M378K Introduction to Mathematical Statistics
Problem Set #14
Confidence intervals.

Problem 14.1. *Suppose that the thumb sizes of the US males are following a normal distribution with an unknown mean μ and standard deviation $\sigma = 20$ on the LDI - scale (*Lauretski's Digital Index - LDI - from 50 to 280*). The US Department of Thumbs and Toes (DTT) reports that the mean thumb size in the country is $\mu = 150$. Being the chairman of the Faculty of Thumbs of the local university you see an excellent opportunity here and decide to conduct your own study of the size of the average American thumb.*

(i) *After carefully collecting a random sample of 100 American thumbs you obtain the following sample mean: $\bar{x} = 153$. This result doesn't seem to be compatible with the DTT report so you decide to construct a 95%-confidence interval for the unknown parameter μ based on your study. What is your confidence interval?*

(ii) *Now, you dream about achieving fame and fortune by being the first person ever to estimate the mean thumb size up to ± 0.1. How large a sample size do you need for that?*

Problem 14.2. Gallup's inaugural measure of global loneliness shows over one in five people worldwide (23%) said they felt loneliness "a lot of the day yesterday."[1] *However, there were considerable variations between countries. For instance, out of* 1000 *individuals polled in Taiwan, 11% reported having felt loneliness "a lot of the day" before. What 90%-confidence would you report for the population proportion of Taiwanese who had felt lonely the day before?*

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[1] https://news.gallup.com/poll/646718/people-worldwide-feel-lonely-lot.aspx

Problem 14.3. *What is the unbiased estimator for* σ^2.

Problem 14.4. *Assume a random sample* $Y_1, Y_2, \ldots, Y_n$ *from a normal distribution with mean* μ *and standard deviation* σ *- both unknown. What's the distribution of*

$$\frac{\sum_{i=1}^{n}(Y_i - \bar{Y}_n)^2}{\sigma^2} = \frac{(n-1)S^2}{\sigma^2}\,?$$

Problem 14.5. *Assume that you are assigned a* confidence level $1-\alpha$. *What does it mean to find a* confidence interval *for* S^2*?*

Problem 14.6. *Are* $\hat{\chi}^2_L$ *and* $\hat{\chi}^2_U$ *as above uniquely defined?*

Problem 14.7. *What's the form of the confidence interval, then?*

Problem 14.8. *Assume the above setting. Let the random sample be of size* $n = 9$*. You do the arithmetic and arrive at the estimate* $s^2 = 7.93$ *(based on the data set). Using the above procedure, find the* $90\%-$*confidence interval for* σ^2*.*

Problem 14.9. *(20 points)*

A random sample of size 10 *is drawn from a normal distribution with* **both** *mean and standard deviation* **unknown.** *The generated sample has the sample mean* $\bar{y}_{10} = 14$ *and the (unbiased) estimate of the variance* $s^2 = 25$.

(i) (10 points) *Construct a (symmetric) 90%-confidence interval for* μ.

(ii) (10 *points) Construct a (symmetric) 90%-confidence interval for* σ^2.
Hint: *Remember that you know the distribution of* $(n-1)S^2/\sigma^2$.