M339D: November 11th, 2022.

The Normal Approximation to the Binomial.

(de Moivre-Laplace)

Consider a sequence of binomial random variables:

$Y_n \sim Binomial(n = \#\text{ of trials}, p = \text{probab. of success})$

Then, $\mathbb{E}[Y_n] = np$

$Var[Y_n] = np(1-p) \Rightarrow SD[Y_n] = \sqrt{np(1-p)}$

$$\boxed{\boxed{\frac{Y_n - np}{\sqrt{np(1-p)}}} \xrightarrow{\mathcal{D}} N(0,1)}$$

Usage:

- Look @ "large" n (rule of thumb: $np \geq 10$ and $n(1-p) \geq 10$).
- $\mathbb{P}[a < Y_n \leq b]$

$$= \mathbb{P}\left[\frac{a-np}{\sqrt{np(1-p)}} < \underbrace{\frac{Y_n - np}{\sqrt{np(1-p)}}}_{\approx N(0,1) \sim Z} \leq \frac{b-np}{\sqrt{np(1-p)}}\right]$$

$$= \mathbb{P}\left[\frac{a-np}{\sqrt{np(1-p)}} < Z \leq \frac{b-np}{\sqrt{np(1-p)}}\right]$$

$$= N\left(\frac{b-np}{\sqrt{np(1-p)}}\right) - N\left(\frac{a-np}{\sqrt{np(1-p)}}\right)$$

N... cumulative dist'n f'tion of N(0,1), i.e.,

$N(z) = \mathbb{P}[Z \leq z]$

- In statistics, we use this theorem as.

$$\boxed{Y_n \approx Normal(mean = np, sd = \sqrt{np(1-p)})}$$

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Problem Set 9

The normal approximation to the binomial.

Problem 9.1. According to the Pew research center, 64% of Americans say that social media have a mostly negative effect on things (see https://pewrsr.ch/3dsV7uR). You take a sample of 1000 randomly chosen Americans. What is the approximate probability that at least 650 of them say that social media have a mostly negative effect on things?

→ : X... a r.v. denoting the # of surveyed people who claim social media are negative.

$$X \sim Binomial(n = \underline{1000}, p = \underline{0.64})$$

$$\mu_X = \mathbb{E}[X] = n \cdot p = 640$$

$$SD[X] = \sqrt{n \cdot p(1-p)} = \sqrt{1000(0.64)(0.36)} = 15.17893$$

R

$$\mathbb{P}[X \geq \underline{650}] = \mathbb{P}\left[\frac{X-640}{15.17893} \geq \frac{650-640}{15.17893}\right] \approx \mathbb{P}[Z \geq \underline{0.658808}]$$

R

R : 1-pnorm(0.6588078) = 0.255