M358K: October 1$^{\text{st}}$, 2021.

UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 5

Normal distribution.

Problem 5.1. Let Z be a standard normal random variable. Find the following probabilities:

i. $\mathbb{P}[-1.33 < Z \le 0.24]$
ii. $\mathbb{P}[0.49 < |Z|]$
iii. $\mathbb{P}[Z^4 < 0.0256]$
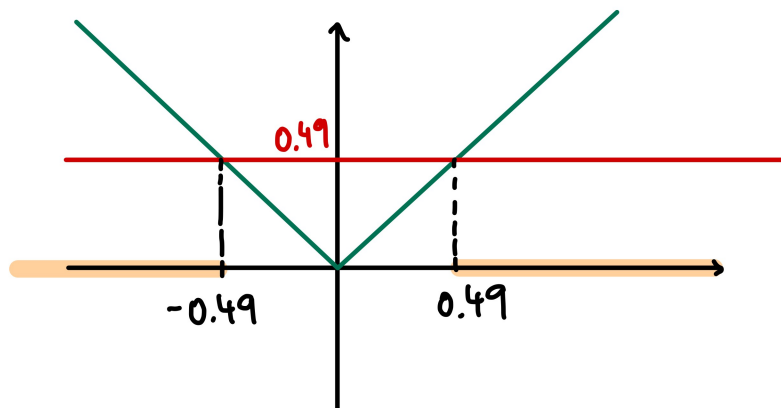iv. $\mathbb{P}[e^{2Z} < 2.25]$
v. $\mathbb{P}[\frac{1}{Z} < 2]$

i. $\mathbb{P}[-1.33 < Z \le 0.24] = \Phi(0.24) - \Phi(-1.33)$

$\text{pnorm}(0.24) - \text{pnorm}(-1.33) = 0.5030757$

ii. $\mathbb{P}[0.49 < |Z|] = ?$

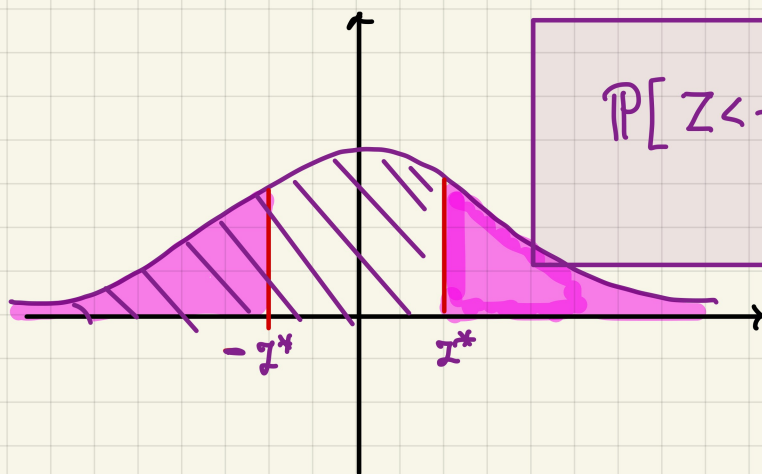$\mathbb{P}[Z < -0.49] + \mathbb{P}[Z > 0.49] = 2 \cdot \mathbb{P}[Z < -0.49]$

$2 * \text{pnorm}(-0.49) = 0.6241339$

iii. $\mathbb{P}[Z^4 < 0.0256] = \mathbb{P}[|Z| < \sqrt[4]{0.0256} = 0.4]$

$= \mathbb{P}[-0.4 < Z < 0.4]$

$= \mathbb{P}[Z < 0.4] - \mathbb{P}[Z < -0.4]$

$$\mathbb{P}[Z<-z^*] = 1 - \mathbb{P}[Z<z^*]$$
$$= \mathbb{P}[Z>z^*]$$

$$= \mathbb{P}[Z<0.4] - (1 - \mathbb{P}[Z<0.4]) =$$
$$= 2 \cdot \mathbb{P}[Z<0.4] - 1$$

2 * pnorm(0.4) − 1 = 0.3108435

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iv. $\mathbb{P}[e^{2Z} < 2.25] = \mathbb{P}[2Z < \ln(2.25)] = \mathbb{P}[Z < 0.5\ln(2.25)]$

↑
ln is an
increasing f^tion

pnorm(0.5 * log(2.25)) =
= 0.6574322

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v. $\mathbb{P}\left[\frac{1}{Z} < \boxed{2}\right] = ?$
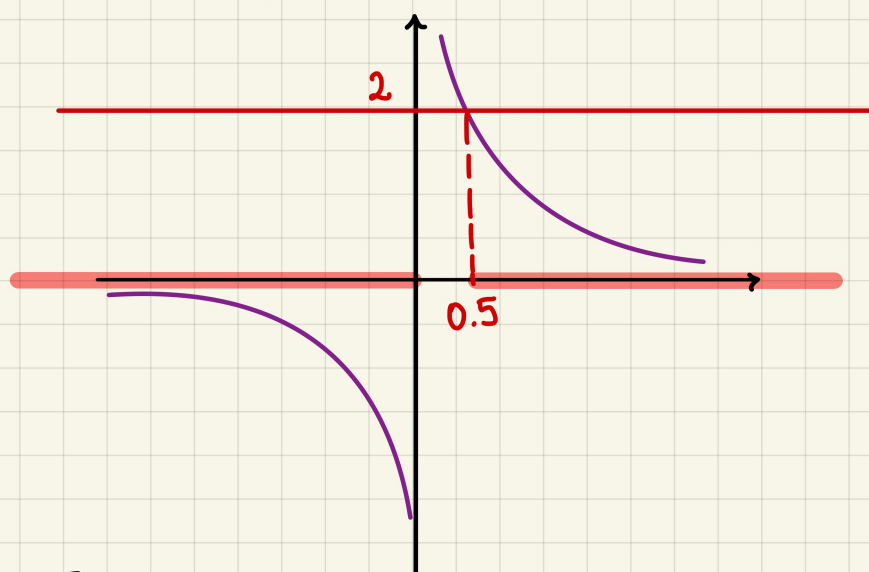
$$\mathbb{P}[Z<0] + \mathbb{P}[Z>0.5] = 0.5 + 1 - \mathbb{P}[Z<0.5] = 1.5 - \mathbb{P}[Z<0.5]$$

1.5 − pnorm(0.5) = 0.8085375