Risk measures [Part I].

W: Feb 15^{th}, 2019.

The Variance

For any random variable X we have the expected value of X is $\mu_X := \mathbb{E}[X]$ (if it exists).

We define the variance of X as

$$Var[X] := \mathbb{E}\left[(X-\mu_X)^2\right] = \mathbb{E}[X^2] - \mu_X^2$$

(if it exists)

Usage: $X \leftrightarrow R$... return of an investment

The Semi-Variance

Define the semi-variance of X as:

$$\sigma_{SV}^2 := \mathbb{E}\left[\left(\min(0, X-\mu_X)\right)^2\right]$$

Value at Risk (VaR).

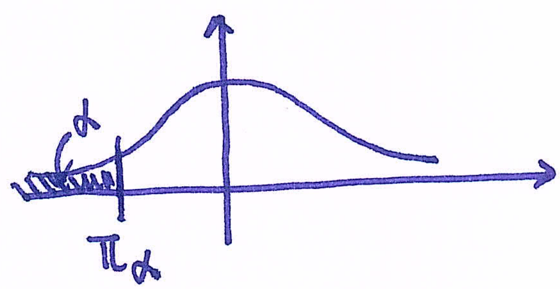
α (p) ... probab. of an adverse event you're willing to live with

R .. is a return random variable, i.e., we benefit if R is high and we have the adverse effect if R is low

Define $VaR_\alpha(R)$ as the value π_α such that

$$\boxed{\mathbb{P}[R \leq \pi_\alpha] = \alpha}$$

If R is a continuous random variable:



$$F_R(\pi_\alpha) = \alpha \iff \pi_\alpha = F_R^{-1}(\alpha)$$

generalizing F_R^{-1}

Consider an R such that its density f_R is always positive (e.g., let R be normally dist'd). For any $a \in \mathbb{R}$:

$$F_R(a) = \int_{-\infty}^{a} f_R(x)\,dx$$

If $f_R(x) > 0$ for all x,

then F_R is strictly increasing

$\Rightarrow F_R$ is one-to-one

$\Rightarrow F_R^{-1}$ exists.

* If we are interested in the upper tail probab. bounds, e.g., if the random variable X signifies a loss, we just look @ $VaR_{1-\alpha}(X)$

the random variable denoting the profits.

34) Let X be the random gain from operations of a company. You are given:

(i) X is normally distributed with mean 42 and variance 6400. $\sigma = 80$

(ii) p is the probability that X is negative.

(iii) K is the amount of capital such that the Value-at-Risk (VaR) at the 5th percentile for $X + K$ is zero.

Calculate p and K.

(A) $p = 0.7$; $K = 157$

(B) $p = 0.7$; $K = 131$

(C) $p = 0.5$; $K = 115$

(D) $p = 0.3$; $K = 115$

(E) $p = 0.3$; $K = 90$

$$X \sim N(\text{mean} = 42, \text{var} = 6400)$$

$$p = \mathbb{P}[X<0] = \mathbb{P}\left[\frac{X-42}{80} < \frac{0-42}{80}\right]$$

$$p = \mathbb{P}[\underbrace{Z}_{N(0,1)} < -0.525] = N(-0.525)$$

$$p = 1 - N(0.525) = 1 - 0.7019$$

rounding up in the std normal tables

$$p = 0.2981 \Rightarrow p \approx 0.3$$

$$\Rightarrow \boxed{(D) \text{ or } (E)}$$

$$VaR_{0.05}(X+K) = 0 = \pi_{0.05}$$

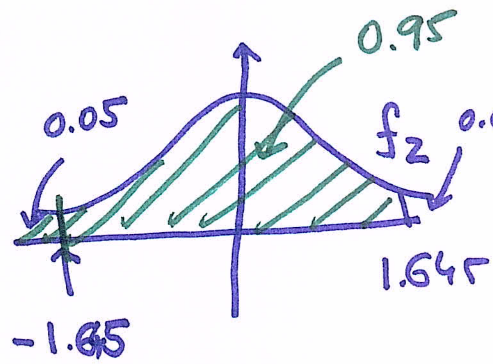
$$\mathbb{P}[X+K \le 0] = 0.05$$

$$X+K \sim \text{Normal}(\text{mean} = 42+K,\ \text{var} = 6400)$$

$$\mathbb{P}[X \le -K] = 0.05$$

in terms of $Z \sim N(0,1)$

$$\mathbb{P}[42+80\cdot Z \le -K] = 0.05$$



$$\underbrace{42+80\cdot(-1.645)}_{= -90} = -K \quad \Rightarrow \quad \boxed{K=90} \quad \Rightarrow (E)$$