M339J: April 15th, 2022.

Compound Poisson.

Homework Problem #6.1.

i. X... severity

Support$(X) = \{1, 2, 3\}$

$p_X(1), p_X(2), p_X(3) = ?$

ii. $\boxed{\mathbb{E}[S] = 56}$ w/ $S = S_1 + S_2 + \dots + S_N$ w/ $N \sim \text{Poisson}(\lambda = 29)$

$$\underbrace{\mathbb{E}[N]}_{29} \cdot \mathbb{E}[X] = 56$$

$$29 \cdot \left(1 \cdot p_X(1) + 2 \cdot p_X(2) + 3 p_X(3)\right) = 56$$

iii. $\boxed{Var[S] = 126}$

$$Var[S] = \underbrace{\mathbb{E}[N]}_{\lambda} \cdot Var[X] + \underbrace{Var[N]}_{\lambda}(\mathbb{E}[X])^2$$

$$Var[S] = \lambda\big(\underbrace{Var[X] + (\mathbb{E}[X])^2}_{\mathbb{E}[X^2]}\big)$$

$\boxed{Var[S] = \lambda \cdot \mathbb{E}[X^2]}$ ALL COMPOUND POISSON!

In this problem:

$$29 \cdot \left(1 \cdot p_X(1) + 4 \cdot p_X(2) + 9 \cdot p_X(3)\right) = 126$$

$p_X(1) + p_X(2) + p_X(3) = 1$ because pmf

280. A compound Poisson claim distribution has $\boxed{\lambda = 5}$ and individual claim amounts distributed as follows:

x	$f_X(x)$	
$\boxed{5}$	0.6	
k	0.4	Where $k > 5$

The expected cost of an aggregate stop-loss insurance subject to a deductible of 5 is 28.03.

Calculate k.

$$\longrightarrow: \ \mathbb{E}[(S-d)_+] = \boxed{\mathbb{E}[S]} - \boxed{\mathbb{E}[S \wedge d]}$$

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

- $\mathbb{E}[S] = \lambda \cdot \mathbb{E}[X] =$

$$= 5 \cdot (5 \cdot (0.6) + k(0.4))$$

$$= 2k + 15$$

281. DELETED

- $\mathbb{E}[S \wedge d] = ?$ $\boxed{d=5}$

$$\text{Support}(S \wedge 5) = \{0, 5\}$$

$$S \wedge 5 \sim \begin{cases} 0 & \text{w/ probab. } p_N(0) = e^{-5} \\ 5 & \text{w/ probab. } 1-e^{-5} \end{cases}$$

$$\mathbb{E}[S \wedge d] = 0 \cdot e^{-5} + 5(1-e^{-5}) = 5(1-e^{-5})$$

$$28.03 = \mathbb{E}[(S-5)_+] = 2k + 15 - 5(1-e^{-5})$$

$$k = \frac{18.03 - 5e^{-5}}{2} = 8.99816 \approx 9$$

289. A compound Poisson distribution has $\lambda = 5$ and claim amount distribution as follows:

x	$p(x)$
100	0.80
500	0.16
1000	0.04

Calculate the probability that aggregate claims will be exactly 600.

(A) 0.022

(B) 0.038

(C) 0.049

(D) 0.060

(E) 0.070

$\longrightarrow$: $\boxed{p_S(600) = ?}$

600

6×100 | 100+500 | 500+100

$$\mathbb{P}[6 \text{ claims of } 100] = \mathbb{P}[N=6, X_1=X_2=X_3=X_4=X_5=X_6=100]$$

$$\underset{\text{independence}}{=} p_N(6)\left(p_X(100)\right)^6 = e^{-5} \cdot \frac{5^6}{6!} (0.80)^6$$

$$= \underline{0.038331}$$

290. DELETED

291. DELETED

292 DELETED

293 DELETED

294 DELETED

295 DELETED

296. DELETED

297. DELETED

298. DELETED

299. DELETED

$$\mathbb{P}[\text{two claims w/ first claim of 100 and second of 500}] =$$

$$= p_N(2) \cdot p_X(100) \cdot p_X(500) =$$

$$= e^{-5} \cdot \frac{5^2}{2!} (0.80)(0.16) = \underline{0.01078}$$

$$\mathbb{P}[\text{two claims w/ first claim of 500 and second of 100}] =$$

$$= \underline{0.01078}$$

answer : $0.038331 + 2(0.01078) = 0.05989$

A Few Compound Poissons.

We start w/ n different streams of losses, each of them a compound Poisson and all independent.

$\{S_j, j = 1..n\}$ are independent compound Poissons, i.e.,

$$\left.\begin{array}{l} \text{for every } j: \quad N_j \sim \text{Poisson}(\lambda_j) \\ \text{and } \quad X^j \sim \text{cdf } F_j \end{array}\right\}$$

More precisely, for stream j, the severity r.v.s are $\{X_1^j, X_2^j, \dots, X_k^j, \dots\}$

$$\Rightarrow \boxed{S_j = X_1^j + X_2^j + \dots + X_{N_j}^j}$$

Set: $\boxed{S = S_1 + S_2 + \dots + S_n}$

Thm. S is itself a compound Poisson w/

$$N \sim \text{Poisson}(\lambda = \lambda_1 + \lambda_2 + \dots + \lambda_n)$$

and w/ the severity r.v.s w/ the cdf

$$\boxed{F_X(x) = \sum_{j=1}^{n} \frac{\lambda_j}{\lambda} F_j(x)}$$

n-point mixture of X_j's.