Marcelo Leszynski
Rootfinding Research Prerequisites
07/30/20

Assignment 2.3
Exercises: 1, 3, 5

Exercise 2.3.1

(2.3.1a): Using grlex order yields

$$f = (x^6 + x^2) \cdot (xy^2 - x) + 0 \cdot (x - y^3) + (x^7 + x^3 - y + 1)$$

whereas using lex order yields

$$\begin{aligned} f =& (x^6 + x^5y + x^4y^2 + x^4 + x^3y + x^2y^2 + 2x^2 + 2xy + 2y^2 + 2) \cdot (xy^2 - x) + \\ & (x^6 + x^5y + x^4 + x^3y + 2x^2 + 2xy + 2) \cdot (x - y^3) + (2y^3 - y + 1) \end{aligned}$$

(2.3.1b): Using grlex order yields

$$f = 0 \cdot (x - y^3) + (x^6 + x^2) \cdot (xy^2 - x) + (x^7 + x^3 - y + 1)$$

whereas using lex order yields

$$\begin{aligned} f =& (x^6y^2 + x^5y^5 + x^4y^8 + x^3y^1 1 + x^2y^1 4 + x^2y^2 + xy^1 7 + xy^5 + y^2 0 + y^8) \cdot (x - y^3) + \\ & 0 \cdot (xy^2 - x) + (y^2 3 + y^1 1 - y + 1) \end{aligned}$$

Exercise 2.3.3 Implementation of the division algorithm can be found in poly_div.py in the Coded Algorithms folder.

Exercise 2.3.5

(2.3.5a): using poly_div.py to calculate r_1 and r_2 yields

$$\begin{aligned} r_1 &= x^3 - x^2z + x - z \\ r_2 &= x^3 - x^2z \end{aligned}$$

The difference between the two calculations occurs during the second step of the algorithm when we are comparing leading terms.

(2.3.5b):

$$r = r_1 - r_2 = (x^3 - x^2z + x - z) - (x^3 - x^2z) = x - z \in \langle f_1, f_2 \rangle$$

because $x - z = f_1 - xf_2$.

(2.3.5c): The remainder is equal to $x - z$. This could be predicted because the leading terms of f_1 and f_2 do not divide any of the terms in r.

(2.3.5d): Let $g = -y \cdot f_1 + (xy + 1) \cdot f_2 = yz - 1 \in \langle f_1, f_2 \rangle$. note that no term in g is divided by the leading terms of f_1 and f_2.

(2.3.5e): The division algorithm does not solve the ideal membership problem since both r and g are contained in $\langle f_1, f_2 \rangle$ but dividing using the algorithm returns nonzero remainders for both r and g.