
UNIVERSITY OF TEXAS AT AUSTINQuiz #4

The exponential distribution.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 4.1. (15 points) The minimum of two independent exponential random variables is also exponential. *True or false? Why?*

Solution: TRUE

Let T_1 and T_2 be two independent exponential random variables with parameters θ_1 and θ_2 , respectively. Define $T = \min(T_1, T_2)$. Obviously, the support of the random variable T is $[0, \infty)$. Let us figure out the survival function of T . For every $t > 0$, we have

$$S_T(t) = \mathbb{P}[T > t] = \mathbb{P}[\min(T_1, T_2) > t] = \mathbb{P}[T_1 > t, T_2 > t].$$

Due to independence of random variables T_1 and T_2 , the above equals

$$\mathbb{P}[T_1 > t]\mathbb{P}[T_2 > t] = S_{T_1}(t)S_{T_2}(t).$$

We now recall the form of the survival function of an exponential random variable (or look into our STAM tables). The above equals

$$e^{-t/\theta_1}e^{-t/\theta_2} = e^{-t(\frac{1}{\theta_1} + \frac{1}{\theta_2})}$$

Finally, we note that T must be exponential with parameter θ satisfying

$$\frac{1}{\theta} = \frac{1}{\theta_1} + \frac{1}{\theta_2}.$$