M378K: February 9th, 2026.

Problem 6.2. *Consider a random variable Y whose cumulative distribution function is given by*

$$F_Y(y) = \begin{cases} 0, & \textit{for } y < 0 \\ y^4, & \textit{for } 0 \leq y < 1 \\ 1, & \textit{for } 1 \leq y \end{cases}$$

Calculate the expectation of the random variable Y.

$4y^3$

$$\int_0^1 y \, 4y^3 \, dy$$

$$\frac{4}{5} \int_0^1 y^4$$

$$\frac{4}{5}$$

The Normal Distribution.

$Y \sim N(\mu, \sigma^2)$

$\begin{cases} \mu \text{... mean} \\ \sigma \text{... standard deviation} \end{cases}$

The pdf: $f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$ for all $y \in \mathbb{R}$

The cdf: for all $y \in \mathbb{R}$:

$$F_Y(y) = \mathbb{P}[Y \leq y]$$
$$= \int_{-\infty}^{y} f_Y(u)\, du$$
$$= \int_{-\infty}^{y} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(u-\mu)^2}{2\sigma^2}}\, du$$

NO ANALYTIC CDF!

Fact.

$$\frac{Y-\mu_Y}{\sigma_Y} \sim N(0,1) \sim Z$$

$$\boxed{Y = \mu_Y + \sigma_Y \cdot Z}$$

$$F_Y(y) = \mathbb{P}[Y \leq y]$$
$$= \mathbb{P}\left[\frac{Y-\mu_Y}{\sigma_Y} \leq \frac{y-\mu_Y}{\sigma_Y}\right]$$
$$= \mathbb{P}\left[Z \leq \frac{y-\mu_Y}{\sigma_Y}\right]$$
$$= \Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right)$$

$$\boxed{\Phi \text{... CDF of } N(0,1)}$$

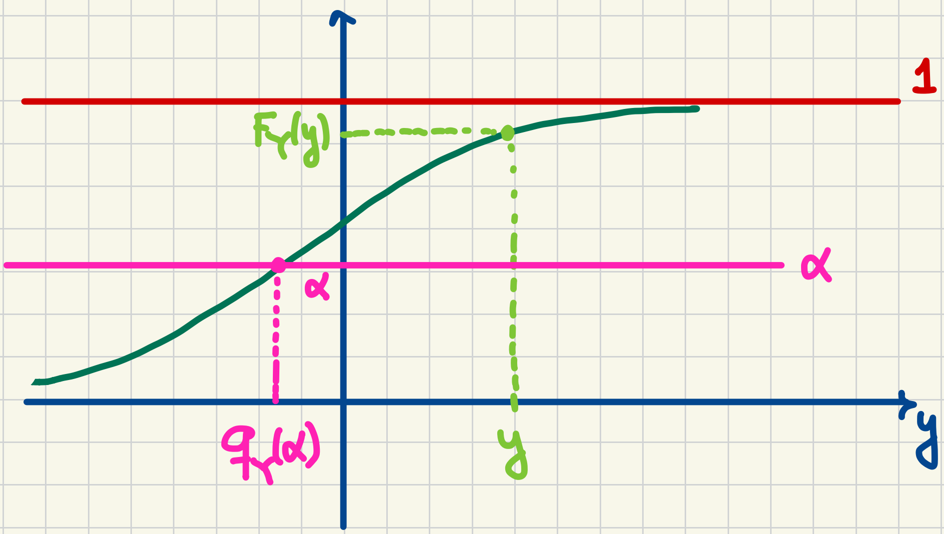
The Quantile Function.

Def'n. Let Y be a r.v. w/ the cdf F_Y.
For $\alpha \in (0,1)$, the α-quantile of the dist'n of Y is defined as the number

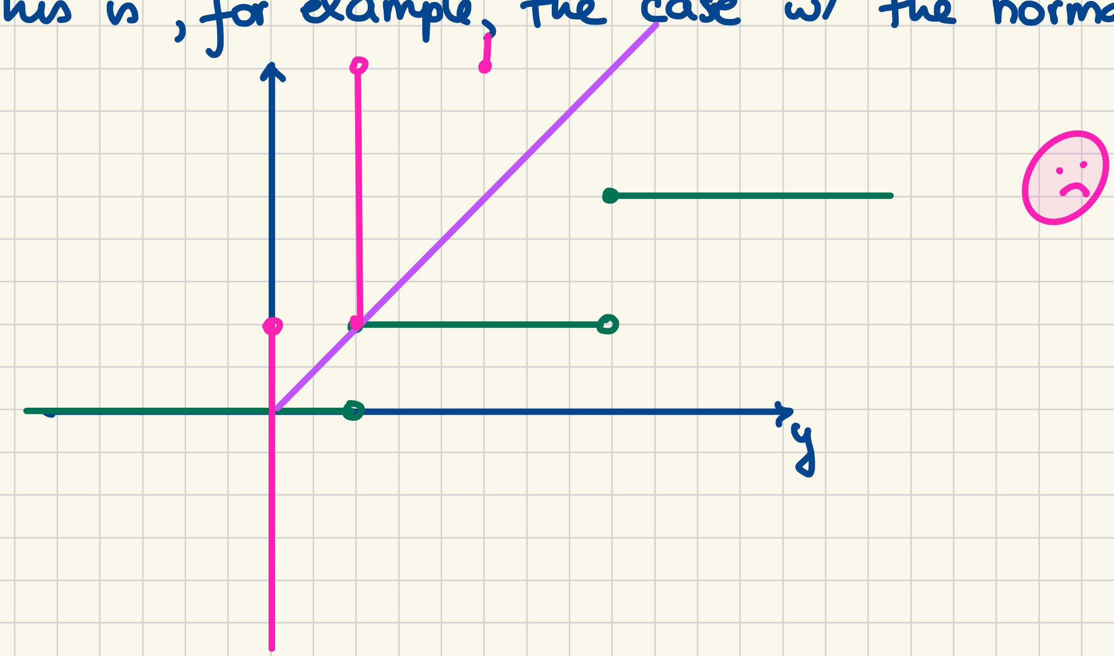
$$q_Y(\alpha) \in \mathbb{R}$$

which satisfied

$$\mathbb{P}[Y \leq q_Y(\alpha)] = \alpha$$
$$\iff$$
$$F_Y(q_Y(\alpha)) = \alpha$$



If F_Y^{-1} exists, then $q_Y(\alpha) = F_Y^{-1}(\alpha)$.
This is, for example, the case w/ the normal.



M378K Introduction to Mathematical Statistics

Problem Set #7

Cumulative distribution functions: Named continuous distributions.

Problem 7.1. Source: Sample P exam, Problem #126.
A company's annual profit is normally distributed with mean 100 and variance 400. Then, we can express the probability that the company's profit in a year is at most 60, given that the profit in the year is positive in terms of the standard normal cumulative distribution function denoted by Φ as

$$1 - \frac{\Phi(2)}{\Phi(5)}$$

True/False? Why?

$Y \sim N(\mu = 100, \sigma^2 = 400)$

:

$$\mathbb{P}[Y \leq 60 \mid Y > 0] = \frac{\mathbb{P}[0 < Y \leq 60]}{\mathbb{P}[Y > 0]}$$

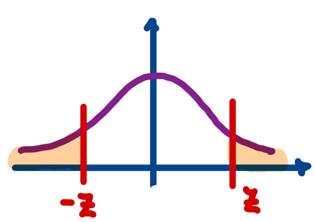
$$= \frac{\mathbb{P}\left[\frac{0-100}{20} < \underbrace{\frac{Y-100}{20}}_{=Z} \leq \frac{60-100}{20}\right]}{\mathbb{P}\left[\frac{Y-100}{20} > \underbrace{\frac{0-100}{20}}_{-5}\right]}$$

$$= \frac{\mathbb{P}[-5 < Z \leq -2]}{\mathbb{P}[-5 < Z]}$$

$$= \frac{\Phi(-2) - \Phi(-5)}{1 - \Phi(-5)}$$

$$= \frac{\cancel{1} - \Phi(2) - (\cancel{1} - \Phi(5))}{\Phi(5)}$$

$$= \frac{\Phi(5) - \Phi(2)}{\Phi(5)} = 1 - \frac{\Phi(2)}{\Phi(5)}$$



$\Phi(-z) = \mathbb{P}[Z > z]$

$= 1 - \mathbb{P}[Z \leq z]$

$= 1 - \Phi(z)$

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