M339W: October 11th, 2021.

Option Elasticity.

Def'n. For any portfolio w/ value function $v(s,t)$, its portfolio elasticity is defined as

$$\boxed{\Omega(s,t) := \frac{\Delta(s,t)\cdot s}{v(s,t)}}$$

In particular, if your portfolio consists of a single option, it's called option elasticity.

Example. *A European call*

Its Black-Scholes price is:

$$v_c(s,t) = s\,\underbrace{e^{-\delta(T-t)}\cdot N(d_1(s,t))}_{\Delta_c(s,t)} - Ke^{-r(T-t)}\cdot N(d_2(s,t))$$

$$\Omega_c(s,t) = \frac{s\cdot\Delta_c(s,t)}{s\cdot\Delta_c(s,t) - Ke^{-r(T-t)}\cdot N(d_2(s,t))} \geq 1$$

Example. *European put*

$$v_p(s,t) = Ke^{-r(T-t)}\cdot N(-d_2(s,t)) - se^{-\delta(T-t)}\cdot N(-d_1(s,t))$$

$$\Delta_p(s,t) = -e^{-\delta(T-t)}N(-d_1(s,t))$$

$$\Rightarrow \Omega_p(s,t) = \frac{\Delta_p(s,t)\cdot s}{Ke^{-r(T-t)}\cdot N(-d_2(s,t)) + \Delta_p(s,t)\cdot s} < 0$$

Use for option elasticity:

σ_S... stock volatility (in the B·S model: constant, deterministic)

We get the option volatility as

$$\boxed{\sigma_{opt}(s,t) = \sigma_S \cdot \left| \Omega_{opt}(s,t) \right|}$$

While σ_S is constant, the option volatility is NOT.

e.g., for a European call:

$$\sigma_C(s,t) = \sigma_S \left| \underbrace{\Omega_C(s,t)}_{\geq 1} \right| \geq \sigma_S$$

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

$\Omega_p(S(0),0) = ?$

$$\Omega_p(S(0),0) = \frac{\boxed{\Delta_p(S(0),0)} \cdot S(0)}{v_p(S(0),0)}$$

↑

by def'n

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

21-24. DELETED

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of \$100.

The chooser option price is \$20 at time $t = 0$.

The stock price is \$95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of \$100.

You are given:

(i) The risk-free interest rate is 0.

(ii) $C(1) = \$4$.

Determine $C(3)$.

(A) \$ 9

(B) \$11

(C) \$13

(D) \$15

(E) \$17

$$\longrightarrow : \quad S(0) = 45$$
$$v_C(0) = 4.45 = v_C(S(0),0)$$
$$v_P(0) = 1.90 = v_P(S(0),0)$$

Investor A: $v_A(s,t) = 2 \cdot v_C(s,t) + v_P(s,t)$

$$\frac{\partial}{\partial s}\Big| \quad \Delta_A(s,t) = 2 \cdot \Delta_C(s,t) + \Delta_P(s,t)$$

$\Rightarrow$ At time 0:

$$5 = \frac{\Delta_A(S(0),0) \cdot S(0)}{v_A(S(0),0)} = \frac{(2 \cdot \Delta_C(S(0),0) + \Delta_P(S(0),0)) \cdot (45)}{2 \cdot (4.45) + 1.9}$$

$\uparrow$ given in problem

$$\left(2 \cdot \Delta_C(S(0),0) + \Delta_P(S(0),0)\right) (\cancelto{9}{45}) = \cancel{5} \cdot (\cancelto{1.2}{10.80})$$

$$2 \cdot \Delta_C(S(0),0) + \Delta_P(S(0),0) = 1.2 \qquad (A)$$

Investor B: $v_B(s,t) = 2 \cdot v_C(s,t) - 3 \cdot v_P(s,t)$

$$\frac{\partial}{\partial s}\Big| \quad \Delta_B(s,t) = 2 \cdot \Delta_C(s,t) - 3 \cdot \Delta_P(s,t)$$

$\Rightarrow$ At time 0:

$$3.4 = 2 \cdot \Delta_C(S(0),0) - 3 \cdot \Delta_P(S(0),0) \qquad (B)$$

$$(A) - (B) \Rightarrow \Delta_P(S(0),0) + 3\Delta_P(S(0),0) = 1.2 - 3.4$$
$$4\Delta_P(S(0),0) = -2.2$$
$$\Delta_P(S(0),0) = -0.55$$

$$\Omega_P(S(0),0) = \frac{\Delta_P(S(0),0) \cdot S(0)}{v_P(S(0),0)} = \frac{-0.55(45)}{1.9} = -13.03$$