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# Standing waves in air columns: Will computers reshape physics courses?

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It is often argued that the introduction of computational physics into undergraduate courses will, in time, change what is taught. This paper presents a supporting example, taken from the field of musical acoustics. The vibration of the air columns inside wind instruments, of any but the most idealized shapes, is a sufficiently difficult analytic problem that the subject is rarely treated in ordinary undergraduate courses. A simple computational treatment of the problem is presented that demonstrates many features of importance to real musicians: the effect of mouthpieces and bells, the positioning and sizing of fingerholes, and the production of multiphonics. The ability to treat previously intractable problems like this may enable acoustics to regain a place in the ordinary curriculum and allow its potential as an intuitive model for related systems to be usefully exploited.

## I. BACKGROUND

Rapid advances in the power and availability of personal computers over the last decade have made computational physics an increasingly important part of undergraduate courses. Our students, at all levels, now have access to a powerful tool with which they can tackle mathematical problems previously considered beyond their reach. As many authors have argued, this could well have a profound effect on what is taught in future physics courses. This article presents an example to support this proposition.

Musical acoustics is a subject which for over 2000 years has had a central place in the teaching of physics (or natural philosophy); both for its own sake, and as an analogy for understanding other fields of knowledge. In the early 19th century, for example, many of the great names in electrical theory—Savart, Ohm, Wheatstone, Kirchhoff—did their early research in this field. Yet in this century, acoustics, as a part of ordinary undergraduate courses, has run up against an intractability barrier. The easier-to-solve problems have been relegated to introductory courses; and generations of teachers have judged that the mathematical difficulty in pursuing the subject further was too high a price for any insights it might bring to their students, except in specialist courses.

However, if the subject were taught using a computational approach, this judgement may be altered, as will be argued by a reconsideration of the old problem of the modes of vibration of a column of air.

The wave equation for the vibrating air in a pipe of variable cross section appeared in many early works, but it is most commonly attributed to Webster (1919).[1] In its simplest form, taking into account only the most important physical processes, it relates the excess pressure $p$ at any point $x$ to the volume rate of flow $U$.

A complete derivation of this relationship may be found in standard textbooks.[2] Basically it is constructed by considering a "plug" of air inside the pipe (see Fig. 1) under steady state conditions when both are oscillating with time at an angular frequency $\omega$. Most pipes of interest are essentially cylindrically symmetric, so the problem is, at most, two-dimensional. Further, although the pressure and flow rate do vary from the center of the pipe to the edge, most of the important physics can be discussed without taking this variation into account, and the problem reduces to one-dimension.

Two distinct physical laws are invoked as follows.

(1) Newton's second law, which says

$$\frac{dp}{dx} = -j\,\frac{\omega\rho_0}{S}\,U, \tag{1}$$

where $S$ is the cross-sectional area of the column and $\rho_0$ the average density; and

(2) the equation of continuity for a gas undergoing adiabatic changes

$$\frac{dU}{dx} = -j\,\frac{\omega S}{\gamma P_0}\,p, \tag{2}$$

where $P_0$ is the average pressure and $\gamma$ the ratio of specific heats.

To find the frequencies of the modes of vibration, the most straightforward analytical approach involves (i) eliminating $U$ by combining these two into a second-order differential equation in $p$; (ii) finding a general solution of this equation; and (iii) imposing boundary conditions at both ends of the column, thereby singling out discrete eigenvalues of $\omega$ which satisfy these conditions. However only a few idealized column shapes can be solved analytically, most notably the cylinder and the cone: and of these, only the former is treated in conventional courses.

Matters become more complicated if other, less important, physical processes are included—such as heat conduction, viscous effects, etc. Then the right-hand sides of Eqs. (1) and (2) are no longer purely imaginary, and it ceases to be a simple eigenvalue problem. Instead the system is investigated by imposing a boundary condition at the far end only, and calculating the ratio of $p$ to $U$ at the near end. For reasons which will become clear in Sec. II, this ratio, of pressure to flow rate, is usually referred to as the *acoustic impedance*; and when the ratio is measured at the near end of a pipe, it may be considered a property of the pipe, called the *input impedance*. The modes of vibration then correspond to extremum values of this quantity. This level of sophistication is certainly never introduced into any but the most serious courses on acoustics.

  

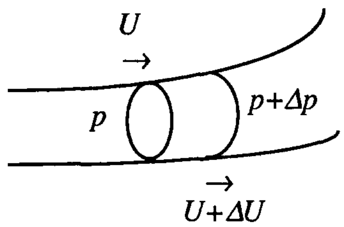


Fig. 1. Variation of the pressure and volume rate of flow along a pipe.

## II. COMPUTATIONAL APPROACH

There are many computing environments available within which students may be guided to approach this problem computationally. The author uses the one called M.U.P.P.E.T.; a scheme in which students are encouraged to write their own programs using PASCAL, and which provides a suite of utilities to help them draw graphs, input data, and make menu choices.[3] What follows will be described exclusively in terms of PASCAL/M.U.P.P.E.T., but it must be stressed that many other approaches could be used to perform the same computations.

The simplest method is to rewrite Eqs. (1) and (2) as two coupled, first-order difference equations,

$$\Delta p = -ZU, \tag{3}$$

$$\Delta U = -Yp, \tag{4}$$

where

$$Z \equiv j\,\frac{\omega\rho_0}{S}\,\Delta x, \quad Y \equiv j\,\frac{\omega S}{\gamma P_0}\,\Delta x. \tag{5}$$

At this point, drawing attention to the fact that an electrical transmission line is an exactly analogous physical system (see Fig. 2) can give students valuable insights.

For this system Kirchhoff's laws read

$$\Delta V = -ZI, \tag{6}$$

$$\Delta I = -YV. \tag{7}$$

Therefore, by analogy, the constant in Eq. (3) plays the role of a distributed *series impedance*, and that in Eq. (4) of a *shunt admittance*.

A skeleton program to solve this problem for an open cylinder is presented in Appendix A. It might easily be set as an exercise for students, though the instructor should be aware of the following general guidelines which went into its construction.

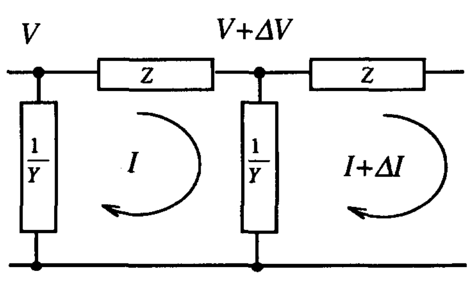


Fig. 2. Variation of the voltage and current along an electrical transmission line.

(1) In any computer program intended to help students solve problems, the mathematical techniques should be as transparent as possible. Further, if at all feasible, the *same* technique should be used for all special cases. By way of an example, if the pipe is conical, a simple transformation of the wave equation makes it analytically soluble; but the present program does not exploit that fact. The perception that each problem always needs its own method of analysis causes many students to lose sight of the physical wood among the mathematical trees.

(2) The numerical approach is to choose a value for $\omega$, after which the functions $Z$ and $Y$ can be calculated. Initial values for $p$ and $U$ are assigned at one end, using the physical boundary conditions, and the difference equations are integrated step by step to the other end. There the values of $p$ and $U$ are checked to see if they match the boundary conditions at that end. If they do not, the value of $\omega$ chosen was not one of the modes of the pipe, and you must go back and choose another value.

In practice the "correct" value might be found by some automated "hunt and shoot" process. A simple binary search procedure can be used (it is one of the standard M.U.P.P.E.T. tools), but this is not explicitly included in the program in Appendix A.

(3) The algorithm used to perform the step-by-step integration is chosen with regard to the accuracy required. Students might usefully be started on Euler's method, but it will soon prove necessary to move to (at least) second-order Runge–Kutta. This is used in the program in Appendix A. Since many personal computers to which students have access are quite slow, it is important to keep the number of integration steps as small as possible. In most cases, 200 steps is quite adequate.

All the time it is worth keeping in mind that, in order to be *musically* useful, the calculation of any eigen-frequency should be accurate to a small fraction of a semitone. This means the ultimate aim is an accuracy of $\sim 0.5\%$. (Of course, the physical assumptions also determine the accuracy—usually to a lot wider range than this.)

(4) The boundary conditions at a closed end are: $U=0$, $p \neq 0$. (The actual value assigned to $p$ does not matter: it merely sets the amplitude of the resulting standing wave.) At an open end they are, to first approximation: $p=0$, $U \neq 0$ (but see Sec. III A below for a more accurate expression).

(5) As long as energy dissipative effects are not included, the program can be written entirely in terms of real numbers. If $p$ is assumed real, then the program variables $U$, $Z$, and $Y$ can be taken to represent the imaginary parts of $U$, $Z$, and $Y$, respectively. This means that Eq. (3), when written as a program statement, effectively changes sign. However it is important to construct the program in such a way that it can eventually be changed to complete complex arithmetic.

## III. RESULTS

The exercise which logically comes first when presenting this material to students—a description of open-open and open-closed cylindrical organ pipes—need not be described here. The well-known results that the former has a full harmonic series as its overtones, while the latter has only the odd harmonics is something that students should know already, and have no difficulty verifying.

Table I. Mode frequencies for a closed/open cylinder.

| Mode number | Frequency (Hz) without end correction | Frequency (Hz) with end correction |
|---|---|---|
| 1 | 107.3 | 106.3 |
| 2 | 321.8 | 318.9 |
| 3 | 536.2 | 531.3 |
| 4 | 750.5 | 743.7 |

### A. End corrections

As is discussed in most standard textbooks,[4] where a standing acoustic wave meets the open end of a pipe, the ratio of pressure to volume rate of flow must match that of a spherical wave spreading out from a flat, circular source. If the radius of the end of the pipe $a$ is very much smaller than the wavelength $\lambda$, then to first order in $a/\lambda$ the two quantities $p$ and $U$ are 90° out of phase with one another; and no energy is radiated. In the electrical analogy, the pipe acts as a transmission line terminated by an inductor.

The effective impedance of the open end depends on the solid angle the spherical wave radiates into (i.e., whether the pipe is flanged or not). For a simple unflanged pipe, the impedance of an open end, following the definitive theoretical anaylsis done by Levine and Schwinger in 1948,[5] is usually taken to be

$$Z_{\text{open}}=\frac{p}{U}\approx j\,\frac{0.6133}{\pi}\,\frac{\rho_0\omega}{a} \tag{8}$$

(an expression which applies only for $\lambda \gg a$). More accurate analysis gives not only deviation from this simple analytic form, but also a *real* part corresponding to non-negligible radiation of energy.

This new boundary condition is straightforwardly incorporated into the computer program by changing the function which must go to zero at the far end of the integration. In terms of ordinary (rather than angular) frequency, this now becomes

```
FinalTestValue:=
   P[n+1]+1.2266*rho0*freq*U[n+1]/
   rMax.
```

With the values for the end radius $r_{\max}=0.012$ m and the length $l=0.800$ m, the frequencies of the first four modes, are shown in Table I.

Most textbooks quote the rough rule of thumb that the effect of the end correction is the same as if the pipe's length were increased by an amount equal to 0.6 $a$. The numbers in Table I are in perfect agreement with this.

### B. Conical pipes

Many real wind instruments are conical in shape, but because the player has to blow into the narrow end, the cone is not complete, i.e., the area at that end is not quite zero.

An easy analytic form for such a cone gives the radius $r$ as a function of $x$ from 0 to $l$ as

$$r=\left(\frac{x+\Delta}{l+\Delta}\right)r_{\max}, \tag{9}$$

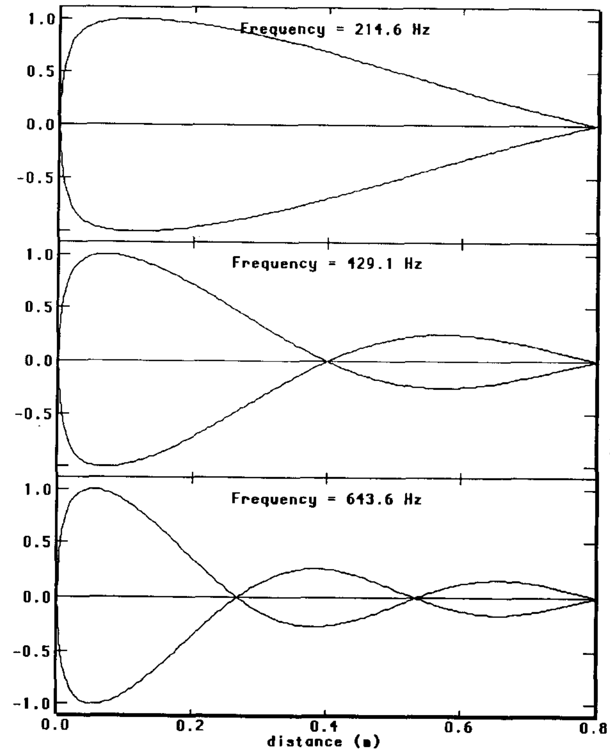


Fig. 3. The first three modes of a nearly complete cone, open at both ends.

where $r_{\max}$ is the radius at the wide end, and $\Delta$ is the distance beyond the narrow end where the cone would come to a point were it extended.

In order to model computationally a nearly complete cone, the quantity $\Delta$ can conveniently be taken to be equal to the integration step; and therefore the only change that needs to be made to the program is to replace the statement line in the function $S$ with

```
S:=pi*sqr(rMax*(x+dX)/(length+dX)).
```

Running the program with both ends of the pipe open verifies the well-known result that the mode frequencies are the same as for an open/open cylinder of the same length. (Analytically the pressure standing waves are spherical Bessel functions which have zeros at the same points as a simple sine wave.) The shape of the first three modes are shown in Fig. 3.

In real instruments the narrow end is always closed by a reed or the player's lips. Common sense says that if the cone is complete there can be no physical difference whether the narrow end is open or closed. Running the program for the nearly complete cone demonstrates this is true (to 1% accuracy), even though the formal boundary conditions are quite different. Compare Figs. 3 and 4.

There are many exercises of interest to musicians that may be set for students using this simple program. One such is the phenomenon of "overblowing." With a conical instrument (an oboe or a bassoon) a skilled player can, merely by blowing differently, change from playing a fundamental note to the first overtone which has twice the frequency (a musical interval of an octave). Such instruments are said to "overblow to the octave." With a closed/open cylinder (a clarinet) the first overtone has a frequency a factor of 3 higher (an octave plus a fifth). It overblows to the twelfth. Students might be asked to model

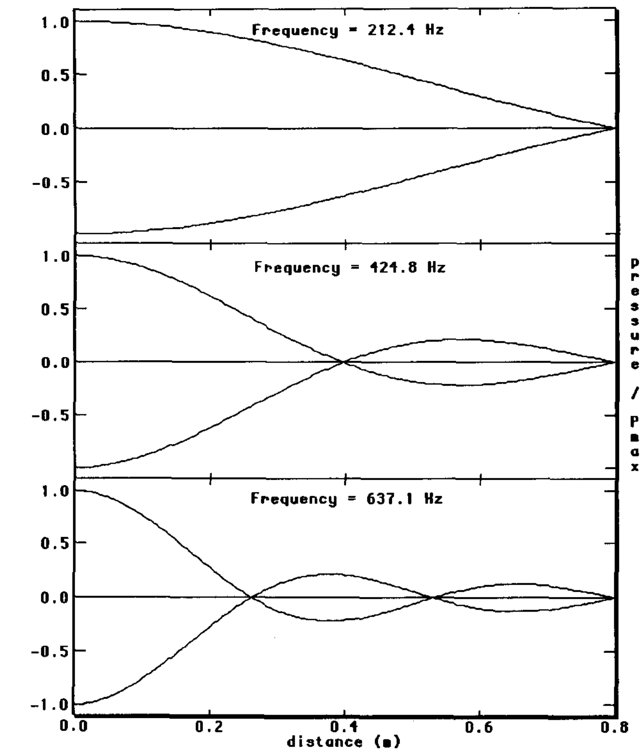


Fig. 4. The first three modes of a nearly complete cone, closed at the narrow end.

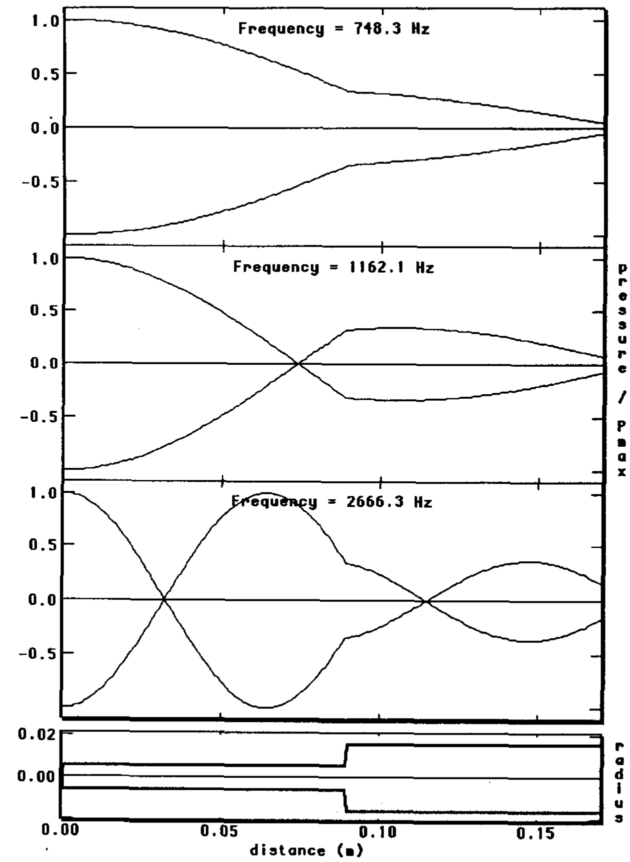


Fig. 5. The three lowest modes of vibration of a two-pipe model for the vowel ah.

an instrument played by Australian aborigines, the didjeridu, which often overblows to round about a tenth (a factor of 2.5).[10]

### C. The human vocal tract

In singing, the vocal folds produce a vibration with a wide range of overtones, usually up to 3000–4000 Hz. The throat and mouth cavity act as a filter, causing the spectrum to peak at several frequencies in the range. These frequencies—called *formants*—are independent of the fundamental pitch of the note; and are correlated with the ear's interpretation of the sound.[7] If the vocal tract is taken to be a one-dimensional pipe about 17 cm in length for adult males or 15 cm for females, closed at one end (the oesophagous) and open at the other (the mouth), the formants correspond to the low standing wave modes of this pipe.

Many textbooks on linguistics, a subject particularly interested in subtle differences between vocal sounds, idealize this model to consist of two cylinders joined together. As an example, the vowel sound "ah" characteristically has its the first three formants near 700, 1100, and 2600 Hz. It is made by opening the mouth wide and constricting the back of the throat, and is modeled by a pipe of length 9 cm, area 1 $cm^2$, joined to another of length 8 cm, area 7 $cm^2$.[9]

This is treated computationally by coding the following statement into the area-calculating procedure of the program:

```
IF x<0.09 THEN S:=0.0001
ELSE S:=0.0007;
```

and the first three modes of vibration are as shown in Fig. 5.

The two-pipe model is a classic example of one which has no real validity other than being easy to analyze algebraically (it just involves matching the amplitude and slope of two sine waves at the junction). But computationally it is no easier or more difficult than any other model, and one could as easily use actual measured cross sections. The seminal work in this area was done by Fant in 1959.[8] His measurements, based on x-ray photographs, give the cross-sectional area $A$ at various distances $d$ along the vocal tract, for the vowel *ah* which are tabulated in Appendix B, Table III.

The following PASCAL statement will calculate a smoothly varying cross section by piece-wise linear interpolation.

```
FOR i:=1 TO 34 DO
  IF (x<=d[i]) AND (x>=d[i+1])
    THEN S:=A[i]+(A[i+1]-A[i])*
         (x-d[i])/(d[i+1]-d[i]);
```

The computed standing waves are then as shown in Fig. 6.

Comparing Figs. 5 and 6 show that both computations place the three formants at more or less the right positions; but that the latter gives a clearer indication why the third formant is intrinsically strong (intuited by comparing the amplitudes at the input and output ends of the pipe). Such calculations are potentially very useful from a pedagogical point of view. There is a wealth of experimental data in the literature (there are at least 12 common vowel sounds in

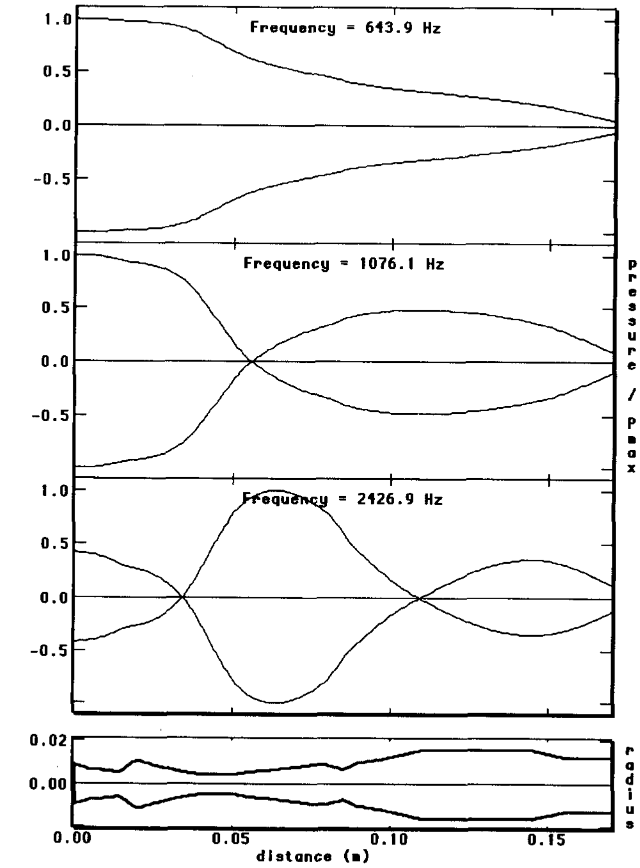


Fig. 6. The three lowest modes of vibration of the measured vocal tract for the vowel ah as in Table III.

English alone) which could be used as exercises for students to test their understanding of and to develop intuition about the relation between the shapes of eigenfunctions and of the pipes that produce them.

## D. Brass instruments

It is not easy to explain simply to students the overtone spectrum of brass instruments. These consist of a pipe, closed at one end by the player's lips and open at the other, often accurately cylindrical for much of their length (like sackbuts or trombones): yet they play a full harmonic series, odd *and even* harmonics. The music written for bugles, alphorns, buisines, etc., shows that since (at least) medieval times, skilled instrument makers have known how to adjust the overtones in this way.[10]

A simple computation demonstrates that the secret lies in the mouthpiece and the flared end (the *bell*) which are integral parts of such instruments. It is best done in three stages.

(1) First calculate the mode frequencies of a bare pipe, 141 cm in length, made up of two gently sloping conical sections from a radius of 5.7 mm at $x=0$, to 88 m at $x=141$ cm; see Appendix B Table IV. (Numerical shapes in this computation are intended to be similar to a standard trumpet, and are adapted from diagrams by Benade.)[11]

(2) Next add a mouthpiece, consisting of a roughly hemispherical cup of radius 9 cm, joined to a short (4 cm) conical pipe at a narrow constriction (radius 2.3 cm). The shape specified in Table V of Appendix B will fit smoothly to the pipe in Table IV.

Table II. Comparison of the first four mode frequencies of an imaginary trumpet.

| Mode number | Frequency (Hz) bare pipe | Frequency (Hz) with mouthpiece | Frequency (Hz) with mouthpiece and bell |
|---|---|---|---|
| 1 | 67.2 | 67.8 | 78.1 |
| 2 | 188.2 | 188.2 | $214.3=2\times 107.2$ |
| 3 | 308.9 | 293.3 | $322.8=3\times 107.6$ |
| 4 | 428.0 | 383.3 | $424.1=4\times 106.0$ |
| 5 | 548.7 | 490.8 | $542.6=5\times 108.5$ |

(3) Add a bell to the other end, which flares rapidly from radius 7.6 to 58 cm in a length of 33.5 cm, constructed so that its radius of curvature continually increases. The shape in Table VI of Appendix B also fits smoothly to the pipe in Table IV of Appendix B.

The results of these three calculations are shown in Table II.

The shapes of the standing waves for the complete trumpet, are shown in Fig. 7. Several features are clear from these calculations.

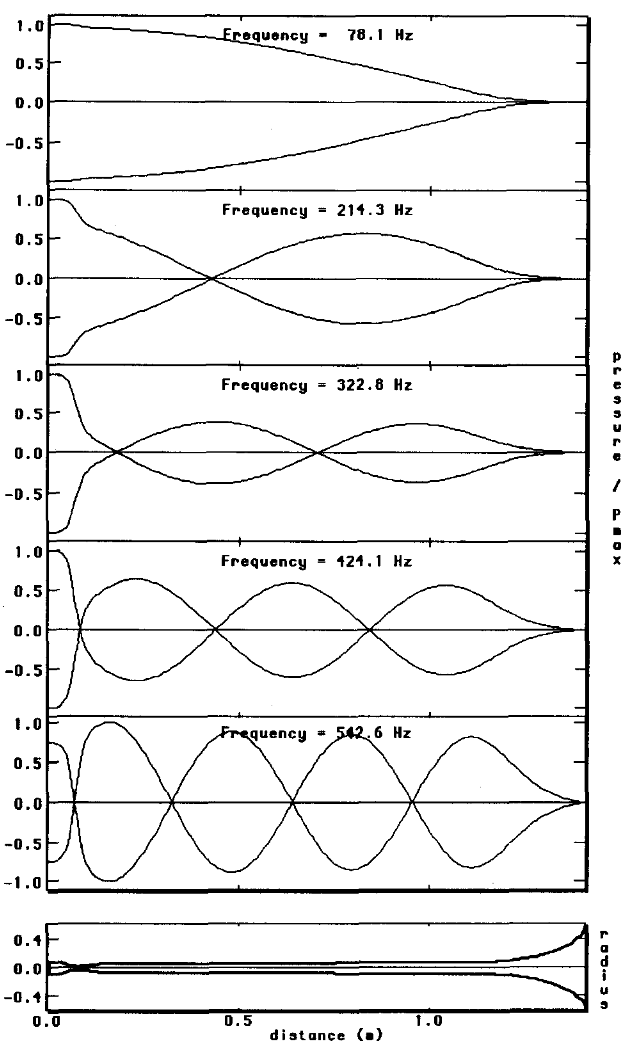


Fig. 7. The five lowest modes of vibration of the imaginary trumpet described by Tables IV–VI.

(1) There is a tendency for a pressure node to occur near the constriction: and the upper modes look more and more like those of an open/open pipe (nodes at two fixed points). Hence the mouthpiece has the greatest effect in tuning the upper overtones. As an example of how near this tuning can be, the French hornist Denis Brain once played the *Aplhorn Concerto* by Leopold Mozart using just a mouthpiece in a length of hose pipe.[12]

(2) The bell has greatest effect on the lower overtones. The rapidly varying curvature acts as a kind of barrier, with the pressure wave being *attenuated* when the curvature becomes too large. This effect is most pronounced for longer wavelengths, and bears a striking analogy to what happens to electron waves obeying the Schrödinger equation (see Sec. IV).

(3) These calculations are schematic rather than physically real. For one thing, since the end radius of the bell is larger than many of the wavelengths in question, the simple end-correction formula, Eq. (8), does not apply and has not been included. For another, the assumption made implicitly right at the beginning, that the wave fronts were plane and perpendicular to the $x$ axis needs to be rethought.[13] (It would make an excellent student exercise to change the program to include this effect.)

Nevertheless, the point of the calculation is to observe that modes 2–5 are harmonics of the same (virtual) fundamental to within $\pm 1.2\%$; and therefore almost perfectly in tune with one another.

(4) The real fundamental however is grossly out of tune: a feature common to all real instruments in this family.

Again the great variety of shapes and sizes of musical instruments can be used as exploratory exercises for students.

### E. Woodwinds

In this class of instrument the pitch of the note is changed by covering or uncovering holes in the side of the pipe. Standard analysis of the effect of a hole is that, provided the hole is small, the laws of continuity say that (1) the pressure is essentially constant across the hole; and (2) the gas flow divides between the hole and the rest of the pipe. In the electrical analogy, the hole is a *parallel* element. Its impedance is like that of an open end, Eq. (8), although the constant is different because the hole is effectively flanged by the pipe wall.[14]

$$Z_{\text{hole}} \approx j\,\frac{1.40}{\pi}\,\frac{\rho_0\omega}{a}. \tag{10}$$

Therefore the hole acts like a *parallel* inductor.

To include this in the mathematical model it is simply necessary to change Eq. (4), at the location of the hole only, to

$$\Delta U = \left(Y + \frac{1}{Z_{\text{hole}}}\right)p \tag{11}$$

and in the computer program to change the procedure which calculates $Y$ to:

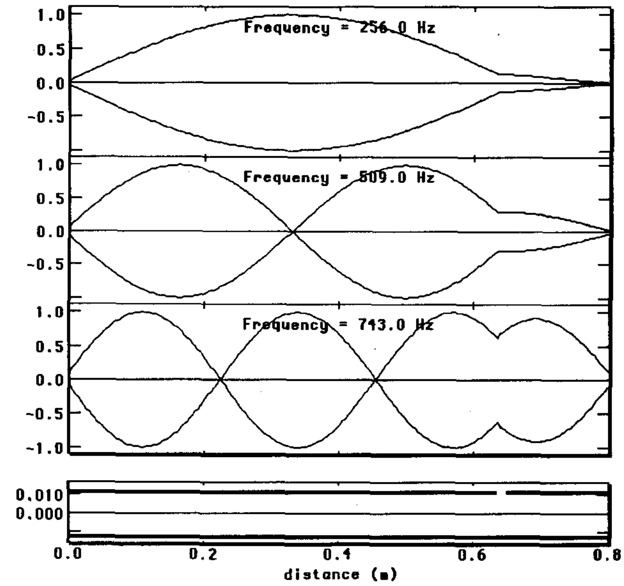


Fig. 8. The three lowest modes of vibration of an open/open cylinder with a single side hole.

```
IF (x>x0-dX/2) AND (X<=x0+dx/2)
  THEN Yh:=rHole/(2.80*rho0*freq)
  ELSE Yh:=0;
Y:=(2*pi*freq*S(x)/P0/gamma*dX)
    +(Yh);
```

Illustrative of the kind of problem students might be asked to tackle is the computation shown in Fig. 8, involving an open/open cylinder of length 80 cm and radius 1.2 cm; with one hole, radius 0.6 cm, located 16 cm from the far end.

The following points can immediately be observed.

(1) The fundamental frequency is close to what it would be if the cylinder were truncated at the 64 cm (262.1 Hz); but only to first approximation. The real frequency is significantly lower.

(2) There is a lot of the standing wave in the shorter part of the pipe. Other holes in this region will significantly raise the mode frequencies.

(3) The ratios of the overtones to the fundamental are 1.99 and 2.90. The latter differs from being harmonic by 3% and is therefore musically unacceptable (because any real instrument would be expected to be able to play in an upper "register"—i.e., using overblown notes).

At a research level, calculations of the tuning of woodwinds were not really feasible before the advent of computers in the late 1950's; and are still appearing in the technical literature today.[15–17] Most use a semianalytical approach—treating the pipe as a number of cylindrical or conical segments, matching known solutions at each junction; with the holes as parallel impedances at these junctions. The present approach could in principle (i.e., without all the physical simplifications) reproduce and even improve these calculations, since it can take into account that the hole might extend over several integration steps (i.e., it is a *distributed* parallel impedance). Be that as it may, what is important is that undergraduate students are given the opportunity, all too rare, of tackling problems which they can compare with currently published research papers.

The same is true in *musical* terms. Most woodwinds can, with special fingerings, play several notes simultaneously, of harmonically unrelated yet musically useful pitches (called *multiphonics*). This fact only came into prominence in the 1960's,[18] and even today performers are still discovering new fingerings.

The program can easily model a simple example, playable on an ordinary recorder (in C), which involves uncovering the C and G holes and almost closing the speaker hole. Appropriate numerical data are tabulated in Appendix B, Table VII. As Fig. 9 shows, the four lowest modes correspond quite closely to the fundamental and second harmonic of two independent notes separated by a major third (a ratio of 1.25).

## IV. DISCUSSION

Ordinarily, it is only upper level (perhaps even postgraduate) acoustics courses which include this kind of material, mainly because of the mathematical maturity its usual exposition demands. The program described here could be used to present the subject matter at a considerably lower level. More advanced courses might then take account of other physical processes and concentrate on calculating input impedances, but so long as the problem remains one-dimensional the same basic program can be used, simply by converting to complex arithmetic. Therefore the computational approach does indeed allow acoustics to be taught at many levels, and can expose students to acoustical concepts which are valuable analogies in the study of other, similar systems.

For example, the formal similarity between the Webster and Schrödinger equations has been known for some time,[19] but has never been fully exploited as a pedagogical tool. With Eqs. (1) and (2) combined, the wave equation can be written

$$\frac{d^2\psi}{dx^2} - \left(\frac{1}{\sqrt{S}}\frac{d^2\sqrt{S}}{dx^2}\right)\psi = -\left(\frac{\omega^2\rho_0}{\gamma P_0}\right)\psi \quad \text{with } \psi \equiv \sqrt{S}p. \quad (12)$$

The coefficient of $\psi$ on the left-hand side, known as the *horn function*, looks like a quantum potential; and at the point where it equals the right-hand side, the standing wave will be reflected (as in the bell of a trumpet). So if students of quantum mechanics have difficulty understanding how electron waves are contained by sloping potentials, they could be told to put their hands inside the bell of a French horn and *feel* where the acoustic wave reflects.

Likewise, since a single hole contributes a localized impedance to Eq. (4), it adds a narrow "potential hill" to the horn function. A *row* of holes will therefore be analogous to a "lattice" of such hills. It has been shown that the study of small finite lattices can give insights into the behavior of semiconductors,[20] so it is a tantalizing thought that teachers might one day use a flute or a recorder as a physical model of a crystalline solid.

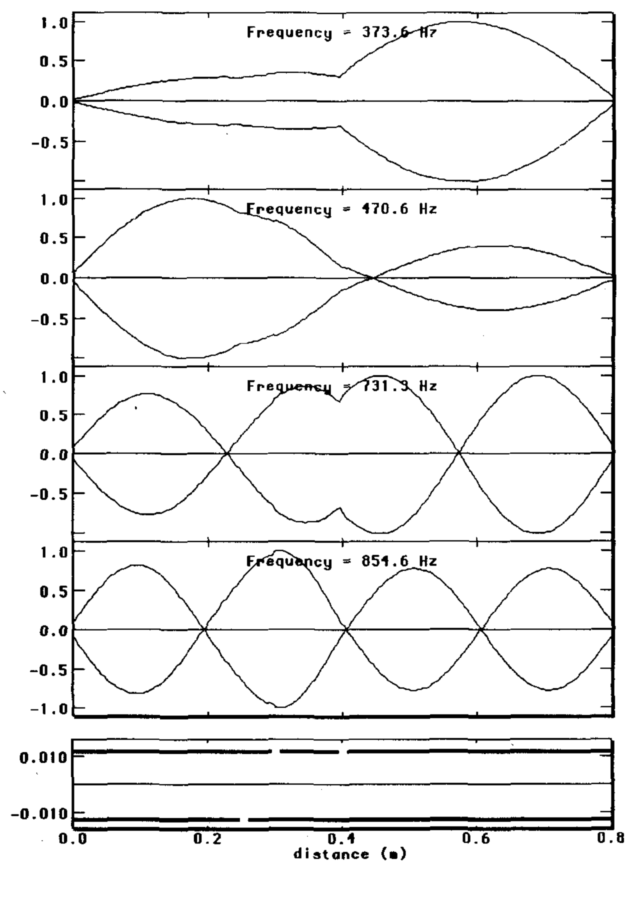


Fig. 9. The four lowest modes of vibration of an open/open cylinder with multiphonic fingering.

## V. CONCLUSION

Acoustics is an interesting subject with a great appeal for many of our students, yet it is not a mainstream subject in physics curriculums largely because of the technical mathematical difficulties in teaching it properly. The work reported in this paper shows clearly how these difficulties might be overcome. It cannot be long before all our students of science own their own computers, so perhaps the time is ripe to rethink the place of acoustics in the typical undergraduate curriculum.

But there is a wider significance than just for acoustics. This is just one example of an important subject, previously thought to be mathematically intractable, being brought within the reach of ordinary students by the computer and M.U.P.P.E.T.-like programming environments. As more of these possibilities are realized by the designers of undergraduate curriculums, the computer can be expected to change, not only *how* physics is taught, but also *what* physics is taught.

## APPENDIX A: THE PROGRAM

The following is a PASCAL computer program which can be extended to reproduce many of the aforementioned results. It is given in its most basic form, with no graphical or search features (and therefore making no explicit use of the M.U.P.P.E.T. utilities).

```
PROGRAM StandingWaves;
CONST
  length=0.800;              {length of the pipe, m}
  rMax=0.100;              {radius of end of pipe, m}
  rho0=1.20;                 {density of air, kg/m-3}
  PO=1.01E5;              {ambient air pressure, Pa}
  gamma=1.4;                {for a diatomic gas}
  n=200;                {number of integration points}

TYPE
  DataVector=ARRAY[1..401] OF Real; {standard MUPPET type}
VAR
  p,U: DataVector;
  dX, freq: Real;

PROCEDURE CalculateGlobalQuantities;
  BEGIN
    dX:=length/n;          {integration step, m}
  END;

FUNCTION S(x:Real): Real;
{** Take the pipe to be a cylinder **}
  BEGIN
    S:= pi*rMax*rMax;
  END;

FUNCTION Z(x:Real): Real;
  BEGIN
    Z:= 2*pi*freq*rho0/S(x)*dX;
  END;

FUNCTION Y(x:Real): Real;
  BEGIN
    Y:= 2*pi*freq*S(x)/P0/gamma*dX;
  END;

PROCEDURE InitializeIntegration;
{** Take the end at x=0 to be closed **}
  BEGIN
    p[1]:=1;
    U[1]:=0;
  END;

PROCEDURE PerformIntegration;
{** Using second order Runge-Kutta **}
  VAR
    x,Phalf, Uhalf: Real;
    i:     Integer;
  BEGIN
    FOR i:=2 TO n+1 DO
    BEGIN
    x:= (i - 1)*dX;
    Phalf:= p[i - 1] + 0.5*U[i - 1]*Z(x);
    Uhalf:= U[i - 1]- 0.5*p[i - 1]*Y(x);

    p[i]:= p[i - 1] + Uhalf*Z(x + dX/2);
    U[i]:= U[i - 1] - Phalf*Y(x+ dX/2);
    END;
 END;

FUNCTION FinalTestValue: Real;
{** Far end open means pressure is zero (within 0.001) **}
```

```
  BEGIN
    FinalTestValue:=p[n+1];

  END;  {------------------MAIN PROGRAM-------------------}

BEGIN
  CalculateGlobalQuantities;
  REPEAT
    Readln(freq);
    InitializeIntegration;
    PerformIntegration;
  UNTIL abs(FinalTestvalue)<0.001;
END.
```

## APPENDIX B: NUMERICAL SHAPES OF VARIOUS PIPES

Table III. Cross-sectional area $A$ vs distance $d$ from the back end of the vocal tract for the vowel ah used in Sec. III D (after Fant).

| $d$ cm | $A$ $cm^2$ | $d$ cm | $A$ $cm^2$ | $d$ cm | $A$ $cm^2$ | $d$ cm | $A$ $cm^2$ | $d$ cm | $A$ $cm^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.6 | 3.5 | 1.0 | 7.0 | 2.0 | 10.5 | 6.5 | 14.0 | 8.0 |
| 0.5 | 1.6 | 4.0 | 0.65 | 7.5 | 2.6 | 11.0 | 8.0 | 14.5 | 8.0 |
| 1.0 | 1.3 | 4.5 | 0.65 | 8.0 | 2.6 | 11.5 | 8.0 | 15.0 | 6.5 |
| 1.5 | 1.0 | 5.0 | 0.65 | 8.5 | 1.6 | 12.0 | 8.0 | 15.5 | 5.0 |
| 2.0 | 4.0 | 5.5 | 1.0 | 9.0 | 3.2 | 12.5 | 8.0 | 16.0 | 5.0 |
| 2.5 | 2.6 | 6.0 | 1.3 | 9.5 | 4.0 | 13.0 | 8.0 | 16.5 | 5.0 |
| 3.0 | 1.6 | 6.5 | 1.6 | 10.0 | 5.0 | 13.5 | 8.0 | 17.0 | 5.0 |

Table IV. Cross-sectional area $A$ vs distance $d$ for the "bare pipe" of a trumpet used in Sec. III E (adapted from Benade).

| | | | | | |
|---|---|---|---|---|---|
| $d$ (cm) | 0 | 12 | 35 | 107.5 | 141 |
| $A$ ($cm^2$) | 1.02 | 1.25 | 1.60 | 1.84 | 2.04 |

Table V. Cross-sectional area $A$ vs distance $d$ for an imaginary mouthpiece used in Sec. III E (adapted from Benade).

| $d$ cm | $A$ $mm^2$ | $d$ cm | $A$ $mm^2$ | $d$ cm | $A$ $mm^2$ | $d$ cm | $A$ $mm^2$ | $d$ cm | $A$ $mm^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 2.54 | 2.5 | 2.20 | 5.0 | 0.640 | 7.5 | 0.172 | 10.0 | 0.515 |
| 0.5 | 2.54 | 3.0 | 2.06 | 5.5 | 0.407 | 8.0 | 0.172 | 10.5 | 0.640 |
| 1.0 | 2.49 | 3.5 | 1.84 | 6.0 | 0.311 | 8.5 | 0.200 | 11.0 | 0.916 |
| 1.5 | 2.39 | 4.0 | 1.83 | 6.5 | 0.245 | 9.0 | 0.245 | 11.5 | 1.08 |
| 2.0 | 2.30 | 4.5 | 1.25 | 7.0 | 0.200 | 9.5 | 0.311 | 12.0 | 1.25 |

Table VI. Cross-sectional area $A$ vs distance $d$ for an imaginary trumpet bell used in Sec. III E (adapted from Benade).

| $d$ cm | $A$ $cm^2$ | $d$ cm | $A$ $cm^2$ | $d$ cm | $A$ $cm^2$ | $d$ cm | $A$ $cm^2$ |
|---|---|---|---|---|---|---|---|
| 107.5 | 1.84 | 117.5 | 2.60 | 127.5 | 7.6 | 137.5 | 40.31 |
| 110.0 | 1.97 | 120.0 | 3.14 | 130.0 | 11.44 | 140.0 | 70.14 |
| 112.5 | 2.11 | 122.5 | 3.91 | 132.5 | 16.94 | 141.0 | 107.51 |
| 115.0 | 2.30 | 125.0 | 5.50 | 135.0 | 25.57 | | |

Table VII. Numerical data for producing a multiphonic in a cylindrical pipe with three holes, used in Sec. III E.

| | | | | |
|---|---|---|---|---|
| Cylinder: | total length (cm) | | 80 | |
| | radius (cm) | | 1.2 | |
| Holes: | position (cm) | 25 | 30 | 40 |
| | radius (cm) | 0.10 | 0.50 | 0.50 |


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