MARCELO LESZYNSKI
Rootfinding Research Prerequisites
07/30/20

Assignment 2.4
Exercises: 1, 3, 4

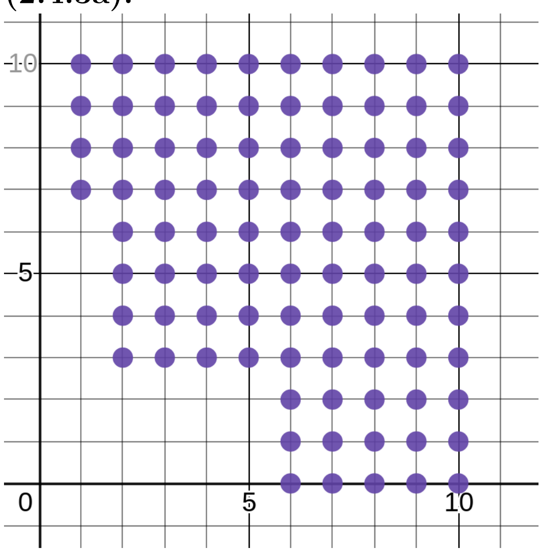
Exercise 2.4.1

Proof. Let $G = \langle x^\alpha | x^\alpha$ can be found in some $f \in I\rangle$ be a monomial ideal. If I is a monomial ideal, then $I = G$ by Lemma 3 and Corollary 4.

To show that $I = G$, we examine $g \in G$ which can be expressed as the sum of terms cx^α where x^α is a monomial contained in some $f \in I$. By construction, we then have that $x^\alpha \in I$, so it follows that $g \in I$ by the definition of an ideal. To show the other inclusion, suppose that $g \in I$. This means that for all terms cx^α contained in g, $x^\alpha \in G$ by the construction of G. It then follows that $g \in G$ since G is an ideal. $\square$

Exercise 2.4.3

(2.4.3a):



(2.4.3b):

$$
\begin{aligned}
&1, x, x^2, x^3, x^4, x^5\\
&y, xy, x^2y, x^3y, x^4y, x^5y\\
&y^2, xy^2, x^2y^2, x^3y^2, x^4y^2, x^5y^2\\
&y^3, xy^3\\
&y^4, xy^4\\
&y^5, xy^5\\
&y^6, xy^6\\
&y^\alpha, \alpha \geq 7
\end{aligned}
$$

Exercise 2.4.4

(2.4.4a): Let $J = \langle x^\alpha | x^\alpha y^\beta \in I \rangle$ for some $\beta \geq 0$. Since $x^3y^6 \in I$, we have that $J = \langle x^3 \rangle$ with $\beta = 6$. This implies that $J_0 = J_1 = J_2 = J_3 = \langle x^6 \rangle$ and $J_4 = J_5 = \langle x^5 \rangle$. Then, by Theorem 5, we have that $I = \langle x^3y^6, x^6, x^6y, x^6y^2, x^6y^3, x^5y^4, x^5y^5 \rangle$.

(2.4.4b): Removing all terms in the previous basis that can be divided by other distinct terms results in the basis $I = \langle x^3y^6, x^6, x^5y^4 \rangle$.