UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 8

Various positions. Binomial asset pricing.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 8.1. (5 points) Which one of the following statements is **TRUE**?

(a) The payoff curve of a call bear spread is never positive.

(b) A straddle has a nonnegative profit function.

(c) A strangle can be replicated with a long put and a short call.

(d) The payoff of the call bull spread is equal to the payoff of the put bull spread.

(e) None of the other statements is TRUE.

Problem 8.2. Let the continuously compounded, risk-free interest rate be 0.05. The current price of a particular stock is \$100 per share. You model the stock price at the end of one year as follows

$$S(1) \sim \begin{cases} 90 & \text{with probability } 2/5 \\ 105 & \text{with probability } 2/5 \\ 125 & \text{with probability } 1/5 \end{cases}$$

The price of the one-year, 100-strike call on the above stock is 13.60, while the price of the one-year, 120-strike call on the above stock equals 8.00. Bertie buys a one-year $(100, 120)$ call bull spread on the above stock. What is Bertie's expected profit?

Problem 8.3. Consider three kinds of European call options on the same underlying asset and with the same exercise date. Their strikes are $100, 110$ and 120. The price of the \$100-strike call is 16.70. The price of the $120-$strike call is 4.50. A butterfly spread is constructed using the above call options. Its price is \$3.00 and the total number of options used to construct it is four. What is the price of the \$110-strike call option?

Problem 8.4. An investor bought a six-month, $(70, 80)$-put bear spread on an index. The \$70-strike, six-month put is currently valued at \$1, while the \$80-strike, six-month put is currently valued at \$8.

Assume that the continuously-compounded, risk-free interest rate equals 0.05.

What is the **break-even** final index price for the above put bear spread?

Problem 8.5. (5 points) Consider the ratio spread consisting of:

- five long \$40-strike, one-year calls on **S**,
- seven short \$60-strike, one-year calls on **S**.

You model the stock price at time-1 using the following model

$$S(1) \sim \begin{cases} \$35, & \text{with probability } 0.15 \\ \$45, & \text{with probability } 0.25 \\ \$55, & \text{with probability } 0.35 \\ \$65, & \text{with probability } 0.25 \end{cases}$$

What is the expected payoff of the ratio spread above?

Problem 8.6. (5 points) An investor buys a two-year ($800, $900)-strangle on gold. The price of gold two years from now is modeled using the following distribution:

$750, with probability 0.45,

$850, with probability 0.4,

$925, with probability 0.15.

What is the investor's expected payoff?

Problem 8.7. A portfolio consists of the following:

- one **short** one-year, 50−strike call option with price equal to $8.50,
- one **long** one-year, 60−strike put option with price equal to $6.75

All of the options are European and with the same underlying asset.

Assume that the continuously compounded, risk-free interest rate equals 0.04.

What is the portfolio's profit is the final price of the underlying asset equals $55?

Problem 8.8. (5 points) Bertie constructs an asymmetric butterfly spread using call options with strikes $75, 78$ and 90. It is constructed using m of the (75, 78) bull spreads and n (78,90) bear spreads. How much is m/n?

Problem 8.9. (10 points) Assume that one of the no-arbitrage conditions in the binomial model for pricing options on a non-dividend paying stock S is violated. Namely, let

$$e^{r\cdot h} \leq d < u.$$

Illustrate that the above inequalities indeed violate the no-arbitrage requirement. In other words, construct an arbitrage portfolio and show that your proposed arbitrage portfolio is, indeed, and arbitrage portfolio.