M339D : September 26^{th}, 2022.

15) You are given the following information about Stock X, Stock Y, and the market:

(i) The expected return and volatility for Stock X, Stock Y, and the market are shown in the table below:

	Required Return	Volatility
Stock X	3.0%	50%
Stock Y	?	35%
Market	6.0%	25%

(ii) The correlation between the returns of stock X and the market is –0.25. $\boxed{\rho_{X,Mkt} = -0.25}$

(iii) The correlation between the returns of stock Y and the market is 0.30. $\boxed{\rho_{Y,Mkt} = 0.3}$

Assume the Capital Asset Pricing Model holds. Calculate the required return for Stock Y.

(A) 1.48%

(B) 2.52%

(C) 3.16%

(D) 4.84%

(E) 6.52%

$\longrightarrow$:

$$\beta_X = \frac{\sigma_X}{\sigma_{Mkt}} \cdot \rho_{X,Mkt} = \frac{0.5}{\cancel{0.25}}(-\cancel{0.25}) = -0.5$$

$$r_X = r_f + \beta_X\left(\mathbb{E}[R_{Mkt}] - r_f\right)$$

$$0.03 = r_f + (-0.5)(0.06 - r_f)$$

$$0.03 = r_f - 0.03 + 0.5\, r_f$$

$$1.5\, r_f = 0.06 \quad \Rightarrow \quad \boxed{r_f = 0.04} \checkmark$$

$$\beta_Y = \frac{\sigma_Y}{\sigma_{Mkt}} \cdot \rho_{Y,Mkt} = \frac{0.35}{0.25}(0.3) = 0.42$$

$$r_Y = r_f + \beta_Y\left(\mathbb{E}[R_{Mkt}] - r_f\right) = 0.04 + 0.42(0.06 - 0.04) = \underline{0.0484}$$

$\square$

Beta of a Portfolio.

Let p be a portfolio such that

$$R_p = \boxed{w_1 \cdot R_1 + w_2 \cdot R_2 + \cdots + w_n \cdot R_n}$$

$$\beta_p = \frac{\sigma_p}{\sigma_{Mkt}} \cdot \rho_{p,Mkt} \cdot \frac{\sigma_{Mkt}}{\sigma_{Mkt}} = \frac{Cov[R_p, R_{Mkt}]}{Var[R_{Mkt}]} =$$

$$= \frac{Cov[w_1 \cdot R_1 + w_2 \cdot R_2 + \cdots + w_n \cdot R_n, R_{Mkt}]}{Var[R_{Mkt}]}$$

$$= \sum_{i=1}^{n} w_i \cdot \underbrace{\boxed{\frac{Cov[R_i, R_{Mkt}]}{Var[R_{Mkt}]}}}_{= \beta_i} = \sum_{i=1}^{n} w_i \cdot \beta_i$$

7) Consider a portfolio of four stocks as displayed in the following table:

Stock	Weight	Beta
1	0.1	1.3
2	0.2	-0.6
3	0.3	β_3
4	0.4	1.1

Assume the expected return of the portfolio is 0.12, the annual effective risk-free rate is 0.05, and the market risk premium is 0.08.

Assuming the Capital Asset Pricing Model holds, calculate β_3 .

$\rightarrow$: $\mathbb{E}[R_P] = r_P = r_f + \beta_P(\mathbb{E}[R_{Mkt}] - r_f)$

$0.12 = 0.05 + \beta_P(0.08)$

$\beta_P = \frac{0.12 - 0.05}{0.08} = 0.875$

$0.875 = 0.1(1.3) + 0.2(-0.6) + 0.3\beta_3 + 0.4(1.1)$

$\beta_3 = \frac{0.875 - 0.13 + 0.12 - 0.044}{0.3} = 1.4167$ $\square$

A) 0.80

B) 1.06

C) 1.42

D) 1.83

E) 2.17

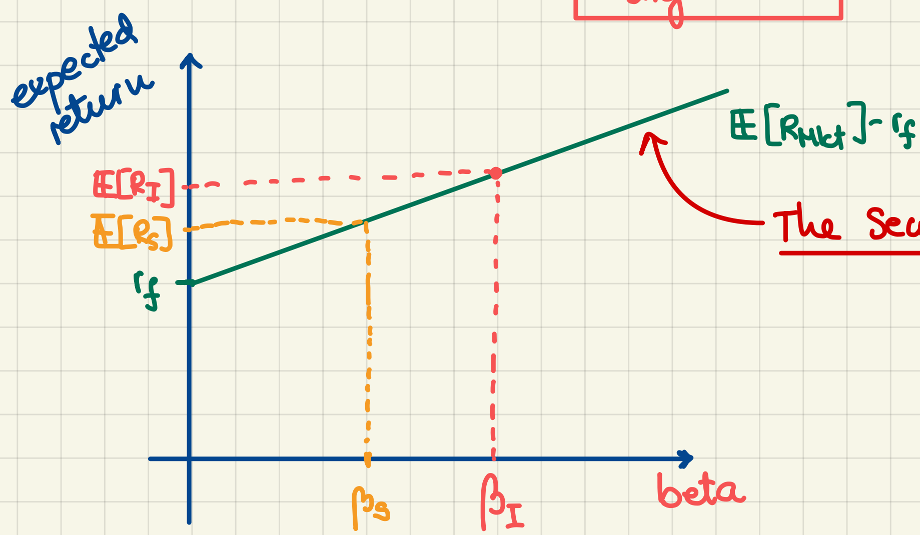
The Equity Cost of Capital.

In CAPM: for all investments I:

independent of investment I

$$\mathbb{E}[R_I] = r_I = \underbrace{r_f}_{\text{intercept}} + \beta_I \underbrace{\left(\mathbb{E}[R_{Mkt}] - r_f\right)}_{\text{the slope}}$$

$\boxed{\text{"independent argument"}}$



The Security Market Line.

Beta Estimation.

Linear Regression.

Explanatory Random Variable: X

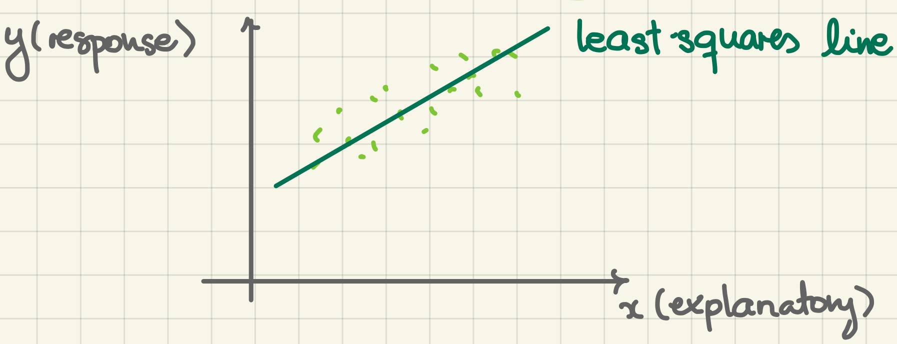
Response Random Variable: Y

Model: ⊛ $\boxed{Y = \alpha + \beta \cdot X + \varepsilon}$ w/ $\varepsilon \sim \text{Normal}(0, \text{variance})$

intercept slope

Assume the same for all values of X.

Observed Values: (x_i, y_i), $i = 1..n$



$$\hat{\beta} = \frac{SD[Y]}{SD[X]} \cdot corr[X, Y]$$

$$Y = \alpha + \beta \cdot X + \varepsilon$$

"Attack" w/ expectation above:

$$\mathbb{E}[Y] = \alpha + \beta \cdot \mathbb{E}[X]$$

They can be estimated using the least-squares line.

In our applications:

excess return of market : EXPLANATORY

$$\underbrace{R_I - r_f}_{\text{excess return of } I} = \alpha_I + \beta_I \overbrace{(R_{Mkt} - r_f)} + \varepsilon_I$$

excess return of I

RESPONSE

The Error Term

the intercept of the linear regression

the slope of the linear regression

Now, we see how we can estimate α_I and β_I from the observed excess returns across different time intervals.

"Attack" with the expectation:

$$\mathbb{E}[R_I] - r_f = \boxed{\alpha_I} + \beta_I \left(\mathbb{E}[R_{Mkt}] - r_f\right) + \overbrace{\mathbb{E}[\varepsilon_I]}^{=0}$$

$$\mathbb{E}[R_I] = \underbrace{r_f + \beta_I \left(\mathbb{E}[R_{Mkt}] - r_f\right)}_{\text{the Security Market Line}} + \alpha_I$$

the distance from SML, i.e., the stock's alpha