Name:

M339D=M389D Introduction to Actuarial Financial Mathematics
University of Texas at Austin
Solution: Practice Problems for In-Term Exam I
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam. The maximal score on this exam is 50 points.
Time: 50 minutes

2.1. **TRUE/FALSE QUESTIONS.**

Problem 2.1. (2 pts) A (long) put is a short position with respect to the underlying asset price.

Solution: TRUE

Problem 2.2. (2 pts) Put-call parity applies to both American and European options.

Solution: FALSE

Problem 2.3. (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date T on the underlying asset S:

- one long call with strike 40,
- two long calls with strike 50,
- one short call with strike 65.

Let $S(T) = 69$. Then, the payoff from the above position at time T is less than 60.

Solution: FALSE
The payoff is

$$(69 - 40) + 2(69 - 50) - (69 - 65) = 63.$$

Problem 2.4. It is possible for the buyer and the writer of the same option to end up having the same profit on the exercise date.

Solution: TRUE
This happens if they both break even, i.e., if both of their profits equal zero.

Problem 2.5. (2 points) Naked writing is the practice of writing options without taking an offsetting position in the underlying asset. *True or false?*

Solution: TRUE

Problem 2.6. (2 points) Consider a one-year, \$45-strike European call option and a one-year, \$45-strike European put option on the same underlying asset. You observe that the time-0 stock price equals \$40 while the time-1 stock price equals \$50. Then, both of the options are out-of-the-money at expiration. *True or false?*

Solution: FALSE

Problem 2.7. (2 points) A covered call is a portfolio consisting of a written call option and the short underlying. *True or false?*

Solution: FALSE

Problem 2.8. (2 points) Derivative securities can only be used for hedging, i.e., they can only be bought and written by agents who already have a position in the underlying asset. *True or false?*

Solution: FALSE

Problem 2.9. (2 points) On the expiration date of your American call option, you look back and you realize that you exercised the option at the optimal time. This means that the time of early exercise was when the price of the underlying asset achieved its maximum during the life of the option. *True or false?*

Solution: TRUE

Problem 2.10. (2 points) The prepaid forward price of a non-dividend-paying stock is strictly decreasing with respect to the delivery date. *True or false?*

Solution: FALSE

Problem 2.11. (2 pts) A non-dividend-paying stock sells today for \$100 per share. The yearly effective interest rate is 0.21. Then, $F_{0,1/2}(S) > 110$.

Solution: FALSE
In fact, $F_{0,1/2}(S) = 100(1.21)^{1/2} = 100 \times 1.1 = 110$.

Problem 2.12. (2 points) A portfolio consisting of a long forward contract and a bond can replicate a long prepaid forward contract. *True or false?*

Solution: TRUE

Problem 2.13. (2 points) Let the current exchange rate of euros (€) to USD (\$) be denoted by $x(0)$, i.e., currently, $1 € = \$X(0)$.

Let $r_\$$ denote the continuously compounded, risk-free interest rate for the \$, and let r_e denote the continuously compounded, risk-free interest rate for the €.

Denote the price of a \$-denominated European call option with strike K and exercise date T by $V_C(0)$ and the price of an otherwise identical put option by $V_P(0)$. Then,

$$V_C(0) - V_P(0) = x(0)e^{-r_\$ T} - Ke^{-r_e T}.$$

True or false?

Solution: FALSE
The two interest rates have switched places.

Problem 2.14. The strike price at which the European call and the otherwise identical European put have the same premiums is the forward price for delivery of the underlying on the exercise date of the two options. *True or false?*

Solution: TRUE

2.2. FREE-RESPONSE PROBLEMS.

Problem 2.15. An investor wants to hold 200 euros two years from today. The spot exchange rate is \$1.31 per euro. If the euro denominated annual interest rate is 3.0% what is the price of a currency prepaid forward?

Solution:

$$F^P_{0,T}(x) = 200e^{-0.03\cdot 2} \cdot 1.31 = 246.74.$$

2.3. MULTIPLE CHOICE QUESTIONS.

Problem 2.16. The initial price of a non-dividend-paying asset is \$100. A six-month, \$95-strike European call option is available at a \$8 premium.

The continuously compounded risk-free interest rate equals 0.04.

What is the break-even point for this call option?

(a) 86.84

(b) 87

(c) 103

(d) 103.16

(e) None of the above.

Solution: (d)
We need to solve for s in

$$(s-95)_+ = 8e^{0.02} \quad \Rightarrow \quad s = 95 + 8e^{0.02} = 103.16$$

Problem 2.17. (5 points) A non-dividend-paying stock sells for \$100 per share today. The one-year forward price is \$110. You short sell the stock and close the short sale in exactly one year. Find your profit if the stock's spot price in one year equals \$130 per share.

(a) 20 loss

(b) 20 gain

(c) 30 loss

(d) 30 gain

(e) None of the above.

Solution: (a)
Because the stock pays no dividends, we have that $FV_{0,T}(S(0)) = F_{0,T}(S)$. So, the profit equals

$$-S(1) - FV_{0,1}(-S(0)) = -S(1) + F_{0,1}(S) = -130 + 110 = -20.$$

Problem 2.18. A market index is currently trading at \$1,000. Which of the following options is/are **in the money**? More than one answer can be true. You get the credit if you circled **all** acceptable answers and **none** of the incorrect ones.

(a) \$1,500-strike put
(b) \$900-strike put
(c) \$1,250 strike call
(d) \$950 strike call
(e) None of the above.

Solution: (a) and (d)

Problem 2.19. For a continuous-dividend-paying stock, the current stock price is observed to be \$80. The forward price for delivery in two years is \$82.44. What is the forward price for delivery in three years?

(a) About 81.36
(b) About 83.67
(c) About 84.32
(d) About 85.91
(e) None of the above.

Solution: (b)
In our usual notation, we are given that

$$F_{0,2}(S) = S(0)e^{2(r-\delta)} = 80e^{2(r-\delta)} = 82.44.$$

So,

$$F_{0,3}(S) = S(0)e^{3(r-\delta)} = 80 \times \left(\frac{82.44}{80}\right)^{3/2} = 83.67.$$

Problem 2.20. The current price of a discrete-dividend-paying stock is \$90 per share. The company projects to pay quarterly dividends starting three months from today to perpetuity. The first dividend amout is \$2 and the dividends are scheduled to increase by a factor of 0.01 every time a dividend is paid.

The continuously compounded risk-free interest rate is 0.06. What is the prepaid forward price of the above stock for delivery in eight months?

(a) \$84.24
(b) \$86.07
(c) \$88.70
(d) \$90.00
(e) None of the above.

Solution: (b)

$$F^P_{0,T}(S) = 90 - 2e^{-0.015} - 2.02e^{-0.03} = 86.0695.$$

Problem 2.21. Let the current price of a non-dividend-paying stock equal 100. The forward price for delivery of this stock in 3 months equals \$101.26

Consider a \$90-strike, six-month put option on this stock whose premium today equals \$2.22.

What will the profit of this long put option be if the stock price at expiration equals \$96?

(a) About \$2.28 loss.
(b) About \$2.22 loss.
(c) About \$2.28 gain.
(d) About \$2.22 gain.
(e) None of the above.

Solution: (a)
The option is out-of-the money at expiration, so its owner suffers a loss of the future value of its premium

$$2.22 \times \left(\frac{101.26}{100}\right)^2 = 2.2763.$$

Problem 2.22. (5 points) You are tasked with buying oranges in the market in grove A, transporting the oranges to a juice factory in the market B, and selling the oranges to the juice factory in the market B. You want to hedge. Which of the following would be a satisfactory hedge?

(a) Long a call in market A and long a put in market B
(b) Short a call in market A and long a put in market B
(c) Long a call in market A and short a put in market B
(d) Short a call in market A and short a put in market B
(e) None of the above.

Solution: (a)

Problem 2.23. The current price of stock **S** is \$50. Stock **S** is scheduled to pay a \$3-dividend in two months.

The current price of stock **Q** is \$60. Stock **Q** is scheduled to pay dividends continuously with the dividend yield 0.03.

A six-month European exchange call option with underlying asset **S** and the strike asset **Q** is sold for \$2.75.

The continuously compounded risk-free interest rate is given to be 0.04.

What is the price of the six-month European exchange put option with underlying asset **S** and the strike asset **Q**?

(a) About \$8.58
(b) About \$9.04
(c) About \$12.75
(d) About \$14.84

(e) None of the above.

Solution: (d)

$$V_{EP}(0,\mathbf{S},\mathbf{Q}) = V_{EC}(0,\mathbf{S},\mathbf{Q}) + F^P_{0,T}(Q) - F^P_{0,T}(S) = 2.75 + 60e^{-0.03\cdot 0.5} - 50 + 3e^{-0.04/6} = 14.84.$$

Problem 2.24. The following nine-month European put options are available in the market:

- a \$120-strike put with the premium of \$12,
- a \$127-strike put with the premium of \$10,

The continuously compounded, risk-free interest rate is 0.04.

You construct a portfolio by buying the \$127-strike put and writing the \$120-strike put. Which of the following statements is correct?

(a) The minimum **profit** of this portfolio is -9.06.
(b) The minimum **profit** of this portfolio is -2.06.
(c) The minimum **profit** of this portfolio is -7.
(d) This is an arbitrage portfolio.
(e) None of the above.

Solution: (d)
The initial cost of this portfolio is $10 - 12 = -2$. The minimum payoff of this portfolio happens for the final asset price below 120. It is equal to 7. So, the minimal gain of this portfolio is

$$7 - (-2)e^{0.03} = 9.06.$$

Problem 2.25. (5 points) Consider the portfolio consisting of these components:

- one long put, and
- one long share of the underlying stock.

This portfolio is referred to as …

(a) a floor.

(b) a cap.

(c) a covered call.

(d) a covered put.

(e) None of the above.

Solution: (a)