University of Texas at Austin

Quiz #21

Required returns. CAPM assumptions.

Provide your **complete solutions** to the following problems:

Problem 21.1. (9 points) State the assumptions of the **Capital Asset Pricing Model.**

Solution:

I. The market is *competitive*, i.e., the securities are bought and sold at the same price. There are no taxes or transaction costs. Both borrowing and lending are at the risk-free interest rate.
II. Investors hold only efficient portfolios.
III. *Homogeneous expectations:* The investors have the same beliefs about the expected values, volatilities, and correlations of returns of securities.

Problem 21.2. (2 points) You are given the following information about stock X and a portfolio P:

- The annual effective risk-free rate is 4%.
- The portfolio's expected return is 0.08 and the its volatility is 0.25.
- The expected return of stock X is 6% and its volatility is 0.4.
- The correlation between the returns of stock X and the portfolio P is -0.2.

Then, the investor holding portfolio P should invest in stock X. *True or false? Why?*

Solution: TRUE
The β for the stock X equals

$$\beta_X = \frac{0.4(-0.2)}{0.25} = -0.32.$$

So, the stock X has a required return equal to

$$r_X = r_f + \beta_X(\mathbb{E}[R_P] - r_f) = 0.04 + (-0.32)(0.08 - 0.04) = 0.0272.$$

Since the expected return exceeds the required return, one should invest in stock X.

Problem 21.3. (2 points) Portfolio P has expected return 0.08 and volatility equal to 12%. Porfolio Q has expected return 0.10 and volatility equal to 12.5%. Then, we can say with certainty that portfolio P is not efficient. *True or false? Why?*

Solution: FALSE
Since both the expected return and the volatility of portfoliio Q are bigger than those of portfolio P, there is no way for us to compare the two with just the information provided.

Problem 21.4. (2 points) You are given the following information about stock X and a portfolio P:

- The annual effective risk-free rate is 3%.
- The portfolio's expected return is 0.10 and the its volatility is 0.20.
- The expected return of stock X is 0.06 and its volatility is 0.30.
- The correlation between the returns of stock X and the portfolio P is -0.25.

Then, the investor holding portfolio P should invest in stock X. *True or false? Why?*

Solution:
The β for the stock X (with respect to portfolio P) equals

$$\beta_X = \frac{0.3(-0.25)}{0.2} = -0.375.$$

So, the stock X has a required return equal to

$$r_X = r_f + \beta_X(\mathbb{E}[R_P] - r_f) = 0.03 + (-0.375)(0.10 - 0.03) = 0.00375.$$

Since the expected return $\mathbb{E}[R_X] = 0.06$ exceeds the required return, one should invest in stock X.
