

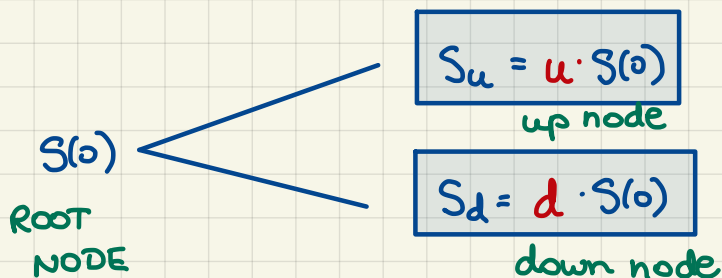
M339D: October 21st, 2022.

The Binomial Asset Pricing Model.

$S(0)$... the observable initial asset price



time horizon (i.e., exercise date of an option)



By convention: $u > d$ ✓

u ... up factor

d ... down factor

(h) ... length of a single period

one period $\Rightarrow S(T) = S(h)$... a r.v. denoting the time T stock price w/ two possible values: S_u and S_d

Returns:

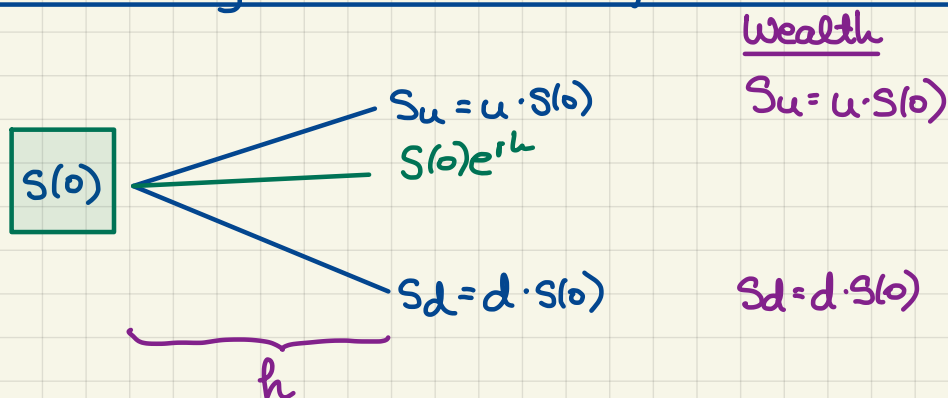
$$\frac{S_u - S(0)}{S(0)} = \frac{S_u}{S(0)} - 1 = u - 1$$

$$\frac{S_d - S(0)}{S(0)} = \frac{S_d}{S(0)} - 1 = d - 1$$

Market Model.

- riskless asset: @ the ccr fir r
- risky asset: non-dividend-paying stock

Imagine investing in one share of stock @ time 0:



At the risk-free rate $S(0)$ accumulates to $S(0)e^{rh}$ @ time T .
" time h .

$$d \cdot \cancel{S(0)} < \cancel{S(0)} e^{rh} < u \cdot \cancel{S(0)}$$

$$\boxed{d < e^{rh} < u} \quad \underline{\text{The no-arbitrage condition.}}$$

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Problem Set #6

Binomial option pricing.

Problem 6.1. In the setting of the one-period binomial model, denote by i the effective interest rate **per period**. Let u denote the “up factor” and let d denote the “down factor” in the stock-price model. If

$$d < u \leq 1 + i$$

then there certainly is no possibility for arbitrage.

(F)

Fixed Statement:

$$d < 1+i < u$$

Problem 6.2. In our usual notation, does this parameter choice create a binomial model with an arbitrage opportunity?

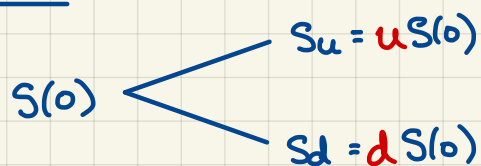
$$u = 1.18, \quad d = 0.87, \quad r = 0.05, \quad \delta \times 0, \quad h = 1/4$$

(NO!)

$$d < e^{rh} = e^{0.05(0.25)} = 1.013 < u$$

Binomial Option Pricing.

Stock Price Tree.



populating
the tree

We want to price a European-style derivative security w/ the exercise date @ the end of the period.

It is completely determined by its payoff function: $v(\cdot)$

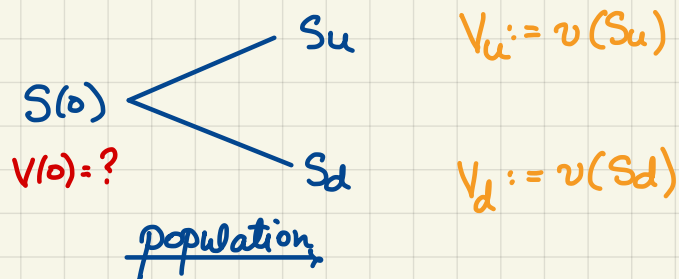
e.g., for a call: $v_c(s) = (s - K)_+$

for a put: $v_p(s) = (K - s)_+$

The payoff of the derivative security is a random variable:

$$V(T) := v(S(T))$$

Payoff



population

← pricing