University of Texas at Austin

Quiz #19

Expected returns. Volatility.

Provide your **complete solution** to the following problems:

Problem 19.1. (10 points) Your model for the economy at the end of your period has three different states *good, so-so* and *bad*. You think that the probability that the economy will be in the *so-so* state is twice the probability that it will be in the *good* state. You also think that the probability that the economy will be in the *good* state is twice the probability that it will be in the *bad* state.

There are two assets in your market model called S and Q. Their returns, depending on the state of the economy are modeled as follows:

Asset	*good*	*so-so*	*bad*
S	10%	2%	-5%
Q	8%	-1%	-4%

Your portfolio is equally weighted between assets S and Q. What is the volatility of this total portfolio?

Solution: The probability of the *good* economy is $2/7$, the probability of the *so-so* economy is $4/7$, and the probability of the *bad* economy is $1/7$. So, the return R_T of the total portfolio has the following distribution

$$R_T = \begin{cases} 0.09 & \text{with probability } 2/7, \\ 0.005 & \text{with probability } 4/7, \\ -0.045 & \text{with probability } 1/7. \end{cases}$$

We get that the mean return of this portfolio equals

$$\mathbb{E}[R_T] = 0.09\left(\frac{2}{7}\right) + 0.005\left(\frac{4}{7}\right) - 0.045\left(\frac{1}{7}\right) = 0.0221429.$$

The second moment of the portfolio's return is

$$\mathbb{E}[R_T^2] = (0.09)^2\left(\frac{2}{7}\right) + (0.005)^2\left(\frac{4}{7}\right) + (-0.045)^2\left(\frac{1}{7}\right) = 0.00261786.$$

Therefore, the variance of the return equals

$$Var[R_T] = \mathbb{E}[R_T^2] - (\mathbb{E}[R_T])^2 = 0.00261786 - (0.0221429)^2 = 0.00212755.$$

Finally, the volatility of the portfolio is $\sigma_T = 0.0461254$.

Problem 19.2. (5 points) According to your model, the economy over the next year could be *good* or *bad*. You believe that *bad* and *good* are equally likely.

Consider two assets, X and Y, existing in this market. If the economy is *good* the return on asset X is 0.12, and the return on asset Y is 0.08. If the economy is *bad* the return on asset X is -0.04 and the return on asset Y is -0.02.

You construct a portfolio P using assets X and Y so that the portfolio's expected return equals 0.0325.

Calculate the volatility of this portfolio's return.

Solution: First, we need to figure out the weight w_X the asset X is given in portfolio P. The expected returns of assets X and Y are

$$\mathbb{E}[R_X] = \frac{1}{2}(0.12) + \frac{1}{2}(-0.04) = 0.06 - 0.02 = 0.04,$$
$$\mathbb{E}[R_Y] = \frac{1}{2}(0.08) + \frac{1}{2}(-0.02) = 0.04 - 0.01 = 0.03.$$

So, the portfolio P must be such that the weight given to the asset X equals $w_X = 0.25$. Hence, the distribution of the portfolio's return can be described as

$$R_P \sim \begin{cases} 0.09, & \text{with probability } 1/3 \\ -0.025, & \text{with probability } 2/3 \end{cases}$$

The second moment of the portfolio's return is

$$\mathbb{E}[R_P^2] = (0.09)^2 \times \frac{1}{2} + (-0.025)^2 \times \frac{1}{2} = 0.0043625.$$

The variance of the portfolio's return is

$$Var[R_P] = 0.0043625 - 0.0325^2 = 0.00330625.$$

Finally, its volatility equals $\sigma = \sqrt{0.00330625} = 0.0575$.