

DOING PHYSICS WITH MATLAB

WAVE MOTION

STANDING WAVES IN AIR COLUMNS

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DOWNLOAD DIRECTORY FOR MATLAB SCRIPTS

air_columns.m

This mscript can be used to model a number of musical instruments and the human voice tract for the sound "ah". For an open end you can select to ignore or include an end correction. A **Graphical User Interface** is used to find a frequency so that the boundary conditions at the end of the tube are satisfied. A pair of coupled first order difference equations are solved by a finite difference method to find the natural frequencies of vibration of the tube and the acoustic pressure along the length of the tube. You can listen to the **sounds** of the natural frequencies. You need to load data files for the profiles of the oboe, trumpet and voice track.

x_oboe.mat y_oboe.mat x_trumpet.mat y_trumpet.mat x_voice.mat x_voicee.mat

wav_matrix_tube.m

This mscript solves a wave equation (Webster's horn equation) with the imposed boundary conditions using a matrix method. A solution gives a set of eigenvalues (natural frequencies) and eigenfunctions (pressure distributions). You need to load data files for the profiles of the oboe, trumpet and voice track.

x_oboe.mat y_oboe.mat x_trumpet.mat y_trumpet.mat x_voice.mat x_voicee.mat

pipe_co.m

The pressure distribution can be visualized along the length a cylindrical tube that has the boundary conditions closed / open. The input parameters are the mode number and length of the tube. *Some bugs in the program that need fixing*

STANDING WAVES IN AIR COLUMNS

The study into the vibrations of air columns is usually restricted to pipes of uniform cross-sectional area with the ends either opened or closed. However, using a numerical approach, the vibrations of the air inside wind instruments and the human vocal tract can be investigated which demonstrates many features of real musical instruments and the human voice.

Many real wind instruments are conical in shape with a narrow end so the player can blow into it. In real instruments, the neck end is always closed by a reed or the player's lips. Examples of conical instruments include the oboe and bassoon - a good musician can change from playing the first harmonic (fundamental) to the second harmonic by simply blowing differently. This is called overblowing to an octave. The clarinet is like a closed / open cylinder.

The brass instruments consist of a tube, closed at one end by the player's lips and open at the other. The trombone is basically a cylinder for most of its length – yet can play a full harmonic series, both the odd and even harmonics. The natural frequencies of the cylindrical tube are modified by the shapes of the mouth piece and bell.

The normal modes of vibration of many wind and brass instrument can be modelled and the pressure distribution along the axis of the tube and the natural frequencies of vibration can be computed. Figure (1) gives a number of sample tube profiles that are modelled using the mscripts **mscripts air_columns.m** or **wav_tubes_matrix.m**. It is a simple task to add profiles for other tubes.

Tube profiles

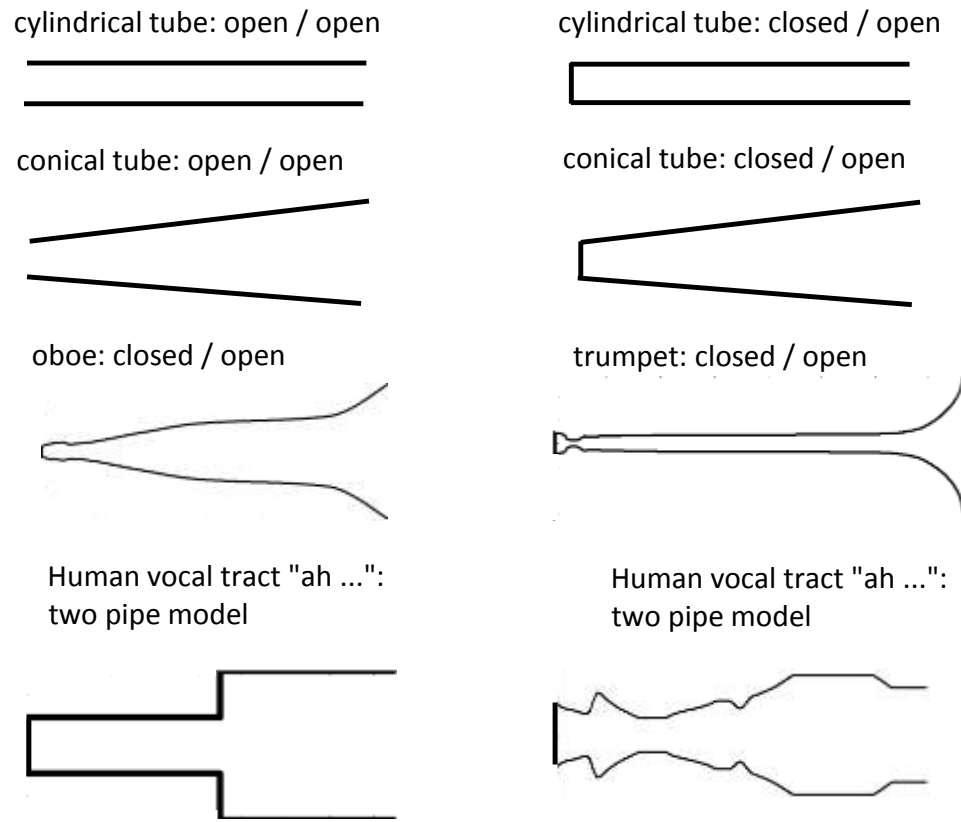


Fig. 1. Types of tubes for modelling the standing waves in air columns.

The data used for the trumpet and human voice track were taken from the paper by Johnston (Appendix). The numerical procedure used 2000 points for the calculations. To obtain 2000 points for the length and radius from the initial data, the following procedure using Matlab was done:

1. The trumpet data was entered into two column vectors for the length and radius of the pipe.
2. The length x was plotted against the radius y .
3. From the menu bar in the plot window: Tools / Basic Fitting / Shape preserving interpolant / 5 significant figures.
4. Find $Y = f(X)$: define X range of values by `linspace(min(L), Max(L), 2000)`.
5. Save data to Workspace as `x_trumpet` and `y_trumpet`.
6. Save variables to files: `save x_trumpet` and `save y_trumpet`.
7. The procedure was repeated for the human voice tract data with the data saved as `x_voice` and `y_voice`.

All the computations are non-realistic and do not apply to real musical instruments because of the many simplifications and assumptions that are made: only a simple end-correction formula is used; all the wave fronts are planar and perpendicular to the length axis of the tube and dissipation effects are ignored.

FINITE DIFFERENCE METHOD

The normal modes of vibration for the air inside various tubes can be found by using a finite difference method. The pressure distribution along the tube and the natural frequencies of vibrating can be computed.

In a simple model where there is no dissipation of energy, all physical quantities are taken as real, the phase remains constant in a plane perpendicular to the axis of the pipe and the motion is sinusoidal with frequency f and angular frequency $\omega = 2\pi f$. It is assumed that the pressure gradient $\Delta p / \Delta x$ is proportional to the volume velocity U of the air and the gradient of the volume velocity $\Delta U / \Delta x$ is proportional to the acoustic pressure p . The volume velocity U is given by

$$(1) \quad U = Au$$

where A is the cross-sectional area of the tube and u is the particle velocity.

The simplest method to find the pressure $p(x)$ along the pipe is by solving the two coupled first order difference equations given by equations (2) and (3) subject to the physical **boundary conditions** at the ends of the pipe.

The boundary conditions at a closed end correspond to a pressure antinode (maximum) and a node for the volume flow rate ($U = 0$). The boundary conditions at an open end correspond to a pressure node ($p = 0$) and an antinode for the volume flow rate (maximum). The value assigned to the maximum values of p or U is not important, it only sets the amplitude of the standing wave.

$$(2) \quad \frac{\Delta p(x)}{\Delta x} = \frac{\rho_0 \omega}{A(x)} U(x)$$

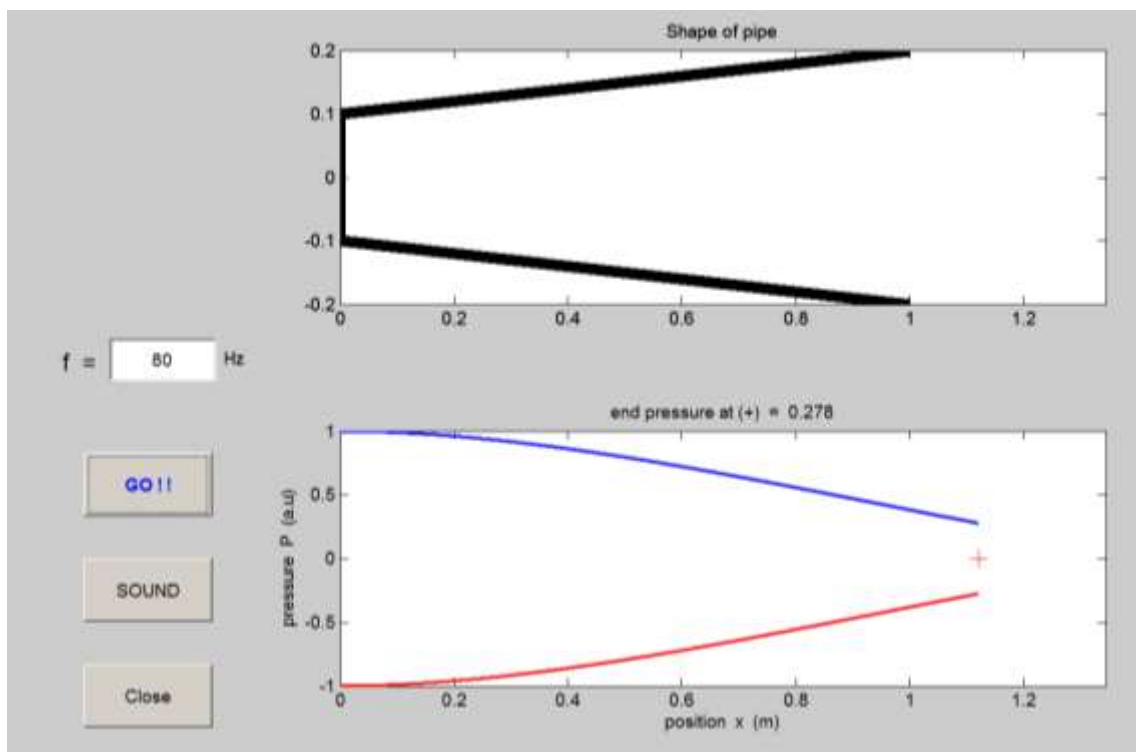
$$(3) \quad \frac{\Delta U(x)}{\Delta x} = -\frac{\omega A(x)}{\rho_0 c^2} p(x)$$

The steps in the finite difference method to solve the pair of coupled first order difference equations using the mscript `air_columns.m` are:

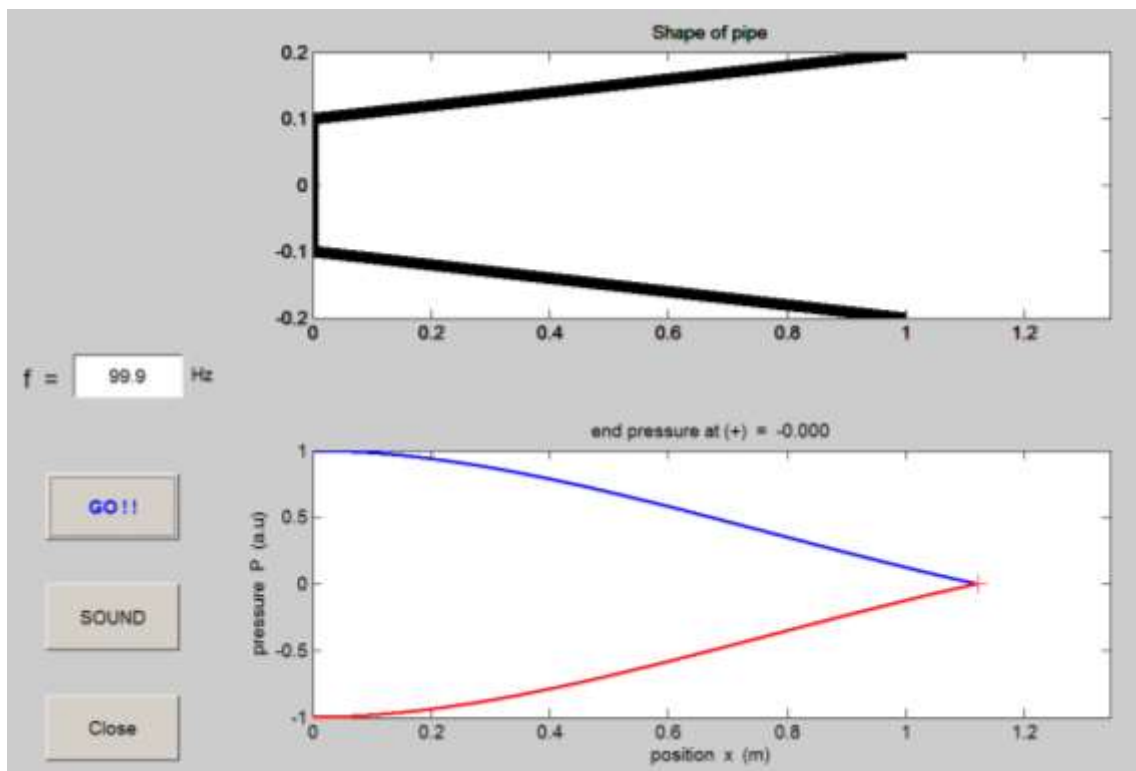
- A frequency f is entered manually in a GUI.
- The initial values for p and U are assigned at one end ($x = 0$) of the air column using the boundary conditions for either a closed or open end.
- The difference equations, equations (2) and (3) are integrated step by step along the pipe to the other end using a second order Runge-Kutta procedure.
- If the boundary conditions are satisfied at the far end of the pipe, the frequency f corresponds to one of the natural frequency of the pipe. If not, another valued of f is entered.

When a standing acoustic wave meets an open end of a pipe, the pressure to volume flow rate must match that of a spherical wave spreading out from a flat circular source. To meet this criteria, a rough rule of thumb is used for an **end correction**. When a pipe has an open end, it is stretch in length by the amount of $0.6 a$, where a is the end radius of the pipe. This is only a crude approximation. You can easily modify the program to use a more accurate estimate of the end correction.

Graphical user interface



Boundary condition at the end of the tube is NOT satisfied. `air_column.m`



Boundary conditions satisfied. For this normal mode, the natural frequency is 99.9 Hz and for the pressure distribution there are zero nodes in the interior of the tube. Due to end correction, the pressure node occurs outside the right hand end of the tube.

air_column.m

Sound

A function is called to produce a sound at the frequency f that has been entered in the GUI. The sound corresponds to the pure tone at frequency f .

```
%*****
% SOUND Do the calculations
% *****
cf2 = boxB;
sf = 22050;           % sample frequency (Hz)
d = 5.0;              % duration (s)
n = sf * d;           % number of samples
s = (1:n) / sf;        % sound data preparation
s = sin(2 * pi * cf2 * s); % sinusoidal modulation
s = s./max(s);
sound(s, sf);          % sound presentation
pause(d + 0.5);        % waiting for sound end
```

Sample Results

The sample results for a trumpet composed of a mouth piece (closed end), pipe and bell (open end) are given below. The first five natural frequencies of vibration in hertz are

69.8 190.5 288.8 388.1 495.2

and the pressure distribution along the pipe for the frequency 388.1 Hz which corresponds to the 3rd overtone is shown in figure 1.

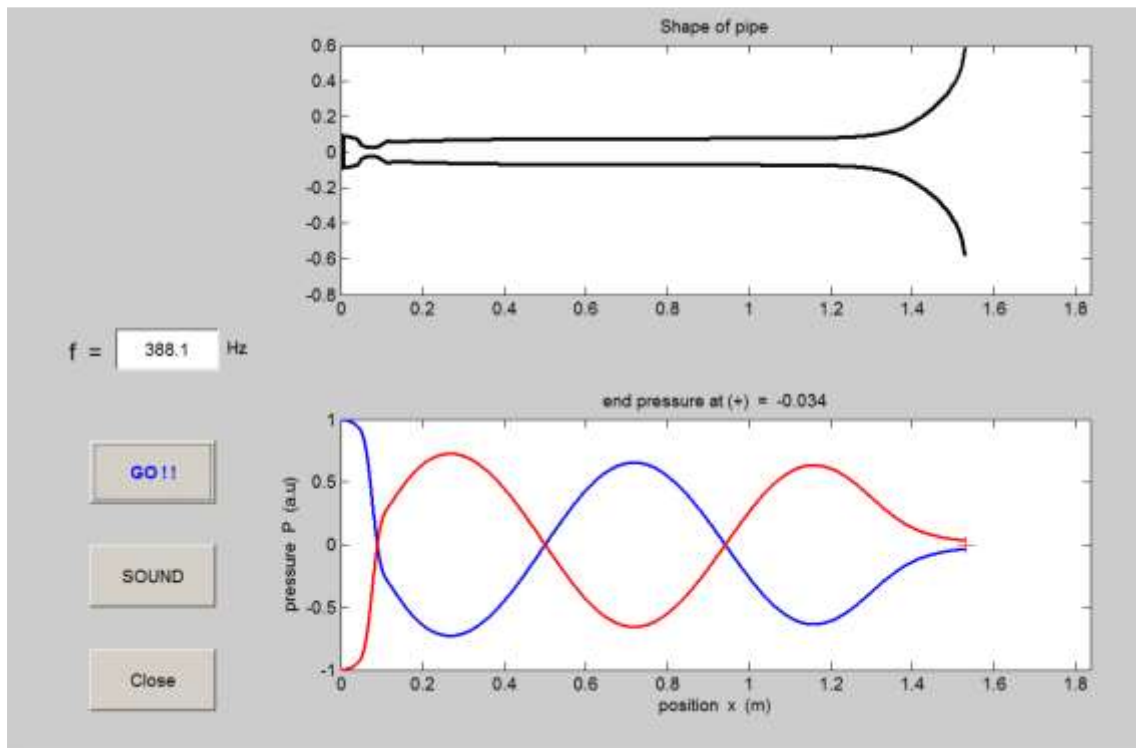


Fig. 2. The shape of the trumpet and the pressure along the pipe for the 3rd overtone at the frequency $f = 388.1$ Hz.

Woodwind with a hole

In many musical wind instruments the pitch of the note can be changed by covering or uncovering holes along the length of the pipe. A hole is effectively a flanged open end and the pressure is essentially constant across the width of the hole. The effect of the hole of radius R_{hole} in a tube can be modelled by changing equation (3) to give equation (4)

$$(4) \quad \frac{\Delta U(x)}{\Delta x} = - \left(\frac{\omega A(x)}{\rho_0 c^2} - \frac{R_{hole}}{2.8 \rho_0 f} \right) p(x)$$

Table 1 gives the results for simulations of a cylindrical pipe of length 0.6253 m and radius 0.9675×10^{-2} m and the same cylinder with a hole of radius 0.02 m using the mscript [air_columns.m](#).

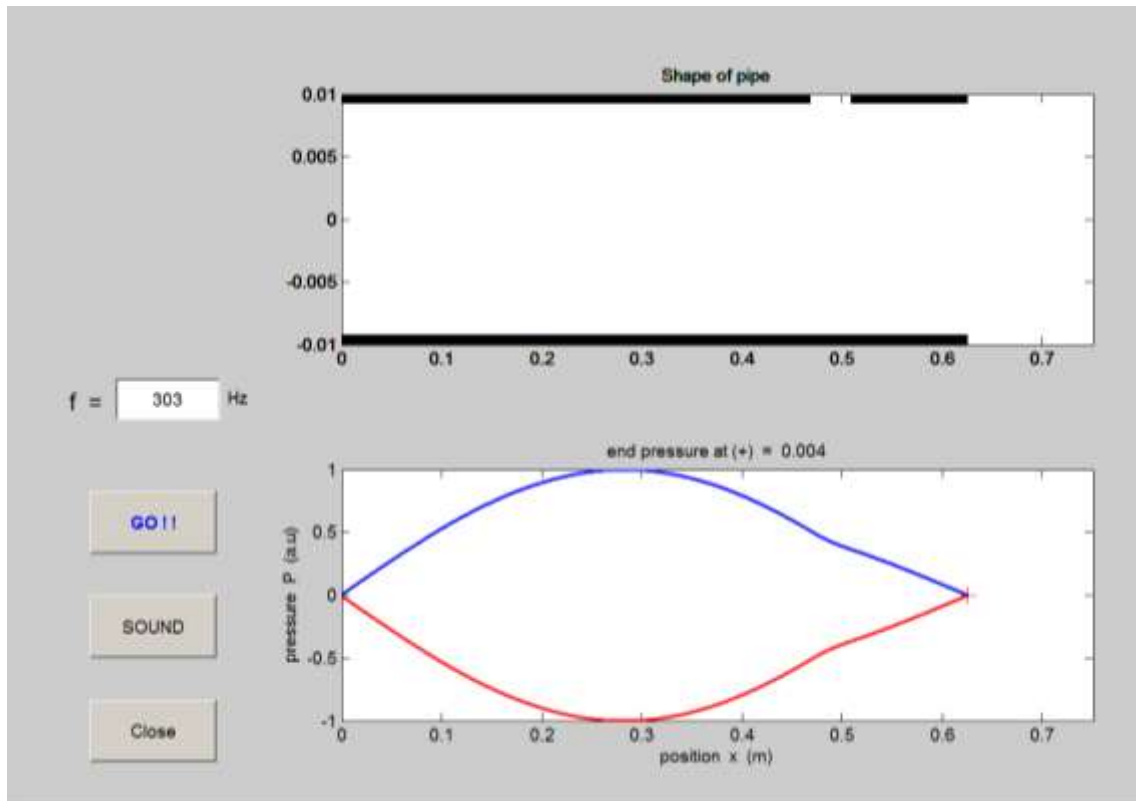


Table 1. Natural frequencies for a cylinder and a cylinder with one side hole.

	f_n (Hz)	f_{hole} (Hz)	f_{hole} / f_1
Fundamental	274 (1.00)	303 (1.00)	1.10
1 st overtone	549 (2.00)	592 (1.95)	2.16
2 nd overtone	822 (3.00)	852 (2.81)	3.11
3 rd overtone	1097 (4.00)	1103 (3.64)	4.02
4 th overtone	1371 (5.00)	1375 (4.53)	5.01
5 th overtone	1646 (6.00)	1657 (5.47)	6.04
6 th overtone	1920 (7.00)	1933 (6.38)	7.05
7 th overtone	2194 (8.00)	2199 (7.26)	8.02

The fundamental frequency for the tube with the single hole is higher than that for complete cylinder because the hole effectively reduces the effective length of the tube. The overtones with frequencies greater 2nd overtone approximately form a harmonic series with the fundamental frequency of the complete cylinder.

Figure 3 shows the acoustic pressure distribution along the tube for the fundamental and the first two overtones for the cylinder with a single side hole. It is as simple matter to modify the msript to change the hole's radius and place more holes along the length of the tube.

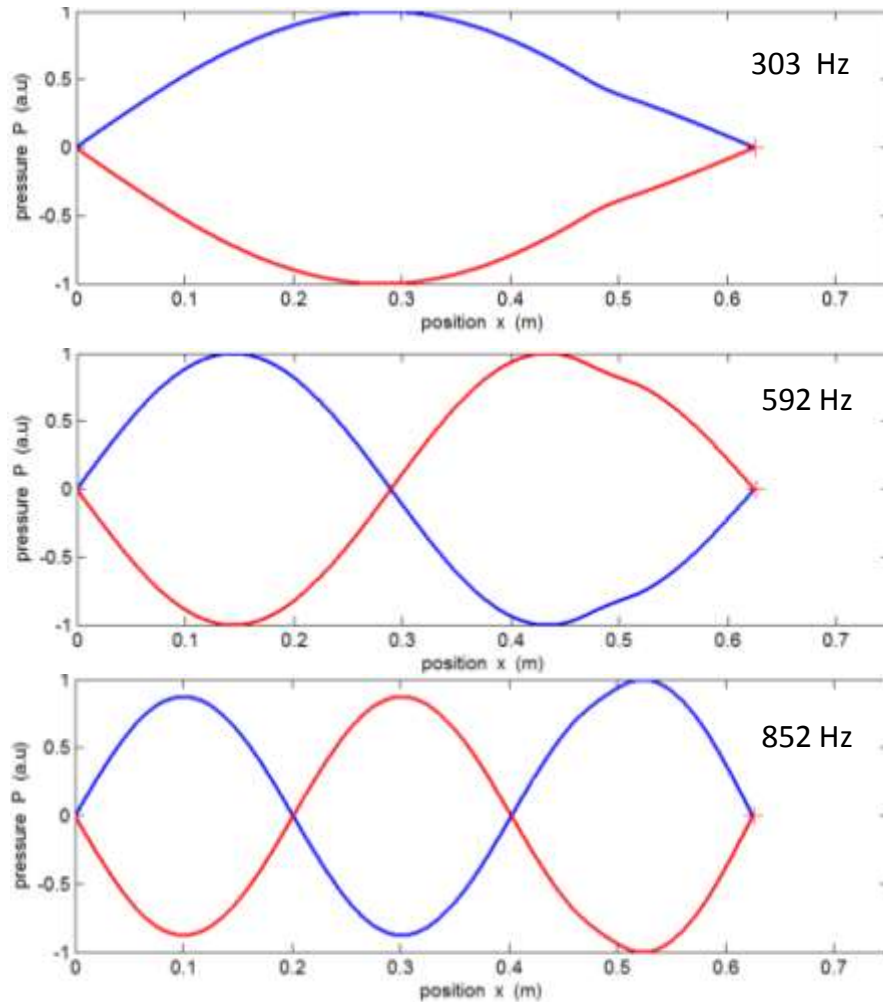


Fig. 3. The three lowest modes of vibration of an open / open cylinder with a single side hole.

MATRIX METHOD

Equations (2) and (3) can be rearranged to give the **Webster's horn equation** (~1919) which gives a one-dimensional approximation for low-frequency sound waves along a rigid tube with a variable cross-sectional area $A(x)$.

$$(5) \quad \frac{d^2 p(x)}{dx^2} + \frac{1}{A(x)} \frac{dA(x)}{dx} \frac{dp(x)}{dx} = -k^2 p(x)$$

$$(6) \quad \left[\frac{d^2}{dx^2} + \frac{1}{A(x)} \frac{dA(x)}{dx} \frac{d}{dx} \right] p(x) = -k^2 p(x)$$

$$(7) \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2\pi f}{c}$$

Equation (6) is very similar to the one-dimensional Schrodinger Equation

$$(8) \quad \left[\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + U(x) \right] \psi(x) = E \psi(x)$$

Equation (6) is an example of an **eigenvalue/eigenfunction** problem. An **eigenvector** of a square matrix $[M]$ is a non-zero vector $[p]$ such that, when the matrix $[M]$ multiplies $[p]$ it yields a constant multiple (K) called the **eigenvalue**

$$(9) \quad [M]p = Kp \quad K \equiv k^2$$

The mscript **wav_tube_matrix.m** is used to find the eigenvalues and eigenvectors of equation (9).

We have to form the matrix to represent the operator acting on $p(x)$. Assume there are N discrete x values (x_1, x_2, \dots, x_N). So we want to compute the acoustic pressure p at each x position. To do this, arrays are constructed for the first and second derivatives.

Consider arrays of size $N \times 1$ representing the known x values, the known cross-sectional area A values, known pressures at the end point p_1 and p_N and the $N-2$ unknown pressures p_i at the interior points

$$\bullet(x_1 \ A_1 \ p_1) \ \bullet(x_2 \ A_2 \ p_2) \ \dots \ \bullet(x_i \ A_i \ p_i) \ \dots \ \bullet(x_{N-1} \ A_{N-1} \ p_{N-1}) \ \bullet(x_N \ A_N \ p_N)$$

The pressures p_1 and p_N at the ends of the tubes are always specified by the **boundary conditions**. The length of the tube is L and Δ is an end correction and so the effective length of the tube is $L_{\text{eff}} = L + \Delta$.

The pressure p_N at the mouth x_N is always a pressure node (open end): $p_N = 0$

The pressure p_1 at the throat x_1 is either a pressure node (open end) or antinode (closed end)

Pressure node (open end): $p_1 = 0$

Pressure antinode (closed end): $p_1 = p_{\text{max}}$ (arbitrary value)

We only have to find $(N-2)$ values of pressure since the two end pressure values are known. Hence, in equation (9), the size of the matrix $[M]$ is $(N-2) \times (N-2)$ and the column matrix $[p]$ is of size $(N-2) \times 1$.

- **Matrix for the term** $\left[\frac{1}{A(x)} \frac{dA(x)}{dx} \right]$

The cross-section area $A(x)$ is calculated from the given radii $R(x)$

$$A = (\pi * R^2) .* \text{ones}(1, \text{num});$$

The term $dA(x)/dx$ is calculated by using the Matlab command **gradient**

```
% Area Matrix
dAdx = gradient(A,dx);           % gradient of A wrt x
dAdx_A = dAdx ./A;
AREA_matrix = zeros(N-2,N-2);
for cc = 1 : N-2;
    AREA_matrix(cc,cc) = dAdx_A(cc+1);
end
```

- **Matrix for the first derivative operator** $\frac{d}{dx}$

Interior points: $\text{pressure} \frac{dp}{dx} \Big|_i = \frac{p_{i+1} - p_{i-1}}{2 \Delta x} \quad \Delta x = x_2 - x_1$

At mouth: $p_N = 0 \quad \frac{dp}{dx} \Big|_{N-1} = \frac{p_N - p_{N-2}}{2 \Delta x} = \frac{0 - p_{N-2}}{2 \Delta x}$

At throat:

pressure node $p_1 = 0 \quad \frac{dp}{dx} \Big|_2 = \frac{p_3 - p_1}{2 \Delta x} = \frac{p_3 - 0}{2 \Delta x}$

pressure antinode $p_1 = p_2 \quad \frac{dp}{dx} \Big|_2 = \frac{p_3 - p_1}{2 \Delta x} = \frac{p_3 - p_2}{2 \Delta x}$

Example for the first derivative matrix with $N = 9$ and $N-2 = 7$

$$\begin{pmatrix} dp_2/dx \\ dp_3/dx \\ dp_4/dx \\ dp_5/dx \\ dp_6/dx \\ dp_7/dx \\ dp_8/dx \end{pmatrix} = \frac{1}{2 \Delta x} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{pmatrix}$$

Element (1,1) = 0 for pressure node at throat and element (1,1) = -1 for pressure antinode.

```
% Pressure gradient matrix
offP = ones(N-3,1);
dP_matrix = (zeros(N-2) + diag(offP,1) - diag(offP,-1));
if flagBC == 1; dP_matrix(1,1) = -1; end;
dP_matrix = dP_matrix ./ (2*dx);

AP_matrix = AREA_matrix * dP_matrix;
```

- **Matrix for the second derivative** $\frac{d^2}{dx^2}$

Interior points: $\text{pressure } \frac{d^2 p}{dx^2} \Big|_i = \frac{p_{i+1} - 2p_i + p_{i-1}}{2\Delta x}$

At mouth: $p_N = p(x_N) = 0 \quad \frac{d^2 p}{dx^2} \Big|_{N-1} = \frac{p_N - 2p_{N-1} + p_{N-2}}{2\Delta x} = \frac{0 - 2p_{N-1} + p_{N-2}}{2\Delta x}$

At throat:

pressure node $p_1 = p(x_1) = 0 \quad \frac{d^2 p}{dx^2} \Big|_2 = \frac{p_3 - 2p_2 + p_1}{2\Delta x} = \frac{p_3 - 2p_2 + 0}{2\Delta x}$

pressure antinode $p_1 = p(x_1) = p(x_2) = p_2$

$$\frac{d^2 p}{dx^2} \Big|_2 = \frac{p_3 - 2p_2 + p_1}{2\Delta x} = \frac{p_3 - 2p_2 + p_2}{2\Delta x} = \frac{p_3 - p_2}{2\Delta x}$$

Example for the second derivative matrix with $N = 9$ and $N-2 = 7$

$$\begin{pmatrix} d^2 p_2 / dx^2 \\ d^2 p_3 / dx^2 \\ d^2 p_4 / dx^2 \\ d^2 p_5 / dx^2 \\ d^2 p_6 / dx^2 \\ d^2 p_7 / dx^2 \\ d^2 p_8 / dx^2 \end{pmatrix} = \frac{1}{\Delta x^2} \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{pmatrix}$$

Element (1,1) = **-2** for pressure node at throat and element (1,1) = **-1** for pressure antinode.

% Make Second Derivative Matrix

off = ones(N-3,1);

SD_matrix = (2*eye(N-2) - diag(off,1) - diag(off,-1));

if flagBC == 1; SD_matrix(1,1) = 1; **end**

SD_matrix = SD_matrix ./ dx2;

Matrix = SD_matrix - AP_matrix ;

where $-\left[\frac{d^2}{dx^2} + \frac{1}{A(x)} \frac{dA(x)}{dx} \frac{d}{dx}\right] \rightarrow \text{Matrix}$

To find the eigenvalues and eigenvectors of the matrix $[M]$ given by equation (9) we can use the Matlab command **eigs**. You can't use the command **eig** because it often produces an aliasing effect for the eigenvectors.

To find the first N_{eig} eigenvalues:

```
SIGMA = 'sm';
[e_funct e_values] = eigs(Matrix,Neig,SIGMA);
```

The **eigs** command **eigs(Matrix,Neig,SIGMA)** finds the smallest ['sm'] N_{eig} eigenvalues and eigenvectors of Matrix .

To extract the eigenvalues $K = k^2$:

```
% All Eigenvalues 1, 2 , ... N-2 wave number k
k2 = zeros(1,Neig);
for cc = 1 : Neig
    k2(cc) = e_values(cc,cc);
end
```

To extract the eigenvectors for the pressure distribution:

```
% Corresponding Eigenfunctions 1, 2, ... ,n
clear y
p = zeros(N,Neig); % axial pressure
for cn = 1 : Neig
    if flagBC == 1; yBC = e_funct(1,cn); else yBC = 0; end;
    p(:,cn) = [yBC; e_funct(:,cn); 0];
end % for
```

When Matlab finds the eigenvalues, the order in which they are assigned is not always from the lowest to the highest value.

The eigenvalues for the frequency are displayed in the Command Window and the eigenvectors for pressure are shown in Figure Windows.

SIMULATIONS: NORMAL MODES

The mscript **wav_tube_matrix.m** can be used to investigate the normal modes of vibration (natural frequencies of vibration and pressure distribution within the tube) of a large variety of tubes of differing lengths and profiles. As an example, we will consider a range of tubes with dimensions related to the **Strasser oboe**. No end correction is used in the simulations.

The length of a Strasser oboe with a reed staple in place but without the reed:

$$L = 62.53 \times 10^{-2} \text{ m}$$

The radius at the throat and radius at the mouth:

$$a_{\text{throat}} = 0.115 \times 10^{-2} \text{ m} \quad a_{\text{mouth}} = 1.82 \times 10^{-2} \text{ m}$$

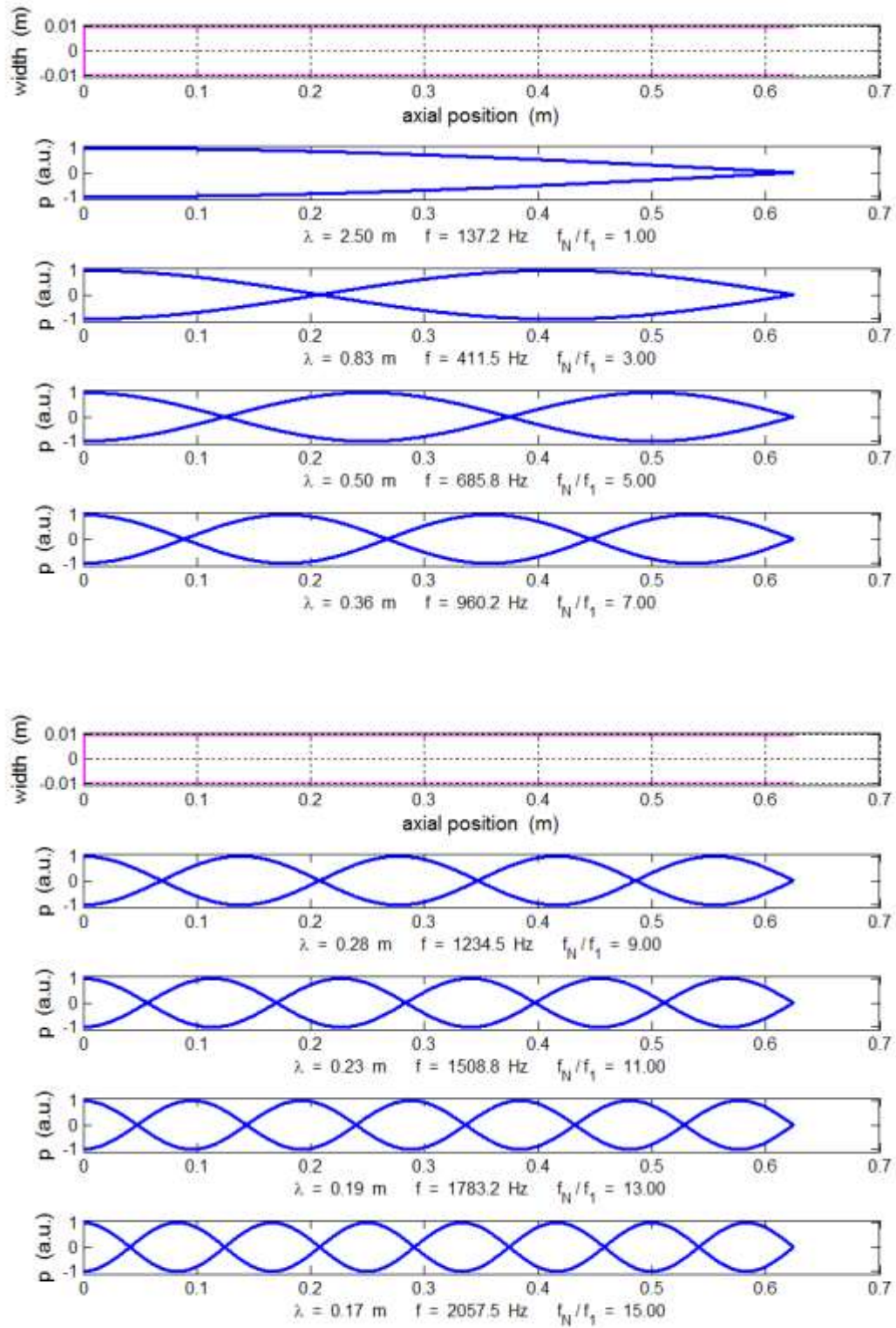
$$\text{average radius } a_{\text{avg}} = 0.9675 \times 10^{-2} \text{ m}$$

Speed of sound $c = 343 \text{ m.s}^{-1}$

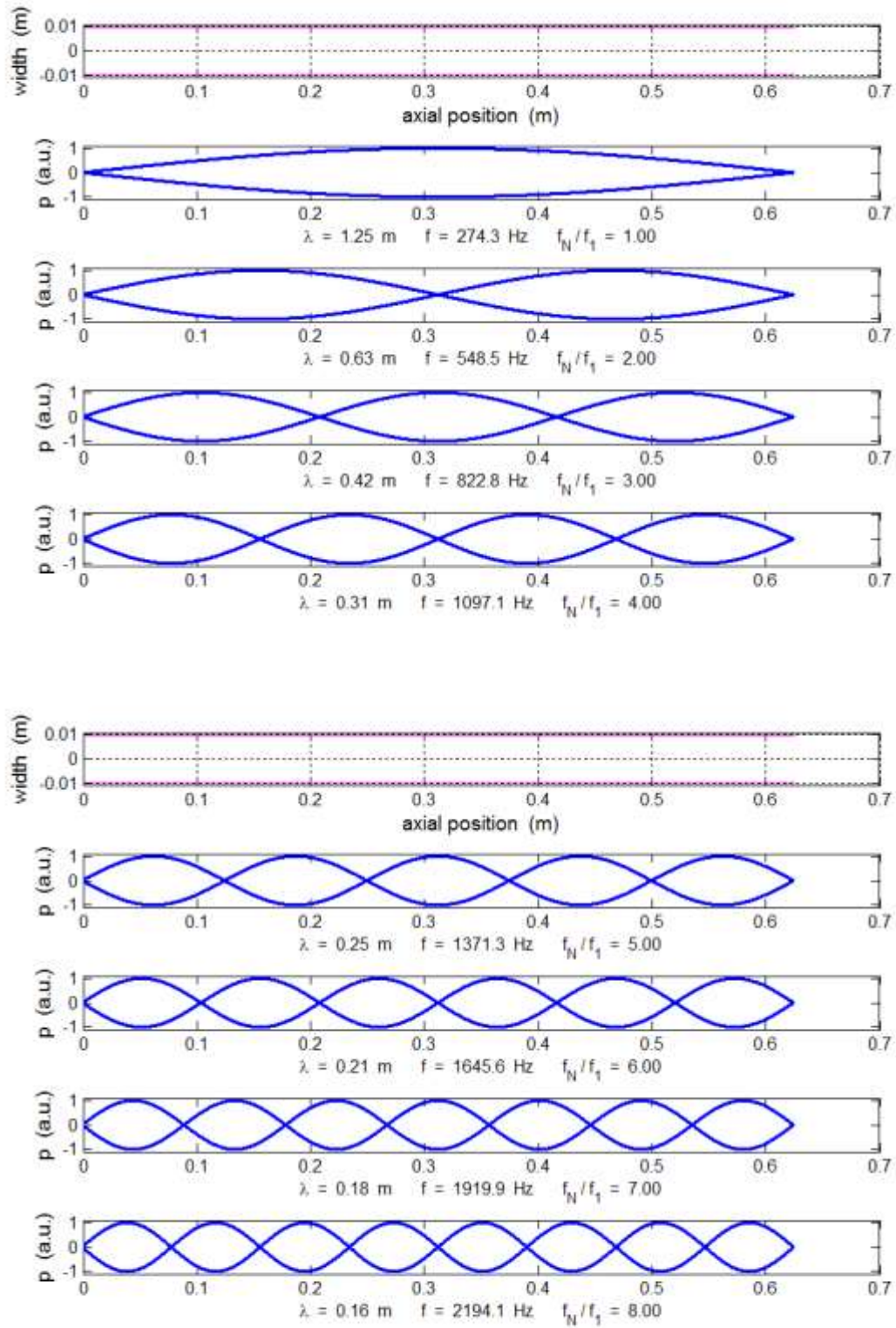
Tube	Natural frequencies [Hz] ratio to fundamental (f/f_1)							
cylinder (c / o)	137.2 (1.00)	411.5 (3.00)	685.8 (5.00)	960.2 (7.00)	1234.5 (9.00)	1508.8 (11.00)	1783.2 (13.00)	2057.5 (15.00)
cylinder (o / o)	274.3 (1.00)	548.5 (2.00)	822.8 (3.00)	1097.1 (4.00)	1371.3 (5.00)	1645.6 (6.00)	1919.9 (7.00)	2194.1 (8.00)
cone (o / o)	274.3 (1.00)	548.5 (2.00)	822.8 (3.00)	1097.1 (4.00)	1371.3 (5.00)	1645.6 (6.00)	1919.9 (7.00)	2194.1 (8.00)
cone (c / o)	257.1 (1.00)	515.5 (2.00)	775.7 (3.02)	1038.2 (4.04)	1302.7 (5.07)	1568.9 (6.10)	1836.6 (7.14)	2105.5 (8.19)
oboe	234.4 (1.00)	547.2 (2.33)	752.9 (3.21)	1002.6 (4.28)	1265.2 (5.40)	1543.9 (6.59)	1824.0 (7.78)	2102.7 (8.97)
trumpet (scaled)	70.5 (1.00)	192.5 (2.73)	292.2 (4.15)	392.1 (5.56)	499.8 (7.09)	609.11 (8.64)	718.1 (10.19)	827.9 (11.75)

The fundamental frequency is f_1 .

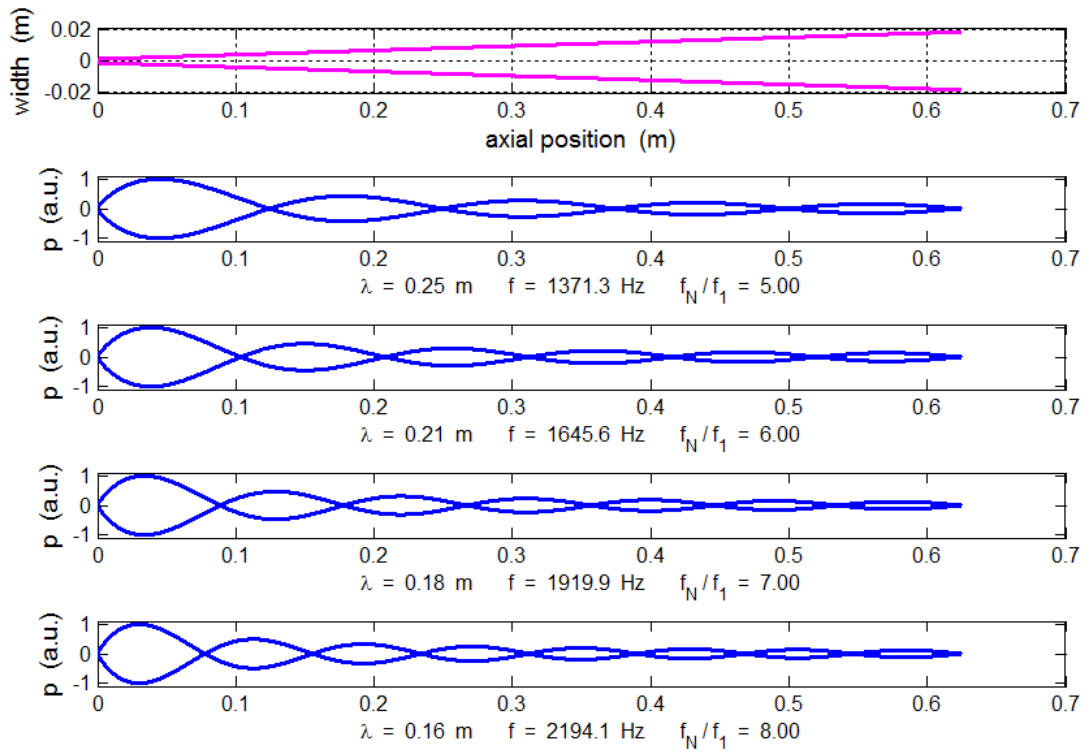
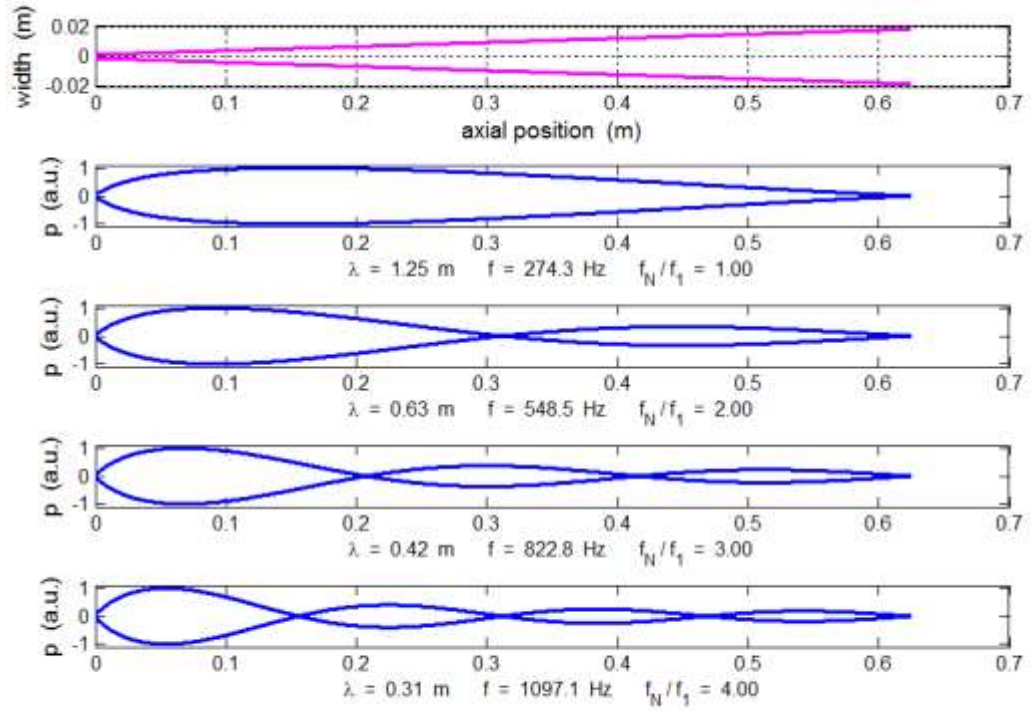
Cylinder (closed / open) $L = 0.6253 \text{ m}$ $a_{\text{avg}} = 0.9675 \times 10^{-2} \text{ m}$



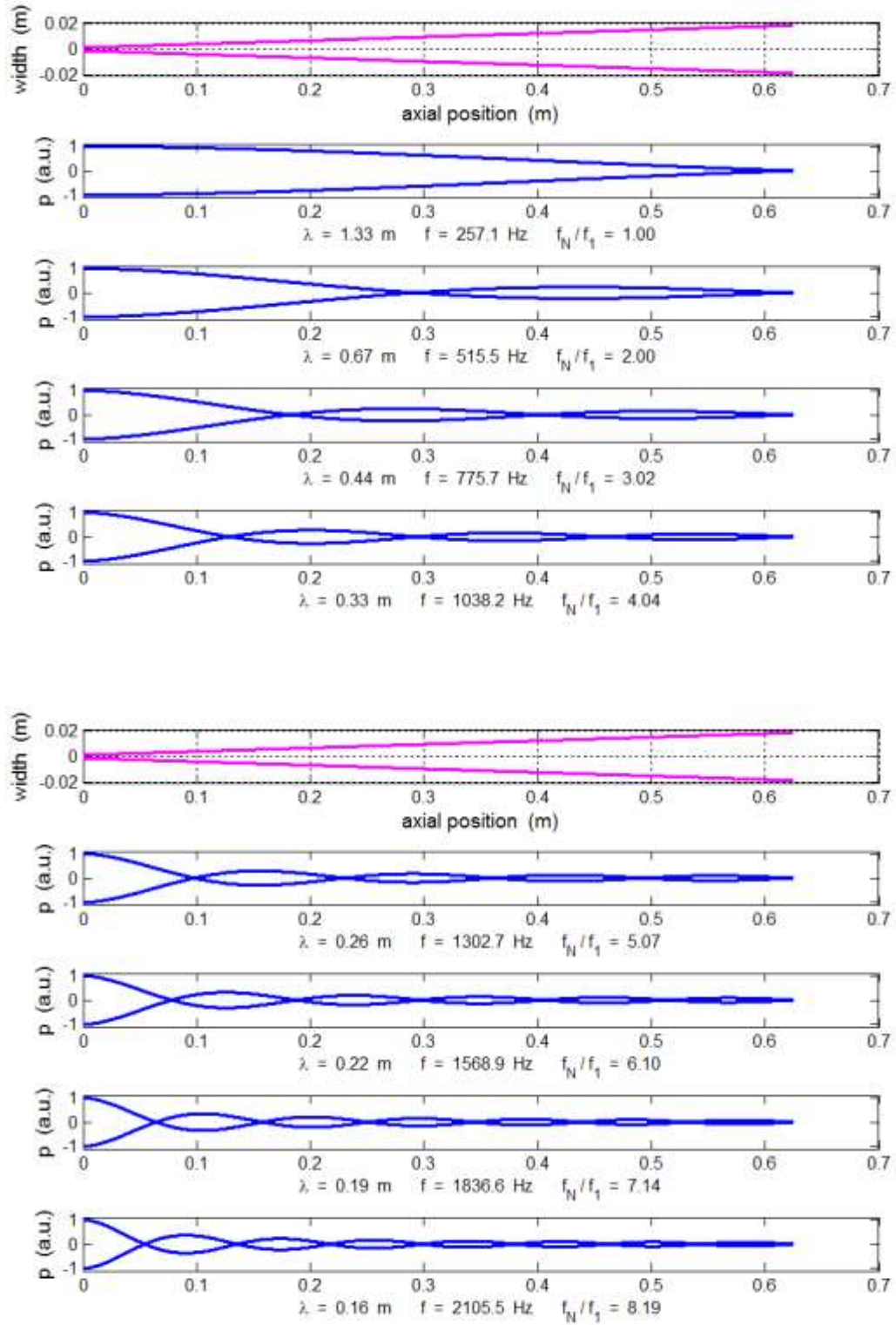
Cylinder (open / open) $L = 0.6253 \text{ m}$ $a_{\text{avg}} = 0.9675 \times 10^{-2} \text{ m}$



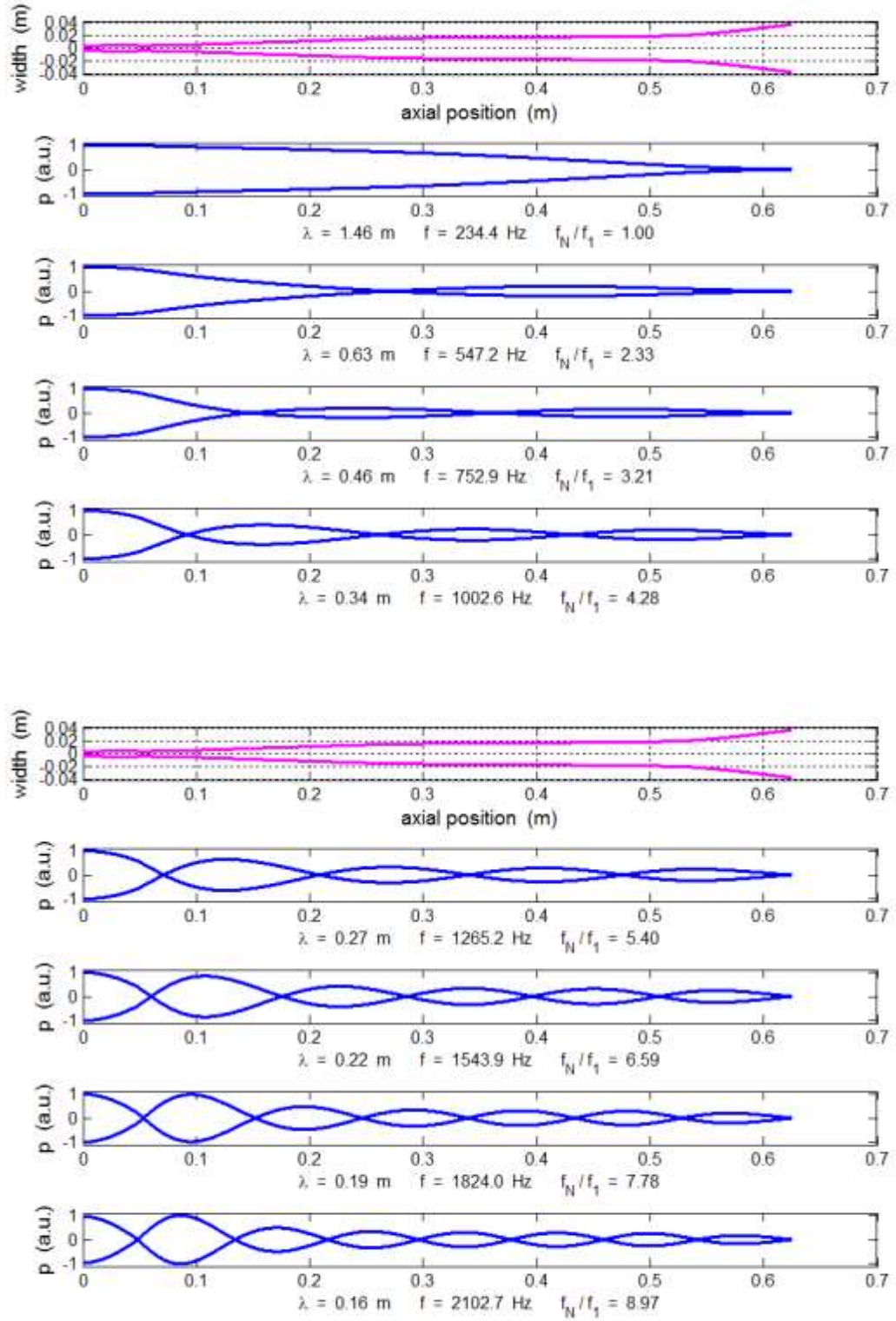
Cone (open / open) $L = 0.6253 \text{ m}$ $a_{\text{throat}} = 0.115 \times 10^{-2} \text{ m}$ $a_{\text{mouth}} = 1.82 \times 10^{-2} \text{ m}$



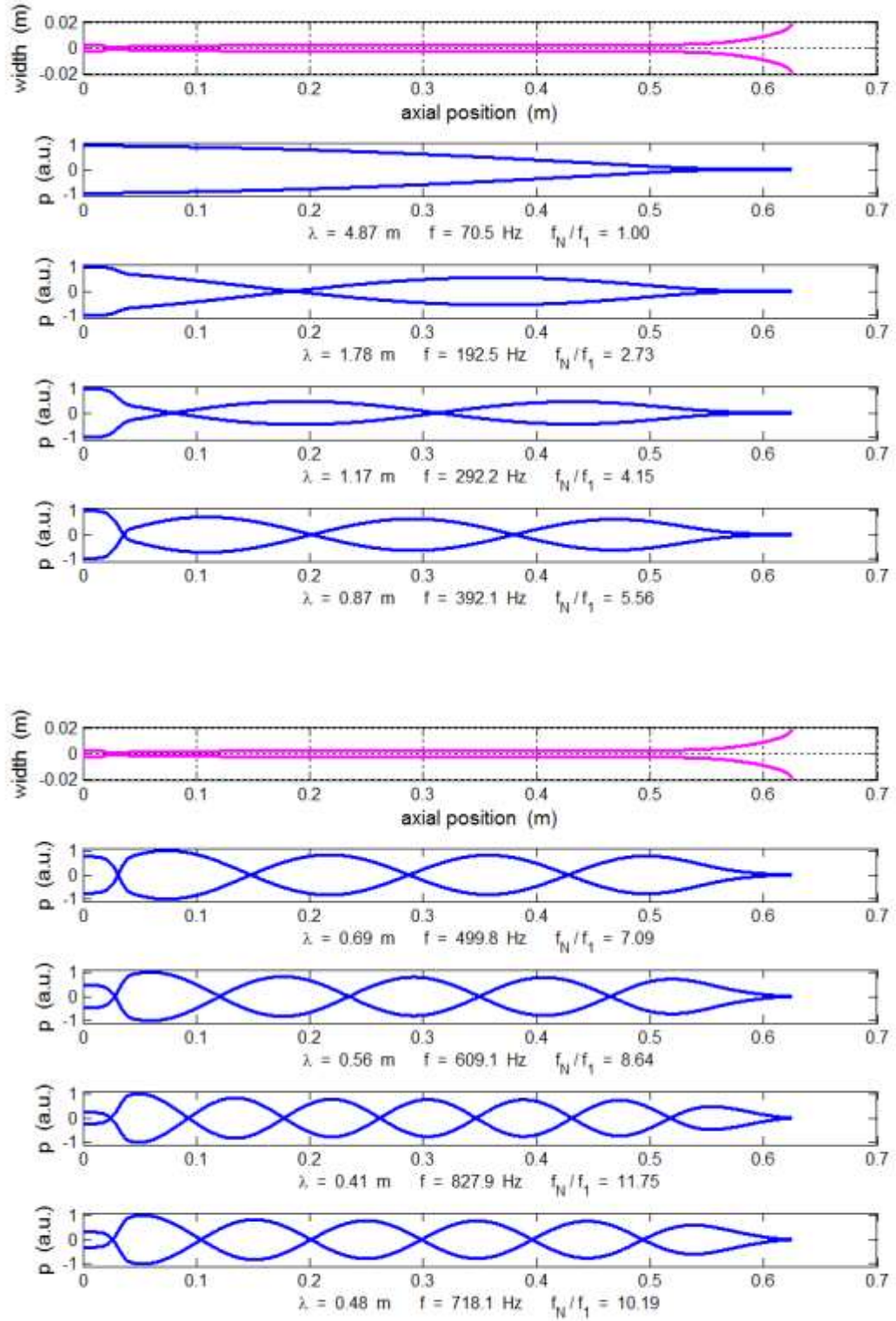
Cone (closed / open) $L = 0.6253 \text{ m}$ $a_{\text{throat}} = 0.115 \times 10^{-2} \text{ m}$ $a_{\text{mouth}} = 1.82 \times 10^{-2} \text{ m}$



Oboe (open / open) $L = 0.6253 \text{ m}$ $a_{\text{throat}} = 0.115 \times 10^{-2} \text{ m}$ $a_{\text{mouth}} = 1.82 \times 10^{-2} \text{ m}$



Scaled Trumpet (open / open) $L = 0.6253 \text{ m}$ $a_{\text{mouth}} = 1.82 \times 10^{-2} \text{ m}$



Cylindrical organ pipes with **both ends open** have a full harmonic series of overtones

$$f_n = n f_1 \quad n = 1, 2, 3, \dots$$

Cylindrical organ pipes with **closed / open** boundary conditions have only the odd harmonics

$$f_{2n-1} = (2n-1) f_1$$

For cylindrical tubes, the pressure varies as a sinusoidal function along the length of the tube

A **cone profile tube** that is **open at both ends** has the same set of natural frequencies as a cylindrical tube of the same length – a full harmonic sequence.

A **cone profile tube** that is **closed / open**, has an approximate harmonic series if the neck region is narrow. The “even” harmonics and “odd” harmonics can be both excited where as in the cylindrical tube which is closed / open only the odd harmonics can be excited. The lower overtones more closely match the harmonic series. The narrower the neck, then the distinction between an open or closed end reduces – there being no physical difference if the cone is complete.

Oboe: for an oboist to achieve good, clear low frequencies tones, it is important that the low frequencies form a harmonic series. Does this model of the oboe achieve this good and clear tone at low frequencies? Our model does not include the reed. When a reed is placed on the oboe, it slightly lowers the natural frequencies of vibration. Is our model oboe well designed to offset the effect of the reed?

The shape of the standing wave patterns for the **trumpet** shows:

- Tendency for pressure nodes to occur near the throat where the tube is most constricted.
- The bell has the greatest effect on the lower overtones – the fundamental frequency of the scaled trumpet being only 70.5 Hz and hence completely out of tune, however, the overtones 1 to 4 are almost harmonic to the same virtual fundamental (96.3 Hz) and therefore, nearly in tune with one another.

$$1^{\text{st}} \text{ overtone: } 2 \times 96.3 \text{ Hz} = 193 \text{ Hz (193 Hz)}$$

$$2^{\text{nd}} \text{ overtone: } 3 \times 96.3 \text{ Hz} = 289 \text{ Hz (292 Hz)}$$

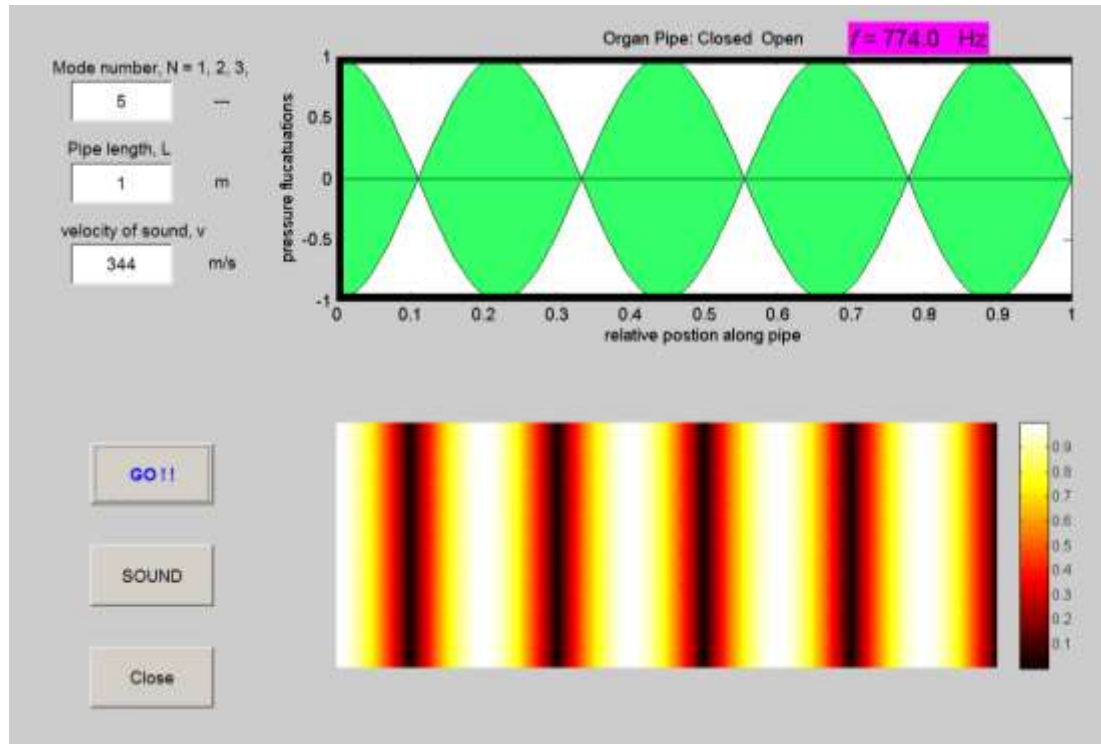
$$3^{\text{rd}} \text{ overtone: } 4 \times 96.3 \text{ Hz} = 385 \text{ Hz (392 Hz)}$$

$$4^{\text{th}} \text{ overtone: } 5 \times 96.3 \text{ Hz} = 482 \text{ Hz (500 Hz)}.$$

The trumpet is an example of a brass instrument that consists of a pipe, closed at one end by the player's lips at the mouth piece and a bell shaped flange at the other. The brass instruments even though they have closed and open boundary conditions at the ends, a full range of even and odd harmonics can be played because of the mouthpiece and bell of the instrument. The brass player's lips act as a controlled driving mechanism. At the normal modal frequencies, the air column vibrations will cooperate with the driving mechanisms to sustained steady oscillations. The data used for the profile is only for an imaginary trumpet and the model we are using is very crude because at the flanged end of the trumpet the wave fronts will no longer be plane and perpendicular to the x -axis. Also, at the bell end, the end correction used is not really valid because of the large radius at the end of the bell.

VISUALIZATION OF THE PRESSURE DISTRIBUTION

The pressure distribution can be visualized along the length a cylindrical tube that has the boundary conditions closed / open. The input parameters are the mode number and length of the tube.



[pipe_co.m](http://pipe.co.m)

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APPENDIX

Original data from Johnston for the length and radius of a trumpet and the human voice tract.

length: trumpet (m)	radius: trumpet (m)	length: voice tract (m)	radius: voice tract (m)
0	0.0899	0	0.0091
0.005	0.0899	0.005	0.0071
0.010	0.0890	0.010	0.0064
0.015	0.0872	0.015	0.0056
0.020	0.0856	0.020	0.0113
0.025	0.0837	0.025	0.0091
0.030	0.0810	0.030	0.0071
0.035	0.0765	0.035	0.0056
0.040	0.0763	0.040	0.0045
0.045	0.0631	0.045	0.0045
0.050	0.0451	0.050	0.0045
0.055	0.0360	0.055	0.0056
0.060	0.0315	0.060	0.0064
0.065	0.0279	0.065	0.0071
0.070	0.0252	0.070	0.0080
0.075	0.0234	0.075	0.0091
0.080	0.0234	0.080	0.0091
0.085	0.0252	0.085	0.0071
0.090	0.0279	0.090	0.0101
0.095	0.0315	0.095	0.0113
0.100	0.0405	0.100	0.0126
0.105	0.0451	0.105	0.0144
0.110	0.0540	0.110	0.0160
0.115	0.0586	0.115	0.0160
0.120	0.0631	0.120	0.0160
0.125	0.0570	0.125	0.0160
0.24	0.0631	0.130	0.0160
0.47	0.0714	0.135	0.0160
1.195	0.0765	0.140	0.0160
1.220	0.0792	0.145	0.0160
1.245	0.0820	0.150	0.0144
1.270	0.085	0.155	0.0126
1.295	0.0910	0.160	0.0126
1.320	0.1000	0.165	0.0126
1.345	0.1116	0.170	0.0126
1.375	0.1323		
1.395	0.1557		
1.420	0.1908		
1.445	0.2322		
1.470	0.2853		
1.52	0.4725		
1.53	0.5850		