UNIVERSITY OF TEXAS AT AUSTIN

Problem set 4

Expectation. Moments. Coefficient of Variation.

Problem 4.1. Let X be a random variable with a finite expectation. Consider the function

$$g(a) = \mathbb{E}[(X-a)^2]$$

defined for all a such that the expectation exists. For which value a does the function g attain its minimum?

Problem 4.2. *Source: "Probability and Statistical Inference" by Hogg, Tanis, and Zimmerman.* An insurance agent receives a bonus if the loss ratio L on the business is less than 0.5 where L is the total losses X divided by the total premiums where the total premiums are equal exactly to 3. The bonus equals

$$\frac{0.5 - L}{10} \tag{4.1}$$

if it occurs (and it is, obviously, zero otherwise). Let X (in 100K) have the probability density function

$$f_X(x) = 3x^{-4}, \quad x > 1.$$

What is the expected value of the bonus?

Problem 4.3. The manufacturer claims that the lifetime of an espresso machine is uniform between 0 and 4. The coffee shop replaces the machine either at the time of failure or at time 3, whichever occurs first. What is the variance of the replacement time?

Problem 4.4. Consider two independent random variables X and Y which have the same mean. You are given that coefficient of variation of X equals 5 and the coefficient of variation of Y equals 12. What is the coefficient of variation of the sum of X and Y?