UNIVERSITEIT TWENTE.

Teaching Quantum Mechanics Using qCraft

Author:
Micha VAN DEN ENK
[s1004654]

Supervisors:
Dr. H. H. LEEMKUIL
Dr. H. VAN DER MEIJ

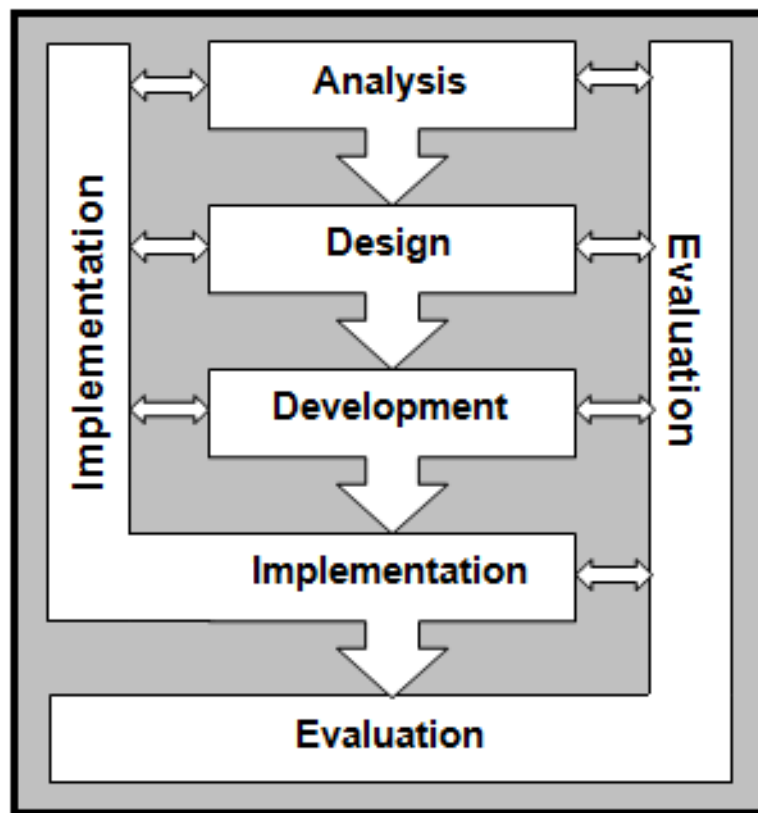August 2, 2015

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Preface

The Generic Model

Figure 1: The generic model by Plomp et al. (1992)

Topics mentioned in literature

The first logical question which should be asked would be the question which topics exist within the domain of introductory quantum mechanics, because only then we can delve into the question how these topics could or should be thought to novice learners. This exploration would have to start with a topic the students are already familiar with. Such a topic is the Rutherford-Bohr Model of the Atom, also known as the Bohr Model, which presents a description of a hydrogen atom (see figure 2). Students in upper secondary education should at least be familiar with this model, especially those with a technical profile. This gives way to introduce the students to the concept of elementary particles, which are the particles which exhibit quantum behaviour. The Bohr-model is also often referred to in the studied literature (Dori, Dangur, Avargil, & Peskin, 2014; McKagan et al., 2008; Müller & Wiesner, 2002; Papaphotis & Tsaparlis, 2008a, 2008b).

Some of the studies (Erduran, 2005; Hubber, 2006; Müller & Wiesner, 2002; Thacker, 2003) then describe properties of specific elementary particles, mostly of electrons or photons. Often used properties are the photoelectric effect or the polarisation of light (Henriksen et al., 2014; McKagan et al., 2008; Müller & Wiesner, 2002). These properties could give more meaning to what the elementary particles are and do. Another benefit would be that these properties are used in the various experiments conducted within the field of quantum mechanics.

The double-slit experiment is the most famous of these experiments, and also the most studied tool for educational purposes (Asikainen & Hirvonen, 2014; Henriksen et al., 2014; Hobson, 2012; Levrini & Fantini, 2013; McKagan et al., 2008; Müller & Wiesner, 2002; Papaphotis & Tsaparlis, 2008a; Singh,

Figure 2: The Rutherford-Borh Model of the Atom

Belloni, & Christian, 2006; Thacker, 2003). A reason why this experiment is famous is because it was the first experiment in history to demonstrate phenomena of quantum mechanics. The experiment entails shooting elementary particles through two narrow slits in a wall, projecting them on a large wall behind these two slits. When the two slits are separated very little, a interference pattern emerges. This is a result expected when the elementary particles would not be particles but waves. However, if the particles are observed and information is available through which slit each particle traversed, a diffraction pattern emerges, which would be the behaviour of particles. The most apparent phenomena demonstrated by this experiment is the wave-particle duality of elementary particles, which then could give way to mathematical descriptions of quantum mechanics like the Schrödinger equation. The Centraal Eindexamen of 2015 also already contained this experiment (Laan, 2013), so educational resources for teaching this experiment already exist.

For understanding the double-split experiment, the concept of superposition is vital. Superposition means that when a particle is not observed, it is in all possible states at the same time. In the case of the double-slit experiment, superposition means that the particle goes through both slits at the same time. It then interferes with itself because of the probability function of where the particle ends up. However, when the particle is observed through which slit it travels, the information through which slit the particle travels is known and forces the probability function of the particle to collapse to either the left or the right slit. Because of this, it behaves like a particle and a diffraction pattern emerges. A video which demonstrates the double-slit experiment can be seen on `https://upload.wikimedia.org/wikipedia/commons/transcoded/e/e4/Wave-particle_duality.ogv/Wave-particle_duality.ogv.480p.webm`. The double-slit experiment can provide the learner with an explanation of the observer dependency of the elementary particles. However, only Müller and Wiesner (2002) mentions this concept in his study, and it also does not appear in the Centraal Eindexamen of 2016 (Groenen et al., 2014).

The concept of superposition also has different cases. There are other properties of elementary particles which can be in superposition, for example the polarity of photons. Upon measurement, the polarisation value of a photon particle collapses to a certain value, but before this collapse it has all the different polarities at the same time. This gives way to the concept of entanglement, mentioned in some studies (Henriksen et al., 2014; Hobson, 2012; Kuttner & Rosenblum, 2010). Entanglement is a phenomenon which occurs between elementary particles, and it has as effect that the collapse of the different particles are interdependent of each other. This entanglement has two forms: boson entanglement and fermion entanglement. When the two particles are bosons, they always collapse to the same state on

observation, and when the two particles are fermions, they always collapse to each others opposite state.

Since the discovery of the phenomena occurring within quantum mechanics, scientists have debated fiercely about how to interpret these phenomena (Barnes, Garner, & Reid, 2004). Roughly speaking, the scientists could be divided intro two camps: the camp of the realists and the camp of the ontologists. The realists thought that there has to be something underneath quantum mechanics which could explain the strange phenomena of superposition and entanglement, whereas the ontologists thought that the phenomena of quantum mechanics stand on its own. The phenomena of entanglement played a huge role in this debate. The realists first thought that they could use entanglement to prove that there is a reality underneath quantum mechanics, but it eventually led mostly to evidence towards the camp of ontologists. One example of this is Bell's inequalities (Kuttner & Rosenblum, 2010; Müller & Wiesner, 2002), which is beyond the scope of this literature study to explain.

Henriksen et al. (2014) writes that there are three main differences between classical mechanics and quantum mechanics. The first difference is the fact that classical mechanics are deterministic and that quantum mechanics are probabilistic, also brought up by Levrini and Fantini (2013) and by Papaphotis and Tsaparlis (2008a). Classical mechanics rely heavily on deterministic causal effects, which can ultimately be explained. This is very apparent in systems of force, where everything moves according to certain laws, take for example the three Newtonian laws. Quantum mechanics however relies heavily on probabilistic models, where certain properties of certain elementary particles collapse to certain values according to probability functions.

A second difference between classical mechanics and quantum mechanics mentioned by Henriksen et al. (2014) is the locality of classical mechanics and the non-locality of quantum mechanics, also mentioned by Hobson (2012). On the scale of classical mechanics, it is possible to determine the exact position of an object, at least it is possible to do this on a significant scale. On a very small scale however, on the scale of quantum mechanics, the exact position of an object cannot be determined. There is an inherent uncertainty about the position of an object, which is very small and insignificant on the scale of classical mechanics, but quite significant on the scale of elementary particles. This is also true for the momentum of an elementary particle, which can be translated to the speed of an elementary particle. This uncertainty can be demonstrated by the uncertainty principle of Heisenberg (Henriksen et al., 2014; Müller & Wiesner, 2002; Velentzas, Halkia, & Skordoulis, 2007), which has as implications that neither the location nor the momentum of an elementary particle can be exactly known and that the more certain the location of an elementary particle is known, the less certain the momentum of an elementary particle can be known and vice versa.

Finally, Henriksen et al. (2014) mentions that classical mechanics are continuous and that quantum mechanics are discrete. This is because of the Planck length, which is the shortest measurable length. In classical mechanics, this length is very insignificant, and because of that the world looks continuous. A fully continuous world would mean that there is no tiniest unit, but that it is always possible to "go smaller". For example, if one had a plank of wood, it could be divided in half infinitesimally. However, on a quantum scale, this is not possible, because it is not possible to have something smaller than the Planck length.

Because of the inherent difficulty with quantum mechanics, some scientists have posited thought experiments, which allows the learner to make a mental model about the different concepts of quantum mechanics. The most famous thought experiment is that of Schrödinger's cat (Müller & Wiesner, 2002; Velentzas et al., 2007), where the life of a cat depends on the collapse of an elementary particle. When the cat is then observed, the state of the elementary is observed indirectly, which causes it to collapse and either kill the cat or let the cat live. This teaches the student about observer dependency, the way observations are linked to the random collapse of an elementary particle. Another thought experiment mentioned in the studied literature is the EPR paradox (Kuttner & Rosenblum, 2010; Müller & Wiesner, 2002; Velentzas et al., 2007), which can be used to teach the student about how entanglement is related to deep questions about the nature of quantum mechanics. This thought experiment however is related to the EPR experiment, which lies beyond the scope of this study to explain.

Finally, there are some studies which recommend certain mathematical approaches to quantum me-

chanics, namely the Schrödinger's equation (Müller & Wiesner, 2002; Singh et al., 2006), the Hermitian operator (Singh et al., 2006), the aforementioned Bell's inequalities (Kuttner & Rosenblum, 2010; Müller & Wiesner, 2002), the eigenvalue equation (Müller & Wiesner, 2002) and the DeBroglie energy levels (Dori et al., 2014; Gianino, 2008; McKagan et al., 2008). However, these mathematical approaches rely on a thorough conceptual understanding of quantum mechanics and are therefore not relevant to this study.

The topics relevant to teaching quantum mechanics can be summed up as the Rutherford-Bohr model of the Atom and elementary particles, the double-slit experiment, superposition, entanglement, the debate between realists and ontologists, the differences with classical mechanics, thought experiment and the mathematical side of quantum mechanics.

Analyses

The first step of the Generic Model by Plomp et al. (1992) (see figure 1 on page 4) is Analysis. Smith and Ragan (2005) give an elaborated description of how to perform these analyses for instructional design. They distinguish three different kinds of analysis: analysing the learner context, analysing the learner and analysing the learning task. The analysis of the learning context can provide the instructional needs and a description of the different factors influencing the instruction. The purpose of the learner analysis is the characterisation of the end user of the instruction, which is in this case the middle school students. Conducting a context analysis and a learner analysis is also in concurrence with the Generic Model, which states that from the very beginning in the analysis phase the designer should already start with the implementation of the instruction (Plomp et al., 1992). In the task analysis the test specifications are written, with which the content of the instruction can be established. These three analyses are executed in the following three sections. As also can be seen in the Generic Model, it is important to start with evaluating the product already in the analysis phase. Therefore, the final section of this chapter describes the evaluation of the test specifications with an expert.

Context Analysis

A learning task always takes place in a certain learning context. In this case this is the middle school. It entails not only the place, but also the temporal and social environment (Smith & Ragan, 2005). The analysis of the learning context can provide the instructional needs and a description of the different factors influencing the instruction. With the instructional needs, the designer can establish the main learning goals for the instruction. The description of the learning environment can provide the learning opportunities and constraints which have to be taken into account for the instruction.

Needs Assessment

The first goal of the need assessment is to investigate whether there exists a need for the instruction. Without a need, it would be a waste of resources to develop the instruction (Smith & Ragan, 2005). Next to this, it is conducted to better specify the need for the instruction. In the context of instruction, the assessment often results in a learning goal, which is the main goal of the instruction. This main goal is needed to continue the rest of the analyses, because all other analyses are conducted in respect to this goal. The goal can also be used to construct the summative evaluation, because when this goal is achieved, the instruction has proved to be successful.

Smith and Ragan (2005) identify three different models for the needs assessment, namely the problem model, the innovation model and the discrepancy model (see figure 3). The problem model is used when there exists a problem in the current system which has to be solved. As can be seen in figure 3, this model is to be used as a prerequisite for the other two models for assessment. With this model, it is determined whether there really is a problem, whether the cause of the problem is related to the performance of employees or to the achievement of learners, whether the solution to the problem is learning and whether instruction for these learning goals is currently offered. After the problem model, the needs assessment splits into the two other models. The innovation model is used when there is a new learning goal that the learners should achieve, and the discrepancy model is used when the already available instruction is

not adequate to achieve the learning goal. The designer should choose one of these models for his needs assessment.

A. Problem model

Figure 3: The three sides of needs assessment (Smith & Ragan, 2005)

In the case of the instruction which will be constructed for this assignment, at first the problem model will be used to investigate the problem, and which of the two follow-up models should be used for the needs assessment.

The problem

In the Netherlands, quantum mechanics always used to be a topic which schools themselves could choose to teach or not to teach. The only skill students had to know for the Centraal Eindexamen (the national central exams at the end of high school) which comes close to quantum mechanics is to elucidate the photoelectric effect and the wave-particle duality, mentioned within point 20 under subdomain E3 (Laan, 2013). However, one of the changes in the Centraal Eindexamen of 2016 was the addition of domain F1, which is called Quantum world (Groenen et al., 2014). For this subdomain the candidate has to be able to apply the wave-particle duality and the uncertainty principle of Heisenberg, and to explain the quantization of energy levels in some examples with a simple quantum physical model. In order to give all candidates a chance of passing this subdomain, schools have to alter their programs in order to meet the expectations of the Centraal Eindexamen.

However, when searching the internet using the search machine Google concerning the implementation of quantum mechanics in Dutch high schools, the quantity and the quality of the results are very low. There are also no results to be found in the Dutch papers. An example is the Dutch site http://www.quantumuniverse.nl/, where teachers can find a small amount of brief courses on fundamental quantum mechanics, and where the forums are very quiet with only 5 discussions, of which 4 are just started threads from the site administrator.

Upon finding this information, an expert was consulted to confirm this conjecture. The expert was researching the implementation of quantum mechanics on in Dutch secondary education, and she also a first degree physics teacher. She stated that within her school there were no initiatives to bring this topic in their classrooms, and that their school was no exception as well.

The fact that next year domain F1 has to be fully implemented and taught to all vwo students who chose physics as an examination subject is therefore slowly turning into a sword of Damocles. This stresses the urgency for the development of new course material. This is an example of extrinsic motivation. However, is there also intrinsic motivation to teach quantum mechanics on high schools? First of

all, there is no article which claimed that quantum mechanics should not be taught on high schools. On the other hand there are but a few authors who did have some arguments in favour of teaching. Müller and Wiesner (2002) and Henriksen et al. (2014) state that quantum mechanics shapes our world view and that educated citizens should therefore become acquainted with the topic. It is also regarded as fundamental and should therefore be taught (Henriksen et al., 2014; Hobson, 2012). Finally, Erduran (2005) states that the teaching of philosophical themes in science education has been advocated for several decades, and quantum mechanics is one of these themes.

Because it involves new instruction, the innovation model will be used for the second part of this needs assessment.

The innovation

The nature of the innovation lies within the change of the Centraal Eindexamen of 2016 in respect to the Centraal Eindexamen of 2015. The new additions within the domain Kwantumwereld outline the new goals of physics education in the Netherlands, and will be the ultimate goals for the students to achieve, and therefore be the ultimate learning goals for the students to achieve. This results in the following learning goals (Groenen et al., 2014):

The candidate can:

- describe quantum phenomena in terms of the enclosure of a particle:

  - estimate whether quantum phenomena are to expected by comparing the debroglie-wavelength with the order of largeness of the enclosure of the particle;

  - apply the uncertainty principle of Heisenberg;

  - describe the quantum model of the hydrogen atom and calculate the possible energies of the hydrogen atom;

  - describe the quantum model of a particle in a one-dimensional energy well and calculate the possible energies of the particle;

  - Bohr radius, zero-point energy.

- describe the quantum-tunnel effect with a simple model and indicate how the chance of tunneling depends on the mass of the particle and the height and width of the energy-barrier,

  - minimal in the contexts of: Scanning Tunneling Microscope, alpha-decay.

These goals confirm what the literature describes about the current appliance of quantum mechanics teaching within secondary, namely that often quantum mechanics is introduced with great emphasis on learning and practising algorithmic skills (Papaphotis & Tsaparlis, 2008a, 2008b). However, it is also found that high school students show higher interest in the conceptual aspects than the algorithmic aspects (Papaphotis & Tsaparlis, 2008a, 2008b; Levrini & Fantini, 2013). When focusing on the conceptual aspects, it engages students (Henriksen et al., 2014) and students start asking fundamental questions (McKagan et al., 2008). Furthermore, mathematical oriented approaches might be more common, however, quantum mechanics is regarded to be mathematically challenging (Gianino, 2008; McKagan et al., 2008), and most high school students lack proper background in mathematics at the required level (Dori et al., 2014). Because the usual focus on the algorithmic aspects, students often do not learn what instructors want them to learn (Asikainen & Hirvonen, 2014; McKagan et al., 2008), and improved student learning is possible by shifting the focus to conceptual understanding (McKagan et al., 2008).

Therefore, the aim of this instruction is to focus on a conceptual approach instead of a mathematical approach. Then, after the students have a sufficiently conceptual understanding of the material, the concurrent instructions in the curriculum can emphasise the goals stated by the Centraal Eindexamen of 2016, which adds the mathematical layer on top of the conceptual layer and can deepen the understanding of quantum mechanics. In summary, the main goal of this instruction is *to provide the student with a conceptual understanding of the different phenomena occurring in the realm of quantum mechanics.*

Learning Environment

The learning environment description is the other major component of the learning context analysis (Smith & Ragan, 2005). The description contains information of all the external factors influencing the instruction. These are the mediators of the instruction, the already existing curricula which takes place in the environment, the available equipment available on the location of the instruction, the characteristics of the facilities at the location of the instruction, the characteristics of the organisation in which the instruction will take place, and the philosophies and taboos of the larger community in which this organisation exists.

A current first degree teacher training (*Cursussen Leraar Natuurkunde (Professional Master) Tilburg — Fontys*, 2015) does encompass quantum mechanics, so teachers which had this training should be familiar to the domain. However, Asikainen and Hirvonen (2014) states that teachers often still possess misconceptions about quantum mechanics, which are comparable to the misconceptions of the students themselves. These misconceptions will be discussed in the learner analysis section on page 12. Experienced teachers who are teaching modern physics are more capable of teaching quantum mechanics (Asikainen & Hirvonen, 2014).

When implementing the instruction, the placing within the already existing curriculum is also important, because the instruction depends on prerequisites from other elements of the curriculum. The main prerequisite is knowledge of Bohr his atom model, because the different particles within this model are the particles on which quantum mechanics apply. This knowledge is taught in Domain E from the centraal eindexamen (Groenen et al., 2014), and because of the prerequisite, it is of upmost importance that this instruction is placed after Domain E in the existing curriculum. As already described in the needs assessment, the conceptual instruction could be followed by instruction of the mathematical aspects of quantum mechanics. Also, various experiments could be taught, which demonstrate the discovery of the various concepts introduced in the instruction and explain the different principles between the concepts. This could for example be the EPR experiment (Kuttner & Rosenblum, 2010; Müller & Wiesner, 2002; Velentzas et al., 2007), which could lead to critical assessment of the realist and ontologist perceptions on quantum mechanics.

Another important aspect of the instructional environment is the method of delivery (Smith & Ragan, 2005). The recommendations for different aspects of the medium used for the delivery of the instruction entail interactivity, visualisation, the combination of different modes of representation, and the use of computation. By making it able to interact with the medium, it is possible for students to experiment with the different concepts, which gives way to inquiry learning (Adegoke, 2012; Asikainen & Hirvonen, 2014; Dori et al., 2014; McKagan et al., 2008). Visualisation is a powerful tool, and can make the matter less abstract (Dori et al., 2014; Henriksen et al., 2014; McKagan et al., 2008). It also is easier to build mental models of quantum mechanics. Levrini and Fantini (2013) warns however against the use of oversimplified visualisations, because pictures are extremely partial and can be misleading. Therefore, it is important that the visualisation does not entail any unnecessary simplified representation of the matter. This combined with other different modes of representation, for example a textual description of the concept, makes it possible for the student to complete their mental model (Dori et al., 2014). Finally, the use of computation makes it possible to take away the mathematical complexity from quantum mechanics, making way for a purely conceptual approach (Barnes et al., 2004; McKagan et al., 2008; Velentzas et al., 2007).

One could combine all these aspects by using simulations, which is also recommended by McKagan et al. (2008). However, teachers often prefer traditional lectures, because that is easier to implement in their classroom (Adegoke, 2012). This difficulty has to be overcome if quantum mechanics is to be implemented successfully in the classroom. Furthermore, it has to be investigated whether the sufficient hardware is available in the learning environment.

Finally, it is important to investigate whether the instruction fits in with the mission and vision of the school, and also the philosophies and taboos that the teachers hold. Therefore, it is advised to find these discrepancies by the means of interviews, in which the school board is asked about their mission and vision, and the teachers about their personal believes in regard to quantum mechanics.

In any case, this assignment does not look into the implementation of the instruction yet, so these factors have to be looked closer at when embedding the instruction in the context of a specific school.

Learner Analysis

The second analysis is that of the learners (Smith & Ragan, 2005). The purpose of this analysis is the characterisation of the end user of the instruction, which is in this case the students of secondary education in the Netherlands, mostly ranging from age 17 to 18. This analysis will focus itself on the general misconceptions held by members of this demographic.

When developing an instruction, it is important to consider the already available conceptions on the topic. In quantum mechanics, these preconceived models often prove to be incorrect (Asikainen & Hirvonen, 2014; Papaphotis & Tsaparlis, 2008b; Thacker, 2003). This partly comes from the nature of quantum theory (Papaphotis & Tsaparlis, 2008b), but also partly from textbooks and instruction (Hubber, 2006; Papaphotis & Tsaparlis, 2008b). The problems often stem from depending on outdated deterministic or realist models (Hubber, 2006; Papaphotis & Tsaparlis, 2008a, 2008b), an often mentioned example of this is that students often mix up the deterministic planetary model with the indeterministic atom model (Dori et al., 2014; Henriksen et al., 2014; Hubber, 2006; Müller & Wiesner, 2002; Papaphotis & Tsaparlis, 2008a, 2008b). McKagan et al. (2008) also mentions that it is difficult for students to recognise the scale in which quantum mechanics take place.

Thacker (2003) describes how much knowledge of students consists out of memorised facts, for example that light is a wave and electrons are particles. When the student then is confronted with new or different information from what they know, they develop new memorised facts instead of creating the right model. This then results in models consistent with fragmented models of microscopic processes, which are often incorrect but self-consistent with a certain experiment (Hubber, 2006; Thacker, 2003). When the student cannot model the fragments anymore, this can result in deep skepticism towards quantum mechanics (Barnes et al., 2004; Henriksen et al., 2014; Levrini & Fantini, 2013). Müller and Wiesner (2002) has created a long list of exact conceptions students hold about microscopic phenomena, which are too detailed to enlist fully in this article.

The misconceptions described in the studied articles mostly are measured in relation to the teaching material which is currently employed within schools or universities. This material is already described in the preface, and can generally described as misconceptions in relation to the Rutherford-Bohr model of the atom, and in relation to experiments like the double-slit experiment. However, it would be interesting to investigate the misconceptions in relation to the fundamental concepts of quantum mechanics themselves, for example in relation to elementary particles, superposition, random collapse of the probability function and entanglement. This would give greater insight in the conceptual understanding of students. However, no research has been conducted in these areas yet.

Task Analysis

The final step is analysing the learning task (Smith & Ragan, 2005). In this analysis the goals from the needs assessment during the analysis of the learning context have to be translated to test specifications, with which the content of the instruction can be established. To establish these test specifications, it is important to investigate the learning goal first. This will be followed by a description of the various strategies mentioned within the literature for how to teach quantum mechanics. The next step is the prerequisite analysis, which is a description of the prerequisite knowledge the students have to possess in order to partake in this instruction in relation to the content, described in the section *Topics mentioned in literature* on page 4. After that, the learning objectives can be written, which model the outline of the instruction. Finally, the test specifications are formulated, which can be used to measure the student his understanding of quantum mechanics after the instruction.

Learning goal

The main learning goal as specified needs assessment using the innovation model from page 10, the instruction will pursue provide the student with a conceptual understanding of the different phenomena occurring in the realm of quantum mechanics. This goal will be the main guideline for the information-processing analysis. However, it will not be possible to provide the students with a complete conceptual understanding of quantum mechanics. First of all, not even the scientific community has a full grasp of quantum mechanics, because it is a relatively new and still developing subject area. Second to that, physics students in secondary education do not have the time needed to gain at least the complete conceptual understanding of quantum mechanics available from the scientific community. Finally, there is not enough time to design an instruction which achieves a complete conceptual understanding, and the time available for this instruction is also limited. Therefore a choice has to be made in the different topics being taught.

Furthermore, there exists a consensus within the studied articles that quantum mechanics is a difficult topic, and this is also a consensus among educators (Gianino, 2008; Papaphotis & Tsaparlis, 2008a, 2008b). There are a couple of reasons mentioned within the articles to explain this topical difficulty. A couple of sources state that quantum mechanics is a very counter intuitive topic (Henriksen et al., 2014; Levrini & Fantini, 2013; McKagan et al., 2008; Singh et al., 2006), because it contradicts many aspects of our daily experience, like locality or determinism. Quantum mechanics is also considered to be a very abstract topic (Barnes et al., 2004; Gianino, 2008; McKagan et al., 2008; Papaphotis & Tsaparlis, 2008a; Singh, 2006). Because quantum mechanics differs a lot from our everyday experiences and because of its abstractness, it is difficult for learners to visualise the concepts of quantum mechanics (Henriksen et al., 2014; McKagan et al., 2008). Another factor contributing to the difficulty of quantum mechanics is that it is mathematically challenging (Gianino, 2008; McKagan et al., 2008), it involves mathematical skills that most high school students — even vwo 6 students — do not possess. Because of the difficulties stemming from teaching quantum mechanics, it would be better to teach concentrate on teaching a few topics of quantum mechanics in a way that it can be understood, rather than trying to teach as much as possible in the limited time available.

Teaching strategies

The literature provides a lot of strategies which can be used to teach quantum mechanics. There are recommendations for which content to use. Some of the strategies focus on meta-cognitive aspects. Finally, there are a few frameworks which can be used to teach modern physics.

Some of the content-related strategies emphasise the importance of embedding the instruction in real-world contexts, for they help with understanding (McKagan et al., 2008; Thacker, 2003; Dori et al., 2014) and help appreciate the relevance of quantum mechanics (Barnes et al., 2004; Henriksen et al., 2014; McKagan et al., 2008). Furthermore, Thacker (2003) suggests introducing microscopic processes as an integral part of a study of electricity and magnetism. This could help demystify the topic, which also would contribute towards a better understanding (Barnes et al., 2004; Müller & Wiesner, 2002). An example of how this can be done is by using the e/m experiment, where the electromagnetic effect is demonstrated by the properties of electrons. Furthermore, the language of physics is important (Henriksen et al., 2014), and should be used carefully (McKagan et al., 2008). The consulted articles all recommend a conceptual approach above a mathematical-oriented approach. Mathematical-oriented approaches might be more common, but most high school students lack proper background in mathematics at the required level (Dori et al., 2014). Barnes et al. (2004) and Henriksen et al. (2014) believe teaching through history of science is believed to be constructive.

Papaphotis and Tsaparlis (2008a) states that critical thinking skills are crucial for understanding quantum mechanics, because students have to investigate the new material in a critical way to build the correct mental models. Active learning also contributes to investigation of the material. Because the students easily build misconception, right feedback is vital to prevent misconceptions and can stimulate students to build correct mental models. Finally, Papaphotis and Tsaparlis (2008a) suggests collabora-

tion, which is also suggested by Adegoke (2012) and Barnes et al. (2004). Collaboration could lead to peers providing each other with critical questions and feedback. Especially in the case of female students this could benefit to learning quantum mechanics (Adegoke, 2012).

The frameworks mentioned by different authors are directly or very similar to thought experiments (Asikainen & Hirvonen, 2014; Erduran, 2005; Levrini & Fantini, 2013; Velentzas et al., 2007). Asikainen describes the most elaborated framework for a well-conducted thought experiment, which includes the steps question and general assumptions, description of the features of the system, performance of the thought experiment itself, extraction of the results and drawing conclusions. Erduran (2005) and Levrini and Fantini (2013) also describe a framework, but the steps they mention already overlap with those of Asikainen and Hirvonen (2014).

Prerequisite analysis

Furthermore, it is important to look at the pre-existing knowledge about quantum mechanics of the students. According to the Centraal Eindexamen 2016 (Groenen et al., 2014), the candidate should already have learned the Rutherford-Bohr Model of the Atom, and this is earlier already specified as prerequisite of the instruction. This means that the students have knowledge of at least two elementary particles, namely the photon and the electron, and their place within the atom. They also should know about the nucleus and the shell of an atom, and about protons and neutrons. Furthermore, it could be that the students already learned about the double-slit experiment (Laan, 2013), although it would be better if the students get instruction about this experiment after this instruction. This is because then they will be familiar with terms like superposition and observer dependency, which are necessary to fully comprehend the double-slit experiment. Finally, the students could have learned about some of the concepts of quantum mechanics outside of the standard curriculum, like for example via science magazines. However, for this instruction it will be presumed that they have no knowledge of these concepts, for the reason that they have not been taught in the standard curriculum.

Learning objectives

Zeilinger (2005) provides a clear conceptual understanding of some of the phenomena occurring within quantum mechanics. The first step was to summarise this book, so learning objectives could be extracted from this book. This summary was then translated into a unidirectional dependency graph, which means that the different elements of the summary were projected into nodes, and the edges between these elements displayed a logical order of teaching the different elements. These elements were then extended by using the taxonomy of Bloom (Bloom, Englehart, Furst, Hill, & Hrathwohl, 1956) (see figure 4). This made the different elements more defined and also displayed better which skills the student should master in order to achieve all the learning objectives. The graph however cannot be easily displayed on a4 format, this is why it was translated to a table which displays each topics and its prerequisite. This table is included in the appendix *Learning Objectives* on page 28.

Figure 4: The taxonomy of Bloom (Bloom et al., 1956)

36 of the learning objectives fall into the knowledge domain within the taxonomy of Bloom, 15 learning objectives fall into the comprehension domain, 3 learning objectives into the application domain, and 2 objectives into the analysis domain. The reason why most of the learning objectives fall into the knowledge domain is because the topic of quantum mechanics is mainly new to the high school student. Because of this, the student first has to learn a lot of new terms and definitions before he can actually comprehend quantum mechanics, for having knowledge is a prerequisite for an eventual understanding of the concepts. However, the main goal of the instruction is comprehension of the concepts, even if there are only 15 learning objectives within this domain, for comprehension leads to conceptual understanding. Application and Analysis is less relevant, this would be more relevant to principle learning, which would be important for learning the algorithmic aspects of quantum mechanics. Any higher domains cannot be reached, for they require mastery of the application and analysis domain. However, this could be achieved next in the curriculum.

Furthermore, the learning objectives are also categorised by topical domains, which are Applications of Quantum MechanicsPre-knowledge, Elementary Particles, Classical Communication, Observation Dependency, Realism and Ontology, Superposition, Entanglement, Uncertainty Principle of Heisenberg and Teleportation. Each instruction should start with a reason why the student might be interested in learning the subject. This is established by stating the relevance of quantum mechanics by enlisting the different applications of quantum mechanics.

When the student knows why he should learn about quantum mechanics, the instruction continues with what the student already might know, because this activates the brain regions containing this information, making it possible for the student to connect the new knowledge with what he already knows. As already stated in the *Learner Analysis* on page 12, the student should already be familiar with the Rutherford-Bohr model of the Atom. This thereupon leads easily into the next domain, namely that of the *Elementary Particles*. Here, the student learns about the particles which demonstrate quantum behaviour.

When learned about the existence of elementary particles, the student can also learn about *Classical Communication*. This needs understanding about elementary particles, for these are the particles which are used to conduct classical communication. Classical communication is relevant to quantum mechanics for two reasons. The first reason is that it connects the mysterious elementary particles to the daily experience. Everyone uses the internet everyday, which uses classical communication. Therefore, the student should already be familiar with the concept. The second reason is that it is a prerequisite for learning about quantum teleportation, which will be explained later on in this section.

It might strike the reader that the famous double-slit experiment is not entailed within the learning objectives. There are two reasons for not including this experiment in the instruction. First of all, the double-slit is already part of the standard curriculum (Laan, 2013), so there is no need to develop educational material for it. The second and more important reason is that the double-slit experiment can be used to demonstrate the relations between the concepts within quantum mechanics, but to fully comprehend these relations the student should first learn the concepts themselves. The term "wave-particle duality" also does not hold any meaning if the student is still unaware of the probabilistic nature of quantum mechanics. However, this instruction could be followed up by instruction covering the double-slit experiment.

The first concept underlying the double-slit experiment is *Observer Dependency*. This is also the first time the student is confronted by one of the more counterintuitive concepts of quantum mechanics. It teaches the student that quantum mechanics cannot only be explained by pure determinism and causal relations, but that quantum mechanics has a probabilistic nature. The student learns this concept by examining what happens if a property of an elementary particle is observed.

The probabilistic nature of quantum mechanics is further explored with the debate between *Realism and Ontology*. This section starts with the realist interpretation of quantum mechanics, which entails that there must be an explanation for the seemingly random behaviour of elementary particles, it is only not possible to measure with the current level of technology. The ontologist explanation however is that there is no explanation of the random behaviour of elementary particles, and it is the inherent nature of

quantum mechanics. With this debate, the student is challenged to evaluate his own believes about the nature of physics. Most likely, he will start at the realistic interpretation, and this is a natural way to teach the student to let him think about the other interpretation.

After this historical and philosophical intermezzo, the student is then introduced to the second concept underlying the double-slit experiment, which is *Superposition*. This concept can only be fully understood when the student is aware of the ontologist interpretation, for it involves comprehension of the fact that quantum mechanics is truly probabilistic. This is because the student has to understand that before measurement, the elementary particle has no true defined state yet, and is in all different states at the same time.

The next concept the student is introduced to is the concept of *Entanglement*, which teaches the student that the behaviour of elementary particles is not entirely random. Although the collapse of two entangled particles is fundamentally random, the two particles are interdependent. This means that the particles can either collapse to the same state on observation in the case of bosons, or that the particles collapse to each others opposite state in the case of fermions. This concept could be used to develop further in the EPR experiment, which was an attempt of realist interpreters to prove that there is an underlying explanation of quantum mechanics, but ultimately led to more evidence for the ontologist interpretation. The EPR experiment makes way for topics like the hypothesis of locality, the hidden variable hypothesis and Bell's inequalities. However, this instruction will limit itself to only explaining the concept of entanglement itself, for the EPR experiment is quite sizeable and fits in an entire separate instruction.

The next topic is the *Uncertainty Principle of Heisenberg*, which stands a bit on its own. It can be used to teach the student about the non-locality of quantum mechanics, because it demonstrates that the specific location or the specific speed of an elementary particle cannot be known exactly, and there is an inherent uncertainty about these two variables. Furthermore, it also has a different realist and ontologist interpretation. To understand the ontologist interpretation, it is important that the student is already taught about superposition. The reason why this topic is included in the instruction is because it is another counterintuitive but central concept within quantum mechanics, and it is regarded as important for teaching by Henriksen et al. (2014). The principle is also one of the few topics directly included in the Centraal Eindexamen 2016 (Groenen et al., 2014) within this instruction. Finally, it also ties in with the next domain.

This last domain is *Teleportation*, which teaches the student about the inner workings of entanglement in the real world. This topic will first explain what it is not, but is easily confused with, namely the teleportation used in science-fiction. To argue that this is actually not possible, the student has to use the uncertainty principle of Heisenberg. This is also the main reason that it is included, all the topics taught before have to be used in the Teleportation experiment, it can therefore serve as a conclusion of the instruction by letting the student go through all the different domains one more time.

Test specifications

Every learning objective has to contain a description of the terminal behaviour or actions that will demonstrate learning, a description of the conditions of demonstration of that action and a description of the standard or criterion (Smith & Ragan, 2005). The description of the terminal behaviour is already specified in the *Learning Objectives* appendix on page 28. If a condition is present for the execution of the learning objective, this is already specified as well, most of the time using the word "given". However, the standards or criteria for measuring specifying how the learning objective has to be achieved by the student are not yet included in the table.

Thinking about the criteria or standards on beforehand can be very useful for the designer, for he then also has an idea for how specific the instruction text should be written. Therefore, a list with the standards given a certain learning objective has also been included, in the *Learning Objectives and Standards* appendix on page 32. The objectives are indicated by numbers, and the standards by letters. The standards are formulated as the behaviour the student should display upon measurement of the learning objective. Some of the learning objectives are formulated in such a way that further specification

by standards is not necessary, for example learning objective 2: "The student can state that everything we observe exists out of molecules".

Evaluation

To ensure that the instruction would not contain errors on the subject of quantum mechanics, an expert was consulted to look at the learning objectives and criteria. The expert consulted is a part-time PhD student which researched the implementation of quantum mechanics in the curriculum of Dutch secondary education, and a part-time first grade physics teacher. The same expert was also consulted during the Needs Assessment (see page 9). For this evaluation, a focus group evaluation was conducted (Nieveen, Folmer, & Vielgen, 2012). With the walkthrough, the design proposal was checked on factual errors, using an interview with the expert. The result of the walkthrough would be an evaluation of the relevancy and the consistency of the product (see the *Evaluation Matchboard* appendix on page 39), which entailed going through all the learning objectives and standards. Although she gave feedback on the specific formulation of some of the learning objectives or criteria, she did not find any factual errors. She did give some feedback on which learning objectives might not be considered to be pre-knowledge, such as learning objective 8: "The student can state the value of the reduced Planck Constant".

Design

The second phase of the Generic Model is the Design phase (Plomp et al., 1992) (see figure 1 on page 4). In this phase, the results from the Analysis phase are translated into design principles and an outline of the instruction that can be used directly for the development of the instruction. This chapter will start out with the relevant conclusions which are drawn from the analysis phase in order to summarise the results and translate them to the context of the product, the instruction. After the summary, an active choice still has to be made about the medium to use for delivering the instruction. This will be done in the next section according to the results relevant to the choice of the medium. The remaining conclusions will then be used to formulate design principles, which facilitate the concrete use of the results from the analysis. Furthermore, design principles can be used after the development for a screening evaluation (Nieveen et al., 2012) (see the Evaluation Matchboard appendix on page 39). Finally, the different learning objectives formulated during the task analysis have to be ordered into an outline of the instruction, so the designer can directly translate them into text or other means of instruction.

Conclusions from the Analysis

There is a wide variety of topics available for quantum mechanics education. It is advisable to start any instruction with the Rutherford-Bohr model of the Atom and its relation to elementary particles, for this is the scale on which quantum mechanics take place. A next step could be introducing the different phenomena which happen on a quantum scale, such as superposition, observer dependency and entanglement. One way to introduce superposition and observer dependency is by using the double-slit experiment. This also teaches about the wave-particle duality of elementary particles. Furthermore, the students could be taught the differences between classical mechanics and quantum mechanics, which are determinism and probabilism, locality and non-locality, and continuous and discrete distances. One way to highlight the strangeness of quantum mechanics is by using the debate between realism and ontology, this emphasises that quantum mechanics are different than a student might think at first. There are also famous thought experiments, for example Schrödinger's cat, which could be used to create different mental models with the students. Finally, there are some mathematical approaches to quantum mechanics, however there is a consensus within educators that a conceptual approach is more effective.

Not a lot of studies write why quantum mechanics should be taught on secondary education. There is however no study which claimed it should not be taught. Reasons why it should be taught are that educated citizens should be acquainted with quantum mechanics and that it is regarded fundamental.

Quantum mechanics is regarded to be a difficult topic, because it is counter intuitive, it contradicts daily experienced locality and determinism, it is regarded to be abstract, it is difficult to visualise and it is mathematically challenging.

Misconceptions about quantum mechanics can arise out of the nature of quantum theory, but also from textbooks and previous instruction. It is important that the information handed to the students are not simplified, because this can lead to misconceptions. Problems also often originate from outdated deterministic or realist models. The students therefore have to be confronted with the fact that their own models cannot be applied in the context of quantum mechanics. Students also find it difficult to recognise the scale in which quantum mechanics take place. Hence, the students should be informed that quantum mechanics take place on the scale of elementary particles. Knowledge of students often consists out

of memorised facts, and when confronted with new or different information, new facts are memorised instead of creating right models. This results in fragmented incorrect but self-consistent models. Because of this, the instruction should provide the student with a coherent description of quantum mechanics.

The literature prescribes different strategies for teaching quantum mechanics, namely recommendations for content, aspects of the medium to use, meta-cognitive aspects and a framework. Content related strategies entail embedding the instruction in real-world contexts, letting the student appreciate the relevance of quantum mechanics, introducing microscopic processes as an integral part of electricity and magnetism, using the language of physics, and using an conceptual approach instead of a mathematical approach. The medium used should be interactive, it should visualise the concepts, it should make use of different modes of representation, and it should make use of computation. It should be noted however that one should be careful with visualisation, for this can easily lead to misconceptions. The use of simulations are suggested. Furthermore, critical thinking skills, active learning, feedback and collaborative learning could contribute to students building the correct mental models of how quantum mechanics work. Finally, thought experiments are regarded to be very effective for learning quantum mechanics, this entails questions and general assumptions, a description of the features of the system, performance of the thought experiment itself, extraction of the results and drawing conclusions.

In secondary education, instructions often emphasise learning and practising algorithmic skills. This however clashes with the recommendations from the literature, which advocates an emphasis on conceptual understanding. It is also found that students show higher interest in conceptual aspects than the algorithmic aspects. Conceptual approaches engage students and probes the student to ask fundamental questions. Because of the focus on algorithmic aspects, students currently often do not learn what is intended by the developer of the instruction, improved student learning might be possible by shifting the focus to conceptual understanding. This is why the learning objectives are all targeted towards a better conceptual understanding of the student, without having to rely on complex mathematics or experiments. The different domains which will be taught are Applications of Quantum Mechanics, Pre-knowledge, Elementary Particles, Classical Communication, Observation Dependency, Realism and Ontology, Superposition, Entanglement, and the Uncertainty Principle of Heisenberg. These Domains will then be combined within the experiment of Quantum Teleportation.

The Medium

Requirements for the Medium

The analysis phase provides a couple of recommendations or requirements for the medium used to deliver the instruction:

- The medium should allow interaction;

- The medium should be able to visualise the content;

- The medium should be able to use different modes of representation;

- The medium should take away the mathematical difficulty, for example by using computation.

As already stated within the Analysis chapter, these requirements cannot be reached by relying on traditional means of instruction such as text books, pictures and lectures from a teacher. It is not possible for a student to interact with the contents of a book or with pictures, and lectures from a teacher often entail the teacher standing in front of the class and telling the students about quantum mechanics in a supplantive way. Of course it would be possible for a student to ask questions, but more room for interaction is needed to be even more successful, and the teacher has not enough time to facilitate interactive learning to every student in a differentiated way.

One of the suggestions from the studied literature entailed using simulations, because these could provide all of the requirements. However, simulations often are related to certain experiments, for example by simulating the double-slit experiment and providing the student with data such as visuals or

numerals. This is in contrast with the requirements from the task analysis, which specify that rather than teaching by means of experiments, the instruction should first teach about the concepts themselves in a more direct way.

Another way of teaching quantum mechanics via electronic media would be to deliver the instruction by using game-based learning. game-based learning refers to the use of games for educational purposes. Relatively speaking, only little research has been carried out to investigate game-based learning, but recent reviews revealed a potential of games being an effective tool for instruction, "even more effective than conventional instruction" (Wouters & van Oostendorp, 2012, p. 1). In order for game-based learning to take place, the game has to supersede sole entertainment purposes by having an educational value, so that the player learns something about the real world by playing the game. Examples of these games are historical games like Assassin's Creed, Age of Empires or Valiant Hearts, in which the player is exposed to certain historical events or figures, or puzzle games like Portal, in which the player develops an understanding about physics. However. these games are often only intended for entertainment purposes and are only tangentially fit for educational purposes. Most research within game-based learning is conducted on games which are specifically designed to have an educational value. These games often contain explicit formulated learning goals which can be tested by pre- and post-testing. They are designed with the primary goal of education rather than entertainment, and are therefore a subset of Serious Games. This term refers to the set of games designed for a primary goal other than pure entertainment. This goal can be teaching, but also raising awareness about a certain topic or persuading the player to believe something.

One disadvantage for using game-based learning is that it takes a lot of time to develop games in general, and there is no game available on the market which can be directly used for teaching the learning objectives specified in the Task Analysis. However, there already exists an easy to use engine which can be used to teach quantum mechanics on a conceptual level. This engine is qCraft, developed as a modification for the game Minecraft.

Minecraft

Minecraft is a game initiated by Marcus Persson (also known as Notch), further developed by Mojang AB, and now property of Microsoft. It is available for Windows, OS X, Linux, Java platform, Java applet, Android, iOS, Windows Phone, Amazon Fire TV, Xbox 360, Xbox One, Rasberry Pi, Playstation 3, Playstation 4, and the Playstation Vita. There are differences between the versions, and the main version is being developed for the PC only. It is the third-best sold game of all time, and the best sold game for PC with 70 million sold copies at the time of writing.

The game is a first person sandbox survival game, available for playing both in single-player and multiplayer. In the game, the player walks around in a procedurally generated world fully made out of blocks which are one cubic meter in size (see figure 5). The player can control his avatar similar to other first person games, by using the WASD keys for moving and using the mouse for looking around. The unique feature of the game is that the player can place and remove blocks for himself. Each block also has a specific block-type, which gives it a certain appearance and sometimes also a special physical attribute. For example, blocks with the sand block-type always fall down unless being on top of another block. Some blocks also emit a signal, which can be used to trigger certain commands or for example open a door to a room.

Minecraft is an easy to use engine for delivering instructions as well. It has support for signs with text and pictures which can be placed in the world, books the player can open and read, and commands which can be used to move the player to certain areas in the world or giving the player certain items. There is even a special version of Minecraft for use in classrooms, which is called MinecraftEdu (`1minecraftedu.com`), and it is used by teachers around the world.

Figure 5: A screenshot within the game Minceraft, where a player created structure is placed within the procedurally generated terrain.

qCraft

In order to use Minecraft to students, a certain modification of the game has to be installed. This modification is also available for MinecraftEdu, which makes it easier to use for teachers. The mod introduces special types of blocks into the game, which are Observer Dependent Blocks and Quantum Blocks. The Quantum Blocks behave in a way similar to elementary particles, which is that every time a Quantum Block is observed, it first is in a green "in-between state" and then takes on one of two block-types at random. An observation takes place every time the player looks at it. For example, if a player looks at a Quantum Block, it can collapse 50% of the time to a gold block-type and 50% of the time to a diamond block-type. The green colour the block takes on before collapsing to either of the two block-types represents the fact that the block is in superposition before measurement.

An Observer Dependent Block has one difference with the Quantum Block, which is that its block-type does not collapse at random, but that the collapse is determined by the angle the block is observed from. This block is added to the game in order to learn the student observer dependency, without also having to explain the random collapse.

Quantum Blocks can also be entangled with each other. This means that every time one of the entangled blocks is observed, all Quantum Blocks their block-type collapse to the same value. Only the boson type of entanglement is implemented yet.

Furthermore, the modification adds a couple of tools to interact with the blocks. It adds two types of goggles, which are the Quantum Goggles and the Anti-Observation Goggles. The Quantum Goggles highlight all of the special blocks by giving it a fluorescent green colour, so the player can easily identify all of the qCraft blocks present around him. The Anti-Observation Goggles prevent the player from generating observations to the qCraft blocks. Another tool provided by qCraft is the Automatic Observer, which can be activated to generate an observation to the qCraft block attached to the Automatic Observer.

Possibilities of using qCraft for teaching quantum mechanics

First of all, qCraft meets all the requirements earlier specified based on recommendations by the studied literature. It is interactive, because the student can move around in the world instead of being a passive observer. This allows for walking around the qCraft block, observing them in different ways, trying to find the patterns, and even trigger mechanisms based on the collapse of the block type of the qCraft block. Furthermore, the mechanics introduced by qCraft make quantum mechanics visible. The player can see what observer dependency means for a Quantum Block, and how boson type entanglement works between multiple Quantum Blocks. One of the advantages of using Quantum Blocks is that they already demonstrate the correct behaviour, so there is no need to rely on thought experiments such as Schrödinger's Cat. The student can observe the behaviour directly, and can therefore experiment with it without needing an external analogy. Minecraft also allows for different modes of representation. Next to the qCraft blocks, it is also possible to hang pictures in the world, to place text on signs, and to add books into the world. This allows for explaining the players the specific behaviour of the blocks and to transfer this to real quantum mechanics by comparing it to the behaviour of elementary particles. Additionally, the only mathematical comprehension the player needs in order to understand the behaviour of the blocks is that the block types collapse to a random state, without having to comprehend difficult probability functions which require mathematical skills which students of secondary education do not possess.

Finally, all of the learning objectives can be achieved using only qCraft, without having to rely on more external sources of delivery. One disadvantages of using qCraft is that it is not possible to visualise the double-slit experiment for example. However, it was already stated in the Analysis phase that this experiment would occur only after a conceptual understanding was achieved with the student, and that it could be introduced later on in the curriculum. The debate between realism and ontology can be described within a book. The uncertainty principle of Heisenberg cannot be emulated within qCraft. However, it is possible to display the different variables within the formula on images situated in the world. This way the student can learn the formula by means of discovery learning. The experiment of teleportation is visualisable in qCraft, for the only concepts needed are random collapse and entanglement.

Design Principles

Outline of the Instruction

Types of Learning

Learning Events of Instruction

Supplantive and Generative Instruction

Development

Formative Evaluation

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Appendices

Learning objectives

#	Name	Prerequisite	Taxonomy of bloom	Domain
1	The student can list the different applications of quantum mechanics		Knowledge	Applications of Quantum Mechanics
2	The student can state that everything we observe exists out of molecules		Knowledge	Pre-knowledge
3	The student can state that molecules exist out of atoms	2	Knowledge	Pre-knowledge
4	The student can state that atoms exist out of protons, neutrons and electrons	3	Knowledge	Pre-knowledge
5	The student can state what a photon is		Knowledge	Pre-knowledge
6	The student can state the speed of light	5	Knowledge	Probably pre-knowledge
7	The student can state the value of the diameter of an atom	3	Knowledge	Might be pre-knowledge
8	The student can state the value of the Planck Constant		Knowledge	Might be pre-knowledge
9	The student can state that protons exist out of 2 up-quarks and 1 down-quark	4	Knowledge	Might be pre-knowledge
10	The student can state that neutrons exist out of 1 up-quark and 2 down-quarks	4	Knowledge	Might be pre-knowledge
11	The student can state that protons and neutrons are about as heavy, and that electrons have an insignificant weight compared to the other two	4	Knowledge	Pre-knowledge
12	The student can state that opposite charged particles attract each other	4	Knowledge	Pre-knowledge
13	The student can state that protons are positively charged	4	Knowledge	Pre-knowledge
14	The student can state that electrons are negatively charged	4	Knowledge	Pre-knowledge

15	The student can state that neutrons do not have any charge	4	Knowledge	Pre-knowledge
16	The student can state that photons, electrons and quarks are elementary particles	9, 10	Knowledge	Elementary Particles
17	The student can state the definition of an elementary particle	16	Knowledge	Elementary Particles
18	The student can explain what an elementary particle is	17	Comprehension	Elementary Particles
19	The student can state that classical Communication happens by sending particles through a channel	18	Knowledge	Classical Communication
20	The student can state that no particle can travel faster than light	6	Knowledge	Classical Communication
21	The student can explain why messages through a classical communication channel cannot travel faster than the speed of light	19, 20	Comprehension	Classical Communication
22	The student can calculate the time needed to send a message from location A to location B using classical communication given the distance between A and B	21	Application	Classical Communication
23	The student can state certain properties of elementary particles	11, 12, 13, 14, 15	Knowledge	Observation Dependency
24	The student can state that a property of an elementary particle collapses to a certain value on observation	23	Knowledge	Observation Dependency
25	The student can state that the collapse of a property of an elementary particle to a certain value is random	24	Knowledge	Observation Dependency
26	The student can explain what happens to the property of an individual elementary particle if it is observed	25	Comprehension	Observation Dependency
27	The student can list the two different interpretations of quantum mechanics		Knowledge	Realism and Ontology
28	The student can state the realist interpretation of the collapse of a property	26, 27	Knowledge	Realism and Ontology
29	The student can state the ontologist interpretation of the collapse of a property	26, 27	Knowledge	Realism and Ontology
30	The student can explain the realist interpretation of quantum mechanics	28	Comprehension	Realism and Ontology
31	The student can explain the ontologist interpretation of quantum mechanics	29	Comprehension	Realism and Ontology

32	The student can differentiate between a realist interpretation and an ontological interpretation of quantum mechanics, given a statement of either a realist interpretation or an ontologist interpretation	30, 31	Comprehension	Realism and Ontology
33	The student can state that before observing a property of an elementary particle, it is in a state of superposition	31	Knowledge	Superposition
34	The student can state the definition of superposition	33	Knowledge	Superposition
35	The student can explain in what state the property of an elementary particle is before observing this property	34	Comprehension	Superposition
36	The student can state the definition of entanglement	26	Knowledge	Entanglement
37	The student can state that entanglement occurs between two elementary particles	27	Knowledge	Entanglement
38	The student can state that entanglement can take place no matter the distance between the two particles	37	Knowledge	Entanglement
39	The student can state that entanglement can take place no matter when each particle is observed	37	Knowledge	Entanglement
40	The student can list the two different types of entanglement	28	Knowledge	Entanglement
41	The student can state that the properties of two boson entangled particles always collapse to the same state	29	Knowledge	Entanglement
42	The student can state that the properties of two fermion entangled particles always collapse to each others opposite state	29	Knowledge	Entanglement
43	The student can explain what happens to the properties of two entangled particles on observation	41, 42	Comprehension	Entanglement
44	The student can use entanglement to predict which state the property of a particle will collapse to, given the state of the property of an entanglement particle and the type of entanglement between the two particles	43	Application	Entanglement
45	The student can deduce the type of entanglement given the states of a common property between two entanglement particles	43	Analysis	Entanglement
46	The student can state the uncertainty principle of Heisenberg	8, 17	Knowledge	Uncertainty Principle of Heisenberg

47	The student can explain the meaning of each variable in the uncertainty principle of Heisenberg	46	Comprehension	Uncertainty Principle of Heisenberg
48	The student can explain why the exact location or momentum cannot be known according to the uncertainty principle of Heisenberg	47	Comprehension	Uncertainty Principle of Heisenberg
49	The student can explain why the position has to be lesser known if the momentum is better known and vice versa according to the uncertainty principle of Heisenberg	47	Comprehension	Uncertainty Principle of Heisenberg
50	The student can explain why the uncertainty principle of Heisenberg is only significant when dealing with very small objects	7, 47	Comprehension	Uncertainty Principle of Heisenberg
51	The student can differentiate between the measurement of the location of a planet and the measurement of the location of an electron	50	Comprehension	Uncertainty Principle of Heisenberg
52	The student can list the steps of teleportation in science-fiction movies		Knowledge	Teleportation
53	The student can explain why science-fiction teleportation is not possible	48	Comprehension	Teleportation
54	The student can list the steps of quantum teleportation	43	Knowledge	Teleportation
55	The student can explain what happens when conducting quantum teleportation	54	Comprehension	Teleportation
56	The student can apply the correct step from the quantum teleportation experiment, given a certain situation	55, 44, 45	Application	Teleportation
57	The student can deduce why quantum teleportation cannot be used to communicate instantly with someone on a different location	55	Analysis	Teleportation

Table 1: A complete outline of the learning objectives used for the instruction. With each learning objective, the name, the direct prerequisites, the category within the taxonomy of Bloom and the domain is provided.

Learning Objectives and Standards

1. The student can list the different applications of quantum mechanics

   (a) The student enlists the transistor as an application of quantum mechanics

   (b) The student enlists the laser as an application of quantum mechanics

   (c) The student enlists quantum computing as a possible future application of quantum mechanics

2. The student can state that everything we observe exists out of molecules

3. The student can state that molecules exist out of atoms

4. The student can state that atoms exist out of protons, neutrons and electrons

5. The student can state what a photon is

   (a) The student states that a photon is a light particle

6. The student can state the speed of light

   (a) The student states that the speed of light is about $3.0 \cdot 10^8$ m/s

7. The student can state the value of the diameter of an atom

   (a) The student states that the value of the diameter of an atom is 0.1 to 0.5 nm

8. The student can state the value of the reduced Planck Constant

   (a) The student states that the Planck Constant is $1.0 \cdot 10^{-34}$ Js

9. The student can state that protons exist out of 2 up-quarks and 1 down-quark

10. The student can state that neutrons exist out of 1 up-quark and 2 down-quarks

11. The student can state that protons and neutrons are about as heavy, and that electrons have an insignificant weight compared to the other two

12. The student can state that opposite charged particles attract each other

13. The student can state that protons are positively charged

14. The student can state that electrons are negatively charged

15. The student can state that neutrons do not have any charge

16. The student can state that photons, electrons and quarks are elementary particles

17. The student can state the definition of an elementary particle

    (a) The student states that it is a particle of which its substructure is not known

18. The student can explain what an elementary particle is

    (a) The student uses the definition of an elementary particle to explain what it is

    (b) The student states that some scientists believe they have no substructure

19. The student can state that classical communication happens by sending particles through a channel

20. The student can state that no particle can travel faster than light

21. The student can explain why messages through a classical communication channel cannot travel faster than the speed of light

    (a) The student states that particles are used in certain pattern for classical communication

    (b) The student states that in order to communicate from one location to another, the particles have to travel between the locations through a communication channel

    (c) The student states that no particles can travel faster than light

22. The student can calculate the time needed to send a message from location A to location B using classical communication given the distance between A and B

    (a) The student divides the distance by the speed of light to derive the time needed

23. The student can state certain properties of elementary particles

    (a) The student states charge as a property of an elementary particle

    (b) The student states polarisation as a property of photons

24. The student can state that a property of an elementary particle collapses to a certain value on observation

25. The student can state that the collapse of a property of an elementary particle to a certain value is random

26. The student can explain what happens to the property of an individual elementary particle if it is observed

    (a) The student states that a property of an individual elementary particle will collapse to a random state upon observation

27. The student can list the two different interpretations of quantum mechanics

    (a) The student enlists realism as an interpretation of quantum mechanics

    (b) The student enlists ontology as an interpretation of quantum mechanics

28. The student can state the realist interpretation of the collapse of a property

    (a) The student states that the realist interpretation of the collapse of a property has an underlying explanation or mechanic which we do not have the technology available for to measure

29. The student can state the ontologist interpretation of the collapse of a property

    (a) The student states that the ontologist interpretation of the collapse of a property has no underlying explanation or mechanic but is inherently random

30. The student can explain the realist interpretation of quantum mechanics

    (a) The student states that the realist interpretation of quantum mechanics is that every physical theory is grounded in reality

    (b) The student states that realists believe that every theory can be explained by classical mechanics

    (c) The student states that realists believe that even the seemingly random phenomena occurring within quantum mechanics can eventually be explained by the rules of classical mechanics

31. The student can explain the ontologist interpretation of quantum mechanics

    (a) The student states that the ontologist interpretation of quantum mechanics is that there is no underlying explanation of quantum mechanics

    (b) The student states that the ontologist interpretation of quantum mechanics is that quantum mechanics is real and happens on its own

32. The student can differentiate between a realist interpretation and an ontological interpretation of quantum mechanics, given a statement of either a realist interpretation or an ontologist interpretation

33. The student can state that before observing a property of an elementary particle, it is in a state of superposition

34. The student can state the definition of superposition

    (a) The student states that if a property is in superposition, its state is in all states at the same time

35. The student can explain in what state the property of an elementary particle is before observing this property

    (a) The student states that the property is in all states at the same time and collapses to a specific state upon observation

36. The student can state the definition of entanglement

    (a) The student states that entanglement entails an interdependence witihin the collapse of two elementary particles

37. The student can state that entanglement occurs between two elementary particles

38. The student can state that entanglement can take place no matter the distance between the two particles

39. The student can state that entanglement can take place no matter when each particle is observed

40. The student can list the two different types of entanglement

    (a) The student enlists boson type entanglement

    (b) The student enlists fermion type entanglement

41. The student can state that the properties of two boson entangled particles always collapse to the same state

42. The student can state that the properties of two fermion entangled particles always collapse to each others opposite state

43. The student can explain what happens to the properties of two entangled particles on observation

    (a) The student state that dependent on the type of entanglement, the property of the particles either collapse to the same state or each others opposite state

    (b) The student states that if the particles are boson type entangled, the properties will always collapse to the same state upon observation

    (c) The student states that if the particles are fermion type entangled, the properties will always collapse to each others opposite state upon observation

44. The student can use entanglement to predict which state the property of a particle will collapse to, given the state of the property of an entanglement particle and the type of entanglement between the two particles

    (a) If the particles are bosons, the student argues that the state to which the property will collapse will be the same as the state of the property of the entangled particle

    (b) If the particles are fermions, the student argues that the state to which the property will collapse will be the opposite of the state of the property of the entangled particle

45. The student can deduce the type of entanglement given the states of a common property between two entanglement particles

    (a) The student argues that if the states of the common property are the same, the two particles must be bosons

    (b) The student argues that if the states of the common property are the opposite, the two particles must be fermions

46. The student can state the uncertainty principle of Heisenberg

    (a) The student states that the uncertainty principle of Heisenberg can be expressed as $\sigma x \cdot \sigma p \geq \frac{\hbar}{2}$

47. The student can explain the meaning of each variable in the uncertainty principle of Heisenberg

    (a) The student states that the σ symbol is an expression of uncertainty

    (b) The student states that the x refers to the location of the particle

    (c) The student states that the p refers to the momentum of the particle

    (d) The student states that the momentum is an expression of the speed of a particle

    (e) The student states that the $\geq$ symbol expresses "greater or equal than"

    (f) The student states that the $\hbar$ symbol refers to the reduced Planck constant

    (g) The student summarises this as that the product of uncertainty in location and uncertainty in momentum has to be greater or equal than the reduced Planck constant divided by 2

48. The student can explain why the exact location or momentum cannot be known according to the uncertainty principle of Heisenberg

    (a) The student argues that if either σx or σp is 0, $\sigma x \cdot \sigma p$ is also 0, and that violates the principle

    (b) The student states that this means that neither the uncertainty in location nor the uncertainty in momentum can be 0

49. The student can explain why the position has to be lesser known if the momentum is better known and vice versa according to the uncertainty principle of Heisenberg

    (a) The student argues that if σx becomes smaller, σp has to become bigger, for else $\sigma x \cdot \sigma p > \frac{\hbar}{2}$

    (b) The student states that this means that if the uncertainty in location decreases, the uncertainty in momentum increases

50. The student can explain why the uncertainty principle of Heisenberg is only significant when dealing with very small objects

    (a) The student argues that $\hbar$ is an insignificantly tiny number when referring to the scale of daily experiences

    (b) The student argues that only when the scale decreases to that of the width of an atom, $\hbar$ becomes significant

51. The student can differentiate between the measurement of the location of a planet and the measurement of the location of an electron
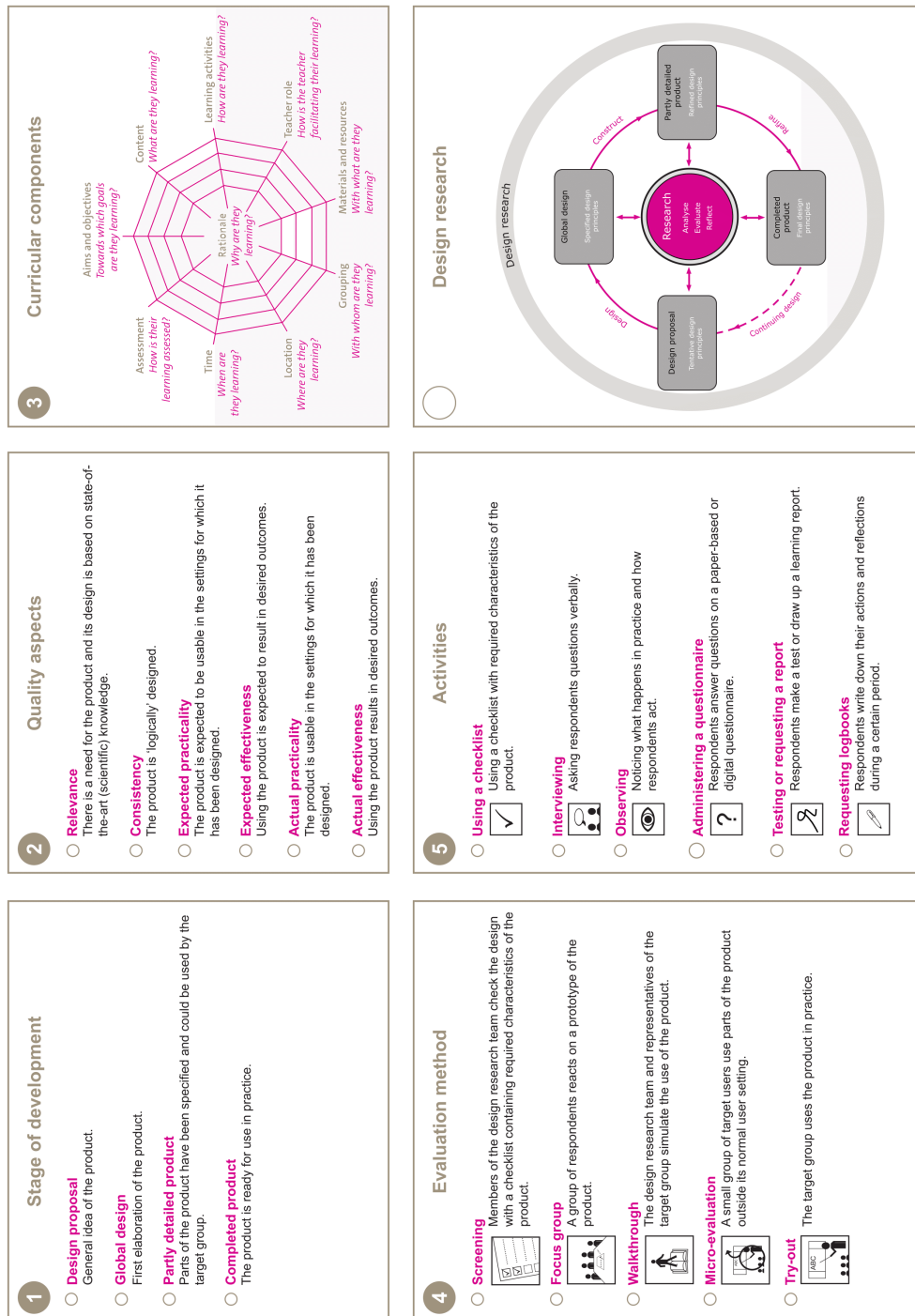
(a) The student argues that because $\hbar$ is insignificant on the scale of a planet, the location of a planet is relatively certain

(b) The student argues that $\hbar$ is significant on the scale of an electron, and because of this the location of an electron is relatively uncertain

52. The student can list the steps of teleportation in science-fiction movies

    (a) The student states that first the object at the first station is scanned

    (b) The student states that by scanning the object, all the information about all of the particles within the object are measured

    (c) The student states that then this information is used at the second station to build an exact copy of the object at the first station

    (d) The student states that the object at the first station is now completely destroyed

53. The student can explain why science-fiction teleportation is not possible

    (a) The student argues that because of the uncertainty principle of Heisenberg, all information about the object at the first station cannot be exactly determined

    (b) The student argues that because the objects at the first station cannot be exactly determined, it is not possible to build an exact copy at the second station

54. The student can list the steps of quantum teleportation

    (a) The student states that Alice has a particle X

    (b) The student states that Alice gets particle A and Bob gets particle B, which are entangled from a quantum channel

    (c) The student states that Alice executes a Bell Measurement with a combination of A and X, this way X loses its individual properties, these properties are given to B

    (d) The student states that Alice now knows the type of entanglement between particle A and particle B

    (e) The student states that particle B can arrive in two different states, because there are two different forms of entanglement possible

    (f) The student states that Alice transmits the type of entanglement to Bob via a classical channel

    (g) The student states that with this information, Bob can determine the state of particle X

    (h) The student states that Bob now transmits the state of particle B and his prediction for the state of particle X to Alice, so she can confirm whether the teleportation was successful

55. The student can explain what happens when conducting quantum teleportation

    (a) The student states that because particle A and B are entangled and particle X loses its individual properties to particle B, particle X and B are entangled

    (b) The student states that because Bob learns the type of entanglement from Alice, he now can derive the state of particle X

    (c) The student states that quantum teleportation means that information about the state of particle X is transmitted to particle B via entanglement

56. The student can apply the correct step from the quantum teleportation experiment, given a certain situation

57. The student can deduce why quantum teleportation cannot be used to communicate instantly with someone on a different location

(a) The student argues that an elementary particle cannot be influenced to collapse to a certain value on observation, but that it collapses to a random state.

(b) The student argues that this would be necessary to send a message from Alice to Bob

(c) Furthermore, the student argues that Bob still needs information from Alice via a classical communication channel about the type of entanglement, for else he cannot determine the state of particle X

Design Principles

1.

The Evaluation Matchboard

③ Curricular components

Aims and objectives — *Towards which goals are they learning?*
Content — *What are they learning?*
Learning activities — *How are they learning?*
Teacher role — *How is the teacher facilitating their learning?*
Materials and resources — *With what are they learning?*
Grouping — *With whom are they learning?*
Location — *Where are they learning?*
Time — *When are they learning?*
Assessment — *How is their learning assessed?*
Rationale — *Why are they learning?*

○ Design research

② Quality aspects

- **Relevance**
  There is a need for the product and its design is based on state-of-the-art (scientific) knowledge.
- **Consistency**
  The product is 'logically' designed.
- **Expected practicality**
  The product is expected to be usable in the settings for which it has been designed.
- **Expected effectiveness**
  Using the product is expected to result in desired outcomes.
- **Actual practicality**
  The product is usable in the settings for which it has been designed.
- **Actual effectiveness**
  Using the product results in desired outcomes.

⑤ Activities

- **Using a checklist**
  Using a checklist with required characteristics of the product.
- **Interviewing**
  Asking respondents questions verbally.
- **Observing**
  Noticing what happens in practice and how respondents act.
- **Administering a questionnaire**
  Respondents answer questions on a paper-based or digital questionnaire.
- **Testing or requesting a report**
  Respondents make a test or draw up a learning report.
- **Requesting logbooks**
  Respondents write down their actions and reflections during a certain period.

① Stage of development

- **Design proposal**
  General idea of the product.
- **Global design**
  First elaboration of the product.
- **Partly detailed product**
  Parts of the product have been specified and could be used by the target group.
- **Completed product**
  The product is ready for use in practice.

④ Evaluation method

- **Screening**
  Members of the design research team check the design with a checklist containing required characteristics of the product.
- **Focus group**
  A group of respondents reacts on a prototype of the product.
- **Walkthrough**
  The design research team and representatives of the target group simulate the use of the product.
- **Micro-evaluation**
  A small group of target users use parts of the product outside its normal user setting.
- **Try-out**
  The target group uses the product in practice.

'Evaluation matchboard'

Explanation: On one horizontal row, combine a stage of development (1) with a quality aspect (2) and find an evaluation method (4) with relevant activities (5)

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