The expected return of a portfolio

Say that your portfolio has n different securities in it,

$i = 1..n$ the indices of the investment components in your portfolio

For every i : R_i .. the realized (simple) return of the i^{th} component over a particular time period (say, a year)

R_P ... the realized return of the entire portfolio

beg — end

$$R_P := \frac{P_P^{end} - P_P^{beg}}{P_P^{beg}}$$

Compare to the notion of the effective interest rate in interest theory.

w/ P_P ... the price of the total portfolio,

i.e.,

$$P_P = \sum_{i=1}^{n} P_i$$

P_i ... value of the component i

$$R_P = \frac{\sum_{i=1}^{n} P_i^{end} - \sum_{i=1}^{n} P_i^{beg}}{P_P^{beg}} = \sum_{i=1}^{n} \frac{P_i^{end} - P_i^{beg}}{P_P^{beg}} \cdot \frac{P_i^{beg}}{P_i^{beg}}$$

$$= \sum_{i=1}^{n} \underbrace{\frac{P_i^{beg}}{P_P^{beg}}}_{\text{!!}\ w_i} \cdot \underbrace{\frac{P_i^{end} - P_i^{beg}}{P_i^{beg}}}_{= R_i}$$

w_i ... portfolio weight of investment i (deterministic)

$$R_P = \sum_{i=1}^{n} w_i \cdot R_i$$

$\Rightarrow$ the expected return : $\boxed{\mathbb{E}[R_P] = \sum_{i=1}^{n} w_i \mathbb{E}[R_i]}$ ✓