
Note: You **must** show all your work. Numerical answers without a proper explanation or a clearly written down path to the solution will be assigned zero points.

Problem 8.1. (10 points) The aggregate loss random variable S has a compound Poisson claims distribution, i.e., let the frequency random variable N have the Poisson distribution. You are given that

i. Individual claim amounts may only be equal to 1, 2, or 3.
ii. $\mathbb{E}[S] = 56$
iii. $Var[S] = 126$
iv. The rate of the Poisson claim count random variable is $\lambda = 29$.

Determine the probability mass function of the claim amounts.

Problem 8.2. (15 points) In the compound model for aggregate claims, let the frequency random variable N be negative binomial with parameters $r = 2$ and $\beta = 4$, and let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be given by the probability (mass) function $p_X(1) = 0.3$ and $p_X(2) = 0.7$.
Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
Calculate $\mathbb{P}[S \leq 3]$.

Problem 8.3. (5 pts) In the compound model for aggregate claims, let the frequency random variable N have the Poisson distribution with mean 5. Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be the two-parameter Pareto with parameters $\alpha = 3$ and $\theta = 10$. Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$.
Define the aggregate loss as $S = \sum_{j=1}^{N} X_j$.
What is the variance of S?

Problem 8.4. (10 points) We are using the aggregate loss model and our usual notation. The frequency random variable N is assumed to be Poisson distributed with mean equal to 1. The severity random variable is assumed to have the following probability mass function:

$$p_X(100) = 3/5, \quad p_X(200) = 3/10, \quad p_X(300) = 1/10.$$

Find the probability that the total aggregate loss **exactly** equals 300.

Problem 8.5. (10 points) In the compound model for aggregate claims, let the frequency random variable N be negative binomial with parameters $r = 15$ and $\beta = 5$.

Moreover, let the common distribution of the i.i.d. severity random variables $\{X_j; j = 1, 2, \dots\}$ be the two-parameter Pareto with $\alpha = 3$ and $\theta = 10$.

Let our usual assumptions hold, i.e., let N be independent of $\{X_j; j = 1, 2, \dots\}$. The insurer is interested in finding the total premium π such that the aggregate losses exceed it with the probability less than or equal to 5%. Using the normal approximation, find π such that

$$\mathbb{P}[S > \pi] = 0.05.$$