Problem set #11: Binomial Monte Carlo

Milica Cudina

2025-10-31

Let the **volatility** of a stock be the standard deviation of its (continuously compounded) realized return on an annual basis. Then, we can define the up and down factors in the so-called *forward binomial tree* for a **non-dividend-paying** stock as

$$\begin{aligned} u &= e^{rh+\sigma\sqrt{h}} \\ d &= e^{rh-\sigma\sqrt{h}} \end{aligned} \tag{1}$$

Let the continuously compounded, risk-free interest rate be 0.04.

```
r=0.04
```

Consider a stock whose current price is \$100 and whose volatility is 0.25. We will be pricing a one-year, at-the-money call option in a variety of ways here.

```
#about the stock
s0=100
sigma=0.25

#about the call
T=1
K=s0
```

Problem #1: Analytic one period

Price the option above using a one period binomial tree.

```
#number of periods
n=1

#length of each period
h=T/n

#define the factors in the binomial tree
(u=exp(r*h+sigma*sqrt(h)))
```

```
## [1] 1.336427
```

```
(d=exp(r*h-sigma*sqrt(h)))
```

```
## [1] 0.8105842
```

```
#the risk-neutral probability
(p=(exp(r*h)-d)/(u-d))
```

```
## [1] 0.4378235
```

```r
#the forward-tree shortcut
(p.dash=1/(1+exp(sigma*sqrt(h))))
```

```
## [1] 0.4378235
```

```r
#the possible stock prices
(s.u=s0*u)
```

```
## [1] 133.6427
```

```r
(s.d=s0*d)
```

```
## [1] 81.05842
```

```r
#the possible payoffs are
(v.u=max(s.u-K,0))
```

```
## [1] 33.64275
```

```r
(v.d=max(s.d-K,0))
```

```
## [1] 0
```

```r
#initial option price
(v.0=exp(-r*T)*(p*v.u+(1-p)*v.d))
```

```
## [1] 14.15203
```

Problem #2: Monte Carlo one period

Price the option above using Monte Carlo a one period binomial tree. Use 10000 simulations.

Problem #3: Analytic two periods

Price the above option using a two-period binomial tree.

Problem #4: Monte Carlo two periods

Price the option above using Monte Carlo a two period binomial tree. Use 10000 simulations.

Problem #5: Analytic one hundred periods

Price the above option using a $100-$period binomial tree.

Problem #6: Monte Carlo with one hundred periods

Price the option above using Monte Carlo with a hundred period binomial tree. Use 10000 simulations.