M378K: February 13th, 2026.

Random Vectors.

Say, we are interested in two (or more) r.v.s as a PAIR (or VECTOR), i.e., we look at

$$(Y_1, Y_2)$$

Then, it's not sufficient to only look @ their "individual" dist'ns, but also their association.

Example. Y_i ... cointoss for $i=1,2$ of fair coins

independence.

$\{Y_1 = H, Y_2 = H\}$ $\{Y_1 = T, Y_2 = H\}$

$\{Y_1 = H, Y_2 = T\}$ $\{Y_1 = T, Y_2 = T\}$

complete dependence

$\{Y_1 = H, Y_2 = H\}$ ✗

✗ $\{Y_1 = T, Y_2 = T\}$

Discrete 2D Envioronment.

The Joint Distribution Table.

X \ Y	y_1	y_2	....	y_j	....	y_l	
x_1							$p_X(x_1) = \sum_{j=1}^{l} p_{1j}$
x_2							$\vdots$
$\vdots$							$\vdots$
x_i				p_{ij}			$p_X(x_i) = \sum_{j=1}^{l} p_{ij}$
$\vdots$							$\vdots$
x_m							$\vdots$
	The Marginal Dist'n of Y						

The MARGINAL dist'n of X

$p_{ij} = \mathbb{P}[X = x_i, Y = y_j]$, i.e., the joint pmf for all i,j

X and Y are independent iff

$$\boxed{p_{ij} = p_X(x_i) \cdot p_Y(y_j) \quad \text{for } i,j}$$

Example. We *independently* throw two fair dice and record the results as Y_1 and Y_2

joint pmf $\quad p_{ij} = \frac{1}{36}$ for all $1 \leq i,j \leq 6$

Define $\boxed{Z = Y_1 + Y_2}$

Q: What is the joint dist'n table for (Y_1, Z)?

Y_1 \ Z	2	3	4	5	6	7	8	9	10	11	12
1							0	0	0	0	0
2	0							0	0	0	0
3	0	0				$\boxed{\frac{1}{36}}$			0	0	0
4	0	0	0							0	0
5	0	0	0	0							0
6	0	0	0	0	0						
	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Joint Dist'ns: The Continuous Case

Recall: For a continuous r.v. Y w/ a pdf f_Y, we can calculate probabilities as

$$\mathbb{P}[Y \in [a,b]] = \mathbb{P}[a \leq Y \leq b]$$
$$= \int_a^b f_Y(y)\,dy \quad \text{for all } a \leq b$$

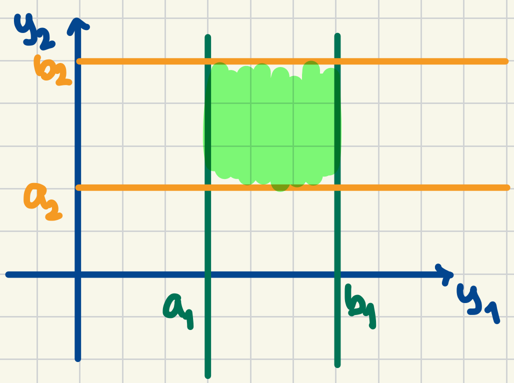
In multiple dimensions.

Say the random vector $(Y_1, Y_2, \ldots, Y_n)$ is jointly continuous w/ density

$$\boxed{f_{Y_1,\ldots,Y_n}}$$

Then,

$$\mathbb{P}[Y_1 \in [a_1,b_1], Y_2 \in [a_2,b_2], \ldots, Y_n \in [a_n,b_n]] =$$
$$= \int_{a_1}^{b_1}\int_{a_2}^{b_2}\cdots\int_{a_n}^{b_n} f_{Y_1,\ldots,Y_n}(y_1,\ldots,y_n)\,dy_n \cdots dy_2\,dy_1$$



For any "nice" region $A \subseteq \mathbb{R}^n$,

$$\mathbb{P}[(Y_1,\dots,Y_n)\in A] = \underbrace{\int\int\cdots\int}_{A} f_{Y_1,\dots,Y_n}(y_1,\dots,y_n)\,dy_n\,dy_{n-1}\cdots dy_1$$

Example. (Y_1,Y_2) ... represents a point chosen a random in a unit square $[0,1]^2$

$$f_{Y_1,Y_2}(y_1,y_2) = \underset{?}{c}\cdot \mathbb{1}_{[0,1]\times[0,1]}(y_1,y_2)$$

$$\boxed{f_{Y_1,Y_2}(y_1,y_2) = 1\cdot \mathbb{1}_{[0,1]\times[0,1]}(y_1,y_2)}$$

$$\mathbb{P}[Y_1>Y_2] = \cancel{?}\ \frac{1}{2}$$