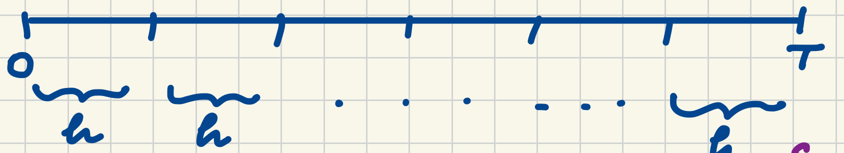
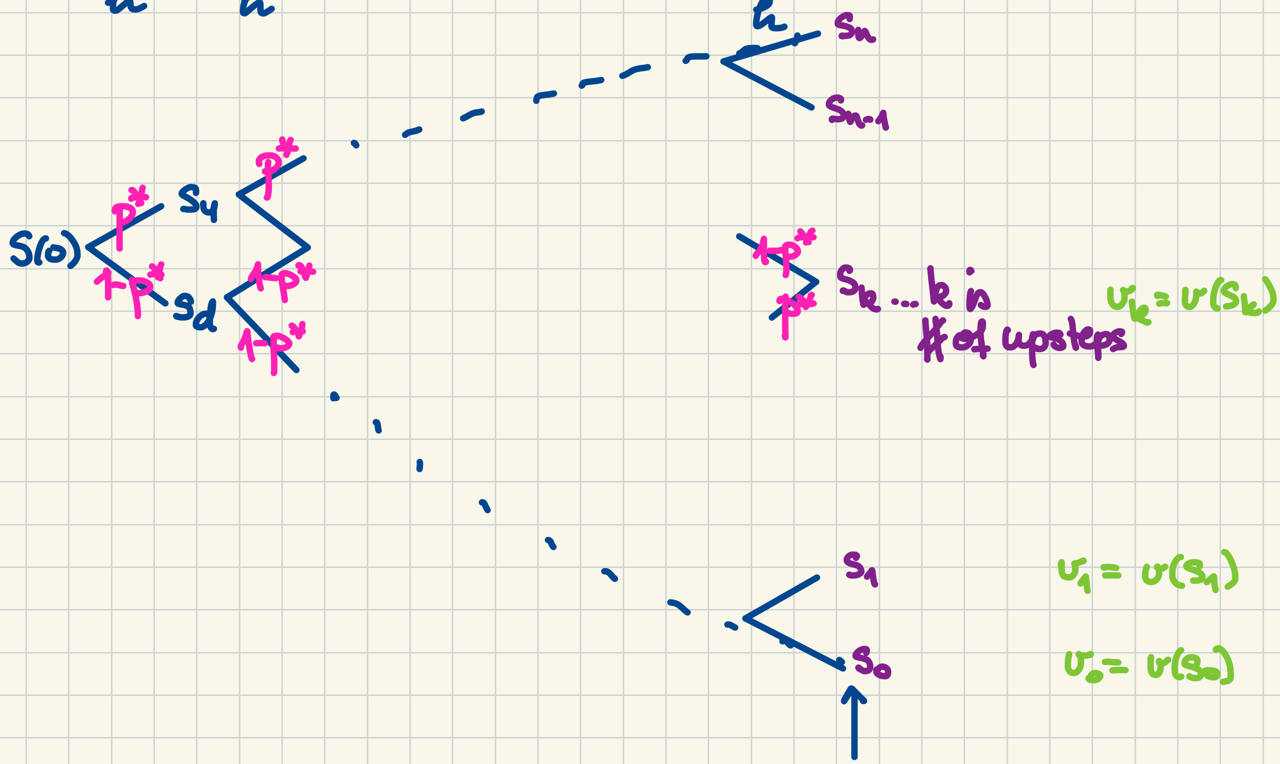
Multiple Binomial Periods.

T ... exercise date of a European option
n ... # of periods

} the length of each period: $\boxed{h = \frac{T}{n}}$





The $n+1$ values are the support of $S(T)$.

$\Rightarrow$ for every $k = 0, 1, \dots, n$:

$$s_k = S(0)u^k \cdot d^{n-k} = S(0)\left(\frac{u}{d}\right)^{k} d^n$$

k ... # of upsteps

Consider a European option w/ payoff f'tion $v(\cdot)$.
Then, the possible values of the payoff will be

$$\boxed{v_k = v(s_k)} \quad \text{for } k = 0, 1, \dots, n$$

Risk-Neutral Pricing.

$$V(0) = e^{-rT}\mathbb{E}^*[V(T)] \quad \text{w/} \quad \boxed{p^* = \frac{e^{rh} - d}{u - d}}$$

$\Rightarrow$ The risk-neutral probability of reaching the payoff v_k is:

$$\boxed{\binom{n}{k}(p^*)^k(1-p^*)^{n-k}} \quad \text{for } k = 0, 1, \dots, n$$

$\Rightarrow$ The risk-neutral option price:

$$\boxed{V(0) = e^{-rT}\sum_{k=0}^{n}\left(\binom{n}{k}(p^*)^k(1-p^*)^{n-k}\cdot v_k\right)}$$

Problem 10.2. Let the continuously compounded risk-free interest rate be 0.10. Let the initial price of a non-dividend-paying stock be $100 per share. You use a five-period binomial tree to model the stock price over the next year. Let $u = 1.04$ and $d = 0.96$.

$h = 1/5 = 0.2$

What is the price of a one-year, at-the-money European call option on the above stock?

→: The Risk-Neutral Probability:

$$p^* = \frac{e^{rh} - d}{u - d} = \frac{e^{0.10 \cdot 0.2} - 0.96}{1.04 - 0.96} = \underline{0.7525}$$

The relevant final stock prices in our tree:

K=100

$S_5 = S(0)u^5 = 100 \cdot (1.04)^5 = \underline{121.67}$ $\quad v_5 = 21.67$

$S_4 = S(0)u^4 \cdot d = 100 \cdot (1.04)^4 (0.96) = \underline{112.31}$ $\quad v_4 = 12.31$

$S_3 = S(0)u^3 \cdot d^2 = 100 (1.04)^3 (0.96)^2 = \underline{103.67}$ $\quad v_3 = 3.67$

The remaining terminal nodes are all out-o-money.

$\Rightarrow$

$\binom{5}{4} = \binom{5}{1}$

$$V(0) = e^{-0.10}\Big(21.67\,(p^*)^5 + 12.31 \cdot 5 \cdot (p^*)^4(1-p^*) + 3.67 \underbrace{\binom{5}{2}}_{10} (p^*)^3 (1-p^*)^2\Big) = \underline{10.01821}$$

$\binom{5}{2} = \binom{5}{3}$

□