M339D: February 20th, 2026.

Law of the Unique Price.

Assume that the payoffs of two static portfolios A and B are equal, i.e.,

$$\boxed{\underbrace{V_A(T)}_{\text{random variables}} = \underbrace{V_B(T)}_{\text{random variables}}}$$

T... time horizon (temporarily fixed)

Claim. $\boxed{V_A(0) = V_B(0)}$

Proof. Assume, to the contrary, that

$$V_A(0) \neq V_B(0)$$

Without loss of generality,

$$\boxed{\underbrace{V_A(0)}_{\text{relatively cheap}} < \underbrace{V_B(0)}_{\text{relatively expensive}}}$$

Diagnosis.

Propose an arbitrage portfolio:

- Long Portfolio A
- Short Portfolio B

} Total Portfolio

Verify that this is, indeed, an arbitrage portfolio.

- $\text{InitialCost(Total Portfolio)} = V_A(0) - V_B(0) < 0$

↑ Initial inflow of money

- $\text{Payoff(Total Portfolio)} = V_A(T) - V_B(T)$
$= 0$

$$\text{Profit} = \text{Payoff} - FV_{0,T}(\text{Initial Cost})$$
$$= 0 - FV_{0,T}(\underbrace{V_A(0) - V_B(0)}_{<0}) > 0$$

Indeed, this is an ARBITRAGE PORTFOLIO!

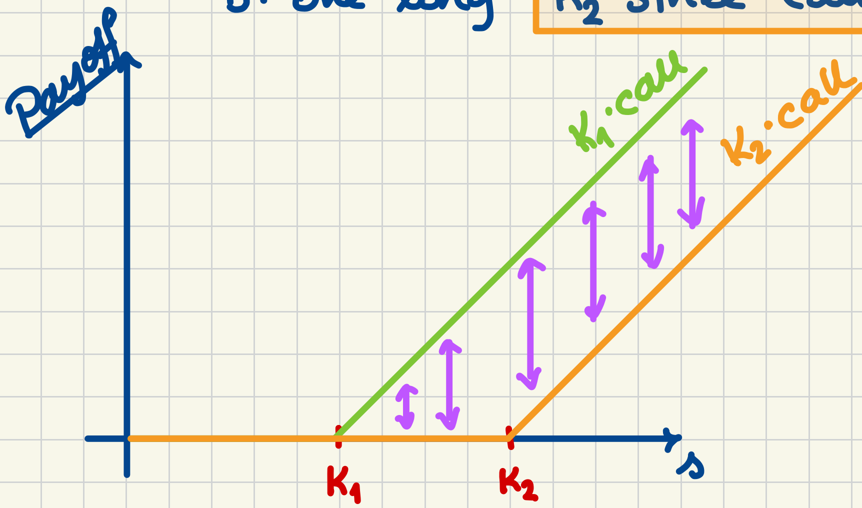
$\Rightarrow\Leftarrow$ $\square$

Corollary. If $\boxed{V_A(T) \geq V_B(T)}$, then $\boxed{V_A(0) \geq V_B(0)}$ $\square$

Example. $K_1 < K_2$

A: one long $\boxed{K_1\text{-strike call}}$ } European and w/ the same underlying asset and exercise date.

B: one long $\boxed{K_2\text{-strike call}}$

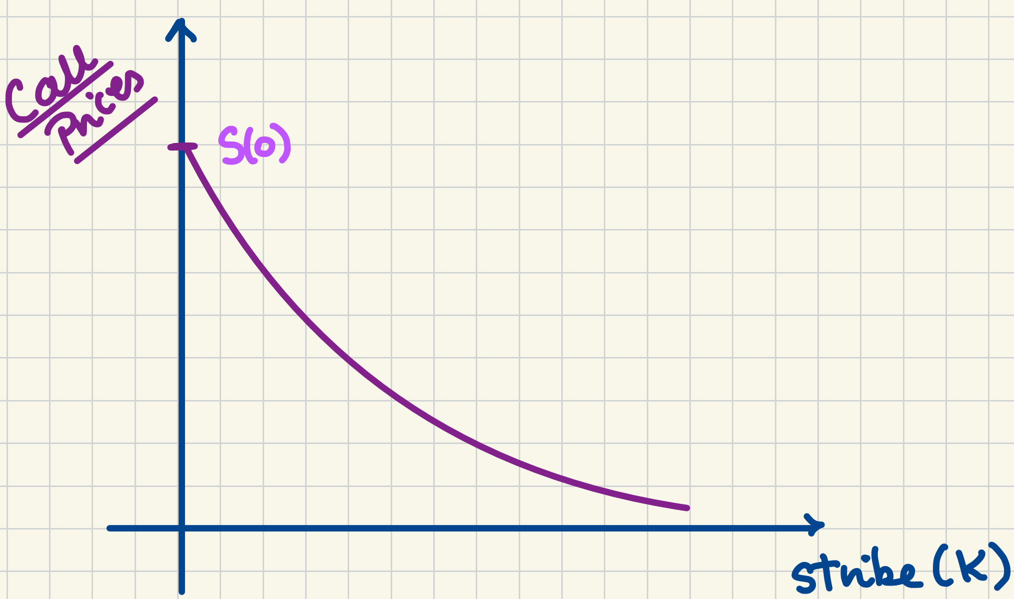


The payoff of the K_1-strike call **dominates** the payoff of the K_2-strike call.

$\Rightarrow$ **The K_1-call costs @ least as much as the K_2-call.**

In Math: $\boxed{K_1 < K_2 \Rightarrow V_C(0, K_1) \geq V_C(0, K_2)}$

As a function of the strike price, call prices are **decreasing**



Replicating Portfolios.

Def'n. Consider a European-style derivative security. A static portfolio w/ the same payoff as that of the derivative security is called its **replicating portfolio**.

Note: Because we assume no arbitrage, the initial price of the derivative security is equal to the initial price of its replicating portfolio.

Example. Consider a forward contract on a non-dividend-paying stock/index.

$$0 \longrightarrow T$$

Forward Contract: $S(T) - F$

Replicating Portfolio:
- long one share of stock
- issue a bond w/ redemption amount F and maturity date T

$$\text{Payoff(Portfolio)} = S(T) - F$$

NO ARBITRAGE!

$\Rightarrow$ The forward contract and its replicating portfolio must have the same initial cost, i.e.,

$$0 = \underbrace{S(0)}_{\text{long stock}} - \underbrace{PV_{0,T}(F)}_{\text{short bond}}$$

$$\Rightarrow \quad PV_{0,T}(F) = S(0)$$

$$\Rightarrow \quad \boxed{F = FV_{0,T}(S(0)) = S(0)e^{rT}}$$