University of Texas at Austin

Problem Set # 4

Problem 4.1. *Source: An old ASM Manual for Exam MFE.*
A one-year American, GBP-denominated put option on euros allows the sale of 100 euros for 90 GBP. The evolution of the exchange rate over the next year is modelled with a two-period forward binomial tree. You have the following:

- The current spot exchange rate is 0.80 GBP per euro.
- The continuously compounded risk-free interest rate for the GBP is 0.06.
- The continuously compounded risk-free interest rate for the euros is 0.04.
- The volatility of the exchange rate of GBP to euros is 0.10.

What is the price of the above put option?

Problem 4.2. *Source: An old ASM Manual for Exam MFE.*
An American call option expiring in one year on a futures contract on a market index is modelled with a two-period binomial tree based on forward prices. You are given that:

- The futures price is currently 500.
- The option's strike price is 500.
- The volatility of the market index is 0.30.
- The continuously compounded risk-free interest rate is 0.06.
- The dividend yield of the market index is 0.02.

What is the amount of money lent in the replicating portfolio for the call at the beginning of the year?

Problem 4.3. You are fiven a TRUE/FALSE exam with 30 questions. Suppose that you need to answer 21 questions correctly in order to pass. You have no idea what the class is about and decide to toss a fair coin to answer all the questions; you circle TRUE if the outcome is tails and you circle FALSE if the outcome is heads. What is your estimate of the probability p that you manage to pass the exam using this strategy?

Hint: It is best to use the **normal approximation** to get the approximate probability. There is no need to use the continuity correction.

Problem 4.4. The current price of a continuous-dividend-paying stock is \$100 per share. The stock's dividend yield is 0.02. According to your model, the expected value of the stock price in one year is \$90 per share. You are also given:

The risk-free interest rate exceeds the dividend yield.

The one-year forward price on a share of this stock is denoted by F. At this price you are willing to enter into the forward. What is the smallest range of values F can take according to the above information?

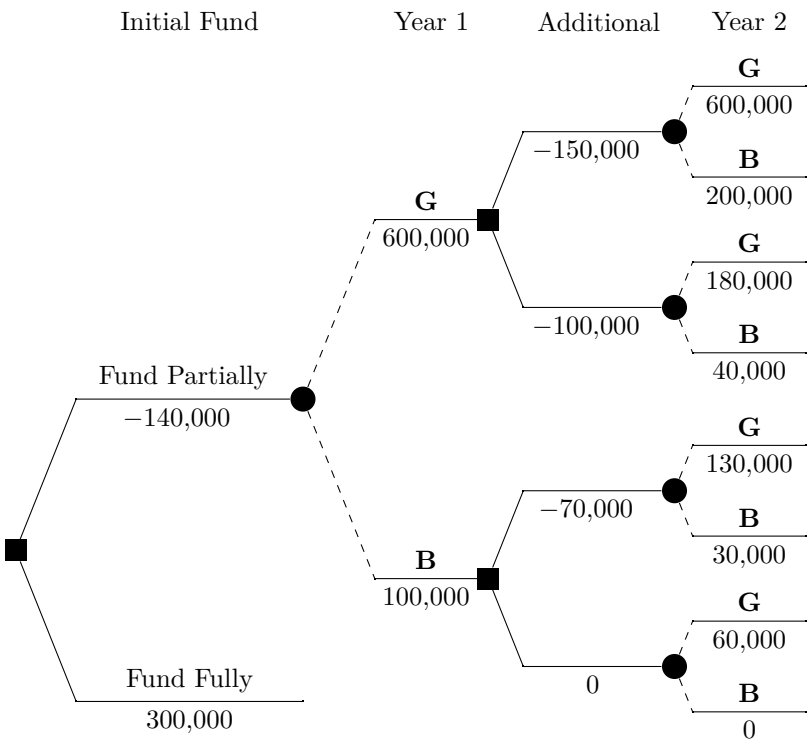
Problem 4.5. A marketing department is considering a marketing campaign. If the marketing campaign is fully funded at time-0 the project has a net present value of 300,000.

The decision tree below shows the cash flows associated with the extra revenue due to the marketing campaign being conducted in two stages. If the funding in the beginning of the Year 1 (i.e., at $t = 0$) is only partial with an option to provide different amounts of funding at the beginning of Year 2 (i.e., at $t = 1$) depending on how well the marketed product is doing.

This tree reflects two possible receptions of the marketed product at each information node (**G** = good, **B** = bad). The probability of the product being a success is given to be 1/2 and the probability of it being mediocre is 1/2. Failure is not an option.

Assume the interest rate is 0%.

Find the **initial** (i.e., at $t = 0$) value of the option to fund partially.



Problem 4.6. For Company A there is a p_A chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean μ_A and standard deviation σ_A.

For Company B there is a p_B chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean μ_B and standard deviation σ_B.

The total claim amounts of the two companies are independent. Calculate the probability that, in the coming year, Company B's total claim amount will strictly exceed Company A's total claim amount. In the expression you get, you are allowed to use the parameters given and the standard normal cumulative distribution function N.

Problem 4.7. *Source: Sample P exam Problem #81.*
Claims filed under auto insurance policies follow a normal distribution with mean $19,400$ and standard deviation $5,000$. Calculate the probability that the average of 25 randomly selected claims exceeds $20,000$.

Problem 4.8. A brand of light bulb has a lifetime (in months) that is modelled as normally distributed with mean 4 and variance 1. Roger wants to purchase a number of these bulbs with the intention of replacing them successively as they burn out. The light bulbs have mutually independent lifetimes.

What the smallest number of bulbs Roger needs to buy so that the succession of light bulbs produces light for at least 96 months with probability at least 0.9772?

Problem 4.9. The profit of an individual franchise which sells Pokemon plushies is normally distributed with mean 100000 and standard deviation 30000. The profits of individual franchises are assumed to be independent.

Calculate the probability that the total profit of two randomly selected franchises will exceed 1.6 times the profit a third randomly selected franchise.