UNIVERSITY OF TEXAS AT AUSTIN

Quiz # 7

The normal distribution. $z-$scores.

Provide your **final answers only** to the following questions.

Problem 7.1. When a variable follows a normal distribution, what percent of observations are contained within 1.75 standard deviations of the mean?

(a) 68.26%
(b) 91.98%
(c) 95.99%
(d) Not enough information is given.
(e) None of the above.

Solution: (b) or (e)
Note: I added (e) as correct above since I had a typo in the original formulation of the problem.
Let $X \sim Normal(mean = \mu, sd = \sigma)$. Then,

$$\mathbb{P}[|X-\mu| \leq 1.75\sigma] = \mathbb{P}[-1.75\sigma < X - \mu \leq 1.75\sigma] = \mathbb{P}\left[-\frac{1.75\sigma}{\sigma} < \frac{X-\mu}{\sigma} \leq \frac{1.75\sigma}{\sigma}\right] = \mathbb{P}[-1.75 < Z < 1.75]$$

where $Z \sim N(0,1)$. Using the symmetry of the bell curve and the standard normal tables, we get

$$\mathbb{P}[-1.75 < Z < 1.75] = 2\mathbb{P}[Z < 1.75] - 1 = 2(0.9599) - 1 = 0.9198.$$

Problem 7.2. Which of the following statements about z-scores is/are <u>true</u>?

(a) Larger z-scores are always better.
(b) The z-score for an observation that is equal to the mean is 1.
(c) If a z-score is 2 that means that the observation is two times the value of the mean.
(d) If a z-score is negative that means that the observation is less than mean.
(e) None of the above are true.

Solution: (d)

Problem 7.3. Heights of boys in a high school are approximately normally distributed with mean of 175 cm standard deviation of 5 cm. What is the 20^{th} percentile of heights?

(a) 165.88 cm
(b) 171.71 cm
(c) 173.32 cm
(d) 181.01 cm
(e) None of the above.

Solution: (e)
The 20^{th} percentile of the standard normal distribution is $z_{0.20} = \Phi^{-1}(0.20) = -0.84$. So, the 20^{th} percentile of the heights is

$$175 + (-0.84)5 = 170.80$$