M339G: March 22nd, 2024.

Any Bivariate Normal.

Random variables U and V are said to have the bivariate normal distribution w/ parameters $\mu_U, \mu_V, \sigma_U, \sigma_V$ and ρ if

$$\boxed{\left(X = \frac{U-\mu_U}{\sigma_U}, Y = \frac{V-\mu_V}{\sigma_V} \right)}$$

has the standard bivariate normal dist'n w/ correlation ρ.

Note: • $\rho(U,V) = ?$

By def'n:

$$\boxed{\begin{aligned} U &= \mu_U + \sigma_U \cdot X \\ V &= \mu_V + \sigma_V \cdot Y \end{aligned}}$$

$$\rho(U,V) = \frac{Cov[U,V]}{SD[U]\cdot SD[V]} = \frac{Cov[\cancel{\mu_U}+\sigma_U\cdot X, \cancel{\mu_V}+\sigma_V\cdot Y]}{SD[\cancel{\mu_U}+\sigma_U\cdot X]\cdot SD[\cancel{\mu_V}+\sigma_V\cdot Y]}$$

μ_U and μ_V deterministic

$$= \frac{Cov[\sigma_U\cdot X, \sigma_V\cdot Y]}{SD[\sigma_U\cdot X]\cdot SD[\sigma_V\cdot Y]}$$

$$= \frac{\cancel{\sigma_U}\cdot\cancel{\sigma_V}\cdot Cov[X,Y]}{\cancel{\sigma_U}\cdot SD[X]\cdot\cancel{\sigma_V}\cdot SD[Y]} = \rho(X,Y) = \rho$$

• U and V are independent $\iff$ $\rho = 0$ (iff)

Example. Midterm and Final.

Midterm and final scores in a large class have an (approximately) bivariate normal distribution w/ parameters:

	mean	sd
midterm scores:	65	18
final scores:	60	20

correlation: 0.76

Q: What is the estimated mean final score of the students who were above the mean on the midterm?

→: Let U be the midterm score and
V be the final score.

Let X and Y be U and V in standard units, resp.

Our first task is to find:

$$\mathbb{E}[Y \mid X>0] = \int_{0 \cancel{-\infty}}^{+\infty} \boxed{\mathbb{E}[Y \mid X=x]} \boxed{f_X(x \mid X>0)\, dx}$$

$\mathbb{E}[Y \mid X=x] = \rho x$

The Law of Total probability

and for $x>0$:

$$f_X(x \mid X>0)\,dx = \mathbb{P}[x \in dx \mid X>0] = \frac{\mathbb{P}[x \in dx \text{ and } X>0]}{\mathbb{P}[X>0]}$$