

UNIVERSITY OF TEXAS AT AUSTIN

Homework assignment 10

Continuous probability. The normal distribution.

Please, provide your final answer only to the following questions:

Problem 10.1. (2 points) A linear combination of two jointly normally distributed random variables is always also normally distributed. Assume that a constant is also considered normally distributed with variance zero. *True or false?*

Problem 10.2. (2 points) If the random variable X has the distribution function F_X , then the distribution function of the random variable $Y = |X|$ equals

$$F_Y(y) = 2F_X(y).$$

True or false?

Problem 10.3. (2 points) Let X_1, \dots, X_n be random variables with finite expectations and let $\alpha_1, \dots, \alpha_n$ be constants. Then, we always have that

$$\mathbb{E}[\alpha_1 X_1 + \dots + \alpha_n X_n] = \sum_{i=1}^n \alpha_i \mathbb{E}[X_i].$$

True or false?

Please, provide your complete solution to the following problems. Only the final answer without justification will receive zero credit.

Problem 10.4. (2 points) It is possible that a cumulative distribution function be even. *True or false? Why?*

Problem 10.5. (2 pts) If the random variable X is standard normal, then the distribution function of the random variable $Y = |X|$ equals

$$F_Y(a) = 2\Phi(a) - 1 \text{ for every } a \geq 0.$$

True or false? Why?

Problem 10.6. (10 pts) Let Z be a standard normal random variable. Find the following probabilities:

- i. $\mathbb{P}[-1.33 < Z \leq 0.24]$
- ii. $\mathbb{P}[0.49 < |Z|]$
- iii. $\mathbb{P}[Z^4 < 0.0256]$
- iv. $\mathbb{P}[e^{2Z} < 2.25]$
- v. $\mathbb{P}\left[\frac{1}{Z} < 2\right]$

Problem 10.7. (5 points) Let the density function of a random variable X be given as

$$f_X(x) = cx, \quad x \in [0, 1],$$

for some constant c . Find $\mathbb{E}[X^3]$.

Problem 10.8. (10 points) Two laser pointers are used to measure the length ℓ of a building. The error made by the less accurate laser pointer is normally distributed with mean 0 and standard deviation 0.0144ℓ . The error made by the more accurate laser pointer is normally distributed with mean 0 and standard deviation 0.0036ℓ . The errors from the two laser pointers are independent of each other. Calculate the probability that the average value of the two measurements is within 0.001ℓ of the true length ℓ of the building.

Problem 10.9. (10 points) An astronomical instrument measures the distance d to a far-away planet. You know that the instrument is calibrated so that its measurement error is normally distributed, centered around zero, and with variance $(0.0001d)^2$. The different measurements using this same instrument are assumed to be independent. How many independent measurements would you have to perform so that your average is within $10^{-6}d$ with probability 99%?

Problem 10.10. (5 points) The profit of a certain company are modelled using a normal distribution with mean 1,000,000 and standard deviation 400,000. Given that the profit is positive, what is the probability that it is below 1,200,000?