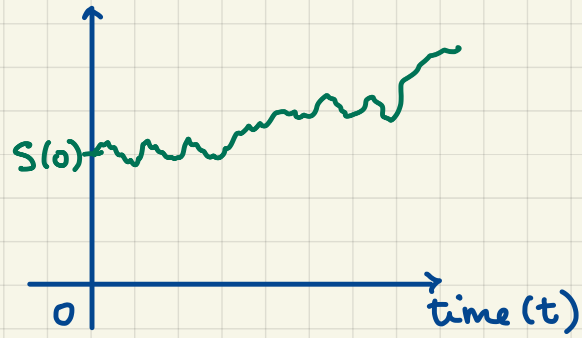
Stock Prices.

$S(t)$, $t \geq 0$ ... time-t stock price

$\boxed{\text{a stochastic process}}$



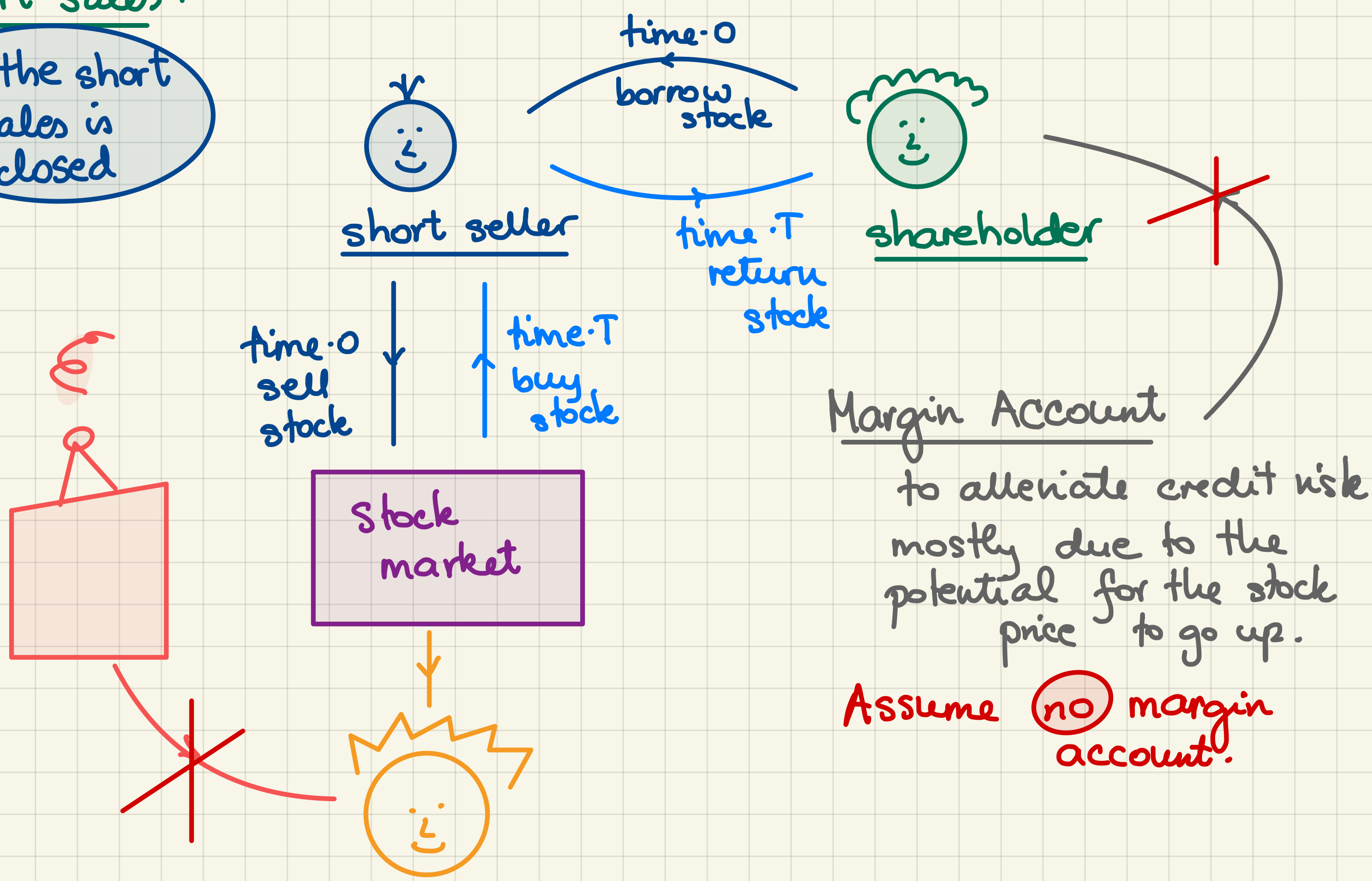
Outright Purchase of one share of stock.

Assume no dividends.

$-S(0)$ at 0; $S(T)$... random variable at T... time horizon

Short sales.

T... the short sales is closed



Assume no margin account.

$+S(0)$ at 0; $-S(T)$ at T

The Return of the Portfolio.

Say that your portfolio has n different securities in it.

$i = 1..n$ ... indices of the investment components in your portfolio

For every i : R_i ... the realized simple return of the i^{th} component over a particular time period (say, a year)

R_p ... the realized simple return of the portfolio

beg end

$$R_p := \frac{P_p^{end} - P_p^{beg}}{P_p^{beg}}$$

w/ P_p ... the price of the total portfolio, i.e., $P_p = \sum_{i=1}^{n} \underbrace{P_i}$ ← value of component i

$$R_p := \frac{\sum_{i=1}^{n} P_i^{end} - \sum_{i=1}^{n} P_i^{beg}}{P_p^{beg}}$$

$$= \sum_{i=1}^{n} \frac{P_i^{end} - P_i^{beg}}{P_p^{beg}} \cdot \frac{P_i^{beg}}{P_i^{beg}}$$

$$= \sum_{i=1}^{n} \underbrace{\frac{P_i^{beg}}{P_p^{beg}}}_{\overset{!!}{=}\, w_i} \cdot R_i$$

w_i ... portfolio weight of investment i

deterministic

$$\boxed{R_p = \sum_{i=1}^{n} w_i \cdot R_i}$$

⇒ The expected return is:

$$\boxed{\mathbb{E}[R_p] = \sum_{i=1}^{n} w_i \, \mathbb{E}[R_i]}$$