M339W: October 18^{th}, 2021.

Delta·Gamma·Theta Approximation.

In our market model, we have:

- a risk-free asset, i.e., borrowing/lending money @ a ccrf ir r

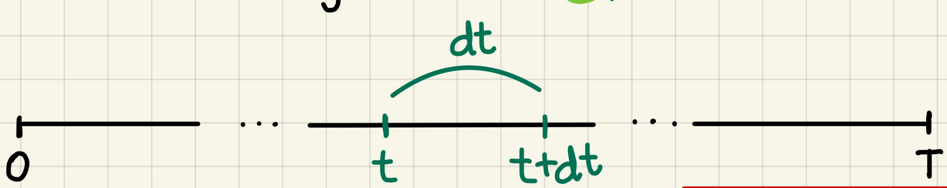
and

- a risky asset, i.e., a continuous dividend paying stock w/ the price denoted by
$$\underbrace{\{S(t), t \geq 0\}}_{\text{stochastic process}}$$

Derivative securities w/ S as the underlying asset are also available (just EUROPEAN).

Assume that we model S using the Black·Scholes framework.

For any portfolio in this market model, we can look @ its value f'tion $v(s,t)$



$$\boxed{ds = S(t+dt) - S(t)}$$

$$S(t) \qquad S(t+dt) = S(t)e^{(r-\delta-\frac{\sigma^2}{2})\cdot dt + \sigma\sqrt{dt}\cdot Z}$$

$$\updownarrow \qquad\qquad \updownarrow$$

$$s \qquad\qquad s+ds$$

$Z \sim N(0,1)$

$$v(s,t) \qquad v(s+ds, t+dt)$$

Taylor-like expansion:

$$v(s+ds,t+dt) \cong v(s,t) + \underbrace{\frac{\partial}{\partial s} v(s,t)}_{=:\,\Delta(s,t)} ds + \frac{1}{2} \boxed{\frac{\partial^2}{\partial s^2} v(s,t)}_{=:\,\Gamma(s,t)} (ds)^2 + \underbrace{\frac{\partial}{\partial t} v(s,t)}_{=:\,\Theta(s,t)} dt$$

Delta-Gamma-Theta Approximation.

$\delta = 0$

19. Assume that the Black-Scholes framework holds. The price of a nondividend-paying stock is \$30.00. The price of a put option on this stock is \$4.00.

$S(0) = 30$ $\qquad$ $v_p(S(0), 0) = 4$

You are given:

(i) $\Delta = -0.28$

(ii) $\Gamma = 0.10$

Using the delta-gamma approximation, determine the price of the put option if the stock price changes to \$31.50.

$S(dt) = 31.50$

(A) \$3.40

(B) \$3.50

(C) \$3.60

(D) \$3.70

(E) \$3.80

$$v_p(S(dt), dt) \cong v_p(S(0), 0) + \Delta_p(S(0), 0)\,ds + \frac{1}{2}\Gamma_p(S(0), 0)(ds)^2$$

w/ $ds = 31.50 - 30 = 1.50$

****END OF EXAMINATION****

$$\text{answer} = 4 + (-0.28)(1.50) + \frac{1}{2}(0.10)(1.5)^2 = 3.69$$

20. Assume that the Black-Scholes framework holds. Consider an option on a stock.

You are given the following information at time 0:

(i) The stock price is $S(0)$, which is greater than 80. $S(0) > 80$

(ii) The option price is 2.34. $\boxed{v(S(0),0) = 2.34}$

(iii) The option delta is –0.181. $\boxed{\Delta(S(0),0) = -0.181}$

(iv) The option gamma is 0.035. $\boxed{\Gamma(S(0),0) = 0.035}$

$S(dt) = 86.$

The stock price changes to 86.00. Using the delta-gamma approximation, you find that the option price changes to 2.21.

$\underline{v(S(dt), dt) = 2.21.}$

Determine $S(0)$.

By the $\Delta \cdot \Gamma$ approximation:

(A) 84.80 : $ds = 1.20$

(B) 85.00 : $ds = 1$

(C) 85.20 : $ds = 0.80$

(D) 85.40 : $ds = 0.60$

(E) 85.80 : $ds = 0.20$

$$v(S(dt), dt) = v(S(0),0) + \Delta(S(0),0)\, ds + \frac{1}{2}\Gamma(S(0),0)\,(ds)^2$$

$$2.21 = 2.34 + (-0.181) \cdot ds + \frac{1}{2}(0.035)(ds)^2$$

Solve the quadratic.

****END OF EXAMINATION****