UNIVERSITY OF TEXAS AT AUSTIN

Lecture 11

Put-call parity: The general case

11.1. **Construction.** Let *Portfolio A* consist of a long European call and a short European put on the same underlying asset S with the same strike K and the same exercise date T. The initial value of this portfolio is

$$V_A(0) = V_C(0) - V_P(0).$$

There are no intermediate cash-flows associated with this portfolio and its payoff at time T is

$$V_C(T) - V_P(T) = S(T) - K.$$

On the other hand, let *Portfolio B* consist of the following:

(1) a **long** prepaid forward contract on S for delivery at time T,
(2) **borrowing** the present value of the strike price to be repaid at time T.

Then, the initial cost of this portfolio equals:

$$F^P_{0,T}(S) - PV_{0,T}(K).$$

Since there are no intermediate cash-flows associated with this portfolio, either, its payoff at time T is

$$S(T) - K.$$

Since the above portfolios have the same final payoff, by the no-arbitrage principle, we conclude that their initial values must also be the same. We get the more general version of put-call parity:

$$V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K).$$

11.2. **Special cases.** Our most common setting is the one with a continuously compounded interest rate r. In that case the put-call parity reads as

$$V_C(0) - V_P(0) = F^P_{0,T}(S) - Ke^{-rT}.$$

With respect to dividends, these are the three cases we will be looking into:

- non-dividend-paying stocks:
$$V_C(0) - V_P(0) = S(0) - Ke^{-rT}$$
- discrete dividends $D_i, i = 1, \dots, n$ at times $0 < t_1 < \dots < t_n \leq T$:
$$V_C(0) - V_P(0) = S(0) - \sum_{i=1}^{n} D_i e^{-rt_i} - Ke^{-rT}$$
- continuous dividends at the rate δ:
$$V_C(0) - V_P(0) = S(0)e^{-\delta T} - Ke^{-rT}$$

11.3. **MFE Exam Spring 2007: Problem #1.** On April 30, 2007, a common stock is priced at \$52.00. You are given that:

(1) Dividends in equal amounts are to be paid on June 30, 2007, and on September 30, 2007.
(2) A European call on the above stock with strike $K = \$50$ and the exercise date in six months sells for \$4.50.
(3) A European put on the above stock with strike $K = \$50$ and the exercise date in six months sells for \$2.45.
(4) The continuously-compounded risk-free interest rate equals 0.06.

Calculate the amount of each dividend.

Solution: In addition to our usual notation, we introduce D to stand for the amount of each dividend payment. Then, the put-call parity reads as

$$V_C(0) - V_P(0) = S(0) - De^{-rt_1} - De^{-rt_2} - Ke^{-rT}$$

with $t_1 = 1/6$ and $t_2 = 5/12$. Solving for D above, we get

$$D = \frac{S(0) - Ke^{-rT} - V_C(0) + V_P(0)}{e^{-rt_1} + e^{-rt_2}} = \frac{52 - 50e^{-0.06\cdot(1/2)} - 4.5 + 2.45}{e^{-0.06\cdot(1/6)} + e^{-0.06\cdot(5/12)}} \approx 0.73.$$