University of Texas at Austin

Subjective probabilities.

Problem 2.1. IFM Sample (Introductory) Problem #6.
The following relates to one share of XYZ stock:

- The current price is 100.
- The forward price for delivery in one year is 105.
- An investor who decides to long the forward contract denotes by P the expected stock price in one year.

Determine which of the following statements about P is **TRUE**.

(A) $P < 100$
(B) $P = 100$
(C) $100 < P < 105$
(D) $P = 105$
(E) $P > 105$

Solution: (e)
Since the investor decided to long the forward contract, the payoff/profit will be

$$S(T) - 105$$

where $S(T)$ denotes the stock price on the delivery date T. The reason the investor chose to long the forward was the belief that the expected profit would be positive, i.e.,

$$\mathbb{E}[S(T)] = P > 105.$$

Problem 2.2. IFM Sample (Introductory) Problem #38.
The current price of a medical companys stock is 75. The expected value of the stock price in three years is 90 per share. The stock pays no dividends. You are also given:

- The risk-free interest rate is positive.
- There are no transaction costs.
- Investors require compensation for risk.

The price of a three-year forward on a share of this stock is X, and at this price an investor is willing to enter into the forward. Determine what can be concluded about X.

(A) $X < 75$
(B) $X = 75$
(C) $75 < X < 90$
(D) $X = 90$
(E) $X > 90$

Solution: (c)
Using the fact that the investor is willing to enter a forward contract, we conclude that $90 > X$. On the other hand, we know that, since there are no dividends,

$$X = S(0)e^{rT} = 75e^{3r} > 75.$$

The last inequality is valid since $r > 0$.

Problem 2.3. IFM Sample (Introductory) Problem #70.
Investors in a certain stock demand to be compensated for risk. The current stock price is 100. The stock pays dividends at a rate proportional to its price. The dividend yield is 2%. The continuously compounded risk-free interest rate is 5%. Assume there are no transaction costs.

Let X represent the expected value of the stock price 2 years from today. Assume it is known that X is a whole number. Determine which of the following statements is true about X.

(A) The only possible value of X is 105.
(B) The largest possible value of X is 106.
(C) The smallest possible value of X is 107.
(D) The largest possible value of X is 110.
(E) The smallest possible value of X is 111.

Solution: (c)
Say that an investor longs one share of stock. Then, with continuous reinvestment of dividends the investor's profit can be expressed, in our usual notation, as

$$S(T)e^{\delta T} - S(0)e^{rT}\,.$$

A rational investor who demands to be compensated for risk would only invest if the expected profit above were positive. So,

$$X = \mathbb{E}[S(T)] > S(0)e^{(r-\delta)T} = 100e^{(0.03)2} = 106.18365.$$