University of Texas at Austin

HW Assignment 6

Please, provide your **complete solutions** to the following problems. Final answers only, even if correct will earn zero points for those problems.

Problem 6.1. (2 points) Consider a one-year, \$55-strike European call option and a one-year, \$45-strike European put option on the same underlying asset.

You observe that the time-0 stock price equals \$40 while the time-1 stock price equals \$50. Then, both of the options are out-of-the-money at expiration.

Solution: TRUE

Problem 6.2. (15 points) You produce cupcakes. You plan to sell 1,000 festive cupcakes in a month. Your (unhedged) payoff will be $\$25,000 - S(1)$, where $S(1)$ denotes the price of the 1,000 fondant reindeer required to decorate the 1,000 cakes.

Assume that the continuously compounded annual risk-free interest rate equals 6%.

Your hedge consists of the following two components:

(1) one **long** one-month, \$10,000-strike call option on the fondant reindeer you need; its premium is $V_C(0) = \$70.00$,
(2) one **written** one-month, \$9,000-strike put option on the fondant reindeer you need; its premium is $V_P(0) = \$150.00$.

Calculate the maximum and the minimum of the profit for the (overall) **hedged** portfolio.

Solution: The hedged portfolio consists of the following components:

(1) the **payoff** from the cupcake sales,
(2) one **long** one-month, \$10,000-strike call option on the fondant reindeer whose premium was \$70.00,
(3) one **written** one-month, \$9,000-strike put option on the fondant reindeer whose premium was \$150.00.

The initial cost for this portfolio is the cost of hedging (all other accumulated production costs are incorporated in the revenue expression $25,000 - S(1)$). Their future value is

$$(70 - 150)e^{0.005} \approx -80.401.$$

As usual, the negative initial cost signifies an initial influx of money for the principal character.

The **hedged** profit is

$$25000 - S(1) - (9000 - S(1))_+ + (S(1) - 10000)_+ + 80.401.$$

The maximum is attained for $S(1) \leq 9,000$. It equals

$$16000 + 80.401 = 16,080.401.$$

The minimum profit is attained for $S(1) \geq 10,000$. It equals

$$15000 + 80.401 = 15,080.401.$$

Problem 6.3. (15 points) The future value in one year of the total aggregate costs of manufacturing a widget is \$100. You will sell a widget in one year at its market price of $S(1)$.

Assume that the continuously compounded, risk-free interest rate equals 5%.

You purchase a one-year, \$120-strike put on one widget for a premium of \$7. You sell some of the potential gain by writing a one-year, \$150-strike call on one widget for a \$3 premium.

What is the **range** of the profit of your total hedged porfolio?

Solution: The payoff, written as a piecewise function, is

$$v(s) = \begin{cases} K_P, & \text{for } 0 \leq s \leq K_P \\ s, & \text{for } K_P \leq s \leq K_C \\ K_C, & \text{for } K_C \leq s \end{cases}$$

where K_P denotes the strike price for the put while K_C denotes the call's strike price. So, the range of the payoff function is $[120, 150]$.

The future value of the total cost of both production and hedging is

$$100 + (7-3)e^{-0.05} = 104.21.$$

So, the range of the profit equals $[15.79, 45.79]$.

Floors. The portfolio consisting of

- the **long** risky asset, and
- a **long** put on that asset

is commonly referred to as the *floor.* It arises naturally when the producer of a commodity or an owner of a risky asset (shares of stock, e.g.) uses puts to hedge his/her exposure to risk.

Provide your final answer only for the following problem.

Problem 6.4. (5 points) **Sample FM(DM) #13.**
Suppose that you short one share of a stock index for 50, and that you also buy a 60−strike European call option that expires in 2 years for 10. Assume the effective annual interest rate is 3%. If the stock index increases to 75 after 2 years, what is the profit on your combined position, and what is an alternative name for the call in this context?

	Profit	**Name**
A.	−22.64	Floor
B.	−17.56	Floor
C.	−22.64	Cap
D.	−17.56	Cap
E.	−22.64	"Written" Covered Call

Solution: First, about the qualitative portion of the answer. A combination of a short asset and a long call is called a **cap**. So, one can eliminate all of the offered answers except for C. and D.

Next, the quantitative component of the problem. The payoff is simply:

$$-75 + (75-60)_+ = 60.$$

In words, as a short seller, you have to purchase the stock index back and you are going to take advantage of owning the call option on that index (as opposed to paying the higher market price). The initial cost is $-50+10=-40$. So, the value of this initial cost in 2 years equals $-40 \cdot (1.03)^2 = -42.436$. The profit is

$$40 - 42.436 = 17.564.$$

We choose $\boxed{\text{D.}}$ as the correct answer.

Covered puts. The writer of a put option might want to hedge his/her exposure to risk by shorting the underlying asset. The position consisting of

- the **short** risky asset, and
- a **written** put on that asset

is commonly referred to as the *covered put.*

Problem 6.5. (2 points) Which one of the following constructs a *covered put*?

(a) Write a one-year, \$100-strike call and buy the underlying.
(b) Write a one-year, \$100-strike put and short the underlying.
(c) Write a one-year, \$100-strike put and buy the underlying.

(d) Write a one-year, \$100-strike put and write a one-year, \$100-strike call.
(e) None of the above.

Solution: (b)

Parallels between put options and classical insurance. Consider *homeowner's insurance.* An insurance policy is there to compensate the homeowner in case there is a financial loss due to physical damage to the home (fire, e.g.). At the time the insurance policy is issued the home is apraised and its **initial value** becomes part of the insurance policy. If the property is damaged, the insurance company is liable to make a benefit payment to the policyholder in the amount needed to bring the home back to its original state. In order for this to happen, however, the policyholder needs to initiate a claim. The homeowner is not required to file a claim, but should the claim be filed, the insurance company is required to proceed according to the contract and make the benefit payment.

So far, we have discussed the use of derivative securities (forward contracts, call options and put options) for hedging. If we draw parallels between classical insurance and use of options, we get the following correspondence:

Classical homeowner's insurance	**Hedging with derivative securities**
Home	Risky asset
Value of home	Market price of the risky asset
Insurance company	Option writer
Policyholder	Option buyer
Benefit payment	Payoff

If we specify the features of the insurance policy, we can see even more precise connections. Most insurance policies include a type of cost-sharing between the insurer and the insured. Most commonly, homeowner's insurance includes a *deductible*. The deductible d is the monetary amount up to which the policyholder pays for the damages. Once the loss exceeds d, the insurer pays for the excess of the loss over the deductible. So, if we denote the loss amount by the random variable X, the amount paid by the insurer and received by the policyholder is $(X-d)_+$.

The loss X can be understood as the reduction in the home's value due to physical damage. If we denote the home's value at time t by $S(t)$, we see that $X = S(0) - S(T)$ with T denoting the end of the insurance period, say. Having observed this, we see that the amount received by the policyholder is

$$(X-d)_+ = (S(0) - S(T) - d)_+ = ((S(0) - d) - S(T))_+$$

The expression above is exactly the payoff of a put option with strike price $S(0) - d$. With this observation, our analogy is complete.

Please, provide the **final solution only** to the following problem(s):

Problem 6.6. (2 points) *Source: Sample FM(DM) Problem #27.*
The position consisting of *one long homeowner's insurance contract* benefits from falling prices in the underlying asset. *True or false?*

Solution: TRUE
Recall our comparison of the homeowner's insurance policy to the put option. The payoff of the put option is decreasing in the price of the underlying asset.

Problem 6.7. (2 points) The owner of a house worth \$180,000 purchases an insurance policy at the beginning of the year for a price of \$1,000. The deductible on the policy is \$5,000.
If after 6 months the homeowner experiences a casualty loss valued at \$50,000, what is the homeowner's net loss? Assume that the continuously compounded interest rate equals 4.0%.

(a) \$6,020
(b) \$11,020
(c) \$50,000
(d) \$51,020

(e) None of the above.

Solution: (a)
The homeowner had to give up the insurance premium of \$1,000, and six months later he/she also had to pay the \$5,000 deductible. Taking into account the interest that the initial premium would have earned over the six-month period, the homeowner's total loss is

$$1,000(1.04)^{1/2} + 5,000 \approx 6,020.$$

Problem 6.8. (7 points) Draw the profit diagram for the homeowner's **complete** position consisting of both the propery and the insurance policy.

Solution: There are two variations of the graph which I am accepting as correct. The first one goes along with a homeowner who has had the property for a while, so that one can choose his/her initial investment in the house to be \$0. It is shown in Figure 1. The second one is the version in which the house is purchased at a price whose time-value at the end of the insurance term equals \$200,000.

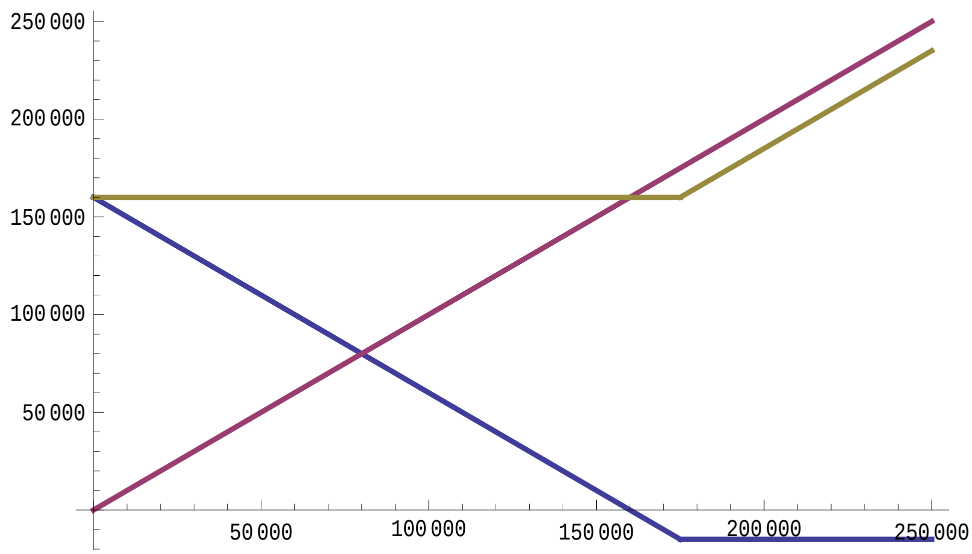

FIGURE 1. Inherited house

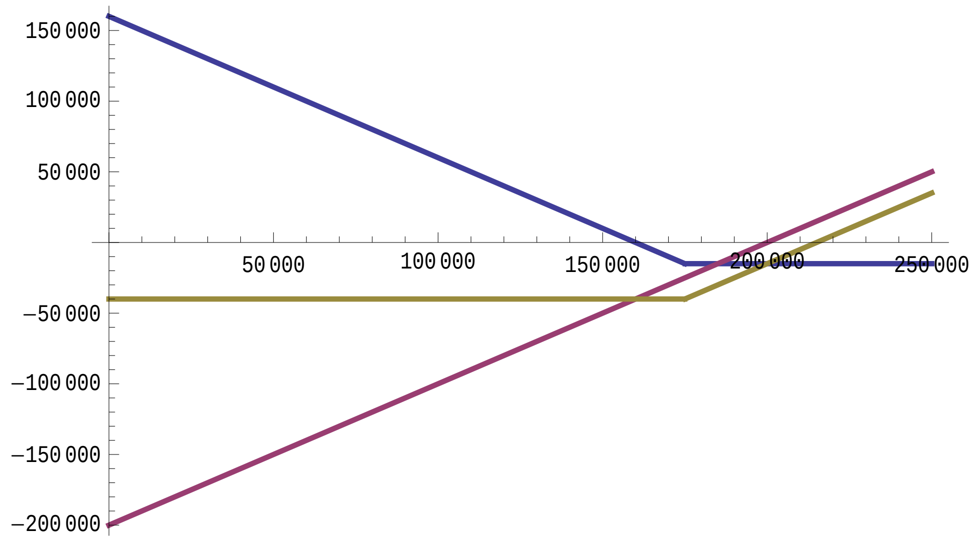

Figure 2. House purchased in the beginning of the insurance period