M339D: September 16^{th}, 2022.

Example.

- Portfolio P w/ components X and Y w/ weights w_X and w_Y.
- Economy can be "good" or "bad" w/ probability p_g and p_b, resp.

	G	B
R_X	x_g	x_b
R_Y	y_g	y_b

$$\boxed{R_P = w_X \cdot R_X + w_Y \cdot R_Y}$$

$$R_P \sim \begin{cases} w_X \cdot x_g + w_Y \cdot y_g & \text{"good" economy w/ probab. } p_g \\ w_X \cdot x_b + w_Y \cdot y_b & \text{"bad" economy w/ probab. } p_b \end{cases}$$

11) You are given the following information about a portfolio that has two equally-weighted stocks, P and Q.

$w_P = w_Q = \frac{1}{2}$

(i) The economy over the next year could be good or bad with equal probability.

$p_g = p_b = \frac{1}{2}$

(ii) The returns of the stocks can vary as shown in the table below:

Stock	Return when economy is good	Return when economy is bad
P	10%	–2%
Q	18%	–5%

Calculate the volatility of the portfolio return.

(A) 1.80%

(B) 6.90%

(C) 7.66%

(D) 8.75%

(E) 13.42%

$\rightarrow$: R_T ... the return of the total portfolio

$\sigma_T = SD[R_T] = \sqrt{Var[R_T]} = ?$

$R_T = \frac{1}{2}(R_P + R_Q)$

$R_T \sim \begin{cases} \underline{0.14} & \text{if "good" w/ prob. } \frac{1}{2} \\ \underline{-0.035} & \text{if "bad" w/ prob } \frac{1}{2} \end{cases}$

$Var[R_T] = \mathbb{E}[R_T^2] - (\mathbb{E}[R_T])^2$

- $\mathbb{E}[R_T] = \frac{1}{2}(0.14 + (-0.035)) = \underline{0.0525}$
- $\mathbb{E}[R_T^2] = \frac{1}{2}((0.14)^2 + (-0.035)^2) = \underline{0.0104125}$

$Var[R_T] = 0.0104125 - (0.0525)^2 = \underline{0.0076563}$

$\sigma_T = \sqrt{0.0076563} = \underline{0.0875}$

Diversification of an Equally Weighted Portfolio.

$w_i = \frac{1}{n}$ for $i = 1..n$

$$\Rightarrow \quad R_p = \frac{1}{n}(R_1 + R_2 + \dots + R_n)$$

$$\Rightarrow \quad Var[R_p] = Var\left[\frac{1}{n}(R_1 + \dots + R_n)\right] =$$

$$= \frac{1}{n^2} Var[R_1 + \dots + R_n]$$

$$= \frac{1}{n^2}\left(\boxed{\sum_{i=1}^{n} Var[R_i]} + \boxed{\sum_{i \neq j} Cov[R_i, R_j]}\right)$$

$$= \underbrace{\frac{1}{n} \cdot \boxed{\frac{1}{n} \cdot \sum_{i=1}^{n} Var[R_i]}}_{\xrightarrow{n \to \infty} 0} + \underbrace{\frac{1}{n^{\cancel{2}}} \cancel{n}(n-1) \boxed{\frac{1}{n(n-1)} \sum_{i \neq j} Cov[R_i, R_j]}}_{\xrightarrow{n \to \infty} \boxed{\text{Average Covariance}}}$$

$$\frac{1}{n^{\cancel{2}}}\cancel{n}(n-1) = \left(1 - \frac{1}{n}\right)$$

Average Variance of the individual components (assume bounded)

Average Covariance between different stocks in the Portfolio

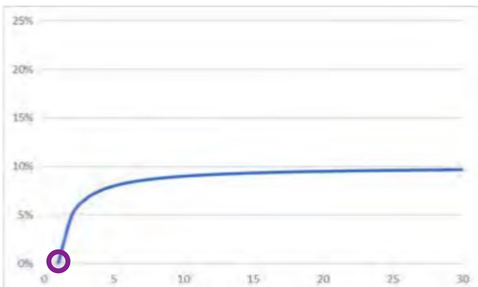
9) You are given the following information about an equally-weighted portfolio of n stocks:

(i) For each individual stock in the portfolio, the variance is 0.20.

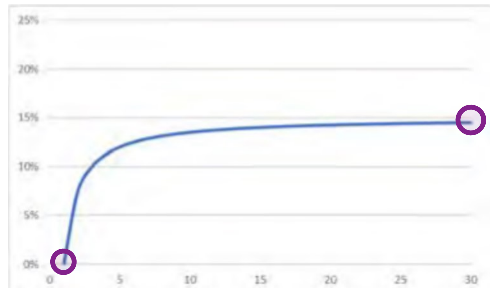
(ii) For each pair of distinct stocks in the portfolio, the covariance is 0.10.

Determine which graph displays the variance of the portfolio as a function of n.

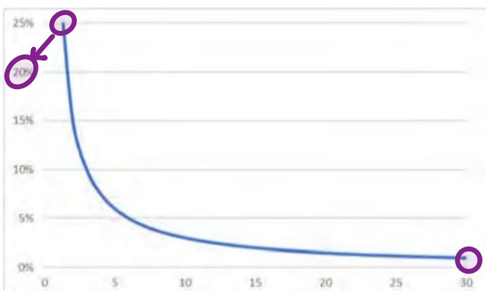
(A)



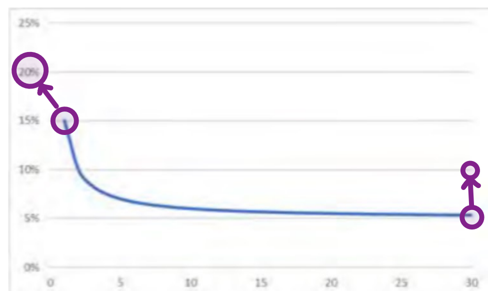
(B)



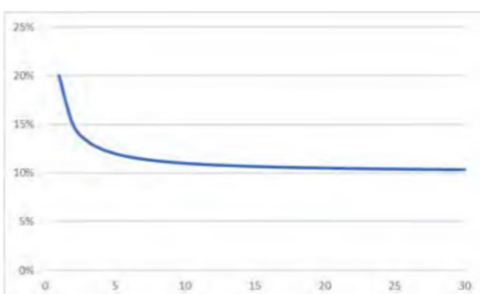
(C)



(D)



(E)



Diversification for a General Portfolio.

Assume: $\boxed{w_i \geq 0}$

Recall:

$$\sigma_P^2 = Var[R_P] = \sum_{i=1}^{n} w_i \cdot Cov[R_i, R_P]$$

$$= \sum_{i=1}^{n} w_i \cdot \sigma_i \, \sigma_P \, \rho_{i,P}$$

$$= \sigma_P \sum_{i=1}^{n} w_i \cdot \sigma_i \cdot \rho_{i,P} \qquad / : \sigma_P$$

$$\sigma_P = \sum_{i=1}^{n} w_i \cdot \sigma_i \cdot \underbrace{\rho_{i,P}}_{\leq 1}$$

$$\boxed{\sigma_P \leq \sum_{i=1}^{n} w_i \cdot \sigma_i}$$

Equality only when all the assets are perfectly positively correlated.

6) You are given the following information about the four distinct portfolios:

Portfolio	Expected Return	Volatility
P	3%	10%
Q	5%	10%
R	5%	15%
S	7%	20%

Determine which two of the four given portfolios are NOT efficient.

(A) P and Q

(B) P and R

(C) P and S

(D) Q and R

(E) Q and S