M339J: May 5th, 2021.

Exam Questions:

Problem.
- 3 insureds
- the common severity X w/

$$p_X(0)=0.4,\quad p_X(1)=0.3,\quad p_X(2)=0.2,\quad p_X(3)=0.1$$

$\mathbb{P}[S=3]=?$

$S = X_1 + X_2 + X_3$ aggregate loss

$$\begin{aligned} 3 &= 0 + 0 + 3 && \text{(3 permutations)} \\ &= 0 + 1 + 2 && \text{(6 permutations)} \\ &= 1 + 1 + 1 && \text{(1 permutation)} \end{aligned}$$

$$3(p_X(0))^2 \cdot p_X(3) + 6 \cdot p_X(0) \cdot p_X(1) \cdot p_X(2) + (p_X(1))^3$$

$$= 3(0.4)^2 \cdot 0.1 + 6 \cdot 0.4 \cdot 0.3 \cdot 0.2 + (0.3)^3 = 0.219$$

Per Loss and Per Payment Random Variables.

Goal: We want to look @ aggregate payments in case that individual policies are modified via an ordinary deductible.

Per loss: $Y^L = (X-d)_+$

Per payment: $Y^P = Y^L \mid Y^L > 0 = X - d \mid X > d$

Our notation was: $v = \mathbb{P}[Y^L > 0]$

$$Y^L = \begin{cases} Y^P & \text{if pmt w/ probab } v \\ 0 & \text{if no pmt w/ probab } 1-v \end{cases}$$

$$M_{Y^L}(t) = v \cdot M_{Y^P}(t) + (1-v)\cdot \underbrace{t^0}_{=1}$$

N^L .. # of losses

N^P .. # of pmts

$$P_{N^P}(z) = P_{N^L}\big((1-v) + v \cdot z\big)$$

pgf applied to

Aggregate Losses:

On the per-loss basis:

$\longrightarrow S = Y_1^L + Y_2^L + \cdots + Y_{N_L}^L$

On the per-payment basis:

$\longrightarrow S = Y_1^P + Y_2^P + \cdots + Y_{N^P}^P$

Q: What's the difference?

✓

$N^L \sim \text{Poisson}(\lambda_L = 20)$

109. A company insures a fleet of vehicles. Aggregate losses have a compound Poisson distribution. The expected number of losses is 20. Loss amounts, regardless of vehicle type, have exponential distribution with $\theta = 200$.

$X \sim \text{Exponential}(\theta = 200)$

To reduce the cost of the insurance, two modifications are to be made:

(i) a certain type of vehicle will not be insured. It is estimated that this will reduce loss frequency by 20%.

Thinning: $\tilde{N}^L$... new loss count

$\tilde{N}^L \sim \text{Poisson}(\tilde{\lambda}_L = 16)$

(ii) a deductible of 100 per loss will be imposed.

$d = 100$ $\quad Y^L = (X - 100)_+$

Calculate the expected aggregate amount paid by the insurer after the modifications.

(A) 1600

(B) 1940

(C) 2520

(D) 3200

(E) 3880

$S = Y_1^L + \cdots + Y_{\tilde{N}^L}^L$

$\Rightarrow \mathbb{E}[S] = \underbrace{\mathbb{E}[\tilde{N}^L]}_{16} \cdot \underbrace{\mathbb{E}[Y^L]}_{?} = 16 \cdot 200 \cdot e^{-\frac{1}{2}} = 1940.898$

$\mathbb{E}[Y^L] = \mathbb{E}[X] - \mathbb{E}[X \wedge 100]$

$= 200 - 200 \cdot F_X(100) = 200 \cdot S_X(100) = 200 e^{-\frac{100}{200}}$

110. You are the producer of a television quiz show that gives cash prizes. The number of prizes, N, and prize amounts, X, have the following distributions:

n	$\Pr(N = n)$
1	0.8
2	0.2

x	$\Pr(X = x)$
0	0.2
100	0.7
1000	0.1

Your budget for prizes equals the expected prizes plus the standard deviation of prizes.

Calculate your budget.

(A) 306

(B) 316

(C) 416

(D) 510

(E) 518

$N^L \sim$ Poisson $(\lambda = 5)$

126. The number of annual losses has a Poisson distribution with a mean of 5. The size of each loss has a two-parameter Pareto distribution with $\theta = 10$ and $\alpha = 2.5$. An insurance for the losses has an ordinary deductible of 5 per loss.

Calculate the expected value of the aggregate annual payments for this insurance.

$\mathbb{E}[S] = ?$

$S = Y_1^L + Y_2^L + \cdots + Y_{N^L}^L$

and w/ $Y^L = (X-5)_+$ w/ $X \sim$ Pareto $(\theta = 10, \alpha = 2.5)$

$\mathbb{E}[S] = \underbrace{\mathbb{E}[N^L]}_{= 5} \cdot \mathbb{E}[Y^L]$

(A) 8

(B) 13

(C) 18

(D) 23

(E) 28

127. Losses in 2003 follow a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 5$. Losses in 2004 are uniformly 20% higher than in 2003. An insurance covers each loss subject to an ordinary deductible of 10.

Calculate the Loss Elimination Ratio in 2004.

(A) 5/9

(B) 5/8

(C) 2/3

(D) 3/4

(E) 4/5

128. DELETED

129. DELETED

$$\mathbb{E}[Y^L] = \mathbb{E}[(X-5)_+] = \mathbb{E}[X] - \mathbb{E}[X \wedge 5]$$

$$= \cancel{\frac{\theta}{\alpha-1}} - \frac{\theta}{\alpha-1}\left[\cancel{1} - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1}\right]$$

$$= \frac{\theta}{\alpha-1} \cdot \left(\frac{\theta}{d+\theta}\right)^{\alpha-1} = \frac{\theta^{\alpha}}{(\alpha-1)(d+\theta)^{\alpha-1}}$$

$$= \frac{10^{2.5}}{(2.5-1)(5+10)^{2.5-1}} = \cdots = 3.629$$

$$\mathbb{E}[S] = 5 \cdot (3.629) = 18.144$$

$\blacksquare$

211. An actuary for a medical device manufacturer initially models the failure time for a particular device with an exponential distribution with mean 4 years.

This distribution is replaced with a spliced model whose density function:

(i) is uniform over [0, 3]

(ii) is proportional to the initial modeled density function after 3 years

(iii) is continuous

Calculate the probability of failure in the first 3 years under the revised distribution.

(A) 0.43

(B) 0.45

(C) 0.47

(D) 0.49

(E) 0.51

212. For an insurance:

(i) The number of losses per year has a Poisson distribution with $\lambda = 10$.

(ii) Loss amounts are uniformly distributed on (0, 10). $X \sim U(0,10)$

(iii) Loss amounts and the number of losses are mutually independent.

(iv) There is an ordinary deductible of 4 per loss.

Calculate the variance of aggregate payments in a year.

(A) 36

(B) 48

(C) 72

(D) 96

(E) 120

Q: What is the probability that a particular loss results in a pmt?

$\longrightarrow$: $v = \mathbb{P}[X > 4] = 0.6$

$\Rightarrow \begin{cases} \bullet\ N^P \sim \text{Poisson}(\text{mean} = \lambda^P = 0.6(10) = 6) \\ \bullet\ Y^P \sim U(0,6) \end{cases}$

Aggregate pmts : $S = Y_1^P + Y_2^P + \cdots + Y_{N^P}^P$

$$\mathrm{Var}[S] = \lambda \cdot \mathbb{E}[(Y^P)^2]$$
$$= \lambda \left(\mathrm{Var}[Y^P] + (\mathbb{E}[Y^P])^2 \right)$$
$$= 6\left(\frac{6^2}{12} + 3^2 \right) = 6(3+9) = 72 \quad \blacksquare$$