UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 3

Prerequisite material. Subjective expectations.

Provide your **complete solution** to the following problems. Final answers only, without appropriate justification, will receive zero points even if correct.

Problem 3.1. (5 points) In the setting of the one-period binomial model, denote by i the **effective** interest rate **per period**. Let u denote the "up factor" and let d denote the "down factor" in the stock-price model for a non-dividend-paying stock. Which of the following statements is the correct no-arbitrage condition for the binomial asset-pricing model?

(a) $d < 1 + i < u$

(b) $d < 1 < u$

(c) $d < e^i < u$

(d) $d = \frac{i}{1+i}$

(e) None of the above.

Problem 3.2. (5 points) The current price of a continuous-dividend paying stock is \$100. Its dividend yield is 0.02. Its evolution over the following year is modeled using a four-period binomial tree under the assumption that the price can increase by 1% or decrease by 0.5% over each period.

The continuously compounded, risk-free interest rate is 0.02.

What is the risk-neutral probability of the stock price going up in a single period?

Problem 3.3. (5 points) The evolution of a market index over the following year is modeled using a four-period binomial tree. We are given that the current value of the market index equals \$144, that its volatility equals 0.25, and that it pays dividends continuously.

You are tasked with constructing a four-period forward tree for the evolution over the following year of the forward price of the above market index with delivery at time-2.

What is the down factor d_F in the forward price tree for the futures prices on the stock?

Problem 3.4. (5 points) The current exchange rate is given to be \$1.25 per Euro and its volatility is given to be 0.15.

The continuously-compounded, risk-free interest rate for the US dollar is 0.03, while the continuously-compounded, risk-free interest rate for the Euro equals 0.06.

The evolution of the exchange rate over the following nine-month period is modeled using a three-period forward binomial tree.

What is the value of the so-called down factor in the above tree?

Problem 3.5. (5 points) The current price of a continuous-dividend-paying stock is \$80 per share. The stock's dividend yield is 0.02. According to your model, the expected value of the stock price in two years is \$90 per share. You are also given:

The risk-free interest rate exceeds the dividend yield.

The two-year forward price on a share of this stock is denoted by F. At this price you are willing to enter into the forward. What is the smallest range of values F can take according to the above information?

Problem 3.6. (5 points) The current price of a non-dividend-paying stock is \$100 per share. According to your model, the expected value of the stock price in two years is \$90 per share. You are also given:

The risk-free interest rate is strictly positive.

The two-year forward price on a share of this stock is denoted by F. At this price you are willing to short the forward contract. What is the smallest range of values F can take according to the above information?

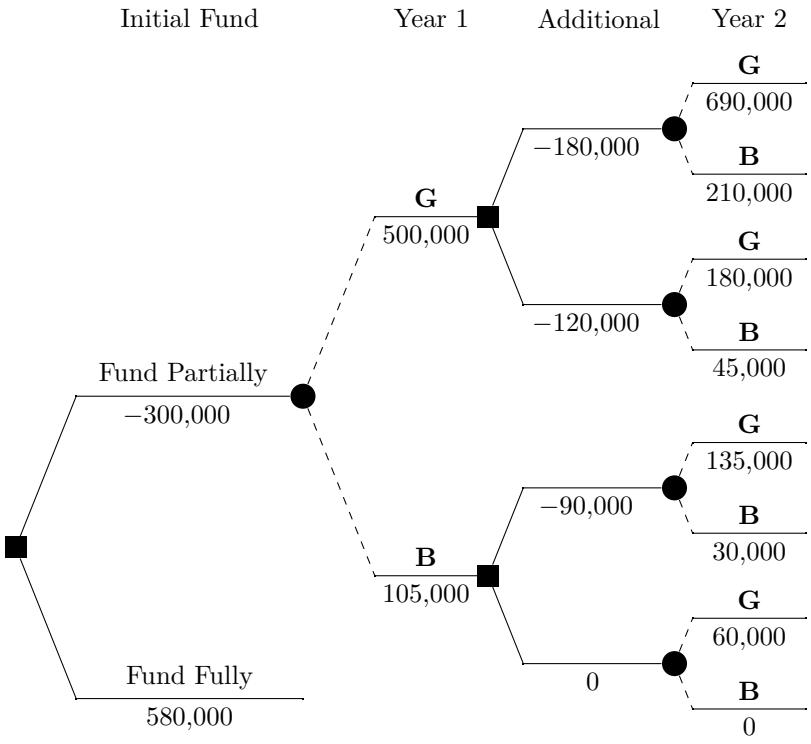
Problem 3.7. (15 points) Netflix is considering a cartoon series. When the production of two seasons is fully funded at time-0 the project has a net present value of 580,000.

The decision tree below shows the cash flows of the series when the promotion at the beginning of the Year 1 (i.e., at $t = 0$) is only partial with an option to provide different amounts of funding at the beginning of Year 2 (i.e., at $t = 1$) depending on how well the first season did.

This tree reflects two possible receptions of the two seasons at each information node (**G** = good, **B** = bad). The probability of the series being a success is given to be 1/2 and the probability of it being merely watchable is 1/2.

Assume the interest rate is 0%.

Find the **initial** (i.e., at $t = 0$) value of the option to fund partially.



Problem 3.8. (5 points) A certain type of lightbulb has a useful life that is normally distributed with mean 4 years and standard deviation of a year. The useful lives of different lightbulbs are independent. Calculate the probability that the total useful life of two randomly selected lightbulbs exceeds 1.2 times the useful life of a third randomly selected light bulb.