UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 2

Prerequisite material.

Please, provide your justification for your response to every question in this subsection. Just the final numerical answer will receive zero credit, even if it is correct. For the graphs, it is sufficient to carefully draw the graph correctly in a clearly labeled coordinate system.

Problem 2.1. (5 points) Let the function f be given by

$$f(x) = \begin{cases} x - 300 & \text{for } x \geq 300 \\ 0 & \text{otherwise} \end{cases}$$

Draw the graph of the function g defined as

$$g(x) = f(x) - 50.$$

Problem 2.2. (5 points) Let the function f be defined as

$$f(x) = x$$

Let the function g be defined as

$$g(x) = \begin{cases} 0 & \text{for } x < 500 \\ x - 500 & \text{for } x \geq 500 \end{cases}$$

Draw the graph of the function $f - g$.

Problem 2.3. (5 points) Let $x > 0$. Then, we always have $e^x > 1 + x$. *True or false? Why?*

Problem 2.4. (5 points)
We define the minimum of two values in the usual way, i.e.,

$$\min(x, y) = \begin{cases} x & \text{if } x \leq y \\ y & \text{if } x \geq y \end{cases}$$

We define the maximum of two values in the usual way, i.e.,

$$\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{if } x \leq y \end{cases}$$

Then, for every x and y we have that

$$x - \min(x - y, 0) = \max(x, y)$$

True or false? Why?

Problem 2.5. (5 points)
We define the maximum of two values in the usual way, i.e.,

$$\max(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{if } x \leq y \end{cases}$$

Then, for every x and y we have that

$$\max(x, y) = \max(x - y, 0) + y$$

True or false? Why?

Problem 2.6. (5 points) Let $\Omega = \{a_1, a_2, a_3, a_4\}$ be an outcome space, and let $\mathbb{P}$ be a probability distribution on Ω. Assume that $\mathbb{P}[\{a_1, a_2\}] = 1/3, \mathbb{P}[\{a_2, a_3\}] = 1/4$ and $\mathbb{P}[\{a_1, a_3\}] = 1/9$. How much is $\mathbb{P}[\{a_4\}]$?

Problem 2.7. (5 points) Let Y be a random variable such that $\mathbb{P}[Y = 2] = 1/2$, $\mathbb{P}[Y = 3] = 1/3$ and $\mathbb{P}[Y = 6] = 1/6$. What is $\mathbb{E}[\min(Y, 5)]$?

Problem 2.8. (5 points) A coin for which *Heads* is twice as likely as *Tails* is tossed twice, independently. If the outcomes are two consecutive *Heads*, Bertie gets a \$15 reward; if the outcomes are different, Bertie gets a \$10 reward; if the outcomes are two consecutive *Tails*, Bertie has to pay \$5. What is the expected value of Bertie's winnings?

Problem 2.9. (5 points) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions given by

$$f(x) = |x - 10|$$

and

$$g(x) = \begin{cases} \min(x, 4) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then, $g(f(7))$ equals ...

(a) 0
(b) 3
(c) 4
(d) 7
(e) None of the above

Problem 2.10. (5 points) Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ be a probability space. We denote by p_k the probability of the elementary outcome ω_k, i.e., $p_k = \mathbb{P}[\{\omega_k\}]$ for $k = 1, \ldots, 5$. You are given that p_k/p_{k-1} is constant for $k = 2, 3, 4, 5$. You are also given that $p_1 = 16/31$. Find p_5.

(a) 1/31
(b) 2/31
(c) 4/31
(d) Not enough information is given.
(e) None of the above.