

UNIVERSITY OF TEXAS AT AUSTIN

## Quiz # 8

Confidence intervals (the normal distribution).

Provide your **final answer only** to the following questions.

**Problem 8.1.** (2 points) The margin of error for a confidence interval for the population mean  $\mu$  increases as the sample size increases. *True or false?*

**Solution: FALSE**

**Problem 8.2.** (2 points) The margin of error for a confidence interval for the population mean  $\mu$ , based on a fixed specified sample size  $n$ , increases as the confidence level decreases. *True or false?*

**Solution: FALSE**

**Problem 8.3.** (2 points) The Midsomer Worthy Middle School has calculated a 95% confidence interval for the population mean height  $\mu$  of 11-year-old boys at their school. They found it to be  $57 \pm 2$  inches.

This means that there is a 95% probability that the population mean  $\mu$  is between 55 and 59. *True or false?*

**Solution: FALSE**

**Problem 8.4.** (2 points) The Midsomer Worthy Middle School has calculated a 95% confidence interval for the population mean height  $\mu$  of 11-year-old boys at their school. They found it to be  $57 \pm 2$  inches.

If we took many additional random samples of the same size and from each computed a 95% confidence interval for  $\mu$ , approximately 95% of these intervals would contain the population mean  $\mu$ . *True or false?*

**Solution: TRUE**

**Problem 8.5.** (2 points)

Resident statistician Margie N. Rivera calculated a confidence interval of  $[-0.56, 0.88]$ . Her assistant boasts: “We should be 95% confident that the **sample average** falls in the provided interval”. This is a valid statement. *True or false?*

**Solution: FALSE**

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Please, provide your **complete solution** to the following problem.

**Problem 8.6.** (5 points) To estimate a population mean, our resident statistician Martyn Rivera plans to pick two independent simple random samples, each of size 100, from the population. He also plans to calculate the confidence interval with level  $C$  for each sample. What is the probability that **exactly one** of his confidence intervals will cover the population mean?

**Solution:** The probability that the first confidence interval covers the population mean is  $C$  while the probability that the second confidence interval does not cover the population mean equals  $(1 - C)$ . The samples are independent, so the probability of both these events happening is  $C(1 - C)$ .

Similarly, the probability that the first confidence interval does not cover the population mean is  $(1 - C)$  while the probability that the second confidence interval does cover the population mean equals  $1 - C$ . Again, the samples are independent, so the probability of both these events happening is  $C(1 - C)$ .

Altogether, the probability we were looking for is  $2C(1 - C)$ .