M339D: March 29th, 2021.

The Relationship between prepaid forwards & forwards.

Forward Contract:

- pmt: $F_{0,T}$
- delivery of stock

Init. Cost = 0

Payoff = $S(T) - F_{0,T}$

Prepaid Forward:

- pmt: $F^P_{0,T}$

Init. Cost = $F^P_{0,T}$

- delivery of stock

Payoff = $S(T)$

I propose the following replicating portfolio for the prepaid forward contract:

PORT
- forward contract ✓
- invest $PV_{0,T}(F_{0,T})$ @ the ccrfir (r) to be withdrawn @ time T

This is a static portfolio, so I just need to check whether the payoffs match.

$$\text{Payoff}(\text{PORT}) = \underbrace{S(T) - F_{0,T}}_{\text{forward contract}} + \underbrace{FV_{0,T}\left(PV_{0,T}(F_{0,T})\right)}_{\text{risk-free investment}}$$

$$= S(T) - F_{0,T} + F_{0,T} = S(T)$$

=> This is, indeed, a replicating portfolio for the prepaid forward ☺

$\Rightarrow$ NO ARBITRAGE!

$$F^P_{0,T} = PV_{0,T}(F_{0,T})$$

$$\Leftrightarrow$$

$$F_{0,T} = FV_{0,T}(F^P_{0,T})$$

Focus on (prepaid) forwards on stocks.

Goal: To figure out the expressions for the prepaid forward price and the forward price in terms of more primitive "ingredients".

Case #1. NO DIVIDENDS.

Prepaid Forward: I.C. $= F^P_{0,T}$

Outright Purchase: I.C. $= S(0)$

$\Leftarrow$ NO ARBITRAGE!

Payoff: $S(T)$ = Payoff: $S(T)$

$\Rightarrow$ $\boxed{F^P_{0,T}(S) = S(0)}$

Q: What is the dependence on the delivery date?

NONE!

$\Rightarrow$

$$F_{0,T}(S) = FV_{0,T}(F^P_{0,T}(S)) = FV_{0,T}(S(0)) = S(0)e^{rT}$$

r... continuously compounded interest rate > 0

- Increasing with the delivery date!

Q: Propose a replicating portfolio for a forward contract? A Fully-Leveraged Purchase is a synthetic forward.

Case #2. CONTINUOUS DIVIDENDS.

δ... dividend yield

Init. Cost: | **Payoff:**

Prepaid Forward: $F^P_{0,T}(S) = S(0)e^{-\delta T}$

Outright Purchase: buy $e^{-\delta T}$ shares | own 1 share

$S(T) = S(T)$

NO ARBITRAGE!

$$\Rightarrow \boxed{F^P_{0,T}(S) = S(0)e^{-\delta T}} \xrightarrow[T \to +\infty]{} 0$$

❗ Decreasing w/ the delivery date ❗

$$\Rightarrow F_{0,T}(S) = FV_{0,T}\left(F^P_{0,T}(S)\right)$$

$$= FV_{0,T}\left(S(0)e^{-\delta T}\right) = e^{rT} \cdot S(0) \cdot e^{-\delta \cdot T}$$

↑ r... ccrfir

$$\Rightarrow \boxed{F_{0,T}(S) = S(0)e^{(r-\delta)\cdot T}}$$

the break-even point for the long stock.

❗ Depends on the order of r & δ whether it's decreasing/increasing/constant w.r.t. T ❗

Case #3. DISCRETE DIVIDENDS.

$k = 1..n$; $t_n \leq T$

by convention, if $t_n = T$, the dividend is paid just prior to the delivery

$$F^P_{0,T}(S) = S(0) - \sum_{k=1}^{n} PV_{0,t_k}(D_k)$$

In words, the investor must be compensated for the "loss" of dividends through the prepaid forward price.

$$F_{0,T}(S) = S(0)e^{rT} - \sum_{k=1}^{n} D_k e^{r(T-t_k)}$$

↑ r... ccrfir