University of Texas at Austin

# HW Assignment 10

The feasible set. Portfolio returns and volatilities. Efficient portfolios.

Provide your final answer only to the following problem(s):

**Problem 10.1.** (2 points) Consider the feasible set for two stocks. The higher the correlation of the two stocks' returns, the higher the curvature of the feasible set. *True or false?*

**Solution:  FALSE**

**Problem 10.2.** (2 points) You are considering equally-weighted large portfolios which you construct so that:
- For each individual stock in the portfolio, the variance is 0.20.
- For each pair of distinct stocks in the portfolio, the covariance is 0.10.

Then, as the number of stocks in the portfolio gets larger, the variance of the portfolio's return approaches 0.10. *True or false?*

**Solution: TRUE**

**Problem 10.3.** (2 points) We call risk that is perfectly correlated across assets **systematic risk.** *True or false?*

**Solution: TRUE**
Also known as **non-diversifiable** or **common** risk.

**Problem 10.4.** (2 points) An efficient portfolio contains only systematic risk. *True or false?*

**Solution:  TRUE**

**Problem 10.5.** Portfolio $P$ has expected return 0.08 and volatility equal to 12%. Porfolio $Q$ has expected return 0.10 and volatility equal to 12.5%. Then, we can say with certainty that portfolio $P$ is not efficient. *True or false?*

**Solution:  FALSE**

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**Problem 10.6.** (5 points) Consider two assets $X$ and $Y$ such that:
- their expected returns are $\mathbb{E}[R_X] = 0.12$ and $\mathbb{E}[R_Y] = 0.15$;
- their volatilities are $\sigma_X = 0.2$ and $\sigma_Y = 0.25$;
- the correlation coefficient of their returns is $\rho_{X,Y} = -1$.

You are tasked with constructing a portfolio consisting of shares of $X$ and $Y$ with a risk-free return. What should the weight $w_X$ given to asset $X$ be?
  (a) 4/9
  (b) 1/2
  (c) 5/9
  (d) Such a weight does not exist.
  (e) None of the above.

**Solution: (c)**

$$w_X = \frac{\sigma_Y}{\sigma_X + \sigma_Y} = \frac{0.25}{0.2 + 0.25} = \frac{5}{9}.$$

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**Problem 10.7.** (5 points) You are given the following information about a portfolio consisting of stocks X, Y, and Z:

| Stock | Investment Amount | Expected Return |
|-------|------------------|-----------------|
| X | 20,000 | 8% |
| Y | 5,000 | 12% |
| Z | 25,000 | 14% |

Calculate the expected return of the portfolio.

  (a) 11.3%
  (b) 11.4%
  (c) 11.5%
  (d) 11.6%
  (e) None of the above.

**Solution: (b)**

The three weights given to the assets X, Y, and Z are $w_X = 0.4, w_y = 0.1$ and $w_Z = 0.5$. So,
$$\mathbb{E}[R_P] = 0.4(0.08) + 0.1(0.12) + 0.5(0.14) = 0.114.$$

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Provide your <u>complete solution(s)</u> to the following problem(s):

**Problem 10.8.** (10 points) You are given the following information about a portfolio with four assets:

| Asset | Market value of asset | Covariance of asset's return with the portfolio return |
|-------|----------------------|--------------------------------------------------------|
| I | 40,000 | 0.2 |
| II | 30,000 | -0.10 |
| III | 20,000 | 0.25 |
| IV | 10,000 | -0.05 |

Calculate the standard deviation of the portfolio return.

**Solution:** Let the weights of the four assets in the portfolio be denoted by $w_I, w_{II}, w_{III}$ and $w_{IV}$. We have established the following in class:
$$Var[R_P] = Cov[R_P, R_P] = Cov[w_I R_I + w_{II} R_{II} + w_{III} R_{III} + w_{IV} R_{IV}, R_P]$$
$$= w_I Cov[R_I, R_P] + w_{II} Cov[R_{II}, R_P] + w_{III} Cov[R_{III}, R_P] + w_{IV} Cov[R_{IV}, R_P]$$

Using the information provided in the table in the problem, we get
$$Var[R_P] = 0.4(0.2) + 0.3(-0.1) + 0.2(0.25) + 0.1(-0.05) = 0.095.$$

So, the portfolio's volatility equals $SD[R_P] = \sqrt{0.095} = 0.3082$.

**Problem 10.9.** (10 points) You are given the following information about the annual returns of two stocks $X$ and $Y$:

- The expected returns of $X$ and $Y$ are $\mathbb{E}[R_X] = 0.08$ and $\mathbb{E}[R_Y] = 0.10$.
- The volatilities of the returns are $\sigma_X = 0.20$ and $\sigma_Y = 0.25$.
- The correlation coefficient of the returns for these two stocks is $-0.4$.
- The expected return for a certain portfolio, consisting only of stocks $X$ and $Y$, is 0.09

Calculate the volatility of this portfolio.

**Solution:** Let the weights of the two stocks in the portfolio be denoted by $w_X$ and $w_Y$. Then,
$$0.09 = \mathbb{E}[R_P] = w_X(0.08) + (1 - w_X)(0.10) \quad \Rightarrow \quad w_x = 0.5.$$

Hence,
$$Var[R_P] = Var[0.5R_X + 0.5R_Y] = Var[0.5(R_X + R_Y)] = 0.25Var[R_X + R_Y].$$

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From the given information, we can calculate

$$Var[R_X + R_Y] = \sigma_X^2 + \sigma_Y^2 + 2\sigma_X\sigma_Y\rho_{X,Y} = (0.2)^2 + (0.25)^2 + 2(0.2)(0.25)(-0.4) = 0.0625.$$

Finally,

$$SD[R_P] = 0.5\sqrt{0.0625} = 0.125.$$

**Problem 10.10.** (10 points) Your model for the economy at the end of your period has two different states *good* and *bad*. You are an optimist and you think that the probability that the economy will be in the *good* state is twice the probability that it will be in the *bad* state.

There are three assets in your market model called $S, T$ and $Q$. Their returns, depending on the state of the economy are modeled as follows:

| Asset | *good* | *bad* |
|-------|--------|-------|
| $S$ | 10% | -4% |
| $T$ | 6% | -5% |
| $Q$ | 8% | -1% |

You put half of your wealth into asset $S$, a quarter into asset $Q$ and the remainder into asset $T$. What is the volatility of this total portfolio?

**Solution:** The probability of the *good* economy is $2/3$ and the probability of the *bad* economy is $1/3$. So, the return $R_W$ of the total portfolio has the following distribution

$$R_W = \begin{cases} 0.5(0.1) + 0.25(0.06) + 0.25(0.08) & \text{with probability } 2/3, \\ 0.5(-0.04) + 0.25(-0.05) + 0.25(-0.01) & \text{with probability } 1/3. \end{cases} = \begin{cases} 0.085 & \text{with probability } 2/3, \\ -0.035 & \text{with probability } 1/3. \end{cases}$$

We get that the mean return of this portfolio equals

$$\mathbb{E}[R_W] = 0.085\left(\frac{2}{3}\right) + (-0.035)\left(\frac{1}{3}\right) = 0.045.$$

The second moment of the portfolio's return is

$$\mathbb{E}[R_W^2] = (0.085)^2\left(\frac{2}{3}\right) + (-0.035)^2\left(\frac{1}{3}\right) = 0.005225.$$

Therefore, the variance of the return equals

$$Var[R_W] = \mathbb{E}[R_W^2] - (\mathbb{E}[R_W])^2 = 0.005225 - (0.045)^2 = 0.0032.$$

Finally, the volatility of the portfolio is $\sigma_W = 0.0565685.$