M339D : November 12th, 2025.

More on Log-Normal Stock Prices.

Under the risk-neutral probability measure $\mathbb{P}^*$,

we model

$$\boxed{R(0,T) \sim Normal\left(mean = \left(r - \frac{\sigma^2}{2}\right)\cdot T,\ var = \sigma^2 \cdot T\right)}$$

Say, $Z \sim N(0,1)$.

Then, we can express $R(0,T)$ as

$$R(0,T) = \left(r - \frac{\sigma^2}{2}\right)\cdot T + \sigma\sqrt{T}\cdot Z$$

$$\mathbb{P}\left[\left(r - \frac{\sigma^2}{2}\right)\cdot T < R(0,T)\right] = \frac{1}{2}$$

$$\mathbb{P}\left[\underbrace{e^{\left(r - \frac{\sigma^2}{2}\right)\cdot T}}_{S(0)} < \underbrace{e^{R(0,T)}}_{S(0)}\right] = \frac{1}{2}$$

Thus,

$$\boxed{S(T) = S(0)e^{R(0,T)} = S(0)e^{\left(r - \frac{\sigma^2}{2}\right)\cdot T + \sigma\sqrt{T}\cdot Z}}$$

Q: What is the median of $S(T)$ under the risk-neutral probability measure $\mathbb{P}^*$?

$\rightarrow$:
$$\boxed{S(0)e^{\left(r - \frac{\sigma^2}{2}\right)\cdot T}}$$

Note:

$$\frac{mean}{median} = \frac{\cancel{S(0)}e^{\cancel{r}T}}{\cancel{S(0)}e^{\left(\cancel{r} - \frac{\sigma^2}{2}\right)\cdot T}} = e^{\frac{\sigma^2}{2}\cdot T}$$