M339D: February 23rd, 2026.

3. An insurance company sells single premium deferred annuity contracts with return linked to a stock index, the time-t value of one unit of which is denoted by $S(t)$. The contracts offer a minimum guarantee return rate of $g\%$. At time 0, a single premium of amount π is paid by the policyholder, and $\pi \times y\%$ is deducted by the insurance company. Thus, at the contract maturity date, T, the insurance company will pay the policyholder

$$\pi \times (1 - y\%) \times \text{Max}[S(T)/S(0), (1 + g\%)^T].$$

You are given the following information:

(i) The contract will mature in one year.

(ii) The minimum guarantee rate of return, $g\%$, is 3%.

(iii) ~~Dividends are incorporated in the stock index. That is, the stock index is constructed with all stock dividends reinvested.~~ NO DIVIDENDS!

(iv) $\boxed{S(0) = 100}$.

(v) The price of a one-year European $\boxed{\text{put option}}$ with strike price of \$103, on the stock index is $\boxed{\$15.21}$.

Determine $y\%$, so that the insurance company does not make or lose money on this contract.

(A) 12.8%.

(B) 13.0%

(C) 13.2%

(D) 13.4%

(E) 13.6%.

The Synopsis:

1. Focus on the insurance company's liability

2. Use our data

3. Algebraically simplify w/ an eye to "creating" the expression for payoff of a $\boxed{\text{put}}$

→ : The Insurance Company's Liability:

$$\underbrace{\boxed{\pi(1-y)}}_{\text{const.}} \cdot \text{Max}\left[\frac{S(T)}{\boxed{S(0)}}, (1+g)^T\right]$$

$$\underbrace{\frac{1}{S(0)}}_{\text{const.}} \text{Max}\left[S(T), \underbrace{\underbrace{S(0)}_{100}\underbrace{(1+g)^{T}}_{1.03}}_{103}\right]$$

$T = 1$

$\text{Max}[S(T), 103] = ?$

$$\boxed{V_P(T) = (103 - S(T))_+}$$

a, b

$$\max(a,b) = a + \max(0, b-a) = a + (b-a)_+$$
$$= b + \max(a-b, 0) = b + (a-b)_+$$

$$\text{Max}[S(T), 103] = \underbrace{\boxed{S(T)}}_{\text{Long stock}} + \underbrace{\boxed{(103 - S(T))_+}}_{\text{Payoff of the put w/ strike 103 and exercise date } T=1.}$$

The insurance company can perfectly hedge by:

- Longing/Buying $\frac{\pi(1-y)}{S(0)}$ units of the stock index;
- Buying $\frac{\pi(1-y)}{S(0)}$ European puts w/ $K = 103$ and $T = 1$.

If they receive the same amount of money @ time 0 as is the cost of this replicating portfolio, they break even.

$$\cancel{\pi} = \frac{\cancel{\pi}(1-y)}{S(0)}\left(S(0) + V_P(0)\right)$$

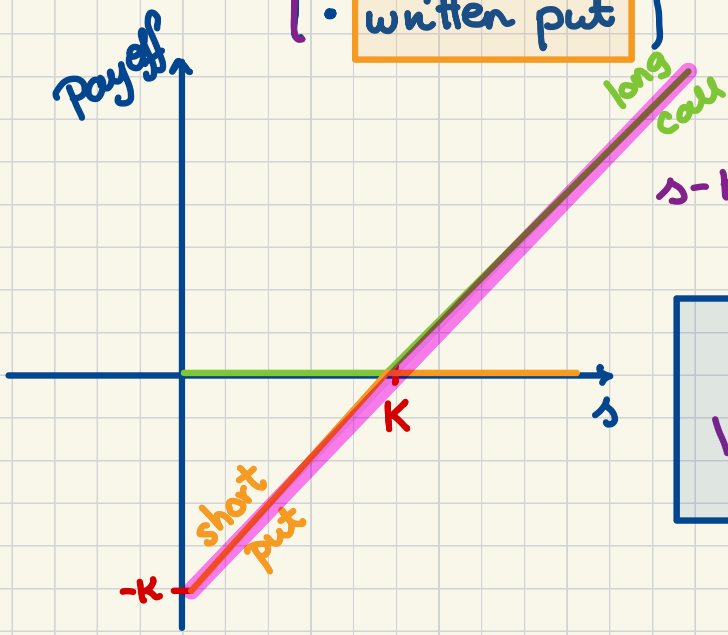
$$100 = (1-y)(100 + 15.21)$$

$$1 - y = \frac{100}{115.21} \Rightarrow y = \frac{15.21}{115.21} = \underline{0.132} \quad \square$$

Put·Call Parity

Portfolio A:
- long call
- written put

both European & otherwise identical



The Payoff:

$$V_A(T) = V_C(T) - V_P(T) = S(T) - K$$

Portfolio B:
- long non·dividend·paying stock
- borrow $PV_{0,T}(K)$ @ the risk·free rate to be repaid @ time·T

$$\Rightarrow \boxed{V_B(T) = S(T) - K}$$

NO ARBITRAGE!

$\Rightarrow$

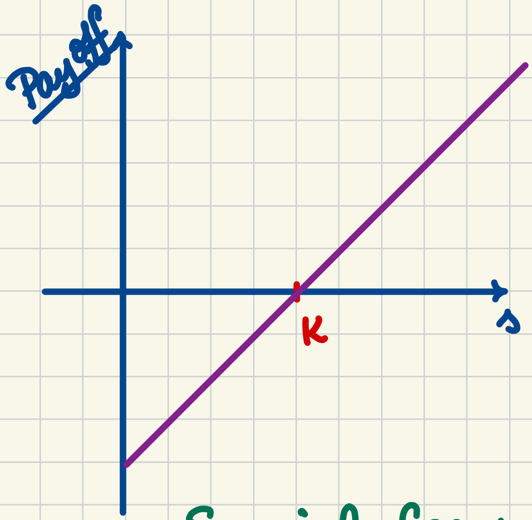
$$V_A(0) = V_B(0)$$

$$\boxed{V_C(0) - V_P(0) = S(0) - PV_{0,T}(K)}$$ Put·Call Parity.

More generally: for all $t \in [0,T]$:

$$\boxed{V_C(t) - V_P(t) = S(t) - PV_{t,T}(K)}$$

Remarks:
- The no·arbitrage assumption is sufficient
- Only works for European options.



With Portfolio A,
we construct a
"synthetic" forward
or
"off-market" forward

Special Case:

strike price K = forward price for the stock F $\circledast$

$$\Longleftrightarrow$$

$$K = F_{0,T}(S) = S(0)e^{rT} = FV_{0,T}(S(0))$$

$$\Longleftrightarrow$$

$$PV_{0,T}(K) = S(0)$$

$$\Longleftrightarrow$$

$$V_C(0) - V_P(0) = 0 = S(0) - PV_{0,T}(K)$$

$$\Longleftrightarrow$$

$$\boxed{V_C(0) = V_P(0)}$$ By put-call Parity