}[R_Q] - 0.04) \\ &\Rightarrow \quad 2\mathbb{E}[R_S] - 3\mathbb{E}[R_Q] = 0.08 - 0.12 = -0.04. \end{aligned}$$

On the other hand, from the given expected return on the equally-weighted portfolio, we know that

$$\frac{1}{2}(\mathbb{E}[R_S] + \mathbb{E}[R_Q]) = 0.10 \quad \Rightarrow \quad \mathbb{E}[R_S] + \mathbb{E}[R_Q] = 0.20.$$

Solving the above system of two equations with two unknowns, we get

$$\mathbb{E}[R_S] = 0.112 \quad \text{and} \quad \mathbb{E}[R_Q] = 0.088.$$