University of Texas at Austin, Department of Mathematics
M358K - Applied Statistics

In-term exam II

Definitions.

Problem 1.1. (5 points) Provide the expression for the *probability density function* of a **standard normal** random variable.

Solution:

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad x \in \mathbb{R}.$$

True/False Questions.

Problem 1.2. (2 points) The margin of error for a confidence interval for the population mean μ, based on a fixed specified sample size n, increases as the confidence level decreases. *True or false?*

Solution: FALSE

Problem 1.3. (3 points) The Midsomer Worthy Middle School has calculated a 95% confidence interval for the population mean height μ of 11−year-old boys at their school. They found it to be 57 ± 2 inches.

This means that there is a 95% probability that the population mean μ is between 55 and 59. *True or false?*

Solution: FALSE

Free-response problems.

Problem 1.4. (10 points) Walter Hingel, the enterprising preschooler, is planning for Halloween. Based on industry standards, the strap on his gigantic plastic pumpkin will snap once the pumpkin is loaded with more than 3lbs of candy. The weight of each piece of candy is assumed to have the mean of 1oz and the standard deviation of 0.1oz. Walter will go to 49 houses politely collecting one piece of candy at each. What is the approximate probability that Walter's pumpkin strap stays intact?

Solution: Let the weight of each piece of candy be denoted by X_i for $i = 1, \dots, 49$. The total weight in Walter's pumpkin at the end of the evening will be $S = X_1 + X_2 + \cdots + X_{49}$. By the Central Limit Theorem, the distribution of S is approximately

$$S \approx Normal(mean = 1(49) = 49, sd = 0.1\sqrt{49} = 0.7).$$

We need to find the probability that S is below $3(16) = 48$. We can use **R** to get

$$pnorm(48, 49, 0.1 * sqrt(49)) = 0.07656373.$$

If I were Walter, I would carry a pillow case.

Problem 1.5. (10 points) **Toddler Bribery!**
Dr. P. Piagette, a developmental psychologist, is trying to figure out what percentage of parents of small children resort to offering candy to their offspring to quiet them down. Realizing that parents might not answer truthfully to an outright question about bribing their little ones, she decides to use the randomized-response method.

She sets up a computer to display the question

"Have you ever offered candy to appease your toddler?"

with probability 0.6. The rest of the time, a virtual spinner spins on the screen. Half of the time, the spinner lands on red, a third of the time, the spinner lands on blue, and one sixth of the time, the spinner lands on yellow. The parent is asked

"Did the spinner land on blue?"

In both cases, the subject is prompted to click the button with **Yes** or **No**. The interviewer did not know the actual question asked, just the ultimate response. So, there was no real reason for the subject to lie, and we assume that the subjects responded truthfully.

It turned out that 53% of the subjects answered "yes". Give an estimate of the proportion of *parents who bribe their children* in this population.

Solution: Now, we are given that $\mathbb{P}[Yes] = 0.53$. Our goal is to figure out $p = \mathbb{P}[Yes \mid Q]$ with the conditioning event Q given by

$$Q = \{\text{the subject was asked the bribery question}\}.$$

We are given that $\mathbb{P}[Q] = 0.6$.

By the *Law of Total Probability*,

$$\begin{aligned}\mathbb{P}[Yes] = \mathbb{P}[Yes \cap Q] + \mathbb{P}[Yes \cap Q^c] &= \mathbb{P}[Yes \mid Q]\mathbb{P}[Q] + \mathbb{P}[Yes \mid Q^c]\mathbb{P}[Q^c] \\ &= p(0.6) + \frac{1}{3}(0.4) = \frac{3}{5}p + \frac{2}{15}.\end{aligned}$$

So,

$$\frac{3}{5}p = 0.53 - \frac{2}{15} \quad \Rightarrow \quad p = 0.6611.$$

Problem 1.6. (10 points) A particular type of wool for clothes manufacturing has to have a specific tensile strength in order to be used in weaving machines without breaking. We model its tensile strength as normally distributed with standard deviation 0.4 MPa. How is the variance of the sample mean changed when the sample size increases from 64 to 196?

Solution: The variance of the sample mean $\bar{X}_{64}$ for the sample of size 64 is

$$Var[\bar{X}_{64}] = \frac{(0.4)^2}{64} = 0.0025.$$

The variance of the sample mean $\bar{X}_{196}$ for the sample of size 196 is

$$Var[\bar{X}_{196}] = \frac{(0.4)^2}{196} = 0.0008.$$

Problem 1.7. (10 points) Let the monthly profit of a local cupcakery be normally distributed with mean \$20,000 and standard deviation of \$4,000. What is the probability that the combined profit in the months of October and November exceeds \$36,000 (assuming that profits over different months are independent)?

Solution: Let X_1 be the profit for October and let X_2 be the profit for November. Then

$$X_1 + X_2 \sim Normal(mean = 40000, variance = 32,000,000).$$

The probability we are looking for is $\mathbb{P}[X_1 + X_2 > 36000]$. Our answer is

1-pnorm(36000,40000,sqrt(32000000))

which is equal to 0.76024991

Multiple-choice problems.

Problem 1.8. (5 points) A political scientist is trying to gauge whether climate change plays a role in the upcoming elections. She plans to collect a sample from those eligible to vote and ask them if they think that climate change is an important factor. What is she trying to estimate?

(a) the population proportion.

(b) the point estimate.

(c) the sample median.

(d) the sample standard deviation.

(e) the sample variance.

Solution: (a)

Problem 1.9. (5 points) Let the population distribution be normal with mean μ and standard deviation σ. Let $\bar{X}$ denote the sample mean of a sample of size n from this population. Then, we know the following about the distribution of $\bar{X}$:

(a) $\bar{X} \sim Normal(mean = \mu, variance = \sigma^2)$

(b) $\bar{X} \sim Normal(mean = \mu, variance = \frac{\sigma^2}{\sqrt{n}})$

(c) $\bar{X} \sim Normal(mean = \frac{\mu}{n}, variance = \frac{\sigma^2}{n})$

(d) $\bar{X} \sim Normal(mean = \mu, variance = \frac{\sigma^2}{n})$

(e) None of the above are correct.

Solution: (d)
For the verification, see class notes.

Problem 1.10. (5 points) A researcher would like to study the effect of eating breakfast on a cognitive function. Volunteers are recruited through the study by posting flyers on campus. He plans to randomly assign subjects to two groups, one told to eat before participating in the study and one asked to eat breakfast following the study. However, he suspects whether or not the person typically eats breakfast affects the suspected relationship. In order to address this, what should he do prior to assigning subjects to experimental groups?

(a) Cluster on typical breakfast habits.

(b) Randomly assign subjects to typical breakfast habits.

(c) Sample from each strata, typical breakfast eater and not.

(d) Block on typical breakfast habits.

(e) There is no way he can address his suspicions in his experimental design.

Solution: (d)