M339D : March 23rd, 2022.

Bounds on call / put prices [cont'd].

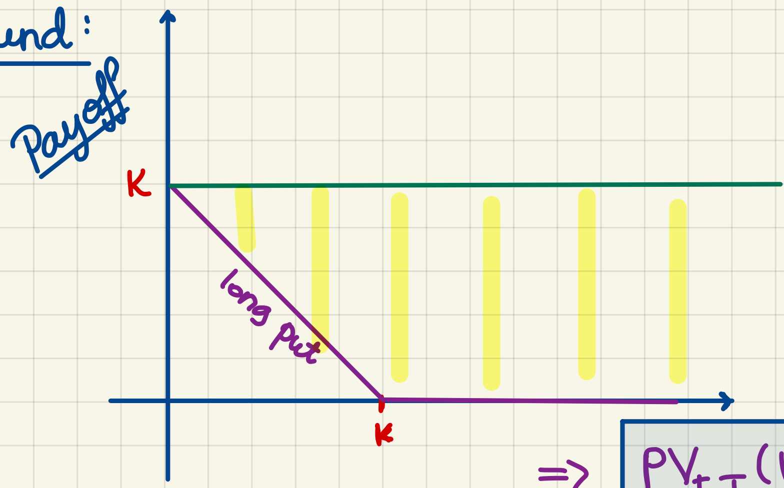
Puts.

Lower bound:

- $V_P(t) \geq 0$
- Put-call Parity: $V_P(t) - \underbrace{V_C(t)}_{\geq 0} = PV_{t,T}(K) - F^P_{t,T}(S)$

$\Rightarrow \boxed{V_P(t) \geq \max\left(PV_{t,T}(K) - F^P_{t,T}(S), 0\right)}$

Upper bound:



zero-coupon bond w/ maturity @ time T & w/ redemption amt K

$\Rightarrow \boxed{PV_{t,T}(K) \geq V_P(t)}$ ✓

Q: Assume $r > 0$. What happens to the put price as the exercise date T goes to $+\infty$?

$\longrightarrow$:

$$Ke^{-r(T-t)} \geq V_P(t, T) \geq 0$$

$$Ke^{-r(T-t)} \xrightarrow{T\to\infty} 0$$

$$V_P(t,T) \xrightarrow[T\to\infty]{} 0$$

□

Call/Put Prices as Functions of the Strike Price.

Assume that all other inputs are fixed.

Fix the valuation date @ time $\cdot 0$ $\Rightarrow$ we supress it from our notation.

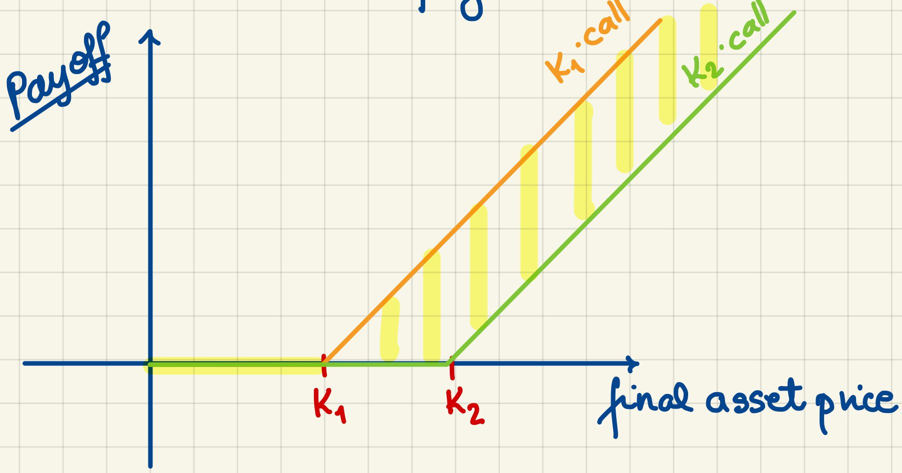
We vary the strike K, so it makes sense to write:

$$\left.\begin{matrix} V_c(K) \dots \text{call} \\ V_p(K) \dots \text{put} \end{matrix}\right\} \text{price w/ strike } K$$

Monotonicity.

$\boxed{\text{Calls.}}$ Let $\boxed{K_1 < K_2}$.

We want to figure out the ordering of $V_c(K_1)$ and $V_c(K_2)$.



The payoff of the K_1-strike call dominates the payoff of the K_2-strike call.

$$\Downarrow$$

$$\boxed{V_c(K_1) \ \underline{\geq} \ V_c(K_2)} \quad \checkmark$$

European call prices are <u>decreasing</u> w/ respect to the strike price.

Q: What would one do if the above inequality is violated?

$\longrightarrow$: Assume, to the contrary, that there exist $K_1 < K_2$ such that

$$V_c(K_1) < V_c(K_2).$$

I. <u>Suspicion</u>. ✓

II. Propose an arbitrage portfolio.

- Long the K_1·strike call
- write the K_2·strike call

} call bull spread

III. Verification.

Initial inflow of money.

Initial Cost: $V_c(K_1) - V_c(K_2) < 0$

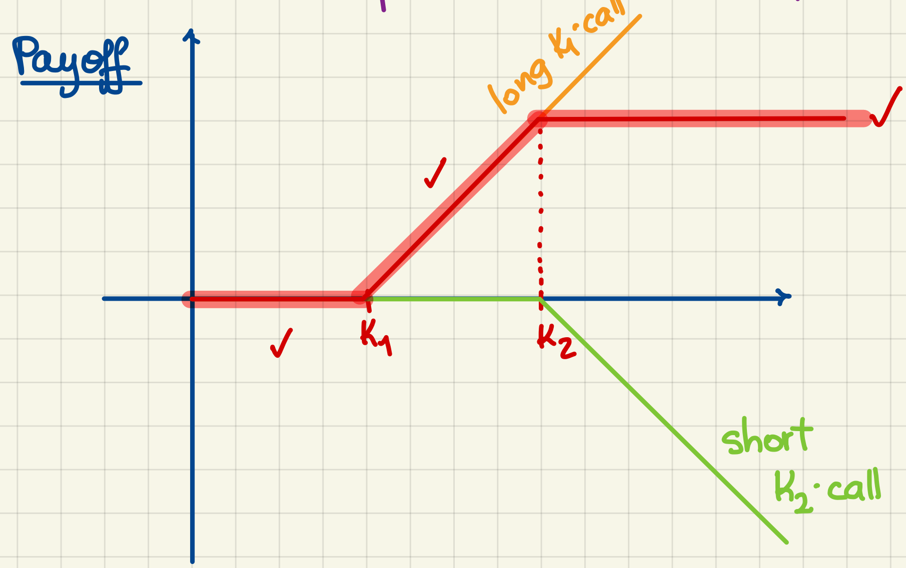
Payoff:

$$(S(T)-K_1)_+ - (S(T)-K_2)_+ =$$

$$= \begin{cases} 0 & \text{if } S(T) < K_1 \quad ✓ \\ S(T) - K_1 & \text{if } K_1 \leq S(T) < K_2 \quad ✓ \\ \cancel{S(T)} - K_1 - (\cancel{S(T)} - K_2) = K_2 - K_1 & \text{if } K_2 \leq S(T) \quad ✓ \end{cases}$$

$\Rightarrow$ Payoff ≥ 0

$\Rightarrow$ Profit > 0 $\Rightarrow$ This is, indeed, an arbitrage portfolio!



call bull spread

Long w/ respect to the underlying

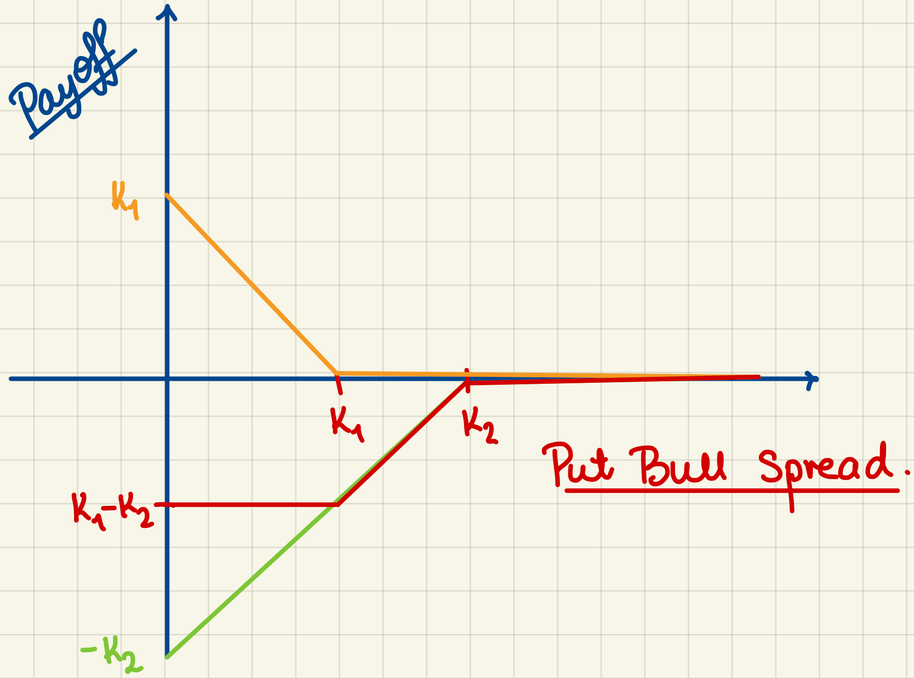
$\Downarrow$

A suitable hedge for a short position.

Q: How would you construct a put bull spread, i.e., the financial position w/ the same shape of the payoff curve consisting of put?

$\longrightarrow$: Let $K_1 < K_2$.

- Long ~~(?)~~ the K_1·strike put
- Write ~~(?)~~ the K_2·strike put



Task: Figure out the difference between the profit of the call bull spread and the profit of the put bull spread!