Types of Errors.

H_0: $\boxed{}$ vs. H_a: ______

α ... significance level

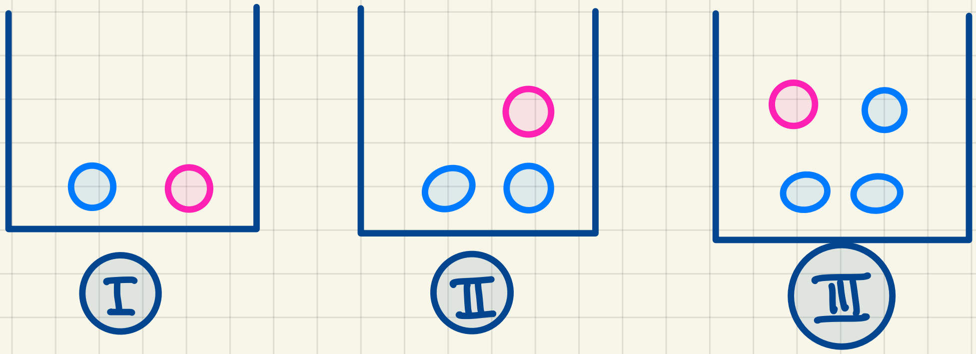
"Truth" / Decision	H_0 true	H_0 not true
Reject H_0	Type I Error	☺
Fail to Reject H_0	☺	Type II Error

$$\mathbb{P}[\text{Type I Error}] = \mathbb{P}_0[\text{Reject } H_0] = \underline{\alpha \text{ (significance level)}}$$

under the null

Bayesian statistics.

Example.



Someone takes out a ball from a box.

You must try to guess its "box of origin".

The prior probabilities would be

$$\boxed{\hat{\pi}_{I} = \frac{2}{9}}, \quad \hat{\pi}_{II} = \frac{3}{9}, \quad \hat{\pi}_{III} = \frac{4}{9}$$

Now, what if the ball is BLUE?

Then,

$$\mathbb{P}[I \mid Y=B] = \frac{\boxed{\mathbb{P}[I]} \cdot \mathbb{P}[Y=B \mid I]}{\mathbb{P}[I]\cdot\mathbb{P}[Y=B \mid I] + \mathbb{P}[II]\cdot\mathbb{P}[Y=B \mid II] + \mathbb{P}[III]\cdot\mathbb{P}[Y=B \mid III]}$$

$$= \frac{\frac{\cancel{2}}{9}\cdot\frac{1}{\cancel{2}}}{\frac{\cancel{2}}{9}\cdot\frac{1}{\cancel{2}} + \frac{\cancel{3}}{9}\cdot\frac{2}{\cancel{3}} + \frac{\cancel{4}}{9}\cdot\frac{3}{\cancel{4}}} = \frac{\frac{1}{9}}{\frac{6}{9}} = \frac{1}{6}$$

$$\mathbb{P}[II \mid Y=B] = \frac{\mathbb{P}[II]\cdot\mathbb{P}[Y=B \mid II]}{\frac{6}{9}} = \frac{\frac{3}{9}\cdot\frac{2}{3}}{\frac{6}{9}} = \frac{1}{3}$$

$$\mathbb{P}[III \mid Y=B] = \frac{1}{2}$$