M378K Introduction to Mathematical Statistics

Homework assignment #7

Please, provide your **final answer only** to the following problems.

Problem 7.1. *(3 × 7 = 21 points) Identify the distributions with the following mgfs:*

- $\frac{2}{2-t}$.
- $e^{2e^t - 2}$,
- $e^{t(t-2)}$,
- $(3 - 2e^t)^{-1}$
- $\frac{1}{9} + \frac{4}{9}e^t + \frac{4}{9}e^{2t}$.
- $\frac{1}{t}(1 - e^{-t})$.
- $\frac{1}{4}(e^{4t} + 3e^{-t})$

If the distribution has a name, give the name and the parameters. If it does not, give the pdf or the pmf (table).

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 7.2. *(30 points) Solve* **Problem 7.6.11** *from the* Lecture notes.

Problem 7.3. *(18 points)* Source: "Mathematical Statistics with Applications" by Wackerly, Mendenhal, Scheaffer.
Suppose that a random variable Y has a probability density function given by

$$f_Y(y) = \kappa y^3 e^{-y/2} \mathbf{1}_{(0,\infty)}(y)$$

(i) *(5 points) Find the value of κ that makes $f_Y(y)$ a density function.*

(ii) *(3 points) Does Y have a χ^2-distribution? If so, how many degrees of freedom?*

(iii) *(5 points) What are the mean and standard deviation of Y?*

(iv) *(5 points) (Extra credit) Using* **R**, *find the probability that Y lies within 2 standard deviations of its mean?*